Civilittee

اللجنة الأكاديمية للهندسة المدنية

سلايدات

هندسة جيوتقنية

د. سامر ربابعة

The Hashemite University
Faculty of Engineering
Course Syllabus
Department of Mechanical Engineering

Course Title:	**Geotechnical Eng. 3 (3,0, 0)**	**Course Number:**	**110401336**
Designation:	Compulsory	**Prerequisite(s):**	**110401212**
Instructor:	Dr. Samer Rabab'ah	**Instructor's e-mail:**	srababah@hu.edu.o
Office Hours:			

Course Description (catalog): Index and classification of soils, water flow in soils (one and two dimensional water flow), soil stresses, soil compaction, distribution of stresses in soil due to external loads, consolidation and consolidation settlement, shear strength of soils, slope stability.

Textbook(s) and/or Other Supplementary Materials:

Braja M. Das and Khaled, Principles of Geo-technical Engineering, 8th Edition, SI Edition, 2014, Cengage Learning ,Stamford, CT 06902, USA.

References:

1. .F. Craig, Soil Mechanics, Spon Press, 2004
2. Budhu M. (2007)"Soil Mechanics and Foundations" Wiley, New York
3. Holtz, R.D. and Kovacs, W.D. (1981). An Introduction to Geotechnical Engineering, Prentice Hall. (Chapter 1 and 2)9.

Major Topics Covered:

Topics	No. of Weeks	Contact hours*
Formation of Soils and Mineralogy of Soil Solids	1	5
Index Properties and Classification of Soils	1	5
Soil Compaction	1	5
water in Soils (permeability, seepage, and effective stresses)	1	5
Stress Distribution in Soils Due to External Loading	1	5
Soil Consolidation, Consolidation Settlement, and Rate of Consolidation	1	5
Shear Strength of Soils	1	5
Stability of Slopes	1	5
Total	**15**	**45**

*Contact hours include lectures, quizzes and exams

Specific Outcomes of Instruction (Course Learning Outcomes):
After completing the course, the student will be able to:

1. Understand the basics properties of soil, and soil formation. (a)
2. Use standards methods to classify soils. (a, e)
3. Determine compaction, permeability of soil. (a)
4. Determine total and effective stresses and pore water pressures and determine how surface stresses are distributed within a soil mass. (a, e)
5. Draw flow net, stability of earth dams due to seepage force (a, e)
6. Recognize soil consolidation and Determine soil settlement due to consolidation(a, e)
7. Recognize soil shear strength and evaluate slope stability (a, e)

Student Outcomes (SO) Addressed by the Course:

#	Outcome Description	Contribution
	General Engineering Student Outcomes	
(a)	an ability to apply knowledge of mathematics, science, and engineering	M (40)
(b)	an ability to design and conduct experiments, as well as to analyze and interpret data	
(c)	an ability to design a system, component, or process to meet desired needs within realistic constraints such as economic, environmental, social, political, ethical, health and safety, manufacturability, and sustainability	
(d)	an ability to function on multidisciplinary teams	
(e)	an ability to identify, formulate, and solve engineering problems	H(60)
(f)	an understanding of professional and ethical responsibility	
(g)	an ability to communicate effectively	

(h)	the broad education necessary to understand the impact of engineering solutions in a global, economic, environmental, and societal context	
(i)	a recognition of the need for, and an ability to engage in life-long learning	
(j)	a knowledge of contemporary issues	
(k)	an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.	
H=High, **M**= Medium, **L**=Low		

Grading Plan:

1st Exam	30 Points
2nd Exam	30 Points
HWs. & Qs	00 points
Final exam	40 points

General Notes:

Beware of Plagiarism: copying and handing in for credit someone else's work
Any plagiarism case will result in an automatic 'F' for the course

Prepared by: Dr. omar hattamleh **Date:** 18th Sep. 2017

1. Origin of Soils and Rocks (Das, Chapter 2)

Introduction

- **The mineral grains that form the solid phase of a soil aggregate are the product of rock weathering.**
- **The physical properties of soil are dictated by the size, shape, and chemical composition of the grains, and hence the rock from which is derived.**
- **Rocks are compact, semi-hard to hard mass composed of one or several minerals.**

On the basis of their mode of origin, rocks can be divided into three basic types:

- *Igneous rocks*
- *Sedimentary rocks*
- *Metamorphic rocks*

Rock Cycle

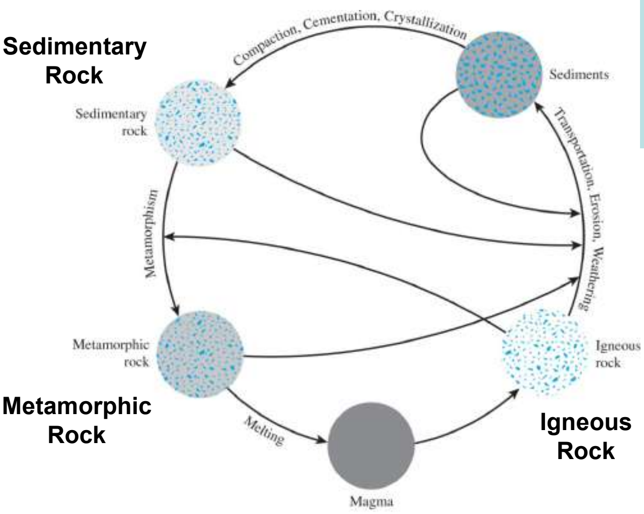


Figure 2.1 Rock cycle

The formation cycle of different types of rock and the processes associated with them. This is called the *rock cycle.*

The rock cycle is important in any discussion of soil formation, especially if we know that soils are nothing but disintegrated rocks.

2. FORMATION OF SOILS

Soil: some definitions

- Soils are formed from rock, loose unconsolidated materials (may be transported), or organic residues.

- The word ‘soil’ means different things to different people but basically it may be defined as the solid material on the earth’s surface that results from the interaction of **weathering** and biological activity on the soil parent material or underlying hard rock.

Weathering

- This refers to the breakdown and decomposition of rocks, soils and minerals into smaller pieces through contact with the atmosphere, biota and waters.

- Weathering include comprise of mechanical and chemical weathering

Weathering

Photos of Mechanical Weathering:

8

Parent Rock

Residual soils

~ in situ weathering (by physical & chemical agents) of parent rock

Transported soils

~ weathered and transported far away

by wind, water, ice, gravity.

9

B. Transported Soils

These are soils which were formed from rock weathering at one site and are now found at another site. The transporting agent may be:

1. Water (Principal transporting agent)
2. Glaciers
3. Wind
4. Gravity

Transported soils are very important in engineering because nearly all major cities are located, at least in part, on flood plains, deltas, and coastal plains.

10

The transported soils may be classified into several groups, depending on their mode of transportation and deposition:

1. **Glacial soils** — formed by transportation and deposition of glaciers
2. **Alluvial soils** —transported by running water and deposited along streams
3. **Lacustrine soils** —formed by deposition in quiet lakes
4. **Marine soils** —formed by deposition in the seas
5. **Aeolian soils** —transported and deposited by wind
6. **Colluvial soils** —formed by movement of soil from its original place by gravity, such as during landslides

11

2. Grain Size Distribution (Das, Chapter 2)

Soils - What are they?

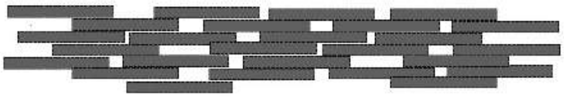
- Soils are natural material that are made up of particles that have different sizes.
- Soils differ from other engineering materials in that one has little control over their properties
- Broad Categories of soil particle sizes are:
  - Coarse grained soils
    - sands, gravels - visible to naked eye
  - Fine grained soils
    - silts, clays, organic soils – not visible to naked eye
- Particle size is related to mineralogy:
  - **Gravelly** and **Sandy** soils are formed due to decomposition of rocks containing quartz with high in silica content.
  - **Silt** and **Clay** formed from rocks which contain iron, magnesium, calcium, or sodium minerals with little silica

Soil Grain Shapes

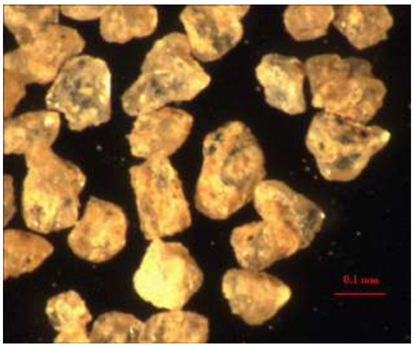
- Soil grains have different *shapes that* somewhat difficult to quantify.
- An infinite number of shapes are possible, a few of which below:

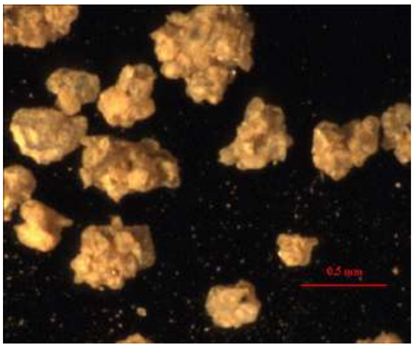
<u>Bulky (sands and gravel)</u>

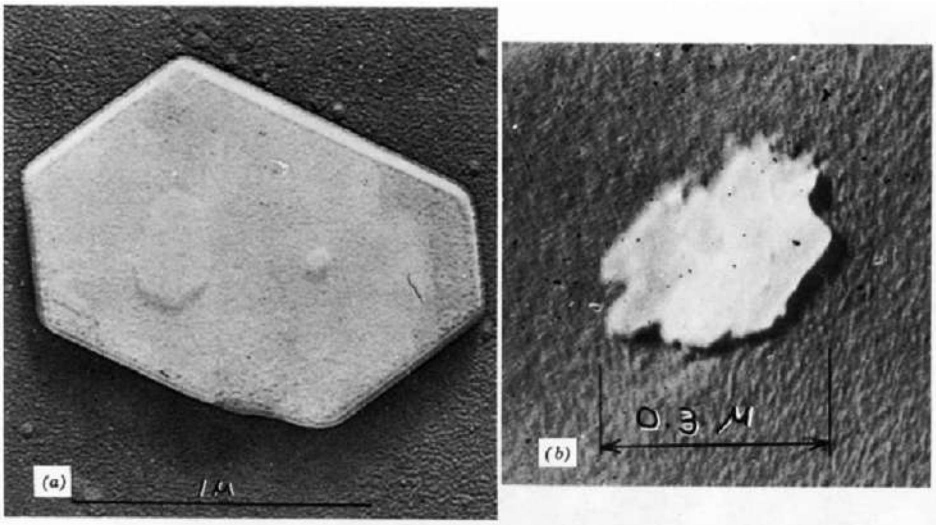
<u>Flaky or Needle shape (clay)</u>



Soil Grain Shapes







Clay particles. (a) Kaolinite (From Lambe, 1951). (b) Illite (by R. T. Martin, M.I.T.)

5

Soil- Grain Size

Grain size of soil refers to the diameters of the soil particles making up the soil mass.

The sizes of the soil particles are important factors which influence soil properties including :

- Strength
- Deformation
- Permeability
- Suitability as a construction materials like in dams and pavements

6

Soil- Grain Size

Depending on the major size of particles within the soil, the sizes of particles that make up soils vary over a wide range. Soils generally are called :

- Gravel
- Sand
- Silt
- Clay

Table 2.3 Particle-Size Classifications

Name of organization	Grain size (mm) Gravel	Sand	Silt	Clay
Massachusetts Institute of Technology (MIT)	>2	2 to 0.06	0.06 to 0.002	<0.002
U.S. Department of Agriculture (USDA)	>2	2 to 0.05	0.05 to 0.002	<0.002
American Association of State Highway and Transportation Officials (AASHTO)	76.2 to 2	2 to 0.075	0.075 to 0.002	<0.002
Unified Soil Classification System (U.S. Army Corps of Engineers, U.S. Bureau of Reclamation, and American Society for Testing and Materials)	76.2 to 4.75	4.75 to 0.075	Fines (i.e., silts and clays) <0.075	

Note: Sieve openings of 4.75 mm are found on a U.S. No. 4 sieve; 2-mm openings on a U.S. No. 10 sieve; 0.075-mm openings on a U.S. No. 200 sieve. See Table 2.5.

Grain Size Distribution of Soils

Grain size distribution is the determination of size of particles in a soil, expressed as a percentage of the total dry weight.

Significance of GSD:

- To know the relative proportions of different grain sizes.
- An important factor influencing the geotechnical characteristics of a coarse grain soil.

Particle size:

Soil particle sizes range from more than 1m. dia. for boulders to 0.001 mm clay size:

Soil type	particle size
Boulder	>0.3 m
Cobble	0.15m-0.3 m
Gravel	4.75 mm- 76.2 mm/0.15m
Sand	.075mm-4.75 mm
Silt	.002-0.075 mm
Clay	<0.002mm

Grain Size Distribution of Soils

Two methods generally are used to find the particle size-distribution of soil:

- Sieve (mechanical) Analysis: for particle size greater than 0.075 mm in diameter.
- Hydrometer (wet) Analysis: for particle size smaller than 0.075 mm in diameter.

Sieve Analysis

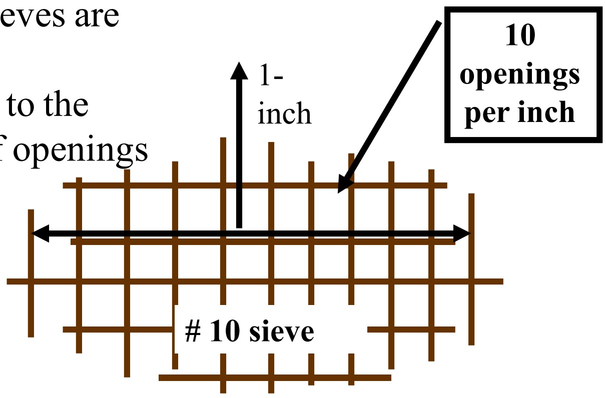
Sieve Designation - Large

Sieves larger than the #4 sieve are designated by the size of the openings in the sieve

13

Sieve Designation - Smaller

Smaller sieves are numbered according to the number of openings per inch



14

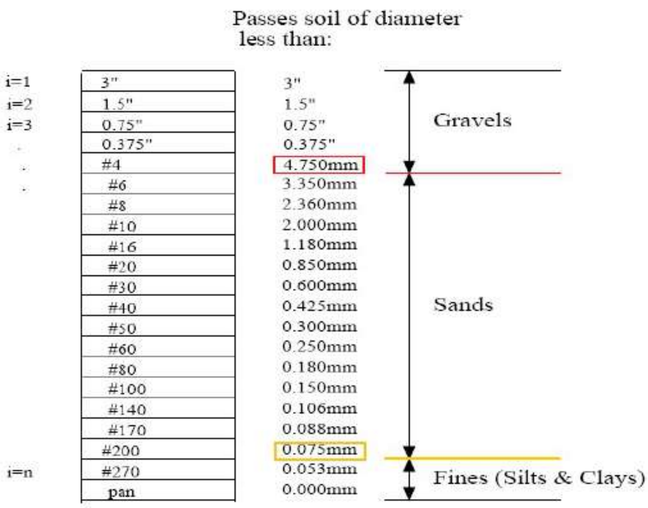
Sieve Analysis

(for coarse-grained soils)

Table 2.5 U.S. Standard Sieve Sizes

Sieve no.	Opening (mm)	Sieve no.	Opening (mm)
4	4.75	35	0.500
5	4.00	40	0.425
6	3.35	50	0.355
7	2.80	60	0.250
8	2.36	70	0.212
10	2.00	80	0.180
12	1.70	100	0.150
14	1.40	120	0.125
16	1.18	140	0.106
18	1.00	170	0.090
20	0.850	200	0.075
25	0.710	270	0.053
30	0.600		

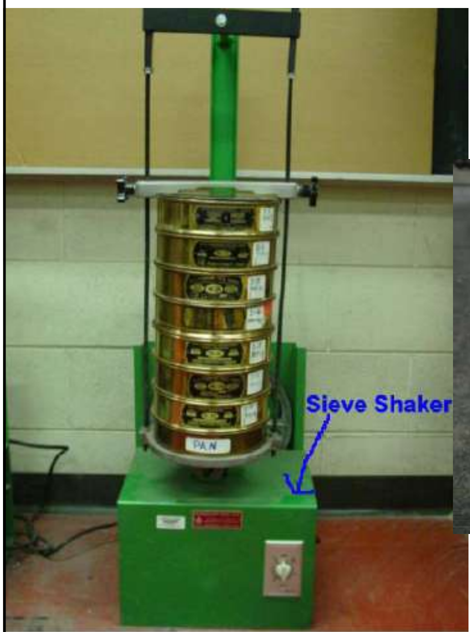
Sieving- (for coarse-grained soils with D > 0.075mm)




Set of Sieves
Soil Specimen
PAN
Balance


PAN

17


Sieve Shaker
PAN

18

Sieving procedure

(1) Write down the weight of each sieve as well as the bottom pan to be used in the analysis.

(2) Record the weight of the given dry soil sample.

(3) Make sure that all the sieves are clean, and assemble them in the ascending order of sieve numbers (#4 sieve at top and #200 sieve at bottom). Place the pan below #200 sieve. Carefully pour the soil sample into the top sieve and place the cap over it.

(4) Place the sieve stack in the mechanical shaker and shake for 10 minutes.

(5) Remove the stack from the shaker and carefully weigh and record the weight of each sieve with its retained soil. In addition, remember to weigh and record the weight of the bottom pan with its retained fine soil.

Data Analysis:

(1) Obtain the mass of soil retained on each sieve by subtracting the weight of the empty sieve from the mass of the sieve + retained soil, and record this mass as the weight retained on the data sheet. The sum of these retained masses should be approximately equals the initial mass of the soil sample. A loss of more than two percent is unsatisfactory.

(2) Calculate the percent retained on each sieve by dividing the weight retained on each sieve by the original sample mass.

(3) Calculate the percent passing (or percent finer) by starting with 100 percent and subtracting the percent retained on each sieve as a cumulative procedure.

Sieve Number	Diameter (mm)	Mass of Empty Sieve (g)	Mass of Sieve+Soil Retained (g)	Soil Retained (g)	Percent Retained	Percent Passing
4	4.75	116.23	166.13	49.9	9.5	90.5
10	2.0	99.27	135.77	36.5	7.0	83.5
20	0.84	97.58	139.68	42.1	8.0	75.5
40	0.425	98.96	138.96	40.0	7.6	67.8
60	0.25	91.46	114.46	23.0	4.4	63.4
140	0.106	93.15	184.15	91.0	17.4	46.1
200	0.075	90.92	101.12	10.2	1.9	44.1
Pan	---	70.19	301.19	231.0	44.1	0.0
			Total Weight=	523.7		

21

Hydrometer (Wet analysis) Analysis

- Based on the principle of sedimentation of soil grains in water.

- It is assumed that all the soil particles are spheres and that the velocity of soil particles can be expressed by *Stokes' law*

$$v = \frac{\rho_s - \rho_w}{18\eta} D^2$$

where v = velocity
ρ_s = density of soil particles
ρ_w = density of water
η = viscosity of water
D = diameter of soil particles

22

Hydrometer (Wet analysis) Analysis

$$v = \frac{\rho_s - \rho_w}{18\eta} D^2$$

$$D = \sqrt{\frac{30\eta}{(G_s - 1)\rho_w}} \sqrt{\frac{L}{t}}$$

$$D\,(\text{mm}) = K\sqrt{\frac{L\,(\text{cm})}{t\,(\text{min})}}$$

Figure 2.23
ASTM 152H hydrometer
(Courtesy of ELE International)

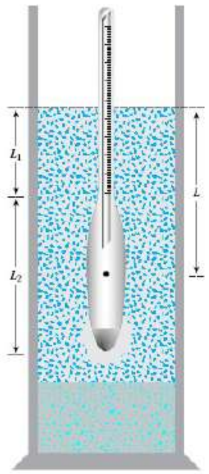
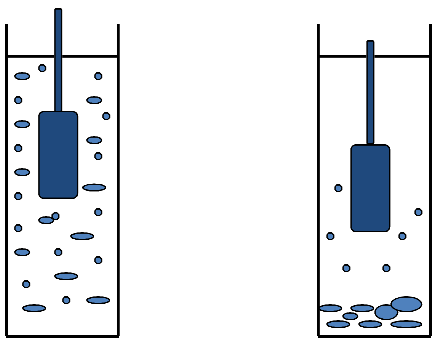


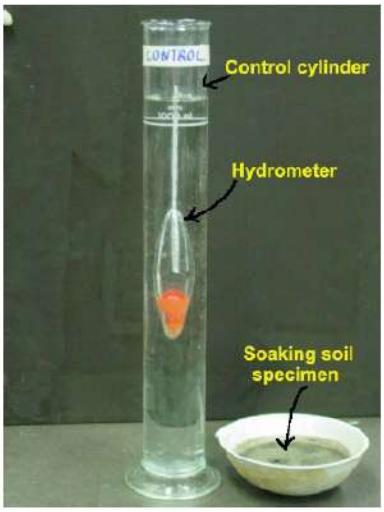
Figure 2.24 Definition of L in hydrometer test

- Hydrometer test - used for smaller particles



Schematic diagram of hydrometer test

Hydrometer Analysis - Procedure:



Hydrometer Analysis

Test Date: September 15, 2002
Tested By: CEMM315 Class, Group A
Hydrometer Number (if known): 152 H
Specific Gravity of Solids: 2.56
Dispersing Agent: Sodium Hexametaphosphate
Weight of Soil Sample: 50.0 gm
Zero Correction: +6
Meniscus Correction: +1

Date	Time	Elapsed Time (min)	Temp. °C	Actual Hydro. Rdg R_a	Hyd. Corr. for Meniscus	L from Table 1	K from Table 2	D mm	C_T from Table 3	a from Table 4	Corr. Hydr. Rdg. R_c	% Finer P	% Adjusted Finer P_A
09/15	4:06 PM	0	25	55	56	7.1	0.01326	0	+1.3	1.018	-	-	-
	4:07	1	25	47	48	8.6	0.01326	0.03029	+1.3	1.018	42.3	86.1	37.8
	4:08	2	25	42	43	9.2	0.01326	0.02844	+1.3	1.018	37.3	75.9	33.3
	4:10	4	25	40	41	9.6	0.01326	0.02054	+1.3	1.018	35.3	71.9	31.6
	4:14	8	25	37	38	10.1	0.01326	0.01490	+1.3	1.018	32.3	65.8	28.6
	4:22	16	25	32	33	10.9	0.01326	0.01094	+1.3	1.018	27.3	55.6	24.1
	4:40	34	25	28	29	11.5	0.01326	0.00771	+1.3	1.018	23.3	47.4	20.8
	6:22	136	23	22	23	12.5	0.01356	0.00411	+0.7	1.018	16.7	34	14.9
09/16	5:24 PM	1518	22	15	16	13.7	0.01366	0.00130	+0.4	1.018	9.4	19.1	8.4

Unified Classification of Soil: SC/SM

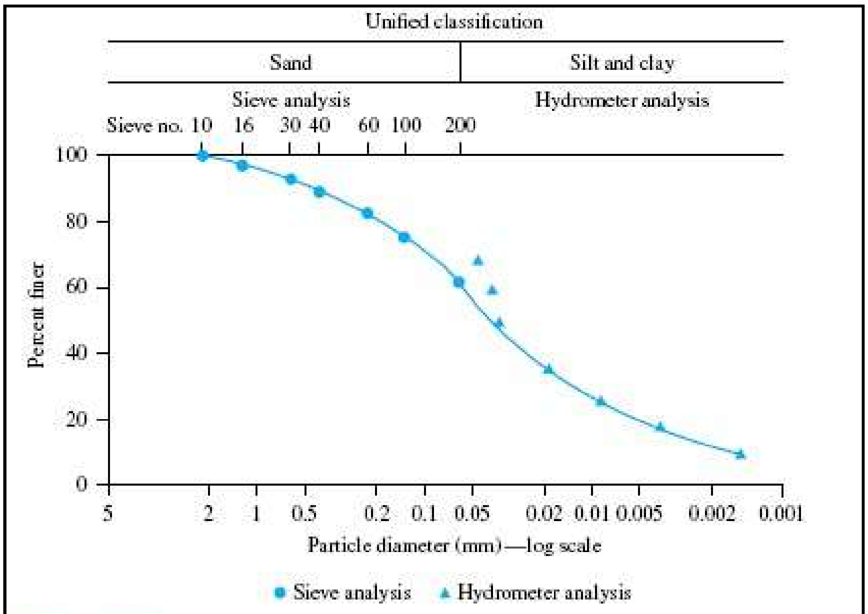


Figure 2.25 Particle-size distribution curve—sieve analysis and hydrometer analysis

27

Example 1

Following are the results of a sieve analysis. Make the necessary calculations and draw a particle-size distribution curve.

U.S. sieve no.	Mass of soil retained on each sieve (g)
4	0
10	40
20	60
40	89
60	140
80	122
100	210
200	56
Pan	12

Solution

The following table can now be prepared.

U.S. sieve (1)	Opening (mm) (2)	Mass retained on each sieve (g) (3)	Cumulative mass retained above each sieve (g) (4)	Percent finer[a] (5)
4	4.75	0	0	100
10	2.00	40	0 + 40 = 40	94.5
20	0.850	60	40 + 60 = 100	86.3
40	0.425	89	100 + 89 = 189	74.1
60	0.250	140	189 + 140 = 329	54.9
80	0.180	122	329 + 122 = 451	38.1
100	0.150	210	451 + 210 = 661	9.3
200	0.075	56	661 + 56 = 717	1.7
Pan	–	12	717 + 12 = 729 = ΣM	0

[a] $\dfrac{\sum M - \text{col. 4}}{\sum M} \times 100 = \dfrac{729 - \text{col. 4}}{729} \times 100$

28

Example 2

For the particle-size distribution curve shown in Figure 2.28, determine the percentages of gravel, sand, silt, and clay-size particles present. Use the Unified Soil Classification System.

Solution

From Figure 2.28, we can prepare the following table.

Size (mm)	Percent finer	
76.2	100	
		100 − 100 = **0% gravel**
4.75	100	
		100 − 1.7 = **98.3% sand**
0.075	1.7	
		1.7 − 0 = **1.7% silt and clay**
–	0	

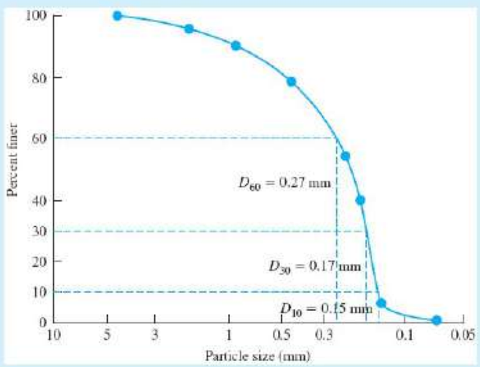


Figure 2.28 Particle-size distribution curve

29

Grain-Size Distribution Curve

The results of mechanical analysis (sieve and hydrometer analysis) are generally presented by semi logarithmic plots known as GRAIN or PARTICLE-SIZE DISTRIBUTION CURVES.

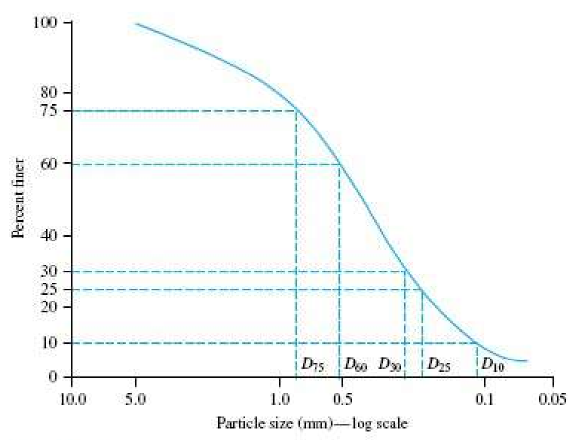


Figure 2.26 Definition of D_{75}, D_{60}, D_{30}, D_{25}, and D_{10}

1. Effective Size

D_{10}

2. Uniformity Coefficient

$$C_u = \frac{D_{60}}{D_{10}}$$

3. Coefficient of Gradation

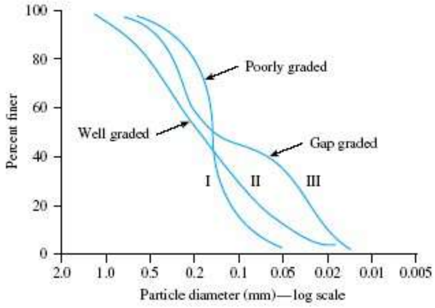
$$C_c = \frac{D_{30}^2}{D_{60} \times D_{10}}$$

Characteristics of the Grain size Distribution Curve

The shape of the particle-size distribution curve depends on the range and amounts of the various sizes of particles in the soil sample.

Types of soils with regard to shape of their distribution curves:

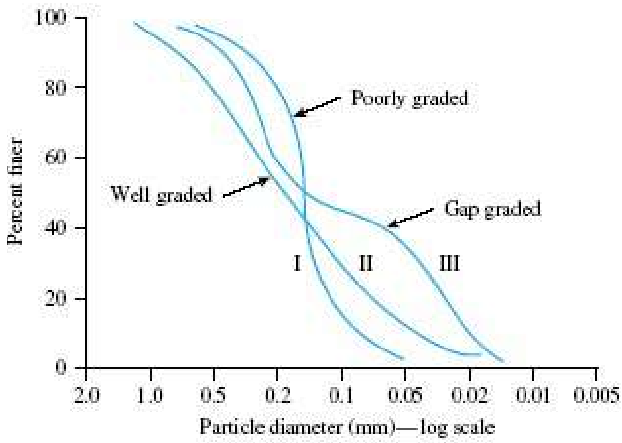
1. Poorly-graded soil: The curve -is nearly vertical. This indicates that the soil is UNIFORM.



31

2. Well-graded soil: The curve is smooth and covers a wide range of sizes. This indicates that the soil is NON-UNIFORM

3. Gap-graded soil: This is the case when intermediate sizes are absent. This could be the case when two separate soils are mixed.



32

Example 1

Example 2.2

For the particle-size distribution curve shown in Figure 2.28 determine

a. D_{10}, D_{30}, and D_{60}
b. Uniformity coefficient, C_u
c. Coefficient of gradation, C_z

Solution

Part a

From Figure 2.28,

$$D_{10} = 0.15 \text{ mm}$$
$$D_{30} = 0.17 \text{ mm}$$
$$D_{60} = 0.27 \text{ mm}$$

Part b

$$C_u = \frac{D_{60}}{D_{10}} = \frac{0.27}{0.15} = 1.8$$

$$C_z = \frac{D_{30}^2}{D_{60} \times D_{10}} = \frac{(0.17)^2}{(0.27)(0.15)} = 0.71$$

Well-Graded?

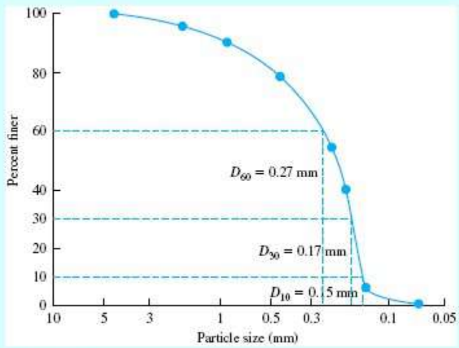


Figure 2.28 Particle-size distribution curve

Clay mineralogy:

✓ Platy shaped particles
✓ Crystalline
✓ Net negative charge at surface

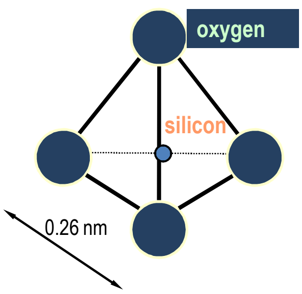
Three most common types :

1. Kaolinites
2. Illites
3. & montmorillonites (smectites).

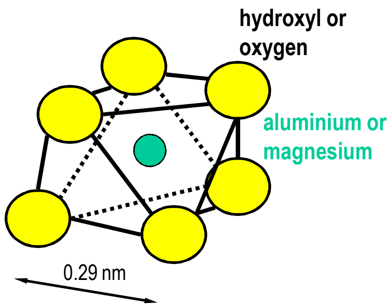
Combined of Two basic crystalline units:
* tetrahedron (silica sheet)
* Octahedron (alumina or gibbsite sheet)

Basic Structural Units

Clay minerals are made of two distinct structural units.



Silicon tetrahedron



Aluminium Octahedron

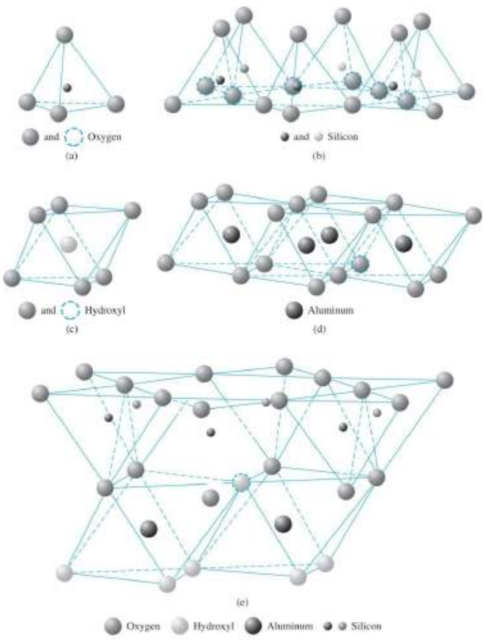
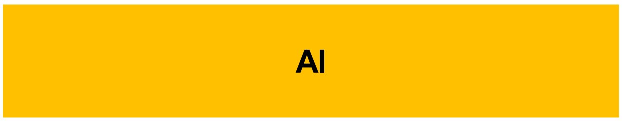


Figure 2.10 (a) Silica tetrahedron; (b) silica sheet; (c) alumina octahedron; (d) octahedral (gibbsite) sheet; (e) elemental silica-gibbsite sheet (*After Grim, 1959. With permission from ASCE.*)

Tetrahedral & Octahedral Sheets

For simplicity, let's represent silica **tetrahedral sheet** by:

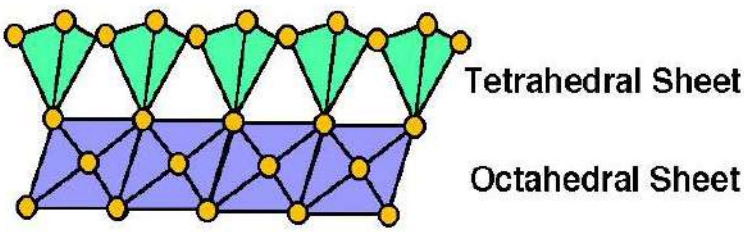
and alumina **octahedral sheet** by:



Different Clay Minerals

Different combinations of tetrahedral and octahedral sheets form different clay minerals:

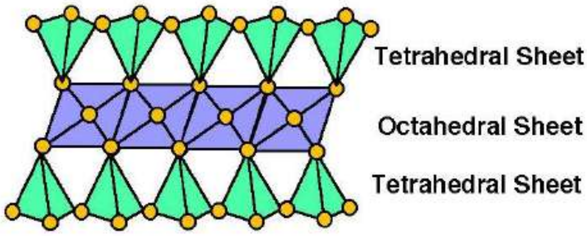
<u>1:1 Clay Mineral</u> (e.g., kaolinite, halloysite):



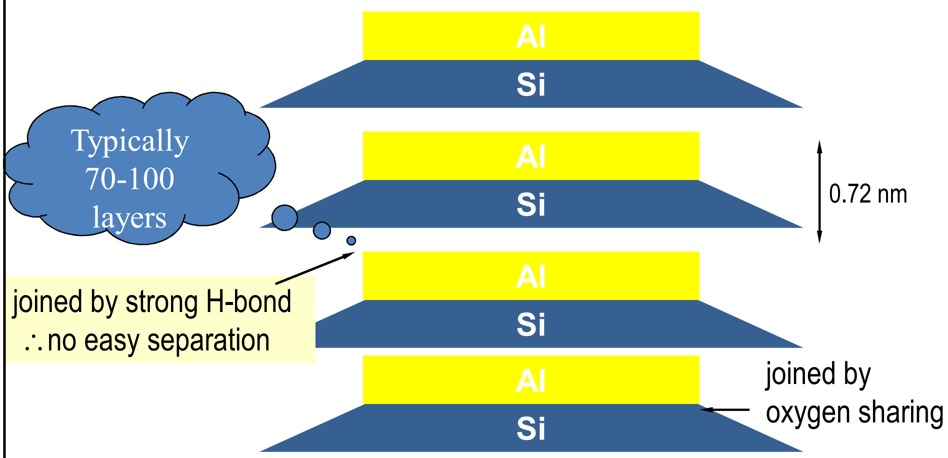
Different Clay Minerals

Different combinations of tetrahedral and octahedral sheets form different clay minerals:

<u>2:1 Clay Mineral</u> (e.g., montmorillonite, illite)



Kaolinite



Kaolinite

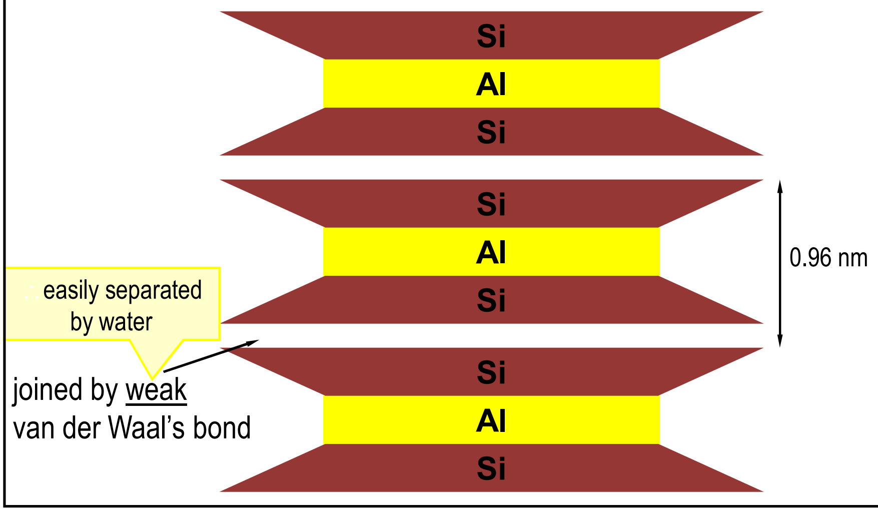
- used in paints, paper and in pottery and pharmaceutical industries
- $(OH)_8Al_4Si_4O_{10}$

Halloysite

- kaolinite family; hydrated and tubular structure
- $(OH)_8Al_4Si_4O_{10}.4H_2O$

Montmorillonite

- also called **smectite**; expands on contact with water



Montmorillonite

- A highly reactive (expansive) clay

swells on contact with water

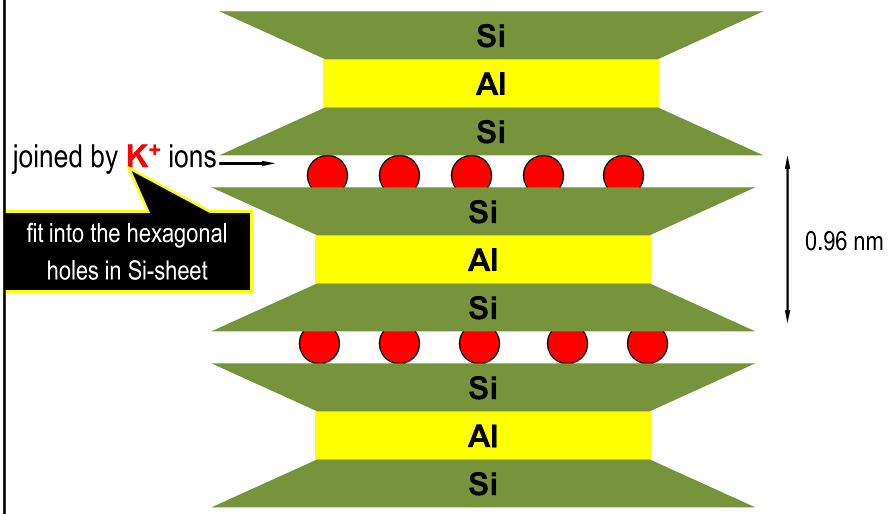
- $(OH)_4Al_4Si_8O_{20}.nH_2O$

high affinity to water

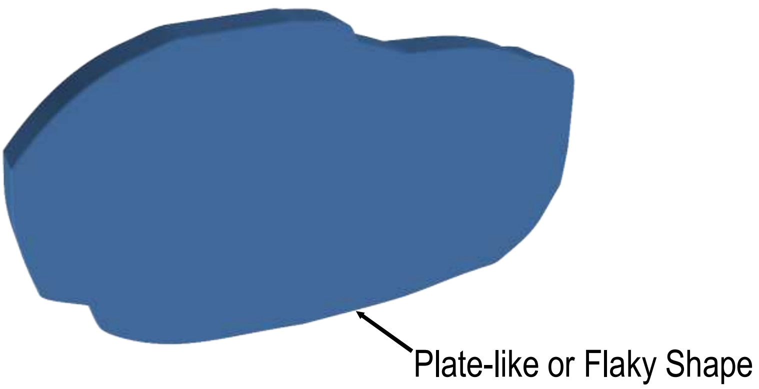
Bentonite

- montmorillonite family
- used as drilling mud, in slurry trench walls, stopping leaks

Illite

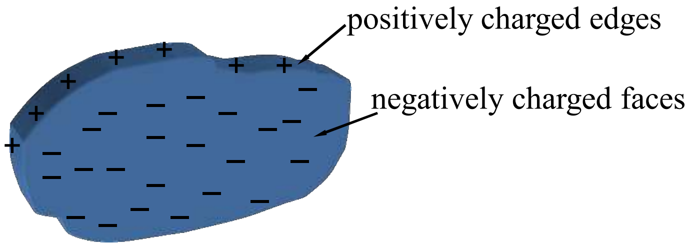


A Clay Particle



Isomorphous Substitution

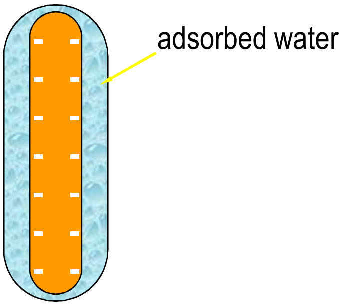
- substitution of Si^{4+} and Al^{3+} by other lower valence (e.g., Mg^{2+}) cations
- results in charge imbalance (net negative)



Clay Particle with Net negative Charge

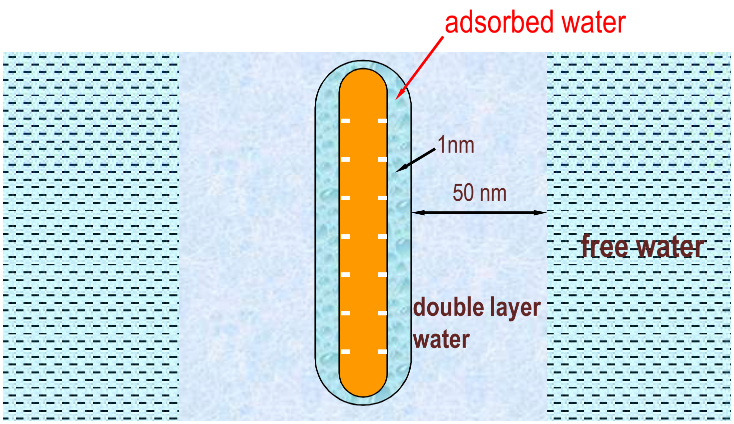
Adsorbed Water

- A thin layer of water tightly held to particle; like a skin
- 1-4 molecules of water (1 nm) thick
- more viscous than free water



47

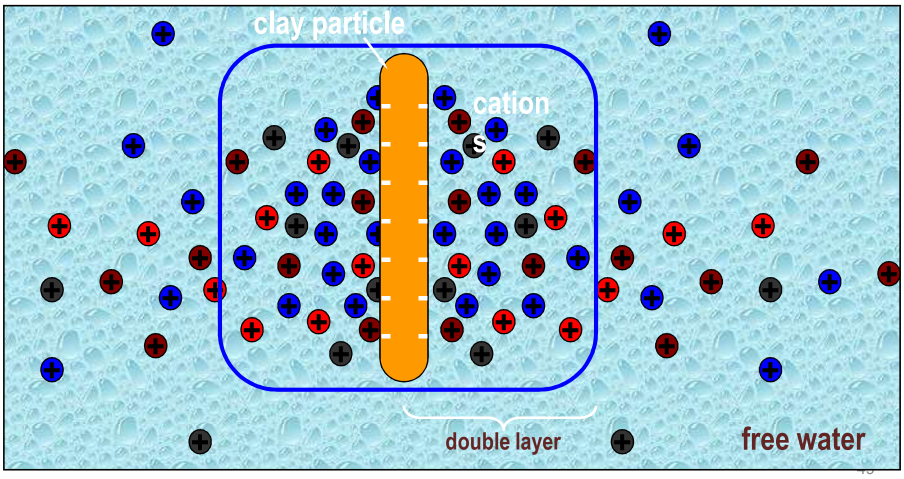
Clay Particle in Water



48

Cation Concentration in Water

- cation concentration drops with distance from clay particle



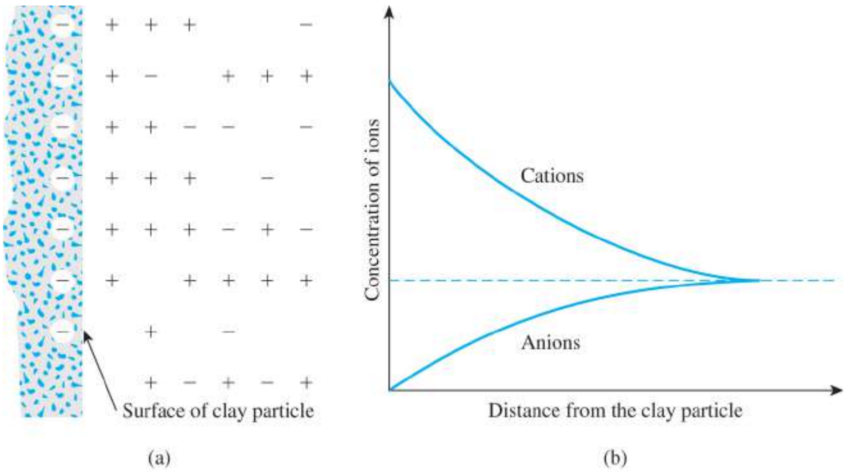


Figure 2.17 Diffuse double layer

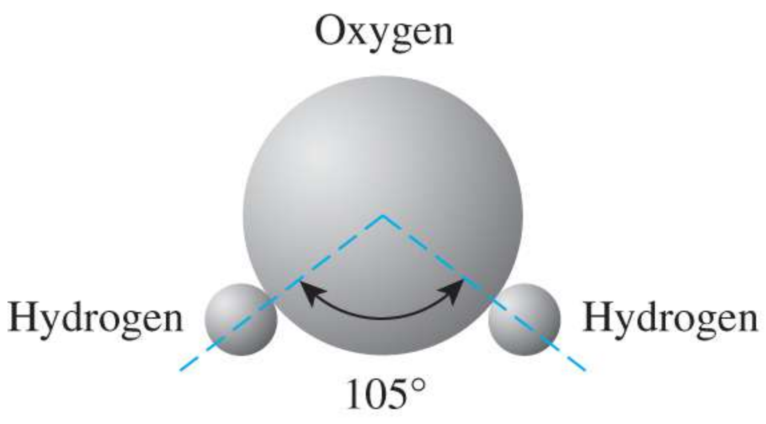


Figure 2.18 Dipolar character of water

3-51

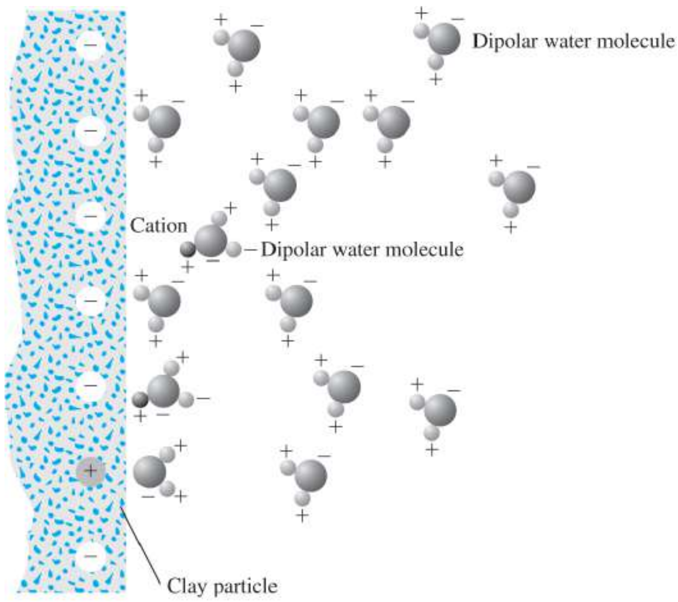
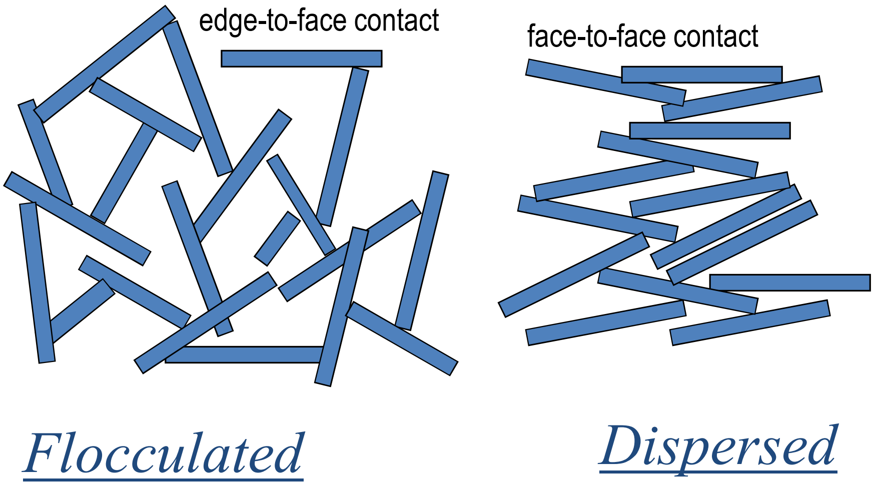


Figure 2.19 Attraction of dipolar molecules in diffuse double layer

3-52

Clay Fabric



53

Cation Exchange Capacity (c.e.c)

known as exchangeable cations

- capacity to attract cations from the water (i.e., measure of the net negative charge of the clay particle)
- measured in meq /100g (net negative charge per 100 g of clay)

  milliequivalents

- The replacement power is greater for higher valence and larger cations.

  $Al^{3+} > Ca^{2+} > Mg^{2+} >> NH_4^+ > K^+ > H^+ > Na^+ > Li^+$

54

3. Phase Relations

For this chapter, we need to know the following

- Mass (M) is a measure of a body's inertia, or its "quantity of matter". Mass does not changed at different places.

- Weight (W) is the force of gravity acting on a body.

$$W = M \cdot g \qquad \text{where } g : acceleration\ due\ to\ gravity = 9.81\ {}^{m}\!/_{\sec^2}$$

$$Density,\ \rho = \frac{Mass}{Volume}$$

$$Unit\ weight,\ \gamma = \frac{Weight}{Volume} = \frac{Mass \cdot g}{Volume}$$

$$So \qquad \gamma = \rho \cdot g$$

- The unit weight is frequently used in geotechnical engineering than the density (e.g. in calculating the overburden pressure).

Units of unit weight and density

✓The SI unit of mass density (ρ) is kilograms per cubic meter (kg/m^3).

✓The SI unit of force is Newton, therefore, the unit weights of soils are typically expressed in kN/m^3

Relationship between unit weight and density

The unit weights of soil in kN/m^3 can be obtained from densities in kg/m^3 as

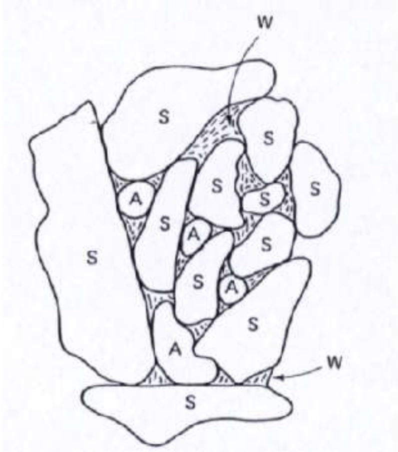
$$\boxed{\gamma\,(\mathrm{kN/m^3}) = \frac{g\rho(\mathrm{kg/m^3})}{1000}}$$

✓The density of water ρ_w varies slightly, depending on the temperature. At 4C^o , water's density is equal to 1000 kg/m^3 or 1 g/cm^3

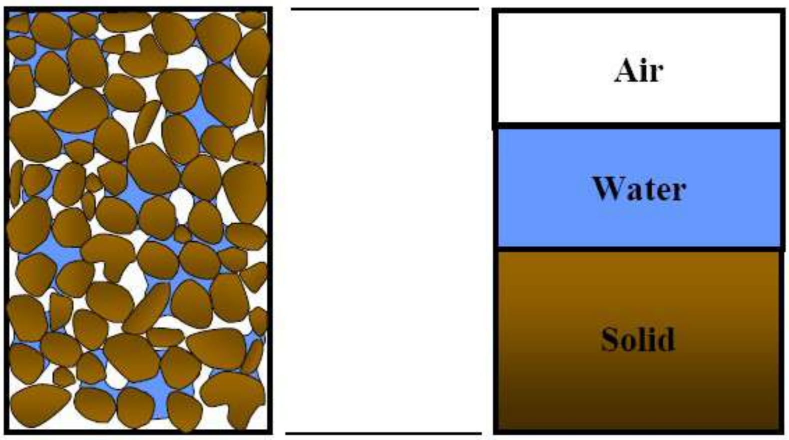
$$\text{unit weight of water}\,, \gamma_w = 9.81\ {kN}/{m^3}$$

Soil Phases

- Soil consists of solid particles plus the void space between the particles
- The void spaces are partially or completely filled with water or other liquid.
- Voids space not occupied by fluid are filled with air or other gas.
- Hence soil are referred to as three-phase system, i.e. Solid + Liquid (water) + Gas (air)

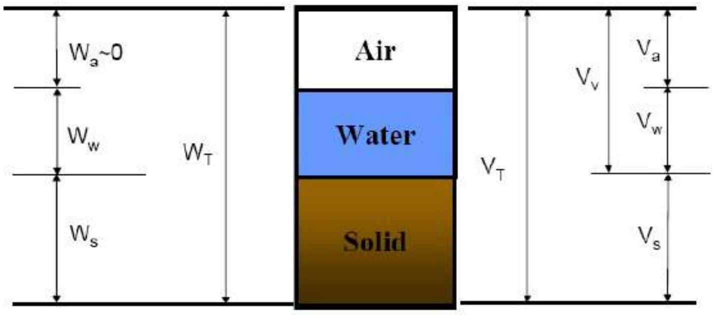


Three Phase Diagram



PHASE DIAGRAM

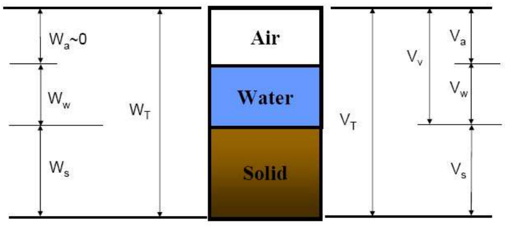
For purpose of study and analysis, it is convenient to represent the soil by a PHASE DIAGRAM, with part of the diagram representing the solid particles, part representing water or liquid, and another part air or other gas.



Wt: total weight
Ws: weight of solid
Ww: weight of water
Wa: weight of air = 0

Vt: total volume
Vs: volume of solid
Vw: volume of water
Vv: volume of the void

Volumetric Ratios



(1) Void ratio **e**

$$e = \frac{Volume\ of\ voids}{Volume\ of\ solids} = \frac{V_v}{V_s}$$

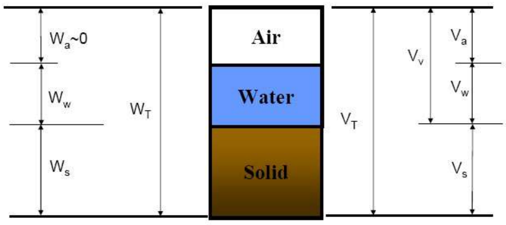
(2) Porosity **n%**

$$n = \frac{Volume\ of\ voids}{Total\ volume\ of\ soil\ sample} = \frac{V_v}{V_t} \times 100$$

(3) Degree of Saturation **S% (0 – 100%)**

$$S = \frac{Total\ volume\ of\ voids\ contains\ water}{Total\ volume\ of\ voids} = \frac{V_w}{V_v} \times 100\%$$

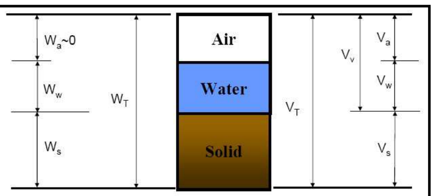
Weight Ratios



(1) Water Content ***w%***

$$w = \frac{Weight\ of\ water}{Weight\ of\ soil\ solids} = \frac{W_w}{W_s} \cdot 100\%$$

Soil unit weights



(1) Dry unit weight

$$\gamma_d = \frac{Weight\ of\ soil\ solids}{Total\ volume\ of\ soil} = \frac{W_s}{V_t}$$

(2) Total, Wet, Bulk, or Moist unit weight

$$\gamma = \frac{Total\ \text{weight}\ of\ soil}{Total\ volume\ of\ soil} = \frac{W_s + W_w}{V_t}$$

Saturated unit weight (considering S=100%, V_a =0)

$$\gamma_{sat} = \frac{Weight\ of\ soil\ solids + water}{Total\ volume\ of\ soil} = \frac{W_s + W_w}{V_t}$$

Specific gravity, G_s

The ratio of the mass of a solid particles to the mass of an equal volume of distilled water at 4°C

$$\boxed{G_s = \frac{w_s}{V_s\ \gamma_w}}$$

i.e., the specific gravity of a certain material is ratio of the unit weight of that material to the unit weight of water at 4° C.

The specific gravity of soil solids is often needed for various calculations in soil mechanics.

$$\boxed{G_s = \frac{\gamma_s}{\gamma_w}}$$

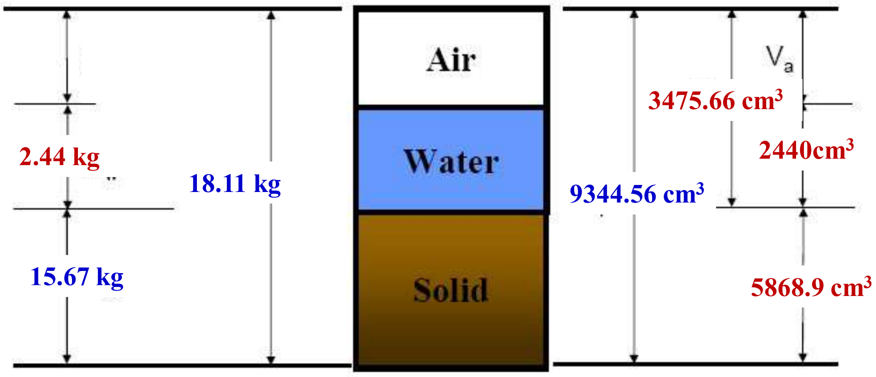
- $G_{w=1}$
- $G_{mercury} = 13.6$

- Expected Value for Gs

Type of Soil	Gs
Sand	2.65 - 2.67
Silty sand	2.67 – 2.70
Inorganic clay	2.70 – 2.80
Soils with mica or iron	2.75 – 3.00
Organic soils	< 2.00

Example 1:

In its natural state, a moist soil has a total volume of 9344.56 cm^3 and a mass 18.11 kg. The oven-dry mass of soil is 15.67 kg. If G_s = 2.67, calculate the moisture content, moist unit weight, void ratio and degree of saturation.



1. Relationship between e and n

$$e = \frac{V_v}{V_s} = \frac{V_v}{V - V_v} = \frac{\left(\frac{V_v}{V}\right)}{1 - \left(\frac{V_v}{V}\right)} = \frac{n}{1-n} \quad (3.6)$$

Also, from Eq. (3.6),

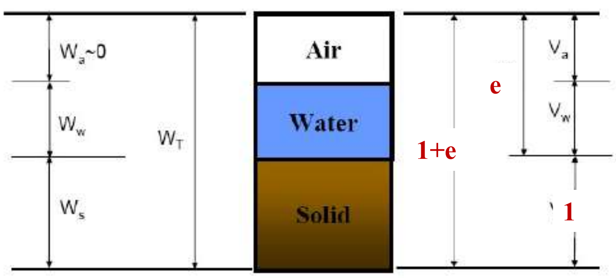
$$n = \frac{e}{1+e} \quad (3.7)$$

Using phase diagram

Given : e

required: n

$$n = \frac{V_v}{V_t} = \frac{e}{1+e}$$



2. Relationship among e, S, w, and Gs

$$w = \frac{W_w}{W_s} = \frac{\gamma_w V_w}{\gamma_s V_s} = \frac{\gamma_w V_w}{\gamma_w G_s V_s} = \frac{V_w}{G_s V_s}$$

•**Dividing the denominator and numerator of the R.H.S. by V_v *yields:***

$$\boxed{Se = wG_s}$$

•This is a very useful relation for solving THREE-PHASE RELATIONSHIPS.

2. Relationship among e, S, w, and Gs

•**<u>Textbook derivation</u>**

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s\gamma_w + wG_s\gamma_w}{1+e} = \frac{(1+w)G_s\gamma_w}{1+e} \qquad (3.15)$$

and

$$\gamma_d = \frac{W_s}{V} = \frac{G_s\gamma_w}{1+e} \qquad (3.16)$$

or

$$e = \frac{G_s\gamma_w}{\gamma_d} - 1 \qquad (3.17)$$

Because the weight of water for the soil element under consideration is $wG_s\gamma_w$, the volume occupied by water is

$$V_w = \frac{W_w}{\gamma_w} = \frac{wG_s\gamma_w}{\gamma_w} = wG_s$$

Hence, from the definition of degree of saturation [Eq. (3.5)],

$$S = \frac{V_w}{V_v} = \frac{wG_s}{e}$$

or

$$\boxed{Se = wG_s} \qquad (3.18)$$

•Various Unit Weight Relationships

Table 3.1 Various Forms of Relationships for γ, γ_d, and γ_{sat}

Moist unit weight (γ)		Dry unit weight (γ_d)		Saturated unit weight (γ_{sat})	
Given	**Relationship**	**Given**	**Relationship**	**Given**	**Relationship**
w, G_s, e	$\frac{(1+w)G_s\gamma_w}{1+e}$	γ, w	$\frac{\gamma}{1+w}$	G_s, e	$\frac{(G_s+e)\gamma_w}{1+e}$
S, G_s, e	$\frac{(G_s+Se)\gamma_w}{1+e}$	G_s, e	$\frac{G_s\gamma_w}{1+e}$	G_s, n	$[(1-n)G_s+n]\gamma_w$
w, G_s, S	$\frac{(1+w)G_s\gamma_w}{1+\frac{wG_s}{S}}$	G_s, n	$G_s\gamma_w(1-n)$	G_s, w_{sat}	$\left(\frac{1+w_{sat}}{1+w_{sat}G_s}\right)G_s\gamma_w$
w, G_s, n	$G_s\gamma_w(1-n)(1+w)$	G_s, w, S	$\frac{G_s\gamma_w}{1+\left(\frac{wG_s}{S}\right)}$	e, w_{sat}	$\left(\frac{e}{w_{sat}}\right)\left(\frac{1+w_{sat}}{1+e}\right)\gamma_w$
S, G_s, n	$G_s\gamma_w(1-n)+nS\gamma_w$	e, w, S	$\frac{eS\gamma_w}{(1+e)w}$	n, w_{sat}	$n\left(\frac{1+w_{sat}}{w_{sat}}\right)\gamma_w$
		γ_{sat}, e	$\gamma_{sat}-\frac{e\gamma_w}{1+e}$	γ_d, e	$\gamma_d+\left(\frac{e}{1+e}\right)\gamma_w$
		γ_{sat}, n	$\gamma_{sat}-n\gamma_w$	γ_d, n	$\gamma_d+n\gamma_w$
		γ_{sat}, G_s	$\frac{(\gamma_{sat}-\gamma_w)G_s}{(G_s-1)}$	γ_d, S	$\left(1-\frac{1}{G_s}\right)\gamma_d+\gamma_w$
				γ_d, w_{sat}	$\gamma_d(1+w_{sat})$

Example 2

The moist unit weight of a soil is 19.2 kN/m³. Given that G_s 2.69 and $w = 9.8\%$, determine

a. Dry unit weight
b. Void ratio
c. Porosity
d. Degree of saturation

a. $\gamma_d = \frac{\gamma}{1+w} = \frac{19.2}{1+\frac{9.8}{100}} = \mathbf{17.5\ kN/m^3}$

b. $\gamma_d = 17.5 = \frac{G_s\gamma_w}{1+e} = \frac{(2.69)(9.81)}{1+e}; \quad e = \mathbf{0.51}$

c. $n = \frac{e}{1+e} = \frac{0.51}{1+0.51} = \mathbf{0.338}$

d. $S = \frac{wG_s}{e} = \frac{(0.098)(2.69)}{0.51} \times 100 = \mathbf{51.7\%}$

Example 3

Field density testing (e.g., sand replacement method) has shown bulk density of a compacted road base to be 2.06 t/m^3 with a water content of 11.6%. Specific gravity of the soil grains is 2.69. Calculate the dry density, porosity, void ratio and degree of saturation.

Solution:

$$w = \frac{Se}{G_s}$$

$$\therefore Se = (0.116)(2.69) = 0.312$$

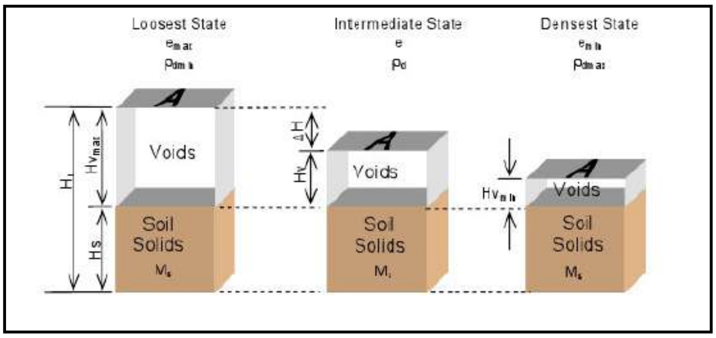
$$\rho_m = \frac{G_s + Se}{1+e}\rho_w$$

$$\therefore 2.06 = \frac{2.69 + 0.312}{1+e} \times 1.0$$

$$\therefore e = 0.457$$

•Relative Density

- The relative density is the parameter that compare the volume reduction achieved from compaction to the maximum possible volume reduction
- The relative density Dr, also called density index is commonly used to indicate the IN SITU denseness or looseness of granular soil.



Volume reduction from compaction of granular soil

D_r can be expressed either in terms of void ratios or dry densities.

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}}$$

where D_r = relative density, usually given as a percentage
e = *in situ* void ratio of the soil
e_{max} = void ratio of the soil in the loosest state
e_{min} = void ratio of the soil in the densest state

$$D_r = \frac{\left[\frac{1}{\gamma_{d(min)}}\right] - \left[\frac{1}{\gamma_d}\right]}{\left[\frac{1}{\gamma_{d(min)}}\right] - \left[\frac{1}{\gamma_{d(max)}}\right]} = \left[\frac{\gamma_d - \gamma_{d(min)}}{\gamma_{d(max)} - \gamma_{d(min)}}\right]\left[\frac{\gamma_{d(max)}}{\gamma_d}\right]$$

where $\gamma_{d(min)}$ = dry unit weight in the loosest condition (at a void ratio of e_{max})
γ_d = *in situ* dry unit weight (at a void ratio of e)
$\gamma_{d(max)}$ = dry unit weight in the densest condition (at a void ratio of e_{min})

•<u>Remarks</u>

- The relative density of a natural soil very strongly affect its engineering behavior.
- The range of values of D_r may vary from a minimum of zero for very LOOSE soil to a maximum of 100% for a very DENSE soil.
- Because of the irregular size and shape of granular particles, it is not possible to obtain a ZERO volume of voids.

- Granular soils are qualitatively described according to their relative densities as shown below

Relative Density (%)	Description of soil deposit
0-15	Very loose
15-50	Loose
50-70	Medium
70-85	Dense
85-100	Very dense

- The use of relative density has been restricted to granular soils because of the difficulty of determining e_{max} in clayey soils. Liquidity Index in fine-grained soils is of similar use as D_r in granular soils.

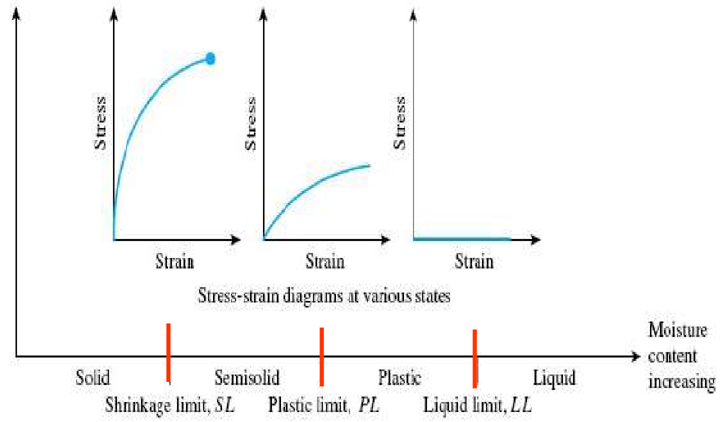
4. Soil Consistency (Plasticity)

1

What is Consistency?

- Consistency is a term used to describe the degree of firmness of fine-grained soils (silt and clay). The consistency of fine grained soils is expressed qualitatively by such terms as very soft , soft, stiff, very stiff and hard.
- Water content significantly affects properties of silty and clayey soils (unlike sand and gravel).
- It has been found that at the same water content, two samples of clay of different origins may possess different consistency.

2



- From the figure above, at a very low moisture content, soil behaves more like a solid. When the moisture content is very high, the soil and water may flow like a liquid.
- Also, soil strength decreases as water content increases.
- Therefore, the soil behavior of the soil, based on is divided into four states based on moisture content: *solid, semi-solid, plastic, liquid*.

Atterberg Limits: are water contents at certain limiting or critical stages in soil behavior. These limits are:

Liquid Limit (LL): The water content, in percent, at the point of transition from plastic to liquid state

Plastic Limit (PL): The water content, in percent, at the point of transition from semisolid to plastic state.

Shrinkage Limit (SL): The water content, in percent, at the point of transition from solid to semisolid state

Importance of Atterberg limits

The Atterberg limits may be used for the following:

1. To obtain general information about a soil and its strength, compressibility, and permeability properties.

2. Empirical correlations for some engineering properties.

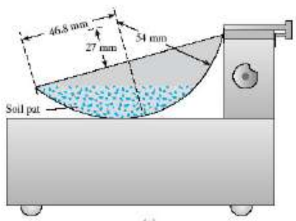
3. Soil classification

5

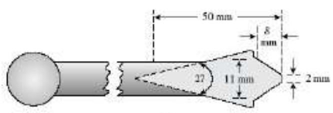
Liquid limit (LL) determination

1. Multi-Point Method (ASTM D-4318)

The water content required to close a distance of ½ inch (12.7 mm) along the bottom of the groove after 25 blows is defined as the Liquid Limit.



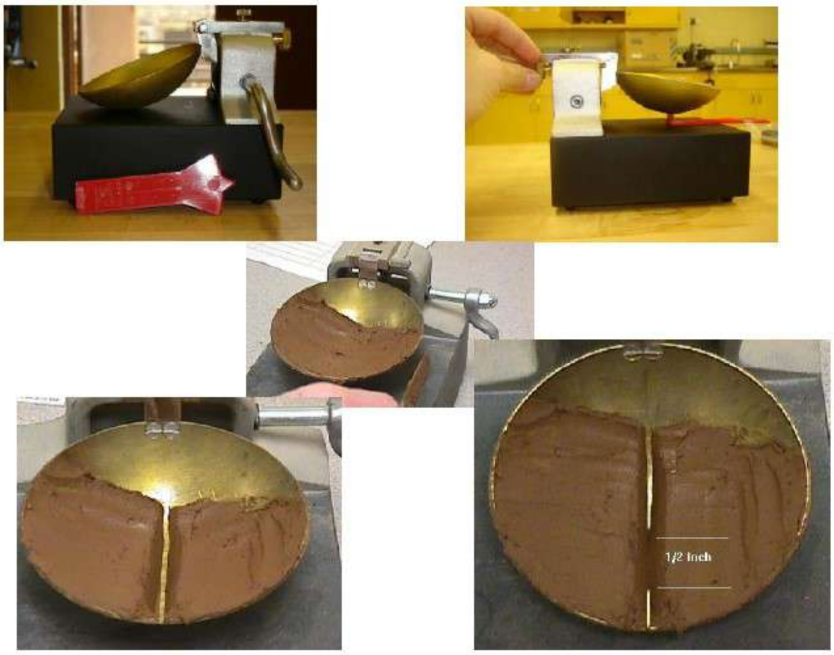
(a)



(b)


Casagrande Cup

6



- Number of tests for the same soil are made at varying w%, and then w% values are plotted against the logarithm of the number of blows, N.

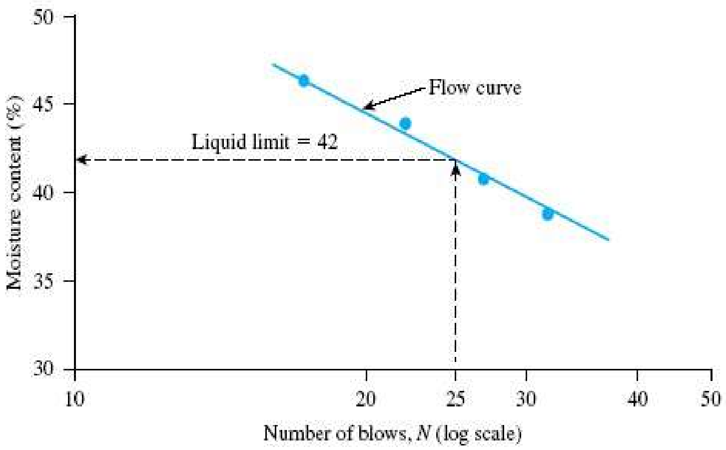


Figure 4.5
Flow curve for liquid limit determination of a clayey silt

2. One-Point Method (ASTM D-4318)

- Proposed by the USACE in 1949 based on the analysis of hundreds of liquid limit tests.

$$\boxed{LL = w_N\left(\frac{N}{25}\right)^{\tan\beta}}$$

$$\boxed{\tan\beta = 0.121}$$

w_N = moisture content of the soil which closed in N blows (N should be between 10 and 40).
N = number of blows required to close the standard groove for a distance of ½ inch (12.7mm)

- This formula generally yields good results for the number of blows between 20 and 30.

9

Plastic Limit (PL) determination

1. Rolling into Thread Method (ASTM D-4318)

The plastic limit is defined as the moisture content in percent, at which the soil crumbles, when rolled into threads of 3.18 mm (1/8 in.) in diameter.

10

Shrinkage Limit (SL) (ASTM D-427, ASTM D-4943)

The shrinkage limit is defined as the moisture content, in percent, at which the volume of the soil mass ceases to change.

$$SL = w_i(\%) - \Delta w(\%)$$

$$SL = \left(\frac{M_1 - M_2}{M_2}\right)(100) - \left(\frac{V_i - V_f}{M_2}\right)(\rho_w)(100)$$

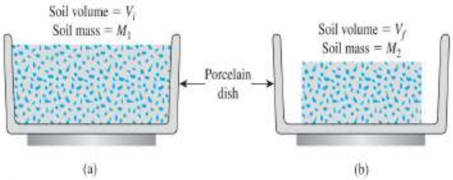


Figure 4.10 Shrinkage limit test: (a) soil pat before drying; (b) soil pat after drying

Mineral	Shrinkage limit
Montmorillonite	8.5–15
Illite	15–17
Kaolinite	25–29

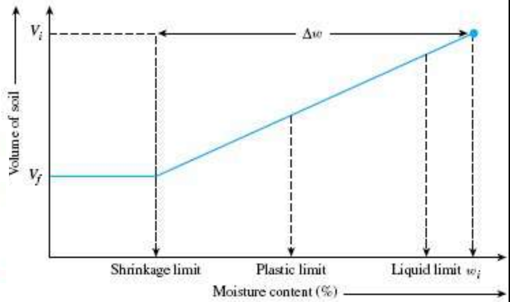


Figure 4.9 Definition of shrinkage limit

SOIL CONSISTENCY INDICES

Various indices have been developed using Atterberg limits.

1. Plasticity Index (PI)

$$\boxed{PI = LL - PL}$$

- This index provides a measure of a soil plasticity, which is the amount of water that must be added to change a soil from its plastic limit to its liquid limit.
- The PI is useful in engineering classification of fine-grained soils, and many engineering properties have been found to correlate with the PI.

Table 4.1 Typical Values of Liquid Limit, Plastic Limit, and Activity of Some Clay Minerals

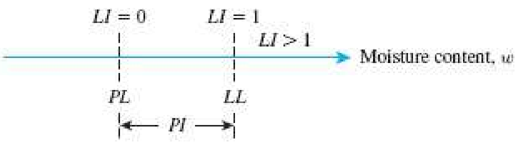
Mineral	Liquid limit, *LL*	Plastic limit, *PL*	Activity, *A*
Kaolinite	35–100	20–40	0.3–0.5
Illite	60–120	35–60	0.5–1.2
Montmorillonite	100–900	50–100	1.5–7.0

PI	Description
0	Nonplastic
1–5	Slightly plastic
5–10	Low plasticity
10–20	Medium plasticity
20–40	High plasticity
>40	Very high plasticity

2. Liquidity Index (LI)

- The relative consistency of a cohesive soil in the natural state can be defined by a ratio called the Liquidity Index, which is given by

$$LI = \frac{w - PL}{LL - PL}$$



- This index provides a clue as the condition of the in situ soil. This index helps us to know if our sample was likely to behave as a plastic, a brittle, or a liquid.

• If LI< 0 Brittle behavior (desiccated (dried) hard soil)
• If 0<LI<1 The soil behave like a plastic
• If LI>1 The soil is a very viscous liquid.

3. Activity

- The presence of even small amounts of certain clay minerals in a soil mass can have a significant effect on the properties of the soil.
- Identifying the type and amount of clay minerals may be necessary in order to predict the soil's behavior or to develop methods for minimizing detrimental effects.
- An indirect method of obtaining information on the type and effect of clay minerals in a soil is to relate plasticity to the quantity of clay–sized particles.
- It is known that for a given amount of clay mineral, the plasticity resulting in a soil will vary for the different types of clays.

15

- A quantity called activity is defined as the slope of the line correlating PI and %finer then 2 micrometer and expressed as:

$$A = \frac{PI}{(\%\text{of clay-size fraction, by weight})}$$

Table 4.1 Typical Values of Liquid Limit, Plastic Limit, and Activity of Some Clay Minerals

Mineral	Liquid limit, *LL*	Plastic limit, *PL*	Activity, *A*
Kaolinite	35–100	20–40	0.3–0.5
Illite	60–120	35–60	0.5–1.2
Montmorillonite	100–900	50–100	1.5–7.0

- **The activity factor gives information on the type and effect of CLAY MINERAL in a soil.**

16

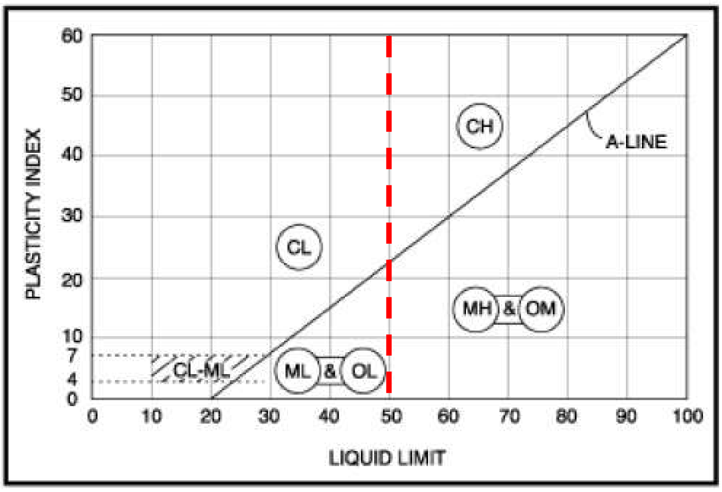
- Clay minerals with KAOLINITE have LOW activity, whereas those soils with MONTMORILLONITE will have a high activity value.
- Activity is used as an index for identifying the <u>swelling potential</u> of clay soils.

Activity	**Classification**
<0.75	Inactive clays
0.75-1.25	Normal Clays
>1.25	Active Clays

17

PLASTICITY CHART

- Casagrande (1932) studied the relationship of the plasticity index to the liquid limit of a wide variety natural soils.



18

- Above A-line Clays
  Below A-line Silts and organic soils (silt and clays)
- Left of B-line --→Low plasticity
  Right of B-line--→ High plasticity
- U-line is approximately the upper limit of the relationship of PI and the LL for any soil found so far. The data plotting above or to the left of U-Line should be considered as likely in error and should be rechecked.
- All the lines (A, U, and B) are empirical.
- The plasticity chart is the basis for the classification of the fine-grained soils according to USCS.

19

5. Soil Classification

(Das, chapter 5)

Purpose

- Classifying soils into groups or sub-groups with similar engineering behavior.
- Classification systems were developed in terms of *simple* indices (GSD and plasticity).
- These classifications can provide geotechnical engineers with general guidance about engineering properties of the soils through the *accumulated experience*.

Communicate between engineers

Simple indices GSD, LL, PI → Classification system (Language) → Estimate engineering properties → Achieve engineering purposes

Use the accumulated experience

Classification Systems

Two commonly classification system used are:

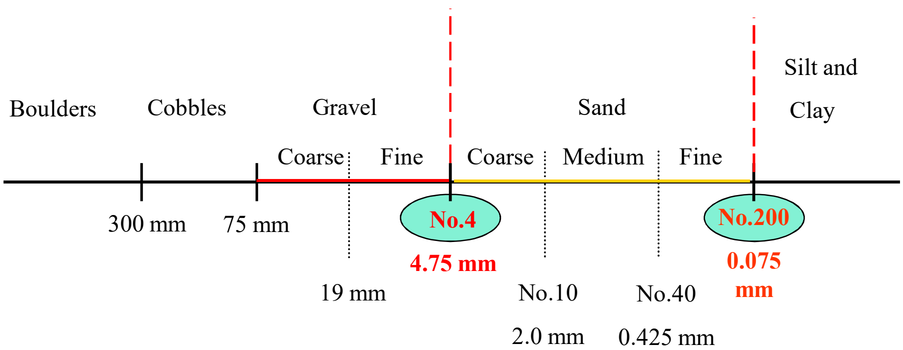
1. Unified Soil Classification System (USCS) (preferred by geotechnical engineers).

2. American Association of State Highway and Transportation Officials (AASHTO) System (preferred by Transportation engineers).

1. Unified Soil Classification System (USCS)

i. Definition of Grain Size

No specific grain size-use Atterberg limits



1. Unified Soil Classification System (USCS)

% Passing sieve No. 200 (0.075 mm)

< 50%

Coarse-grained soils

•Grain size distribution

> 50%

Fine-grained soils
Silt (M)
Clay (C)

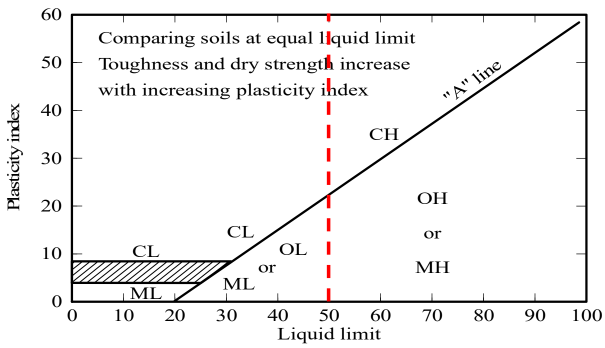
•Use Plasticity chart

•LL, PL

Required tests: Sieve analysis
Atterberg limit

1. Unified Soil Classification System (USCS)

Used for Fine grained soils to determine whether silt (M) or clay (C)



Plasticity chart

Below A-line is silt – use symbol M
Above A-line is clay – use symbol C

LL > 50 → High plasticity
LL< 50 → low plasticity

1. Unified Soil Classification System (USCS)

% Passing sieve No. 200 (0.075 mm)

< 50%

Coarse-grained soils

•Grain size distribution

> 50%

Fine-grained soils
Silt (M)
Clay (C)

•Use Plasticity chart

ML, MH, CL, CH

Required tests: Sieve analysis
Atterberg limit

1. Unified Soil Classification System (USCS)

% Passing sieve No. 200 (0.075 mm)

< 50%

Coarse-grained soils

% Coarse soil (Co) = 100 - % Passing # 200
% Gravel (G) = 100 - % Passing # 4

G > 1/2 Co
Gravel (G)

G < 1/2 Co
Sand (S)

% Passing sieve No. 200

- < 5% — GW, GP, SW or SP — Use → C_u, C_c — W : well graded P: poorly graded
- 5% -12 % — GW-GM, GW-GC, GP-GM, GP-GC, SW-SM, SW-SC, SP-SM, SP-SC
- > 12% — GM, GC, SM, SC — Use → plasticity charts

> 50%

Fine-grained soils
Silt (M)
Clay (C)

•Use Plasticity chart

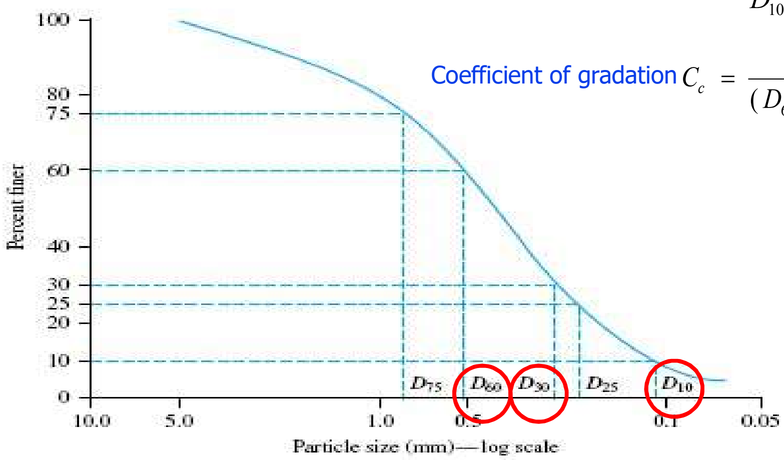
•LL, PL

1. Unified Soil Classification System (USCS)

To determine if well graded (W) or poorly graded (P), calculate C_u and C_c

Coefficient of uniformity $C_u = \dfrac{D_{60}}{D_{10}}$

Coefficient of gradation $C_c = \dfrac{D_{30}^2}{(D_{60} \times D_{10})}$



1. Unified Soil Classification System (USCS)

Coefficient of uniformity $C_u = \dfrac{D_{60}}{D_{10}}$

Coefficient of gradation $C_c = \dfrac{D_{30}^2}{(D_{60} \times D_{10})}$

Conditions for Well-graded soils

For gravels → $C_u > 4$ **and** C_c is between 1 and 3

For Sand → W if $C_u > 6$ **and** C_c is between 1 and 3

1. Unified Soil Classification System (USCS)

Table 5.2 Unified Soil Classification System (Based on Material Passing 76.2-mm Sieve)

Criteria for assigning group symbols				Group symbol
Coarse-grained soils More than 50% of retained on No. 200 sieve	**Gravels** More than 50% of coarse fraction retained on No. 4 sieve	Clean Gravels Less than 5% fines[a]	$C_u \geq 4$ and $1 \leq C_c \leq 3$[c]	GW
			$C_u < 4$ and/or $1 > C_c > 3$[c]	GP
		Gravels with Fines More than 12% fines[a,d]	$PI < 4$ or plots below "A" line (Figure 5.3)	GM
			$PI > 7$ and plots on or above "A" line (Figure 5.3)	GC
	Sands 50% or more of coarse fraction passes No. 4 sieve	Clean Sands Less than 5% fines[b]	$C_u \geq 6$ and $1 \leq C_c \leq 3$[c]	SW
			$C_u < 6$ and/or $1 > C_c > 3$[c]	SP
		Sands with Fines More than 12% fines[b,d]	$PI < 4$ or plots below "A" line (Figure 5.3)	SM
			$PI > 7$ and plots on or above "A" line (Figure 5.3)	SC
Fine-grained soils 50% or more passes No. 200 sieve	**Silts and clays** Liquid limit less than 50	Inorganic	$PI > 7$ and plots on or above "A" line (Figure 5.3)[e]	CL
			$PI < 4$ or plots below "A" line (Figure 5.3)[e]	ML
		Organic	$\frac{\text{Liquid limit — oven dried}}{\text{Liquid limit — not dried}} < 0.75$; see Figure 5.3; OL zone	OL
	Silts and clays Liquid limit 50 or more	Inorganic	PI plots on or above "A" line (Figure 5.3)	CH
			PI plots below "A" line (Figure 5.3)	MH
		Organic	$\frac{\text{Liquid limit — oven dried}}{\text{Liquid limit — not dried}} < 0.75$; see Figure 5.3; OH zone	OH
Highly Organic Soils	Primarily organic matter, dark in color, and organic odor			Pt

[a] Gravels with 5 to 12% fine require dual symbols: GW-GM, GW-GC, GP-GM, GP-GC.

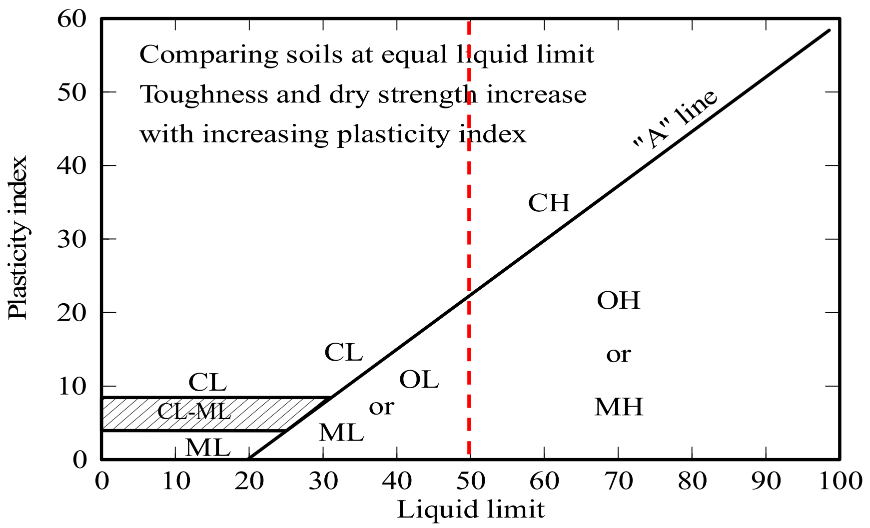
[b] Sands with 5 to 12% fines require dual symbols: SW-SM, SW-SC, SP-SM, SP-SC.

[c] $C_u = \frac{D_{60}}{D_{10}}$; $C_c = \frac{(D_{30})^2}{D_{60} \times D_{10}}$

[d] If $4 \leq PI \leq 7$ and plots in the hatched area in Figure 5.3, use dual symbol GC-GM or SC-SM.

[e] If $4 \leq PI \leq 7$ and plots in the hatched area in Figure 5.3, use dual symbol CL-ML.

1. Unified Soil Classification System (USCS)

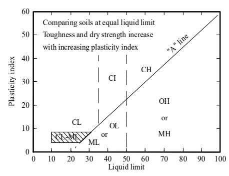


Plasticity chart

Unified soil classification (including identification and description)

Field identification procedures (Excluding particles larger than 75mm and basing fractions on estimated weights)							Group symbols 1	Typical names	Information required for describing soils	Laboratory classification criteria
Coarse grained soils More than half of material is larger than .075mm sieve size	Gravels More than half of coarse fraction is larger than 2.36mm	Clean gravels (little or no fines)	Wide range of grain size and substantial amounts of all intermediate particle sizes				GW	Well graded gravels, gravel-sand mixtures, little or no fines	Give typical names: indicate approximate percentages of sand and gravel: maximum size: angularity, surface condition, and hardness of the coarse grains: local or geological name and other pertinent descriptive information and symbol in parentheses.	Use grain size curve in identifying the fractions as given under field identification. Determine percentages of gravel and sand from grain size curve. Depending on percentages of fines (fraction smaller than .075mm sieve size) coarse grained soils are classified as follows: Less than 5% GW, GP, SW, SP; More than 12% GM, GC, SM, SC; 5% to 12% Borderline case requiring use of dual symbols. $C_U = \frac{D_{60}}{D_{10}}$ Greater than 4; $C_c = \frac{(D_{30})^2}{D_{10} \times D_{60}}$ Between 1 and 3
			Predominantly one size or a range of sizes with some intermediate sizes missing				GP	Poorly graded gravels, gravel-sand mixtures, little or no fines		Not meeting all gradation requirements for GW
		Gravels with fines (appreciable amount of fines)	Non-plastic fines (for identification procedures see ML below)				GM	Silty gravels, poorly graded gravel-sand-silt mixtures	For undisturbed soils add information on stratification, degree of compactness, cementation, moisture conditions and drainage characteristics.	Atterberg limits below "A" line or PI less than 4. Above "A" line with PI between 4 and 7 are borderline cases requiring use of dual symbols
			Plastic fines (for identification procedures see CL below)				GC	Clayey gravels, poorly graded gravel-sand-clay mixtures		Atterberg limits above "A" line with PI greater than 7
	Sands More than half of coarse fraction is smaller than 2.36mm	Clean sands (little or no fines)	Wide range in grain sizes and substantial amounts of all intermediate particle sizes				SW	Well graded sands, gravelly sands, little or no fines	*Example:* Silty sand, gravelly; about 20% hard angular gravel particles 12.5mm maximum size; rounded and subangular sand grains coarse to fine, about 15% non-plastic lines with low dry strength; well compacted and moist in places; alluvial sand; (SM)	$C_U = \frac{D_{60}}{D_{10}}$ Greater than 6; $C_c = \frac{(D_{30})^2}{D_{10} \times D_{60}}$ Between 1 and 3
			Predominantely one size or a range of sizes with some intermediate sizes missing				SP	Poorly graded sands, gravelly sands, little or no fines		Not meeting all gradation requirements for SW
		Sands with fines (appreciable amount of fines)	Non-plastic fines (for identification procedures, see ML below)				SM	Silty sands, poorly graded sand-silt mixtures		Atterberg limits below "A" line or PI less than 4. Above "A" line with PI between 4 and 7 are borderline cases requiring use of dual symbols
			Plastic fines (for identification procedures, see CL below)				SC	Clayey sands, poorly graded sand-clay mixtures		Atterberg limits above "A" line with PI greater than 7
Fine grained soils More than half of material is smaller than .075mm sieve size	Identification procedure on fraction smaller than .425mm sieve size			*Dry strength crushing character-istics*	*Dilatency (reaction to shaking)*	*Toughness (consistency near plastic limit)*				
	Silts and clays liquid limit less than 50			None to slight	Quick to slow	None	ML	Inorganic silts and very fine sands, rock flour, silty or clayey fine sands with slight plasticity	Give typical name; indicate degree and character of plasticity, amount and maximum size of coarse grains: colour in wet condition, odour if any, local or geological name, and other pertinent descriptive information, and symbol in parentheses	
				Medium to high	None to very slow	Medium	CL,CI	Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silty clays, lean clays		
				Slight to medium	Slow	Slight	OL	Organic silts and organic silt-clays of low plasticity		
	Silts and clays liquid limit greater than 50			Slight to medium	Slow to none	Slight to medium	MH	inorganic silts, micaceous or dictomaceous fine sandy or silty soils, elastic silts	For undisturbed soils add information on structure, stratification, consistency and undisturbed and remoulded states, moisture and drainage conditions	
				High to very high	None	High	CH	Inorganic clays of high plasticity, fat clays		
				Medium to high	None to very high	Slight to medium	OH	Organic clays of medium to high plasticity	*Example* Clayey silt, brown: slightly plastic: small percentage of fine sand: numerous vertical root holes: firm and dry in places; loess; (ML)	
Highly organic soils			Readily identified by colour, odour spongy feel and frequently by fibrous texture				Pt	Peat and other highly organic soils		

The .075mm sieve size is about the smallest particle visible to the naked eye



Plasticity chart
for laboratory classification of fine grained soils

Organic Soils

- Highly organic soils- Peat (Group symbol PT)
  - A sample composed primarily of vegetable tissue in various stages of decomposition and has a fibrous to amorphous texture, a dark-brown to black color, and an organic odor should be designated as a highly organic soil and shall be classified as peat, PT.
- Organic clay or silt (group symbol OL or OH):
  - “The soil’s liquid limit (LL) after oven drying is less than 75 % of its liquid limit before oven drying.” If the above statement is true, then the first symbol is O.
  - The second symbol is obtained by locating the values of PI and LL (not oven dried) in the plasticity chart.

Borderline Cases (Dual Symbols)

Coarse-grained soils with 5% - 12% fines.

- About 7 % fines can change the hydraulic conductivity of the coarse-grained media by orders of magnitude.
- The first symbol indicates whether the coarse fraction is well or poorly graded. The second symbol describe the contained fines. For example: SP-SM, poorly graded sand with silt.

Fine-grained soils with limits within the shaded zone. (PI between 4 and 7 and LL between about 12 and 25).

- It is hard to distinguish between the silty and more claylike materials.
- CL-ML: Silty clay, SC-SM: Silty, clayed sand.

Soil contain similar fines and coarse-grained fractions.

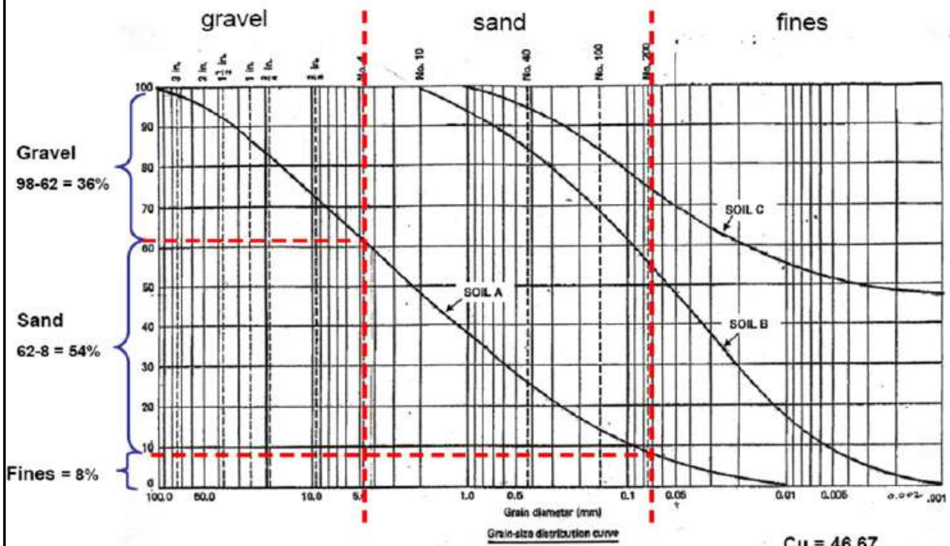
- possible dual symbols GM-ML

Example 1

Classify the following soils Using Unified Classification System.

Soil	No. 4 Sieve	No. 200 Sieve	LL	PI
(cumulative % passing)				
A	92	48	30	10
B	99	76	60	32
C	80	35	24	2

Example 1 – Soil A



Soil A: D_{60} = 4.2 mm , D_{30} = 0.6 mm, D_{10} = 0.09 mm

Cu = 46.67
Cc = 0.95

Summary of the USCS

coarse grain soils ←|→ fine grain soils

[>50% larger than 0.075 mm] [>50% smaller than 0.075 mm]

% of fines

0 5 12 50 100

CoF
e.g., SM

FP
e.g., CH

CoGr
e.g., GP

CoGr - CoF
e.g., GP-GC

Co: Coarse
G = Gravel
S = Sands

F: Fines
M = Silts
C = Clays

Gr: Gradation
W = well graded
P = poorly graded

P: Plasticity
H = LL > 50
L = LL < 50

Symbols

Soil symbols:

G: Gravel

S: Sand

M: Silt

C: Clay

O: Organic

Pt: Peat

Liquid limit symbols:

H: High LL (LL>50)

L: Low LL (LL<50)

Gradation symbols:

W: Well-graded

P : Poorly-graded

Example: SW, Well-graded sand
SC, Clayey sand
SM, Silty sand,
MH, Elastic silt

Well – graded soil

$1 < C_c < 3$ and $C_u \geq 4$

(for gravels)

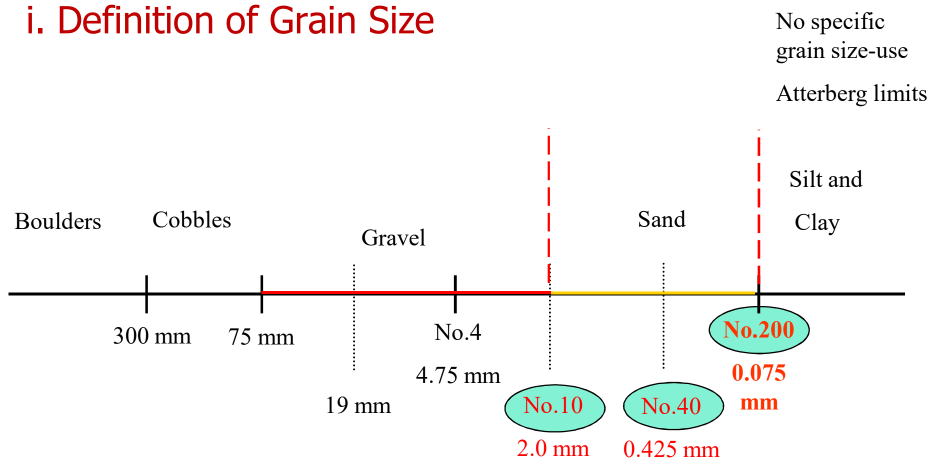
$1 < C_c < 3$ and $C_u \geq 6$

(for sands)

2. American Association of State Highway and Transportation Officials system (AASHTO)

2. AASHTO

i. Definition of Grain Size



2. AASHTO

ii. General guidance

- 8 major groups: A1~ A7 (with several subgroups) and organic soils A8
- The required tests are sieve analysis and Atterberg limits.
- The group index, an empirical formula, is used to further evaluate soils within a group (subgroups).

A1 ~ A3	A4 ~ A7
Granular Materials	Silt-clay Materials
≤ 35% pass No. 200 sieve	≥ 36% pass No. 200 sieve
Using LL and PI separates silty materials from clayey materials (only for A2 group)	Using LL and PI separates silty materials from clayey materials

- The original purpose of this classification system is used for road construction (subgrade rating).

2. AASHTO

iii. Classification

General classification	Granular materials (35% or less of total sample passing No. 200)						
	A-1			A-2			
Group classification	A-1-a	A-1-b	A-3	A-2-4	A-2-5	A-2-6	A-2-7
Sieve analysis (percentage passing)							
No. 10	50 max.						
No. 40	30 max.	50 max.	51 min.				
No. 200	15 max.	25 max.	10 max.	35 max.	35 max.	35 max.	35 max.
Characteristics of fraction passing No. 40							
Liquid limit				40 max.	41 min.	40 max.	41 min.
Plasticity index	6 max.		NP	10 max.	10 max.	11 min.	11 min.
Usual types of significant constituent materials	Stone fragments, gravel, and sand		Fine sand	Silty or clayey gravel and sand			
General subgrade rating	Excellent to good						

Classification starts from left to right

2. AASHTO

iii. Classification

General classification	Silt-clay materials (more than 35% of total sample passing No. 200)			
Group classification	A-4	A-5	A-6	A-7 A-7-5[a] A-7-6[b]
Sieve analysis (percentage passing)				
No. 10				
No. 40				
No. 200	36 min.	36 min.	36 min.	36 min.
Characteristics of fraction passing No. 40				
Liquid limit	40 max.	41 min.	40 max.	41 min.
Plasticity index	10 max.	10 max.	11 min.	11 min.
Usual types of significant constituent materials	Silty soils		Clayey soils	
General subgrade rating	Fair to poor			

[a]For A-7-5, $PI \le LL - 30$
[b]For A-7-6, $PI > LL - 30$

Note:

The first group from the left to fit the test data is the correct AASHTO classification.

Group Index

- Used to evaluate the quality of a soil as a highway subgrade material.
- This index is written in parentheses after the group or subgroup designation [e.g. A-4(3)].

The first term is determined by the LL

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10) \qquad (1)$$

The second term is determined by the PI

For Groups A-2-6 and A-2-7

$GI = 0.01(F_{200} - 15)(PI - 10)$ use the second term only

F200: percentage passing through the No.200 sieve

In general, the rating for a pavement subgrade is inversely proportional to the group index, GI.

Some rules of Group Index GI

1. If Eq. (1) yields a negative value for GI, it is taken as 0.
2. The group index is rounded off to the nearest whole number (for example, GI 3.4 is rounded off to 3; GI 3.5 is rounded off to 4).
3. There is no upper limit for the group index.
4. The group index of soils belonging to groups A-1-a, A-1-b, A-2-4, A-2-5, and A-3 is always 0.

Example 1

Classify the following soils by the AASHTO classification system.

Description	A	B	C	D	E
			Soil		
Percent finer than No. 10 sieve	83	100	48	90	100
Percent finer than No. 40 sieve	48	92	28	76	82
Percent finer than No. 200 sieve	20	86	6	34	38
Liquid limit[a]	20	70	—	37	42
Plasticity index[a]	5	32	Nonplastic	12	23

Example 1 [Soil B]

Passing No.200 86%

LL=70, PI=32

LL-30=40 > PI=32

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10) = 33.47 \cong 33 \quad \textit{Round off}$$

A-7-5(33)

General classification	Silt-clay materials (more than 35% of total sample passing No. 200)			
Group classification	*A-4*	*A-5*	*A-6*	*A-7* *A-7-5*[a] *A-7-6*[b]
Sieve analysis (percentage passing)				
No. 10				
No. 40				
No. 200	36 min.	36 min.	36 min.	36 min.
Characteristics of fraction passing No. 40				
Liquid limit	40 max.	41 min.	40 max.	41 min.
Plasticity index	10 max.	10 max.	11 min.	11 min.
Usual types of significant constituent materials	Silty soils		Clayey soils	
General subgrade rating	Fair to poor			

[a] For A-7-5, $PI \le LL - 30$

[b] For A-7-6, $PI > LL - 30$

Example 2

- Classify the following soil Using AASHTO System.
- Given:
- % passing No. 10 = 100;
- % passing No. 40 = 80;
- '% passing No.200 = 58
- LL = 30; PI = 10.

Example 2

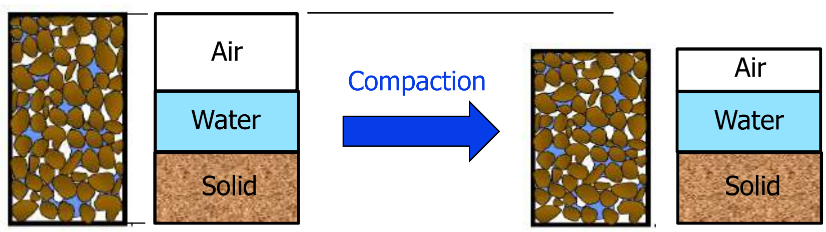
From Table, the group classification is A-4.

Now we calculate the Group Index
GI= (F-3S) [0.2 + 0.005(LL-40)] + 0.01(F-1)(PI-10)
Using the given data, F= 58, LL=30 & PI=10
GI= (58-23) [0.2 + 0.005(30-40)] + 0.01(58-15) (10-10)
GI= (35) [0.2 + 0.005(-10)] + 0.01(58-15) (0)
GI= 5.25
Then GI= 5.25 - - > 5
Thus, the AASHTO Classification is A- 4 (5); Silty Soil;
Fair to poor as subgrade material

6. Soil Compaction

What is Compaction?

- Compaction- Densification of soil by reducing air voids by application of mechanical energy.
- Compaction is used in construction of highway embankments, earth dams and many other engineering structures, loose soils must be compacted to improve their strength by increasing their unit weight
- The degree of compaction is measured in terms of its dry unit weight.



Purpose of Compaction

- Increases shear strength of soils
- Increases the bearing capacity of foundations
- Decreases the undesirable settlement of structures
- Reduction in hydraulic conductivity
- Increasing the stability of slopes on embankments.

Role of Water in Compaction

- Water is added to the soil during compaction which acts as a lubricating agent on the soil particles. The soil particles slip over each other and move into a densely packed position.

- In soils, compaction is a function of water content
  - Consider 0% moisture -Only compact so much
  - Add a little water - compacts better
  - A little more water -a little better compaction
  - Even more water – ???

Role of Water in Compaction

- For a given soil, the dry unit weight increases as water is added to the soil. This continues up to certain moisture content.

- Beyond this moisture content, more water added will fill the void space with water so further compaction is not possible.

- The moisture content at which the maximum dry unit weight is attained is generally referred to as the optimum moisture content.

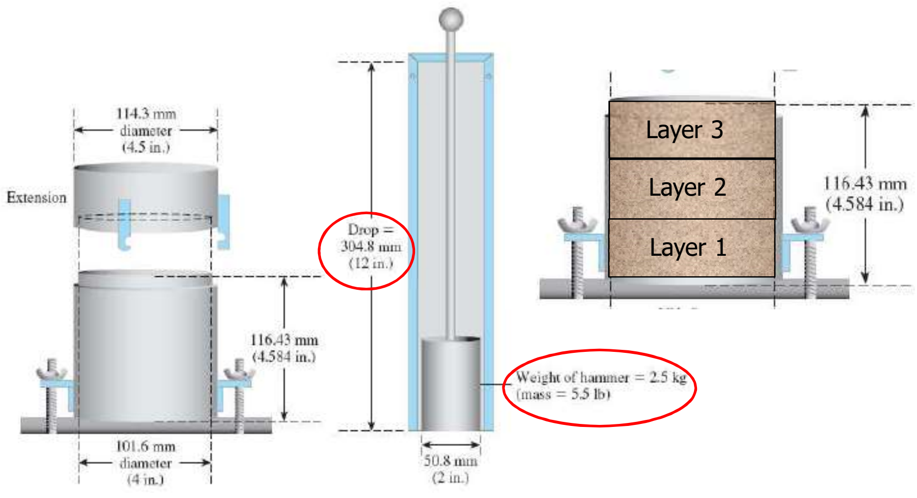
Another Definition of Compaction

Compaction is the densification of soil that is realized by re-arrangement of soil particles with outflow of air only. It is realized by application of mechanic energy. It does not involve fluid flow, but moisture content change.

Laboratory Compaction tests

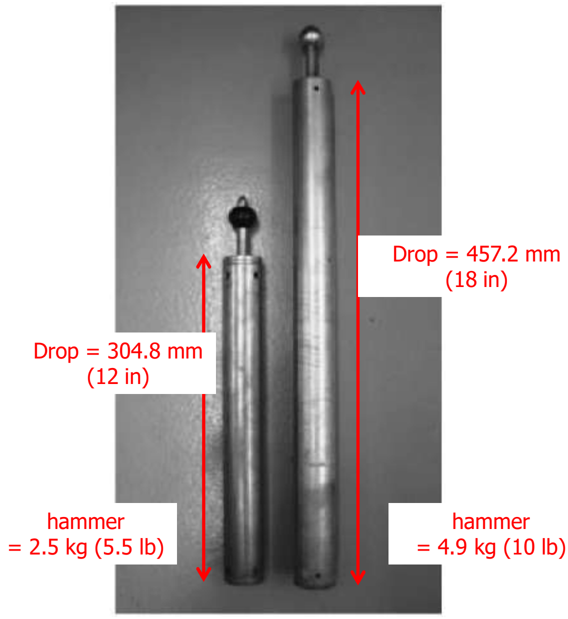
- Purpose: Find the maximum dry unit weight and corresponding optimum moisture content.
- The standard was originally developed to simulate field compaction in the laboratory
- There are two types of standard tests (ASTM – 698) :
  1. Standard Proctor test
  2. Modified Proctor test

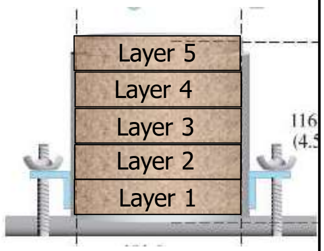
Standard Proctor Test



No. of blows = 25

Modified Proctor Test





No. of blows = 25

Summary of Laboratory Compaction Tests

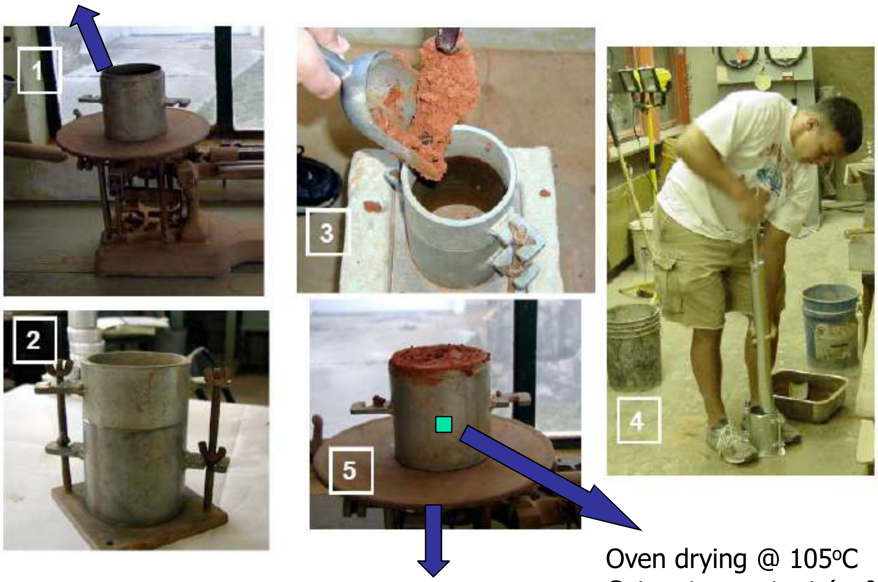
Test	Hammer weight	Height of drop	#Blows per layer	# Layers	Volume of Mold
Standard proctor Test	2.5 kg (5.5 lb)	304.8 mm (12 in)	25	3	944 cm^3 (1/30 ft^3)
Modified Proctor test	4.9 kg (10 lb)	457.2 mm (18 in)	25	5	944 cm^3 (1/30 ft^3)

$$Compaction\ effort\ per\ unit\ volume = \frac{(Hammer\ weight) \times (height\ of\ drop) \times (\#blows\ per\ layer) \times (\#\ layer)}{(volume\ of\ mold)}$$

Standard test compaction effort = 600 kN-m/m^3 (12,400 lb-ft/ft^3)
Modified test compaction effort = 2700 kN-m/m^3 (56,000 lb-ft/ft^3)

Compaction Test- Procedure

W_1



W_2

Oven drying @ 105°C
Get water content (w %)

Presentation of Results

For each test, the moist unit weight of compacted soil is

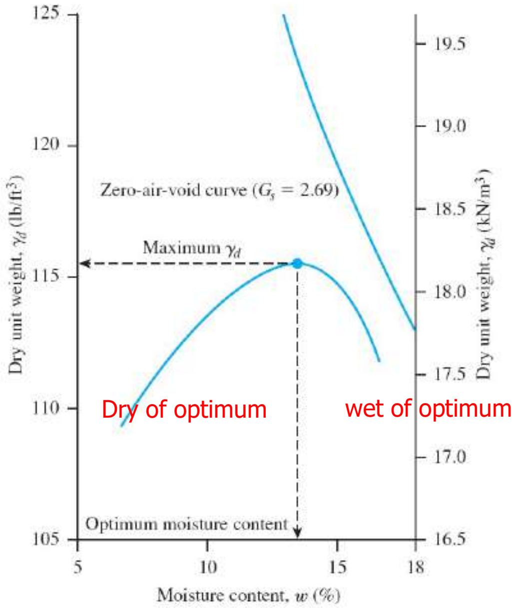
$$\gamma = \frac{W_2 - W_1}{V_{mold}}$$

Then the dry unit weight is calculated as

$$\gamma_d = \frac{\gamma}{\left(1 + \frac{w\%}{100}\right)}$$

Repeat the previous procedure for several water content and calculate corresponding γ_d.

Presentation of Results



Presentation of Results

To better understand and check the compaction curve, it is helpful to plot the **zero air void curve**.

The **zero air void curve** is a plot of the dry unit weight against water content at 100% degree of saturation.

This variation of γ_{dry} as function of water content and degree of saturation can be calculated using:

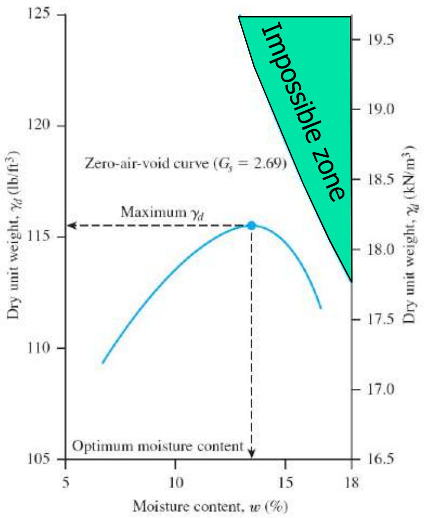
$$\gamma_d = \frac{G_s \gamma_w}{1 + \frac{G_s w}{S}}$$

Form definition of zero air voids, S =1

$$\gamma_{d\,zav} = \frac{G_s \gamma_w}{1 + G_s w}$$

Presentation of Results

To plot the zero air void curves, assume values for w (say 5%, 10%. 15%, etc) and calculate $\gamma_{d\ zav}$



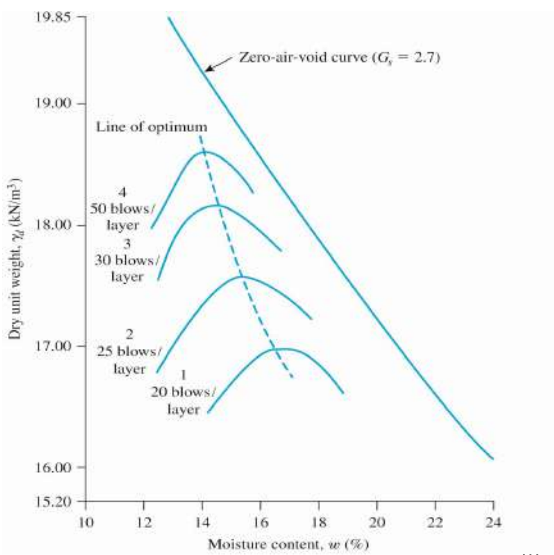
$$\gamma_{d\,zav} = \frac{G_s\,\gamma_w}{1 + G_s\,w}$$

The actual compaction curve will be always below the zero air void curve.

Factors Effecting Compaction

Compaction effort

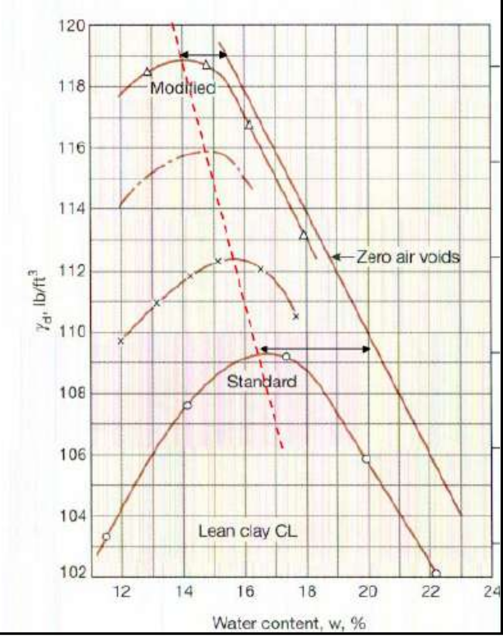
- Compaction effort is the energy per unit volume
- As the compaction effort increases:
  - ➢ The maximum dry unit weight of compaction increase.
  - ➢ The optimum moisture content decreases.



Factors Effecting Compaction

Compaction Effort

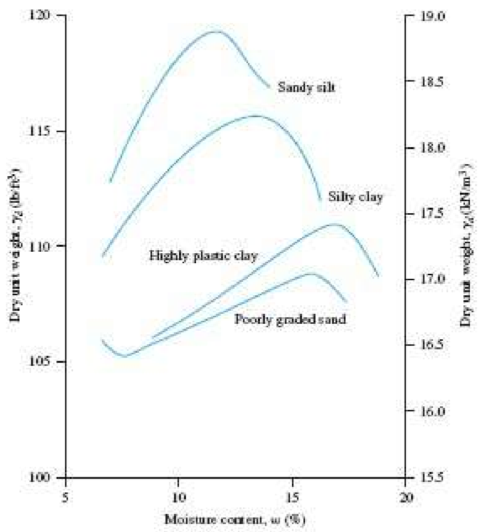
- Compaction effort is the energy per unit volume
- As the compaction effort increases:
  - The maximum dry unit weight of compaction increase.
  - The optimum moisture content decreases.



Factors Effecting Compaction

Soil Type

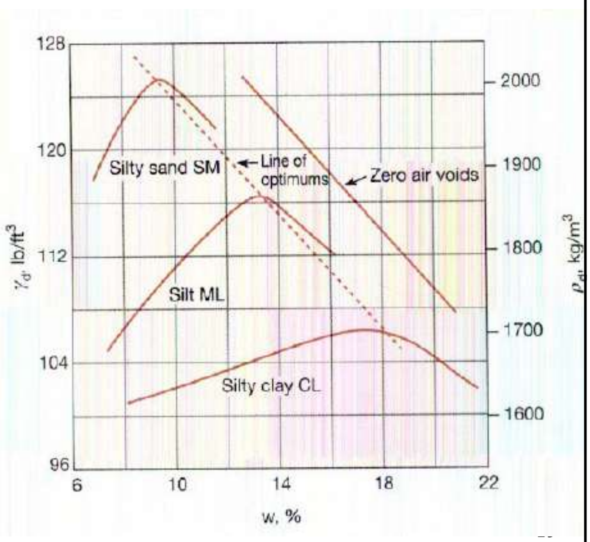
- Grain-size distribution, shape of the soil grains, specific gravity of soil solids, and amount and type of clay minerals present
- Fine grain soil needs more water to reach optimum; while coarse grain soil needs less water to reach optimum.



Factors Effecting Compaction

Soil Type

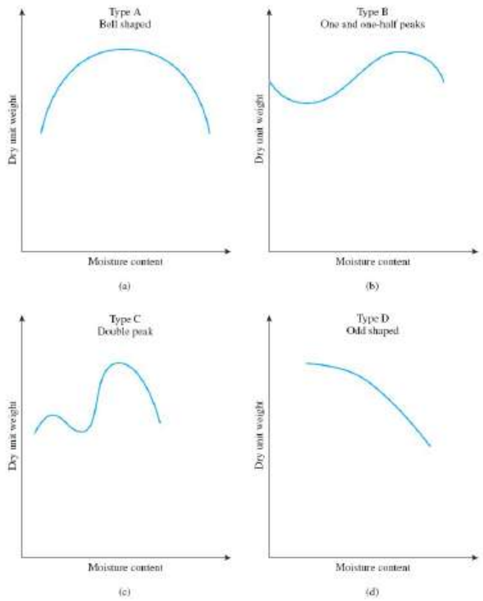
- Grain-size distribution, shape of the soil grains, specific gravity of soil solids, and amount and type of clay minerals present
- Fine grain soil needs more water to reach optimum; while coarse grain soil needs less water to reach optimum



Factors Effecting Compaction

Soil Plasticity

Type of compaction curve (Figure 6.5)	Description of curve	Liquid limit
A	Bell shaped	Between 30 to 70
B	1-1/2 peak	Less than 30
C	Double peak	Less than 30 and those greater than 70
D	Odd shaped	Greater than 70



Factors Effecting Compaction

Soil Plasticity

Gurtug and Sridharan (2004) proposed correlations for optimum moisture content and maximum dry unit weight with the plastic limit (PL) of cohesive soils. These correlations can be expressed as:

$$w_{opt}(\%) = [1.95 - 0.38(\log CE)](PL) \tag{6.8}$$

$$\gamma_{d(max)}\,(kN/m^3) = 22.68e^{-0.0183 w_{opt}(\%)} \tag{6.9}$$

where PL = plastic limit (%)
CE = compaction energy (kN-m/m^3)

For modified Proctor test, CE = 2700 kN/m^3. Hence,

$$w_{opt}(\%) \approx 0.65(PL)$$

and

$$\gamma_{d(max)}\,(kN/m^3) = 22.68e^{-0.012(PL)}$$

Example

- Example 6.1 → Das, Chapter 6.

Field Compaction Equipment

1. Smooth wheel rollers

- Suitable for proof rolling subgrades and for finishing operation of fills with sandy and clayey soils.
- They are not suitable for producing high unit weights of compaction when used on thicker layers.

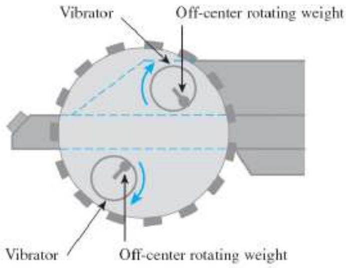
2. Pneumatic rubber-tired rollers

- Better than the smooth-wheel rollers.
- can be used for sandy and clayey soil compaction.
- Compaction is achieved by a combination of pressure and kneading action.

Field Compaction Equipment

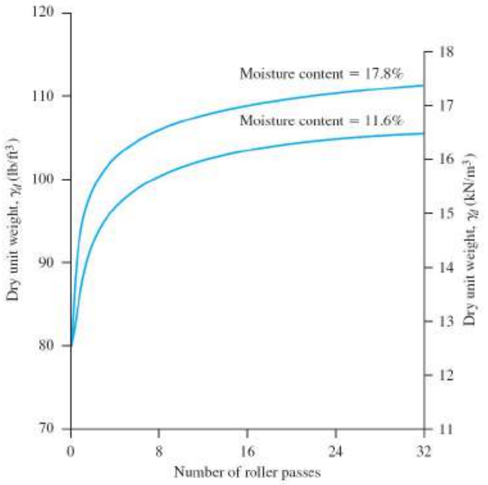
3. Sheepsfoot roller

- most effective in compacting clayey soils.

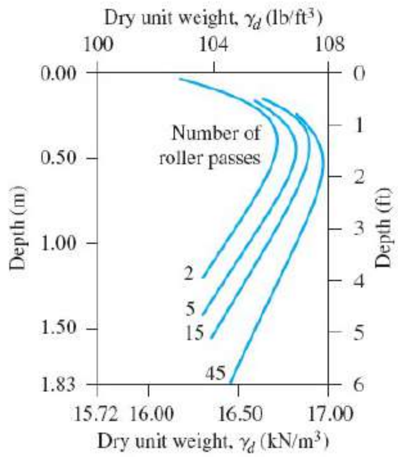


6. Vibratory rollers

Factors affecting Field Compaction



Effect of No. of roller Passes



Thickness of lift

Table 6.2 Requirements to Achieve R = 95 to 100% (based on standard Proctor maximum dry unit-weight)*

Equipment type	Applicability	Requirements for compaction of 95 to 100% standard Proctor maximum dry unit weight					Possible variations in equipment
		Compacted lift thickness	Passes or coverages	Dimensions and weight of equipment			
				Soil type	*Foot contact area*	*Foot contact pressures*	
Sheepsfoot rollers	For fine-grained soils or dirty coarse-grained soils with more than 20% passing the No. 200 sieve. Not suitable for clean, coarse-grained soils. Particularly appropriate for compaction of impervious zone for earth dam or linings where bonding of lifts is important.	150 mm (6 in.)	4 to 6 passes for fine-grained soil	Fine-grained soil, $PI > 30$	30 to 80 cm^2 (5 to 12 in^2)	1700 to 3400 kN/m^2 (250 to 500 psi)	For earth dam, highway, and airfield work, a drum of 1.5 m (60-in.) diameter, loaded to 45 to 90 kN per linear meter (1.5 to 3 tons per linear ft) of drum is generally utilized. For smaller projects, a 1 m (40-in.) diameter drum, loaded to 22 to 50 kN per linear meter (0.75 to 1.75 tons per linear ft) of drum, is used. Foot contact pressure should be regulated to avoid shearing the soil on the third or fourth pass.
				Fine-grained soil, $PI < 30$	45 to 90 cm^2 (7 to 14 in^2)	1400 to 2800 kN/m^2 (200 to 400 psi)	
			6 to 8 passes for coarse-grained soil	Coarse-grained soil	65 to 90 cm^2 (10 to 14 in^2)	1000 to 1800 kN/m^2 (150 to 250 psi)	
				Efficient compaction of soils on the wet side of the optimum requires less contact pressure than that required by the same soils at lower moisture contents.			
Rubber-tired rollers	For clean, coarse-grained soils with 4 to 8% passing the No. 200 sieve.	250 mm (10 in)	3 to 5 coverages	Tire inflation pressures of 400 to 550 kN/m^2 (60 to 80 psi) for clean granular material or base course and subgrade compaction. Wheel load, 80 to 110 kN (18,000 to 25,000 lb.)			A wide variety of rubber-tired compaction equipment is available. For cohesive soils, light-wheel loads, such as provided by wobble-wheel equipment, may be substituted for heavy-wheel loads if lift thickness is decreased. For cohesionless soils, large-size tires are desirable to avoid shear and rutting.
	For fine-grained soils or well-graded, dirty, coarse-grained soils with more than 8% passing the No. 200 sieve.	150 to 200 mm (6 to 8 in)	4 to 6 coverages	Tire inflation pressures in excess of 450 kN/m^2 (65 psi) for fine-grained soils of high plasticity. For uniform clean sands or silty fine sands, use larger-size tires with pressures of 280 to 350 kN/m^2 (40 to 50 psi.)			

Smooth-wheel rollers	Appropriate for subgrade or base course compaction of well-graded sand-gravel mixtures.	200 to 300 mm (8 to 12 in.)	4 coverages	Tandem-type rollers for base-course or sub-grade compaction, 90 to 135 kN (10 to 15 ton) weight, 53 to 88 kN per linear meter (300 to 500 lb per linear in.) of width of rear roller.	Three-wheel rollers are obtainable in a wide range of sizes. Two-wheel tandem rollers are available in the 9 to 180 kN (1- to 20-ton) weight range. Three-axle tandem rollers are generally used in the 90 to 180 kN (10- to 20-ton) weight range. Very heavy rollers are used for proof rolling of subgrade or base course.
	May be used for fine-grained soils other than in earth dams. Not suitable for clean, well-graded sands or silty, uniform sands.	150 to 200 mm (6 to 8 in.)	6 coverages	Three-wheel roller for compaction of fine-grained soil; weights from 45 to 55 kN (5 to 6 tons) for materials of low plasticity to 90 kN (10 tons) for materials of high plasticity.	
Vibrating baseplate compactors	For coarse-grained soils with less than about 12% passing the No. 200 sieve. Best suited for materials with 4 to 8% passing the No. 200 sieve, placed thoroughly wet.	200 to 250 mm (8 to 10 in.)	3 coverages	Single pads or plates should weigh no less than 0.9 kN (200 lb). May be used in tandem where working space is available. For clean, coarse-grained soil, vibration frequency should be no less than 1600 cycles per minute.	Vibrating pads or plates are available, hand-propelled or self-propelled, single or in gangs, with width of coverage from 0.45 to 4.5 m ($1\frac{1}{2}$ to 15 ft). Various types of vibrating-drum equipment should be considered for compaction in large areas.
Crawler tractor	Best suited for coarse-grained soils with less than 4 to 8% passing the No. 200 sieve, placed thoroughly wet.	250 to 300 mm (10 to 12 in.)	3 to 4 coverages	No smaller than D8 tractor with blade, 153 kN (34,500 lb) weight, for high compaction.	Tractor weights up to 265 kN (60,000 lb.)
Power tamper or rammer	For difficult access, trench backfill. Suitable for all inorganic soils.	100 to 150 mm (4 to 6 in.) for silt or clay; 150 mm (6 in.) for coarse-grained soils	2 coverages	130 N (30 lb) minimum weight. Considerable range is tolerable, depending on materials and conditions.	Weights up to 1.1 kN (250 lb); foot diameter, 100 to 250 mm (4 to 10 in.)

* *After U.S. Navy (1971). Published by U.S. Government Printing Office*

Specification of Field Compaction

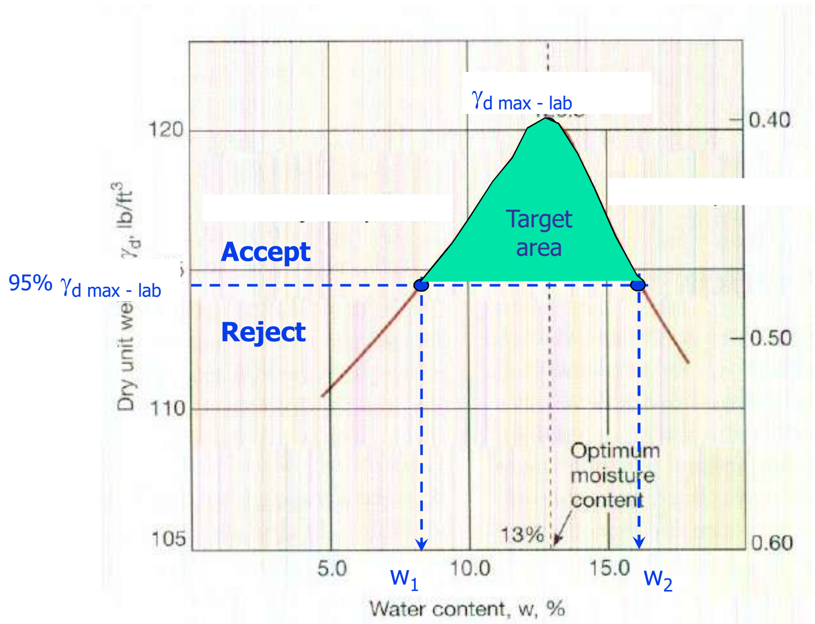
- During construction of soil structures (dams, roads), there is usually a requirement to achieve a specified dry unit weight.
- In most specifications, it is required to achieve a dry density of 90 to 98% of maximum dry density obtained from standard or modified Proctor tests.
- This is expressed as relative compaction as follows:

$$R = \frac{\gamma_{\text{d field}}}{\gamma_{\text{d max-lab}}} \times 100$$

$\gamma_{\text{d field}}$ = dry unit weight in the field

$\gamma_{\text{d max-lab}}$ = max. dry unit weight from laboratory test

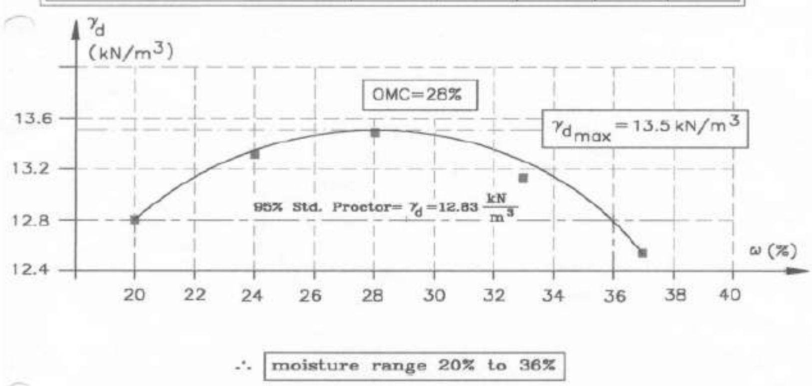
Specification of Field Compaction



Example 1

Find (a) the maximum dry unit weight of crushed limestone fill to be used as road base material, (b) its OMC, and (c) the moisture range for 95% of Standard Proctor.

TRIAL NO.	1	2	3	4	5
W: Wt. of wet soil (Newtons)	14.5	15.6	16.3	16.4	16.1
ω: water content (%)	20	24	28	33	37
$\gamma = \frac{W}{V}$ (kN/m^3)	15.4	16.5	17.3	17.4	17.1
$\gamma_d = \frac{\gamma}{1+\omega}$ (kN/m^3)	12.8	13.3	13.5	13.1	12.5



Example 2

Find (a) The dry unit weight and water content at 95%Std. Proctor; (b) The degree of saturation at maximum dry density, and (c) plot the zero air voids line.

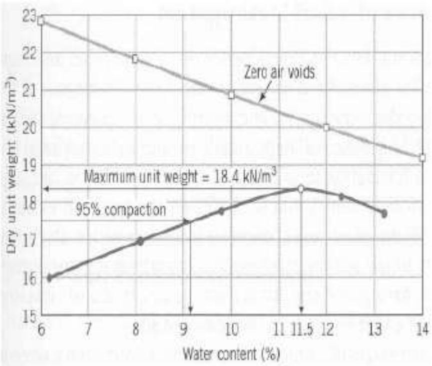
		1	2	3	4	5	6
Given: Water content w (%)		6.2	8.1	9.8	11.5	12.3	13.2
Unit weight γ (kN/m³)		16.9	18.7	19.5	20.5	20.4	20.1
Solution: Dry unit weight	$\gamma_d = \dfrac{\gamma}{1+w}$	15.9	17.3	17.8	18.4	18.2	17.8
Zero air voids	$\gamma_{zav} = \dfrac{G_s\gamma_w}{1+wG_s}$	22.8	21.8	20.8		20.0	19.2

From graph, $(\gamma_d)_{max} = 18.4$ kN/m³ at $w_{omc} = 11.5\%$

At 95% compaction $\gamma_d = 18.4 \times 0.95 = 17.5$ kN/m³ and $w = 9.2\%$

The degree of saturation at the OMC is,

$$S = \frac{wG_s(\gamma_d)_{max}/\gamma_w}{G_s - (\gamma_d)_{max}/\gamma_w} = \frac{0.115 x 2.7 x (18.4/9.8)}{2.7 - (18.4/9.8)} = 0.71 = 71\%$$



Example 3

The results of a standard Proctor test are given in the following table. Determine the maximum dry unit weight of compaction and the optimum moisture content. Also, determine the moisture content required to achieve 95% of $\gamma_{d(max)}$.

Volume of Proctor mold (cm³)	Mass of wet soil in mold (kg)	Moisture content (%)
943.3	1.47	10.0
943.3	1.83	12.5
943.3	2.02	15.0
943.3	1.95	17.5
943.3	1.73	20.0
943.3	1.69	22.5

Volume of mold, V (cm^3)	Weight of wet soil, W (N)	Moist unit weight, γ (kN / m^3)	w (%)	γ_d (kN / m^3)
943.3	14.42	15.29	10.0	13.9
943.3	17.95	19.03	12.5	16.92
943.3	19.82	21.01	15.0	18.27
943.3	19.13	20.28	17.5	17.26
943.3	16.97	17.99	20.0	14.99
943.3	16.58	17.58	22.5	14.35

W = mass in kg × 9.81; $\gamma = W/V$

The plot of γ_d vs. w is shown.

From the graph,

$\gamma_{d(\max)}$ = **18.3 kN / m^3**

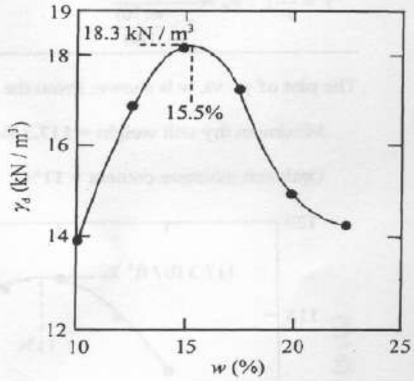
w_{opt} = **15.5%**

$\gamma_d = 0.95\,\gamma_{d(\max)} = (0.95)(18.3)$

$= 17.39$ kN / m^3

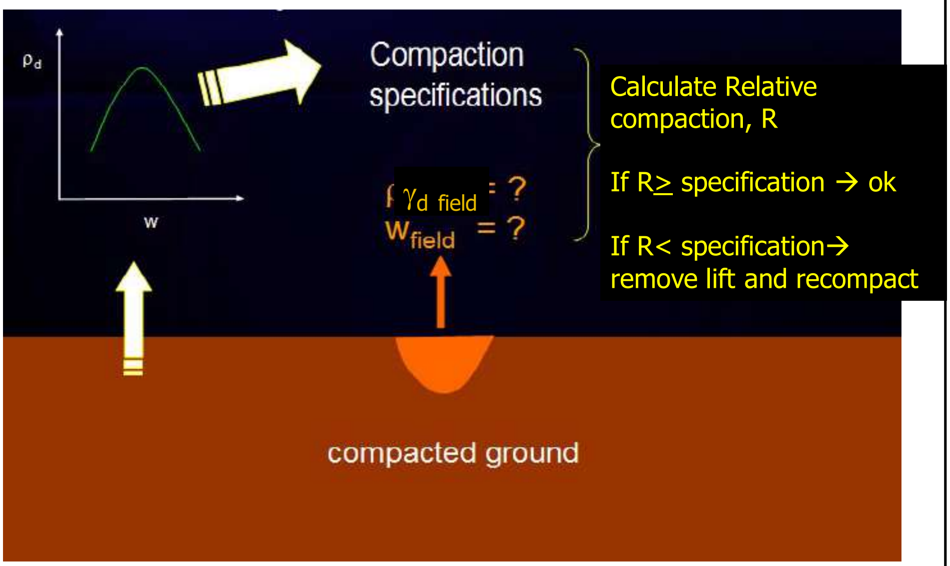
From the graph,

w = **13% at 0.95 $\gamma_{d(\max)}$**



Remember the dry side is better in the field. This is why we talk about 95%.

Compaction Field Control



Compaction Field Control

The standard procedures for determining the field unit weight of compaction include:

1. Sand cone method
2. Rubber balloon method
3. Nuclear method

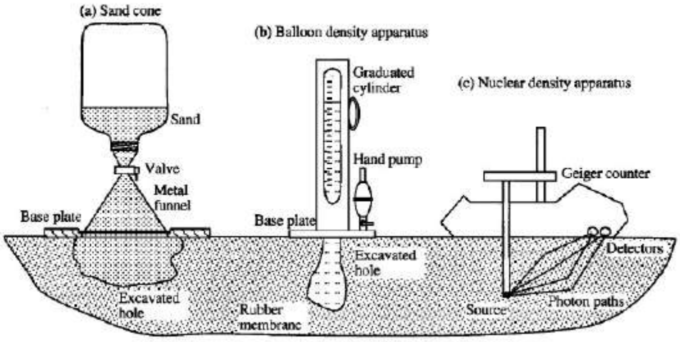
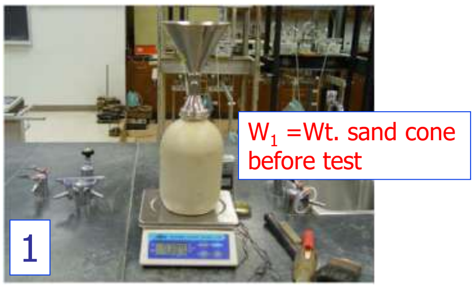


Figure 1. Methods of determining the unit weight of soils in the **field**

Determination of Field unit weight

Sand Cone Test Procedure



Determination of Field unit weight

Sand Cone Test Procedure

Oven dry @ 105°C → w%

W_2=moist wt. of soil from hole

W_4 =Wt. sand cone after test

Weight of sand to fill the cone and hole,

$$W_5 = W_1 - W_4$$

Volume of soil $= V = \dfrac{W_5 - Wc}{\gamma_{d(sand)}}$

Wc = weight of sand to fill the cone

γ_{dsand} = unit weight of sand

Both obtained from laboratory calibration of sand cone

Dry weight of soil $= W_3 = \dfrac{W_2}{(1 + w/100)}$

$$\gamma_{d\ \text{field}} = \frac{W_3}{V}$$

Now you can calculate:

$$R = \frac{\gamma_{d\ \text{field}}}{\gamma_{d\ \text{max-lab}}} \times 100$$

Example

- Example 6.3 → Das, Chapter 6.

7.Permeability

1

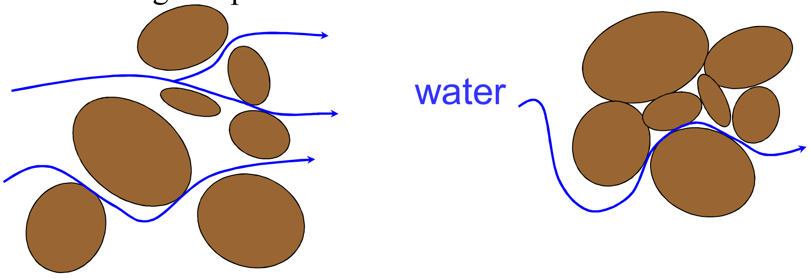
Soil Permeability- Definition

- It is a measure of how easily a fluid (e.g. water) can pass through the soil
- Different soil has different permeabilities.

2

What is Permeability?

- Soils consists of solid particles with interconnected voids where water can flow from a point of **high energy** to a point of **low energy**
- Permeability is the measure of the soil's ability to permit water to flow through its pores or voids



Loose soil

- easy to flow
- **high** permeability

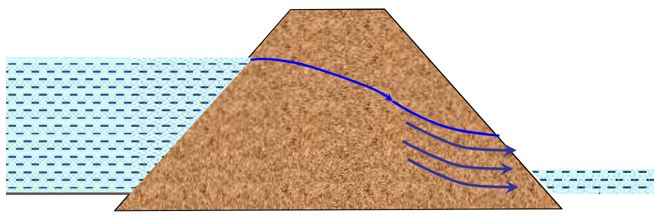
Dense soil

- difficult to flow
- **low** permeability

Importance of Permeability

The following applications illustrate the importance of permeability in geotechnical design:

- The design of earth dams is very much based upon the permeability of the soils used.
- The stability of slopes and retaining structures can be greatly affected by the permeability of the soils involved.



Bernoulli's Equation

The energy of fluid comprise of:

1. Potential energy

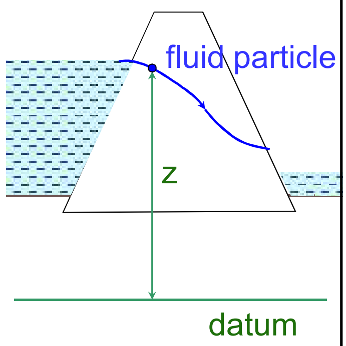
   - due to **elevation** (z) with respect to a datum

2. Pressure energy

   - due to **pressure**

3. Kinetic energy

   - due to **velocity**



Bernoulli's Equation

Then at any point in the fluid, the total energy is equal to

Total Energy = Potential energy + Pressure energy + Kinetic energy

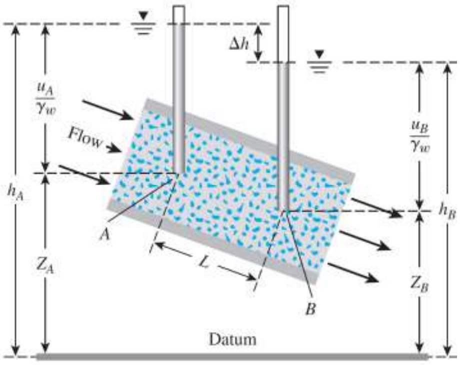
$= \rho_w g h + P + \frac{1}{2}\rho_w v^2$

Expressing the total energy as head (units of length)

Total Head = Elevation Head + Pressure Head + Velocity Head

$$h = Z + \frac{u}{\gamma_w} + \cancelto{0}{\frac{v^2}{2g}}$$

Bernoulli's Equation



At any point

$$h = Z + \frac{u}{\gamma_w}$$

The head loss between A and B

$$\Delta h = h_A - h_B = \left(Z_A + \frac{u_A}{\gamma_w}\right) - \left(Z_B + \frac{u_B}{\gamma_w}\right)$$

Head loss in **non-dimensional** form

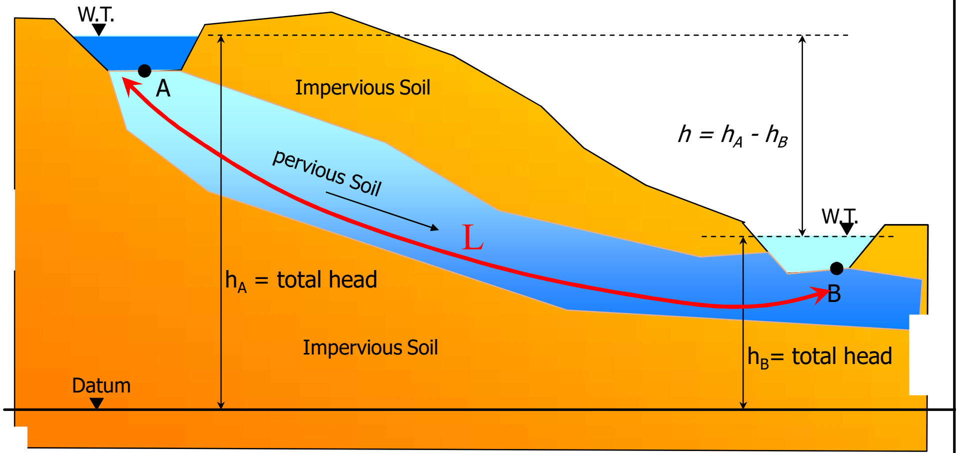
$$i = \frac{\Delta h}{L}$$

Hydraulic gradient ← i

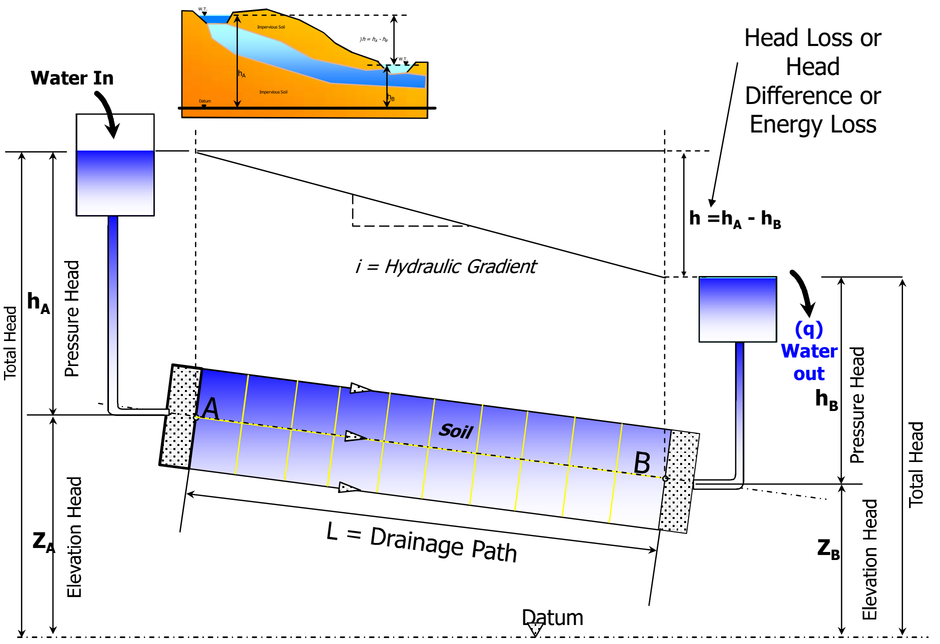
Δh → Difference in total head

L → Distance between points A and B

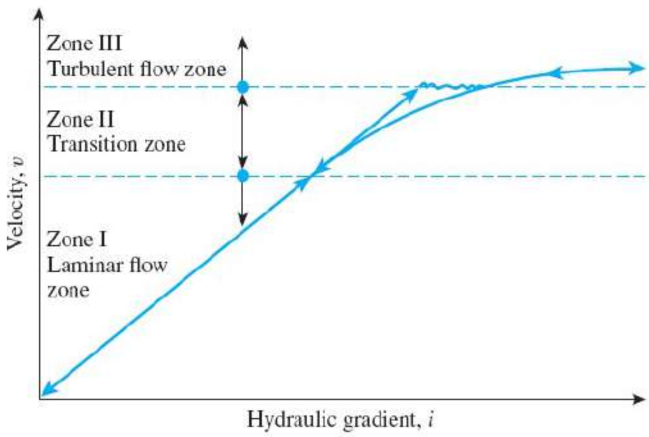
Hydraulic Gradient



$$i = \frac{(h_A - h_B)}{L} = \frac{\Delta h}{L}$$



Darcy's Law:



Since velocity in soil is small, flow can be considered laminar

$$v \propto i$$

v = discharge velocity =

i = hydraulic gradient

$v = k\,i$ k = coefficient of permeability

Darcy's Law:

Since velocity in soil is small, flow can be considered laminar

$$v = k.i$$

Where:

v = discharge velocity which is the quantity of water flowing in unit time through a unit gross cross-sectional area of soil at right angles to the direction of flow.

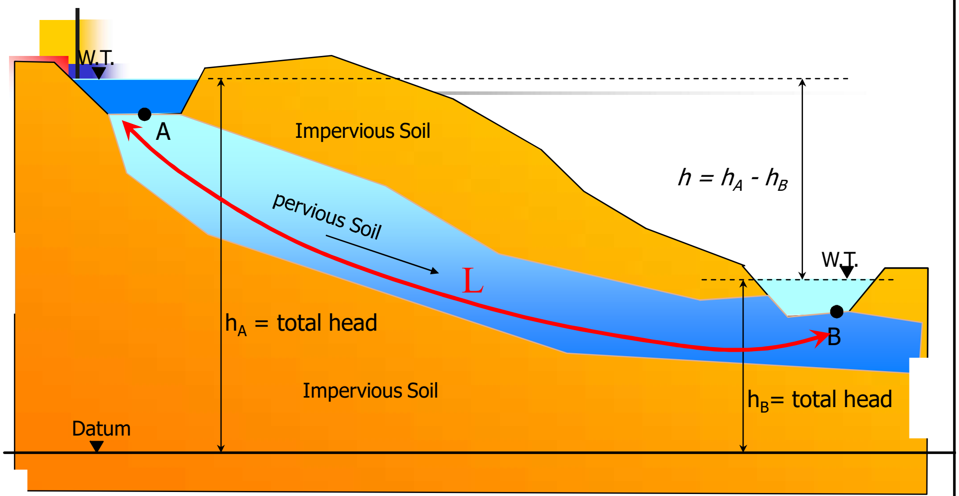
k = hydraulic conductivity (has units of L/T)

i = hydraulic gradient = h/L

Then the quantity of water flowing through the soil per unit time is

Discharge = Q = v. A = k (h/L). A

Flow in Soil



$$i = \frac{(h_A - h_B)}{L} = \frac{\Delta h}{L} \Rightarrow Q = k\ i\ A = k\ \frac{\Delta h}{L}\ A$$

To determine the quantity of flow, two parameters are needed

* k = hydraulic conductivity **(how permeable the soil medium)**
* i = hydraulic gradient **(how large is the driving head)**

k can be determined using
1- Laboratory Testing → [constant head test & falling head test]
2- Field Testing → [pumping from wells]
3- Empirical Equations

i can be determined
1- from the head loss and geometry
2- flow net (chapter 8)

Hydraulic Conductivity

- The hydraulic conductivity k is a measure of how easy the water can flow through the soil.
- The hydraulic conductivity is expressed in the units of velocity (such as cm/sec and m/sec).

Table 7.1 Typical Values of Hydraulic Conductivity of Saturated Soils

Soil type	k (cm/sec)	k (ft/min)
Clean gravel	100−1.0	200−2.0
Coarse sand	1.0−0.01	2.0−0.02
Fine sand	0.01−0.001	0.02−0.002
Silty clay	0.001−0.00001	0.002−0.00002
Clay	<0.000001	<0.000002

Hydraulic Conductivity

- Hydraulic conductivity of soils depends on several factors:
  - Fluid viscosity (η): as the viscosity increases, the hydraulic conductivity decreases
  - Pore size distribution
  - Temperature
  - Grain size distribution
  - Degree of soil saturation

It is conventional to express the value of k at a temperature of 20^{o}C.

$$k_{20^0 C} = k_{T^0 C} \frac{\eta_{T^0 C}}{\eta_{20^0 C}}$$

Laboratory Testing of Hydraulic Conductivity

Two standard laboratory tests are used to determine the hydraulic conductivity of soil

- The constant-head test
- The falling-head test.

Constant Head Test

- The constant head test is used primarily for coarse-grained soils.
- This test is based on the assumption of laminar flow (Darcy's Law apply)

From Darcy's Law

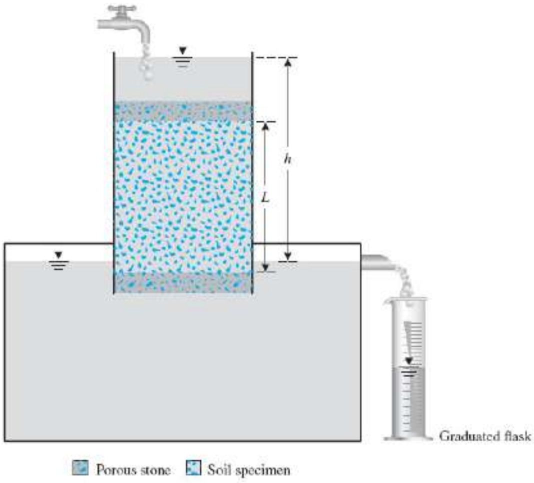
$$Q = k \cdot i \cdot A \cdot t = k \cdot \frac{h}{L} \cdot A \cdot t$$

Where:
Q = volume of water collection
A = cross section area of soil specimen
T = duration of water collection

$$k = \frac{V \cdot L}{h \cdot A \cdot t}$$

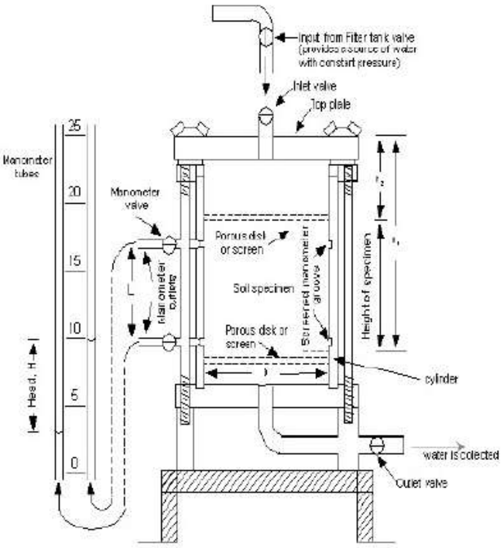
Then compute:

$$k_{20^0 C} = k_{T^0 C} \frac{\eta_{T^0 C}}{\eta_{20^0 C}}$$



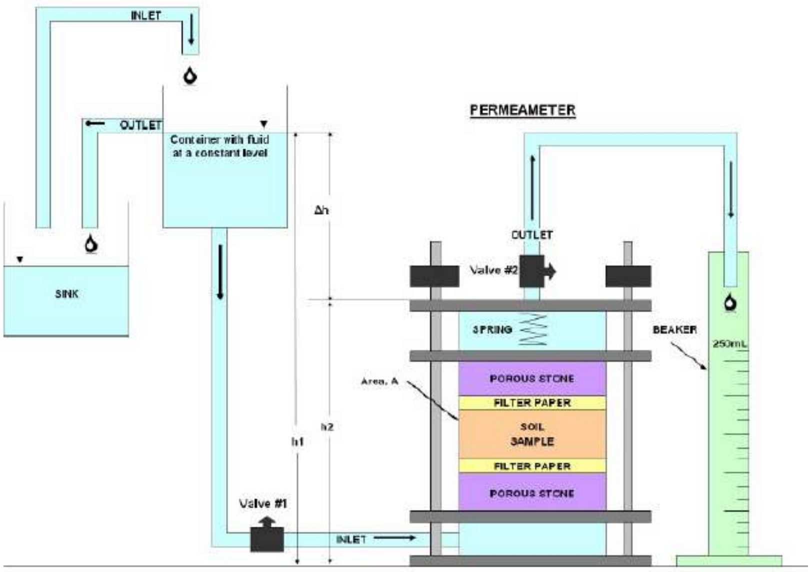
Constant Head Test

Constant Head Test



Constant-Head Permeameter

Constant Head Test

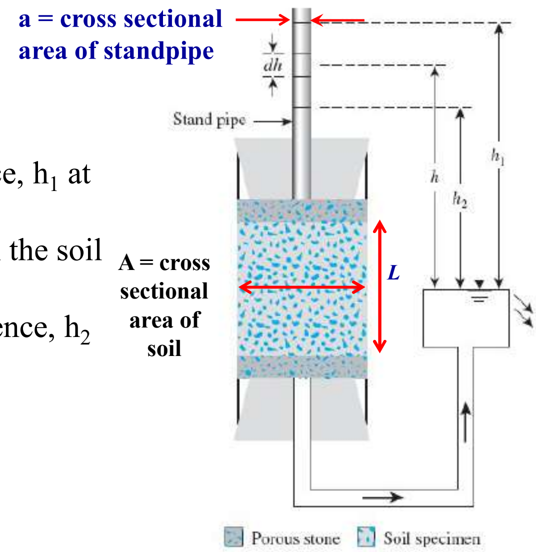


Falling Head Test

• The falling head test is mainly for fine-grained soils.

Simplified Procedure:

– Record initial head difference, h_1 at $t_1 = 0$
– Allow water to flow through the soil specimen
– Record the final head difference, h_2 at time $t = t_2$



Falling Head Test

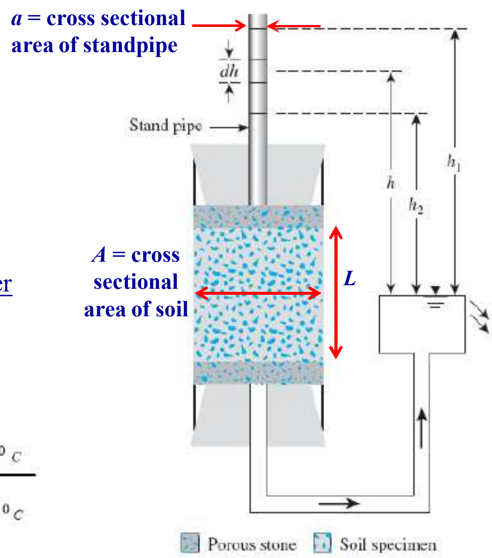
Calculations:

$$k = \frac{aL}{At}\ln\left(\frac{h_1}{h_2}\right)$$

Where:
A = cross sectional area of the soil
a = cross sectional area of the standpipe
h_1 = distance to bottom of the beaker before the test
h_2 = distance to bottom of the beaker after the test
L = length of the sample
$t = t_2 - t_1$

Then compute:

$$k_{20^0 C} = k_{T^0 C}\frac{\eta_{T^0 C}}{\eta_{20^0 C}}$$



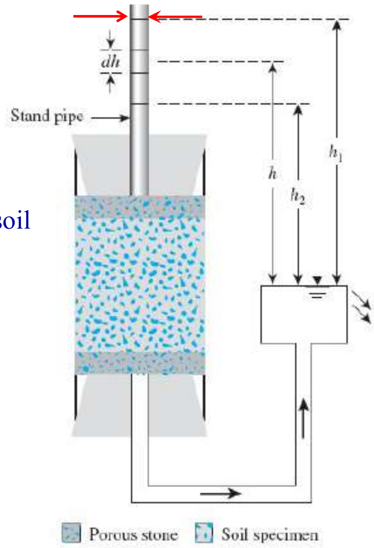
Falling Head Test

Calculations:

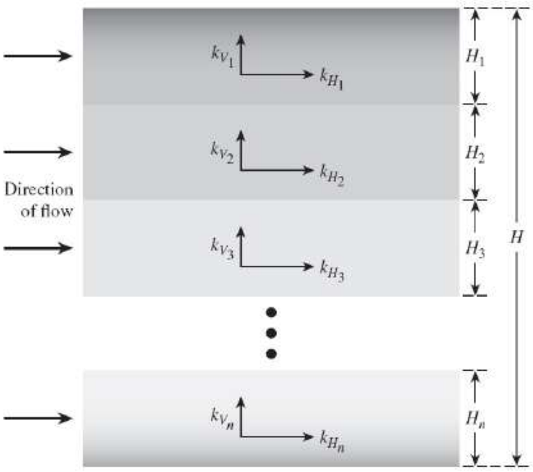
$$k = \frac{aL}{At}\ln\left(\frac{h_1}{h_2}\right)$$

The above equation is derived assuming:

The flow through the standpipe = flow through the soil



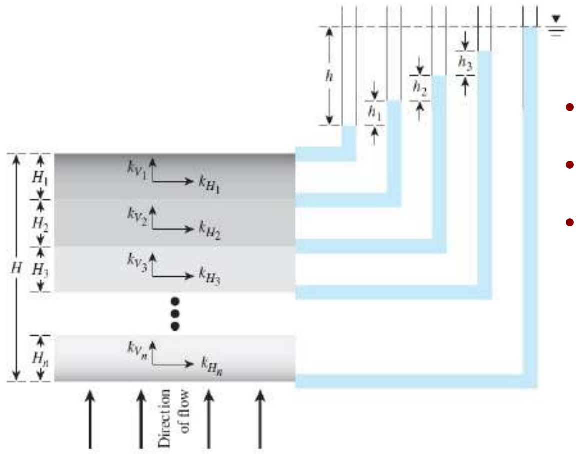
Equivalent Hydraulic Conductivity on Stratified Soils



- Horizontal flow
- Constant hydraulic gradient conditions
- Analogous to resistors in series

$$k_{H(eq)} = \frac{1}{H}(k_{H_1}H_1 + k_{H_2}H_2 + k_{H_3}H_3 + \cdots + k_{H_n}H_n)$$

Equivalent Hydraulic Conductivity on Stratified Soils



- Vertical flow
- Constant velocity
- Analogous to resistors in parallel

$$k_{V(\mathrm{eq})} = \frac{H}{\left(\frac{H_1}{k_{V_1}}\right) + \left(\frac{H_2}{k_{V_2}}\right) + \left(\frac{H_3}{k_{V_3}}\right) + \cdots + \left(\frac{H_n}{k_{V_n}}\right)}$$

Limitations of Laboratory tests for Hydraulic Conductivity

i. It is generally hard to duplicate in-situ soil conditions (such as stratification).

ii. The structure of in-situ soils may be disturbed because of sampling and test preparation.

iii. Small size of laboratory samples lead to effects of boundary conditions.

Determination of Hydraulic conductivity in the Field

1. Pumping Wells with observation holes

2. Borehole test.

3. Packer Test.

Permeability Tests using Pumping Wells

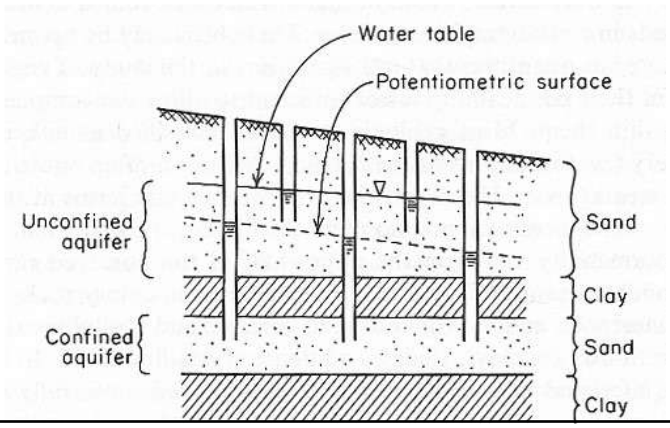
- Used to determine the hydraulic conductivity of soil in the field.

- During the test, water is pumped out at a constant rate from a test well that has a perforated casing. Several observation wells at various radial distances are made around the test well. Continuous observations of the water level in the test well and in the observation wells are made after the start of pumping, until a steady state is reached. The steady state is established when the water level in the test and observation wells becomes constant.

Pumping Well with Observation holes

Definitions

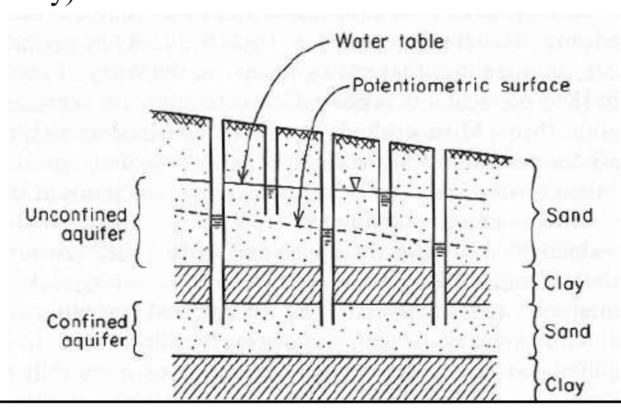
- <u>Aquifer:</u> Soil or rock forming stratum that is saturated and permeable enough to yield significant quantities of water *(e.g. sands, gravels, fractured rock)*



Pumping Well with Observation holes

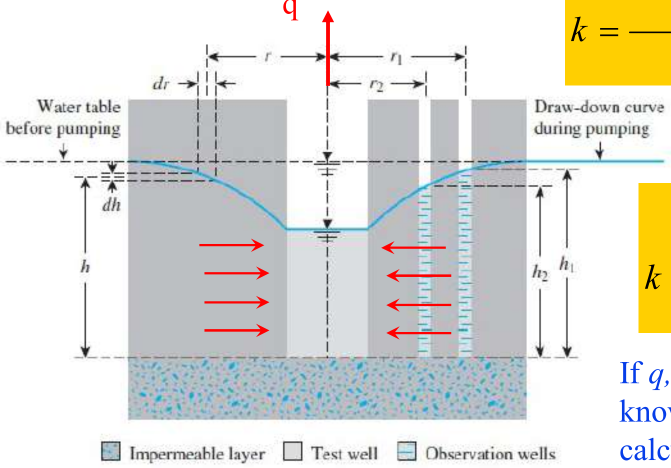
Definitions (cont.)

- <u>Unconfined Aquifer</u> (water table aquifer) is an aquifer in which the water table forms the upper boundary.
- <u>Confined Aquifer</u> is an aquifer confined between two impervious layers (e.g. clay).



Pumping Well with Observation holes

Pumping Well in an Unconfined Aquifer



$$k = \frac{2.303 \cdot q \,.\log_{10} \dfrac{r_2}{r_1}}{\pi (h_2^2 - h_1^2)}$$

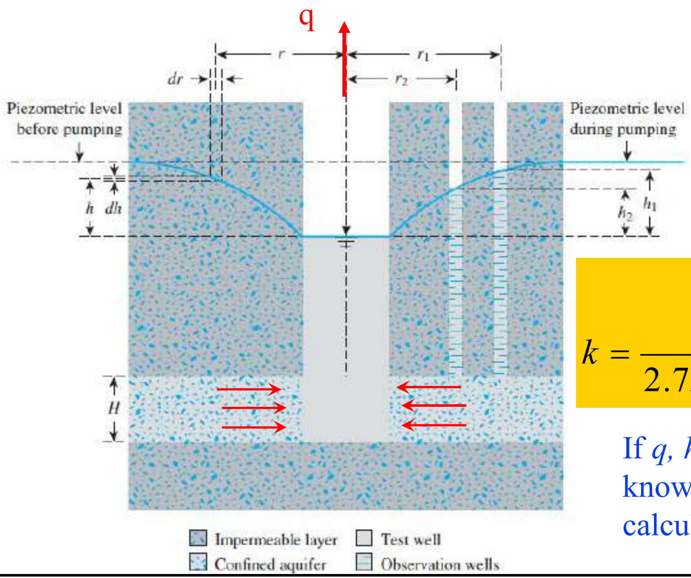
OR

$$k = \frac{q \,.\ln \dfrac{r_2}{r_1}}{\pi (h_2^2 - h_1^2)}$$

If q, h_1, h_2, r_1, r_2 are known , k can be calculated

Pumping Well with Observation holes

Pumping Well in a Confined Aquifer

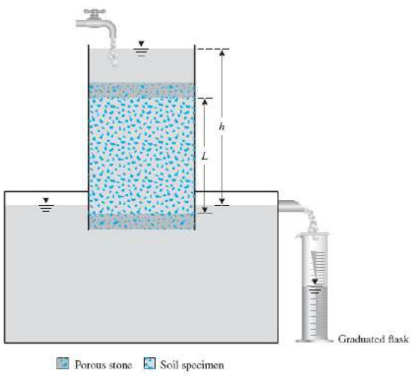


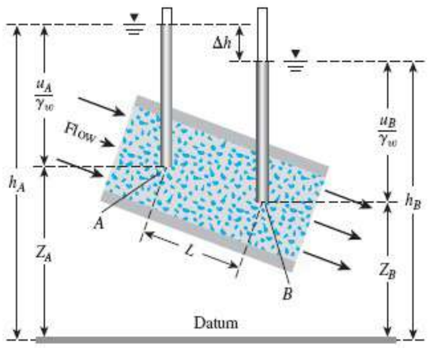
$$k = \frac{q}{2.727 \cdot H} \frac{\log_{10} \dfrac{r_2}{r_1}}{(h_2 - h_1)}$$

If q, h_1, h_2, r_1, r_2 are known , k can be calculated

Seepage

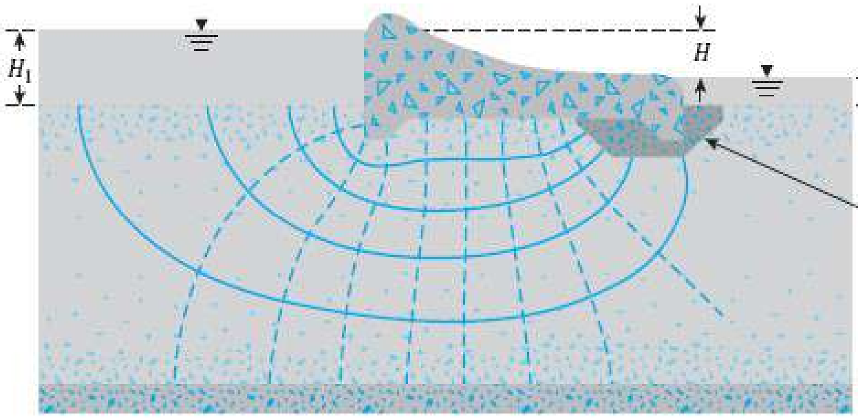
One-dimensional flow





Discharge = Q = v. A = k . i . A= k (h/L). A

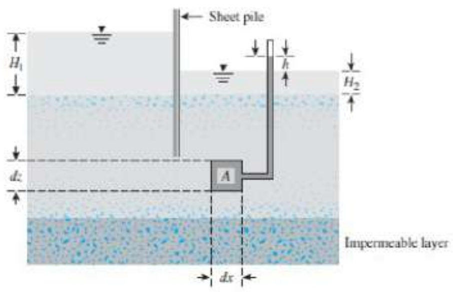
Two-dimensional flow

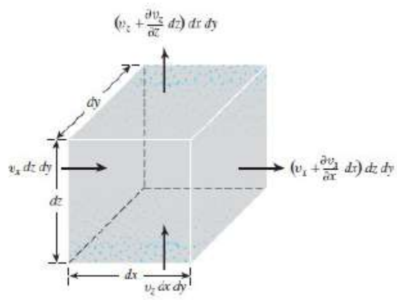


3

Laplace equation of Continuity

- In reality, the flow of water through soil is not in one direction only, nor is it uniform over the entire area perpendicular to the flow.
- The flow of water in two dimensional is described using Laplace equation.
- Laplace equation is the combination of the equation of continuity and Darcy's law.



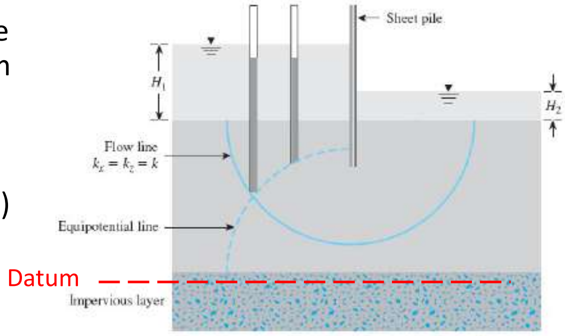


Flow nets

- Flow nets are a graphical solution method of Laplace equation for 2D flow in a homogeneous, isotropic aquifer.

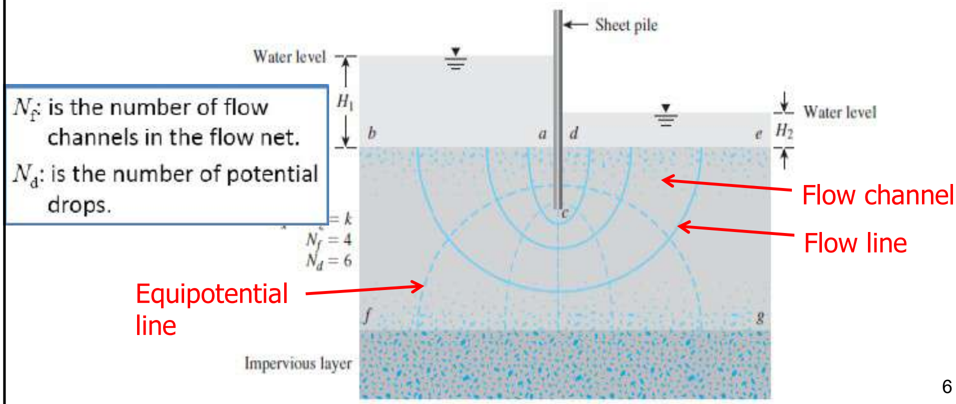
1. Flow lines: the line along which a water particle will travel from upstream to the downstream side in the permeable soil medium

2. Equipotential lines: the line along which the potential (pressure) head at all points is equal.



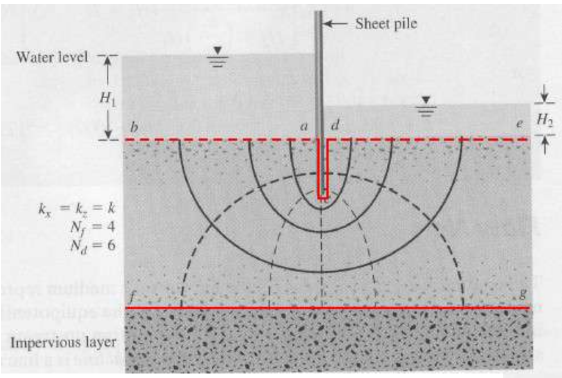
Flow nets

- Flow nets are the combination of flow lines and equipotential lines.
- To complete the graphic construction of a flow net, one must draw the flow and equipotential lines in such away that:
  1. The equipotential lines intersect the flow lines at right angles.
  2. The flow elements formed are approximate squares.

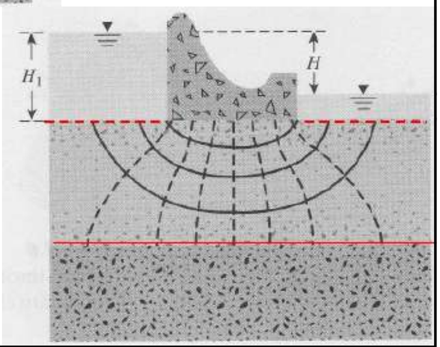


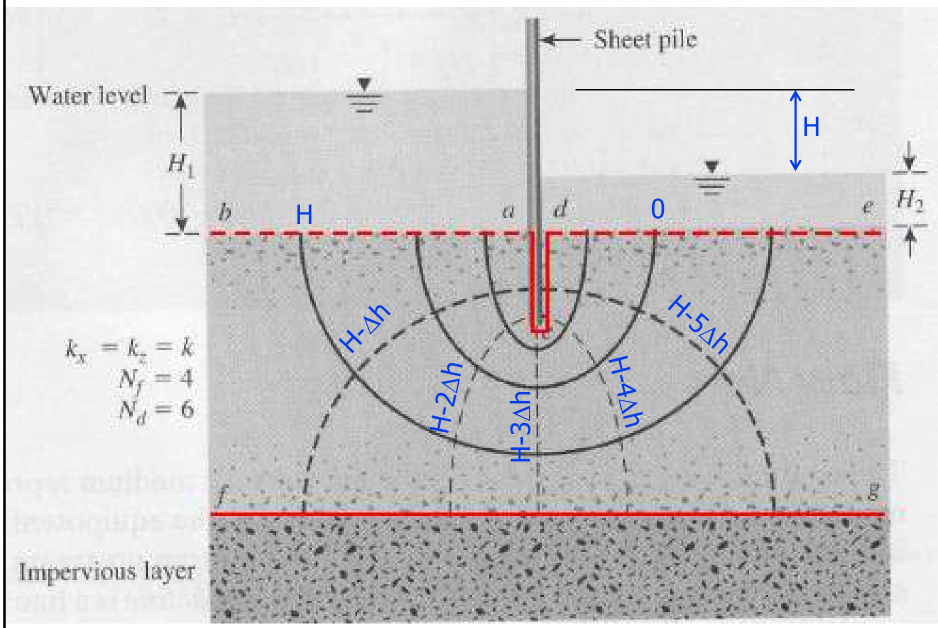
Flow Net Drawing Technique

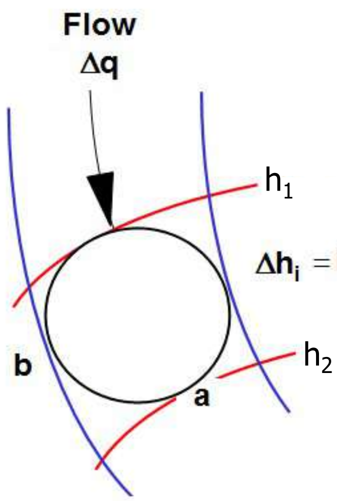
1. Draw to a convenient scale the geometry of the problem.
2. Establish constant head and no flow boundary conditions and draw flow and equipotential lines near boundaries.
   - Constant head boundaries (water levels) represent initial or final equipotentials
   - Impermeable (no-flow) boundaries are flow lines
3. Sketch flow lines by smooth curves (3 to 5 flow lines).
   - Flow lines should not intersect each other or impervious boundary
4. Draw equipotential lines by smooth curves adhering to right angle intersections and square grids conditions (aspect ratio =1).
5. Continue sketching and re-adjusting until you get **squares almost everywhere**. Successive trials will result in a reasonably consistent flow net.



Boundary Conditions







Δh_i = head lost between equipotentials

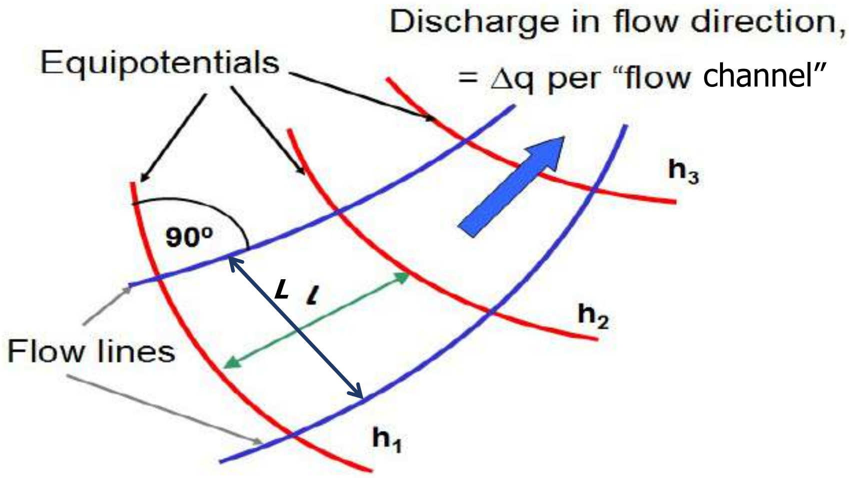
$= h_1 - h_2$

“square, M”, a x b

Common convention

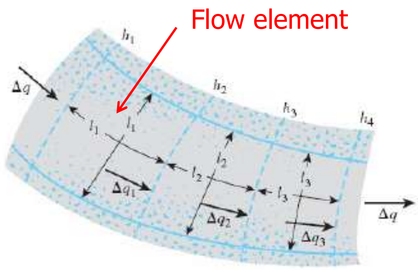
draw “squares” with a = b

Seepage Flow



Seepage Calculation from Flow Net

- In a flow net, the strip between any two adjacent flow lines is called a flow channel.
- The drop in the total head between any two adjacent equipotential lines is called the potential drop.



- If the ratio of the sides of the flow element are the same along the flow channel, then:

1. Rate of flow through the flow channel per unit width perpendicular to the flow direction is the same.

$$\Delta q_1 = \Delta q_2 = \Delta q_3 = \Delta q$$

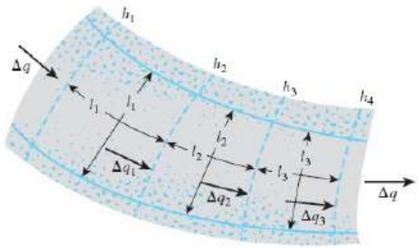
2. The potential drop is the same and equal to:

$$h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = \frac{H}{N_d}$$

Where H: head difference between the upstream and downstream sides.
N_d: number of potential drops.

Seepage Calculation from Flow Net

From Darcy's Equation, the rate of flow is equal to:



$$\Delta q = \left(\frac{h_1 - h_2}{l_1}\right) l_1 = \left(\frac{h_2 - h_3}{l_2}\right) l_2 = \left(\frac{h_3 - h_4}{l_3}\right) l_3$$

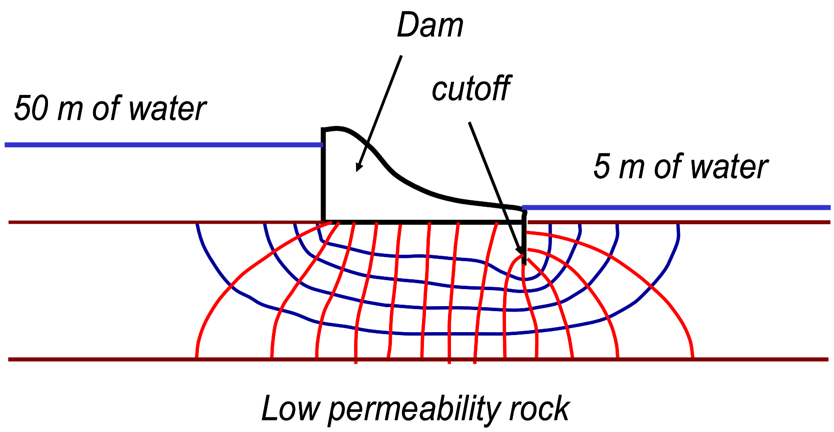
$$\Delta q = k\ H\left(\frac{1}{N_d}\right)$$

- If the number of flow channels in a flow net is equal to N_f, the total rate of flow through all the channels per unit length can be given by:

$$\Delta q = k\ H\left(\frac{N_f}{N_d}\right)$$

Example on estimating the total flow under dam

Example: if k = 10^{-7} m/sec, what would be the flow per day over a 100 m length of wall?



Calculations

$N_f = 5$
$N_d = 14$
$\Delta h = 45$ m
$k = 10^{-7}$ m/sec

$$q = k \frac{N_f}{N_d} \Delta h$$

Answer:

= 10^{-7}(5/14) 45 x 100 m length
= 0.000161 m^3/sec
= 13.9 m^3/day

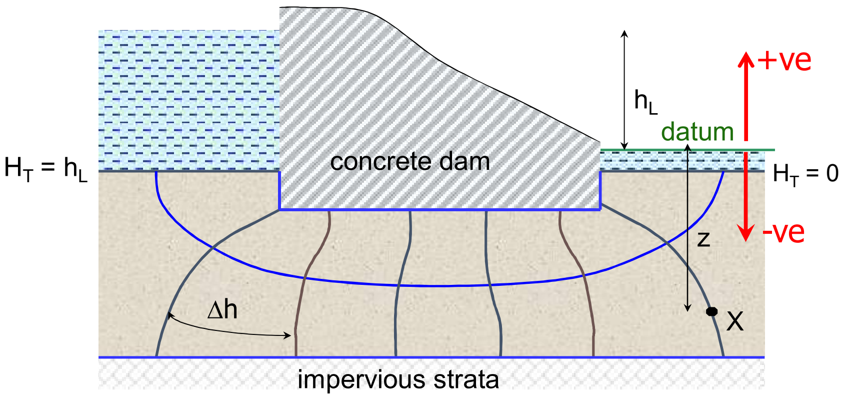
Pressure head at any point

Total head = h_L - # of drops from upstream x Δh

Elevation head = -z

Pressure head = Total head – Elevation head

$$\Delta h = \frac{h_L}{N_d}$$



Uplift Pressure under Hydraulic structures

<u>At point a:</u>

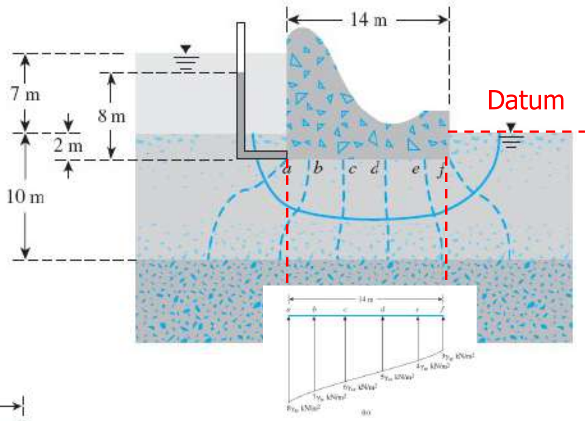
$u_a/\gamma_w = (7 - 1x1) - (-2)$
$= 8\ kN/m^2$

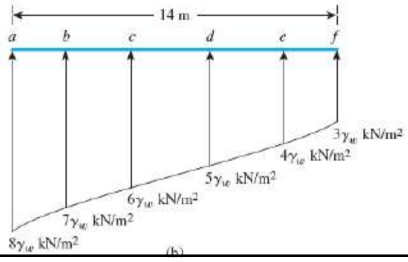
<u>At point b:</u>

$u_a/\gamma_w = (7 - 2x1) - (-2)$
$= 7\ kN/m^2$

<u>At point f:</u>

$u_a/\gamma_w = (7 - 6x1) - (-2)$
$= 3\ kN/m^2$





$\Delta h = h_L/N_d = 7/7=1$

Example of Dam Failure

Around 7:00 am on June 5, 1976 a leak about 30 m from the top of Teton dam was observed.

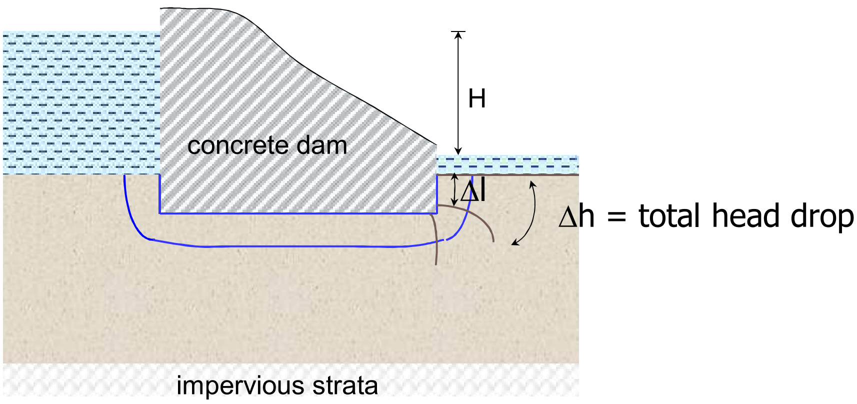
Example of Dam Failure

The Dam Broke at 11:59 AM

Piping in Granular Soils

At the downstream, near the dam,

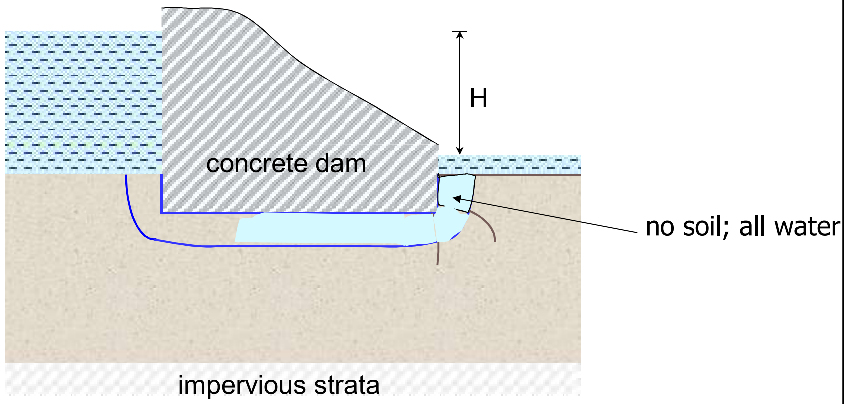
the exit hydraulic gradient $i_{exit} = \dfrac{\Delta h}{\Delta l}$



Piping in Granular Soils

If i_{exit} exceeds the critical hydraulic gradient (i_c), first the soil grains at exit get washed away.

This phenomenon progresses towards the upstream, forming a free passage of water ("pipe").



Critical hydraulic gradient, i_c

The critical hydraulic gradient (i_c),

$$i_c = \frac{\gamma'}{\gamma_w} = \frac{G_s - 1}{1 + e} \qquad \gamma' = \gamma_{sat} - \gamma_w$$

Consequences:

no stresses to hold granular soils together

$\therefore$ soil may flow $\Rightarrow$

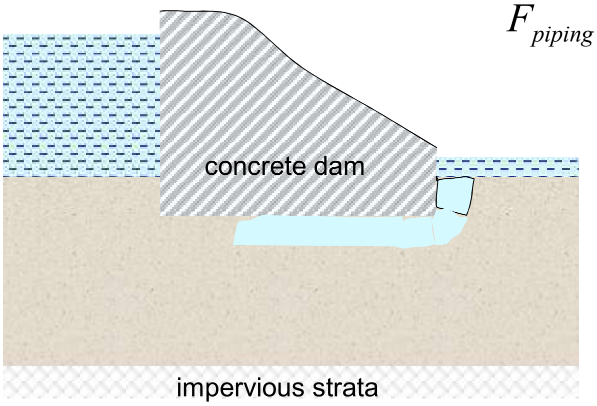
"boiling" or "piping" = EROSION

Piping in Granular Soils

Piping is a very serious problem. It leads to downstream flooding which can result in loss of lives.

Therefore, provide adequate safety factor against piping.

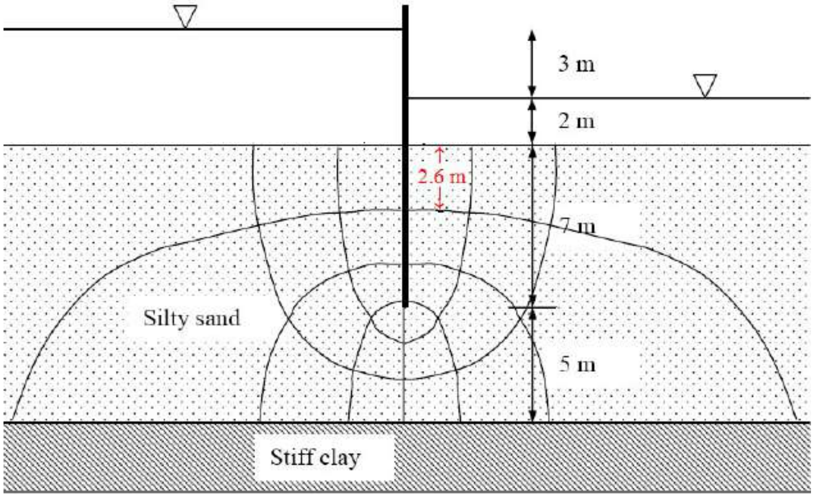
$$F_{piping} = \frac{i_c}{i_{exit}} > 3$$



EXAMPLE

- A stiff clay layer underlies a 12 m thick silty sand deposit. A sheet pile is driven into the sand to a depth of 7 m, and the upstream and downstream water levels are as shown in the figure. Permeability of the silty sand is 8.6×10^{-4} cm/s. The stiff clay can be assumed to be impervious. The void ratio of the silty sand is 0.72 and the specific gravity of the grains is 2.65.
- (a) Estimate the seepage beneath the sheet pile in m^3/day per meter.
- (b) What is the pore water pressure at the tip of the sheet pile?
- (c) Is the arrangement safe against piping?

EXAMPLE



Solution

(a) In the flow net, $N_f = 3$; $N_d = 8$; $\Delta h = 3$ m. The flow (Q) is given by:

$$q = kh_L \frac{N_f}{N_d} = (8.6 \times 10^{-6})(3)\left(\frac{3}{8}\right)(24 \times 3600) = 0.836\,\text{m}^3/day \text{ per metre}$$

(b) Taking downstream water level as the datum, at the tip of the sheet pile,

Total head = 1.5 m

Elevation head = -9 m

$\therefore$ Pressure head = 1.5 – (-9) = 10.5 m

Pore water pressure = (10.5)(9.81) = 103.0 kPa

(c) Head loss per equipotential drop, $\Delta h = 3/8 = 0.375$ m

The maximum exit hydraulic gradient (near the sheet pile) = 0.375/2.6 = 0.144

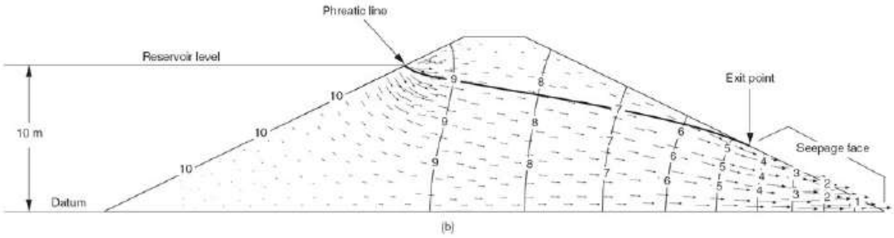
The critical hydraulic gradient (i_c) is given by:

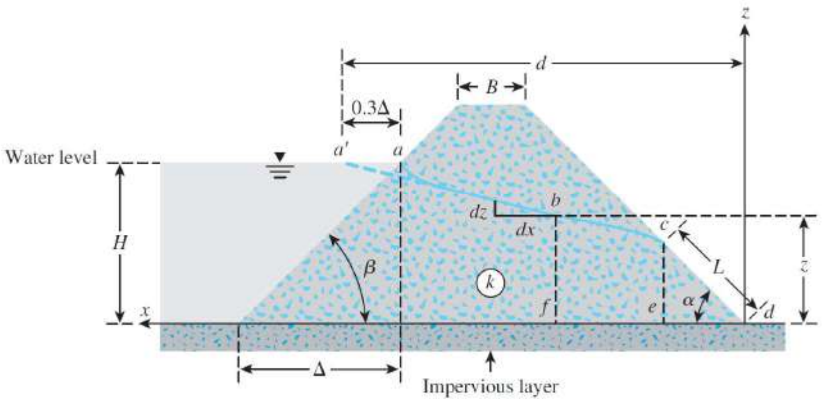
$$i_c = \frac{G_s - 1}{1 + e} = \frac{2.65 - 1}{1 + 0.72} = 0.96$$

$\therefore$ Safety factor with respect to piping F = 0.96/0.144 = 6.7 > 5

The arrangement is quite safe with respect to piping.

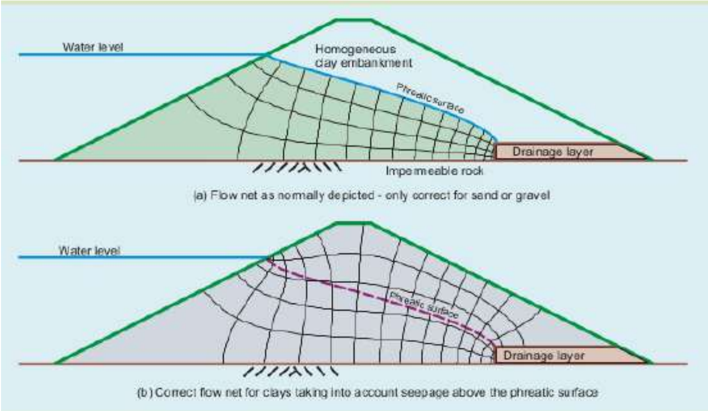
Seepage Through an Earth Dam





$$L = \frac{d}{\cos\alpha} - \sqrt{\frac{d^2}{\cos^2\alpha} - \frac{H^2}{\sin^2\alpha}}$$

$$q = k(\tan\alpha)(L\sin\alpha) = kL\tan\alpha\sin\alpha$$

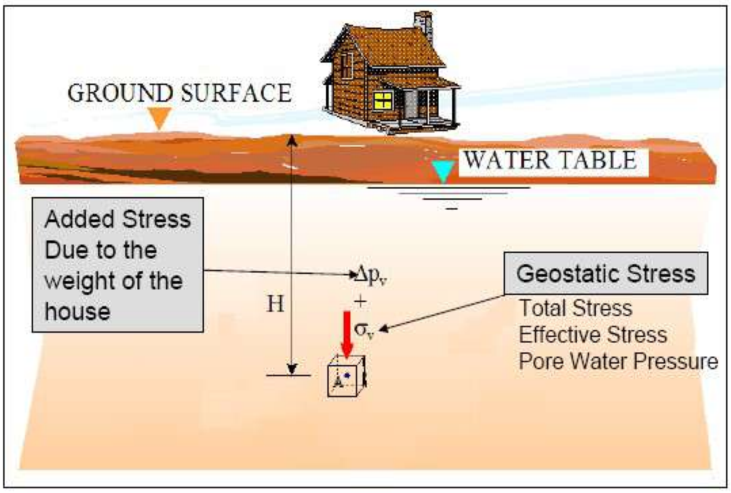


Flow net in homogeneous earth dams

In-Situ Stress

Introduction

There are two types of stresses in soil.



2

Introduction

- The principal source of soil stress is caused by the weight of the soil (adjusted usually by the pore water pressure) above the point in the soil under examination
- Additional stresses can and are induced by load from the soil surface, such as footings, deep foundations, embankments and other weight-bearing structures

3

Hydrostatic and Total stress

- Hydrostatic Stress
  - The stress induced by the weight of the water at a given depth

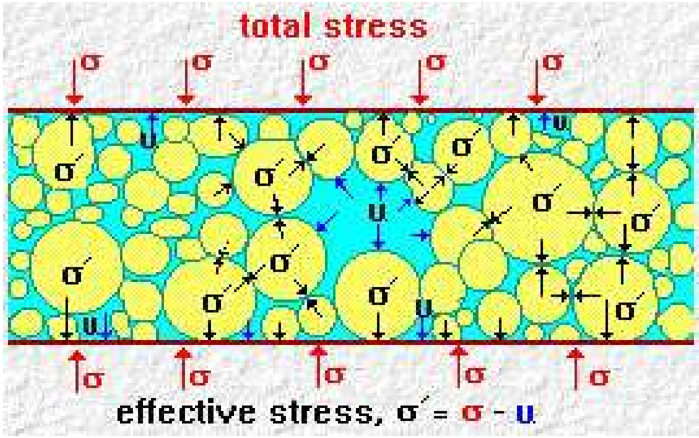
$$u = \gamma_w z$$

  - Assumes water is at rest and not experiencing forces other than static gravity
  - Also exists in the pores of the soil as it does in free water, thus the name "pore water pressure" (but pore water pressure doesn't have to be hydrostatic)
- Total Stress
  - The stress produced at any point by the overburden pressure of the soil plus any applied loads (equation does not include loads)

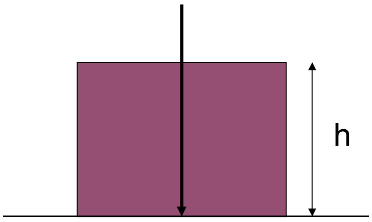
$$\sigma_{total} = \sum_{i=1}^{n} H_i \gamma_{t_i}$$

- Generally ignores the effect of pore water pressure
- Applies in general to dry or unsaturated soils

4



5



The density of the block = γ

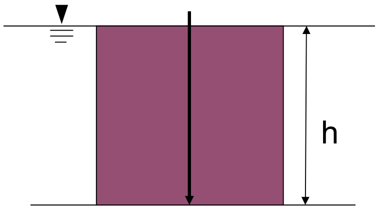
Volume (V) = Area (A) x height (h);

The weight of the block = density x volume = (γ x A x h)

The total Vertical stress at the bottom = weight / area = (γ x A x h) / A = γ x h

When water is present the effective vertical stress is less due to buoyancy.

6



The density of the block = γ

The water stress= u= $\gamma_w h$
The effective vertical stress at the bottom = $\gamma h - \gamma_w h = (\gamma - \gamma_w) \times h$

7

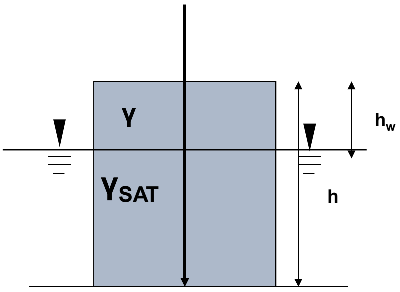
The stress has two components. The stress due to pore pressure (u) ;
$u = h\ \gamma_w$ → γ_w = density of water; h = depth (ft)

Stress due to the weight of the rectangle; **$\sigma = h\ \gamma$**

The principle effective stress is
$\sigma' = \sigma - u$

Total stress (σ) is equal to the sum of effective stress (σ') and pore water pressure (u) or, alternatively, effective stress is equal to total stress minus pore water pressure.

8



Now assume the block is sand, what if the ground water level is at a depth h_w below the ground level,

$$\sigma = \gamma h_w + \gamma_{SAT} (h - h_w)$$

The pore pressure is $u = \gamma_w(h - h_w)$

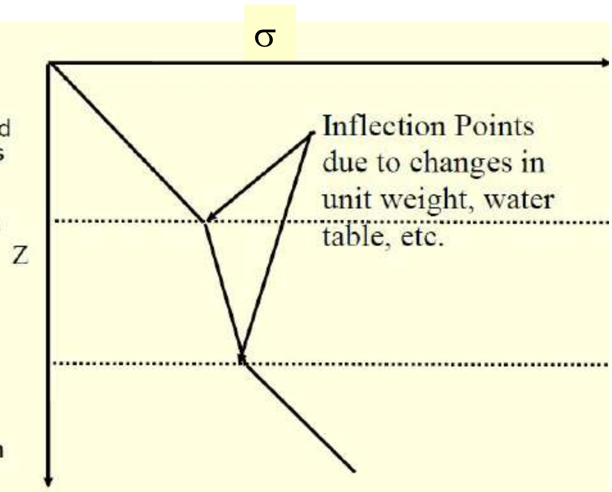
So the effective stress is;

$$\sigma' = \sigma - u = \gamma h_w + \gamma_{SAT}(h - h_w) - \gamma_w(h - h_w)$$

9

σ Diagram

- Useful tool to visualise the increase of effective stress/overburden pressure as a function of depth and to clearly see the effect of the pore water pressure
- Can also be used in some cases to illustrate the changes that take place with applied loads and other conditions



10

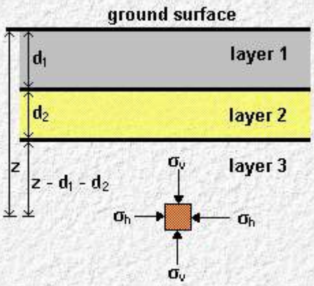
Total stress in multi-layered soil

The total stress at depth z is the sum of the weights of soil in each layer thickness above.
Vertical total stress at depth z,

$$\sigma_v = \gamma_1 d_1 + \gamma_2 d_2 + \gamma_3 (z - d_1 - d_2)$$

where

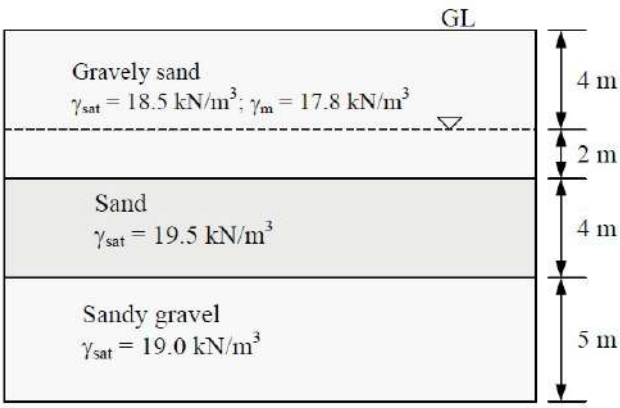
γ_1, γ_2, γ_3, etc. = unit weights of soil layers 1, 2 , 3, etc. respectively



11

EXAMPLE

Plot the variation of total and effective vertical stresses, and pore water pressure with depth for the soil profile shown below in Fig.



12

Solution:

Within a soil layer, the unit weight is constant, and therefore the stresses vary linearly. Therefore, it is adequate if we compute the values at the layer interfaces and water table location, and join them by straight lines.

At the ground level,
$\sigma_v = 0$; $\sigma_v' = 0$; and $u=0$

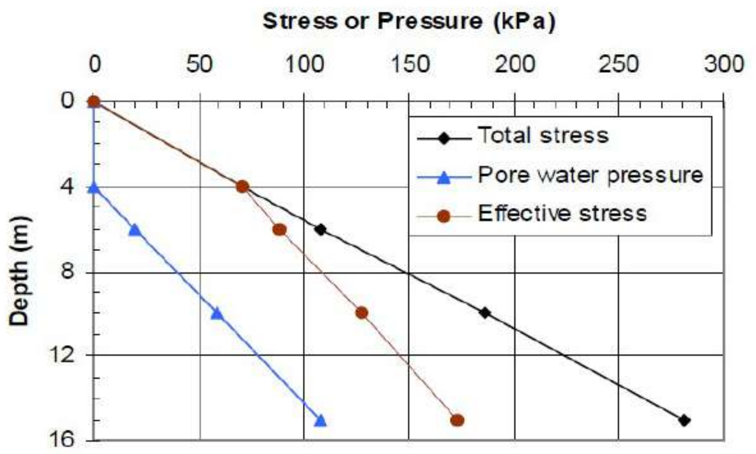
At 4 m depth,
$\sigma_v = (4)(17.8) = 71.2$ kPa; $u = 0$
$\therefore \sigma_v' = 71.2$ kPa

At 6 m depth,
$\sigma_v = (4)(17.8) + (2)(18.5) = 108.2$ kPa
$u = (2)(9.81) = 19.6$ kPa
$\therefore \sigma_v' = 108.2 - 19.6 = 88.6$ kPa

At 10 m depth,
$\sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) = 186.2$ kPa
$u = (6)(9.81) = 58.9$ kPa
$\therefore \sigma_v' = 186.2 - 58.9 = 127.3$ kPa

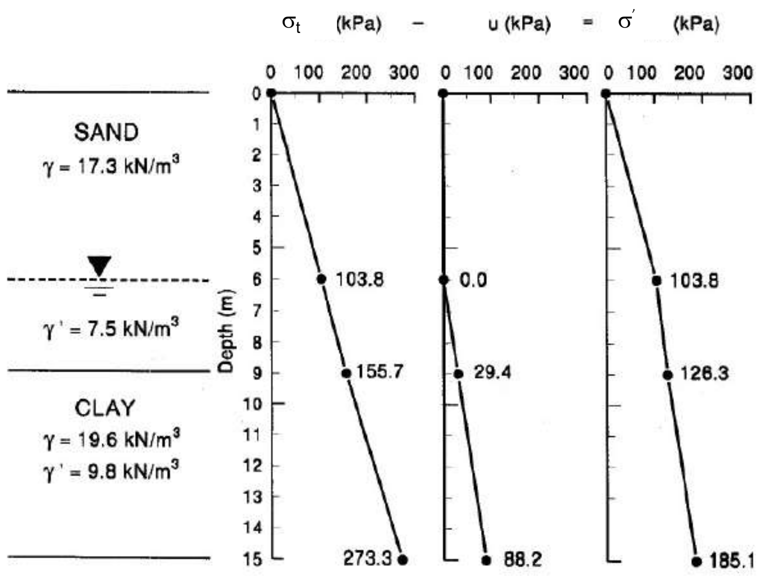
At 15 m depth,
$\sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) + (5)(19.0) = 281.2$ kPa
$u = (11)(9.81) = 107.9$ kPa
$\therefore \sigma_v' = 281.2 - 107.9 = 173.3$ kPa

13



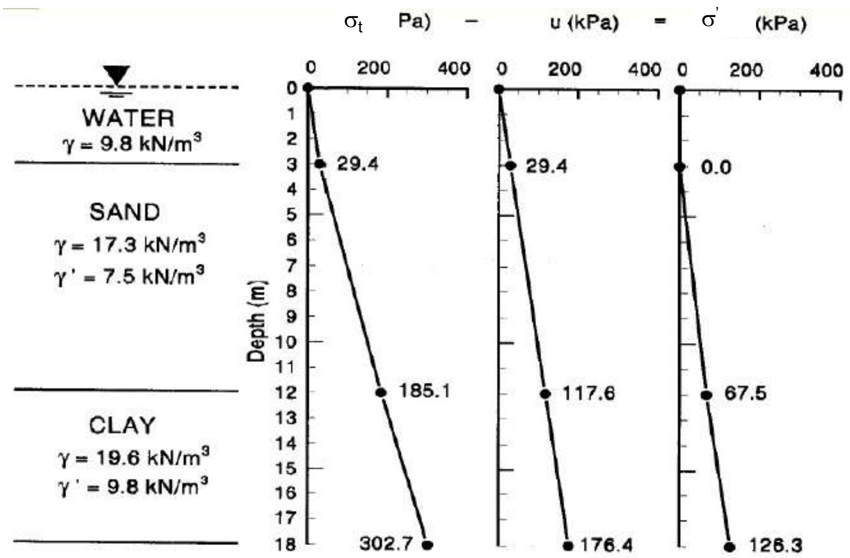
14

Example 2



15

Example 3



16

Stresses in Saturated Soils without Seepage (No flow)

At A,

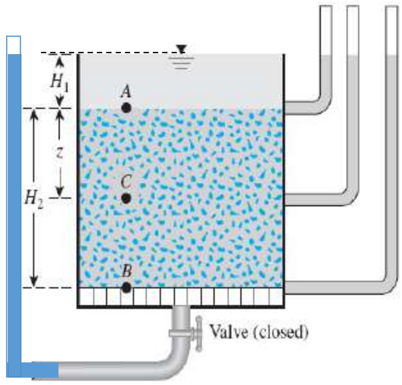
- Total Stress: $\sigma_A = H_1\,\gamma_w$
- Pore water pressure: $u_A = H_1\,\gamma_w$
- Effective stress: $\sigma'_A = 0$

At B,

- Total Stress: $\sigma_B = H_1\,\gamma_w + H_2\,\gamma_{sat}$
- Pore water pressure: $u_B = (H_1 + H_2)\,\gamma_w$
- Effective stress: $\sigma'_B = H_2(\gamma_{sat} - \gamma_w) = H_2\,\gamma'$

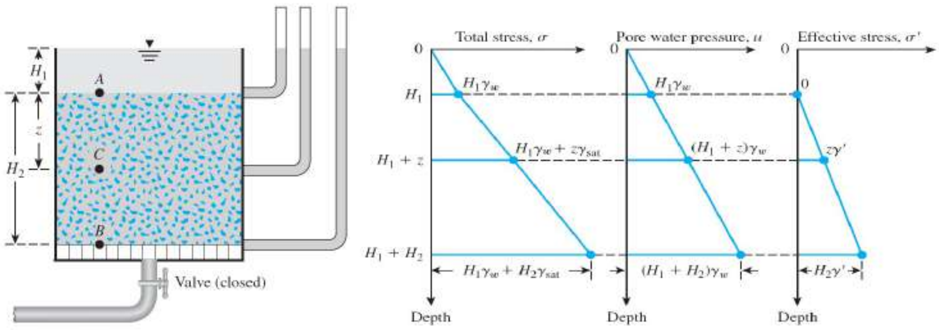
At C,

- Total Stress: $\sigma_C = H_1\,\gamma_w + z\,\gamma_{sat}$
- Pore water pressure: $u_C = (H_1 + z)\,\gamma_w$
- Effective stress: $\sigma'_C = z(\gamma_{sat} - \gamma_w) = z\,\gamma'$



Stresses in Saturated Soils without Seepage (No flow)

Variations of the total stress, pore water pressure, and effective stress, respectively, with depth for a soil layer without seepage



Stresses in Saturated Soils with Upward Seepage

If water is seeping, the effective stress at any point in a soil mass will differ from that in the static case. It will increase or decrease, depending on the direction of seepage.

At A,

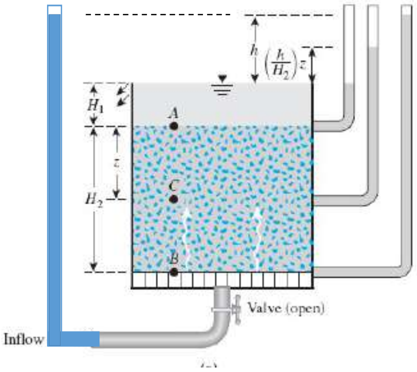
- Total stress: $\sigma_A = H_1\gamma_w$
- Pore water pressure: $u_A = H_1\gamma_w$
- Effective stress: $\sigma'_A = \sigma_A - u_A = 0$

At B,

- Total stress: $\sigma_B = H_1\gamma_w + H_2\gamma_{sat}$
- Pore water pressure: $u_B = (H_1 + H_2 + h)\gamma_w$
- Effective stress: $\sigma'_B = \sigma_B - u_B$
$$= H_2(\gamma_{sat} - \gamma_w) - h\gamma_w$$
$$= H_2\gamma' - h\gamma_w$$

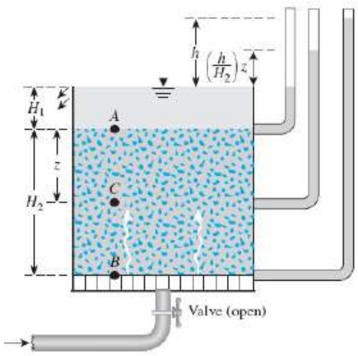
At C,

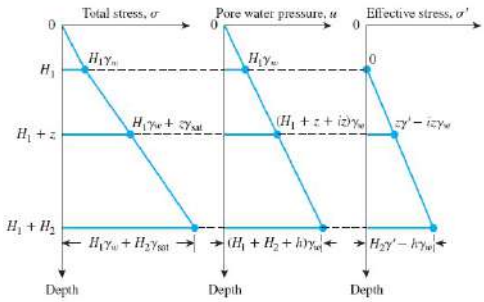
- Total stress: $\sigma_C = H_1\gamma_w + z\gamma_{sat}$
- Pore water pressure: $u_C = \left(H_1 + z + \frac{h}{H_2}z\right)\gamma_w$
- Effective stress: $\sigma'_C = \sigma_C - u_C$
$$= z(\gamma_{sat} - \gamma_w) - \frac{h}{H_2}z\gamma_w$$
$$= z\gamma' - \frac{h}{H_2}z\gamma_w$$



Stresses in Saturated Soils with Upward Seepage

Variations of the total stress, pore water pressure, and effective stress, respectively, with depth for a soil layer with upward seepage





Stresses in Saturated Soils with Upward Seepage

- Note that h/H_2 *is the hydraulic gradient I caused by the flow, and therefore:*

$$\sigma' = z\gamma' - iz\gamma_w$$

- If the rate of seepage and thereby the hydraulic gradient gradually are increased, a limiting condition will be reached, at which σ' is zero:

$$0 = z\gamma' - i_{cr} z\gamma_w$$

Where i_{cr}: *critical hydraulic gradient (for zero effective stress).*

$$i_{cr} = \frac{\gamma'}{\gamma_w}$$

- Under such a situation, soil stability is lost. This situation generally is referred to as boiling, or a quick condition.

- For most soils, the value of *icr varies from 0.9 to 1.1, with an average of 1.*

Stresses in Saturated Soils with Downward Seepage

At *A*,

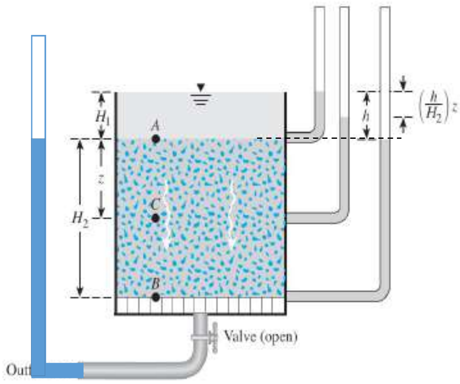
- Total stress: $\sigma_A = H_1\gamma_w$
- Pore water pressure: $u_A = H_1\gamma_w$
- Effective stress: $\sigma'_A = \sigma_A - u_A = 0$

At B,

- Total Stress: $\sigma_B = H_1\gamma_w + H_2\gamma_{sat}$
- Pore water pressure: $u_B = (H_1 + H_2 - h)\gamma_w$
- Effective stress: $\sigma'_B = H_2(\gamma_{sat} - \gamma_w) + h\gamma_w$

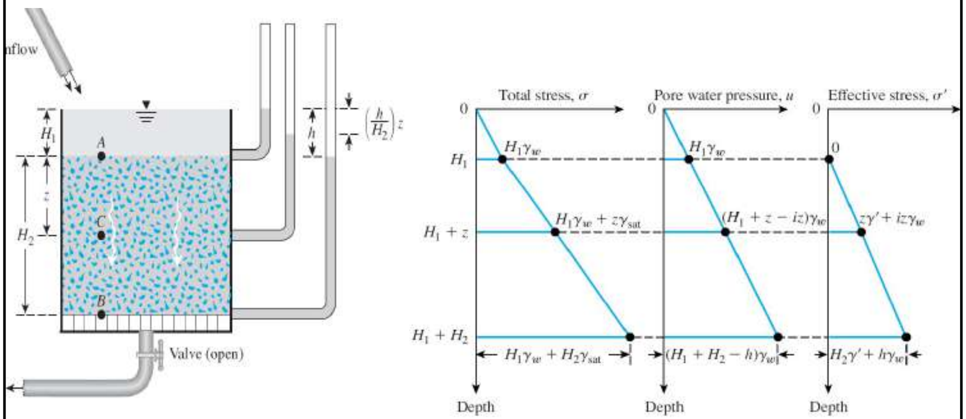
At C,

- Total Stress: $\sigma_C = H_1\gamma_w + z\gamma_{sat}$
- Pore water pressure: $u_C = (H_1 + z - h/H_2\, z)\gamma_w$
  $= (H_1 + z - i\, z)\gamma_w$
- Effective stress: $\sigma'_C = z(\gamma_{sat} - \gamma_w) + iz\gamma_w$
  $= z\gamma' + iz\gamma_w$



Stresses in Saturated Soils with Downward Seepage

Variations of the total stress, pore water pressure, and effective stress, respectively, with depth for a soil layer with downward seepage



10. Stresses in Soil Mass
(Das, Chapter 10)

Sections: All except 10.1, 10.2, 10.5, 10.11

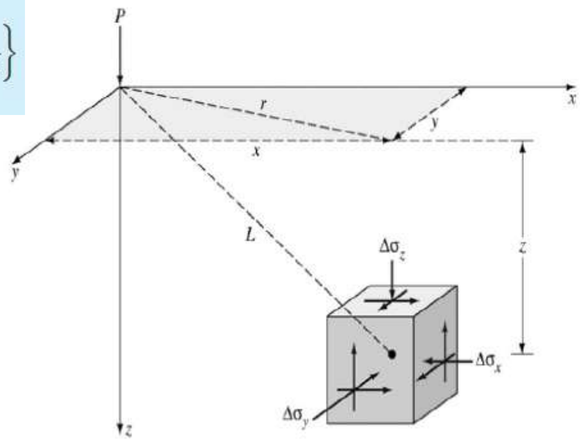
Stress in a Soil Mass

- ❑ Vertical stress due to a point load
- ❑ Vertical stress due to a vertical line load
- ❑ Vertical stress due to a vertical strip load
- ❑ Vertical stress due to embankment load
- ❑ Vertical stress below a center of a uniform loaded Circular area
- ❑ Vertical stress at any point below a uniformly loaded Circular area
- ❑ Vertical stress caused by rectangular load
- ❑ Influence chart for vertical pressure

Vertical Stress Due to a Point Load

$$\Delta\sigma_z = \frac{P}{z^2}\left\{\frac{3}{2\pi}\frac{1}{[(r/z)^2 + 1]^{5/2}}\right\}$$

$$= \frac{P}{z^2} I_1$$

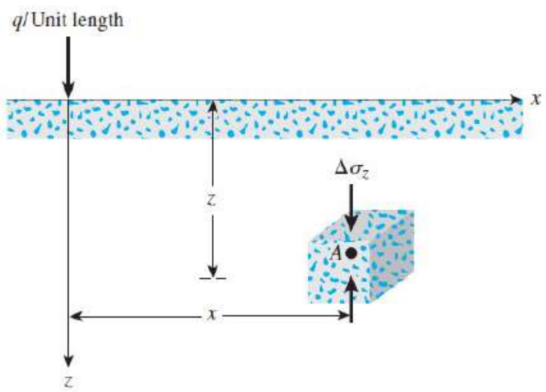


Vertical Stress Due to a Point Load

Table 10.1 Variation of I_1 for Various Values of r/z [Eq. (10.14)]

r/z	I_1	r/z	I_1	r/z	I_1
0	0.4775	0.36	0.3521	1.80	0.0129
0.02	0.4770	0.38	0.3408	2.00	0.0085
0.04	0.4765	0.40	0.3294	2.20	0.0058
0.06	0.4723	0.45	0.3011	2.40	0.0040
0.08	0.4699	0.50	0.2733	2.60	0.0029
0.10	0.4657	0.55	0.2466	2.80	0.0021
0.12	0.4607	0.60	0.2214	3.00	0.0015
0.14	0.4548	0.65	0.1978	3.20	0.0011
0.16	0.4482	0.70	0.1762	3.40	0.00085
0.18	0.4409	0.75	0.1565	3.60	0.00066
0.20	0.4329	0.80	0.1386	3.80	0.00051
0.22	0.4242	0.85	0.1226	4.00	0.00040
0.24	0.4151	0.90	0.1083	4.20	0.00032
0.26	0.4050	0.95	0.0956	4.40	0.00026
0.28	0.3954	1.00	0.0844	4.60	0.00021
0.30	0.3849	1.20	0.0513	4.80	0.00017
0.32	0.3742	1.40	0.0317	5.00	0.00014
0.34	0.3632	1.60	0.0200		

Vertical Stress Due to a Vertical line Load



$$\frac{\Delta\sigma_z}{(q/z)} = \frac{2}{\pi[(x/z)^2 + 1]^2}$$

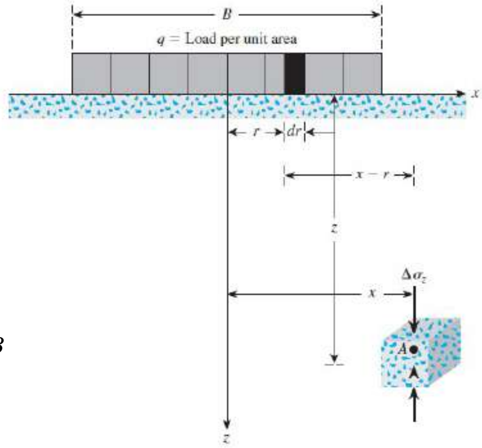
Vertical Stress Due to a Vertical Line Load

Table 10.2 Variation of $\Delta\sigma_z/(q/z)$ with x/z [Eq. (10.16)]

x/z	$\Delta\sigma_z/(q/z)$	x/z	$\Delta\sigma_z/(q/z)$
0	0.637	1.3	0.088
0.1	0.624	1.4	0.073
0.2	0.589	1.5	0.060
0.3	0.536	1.6	0.050
0.4	0.473	1.7	0.042
0.5	0.407	1.8	0.035
0.6	0.344	1.9	0.030
0.7	0.287	2.0	0.025
0.8	0.237	2.2	0.019
0.9	0.194	2.4	0.014
1.0	0.159	2.6	0.011
1.1	0.130	2.8	0.008
1.2	0.107	3.0	0.006

Vertical Stress Due to a Vertical Strip Load
(Finite width and infinite length)

$$\Delta\sigma_z = \frac{q}{\pi}\left\{\tan^{-1}\left[\frac{z}{x-(B/2)}\right] - \tan^{-1}\left[\frac{z}{x+(B/2)}\right] - \frac{Bz[x^2 - z^2 - (B^2/4)]}{[x^2 + z^2 - (B^2/4)]^2 + B^2 z^2}\right\}$$



Calculate $\Delta\sigma_z/q$ from $2z/B$ and $2x/B$

Vertical Stress Due to a Vertical Strip Load

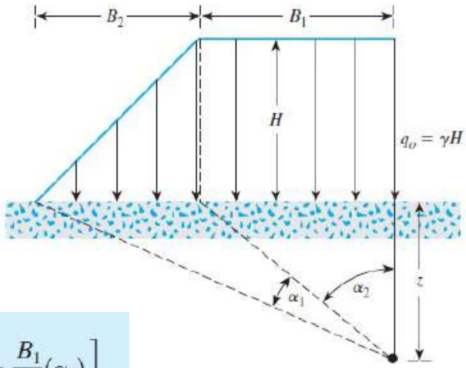
Table 10.4 Variation of $\Delta\sigma_z/q$ with $2z/B$ and $2x/B$ [Eq. (10.19)]

2z/B	2x/B: 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.000
0.10	1.000	1.000	0.999	0.999	0.999	0.998	0.997	0.993	0.980	0.909	0.500
0.20	0.997	0.997	0.996	0.995	0.992	0.988	0.979	0.959	0.909	0.775	0.500
0.30	0.990	0.989	0.987	0.984	0.978	0.967	0.947	0.908	0.833	0.697	0.499
0.40	0.977	0.976	0.973	0.966	0.955	0.937	0.906	0.855	0.773	0.651	0.498
0.50	0.959	0.958	0.953	0.943	0.927	0.902	0.864	0.808	0.727	0.620	0.497
0.60	0.937	0.935	0.928	0.915	0.896	0.866	0.825	0.767	0.691	0.598	0.495
0.70	0.910	0.908	0.899	0.885	0.863	0.831	0.788	0.732	0.662	0.581	0.492
0.80	0.881	0.878	0.869	0.853	0.829	0.797	0.755	0.701	0.638	0.566	0.489
0.90	0.850	0.847	0.837	0.821	0.797	0.765	0.724	0.675	0.617	0.552	0.485
1.00	0.818	0.815	0.805	0.789	0.766	0.735	0.696	0.650	0.598	0.540	0.480
1.10	0.787	0.783	0.774	0.758	0.735	0.706	0.670	0.628	0.580	0.529	0.474
1.20	0.755	0.752	0.743	0.728	0.707	0.679	0.646	0.607	0.564	0.517	0.468
1.30	0.725	0.722	0.714	0.699	0.679	0.654	0.623	0.588	0.548	0.506	0.462
1.40	0.696	0.693	0.685	0.672	0.653	0.630	0.602	0.569	0.534	0.495	0.455
1.50	0.668	0.666	0.658	0.646	0.629	0.607	0.581	0.552	0.519	0.484	0.448
1.60	0.642	0.639	0.633	0.621	0.605	0.586	0.562	0.535	0.506	0.474	0.440
1.70	0.617	0.615	0.608	0.598	0.583	0.565	0.544	0.519	0.492	0.463	0.433
1.80	0.593	0.591	0.585	0.576	0.563	0.546	0.526	0.504	0.479	0.453	0.425
1.90	0.571	0.569	0.564	0.555	0.543	0.528	0.510	0.489	0.467	0.443	0.417
2.00	0.550	0.548	0.543	0.535	0.524	0.510	0.494	0.475	0.455	0.433	0.409
2.10	0.530	0.529	0.524	0.517	0.507	0.494	0.479	0.462	0.443	0.423	0.401
2.20	0.511	0.510	0.506	0.499	0.490	0.479	0.465	0.449	0.432	0.413	0.393
2.30	0.494	0.493	0.489	0.483	0.474	0.464	0.451	0.437	0.421	0.404	0.385
2.40	0.477	0.476	0.473	0.467	0.460	0.450	0.438	0.425	0.410	0.395	0.378
2.50	0.462	0.461	0.458	0.452	0.445	0.436	0.426	0.414	0.400	0.386	0.370
2.60	0.447	0.446	0.443	0.439	0.432	0.424	0.414	0.403	0.390	0.377	0.363
2.70	0.433	0.432	0.430	0.425	0.419	0.412	0.403	0.393	0.381	0.369	0.355
2.80	0.420	0.419	0.417	0.413	0.407	0.400	0.392	0.383	0.372	0.360	0.348
2.90	0.408	0.407	0.405	0.401	0.396	0.389	0.382	0.373	0.363	0.352	0.341
3.00	0.396	0.395	0.393	0.390	0.385	0.379	0.372	0.364	0.355	0.345	0.334
3.10	0.385	0.384	0.382	0.379	0.375	0.369	0.363	0.355	0.347	0.337	0.327
3.20	0.374	0.373	0.372	0.369	0.365	0.360	0.354	0.347	0.339	0.330	0.321
3.30	0.364	0.363	0.362	0.359	0.355	0.351	0.345	0.339	0.331	0.323	0.315
3.40	0.354	0.354	0.352	0.350	0.346	0.342	0.337	0.331	0.324	0.316	0.308
3.50	0.345	0.345	0.343	0.341	0.338	0.334	0.329	0.323	0.317	0.310	0.302
3.60	0.337	0.336	0.335	0.333	0.330	0.326	0.321	0.316	0.310	0.304	0.297
3.70	0.328	0.328	0.327	0.325	0.322	0.318	0.314	0.309	0.304	0.298	0.291
3.80	0.320	0.320	0.319	0.317	0.315	0.311	0.307	0.303	0.297	0.292	0.285
3.90	0.313	0.313	0.312	0.310	0.307	0.304	0.301	0.296	0.291	0.286	0.280
4.00	0.306	0.305	0.304	0.303	0.301	0.298	0.294	0.290	0.285	0.280	0.275
4.10	0.299	0.299	0.298	0.296	0.294	0.291	0.288	0.284	0.280	0.275	0.270
4.20	0.292	0.292	0.291	0.290	0.288	0.285	0.282	0.278	0.274	0.270	0.265
4.30	0.286	0.286	0.285	0.283	0.282	0.279	0.276	0.273	0.269	0.265	0.260
4.40	0.280	0.280	0.279	0.278	0.276	0.274	0.271	0.268	0.264	0.260	0.256
4.50	0.274	0.274	0.273	0.272	0.270	0.268	0.266	0.263	0.259	0.255	0.251
4.60	0.268	0.268	0.268	0.266	0.265	0.263	0.260	0.258	0.254	0.251	0.247
4.70	0.263	0.263	0.262	0.261	0.260	0.258	0.255	0.253	0.250	0.246	0.243
4.80	0.258	0.258	0.257	0.256	0.255	0.253	0.251	0.248	0.245	0.242	0.239
4.90	0.253	0.253	0.252	0.251	0.250	0.248	0.246	0.244	0.241	0.238	0.235
5.00	0.248	0.248	0.247	0.246	0.245	0.244	0.242	0.239	0.237	0.234	0.231

Table 10.4 (continued)

2z/B	2x/B: 1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.10	0.091	0.020	0.007	0.003	0.002	0.001	0.001	0.000	0.000	0.000
0.20	0.225	0.091	0.040	0.020	0.011	0.007	0.004	0.003	0.002	0.002
0.30	0.301	0.165	0.090	0.052	0.031	0.020	0.013	0.009	0.007	0.005
0.40	0.346	0.224	0.141	0.090	0.059	0.040	0.027	0.020	0.014	0.011
0.50	0.373	0.267	0.185	0.128	0.089	0.063	0.046	0.034	0.025	0.019
0.60	0.391	0.298	0.222	0.163	0.120	0.088	0.066	0.050	0.038	0.030
0.70	0.403	0.321	0.250	0.193	0.148	0.113	0.087	0.068	0.053	0.042
0.80	0.411	0.338	0.273	0.218	0.173	0.137	0.108	0.086	0.069	0.056
0.90	0.416	0.351	0.291	0.239	0.195	0.158	0.128	0.104	0.085	0.070
1.00	0.419	0.360	0.305	0.256	0.214	0.177	0.147	0.122	0.101	0.084
1.10	0.420	0.366	0.316	0.271	0.230	0.194	0.164	0.138	0.116	0.098
1.20	0.419	0.371	0.325	0.282	0.243	0.209	0.178	0.152	0.130	0.111
1.30	0.417	0.373	0.331	0.291	0.254	0.221	0.191	0.166	0.143	0.123
1.40	0.414	0.374	0.335	0.298	0.263	0.232	0.203	0.177	0.155	0.135
1.50	0.411	0.374	0.338	0.303	0.271	0.240	0.213	0.188	0.165	0.146
1.60	0.407	0.373	0.339	0.307	0.276	0.248	0.221	0.197	0.175	0.155
1.70	0.402	0.370	0.339	0.309	0.281	0.254	0.228	0.205	0.183	0.164
1.80	0.396	0.368	0.339	0.311	0.284	0.258	0.234	0.212	0.191	0.172
1.90	0.391	0.364	0.338	0.312	0.286	0.262	0.239	0.217	0.197	0.179
2.00	0.385	0.360	0.336	0.311	0.288	0.265	0.243	0.222	0.203	0.185
2.10	0.379	0.356	0.333	0.311	0.288	0.267	0.246	0.226	0.208	0.190
2.20	0.373	0.352	0.330	0.309	0.288	0.268	0.248	0.229	0.212	0.195
2.30	0.366	0.347	0.327	0.307	0.288	0.268	0.250	0.232	0.215	0.199
2.40	0.360	0.342	0.323	0.305	0.287	0.268	0.251	0.234	0.217	0.202
2.50	0.354	0.337	0.320	0.302	0.285	0.268	0.251	0.235	0.220	0.205
2.60	0.347	0.332	0.316	0.299	0.283	0.267	0.251	0.236	0.221	0.207
2.70	0.341	0.327	0.312	0.296	0.281	0.266	0.251	0.236	0.222	0.208
2.80	0.335	0.321	0.307	0.293	0.279	0.265	0.250	0.236	0.223	0.210
2.90	0.329	0.316	0.303	0.290	0.276	0.263	0.249	0.236	0.223	0.211
3.00	0.323	0.311	0.299	0.286	0.274	0.261	0.248	0.236	0.223	0.211
3.10	0.317	0.306	0.294	0.283	0.271	0.259	0.247	0.235	0.223	0.212
3.20	0.311	0.301	0.290	0.279	0.268	0.256	0.245	0.234	0.223	0.212
3.30	0.305	0.296	0.286	0.275	0.265	0.254	0.243	0.232	0.222	0.211
3.40	0.300	0.291	0.281	0.271	0.261	0.251	0.241	0.231	0.221	0.211
3.50	0.294	0.286	0.277	0.268	0.258	0.249	0.239	0.229	0.220	0.210
3.60	0.289	0.281	0.273	0.264	0.255	0.246	0.237	0.228	0.218	0.209
3.70	0.284	0.276	0.268	0.260	0.252	0.243	0.235	0.226	0.217	0.208
3.80	0.279	0.272	0.264	0.256	0.249	0.240	0.232	0.224	0.216	0.207
3.90	0.274	0.267	0.260	0.253	0.245	0.238	0.230	0.222	0.214	0.206
4.00	0.269	0.263	0.256	0.249	0.242	0.235	0.227	0.220	0.212	0.205
4.10	0.264	0.258	0.252	0.246	0.239	0.232	0.225	0.218	0.211	0.203
4.20	0.260	0.254	0.248	0.242	0.236	0.229	0.222	0.216	0.209	0.202
4.30	0.255	0.250	0.244	0.239	0.233	0.226	0.220	0.213	0.207	0.200
4.40	0.251	0.246	0.241	0.235	0.229	0.224	0.217	0.211	0.205	0.199
4.50	0.247	0.242	0.237	0.232	0.226	0.221	0.215	0.209	0.203	0.197
4.60	0.243	0.238	0.234	0.229	0.223	0.218	0.212	0.207	0.201	0.195
4.70	0.239	0.235	0.230	0.225	0.220	0.215	0.210	0.205	0.199	0.194
4.80	0.235	0.231	0.227	0.222	0.217	0.213	0.208	0.202	0.197	0.192
4.90	0.231	0.227	0.223	0.219	0.215	0.210	0.205	0.200	0.195	0.190
5.00	0.227	0.224	0.220	0.216	0.212	0.207	0.203	0.198	0.193	0.188

Vertical Stress Due to Embankment Loading

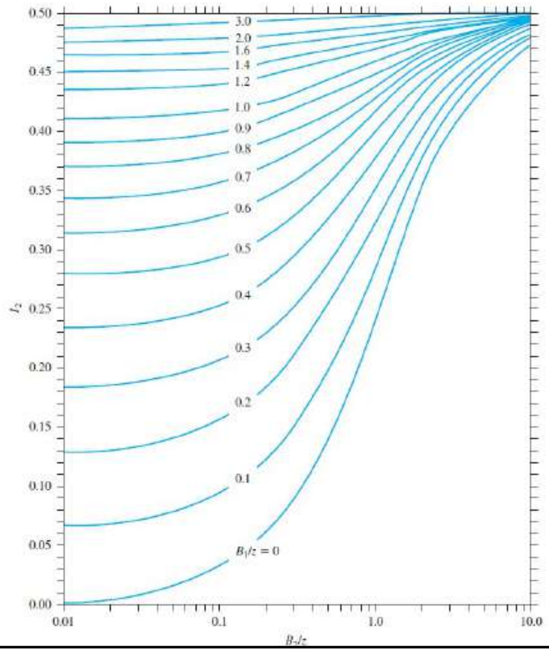


$$\Delta\sigma_z = \frac{q_o}{\pi}\left[\left(\frac{B_1 + B_2}{B_2}\right)(\alpha_1 + \alpha_2) - \frac{B_1}{B_2}(\alpha_2)\right]$$

$$\Delta\sigma_z = q_o I_2$$

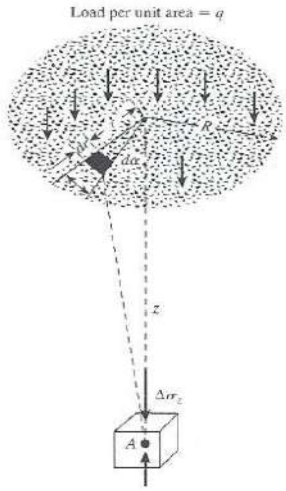
I_2 is a function of B_1/z and B_2/z

Vertical Stress Due to Embankment Loading

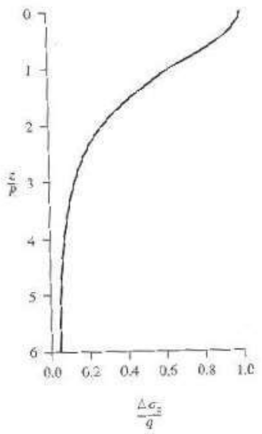


Vertical Stress Below Center of Uniform Loaded Circular Area

$$\Delta\sigma_z = q\left(1 - \frac{1}{\left[(R/z)^2+1\right]^{3/2}}\right)$$



Vertical Stress Below Center of Uniform Loaded Circular Area



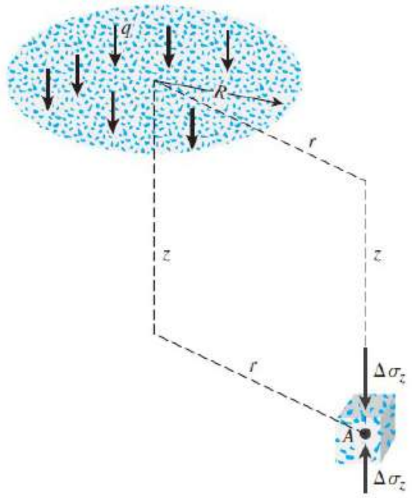
Variation of $\Delta\sigma_z/q$ with z/R [Eq. (5.32)]

z/R	$\Delta\sigma_z/q$	z/R	$\Delta\sigma_z/q$
0	1	1.0	0.6465
0.02	0.9999	1.5	0.4240
0.05	0.9998	2.0	0.2845
0.10	0.9990	2.5	0.1996
0.2	0.9925	3.0	0.1436
0.4	0.9488	4.0	0.0869
0.5	0.9106	5.0	0.0571
0.8	0.7562		

Vertical Stress at Any Point Below a Uniformly Loaded Circular Area

$$\Delta\sigma_z = q(A' + B')$$

A' and B' are functions of z/R and r/R.



Vertical Stress at Any Point Below a Uniformly Loaded Circular Area

Table 10.6 Variation of A' with z/R and r/R*

	r/R								
z/R	**0**	**0.2**	**0.4**	**0.6**	**0.8**	**1**	**1.2**	**1.5**	**2**
0	1.0	1.0	1.0	1.0	1.0	0.5	0	0	0
0.1	0.90050	0.89748	0.88679	0.86126	0.78797	0.43015	0.09645	0.02787	0.00856
0.2	0.80388	0.79824	0.77884	0.73483	0.63014	0.38269	0.15433	0.05251	0.01680
0.3	0.71265	0.70518	0.68316	0.62690	0.52081	0.34375	0.17964	0.07199	0.02440
0.4	0.62861	0.62015	0.59241	0.53767	0.44329	0.31048	0.18709	0.08593	0.03118
0.5	0.55279	0.54403	0.51622	0.46448	0.38390	0.28156	0.18556	0.09499	0.03701
0.6	0.48550	0.47691	0.45078	0.40427	0.33676	0.25588	0.17952	0.10010	
0.7	0.42654	0.41874	0.39491	0.35428	0.29833	0.21727	0.17124	0.10228	0.04558
0.8	0.37531	0.36832	0.34729	0.31243	0.26581	0.21297	0.16206	0.10236	
0.9	0.33104	0.32492	0.30669	0.27707	0.23832	0.19488	0.15253	0.10094	
1	0.29289	0.28763	0.27005	0.24697	0.21468	0.17868	0.14329	0.09849	0.05185
1.2	0.23178	0.22795	0.21662	0.19890	0.17626	0.15101	0.12570	0.09192	0.05260
1.5	0.16795	0.16552	0.15877	0.14804	0.13436	0.11892	0.10296	0.08048	0.05116
2	0.10557	0.10453	0.10140	0.09647	0.09011	0.08269	0.07471	0.06275	0.04496
2.5	0.07152	0.07098	0.06947	0.06698	0.06373	0.05974	0.05555	0.04880	0.03787
3	0.05132	0.05101	0.05022	0.04886	0.04707	0.04487	0.04241	0.03839	0.03150
4	0.02986	0.02976	0.02907	0.02802	0.02832	0.02749	0.02651	0.02490	0.02193
5	0.01942	0.01938				0.01835			0.01573
6	0.01361					0.01307			0.01168
7	0.01005					0.00976			0.00894
8	0.00772					0.00755			0.00703
9	0.00612					0.00600			0.00566
10								0.00477	0.00465

Vertical Stress at Any Point Below a Uniformly Loaded Circular Area

Table 10.7 Variation of B' with z/R and r/R*

	r/R								
z/R	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2
0	0	0	0	0	0	0	0	0	0
0.1	0.09852	0.10140	0.11138	0.13424	0.18796	0.05388	−0.07899	−0.02672	−0.00845
0.2	0.18857	0.19306	0.20772	0.23524	0.25983	0.08513	−0.07759	−0.04448	−0.01593
0.3	0.26362	0.26787	0.28018	0.29483	0.27257	0.10757	−0.04316	−0.04999	−0.02166
0.4	0.32016	0.32259	0.32748	0.32273	0.26925	0.12404	−0.00766	−0.04535	−0.02522
0.5	0.35777	0.35752	0.35323	0.33106	0.26236	0.13591	0.02165	−0.03455	−0.02651
0.6	0.37831	0.37531	0.36308	0.32822	0.25411	0.14440	0.04457	−0.02101	
0.7	0.38487	0.37962	0.36072	0.31929	0.24638	0.14986	0.06209	−0.00702	−0.02329
0.8	0.38091	0.37408	0.35133	0.30699	0.23779	0.15292	0.07530	0.00614	
0.9	0.36962	0.36275	0.33734	0.29299	0.22891	0.15404	0.08507	0.01795	
1	0.35355	0.34553	0.32075	0.27819	0.21978	0.15355	0.09210	0.02814	−0.01005
1.2	0.31485	0.30730	0.28481	0.24836	0.20113	0.14915	0.10002	0.04378	0.00023
1.5	0.25602	0.25025	0.23338	0.20694	0.17368	0.13732	0.10193	0.05745	0.01385
2	0.17889	0.18144	0.16644	0.15198	0.13375	0.11331	0.09254	0.06371	0.02836
2.5	0.12807	0.12633	0.12126	0.11327	0.10298	0.09130	0.07869	0.06022	0.03429
3	0.09487	0.09394	0.09099	0.08635	0.08033	0.07325	0.06551	0.05354	0.03511
4	0.05707	0.05666	0.05562	0.05383	0.05145	0.04773	0.04532	0.03995	0.03066
5	0.03772	0.03760				0.03384			0.02474
6	0.02666					0.02468			0.01968
7	0.01980					0.01868			0.01577
8	0.01526					0.01459			0.01279
9	0.01212					0.01170			0.01054
10								0.00924	0.00879

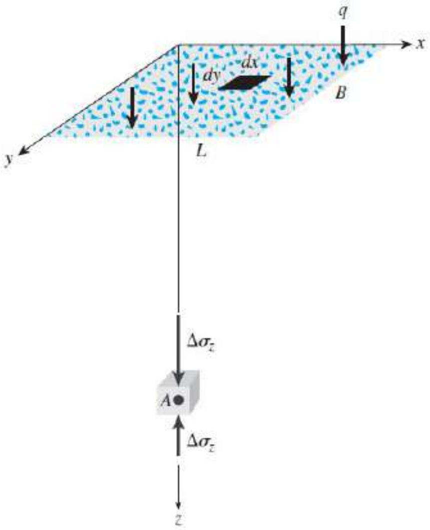
Vertical Stress Caused by Rectangular Load (under corner of the loading area)

$$\Delta\sigma = q \times I_3$$

$$I_3 = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right) + \tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right]$$

$$m = \frac{B}{Z}$$

$$n = \frac{L}{Z}$$



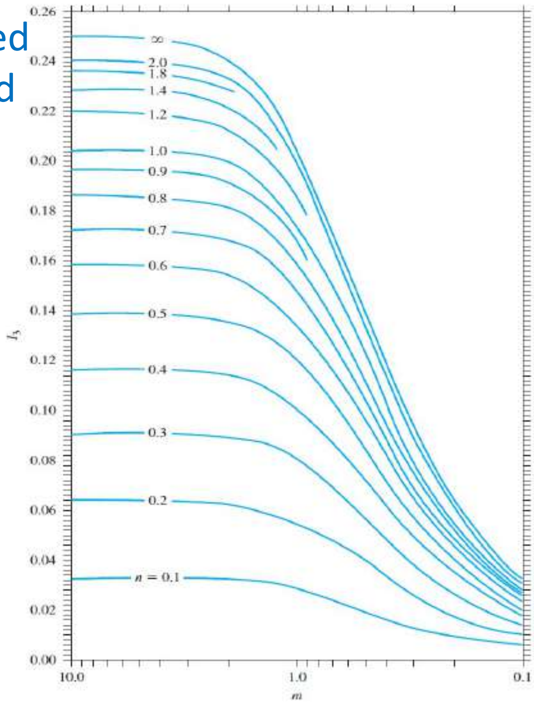
I_3 is a function of m and n

Vertical Stress Caused by Rectangular Load

Table 10.8 Variation of I_3 with m and n [Eq. (10.30)]

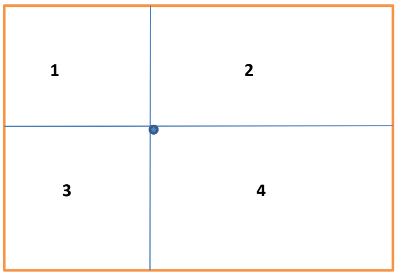
	m									
n	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
0.1	0.0047	0.0092	0.0132	0.0168	0.0198	0.0222	0.0242	0.0258	0.0270	0.0279
0.2	0.0092	0.0179	0.0259	0.0328	0.0387	0.0435	0.0474	0.0504	0.0528	0.0547
0.3	0.0132	0.0259	0.0374	0.0474	0.0559	0.0629	0.0686	0.0731	0.0766	0.0794
0.4	0.0168	0.0328	0.0474	0.0602	0.0711	0.0801	0.0873	0.0931	0.0977	0.1013
0.5	0.0198	0.0387	0.0559	0.0711	0.0840	0.0947	0.1034	0.1104	0.1158	0.1202
0.6	0.0222	0.0435	0.0629	0.0801	0.0947	0.1069	0.1168	0.1247	0.1311	0.1361
0.7	0.0242	0.0474	0.0686	0.0873	0.1034	0.1169	0.1277	0.1365	0.1436	0.1491
0.8	0.0258	0.0504	0.0731	0.0931	0.1104	0.1247	0.1365	0.1461	0.1537	0.1598
0.9	0.0270	0.0528	0.0766	0.0977	0.1158	0.1311	0.1436	0.1537	0.1619	0.1684
1.0	0.0279	0.0547	0.0794	0.1013	0.1202	0.1361	0.1491	0.1598	0.1684	0.1752
1.2	0.0293	0.0573	0.0832	0.1063	0.1263	0.1431	0.1570	0.1684	0.1777	0.1851
1.4	0.0301	0.0589	0.0856	0.1094	0.1300	0.1475	0.1620	0.1739	0.1836	0.1914
1.6	0.0306	0.0599	0.0871	0.1114	0.1324	0.1503	0.1652	0.1774	0.1874	0.1955
1.8	0.0309	0.0606	0.0880	0.1126	0.1340	0.1521	0.1672	0.1797	0.1899	0.1981
2.0	0.0311	0.0610	0.0887	0.1134	0.1350	0.1533	0.1686	0.1812	0.1915	0.1999
2.5	0.0314	0.0616	0.0895	0.1145	0.1363	0.1548	0.1704	0.1832	0.1938	0.2024
3.0	0.0315	0.0618	0.0898	0.1150	0.1368	0.1555	0.1711	0.1841	0.1947	0.2034
4.0	0.0316	0.0619	0.0901	0.1153	0.1372	0.1560	0.1717	0.1847	0.1954	0.2042
5.0	0.0316	0.0620	0.0901	0.1154	0.1374	0.1561	0.1719	0.1849	0.1956	0.2044
6.0	0.0316	0.0620	0.0902	0.1154	0.1374	0.1562	0.1719	0.1850	0.1957	0.2045

Vertical Stress Caused by Rectangular Load

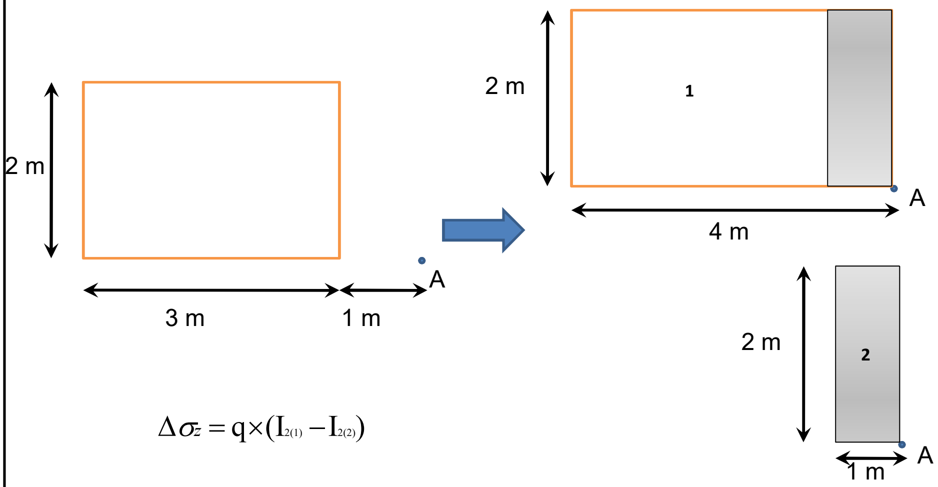


Vertical Stress Caused by Rectangular Load

$$\Delta\sigma_z = q\times(I_{2(1)} + I_{2(2)} + I_{2(3)} + I_{2(4)})$$



Example 10.9



$$\Delta\sigma_z = q\times(I_{2(1)} - I_{2(2)})$$

Chapter 11
Compressibility of Soil

Compressibility and Consolidation

- Structures are built on soils. They transfer loads to the subsoil through the foundations. The effect of the loads is felt by the soil normally up to a depth of about four times the width of the foundation. The soil within this depth gets **compressed** due to the imposed stresses. The compression of the soil mass leads to the decrease in the volume of the mass which results in the **settlement** of the structure.

Compressibility and Consolidation

- Compressibility :

Volume changes in a soil when subjected to pressure –giving ***AMOUNT*** of settlement.

- *Consolidation:*

Rate of volume change with time –giving **TIME** to produce an amount of settlement.

These are different from:

1. Compaction which is the removal of air from a soil by applying compaction energy.
2. Immediate or undrained settlement which is the resultant deformation of a soil under applied stresses without any volume change taking place.

3

Soil Settlement

Soil settlement can be divided into three categories:

1. **Elastic Settlement (S_e)** (immediate settlement) :can be described by the Hooke's law. Elastic deformation of soil without any change in M/C

2. **Primary Consolidation (S_c) :** due to volume change in saturated cohesive soil due to expulsion of water from voids.

3. **Secondary Consolidation (S_s) :** due to plastic adjustment of soil skeletons in saturated cohesive soil.

Total Settlement (S_T)= S_e+ S_c+ S_s

4

Significance

1. The amount of soil volume change that will occur is often one of the governing design criteria of a project;
2. If the settlement is not kept to tolerable limit, the desire use of the structure may be impaired and the design life of the structure may be reduced;
3. It is important to have a mean to predict the total amount of soil settlement;
4. It is important to know the rate of consolidation as well as the total consolidation to be expected.

5

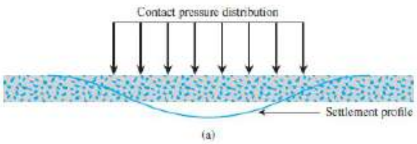
Elastic Settlement

- It occurs instantaneously when the load (weight of the foundations) exerts on the soil. This is why the elastic settlement is also called immediate settlement;
- No alternation of the moisture content of the soil by elastic settlement;
- The magnitude of the contact settlement depends on the flexibility of the foundation and the type of soil.

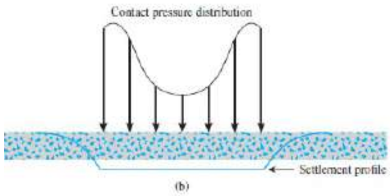
6

Elastic Settlement Profile and Contact Pressure in Clay and Sand

1. **Clay**: Elastic saturated clay, modulus of elasticity is fairly constant with depth.



Flexible Foundation-Clay
Uniform contact pressure with sagging settlement profile.

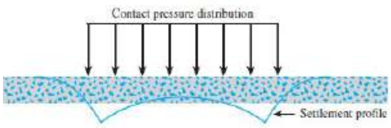


Rigid Foundation-Clay
Uniform settlement with redistribution of contact pressure.
The contact pressure will not be distributed uniformly.

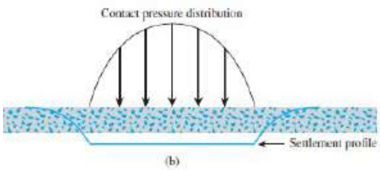
7

Elastic Settlement Profile and Contact Pressure in Clay and Sand

2. **Sand**: Cohesionless soil, modulus of elasticity increases with depth.



Flexible Foundation-Sand
Sand at the edge is pushed outward. Settlement profile in concave downward.



Rigid Foundation-Sand
Uniform settlement with redistribution of contact pressure.

$$S_{e(\text{rigid})} = 0.93 S_{e(\text{flexible, center})}$$

8

Elastic Settlement Calculations

$$S_e = \Delta\sigma(\alpha B')\frac{1-\mu_s^2}{E_s}I_s I_f$$

where $\Delta\sigma$ = net applied pressure on the foundation
μ_s = Poisson's ratio of soil
E_s = average modulus of elasticity of the soil under the foundation measured from $z = 0$ to about $z = 4B$
B' = $B/2$ for center of foundation
= B for corner of foundation
I_s = shape factor (Steinbrenner, 1934)

$$= F_1 + \frac{1-2\mu_s}{1-\mu_s}F_2 \quad (11.2)$$

$$F_1 = \frac{1}{\pi}(A_0 + A_1) \quad (11.3)$$

$$F_2 = \frac{n'}{2\pi}\tan^{-1}A_2 \quad (11.4)$$

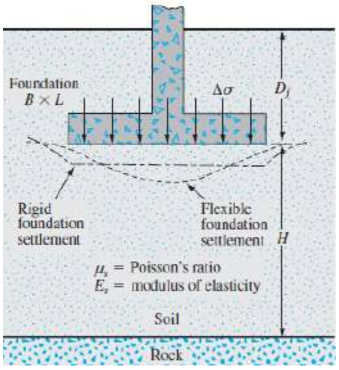
$$A_0 = m'\ln\frac{(1+\sqrt{m'^2+1})\sqrt{m'^2+n'^2}}{m'(1+\sqrt{m'^2+n'^2+1})} \quad (11.5)$$

$$A_1 = \ln\frac{(m'+\sqrt{m'^2+1})\sqrt{1+n'^2}}{m'+\sqrt{m'^2+n'^2+1}} \quad (11.6)$$

$$A_2 = \frac{m'}{n'\sqrt{m'^2+n'^2+1}} \quad (11.7)$$

$$I_f = \text{depth factor (Fox, 1948)} = f\left(\frac{D_f}{B}, \mu_s, \text{and } \frac{L}{B}\right) \quad (11.8)$$

α = factor that depends on the location on the foundation where settlement is being calculated

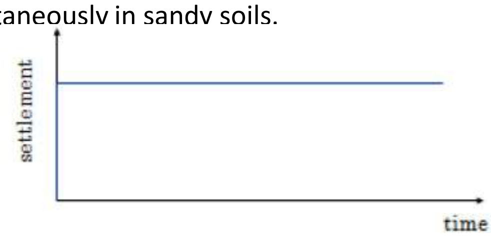


See Example 11.1

The variations of F_1 and F_2 [Eqs. (11.3) and (11.4)] with m' and n' given in Tables 11.1 and 11.2. Also the variation of I_f with D_f/B and μ_s is given in Table 11.3. Note that when $D_f = 0$, *the value of $I_f = 1$ in all cases.*

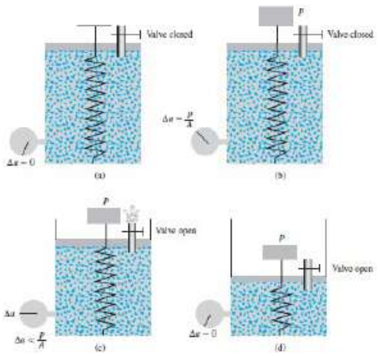
11.4 Fundamentals of Consolidation

- When a saturated soil layer is subjected to a stress increase, the pore water pressure increases suddenly.
- For highly permeable sandy soils, this increase in pressure will cause drainage immediately.
- The drainage leads to a reduction of the soil volume, leading to settlement.
- Due to this rapid drainage, elastic settlement and consolidation occur simultaneously in sandy soils.



11.4 Fundamentals of Consolidation (Cont.)

- For clay layers with low hydraulic conductivities, the consolidation continues after the elastic settlement.
- This may be visualized by a spring-cylinder model.



a) The cylinder is filled with water and has a frictionless water tight piston and valve.

b) If we place a load "**P**" on the piston and keep the valve closed. The entire load will be taken by the water in the cylinder because water is incompressible. The spring will not deform

$\Delta u = P/A$, $P = P_s + P_w$

P_s= Load carried by the Spring.

P_w= Load carried by the water.

P_s= 0 and P_w=P

11.4 Fundamentals of Consolidation (Cont.)

c) If the valve is opened, the water will flow outward. This flow will be accompanied by a reduction of the excess hydrostatic pressure and an increase in the compression of the spring.

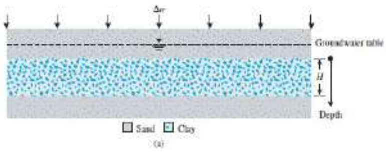
$P_s > 0$ and $P_w < P$ and $\Delta u < P/A$

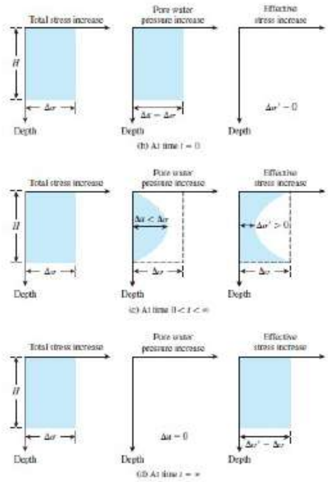
d) After some time, the excess hydrostatic pressure will become zero and the system will reach a state of equilibrium.

$P_s = P$ and $P_w = 0$

11.4 Fundamentals of Consolidation (Cont.)

Variation of total stress, pore water pressure, and effective stress in a clay layer drained at top and bottom as the result of an added stress $\Delta\sigma$





-Clay has a very low hydraulic conductivity and water is incompressible.

- At time (t=0), the entire incremental stress, $\Delta\sigma$, will be carried by the water ($\Delta\sigma = \Delta U$) at all depths. None will be carried by the soil ($\Delta\sigma'$)=0.

- After application of the $\Delta\sigma$ to the clay layer, water in the voids will start to be squeezed out and will drain in both directions in the sand. The excess pore water pressure at any depth in the clay layer gradually will decrease, and the stress carried by the soil (effective stress) will increase.

13

11.5 One-Dimensional Laboratory Consolidation Test

-A Consolidometer, or *Oedometer*, is used.

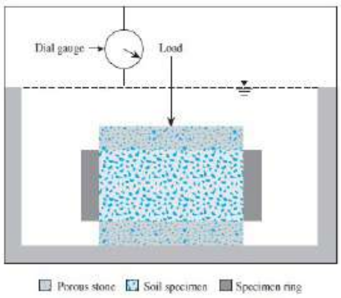
-The soil is placed in a metal ring, with porous stones at the top and bottom of the specimen.

-A micrometer measures the compression.

-A load is applied by a lever arm, and is typically doubled every 24 hours.

-The specimen is kept underwater during the test.

- The dry weight of specimen is determined at the end of the test.



14

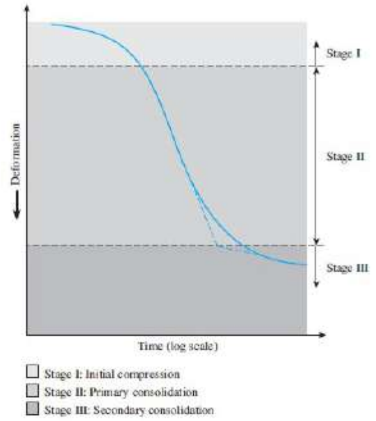
11.5 One-Dimensional Laboratory Consolidation Test (cont.)

The deformation occurs in three stages:

1- Initial compression (I)

2. Primary consolidation as pore water is expelled (II)

3. Secondary consolidation as the soil fabric readjusts plastically (III)



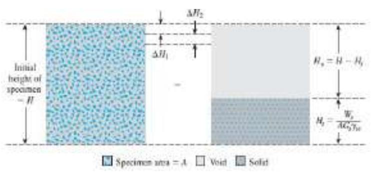
11.6 Void Ratio-Pressure Plots

- Calculate the height of solids, $\boldsymbol{H_s}$*, in the soil specimen using* the equation

$$H_s = \frac{W_s}{AG_s\gamma_w} = \frac{M_s}{AG_s\rho_w}$$

$$H_v = H - H_s$$

where H = initial height of the specimen.



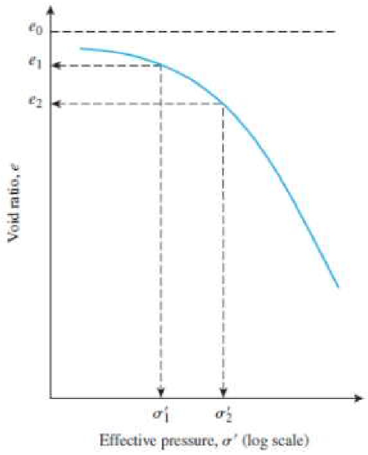
$$e_O = \frac{V_v}{V_s} = \frac{H_v}{H_s}\frac{A}{A} = \frac{H_v}{H_s}$$

$$\Delta e_1 = \frac{\Delta H_1}{H_s}$$

$$e_1 = e_O - \Delta e_1$$

$$e_2 = e_1 - \frac{\Delta H_2}{H_s}$$

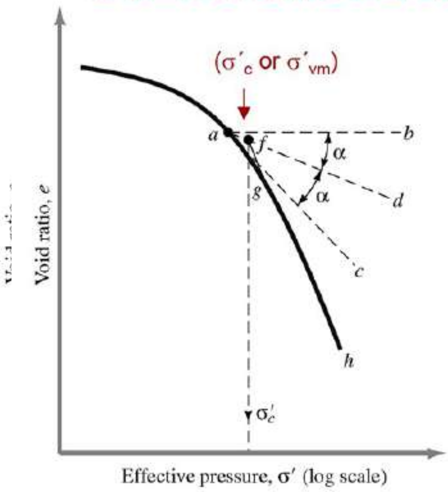
The effective stress σ’ and the corresponding void ratios *e* at the end of consolidation are plotted on semi-logarithmic graph:



In the initial phase, relatively great change in pressure only results in less change in void ratio e. The reason is part of the pressure got to compensate the expansion when the soil specimen was sampled. In the following phase e changes at a great rate

17

OVERCONSOLIDAITON RATIO (OCR)



$$OCR = \frac{\sigma'_c}{\sigma'}$$

Where:

σ´c (a.k.a. σ´vm) = Preconsolidation Pressure (a.k.a Maximum Past Pressure).

σ´ = Present Effective Vertical Stress

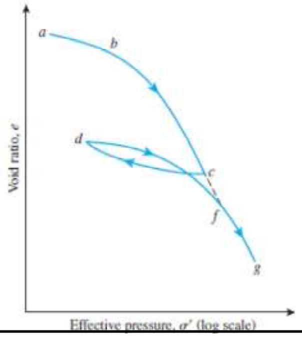
General Guidelines:
NC Soils: 1 = OCR
OC Soils : OCR > 1

Possible Causes of OC Soils:
Preloading (thick sediments, glacial ice); fluctuations of GWT, underdraining, light ice/snow loads, desiccation above GWT, secondary compression.

18

11.7 Normally Consolidated and Overconsolidated Clays

- **Normally consolidated**: the present effective overburden pressure is the max. pressure that the soil was subjected to in the past.
- **Overconsolidated**: the present effective overburden pressure is less than that which the soil experience in the past. The max. effective past pressure is called the **preconsolidation** pressure.
- The reduction of effective pressure in the field may be caused by natural geologic processes or human processes.



$$OCR = \frac{\sigma'_c}{\sigma'}$$

where σ'_c = preconsolidation pressure of a specimen
σ' = present effective vertical pressure

- Loading the specimen to exceed the max. effective overburden pressure.
- Then unloading (*cd*) and reloading again (*dfg*).

19

$$\Delta V = V_0 - V_1 = HA - (H - S_c)A = S_c A$$

$$\Delta V = S_c A = V_{v_0} - V_{v_1} = \Delta V_v$$

$$\Delta V_v = \Delta e V_s$$

where Δe = change of void ratio. But

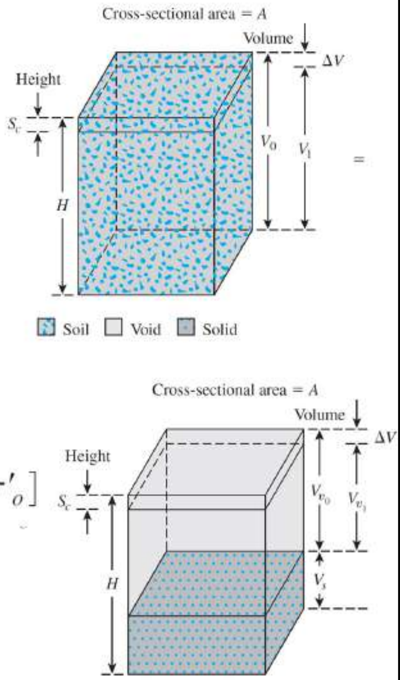
$$V_s = \frac{V_0}{1 + e_o} = \frac{AH}{1 + e_o}$$

$$\Delta V = S_c A = \Delta e V_s = \frac{AH}{1 + e_o} \Delta e$$

$$S_c = H \frac{\Delta e}{1 + e_o}$$

$$\Delta e = C_c[\log(\sigma'_o + \Delta\sigma') - \log \sigma'_o]$$

$$S_c = \frac{C_c H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$



11.10 Calculation of Settlement from One-Dimensional Primary Consolidation

- Normally consolidated clays: $\sigma'_o \approx \sigma'_c$ (OCR ≈1) exhibit a linear e-log σ' relationship

C_c = slope of the e–log σ' plot and is defined as the compression index.

σ'_o = Effective soil pressure at the middle of the layer.

$\Delta\sigma'$ = Increase in the effective pressure (load).

$$S_c = \frac{C_c H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$

H= Thickness of clay layer.

e_o =Initial void ratio (in the field).

$$C_c = 0.009(LL - 10)$$

11.10 Calculation of Settlement from One-Dimensional Primary Consolidation

- Overconsolidated clays:

C_c = slope of the e–log σ' plot and is defined as the compression index.

A) $\sigma'_o + \Delta\sigma' \leq \sigma'_c$

σ'_o = Effective soil pressure at the middle of the layer.

$$S_c = \frac{C_s H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$

σ'_c = maximum past pressure.

B) $\sigma'_o + \Delta\sigma' > \sigma'_c$

$\Delta\sigma'$ = Increase in the effective pressure (load).

H= Thickness of clay layer.

Cs= Swell index (slope of the rebound curve)

$$S_c = \frac{C_s H}{1 + e_o} \log\frac{\sigma'_c}{\sigma'_o} + \frac{C_c H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right)$$

$$C_s \simeq \tfrac{1}{5} \text{ to } \tfrac{1}{10} C_c$$

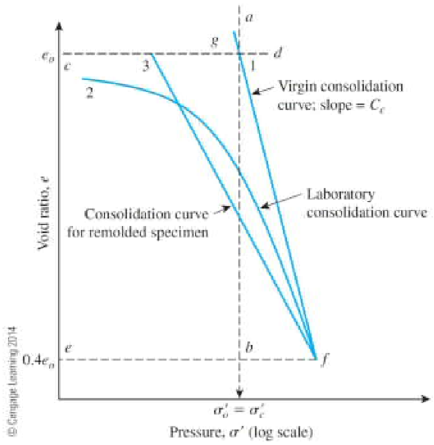


Figure 11.17 Consolidation characteristics of normally consolidated clay of low to medium sensitivity

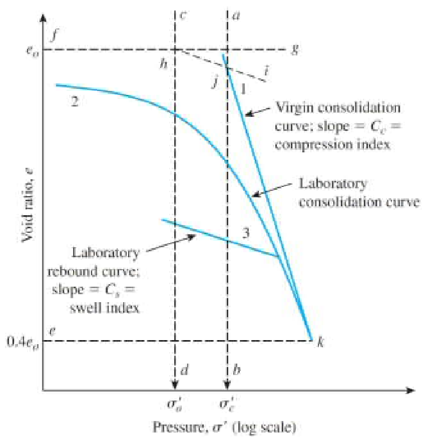


Figure 11.18 Consolidation characteristics of overconsolidated clay of low to medium sensitivity

Sensitivity (S_t) $S_t = \dfrac{q_{u(undisturbed)}}{q_{u(remolded)}}$

Where q_u = Unconfined Compressive Strength

11.13 Secondary Consolidation Settlement

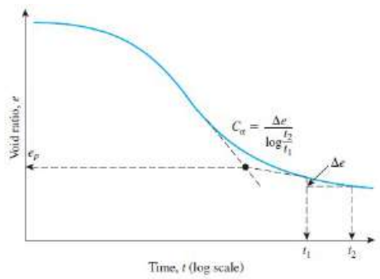
- Secondary consolidation is observed after the excess pore water pressure is completely dissipated.
- The plot of deformation against the logarithm of time is almost linear during secondary consolidation.
- The secondary compression index is:

$$C_\alpha = \frac{\Delta e}{\log t_2 - \log t_1} = \frac{\Delta e}{\log(t_2/t_1)}$$

where C_α = secondary compression index
Δe = change of void ratio
t_1, t_2 = time

$$S_s = C'_\alpha H \log\left(\frac{t_2}{t_1}\right)$$

$$C'_\alpha = \frac{C_\alpha}{1 + e_p}$$



e_p = void ratio at the end of primary consolidation
H = thickness of clay layer

11.13 Secondary Consolidation Settlement (Cont.)

The general magnitudes of C'_α as observed in various natural deposits

Overconsolidated clays = 0.001 or less
Normally consolidated clays = 0.005 to 0.03
Organic soil = 0.04 or more

- Secondary consolidation is more important than primary consolidation for organic and highly compressible inorganic soils.
- In Overconsolidated inorganic clays, the secondary compression index is very small and of less practical significance.

See Example 11.6

11.14 Time Rate of Consolidation

Terzaghi proposed a theory to predict the rate of consolidation for saturated clay soils, assuming:

- The system of clay and water is homogenous.
- The soil is completely saturated.
- Water is incompressible.
- The soil grains are incompressible but may rearrange.
- Water only flows in the direction of compression.
- Darcy's law is valid.

$$T_v = \frac{c_v t}{H_{dr}^2} = \text{time factor}$$

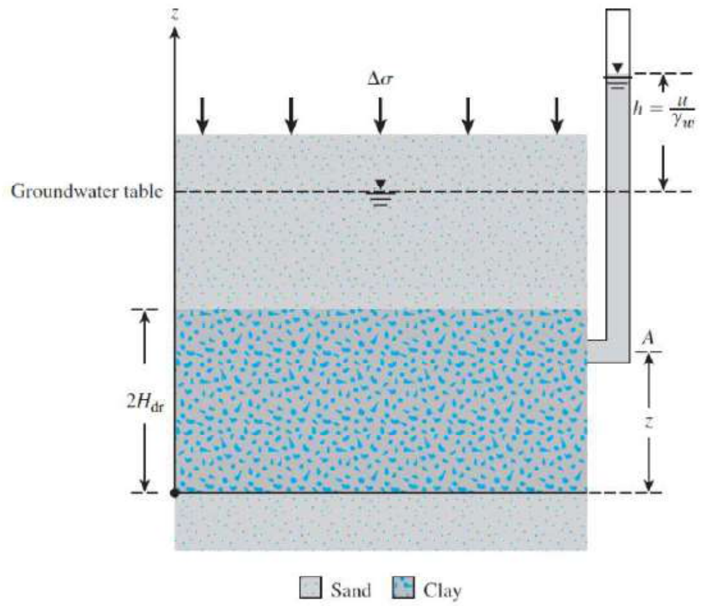
C_v= Coefficient of consolidation.
U= Degree of consolidation.
H_{dr}= is the max. drainage path (half the thickness of the clay layer).

The average degree of consolidation

Table 11.7 and Fig. 11.24 variation of T_v with U

$$U = \frac{S_{c(t)}}{S_c}$$

U = average degree of consolidation
Sc(t)= settlement of the layer at time *t*
Sc = ultimate settlement of the layer from primary



(a)

Table 11.7 Variation of T_v with U

U (%)	T_v	U (%)	T_v	U (%)	T_v	U (%)	T_v
0	0	26	0.0531	52	0.212	78	0.529
1	0.00008	27	0.0572	53	0.221	79	0.547
2	0.0003	28	0.0615	54	0.230	80	0.567
3	0.00071	29	0.0660	55	0.239	81	0.588
4	0.00126	30	0.0707	56	0.248	82	0.610
5	0.00196	31	0.0754	57	0.257	83	0.633
6	0.00283	32	0.0803	58	0.267	84	0.658
7	0.00385	33	0.0855	59	0.276	85	0.684
8	0.00502	34	0.0907	60	0.286	86	0.712
9	0.00636	35	0.0962	61	0.297	87	0.742
10	0.00785	36	0.102	62	0.307	88	0.774
11	0.0095	37	0.107	63	0.318	89	0.809
12	0.0113	38	0.113	64	0.329	90	0.848
13	0.0133	39	0.119	65	0.340	91	0.891
14	0.0154	40	0.126	66	0.352	92	0.938
15	0.0177	41	0.132	67	0.364	93	0.993
16	0.0201	42	0.138	68	0.377	94	1.055
17	0.0227	43	0.145	69	0.390	95	1.129
18	0.0254	44	0.152	70	0.403	96	1.219
19	0.0283	45	0.159	71	0.417	97	1.336
20	0.0314	46	0.166	72	0.431	98	1.500
21	0.0346	47	0.173	73	0.446	99	1.781
22	0.0380	48	0.181	74	0.461	100	∞
23	0.0415	49	0.188	75	0.477		
24	0.0452	50	0.197	76	0.493		
25	0.0491	51	0.204	77	0.511		

28

11.14 Time Rate of Consolidation (Cont.)

$$c_v = \frac{k}{\gamma_w m_v} = \frac{k}{\gamma_w \left(\frac{a_v}{1 + e_o} \right)}$$

m_v = coefficient of volume compressibility = $a_v/(1 + e_o)$
a_v = coefficient of compressibility (a_v can be considered constant for a narrow range of pressure increase)

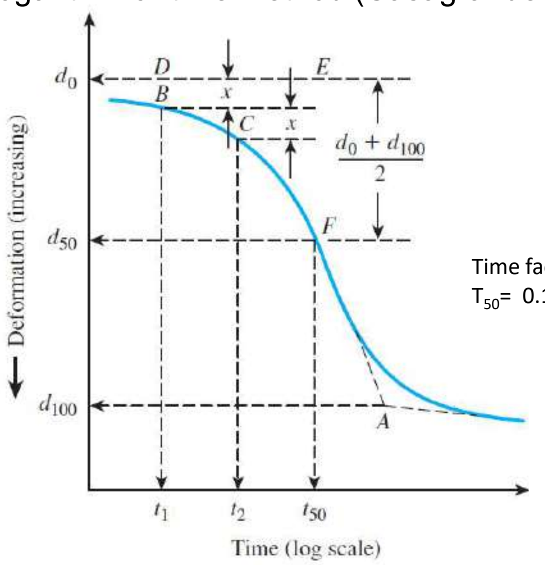
For U = 0 to 60%, $T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2$ → $\frac{c_v t_{\text{field}}}{H^2_{\text{dr(field)}}} = T_v \propto U^2$ → $t \propto U^2$, $\frac{t_1}{t_2} = \frac{U_1^2}{U_2^2}$

See Example 11.8, 9, and 10

For $U > 60\%$, $T_v = 1.781 - 0.933 \log(100 - U\%)$

11.15 Determination of Coefficient of Consolidation

logarithm-of-time method (Casagrande)



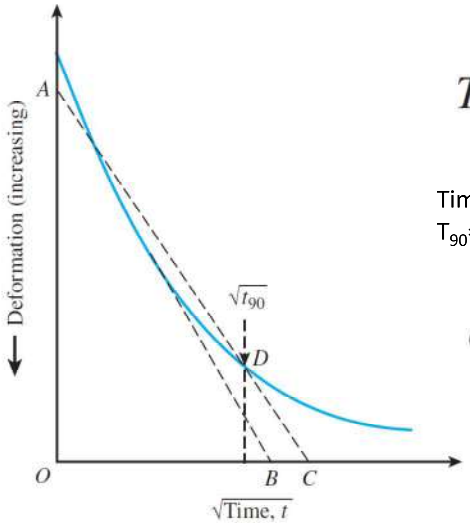
$$T_{50} = \frac{c_v t_{50}}{H^2_{dr}}$$

Time factor to reach 50% consolidation
T_{50}= 0.197 as shown in Table 11.7

$$c_v = \frac{0.197 H^2_{dr}}{t_{50}}$$

11.15 Determination of Coefficient of Consolidation

Square-root-of-time method (Taylor)



$$T_{90} = 0.848 = \frac{c_v t_{90}}{H_{dr}^2}$$

Time factor to reach 90% consolidation, T_{90}= 0.848 as shown in Table 11.7

$$c_v = \frac{0.848 H_{dr}^2}{t_{90}}$$

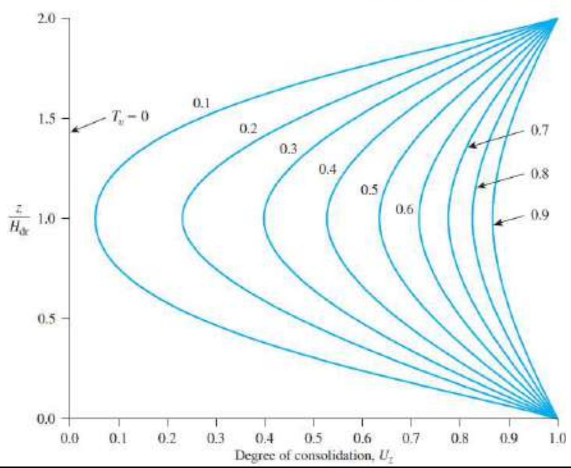
31

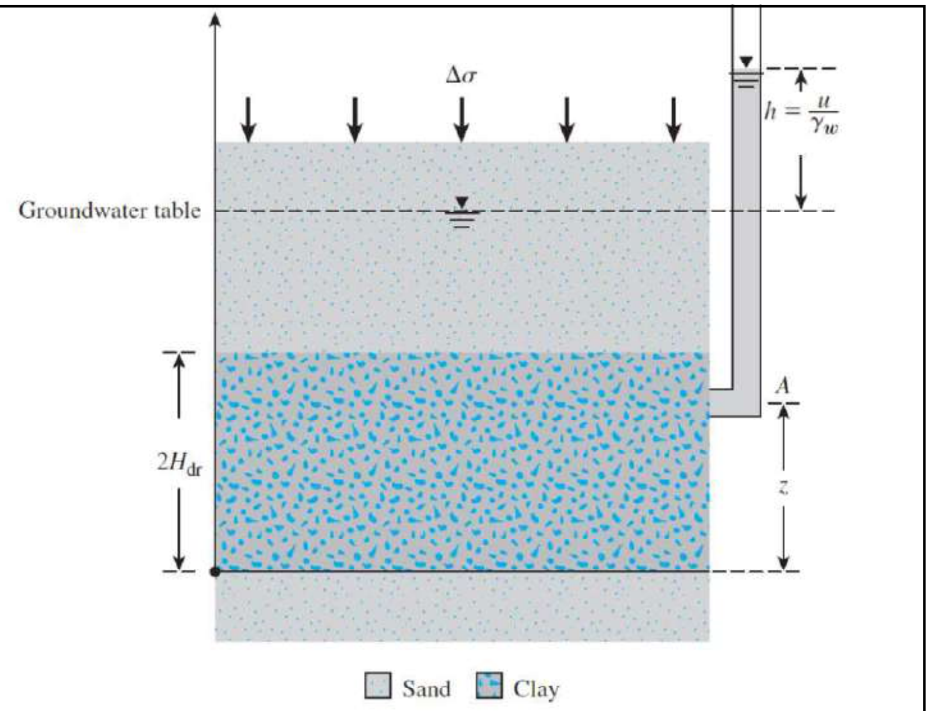
Because consolidation progresses by the dissipation of excess pore water pressure, the degree of consolidation at a distance *z* at any time *t* is

$$U_z = \frac{u_o - u_z}{u_o} = 1 - \frac{u_z}{u_o}$$

u_o =initial excess pore water pressure

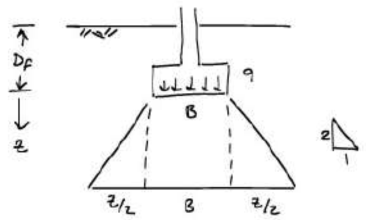
u_z =excess pore water pressure at time *t*.

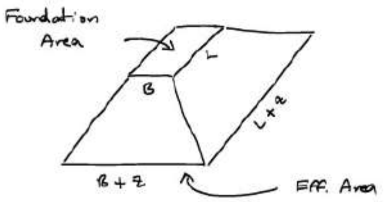




Vertical Stress increase Caused by Foundation Load using 2:1 method

- Rectangular Areas
  - 2:1 Method





$$\Delta\sigma_z = \frac{\text{load}}{\text{Eff. area}} = \frac{qBL}{(B+z)(L+z)}$$

34

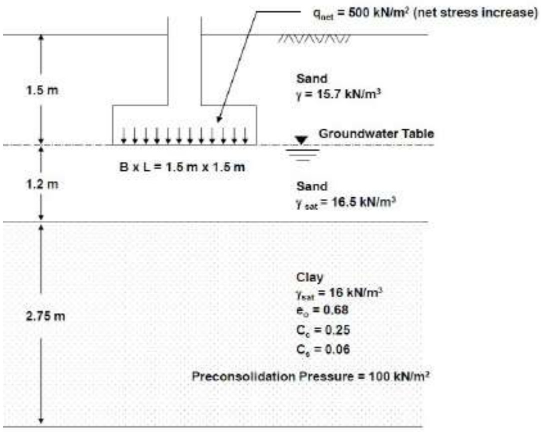
Practice Problem

a) Calculate the effective stress at the middle of the clay layer prior to the construction of the foundation.
(**40.1 kN/m^2**)

b) Determine whether the clay is normally consolidated or overconsolidated. **(Overconsolidated)**

c) Using the 2:1 method, determine the average increase in pressure in the clay layer below the center of the foundation. **(77.2 kN/m^2)**

d) Estimate the primary consolidation settlement under the foundation.
(67.3 mm)



e) If cv= 0.36 m^2/mo. Estimate the time needed for 70% settlement.

Example 11.14

Calculate the settlement of the 3-m-thick clay layer (Figure 11.33) that will result from the load carried by a 1.5-m-square footing. The clay is normally consolidated. Use the weighted average method [Eq. (11.68)] to calculate the average increase of effective pressure in the clay layer.

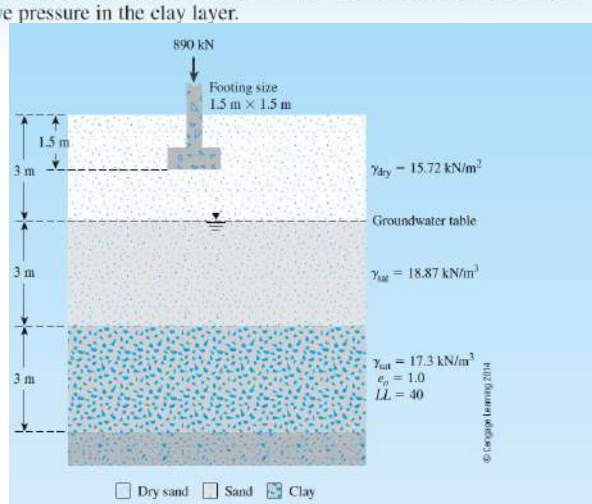


Figure 11.33

Solution

For normally consolidated clay, from Eq. (11.29),

$$S_c = \frac{C_c H}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o}$$

where

$$C_c = 0.009(LL - 10) = 0.009(40 - 10) = 0.27$$
$$H = 3 \times 1000 = 3000 \text{ mm}$$
$$e_o = 1.0$$
$$\sigma'_o = 3\text{ m} \times \gamma_{\text{dry(sand)}} + 3\text{ m}[\gamma_{\text{sat(sand)}} - 9.81] + \frac{3}{2}[\gamma_{\text{sat(clay)}} - 9.81]$$
$$= 3 \times 15.72 + 3(18.87 - 9.81) + 1.5(17.3 - 9.81)$$
$$= 85.58 \text{ kN/m}^2$$

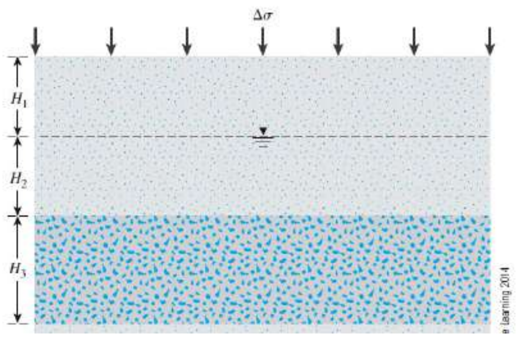
$L/B = 1.5/1.5 = 1)$

m_1	z (m)	$b = B/2$ (m)	$n_1 = z/b$	q (kN/m²)	I_4	$\Delta\sigma' = qI_4$ (kN/m²)
1	6.0	0.75	8	$\frac{890}{1.5 \times 1.5} = 395.6$	0.029	$11.47 = \Delta\sigma'_m$

$$S_c = \frac{(0.27)(3000)}{1 + 1} \log \frac{85.58 + 12.26}{85.58} \approx \mathbf{23.6\ mm}$$

11.9 Refer to Figure 11.44. Estimate the primary consolidation settlement in the clay layer. Given: $\Delta\sigma = 85$ kN/m²; $H_1 = 2$ m; $H_2 = 4$ m; and $H_3 = 6$ m. Soil characteristics are as follows:

- Sand: $e = 0.65$; $G_s = 2.66$
- Clay: $LL = 54$; $e = 0.98$; $G_s = 2.74$; $\sigma'_c = 150$ kN/m²; $C_s = \frac{1}{6} C_c$



11.9 $\gamma_{d(\text{sand})} = \frac{G_s\gamma_w}{1+e} = \frac{(2.66)(9.81)}{1+0.65} = 15.81\,\text{kN/m}^3$

$$\gamma'_{\text{sand}} = \frac{(G_s-1)\gamma_w}{1+e} = \frac{(2.66-1)(9.81)}{1.65} = 9.87\,\text{kN/m}^3$$

$$\gamma'_{\text{clay}} = \frac{(G_s-1)\gamma_w}{1+e} = \frac{(2.74-1)(9.81)}{1+0.98} = 8.62\,\text{kN/m}^3$$

$$\sigma'_o = (2)(15.81) + (4)(9.87) + \left(\frac{6}{2}\right)(8.62) = 96.96\,\text{kN/m}^2$$

$$C_c = 0.009(LL-10) = (0.009)(54-10) = 0.396$$

$$S_c = \frac{C_sH}{1+e_o}\log\left(\frac{\sigma'_c}{\sigma'_o}\right) + \frac{C_cH}{1+e_o}\log\left(\frac{\sigma'_o+\Delta\sigma'}{\sigma'_c}\right)$$

$$= \frac{\left(\frac{0.396}{6}\right)(6)}{1.98}\log\left(\frac{150}{96.96}\right) + \frac{(0.396)(6)}{1.98}\log\left(\frac{96.96+85}{150}\right)$$

$= 0.138\text{ m} \approx$ **13.8 cm**

11.13 Refer to Problem 11.9. How long will it take for 75% consolidation to be over in the field? Given: $c_v = 0.24\text{ cm}^2/\text{min}$.

11.13 $T_v = \frac{c_vt}{H_{dr}^2}$; $U = 75\%$; $T_v = 0.477$ (Table 11.7)

$$0.477 = \frac{(0.24\text{ cm}^2/\text{min})(t)}{\left(\frac{600}{2}\text{cm}\right)^2};\ t = 178{,}875\text{ min} = \mathbf{124.2\ days}$$

11.15 The time for 65% consolidation of a 19-mm clay specimen (drained at top and bottom) in the laboratory is 10 minutes. How long will it take for a 4-m-thick clay layer in the field to undergo 40% consolidation under the same pressure increment? In the field, there is a rock layer at the bottom of the clay.

11.15 In the laboratory:

$$T_{65} = \frac{c_vt_{65}}{H_{dr}^2} \qquad 0.304 = \frac{(c_v)(10\text{ min})}{\left(\frac{0.019}{2}\text{m}\right)^2};\ c_v = 2.74\times10^{-6}\text{ m}^2/\text{min}$$

In the field: $T_{40} = \frac{c_vt_{40}}{H_{dr}^2}$ $\quad U = 40\%$; $T_{40} = 0.126$ (Table 11.7)

$$0.126 = \frac{(2.74\times10^{-6}\text{ m}^2/\text{min})(t_{40})}{(4\text{ m})^2} \qquad t_{40} = 735{,}766\text{ min} = \mathbf{511\ days}$$

Shear Strength of Soil

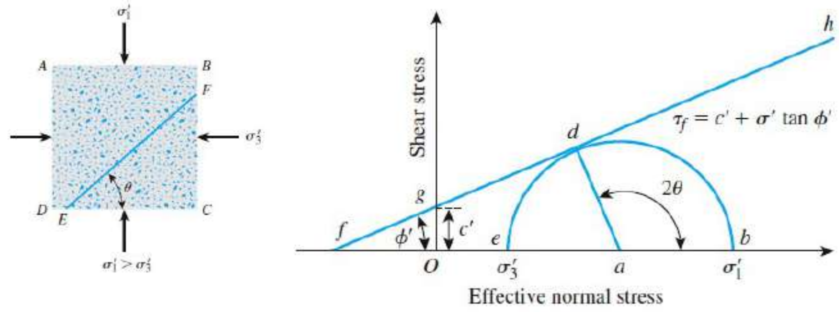
Introduction

- Strength of Different Materials:
  - **Steel**: Tensile strength.
  - **Concrete**: Compression strength.
  - **Soil**: Shear strength.

  Studying the soil shear strength in the presence of pore water pressure becomes more complicated.

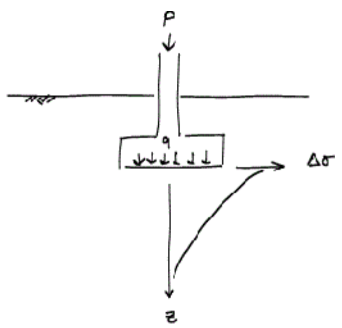
- Soils are essentially frictional materials. They are comprised of individual particles that can slide and roll relative to one another.

- ***Shear Strength*** of a soil mass is the internal resistance per unit area that the soil mass can offer to resist failure and sliding along any plane inside it.

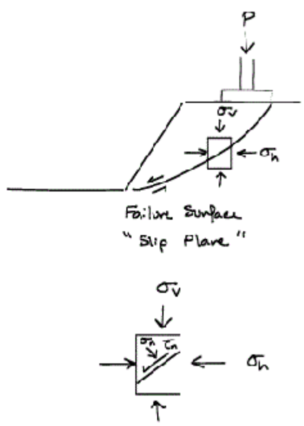


$$\sigma'_1 = \sigma'_3 \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

Introduction (Cont.)



Failure due to Shear Stresses



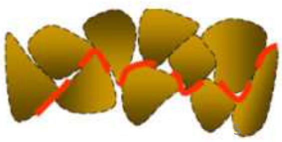
Increase load

→ Deformation may become very large

→ Failure

- Failure is influenced by shear stresses ⇒ Shear strength of soils

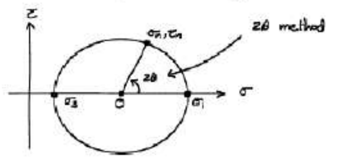
- Shear strength in soils depends primarily on interactions between particle.
- Shear failure occurs when the stresses between the particles are such that they slide or roll past each other.
- Soil derives its shear strength from two sources:
  - **Cohesion** between particles (stress independent component):
    - a measure of the forces that cement particles of soils
    - Cementation between sand grains
    - Electrostatic attraction between clay particles
  - **Frictional resistance between particles (stress dependent component):** is the measure of the shear strength of soils due to friction



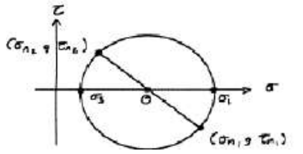
Introduction (Cont.)

Mohr Circle

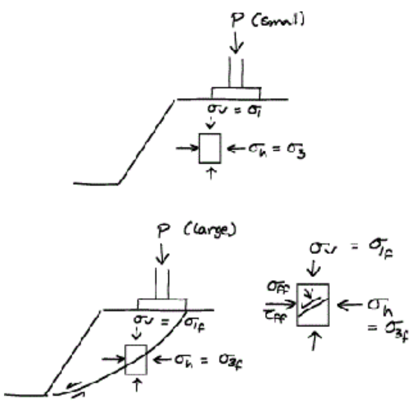
- Given σ_1 and σ_3, determine σ_n and τ_n at an angle θ.



- Given σ_n and τ_n at any two planes, find σ_1 and σ_3.



Mohr-Coulomb Failure Criteria



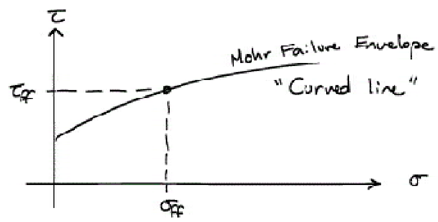
- What are the combination of σ_1 and σ_3 that cause failure?

6

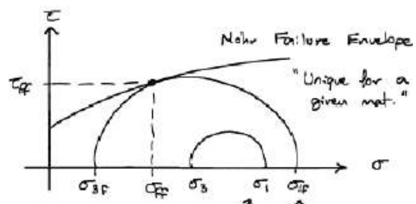
Introduction (Cont.)

Mohr - Coulomb Failure Criteria

- Mohr (1800's) based on many tests noticed that $\tau_{ff} = fn(\sigma_{ff})$

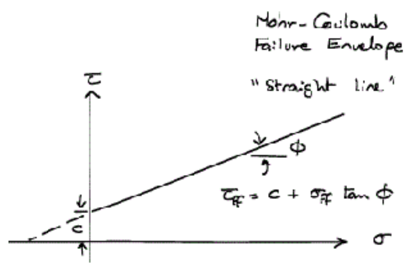


and that failure occurs when the combination of σ_1 and σ_3 form a Mohr circle that is tangent to the Mohr failure envelope.



Introduction (Cont.)

- Coulomb suggested using a straight line to define failure

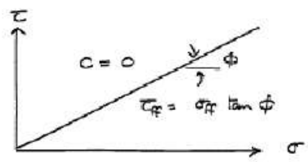


c = cohesion
"implies that soil can withstand tension"

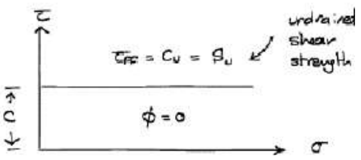
ϕ = angle of internal friction

Special cases:

- Cohesionless soils (e.g. sand)



- Saturated cohesive soils (e.g. clays) in unconsolidated-undrained conditions (later)



Introduction (Cont.)

$$\tau_f = c + \sigma \tan \phi$$

where c = cohesion
ϕ = angle of internal friction
σ = normal stress on the failure plane
τ_f = shear strength

Table 12.1 Typical Values of Drained Angle of Friction for Sands and Silts

Soil type	ϕ' (deg)
Sand: Rounded grains	
Loose	27–30
Medium	30–35
Dense	35–38
Sand: Angular grains	
Loose	30–35
Medium	35–40
Dense	40–45
Gravel with some sand	34–48
Silts	26–35

Introduction (Cont.)

$$\sigma = \sigma' + u$$

The effective stress σ' is carried by the soil solids. The Mohr–Coulomb failure criterion, expressed in terms of effective stress, will be of the form

$$\tau_f = c' + \sigma' \tan \phi'$$

where c' = cohesion and ϕ' = friction angle, based on effective stress.

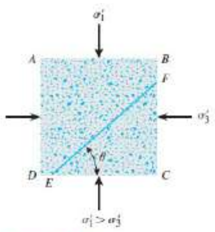


Figure 12.2 Inclination of failure plane in soil with major principal plane

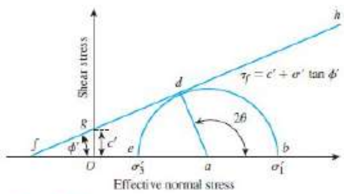
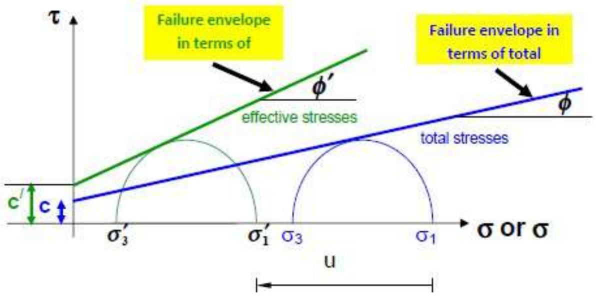


Figure 12.3 Mohr's circle and failure envelope

Introduction (Cont.)

Failure envelopes in terms of total & effective stresses



$$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

$$\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right) + 2c \tan\left(45 + \frac{\phi}{2}\right)$$

11

12.4 Laboratory Test for Determination of Shear Strength Parameters

Laboratory methods to determine the shear strength parameters of a soil specimen include:

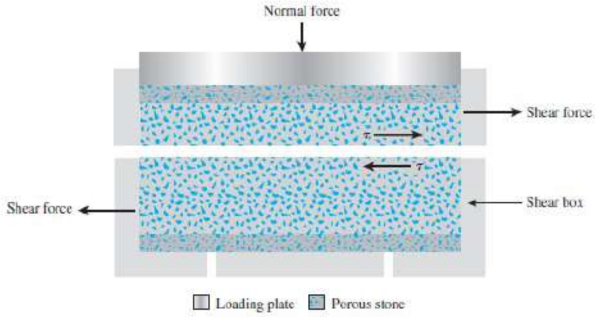
1. **The direct shear test**
2. **The triaxial test**
3. The direct simple shear test
4. The plane strain triaxial test
5. The torsional ring shear test

12

12.5 Direct Shear Test

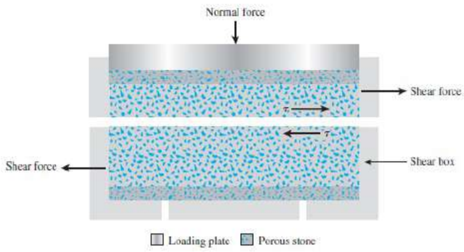
• Direct shear test is most suitable for consolidated drained tests specially on granular soils or stiff clay.
•The soil specimen is placed in a metal shear box that is split horizontally in halves.
• A normal force is applied to the top of the shear box.
• The specimen is sheared by moving one half of the box relative to the other.

• The test may be either stress-controlled or strain-controlled.



13

12.5 Direct Shear Test



•For a stress-controlled test, the shear force is applied in equal increments until the specimen fails.

• The displacement is measured at each load by a dial gauge.
• For a strain-controlled test, a constant rate of shear displacement is applied by a motor.
• The resisting shear force is measured by a load cell.

14

12.5 Direct Shear Test (Cont.)

For a given test, the normal stress can be calculated as

$$\sigma = \text{Normal stress} = \frac{\text{Normal force}}{\text{Cross-sectional area of the specimen}}$$

The resisting shear stress for any shear displacement can be calculated as

$$\tau = \text{Shear stress} = \frac{\text{Resisting shear force}}{\text{Cross-sectional area of the specimen}}$$

regarding the variation of resisting shear stress with shear displacement:

1. In loose sand, the resisting shear stress increases with shear displacement until a failure shear stress of τ_f is reached. After that, the shear resistance remains approximately constant for any further increase in the shear displacement.
2. In dense sand, the resisting shear stress increases with shear displacement until it reaches a failure stress of τ_f. This τ_f is called the *peak shear strength*. After failure stress is attained, the resisting shear stress gradually decreases as shear displacement increases until it finally reaches a constant value called the *ultimate shear strength*.

15

12.5 Direct Shear Test (Cont.)

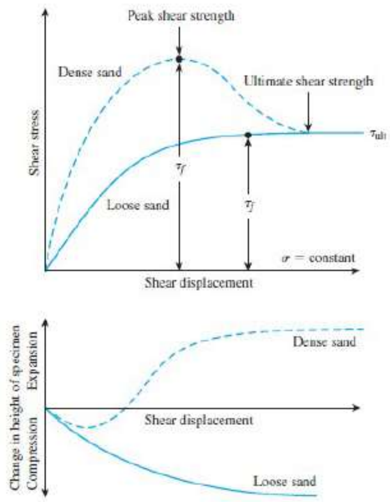


Figure 12.7 Plot of shear stress and change in height of specimen against shear displacement for loose and dense dry sand (direct shear test)

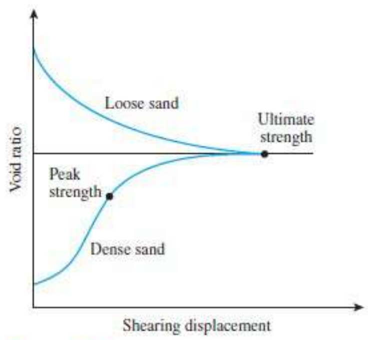


Figure 12.8 Nature of variation of void ratio with shearing displacement

- The void ratio changes as the specimen is displaced.
- At large displacements, the void ratio is the same for both dense and loose sand.
- This value is the *critical void ratio*.

16

12.5 Direct Shear Test (Cont.)

For dry sand

$\sigma = \sigma'$

$c' = 0$

$$\tau_f = \sigma' \tan \phi'$$

$$\phi' = \tan^{-1}\left(\frac{\tau_f}{\sigma'}\right)$$

$$\tau_{ult} = \sigma' \tan \phi'_{ult}$$

$$\phi'_{ult} = \tan^{-1}\left(\frac{\tau_{ult}}{\sigma'}\right)$$

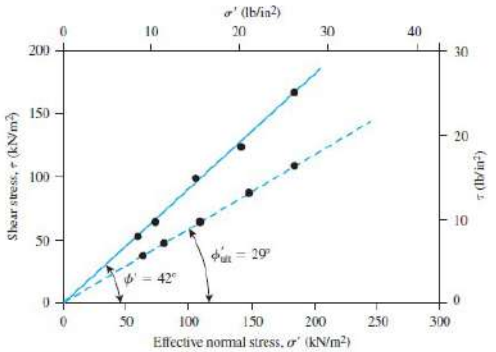


Figure 12.9 Determination of shear strength parameters for a dry sand using the results of direct shear tests

17

12.6 Drained Direct Shear Test on Saturated Sand and Clay

- A *drained test* is made on saturated soil.
- The shear box that contains the soil specimen is kept inside a container filled with water to saturate the specimen.
- The rate of loading is kept slow enough to completely dissipate the excess pore water pressure.
- Sand has a high hydraulic conductivity, so ordinary loading rates allow for essentially complete drainage.
- Clay requires a very slow loading rate due to its low hydraulic

Similar to the ultimate shear strength in the case of sand (Figure 12.8), at large shearing displacements, we can obtain the *residual shear strength* of clay (τ_r) in a drained test. This is shown in Figure 12.10. Figure 12.11 shows the plot of τ_r versus σ'. The average plot *will pass through the origin* and can be expressed as

$$\tau_r = \sigma' \tan \phi'_r$$

18

12.6 Drained Direct Shear Test on Saturated Sand and Clay (Cont.)

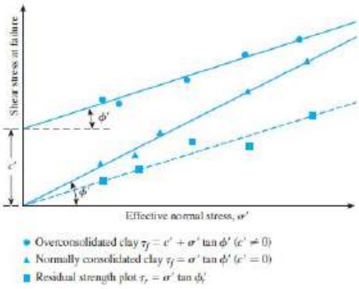


Figure 12.11
Failure envelope for clay obtained from drained direct shear tests

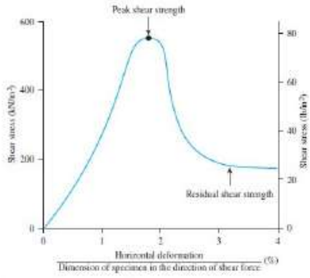


Figure 12.10 Results of a drained direct shear test on an overconsolidated clay [*Note:* Residual shear strength in clay is similar to ultimate shear strength in sand (see Figure 12.7)]

19

12.7 General Comments on Direct Shear Test

- The soil is not allowed to fail on its weakest plane.
- Instead, it must fail along the split of the shear box.
- The shear stress is not distributed uniformly over the shear surface of the specimen.
- Despite this, the direct shear test is the simplest and cheapest test for dry or saturated sandy soil.
- The direct shear test may be used to determine interface friction.
- This is useful to determine the angle of friction between the soil and a foundation material.

See Example 12. 1 and 12.2

20

Example 13.9

A series of three direct shear tests has been conducted on a certain saturated soil. Each test was performed on a 2.375 inch diameter, 1.00 inch tall sample. The test has been performed slowly enough to produce drained conditions. The results of these tests are as follows:

Test Number	Normal Load (lb)	Shear Load at Failure (lb)
1	75	51
2	150	110
3	225	141

Determine c' and ϕ'.

Solution

$$A = \frac{\pi D^2}{4} = \frac{\pi (2.375\ \text{in})^2}{4}\left(\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right) = 0.0308\ \text{ft}^2$$

Based on this area and the measured forces:

Test Number	σ' (lb/ft²)	τ (lb/ft²)
1	2438	1665
2	4876	3576
3	7314	4545

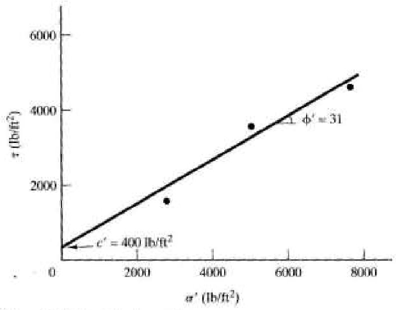


Figure 13.26 Direct shear test results.

This data is plotted in Figure 13.26. It does not form a perfect line. This is due to experimental error, slight differences in the three samples, true nonlinearity, and other factors. We have drawn a best-fit line through these three points to obtain $c' = 400$ lb/ft² and $\phi' = 31°$.

(Coduto, 1999)

12.1 a. $c' = 0$. From Eq. (12.3): $\tau_f = \sigma' \tan \phi'$

$$\tau = \frac{300}{(1000)(0.063)^2} = 75\ \text{kN/m}^2$$

So, $75 = 105 \tan \phi'$

$$\phi' = \tan^{-1}\left(\frac{75}{105}\right) = \mathbf{35.5°}$$

b. For $\sigma' = 180$ kN/m², $\tau_f = 180 \tan 35.5° = 128.39$ kN/m²

Shear force, $S = (128.39)(1000)(0.063)^2 = \mathbf{509.5\ N}$

22

12.8 Triaxial Shear Test-General

- The specimen is enclosed in a thin rubber membrane.
- The specimen is then placed in a chamber confined by compressing the fluid in the chamber
- Axial stress is applied through a vertical ram

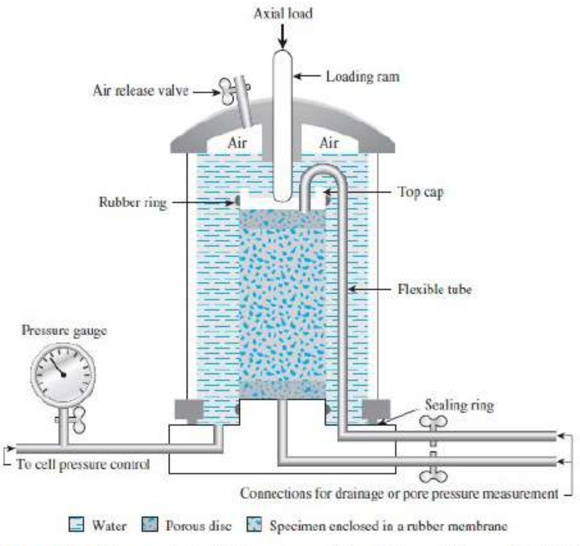
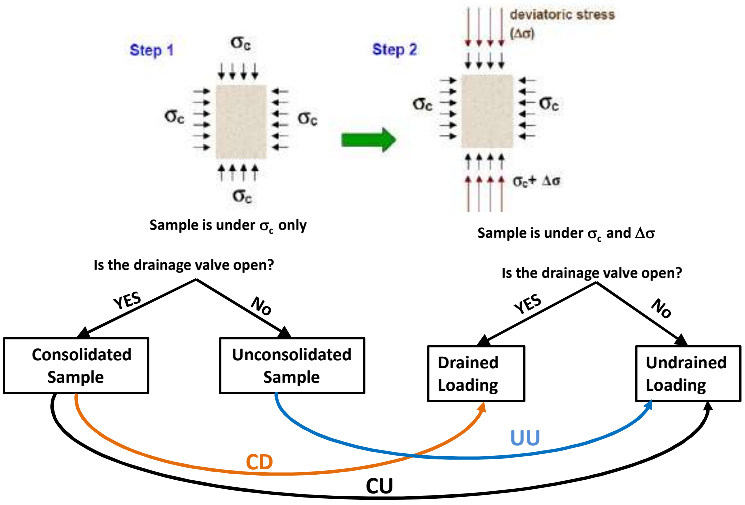


Figure 12.19 Diagram of triaxial test equipment (*After Bishop and Bjerrum, 1960. With permission from ASCE.*)

23

12.8 Triaxial Shear Test-General (Cont.)



σ_C = confining pressure or all around pressure or cell pressure = σ_3

$\Delta\sigma$ = deviatoric stress = $\sigma_1 - \sigma_3$

24

12.9 Consolidated-Drained (CD) Triaxial Test

- The confining pressure leads to an increase of the pore water pressure of the specimen, U_c.

- **B** is Skempton's pore pressure parameter, and expresses the pore water pressure increase nondimensionally:

$$B = \frac{u_c}{\sigma_3}$$

- B is approximately equal to 1.0 for saturated soft soils.

- A connection for drainage is opened, allowing U_c to fall to "0" over time.

- For saturated soil, the change in volume during onsolidation may be determined from the volume of water drained.

25

12.9 CD Test (Cont.)

- The pore water pressure developed during the test is completely dissipated, so:

$\sigma_3 = \sigma'_3$

- At failure, the total and effective axial stress will be:

$\sigma_3 + (\Delta\sigma_d)_f = \sigma_1 = \sigma'_1$

In a triaxial test, σ'_1 is the major principal effective stress at failure and σ'_3 is the minor principal effective stress at failure.

$$\sin\phi' = \frac{\left(\frac{\sigma'_1 - \sigma'_3}{2}\right)}{\left(\frac{\sigma'_1 + \sigma'_3}{2}\right)}$$

$$\phi' = \sin^{-1}\left(\frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3}\right)$$

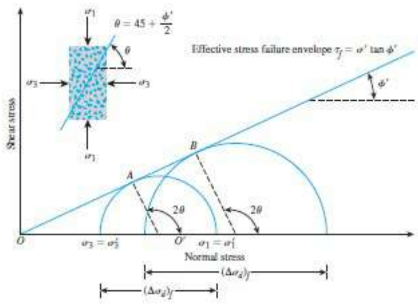


Figure 12.22 Effective stress failure envelope from drained tests on sand and normally consolidated clay

26

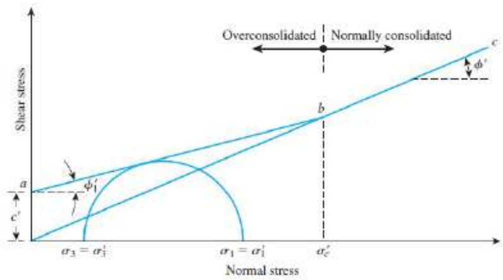
12.9 CD Test (Cont.)

-Overconsolidation results when a clay initially is consolidated under an all-around chamber pressure of σ_c (=σ'_c) and is allowed to swell by reducing the chamber pressure to σ_3 (σ'_3).

- The failure envelope obtained from drained triaxial tests of such overconsolidated clay specimens shows two distinct branches.

- The portion ***ab*** *has a flatter slope with a cohesion intercept, and the shear strength equation for* this branch can be written as

$$\tau_f = c' + \sigma' \tan \phi'_1$$



12.9 CD Test (Cont.)

- The portion ***bc*** *of the failure envelope represents a normally consolidated stage of soil and* follows the equation $\tau f = \sigma' \tan \phi'$.

$$c' = \frac{\sigma'_{1(1)} - \sigma'_{3(1)} \tan^2\left(45 + \frac{\phi'_1}{2}\right)}{2 \tan\left(45 + \frac{\phi'_1}{2}\right)}$$

- CD test simulates the long term condition in the field.

Example 12.3, 12.4, 12.5, and 12.6

12.10 Consolidated-Undrained Triaxial Test

- The saturated soil specimen is first consolidated by an all-around chamber fluid pressure, σ_3.

- Pore water pressure is equal to zero at the end of the consolidation because the valve is open.

- After dissipation of the pore water pressure, the deviator stress is increased on the sample with closed valve to cause failure.

- Because drainage is not permitted, the pore water pressure, Δu_d *will increase.*

$\overline{A}$ – Skempton's pore pressure parameter (Skempton, 1954).

$$\overline{A} = \frac{\Delta u_d}{\Delta \sigma_d}$$

- During the test, simultaneous measurements of $\Delta\sigma_d$ *and* ΔU_d *are made*

29

12.10 Consolidated-Undrained Triaxial Test (Cont.)

- Major principal stress at failure (total): $\sigma_3 + (\Delta\sigma_d)_f = \sigma_1$
- Major principal stress at failure (effective): $\sigma_1 - (\Delta u_d)_f = \sigma'_1$
- Minor principal stress at failure (total): σ_3
- Minor principal stress at failure (effective): $\sigma_3 - (\Delta u_d)_f = \sigma'_3$

In these equations, $(\Delta u_d)_f$ = pore water pressure at failure. The preceding derivations show that

$$\sigma_1 - \sigma_3 = \sigma'_1 - \sigma'_3$$

$$\tau_f = \sigma \tan \phi$$

where σ = total stress
ϕ = the angle that the total stress failure envelope makes with the normal stress axis, also known as the *consolidated-undrained angle of shearing resistance*

$$\phi = \sin^{-1}\left(\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}\right)$$

Example 12.7

$$\phi' = \sin^{-1}\left(\frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3}\right)$$

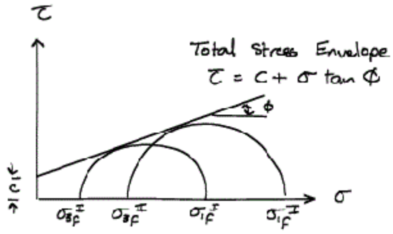
30

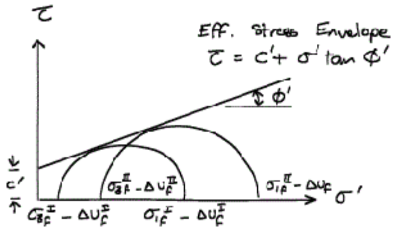
12.10 Consolidated-Undrained Triaxial Test (Cont.)

Consolidated - Undrained Triaxial Test

$\Delta u = 0$ at the end of consolidation

$\Delta u \neq 0$ during shearing ($\sigma' \neq \sigma$)





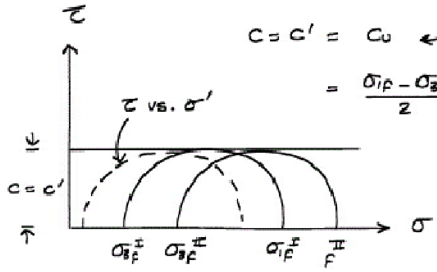
12.11 Unconsolidated-Undrained Test

Unconsolidated - Undrained Triaxial Test

undrained shear strength

$c = c' = c_u$

$= \dfrac{\sigma_{1f} - \sigma_{3f}}{2}$



(Fastest , most conservative

no increase in strength with

increase in confining pressure)

12.11 Unconsolidated-Undrained Test (Cont.)

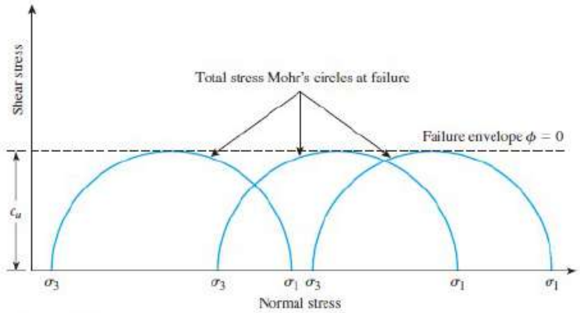


Figure 12.31 Total stress Mohr's circles and failure envelope ($\phi = 0$) obtained from unconsolidated-undrained triaxial tests on fully saturated cohesive soil

$$\tau_f = c = c_u$$

where c_u is the undrained shear strength and is equal to the radius of the Mohr's circles. Note that the $\phi = 0$ concept is applicable to only saturated clays and silts.

12.12 Unconfined Compression Test on Saturated Clay

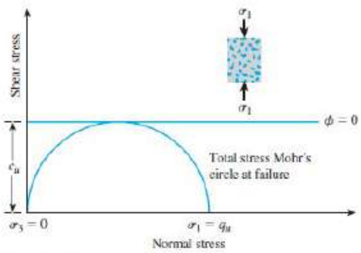


Figure 12.33 Unconfined compression test

Table 12.4 General Relationship of Consistency and Unconfined Compression Strength of Clays

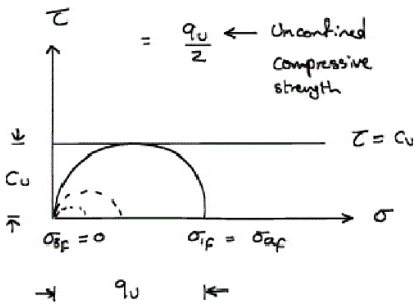
	q_u	
Consistency	kN/m²	ton/ft²
Very soft	0–25	0–0.25
Soft	25–50	0.25–0.5
Medium	50–100	0.5–1
Stiff	100–200	1–2
Very stiff	200–400	2–4
Hard	>400	>4

$\phi = 0$

$c = c_u$ ← undrained shear strength

$= \dfrac{\sigma_{1f} - \cancel{\sigma_{3f}}}{2}$ zero

$= \dfrac{q_u}{2}$ ← Unconfined compressive strength



Summary

Summary

CD — c, ϕ / $c' = c$, $\phi' = \phi$ — long-term

CU — c, ϕ / c', ϕ' — short-term

UU — $c = 0, \phi$ / $c' = c = 0$, $\phi' = \phi$ — most conservative

Practice Problems

For a direct shear test on a dry sand, the following are given:

- Specimen size: 75 mm × 75 mm × 30 mm (height)
- Normal stress: 200 kN/m^2
- Shear stress at failure: 175 kN/m^2

a. Determine the angle of friction, ϕ'

b. For a normal stress of 150 kN/m^2, what shear force is required to cause failure in the specimen?

$$\tau_f = \sigma' \tan \phi'$$

a. $\phi' = 41.2^\circ$

b. $\tau_f = 131.3$ kN/m^2

Shear force= 0.74 kN

A sandy soil has a drained angle of friction of 38°. In a drained triaxial test on the same soil, the deviator stress at failure is 175 kN/m^2. What is the chamber-confining pressure?

$$c' = 0. \quad \sigma_1' = \sigma_3' + \Delta\sigma_{d(f)} = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$$

The shear strength of a normally consolidated clay can be given by the equation $\tau_f = \sigma' \tan 31^\circ$. A consolidated-undrained triaxial test was conducted on the clay. Following are the results of the test:

- Chamber confining pressure = 112 kN/m^2
- Deviator stress at failure = 100 kN/m^2

Determine:

a. Consolidated-undrained friction angle

b. Pore water pressure developed in the clay specimen at failure

$$\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right)$$

$$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\frac{\sigma_1 - \Delta u_{d(f)}}{\sigma_3 - \Delta u_{d(f)}} = \tan^2\left(45 + \frac{31}{2}\right) = 3.124$$

12.5 Draw a graph for shear stress at failure against the normal stress and determine the drained angle of friction from the graph. Given: Specimen diameter = 50 mm; specimen height = 25 mm.

Test no.	Normal force (N)	Shear force at failure (N)
1	250	139
2	375	209
3	450	250
4	540	300

12.6 Consider the clay soil in Problem 12.5. If a drained triaxial test is conducted on the same soil with a chamber confining pressure of 208 kN/m^2, what would be the deviator stress at failure?

12.7 For the triaxial test on the clay specimen in Problem 12.6,

a. What is the inclination of the failure plane with the major principal plane?

b. Determine the normal and shear stress on a plane inclined at 30° with the major principal plane at failure. Also explain why the specimen did not fail along this plane.

12.8 The relationship between the relative density, D_r, and the angle of friction, ϕ', of a sand can be given as $\phi' = 28 + 0.18D_r$ (D_r in %). A drained triaxial test was conducted on the same sand with a chamber-confining pressure of 150 kN/m^2. The sand sample was prepared at a relative density of 68%. Calculate the major principal stress at failire.

37

12.5 Area of specimen $A = \left(\frac{\pi}{4}\right)(0.05)^2 = 0.00196\,\text{m}^2$

Test No.	Normal force N (N)	$\sigma' = \frac{N}{A}$ (N/m^2)	Shear force S (N)	$\tau_f = \frac{S}{A}$ (N/m^2)	$\phi' = \tan^{-1}\left(\frac{\tau_f}{\sigma'}\right)$ (deg)
1	250	79.6	139	44.26	29.07
2	375	119.4	209	66.56	29.13
3	450	143.3	250	79.61	29.05
4	540	171.9	300	95.54	29.06

A graph of τ_f vs. σ' will yield $\phi' \approx$ **29°**.

12.6 $c' = 0$. From Eq. (12.8): $\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$; $\phi' = 30°$

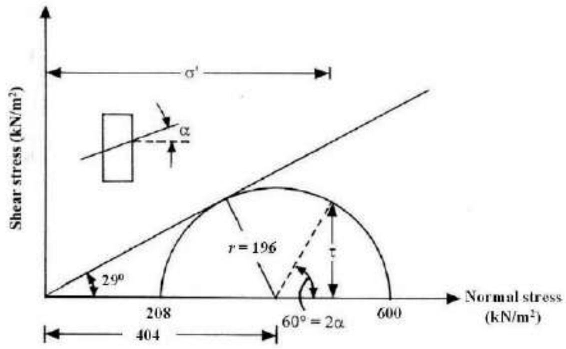
$$\sigma_1' = 208\tan^2\left(45 + \frac{29}{2}\right) \approx 600\ \text{kN/m}^2$$

$$\Delta\sigma_{d(\text{failure})} = \sigma_1' - \sigma_3' = 600 - 208 = \mathbf{392\ kN/m^2}$$

12.7 a. From Eq. (12.4): $\theta = 45 + \frac{\phi'}{2} = 45 + \frac{29}{2} = \mathbf{59.5°}$

b. Refer to the figure.

38



$\tau = 196 \sin 60° = \mathbf{169.7\ kN/m^2}$

$\sigma' = 404 + r\cos 60 = 404 + 196\cos 60 = \mathbf{502\ kN/m^2}$

For failure, $\tau_f = \sigma' \tan \phi' = 502 \tan 29 = 278.26\ kN/m^2$. Since the developed shear stress = $169.5\ kN/m^2$ (which is less than $278.26\ kN/m^2$), the specimen did not fail along this plane.

12.8 $\phi' = 28 + 0.18D_r = 28 + (0.18)(68) = 40.24°$

$$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right) = 150\tan^2\left(45 + \frac{40.24}{2}\right) = \mathbf{697.43\ kN/m^2}$$