اللجنة الأكاديمية لقسم الهندسة المدنية

دفتر

تحليل إنشائي

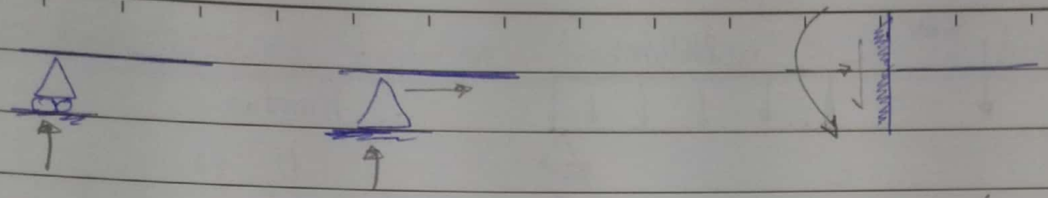
اعداد الطالب

هاني الجليلاتي

/groups/Civilittee

civilittee-hu.com

29/9/2015



• وجدت (reaction) لأنها حدثت بسبب رد فعل للقوة المؤثرة على السطح

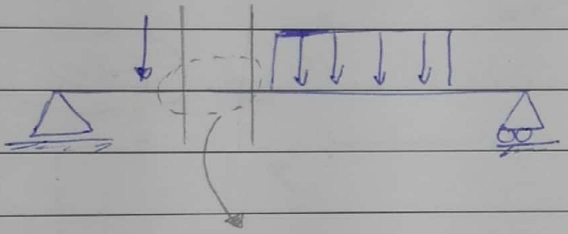
- Sign convention

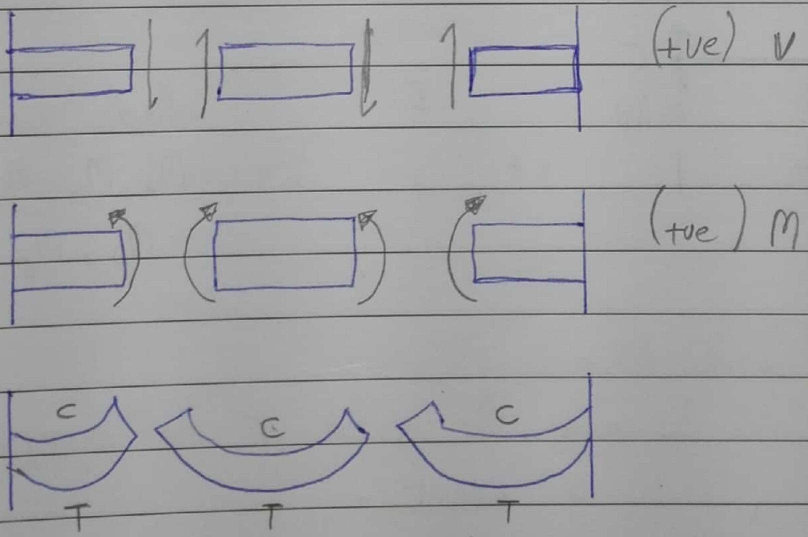
Positive direction

- internal shear force that causes clockwise rotation of member on wich of acts.

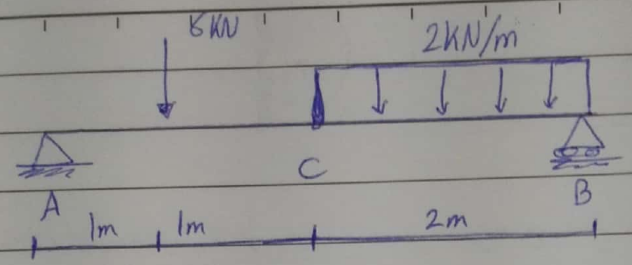
- internal moment that causes compression on the upper part of the member (concave upward)

-





Ex



Raction
$V_c = ??$
$M_c = ??$

$\Sigma M_A = 0 \longrightarrow B_y = 4.5\ kN \uparrow$
$\Sigma F_y = 0 \longrightarrow A_y = 5.5\ kN \uparrow$

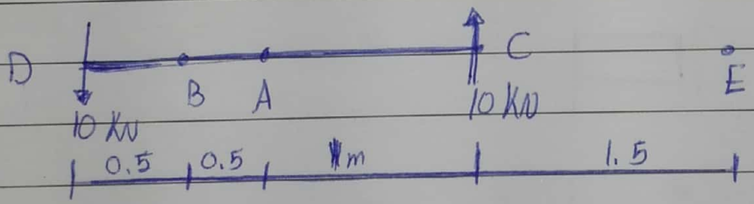
$+\uparrow V_c = ?? \Longrightarrow -5.5 + 6 + V_c = 0$
$V_c = -0.5\ kN$

انتبه للقاعدة السابقة
فعليها تم الحل واخراج الاجابات

$\circlearrowright + M_c = ?? \Longrightarrow -5,5(2) + 6(1) + M_c = 0$
$M_c = 5\ kN.m$

ملاحظة مجموع M عند أي نقطة على الطرف سواء كانت (Roller)(hing)
يساوي صفر

Ex

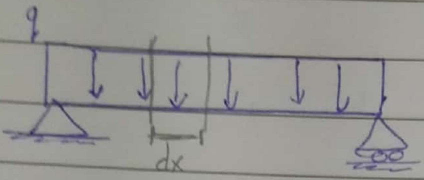


find
$M_A, M_B, M_C,$
M_D, M_E

$M_A, M_B, M_C, M_D, M_E = 20\ kN.m$
Because the system or this example
is couple.

1/10/2015

Relationships between loads, shear force & bending moment



$$\frac{dv}{dx} = -q$$

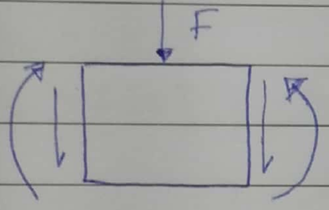
slope of shear diagrams = -ve of disributed load intensity

- change in shear = -ve area under loading curves

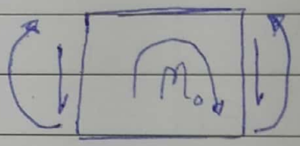
$$\frac{dM}{dx} = V$$

slope of moment diagram = shear

- when $v=0$ → point of max moment change in moment = area under shear diagram

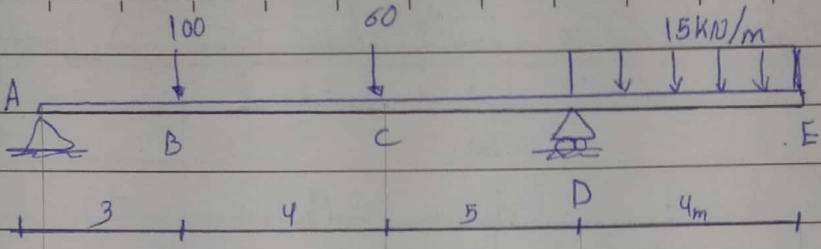


the shear diagram will jump downward



the moment diagram will jump downward if M_o is clockwise

Ex

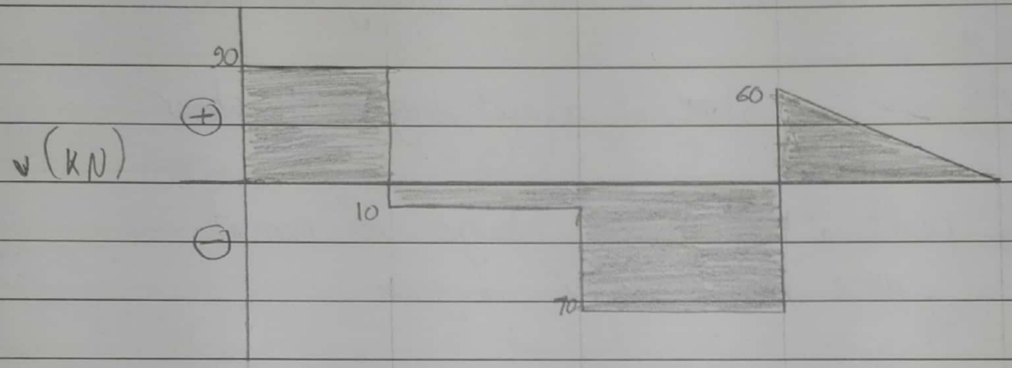


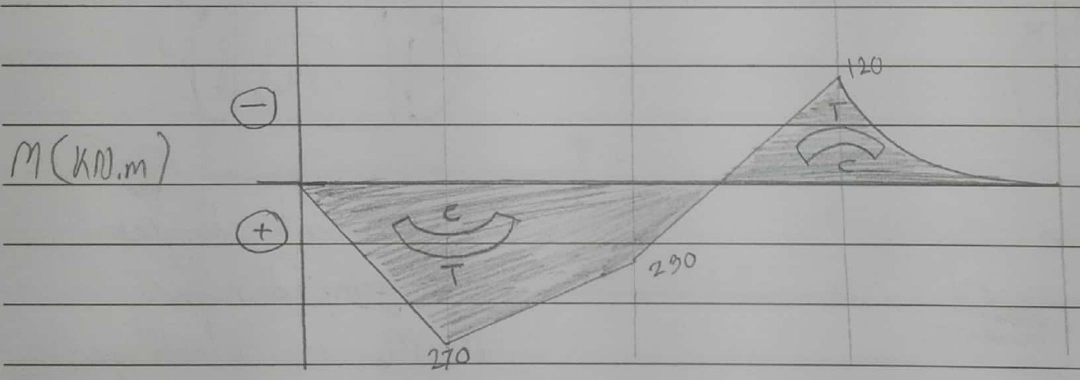
$\cdot \Sigma M_A = 0$

$-100(3) - 60(7) - 60(14) + D_y(12) = 0 \rightarrow D_y = 130\ kN\uparrow$

$\cdot \Sigma F_y = 0$

$A_y = 90KN\uparrow$





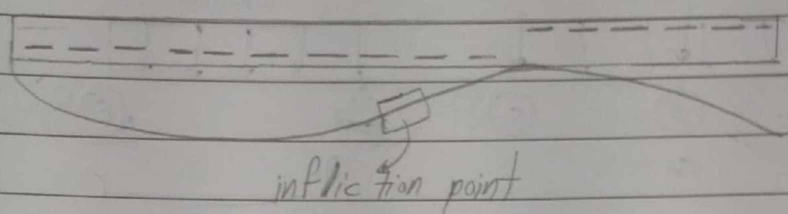
$M_A = 0$

$M_B - M_A = (90 * 3) \rightsquigarrow M_B = 270$

$M_C - M_B = (-10 * 4) \rightsquigarrow M_C = 230$

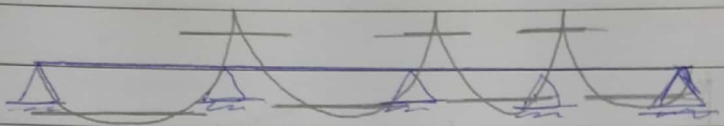
$M_D - M_C = (-70 * 5) \rightsquigarrow M_D = -120$

$M_E - M_D = (60 * \frac{1}{2} * 4) \rightsquigarrow M_E = 0$



• أينما يتواجد M يتواجد T ، لذلك يوضع حديد التسليح عند T لأن الخرسانة
قليلة T كثيرة C ، أما الحديد فكثير T و C ، ولذلك يرسم (+M) للأسفل و (-M) للأعلى

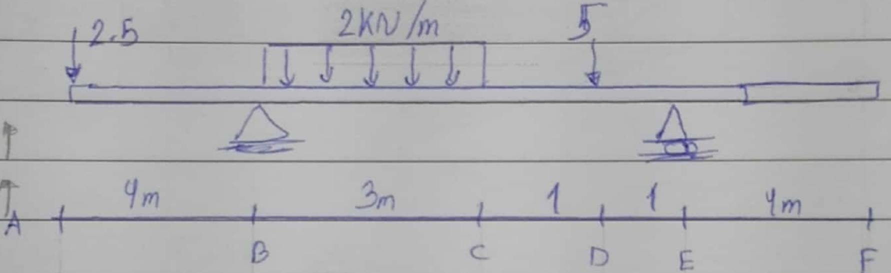
Ex



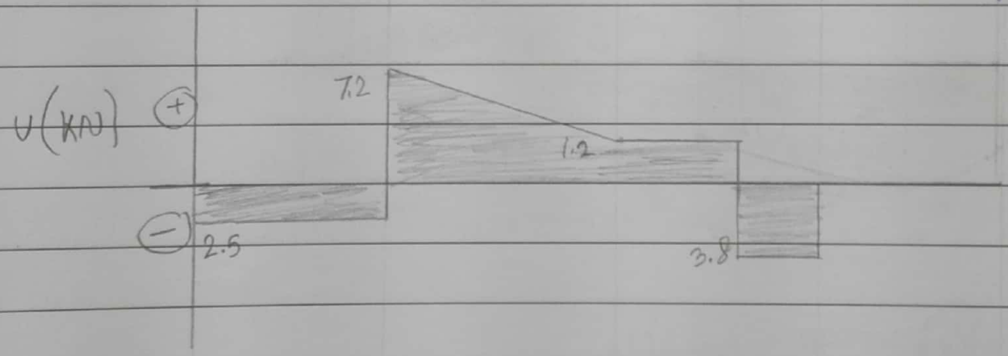
أين ممكن أن يتواجد حديد التسليح ؟؟

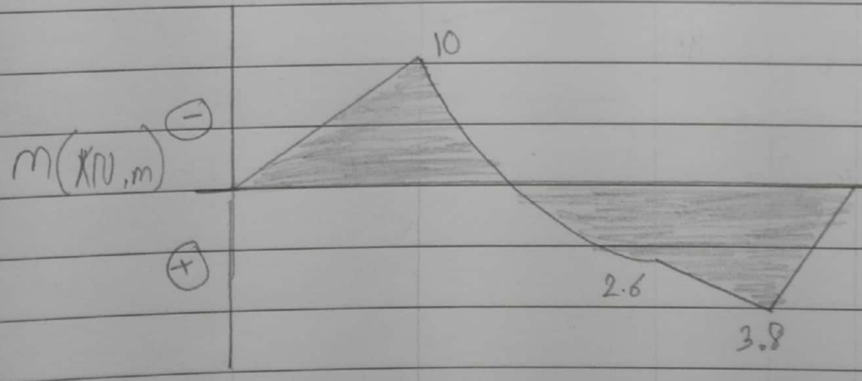
Ex

$B_y = 9.7$ KN ↑
$E_y = 3.8$ KN ↑



المنطقة بين E و F cantilevar beam ولو حذفت في السؤال لن تؤثر على الحل

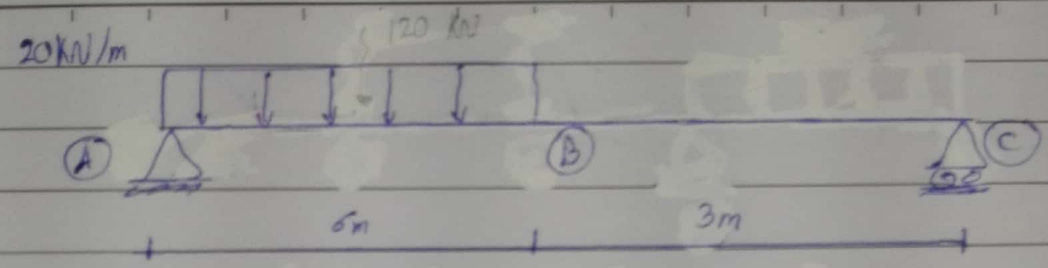




لا داعي لو عملت roller أو hing (M) إن كانت في منتصف beam

4/10/2015

Ex

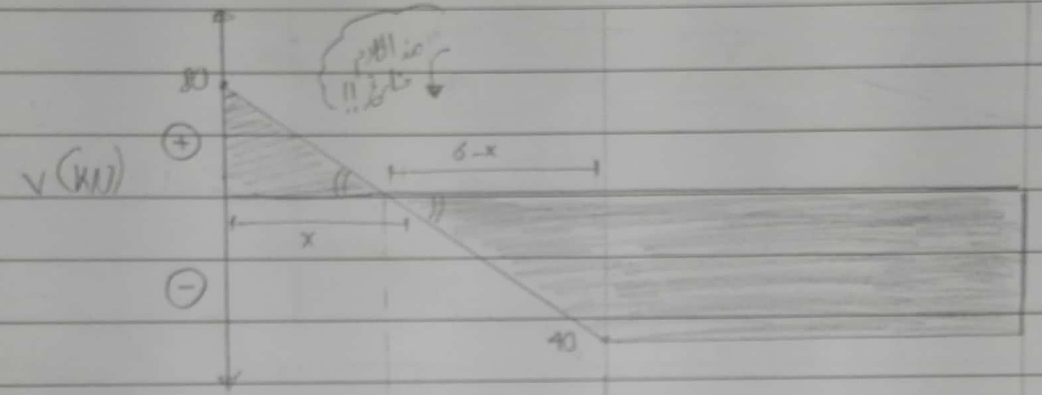


$\circlearrowleft + \; \Sigma M_A = 0 \longrightarrow -120(3) + C_y(9) = 0 \longrightarrow C_y = 40 \text{ kN} \uparrow$

$+\uparrow \; \Sigma F_y = 0 \longrightarrow A_y = 80 \text{ kN} \uparrow$

. المنطقة اللي تحمل قوة أكبر

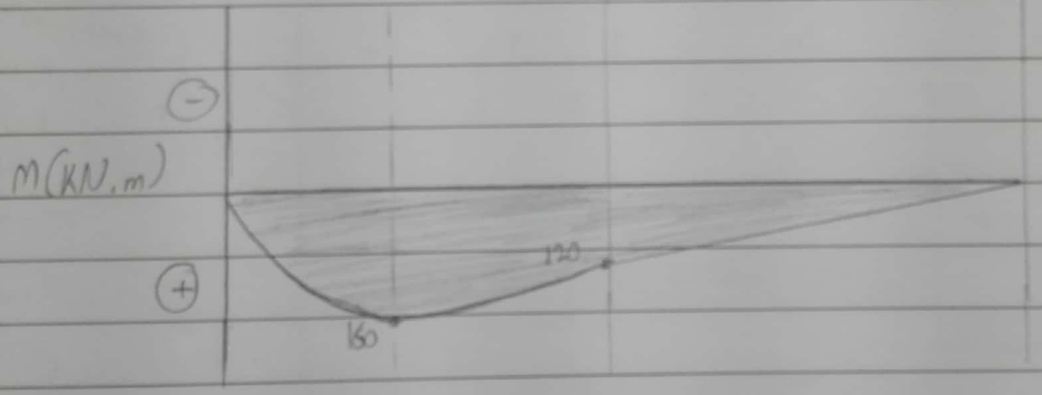
ستعكس قوة أكبر من التي ليس عليها أي قوة



. لإيجاد x بسهولة ↓

$80 - 20x = 0$

$x = 4_m$



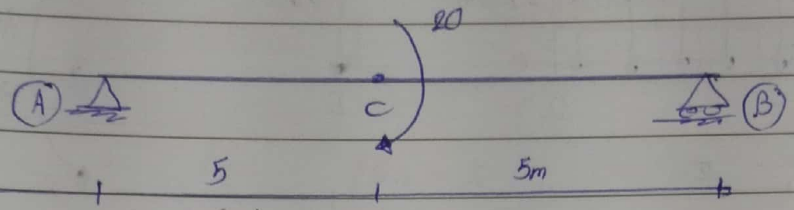
$M_A = 0$

$M_{4m} - M_A = \frac{1}{2}(80)(4) \longrightarrow M_{4m} = 160 \text{ kN.m}$

$M_B - M_{4m} = \frac{1}{2}(-40)(2) \longrightarrow M_B = 120 \text{ kN.m}$

$M_C - M_B = -40(3) \longrightarrow M_C = 0$

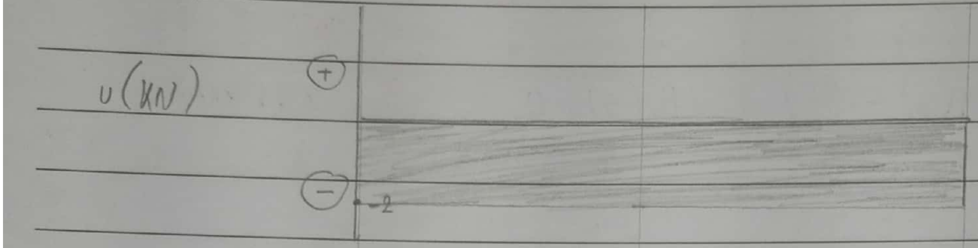
Ex



The M diagram will jump downward if M is clockwise

$\circlearrowleft + \Sigma M_A = 0 \longrightarrow -20 + B_y(10) = 0 \longrightarrow B_y = 2\ kN$

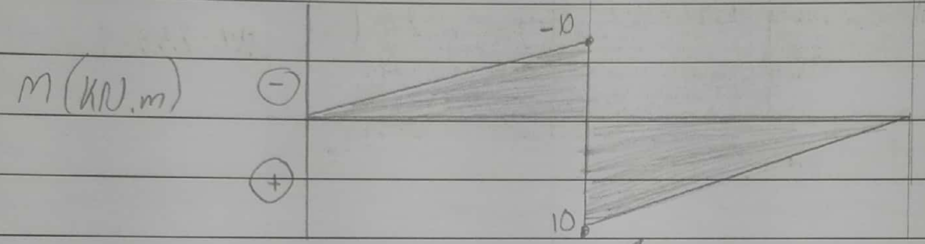
$+\uparrow \Sigma F_y = 0 \longrightarrow A_y = -2\ kN$



- $M_A = 0$
- $M_C - M_A = -2(5)$

  $M_C = -10$
- $M_B - M_C = -2(5)$

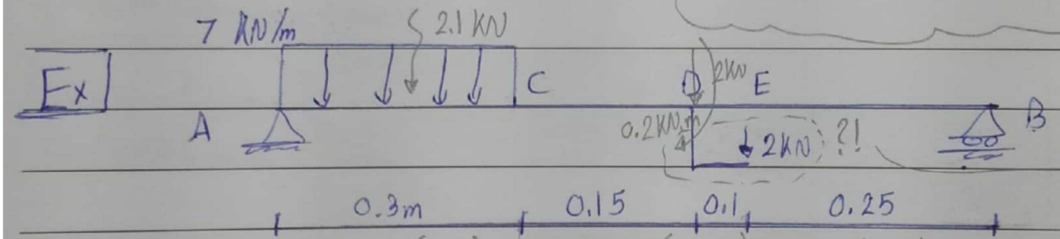
  $M_B = 0$



نأخذ آخر قيمة

وهنا الفرق حدث بسبب وجود M ع النقطة C

Ex

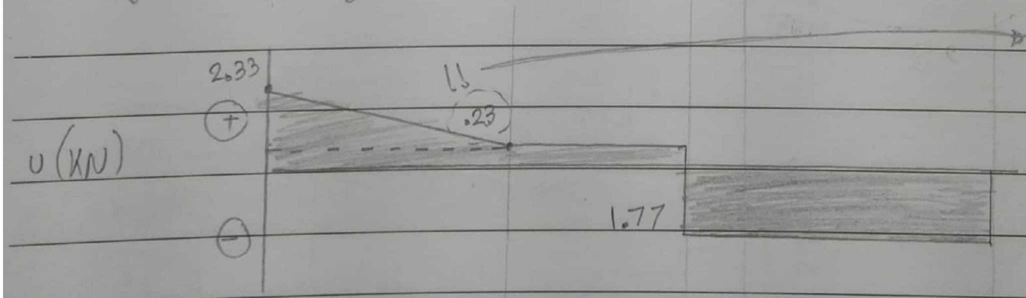


تحول لـ M&F

$\circlearrowleft + \Sigma M_A = 0 \longrightarrow -2.1(0.15) - 0.2 - 2(0.45) + B_y(0.8) = 0 \longrightarrow B_y = 1.77 \uparrow kN$

$+\uparrow \Sigma F_y = 0 \longrightarrow A_y = 2.33\ kN$

!! → 2.33 − 2.1 = 0.23

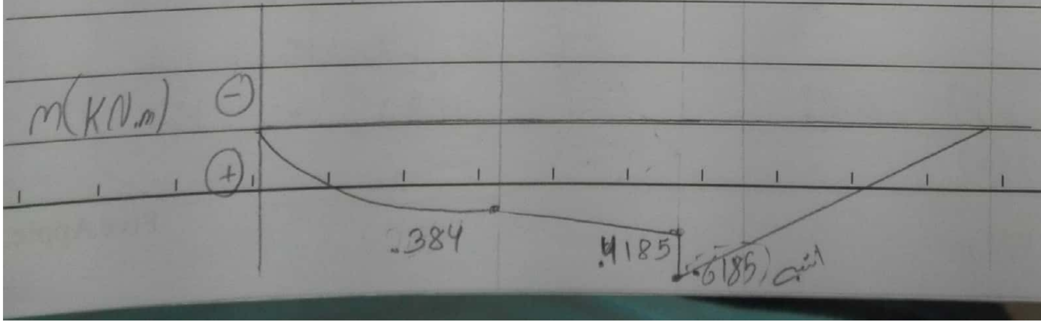


- $M_A = 0$
- $M_C - M_A = \frac{1}{2}(2.1)(0.3) + (.23)(0.3)$

  $M_C = 0.384$
- $M_D - M_C = (.23)(0.15)$

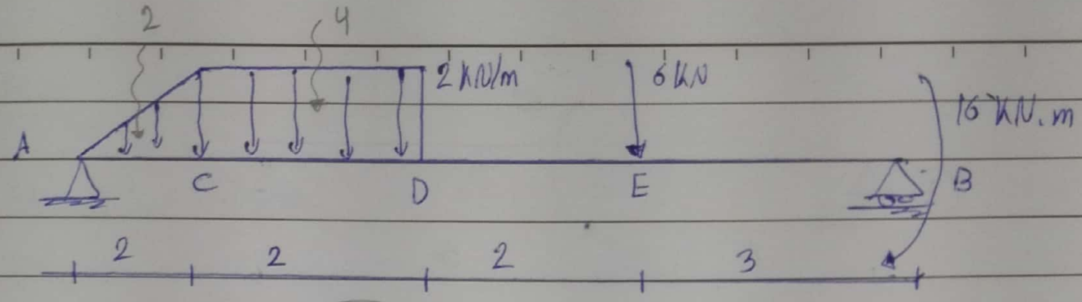
  $M_D = 0.4185$
- M_B الثني $- M_D = (-1.77)(0.25)$

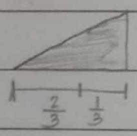
  $M_B \simeq 0$

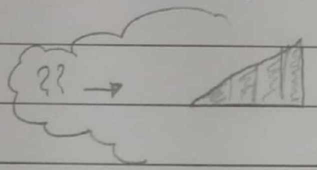


Five Apple

Ex



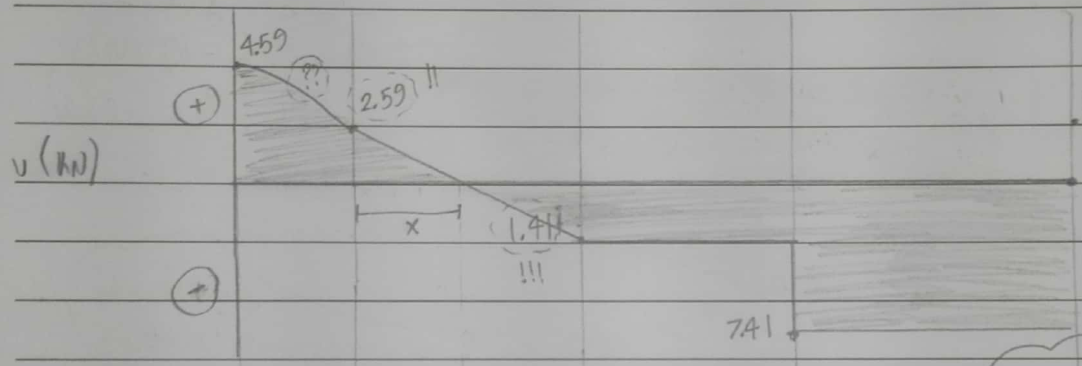




concave down shear

$\curvearrowleft + \Sigma M_A = 0 \longrightarrow -2(2)(\frac{2}{3}) - 4(3) - 6(6) - 16 + B_y(9) = 0 \longrightarrow B_y = 7.41$ KN

$+\uparrow \ \Sigma F_y = 0 \longrightarrow A_y = 4.59 \uparrow$



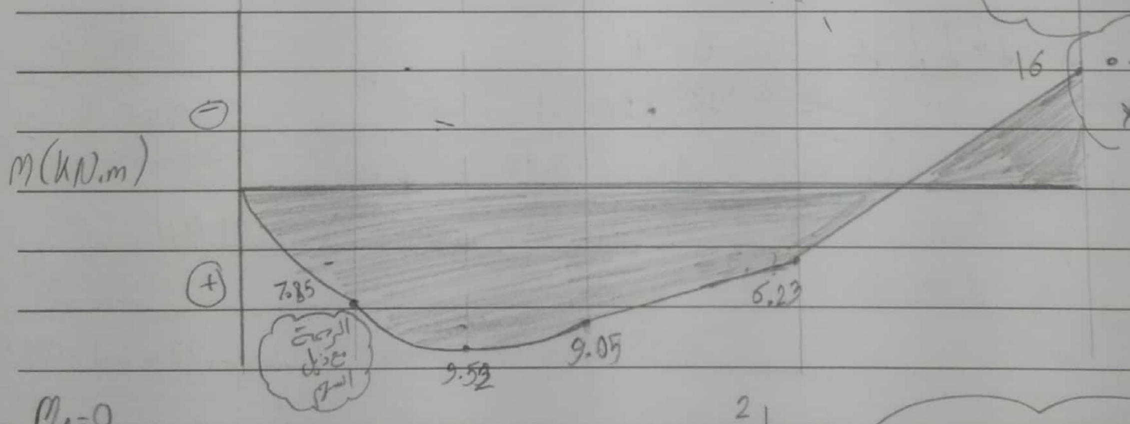
!! 4.59 - 2 = 2.59

!!! 2.59 - 4 = 1.41

ـلـ M القصوى ← V=0

$\circ \ 2.59 - 2x = 0$

$x = 1.295$



$\circ \ M_A = 0$

$\circ \ M_c = ??$ مساحة صعب حسابها →

$\circ \ M_1 - M_C = \frac{1}{2}(2.59)(1.295)$

$M_1 = 9.52$

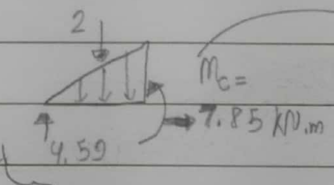
$\circ \ M_2 - M_1 = -1.41(0.705)$

$M_2 = 9.05$

$\circ \ M_E - M_2 = -1.41(2) \rightarrow M_E = 6.23$

$\circ \ M_B - M_E = -(7.41)(3)$

$M_B = -16$ kN.m



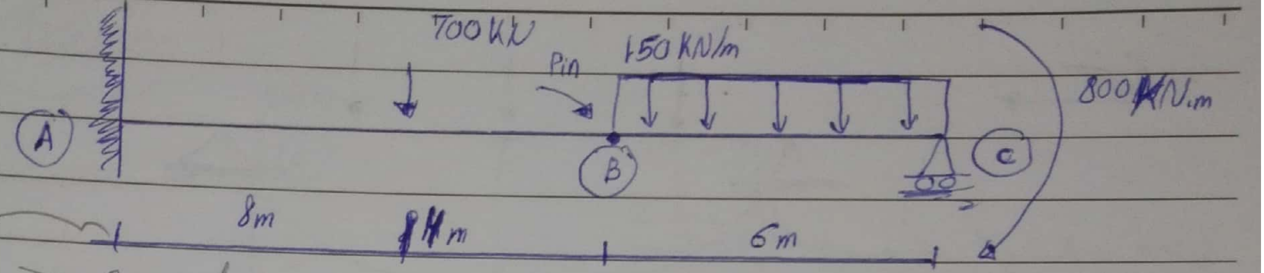
$M_C = 7.85$ kN.m

الاتجاه عكس عقارب الساعة
لأن الغلبة للأقوى
والمسافة أكبر

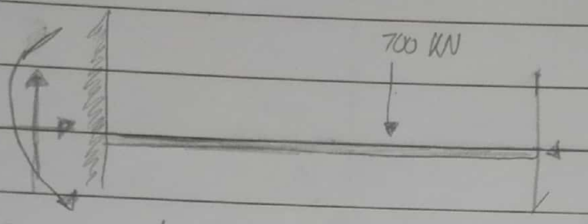
أما عن concave فهو غالباً
للأعلى وللتأكد
يكفي بأخذ ثلاث مقاطع ونظهر
النقاط .. بهذا الشكل

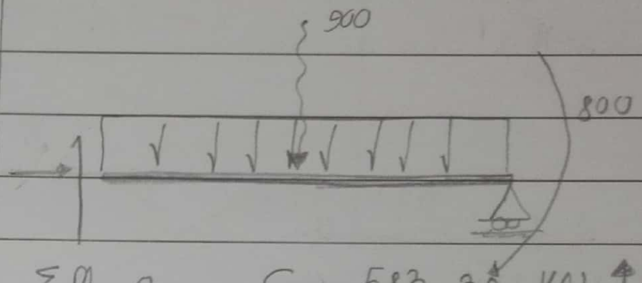
6/10/2015

Ex



4 unknown > 3 equation





$\Sigma F_y = 0 \longrightarrow A_y = 1016.67$ KN ↑+

$\Sigma M_B = 0 \longrightarrow M_A = 9400.04$ KN.m ↺+

$A_x = 0$

$\Sigma M_B = 0 \longrightarrow C_y = 583.33$ KN ↑

$\Sigma F_y = 0 \longrightarrow B_y = 316.67$ KN ↑

$B_x = 0$

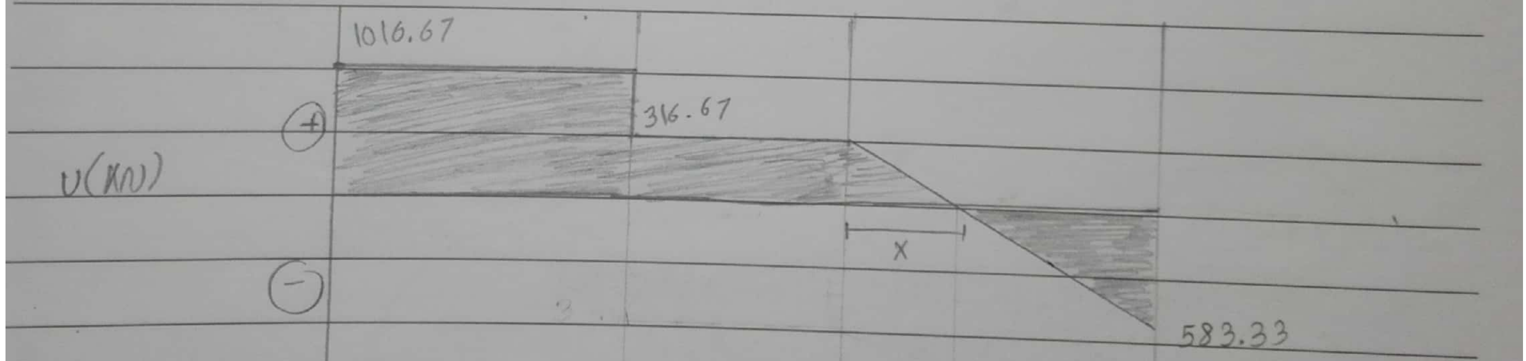
internal hing or pin

$M = 0 \longrightarrow M_B = 0$

• نبدأ بالجزء الثاني لإيجاد (reaction)

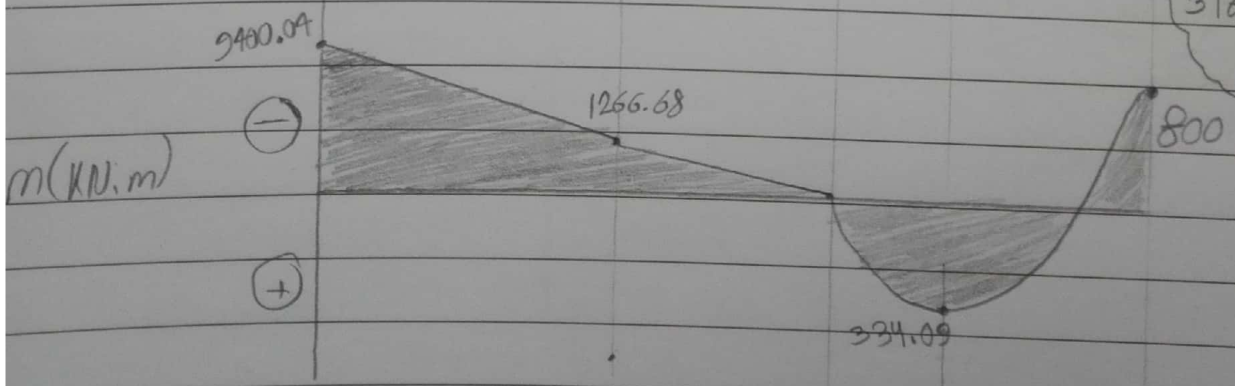
لأنه 3unknown = 3 equation

بينما الجزء الأول 5unknown > 3 equation

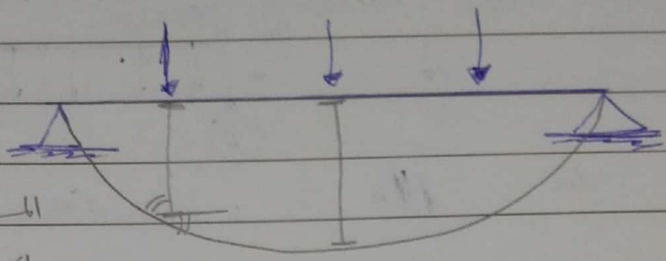


316.67 − 150 x = 0

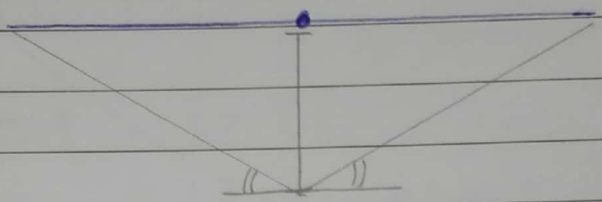
x = 2.11 m



شرح



المسافات لا تساوي بعضها
لكن مقدار الزوايا متساوية



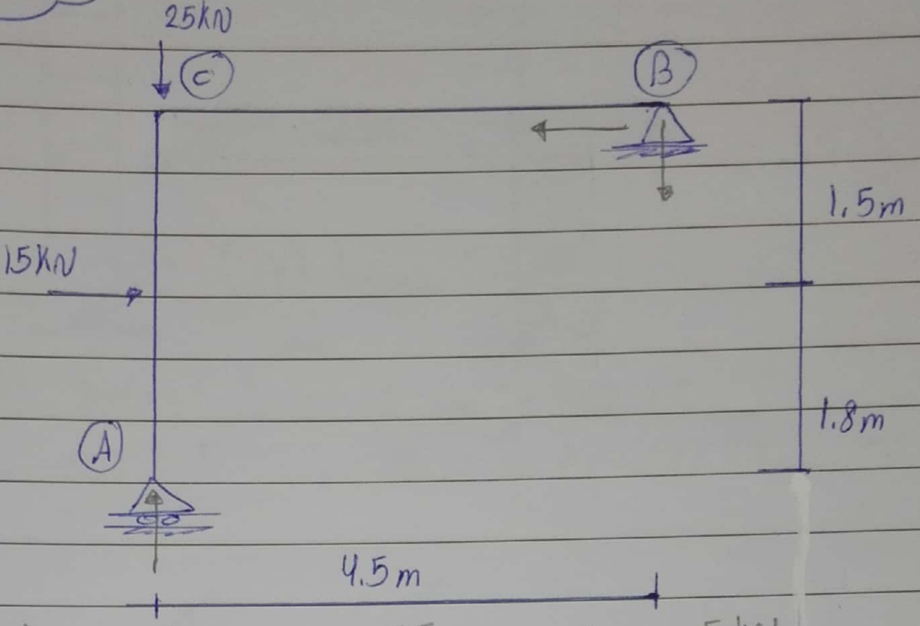
المسافات متساوية
لكن مقدار الزوايا لا
تساوي بعضها البعض

- الفرق بين (pin) و (internal hing)

أن pin يجعل الجسم كله يتحرك حركة واحدة
لكن internal hing تتحرك قطعة منها ويمكن للأخرى أن تتحرك بمقدار مختلف
أو تبقى ثابتة ومثال (internal hing) هو جسم الانسان

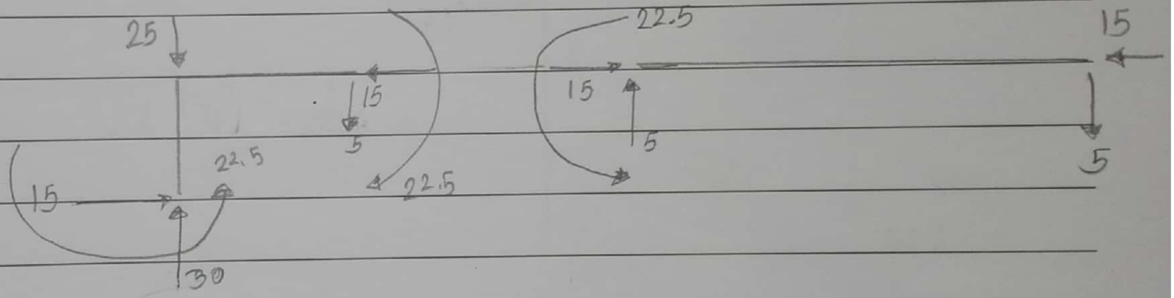
Frames

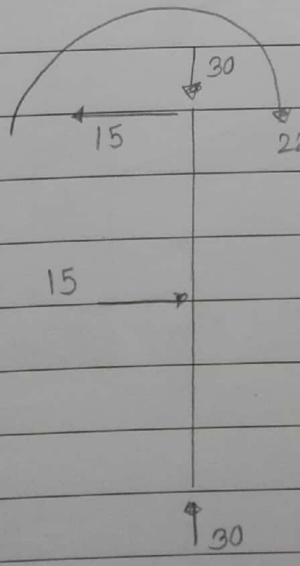
Ex



$\Sigma M_B = 0 \longrightarrow A_y = 30\ kN$; $\Sigma F_y = 0 \longrightarrow B_y = -5\ kN$

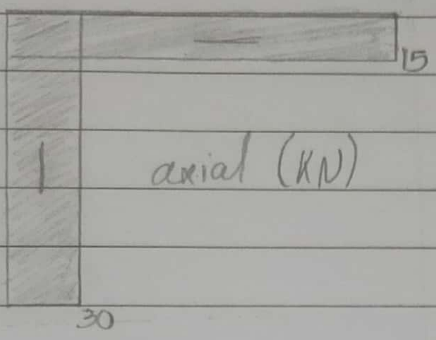
$\Sigma F_x = 0 \longrightarrow B_x = -15\ kN$

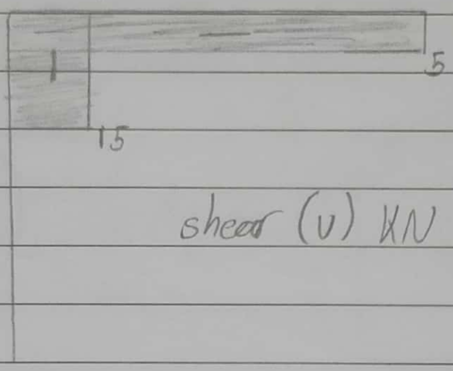


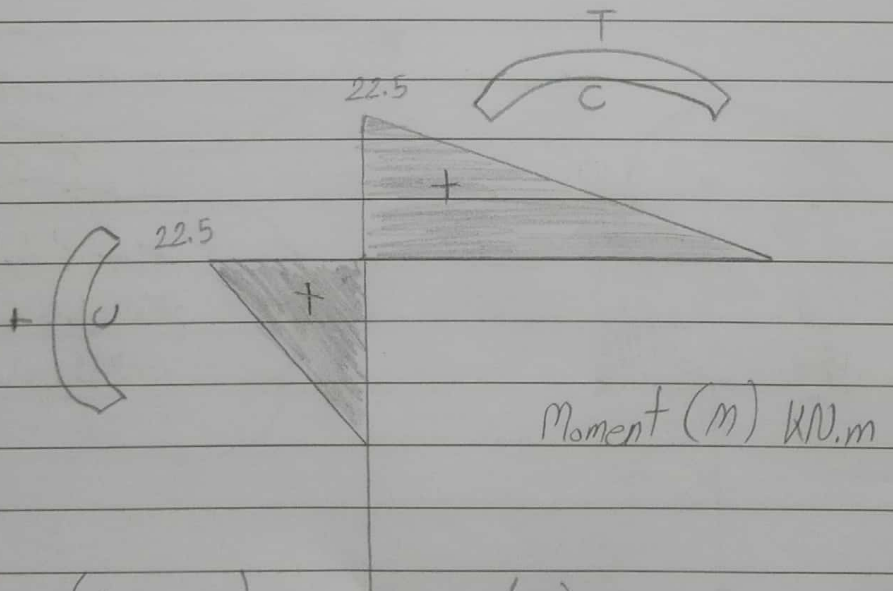


لايجاد M

اذا لم يتواجد F
على طول القطّع
لا داعي لفصله







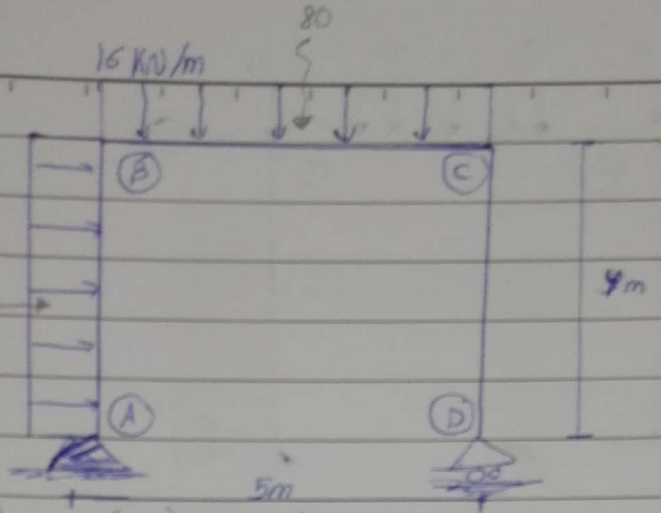
• نطبّق قاعدة ذيل السهم للـ (M) عندما يكون (beam)

• نرسم الـ (M) عند (Tension side)

8/10/2019

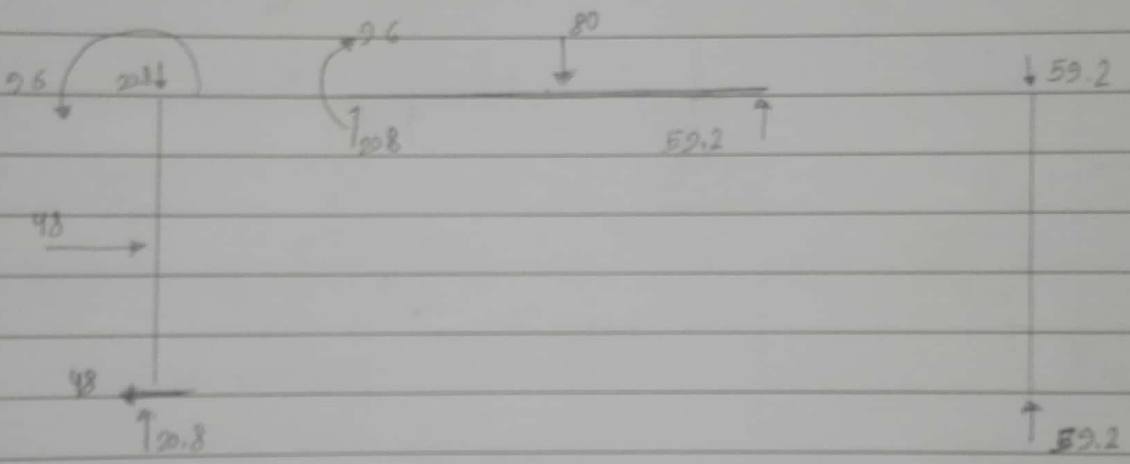
Ex

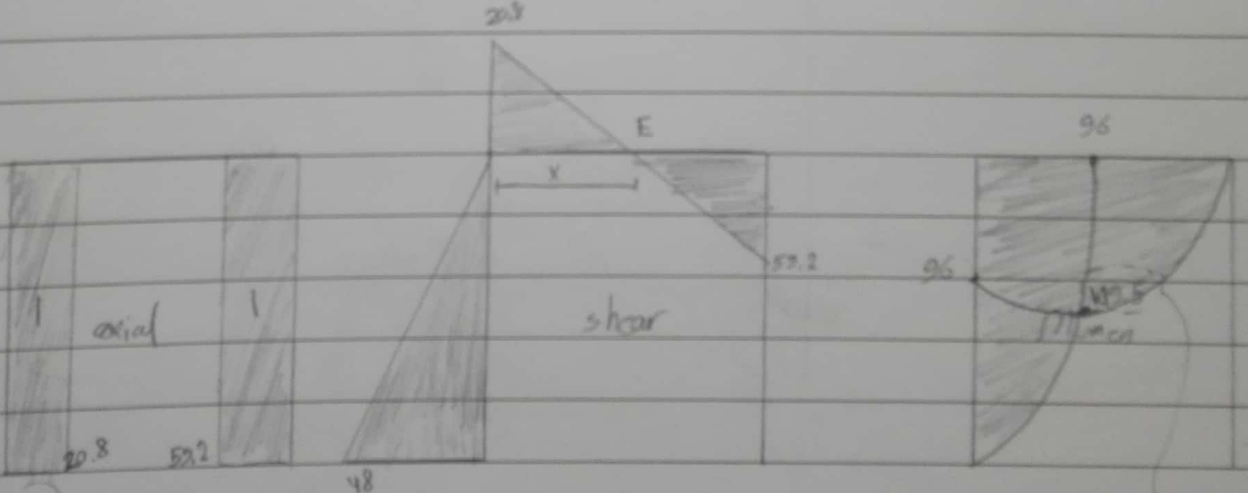
Frame :
جأ بحص من الشمال لليمين ومن تحت لفوق



$\Sigma M_A = 0 \rightarrow -48(2) - 80(2.5) + D_y(5) = 0 \rightarrow D_y = 59.2 \uparrow$

$\Sigma F_y = 0 \rightarrow A_y = 20.8 \uparrow$ $\qquad \Sigma F_x = 0 \rightarrow A_x = 48\ KN \leftarrow$



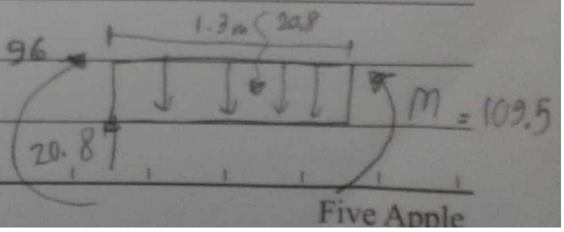


axial في
سواء كانت
الرسمة للداخل
أو الخارج طا
نزل طالما
وضعنا اشارة
negative or positive

$20.8 - 16x = 0$

$x = 1.3m$

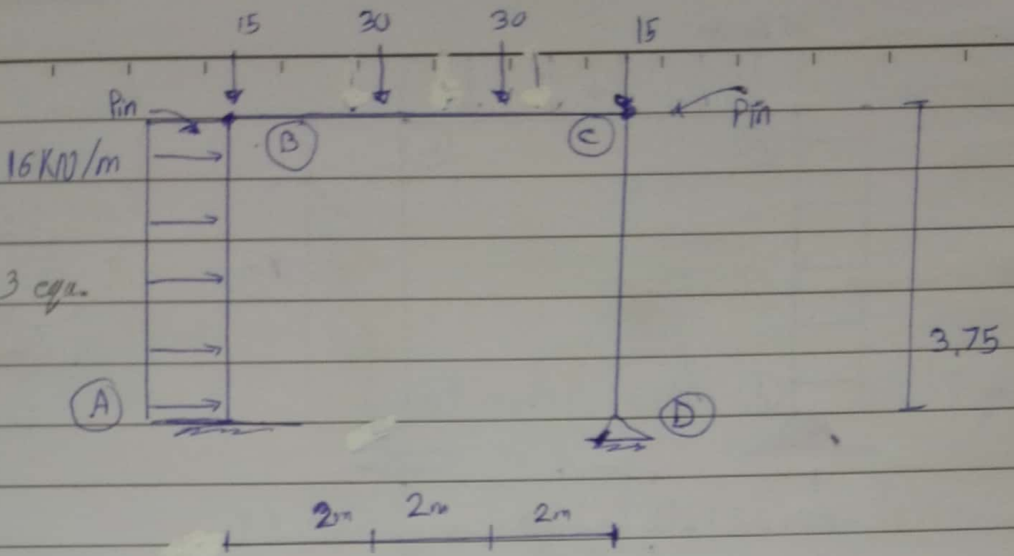
إذا ما عرفت نحسبها
بالمساحة احسبها
section

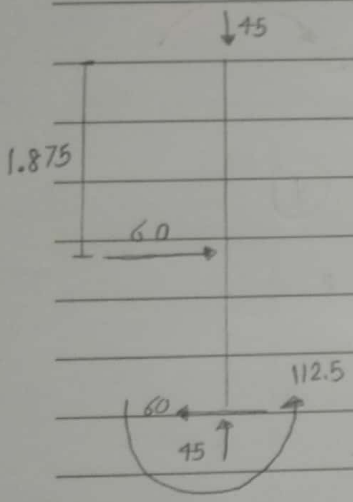


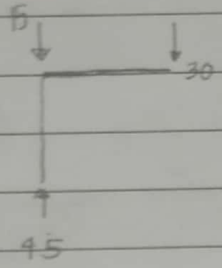
$M = 109.5$

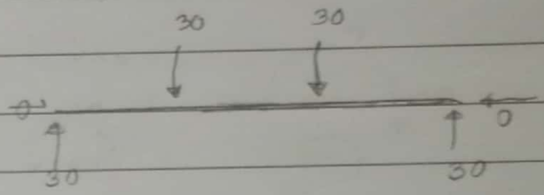
Ex

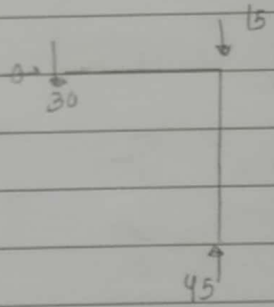
5 unknown >3 eqa.

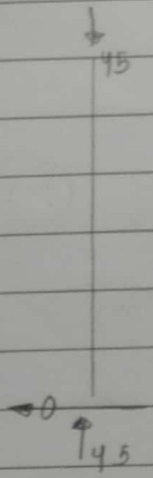


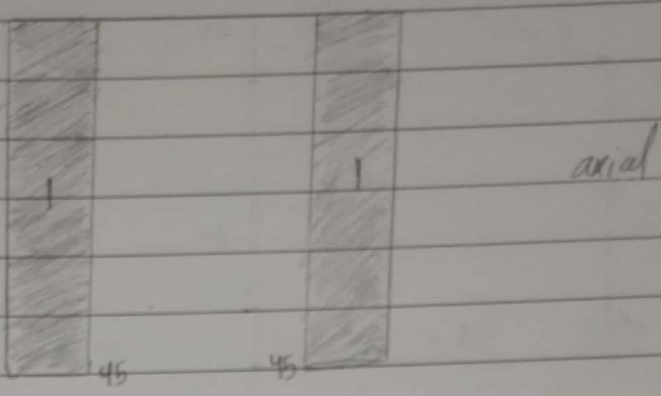


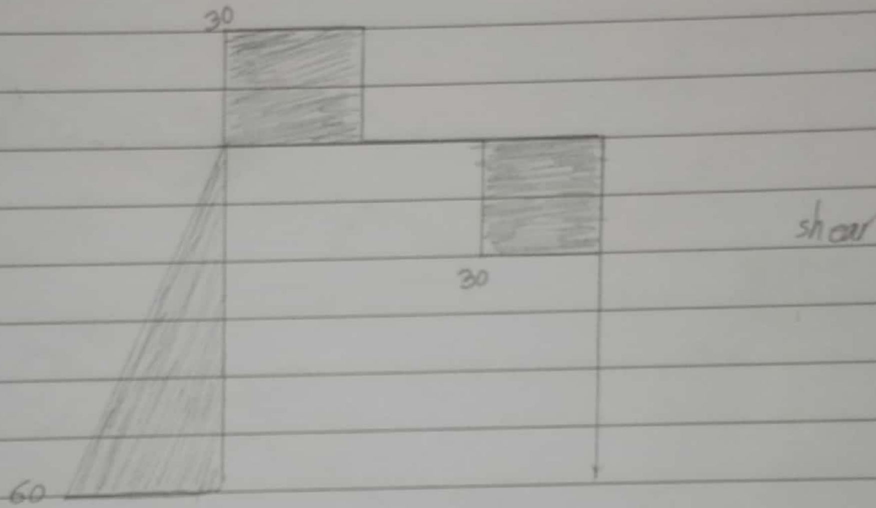


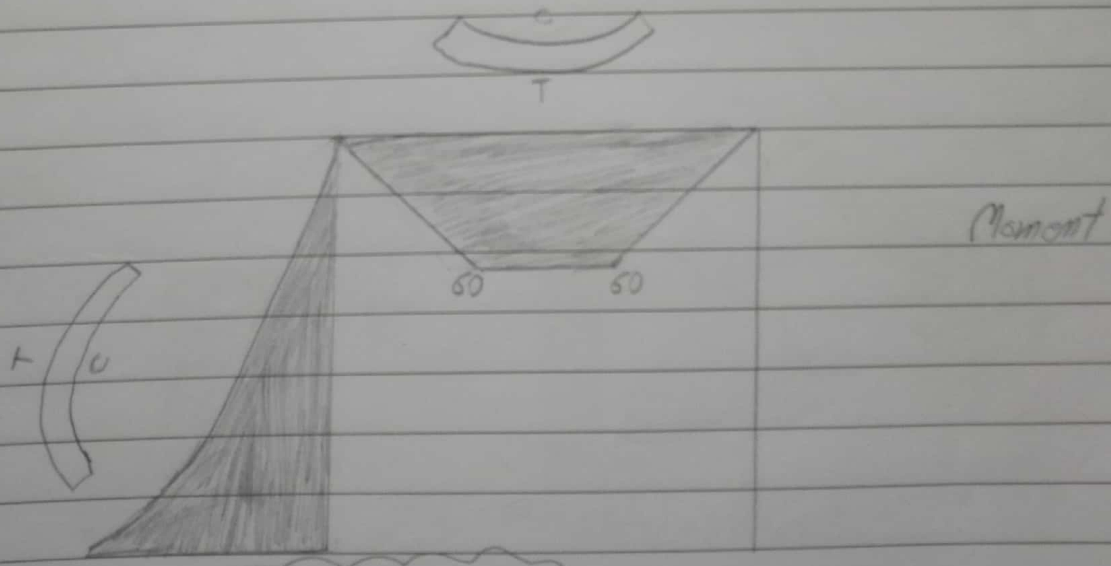








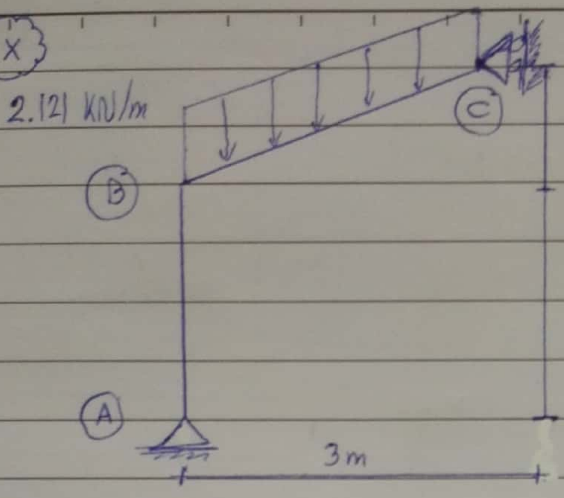


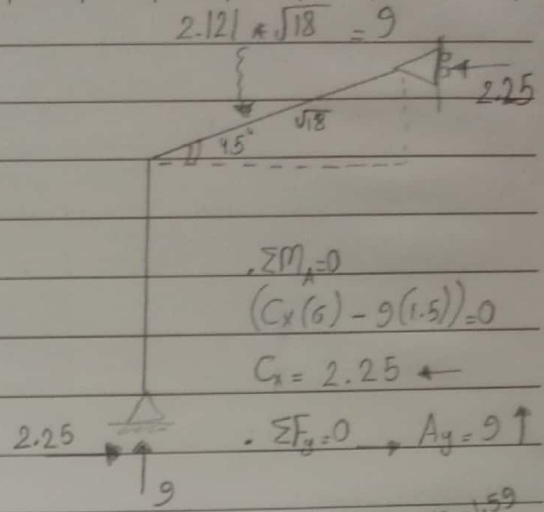


Load يخش M دائماً.

11/10/2015

Ex



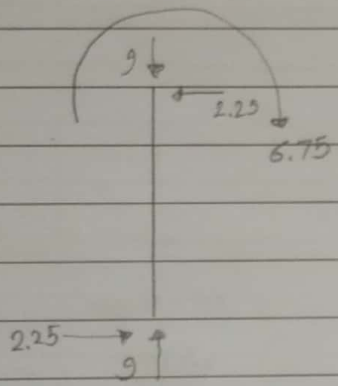


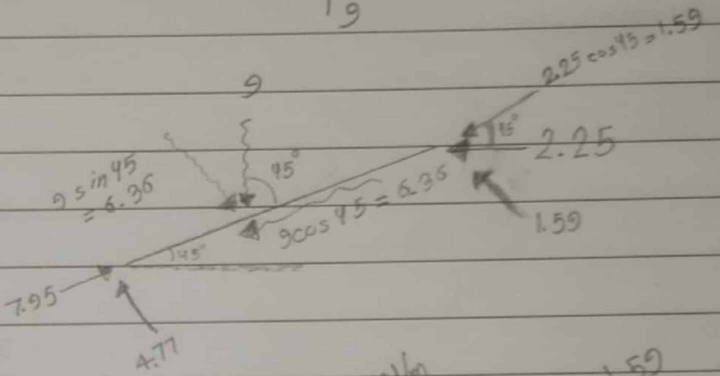
$\Sigma M_A = 0$

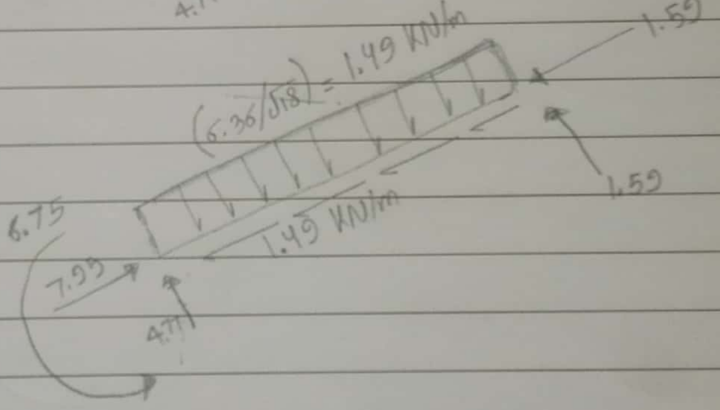
$(C_x(6) - 9(1.5)) = 0$

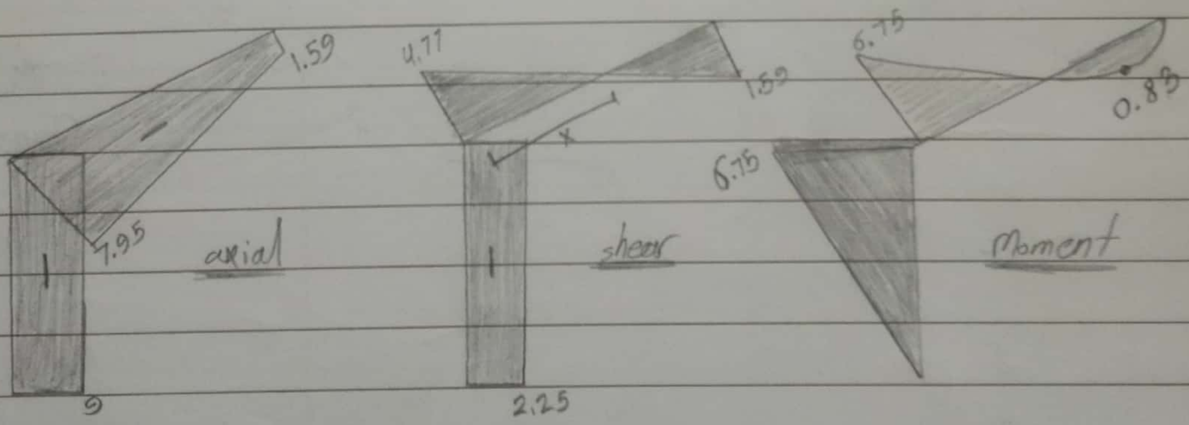
$C_x = 2.25 \leftarrow$

$\Sigma F_y = 0 \rightarrow A_y = 9 \uparrow$







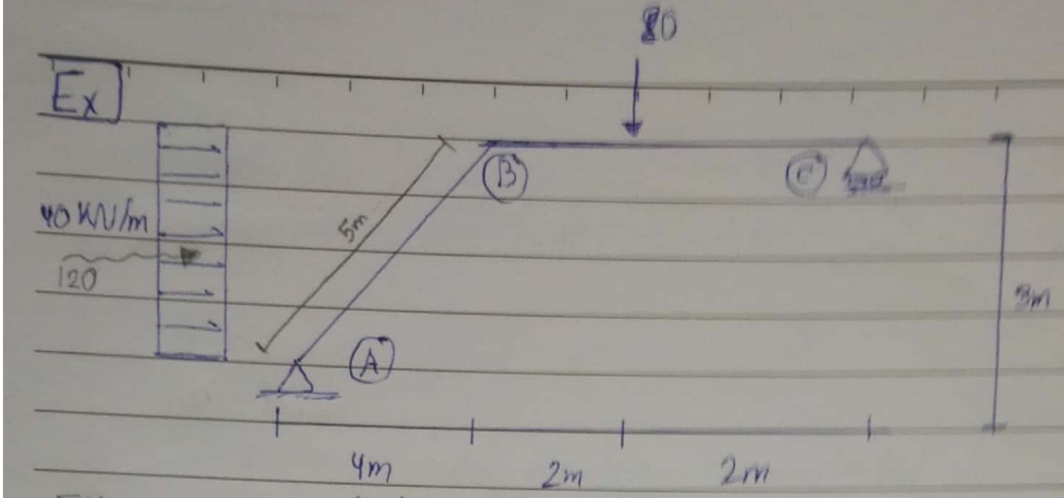


axial shear Moment

$4.77 - 1.49(x) = 0 \rightarrow \boxed{x = 3.2 \text{ m}}$

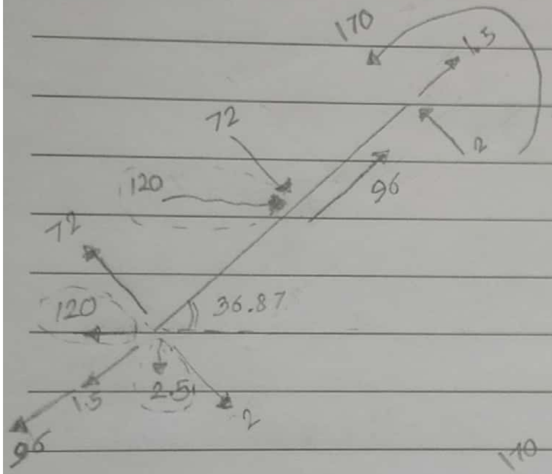
13/10/2015

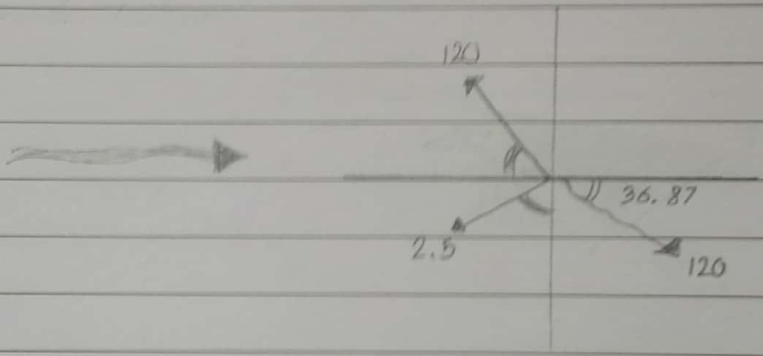
Ex

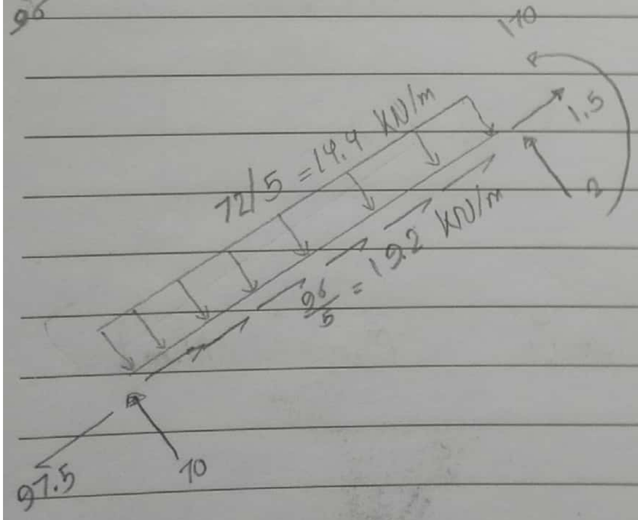


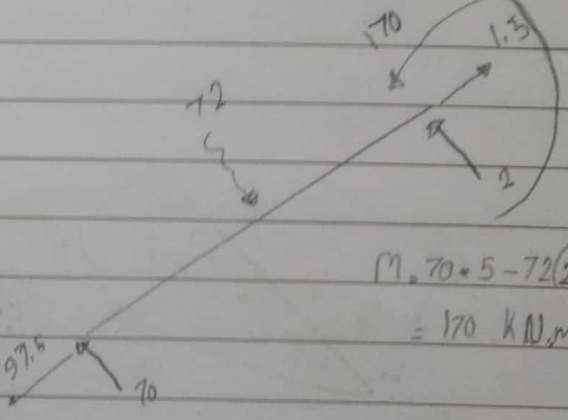
$\Sigma M_A = 0 \rightarrow -120(1.5) - 80(6) + C_y(8) = 0 \rightarrow C_y = 82.5\ kN$

$\Sigma F_y = 0 \rightarrow A_y = -2.5\ kN \qquad \Sigma F_x = 0 \rightarrow A_x = -120\ kN$

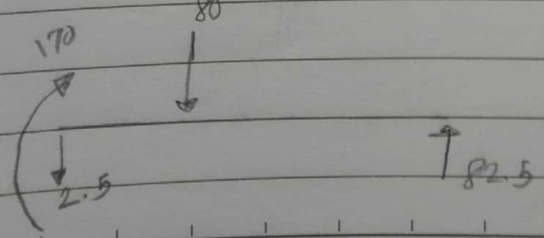




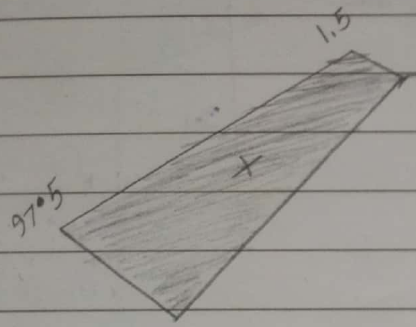


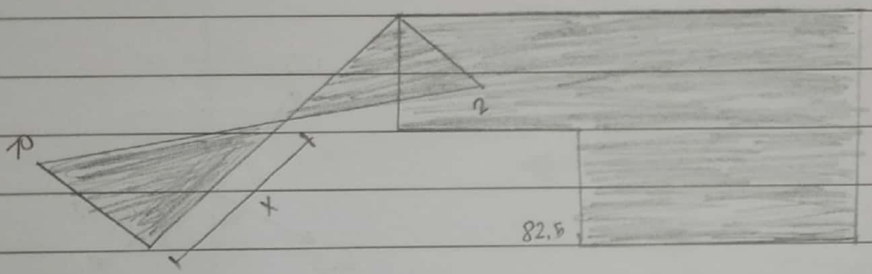


$M = 70 * 5 - 72(2.5)$
$= 170\ kN.m$



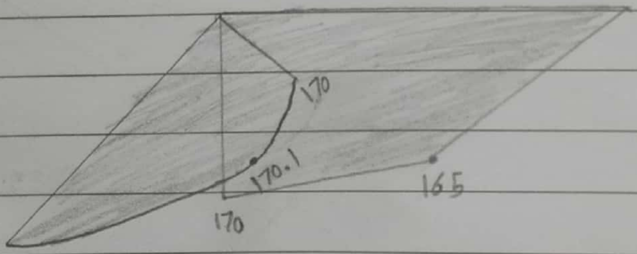
$-82.5(4) + 80(2) = M$





$70 - 14.4\ x = 0$

$x = 4.86$ m



18/10/2015

Truss

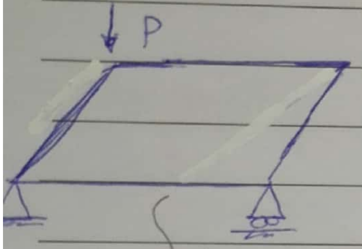
Assumption for Design

- The members are joint togother by smoth pins.
- All loading are applied at the joints
- The weight of the members is neglected

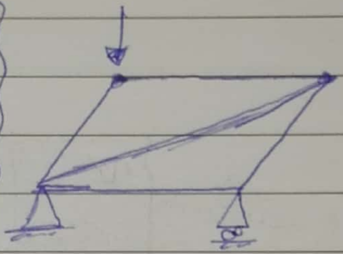
Truss لا يحمل V & M
وإذا حصل تقوس في member
يعني أن هناك V & M
لذلك توزع Load على
joint

classification of coplanar trusses

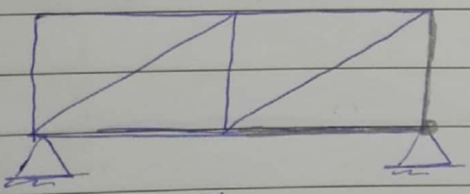
simple truss



this truss will collapse

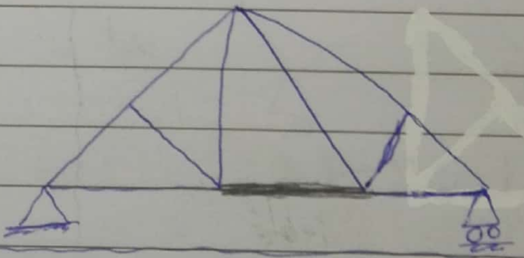


The smplest frame work that is rigid or stable is a triangle

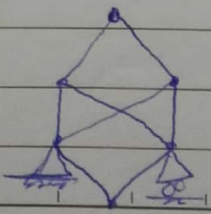


told 2 element and 1 joint

compound truss:
is formed by connecting two or more simple truss together



complex truss:
is one that cannot be classified as being either simple or compound.



Determinacy

b: # member
r: # reaction
j: # joint

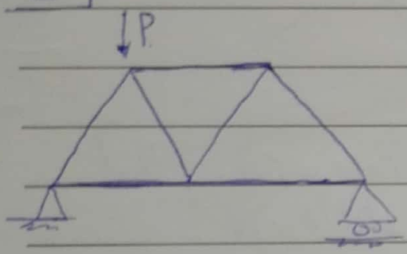
$2j = b+r$ statically determinate
$2j < b+r$ Statically indeterminate
$2j > b+r$ unstable

$b+r \geq 2j$

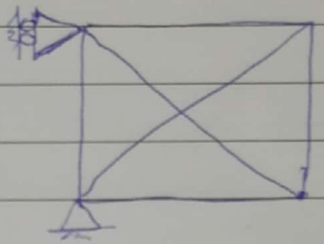
unstable if truss support reactions are concurrent (متلاقي) or parallel or if some of the components of the truss form a collapsible mechanism.

(Look @ figure (3.3-3.5))

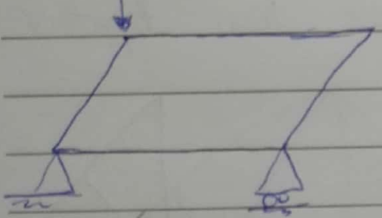
Ex



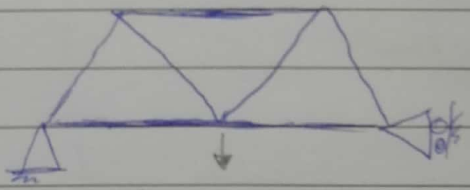
b = 7 j = 5 r = 3
$7+3 = 2*5$
statically determinate



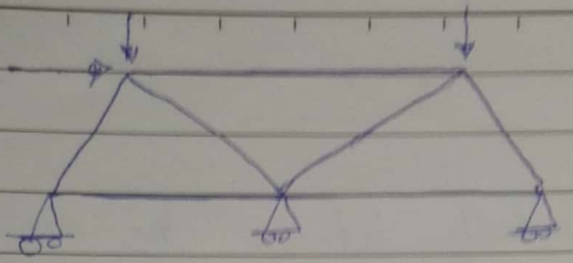
b = 6
r = 3
j = 4
$6+3 > 2*4$ (statically indeterminate)



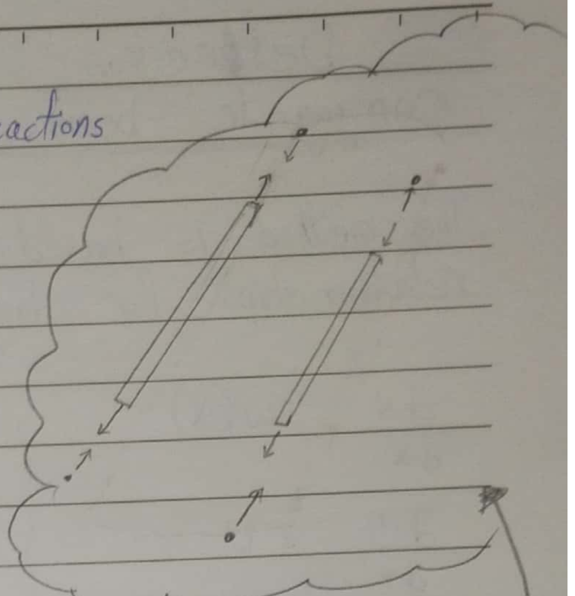
b = 4
j = 4
r = 3
$7 < 8$
unstable

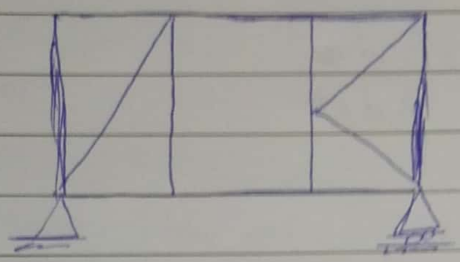


b = 7
r = 3
j = 4.
$10 < 12$
unstable
concurrent reactions



unstable // reactions



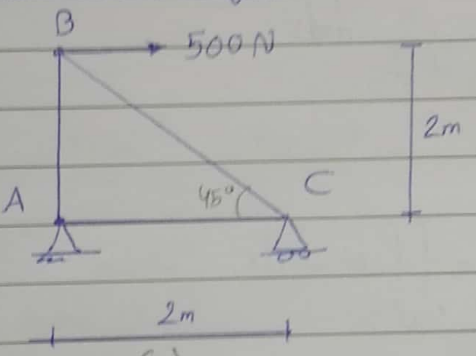


unstable

20/10/2015

The method of joint

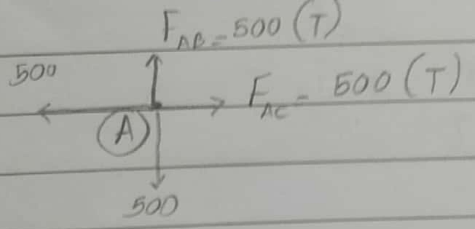
Ex



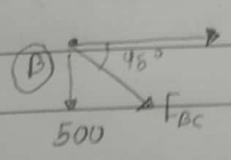
$\Sigma F_x = 0 \longrightarrow A_x = 500 \leftarrow$

$\Sigma F_y = 0 \longrightarrow A_y = 500 \downarrow$

$\Sigma M_A = 0 \longrightarrow C_y = 500 \uparrow$



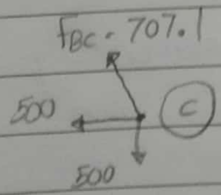
أي سهم ضاغط على joint
نوع (C) أما منطلق نوع (T)

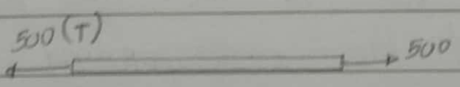


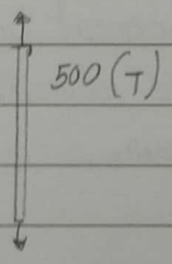
$\Sigma F_x = 0$

$\Sigma F_y = 0 \longrightarrow F_{BC} = -707.1$ KN

$= 707.1$ KN (C)







وإذا أثرت Force على member (T).
وإذا كانت راجعة فهي C
Truss لا يوجد فيها V, δ و M.

Deflection

Conjugate beam method

The method is based on the similarity between the relationships for loading, shear and moment.

$$\frac{dV}{dx} = w(x)$$

$$\frac{dM}{dx} = V$$

$$\frac{d^2M}{dx^2} = w(x)$$

$$\frac{d\theta}{dx} = \frac{M}{EI} = \frac{dv^2}{dx^2}$$

$$\theta = \int \frac{M}{EI}\, dx$$

$$v = \iint \frac{M}{EI}\, dxdx$$

OR

$$V = \int w \, dx$$

$$\Updownarrow$$

$$\theta = \int \frac{M}{EI}\, dx$$

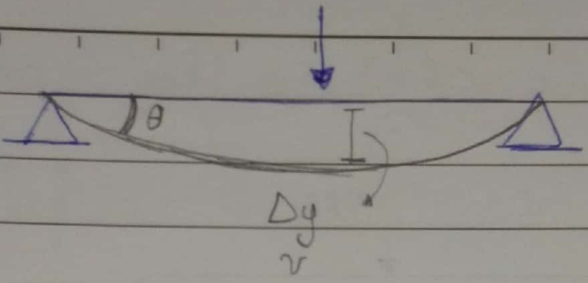
$$M = \iint w \, dxdx$$

$$\Updownarrow$$

$$v = \iint \frac{M}{EI}\, dxdx$$

- The slope at a point in the real beam is numerically equal to the shear at the corresponding point into the conjugate beam.

- The displacement of a point in the real beam is numerically equal to the moment at the corresponding point into the conjugate beam.



Real	conjugate
θ	V
Δ	M

• دائمًا (M=0) عند الأطراف إلّا إذا كان (free end ، hing ، roller)

• خطوات إيجاد الـ (Defleaction):

① نرسم (Moment Diagram)

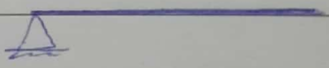
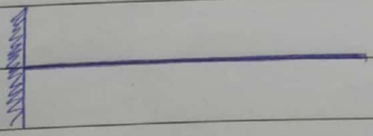
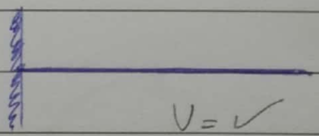
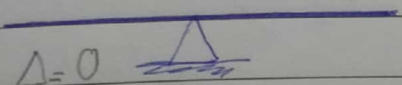
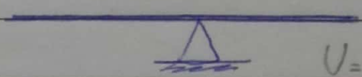
② تحويل (Moment Diagram) لـ (distributed load)

③ تحويل (real support) من حقيقة لـ (conjugate support)

④ $\theta \leftarrow V$

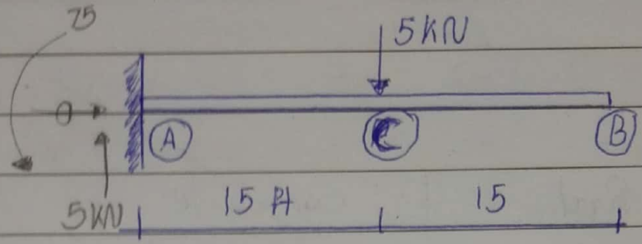
$\Delta \leftarrow M$

look at table 8.2

	Real	conjugate	
θ=✓ Δ=0			V=✓ M=0
θ=0, Δ=0			V=0 , M=0
θ=✓ Δ=✓			V=✓ M=✓
θ=✓ Δ=0			V=✓ M=0
θ=✓ Δ=✓			V=✓ M=✓

• أي (hing , roller) تكون في المنتصف ماشي مشكلة لو أخذت (Moment) وخلّ (shear)

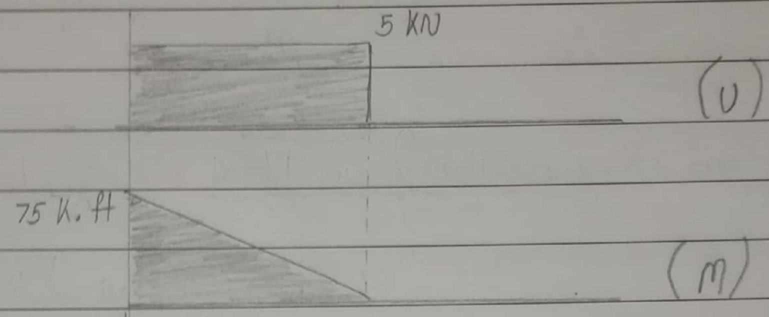
Ex

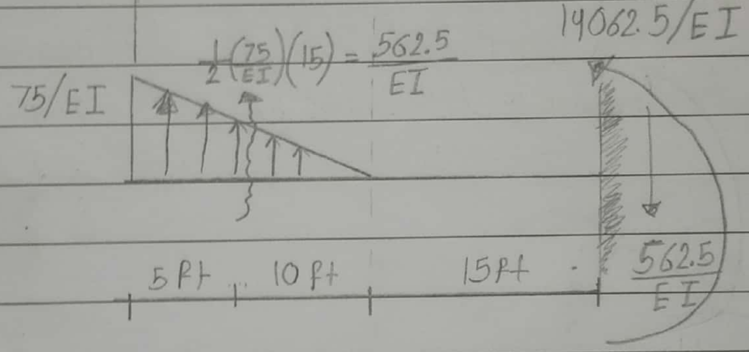


$\theta_B = ??$

$\Delta_B = ??$

$\Delta_C = ??$





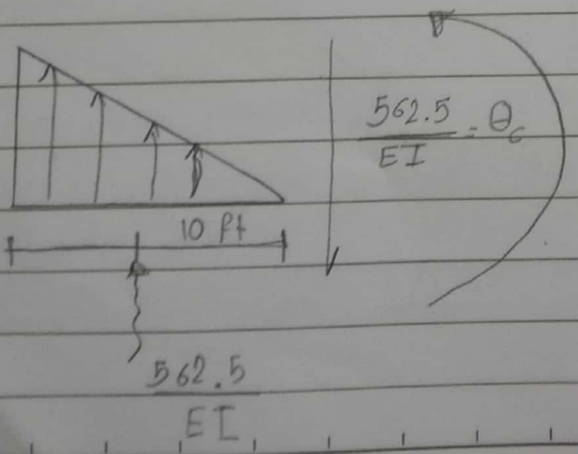
الأسهم منطلقة من جسم (beam)

$\theta_B = \frac{562.5}{EI}$ rad

$\Delta_B = \frac{14062.5}{EI} \downarrow$

• (Deg.) < (rad.)

$rad \ast \frac{180}{\pi} = Deg$



$\frac{562.5}{EI} = \theta_C$

$\frac{5625}{EI} = \Delta_C$

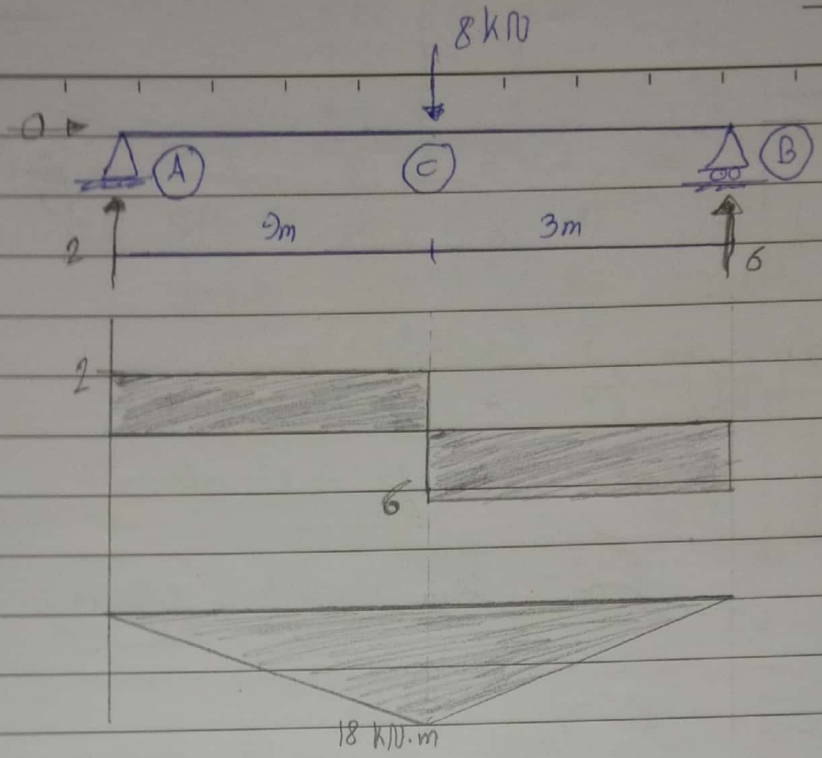
I_{grass} (I_G) هو نفسه بعد التحميل

Five Apple

22/10/2015

8.14

Ex



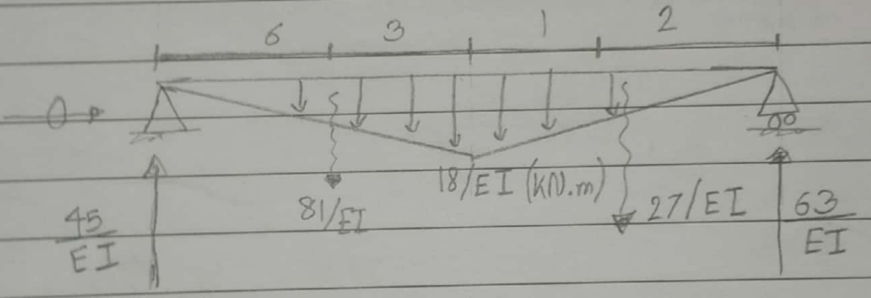
$\Delta_C = ??$

$\theta_C = ??$

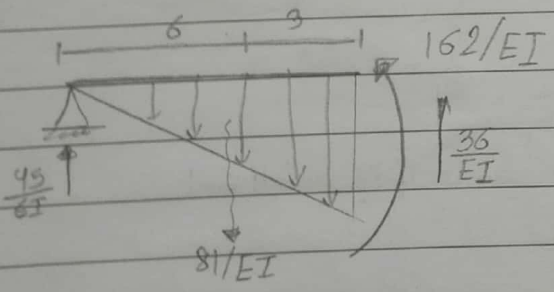
$\Delta_{max} = ??$

$E = 200 \overset{10^6}{G} Pa$

$I = 60 * 10^6 mm^4$



أصبح beam جديد له moment و reaction و shear ⑨



$\theta_C = 36/EI$

$= 3 * 10^{-3}$ rad

$= 0.1718°$

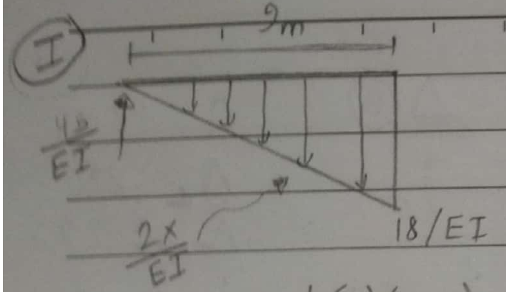
$\Delta_C = \frac{162}{EI} = 0.0135$ m

$= 13.5$ mm

$\Delta_{max} = ?? = M_{max} = V = 0$

Five Apple

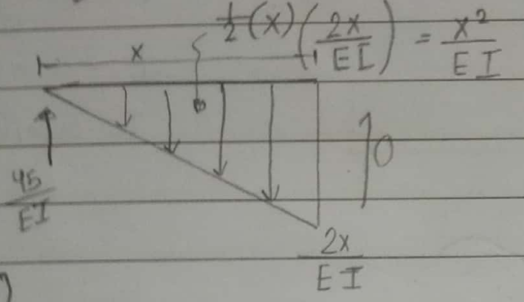
I



$9 \rightarrow 18$

$x \rightarrow 2x$

التحليل لكل مقطع

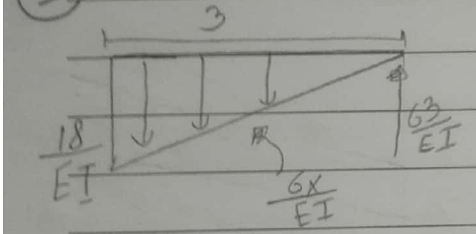


$\frac{1}{2}(x)\left(\frac{2x}{EI}\right) = \frac{x^2}{EI}$

$$\frac{45}{EI} - \frac{x^2}{EI} = 0$$

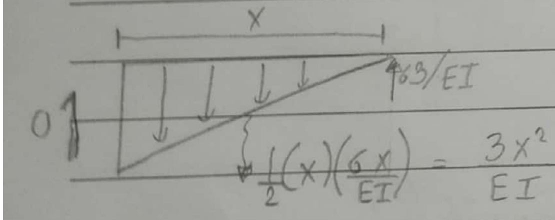
$$\boxed{x = \sqrt{45} = 6.71 \text{ m}}$$

II



$3 \rightarrow 18$

$x \rightarrow 6x$

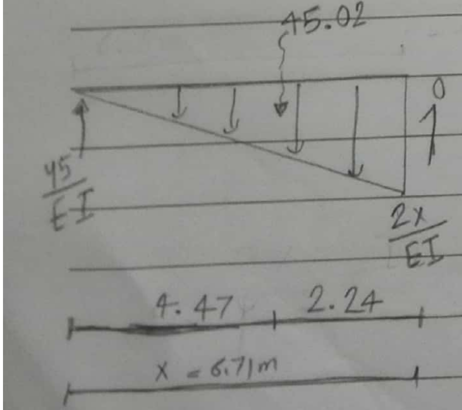


$\frac{1}{2}(x)\left(\frac{6x}{EI}\right) = \frac{3x^2}{EI}$

$$\frac{63}{EI} - \frac{3x^2}{EI} = 0$$

$$\boxed{x = 4.58 \text{ m}}$$

• مسافة المقطع الثاني خاطئة لأن طول المقطع الثاني يساوي (3m)
فنأخذ المقطع الأول لأن طول المقطع الأول يساوي (9m)
و(zero shear) كانت عند مسافة (6.71m) أي في المقطع الأول



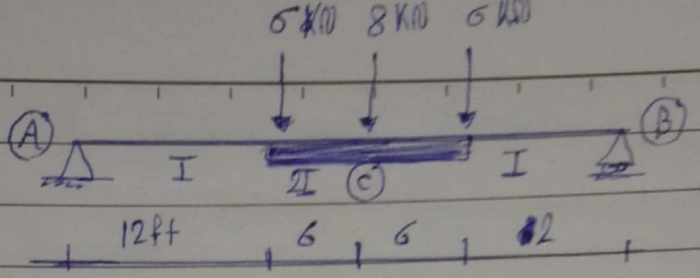
$$\frac{201.11}{EI} \text{ (KN.m)} = \Delta_{max}$$

$$\frac{201.11 * 10^3}{200*10^9 * 60*10^6 * 10^{-12}} = 0.0168 \text{ m}$$

$$= \boxed{16.8 \text{ mm}}$$

25/10/2015

Ex



$\Delta_C = ??$

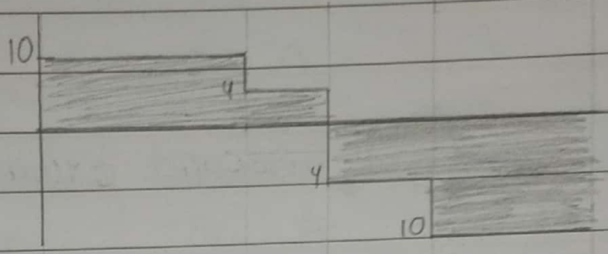
$\theta_C = ??$

• يمكن إيجاد reaction بمجرد النظر وذلك لأنه symmetry
أي أن reaction تتوزع على (hing)(roller) بالتساوي

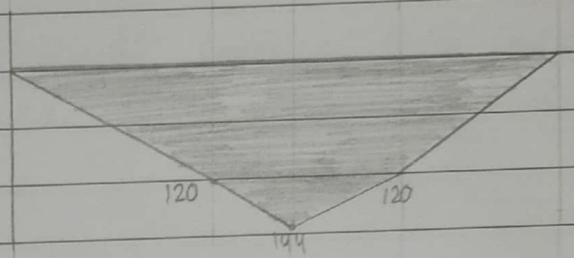
$A_y = 10$ kN ↑

$B_y = 10$ kN ↑

V(kN)

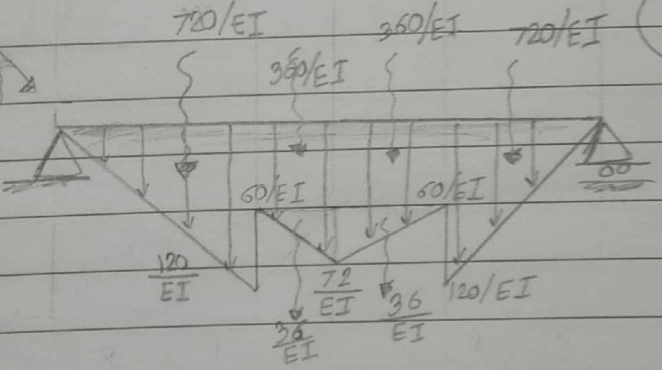


M (kN.m)



beam أخرج
حيث يوجد له
reaction و moment

• ثم جعل Beam كله (I)
وذلك بتحويل الجزء الخبي لعنيره
من أجل ذلك اختلفت
قيمة M

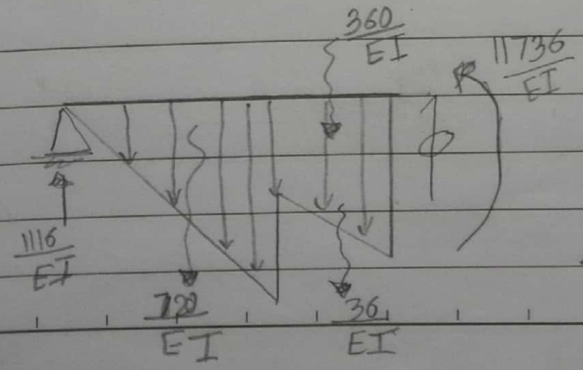


• لأن (beam) symmetry أيضا (تتوزع reaction) على (roller)(hing) بالتساوي

$A_y = \frac{1116}{EI}$ kN , $B_y = \frac{1116}{EI}$ kN

• لإيجاد Δ_C و θ_C نأخذ (section):

وذلك لأن reaction توزعت بالتساوي على B و A فإن (V) منتصف beam سيكون صفر $\rightsquigarrow V_C = 0 = \theta_C$



$\Delta_C = M_C = 11736/EI$ kN.m

$\Sigma M_A = 0$

$-720(8) - 360(15) - 36(16) + M = 0$

⇝ second exam

29/10/2015

Method of virtual work (Beams and Frames)

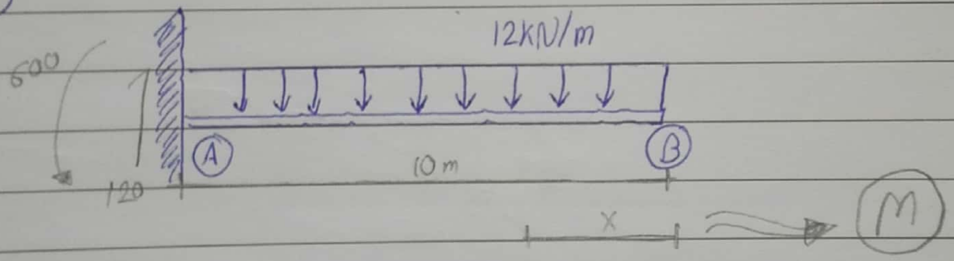
$$1.\ \Delta = \int_0^L \frac{mM}{EI}\,dx$$

1- external virtual unit load acting on the beam or frame in the direction of Δ

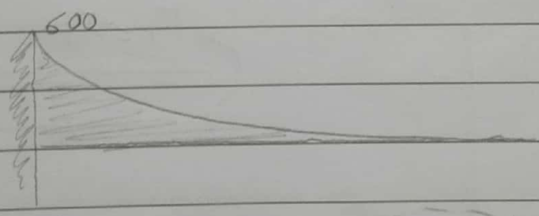
m- internal virtual moment in the beam expressed as function of x and caused by the external virtual unit load.

M- internal moment in the beam or frame expressed as a function of x and caused by the real loads

Ex

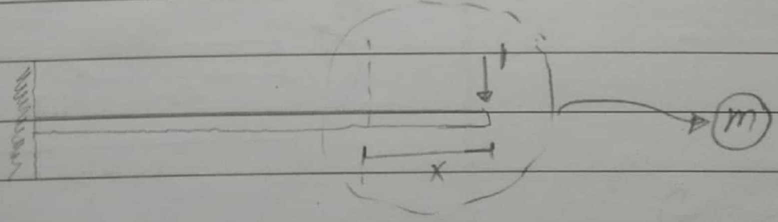


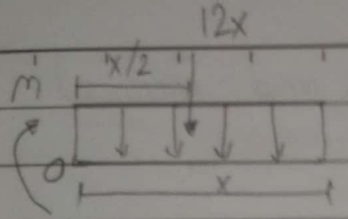
$\Delta_B = ??$

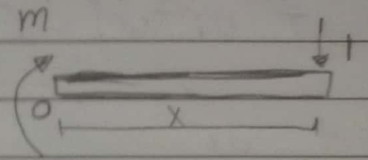


$\Delta_B = ??$

يصعب حلّها
بطريقة conjugate







$\circlearrowleft + \ \Sigma M_o = 0$

$M = -6x^2$

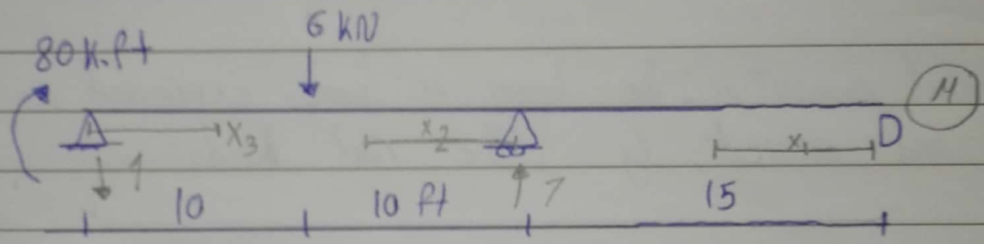
$\circlearrowleft + \ \Sigma M_o = 0$

$-m - x = 0$

$m = -x$

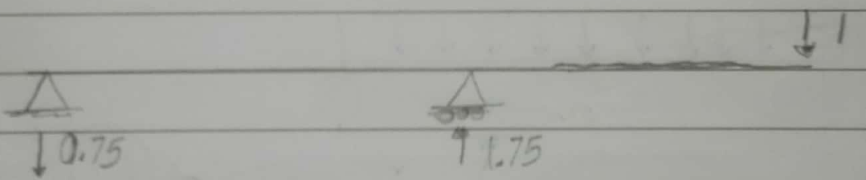
$$\Delta = \int_0^{10} \frac{(-x)(-6x^2)}{EI}\,dx = \left.\frac{6x^4}{4EI}\right]_0^{10} = \frac{15000}{EI} \downarrow$$

Ex

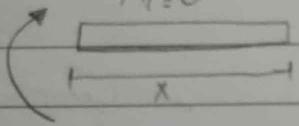


EI const

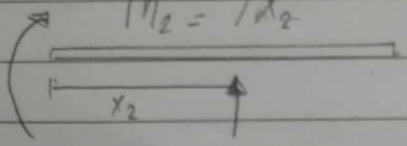
$\Delta_D = ??$

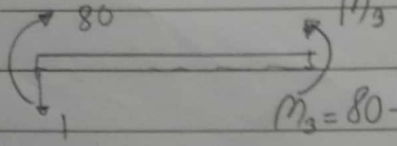


$M_1 = 0$



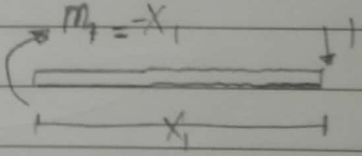
$M_2 = 7X_2$

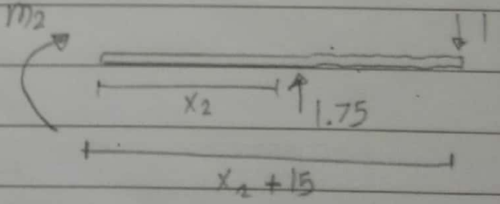


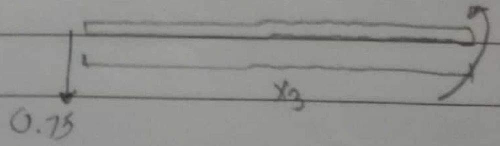


$M_3 = 80 - X_3$

$m_1 = -X_1$







$m_3 = -0.75x_3$

$$\Delta = \int_{0}^{15} \frac{(-x)(0)}{EI}\, dx$$

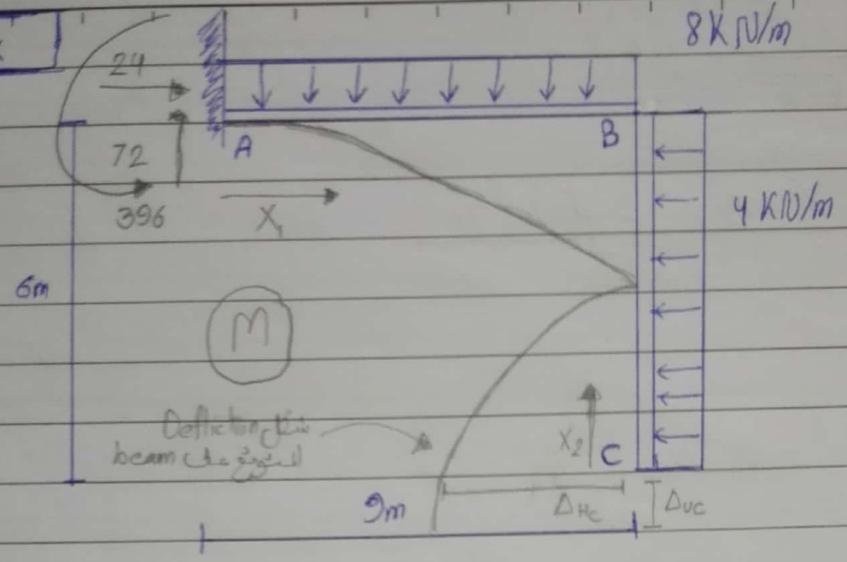
$$+ \int_{0}^{10} \frac{(0.75x - 15)(7x)}{EI}\, dx$$

$$+ \int_{0}^{10} \frac{(-0.75x)(80-x)}{EI}\, dx$$

$$= \frac{-6250}{EI} - \frac{6250}{EI} \uparrow$$

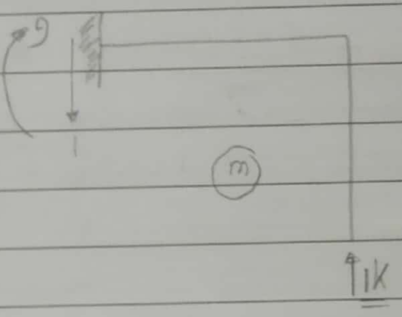
1/11/2015

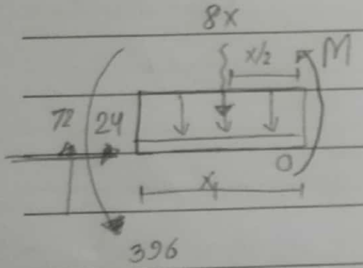
Ex

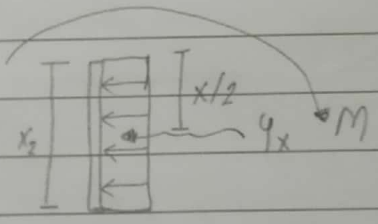


$\Delta_{v_C} = ??$

$$\Delta = \int_0^L \frac{m\,M}{EI}\,dx$$

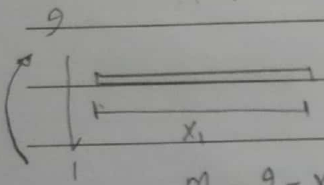




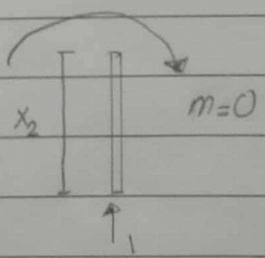


$\overset{+}{\circlearrowleft}\ \Sigma M_0 = 0 \longrightarrow M = 72x - 4x^2 - 396$

$M = -2x^2$



$m = 9 - x$



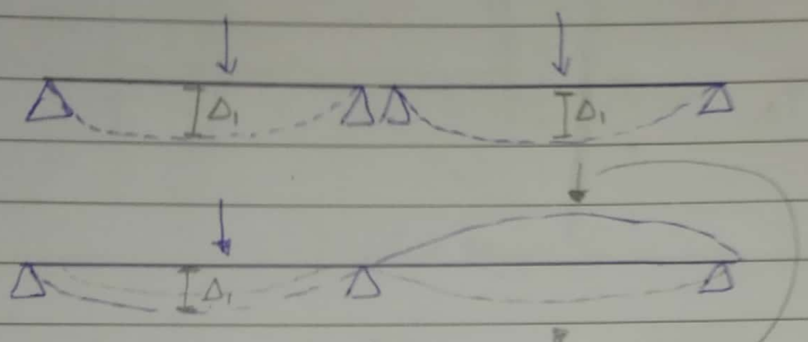
$$\Delta_{vc} = \int_0^9 \frac{(9-x)(72x-4x^2-396)dx}{EI} + \int_0^6 \frac{(0)(-2x^2)dx}{EI} = \frac{-9477}{EI}$$

$$= \frac{9477}{EI} \downarrow$$

④

Inderterminate structure

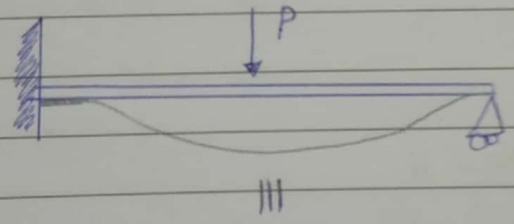
when the # of unknown reaction or internal forces exceed, the # of equilibrium equation a vailable for its analysis → indeterminate

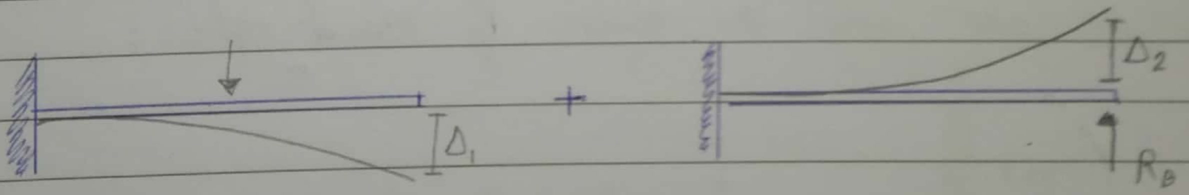


عندما أزنا Load
على الطرف الأخر تغير اتجاه Deflection

كلما كانت علاقة
الانشاء طبقة
(معقدة) وكلما الانشاء
(continous) كلما كان
الانشاء أقل،
لأنه يحتاج لـ M أقل
ويعطي أقل deflection
ويوفر المال من حيث العمالة
والمصانع

الشكل (٢)

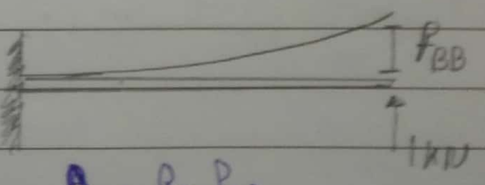




$$\Delta_1 - \Delta_2 = 0$$

$$\Delta_1 - R_B f_{BB} = 0$$

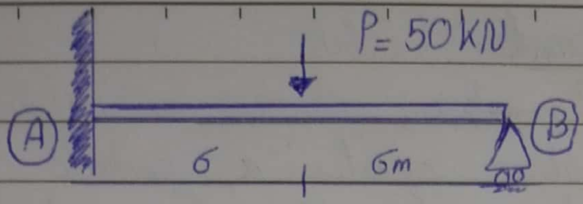
$$R_B = \frac{\Delta_1}{f_{BB}}$$

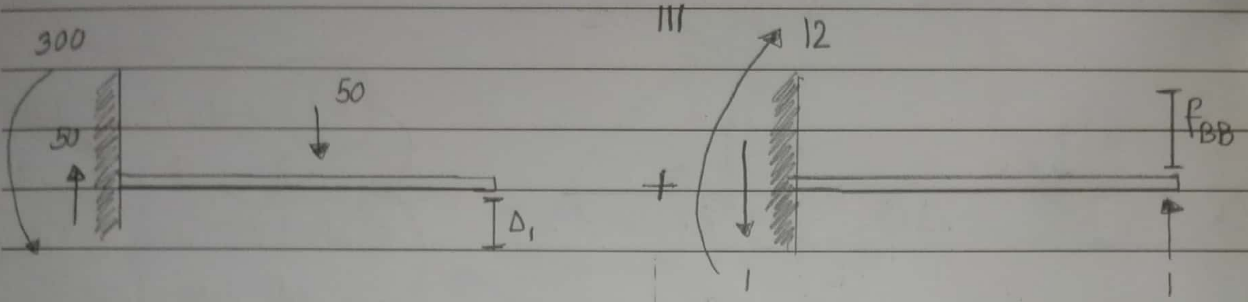


$$\Delta_2 = R_B f_{BB}$$

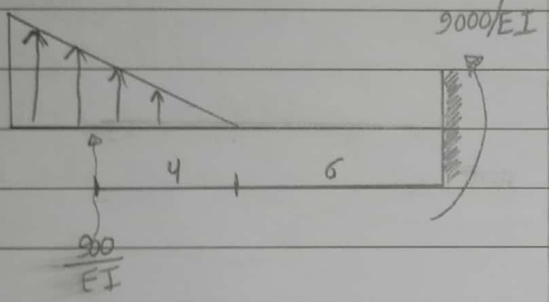
3/11/2015

Ex

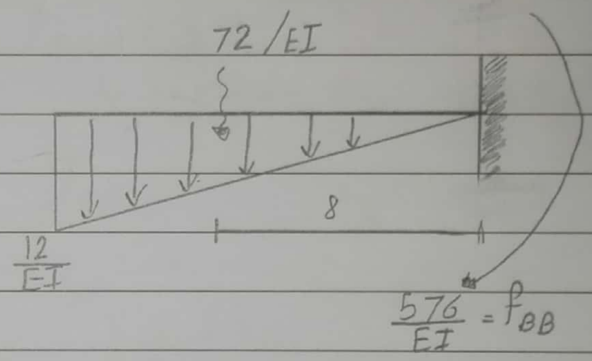




By conjugate method :



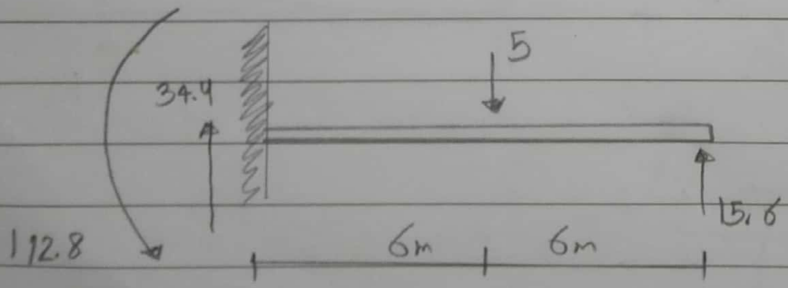
By conjugate method :

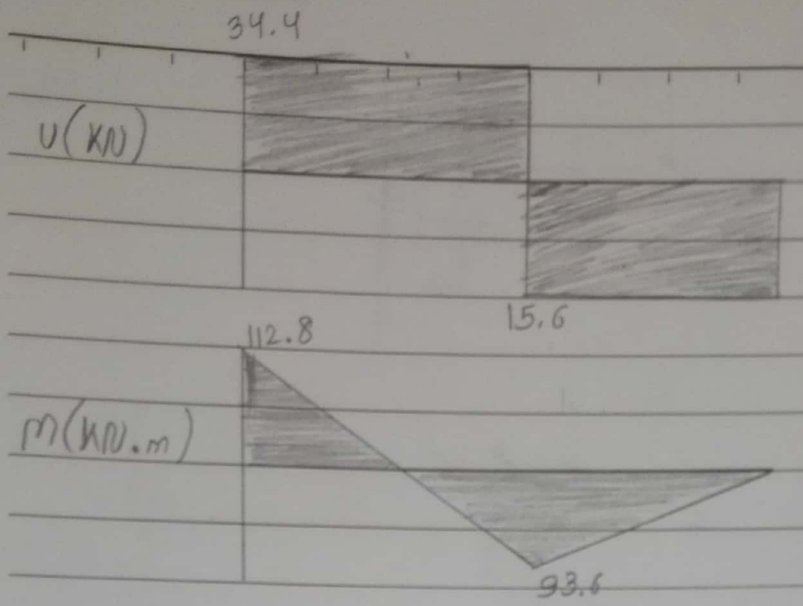


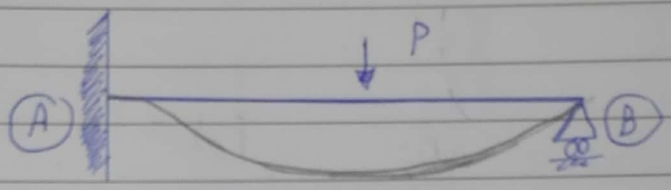
- $B_y = \Delta_1 / f_{BB}$

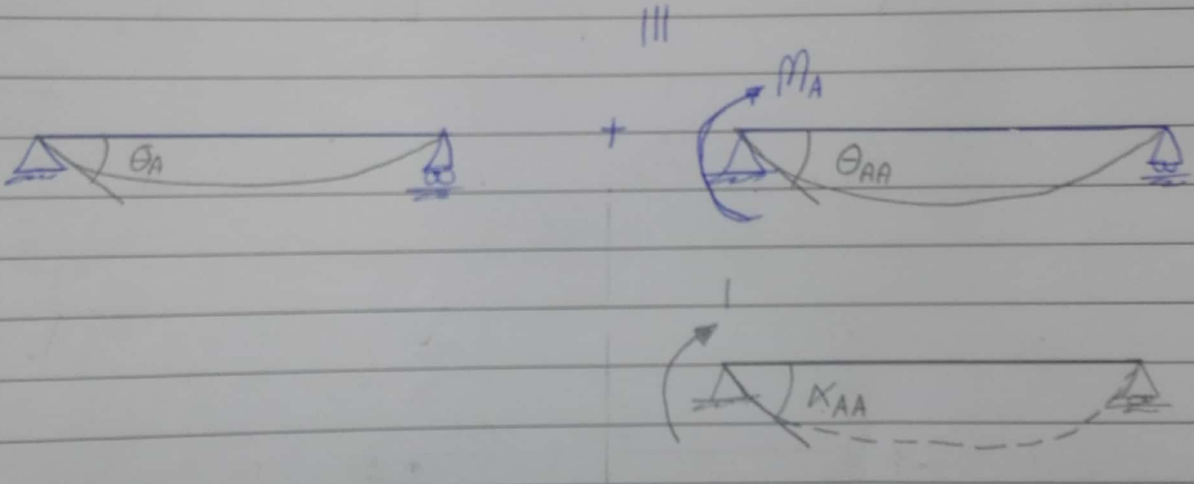
$= 9000/EI / 576/EI = 15.6$ k

- Now we can solve this problem by Normal statics :









$$\theta_{AA} = M_A \alpha_{AA}$$

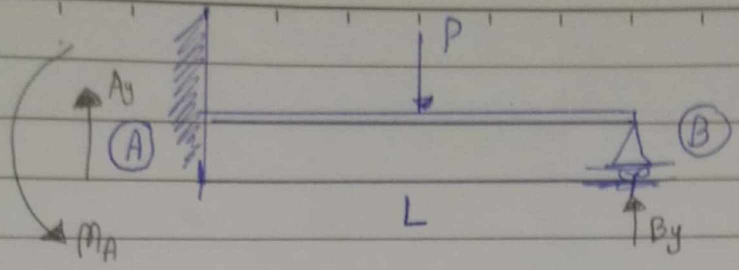
$$\theta_A + \theta_{AA} = 0$$

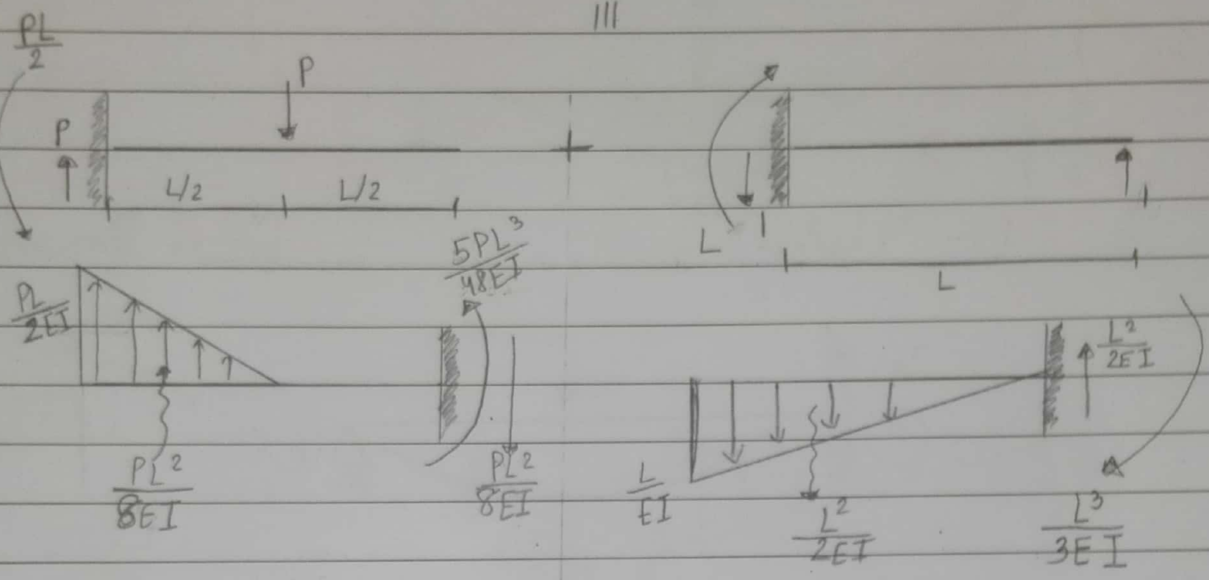
$$M_A = -\theta_A / \alpha_{AA}$$

دلالة على ان

اتجاه M المفروض بالعكس

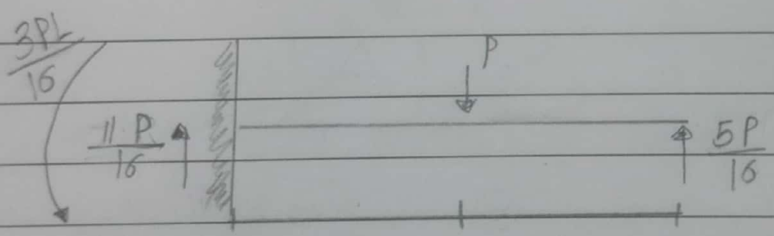
Ex



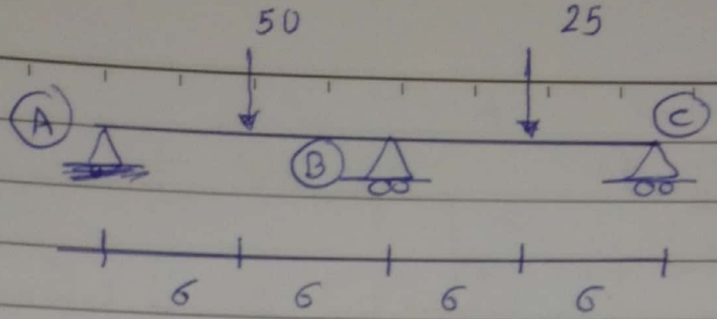


$$B_y = \Delta_1 / f_{BB}$$

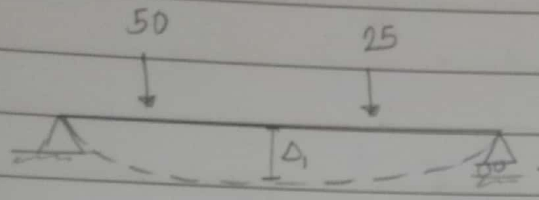
$$= \frac{5PL^3}{48EI} * \frac{3EI}{L^3} = \boxed{\frac{5P}{16}}$$

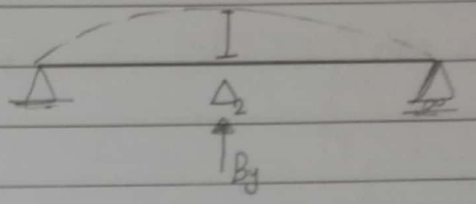


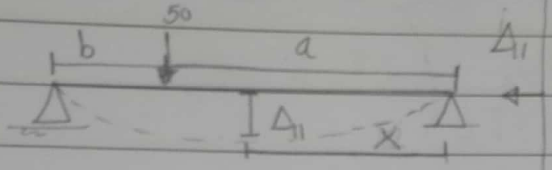
Ex

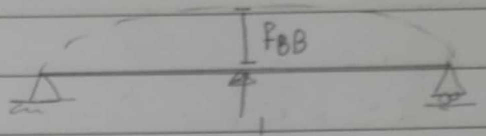


EI-constant







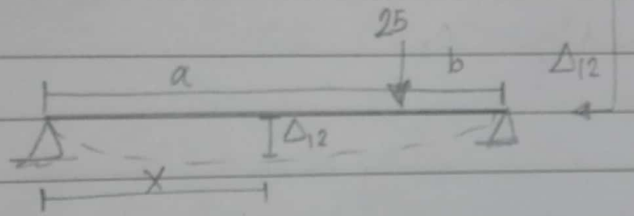


$a = 18$, $b = 6$, $P = 50$, $x = 12$, $L = 24$

$\Delta_{11} = \frac{9900}{EI} \downarrow$

$a = b = x = 12$, $P = 1$, $L = 24$

$f_{BB} = \frac{288}{EI}$



$a = 18$, $b = 6$, $x = 12$, $P = 25$, $L = 24$

$\Delta_{12} = \frac{4950}{EI} \downarrow$

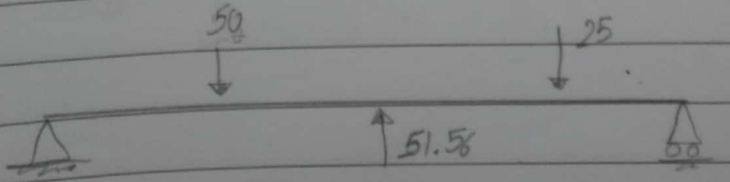
$\Delta_1 = \Delta_{11} + \Delta_{12}$

$$\left(\frac{9900}{EI} + \frac{4950}{EI}\right) - B_y\left(\frac{288}{EI}\right) = 0$$

$B_y = 51.56$ KN

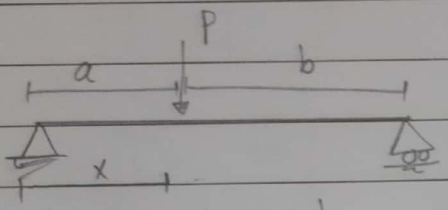
لمعرفة كيفية الحل

إضغط



١- قمنا بتطبيق القانون $\Delta_1 + \Delta_2 = 0$

2- عند حسابنا لـ Δ_2 قمنا بحسابه وفق $\Delta_2 = f_{BB} B_y$
وفرضنا B_y يساوي 1KN
ولكن تم حساب
f_{BB} بقانون يتم إدخاله في الامتحان وهو



$$\Delta = \frac{Pbx}{6EIL}\left(L^2 - b^2 - x^2\right)$$

$a \le x < b$

٣- عند حسابنا لـ Δ_1 تم تقسيمه لجزئين وذلك لأنه تحوي 2 Load
وحسبنا Δ_{11} من جهة اليسار بـ 1 Load واحد فقط
وحسبنا Δ_{12} من جهة اليمين بـ 1 Load واحد فقط
وذلك باستخدام القانون المستخدم في النقطة السابقة

٤- بعد حسابنا لـ Δ_{11} و Δ_{12}
تم تطبيق

$$\Delta_1 = \Delta_{11} + \Delta_{12}$$

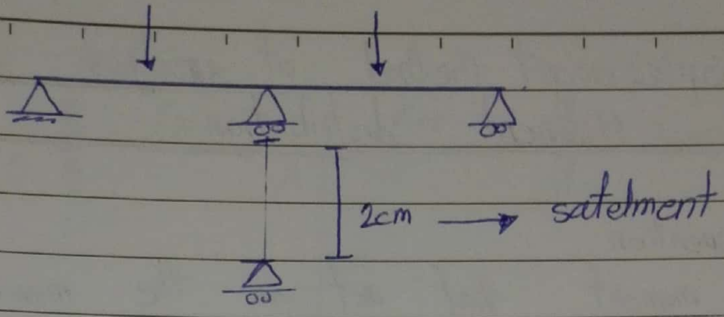
٥- قمنا بتطبيق قانون نقطة رقم ①
لإيجاد B_y وذلك من خلال

$$B_y = \Delta_1 / f_{BB}$$

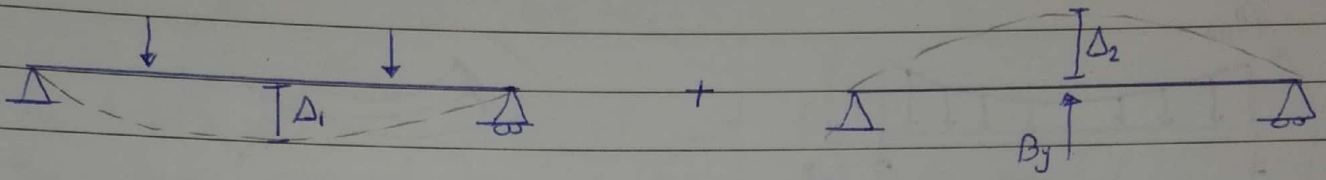
٦- أصبح لدينا سؤال جديد تم حله في slow static
statically intederminate

5/11/2015

Ex}

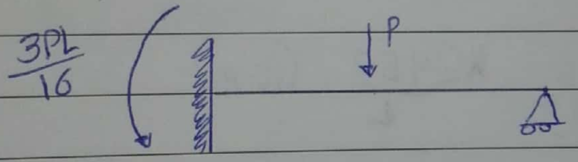


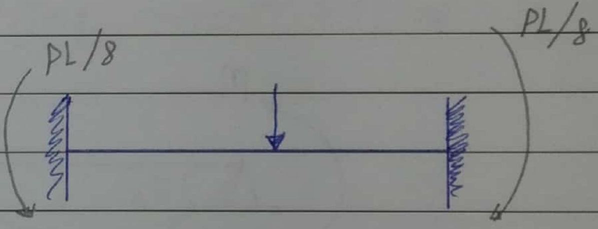
|||

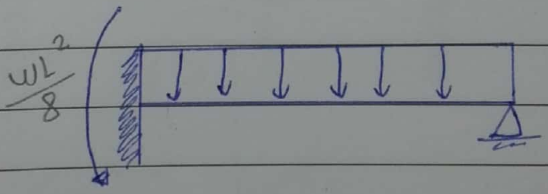


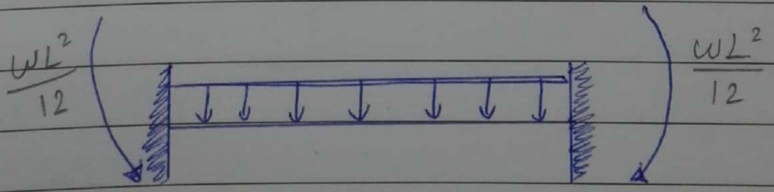
$$\Delta_1 - \Delta_2 \neq 0 \rightsquigarrow \Delta_1 - \Delta_2 = 0.02$$

Fixed End Moment (آخر صفحة في الكتاب)





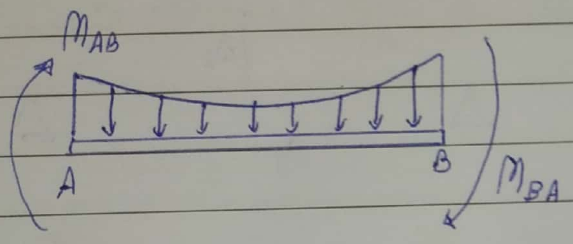


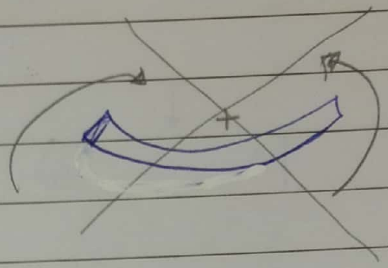


Displacement method of analysis
Moment distribution

- sign convention

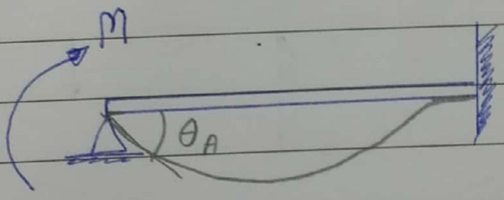
clock wise moment that act on the member are considered positive





- Members stiffness factor

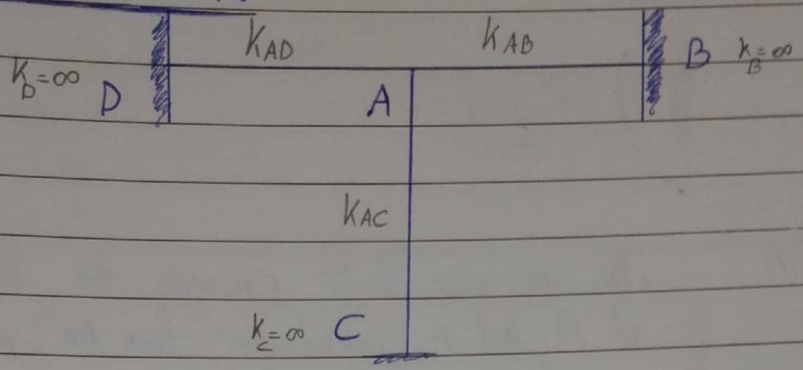
The amount of moment M required to rotate the end A of the beam . $\theta_A = 1rad$



$K = \frac{4EI}{L}$ (N.m)

$\theta_A = 1rad$

Joint stiffness factor



K_T total stiffness factor of joint A,
this value represent the amount of moment needed to rotate the joint A through an angle of 1rad

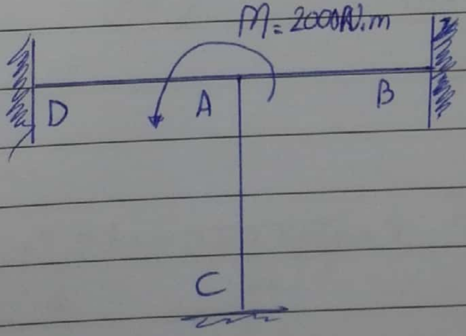
$$K_T = K_{AD} + K_{AB} + K_{AC} = \Sigma K$$

Distribution Factor (DF)

the fraction of the total resisting moment supplied by the member.

$$D_F = \frac{K}{\Sigma K}$$

Ex



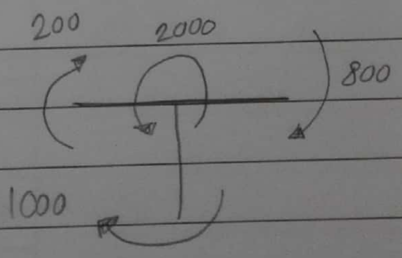
$K_{AD} = 1000$ N.m
$K_{AB} = 4000$ N.m
$K_{AC} = 5000$ N.m

$\Sigma K = 10000$ N.m

$D_{AD} = \frac{1000}{10000} = 0.1$

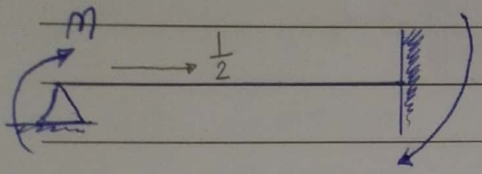
$DF_{AB} = 0.4$

$D_{AC} = 1 - 0.1 - 0.4 = 0.5$



8/11/2015

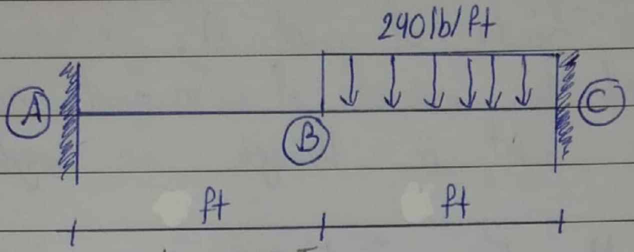
Carry over factor



$M' = \frac{1}{2}M$

$M' = \frac{1}{2}M$. The carry-over factor represents the fraction of M that is carried-over from the pin to the wall

Ex

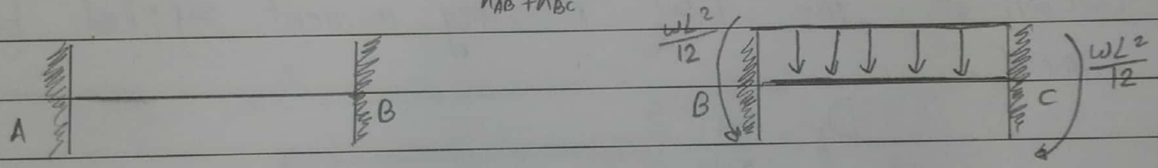


$I_{AB} = 300 \text{ in}^4$

$I_{BC} = 600 \text{ in}^4$

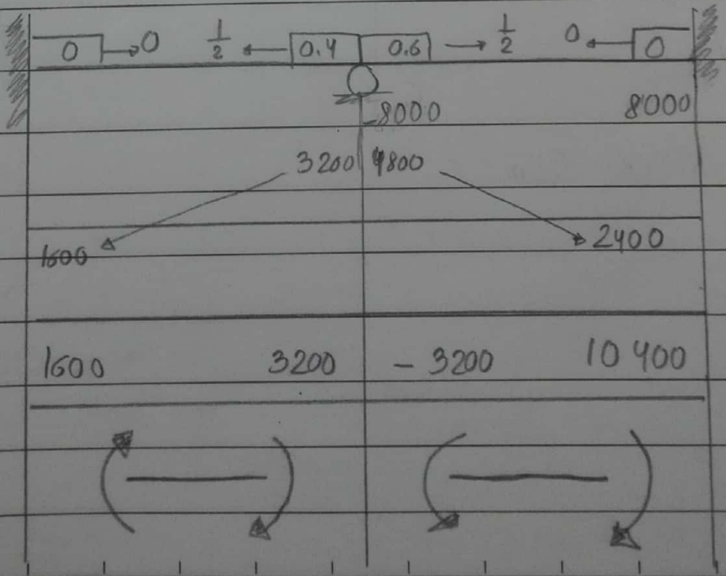
$K_{AB} = \frac{4EI_{AB}}{L_{AB}}$, $K_{BC} = \frac{4EI_{BC}}{L_{BC}}$

$DF_{AB} = 0$, $DF_{CB} = 0$, $DF_{BA} = \frac{K_{AB}}{K_{AB}+K_{BC}} \simeq 0.4$, $DF_{BC} = 1 - 0.4 \simeq 0.6$



$M^f_{BC} = \frac{-240 * 20^2}{12} = -8000 \text{ lb.ft}$

$M^f_{CB} = 8000 \text{ lb.ft}$

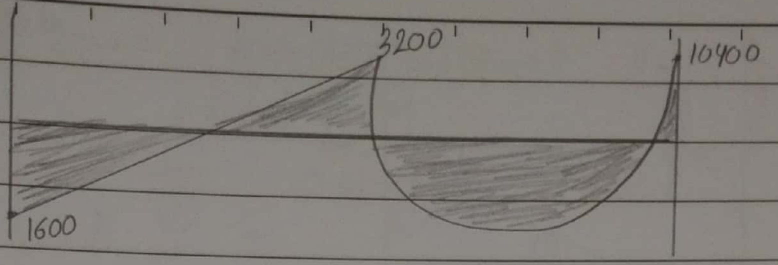


	0	0.4	0.6	0
			-8000	8000
		3200	4800	
	1600			2400
Σ	1600	3200	-3200	10 400

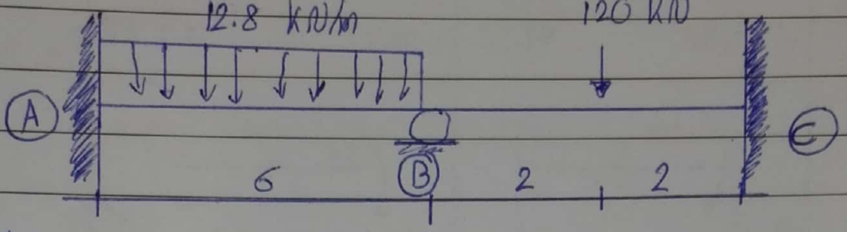
Five Apple

Scanned by CamScanner

10/11/2015



Ex



E = 200 Gpa

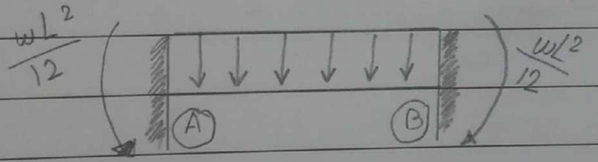
$I_{AB} = 270 * 10^6 \; mm^4$

$I_{BC} = 480 * 10^6 \; mm^4$

$K_{BA} = \frac{4EI}{6}$, $K_{BC} = \frac{4EI}{4}$

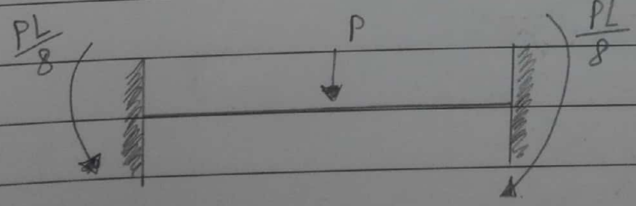
$DF_{AB} = DF_{CB} = 0 \rightarrow DF_{BA} = \frac{K_{AB}}{K_{AB} + K_{BC}} = 0.272$

$DF_{BC} = 1 - 0.272 = 0.728$



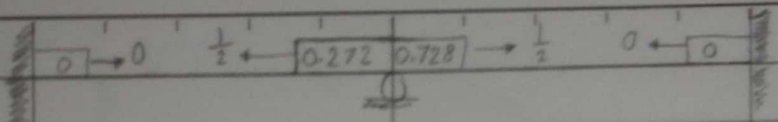
$M^F_{AB} = \frac{-12.8 * 6^2}{12} = -38.4 \; kN.m$

$M^F_{BA} = 38.4 \; kN.m$

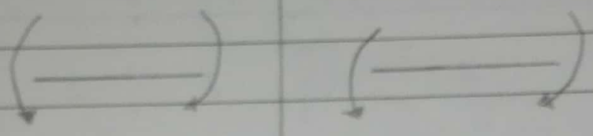


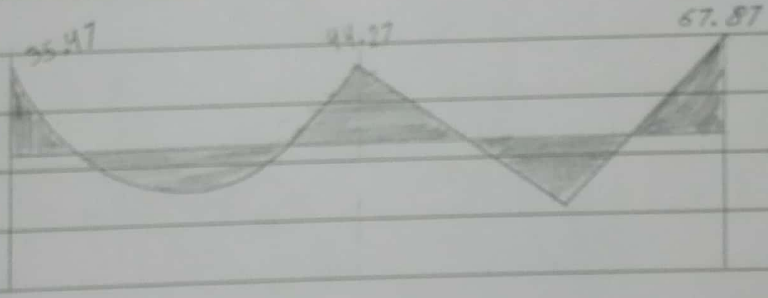
$M^F_{BC} = -60 \; kN.m = \frac{-120 * 4}{8}$

$M^F_{CB} = 60 \; kN.m$

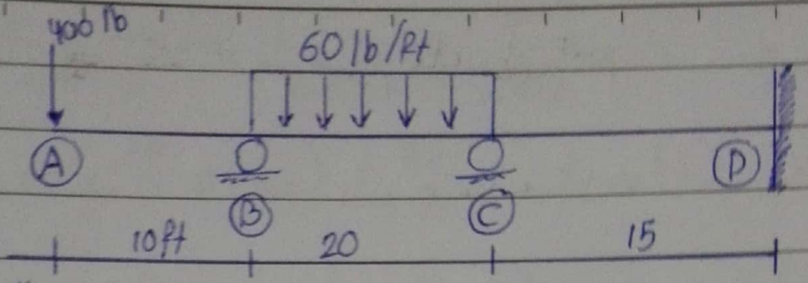


	-38.4	38.4 5.87	-60 15.73	60
	2.935			7.865
Σ	-35.47	44.27	-44.27	67.87





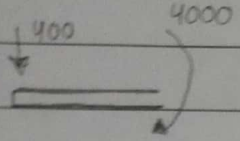
Ex



$I_{AB} = 500\ in^4$

$I_{BC} = 750\ in^4$

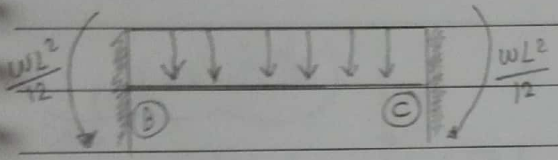
$I_{CD} = 600\ in^4$



$K_{BC} = \frac{4EI_{BC}}{20}$, $K_{CD} = \frac{4EI_{CD}}{15}$

$DF_{BC} = 1.0$, $DF_{CB} = \frac{K_{BC}}{K_{BC} + K_{CD}} = \frac{(750)(15*20)}{20((750*20)+(600*15))} = 0.484$

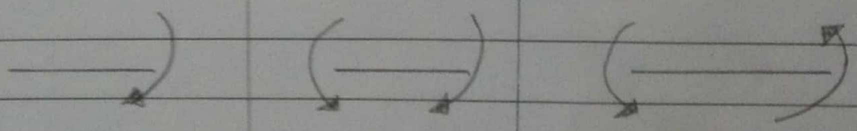
$DF_{CD} = 1 - 0.484 = 0.516$



$m^F_{BC} = -2000\ lb.ft$

$m^F_{CB} = 2000\ lb.ft$

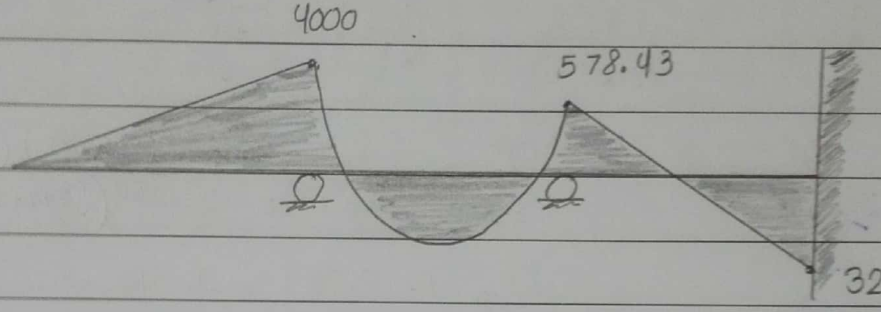
		1.0 → $\frac{1}{2}$	$\frac{1}{2}$ ← 0.484	0.516 → $\frac{1}{2}$	0 ← 0
	4000	-2000	2000		
		-2000	-968	-1032	
		-484	-1000		-516
		484	+484	516	
		+242	242		+258
		-242	-117.13	-124.87	
		-58.57	-121		-62.44
		58.57	58.56	62.44	
Σ	4000	-4000	578.43	-578.43	-320.44



• نشنيه عند الرسم نرسم على ذيل السهم

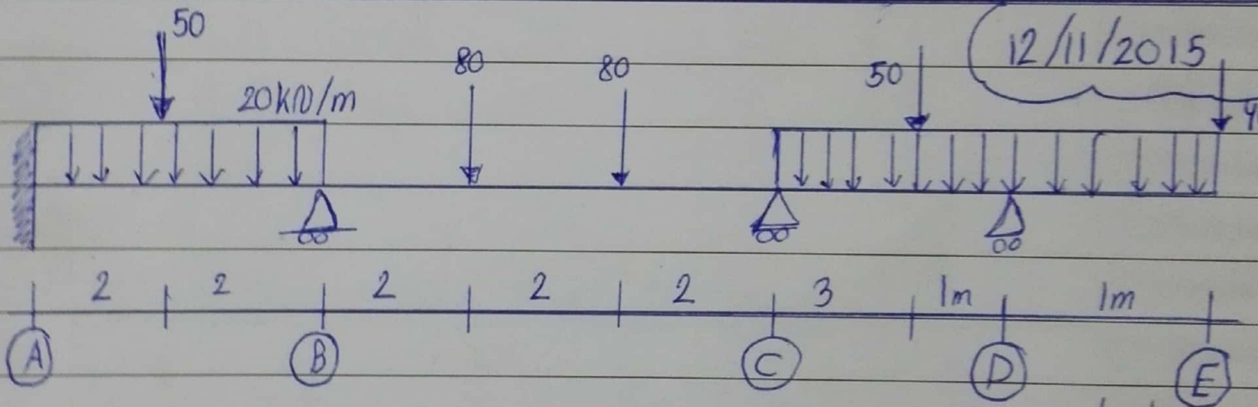
• في هذا (chapter) $\underline{M}$ مع عقارب الساعة (+) وعكس عقارب الساعة (−)

• ننتهي من عمل (iteration) عندما أصل إلى (1%) من قيمة M^f



Ex

12/11/2015



EI − constant

$K_{AB} = \frac{4EI}{4}$, $K_{BC} = \frac{4EI}{6}$, $K_{CD} = \frac{4EI}{4}$ ← member Not joint

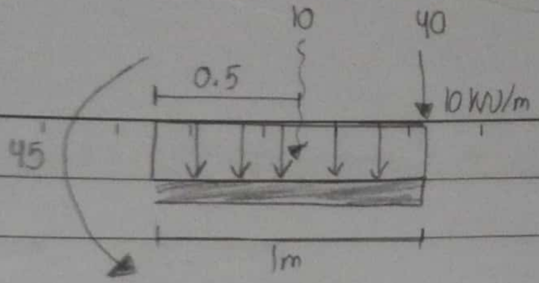
$DF_{AB} = 0$, $DF_{DC} = 1.0$, $DF_{BA} = \frac{K_{AB}}{K_{AB} + K_{BC}}$ ← member Not joint

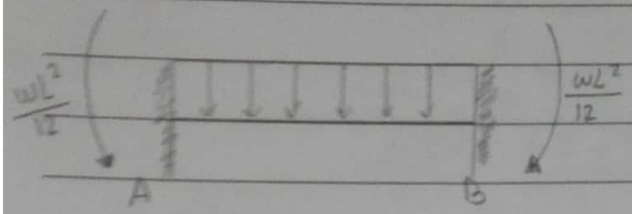
$= 0.6$

$DF_{BC} = 1 - 0.6 = 0.4$

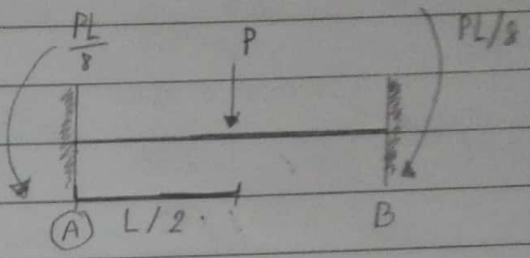
$DF_{CB} = \frac{K_{BC}}{K_{BC} + K_{CD}} = 0.4 \longrightarrow DF_{CD} = 1 - 0.4$

$= 0.6$



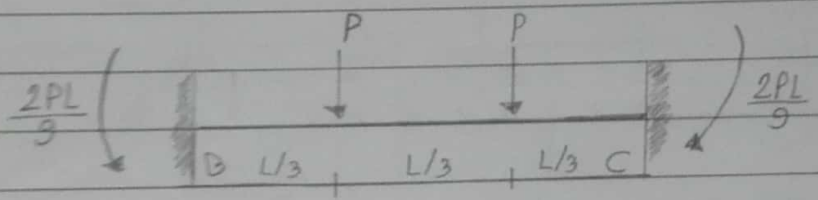


+

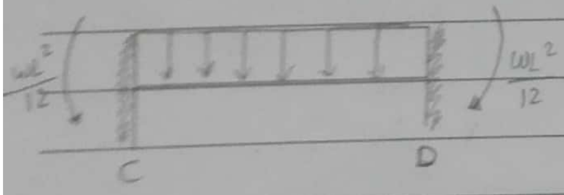


$M^f_{AB} = -\left(\frac{20*4^2}{12} + \frac{50*4}{8}\right) = -51.7$ kN.m

$M^f_{BA} = 51.7$ kN.m



$M^f_{BC} = -(106.7)$ kN.m , $M^f_{CB} = 106.7$ kN.m



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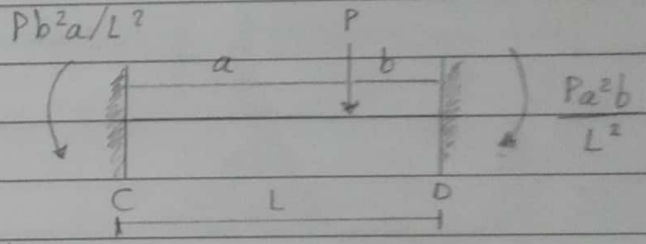
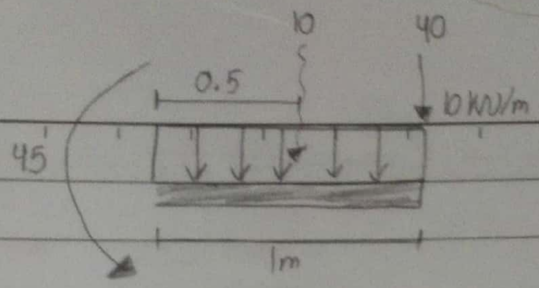
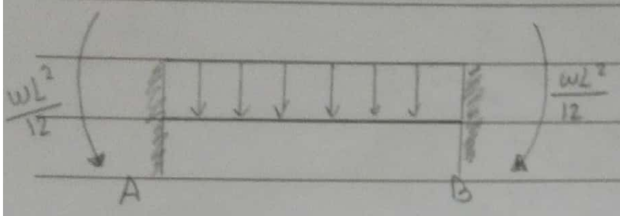


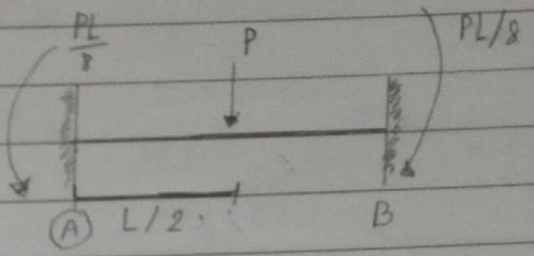
$M^f_{CD} = -\left(\frac{10*4^2}{12} + \frac{50*1^2*3}{4^2}\right) = -22.71$ kN.m

$M^f_{DC} = +\left(\frac{10*4^2}{12} + \frac{50*3^2*1}{4^2}\right) = 41.96$ kN.m



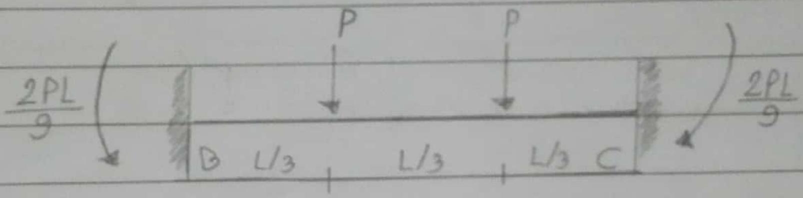


+

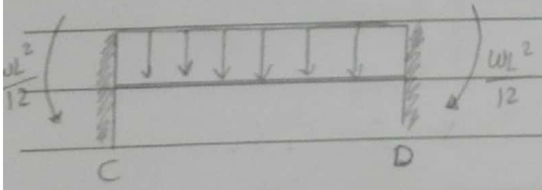


$$M^f_{AB} = -\left(\frac{20 * 4^2}{12} + \frac{50 * 4}{8}\right) = -51.7 \text{ kN.m}$$

$$M^f_{BA} = 51.7 \text{ kN.m}$$



$$M^f_{BC} = -(106.7) \text{ kN.m} \quad , \quad M^f_{CB} = 106.7 \text{ kN.m}$$



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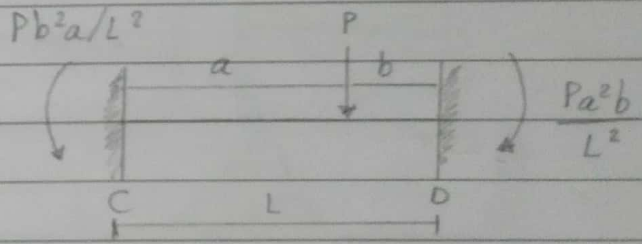
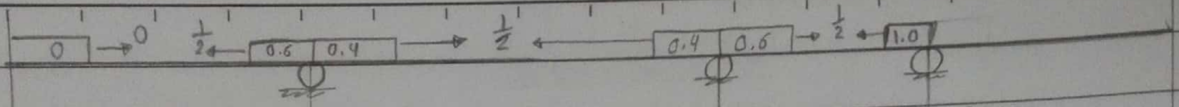
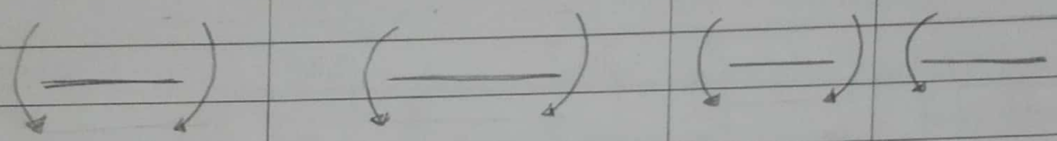


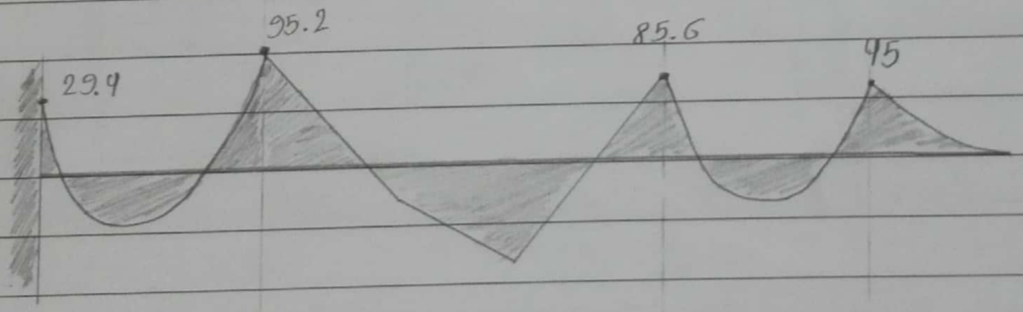
$$M^f_{CD} = -\left(\frac{10 * 4^2}{12} + \frac{50 * 1^2 * 3}{4^2}\right) = -22.71 \text{ kN.m}$$

$$M^f_{DC} = +\left(\frac{10 * 4^2}{12} + \frac{50 * 3^2 * 1}{4^2}\right) = 41.96 \text{ kN.m}$$



	-51.7	51.7	-106.7	106.7	-22.71	41.46	-45
		+33	+22	-33.6	-50.4	3.54	
	16.5		-16.8	11	1.77	25.2	
		+10.08	6.72	-5.1	-7.66	-25.2	
	5.04		-2.55	3.36	-12.6	-3.83	
		1.53	1.02	3.7	5.54	3.83	
	0.765		1.85	0.51	1.915	2.77	
		-1.11	-0.74	-0.97	-1.46	-2.77	
Σ	-29.4	95.2	-95.2	85.6	-85.6	+45	-45





• نرسم على ذيل السهم

ونلتزم بقاعدة M^f أنّ مع عقارب الساعة (+) وعكس (-)

• عادةً بعد 3(itteration) نكون قد وصلنا للنتيجة الصحيحة

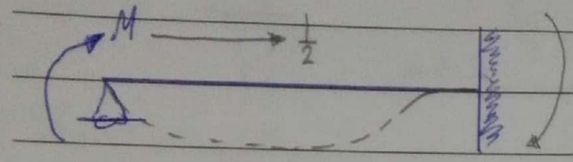
لأن بعد ذلك تصبح الأرقام بالأعشار،

وعند النزول للميدان لن تقول للطوبرجي بدي طوبة (50.004 x 6.77)

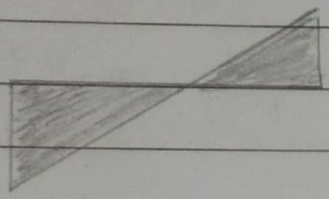
بل ستقول له (51 x 7.00)

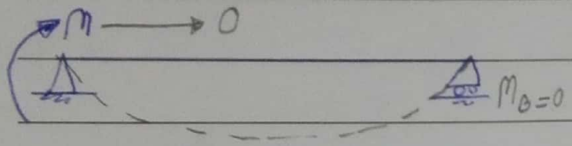
15/11/2015

Stiffness factor Modification

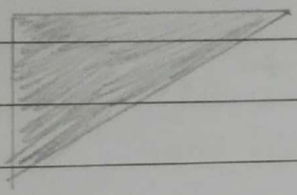


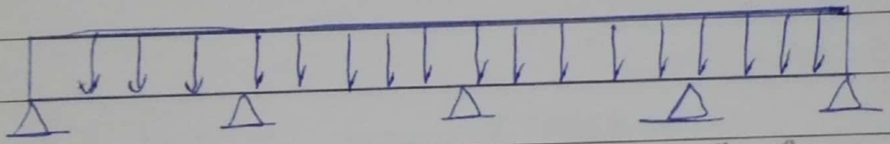
$$K = \frac{4EI}{L}$$

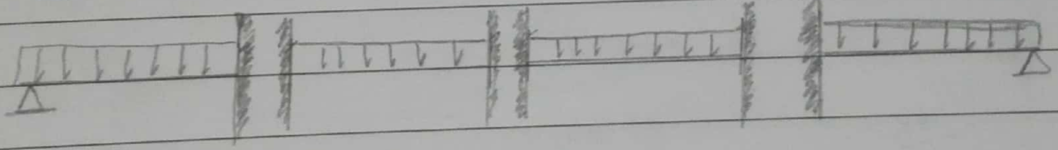




$$K = \frac{3EI}{L}$$

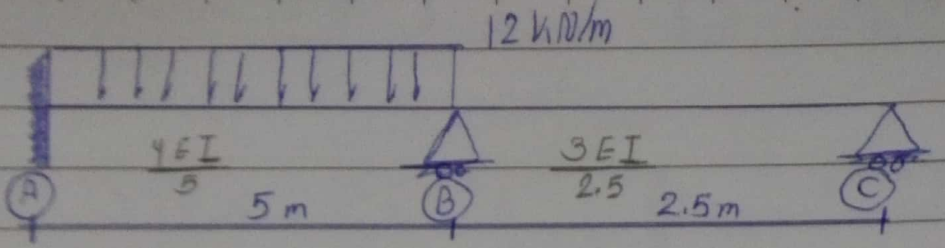






$\boxed{Ex}$

- Max # of It. 3

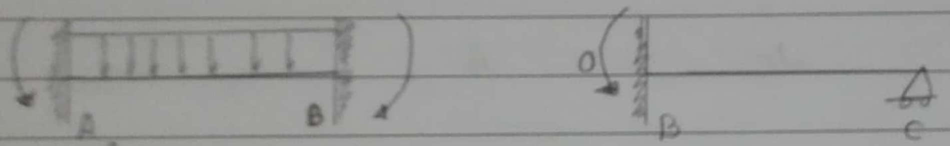


- EI-constant

$$DF_{BA} = \frac{K_{AB}}{K_{AB}+K_{BC}} = \frac{\frac{4EI}{5}}{\frac{4EI}{5}+\frac{3EI}{2.5}} = \frac{4EI}{5} \cdot \frac{5}{10EI} = 0.4$$

$DF_{BC} = 1 - 0.4 = 0.6$

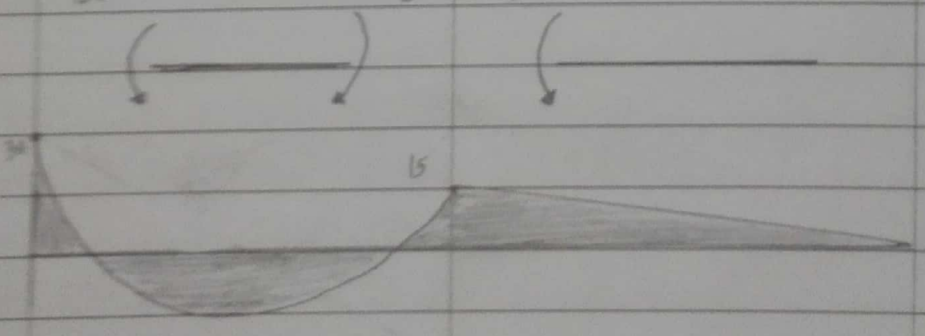
$DF_{AB} = 0$ $\qquad$ $DF_{CB} = 1.0$



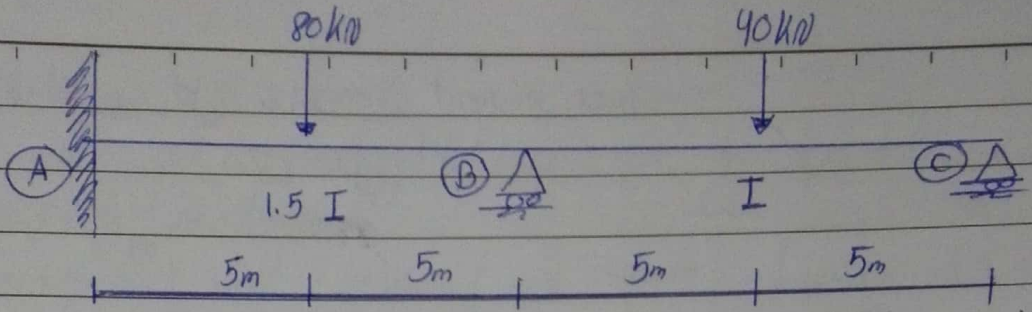
$$M^F_{AB} = \frac{-12 * 5^2}{12} = -25\ kN.m$$

$$M^F_{BA} = 25\ kN.m$$

	0 → 0 $\frac{1}{2}$ ←	0.4	0.6 → 0	1.0
	-25	25		
		-10	-15	
	-5			
Σ	-30	15	-15	



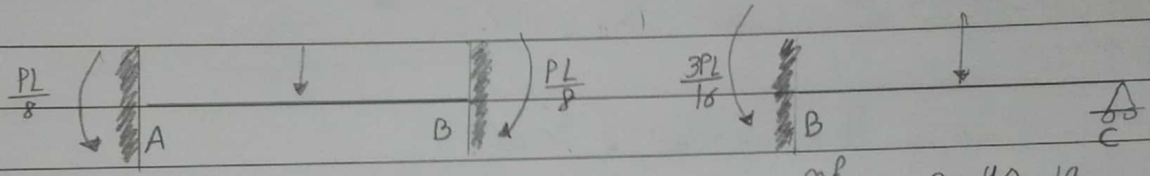
Ex



$$DF_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{\frac{4E1.5I}{10}}{\frac{4E1.5I}{10}+\frac{3EI}{10}} = \frac{6EI}{10} * \frac{10}{9EI} = 0.67$$

$$DF_{BC} = 1-0.67 = 0.33$$

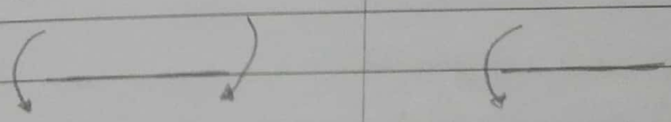
$$DF_{AB} = 0 \qquad DF_{CB} = 1.0$$

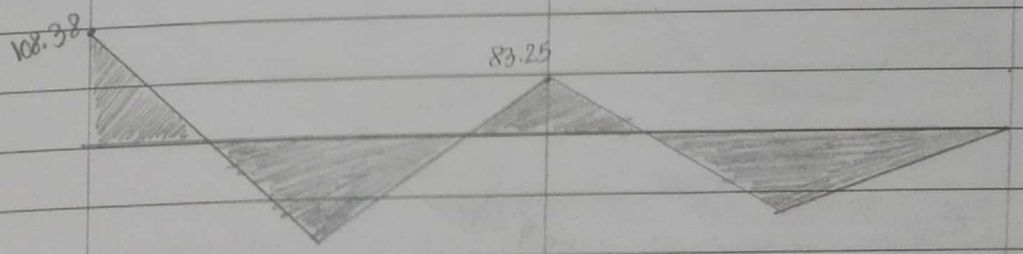


$$M^f_{AB} = \frac{-80*10}{8} = -100\,kN.m \qquad M^f_{BC} = \frac{-3*40*10}{16} = -75\,kN.m$$

$$M^f_{BA} = 100\,kN.m$$

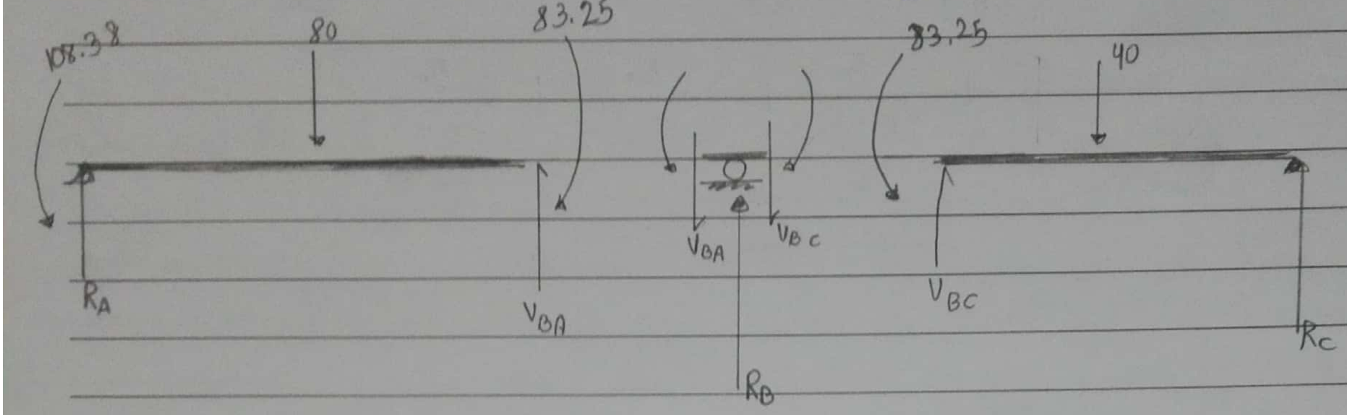
	0 → 0	½ ← 0.67	0.33 → 0	1.0
	-100	100	-75	
		-16.75	-8.25	
	-8.38			
Σ	-108.38	83.25	-83.25	





• لتحديد كل نقاط max. moment

نقوم بعمل الآتي :



$\Sigma M_A = 0$

$-80(5) + V_{BA}(10) - 83.25 = 0 \longrightarrow V_{BA} = 48.33 \uparrow$

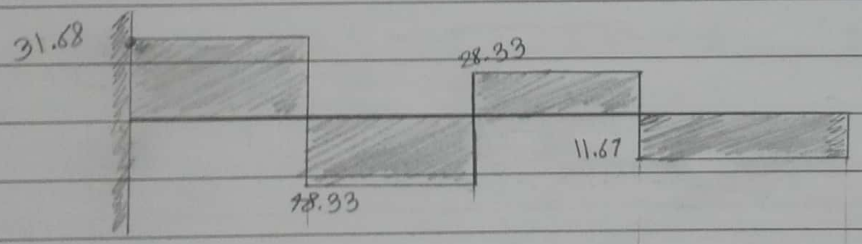
$R_A = 31.68 \uparrow$

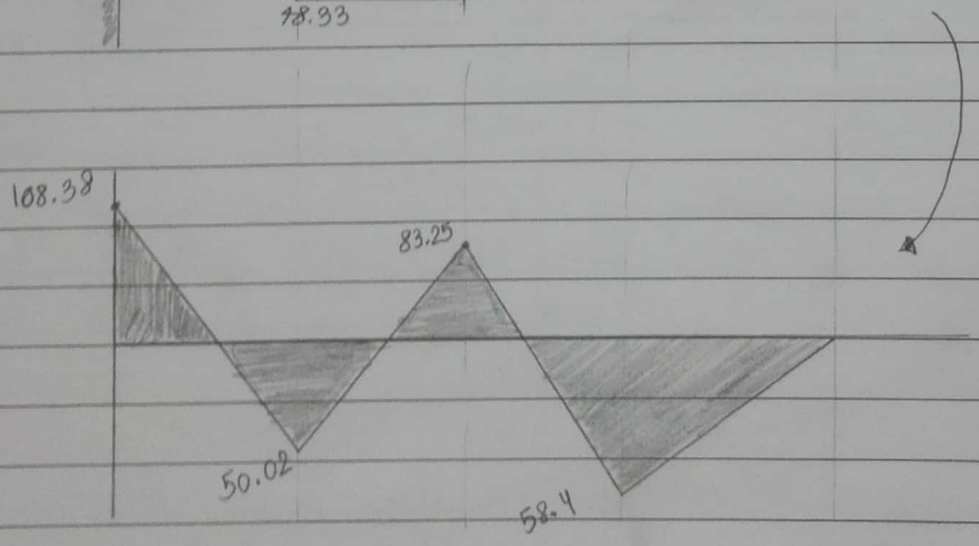
$R_B = V_{BA} + V_{BC} = 76.66 \uparrow$

$\Sigma M_C = 0$

$40(5) - V_{BC}(10) + 83.25 = 0 \longrightarrow V_{BC} = 28.33 \uparrow$

$R_C = 11.68 \uparrow$





مثل
ما تعلمنا
سابقا

$M_2 - M_1 = \text{Area}$

17/11/2015

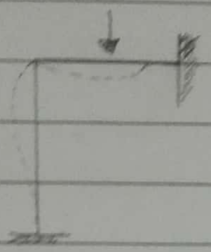
Analysis of frames : No sidesway

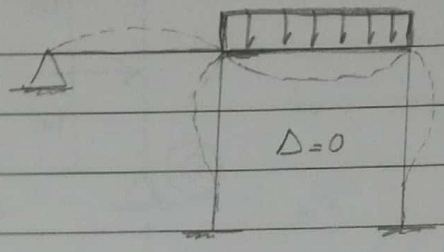
Frame will not sidesway (displaced to the left or righ) if:

ⓐ properly restrained

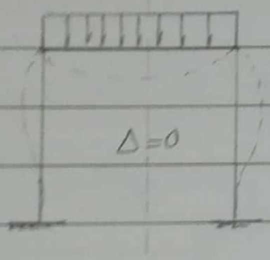
ⓑ symmetric with respect to both loading & geometry.

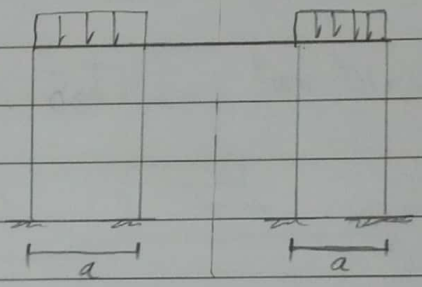
ⓐ



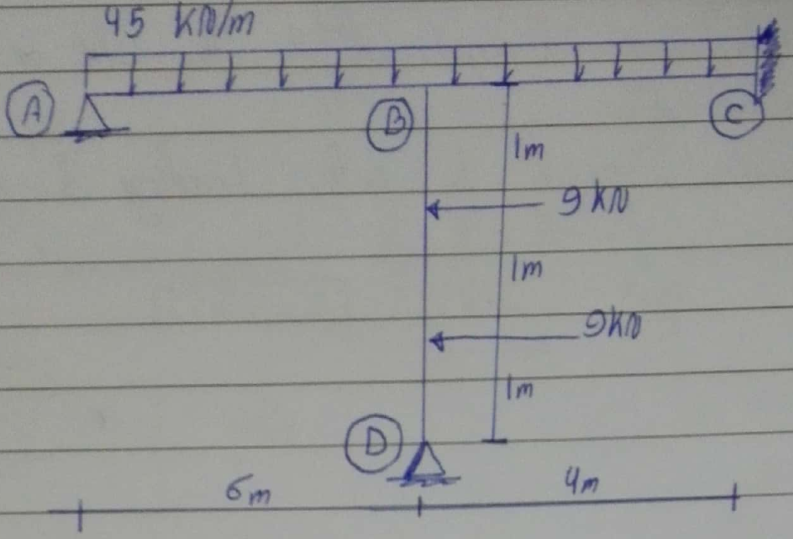


ⓑ



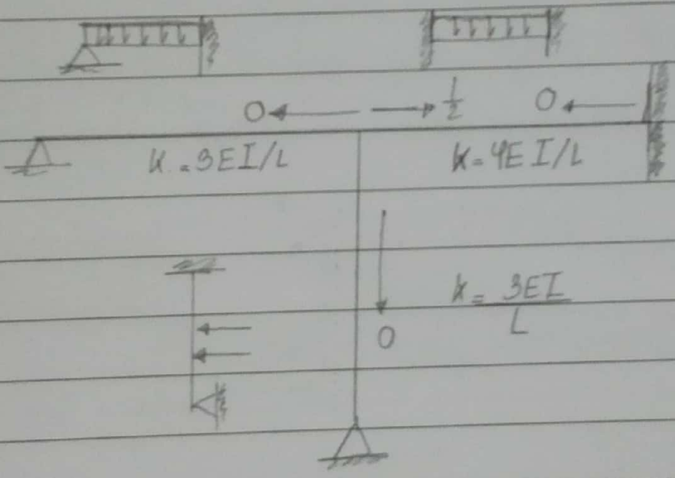


Ex



$E = 210\ Gpa$, $I_{AB} = 280 * 10^{6}\ mm^{4}$

$I_{BC} = 280 * 10^{6}\ mm^{4}$, $I_{BD} = 440 * 10^{6}\ mm^{4}$



$DF_{AB} = 1.0$, $DF_{DB} = 1.0$, $DF_{CB} = 0$

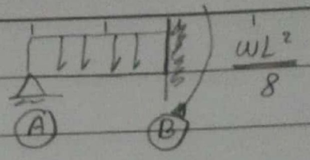
$K_{AB} = \frac{3E\,I_{AB}}{6}$ $K_{BC} = \frac{4E\,I_{BC}}{4}$ $K_{BD} = \frac{3E\,I_{BD}}{3}$

$$DF_{BA} = \frac{K_{AB}}{K_{AB} + K_{BC} + K_{BD}} = 0.1628$$

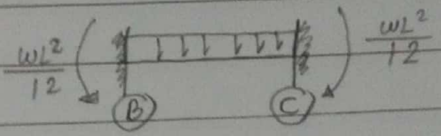
$$DF_{BC} = \frac{K_{BC}}{K_{AB} + K_{BC} + K_{BD}} = 0.3256$$

$$DF_{BD} = 1 - DF_{BC} - DF_{BA} = 0.5116$$

$M^{f}_{BA} = \frac{wL^2}{8} = \frac{45 * 6^2}{8} = 202.5$ kN.m



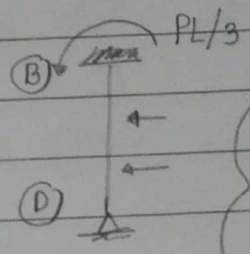
$M^{f}_{BC} = \frac{wL^2}{12} = \frac{45 * 4^2}{12} = -60$ kN.m



$M^{f}_{CB} = 60$ kN.m

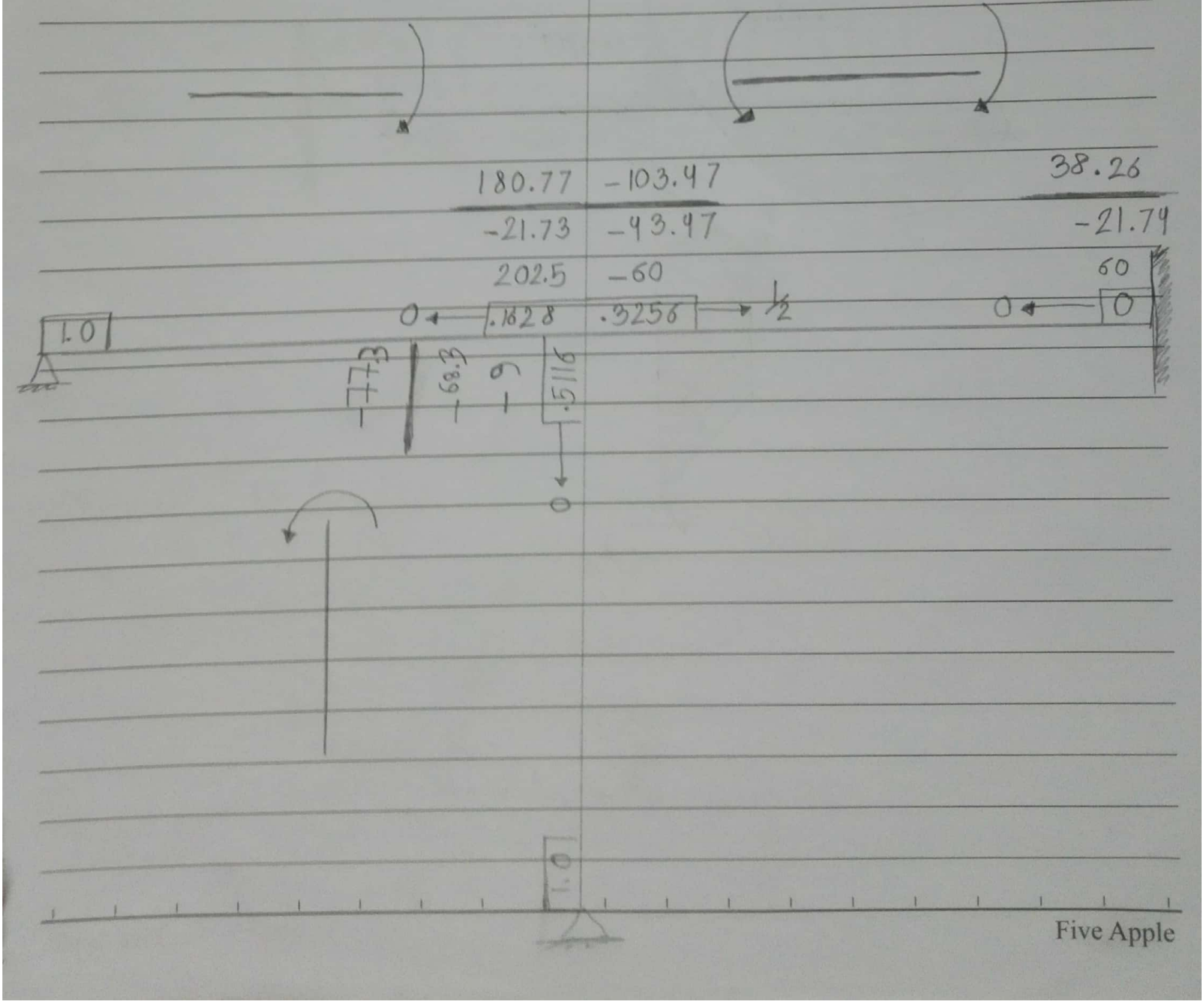
$M_A = 0$

$M_D = 0$



دائماً
رأس
M
يكون باتجاه
Load

$M^{f}_{BD} = \frac{PL}{3} = \frac{9 * 3}{3} = -9$ kN.m



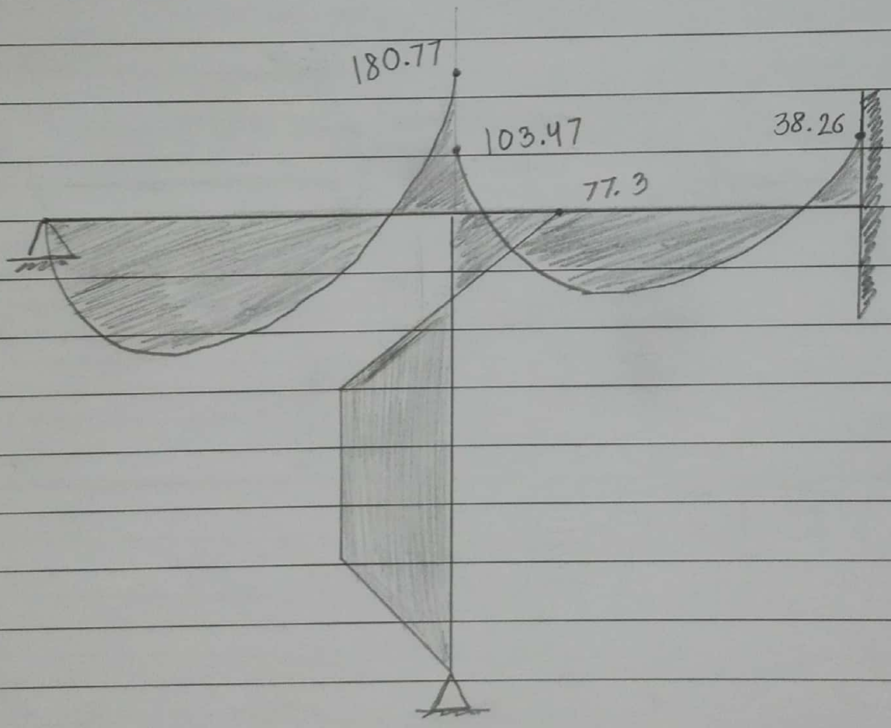
الخطوات : ① نقوم بحساب K

② DF

③ m^f

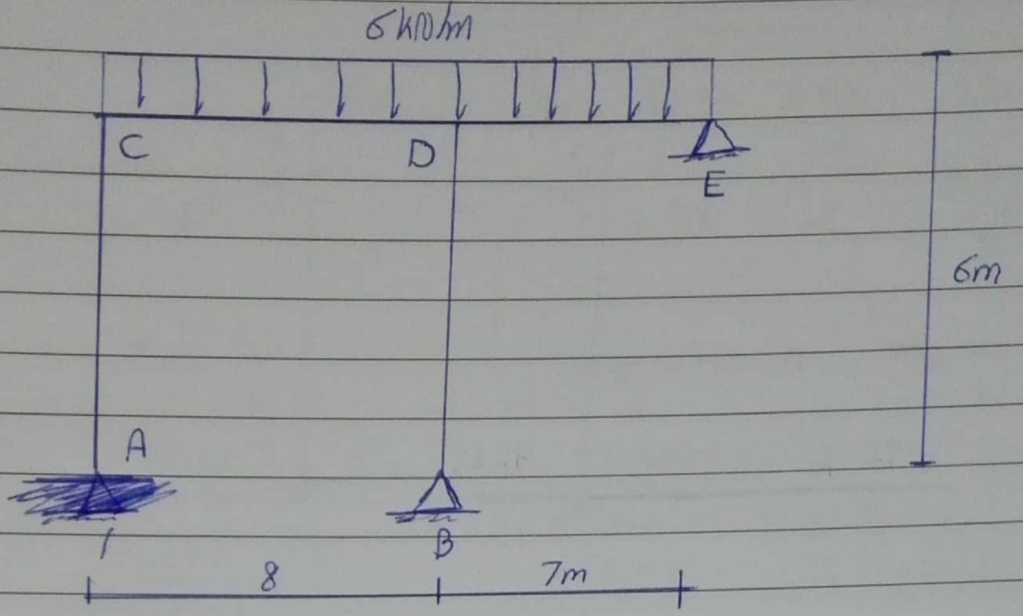
④ نعمل عل اتزان (joint) (Roller or hing)

⑤ بعد الاتزان بشتغل carry over factor



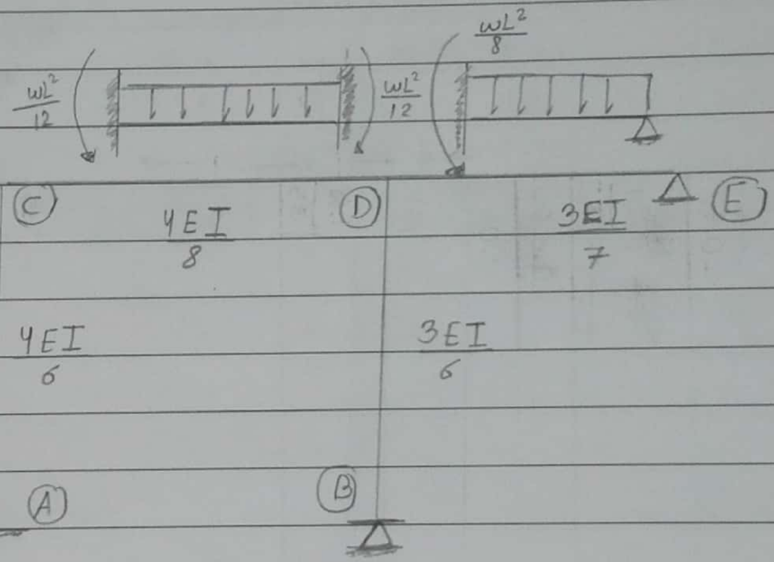
19/11/2015

Ex



EI -constant

$M_B = M_E = 0$



$DF_{AC} = 0$, $DF_{ED} = DF_{BD} = 1.0$

$DF_{CA} = \frac{K_{AC}}{K_{AC} + K_{CD}} = 0.571$

$DF_{CD} = 1- 0.429$

$DF_{DC} = \frac{K_{CD}}{K_{CD} + K_{DE} + K_{DB}} = 0.35$

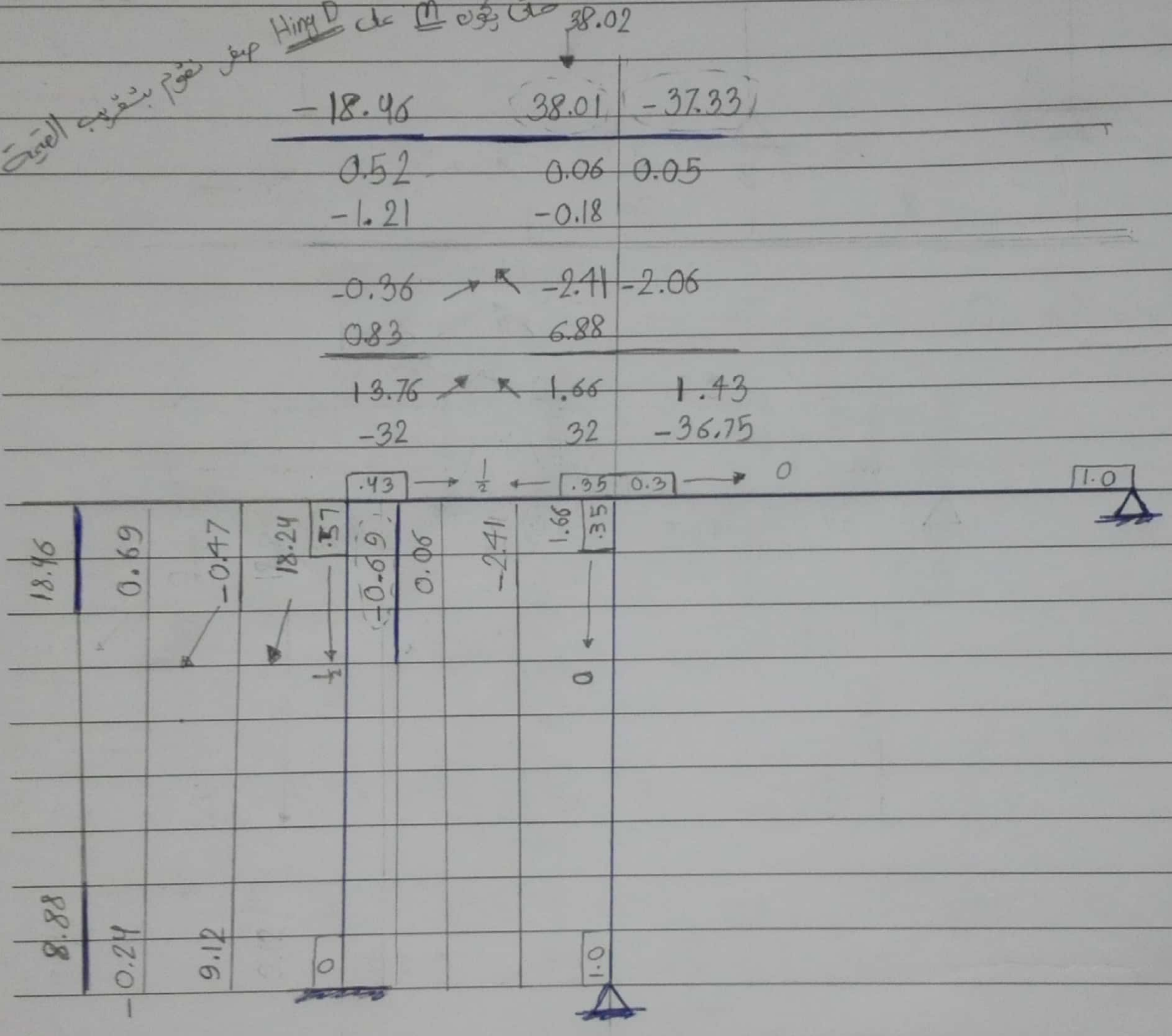
$DF_{DB} = \frac{K_{DB}}{K_{CD} +, K_{DE}, + K_{DB}} = 0.3$ $DF_{DB} = 0.35$

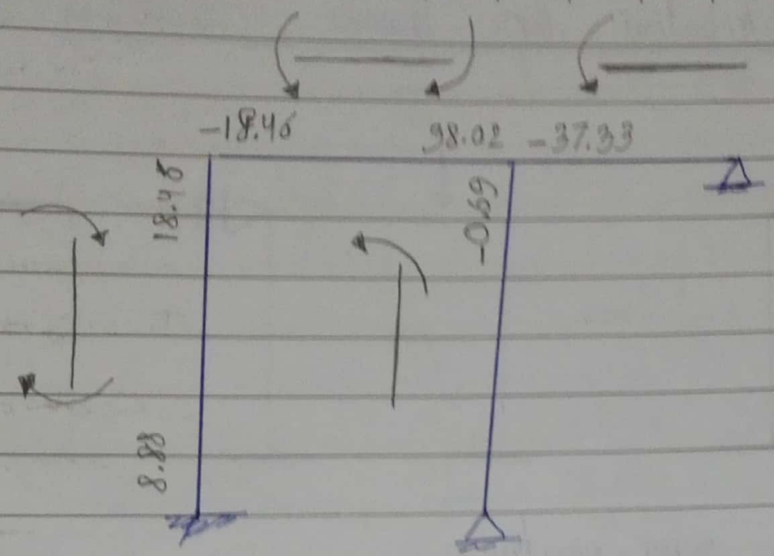
$M^f_{CD} = \frac{-wL^2}{12} = -32$ KN.m

$M^f_{DC} = 32$ KN.m

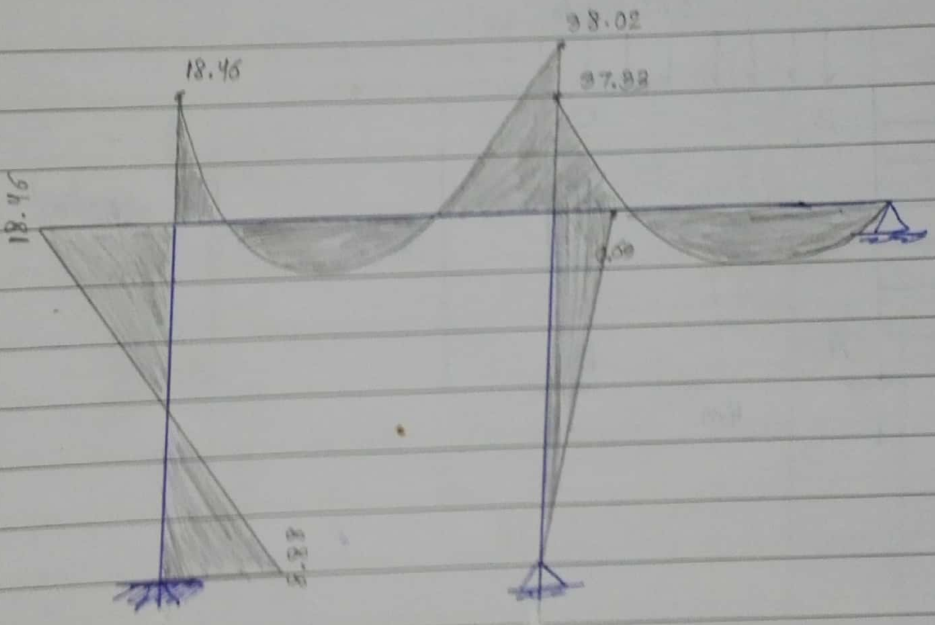
$M^f_{DE} = \frac{-wL^2}{8} = -36.75$ KN.m

حتى يكون m على Hing D صفر نقوم بتقريب القيمة



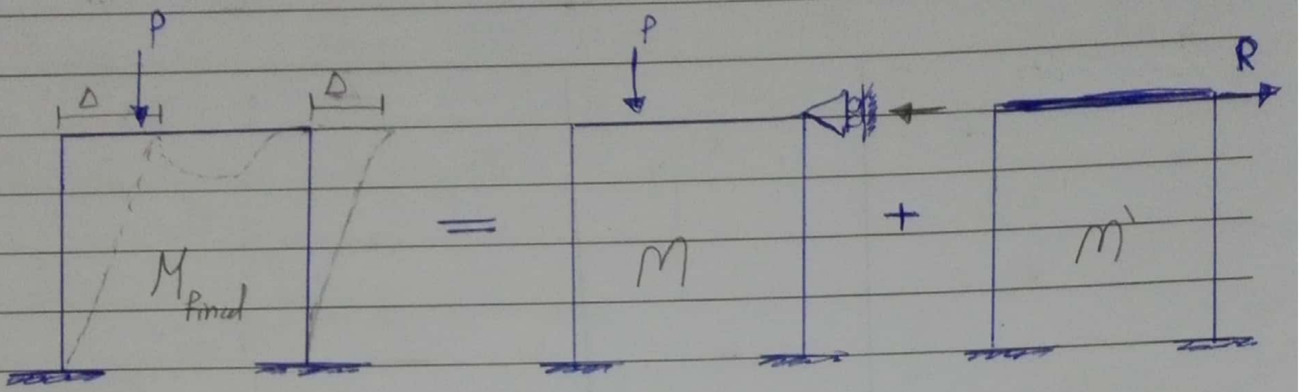


- نرسم على ذلك الرسم



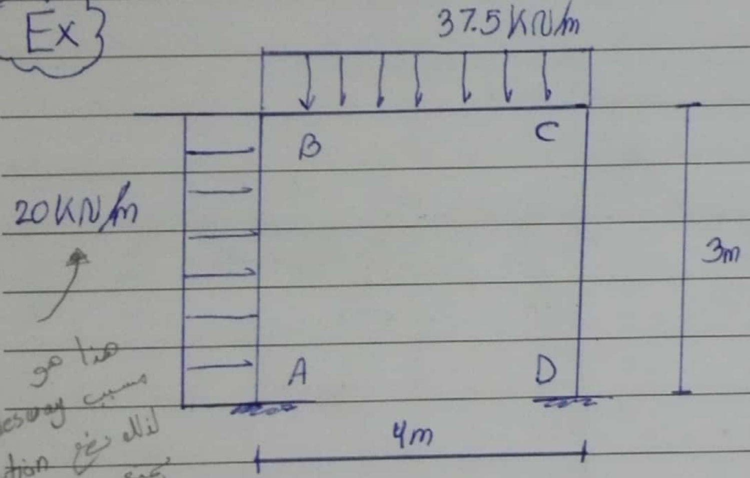
22/11/2015

Moment Distribution for frames : sidesway



The principle of super-position.

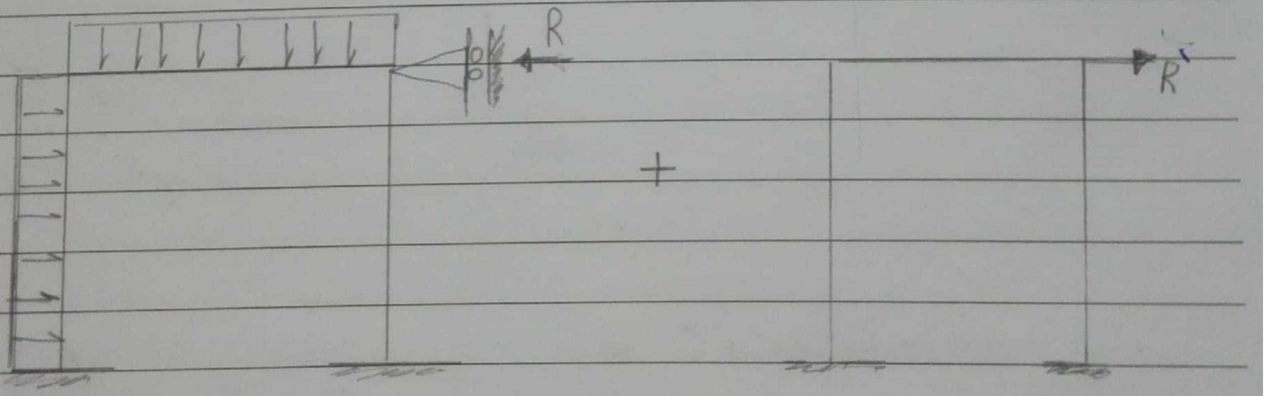
Ex



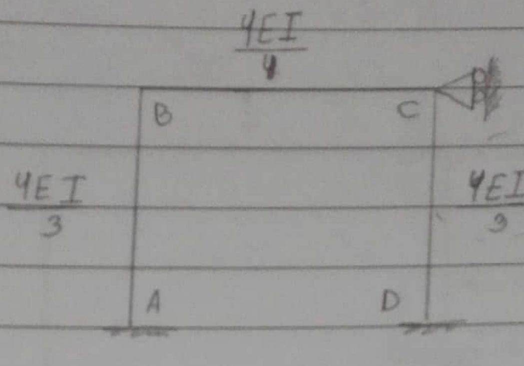
EI-constant

هذا هو مسبب sidesway لذلك نضع reaction تمنع هذا الحركة

|||



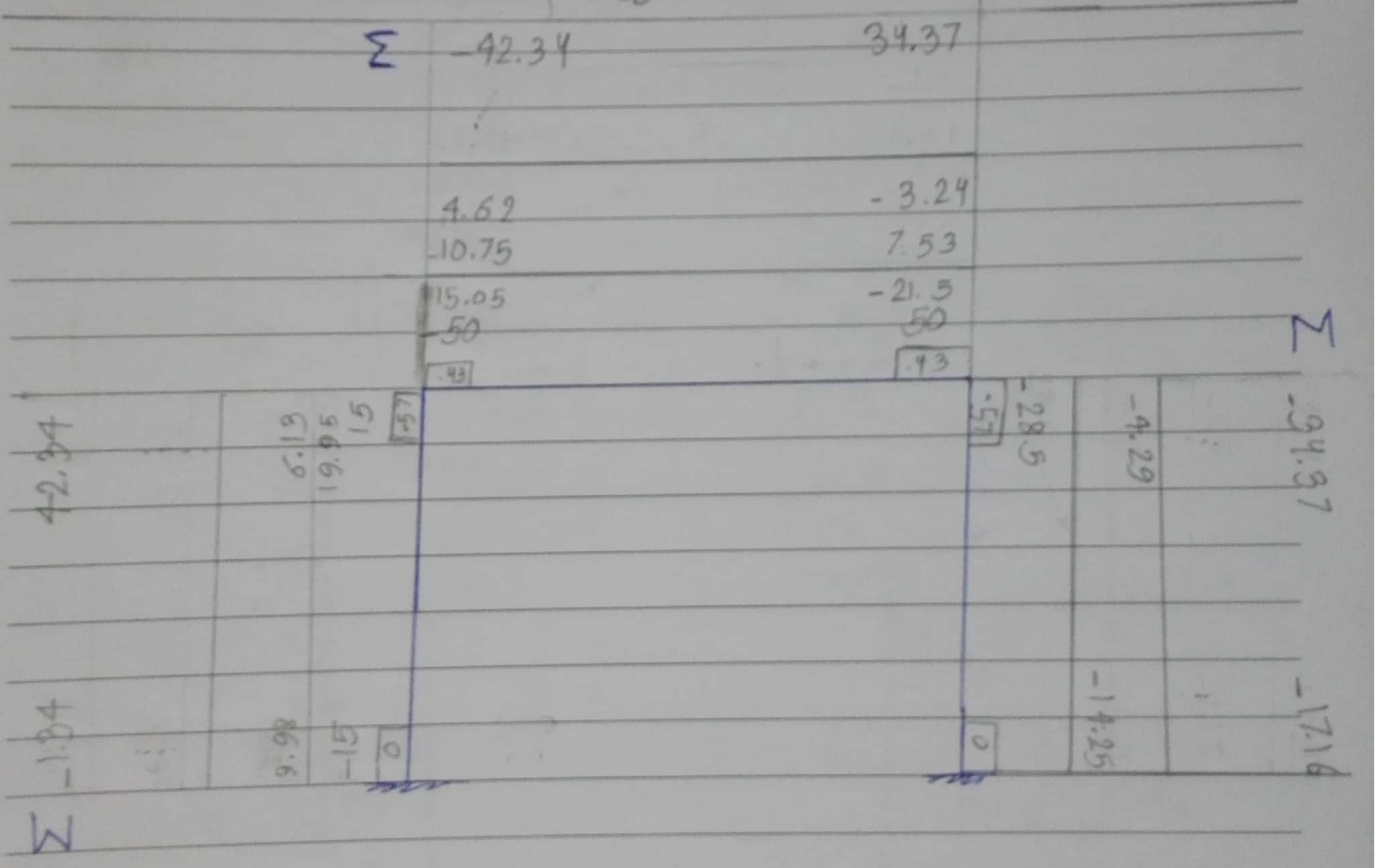
Now Frame ①:



→ انتبه roller
في المنتصف وليست
على الطرف لذلك
اخذنا $\frac{4EI}{L}$

$DF_{AB} = DF_{DC} = 0$
$DF_{BA} = 0.5714 = DF_{CD}$
$DF_{BC} = 0.4286 = DF_{CB}$

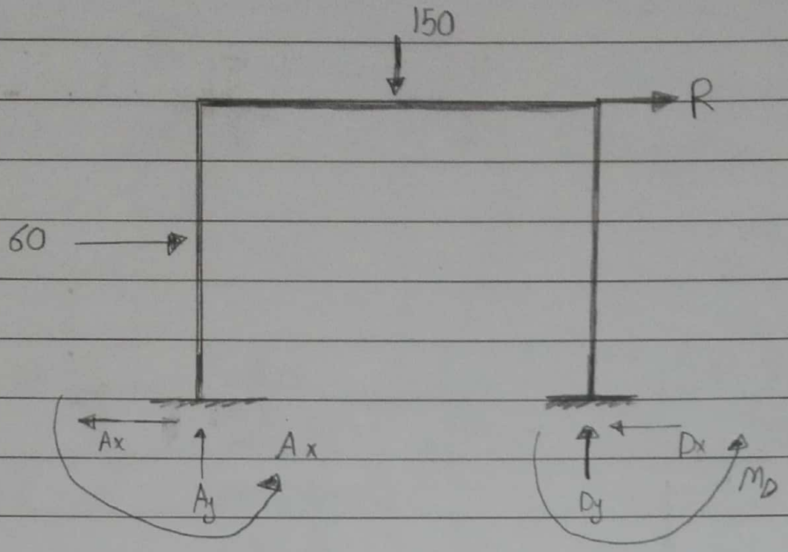
$M^F_{AB} = \frac{-20 * 3^2}{12} = -15$ KN.m
$M^F_{BA} = 15$ KNm
$M^F_{BC} = -50$ KNm
$M^F_{CB} = 50$ KN.m



Note: Σ then 4 itteration
انتبه: لا نعمل carry over factor ونوقف
للجمع لأنه لازم نطلع العزوم M
صفر

• هنا نقف هنا
بعد 2 itter.
لكن ΣM كان بعد 4 itter.
ولكن حتى لو جمعت بعد 2 itter. أو آخر
يجب أن ΣM (continous ←)

Now we calculated R :

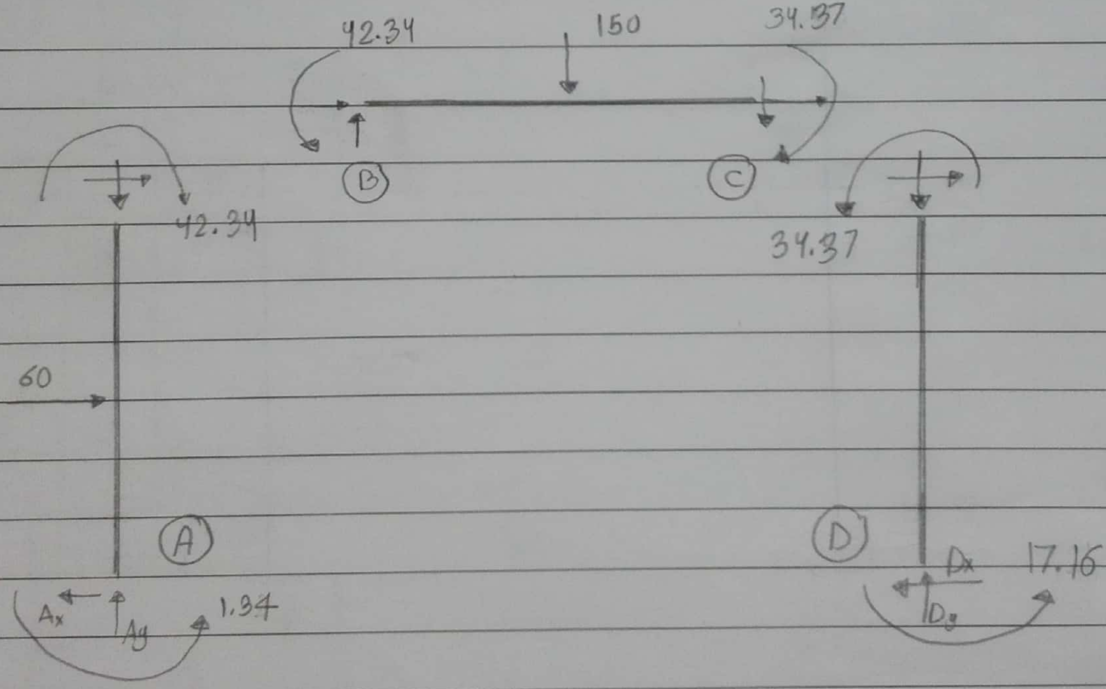


$\Sigma F_x = 0 \Longrightarrow R - A_x - D_x + 60 = 0$

$R - 16.33 - 17.18 + 60 = 0$

$R = -26.49$ KN

حدّد
مدفله



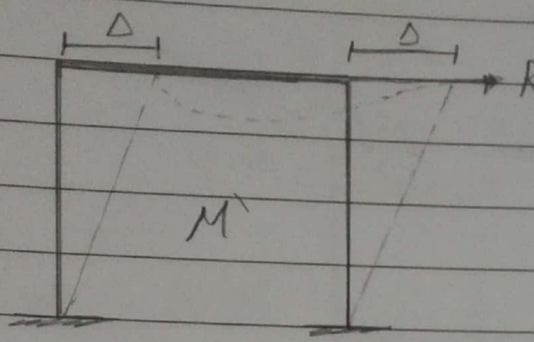
$\circlearrowleft + \ \Sigma M_B = 0 \Longrightarrow -42.34 + 1.34 - A_x (3) + 60 (1.5) = 0$

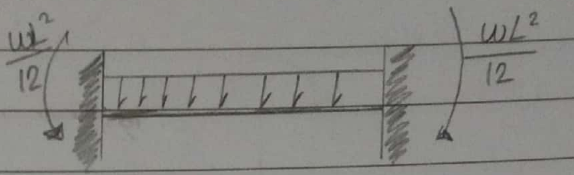
$\circlearrowleft + \ \Sigma M_C = 0 \Longrightarrow +34.37 + 17.16 - D_x (3) = 0$

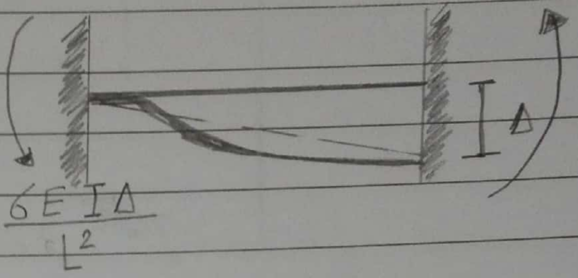
$A_x = 16.33$ KN $\qquad D_x = 17.18$ KN

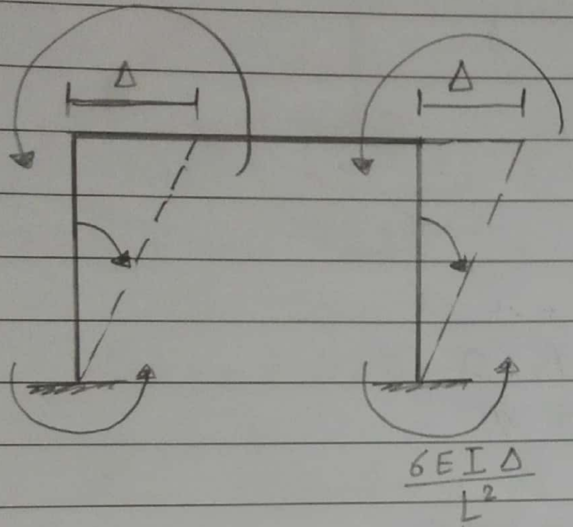
Frame ② :

24/11/2015







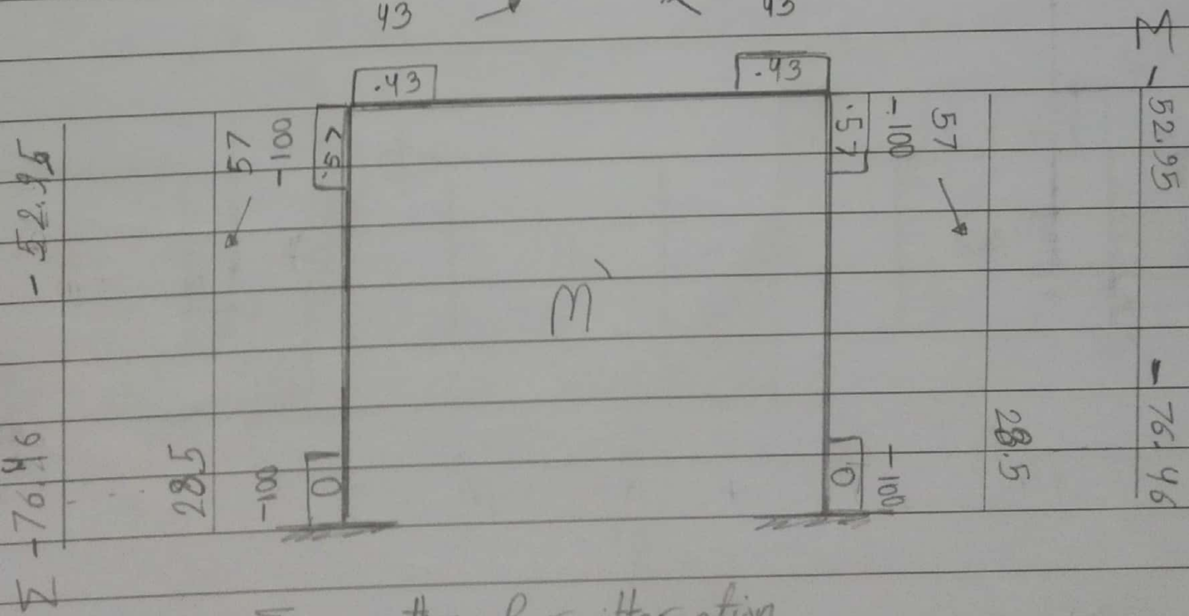


assume → $\frac{6EI\Delta}{L^2} = 100$ so $\Delta = \frac{100 * L^2}{6EI} = \frac{150}{EI}$

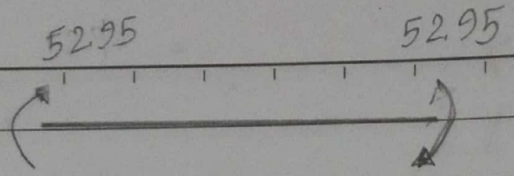
Σ 52.95 52.95

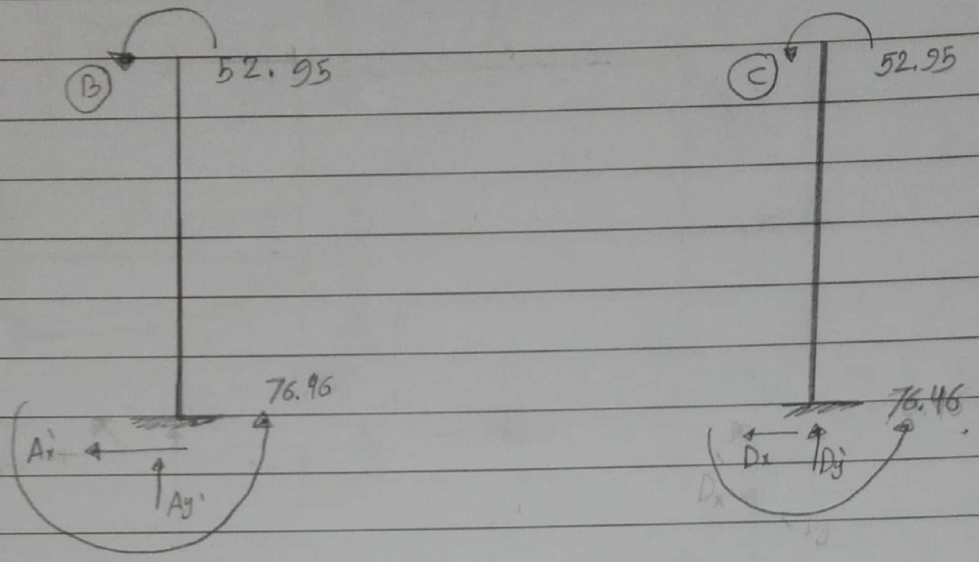
21.5 21.5

43 → ← 43



Σ ..... then four ittereation





$$\Sigma M_B = 0 \rightarrow 52.95 - 76.46 - A'_x(3) = 0$$

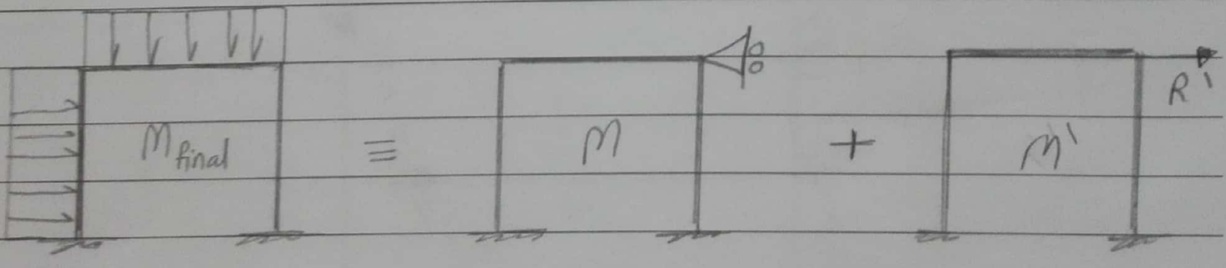
$$A'_x = 43.14 \text{ kN}$$

$$\Sigma M_C = 0 \rightarrow 52.95 - 76.46 - D'_x(3) = 0$$

$$D'_x = 43.14 \text{ kN}$$

$$R' - A'_x - D'_x = 0$$

$$R' = 86.28 \text{ kN} \rightarrow$$



$$M^{final} = M + \frac{R}{R'} M'$$

$$\Delta = \frac{R}{R'} * M'$$

$$= \frac{26.48}{86.28} * \frac{150}{EI} = \frac{46.04}{EI}$$

but M^{final} nessesary in all joint :

$$M_{AB} = -1.34 + \frac{26.49}{86.28}(-76.46) = -24.81$$

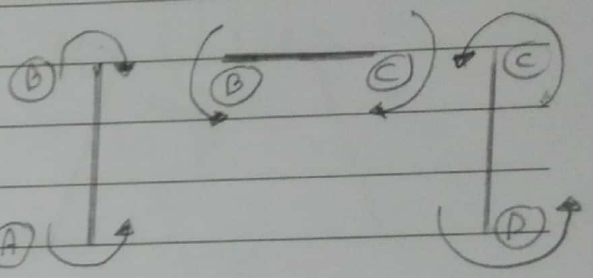
$$M_{BA} = 42.34 + \frac{26.49}{86.28} * (-52.95) = 26.08 \text{ kN.m}$$

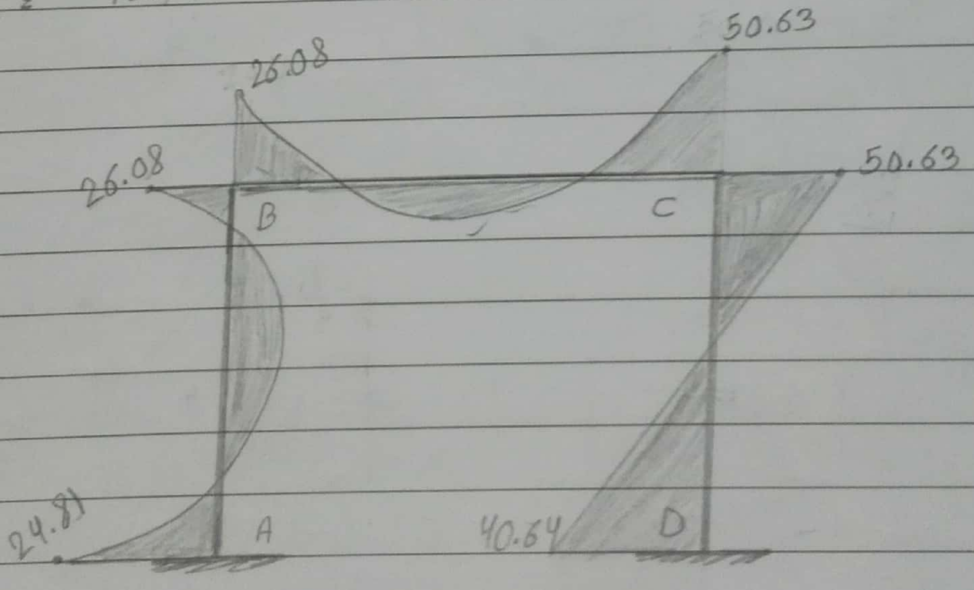
$M_{BC} = -26.08$

$M_{CB} = 50.63$

$M_{CD} = -50.63$

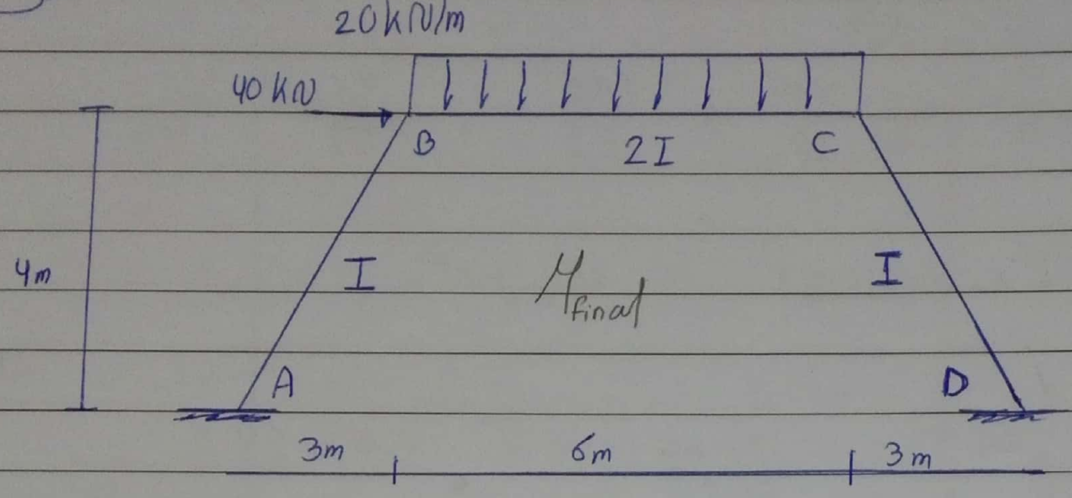
$M_{DC} = -40.64$

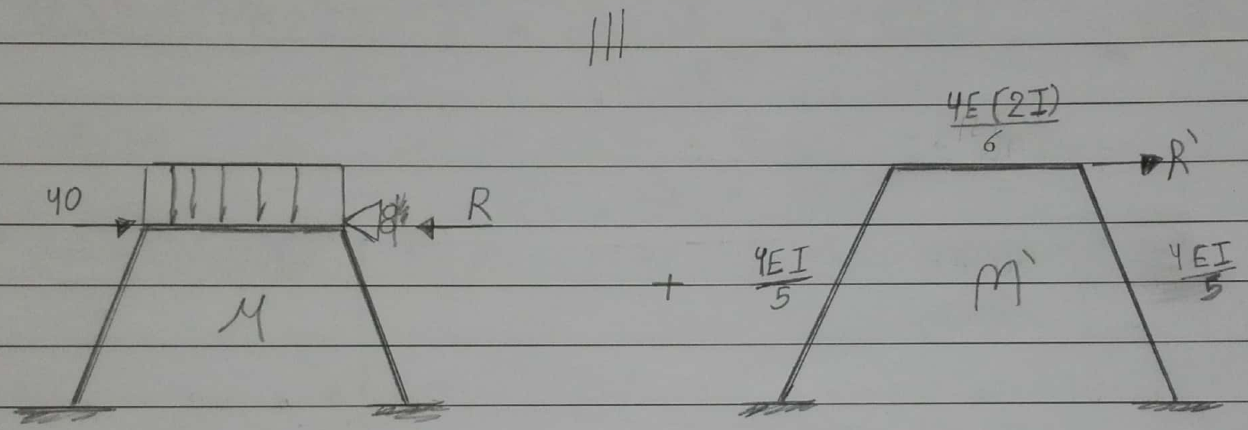




26/11/2015

Ex

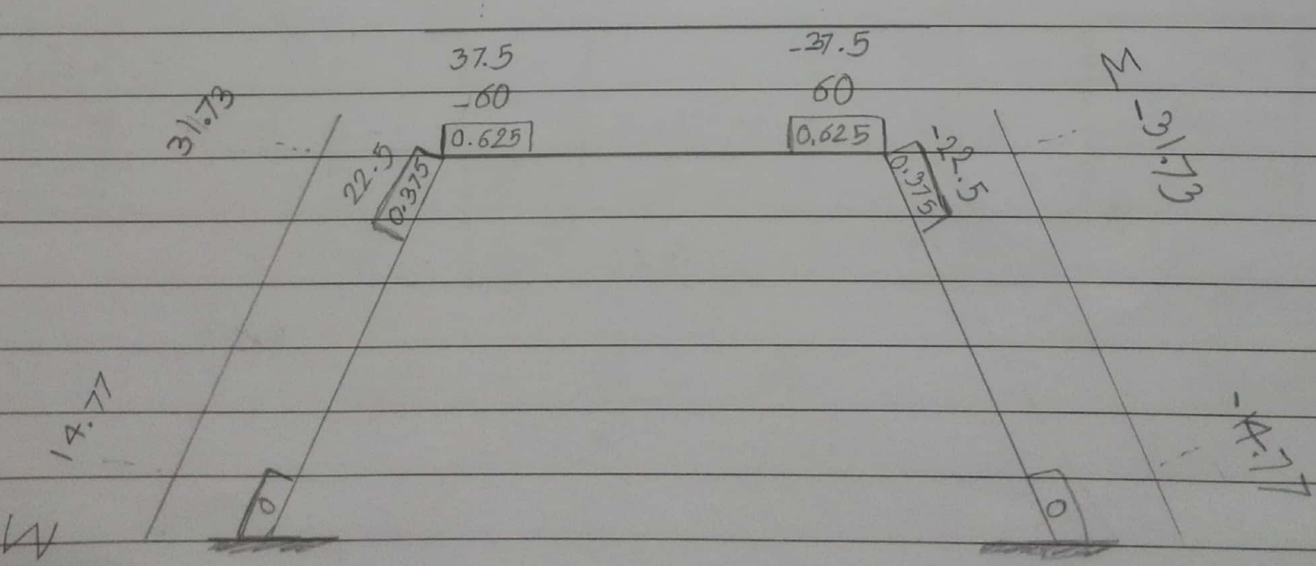




R = 40 kN (بالسالب لأننا نعاكس Load الخارجي)

$M^f_{BC} =$ −60 kN.m $M^f_{CB} =$ 60 kN.m

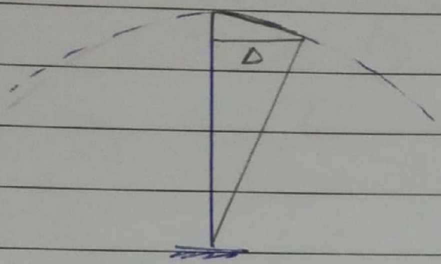
Σ +31.73 31.73

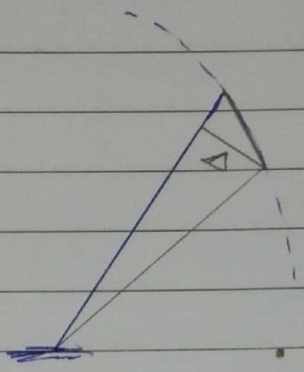


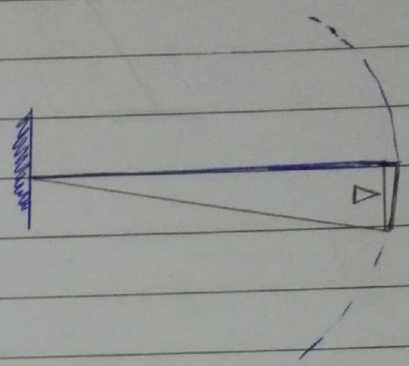
Five Apple

. Σ .... then 3 itteration

Now Fram ② :



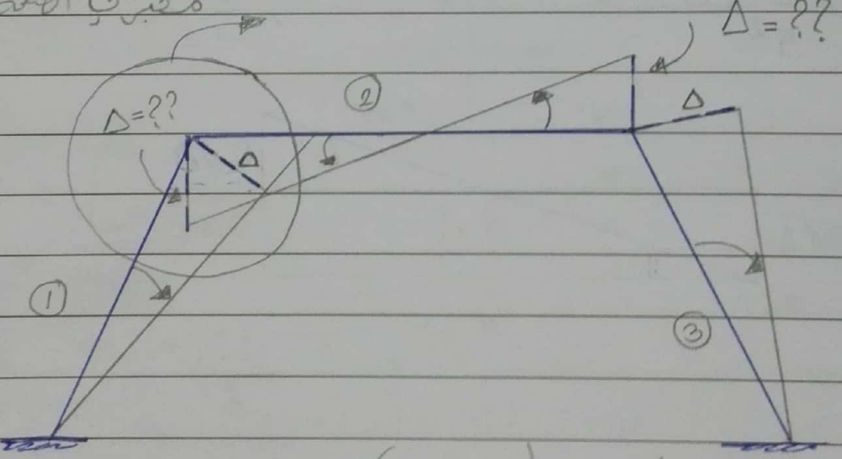




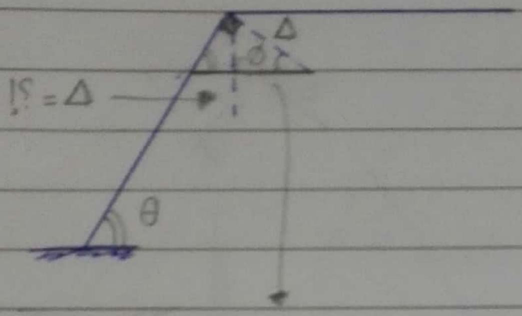
• Beam يمثل حركة دائرية
حسب التأثير المؤثر عليه إما (cw) (c.c.w)

• Δ (Deflection) دائماً نرسم بالعمودي على member

مكتوب في الصفحة التالية



. حركة member (1 و 3) كانت مع عقارب الساعة (افتراض)
. حركة member (2) كانت عكس عقارب الساعة (افتراض)
. فرضنا ان member (1 و 3) تحركوا بمقدار Δ معين
لكن ماذا عن member 2 ؟!

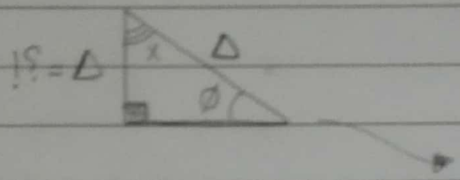


$\theta = \tan^{-1} \frac{4}{3}$

$\theta = 53.13°$

$\phi = 90 - 53.13°$

$\phi = 36.87°$

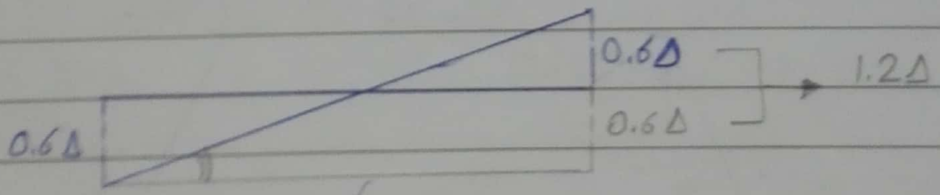


$\sin 36.87 = \frac{(\Delta = ?1)}{\Delta}$

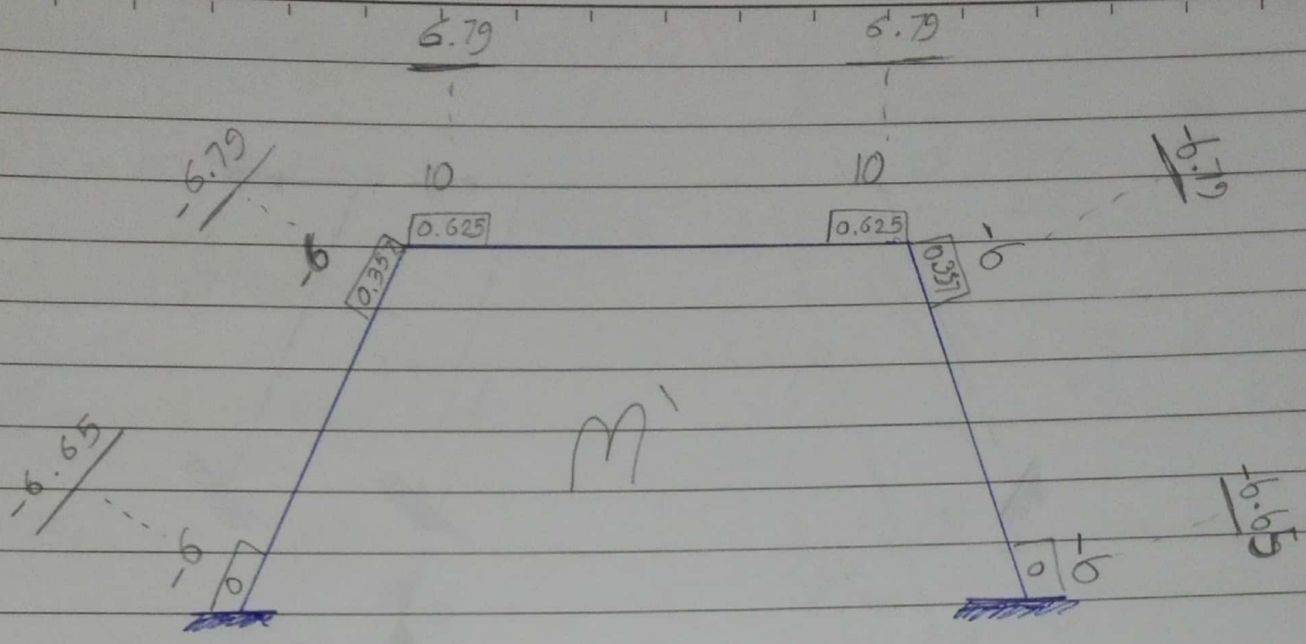
$(\Delta = ?1) = 0.6\,\Delta$

Now assume :

$\frac{6EI\Delta}{5^2} = 6 \longrightarrow \Delta = \frac{25 * 6}{6EI} = \frac{25}{EI}$



$M = \frac{6E(2I)(1.2(\frac{25}{EI})}{5^2} = 10$ kN.m

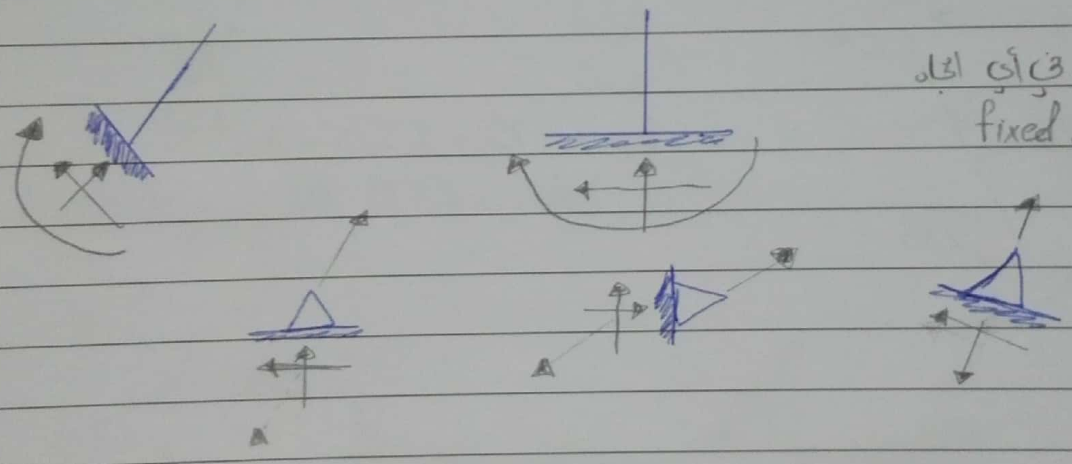


- mf عكس اتجاه حركة member .

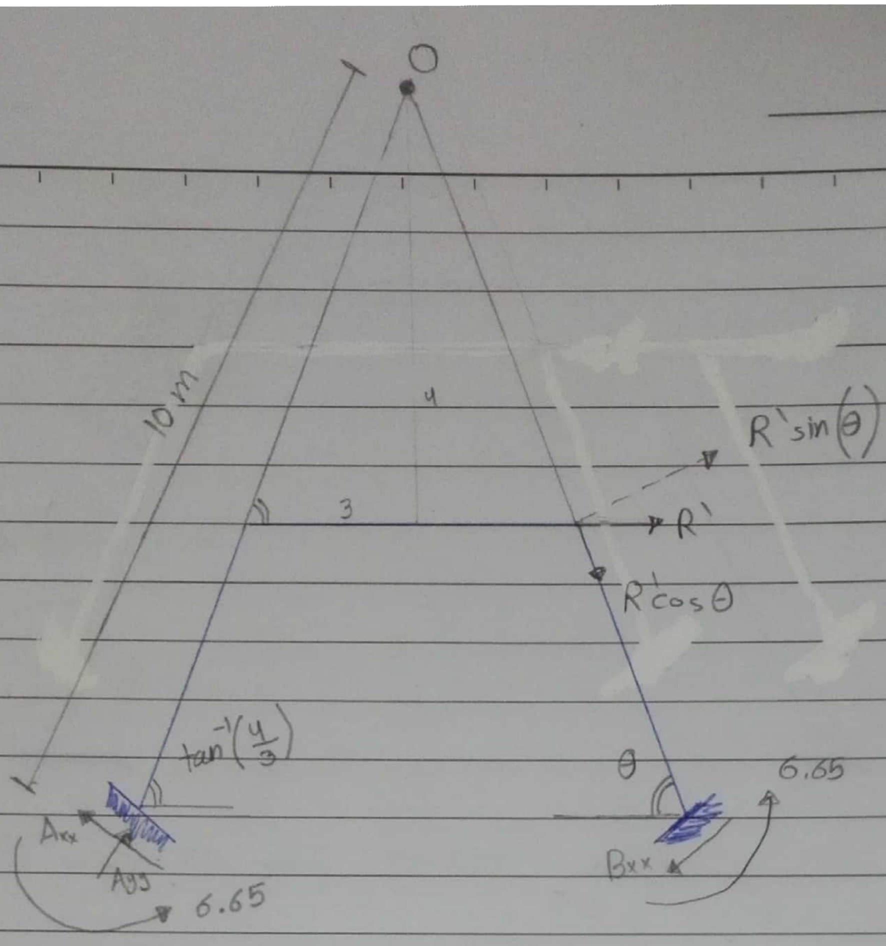
R` = ?! → R` = 10 KN

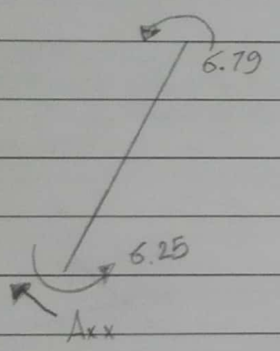
29/11/2015

- تمنع الحركة في أي اتجاه
fixed support



- roller هي الأداة الوحيدة التي تختلف معي إن غيرنا الاتجاه لها
لأن لها (one reaction) .

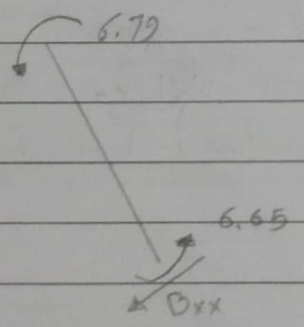




$M_B = 0$

$0 = +6.79 + 6.65 - A_{xx}(5)$

$A_{xx} = 2.688$ kN



$M_C = 0$

$0 = +6.79 + 6.65 - B_{xx}(5)$

$B_{xx} = 2.688$ kN

$M_0 = 0,$

$0 = -A_{xx}(2.688) - B_{xx}(2.688) + 6.65 + 6.65 + R' \sin(\theta)(5) = 0$

$R' = 10.11 \simeq 10$ kN.

$M_{final} = M + \frac{R}{R'} M'$

$M_A = -11.83$ KN.m

$M_{B_1} = 4.57$ KN.m

$M_{C_2} = -58.89$ KN.m

$M_D = -41.37$ KN.m

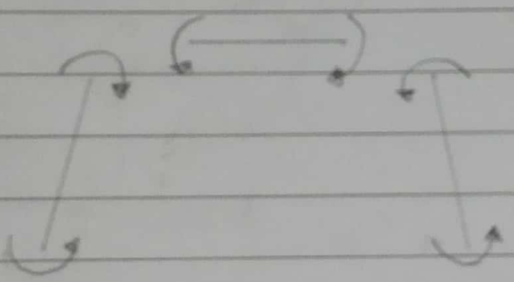
$\frac{6EI\Delta}{L^2 = 5^2} = 6$

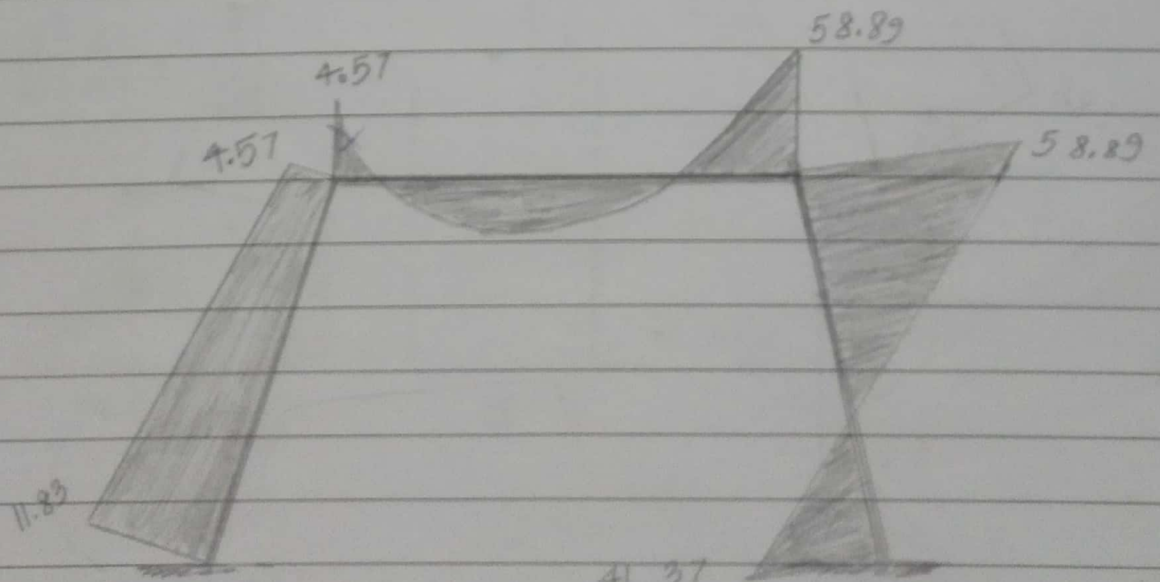
$\Delta = \frac{25}{EI}$

$R' = 10 \longrightarrow R = 40$

$\Delta = \frac{R}{R'} * \frac{25}{EI} = \frac{100}{EI}$

. $R/R' \longrightarrow$ abs. value ناخذهم
لازم دائماً يطلع موجب



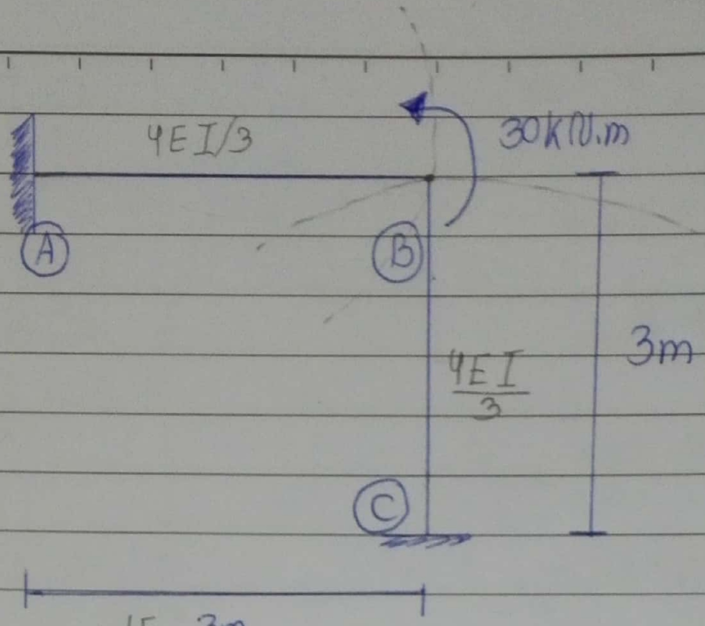


Ex

EI- const

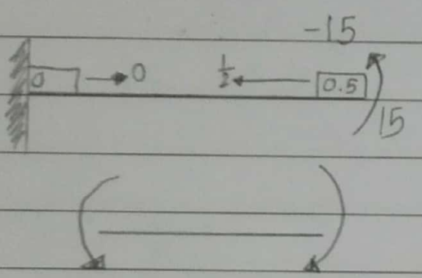
Δ=0

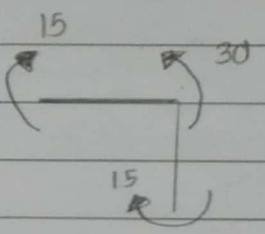
(No sides way)

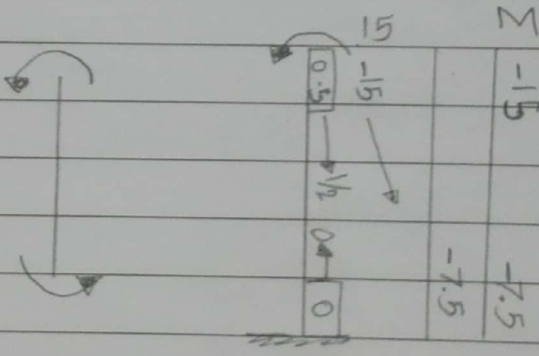


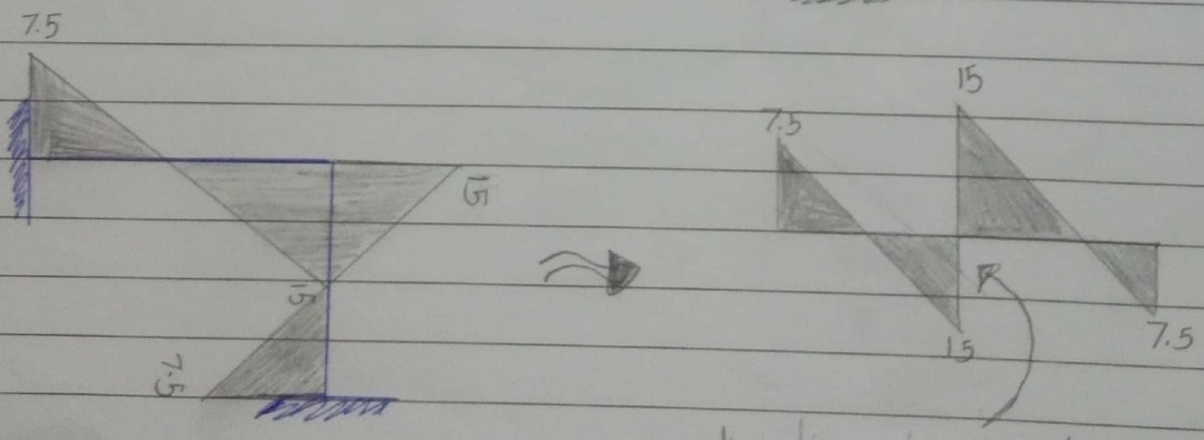
Σ -7.5 -15 3m

-7.5



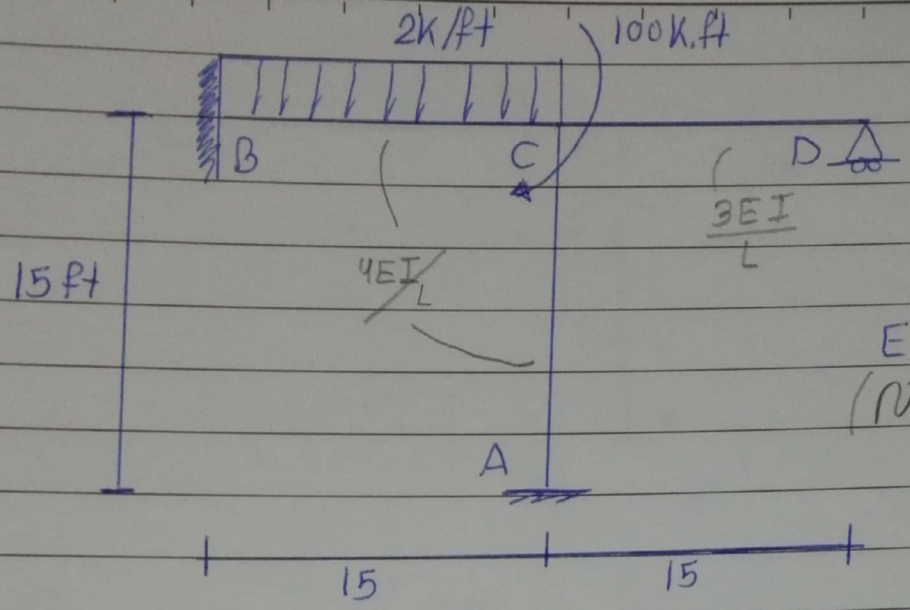






discontinous because here conentrated moment

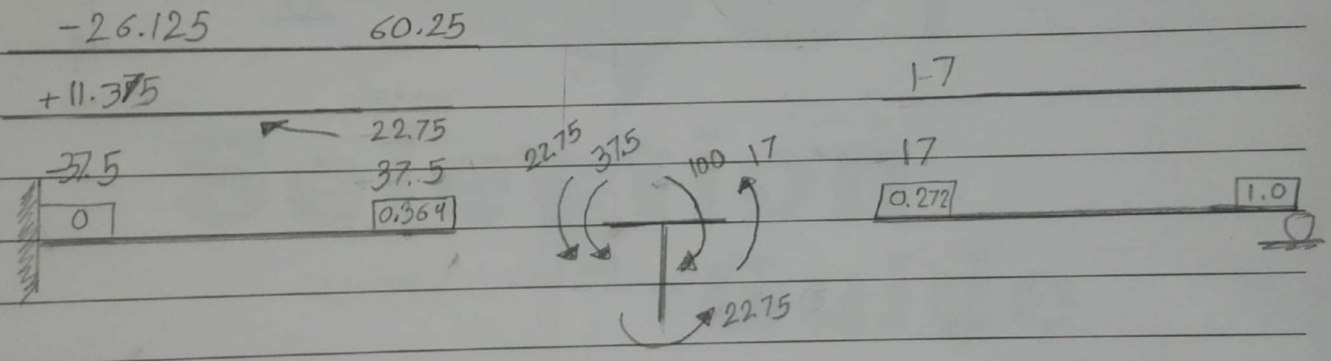
Ex

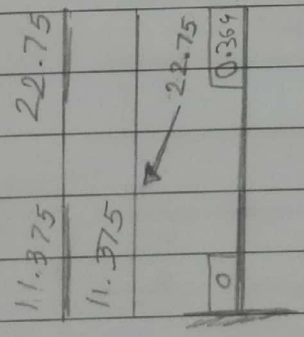


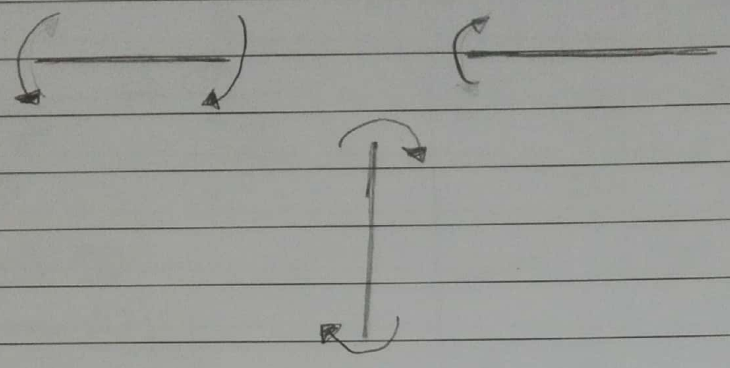
EI- const
(No sides way)

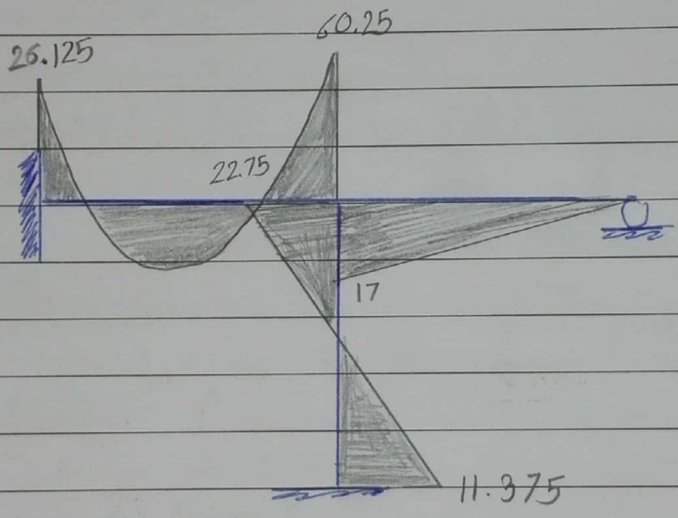
$DF_{CB} = 0.364$ $DF_{CA} = 0.364$ $DF_{CD} = 0.272$

$M^f_{CB} = 37.5$ k.ft $M^f_{BC} = -37.5$ kft









• دائماً منعكس M عند أي joint

Truss Analysis Using the Stiffness Method :-

Determine :-
① unknown displacement
② reaction
③ internal forces for all element

Member & node identification

(العقد over "node"; تحديد over "identification")

○ node number (joint)

□ element number (member)

near ⟶ far end's (من البداية للنهاية)

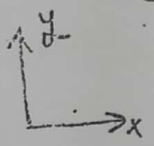
$$P = K\delta$$

$$P = \frac{AE}{L}\,\delta$$

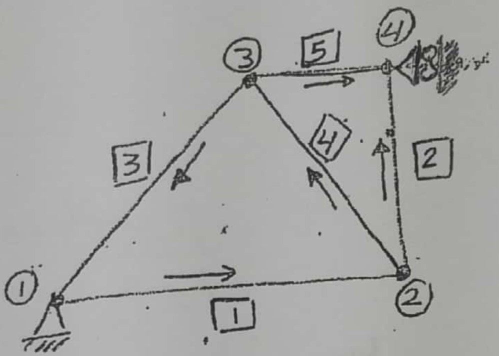
$$\sigma = \frac{P}{A} = E\varepsilon$$

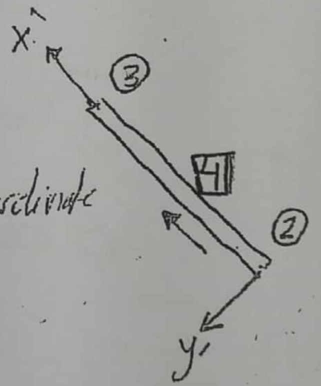
$$\frac{P}{A} = E\,\frac{\delta}{L}$$

$$P = \frac{AE}{L}\,\delta$$



Global coordinate system





Local coordinate system ⟵ (مع اتجاه السهم المرسوم يكون x́
ويكون ý عمودية عليه)

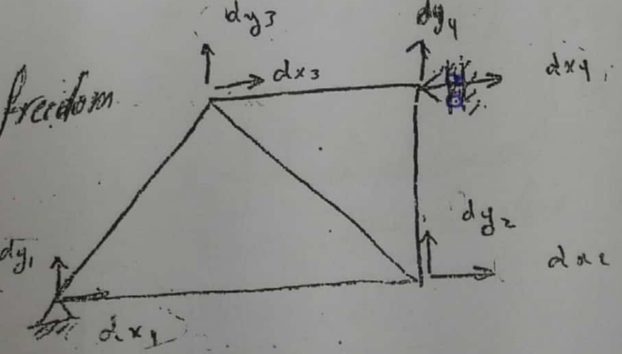
Degrees of Freedom

x_2 dy_2
x_3 dy_3
y_4

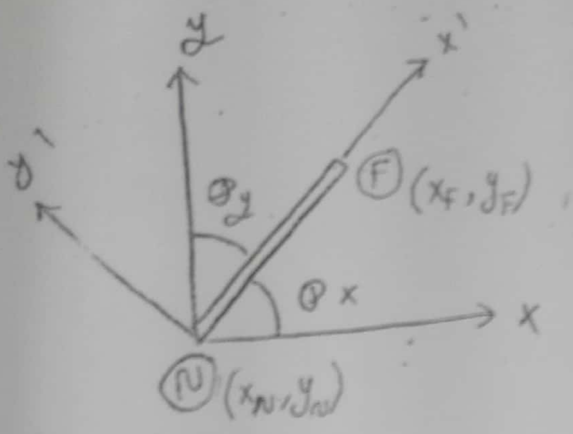
(5) unknown degrees of freedom

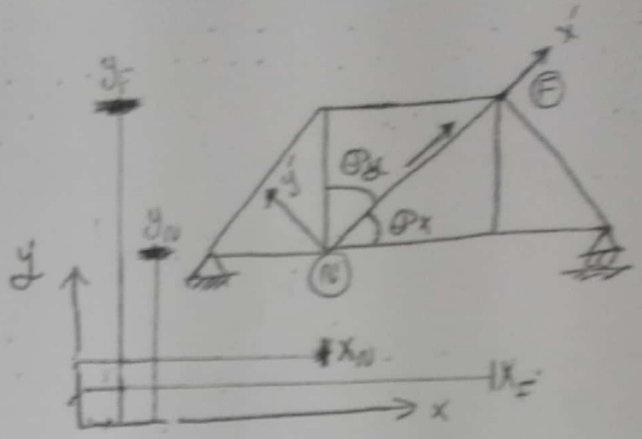
(3) known degrees of freedom

dx_1
$dy_1 = 0$
dx_4



Displacement & Force Transformation Matrix





عكس عقارب الساعة
من X إلى x' ⇐ θ_x

$$\lambda_x = \underline{\cos\theta_x} = \frac{X_F - X_N}{L}$$

$$\lambda_y = \sin\theta_x = \cos\theta_y = \frac{Y_F - Y_N}{L}$$

الإزاحة

Displacement Transformation Matrix

$$d_N = D_{nx}\cos\theta_x + D_{ny}\cos\theta_y$$

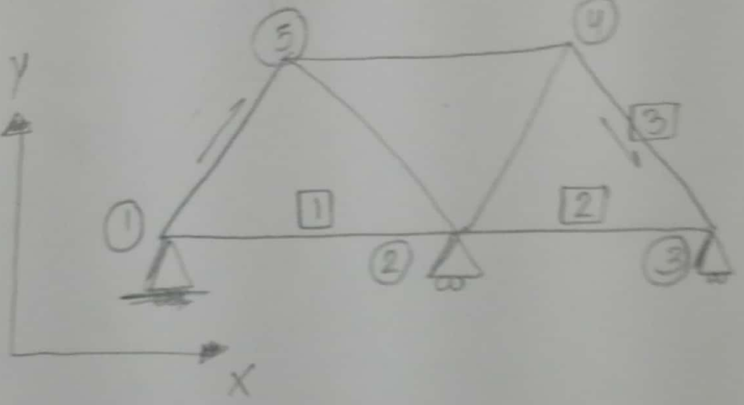
$$d_F = D_{Fx}\cos\theta_x + D_{Fy}\cos\theta_y$$

$$\Rightarrow \begin{bmatrix} d_N \\ d_F \end{bmatrix} = \begin{bmatrix} \lambda_x & \lambda_y & 0 & 0 \\ 0 & 0 & \lambda_x & \lambda_y \end{bmatrix} \begin{bmatrix} D_{Nx} \\ D_{Ny} \\ D_{Fx} \\ D_{Fy} \end{bmatrix}$$

$$T = \begin{bmatrix} \lambda_x & \lambda_y & 0 & 0 \\ 0 & 0 & \lambda_x & \lambda_y \end{bmatrix} = \text{displacement Transformation matrix}$$

$$\Rightarrow [d] = [T][D]$$

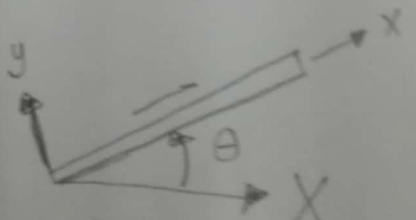
- كل joint عنده حركتين
- وكل member عنده ٤ حركات



10 DOF

4 known

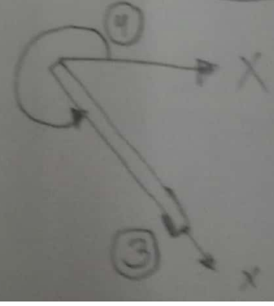
6 unknown



$$[F] = [K][d]$$

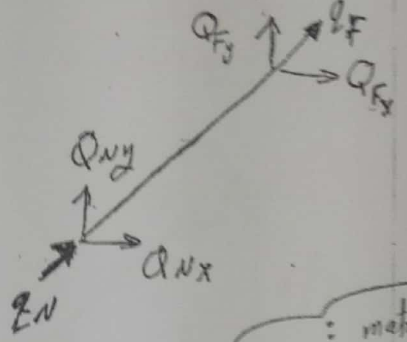
$\lambda_x = \cos\theta$

$\lambda_y = \sin\theta$



Force Transformation Matrix

$$\begin{bmatrix} Q_{Nx} \\ Q_{Ny} \\ Q_{Fx} \\ Q_{Fy} \end{bmatrix} = \begin{bmatrix} \lambda_x & 0 \\ \lambda_y & 0 \\ 0 & \lambda_x \\ 0 & \lambda_y \end{bmatrix} \begin{bmatrix} q_N \\ q_F \end{bmatrix}$$



$$[Q] = [T^T][q]$$

صفات matrix :
① symmetric
② diagonal positive
مهم

كانت هذه الطريقة
تستخدم الآن
حتى ظهرت أجهزة
الحاسوب

$$[k] = \frac{AE}{L} \begin{bmatrix} \lambda_x^2 & \lambda_x\lambda_y & -\lambda_x^2 & -\lambda_x\lambda_y \\ \lambda_x\lambda_y & \lambda_y^2 & -\lambda_x\lambda_y & -\lambda_y^2 \\ -\lambda_x^2 & -\lambda_x\lambda_y & \lambda_x^2 & \lambda_x\lambda_y \\ -\lambda_x\lambda_y & -\lambda_y^2 & \lambda_x\lambda_y & \lambda_y^2 \end{bmatrix} \rightarrow$$

تحفظ في الامتحان

OR

$$\lambda = \begin{bmatrix} c^2 & cs \\ cs & s^2 \end{bmatrix}$$

C – $\cos\theta_x$
S – $\sin\theta_x$

$$\underset{4\times4}{[K]} = \begin{bmatrix} [\lambda] & [-\lambda] \\ [\lambda] & [-\lambda] \end{bmatrix}$$

$$\begin{bmatrix} q_N \\ q_F \end{bmatrix} = \frac{AE}{L} \begin{bmatrix} -1 & 1 \\ +1 & -1 \end{bmatrix} \begin{bmatrix} \lambda_x & \lambda_y & 0 & 0 \\ 0 & 0 & \lambda_x & \lambda_y \end{bmatrix} \begin{bmatrix} D_{Nx} \\ D_{Ny} \\ D_{Fx} \\ D_{Fy} \end{bmatrix}$$

since $q_N = -q_f$

internal force

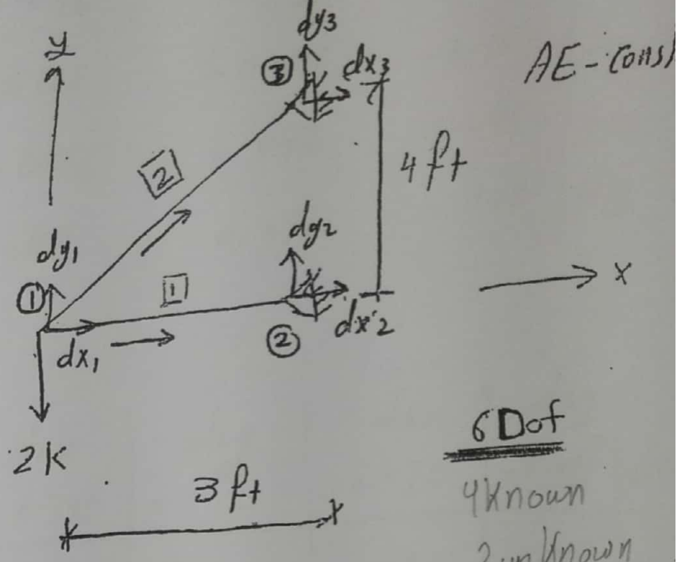
$$q_f = \frac{AE}{L}\begin{bmatrix} -\lambda_x & -\lambda_y & \lambda_x & \lambda_y \end{bmatrix}\begin{bmatrix} D_{Nx} \\ D_{Ny} \\ D_{Fx} \\ D_{Fy} \end{bmatrix}$$

D_{Nx} → Near (النقطة القريبة البداية)

D_{Fy} → Far (النقطة البعيدة النهاية)

3/12

Ex 14-1 14-3



AE - cons

6 Dof

4 known

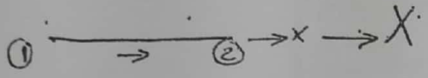
2 unknown

member 1

$\theta = 0$ (تذكر : منافي عكس عقارب الساعة)

$\lambda_x = \cos\theta = 1$

$\lambda_y = \sin\theta = 0$



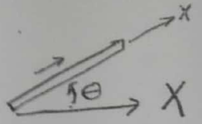
$$k_1 = AE \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{cccc|c} d_{1x} & d_{1y} & d_{2x} & d_{2y} & \\ \frac{1}{3} = 0.333 & 0 & -0.333 & 0 & d_{1x} \\ 0 & 0 & 0 & 0 & d_{1y} \\ -0.333 & 0 & 0.333 & 0 & d_{2x} \\ 0 & 0 & 0 & 0 & d_{2y} \end{array}$$

Local stiffness matrix

ثم إدخال L إلى matrix : مباشرة

لأن طول كل member تختلف عن الأخرى

member 2



$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\lambda_x = \cos\theta = 0.6$

$\lambda_y = 0.8$

local stiffnes Matrix

$$[k_2] = AE \begin{array}{c|cccc|c} & d_{1x} & d_{1y} & d_{3x} & d_{3y} & \\ \hline & \frac{0.6^2}{5} = 0.072 & 0.096 & -0.072 & -0.096 & d_{1x} \\ & 0.096 & 0.128 & -0.096 & -0.128 & d_{1y} \\ & -0.072 & -0.096 & 0.072 & 0.096 & d_{3x} \\ & -0.096 & -.128 & 0.096 & 0.128 & d_{3y} \end{array}$$

طول member 2 من خلال فيثاغورس
يساوي 5ft

$$[K]_{6\times 6} = AE \begin{array}{c|c|c|c|c|c} d_{1x} & d_{1y} & d_{2x} & d_{2y} & d_{3x} & d_{3y} \\ \hline 0.333+.072 = .405 & .096 & -0.333 & 0 & -0.072 & -0.096 \\ 0+0.096 & 0+0.128 & 0 & 0 & -0.096 & -.128 \\ -.333 & 0 & 0.333 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.072 & -0.096 & 0 & 0 & 0.072 & 0.096 \\ -0.096 & -0.128 & 0 & 0 & 0.096 & 0.128 \end{array}$$

تحدد من قبل
Dof

أي شي فارغ عند تعبئة
matrix نضع صفر

Global
Stiffness
Matrix

ternal
ces
بماضرب

$\leftarrow [F] = [K][d]$

$$\begin{bmatrix} F_{1x} = 0 \\ F_{1y} = -2 \\ F_{2x} = ? \\ F_{2y} = ? \\ F_{3x} = ? \\ F_{3y} = ? \end{bmatrix} = A\dot{E} \left[\begin{array}{cccccc} \overset{\sim}{\underset{\shortparallel}{d_{1x}}} & \overset{\sim}{\underset{\shortparallel}{d_{1y}}} & \overset{0}{\underset{\shortparallel}{d_{2x}}} & \overset{0}{\underset{\shortparallel}{d_{2y}}} & \overset{0}{\underset{\shortparallel}{d_{3x}}} & \overset{0}{\underset{\shortparallel}{d_{3y}}} \\ .405 & .096 & & & & \\ .096 & 0.128 & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{array}\right] \begin{bmatrix} d_{1x} \\ d_{1y} \\ d_{2x} \\ d_{2y} \\ d_{3x} \\ d_{3y} \end{bmatrix}$$

نلغي جميع الأسطر التي تحوي أصفار، لأننا لسنا بحاجة لها

$$\Rightarrow \begin{bmatrix} 0 \\ -2 \end{bmatrix} = AE \begin{bmatrix} .405 & 0.096 \\ 0.096 & 0.128 \end{bmatrix} \begin{bmatrix} d_{1x} \\ d_{1y} \end{bmatrix} \xrightarrow{\text{الطريقة}} \begin{cases} 0 = AE\,(.405\, d_{x_1} + 0.096\, d_{y_1}) \\ -2 = AE\,(0.096\, d_{x_1} + 0.128\, d_{y_1}) \end{cases}$$

$$\Rightarrow d_{1x} = \frac{4.505}{AE} \qquad d_{1y} = -\frac{19.003}{AE}$$

Globall y عكس

→ to Find the support reactions

$$\begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{bmatrix} = AE \begin{bmatrix} \\ & 6\times6 & \\ \\ \end{bmatrix} \begin{bmatrix} \frac{4.505}{AE} \\ \frac{-19.003}{AE} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Member ①

$\theta_x = 0 \Rightarrow \lambda_x = 1 \quad \lambda_y = 0 \quad L = 3\,\text{ft}$

$$q_f = \frac{AE}{L}\begin{bmatrix} -\lambda_x & -\lambda_y & \lambda_x & \lambda_y \end{bmatrix}\begin{bmatrix} D_{Nx} \\ D_{Ny} \\ D_{Fx} \\ D_{Fy} \end{bmatrix}$$

$$q = \frac{AE}{3}\begin{bmatrix} -1 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 4.505/AE \\ -19.005/AE \\ 0 \\ 0 \end{bmatrix}$$

$= -1.5\,k \longrightarrow$ السالب دلالة على compnision.

Member ②

$\lambda_x = 0.6 \quad \lambda_y = 0.8$

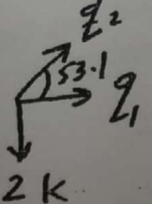
$L = 5\,\text{ft}$

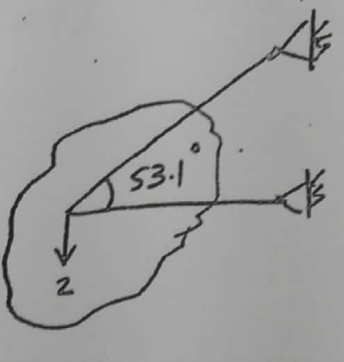
$$q = \frac{AE}{5}\begin{bmatrix} -0.6 & -0.8 & 0.6 & 0.8 \end{bmatrix}\begin{bmatrix} 4.505/AE \\ -19.005/AE \\ 0 \\ 0 \end{bmatrix}$$

$= 2.5\,k$

OR

(للتحقق ب
nt method)





$\Sigma F_y = 0$

$q_2 \sin 53.1 = 2\,k$

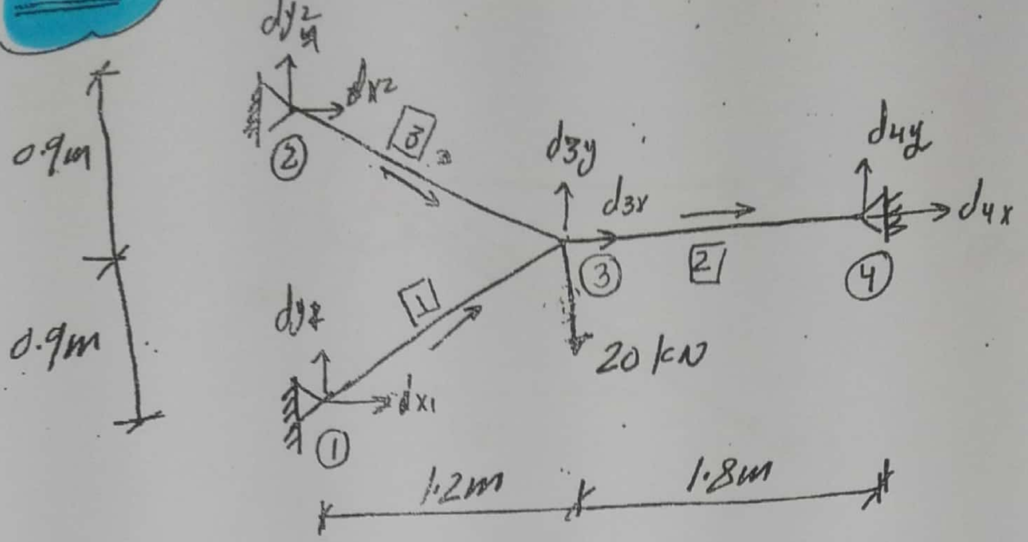
$\Rightarrow q_2 = 2.5\,k$

$\Sigma F_x = 0 \Rightarrow q_1 = -1.5\,k$

26/4

14-3

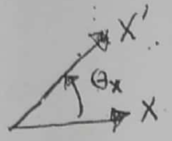
8/12



$A = 300\ mm^2$
$E = 200\ Gpa$

8 Dof
6 → Known
2 → UNKnown

member 1



$\lambda_x = \cos\theta_x = \frac{1.2}{1.5} = 0.8$; $\lambda_y = 0.6$

$$\Longrightarrow [k]^{(1)} = \frac{AE}{1.5}\begin{array}{c} \begin{array}{cccc} 0= & 0= & \sim= & \\ d_{1x} & d_{1y} & d_{3x} & d_{3y} \end{array} \\ \left[\begin{array}{cccc} 0.64 & 0.48 & -0.64 & -0.48 \\ 0.48 & 0.36 & -0.48 & -0.36 \\ -0.64 & -0.48 & \boxed{0.64} & \boxed{0.48} \\ -0.48 & -0.36 & \boxed{0.48} & \boxed{0.36} \end{array}\right] \end{array} \begin{array}{c} \\ d_{1x} \\ d_{1y} \\ d_{3x} \\ d_{3y} \end{array}$$

↓

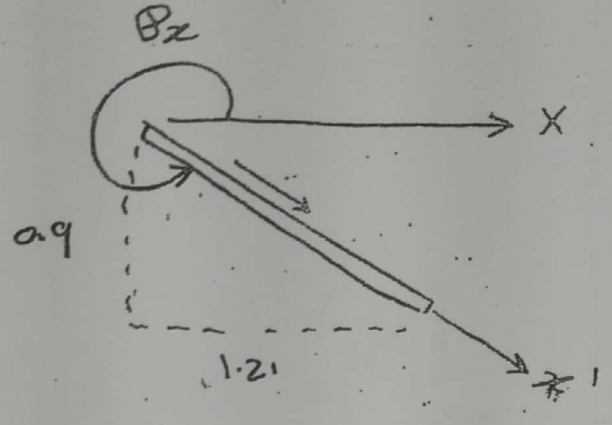
سأستفيد من هذه

member 2

$\theta_x = 0$ $\lambda_x = 1$ $\lambda_y = 0$

$$[k]^{(2)} = \frac{AE}{L}\begin{array}{c} \begin{array}{cccc} d_{3x} & d_{3y} & d_{4x} & d_{4y} \end{array} \\ \left[\begin{array}{cccc} \boxed{1} & \boxed{0} & -1 & 0 \\ \boxed{0} & \boxed{0} & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{c} \\ d_{3x} \\ d_{3y} \\ d_{4x} \\ d_{4y} \end{array}$$

member $\boxed{3}$



$$\theta x = 270 + 53.1$$
$$= 323.1^{\circ}$$

$$\lambda x = \cos \theta x = 0.8$$
$$\lambda y = \sin \theta x = -0.6$$

$$[K]^{(3)} = \frac{AE}{1.5} \left[\begin{array}{cccc|l} dx_2 & dy_2 & dx_3 & dy_3 & \\ 0.64 & -.48 & -.64 & .48 & dx_2 = 0 \\ -0.48 & 0.36 & .48 & -0.36 & dy_2 = 0 \\ -0.64 & 0.48 & 0.64 & -.48 & dx_3 = ? \\ 0.48 & -0.36 & -0.48 & 0.36 & dy_3 = ? \end{array}\right]$$

عامل مشترك??

$$[K] = \boxed{1 * 10^{3}}$$

dx_1	dy_1	dx_2	dy_2	dx_3	dy_3	dx_4	dy_4
25.6	19.2	0	0	-25.4	-19.2	0	0
19.2	14.4						
0	0	25.6		symm			
0	0	-19.2	14.4				
-25.4	-19.2	-25.4	19.2	25.4 + 33.3+25.4	0		
-19.2	-14.4	19.2	-14.4	19.2 -19.2	14.4 +14.4		
0	0	0	0	-33.3	0	33.3	
0	0	0	0	0	0	0	0

- Global matrix ⟶ DoF. بعدد

← displacment

$$\begin{bmatrix} \cancel{F_{1x}} \\ \cancel{F_{1y}} \\ \cancel{F_{2x}} \\ \cancel{F_{2y}} \\ F_{3x} = 0 \\ F_{3y} = -20 \text{ KN} \\ \cancel{F_{4x}} \\ \cancel{F_{4y}} \end{bmatrix} = [K]_{8\times 8} \begin{bmatrix} dx_1 = 0 \\ dy_1 = 0 \\ dx_2 = 0 \\ dy_2 = 0 \\ dx_3 \\ dy_3 \\ dx_4 = 0 \\ dy_4 = 0 \end{bmatrix}$$

• دائماً F معروف بالمقابل displacment مجهول

$$\begin{bmatrix} 0 \\ -20 \end{bmatrix} = (10^{3}) \begin{bmatrix} 34.83 & 0 \\ 0 & 28.8 \end{bmatrix} \begin{bmatrix} d_{3x} \\ d_{3y} \end{bmatrix}$$

$$\Rightarrow d_{3x} = 0$$

$$d_{3y} = -0.6944 * 10^{-3} \ \underline{m}$$

$$= -0.6944 \text{ mm}$$

member ①

$$q_f = q_1 = \frac{AE}{L}\begin{bmatrix}-0.8 & -0.6 & 0.8 & 0.6\end{bmatrix}\begin{bmatrix}0\\0\\0\\-0.6944*10^{-3}\end{bmatrix}$$

$$= -13.89\ kN = 13.9\ kN\ (comp)$$

<u>member ②</u>

$$q_2 = \frac{AE}{L}\begin{bmatrix}-1 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}0\\-0.6944*10^{-3}\\0\\0\end{bmatrix}$$

$$q_2 = 0$$

member ③

$$q_3 = \frac{AE}{L}\begin{bmatrix}-0.8 & 0.6 & 0.8 & -0.6\end{bmatrix}\begin{bmatrix}0\\0\\0\\-0.6944*10^{-3}\end{bmatrix}$$

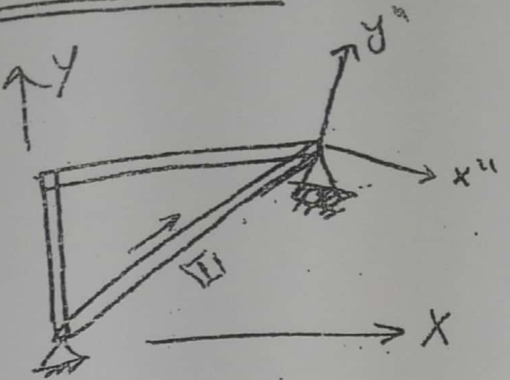
$$= 13.9\ kN\ (Ten)$$

نجد (displacment)

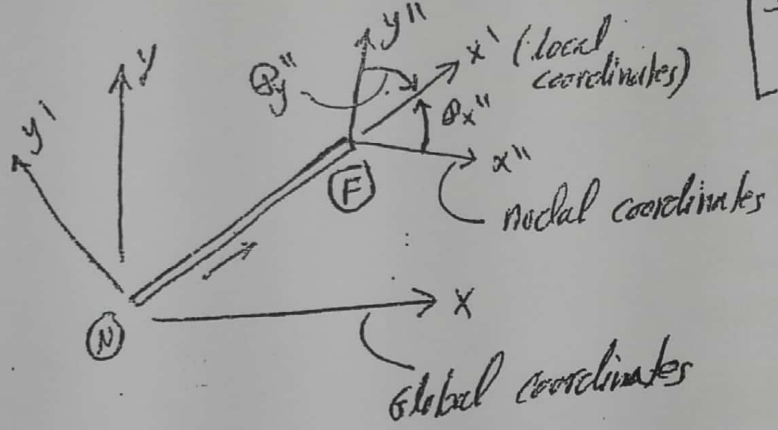
ونجد (reaction) بما أنه اوجدنا (internal force)

غير مطلوب

Nodal Coordinates



$$\begin{bmatrix} d_N \\ d_F \end{bmatrix} = \begin{bmatrix} \lambda_x & \lambda_y & 0 & 0 \\ 0 & 0 & \lambda_{x''} & \lambda_{y''} \end{bmatrix} \begin{bmatrix} D_{Nx} \\ D_{Ny} \\ D_{Fx''} \\ D_{Fy''} \end{bmatrix}$$



$$\begin{bmatrix} Q_{Nx} \\ Q_{Ny} \\ Q_{Fx''} \\ Q_{Fy''} \end{bmatrix} = \begin{bmatrix} \lambda_x & 0 \\ \lambda_y & 0 \\ 0 & \lambda_{x''} \\ 0 & \lambda_{y''} \end{bmatrix} \begin{bmatrix} q_N \\ q_F \end{bmatrix}$$

$$[K] = \frac{AE}{L} \begin{bmatrix} \lambda_x^2 & & \text{Symm} & \\ \lambda_x\lambda_y & \lambda_y^2 & & \\ -\lambda_x\lambda_{x''} & -\lambda_y\lambda_{x''} & \lambda_{x''}^2 & \\ -\lambda_x\lambda_{y''} & -\lambda_y\lambda_{y''} & \lambda_{x''}\lambda_{y''} & \lambda_{y''}^2 \end{bmatrix}$$

see exampl 14-6 Page 552

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⑫ غير مطلوب

Beam analysis using the stiffness Method

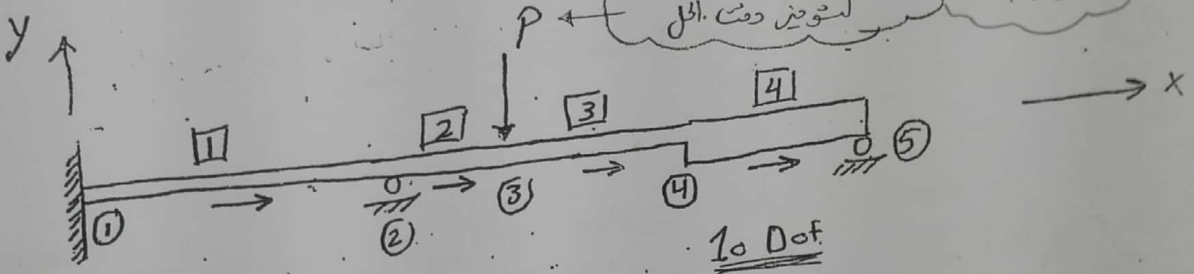
10/12

In order to apply the stiffness Method to beams

- each element must be free from load & have a prismatic cross-section.
- the nodes of each element are located at a support or at points where members are connected together, where an external force is applied, where the cross-sectional are suddenly changes, where the vertical or rotational displacement at a point is to be determined.

يمكن نقلها الى nodes قريب
لتوفير دقت الحل.

مع
Moment



Degrees of freedom

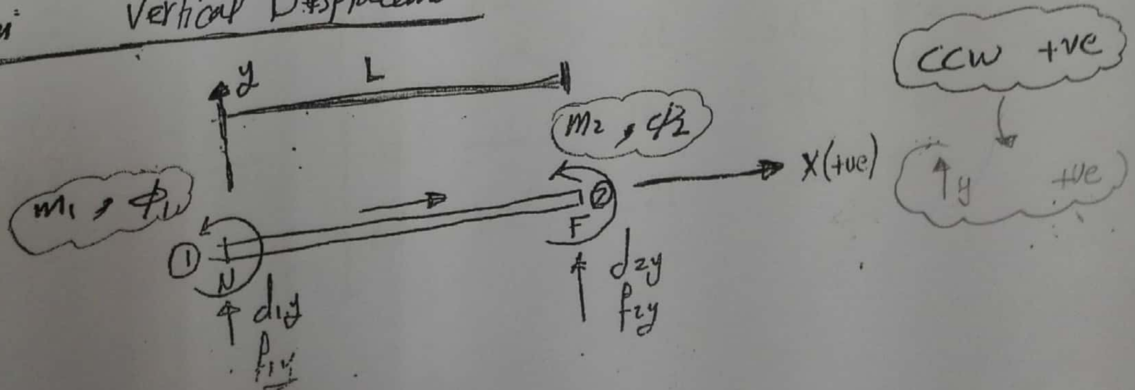
لها علاقة بالحر
العمودية

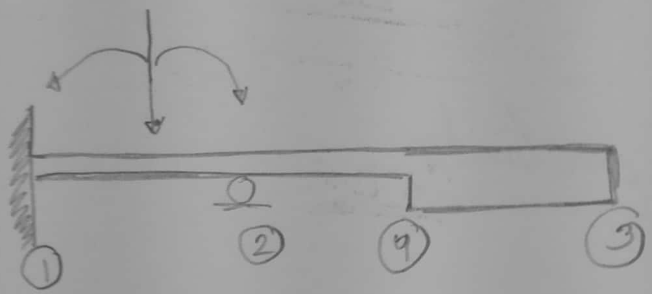
لها علاقة :
بالدورا

bending & shear → each node have two degrees of freedom

bending ⇓ Rotation

shear ⇓ Vertical Displacement





8 DOF
4 Known

هذه المعادلة
حفظ

$$\begin{Bmatrix} f_{1y} \\ m_1 \\ f_{2y} \\ m_2 \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{bmatrix} d_{1y} \\ \phi_1 \\ d_{2y} \\ \phi_2 \end{bmatrix}$$

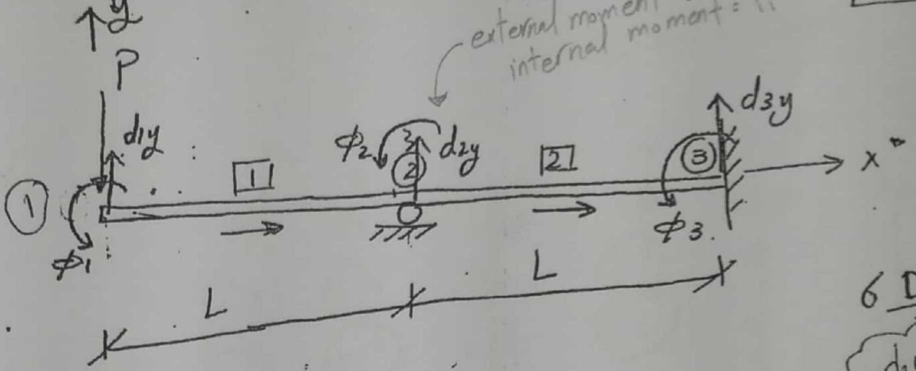
اشتقناها
عن طالب

$$[q] = [K][d]$$

28/4

Ex

EI - constant



external moment = 0
internal moment = ??

6 DoF

$d_{2y} = d_{3y} = \emptyset_3 = 0$

$d_{1y}, \emptyset_1, \emptyset_2 = ?$

element 1

$$[K]^{(1)} = \frac{EI}{L^3} \begin{array}{c} \begin{array}{cccc} d_{1y} & \phi_1 & d_{2y} & \phi_2 \end{array} \\ \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \end{array} \begin{array}{c} d_{1y} \\ \phi_1 \\ d_{2y} \\ \phi_2 \end{array}$$

element 2

$$[K]^{(2)} = \frac{EI}{L^3} \begin{array}{c} \begin{array}{cccc} d_{2y} & \phi_2 & d_{3y} & \phi_3 \end{array} \\ \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \end{array} \begin{array}{c} d_{2y} \\ \phi_2 \\ d_{3y} \\ \phi_3 \end{array}$$

structure stiffness matrix

$$[K]_{6\times 6} = \frac{EI}{L^3}\left[\begin{array}{cccccc|c} d_{1y} & \phi_1 & d_{2y} & \phi_2 & d_{3y} & \phi_3 & \\ 12 & 6L & -12 & 6L & 0 & 0 & d_{1y} \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 & \phi_1 \\ -12 & -6L & 12+12 & -6L+6L & -12 & 6L & d_{2y} \\ 6L & 2L^2 & -6L+6L & 4L^2+4L^2 & -6L & 2L^2 & \phi_2 \\ 0 & 0 & -12 & -6L & 12 & -6L & d_{3y} \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 & \phi_3 \end{array}\right]$$

نعيشوا أسهل من Matrix

$$[Q] = [K][d]$$

المجهول هنا في المقابل وبالعكس

$$\begin{bmatrix} -P \\ 0 = M_1 \\ F_{2y} \\ 0 = M_2 \\ F_{3y} \\ M_3 \end{bmatrix} = \frac{EI}{L^3}\left[\begin{array}{cccccc} \overset{?}{\overset{\shortparallel}{d_{1y}}} & \overset{?}{\overset{\shortparallel}{\phi_1}} & \overset{0}{\overset{\shortparallel}{d_{2y}}} & \overset{?}{\overset{\shortparallel}{\phi_2}} & \overset{0}{\overset{\shortparallel}{d_{3y}}} & \overset{0}{\overset{\shortparallel}{\phi_3}} \\ 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & 12 & -6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{array}\right]\begin{bmatrix} d_{1y} \\ \phi_1 \\ d_{2y} \\ \phi_2 \\ d_{3y} \\ \phi_3 \end{bmatrix}$$

$$\begin{bmatrix} -P \\ 0 \\ 0 \end{bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & 6L \\ 6L & 4L^2 & 2L^2 \\ 6L & 2L^2 & 8L^2 \end{bmatrix}\begin{bmatrix} d_{1y} \\ \phi_1 \\ \phi_2 \end{bmatrix}$$

$$\Rightarrow d_{1y} = \frac{-7PL^3}{12EI}$$

$$\phi_1 = \frac{3PL^2}{4EI}$$

$$\phi_2 = \frac{PL^2}{4EI}$$

$$\Rightarrow$$

$$\begin{array}{c} \\ M_1 \\ \\ M_2 \\ \\ \\ \end{array}\begin{bmatrix} -P \\ 0 \\ F_{2y} \\ 0 \\ F_{3y} \\ M_3 \end{bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & 12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{bmatrix} -7PL^3/12EI \\ 3PL^2/4EI \\ 0 \\ \frac{PL^2}{4EI} \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow F_{2y} = \frac{5}{2}P$$

$$F_{3y} = -\frac{3}{2}P$$

$$M_3 = \frac{1}{2}PL$$

• كل مازادت $\underline{\underline{I}}$

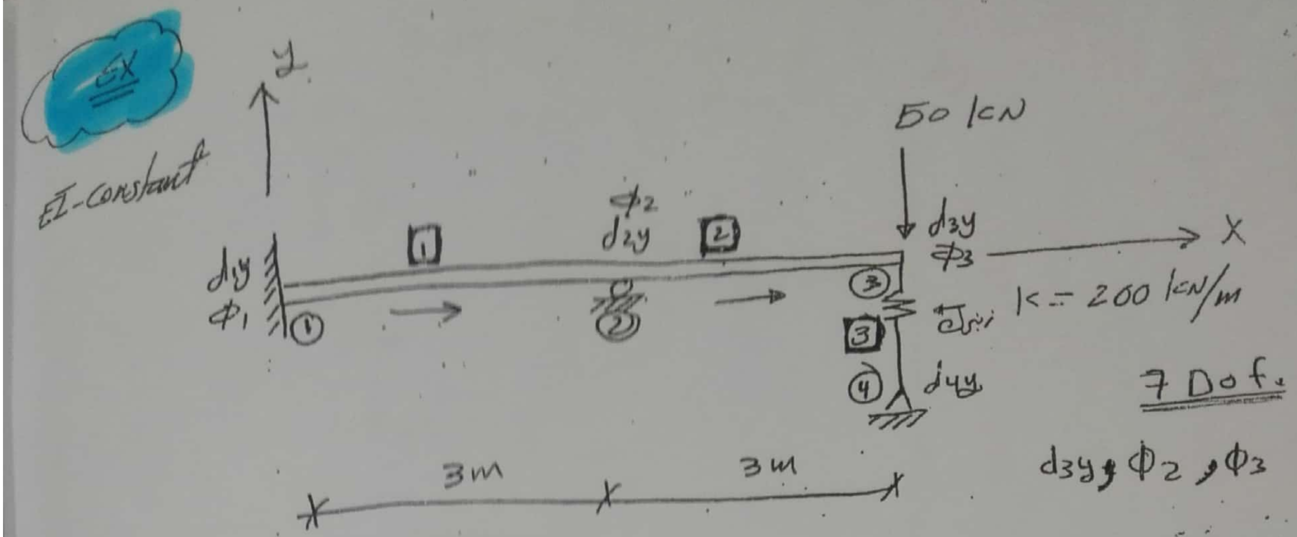
زادت $\underline{\underline{K}}$

• beam عنه صنعت

material : يعني علي

E

I , L



7 Dof.

d_{3y} و ϕ_2 و ϕ_3 ??

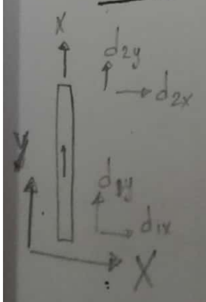
Element 1

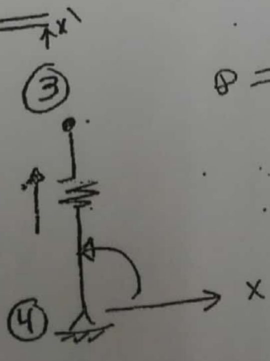
$$[K^{(1)}] = \frac{EI}{27}\begin{array}{cccc|c} d_{1y} & \phi_1 & d_{2y} & \phi_2 & \\ 12 & 18 & -12 & 18 & d_{1y} \\ 18 & 36 & -18 & 18 & \phi_1 \\ -12 & -18 & 12 & -18 & d_{2y} \\ 18 & 18 & -18 & 36 & \phi_2 \end{array}$$

Element 2

$$[K^{(2)}] = \frac{EI}{27}\begin{array}{cccc|c} d_{2y} & \phi_2 & d_{3y} & \phi_3 & \\ 12 & 18 & -12 & 18 & d_{2y} \\ 18 & 36 & -18 & 18 & \phi_2 \\ -12 & -18 & 12 & -18 & d_{3y} \\ 18 & 18 & -18 & 36 & \phi_3 \end{array}$$

Element 3





$\theta = 90° \Rightarrow \lambda x = 0$

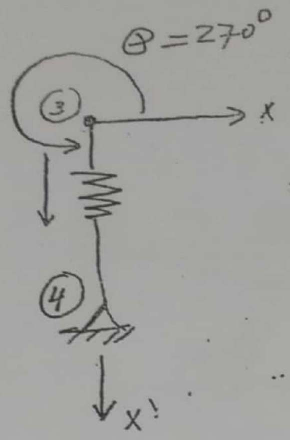
$\lambda y = 1$

$$\Rightarrow [K^{(3)}] = 200 \begin{array}{cc|c} d_{4y} & d_{3y} & \\ 1 & -1 & d_{4y} \\ -1 & 1 & d_{3y} \end{array}$$

- الزنبرك بينزل فقط على امتداده و لا يحدث فيه دوران ويمكن أن نعامل الزنبرك كما نعامل Truss أي نأخذ (4 degrees)

$$\begin{array}{cc} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} & \begin{matrix} d_{1x} \\ d_{1y} \\ d_{2x} \\ d_{2y} \end{matrix} \end{array} \longrightarrow \underset{\substack{\downarrow \\ K=200}}{\boxed{\frac{AE}{L}}} \begin{array}{cc} \begin{bmatrix} +1 & -1 \\ -1 & 1 \end{bmatrix} & \begin{matrix} d_{1y} \\ d_{2y} \end{matrix} \end{array}$$

OR



$\lambda_x = 0$

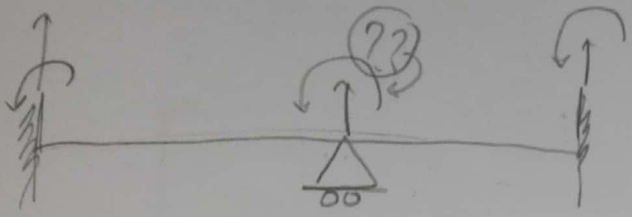
$\lambda_y = -1$

$$\Rightarrow \left[k^{(3)}\right] = 200 \begin{array}{c} \\ \\ \end{array} \begin{array}{cc|c} d_{3y} & d_{4y} & \\ \hline 1 & -1 & d_{3y} \\ -1 & 1 & d_{4y} \end{array}$$

** assume $\boxed{k' = \frac{EI}{27}}$

$$\Rightarrow [K] = \begin{array}{ccccccc|l} \overset{=0}{d_{1y}} & \overset{=0}{\phi_1} & \overset{=0}{d_{2y}} & \phi_2 & d_{3y} & \phi_3 & \overset{=0}{d_{4y}} & \\ \hline 12k' & 18k' & -12k' & 18k' & 0 & 0 & 0 & d_{1y} = 0 \\ 18k' & 36k' & -18k' & 18k' & 0 & 0 & 0 & \phi_1 = 0 \\ -12k' & -18k' & 12k' + 12k' & -18k' + 18k' & -12k' & 18k' & 0 & d_{2y} = 0 \\ 18k' & 18k' & -18k' + 18k' & 36k' + 36k' & -18k' & 18k' & 0 & \phi_2 \\ 0 & 0 & -12k' & -18k' & 12k' + 200 & -18k' & -200 & d_{3y} \\ 0 & 0 & 18k' & 18k' & -18k' & 36k & & \phi_3 \\ 0 & 0 & 0 & 0 & -200 & 0 & 200 & d_{4y} = 0 \end{array}$$

$$\begin{array}{r} M_2 \\ \text{مقدار Load عند النقطة ٣} \\ \text{(مقدار M عند النقطة} \; M_3 \end{array} \begin{bmatrix} 0 \\ -50 \\ 0 \end{bmatrix} = \begin{bmatrix} 72k' & -18k' & 18k' \\ -18k' & 12k' + 200 & -18k' \\ 18k' & -18k' & 36k' \end{bmatrix} \begin{bmatrix} \phi_2 \\ d_{3y} \\ \phi_3 \end{bmatrix}$$

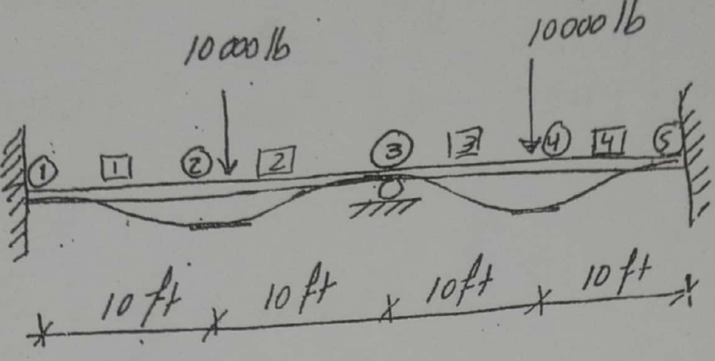


1 unknown

Ex

$E = 30 * 10^6$ psi

$I = 500$ in^4



الدورات عند 2 و 4 = 0
بسبب التماثل .

10 Dof

$d_{1y} = \phi_1 = d_{3y} = d_{5y} = \phi_5 = 0$ → because symmytreic

$[K^{(1)}] = [K]^{(2)} = [K]^{(3)} = [K]^{(4)}$

$[K]_{10 \times 10}$

$$\begin{bmatrix} -10000 \\ 0 \\ 0 \\ -10000 \\ 0 \end{bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 24 & 0 & 6L & 0 & 0 \\ 0 & 8L^2 & 2L^2 & 0 & 0 \\ 6L & 2L^2 & 8L^2 & -6L & 2L^2 \\ & & -6L^2 & 24 & 0 \\ 0 & 0 & 2L^2 & 0 & 8L^2 \\ 0 & 0 & & & \end{bmatrix} \begin{Bmatrix} d_{2y} \\ \phi_2 \\ \phi_3 \\ d_{4y} \\ \phi_4 \end{Bmatrix}$$

the rotation (slope) at nodes 2, 3, and 4 are equal to zero because of symmetry in loading, geometry and material propert about a plane perpendicular to the beam length and passi through node 3.

$\Rightarrow \phi_2 = \phi_3 = \phi_4 = 0$

$$\begin{bmatrix} -10000 \\ -10000 \end{bmatrix} = \frac{EI}{120^3} \begin{bmatrix} 24 & 0 \\ 0 & 24 \end{bmatrix} \begin{bmatrix} d_{2y} \\ d_{4y} \end{bmatrix}$$

$\Rightarrow d_{2y} = d_{4y} = -0.048$ in

$[F] = [k][d]$
$10\times1 \quad 10\times8 \quad 10\times1$

$$\begin{bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \\ F_{4y} \\ M_4 \\ F_{5y} \\ M_5 \end{bmatrix} = \underset{10\times10}{[K]} \begin{bmatrix} d_{1y}=0 \\ \phi_1=0 \\ d_{2y}=-0.048 \\ \phi_2=0 \\ d_{3y}=0 \\ \phi_3=0 \\ d_{4y}=-0.048 \\ \phi_4=0 \\ d_{5y}=0 \\ \phi_5=0 \end{bmatrix}$$

$F_{1y} = 5000$ lb

$F_{2y} = -10000$ lb

$F_{3y} = 10000$ lb

$F_{4y} = -10000$ lb

$F_{5y} = 5000$ lb

$M_1 = 25000$ lb·ft

$M_2 = 0$

$M_3 = 0$

$M_4 = 0$

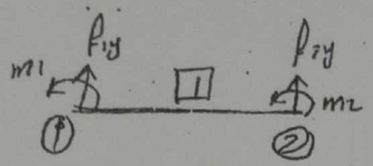
$M_5 = -25000$ lb·ft

??

Scanned by CamScanner

جيلان اشارة : سابية

element $\boxed{1}$



$$\begin{Bmatrix} f_{1y} \\ m_1 \\ f_{2y} \\ m_2 \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} d_{1y} = 0 \\ \phi_1 = 0 \\ d_{2y} = -0.048 \\ \phi_2 = 0 \end{Bmatrix}$$

$$\Rightarrow \quad f_{1y} = 5000 \text{ lb}$$

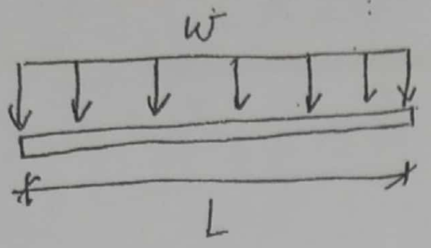
$$m_1 = 25000 \text{ lb}\cdot\text{ft}$$

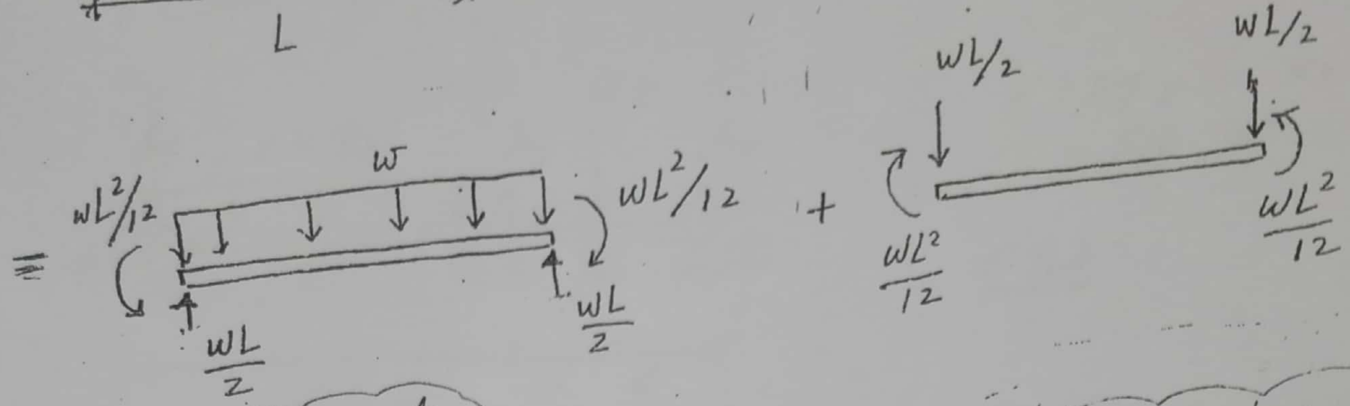
$$f_{2y} = -5000 \text{ lb}$$

$$m_2 = 25000 \text{ lb}\cdot\text{ft}$$

هنا يجب
استخدام طريقة أن نحل
على Local لكل من **beam**
حتى نجد **internal force**.

Intermediate loading تحميل القوى



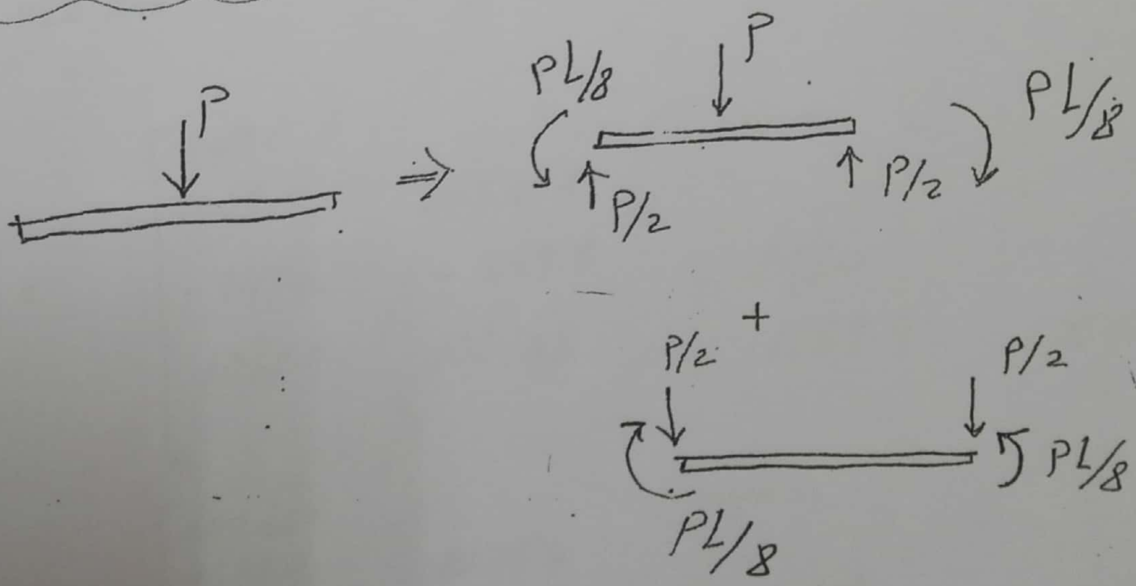


actual loading + Reactions

Fixed-end element loading on joints

هنا دايمًا نأخذ member
Fixed Fixed → radius formula لا يوجد

$$[q] = [k][d] + [q_0]$$



$$[F] = [k][d] - [F_0]$$

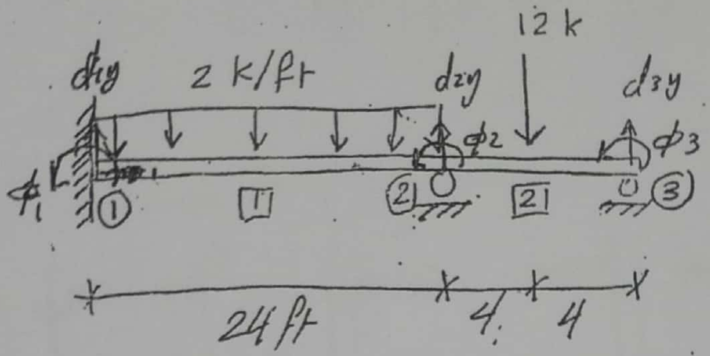
$$[f] = [k][d] - [f_0]$$

---

look at Ex 15.4

13/12

Ex



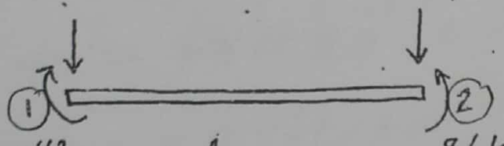
$E = 29 \times 10^3$ ksi

$I = 510$ in^4

6 Dof

element [1]

$wL/2 = 24k$ 24k



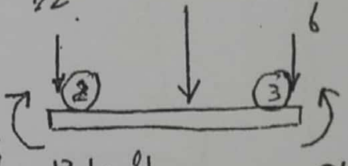
$\frac{wL^2}{12} = 96$ k·ft ÷12 96 k·ft

= 1152 k·in = 1152

ft = 12 insh

element [2]

$P/2 = 6$ 6



12 k·ft

= 144 k.in

$PL/8 = 12$ k·ft

= 144 k·in

$$[k^{(1)}] = \begin{bmatrix} 7430 & & & \text{symm} \\ 1069.9 & 205417 & & \\ -7430 & -1069.9 & 7430 & \\ 1069.9 & 102708 & -1069.9 & 205417 \end{bmatrix}$$

$$[k^{(2)}] = \begin{bmatrix} 200.602 & & & \text{symm} \\ 9628.91 & 616250 & & \\ -200.602 & -9628.91 & 200.602 & \\ 9628.91 & 308125 & -9628.91 & 616250 \end{bmatrix}$$

$$[K] = \begin{array}{cccccc|l} d_{1y} & \phi_1 & d_{2y} & \phi_2 & d_{3y} & \phi_3 & \\ 0 & 0 & 0 & & 0 & & \\ \hline 7430 & & & & & & d_{1y} = 0 \\ 1069.1 & 205417 & & & & & \phi_1 = 0 \\ -7430 & -1069.9 & \begin{matrix} 7430 \\ +200.6 \end{matrix} & & \text{Symm} & & d_{2y} = 0 \\ 1069.9 & 102708 & \begin{matrix} -1069.9 \\ +9628.9 \end{matrix} & \begin{matrix} 205417 \\ +616250 \end{matrix} & & & \phi_2 \\ 0 & 0 & 200.6 & -9628.9 & 200.6 & & d_{3y} = 0 \\ 0 & 0 & 96.28.9 & 308125 & -9628.9 & 616250 & \phi_3 \end{array}$$

$$\begin{bmatrix} 1152-144 \\ 144 \end{bmatrix} = \begin{bmatrix} 821667 & 308125 \\ 308125 & 616250 \end{bmatrix} \begin{bmatrix} \phi_2 \\ \phi_3 \end{bmatrix}$$

$$\Rightarrow \phi_2 = 1.402 * 10^{-3}$$

$$\phi_3 = -0.4673 * 10^{-3}$$

$$[F_0] = \begin{bmatrix} -24 \\ -115.2 \\ -24-6 \\ 1152-144 \\ -6 \\ 144 \end{bmatrix} \begin{matrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{matrix}$$

للتبسيط

$$\begin{bmatrix} F_{1y}' \\ M_1' \\ F_{2y}' \\ M_2' \\ F_3' \\ M_3' \end{bmatrix} = [K]_{6\times6} \begin{bmatrix} d_{1y} = 0 \\ \phi_1 = 0 \\ d_{2y} = 0 \\ 1.402 * 10^{-3} \\ 0 \\ -0.4673 * 10^{-3} \end{bmatrix}$$

كل الحل يكون على (full formula)

$$\begin{bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \quad & \quad & \quad & \quad & \quad & \quad \\ \\ \\ \\ \\ \\ \end{bmatrix} \begin{bmatrix} d_{1y}=0 \\ \phi_1=0 \\ d_{2y}=0 \\ \phi_2=? \\ d_{2y}=0 \\ \phi_3 \end{bmatrix}}_{F'} - \begin{bmatrix} -24 \\ -1152 \\ -30 \\ 1152-144 \\ -6 \\ 144 \end{bmatrix}$$

$F_{1y}' = 1069.9 * 1.402 * 10^{-3} = 1.5\ k$

$M_1' = 102708 * 〃 = 144\ k \cdot in$

$F_{2y}' = 8559 * 〃 + 9628.9 * -0.4673 * 10^{-3} = 7.5\ k$

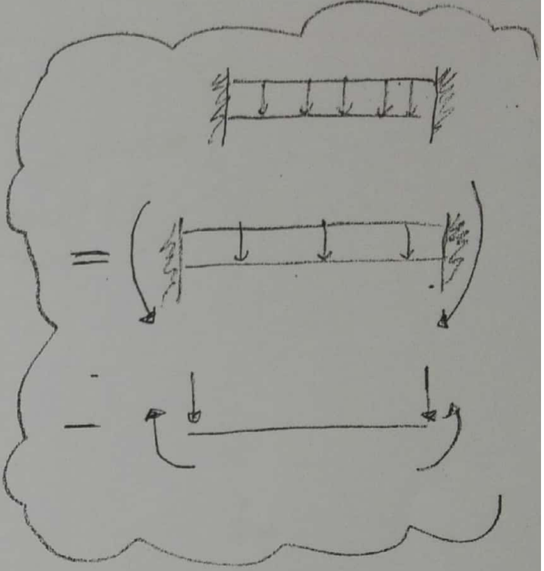
$M_2' = 1008\ k \cdot in$

$F_{3y}' = -9\ k$

$M_3' = 144\ k \cdot in$

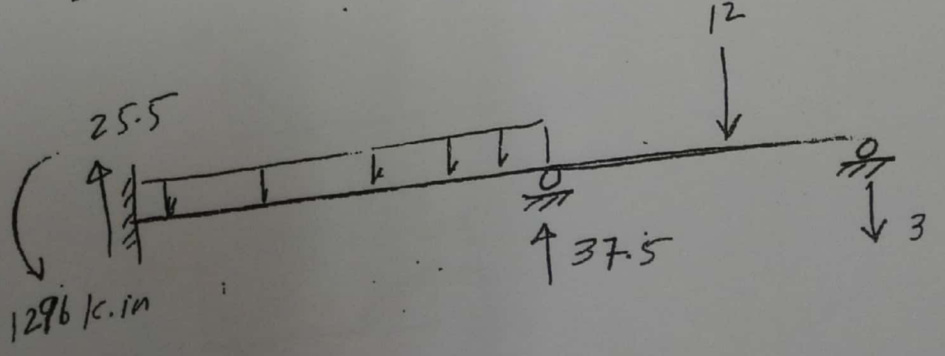
لماذا السالب ؟؟

$$[F] = \underbrace{[F']}_{[k][d]} - [F_0]$$



$$\begin{bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 144 \\ 7.5 \\ 1008 \\ -9 \\ 144 \end{bmatrix} - \begin{bmatrix} -24 \\ -1152 \\ -24-6 \\ 1152-144 \\ -6 \\ 144 \end{bmatrix} = \begin{bmatrix} 25.5 \\ 1296 \\ 37.5 \\ 0 \\ -3 \\ 0 \end{bmatrix}$$

يجب أن يساوي صفر



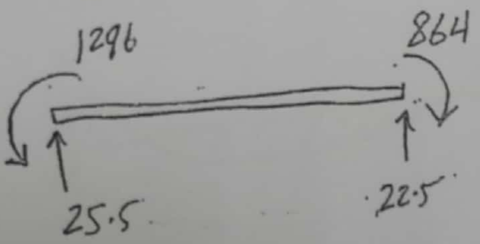
internal forces in member $\boxed{1}$

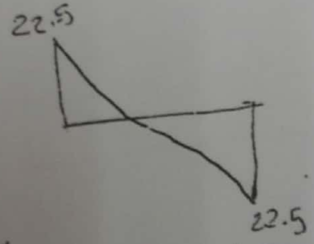
nal force

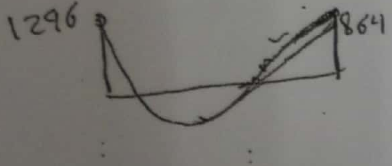
$$\begin{Bmatrix} f_{1y} \\ m_1 \\ f_{2y} \\ m_2 \end{Bmatrix} = \begin{bmatrix} 7430 & & \text{symm} & \\ 1069.9 & 205417 & & \\ -7430 & -1069.9 & 7430 & \\ 1069.9 & 102708 & -1069.9 & 205417 \end{bmatrix} \begin{bmatrix} d_{1y}=0 \\ \phi_1=0 \\ d_{2y}=0 \\ \phi_2=1.4\times10^{-3} \end{bmatrix} - \begin{bmatrix} -24 \\ -1152 \\ -24 \\ +1152 \end{bmatrix}$$

F° خاصة في element $\boxed{1}$

$$\begin{Bmatrix} f_{1y} \\ m_1 \\ f_{2y} \\ m_2 \end{Bmatrix} = \begin{bmatrix} 25.5 \\ 1296 \\ 22.5 \\ -864 \end{bmatrix}$$





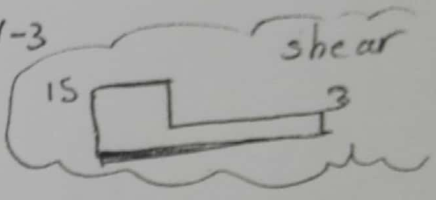


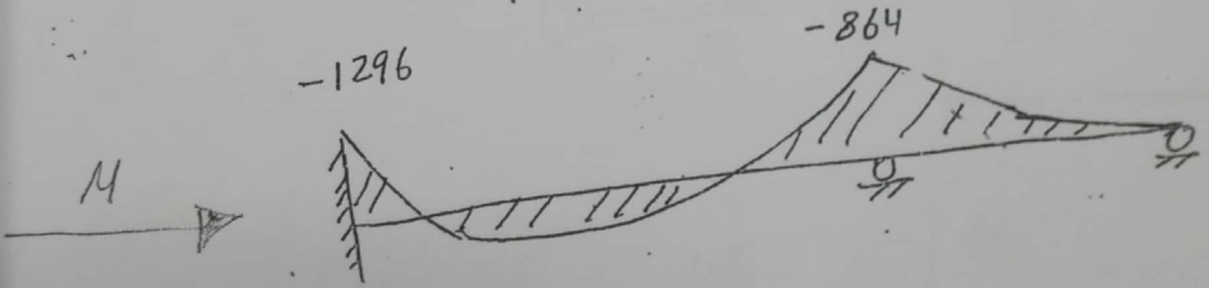
member $\boxed{2}$

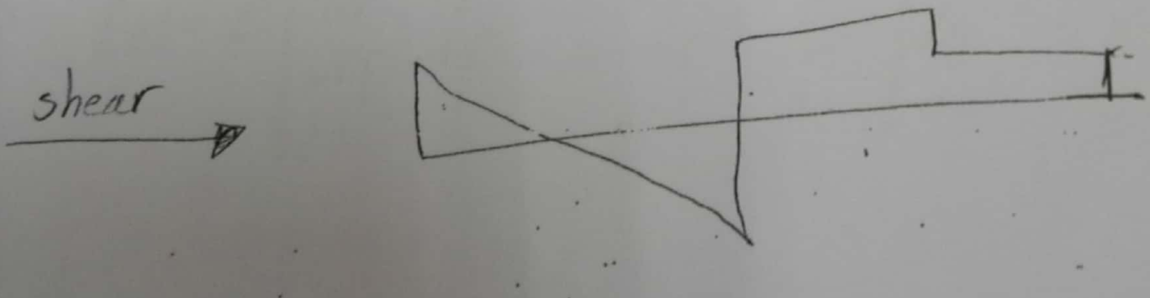
$$\begin{bmatrix} f_{2y} \\ m_2 \\ f_{3y} \\ m_3 \end{bmatrix} = \begin{bmatrix} 200.602 & & & \text{symm} \\ 9628.91 & 616250 & & \\ -200.6 & -9628.9 & 200.6 & \\ 9628.91 & 308125 & -9628.91 & 616250 \end{bmatrix} \begin{bmatrix} d_{2y} = 0 \\ \phi_2 = 1402\times10^{-3} \\ d_{3y} = 0 \\ \phi_3 = -0.457 \end{bmatrix}$$

$$- \begin{bmatrix} -6 \\ -144 \\ -6 \\ 144 \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 864 \\ -3 \\ 0 \end{bmatrix}$$





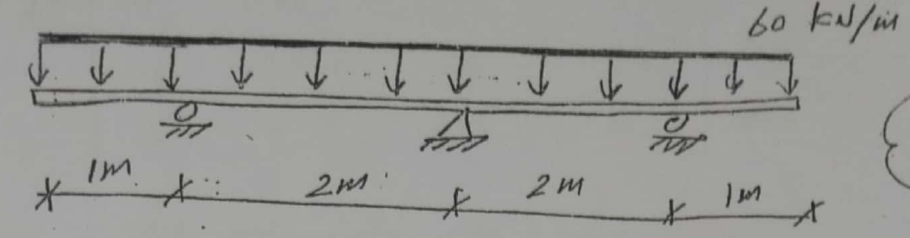


Ex

17/12

5/5

EI - constant



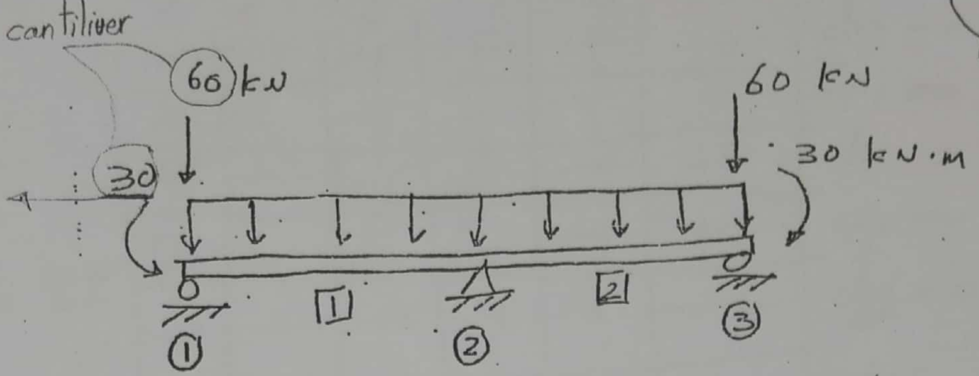
D of 6

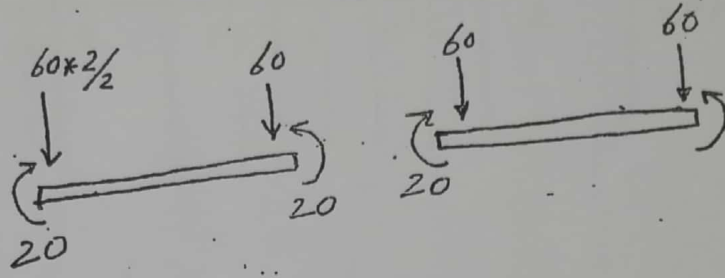
$d_{1y}=0$

$d_{2y}=0$

$d_{3y}=0$

$\phi_2=0$



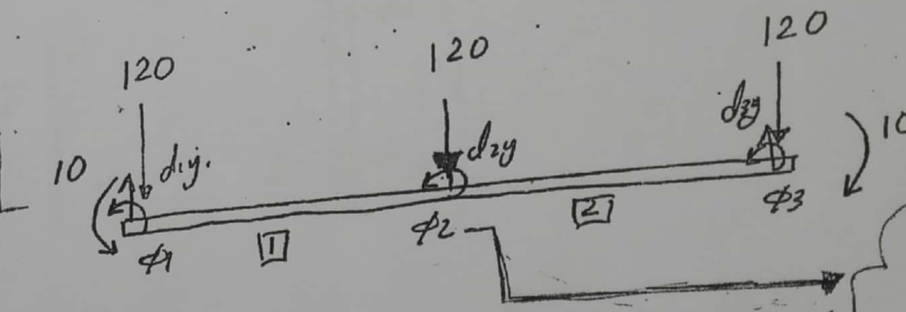


$\frac{wL^2}{12} = \frac{60*4}{12} = 20$

$\equiv$

-60 -60
30 -20
-60 -60
20 -20
-60 -60
20 + 30

$$= \begin{array}{cc} \begin{bmatrix} -120 \\ 10 \\ -120 \\ 0 \\ -120 \\ -10 \end{bmatrix} & \begin{array}{c} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{array} \end{array}$$



$\phi_2 = 0$

Symm.

Member 1

$\frac{12}{8}$

$\frac{3}{2}$ ←

$$[K]^{(1)} = EI \begin{array}{cc} \begin{bmatrix} 1.5 & 1.5 & -1.5 & 1.5 \\ 1.5 & 2 & -1.5 & 1 \\ -1.5 & -1.5 & 1.5 & -1.5 \\ 1.5 & 1 & -1.5 & 2 \end{bmatrix} & \begin{array}{c} d_{1y} \\ \phi_1 \\ d_{2y} \\ \phi_2 \end{array} \end{array}$$

$$[K^{(2)}] = EI \left[\begin{array}{cccc|c} 1.5 & 1.5 & -1.5 & 1.5 & d_{2y} \\ 1.5 & 2 & -1.5 & 1 & \phi_2 \\ -1.5 & -1.5 & 1.5 & -1.5 & d_{3y} \\ 1.5 & 1 & -1.5 & 2 & \phi_3 \end{array}\right]$$

$$[K] = EI \left[\begin{array}{cccccc|l} d_{1y} & \phi_1 & d_{2y} & \phi_2 & d_{3y} & \phi_3 & \\ 1.5 & 1.5 & -1.5 & 1.5 & 0 & 0 & d_{1y}=0 \\ 1.5 & 2 & -1.5 & 1 & 0 & 0 & \phi_1 \\ -1.5 & -1.5 & \begin{matrix}1.5\\+1.5\end{matrix} & \begin{matrix}-1.5\\+1.5\end{matrix} & -1.5 & 1.5 & d_{2y}=0 \\ 1.5 & 1 & \begin{matrix}-1.5\\+1.5\end{matrix} & \begin{matrix}2\\+2\end{matrix} & 1.5 & 1 & \phi_2=0 \\ 0 & 0 & -1.5 & -1.5 & 1.5 & -1.5 & d_{3y}=0 \\ 0 & 0 & 1.5 & 1 & -1.5 & 2 & \phi_3 \end{array}\right]$$

$$\begin{bmatrix} 10 \\ -10 \end{bmatrix} = EI \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_3 \end{bmatrix}$$

$\Rightarrow \phi_1 = \dfrac{5}{EI} \longrightarrow$ عكس عقارب الساعة

$\phi_3 = \dfrac{-5}{EI} \longrightarrow$ مع عقارب الساعة

$$\begin{bmatrix} F_{1y}' \\ M_1' \\ F_{2y}' \\ M_2' \\ F_{3y}' \\ M_3' \end{bmatrix} = EI \begin{bmatrix} 1.5 & 1.5 & -1.5 & 1.5 & 0 & 0 \\ 1.5 & 2 & -1.5 & 1 & 0 & 0 \\ 1.5 & -1.5 & 3 & 0 & -1.5 & 1.5 \\ 1.5 & 1 & 0 & 4 & -1.5 & 1 \\ 0 & 0 & -1.5 & -1.5 & 1.5 & -1.5 \\ 0 & 0 & 1.5 & 1 & -1.5 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 5/EI \\ 0 \\ 0 \\ -5/E \\ 0 \end{bmatrix}$$

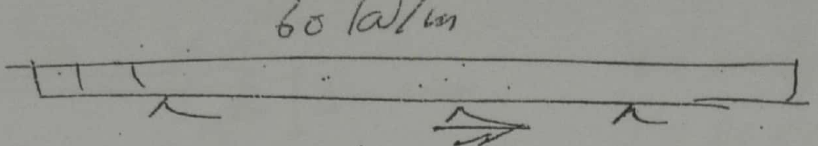
$$\begin{bmatrix} F_{1y}' \\ M_1' \\ F_{2y}' \\ M_2' \\ F_{3y}' \\ M_3' \end{bmatrix} = \begin{bmatrix} 7.5 \\ 10 \\ -15 \\ 0 \\ 7.5 \\ -10 \end{bmatrix}$$

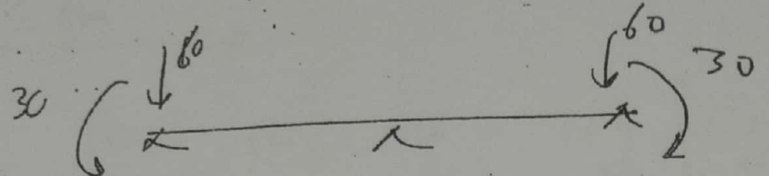
$$[F] = [F'] - [F_0]$$

$$\begin{bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{bmatrix} = \begin{bmatrix} 7.5 \\ 10 \\ -15 \\ 0 \\ 7.5 \\ -10 \end{bmatrix} - \begin{bmatrix} -120 \\ 10 \\ -120 \\ 0 \\ -120 \\ -10 \end{bmatrix} = \begin{bmatrix} 127.5 \\ 0 \\ 105 \\ 0 \\ 127.5 \\ 0 \end{bmatrix}$$

هذه الورقة هي زابيـ
لكن تحتوي حل nal force
لـ ment $\boxed{1}$

$\phi = 5/EI$

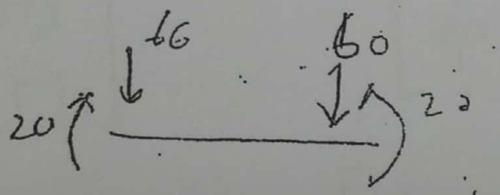




$$k_1 = EI \begin{array}{c|c} \begin{bmatrix} 1.5 & \boxed{1.5} & -1.5 & 1.5 \\ 1.5 & 2 & -1.5 & 1 \\ -1.5 & -1.5 & 1.5 & -1.5 \\ 1.5 & 1 & -1.5 & 2 \end{bmatrix} & \begin{matrix} d_{1y} = 0 \\ \phi_1 \\ d_{2y} = 0 \\ \phi_2 = 0 \end{matrix} \end{array}$$

$$\begin{bmatrix} f_1 \\ m_1 \\ f_2 \\ m_2 \end{bmatrix} = \underset{4\times4}{[k]} [d] - \begin{bmatrix} -60 \\ -20 \\ -60 \\ 20 \end{bmatrix}$$

$$\boxed{\begin{matrix} 127.5 \\ 105 \\ 127.5 \end{matrix}}$$

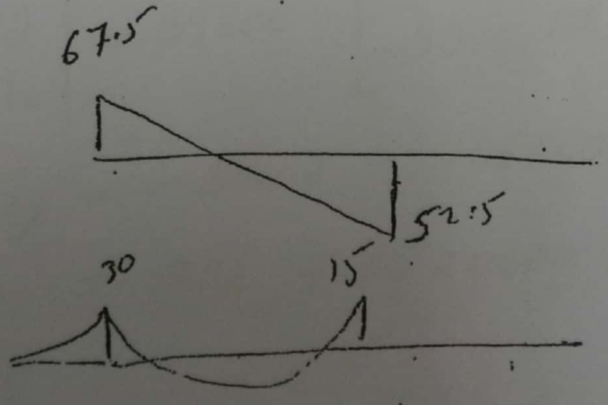


$f_1 = 1.5 * 5 + 60 = 67.5$

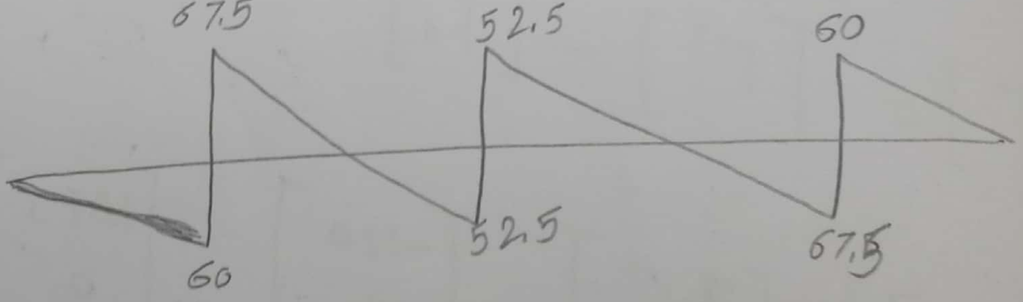
$m_1 = 30$

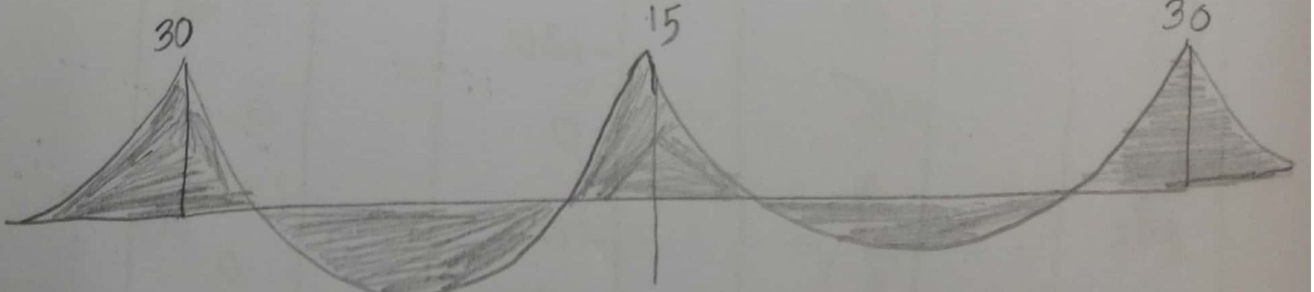
$f_2 = 52.5$

$m_2 = -15$

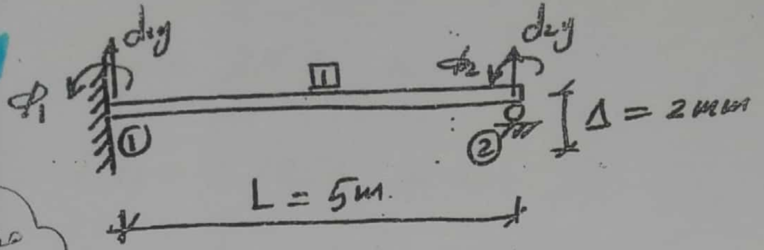


من السهل.
أن نقوم بحل
كل شيء وفق Local
من غير أن نبني Global
أي أن نجد internal force
ومن ثم نرسم M و shear
ونجد من خلالهم reaction


67.5
52.5
60
60
52.5
67.5


30
15
30

Ex



$E = 200 Gpa$

$I = 22 * 10^{-6}\ m^4$

عند المثال
$y \neq 0$
$y = 2mm$

Local
مع نسميها Global
هنا لانه element
واحد

فقط في هذا المثال

$$[k^{(1)}] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$[K] = 35200 \begin{array}{c} \begin{array}{cccc} d_{1y} & \phi_1 & d_{2y} & \phi_2 \end{array} \\ \begin{bmatrix} 12 & 30 & -12 & 30 \\ 30 & 100 & -30 & 50 \\ -12 & -30 & 12 & -30 \\ 30 & 50 & -30 & 100 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \end{bmatrix} = 35200 \begin{bmatrix} 12 & 30 & -12 & -30 \\ 30 & 100 & -30 & 50 \\ -12 & -30 & 12 & -30 \\ 30 & 50 & -30 & 100 \end{bmatrix} \begin{bmatrix} d_{1y} = 0 \\ \phi_1 = 0 \\ d_{2y} \\ \phi_2 \end{bmatrix}$$

$-0.002m$

هنا لا
سوى أن
معادلتين

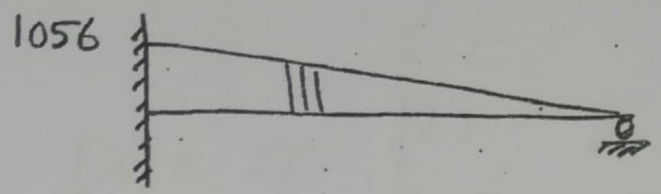
$$\begin{bmatrix} F_{2y} \\ 0 = M_2 \end{bmatrix} = 35200 \begin{bmatrix} 12 & -30 \\ -30 & 100 \end{bmatrix} \begin{bmatrix} -2 * 10^{-3} \\ \phi_2 \end{bmatrix}$$

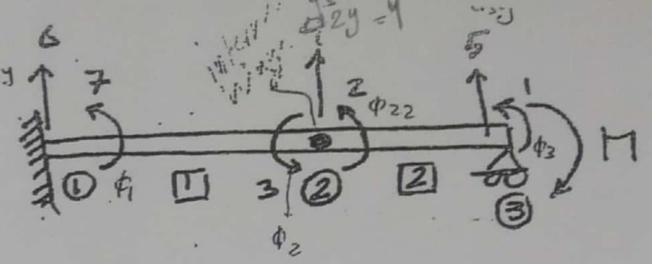
$\Rightarrow F_{2y} = -844.8 - 1056000\,\phi_2 \Rightarrow F_{2y} = -211.2\ N.$

$0 = 2112 + 3520000\,\phi_2 \Rightarrow \phi_2 = -6 * 10^{-4}$ Rad

$F_{1y} = 211.2$ N

$M_1 = 1056$ N·m





7 DoF

$d_{1y} = 0$

$\phi_1 = 0$

$d_{3y} = 0$

Member $\boxed{1}$

$$[k_1] = EI \begin{array}{c|cccc|c} & 6 & 7 & 4 & 3 & \\ \hline & 1.5 & & & & 6 \\ & 1.5 & 2 & \text{symm} & & 7 \\ & -1.5 & -1.5 & 1.5 & & 4 \\ & 1.5 & 1 & -1.5 & 2 & 3 \end{array}$$

Member $\boxed{2}$

$$[k_2] = EI \begin{array}{c|cccc|c} & 4 & 2 & 5 & 1 & \\ \hline & 12 & & & & 4 \\ & 6 & 4 & \text{symm} & & 2 \\ & -12 & -6 & 12 & & 5 \\ & 6 & 2 & -6 & 4 & 1 \end{array}$$

$$\begin{bmatrix} F_1 = -M \\ F_2 = 0 \\ F_3 = 0 \\ F_4 = 0 \\ F_5 = F_{y3} \ ? \\ F_6 = F_{y1} \ ? \\ F_7 = M_x \ ? \end{bmatrix} = EI \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & & & & & & \\ 2 & 4 & & & \text{symm} & & \\ 0 & 0 & 2 & & & & \\ 6 & 6 & -1.5 & 13.5 & & & \\ -6 & -6 & 0 & -12 & 12 & & \\ 0 & 0 & 1.5 & -1.5 & 0 & 1.5 & \\ 0 & 0 & 1 & -1.5 & 0 & 1.5 & 2 \end{array} \begin{bmatrix} D_1 = \phi \\ D_2 = \phi \\ D_3 = \phi \\ D_4 = d \\ D_5 = \\ D_6 = \\ D_7 = \end{bmatrix}$$

Scanned by CamScanner

$\Rightarrow$

$$\frac{-M}{EI} = 4D_1 + 2D_2 + 6D_4$$

$$0 = 2D_1 + 4D_2 + 6D_4$$

$$0 = 2D_3 - 1.5D_4$$

$$0 = 6D_1 + 6D_2 - 1.5D_3 + 13.5D_4$$

solving the above equations yields

$$D_1 = \frac{-3M}{EI} \qquad D_2 = \frac{-2.5M}{EI}$$

$$D_3 = \frac{2M}{EI} \qquad D_4 = \frac{2.667M}{EI}$$

$$\Rightarrow F_5 = -6EI\left(\frac{-3M}{EI}\right) - 6EI\left(\frac{-2.5M}{EI}\right) + 0 - 12EI\left(\frac{2.667M}{EI}\right)$$

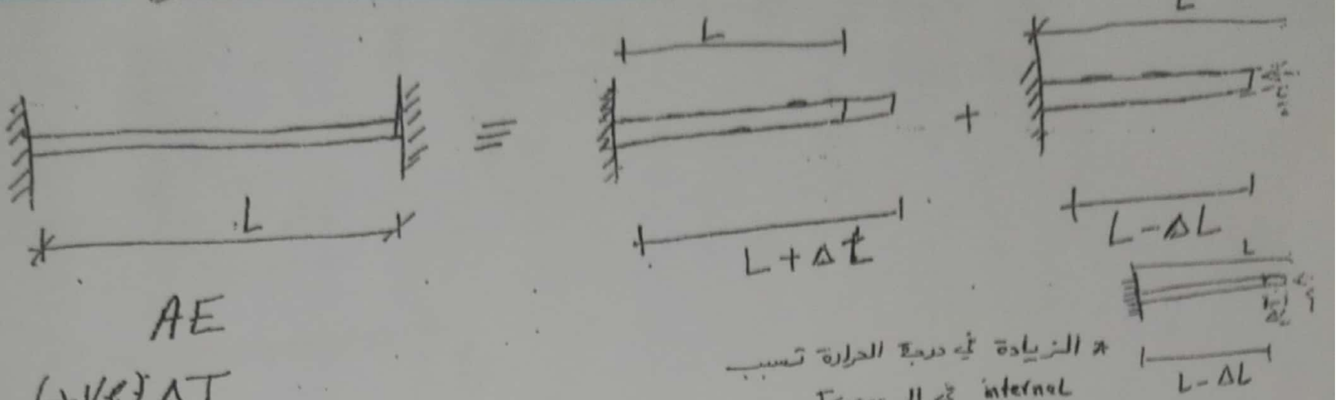
$$\Rightarrow F_5 = M$$

$$F_6 = -M$$

$$Q_7 = -2M$$

(14.8) Trusses Having Thermal changes and Fabricatio Errors

Trusses Having Thermal Changes & fabrication errors



AE

(+ve) ΔT

* الزيادة في درجة الحرارة تسبب internal forces في ال Truses

$$\boxed{\Delta L = \alpha \, \Delta T \, L}$$

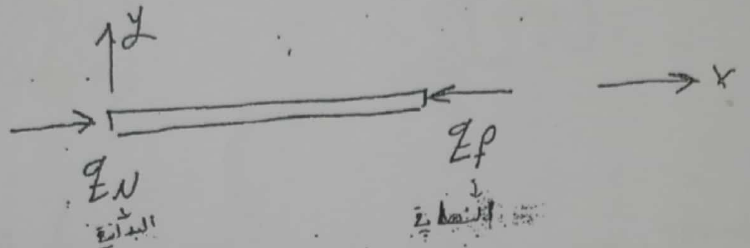
→ نحن نحصل ب ΔT وليس ب T

q

$$\boxed{\Delta L = \frac{q L}{AE}} \Rightarrow q = AE \frac{\Delta L}{L} = AE \, \alpha \, \Delta T \frac{L}{L}$$

(comp force)

$$\Rightarrow \boxed{(q_f)_0 = AE \, \alpha \, \Delta T}$$



$$\boxed{q_N = AE \, \alpha \, \Delta T}$$

$$\boxed{q_f = - AE \, \alpha \, \Delta T}$$

ال Forces اللي ظهرت بسبب تغير درجة الحرا

$$\begin{bmatrix} (Q_{Nx})_0 \\ (Q_{Ny})_0 \\ (Q_{Fx})_0 \\ (Q_{Fy})_0 \end{bmatrix} = AE \, \alpha \, \Delta T \begin{bmatrix} \lambda_x \\ \lambda_y \\ -\lambda_x \\ -\lambda_y \end{bmatrix}$$

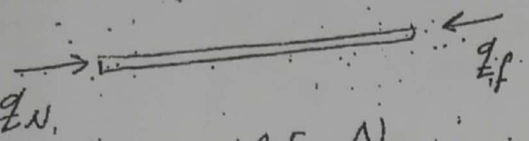
تمدد +ve

تقلص -ve

Fabrication Error $\longrightarrow$ in steel structure

$$\Delta L = \frac{ZL}{AE} \Rightarrow Z = \frac{AE\,\Delta L}{L}$$

increase by ΔL



$$(q_N)_0 = \frac{AE\,\Delta L}{L}$$

$$(q_f)_0 = -\frac{AE\,\Delta L}{L}$$

تحفظ

$$\Rightarrow \begin{bmatrix} (Q_{Nx})_0 \\ (Q_{Ny})_0 \\ (Q_{Fx})_0 \\ (Q_{Fy})_0 \end{bmatrix} = \frac{AE\Delta L}{L} \begin{bmatrix} \lambda_x \\ \lambda_y \\ -\lambda_x \\ -\lambda_y \end{bmatrix}$$

Matrix Analysis

$$\overset{F}{[Q]} = [K][D] + \overset{\nearrow}{[Q_0]}$$

(fabrication error + Thermal changes)
أو أحدهما حسب السؤال .

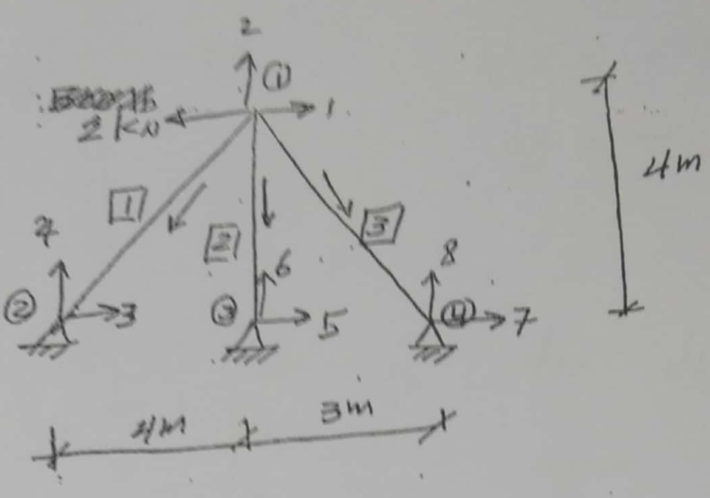
$$q_f = \frac{AE}{L}\begin{bmatrix} -\lambda_x & -\lambda_y & \lambda_x & \lambda_y \end{bmatrix}\begin{bmatrix} D_{Nx} \\ D_{Ny} \\ D_{Fx} \\ D_{Fy} \end{bmatrix} - (q_f)_0$$

Ex 14-6

** مهم **

(سؤال فاينل)

20/12



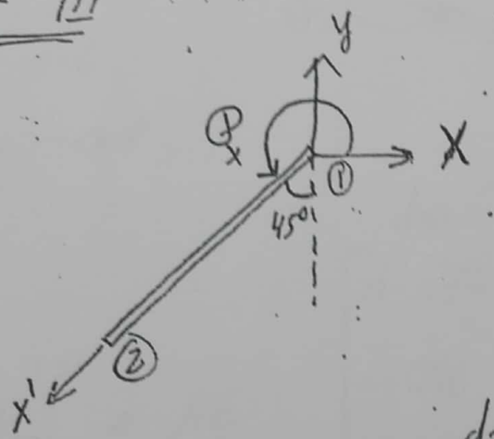
ΔT in member $\boxed{2} = +55^\circ C$

$\alpha = 11.7 * 10^{-6} / C^\circ$

$A = 1000 \text{ mm}^2$

$E = 200$ Gpa

member $\boxed{1}$



$\theta_x = 225^\circ$

$\lambda_x = \cos\theta_x = -0.707$

$\lambda_y = \sin\theta_x = -0.707$

$$[K]^{(1)} = AE \begin{array}{cccc} dx_1 & dy_1 & dx_2 & dy_2 \\ \left[\begin{array}{cccc} 0.08839 & & \text{symm} & \\ 0.08839 & 0.08839 & & \\ -0.08839 & -0.08839 & 0.08839 & \\ -0.08839 & -0.08839 & 0.08839 & 0.0883 \end{array}\right. \end{array}$$

$$\begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \\ F_{4x} \\ F_{4y} \end{bmatrix} = \begin{bmatrix} \\ \\ & K_{8x8} & \\ \\ \\ \end{bmatrix} \begin{bmatrix} d_{1x} = ? \\ d_{1y} = ? \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} +$$

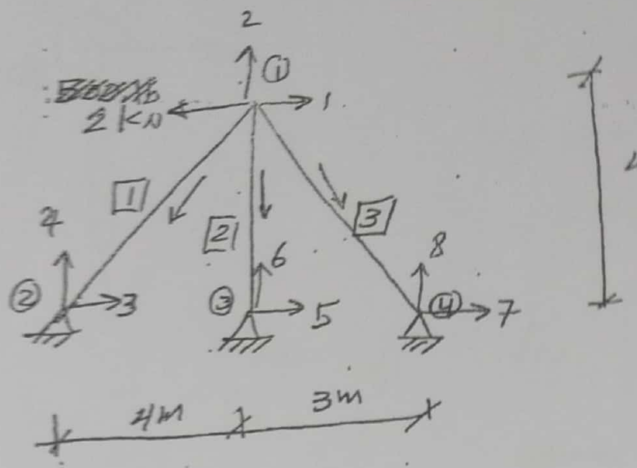
13-12

Ex 14-6

** مهم **

(سؤال فاينل)

20/12



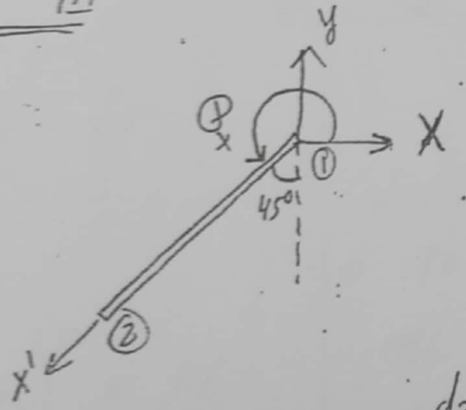
ΔT in member $\boxed{2} = +55^{\circ}C$

$\alpha = 11.7 * 10^{-6} / C^{\circ}$

$A = 1000\ mm^2$

$E = 200\ Gpa$

member $\boxed{1}$



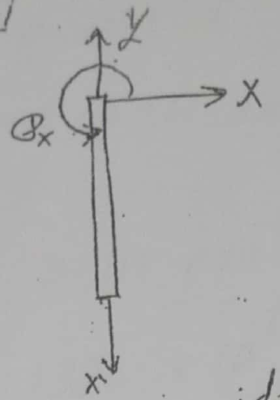
$\theta_x = 225^{\circ}$

$\lambda_x = \cos\theta_x = -0.707$

$\lambda_y = \sin\theta_x = -0.707$

$$[K]^{(1)} = AE \begin{array}{cccc} d_{x1} & d_{y1} & d_{x2} & d_{y2} \\ \left[\begin{matrix} 0.08839 & & \text{symm} & \\ 0.08839 & 0.08839 & & \\ -0.08839 & -0.08839 & 0.08839 & \\ -0.08839 & -0.08839 & 0.08839 & 0.0883 \end{matrix}\right. \end{array}$$

member $\boxed{2}$

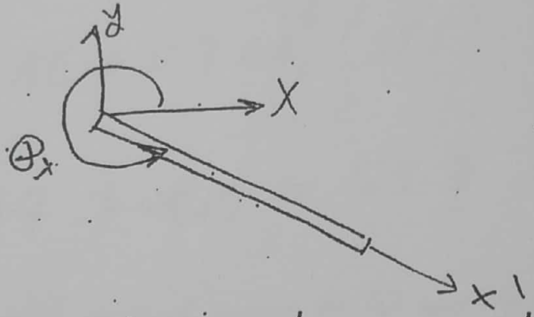


$\lambda_x = \cos 270 = 0$

$\lambda_y = -1$

$$[K]^{(2)} = AE \begin{array}{c} \begin{matrix} d_{x1} & d_{y1} & d_{3x} & d_{3y} \end{matrix} \\ \left[\begin{matrix} 0 & & \text{symm} & \\ 0 & 0.25 & & \\ 0 & 0 & 0 & \\ 0 & -0.25 & 0 & 0.25 \end{matrix}\right] \end{array}$$

member $\boxed{3}$



$\lambda_x = 0.6$

$\lambda_y = -0.8$

$$[K]^{(3)} = AE \begin{array}{c} \begin{matrix} d_{x1} & d_{y1} & d_{4x} & d_{4y} \end{matrix} \\ \left[\begin{matrix} 0.072 & & \text{Symm} & \\ -.096 & 0.128 & & \\ -.072 & 0.096 & 0.072 & \\ 0.096 & -0.128 & -0.096 & 0.128 \end{matrix}\right] \end{array}$$

$$\Rightarrow \quad -2 = 0.16039\,AE\,dx_1 + -0.00761\,AE\,dy_1 + 0$$

$$0 = (-0.00761\,dx_1 + 0.48639\,dy_1 - 643.5 * 10^{-6})\,AE$$

$$\Rightarrow dx_1 = 3.119 * 10^{-6} \text{ m}$$

$$dy_1 = 1380 * 10^{-6} \text{ m}$$

Force in member $\boxed{2}$

$$q_f = q_2 = \frac{AE}{L}\begin{bmatrix} 0 & 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} 3.119 * 10^{-6} \\ 1380 * 10^{-6} \\ 0 \\ 0 \end{bmatrix}$$

$$- AE * 11.7 * 10^{-6}\,(55)$$

$$= -59.7 \text{ kN} = 59.7 \text{ comp}$$

* إذا تغير حرارة 2 (members) فإننا نبقي Q لكل member ثم نجمعهم أو نطرحهم من بعض في المصفوفة النهائية مثل بالضبط مال (k).

$$[K] = AE \begin{array}{c} \\ \\ \end{array}\left[\begin{array}{cccccccc} d_{x1} & d_{y1} & d_{x2} & d_{y2} & d_{x3} & d_{y3} & d_{x4} & d_{y4} \\ 0.16039 & & & & & & & \\ -0.00761 & 0.46639 & & & \text{symm} & & & \\ -0.08839 & -0.08839 & 0.08839 & & & & & \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 & & & & \\ 0 & 0 & 0 & 0 & 0 & & & \\ 0 & -0.25 & 0 & 0 & 0 & 0.25 & & \\ -0.072 & 0.096 & 0 & 0 & 0 & 0 & 0.072 & \\ 0.096 & -.128 & 0 & 0 & 0 & 0 & -.096 & \end{array}\right]$$

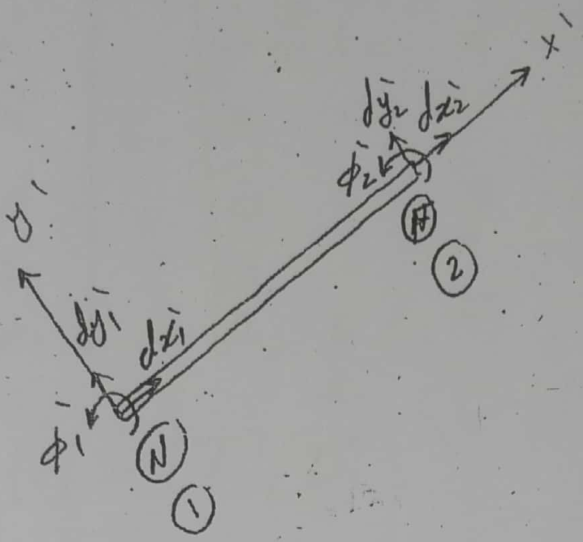
$$\begin{bmatrix} (Q_{1x})_0 \\ (Q_{2y})_0 \\ (Q_{5x})_0 \\ (Q_{6y})_0 \end{bmatrix} = \frac{AE\,(11.7)(10^{-6})(+55)}{643.5\times10^{-6}\ AE}\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} = AE\,(10^{-6})\begin{bmatrix} 0 \\ -643.5 \\ 0 \\ 643.5 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 0 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{bmatrix} = [K]_{8\times8}\begin{bmatrix} d_{x1} \\ d_{y1} \\ d_{x2}=0 \\ d_{y2}=0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + AE(10^{-6})\begin{bmatrix} 0 \\ -643.5 \\ 0 \\ 0 \\ 0 \\ 643.5 \\ 0 \\ 0 \end{bmatrix}\begin{array}{c} Q_{1x} \\ Q_{1y} \\ \vdots \\ \\ \\ \\ \\ \\ \end{array}$$

فرضنا لو في تغير
في الحرارة بكل ال Trusses

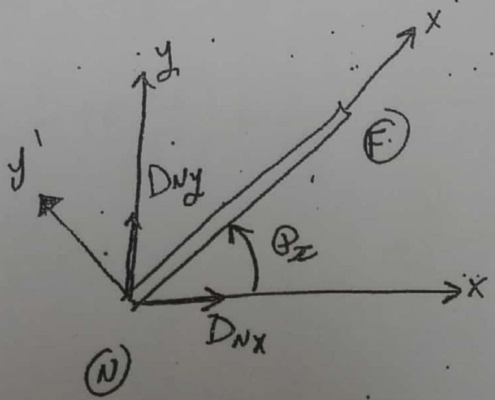
Plane Frame Analysis Using the Stiffness Method

Beam + Truss = Frame



$$[K'] = \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}\left[\begin{array}{cccccc} d_{x1}' & d_{y1}' & \phi_1' & d_{x2}' & d_{y2}' & \phi_2' \\ \frac{AE}{L} & & & & & \\ 0 & \frac{12EI}{L^3} & & & \text{symm} & \\ 0 & 6EI/L^2 & 4EI/L & & & \\ -\frac{AE}{L} & 0 & 0 & AE/L & & \\ 0 & -\frac{12EI}{L^3} & -6EI/L^2 & 0 & 12EI/L^3 & \\ 0 & 6EI/L^2 & 2EI/L & 0 & -6EI/L^2 & 4EI/L \end{array}\right.$$

في هذا الموضوع
فقط . اطلع على
صفحة 44 .



$$\begin{bmatrix} dx_1' \\ dy_1' \\ \phi_1' \\ dx_2' \\ dy_2' \\ \phi_2' \end{bmatrix} = \begin{bmatrix} \lambda_x & \lambda_y & 0 & 0 & 0 & 0 \\ -\lambda_y & \lambda_x & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_x & \lambda_y & 0 \\ 0 & 0 & 0 & -\lambda_y & \lambda_x & 1 \\ 0 & 0 & 0 & & & \end{bmatrix} \begin{bmatrix} \cancel{Dx_1}\ Dx_1 \\ Dy_1 \\ \phi_1 \\ Dx_2 \\ Dy_2 \\ \phi_2 \end{bmatrix}$$

$$[\hat{d}] = [T][D]$$

$$[K] = [T]^T[\hat{k}][T]$$

$$[K] = \left[\begin{array}{cccccc} \frac{AE}{L}\lambda_x^2 + \frac{12EI}{L^3}\lambda_y^2 & - & - & - & - & - \\ \left(\frac{AE}{L} - \frac{12EI}{L^3}\right)\lambda_x\lambda_y & - & - & - & - & - \\ -\frac{6EI}{L^2}\lambda_y & - & - & - & - & - \\ -\left(\frac{AE}{L}\lambda_x^2 + \frac{12EI}{L^3}\lambda_y^2\right) & - & - & - & - & - \\ -\left(\frac{AE}{L} - \frac{12EI}{L^3}\right)\lambda_x\lambda_y & - & - & - & - & - \\ -\frac{6EI}{L^2}\lambda_y & - & - & - & - & - \end{array}\right]_{6\times 6} \begin{array}{ll} \cancel{dx_1} & dx_1 \\ \cancel{dy_1} & dy_1 \\ \cancel{\phi_1} & \phi_1 \\ \cancel{dx_2} & dx_2 \\ \cancel{dy_2} & dy_2 \\ & \phi_2 \end{array}$$

(page 557)

$$\begin{bmatrix} F_{1x} \\ F_{1y} \\ M_1 \\ F_{2x} \\ F_{2y} \\ M_2 \end{bmatrix} = [k]_{6\times6} \begin{bmatrix} d_{1x} \\ d_{1y} \\ \phi_1 \\ d_{2x} \\ d_{2y} \\ \phi_2 \end{bmatrix}$$

element forces

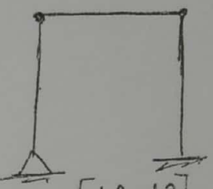
$$[\hat{f}] = [\hat{k}][\hat{d}]$$

$D_x, d_y, \emptyset$ كل نقطة عليها ٣ حركات •

in 3D frames all nodes has 6 DoF [3 motion and 3 rotation and 6 Forces]

Ex

$K_{Global} = [9\times9]$



$K_{Global} = [12\times12]$

$K_{Local} = [6\times6]$

ملاحظات :

(أي أن (Local coord.) هي: size 6x6) rotation، axial، shear ← (بيشيل) frame ①

L، I، E، A ← (بتأثر)

shear، rotation ← (بيشيل) Beam أما

axial، shear ← (بيشيل) Truss

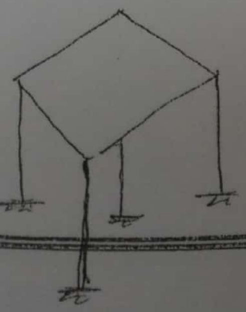
(size 6x6) frame في (Local coord) فإن 2D إذا تحدثنا عن ②

(size 12x12) 3D أما

لأن هناك ثلاث حركات على (z، y، x)

وثلاث دوران على (z، y، x)

Ex

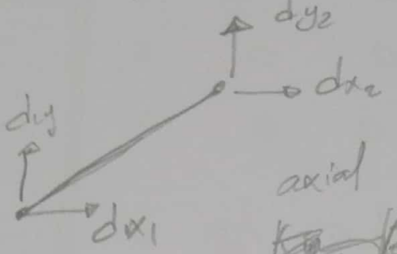


→ Local coord. (12x12)

Global coord. (48x48) → $\underset{joint}{8} \times \underset{Dof}{6} = 48$

Truss

$\frac{AE}{L}$

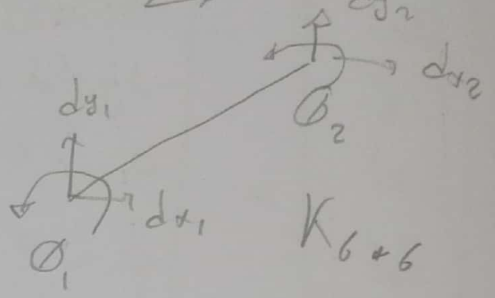


axial

K_{4*4}

Frame

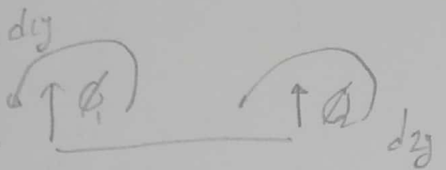
2D



K_{6*6}

E, I, A, L

EI, L

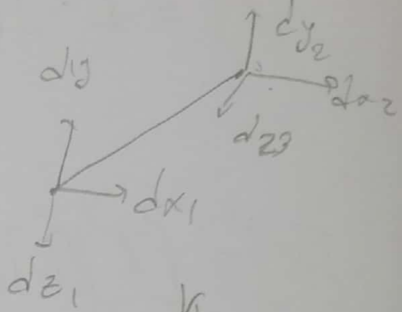


shear
moment

K_{4*4}

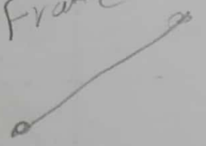
3D

truss



K_{6*6}

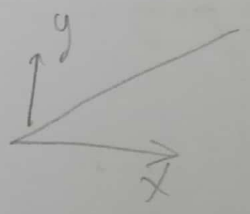
Frame 3D



$K = 12*12$

d_{x1}	d_{x2}
d_{y1}	d_{y2}
d_{z1}	d_{z2}
ϕ_{1x}	ϕ_{2x}
ϕ_{1y}	ϕ_{2y}
ϕ_{1z}	ϕ_{2z}

Frame 2D



d_{x1} = ✓

d_{y1} = ✓

$d_{z3} = 0$

$\phi_x = 0$

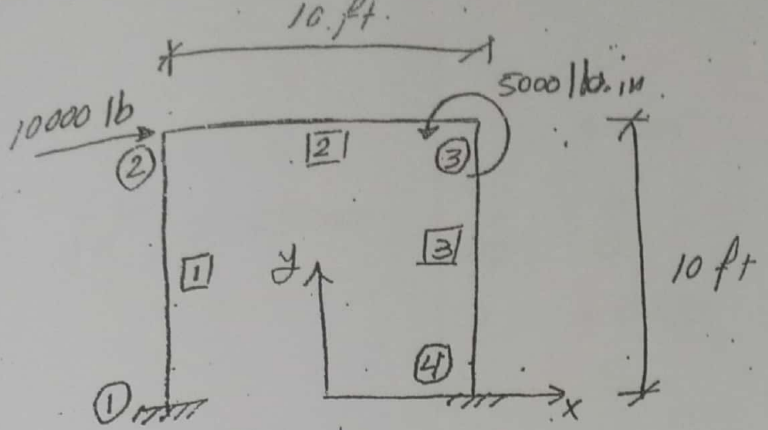
$\phi_y = 0$

ϕ_z = ✓

$$\begin{bmatrix} 10000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 5000 \end{bmatrix} = 250000 \begin{bmatrix} 10.167 & & & & & \\ 0 & 10.0835 & & & & \\ 10 & 5 & 1200 & & & \\ -10 & 0 & 0 & 10.167 & & \\ 0 & 0.0835 & -5 & 0 & 10.0835 & -5 \\ 0 & 5 & 200 & 10 & -5 & 120 \end{bmatrix}$$

$$\times \begin{bmatrix} d_{2x} \\ d_{2y} \\ \phi_2 \\ d_{3x} \\ d_{3y} \\ \phi_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} d_{2x} \\ d_{2y} \\ \phi_2 \\ d_{3x} \\ d_{3y} \\ \phi_3 \end{bmatrix} = \begin{bmatrix} 0.211 \text{ in} \\ 0.00148 \text{ in} \\ -0.00153 \text{ rad} \\ 0.209 \text{ in} \\ -0.00148 \text{ in} \\ -0.00149 \text{ rad} \end{bmatrix}$$



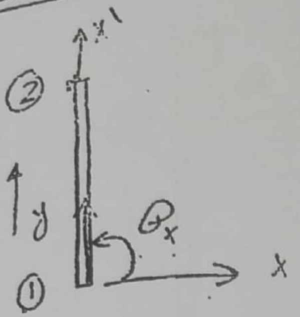
$E = 30 * 10^6$ psi

$A = 10\ in^2$

$I = 200\ in^4$ for element $\boxed{1}$ & $\boxed{3}$

$I = 100\ in^4$ for element $\boxed{2}$

Element $\boxed{1}$

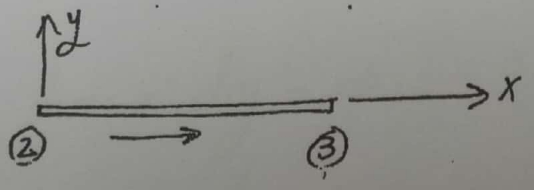


$\theta_x = 90^\circ \Rightarrow \lambda_x = 0$

$\lambda_y = \sin 90 = 1$

$$[K^{(1)}] = 250000 \begin{array}{cccccc|c} d_{1x} & d_{1y} & \phi_1 & d_{2x} & d_{2y} & \phi_2 & \\ 0.167 & 0 & -10 & -0.167 & 0 & -10 & d_{1x} \\ 0 & 10 & 0 & 0 & -10 & 0 & d_{1y} \\ -10 & 0 & 800 & 10 & 0 & 400 & \phi_1 \\ -0.167 & 0 & 10 & 0.167 & 0 & 10 & d_{2x} \\ 0 & -10 & 0 & 0 & 10 & 0 & d_{2y} \\ 10 & 0 & 400 & 10 & 0 & 800 & \phi \end{array}$$

Element $\boxed{2}$

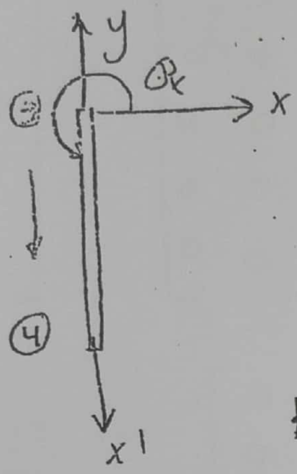


$\theta_x = 0 \Rightarrow \lambda_x = 1$

$\lambda_y = 0$

$$[K^{(2)}] = 250000 \left[\begin{array}{cccccc} 10 & 0 & 0 & -10 & 0 & 0 \\ 0 & 0.0835 & 5 & 0 & -0.0835 & 5 \\ 0 & 5 & 400 & 0 & -5 & 200 \\ -10 & 0 & 0 & 10 & 0 & 0 \\ 0 & -0.0835 & -5 & 0 & 0.0835 & -5 \\ 0 & 5 & 200 & 0 & -5 & 400 \end{array}\right] \begin{array}{c} d_{2x} \\ d_{2y} \\ \phi_2 \\ d_{3x} \\ d_{3y} \\ \phi_3 \end{array}$$

Element $\boxed{3}$



$\theta_x = 270°$

$\lambda x = 0$

$\lambda y = -1$

$$[K^{(3)}] = 250000 \left[\begin{array}{cccccc} 0.167 & & & & \text{symm} & \\ 0 & 10 & & & & \\ 10 & 0 & 800 & & & \\ -0.167 & 0 & -10 & 0.167 & & \\ 0 & -10 & 0 & 0 & 10 & \\ 10 & 0 & 400 & -10 & 0 & 800 \end{array}\right] \begin{array}{c} d_{3} \\ d_{3} \\ \phi \\ d_{4x} \\ d_{4y} \\ \phi \end{array}$$

$d_{1x} = d_{1y} = \phi_1 = 0$

$d_{4x} = d_{4y} = \phi_4 = 0$

element forces

element $\boxed{1}$.

$$[\hat{d}] = [T][D]$$

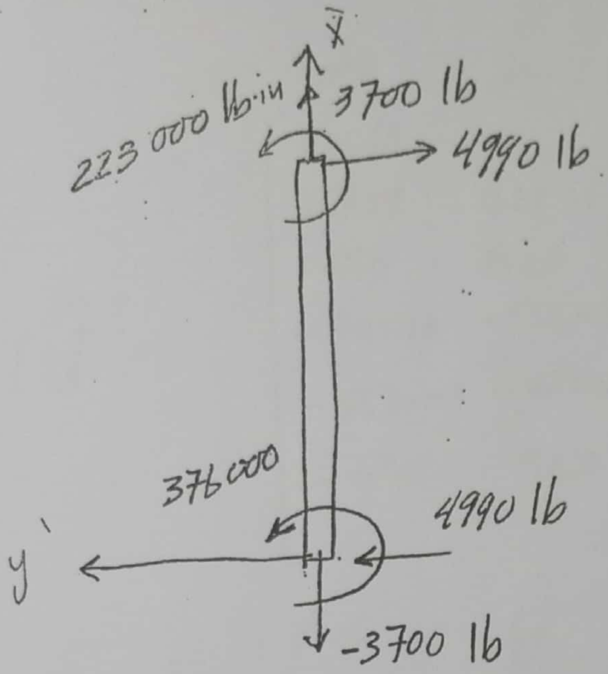
$$\begin{bmatrix} dx_1' \\ dy_1' \\ \phi_1 \\ dx_2' \\ dy_2' \\ \phi_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d_{1x} = 0 \\ d_{1y} = 0 \\ \phi_1 = 0 \\ d_{2x} = 0.211 \\ d_{2y} = 0.00148 \\ \phi_2 = -0.00153 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.00148 \\ -0.211 \\ -0.00153 \end{bmatrix}$$

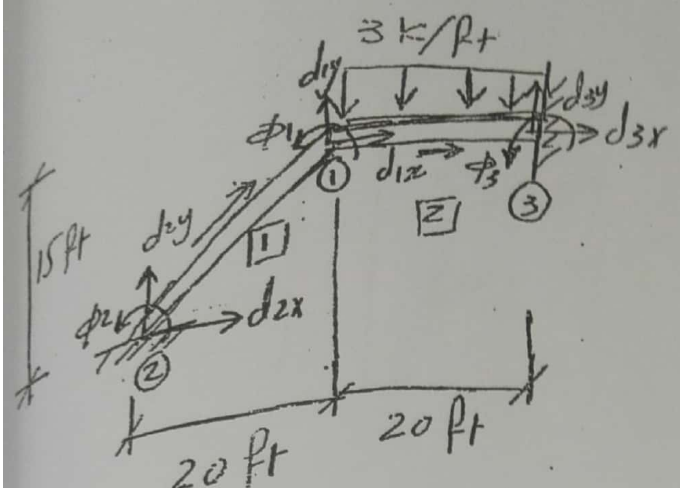
$$[\hat{P}] = [\hat{k}][\hat{d}]$$

$$= 250000 \begin{bmatrix} 10 & 0 & 0 & -10 & 0 & 0 \\ 0 & 0.167 & 10 & 0 & -0.167 & 10 \\ 0 & 10 & 800 & 0 & -10 & 400 \\ -10 & 0 & 0 & 10 & 0 & 0 \\ 0 & -0.167 & -10 & 0 & 0.167 & -10 \\ 0 & 10 & 400 & 0 & -10 & 800 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.0011 \\ -0.211 \\ -0.00\text{[illegible]} \end{bmatrix}$$

$$\begin{bmatrix} \hat{f}_{1x} \\ \hat{f}_{1y} \\ \hat{m}_1 \\ \hat{f}_{2x} \\ \hat{f}_{2y} \\ \hat{m}_2 \end{bmatrix} = \begin{bmatrix} -3700 \text{ lb} \\ 4990 \text{ lb} \\ 376\,000 \text{ lb·in.} \\ 3700 \text{ lb} \\ -4990 \text{ lb} \\ 223\,000 \text{ lb·in} \end{bmatrix}$$



Ex 15-2



$I = 600 \text{ in}^4$

$A = 12 \text{ in}^2$

$E = 29 \times 10^3$ ksi

$d_{2x} = d_{2y} = \phi_2 = 0$

$d_{3x} = d_{3y} = \phi_3 = 0$

$\lambda_x = 0.8 \qquad \lambda_y = 0.6$

$$[k^{(1)}] = \begin{array}{c} \begin{array}{cccccc} d_{2x} & d_{2y} & \phi_2 & d_{1x} & d_{1y} & \phi_1 \end{array} \\ \left[\begin{array}{cccccc} 745.18 & & & & & \\ 553.09 & 422.55 & & & \text{symm} & \\ -696 & 928 & 232000 & & & \\ -745.18 & -553.09 & 886 & 745.18 & & \\ -553.09 & -422.55 & -928 & 553.09 & 422.55 & \\ -696 & 928 & 116000 & 696 & -928 & 232000 \end{array}\right] \end{array}$$

$$[k^{(2)}] = \begin{array}{c} \begin{array}{cccccc} d_{1x} & d_{1y} & \phi_1 & d_{3x} & d_{3y} & \phi_3 \end{array} \\ \left[\begin{array}{cccccc} 1450 & & & & & \\ 0 & 15.1 & & & \text{symm} & \\ 0 & 1812.5 & 1812.5 & & & \\ -1450 & 0 & 290000 & - & - & - \\ & & 0 & & & \\ 0 & -15.1 & & - & - & - \\ 0 & 1812.5 & & & & \end{array}\right] \end{array}$$

$$\begin{bmatrix} 0 \\ -30 \\ -1200 \end{bmatrix} = \begin{bmatrix} 2195.18 & & \\ 553.09 & 437.65 & \\ 696 & 884.5 & 522000 \end{bmatrix} \begin{bmatrix} d_{1x} \\ d_{1y} \\ \phi_1 \end{bmatrix}$$

$\Rightarrow$ $d_{1x} = 0.0247$ in

$d_{1y} = -0.0945$ in

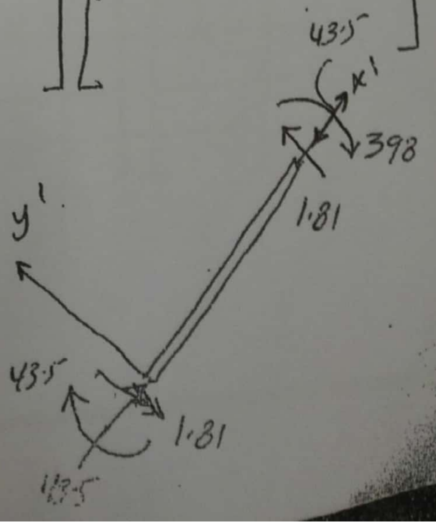
$\phi_1 = -0.00217$ rad.

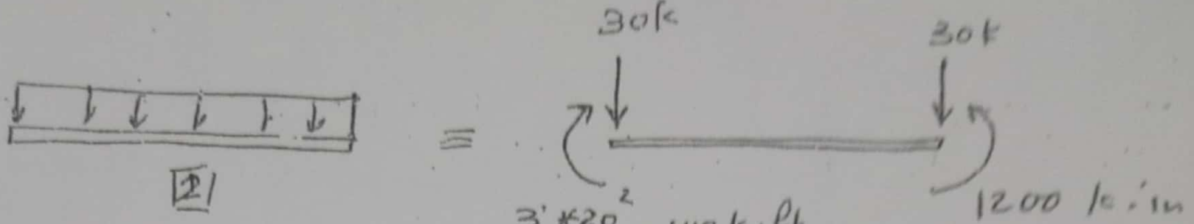
internal force in member $\boxed{1}$

$$[q] = [k_1'][T_1][D]$$
$$= [k_1'][d']$$

$$\begin{bmatrix} q_{2x} \\ q_{2y} \\ m_2 \\ q_{1x} \\ q_{1y} \\ m_1 \end{bmatrix} = \begin{bmatrix} 1160 & & \\ 0 & \cdots & \\ 0 & & \\ -1160 & \cdots & \\ 0 & & \\ 0 & \cdots & \end{bmatrix} \begin{bmatrix} 0.8 & 0.6 & 0 & 0 & 0 & 0 \\ -0.6 & & & & & \\ \vdots & & & & & \\ \vdots & & & & & \end{bmatrix} [\;]$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.0247 \\ -0.0954 \\ -0.00217 \end{bmatrix} = \begin{bmatrix} 43.5 \\ -1.81 \\ -146 \\ -43.5 \\ 1.81 \\ -398 \end{bmatrix}$$





$$\frac{3' * 20^2}{12} = 100\ k.ft$$
$$= 1200\ k.in$$

$$[K] = \begin{array}{c} d_{1x} \quad d_{1y} \quad \phi_1 \quad \cdots \quad d_{3x} \quad d_{3y} \quad \phi_3 \\ \left[\quad \cancel{\phi}\ 9 \times 9 \quad \right] \end{array}$$

$$[Q] = [K][D]$$

$$\begin{bmatrix} 0 \\ -30 \\ -1200 \\ \cancel{\phi_i} \\ \vdots \\ \vdots \\ \vdots \end{bmatrix} = \begin{bmatrix} \quad 9 \times 9 \quad \end{bmatrix} \begin{bmatrix} d_{1x} \\ d_{1y} \\ \phi_1 \\ \cancel{d_{2x}} \\ \cancel{d_{2y}} \\ \vdots \\ \vdots \end{bmatrix}$$