New
2018

Civilittee

اللجنة الأكاديمية للهندسة المدنية

دفتر

تحليل إنشائي

هناء أبو علّوش

Contact us :

Civilittee HU | لجنة المدني

Civilittee Hashemite

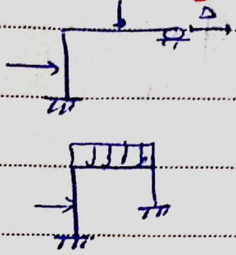
www.civilittee-hu.com

+ Moment Distribution frames.

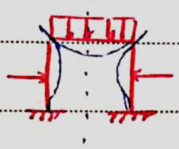
لا يتحرك في حالتين . without side sway

with side sway

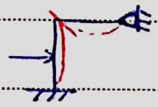
لا يوجد تماثل سوف ينزاح



1 symetey in shape and load



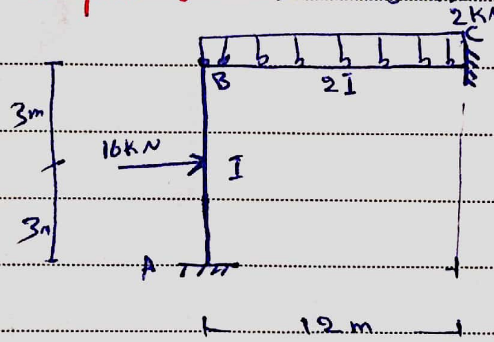
عندما يحدث إزاحة نضع support 2 supported againest sway.



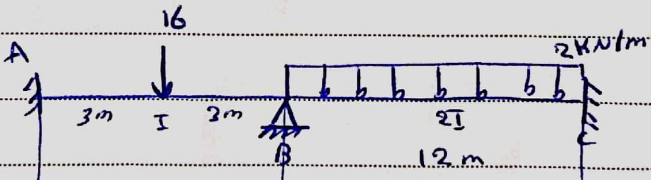
without sway

Example :- E: constant S.D and M.D ??

← اول طريقة انه نفرد الـ frame ومكان كل
corner نضع support

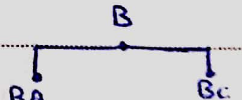


ind truss
+4 Beam
[illegible]



D.F	0	0.5	0.5	0
F.E.M	-12	12	-24	24
Bal	0	+6	+6	0
C.O.M	3	0	0	3
Bal	0	0	0	0
Σ Moments	-9	18	-18	27

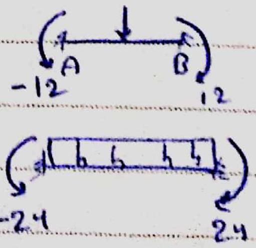
$\frac{I}{L}$

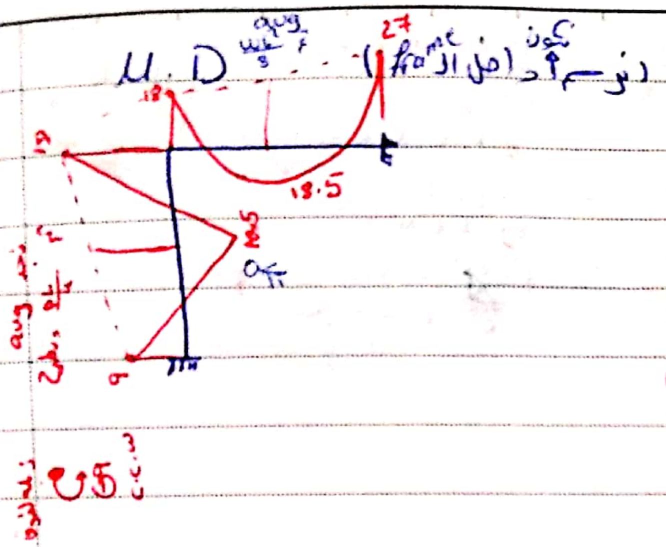


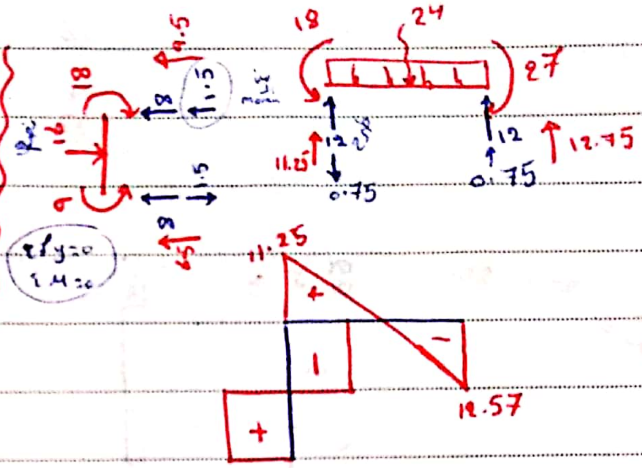
$\frac{1}{6}$ $\frac{2}{12}$

D.F 0.5 0.5

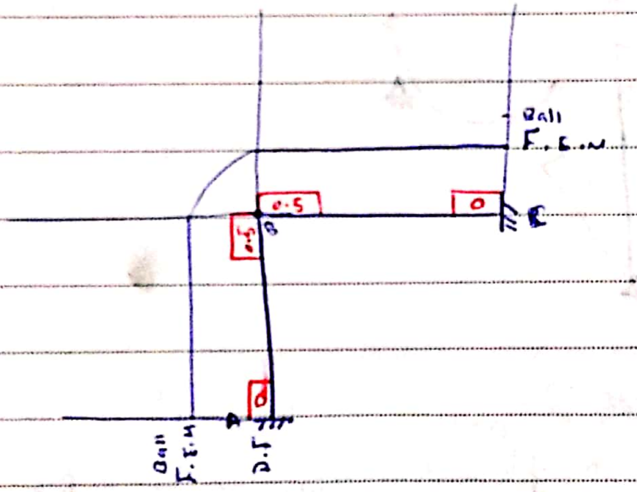
F.E.M



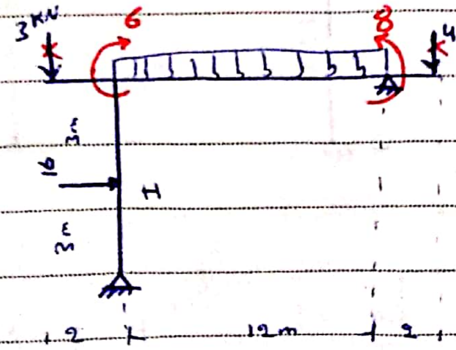


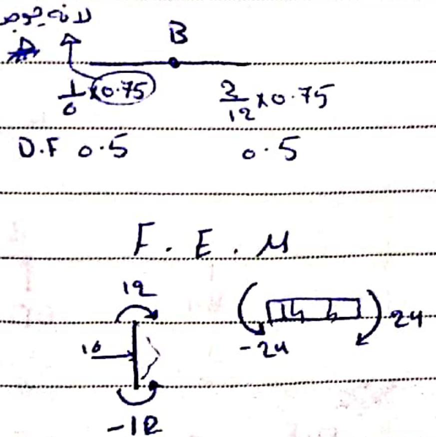


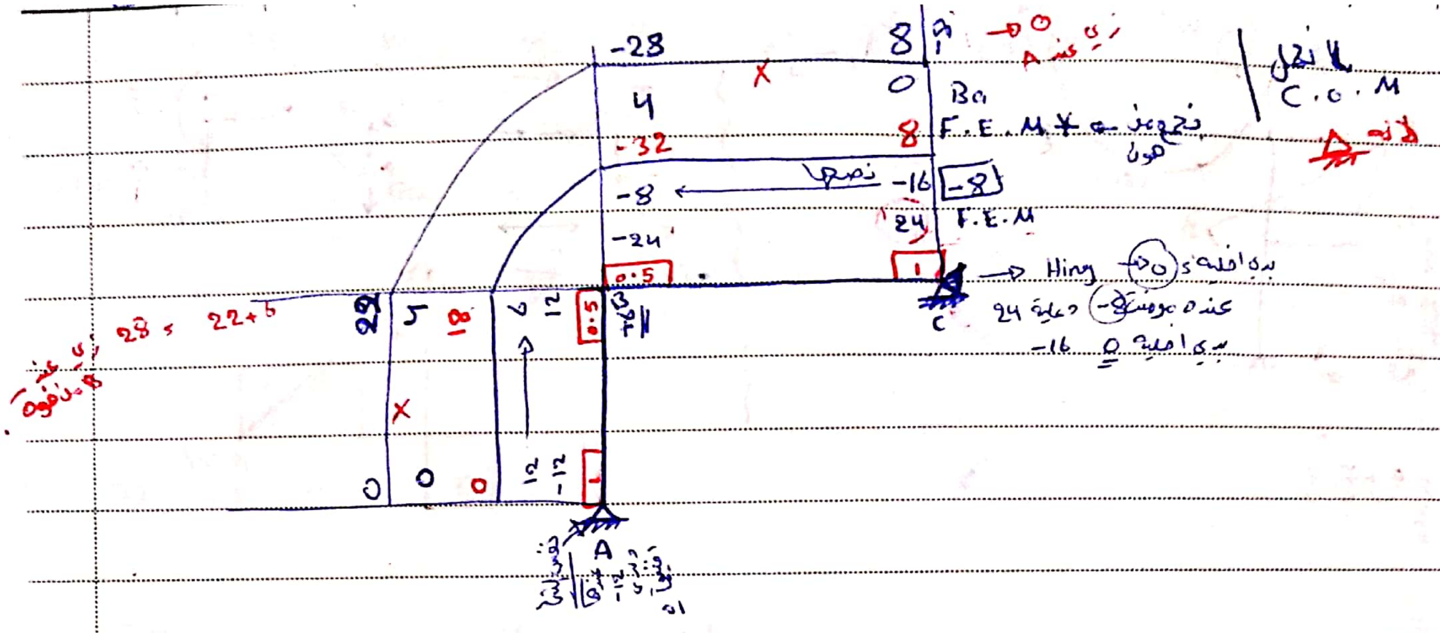
* طريقة ثانية على نفس الـ frame.

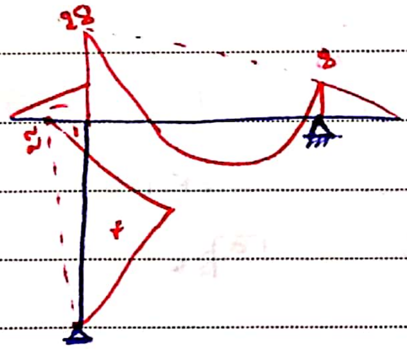


Example :-

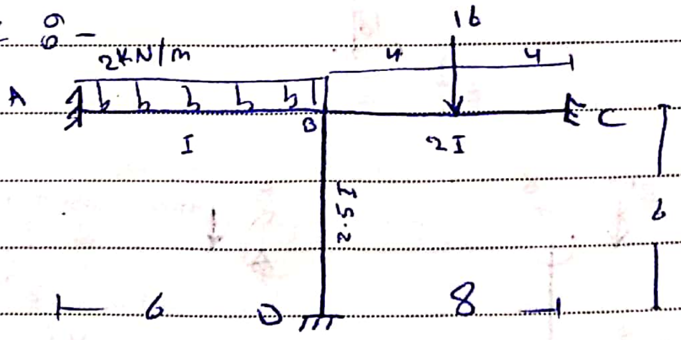








Example ٥:-



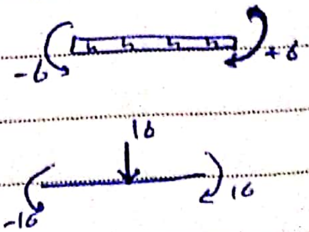
B

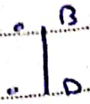
BA	BD	BC
$\frac{1}{6}$	$\frac{2.5}{6}$	$\frac{2}{8}$

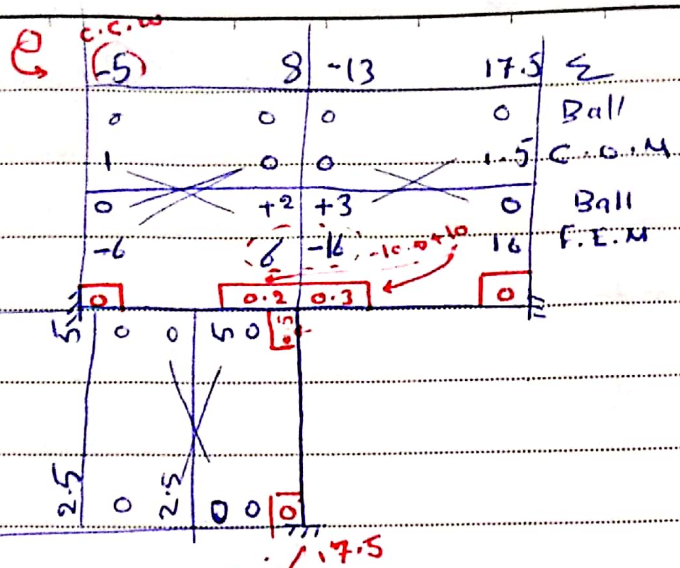
$\frac{K}{\Sigma K}$

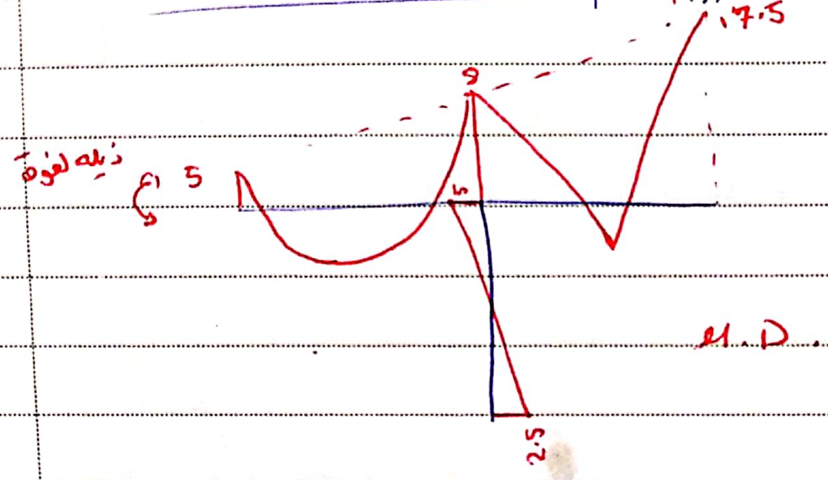
D.F 0.2 : 0.5 : 0.3

F . E . M

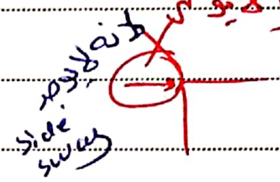




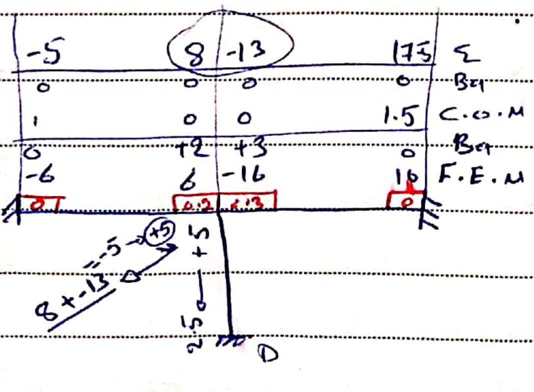




ال load الذي
يؤثر في F.E.M
يكون في member
اما الافقي لا يؤثر



نفس ال frame ⇐ اي ه member ما عليه load ممكن نستخدمه في حساب D.F لكن بحساب M لا ندخله ثم عندما ننتهي من ال members الثانية نأخذ ال moment عند النقطة المتصلة معه ونعكس إشارته وتكون قيمة ال moment على الجهة الثانية منه نصها.



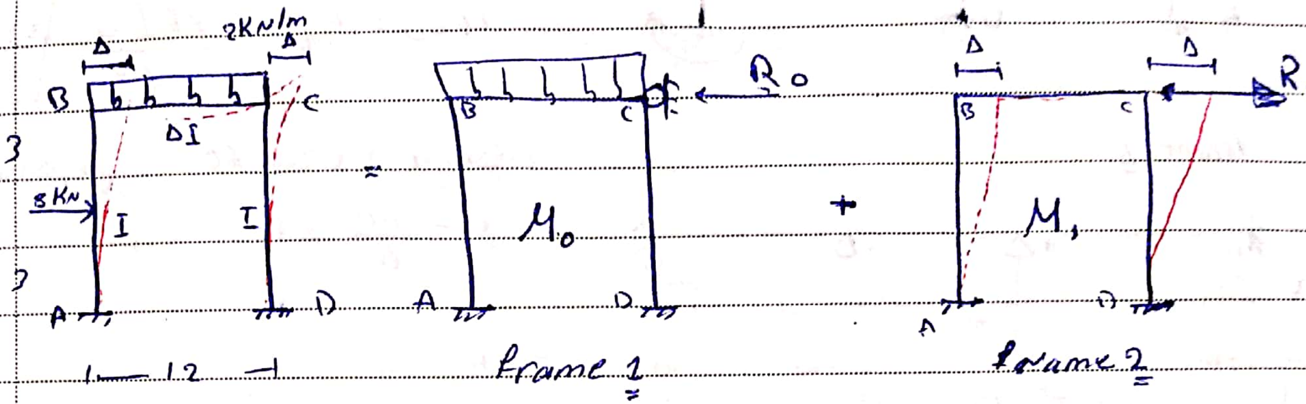
اذا كان fix End نرجع ونعطي نصه
اما اذا كان pin نغير 0
لا يعطي

Day: Monday NO. Date: 18/3/2019

Moment Distribution

Frames with sidesway

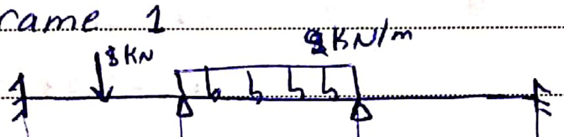
No symetry or No support against sway



* نفرض انه في سند يمنع الحركة

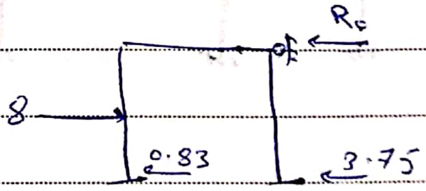
$-R_o + \Delta R_1 = 0$

Frame 1



[illegible] corner support

D.F	0	0.5	0.5	0.5	0.5	0
F.EM	-6	6	-24	24	0	0
Bal	0	+9	+9	-12	-12	0
C.O.M	4.5	0	-6	4.5	0	-6
Bal	0	+3	+3	-2.25	-2.25	0
C.O.M	1.5	0	-1.125	1.5	0	-1.125
Bal	0	0.562	0.562	-0.75	-0.75	0
C.O.M	0.281	0	-0.375	0.281	0	-0.375
Bal	0	-0.1875	-0.1875	-0.1405	-0.1405	0
	6.28	1.96	-1.876	15.14	-15.14	-7.4

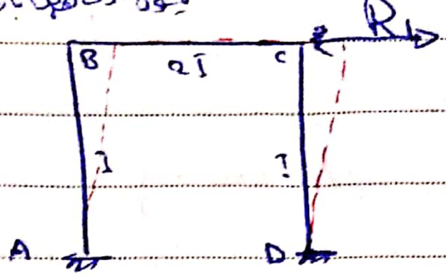


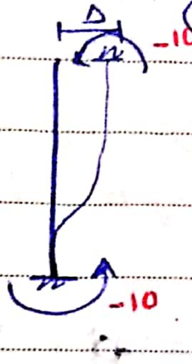
$\Sigma F_x = 0$

$8 - 0.83 - 3.75 - R_o = 0$

$R_o = 3.42$

Δ يكون ازاحتهم باتجاه





F.EM

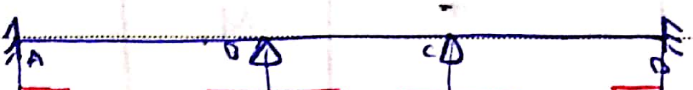
$M_{AB} = M_{BA} = \frac{6EI\Delta}{6^2} = \boxed{\frac{EI\Delta}{6}}$

$M_{CD} = M_{DC} = \frac{6EI\Delta}{6^2} = \boxed{\frac{EI\Delta}{6}}$

frame 2

Assume $EI\Delta = 60$ نفرض أي رقم

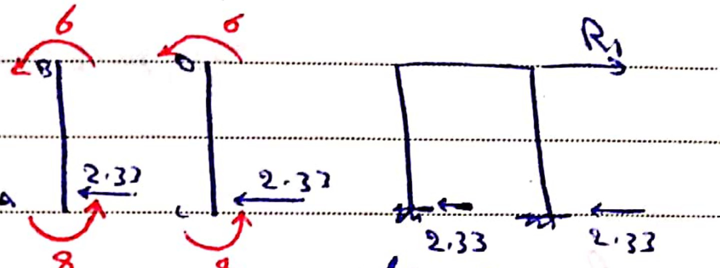
$M = \frac{60}{6} = 10$



	A	B		C		D
D.F	0	0.5	0.5	0.5	0.5	0
F.E.M	-10	-10	0	0	-10	-10
Ball	0	+5	+5	+5	+5	0
C.O.M	2.5	0	2.5	2.5	0	2.5
Ball	0					
C.O.M						
Ball						
$M_1 = \Sigma M$	-8	-6	6	+6	-6	-8

+10

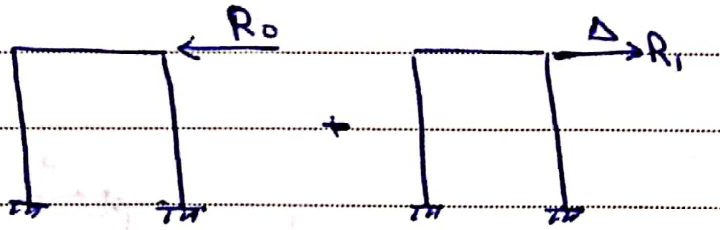
Reaction نجيب



$\Sigma F_x = 0$

$R_1 - 2.33 - 2.33 = 0$

$R_1 = 4.66$

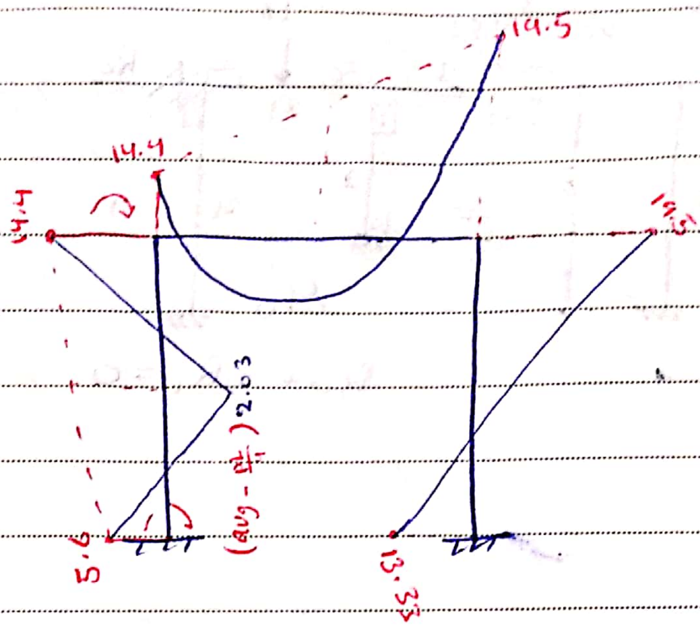


$\Rightarrow \ominus R_0 + \Delta R_1 = 0$

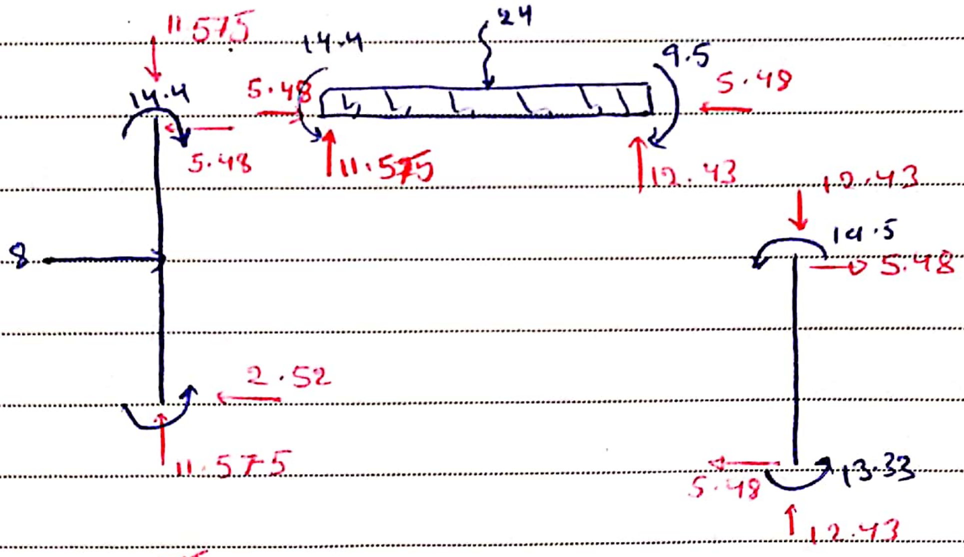
$-3.42 + \Delta \cdot 4.66 = 0$

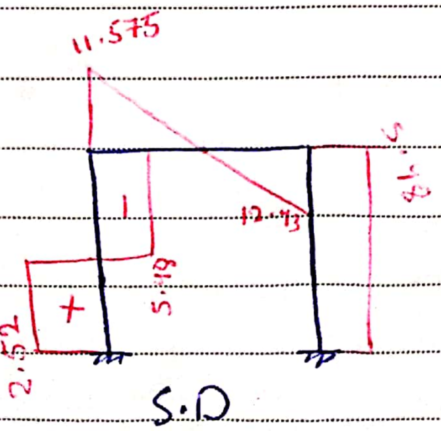
$\boxed{\Delta = +0.724}$

Moment	M_0	M_1	$M_{final} = M_0 + \Delta M_1$	M_{final} c.w
M_{AB}	0.28	-8	0.28 + (0.724) · -8	⊖5.6
M_{BA}	18.76	-6		14.4
M_{BC}	-18.76	+6		-14.4
M_{CB}	15.14	+6		19.5
M_{CD}	-15.14	-6		-19.5
M_{DC}	-7.4	-8		-13.33

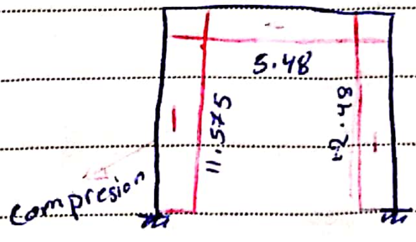

19.5
14.4
14.4
14.5
2.03
(avg - $\frac{wl^2}{8}$)
5.6
13.33

M.D


11.575
14.4
24
9.5
5.48
5.48
14.4
5.48
11.575
12.43
8
2.52
11.575
12.43
14.5
5.48
5.48
13.33
12.43


11.575
12.43
5.48
+
2.52
5.48

S.D

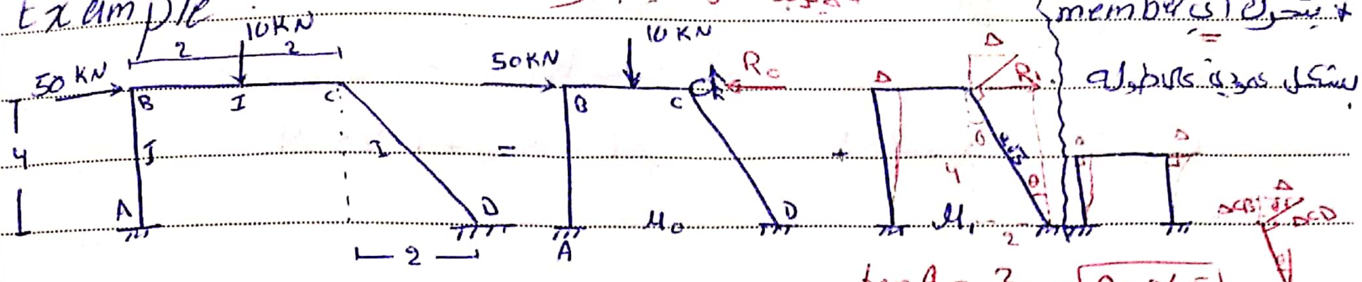

5.48
11.575
12.48
Compresion

M.F.D

Example

sway يوجد symmetry لا يوجد

* يتحرك أي members بشكل عمودي على طوله



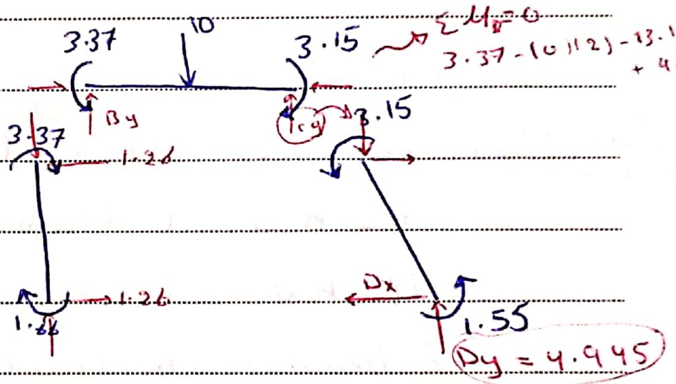
$\tan\theta = \frac{2}{4} \Rightarrow \boxed{\theta = 26.5}$

$* \; \Delta_{CB} = \Delta \tan\theta \qquad \boxed{\Delta_{CB} = 0.5\,\Delta}$

$$\frac{\Delta_{CB}}{\Delta} = \tan\theta$$

$* \; \Delta_{CD} = \frac{\Delta}{\cos\theta} \qquad \boxed{\Delta_{CD} = 1.1\,\Delta}$

	A	B		C		D
D.F	0	0.5	0.5	0.58	0.42	0
F.E.M	0	0	-5	5	0	0
Ball						
C.O.M						
Ball						
$M_0 = \Sigma M$	1.66	3.37	-3.37	3.5	-3.15	-1.56



$\Sigma M_B = 0$

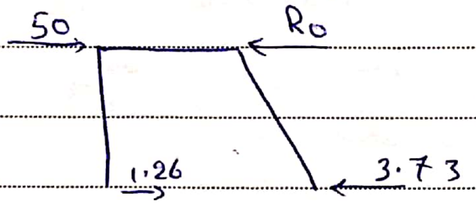
$3.37 - (10)(2) - 3.15 + 4Cy = 0$

$\boxed{Dy = 4.945}$

$\Sigma M_C = 0$

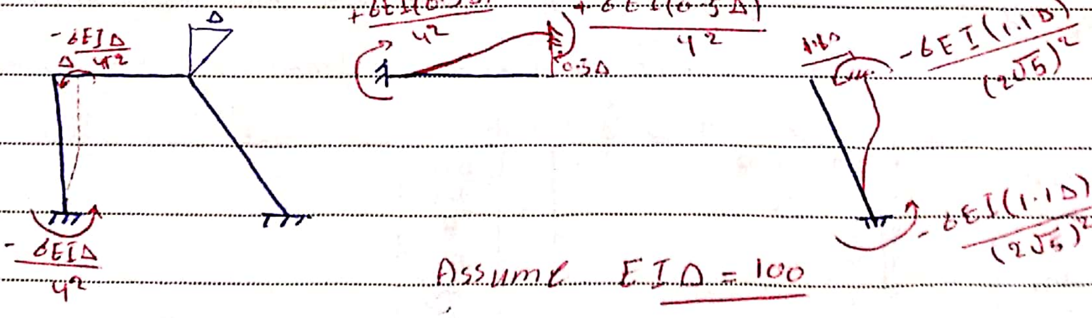
$3.15 + 1.55 + -4Dx + 2 \; ---$

$Dx = ---$



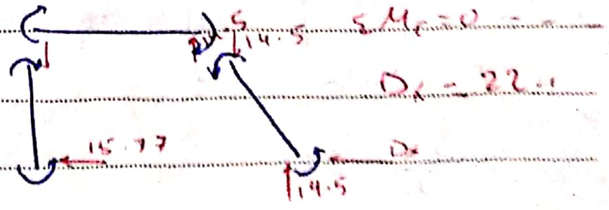
$\Sigma F_x = 0 \qquad 50 - R_0 + 1.26 - 3.73 = 0$

$\boxed{R_0 = 47.53}$



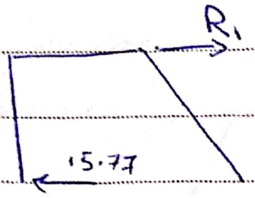
Assume $EI\Delta = 100$

D.F	0	0.5	0.5	0.58	0.42	0
F.E.M						
M_1	-37.8	-29.69	+29.69	29.46	-29.46	-30.48



$\Sigma M_c = 0 \; --$

$D_x = 22.1$



$R_1 = 37.8$

$R_0 = \Delta R_1$ . . . $\Delta = 1.26$

$M = M_0 + 1.26\,M_1$

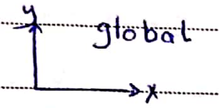
#14 , 15

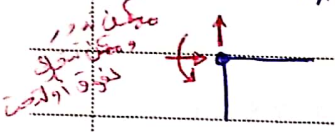
Displacement method ⇒ Stiffness Method

$F = K\Delta$ (Normal or shear)

$M = K\theta$ (Moment)

if Beam is horizantal ⇒ gloBal and local ⇒ نفسهم



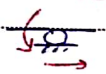


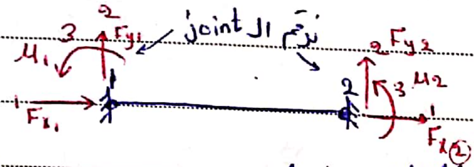
3.Degree of Redam (2 يدور يتحرك)

1.D of Redam

فقط تدور لأنه لا تتحرك على x أو y

2.D of Redam





عكس ال Destribution moment

$\boxed{+ve\ M \Rightarrow C.C.W}$ هنا

each fix has 3 unknown. Deggree of redam

لو ما في Normal بيروحوا

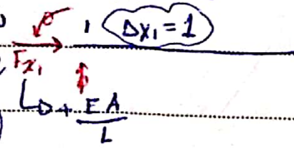
1 عند Stiffness Matrix

Beam ⇒ نادرا ما يوجد Normal فتروح القوى الأفقية ولا يحسب الا اذا في deflection اللاتالية لذلك بنشطب القوى الجوية.

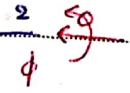
$$\begin{bmatrix} F_{1x} \\ F_{1y} \\ M_1 \\ F_{2x} \\ F_{2y} \\ M_2 \end{bmatrix} = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & \frac{EA}{L} & 0 & 0 & & & \\ 2 & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & & & \\ 3 & 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & & & \\ 4 & -\frac{EA}{L} & 0 & & & & \\ 5 & 0 & \frac{-12EI}{L^3} & 0 & & & \\ 6 & 0 & +\frac{6EI}{L^2} & -\frac{6EI}{L^2} & & & \\ & & & \frac{2EI}{L} & & & \end{array} \begin{bmatrix} \Delta x_1 \\ \Delta y_1 \\ \theta_1 \\ \Delta x_2 \\ \Delta y_2 \\ \theta_2 \end{bmatrix}$$

كله يتغير Joint 1

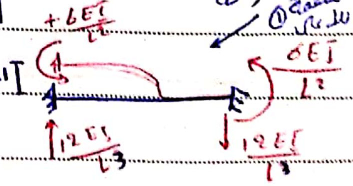
D1 شتحصل ينتج قوة $\boxed{+\frac{EA}{L}}$



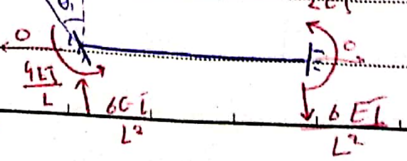
$L \Rightarrow +\frac{EA}{L}$



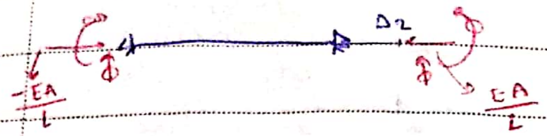
$F_{x_2} = -\frac{EA}{L}$

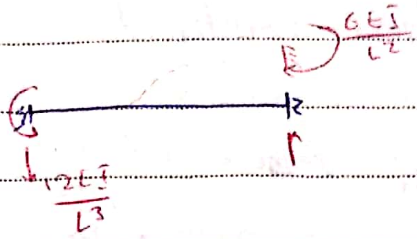


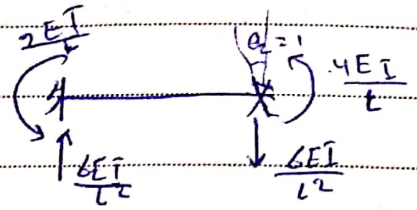
دوران النقطة $\theta = 1$ ①



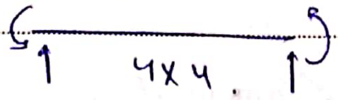
اذا طبقت moment عند نقطة يكون نصها عند الثانية







⇒ Normal لوحاي



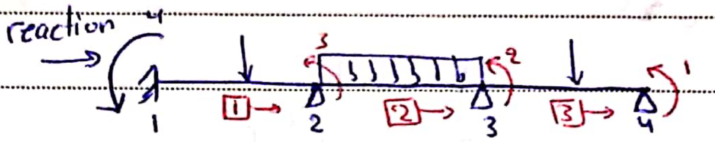
$$\begin{bmatrix} \frac{12EI}{L^3} & \\ \vdots & \\ \vdots & \end{bmatrix}$$

⇒ y↑ shear لوحاي

2 X 2

$$\begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix} = \frac{2EI}{L}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

reaction →



① نرقم ال member

② نرقم ال joint

③ نضع ال redum

K member

member is الاتجاه ④ 3

$$\boxed{1}\quad \frac{2EI}{L}\begin{array}{cc} & \begin{array}{cc} \textcircled{4} & 3 \end{array} \\ \begin{array}{c} \textcircled{4} \\ 3 \end{array} & \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \end{array}$$

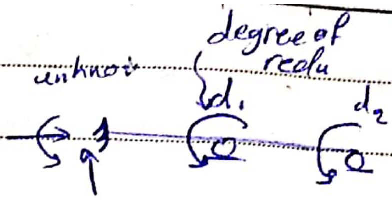
$$\boxed{2}\quad \frac{2EI}{L}\begin{array}{cc} & \begin{array}{cc} 3 & 2 \end{array} \\ \begin{array}{c} 3 \\ 2 \end{array} & \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \end{array}$$

⇒ structral stiffness $\boxed{3}$

$$[F] = [K][\Delta] + [F.E.M]$$

force member

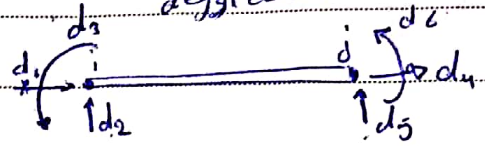
+ stiffness Method



Beams:

نفهمهم في الاتجاه الموجب

degree of redum



$$K_{2\times2} = \frac{2EI}{L}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$$

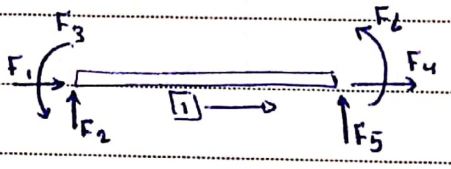
K 6 X 6

البيم نادر وجود قوى افقية فيه

K 4 X 4 → No axial

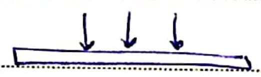
تكون موجودة فقط اذا كان في self

K 2 X 2 → if No axial and No shear



+

member على ال load



بعمل

Fixed end moment

f

force vector on joints

Σ [K] member

Σ [f]

$$[F] = [K][d] + [f]$$

structure stiffness

$$F = KD$$

$$D = [K]^{-1} . F$$

نبدلهم

نغير اشارة بعض

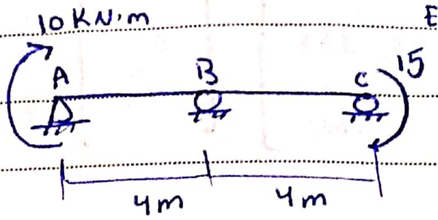
$$A = \begin{bmatrix} a & b\\ c & d\end{bmatrix} \Rightarrow A^{-1} = \frac{1}{\text{Deter}}\begin{bmatrix} d & -b\\ -c & a\end{bmatrix}$$

(ad - bc]

$$\begin{bmatrix}2 & 3 & 1\\ 1 & 4 & 5\\ 2 & 1 & 1\end{bmatrix} + \begin{bmatrix}3\\ 5\\ 1\end{bmatrix} = \Big[$$

المهم شو نفو الناتج

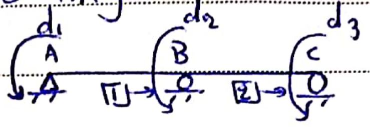
Example.



EI constant ⇒ Draw S.D and M.D

→ Modeling

D. of. Re



displacement matrix

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

الناتجة لل الستركشر كاملة (K 3x3)

$$[F] = [K] \cdot [d]$$

?

عند A

$$[F] = \begin{bmatrix} -10 \\ 0 \\ -15 \end{bmatrix}$$

force vector

القوى على الـ joints

load على الـ member لا يوجد

$$[f] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

fix end moment

member stiffness matrix 2X2 (لأن ال Beam نادر وجود axial ولا يوجد sett لذلك لا يوجد shear)

$$K_1 = \frac{2EI}{4}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = EI \begin{array}{c|c} \begin{matrix} 1 & 2 \end{matrix} & \\ \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} & \begin{matrix} 1 \\ 2 \end{matrix} \end{array}$$

$$K_2 = \frac{2EI}{4}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = EI \begin{array}{c|c} \begin{matrix} 2 & 3 \end{matrix} & \\ \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} & \begin{matrix} 2 \\ 3 \end{matrix} \end{array}$$

structure stiffness matrix $\Sigma [K]$

من التنين 1 + 1

$$[K]_{struc} = \begin{array}{c|c} \begin{matrix} 1 & 2 & 3 \end{matrix} & \\ \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 2 & 0.5 \\ 0 & 0.5 & 1 \end{bmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{array}$$

structure stiffness equation

$$F = Kd + \cancel{f^{0}}$$

$$\begin{bmatrix} -10 \\ 0 \\ -15 \end{bmatrix} = EI \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 2 & 0.5 \\ 0 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 2 & 0.5 \\ 0 & 0.5 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -10 \\ 0 \\ -15 \end{bmatrix}$$

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 1.1666 & -0.333 & 0.1666 \\ -0.333 & 0.666 & -0.333 \\ 0.1666 & -0.333 & 1.1666 \end{bmatrix} \begin{bmatrix} -10 \\ 0 \\ -15 \end{bmatrix}$$

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \frac{1}{EI} \begin{array}{l} \begin{bmatrix} -14.16 \\ 8.33 \\ -19.17 \end{bmatrix} \end{array} \begin{array}{l} A \\ B \\ C \end{array} \text{ rad.}$$

member equation

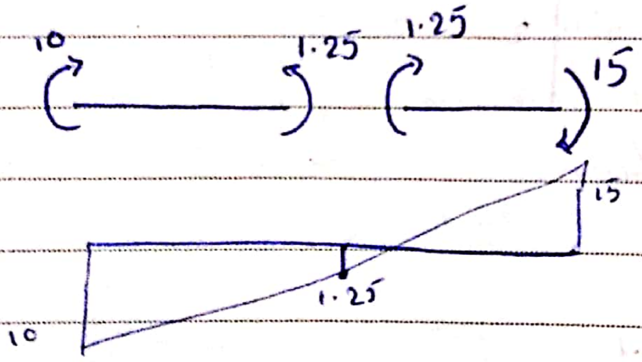
$F = Kd + f^{o}$

$\boxed{1}$

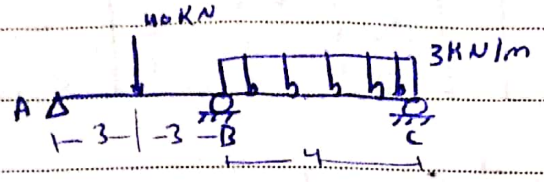
$$\begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = EI \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} * \frac{1}{EI} \begin{bmatrix} -14.16 \\ 8.33 \end{bmatrix} = \begin{bmatrix} \overset{\text{must be}}{\boxed{-10}} \\ 1.25 \end{bmatrix}$$

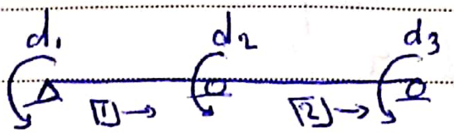
$\boxed{2}$

$$\begin{bmatrix} M_{BC} \\ M_{CB} \end{bmatrix} = EI \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \begin{bmatrix} 8.33 \\ 19.17 \end{bmatrix} = \begin{bmatrix} -1.25 \\ -15 \end{bmatrix}$$



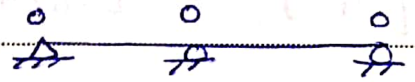
Example



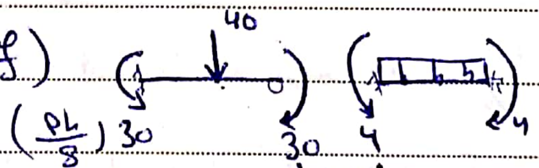


$F = Kd + f$

3 degree of redum K_{stu} 3X3

(F) 

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(f) 

$$\underbrace{30 \quad 4}$$

$$\text{c.w}\quad -30 \overset{+}{+} \overset{-}{4} = -26$$

$$(f) = \begin{bmatrix} 30 \\ -26 \\ -4 \end{bmatrix}$$

$[K]_{member}$

$$[K_1] = \frac{2EI}{L\ \textcircled{6}} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = EI \begin{array}{c} \\ \\ \end{array}\begin{array}{ccc} & 1 & 2 \\ & \left[\begin{matrix} 0.67 \\ \boxed{0.33} \end{matrix}\right. & \left.\begin{matrix} 0.33 \\ 0.67 \end{matrix}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \end{array}$$

$$[K_2] = \frac{2EI}{4} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = EI \begin{array}{ccc} & 2 & 3 \\ & \left[\begin{matrix} 1 \\ 0.5 \end{matrix}\right. & \left.\begin{matrix} 0.5 \\ 1 \end{matrix}\right] \end{array}\begin{array}{c} \\ 2 \\ 3 \end{array}$$

$$K = \Sigma K = \begin{array}{cccc} 1 & 2 & 3 & \\ 0.67 & 0.33 & 0 & 1 \\ 0.33 & 1.62 & 0.5 & 2 \\ 0 & 0.5 & 1 & 3 \end{array}$$

structure equation $\quad F = Kd + f$

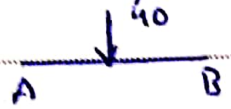
$$\begin{bmatrix}0\\0\\0\end{bmatrix} = EI\begin{bmatrix}0.67 & 0.33 & 0\\0.33 & 1.67 & 0.5\\0 & 0.5 & 1\end{bmatrix}\begin{bmatrix}d_1\\d_2\\d_3\end{bmatrix} + \begin{bmatrix}30\\-26\\-4\end{bmatrix}$$

$$\begin{bmatrix}d_1\\d_2\\d_3\end{bmatrix} = \frac{1}{EI}\begin{bmatrix}0.67 & 0.33 & 0\\0.33 & 1.67 & 0.5\\0 & 0.5 & 1\end{bmatrix}^{-1}\begin{bmatrix}-30\\26\\4\end{bmatrix}$$

$$\begin{bmatrix}d_1\\d_2\\d_3\end{bmatrix} = \frac{1}{EI}\begin{bmatrix}1.68 & -0.39 & 0.19\\-0.39 & 0.79 & -0.39\\0.19 & -0.39 & 1.19\end{bmatrix}\begin{bmatrix}-30\\26\\4\end{bmatrix} = \frac{1}{EI}\begin{bmatrix}-59.96\\30.84\\-11.4\end{bmatrix}$$

member equation

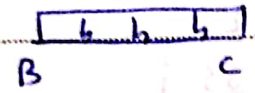
$\boxed{1}$



ن لازم 0 لانه عند A مومنت = صفر A

$$\begin{bmatrix}M_{AB}\\M_{BA}\end{bmatrix} = EI\begin{bmatrix}0.67 & 0.33\\0.33 & 0.67\end{bmatrix}\cdot\frac{1}{EI}\begin{bmatrix}-59.96\\30.84\end{bmatrix} + \begin{bmatrix}30\\-30\end{bmatrix} = \begin{bmatrix}\textcircled{0}\\-30\end{bmatrix}$$

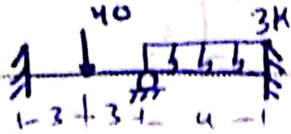
$\boxed{2}$



$$\begin{bmatrix}M_{BC}\\M_{CB}\end{bmatrix} = EI\begin{bmatrix}1 & 0.5\\0.5 & 1\end{bmatrix} + \frac{1}{EI}\begin{bmatrix}30.84\\-11.4\end{bmatrix} + \begin{bmatrix}4\\-4\end{bmatrix} = \begin{bmatrix}30\\\textcircled{0}\end{bmatrix}$$

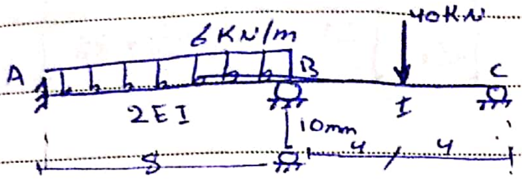
ه لازم صفر لانه عند ~~C~~ مومنت = صفر

$\Rightarrow$

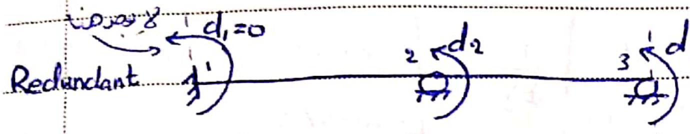


$\Rightarrow$ D. of Redum

Example. $E = 200\ GPa$ $I = 200 \times 10^{6}\ m^4$



* لو في settlement هتحلها 4x4
لكن ممكن نحلها 2x2 ، اهمل
ونعوض السلتمنت في fix e



$$K = \frac{2EI}{L}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

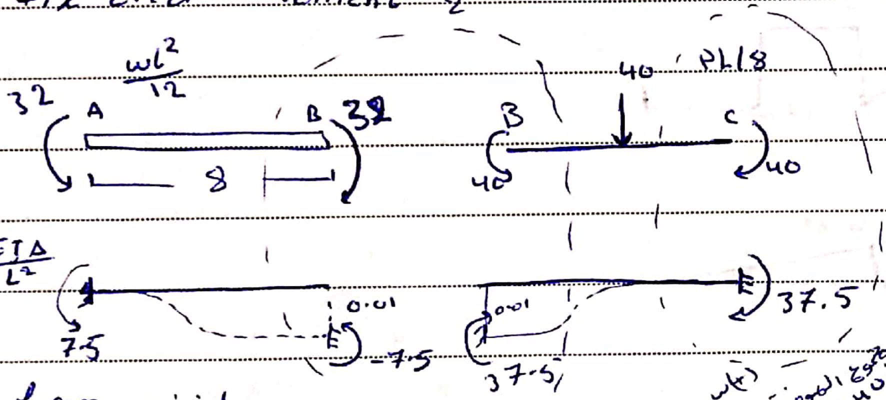
$$* \quad K_1 = EI \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \end{array}$$

redundant قطعه لك شكل

$$K_2 = EI \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} 0.5 & 0.25 \\ 0.25 & 0.5 \end{bmatrix} \end{array} \begin{array}{c} 2 \\ 3 \end{array}$$

$$K = EI \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 1.5 & 0.25 \\ 0 & 0.25 & 0.5 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

Fix End moment



$$K = EI \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} 1.5 & 0.25 \\ 0.25 & 0.5 \end{bmatrix} \end{array} \begin{array}{c} 2 \\ 3 \end{array}$$

forces on joint

$$F = Kd + f$$

c.c.w(+)
مجموع العزوم عند (B)
$= 32 + 75 + 40 - 37.5$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = EI \begin{bmatrix} 1.5 & 0.25 \\ 0.25 & 0.5 \end{bmatrix} \cdot \begin{bmatrix} d_2 \\ d_3 \end{bmatrix} + \begin{bmatrix} 45.5 \\ -77.5 \end{bmatrix}$$

$$\begin{bmatrix} d_2 \\ d_3 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} K^{-1} \end{bmatrix} \cdot \begin{bmatrix} -45.5 \\ 77.5 \end{bmatrix} \Rightarrow \begin{bmatrix} d_2 \\ d_3 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -61 \\ 185.45 \end{bmatrix} \text{rad}$$

AB

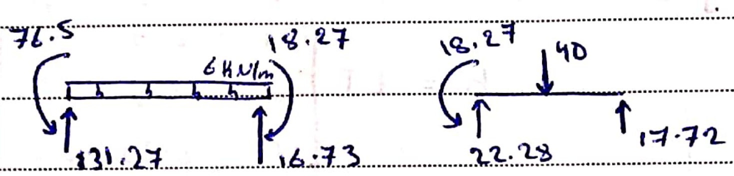
d_1 d_2 fix لا يوجد

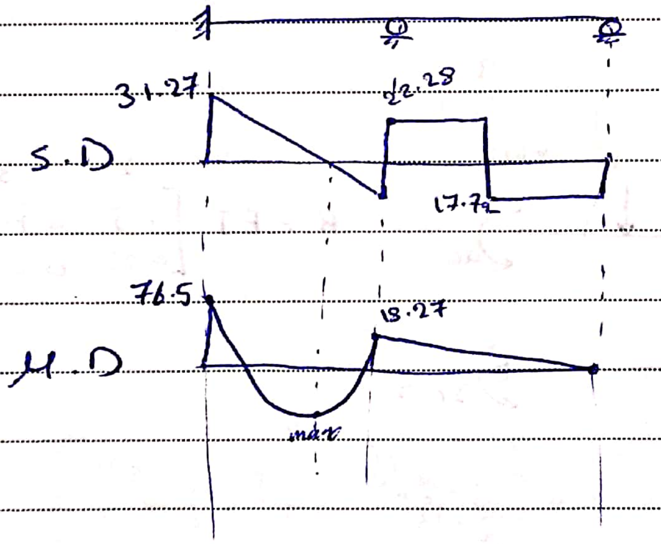
$$\begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = EI \begin{array}{c|c} \begin{array}{cc} 1 & 2 \end{array} & \\ \begin{bmatrix} 1 & 0.5 \\ 0.6 & 1 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \end{array} \end{array} \begin{bmatrix} 0 \\ -61 \end{bmatrix} \frac{1}{EI} + \begin{bmatrix} 32+75 \\ -32+75 \end{bmatrix} = \begin{bmatrix} 76.5 \\ -18.27 \end{bmatrix}$$

BC

$$\begin{bmatrix} M_{BC} \\ M_{CB} \end{bmatrix} = EI \begin{bmatrix} 0.5 & 0.25 \\ 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} -61 \\ 185.45 \end{bmatrix} \frac{1}{EI} + \begin{bmatrix} 40-37.5 \\ -40-37.5 \end{bmatrix} = \begin{bmatrix} 18.27 \\ 0 \end{bmatrix}$$

لازم تطلع 0 لأنه pin المومنت عنده 0





Example

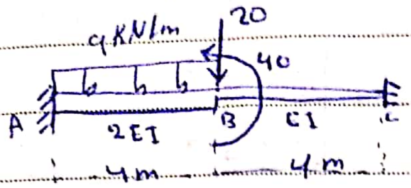
السؤال الي قبل اعطاني الـ settlment وهلا ما اعطاني مساحة

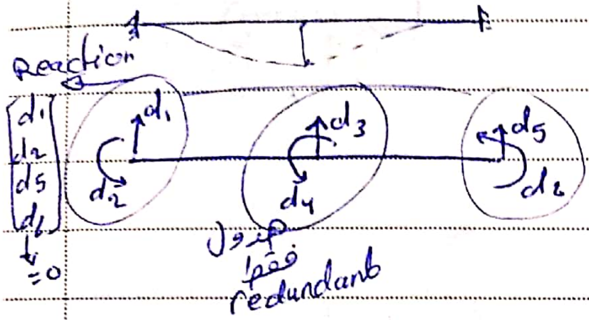
الـ fixed moment

لكن هون لم يعطي ويحدث ماله العناصر يعطى

شـ على matrix

moments virtical لانو فيها 4x4 .





$$K' = EI\begin{array}{c} \\ \left[\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 12/L^3 & 6/L^2 & -12/L^3 & 6/L^2 \\ 6/L^2 & 4/L & -6/L^2 & 2/L \\ -12/L^3 & -6/L^2 & 12/L^3 & -6/L^2 \\ 6/L^2 & 2/L & -6/L^3 & 4/L \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

نعوض في الماتريكس وعنس 2EI

$$\begin{array}{c} \\ \\ K_1 = EI \end{array}\begin{array}{cccc} \multicolumn{2}{c}{Ⓐ} & \multicolumn{2}{c}{Ⓑ} \\ 1 & 2 & 3 & 4 \\ \left[\begin{array}{cc|cc} 0.375 & 0.75 & -0.375 & 0.75 \\ 0.75 & 2 & -0.75 & 1 \\ \hline -0.375 & -0.75 & 0.375 & -0.75 \\ 0.75 & 1 & -0.25 & 2 \end{array}\right] \end{array}\begin{array}{cc} & \\ & \\ 1 & Ⓐ \\ 2 & \\ 3 & Ⓑ \\ 4 & \end{array}$$

$$\begin{array}{c} \\ \\ K_2 = EI \end{array}\begin{array}{cccc} \multicolumn{2}{c}{Ⓑ} & \multicolumn{2}{c}{Ⓒ} \\ 3 & 4 & 5 & 6 \\ \left[\begin{array}{cc|cc} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1 & -0.375 & 0.5 \\ \hline -0.1875 & -0.375 & 0.1875 & 0.375 \\ 0.375 & 0.5 & -0.375 & 1 \end{array}\right] \end{array}\begin{array}{cc} & \\ & \\ 3 & Ⓑ \\ 4 & \\ 5 & Ⓒ \\ 6 & \end{array}$$

تجميع

$$K = \begin{array}{c} \begin{array}{cccccc} \multicolumn{2}{c}{A} & \multicolumn{2}{c}{B} & \multicolumn{2}{c}{C} \\ 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cc|cc|cc} 0.375 & 0.75 & -0.375 & 0.75 & 0 & 0 \\ 0.75 & 2 & -0.75 & 1 & 0 & 0 \\ \hline -0.375 & -0.75 & 0.5625 & -0.375 & -0.1875 & 0.375 \\ 0.75 & 1 & 0.375 & 3 & & 0.5 \\ \hline 0 & 0 & -0.375 & -0.375 & -0.1875 & -0.375 \\ 0 & 0 & 0.375 & 0.5 & 0.325 & 1 \end{array}\right] \end{array}\begin{array}{cc} & \\ & \\ 1 & A \\ 2 & \\ 3 & B \\ 4 & \\ 5 & C \\ 6 & \end{array}$$

الحاصل d3 و d4

لذلك احتاج فقط 3 4

3
4

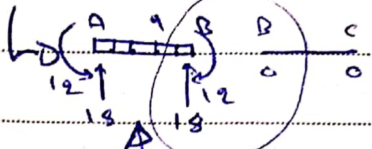
FIVE APPLE

forces in joint → $\boxed{F} = Kd + f$

virtical موجوده عند redundant ال vertical

نجد ال → fix end

$$\begin{bmatrix} -20 \\ 40 \end{bmatrix} = EI \begin{array}{cc|c} 3 & 4 & \\ \hline 0.5625 & -0.375 & 3 \\ -0.375 & 3 & 4 \end{array} \begin{bmatrix} d_3 \\ d_4 \end{bmatrix} + \begin{bmatrix} \boxed{18} \\ -12 \end{bmatrix} \begin{matrix} 3 \\ 4 \end{matrix}$$



$$\frac{1}{EI}\left[K^{-1}\right]\begin{bmatrix} -38 \\ 52 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -61.1 \\ 9.7 \end{bmatrix} \begin{matrix} \to \text{deflection} \\ \to \text{Rotation} \end{matrix}$$

AB

عند A لايوجد def ولايوجد rotation

$$\begin{bmatrix} A_y \\ M_{AB} \\ B_y \\ M_{BA} \end{bmatrix} = EI \begin{array}{cc|cc} A & & B & \\ \hline 0.375 & 0.75 & -0.375 & 0.75 \\ 0.75 & 2 & -0.75 & 1 \\ -0.375 & -0.75 & 0.375 & -0.75 \\ 0.75 & 1 & -0.75 & 2 \end{array} \cdot \frac{1}{EI}\begin{bmatrix} 0 \\ 0 \\ -61.1 \\ 9.7 \end{bmatrix} + \begin{bmatrix} 18 \\ 12 \\ 18 \\ -12 \end{bmatrix} \begin{matrix} A \\ \\ B \\ \end{matrix}$$

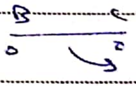
MAB

virtical

$$AB \Rightarrow \begin{bmatrix} 48.2 \\ 67.2 \\ -12.2 \\ 53.2 \end{bmatrix}$$

MB ← 53.2

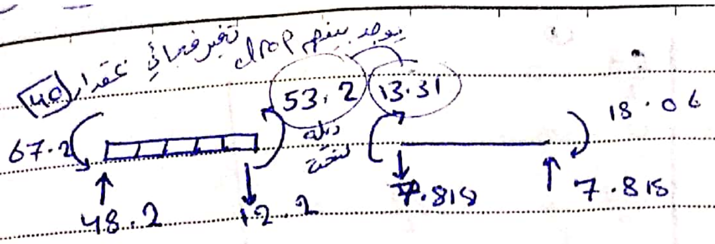
$$BC \Rightarrow \begin{bmatrix} B_y \\ M_{BC} \\ C_y \\ M_{CB} \end{bmatrix} = EI \begin{array}{cc|cc} B & & C & \\ \hline 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1 & -0.375 & 2 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1 \end{array} \cdot \frac{1}{EI}\begin{bmatrix} -61.7 \\ 9.7 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



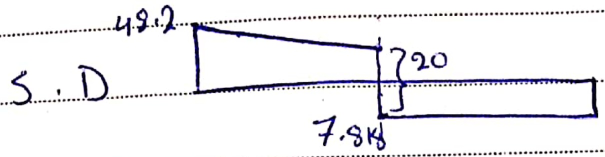
ليسو متساويين لان دي ده consentrated moment [illegible]

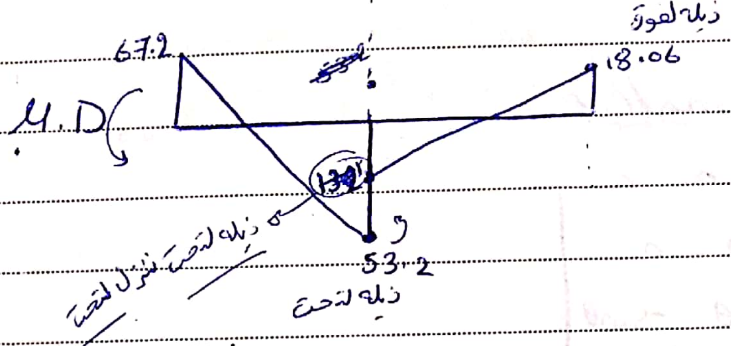
$$BC \Rightarrow \begin{bmatrix} -7.818 \\ -13.31 \\ 7.818 \\ -18.06 \end{bmatrix}$$

MB ← -13.31



اذا طلعت M_A ، A_y
M_{BA} ، B_y
وما بدي ارجع اعمل ماتريكس
لإيجاد عكس
Reaction
فلهون بنحل $\Sigma F_y = 0$ بطلع C_y و ،
M_C بطلع $\Sigma M = 0$

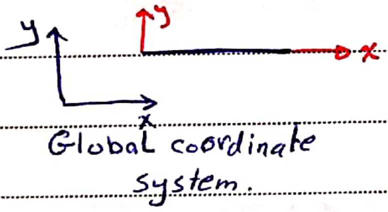


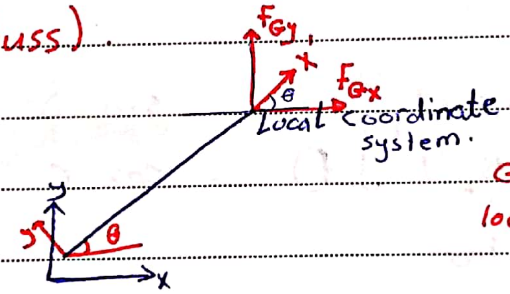


Day: Monday

Date: 1/4/2019

#14 * Stiffness method (Truss).





هون ال Global
local. ليست نفسها

for horizenal member
local = Global

$$F_i = \frac{EA}{L} \qquad F_j = -\frac{EA}{L}$$

Truss stiffness matrix

$$K = \left|\begin{array}{cc|cc} \multicolumn{2}{c}{\textcircled{1}} & \multicolumn{2}{c}{\textcircled{2}} \\ \frac{EA}{L} & 0 & -\frac{EA}{L} & 0 \\ 0 & 0 & 0 & 0 \\ \hline -\frac{EA}{L} & 0 & \frac{EA}{L} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right|$$

$$= \frac{EA}{L}\begin{vmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

T : Transformation matrix.

$$T = \begin{vmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & -\sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{vmatrix}$$

$F_G = k_G D$ sin S cos.sin CS

$F_G = \underbrace{\left[T\, k_L\, T^T\right]}_{k_G} D$ cos C

$$k_G = \frac{EA}{L}\left|\begin{array}{cc|cc} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ \hline -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & +S^2 \end{array}\right|$$

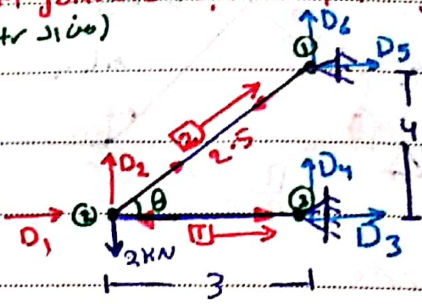
E 14-3^{2} EA constant

نفرض الـ D بالاتجاه الموجب ونبدأ من الـ joint اللي بتتحرك.

(من الـ unrestr)

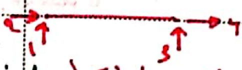
عند 3 و 1 صفر لانها ما بتتحرك

- Find Displacement at joint 2.
- find support reaction
- find forces in all member.



$\boxed{1}$ $L=3$, $\theta=0$ $c=1$ $s=0$ $cs=0$ $c^2=1$ $S^2=0$

$$K_{G_1} = EA\begin{array}{c} \frac{c^2}{L} \end{array}\left|\begin{array}{cccc|c} 1 & 2 & 3 & 4 & \\ 0.333 & 0 & -0.333 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 \\ -0.33 & 0 & 0.333 & 0 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{array}\right|$$



نحنا بنحتاج بس joint 2 3

هدول بينجمعوا فقط

$\boxed{2}$ $L=5$ $\theta=53.8$ $c=0.6$ $S=0.8$ $CS=48$ $c^2=0.36$ $S^2=0.64$

$$K_{G_2} = EA\left|\begin{array}{c|cccc|c} & 1 & 2 & 5 & 6 & \\ 1 & 0.072 & 0.096 & -0.072 & -0.096 & 1 \\ & 0.096 & 0.128 & -0.096 & -0.128 & 2 \\ 3 & -0.072 & -0.096 & 0.072 & 0.096 & 5 \\ 4 & -0.096 & -0.128 & 0.096 & 0.128 & 6 \end{array}\right|$$

للـ support

Structure $K = \sum K$.

$$K_G = EA\left|\begin{array}{cc|cc|cc} 1 & 2 & 3 & 4 & 5 & 6 \\ 0.405 & 0.096 & -0.33 & 0 & -0.072 & -0.096 \\ 0.096 & 0.128 & 0 & 0 & - & - \\ \hline - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \end{array}\right|$$

K_{uu} K_{ur} K_{ur} K_{rr}

القوى على الـ joint

$$F = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{bmatrix} 0 \\ -2 \\ \hline F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} \leftarrow F_u$$

$$D = \begin{bmatrix} D_1 \\ D_2 \\ \hline 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leftarrow D_u$$

unknown displacemenb $F = KD$.

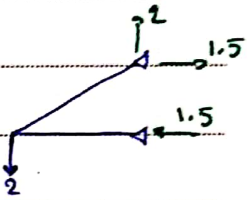
$$\begin{bmatrix} 0 \\ -2 \end{bmatrix} = EA \begin{array}{c|cc|c} & 1 & 2 & \\ & 0.405 & 0.096 & 1 \\ & 0.096 & 0.128 & 2 \end{array} \begin{vmatrix} D_1 \\ D_2 \end{vmatrix}$$

$$\begin{vmatrix} D_1 \\ D_2 \end{vmatrix} = K^{-1} \cdot F \Rightarrow \cancel{0-4508} \quad \frac{1}{EA} \begin{bmatrix} 4.505 \\ -19.03 \end{bmatrix}$$

support reaction:

Kur Du + Krr Dr

$$\begin{bmatrix} F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} = \cancel{EA} \begin{array}{c|cc} & 1 & 2 \\ 3 & -0.33 & 0 \\ 4 & 0 & 0 \\ 5 & -0.072 & -0.96 \\ 6 & -0.96 & -0.128 \end{array} \; \frac{1}{\cancel{EA}} \begin{bmatrix} 4.505 \\ -19.03 \end{bmatrix} = \begin{bmatrix} -1.5 \\ 0 \\ 1.5 \\ 2 \end{bmatrix}$$



$(-0.072)(4.505) - (0.096)(-19.03) = -1.5$

$(-0.092)(4.505) + (0.128)(19.03) = 2$.

force in all memble.

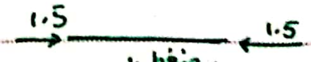
$F_G = KD$.

خذوا member $\boxed{1}$

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = KG_1 \cdot D \Rightarrow \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = K_{G_1} \cdot \begin{bmatrix} +4.505 \\ -19.03 \\ 0 \\ 0 \end{bmatrix}$$

هاي زيرو صفار لكن كتبتها قبل ما التزم
كلهم باخذ اول
تنتين.

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} 0.33 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 4.505 \\ -19.03 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}$$

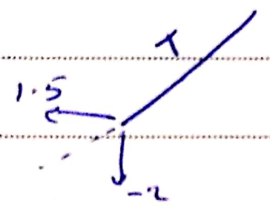


بيضغط ال mem
يعني بيضغطها joint

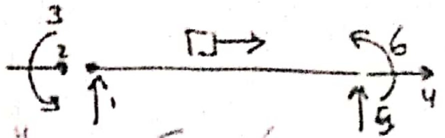
member 2

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{array}{c|cc|} & 1 & 2 \\ 1 & 0.072 & 0.096 \\ 2 & 0.096 & 0.128 \end{array} \cdot \begin{bmatrix} 4.504 \\ -14.03 \end{bmatrix}$$

$$\begin{bmatrix} -1.5 \\ -2 \end{bmatrix} \xrightarrow[\text{في الممبر}]{\text{لإيجاد القوة}} f = F_e \cdot T \quad or \quad F_2 = \sqrt{1.5^2 + 2^2} = 2.5$$



Frames stiffness Matrix



$$[K^l] = \begin{array}{cccccc|c} 1 & 2 & 3 & 4 & 5 & 6 & \\ \hline EA/L & 0 & 0 & -EA/L & 0 & 0 & 1 \\ 0 & 12EI/L^3 & 6EI/L^2 & 0 & -12^{EI}/_{L^3} & 6EI/L^2 & 2 \\ 0 & 6EI/L^2 & 4EI/L & 0 & -6EI/L^2 & 2EI/L & 3 \\ -EA/L & 0 & 0 & EA/L & 0 & 0 & 4 \\ 0 & -12EI/L^3 & -6EI/L^2 & 0 & 12EI/L^3 & -6EI/L^2 & 5 \\ 0 & 6EI/L^2 & 2EI/L & 0 & -6EI/L^2 & 4EI/L & 6 \end{array}$$

$$[T] = \left| \begin{array}{ccc|ccc} c & -s & 0 & & & \\ s & c & 0 & & 0 & \\ 0 & 0 & 1 & & & \\ \hline & & & c & -s & 0 \\ & 0 & & s & c & 0 \\ & & & 0 & 0 & 1 \end{array} \right| \qquad [T]^{-1} = \left| \begin{array}{ccc|ccc} c & s & 0 & & & \\ -s & c & 0 & & 0 & \\ 0 & 0 & 1 & & & \\ \hline & & & c & s & 0 \\ & 0 & & -s & c & 0 \\ & & & 0 & 0 & 1 \end{array} \right|$$

Memb Global stiffness Matrix

$$[K_G] = [T][K^l][T^T]$$

Structure $K_G = \Sigma K$

$$\Rightarrow K_G = \left| \begin{array}{c|c} K_{uu} & K_{ur} \\ \hline K_{rr} & K_{ru} \end{array} \right|$$

Structure stiffness equation:

$$[F_G]_u = [K_G]_{uu} [D_u] + [f_G]_u$$

fixed end forces

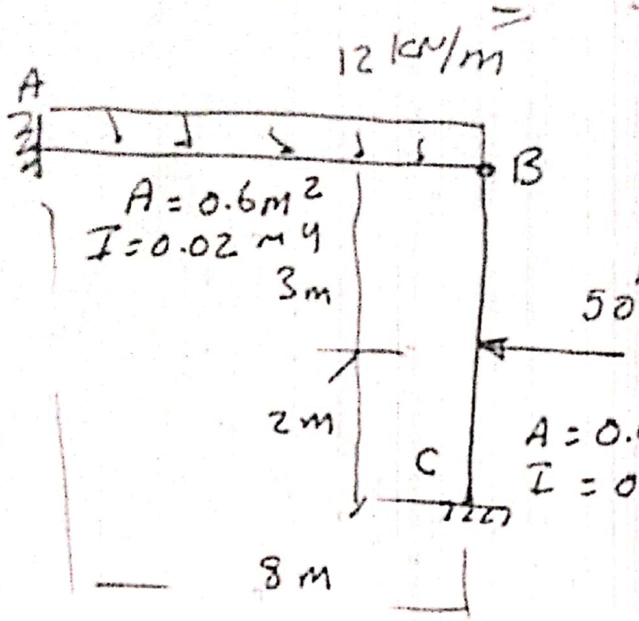
Member forces.

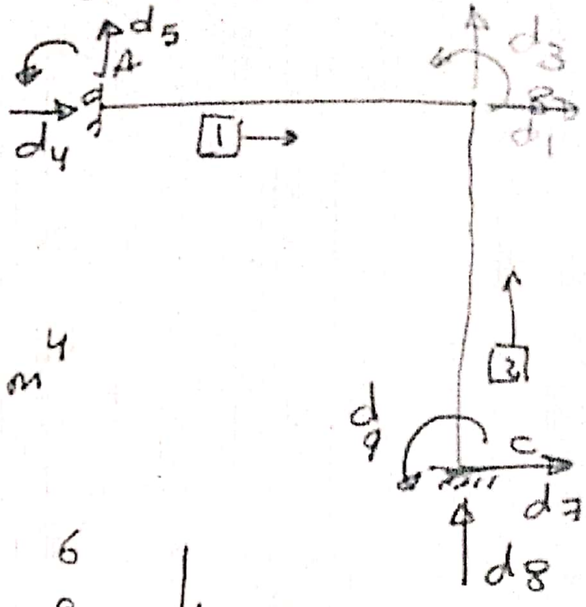
$$[F^l] = [K_m^l][T^T][D_G] + [T^T][f_G]$$

. on local coordinate system.

Draw S.D , M.D.

$E = 10^6 \ KN/m^2$





$$K_l = \begin{array}{|cccccc|c} 1 & 2 & 3 & 4 & 5 & 6 & \\ EA/L & 0 & 0 & -EA/L & 0 & 0 & 1 \\ 0 & 12EI/L^3 & 6EI/L^2 & 0 & -12EI/L^3 & 6EI/L^2 & 2 \\ 0 & 6EI/L^2 & 4EI/L & 0 & -6EI/L^2 & 2EI/L & 3 \\ -EA/L & 0 & 0 & EA/L & 0 & 0 & 4 \\ 0 & -12EI/L^3 & -6EI/L^2 & 0 & 12EI/L^3 & -6EI/L^2 & 5 \\ 0 & 6EI/L^2 & 2EI/L & 0 & -6EI/L^2 & 4EI/L & 6 \end{array}$$

Local

AB : $L = 8$, $\theta = 0$, $EA/L = 75000$, $\frac{12EI}{L^3} = 469$ $\frac{6EI}{L^2} = 1875$, $\frac{4EI}{L} = 10,000$, $\frac{2EI}{L} = 5000$

$c = 1$
$s = 0$

$$K_{1_{AB}} = \begin{array}{|ccc|ccc|c} \multicolumn{3}{c}{Ⓐ} & \multicolumn{3}{c}{Ⓑ} & \\ 1 & 2 & 3 & 4 & 5 & 6 & \\ 75000 & 0 & 0 & -75000 & 0 & 0 & \\ 0 & 469 & 1875 & 0 & -469 & 1875 & Ⓐ \\ 0 & 1875 & 10,000 & 0 & -1875 & 5000 & \\ \hline -75000 & 0 & 0 & 75000 & 0 & 0 & \\ 0 & -469 & -1875 & 0 & 469 & -1875 & Ⓑ \\ 0 & 1875 & 5000 & 0 & -1875 & 10,000 & \end{array}$$

CB $L = 5$, $\theta = 90$. EA/L $12EI/L^3$ $6EI/L^2$ $4EI/L$ $2EI/L$

$c = 0$ $80,000$ 480 1200 4000 2000
$s = 1$

$$K_{2_{CB}} = \begin{array}{|ccc|ccc|c} \multicolumn{3}{c}{Ⓒ} & \multicolumn{3}{c}{Ⓑ} & \\ 80,000 & 0 & 0 & -80000 & 0 & 0 & \\ 0 & 480 & 1200 & 0 & -480 & 1200 & Ⓒ \\ 0 & 1200 & 4000 & 0 & -1200 & 2000 & \\ \hline -80000 & 0 & 0 & 80,000 & 0 & 0 & \\ 0 & -480 & -1200 & 0 & 480 & -1200 & Ⓑ \\ 0 & 1200 & 2000 & 0 & -1200 & 4000 & \end{array}$$

Global stiffness Matrix

member 1

$$K_{1G} = K_{1L}$$

member 2. <u>CB</u>

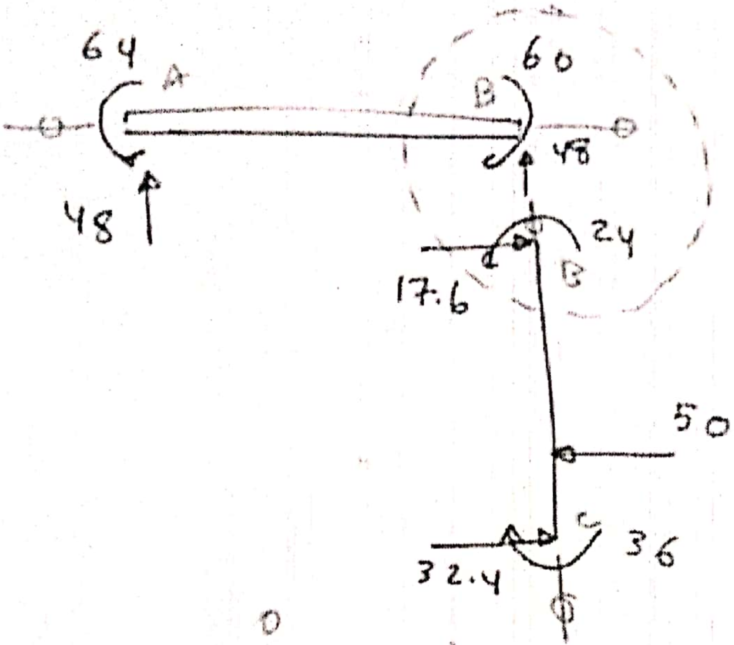
$$[K_G] = \left[\begin{array}{cc} \begin{matrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{matrix} & 0 \\ 0 & \begin{matrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{matrix} \end{array}\right] \left[\begin{array}{ccc|ccc} 80000 & 0 & 0 & -80000 & 0 & 0 \\ 0 & 480 & 1200 & 0 & -480 & 1200 \\ 0 & 1200 & 4000 & 0 & -1200 & 2000 \\ \hline -80000 & 0 & 0 & 80000 & 0 & 0 \\ 0 & -480 & -1200 & 0 & 480 & -1200 \\ 0 & 1200 & 2000 & 0 & -1200 & 4000 \end{array}\right] \times$$

$$\left[\begin{array}{c|c} \begin{matrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{matrix} & 0 \\ \hline 0 & \begin{matrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{matrix} \end{array}\right] = \begin{array}{c} \\ Ⓒ \\ \\ \\ Ⓑ \\ \\ \end{array} \left[\begin{array}{ccc|ccc} \multicolumn{3}{c}{Ⓒ} & \multicolumn{3}{c}{Ⓑ} \\ 480 & 0 & -1200 & -480 & 0 & -1200 \\ 0 & 80000 & 0 & 0 & -80000 & 0 \\ -1200 & 0 & 4000 & 1200 & 0 & 2000 \\ \hline -480 & 0 & 1200 & 480 & 0 & 1200 \\ 0 & -80,000 & 0 & 0 & 80,000 & 0 \\ -1200 & 0 & 2000 & 1200 & 0 & 4000 \end{array}\right]$$

Assembly $K_G = \Sigma K_c$

$$K_S = \begin{array}{cc|ccccccccc} & & \multicolumn{3}{c}{Ⓑ} & \multicolumn{3}{c}{Ⓐ} & \multicolumn{3}{c}{Ⓒ} \\ & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline & 1 & 75480 & 0 & 1200 & -75000 & 0 & 0 & -480 & 0 & 1200 \\ Ⓑ & 2 & 0 & 80469 & -1875 & 0 & -469 & -1875 & 0 & -80000 & 0 \\ & 3 & 1200 & -1875 & 14000 & 0 & 1875 & 5000 & -1200 & 0 & 2000 \\ & 4 & -75000 & 0 & 0 & 75000 & 0 & 0 & & & \\ Ⓐ & 5 & 0 & -649 & 1875 & 0 & 469 & 1875 & & 0 & \\ & 6 & 0 & -1875 & 5000 & 0 & 1875 & 10,000 & & & \\ & 7 & -480 & 0 & -1200 & & & & 480 & 0 & -1200 \\ Ⓒ & 8 & 0 & -80000 & 0 & & 0 & & 0 & 80000 & 0 \\ & 9 & 1200 & 0 & 2000 & & & & -1200 & 0 & 4000 \end{array}$$

fixed End forces.



$$F_u = K_{uu} D_u + f_u$$

$$\begin{bmatrix} \overset{0}{\cancel{F_1}} \\ F_2 \\ F_3 \end{bmatrix} = \begin{array}{c|ccc|} & 1 & 2 & 3 \\ 1 & 75480 & 0 & 1200 \\ 2 & 0 & 80469 & -1875 \\ 3 & 1200 & -1875 & 14000 \end{array} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} + \begin{bmatrix} 17.6 \\ 48 \\ -40 \end{bmatrix} \Longrightarrow \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} -0.00028 \\ -0.00053 \\ 0.00281 \end{bmatrix}$$

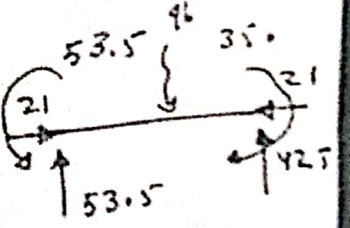
No forces at joint B

---

member forces. $F_m^l = [K_M^l][T^T][D_G] + [T^T][f_G]$

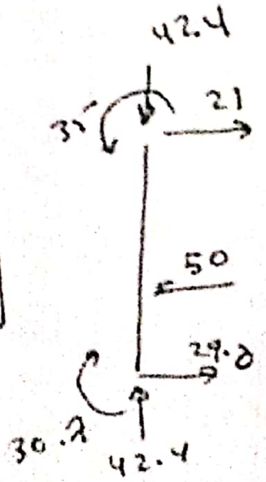
member 1

$$\begin{bmatrix} F_4 \\ F_5 \\ F_6 \\ F_1 \\ F_2 \\ F_3 \end{bmatrix} = [K_1^l] \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.00028 \\ -0.00053 \\ 0.00281 \end{bmatrix} + \begin{bmatrix} 0 \\ 49 \\ 64 \\ 0 \\ 48 \\ -64 \end{bmatrix} = \begin{bmatrix} 21 \\ 53.52 \\ 73.42 \\ -21 \\ 42.5 \\ -35 \end{bmatrix}$$



Member 2. CB

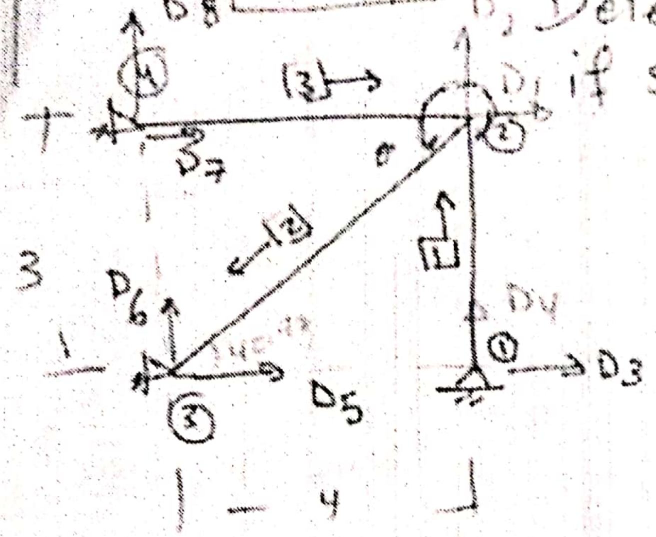
$$\begin{bmatrix} F_7 \\ F_8 \\ F_9 \\ F_1 \\ F_2 \\ F_3 \end{bmatrix} = [k_2^l][T^T] \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.00028 \\ -0.00053 \\ 0.00281 \end{bmatrix} + \begin{bmatrix} 0 \\ -32 \\ -36 \\ 0 \\ 17.6 \\ -24 \end{bmatrix} = \begin{bmatrix} 42.4 \\ -29.0 \\ -30.7 \\ -42.4 \\ -21 \\ 35 \end{bmatrix}$$



EX 14-5

Determine force in member 2 if support at ① settles 25 mm

$EA : 5 \times 10^3 \text{ KN}$



$$D = \begin{array}{c|c|} 1 & D_1 \\ 2 & D_2 \\ 3 & 0 \\ 4 & -0.025 \\ 5 & 0 \\ 6 & 0 \\ 7 & 0 \\ 8 & 0 \end{array}$$

Member 1 $L = 3$, $\theta : 90$ $C = 0$ $S = 1$ $CS : 1$ $C^2 = 0$ $S^2 = 1$

$$K_1 = EA \begin{array}{|cccc|c} 3 & 4 & 1 & 2 & \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0.333 & 0 & -0.333 & 4 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -0.333 & 0 & 0.333 & 2 \end{array}$$

Member 2 : $L = 5$ $\theta = 216.87$ $c = -0.8$, $S = -0.6$ $CS = 0.48$

$c^2 = 0.64$, $S^2 = 0.36$

$$K_2 = EA \begin{array}{|cccc|c} 1 & 2 & 5 & 6 & \\ 0.128 & 0.096 & -0.128 & -0.096 & 1 \\ 0.096 & 0.072 & -0.096 & -0.072 & 2 \\ -0.128 & -0.096 & 0.128 & 0.096 & 5 \\ -0.096 & -0.072 & 0.096 & 0.072 & 6 \end{array}$$

Member 3. $L = 4$, $\theta = 0$ $C = 1$ $S : 0$ $CS = 0$ $S^2 = 0$ $C^2 = 1$

$$K_3 = \begin{array}{|cccc|c} 7 & 8 & 1 & 2 & \\ 0.25 & 0 & -0.25 & 0 & 7 \\ 0 & 0 & 0 & 0 & 8 \\ -0.25 & 0 & 0.25 & 0 & 1 \\ 0 & 0 & 0.\text{[illegible]} & 0 & 2 \end{array}$$

Assembly.

$$K = EA \begin{array}{c|cccccccc|c} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \\ 1 & 0.378 & 0.096 & 0 & 0 & -0.128 & -0.096 & -0.25 & 0 & 1 \\ 2 & 0.096 & 0.405 & 0 & -0.333 & -0.096 & -0.072 & 0 & 0 & 2 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 4 & 0 & -0.333 & 0 & 0.333 & 0 & 0 & 0 & 0 & 4 \\ 5 & -0.128 & -0.096 & 0 & 0 & 0.128 & 0.096 & 0 & 0 & 5 \\ 6 & -0.096 & -0.072 & 0 & 0 & 0.096 & 0.072 & 0 & 0 & 6 \\ 7 & -0.25 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 7 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 \end{array}$$

$$F = KD$$

$$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{bmatrix} 0 \\ 0 \\ \hline F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \end{bmatrix} = [k] \begin{bmatrix} D_1 \\ D_2 \\ \hline 0 \\ -0.25 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array}$$

8×10^{3}

Displacements.

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = EA \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 0.378 & 0.096 \\ 0.096 & 0.405 \end{bmatrix} \end{array} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} + EA \begin{array}{c} \begin{array}{cccccc} 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 0 & 0 & -0.128 & -0.096 & -0.25 & 0 \\ 0 & -0.33 & -0.096 & -0.072 & 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} 0 \\ -0.025 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$D_1 = 0.00556$ m

$D_2 = -0.021875$ m

forces in Member 2.

$$\begin{bmatrix} F_1 \\ F_2 \\ F_5 \\ F_6 \end{bmatrix} = EA \begin{bmatrix} 1 & 2 & 5 & 6 \\ 0.129 & 0.096 & -0.128 & -0.096 \\ 0.096 & 0.072 & -0.096 & -0.072 \\ \hline -0.128 & -0.096 & 0.129 & 0.096 \\ -0.096 & -0.072 & 0.096 & 0.072 \end{bmatrix} \begin{bmatrix} 0.0056 \\ -0.021875 \\ \hline 0 \\ 0 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 5 \\ 6 \end{array}$$

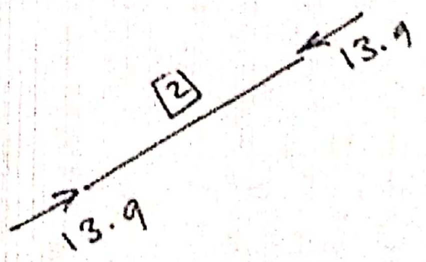
$F_1 = -11.066$

$F_2 = -8.3$

$F_5 = 11.066$

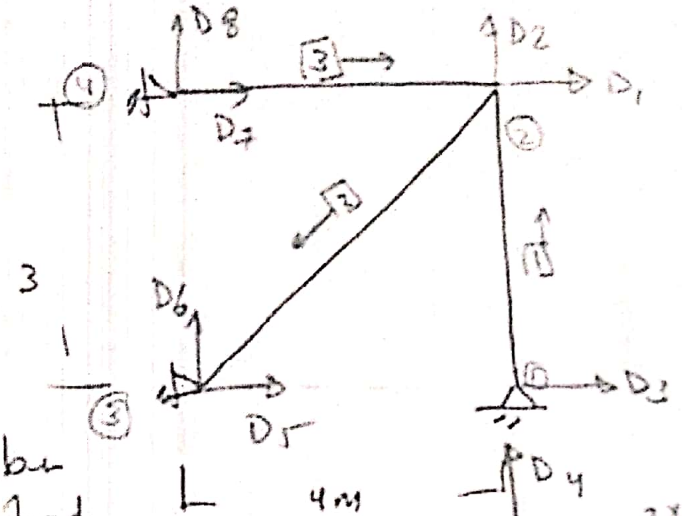
$F_6 = 8.3$

$$\Longrightarrow F^{l} = \sqrt{11.66^2 + 8.3^2} = 13.9 \text{ KN}$$



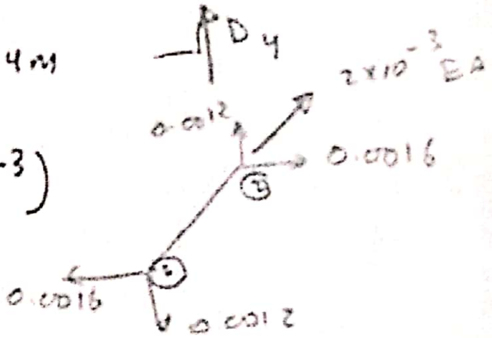
Trusses with Thermal changes and fabrication Errors

$$\boxed{\begin{aligned} F_{Thermal} &= (\alpha \Delta T) EA \\ F_{fabrication} &= \left(\frac{\Delta L}{L}\right) EA \end{aligned}}$$



$EA = 8 \times 10^3$

Determine the forces in member 1 and 2 if member 2 was made 0.01 m too short



$$f_{fabrication} = \underset{EA}{(8\times10^3)}\left(\frac{-0.01}{5}\right) = EA(2\times10^{-3})$$

$$\begin{bmatrix} f_1 \\ f_2 \\ f_5 \\ F_6 \end{bmatrix} = EA \begin{bmatrix} 0.0016 \\ 0.0012 \\ -0.0016 \\ -0.0012 \end{bmatrix}$$

$$K_1 = EA \left|\begin{array}{cc|cc|c} 3 & 4 & 1 & 2 & \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0.33 & 0 & -0.33 & 4 \\ \hline 0 & 0 & 0 & 0 & 1 \\ 0 & -0.33 & 0 & 0.73 & 2 \end{array}\right| \quad K_2 = EA \left|\begin{array}{cccc} 1 & 2 & 5 & 6 \\ 0.178 & 0.096 & -0.129 & -0.096 \\ 0.096 & 0.072 & -0.096 & -0.072 \\ -0.128 & -0.096 & 0.129 & 0.096 \\ -0.096 & -0.072 & 0.096 & 0.072 \end{array}\right| \quad K_3 = EA \left|\begin{array}{cccc} 7 & 8 & 1 & 2 \\ 0.25 & 0 & -0.25 & 0 \\ 0 & 0 & 0 & 0 \\ -0.25 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right|$$

Assemby. $K_G = \Sigma K$

$$K_G = EA \left|\begin{array}{cc|cccccc|c} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \\ 0.375 & 0.096 & & & & & & & 1 \\ 0.096 & 0.405 & & & & & & & 2 \\ \hline & & & & & & & & 3 \\ & & & & & & & & 4 \\ & & & & & & & & 5 \\ & & & & & & & & 6 \\ & & & & & & & & 7 \\ & & & & & & & & 8 \end{array}\right|$$

$$F_u = K_{uu} D_u + f_u$$

$$\Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} = EA \begin{bmatrix} 0.378 & 0.096 \\ 0.096 & 0.405 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} + EA \left[\begin{matrix} 0. \\ 0.00 \end{matrix}\right.$$

$$\Rightarrow D_1 = -0.003704 \text{ m}$$
$$D_2 = -0.002084 \text{ m}$$

forces in Member 2

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = EA \begin{vmatrix} 0.128 & 0.096 \\ 0.096 & 0.072 \end{vmatrix} \begin{bmatrix} -0.003704 \\ -0.002084 \end{bmatrix} + EA \begin{bmatrix} 0.0016 \\ 0.0012 \end{bmatrix} = \begin{bmatrix} 7.41 \\ 5.55 \end{bmatrix}$$

$$f = \sqrt{7.41^2 + 5.55^2} = 9.25 \text{ kN}$$