

* Structural Analysis *

Dr-Ra'ed AL-Hazaidh

Chapter 1: Types of structures and loads

* What is structural Analysis?!

it is system of connected parts used to support a load
 النظام المتصل الأجزاء المستخدمة لدعم الأحمال.

* Examples related to civil engineering :-

- | | | | |
|--------------|---------|------------|--------|
| 1- Buildings | المباني | 3- Bridges | الجسور |
| 2- Towers | الأبراج | * * | |

* The structure is not related to civil engineering, it related in other Branches of engineering

- | | |
|------------------------------------|------------------------|
| → Ship and aircraft frames | إطارات السفن والطائرات |
| → Tanks & Pressure vessels | الخزانات وأوعية الضغط |
| → Mechanical systems | الأنظمة الميكانيكية |
| → Electrical supporting structures | الهيكل الداعم للتمهات |

* The design of a structure involves many considerations, among which are four major objectives that must be satisfied

- 1- The structure must meet the performance requirement (utility)
 - 2- The structure must carry loads safely (Safety)
 - 3- The structure should be economical in material and cost (economy)
 - 4- The structure should have a good appearance (aesthetics)
- يجب تحقيق النفع التي من أجلها يكون آمن و اقتصادي ، يكون بطرق جيدة .

* يكون دور المهندس المعماري باقتراح التصميم ومن ثم يتولى دور المهندس المدني الذي يقوم بتحليل الهيكل للتأكد من أنه يتبع بالصلابة والقوة المطلوبة (stiffness and strength)

* Classification of structures.

□ structural elements → العنصر الإنشائي

1- Tie rods → Tension

(compression % 90 , Tension % 10) ← Concrete * صلب

لأنه لا يتحمل الضغط مستوي

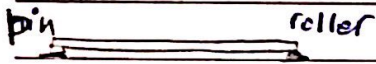


rod bar angle channel

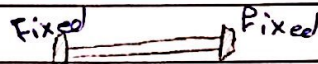
2- Beams: → vertical loads

horizontal members ($\theta = 0$ with horizontal direction)

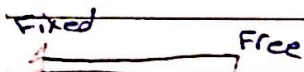
and they are classified according to the way they are supported



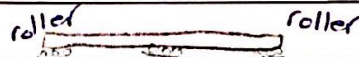
Simply supported beam (تدوير الاستناد)



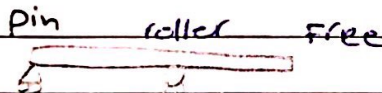
Fixed-supported beam (كثيرة قوى)



cantilevered beam



continuous beam



overhanging beams

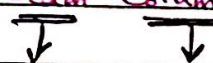
* ملاحظة *

① Shear & Bending & internal Forces → Beams

② Beams لا تتحمل القص

3- columns: → axial compressive loads

95% of columns → axial and Bending → Beam columns.



Moment compression

* لا يتحمل القليل بدون محدد

Loads

* بعد القليل تحلل على:

Force s

① displacement

* كيد الأحمال

codes and local specification

2

* Types of structures :-1- Trusses :- Axial Force (T, C)

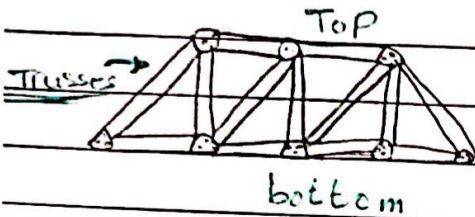
"إلخ الوصلات"

* لا يتحمل Moment لذلك يتركز الإجهاد المحوري

عن مساحة مشتركة بين الجوار (large spans)

* دقات وجود الأعمدة

* تتكون من ضغط (Compression) وشد (Tension) فقط

From 30ft (9m) $\rightarrow$ 400ft (122m)From support to support
 $\rightarrow$ chord.2- Cables & ArchesCables $\rightarrow$ only TensionArches $\rightarrow$ only compression3- Frames $\rightarrow$ composed of beams and columns.Bending, Normal, shear $\leftarrow$ cross section

* Frames member are subjected to internal axial, shear and moment loadings.

4- Surface structures $\rightarrow$ (Thin plates or shells)

* Codes interested (loads, design).

* There are 2 types of codes:

1- General

 $\rightarrow$ American society of civil engineering (ASCE) $\rightarrow$ International Building code (IBC)

2- Design codes (structural codes)

① - AASHTO $\rightarrow$ highway bridges

2 - AREMA $\rightarrow$ railroad bridges

② - ACI 318 $\rightarrow$ concrete structures

④ - AISC $\rightarrow$ steel structures

5 - AFPA $\rightarrow$ wood structures

(1,3,4) use in Jordan.



*Types of Loads:-

- 1- Dead load 2- Live load

*Dead Load:-

Examples of dead load:- columns, beams and girders, the floor slab, roofing, walls, windows, Plumbing, electrical fixtures

→ We know the value about Dead load from ASCE 7-16

القيمة التي يجب استخدامها

Example 1:- A three-ply asphalt felt and gravel roof over 2-in-thick insulation board is supported by 18-in-deep precast reinforced concrete beams with 3-ft-wide flanges (see figure 2-2). If the insulation weighs 3 lb/ft^2 and the asphalt roofing weighs $5 \frac{1}{2} \text{ lb/ft}^2$, determine the total dead load per foot of length each beam must support.

Solution →

$$1\text{-Flang} \rightarrow \frac{3' \times 4''}{12''} \times 150 \frac{\text{lb}}{\text{ft}^3} = 150 \text{ lb/ft}$$

$$2\text{-Stem} \rightarrow \frac{14}{12} \times \frac{10}{12} \times 150 \text{ lb/ft}^3 = 145.8 \text{ lb/ft}$$

* 10 in في 14 in
تقسم على 12

$$3\text{-Insulation} \rightarrow \frac{3 \text{ lb}}{\text{ft}^2} \times 3 \text{ ft} \times 1 \text{ ft} = 9 \text{ lb/ft}$$

$$4\text{-Asphalt} \rightarrow \frac{5.5 \text{ lb}}{\text{ft}^2} \times 3' \times 1' = 16.5 \text{ lb/ft}$$

$$\text{*Total weight} = 150 + 145.8 + 9 + 16.5 = 321.3 \text{ lb/ft} \text{ or } 0.321 \text{ k/ft}$$

لو قطعنا طول متر وعرضه 36 يكون الوزن 321.3

* Example 2:- The floor beam in the figure used to support the 6-ft width of a light weight plain concrete slab having a thickness of 4 in. The slab serves as a portion of the ceiling for the floor below, and therefore its bottom is coated with plaster. Furthermore, an 8-ft-high, 12-in-thick light weight solid concrete block wall is directly over the top flange of the beam. Determine the loading on the beam measured per foot of length of the beam.

solution →

1 * concrete slab light weight Plain Concrete → $\gamma = 8 \text{ lb/ft}^3$

→ From ASCE 7-16 Table C3.1-1a → page (426)

$$\text{load per foot} = \frac{8 \text{ lb}}{\text{ft}^3} \times 4' \times 6' = \boxed{192 \text{ lb/ft}}$$

2 * Plaster ceiling

$$\text{Plaster on tile or concrete} = 5 \text{ lb/ft}^2$$

→ From ASCE 7-16, Table C3.1-1a → page (426)

$$\text{load per foot} = \frac{5 \text{ lb}}{\text{ft}^2} \times 6' \times 1' = \boxed{30 \text{ lb/ft}}$$

3 * Block wall

$$\text{Masonry, concrete light weight unit (Density)} = \frac{105}{150} \text{ lb/ft}^3$$

→ From Table C3.1-1a → ASCE 7-16 → page (430)

$$\Rightarrow \text{Block load per foot} = \frac{105 \text{ lb}}{\text{ft}^3} \times 8' \times \frac{12''}{12''} = \boxed{840 \text{ lb/ft}}$$

$$\Rightarrow \text{Total} = 192 + 30 + 840$$

$$= 1062 \text{ lb/ft}$$

1062 lb/ft هو الحمل الكلي

* Live Loads :- → occupancy etc

→ The minimum live loads for which the floors and roof of a building should be designed are usually specified in the building code

* Live Loads $\Rightarrow$ ASCE 7-16 (غير ثابتة في الموقع ولا القيمة)

example: office use = 50 (2.40)
uniformly dist (KN/m²)

$\rightarrow$ Live load Reduction

$$* L = L_o \left(0.25 + \frac{15}{\sqrt{K_{LL} A_T}} \right) \quad \text{U.S. Customary units}$$

$$* L = L_o \left(0.25 + \frac{4.57}{\sqrt{K_{LL} A_T}} \right) \quad \text{SI units.}$$

L reduced design live load per square foot or square meter of area supported by the member

L_o unreduced design live load per square foot or square meter of area supported by the member

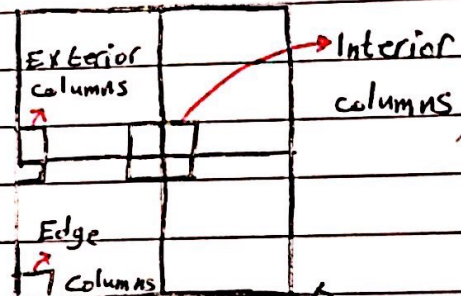
K_{LL} Live Load element factor. For interior columns $K_{LL} = 4$. (Table 4.7-1 ASCE 7-16 pp. 17)

A_T tributary area in square feet or square meters

* ASCE 7-16 allows Live reduction $\Rightarrow$

$$1 - K_{LL} + A_T \geq 400 \text{ ft}^2 (37.2 \text{ m}^2)$$

$$2 - L \geq \begin{cases} 50\% L_o & \text{For members supporting one floor} \\ 40\% L_o & \text{For members supporting more than one floor} \end{cases}$$



جزء من مبنى (مخزن)

3 - No reduction is allowed:-

$$\text{if } L_o \geq 100 \text{ lb/ft}^2 (24.79 \text{ kN/m}^2)$$

or if The structures used for Public assembly, garages, or roofs.

* Roof Live Load reduction for ordinary flat, pitched, and curved roofs $\rightarrow$

The minimum uniformly distributed roof live loads are permitted to be reduced by ASCE standard as follows:

$$L_r = L_o R_1 R_2$$

Where $\rightarrow$ L_o = design roof live load

L_r = reduced roof live load, with minimum of:

$$12 \text{ psf} \leq L_r \leq 20 \text{ psf} \quad (0.58 \text{ kN/m}^2 \text{ in SI units})$$

for ordinary flat, pitched, and curved roofs.

$R_1 = 1$ For $A_T \leq 200 \text{ ft}^2$ (18.58 m^2) and $R_1 = 0.6$ For $A_T \geq 600 \text{ ft}^2$ (55.74 m^2); $R_1 = 1.2 - 0.001 A_T$ ($R_1 = 1.2 - 0.011 A_T$ in SI units)

For $200 \text{ ft}^2 < A_T < 600 \text{ ft}^2$ ($18.58 \text{ m}^2 < A_T < 55.74 \text{ m}^2$)

$R_2 = 1.0$ For flat roofs $F \leq 4$; $R_2 = 1.2 - 0.05 F$ For $A_T \geq 600 \text{ ft}^2$ (55.74 m^2); $R_2 = 1.2 - 0.001 A_T$ ($R_2 = 1.2 - 0.011 A_T$ in SI units) for $200 \text{ ft}^2 < A_T < 600 \text{ ft}^2$ ($18.58 \text{ m}^2 < A_T < 55.74 \text{ m}^2$)

$\hookrightarrow$ * For a column or beam supporting more than one floor, the term A_T represents the sum of the tributary areas from all floors.

* **Example 3.30** A Two-story office building shown in the photo has interior columns that are spaced 6.71 m apart in two perpendicular directions. If the (flat) roof loading is 0.96 kN/m^2 . Determine the reduced live load supported by a typical interior column located at ground level.

solution $\rightarrow$

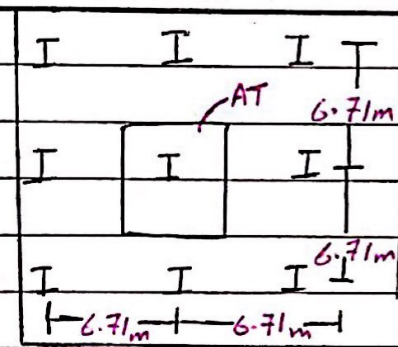
$$A_T = 6.71 \text{ m} \times 6.71 \text{ m} = 45 \text{ m}^2$$

1st floor : live load =

$$0.96 \times 45 = 43.2 \text{ kN}$$

$\rightarrow$ This load can't be reduced

since it is roof load



solution Example 3 or First Floor →

Table 4.3-1 ASCE 7-16 Page 14

$$L_o = 2.4 \text{ kN/m}^2 < 24.79 \text{ kN/m}^2 \rightarrow \text{reduction}$$

$$K_{LL} \rightarrow 4.7-1 \text{ ASCE 7-16}$$

$$K_{LL} = 4 \rightarrow \text{Interior columns}$$

$$K_{LL} \times A_T = 4 \times 45 = 180 \text{ m}^2 > 37.2 \text{ m}^2 \rightarrow \text{ok}$$

$$L = L_o \left(0.25 + \frac{4.57}{\sqrt{K_{LL} A_T}} \right) \rightarrow \text{SI units}$$

من الارتفاعات
من الامكان

$$L = 2.4 \left(0.25 + \frac{4.57}{\sqrt{180}} \right) \Rightarrow L = 1.42 \text{ kN/m}^2$$

$$\text{The load reduction} = \frac{1.42}{2.4} \times 100\% = 59.1\%$$

→ one floor → must be $L \geq 50\% L_o$

$$\text{Floor Load} = \underset{L}{1.42} \times \underset{A_T}{45} = 63.9$$

$$\text{Roof Load} = 0.96 \times 45 = 43.2 \text{ (No reduction)}$$

$$\text{Total load} = 63.9 + 43.2 = \boxed{107.1 \text{ kN}}$$

Highway Bridge Loads → axial load

primary live loads are those due to traffic

specifications for truck loadings are reported in AASHTO (American Association of State Highway and Transportation Officials)

For 2-axle truck, these loads are designated with H followed by the weight of truck in tons and another no. gives the year of the specifications that the loads was reported

* Railway Bridge Loads $\rightarrow$ axle loads

- Loadings are specified in AREMA (American Railway Engineering and Maintenance of way association)

- A modern train having a 320 kN (72 k) loading on the driving axle of the engine is designated as an E-72 loading

* Impact loads: Due to moving vehicles $\rightarrow$ * High stress *

(The % increase of the live loads due to impact is called the impact factor (suddenly $\rightarrow$ live loads $\rightarrow$))

$$\rightarrow I = \frac{50}{L + 125} < 0.3 \text{ (US units)}$$

$\rightarrow$ impact factor

$$\rightarrow I = \frac{15.24}{L + 38.1} < 0.3 \text{ (SI units)}$$

L = length of the span in (ft, m) that is subjected to the live loads.

* wind loads:

kinetic energy $\rightarrow$ potential energy

when $\rightarrow$ structures block the flow of wind

- Effects of wind depends on density of flow of air, angle of incidence, shape & stiffness of the structure roughness of surface.

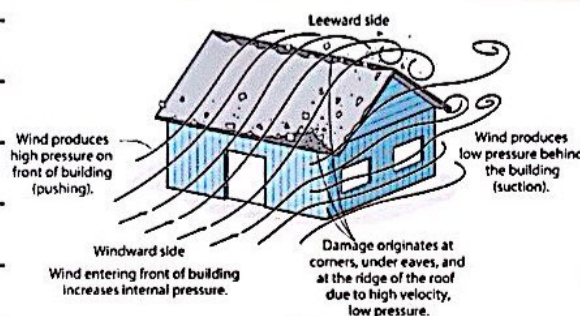
- For design, wind loading can be treated as static or dynamic approach $\rightarrow$ example 1.3.

High pressure

$\rightarrow$ pushing

low pressure

$\rightarrow$ suction



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Area roof > Area side

نادر مقدار ترقص
نرياح

Area roof > Area side

نادر مقدار ترقص

نرياح

10

Five Apple

* Lateral → wind, Earthquake

* Vertical → dead load, Live load

* كيف يقام التحليل lateral load التحليل

① shear walls هي جدران المقوم مع الأحمال الجانبية بتأثير من القاعدة

shear and في الأسفل أعلى شدة

cross bracing (٢)

Knee bracing (٣)

Diagonal bracing (٤)

$$* q = \frac{1}{2} \rho v^2$$

→ ρ : density
→ v : velocity

* Show example 1-3 to know more details.

* ASCE 7-16.

Factor

$$q_z = 0.00256 \left[\frac{K_z}{K_d} \frac{K_{zt}}{K_d} \right] v^2 (16/ft^2)$$

$$q_z = 0.613 K_z K_{zt} K_d v^2 (N/m^2)$$

* دقة هذه القوانين بناءً على

① structure's height ارتفاع المبنى

② the terrain in which it is located ← التضاريس التي يقع فيها

* snow loads:

→ Design loading depend on ① Building's general shape استكمال المبنى

② roof geometry هندسة السطح ③ wind exposure التعرض للرياح ④ location الموقع

→ snow loads are determine from a zone map reporting 50-years recurrence interval

* Earthquake loads: - (أحمال الزلازل) نتيجة عن تحرك الصانع

Their magnitude depends on amount & type of ground acceleration, mass & stiffness of structure.

موجب زيادة stiffness

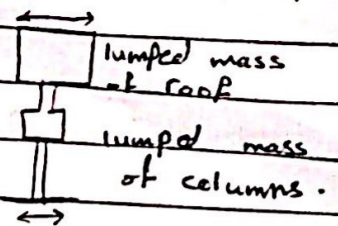
(هو رتبة حجمها على مقدار ارتفاع وسارع الأرض والكتلة)

(Loads) الأحمال

صلاية الرباط

#Lecture 5

→ The ground vibrates both horizontally & vertically
تتأرجح الأرض أفقياً وعمودياً



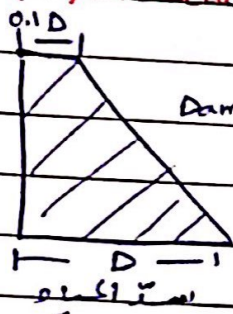
* If the column is stiff & the block has a small mass, the period of vibration of the block will be short, the block will accelerate with the same motion as the ground & undergo slight relative displacement.

إذا كان العمود صلباً والكتلة قليلة، في تكون فترة اهتزاز الكتلة قصيرة وبع تسارع بنفس حركة الأرض.

* If the column is very flexible & the block has a large mass, induced motion will cause small accelerations of the block & large relative displacement.

إذا كان العمود مرناً جداً والكتلة كبيرة الحجم، في تكون التسارع صغيراً والازاحة كبيرة.

* Hydrostatic & Soil Pressure

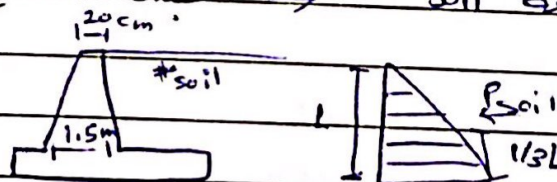


Dam

← كل ضغط المياه في الأسفل

retaining walls (الحيطان الاستنادية) → Soil Pressure

→ retaining walls.



← ضغط يكون بالشد القريب من القاعدة

Other natural loads:-

- 1) Effect of blast → الانفجار
- 2) Temperature changes → تغيرات درجة الحرارة
- 3) Differential settlement of foundations → التسوية التفاضلية للأساس

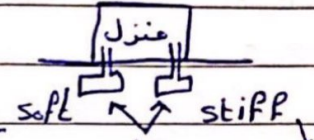
① * effect of blast من عند تصميم مستوي للأساس أو الزخيرة يجب أن تأخذ

بجانب الاعتبار أنه عيّن أن يحصل انفجار

② Temperature changes من عند التعرض لتغيير في الحرارة حين عدد متقابل هذه

Loads تتحول إلى stress

③ Differential settlement of foundation

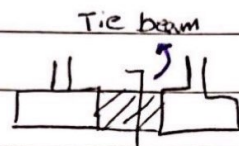


وكانت قوة 3 Kg/cm² ← 1.5 Kg/cm² ← صافي القاعدة ← القاعدة soft فلا لم يثبت

مع تدرج عقبار Δ بمقدار 5 خان القاعدة stiff بتدرج

عقدار 1 فرع ينهار المبني من صفة

القاعدة soft



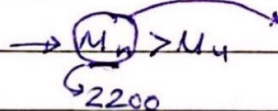
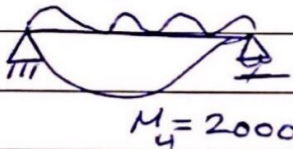
① تكبير القاعدة

② Tie beams (زئار الربط)

* Design Methods:

① Allowable-stress design (ASD) Method: → أكثر تكلفة

→ working stress Design Method طريقة أقل شيئا



حسابها في ارضي

$$\text{required strength} \leq \text{allowable strength}$$

$$R_a \leq \frac{R_n}{\Omega} \rightarrow \text{Nominal strength}$$

$$\text{required strength} \rightarrow \text{Factor of safety}$$

(ASD)

* تتكون الحملات الأكثر دقة من دمج القوى مثل الرياح والزلزال
(في لحظة معينة وقتاً معيناً)

* Load combinations specified by the ASCE 7-16 standard

- 1) D
- 2) D+L
- 3) D+(Lr or S or R)
- 4) D+(0.75L+0.75(Lr or S or R))
- 5) D+(0.6w)
- 6) D+0.75L+0.75(0.6w)+0.75(Lr or S or R)
- 7) 0.6D+0.6w

② ultimate strength Design method: (LRFD) الطريقة الأكثر شيوعاً

required strength $\leq$ design strength

Load Factor > 1

$$R_u = \sum \gamma_i P_{ni} \leq \phi R_n$$

Required strength Load Factor Resistance Nominal strength / Resistance

< 1

* زيادة الحملات (زيادة) strength

↳ This method uses load factors to the loads or combination of loads

- 1) 1.4 D
- 2) 1.2D+1.6L+0.5(Lr or S or R)
- 3) 1.2D+1.6(Lr or S or R)+(L or 0.5w)
- 4) 1.2D+1.0w+L+0.5(Lr or S or R)
- 5) 0.9D+1.0w
- 6) 1.2D+Fv+Eh+L+0.2S
- 7) 0.9D+Fv+Eh

↳ Dead load $^{+}(20\%)$, Live load $^{+}(60\%)$

* Chapter 2: Analysis of statically Determinate structures.

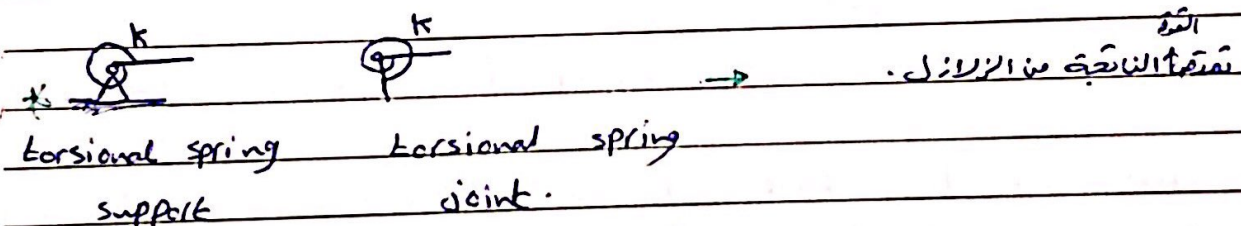
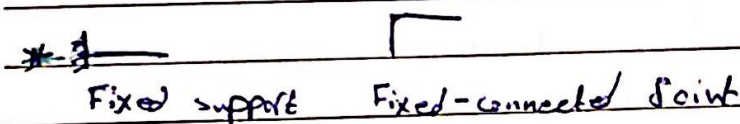
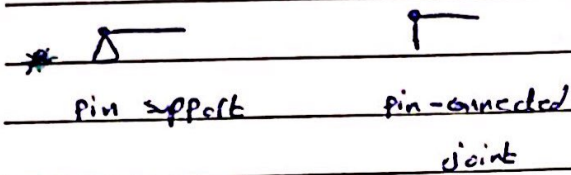
Can this chapter we will learn about idealized structures.

* support connections :-

1) Pin connection $\rightarrow$ (تأثير) $\begin{matrix} \uparrow \\ \times \end{matrix}$ $\begin{matrix} \rightarrow \\ \times \end{matrix}$ $\rightarrow$ مرفوع من قبل المهندس بسبب قلة تكلفتها

2) Roller support $\rightarrow$ (تأثير) $\rightarrow$ يمنع الحركة بالاتجاه العادي ويسمح بالحركة الأفقية ويسمح للدوران

3) Fixed joint $\rightarrow$ expensive (تأثير كبير)



$K=0$, the joint is pin $K=\infty$, the joint is fixed

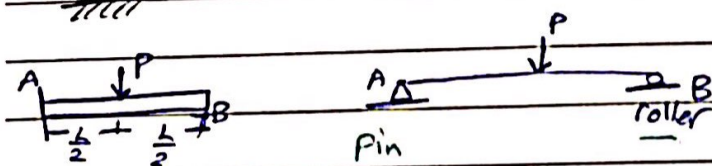
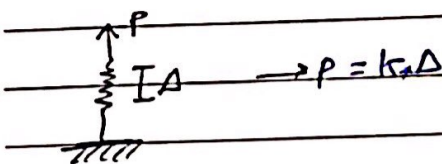


TABLE 2.1 Supports for Coplanar Structures

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1) weightless cable			One unknown. The reaction is a force that acts in the direction of the cable or link.
weightless link			
(2) rollers			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
rockers			
(3) smooth contacting surface			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(4) smooth pin connected collar			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.

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4

(5) 			Two unknowns. The reactions are two force components.
smooth pin or hinge			

(6) 			Two unknowns. The reactions are a force and a moment.
slider			
fixed connected collar			

(7) 			Three unknowns. The reactions are the moment and the two force components.
fixed support			

CE 315 Spring 2021

Dr. Raed Al Mazaidi

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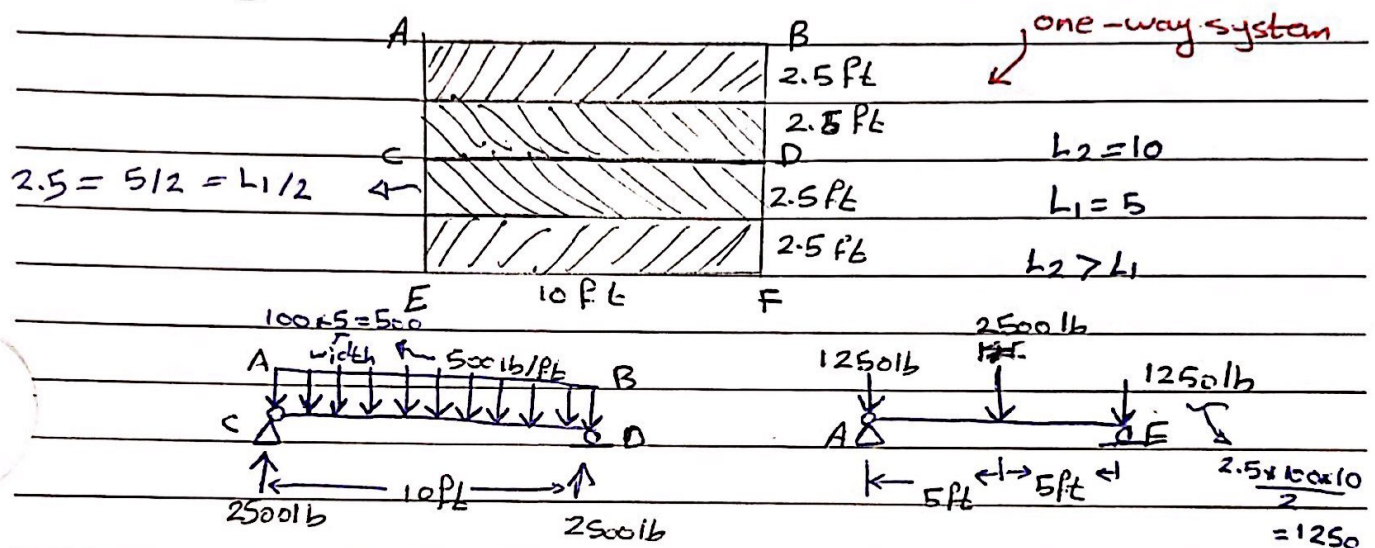
* load path:-

slab (الرقبة) → beam → girder → column → foundation → soil

→ There are two ways depends on the geometry of the structural system and the material from which it is made and the method of its construction.

▶ one-way system

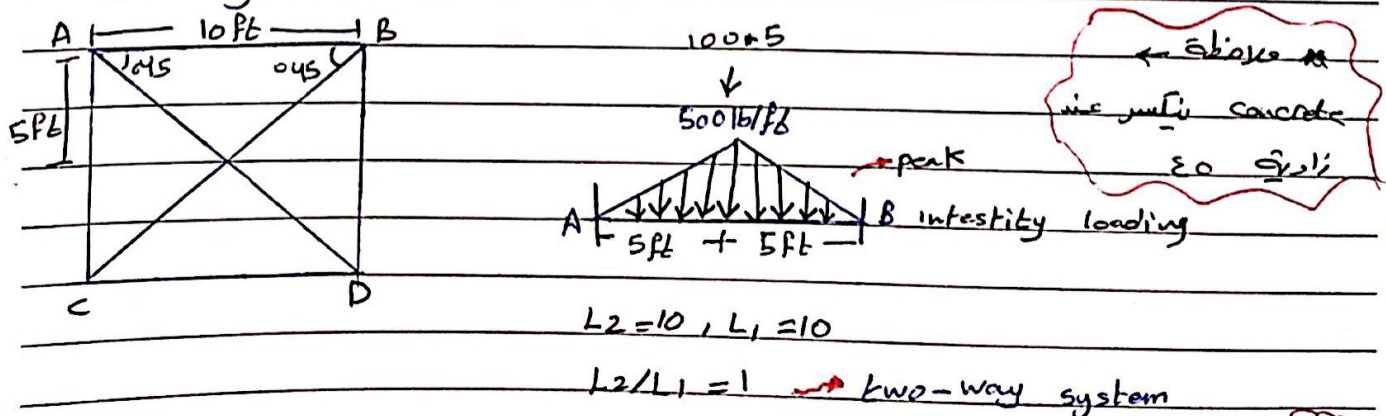
Two-way system

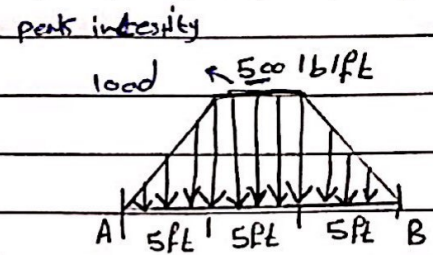
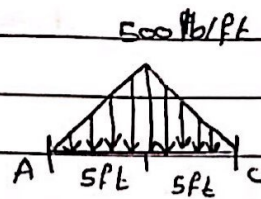
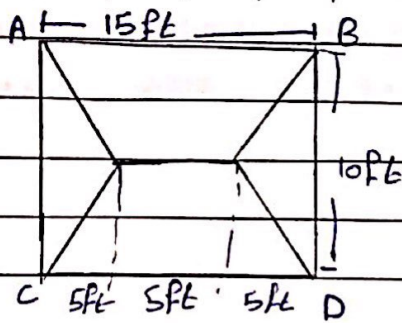


* If $L_2 > L_1$ and if the span ratio $(L_2/L_1) > 2$, the slab will behave as a one-way slab.

→ Two-way system

* If $L_2 > L_1$ and if the $(L_2/L_1) \leq 2$, the slab will behave as a two-way slab





* Example 1:-

The floor of a classroom is to be supported by the bar joists shown in the photo. Each joist is 15 ft long and they are spaced 2.5 ft apart. The floor itself is to be made from lightweight concrete that is 4 in. thick. Neglect the weight of the joists and the corrugated metal deck and determine the load that acts along each joist.

→ Solution:-

① Dead load on the floor:

light weight concrete → Table C3.1-1a (ASCE 7-16) → 8 lb/ft².in

$$\Rightarrow DL = 4 \text{ in} \times 8 \text{ lb/ft}^2 \cdot \text{in} = 32 \text{ lb/ft}^2$$

② Live load on the floor:

class room → Table 4.3-1 (ASCE 7-16) → 40 lb/ft²

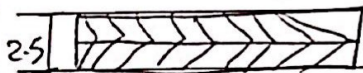
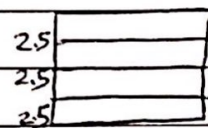
$$LL = 40 \text{ lb/ft}^2$$

⇒ Total load = DL + LL

$$T_{\text{load}} = 32 \text{ lb/ft}^2 + 40 \text{ lb/ft}^2 = 72 \text{ lb/ft}^2$$

$$L_1 = 2.5', L_2 = 15'$$

$$L_2 / L_1 = \frac{15'}{2.5'} = 6 > 2 \rightarrow \text{one-way slab}$$



$$\rightarrow \text{Tributary area} = 15' \times 2.5' = 37.5 \text{ ft}^2$$

$$\rightarrow \text{The uniform load along its length} = 72 \text{ lb/ft}^2 \times 2.5 \text{ ft} = 180 \text{ lb/ft}$$

Shear connector (stud) ← يثبت على concrete لزيادة
 قوة stud و wetting مع الخرسانة

Example 2

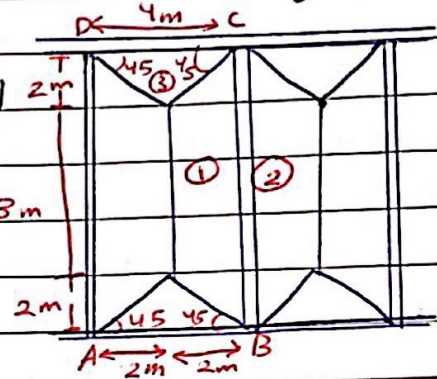
The flat roof of the steel-frame building shown in the photo is interested to support a total load of 2 kN/m^2 over its surface. Determine the roof load within region ABCD that is transmitted to beams BC and DC. The dimensions are shown in Fig 2-6a.

Solution:-

Total load = 2 kN/m^2 (Given in the problem)

$$L_1 = 4.0 \text{ m}, L_2 = 7.0 \text{ m}$$

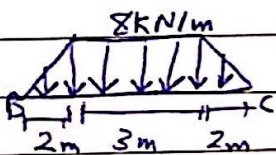
$$\frac{L_2}{L_1} = \frac{7}{4} = 1.75 < 2.0 \rightarrow \text{Two-way slab}$$



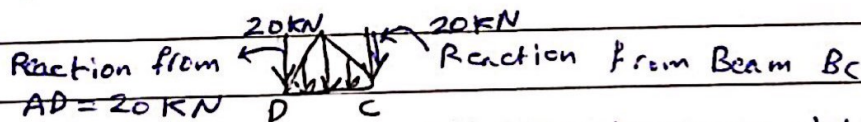
→ Peak intensity of loading on beam BC

$$\text{Area 1} = 2 \text{ kN/m}^2 \times 2 = 4 \text{ kN/m}$$

$$\text{Area 2} = 2 \text{ kN/m}^2 \times 2 = 4 \text{ kN/m} \rightarrow \text{Total Peak intensity of loading} = 4 + 4 = 8 \text{ kN/m}$$



Peak intensity of loading on DC = $2 \text{ kN/m}^2 \times 2 = 4 \text{ kN/m}$



$$= \frac{1}{2} (7 + 3) \times 8 \text{ kN/m} / 2 = 40 / 2 = 20 \text{ kN}$$

$$\frac{1}{2} (B_1 + B_2) \times h$$

* Example 3 :-

The concrete girders shown in the photo of the passenger car parking garage span 30 ft and are spaced 15 ft on center. If the floor slab is 5 in. thick and made of reinforced stone concrete and the specified live load is 50 lb/ft². determine the distributed load the floor system transmits to each interior grid.

Solution →

① Dead load on the floor :-

Reinforced stone concrete → Table C3.1-1a (ASCE 7-16) → 150 lb/ft³

$$\text{Dead load} = 150 \frac{\text{lb}}{\text{ft}^3} \times \left(\frac{5}{12} \right) \text{ ft} = 62.5 \frac{\text{lb}}{\text{ft}^2}$$

② Live load = 50 lb/ft² (Given by problem)

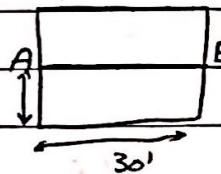
$$\rightarrow \text{Total load} = \text{DL} + \text{LL} = 62.5 \frac{\text{lb}}{\text{ft}^2} + 50 \frac{\text{lb}}{\text{ft}^2} = 112.5 \frac{\text{lb}}{\text{ft}^2}$$

$$\frac{L_2}{L_1} = \frac{30}{15} = 2 \quad \text{Two way slab}$$

peak intensity of loading of girder AB

$$L_1 = 15'$$

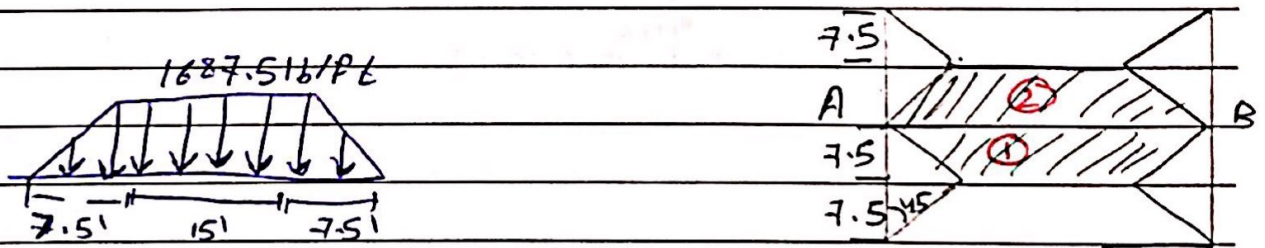
$$L_2 = 30' \quad 15'$$



$$\text{Area 1} = 112.5 \frac{\text{lb}}{\text{ft}^2} \times 7.5 \text{ ft} = 843.75 \frac{\text{lb}}{\text{ft}}$$

$$\text{Area 2} = 112.5 \frac{\text{lb}}{\text{ft}^2} \times 7.5 \text{ ft} = 843.75 \frac{\text{lb}}{\text{ft}}$$

$$\therefore \text{Total peak intensity of loading} = 2 \times 843.75 = 1687.5 \frac{\text{lb}}{\text{ft}}$$



* Principle of superposition :- إذا كان الهيكل يتحمل عدة أحمال خارجية

"The total displacement or internal loadings (stress) at a point in a structure subjected to several external loadings can be determined by adding together the displacements or internal loadings (stress) caused by each of the external loads acting separately"



$$P_1 \Delta_1 + P_2 \Delta_2 = \Delta$$

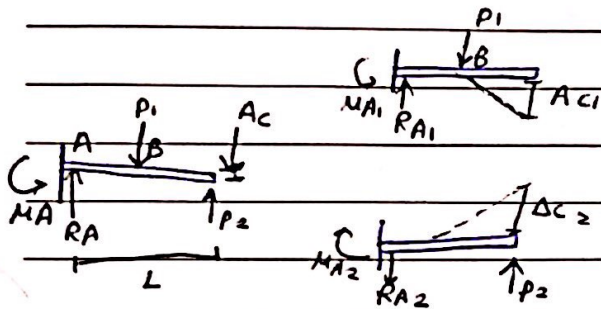
→ For this statement to be valid it is necessary that a linear relationship exist among the loads, stresses, and displacement.

*Two requirements for the principle to apply.

1- The material must behave in a linear-elastic manner, so that

Hooke's law is valid, and therefore the load will be proportional to displacement. $\text{displacement} \propto \text{load}$

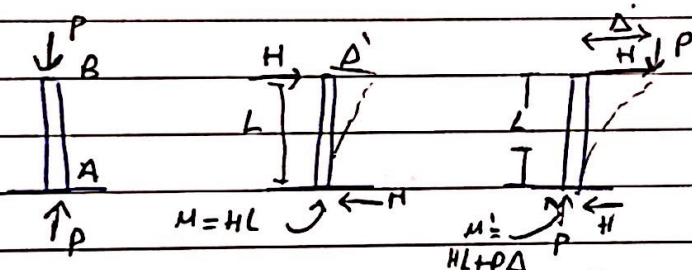
2- The geometry of the structure must not undergo significant change when the loads are applied, small displacement will significantly change the position and orientation of loads.



$$R_A = R_{A1} + R_{A2}$$

$$M_A = M_{A1} + M_{A2}$$

$$\Delta_C = \Delta_{C1} + \Delta_{C2}$$



superposition not applicable.

Equations of Equilibrium

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$



internal loading $\rightarrow$ shear normal

*Determinacy

No. of unknown forces $>$ equilibrium equations $\Rightarrow$ statically indeterminate.

$r = 3n$, statically determinate.

$r > 3n$, statically indeterminate.

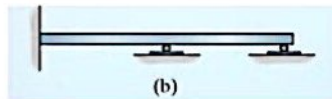
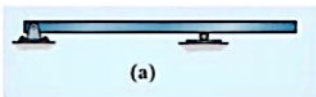
$r \rightarrow$ number force and moment reaction components

$n \rightarrow$ number of parts.

مستقر
غير مستقر

*Example 4:-

number of degree of indeterminacy $= r - 3n$



$n = 1$ $r = ?$

FBD

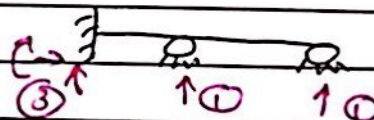


$$r = 3 \rightarrow r = 3n = 3 \rightarrow \text{det}$$

$$r - 3n = 0$$

$n = 1$

FBD

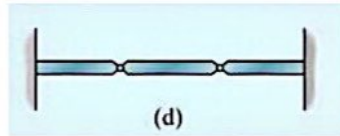
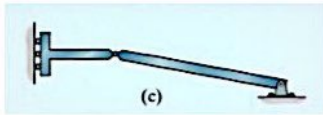


$$r = 5$$

$$3n = 3(1) = 3$$

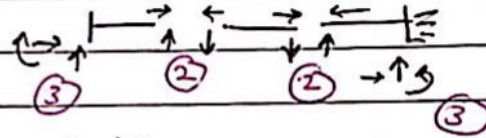
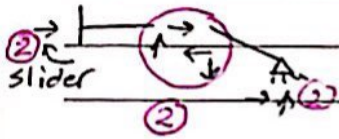
$$r > 3n \quad \text{Ind}$$

$$r - 3n = 5 - 3 = 2$$



$n=2$

$n=3$



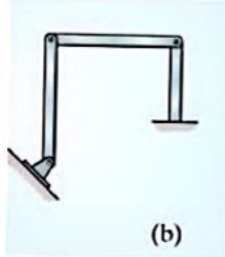
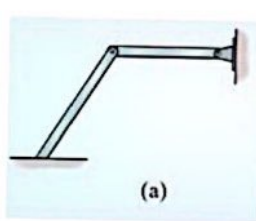
$r=6$ det

$r=10$

$3n = 3(3) = 9$ ind

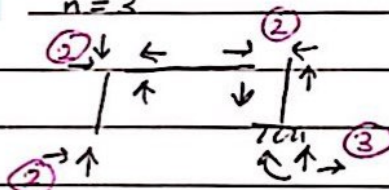
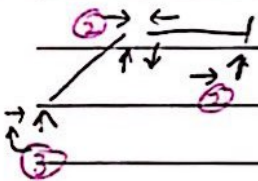
$r - 3n = 10 - 9 = 1^{\text{st}} \text{ degree}$

* Example 5 :-



$n=2$

$n=3$

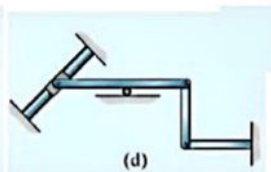
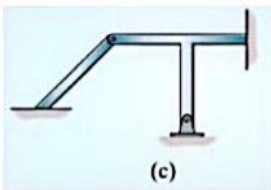


$r=7$

$r=9$

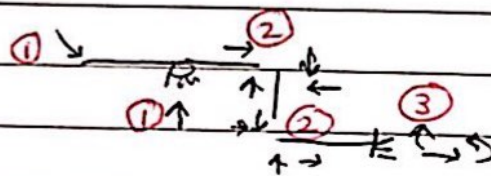
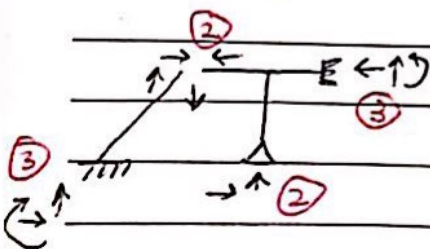
$3n = r = 9$ det.

$r - 3n = 7 - 6 = 1^{\text{st}} \text{ degree}$



$n=2$

$n=3$

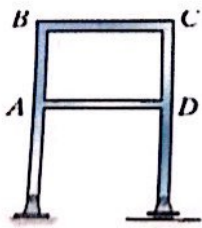


$r=10 \rightarrow 3n=6$

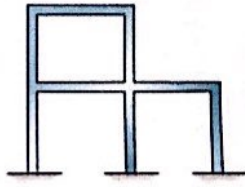
$r=9$ det.

$10 - 6 = 4 \text{ degree}$

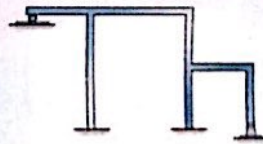
* Example 5:-



(a)

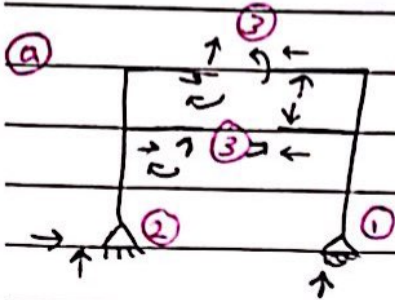


(b)



(This frame has no closed loops.)

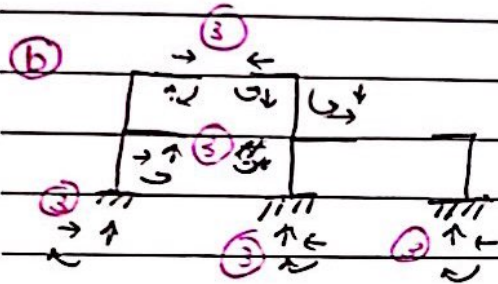
(c)



$$r = 9, n = 2$$

$$r = 9 > 3n = 6 \rightarrow \text{ind}$$

$$r - 3n = 9 - 6 = 3^{\text{rd}} \text{ degree}$$

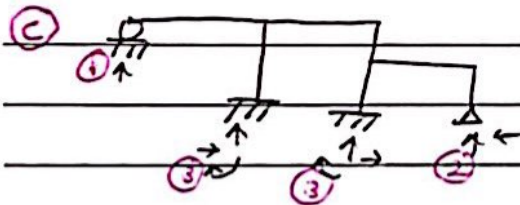


$$r = 3, n = 18$$

$$3n = 3(18) = 54$$

$$r = 3 < 3n = 54 \rightarrow \text{ind}$$

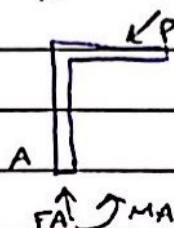
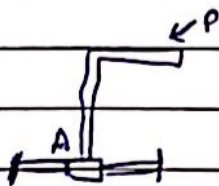
$$r - 3n = 3 - 54 = -51^{\text{th}} \text{ degree}$$



$$r = 9, n = 1$$

$$r = 9 > 3n = 3 \rightarrow \text{ind}$$

$$r - 3n = 9 - 3 = 6^{\text{th}} \text{ degree}$$

stability $\rightarrow$ * 2 situations may occur $\rightarrow$ ① Partial constraints $\rightarrow$ support is not correct

$$r = 2, n = 1$$

$$3n = 3(1) = 3$$

$$r - 3n = 2 - 3(1) = -1 \text{ unstable}$$

غير مستقر

إذا لم يكن الدعم صحيحاً
فإنه غير مستقر

2- Improper constraints:

$r=3, n=1$

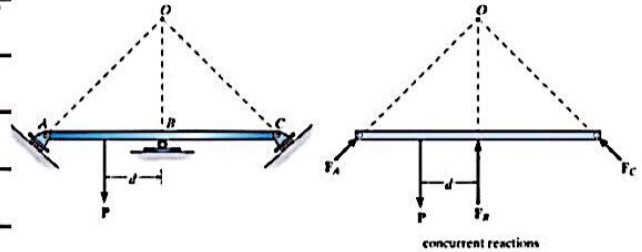
① This can occur if all the support reaction are concurrent at point

$3n=3(1)=3$

$r=3n=3$

det. $\neq 0$

det.

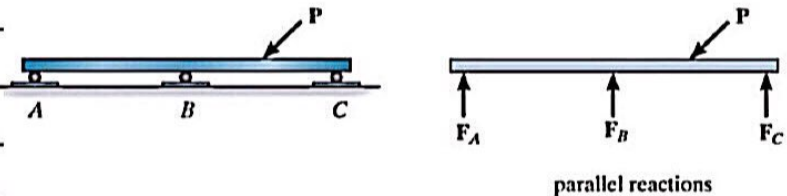


02_024

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② another way in which improper constraining leads to instability occurs when the reactive forces are all parallel

The summation of forces in the horizontal direction will not equal zero.

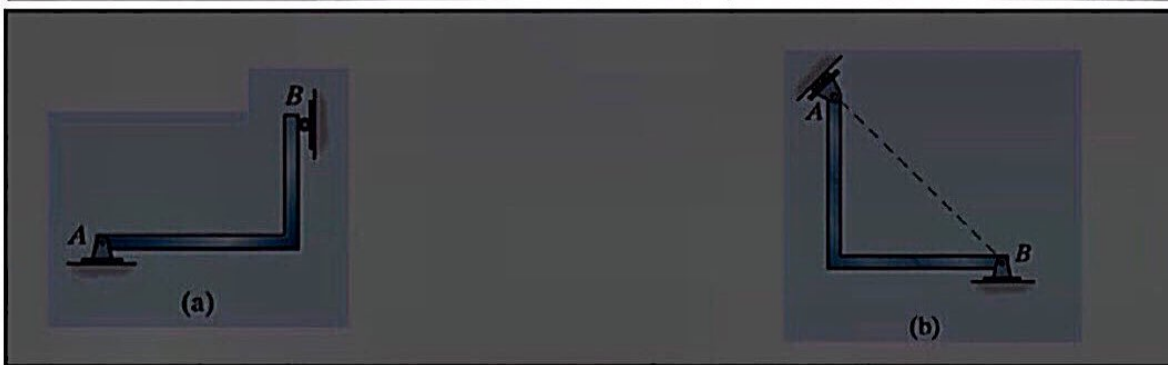


02_025

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 $r < 3n \rightarrow$ unstable

$r \geq 3n \rightarrow$ unstable if member reaction are concurrent or parallel or some of the components form a collapsible mechanism



$r=3, n=1$

$3n=3$

$r=3n$

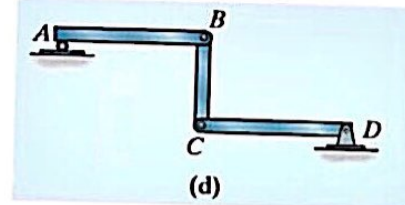
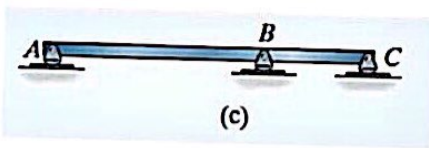
stable

$n=1, r=3$

$3n=3$ det

unstable

concurrent at point B



$$n = 1$$

$$r = 3$$

$$3n = 3$$

$$r = 3n \text{ det}$$

unstable $\rightarrow$ parallel

$$r = 7$$

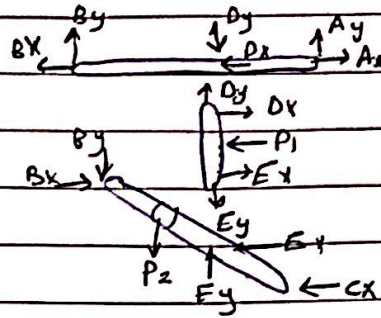
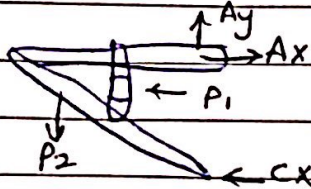
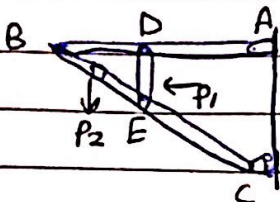
$$n = 3$$

$$3n = 9$$

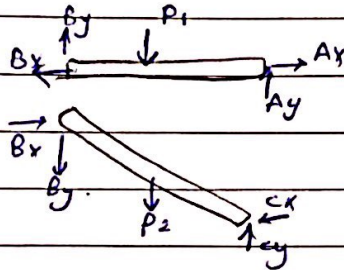
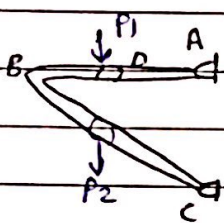
$$r = 7 < 3n = 9$$

unstable.

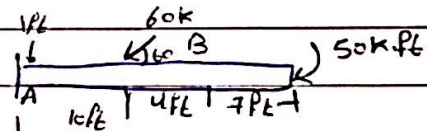
unstable ← إذا طلع معك ind. or det



$$n=3 \quad r=4 \quad c=9 \rightarrow 3n=3(3)=r \rightarrow \text{det.}$$



Example 8: Determine the reactions



$$\rightarrow \sum F_x = 0 \rightarrow A_x - 60 \cos 60 = 0$$

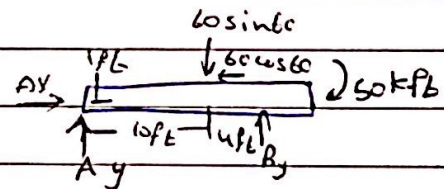
$$\uparrow \sum M_A = 0$$

$$-60 \sin 60 (10) + 60 \cos 60 (1) + B_y (14) - 50 = 0$$

$$B_y = 38.5 \text{ k}$$

$$\uparrow \sum F_y = 0 \rightarrow -60 \sin 60 + 38.5 + A_y = 0$$

$$A_y = 13.4 \text{ k}$$

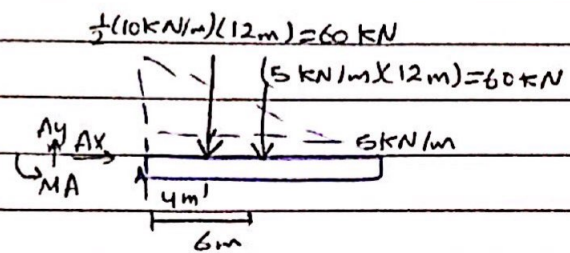
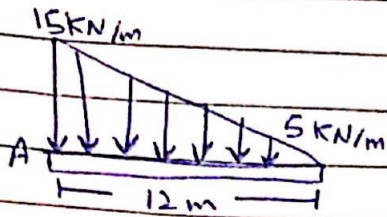


$\cos \theta \rightarrow$ القريب
للزاوية

$\sin \theta \rightarrow$ البعيد

Example 9

Determine the reactions



$$\rightarrow \sum F_x = 0 \rightarrow A_x = 0$$

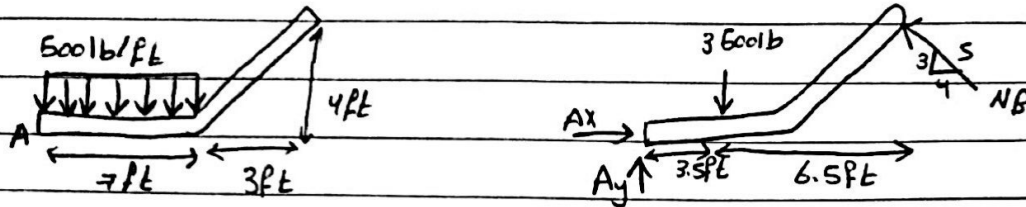
$$\uparrow \sum F_y = 0 \rightarrow A_y - 60 - 60 = 0 \rightarrow A_y = 120 \text{ k}$$

$$\circlearrowleft \sum M_A = 0 \rightarrow -60(4) - 60(6) + M_A = 0 \rightarrow M_A = 600 \text{ kN}\cdot\text{m}$$

* إذا فرضت حمولة x للأعلى
وطلعت القيمة سالبة يعني
فرضتي خطأ بعكس الاتجاه

* Example 10:-

Determine the reaction on the beam in Fig 2-32a. Assume A is a pin and the support at B is a roller (smooth surface).



Solution:-

$$n=1 \rightarrow 3n=3(1)=3 \rightarrow r=3n=3 \rightarrow \text{det} \rightarrow \text{كل على معادلة الحركة}$$

$$+\circlearrowleft \Sigma M_A = 0$$

$$-3500(3.5) + \left(\frac{4}{5}\right) N_B(4) + \left(\frac{3}{5}\right) N_B(10) = 0$$

$$N_B = 1331.516 = 1.33 \text{ k}$$

$$\rightarrow \Sigma F_x = 0$$

$$A_x - \frac{4}{5}(1331.5) = 0$$

$$A_x = 1.07 \text{ k}$$

$$+\uparrow \Sigma F_y = 0$$

$$A_y - 3500 + \frac{3}{5}(1331.5) = 0$$

$$A_y = 2.70 \text{ k}$$

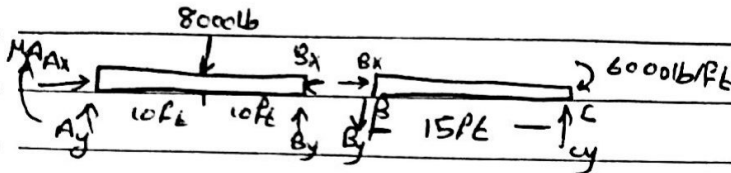
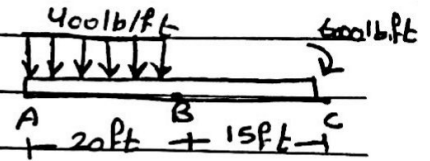
Example 11:-

The compound beam in Fig 2-33a is Fixed at A. determine the reactions at A, B and C. Assume that the connection at B is a pin and C is a roller.

Solution:-

$$n=2, r=6, 3n=3(2)=6$$

$$r=3n=6 \rightarrow \text{indet}$$



segment BC

$$+\circlearrowleft M_C = 0 \rightarrow -6000 + B_y(15) = 0 \rightarrow B_y = 400 \text{ lb}$$

$$+\uparrow F_y = 0 \rightarrow -400 + C_y = 0 \rightarrow C_y = 400 \text{ lb}$$

$$+\rightarrow F_x = 0 \rightarrow B_x = 0$$

segment AB

$$+\circlearrowleft M_A = 0 \rightarrow M_A - 8000(10) + 400(20) = 0$$

$$M_A = 72.0 \text{ K} \cdot \text{ft}$$

$$+\uparrow F_y = 0 \rightarrow A_y - 8000 + 400 = 0$$

$$A_y = 7.60 \text{ K}$$

$$+\rightarrow F_x = 0 \rightarrow A_x - 0 = 0 \rightarrow A_x = 0$$

Example 12:-

Determine the horizontal and vertical components of reaction at the pins A, B, and C of the two-member frame shown in Fig. 2-34a.

Solution:-

$$n = 2, r = 6$$

$$6 - 3n = 0 \text{ det.}$$

Member BC

$$+\circlearrowleft \sum M_C = 0$$

$$-B_y(2) + 6(1) = 0$$

$$B_y = 3 \text{ kN}$$

Member AB

$$+\circlearrowleft \sum M_A = 0$$

$$-8(2) - 3(2) + B_x(1.5) = 0$$

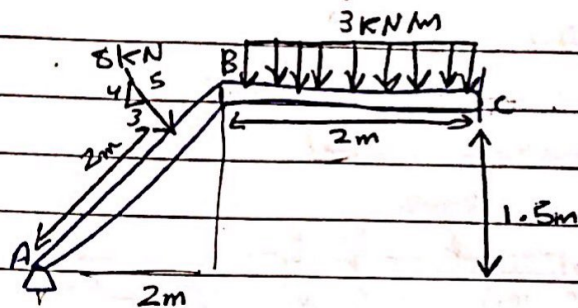
$$B_x = 14.7 \text{ kN}$$

$$+\rightarrow \sum F_x = 0$$

$$A_x + \frac{3}{5}(8) - 14.7 = 0 \rightarrow A_x = 9.87 \text{ kN}$$

$$+\uparrow \sum F_y = 0$$

$$A_y - \frac{4}{5}(8) - 3 = 0 \rightarrow A_y = 9.40 \text{ kN}$$

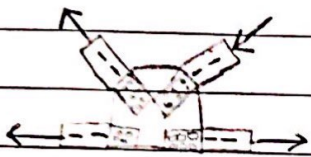


Chapter 2 - Analysis of statically Determinate Trusses

← الجداول

Use in large spans (البجور الطويلة) →
 joints are (axial force) ←
 X ← Moment

* The joint connections are usually formed by bolting or welding the ends of the members to a common plate called gusset plate.



((gusset plate)) → steel

* Planar trusses lie in a single plane & is often used to support roof or bridges

* إذا ما وجد قوس يحمل سقفية آخر طابق trusses لا يكون عنه أي قوة مستقبلية يحمل طابق فوقه. ← النسبة لأن الأحمال التي يتحملها مع تكون moment
 trusses ما يحمل moment مع يتركه المبنى
 لذلك trusses يستخدم ك roof

* Roof Trusses

→ كيف تنتقل الأحمال

الأحمال يجب أن تنتقل في طابق trusses على joints غير ذلك لا يعتبر trusses

هذا طريقين لنقل الأحمال:

① one way

② two way

* إذا اجبة الريح من المرفق فقامها

bracing على شكل cross (X) و شيل

compression and Tension

← حسب اتجاه الريح

* لما تدرس area - يجب أن

تكون مرفقة بين جسر ومرفقة

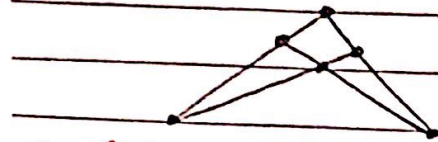
من الأربع جهات ($L_2 > L_1$)

$\frac{L_2}{L_1} > 2 \rightarrow$ one way

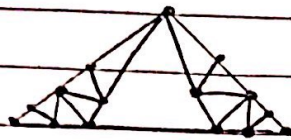
beam مع يأخذ نفس من يمينه ونفس من شماله (purliss) اسرقي trusses

* إذا حادة الرياح من الجانب يقامها **Knee braces** ← نقل bracing من الزاوية

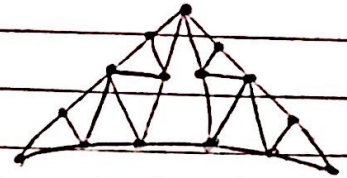
↓ أشكال Trusses ↓



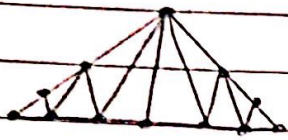
scissors ← على شكل مقص



Fink



cambered Fink ← انحناء

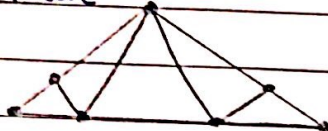


Howe



Warren

مقوس

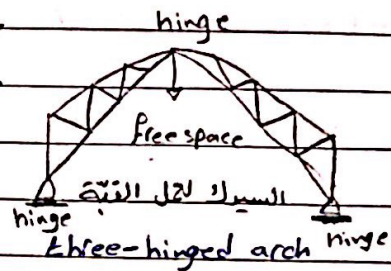


Fan

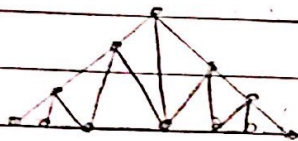


bowstring

← شكل القوس

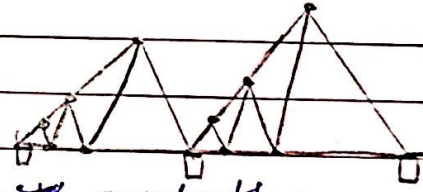


Three-hinged arch hinge hinge hinge
Free space
السيارة لتقل القبة



Pratt

swallowtail ← مثل أسنان أسماك



capital ← اسم المبنى

small ← اسم Trusses

* إذا كان سمي Trusses بالبابية

* إذا كان سمي Trusses بالبابية small ← اسم Trusses

* Bridge trusses: one-way ($L_2 > 2L_1$) and $L_2 > L_1$

deck → stringers (beams) → Floor beams → joints

← reaction

bracing (تدعيم)

* كيف يفهم انحاء يتحرك جانبياً

Dangolas ← تقامها

إذا اجبت الريح من L_1 (المسافة الصغيرة) ← تقامها Sway bracing

← الجزء العلوي (Top cord) و الجزء السفلي (bottom cord)

* يجب على bracing في كل مكان حيث لا يكون أي انضغاط نتيجة الأحمال الجانبية

* لا يتم bridges بغير hinge roller حيث أي قدر أو نقل
يتم بوجه roller (تلقا قوتها المتحد والتقليل من قوة صغيرة جدًا نتيجة الحرارة)

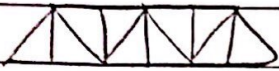
أنواع bridge trusses



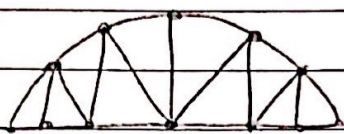
pratt



Howe



warren (with verticals)



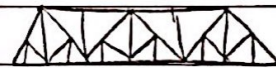
parker



Baltimore



K-truss



subdivided warren

* spans $\rightarrow$ 61 m in length

* ينقل كل الأحمال إلى سائل compression and Tension (axial) $\leftarrow$

* Large spans $\rightarrow$ Parker trusses

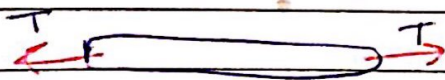
* The warren truss $\rightarrow$ 91 m.

* Assumptions for design $\rightarrow$

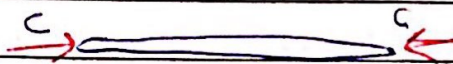
The members are joined together by smooth pin

All loadings are applied at the joints

$\rightarrow$ Due to the 2 assumptions, each truss member acts as an axial force member.



Tension



compression

* Classification of coplanar Trusses .

1 simple 2 compound or complex Truss

→ Triangular (مُثلث) → أكثر استقراراً .

Trusses هي
beams

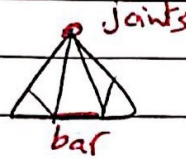
→ عناصر متصلة

compound → simple من

Type 1

joint → مفصل (simple) . Trusses من

1 bar



Type 2

the trusses may be joined by 3 bars



Type 3

The trusses may be joined where bars of a large simple truss, called the main truss, have been substituted by simple truss, called secondary trusses

* Complex Truss .

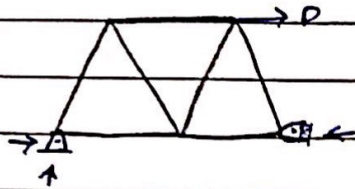
complex truss → compound of simple

* $b + r = 2j$ statically determinate $b + r > 2j$ statically indeterminate→ Degree of indeterminacy : $(b + r - 2j)$ b : The total number of bars of the trussr : The total number of external support reactions .j : The total number of joints .

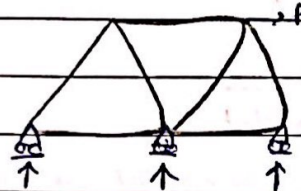
* External stability.

→ A structure is externally unstable if all of its reactions are concurrent or parallel

→ The trusses are externally unstable since the support reactions have lines of action that are either



unstable - concurrent
reactions



unstable - parallel
reactions

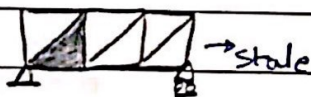
* Internal stability.

• The internal stability can be checked by careful inspection of the arrangement of its member

• if it can be determined that each joint is held fixed so that it cannot move in a "rigid body" sense with respect to the other joints, then the truss will be stable

• A simple truss will always be internally stable.

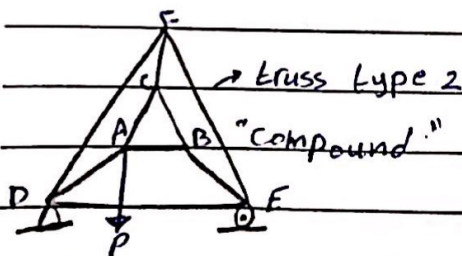
• if a truss is constructed so that it does not hold its joints in a fixed position, it will be unstable or have a "critical form".



$$b=13, r=3, j=8$$

$$b+r=2j$$

$$16=16 \rightarrow \text{det.}$$



unstable.

$$b+r < 2j \text{ unstable.}$$

$$b+r \geq 2j \text{ unstable if truss}$$

support reactions are concurrent

or parallel or if some of the components

of the truss form a collapsible mechanism

* Example 1

Classify each of the trusses in Fig 3-18 as stable, unstable, statically determinate or statically indeterminate. The trusses are subjected to arbitrary external loadings that are assumed to be known and can act anywhere on the trusses.

a) $r=3, b=19, j=11$

$$2j = 2(11) = 22$$

$$2j = b + r = 22 \rightarrow \text{det}$$

External stability $\rightarrow$ concurrent $\rightarrow$ unstable.

$\rightarrow$ parallel $\rightarrow$ unstable

Not parallel and not concurrent $\rightarrow$ Stable

b) $r=4, b=15, j=9$

$$2j = 2(9) = 18$$

$$b + r = 19 > 2j = 18 \rightarrow \text{ind}$$

External stability $\rightarrow$ Stable

Internal stability $\rightarrow$ Stable

c) Compound truss (Type 2)

$$r=3, b=9, j=6$$

$$2j = 2(6) = 12$$

$$r + b = 12 = 2j \rightarrow \text{det}$$

External stability $\rightarrow$ Stable

Internal stability $\rightarrow$ Stable

d) $r=3, b=12, j=8$

$$2j = 2(8) = 16$$

$$r + b = 15 < 2j = 16 \rightarrow \text{unstable}$$

$\hookrightarrow$ one member $\leftarrow$ unstable

* Determination of the member Forces

- ▶ The method of joints
- ▶ The Method of sections.

* The Method of joints.

Compression ← joints إذا كان السهم خارجاً ← * 15

Tension ← joints إذا كان السهم داخلياً ←

Tension ← member إذا كان السهم خارجاً ← *

Compression ← member إذا كان السهم داخلياً ←

Tension ← Positive الإيجابية ←

Tension إذا كان السهم خارجاً ← *

Compression ← Negative الإيجابية ←

* Example 2 :-

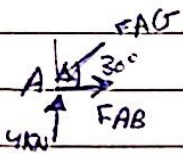
Truss → compound → Type 1 → * connected with joint and bar

Symmetry → ① Geometry, ② load.

* التناظر في الشكل 15

* Example 2 :-Solution $\rightarrow$

symmetric with respect to both loading and geometry.

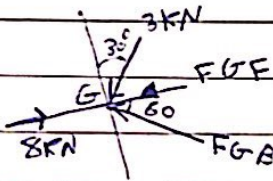
start the analysis at joint A $\rightarrow$ two unknown unknown

$$+\uparrow \sum F_y = 0$$

$$4 - F_{AG} \sin 30^\circ = 0 \rightarrow F_{AG} = 8 \text{ kN (C)}$$

$$+\rightarrow \sum F_x = 0$$

$$F_{AB} - 8 \cos 30^\circ = 0 \rightarrow F_{AB} = 6.928 \text{ kN (T)}$$

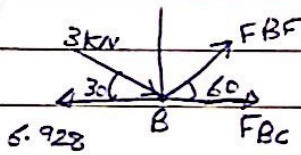


$$+\uparrow \sum F_y = 0$$

$$F_{GB} \sin 60^\circ - 3 \cos 30^\circ = 0 \rightarrow F_{GB} = 3 \text{ kN (C)}$$

$$+\rightarrow \sum F_x = 0$$

$$8 - 3 \sin 30^\circ - 3 \cos 60^\circ - F_{GF} = 0 \rightarrow F_{GF} = 5 \text{ kN (C)}$$



$$+\uparrow \sum F_y = 0$$

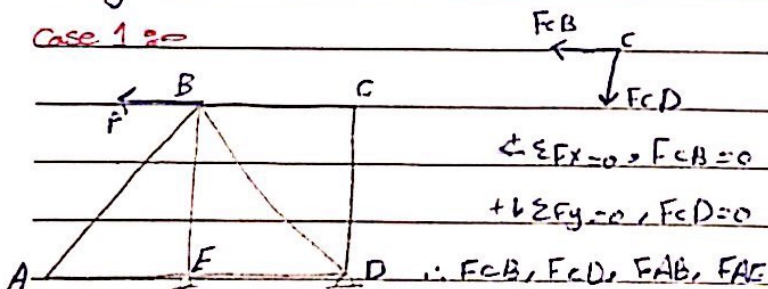
$$F_{BF} \sin 60^\circ - 3 \sin 30^\circ = 0 \rightarrow F_{BF} = 1.73 \text{ kN (T)}$$

$$+\rightarrow \sum F_x = 0$$

$$F_{BC} + 1.73 \cos 60^\circ + 3 \cos 30^\circ - 6.928 = 0 \rightarrow F_{BC} = 3.46 \text{ kN (T)}$$

* Zero-Force Members.

These zero-force members may be necessary for the stability of the truss during construction & to provide support if the applied loading is changed.

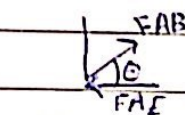
Case 1 :-

$$+\leftarrow \sum F_x = 0 \rightarrow F_{CB} = 0$$

$$+\uparrow \sum F_y = 0 \rightarrow F_{CD} = 0$$

$$\therefore F_{CB}, F_{CD}, F_{AB}, F_{AE}$$

are zero force member



$$+\uparrow \sum F_y = 0 \rightarrow F_{AB} \sin \theta = 0$$

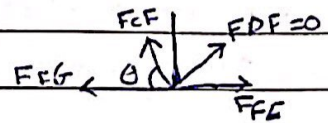
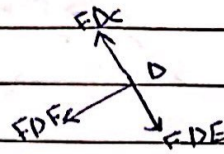
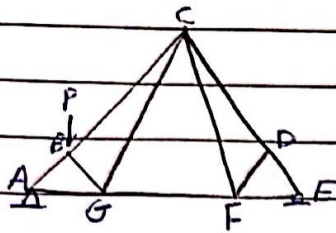
$$F_{AB} = 0 \text{ (since } \sin \theta \neq 0)$$

$$+\rightarrow \sum F_x = 0 \rightarrow -F_{AE} \cos \theta = 0$$

$$F_{AE} = 0$$

Case 2:-

Zero force members also occur at joints having a geometry as joint D.



$$\rightarrow \sum F_y = 0, F_{DF} = 0$$

$$\uparrow \sum F_y = 0, F_{CF} \sin \theta + 0 = 0$$

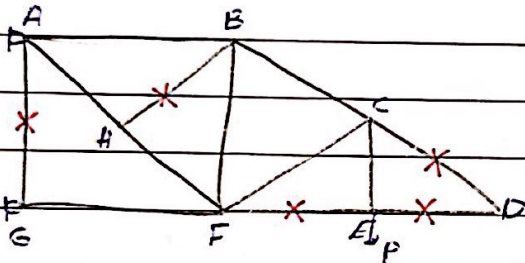
$$F_{CF} = 0 \text{ (since } \sin \theta \neq 0 \text{)}$$

→ F_{DF}, F_{CF}

are zero force member.

*Example 3:-

Find the zero-force members of the truss shown in Fig 3-24a.



*The Method of sections:-

*Example 4:-

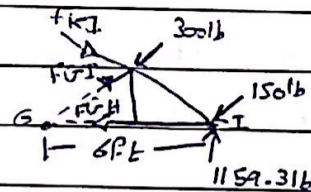
Solution

$$\cos 30^\circ = \frac{3}{5}$$

$$\uparrow \sum M_I = 0$$

$$-FGI \sin 30^\circ (6) + 300(3.464) = 0$$

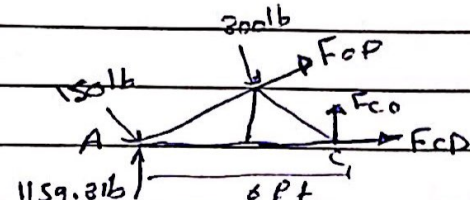
$$\rightarrow FGI = 346.4 \text{ lb (C)}$$



$$\uparrow \sum M_A = 0$$

$$-300(3.464) + F_{CD}(6) = 0$$

$$\rightarrow F_{CD} = 173.1 \text{ lb (T)}$$

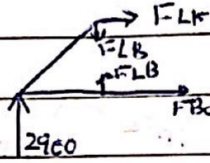


*** Example 5:-**

$$\sum M_L = 0$$

$$-2900(15) + F_{BC}(20) = 0$$

$$F_{BC} = 2175 \text{ lb (T)}$$

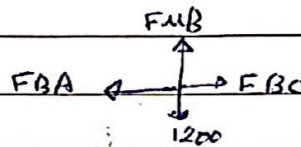


$$+\uparrow \sum F_y = 0$$

$$2900 - 1200 + 1200 - F_{ML} = 0$$

$$F_{ML} = 2900 \text{ lb (T)}$$

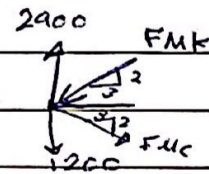
$$\text{and } F_{MB} = 1200 \text{ lb (T)}$$



$$+\rightarrow \sum F_x = 0 \rightarrow \left(\frac{3}{\sqrt{13}}\right) F_{MC} - \left(\frac{3}{\sqrt{13}}\right) F_{MK} = 0$$

$$+\uparrow \sum F_y = 0 \rightarrow 2900 - 1200 - \left(\frac{2}{\sqrt{13}}\right) F_{MC} - \left(\frac{2}{\sqrt{13}}\right) F_{MK} = 0$$

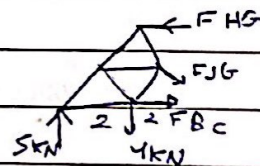
$$F_{MK} = 1532 \text{ lb (C)}, \quad F_{MC} = 1532 \text{ lb (T)}$$

*** Example 6:-**

$$\sum M_C = 0$$

$$-5(4) + 4(2) + F_{HG}(4 \sin 60^\circ) = 0$$

$$F_{GH} = 3.46 \text{ kN (C)}$$



* Chapter 4: Internal loadings Developed in Structural Members.

The internal load at a specified point in a member can be determined by using the method of section.

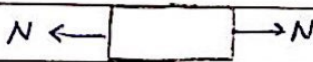
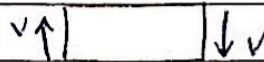
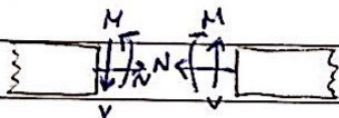
→ This consists of:-

* N. Normal Force

* V. shear Force

* M. bending moment

→ sign convention.



* Example 1:-

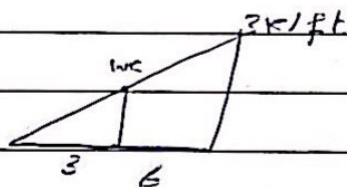
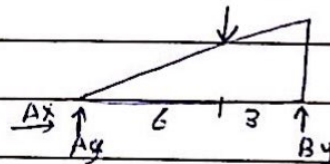
$$\sum \mathcal{M}_A = 0$$

$$-13.5(6) + B_y(9) = 0 \rightarrow B_y = 9 \text{ k}$$

$$\frac{1}{2} \times 3 \frac{\text{k}}{\text{ft}} \times 9 \text{ ft} = 13.5 \text{ k}$$

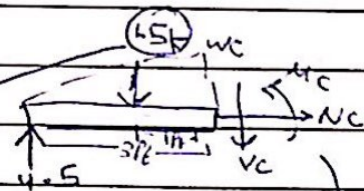
$$\sum F_y = 0$$

$$9 + A_y - 13.5 = 0 \rightarrow A_y = 4.5 \text{ k}$$



$$\frac{3}{9} = \frac{w_c}{3} \rightarrow w_c = 1 \text{ k/ft}$$

$$= \frac{1}{2} \times 1 \times 3 = 1.5 \text{ k}$$



هذا الفرق

$$\sum F_y = 0 \quad 4.5 - 1.5 - V_c = 0 \rightarrow V_c = 3 \text{ k}$$

$$\sum \mathcal{M}_C = 0 \quad -4.5(3) + 1.5(1) + M_c = 0$$

$$M_c = 12 \text{ k.ft.}$$

sign convention

راجع لسلالات الكتل رانث الزاوية (7, 8, 9)

$$\frac{dM}{dx} = V \quad \rightarrow \quad \frac{dV}{dx} = w$$

Moment $\rightarrow$ Shear
Shear $\rightarrow$ load

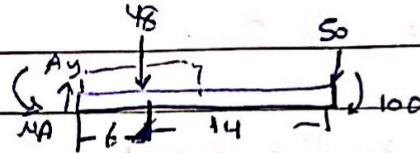
*Example 2:-

$$+\circlearrowleft \sum M_A = -48(6) - 60(20) - 100 + M_A = 0$$

$$M_A = 1588 \text{ k}\cdot\text{ft}$$

$$+\uparrow \sum F_y = 0$$

$$-48 - 60 + A_y = 0 \rightarrow A_y = 108 \text{ k}$$



$$0 \leq x_1 \leq 12 \text{ ft}$$

$$+\uparrow \sum F_y = 0 \rightarrow 108 - 48 - V = 0 \rightarrow V = 108 - 4x_1$$

$$+\circlearrowleft \sum M_S = 0 \rightarrow 1588 - 108x_1 + 48\left(\frac{x_1}{2}\right) + M = 0$$

$$M = -1588 + 108x_1 - 2x_1^2$$

$$\frac{dM}{dx} = 108 - 4x_1 = V$$

$$\frac{dV}{dx} = -4x_1 = w$$

$\rightarrow$ Lecture 17

$$12 \text{ ft} \leq x_2 \leq 20 \text{ ft}$$

$$+\uparrow \sum F_y = 0 \rightarrow 108 - 48 - V = 0 \rightarrow V = 60$$

$$+\circlearrowleft \sum M_S = 0 \rightarrow 1588 - 108x_2 + 48(x_2 - 6) + M = 0 \rightarrow M = 60x_2 - 1300$$

section

$$\frac{dM}{dx} = 60 = V, \quad \frac{dV}{dx} = 0 = w \leftarrow \text{No load!}$$

* Example 3:

$$\sum M_A = 0$$

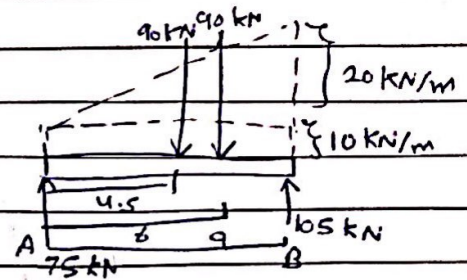
$$-90(4.5) - 90(6) + 9B_y = 0$$

$$\rightarrow B_y = 105 \text{ kN}$$

$$\sum F_y = 0$$

$$A_y - 90 - 90 + 105 = 0$$

$$\rightarrow A_y = 75 \text{ kN}$$

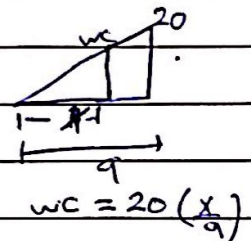


$$+\uparrow \sum F_y = 0, 75 - 10x - \left[\frac{1}{2}(20)\left(\frac{x}{9}\right)x \right] - V = 0$$

$$V = 75 - 10x - 1.11x^2$$

$$(\sum M_s = 0, -75x + (10x)\left(\frac{x}{2}\right) + \left[\frac{1}{2}(20)\left(\frac{x}{9}\right)x \right]\frac{x}{3} + M = 0$$

$$M = 75x - 0.370x^3 - 5x^2$$



$$\begin{aligned} & \text{Shear Force Diagram (V)} \\ & \text{Shear Force (V)} \end{aligned}$$

TABLE 4.1 Relationship between Loading, Shear, and Moment

Loading	Shear Diagram, $\frac{dV}{dx} = -w$	Moment Diagram, $\frac{dM}{dx} = V$
	zero slope	slope = V_L slope = V_R positive constant slopes
	zero slope	zero slope
	constant slope	slope = V_L slope = V_R positive decreasing slope
	slope = $-w_1$ slope = $-w_2$ negative increasing slope	slope = V_L slope = V_R positive decreasing slope
	slope = $-w_1$ slope = $-w_2$ negative decreasing slope	slope = V_L slope = V_R positive decreasing slope

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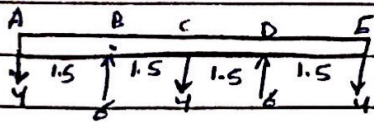
Geometric Properties of Areas

Triangle	$A = \frac{1}{2}bh$ $\bar{x} = \frac{1}{3}b$
Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$ $\bar{x} = \frac{h(2b_1 + b_2)}{3(b_1 + b_2)}$
Semi Parabola	$A = \frac{2}{3}bh$ $\bar{x} = \frac{1}{8}b$
Parabolic spandrel	$A = \frac{1}{3}bh$ $\bar{x} = \frac{1}{4}b$
Semi-segment of nth degree curve	$A = bh \left(\frac{n}{n+1} \right)$ $\bar{x} = \frac{b(n+1)}{2(n+2)}$
Spandrel of nth degree curve	$A = bh \left(\frac{1}{n+1} \right)$ $\bar{x} = \frac{b}{n+2}$

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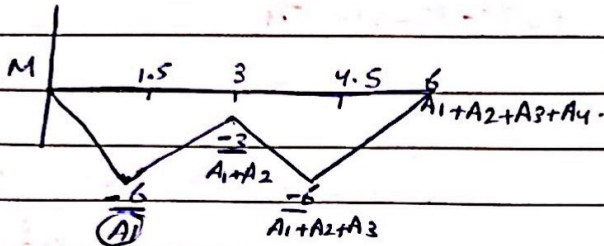
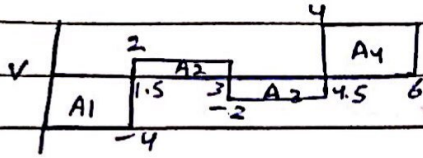
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#Example 4:-

 $w=0 \rightarrow v \text{ slope}=0$

$$A_1 = -4 + 1.5 = -2.5, \quad A_3 = -2 + 1.5 = -0.5$$

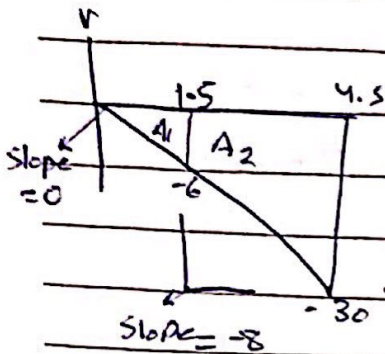
$$\sim A_2 = 2 + 1.5 = 3.5, \quad A_4 = 4 + 1.5 = 5.5$$



***Example 5.**

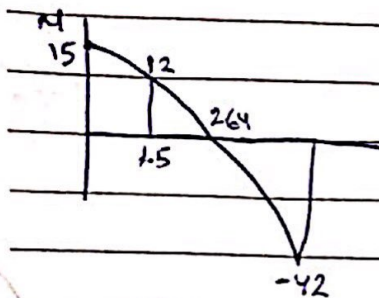
$$R_1 = \frac{1}{2} \times 8 \times 1.5 = 6$$

$$R_2 = -8 \times 3 = -24$$



$$A_1 = \frac{1}{3} \times b \times h = \frac{1}{3} \times 1.5 \times 6 = -3$$

$$A_2 = \frac{1}{2} b (h_1 + h_2) = \frac{1}{2} \times 3 \times (-30 - 6) = -54$$



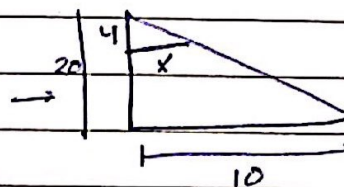
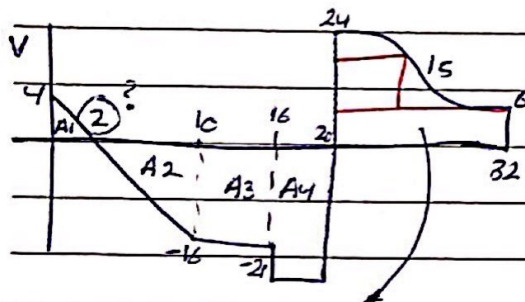
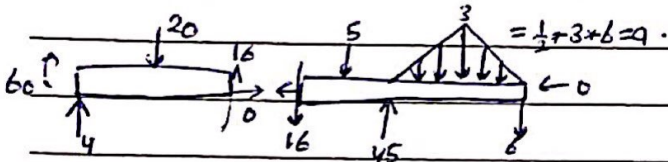
↻ clockwise (+value)

↻ counter-clockwise (-value)

***Example 6:**

$$n=2 \rightarrow 3n=6$$

$$r=6 \rightarrow r=3n=6 \rightarrow \text{det}$$



$$\frac{20}{10} = \frac{4}{x} \rightarrow \boxed{x=2}$$

$$A_1 = \frac{1}{2} \times 4 \times 2 = 4$$

$$A_2 = \frac{1}{2} \times 8 \times -16 = -64$$

$$A_3 = -16 \times 6 = -96$$

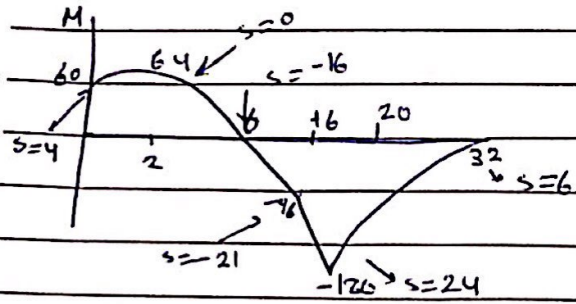
$$A_4 = -21 \times 4 = -84$$

$$a_1 = 6 \times 12 = 72$$

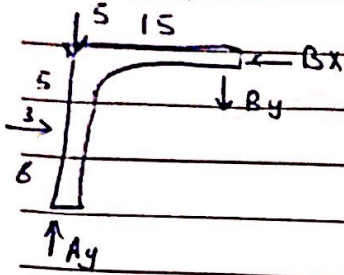
$$a_2 = 6 \times 9 = 54 \quad a_4 = \frac{2 \times 6 \times 9}{3} = 36$$

$$a_3 = \frac{1}{3} \times 6 \times 9 = 18$$

$$\text{Total } A = 72 + 54 + 18 + 36 = 180$$



* Example 7 :-

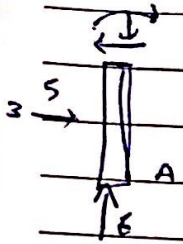


$$(+\sum M_B = 5 \times 15 + 3 \times 5 - A_y(5) = 0$$

$$A_y = 6 \text{ k}$$

$$+\uparrow \sum F_y = 6 - 5 - B_y = 0 \rightarrow B_y = 1 \text{ k}$$

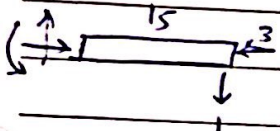
$$\rightarrow \sum F_x = -B_x + 3 = 0 \rightarrow B_x = 3 \text{ k}$$



$$\sum M_C = 3(5) - M_{CL} = 0 \rightarrow M_{CL} = 15 \text{ k}\cdot\text{ft}$$

$$\sum F_x = 3 - C_x L = 0 \rightarrow C_x L = 3 \text{ k}$$

$$\sum F_y = 6 - C_y L = 0 \rightarrow C_y L = 6 \text{ k}$$



$$(+\sum M_C = -1(15) + M_{CR} = 0 \rightarrow M_{CR} = 15 \text{ k}\cdot\text{ft}$$

$$\rightarrow \sum F_x = -3 + C_x R = 0 \rightarrow C_x R = 3 \text{ k}$$

$$+\uparrow \sum F_y = -1 + C_y R = 0 \rightarrow C_y R = 1 \text{ k}$$

