اللجنة الأكاديمية للهندسة المدنية

دفتر

تحليل إنشائي

رهف عيد

Contact us :

Civilittee HU | لجنة المدني

Civilittee Hashemite

www.civilittee-hu.com

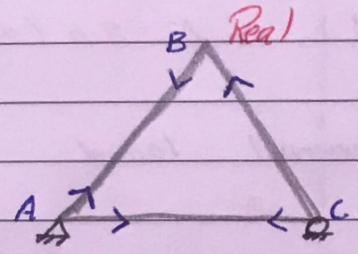
بداية مادة لسكند !!

13/2/2019

Virtual work Method

- frames
- trusses ✓

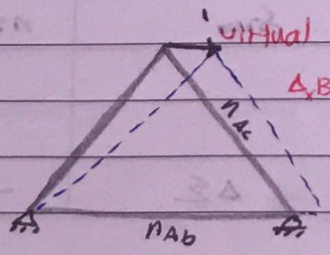
$$\Delta_\theta = \int \frac{Mm}{EI}\, d\theta$$



Normal force in ← N ±
member due to
External load

$$\sigma = E\epsilon$$

$$\frac{N}{A} = E\frac{\delta}{L} \Rightarrow \delta = \frac{NL}{EA}$$



n: normal force in member due to unit load

$$(1)(\Delta) = \Sigma n_i \delta_i$$

$$\Delta = \frac{n_i N_i l_i}{EI}$$

Five Apple

δ in member

δ_N : Due to external load (N)

δ_T : Due to Tempreture : $\alpha \Delta T L \Rightarrow \Delta = \Sigma n (\alpha \Delta T L)$

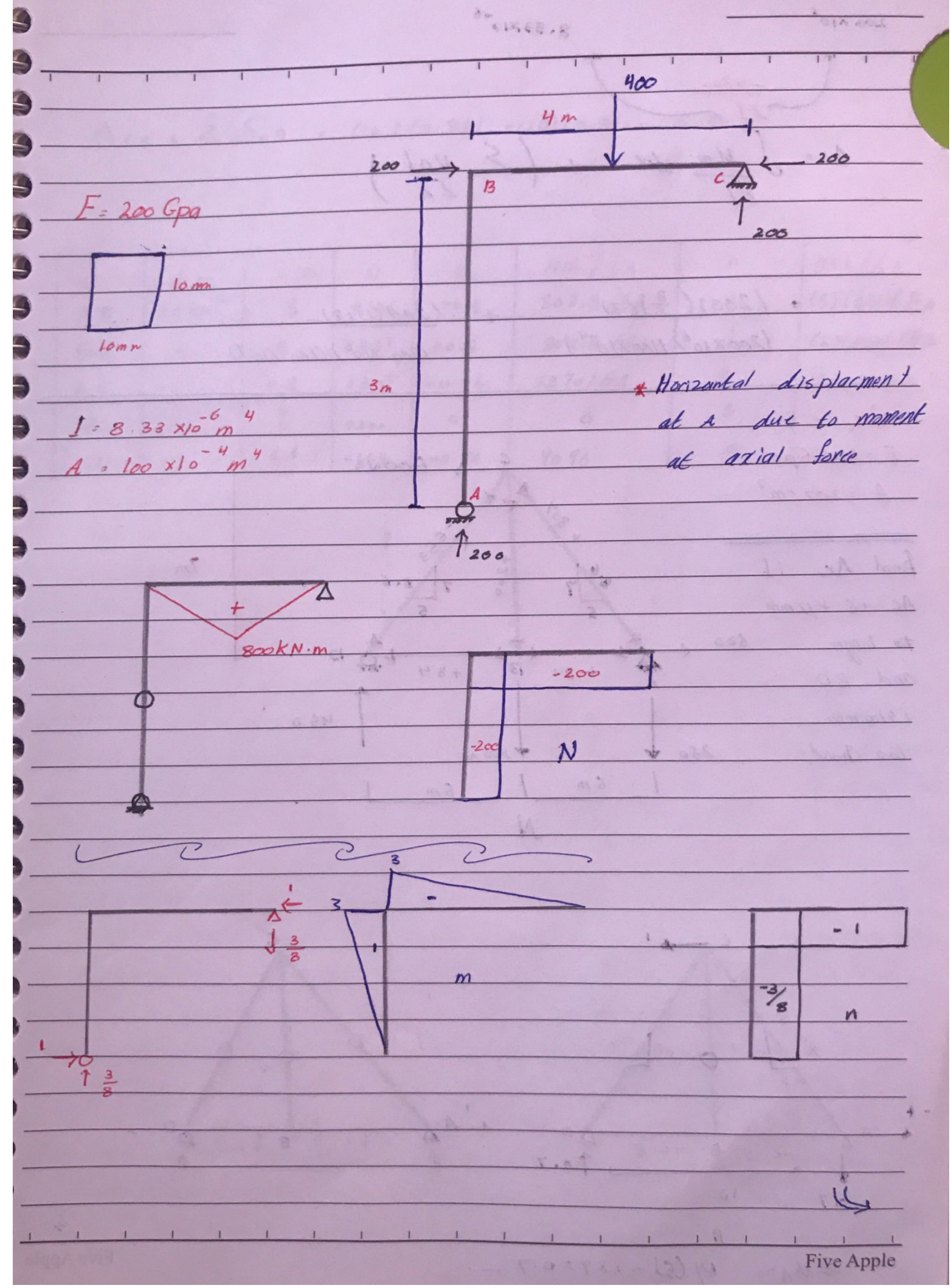
δ_e : Due to fabrication due to external load

$\Delta = \Sigma \delta_{ein}$

member	Li	Ai	N	n	NNL/EA
AB					
BC					
AC				$\Delta\Sigma$	—

member	L	A	n	δ_{tem}	ns	e
AB						
BC						
AC				$\Delta\Sigma$	—	

Five Apple



200×10^6 8.33×10^{-6}

$\frac{-4800}{EI}$

$$\Delta = \int \frac{Mm}{EI} dx + \left(\frac{\sum NnL}{EA} \right)$$

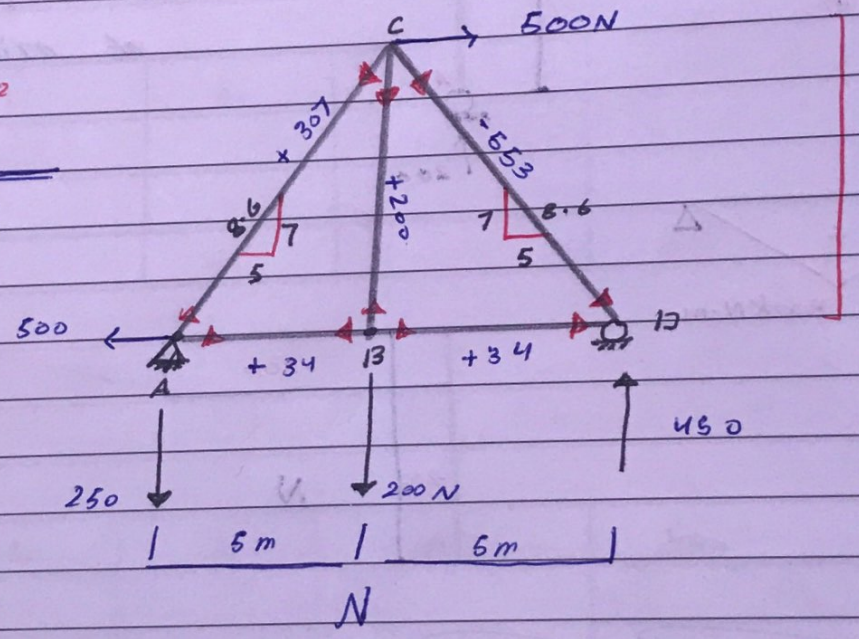
$$= \frac{(200)\left(\frac{3}{8}\right)(3)}{(200 \times 10^6)(1100 \times 10^{-4})} + \frac{(200)(1)(8)}{(200 \times 10^6)(160 \times 10^{-4})}$$

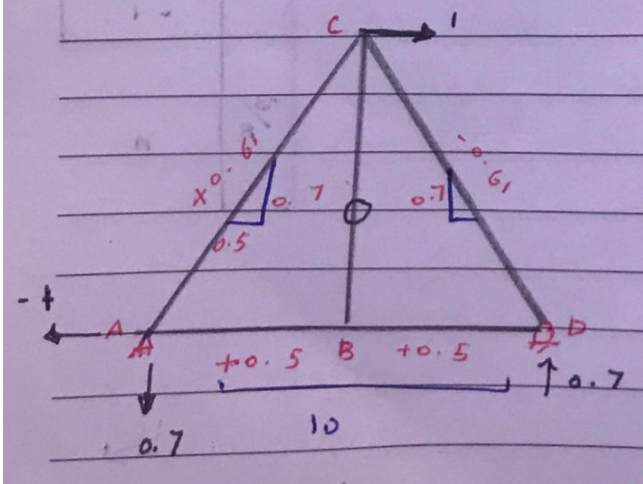
$Ax_c = ?$

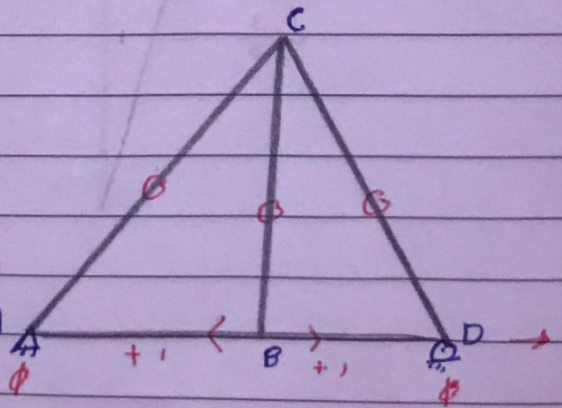
$E = 200$ Gpa

$A = 200\ cm^2$

Find Δx_c if Ac is +12 mm too longer and BD is 16 mm too short.







$\Sigma M_A = 0 \quad D_y(5) - 1 \times 7 = 0.7 \ldots$

Five Appl

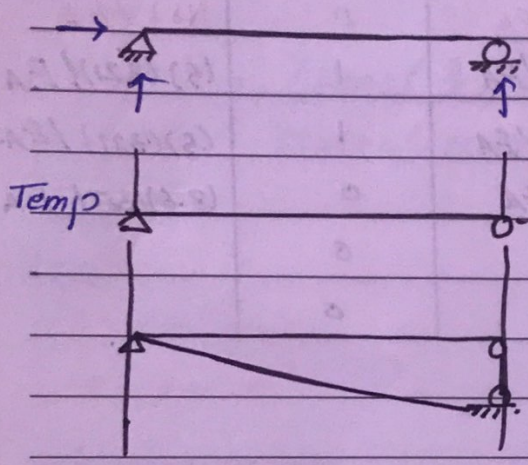
$$\Delta_{Cx} = \Sigma S_{en} = (12)(0.86) - (10)(0.5) = 5.32$$

member	A m^4	L (m)	N	n_c	NnL / EA	n	NnL / EA
AB	20×10^{-4}	5	+321	+0.5	802.5 / EA	1	(5)(321) / EA
BD	"	5	+321	+0.5	802.5 / EA	1	(5)(321) / EA
AC		8.6	+307	+0.86	2270 / EA	0	
BC		7	+200	0	0	0	
CD		8.6	-553	-0.86	4096	0	

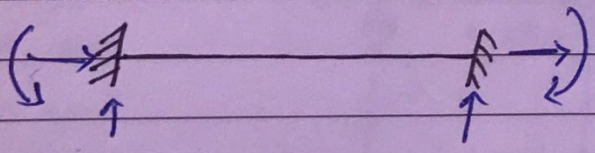
25/2/2019

10 Indeterminate structures

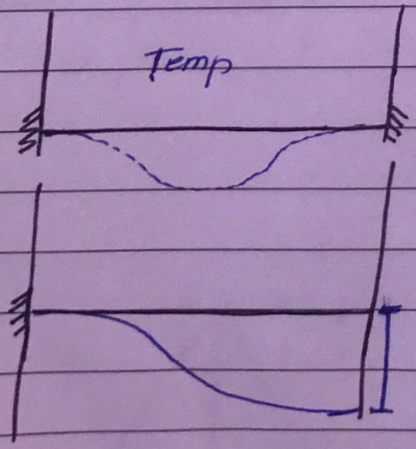
- force Method
- unit load method.

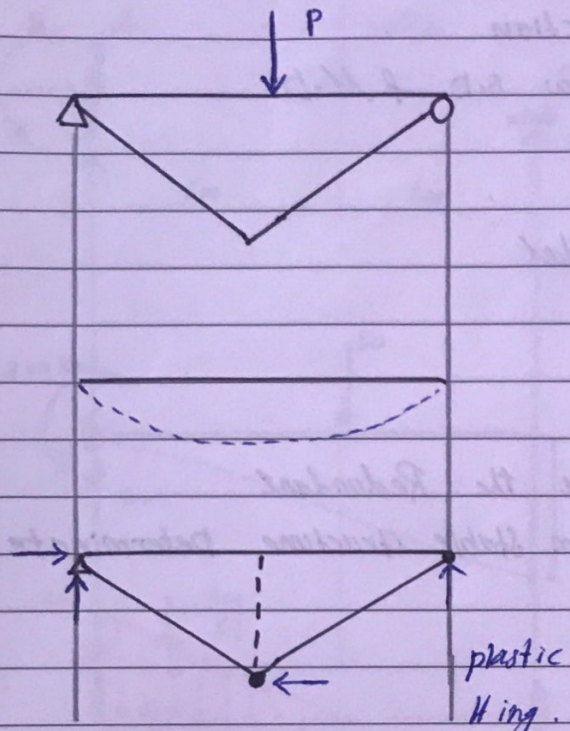


Determinate
equ. equ

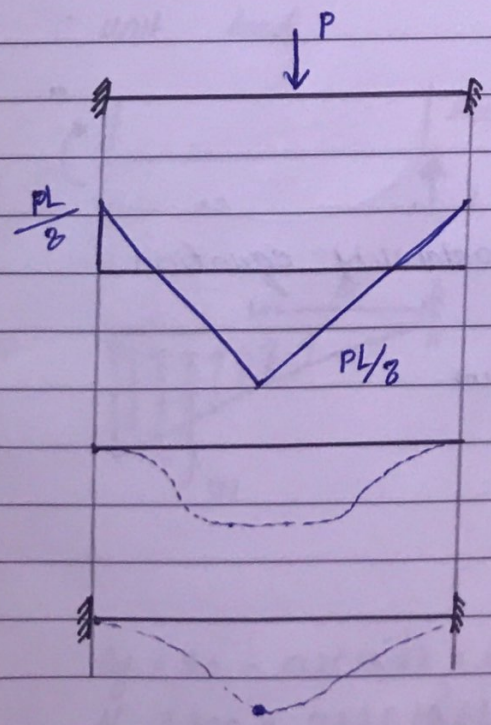


Indeterminate
unknowns > equ equ
use compatability equ





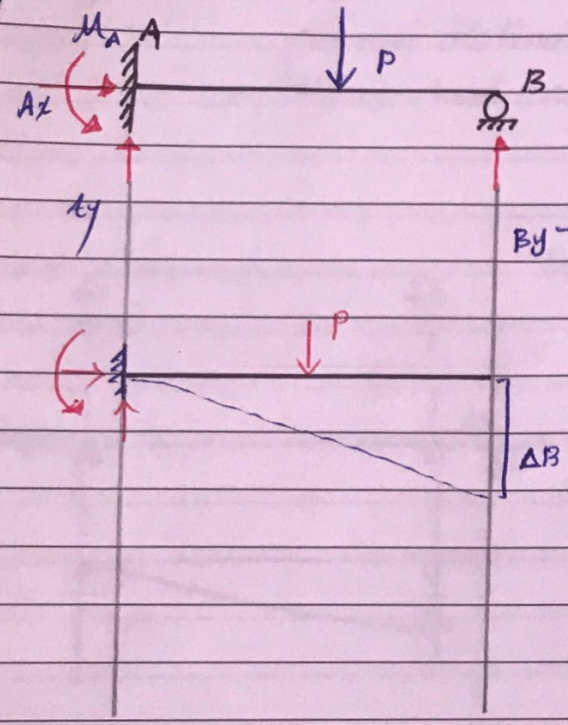
unstable



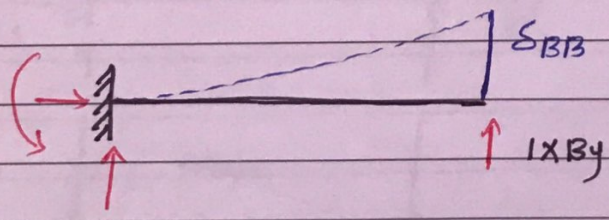
stable

Reaction

Draw S.D & M.D



Remove the Redundant from stable structure determinate



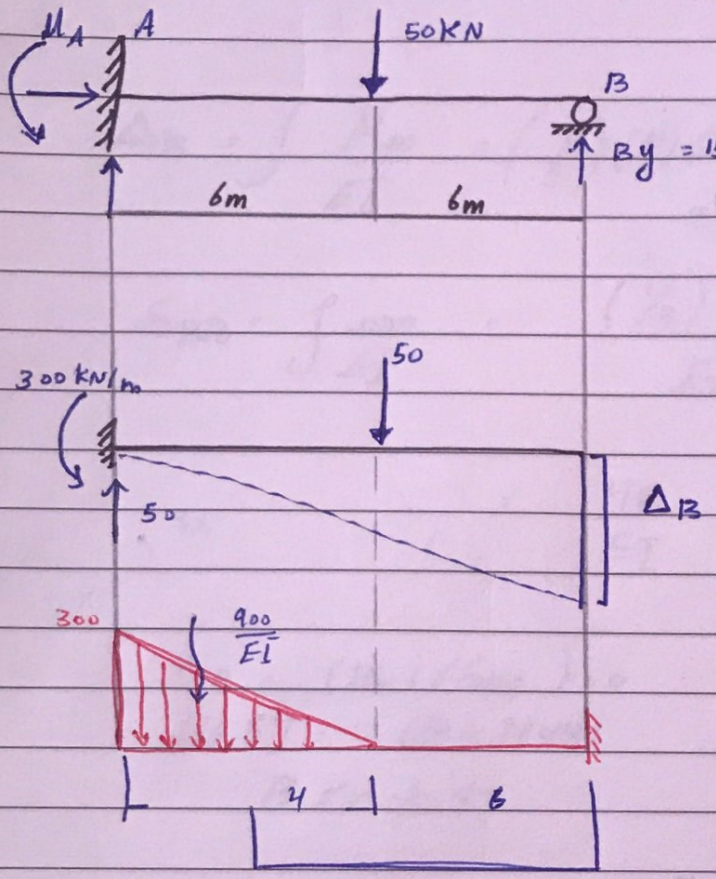
$\Delta_B + B_y \delta_{BB} = 0$: compatability equation

By => kmw => Apply equilibrium equation

$Reaction = R_0 + B_y R_1$

$M_{actual} = M_0 + B_y M_1$

Five Appl



*using equ equ

Reaction

$\uparrow \Sigma F_y = 0$

$Ay - 50 + 15.6 = 0$

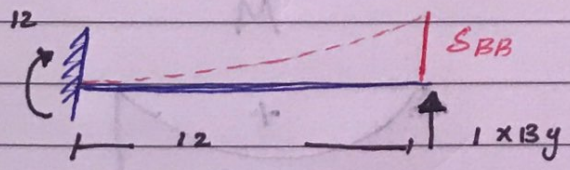
$Ay = 34.4$

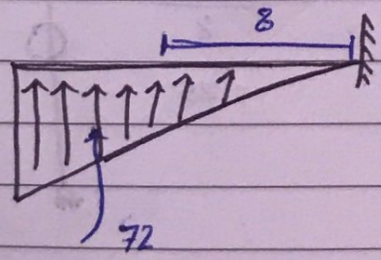
$\circlearrowleft + \Sigma M = 0$

$M_A - 50(6) + (15.6)(12) = 0$

$M_A = 112.8$

unit load





$\Delta_B = \frac{9000}{EI}$

$\delta_{BB} = \frac{576}{EI}$

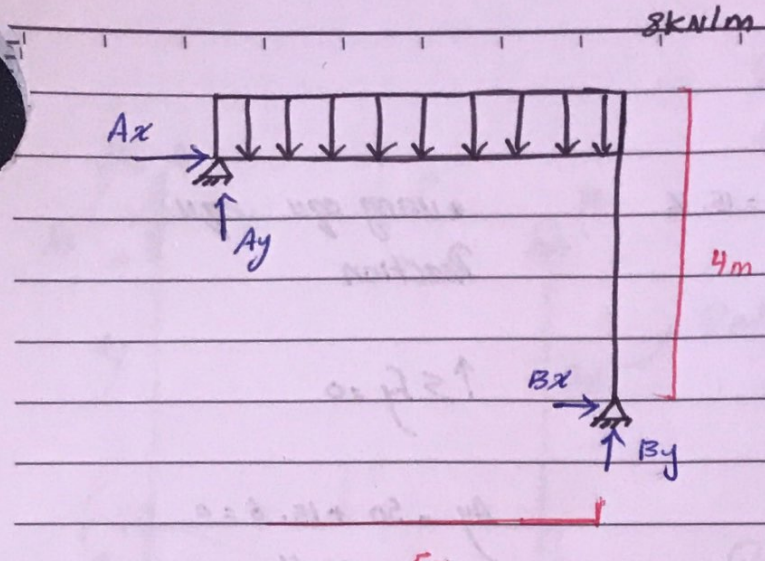
$\Delta_B + By\,\delta_{BB} = 0$

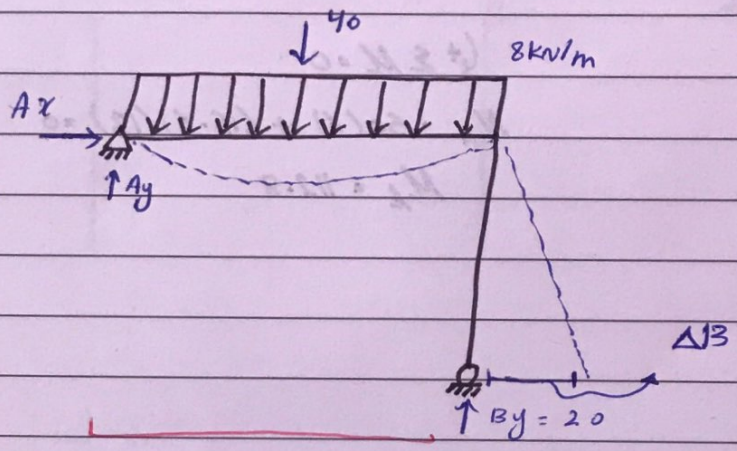
$-9000 + (576)(By) = 0$

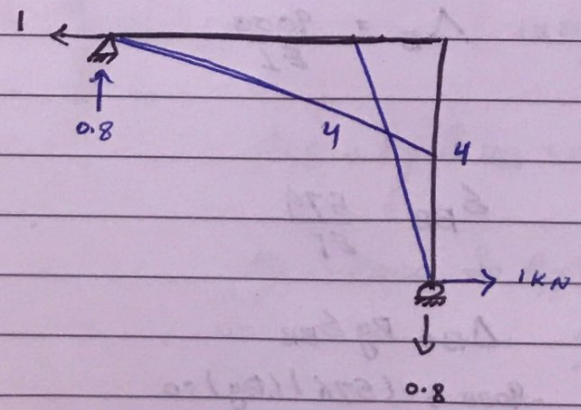
$By = 15.6\,kN$

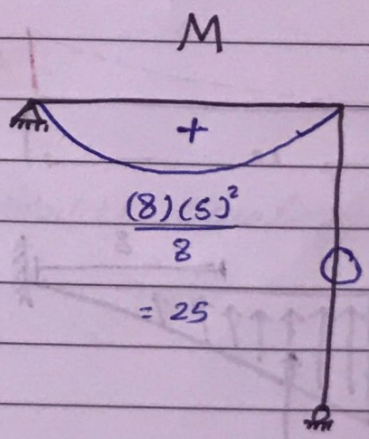
$Ay = 50 - (1)(15.6) = 34.4$

$M_A = 300 - (12)(15.6) = 112$









$$\Delta_B = \int \frac{M_m}{EI} = \left(\frac{1}{3}\right)(4)(25)(5) = 166.67$$

$$\delta_{BB} = \int \frac{mm}{EI} = \frac{(1/3)(4)(4)(4)}{EI} + \frac{(1/3)(4)(4)(5)}{EI}$$

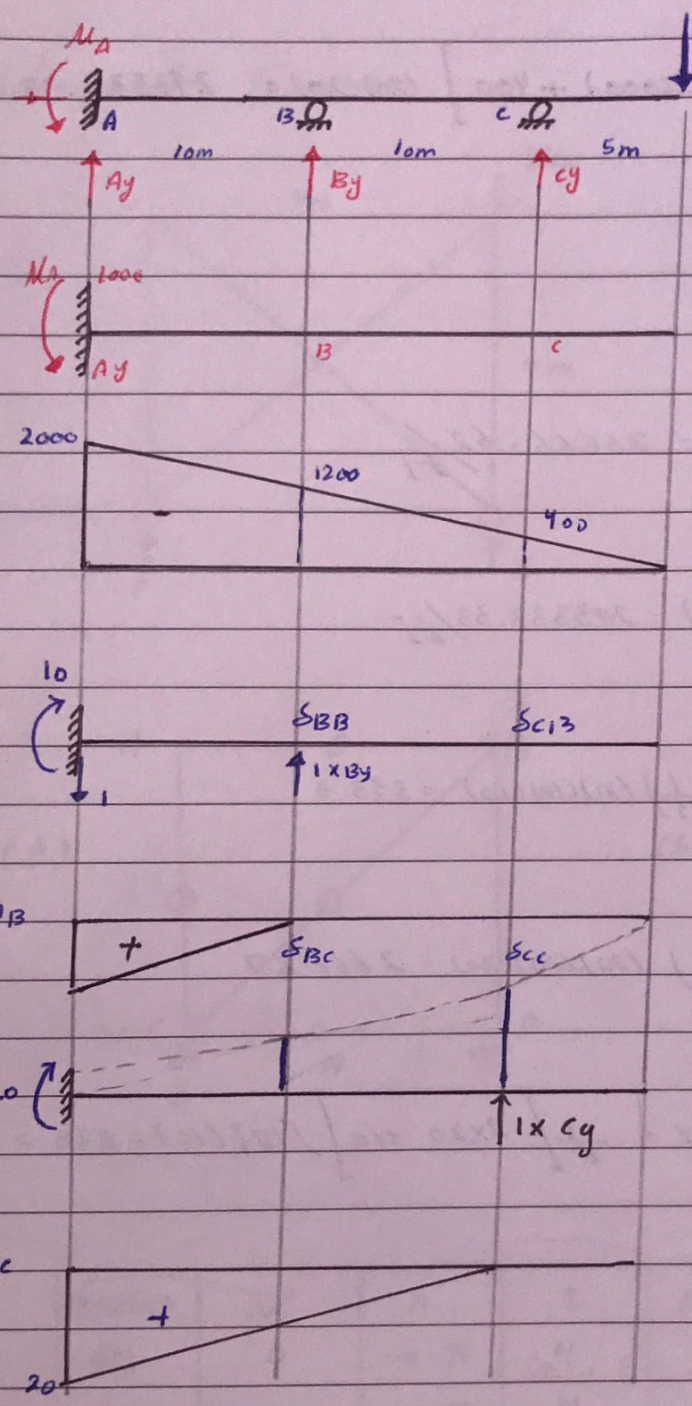
$$= \frac{48}{EI}$$

$$\Delta_B + (B_x)(\delta_{BB}) = 0$$

$$166.67 = -(B_x)(48)$$

$$B_x = -3.47$$

27/2/2019



Δ_B = ? 26666.67

Δ_C = ? 293333.33

δ_{BB} , δ_{BC} ?

δ_{CC} , δ_{BC}

$$\Delta_B = By\,\delta_{BB} + Cy\,\delta_{CB}$$

$$\Delta_C = By\,\delta_{BC} + Cy\,\delta_{CC}$$

$$\Delta_B = \int \frac{Mm}{EI}\,dx$$

$$= \frac{1}{6}\left[(2)(2000) + 1200\right](10)(10)$$

$$= 86666.67/EI$$

Five Apple

$$\Delta_C = \int \frac{Mm}{EI}\,dx = \frac{1}{6}\left[(2)(2000)+400\right](20)(20) = 293333.33$$

$$\frac{Px^2}{6EI}(3L-x)$$

$$\Delta_B = \frac{(80)(10)^2}{6EI}(3(25)-10) = 86666.62/EI$$

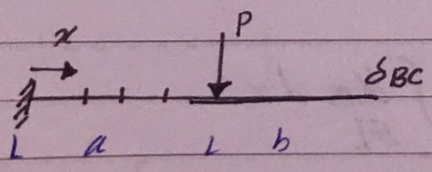
$$\Delta_C = \frac{(80)(20)^2}{6EI}((3)(25)-20) = 293333.33/EI$$

$$\delta_{BB} = \int \frac{m_B m_B}{EI}\,dx = \left(\frac{1}{3}\right)(10)(10)(10) = 333.3$$

$$\delta_{CC} = \int \frac{m_C m_C}{EI}\,dx = \left(\frac{1}{3}\right)(20)(20)(20) = 2666.67$$

$$\delta_{BC} = \delta_{CB} = \int \frac{m_B m_C}{EI}\,dx = \frac{1}{6}\left[2x20+40\right][18](10) = 833.3$$

$0 < x < L$ $\quad \Delta = \frac{Pa^2}{6EI}(3-a)$



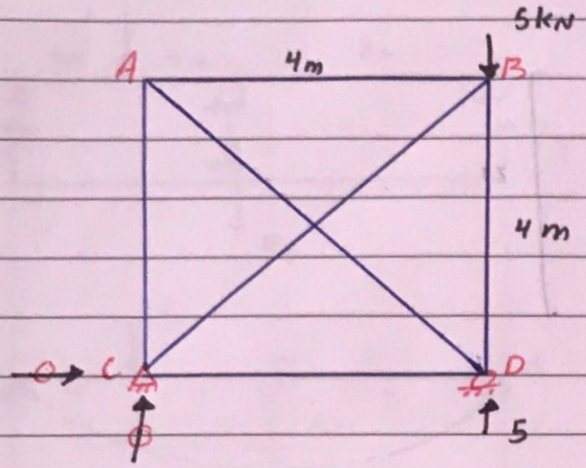
$0 \le x \le a$

$$\Delta = \frac{Px^2}{6EI}(3a-x)$$

$$= \frac{(1)(10)^2}{6}((3)(10)-10)$$

Five Apple

Indeteminate Trusses

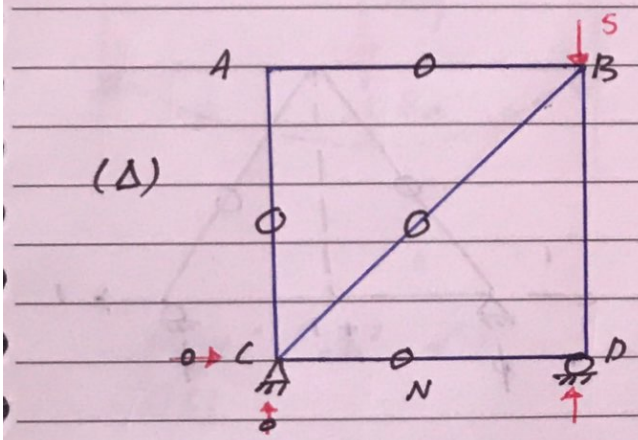


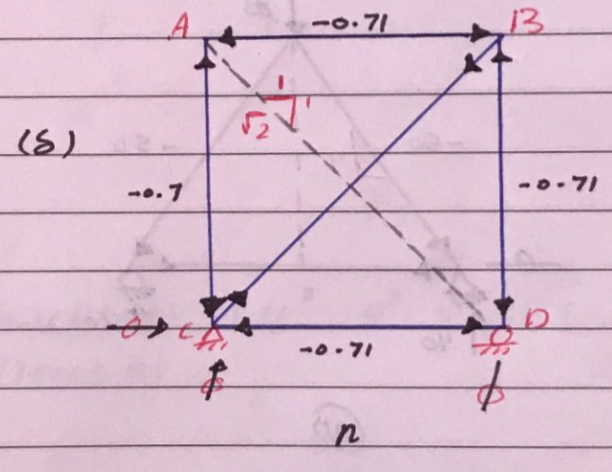
9 > 2x4

1st degree

$$\Delta = \frac{\Sigma NnL}{EA}$$

$$\delta = \frac{\Sigma nnL}{EA}$$





member	N	n	L	nNL	nnL/EI	Nf = N + χn
AB	0	-0.71	4	0	-	
AC	0	-0.71	4	0	-	
AD	0	1	$4\sqrt{2}$	0	-	
BD	-5	-0.71	4	14.2	-	
BC	0	1	$4\sqrt{2}$	0	-	
CD	0	-0.71	4	0	-	

$\Delta\Sigma = 14.2$

$\delta\Sigma = 19.5$

$\Delta + \chi\delta = 0 \rightarrow \chi$

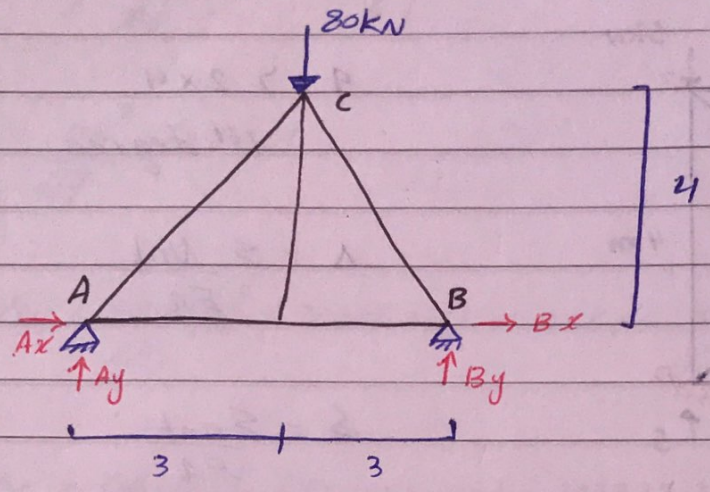
$N_f = N + \chi n$

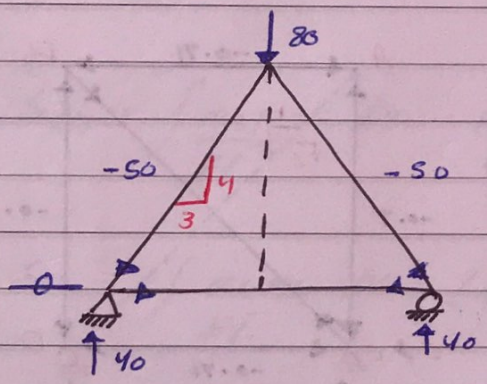
$14.2 + (\chi)(19.5) = 0$

$\chi = -0.72$

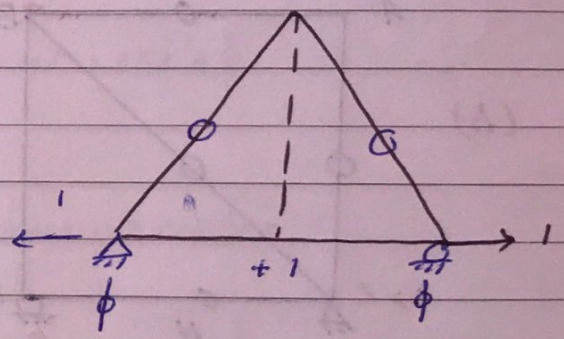
Five Apple

Externally to the 1st degree





(N)



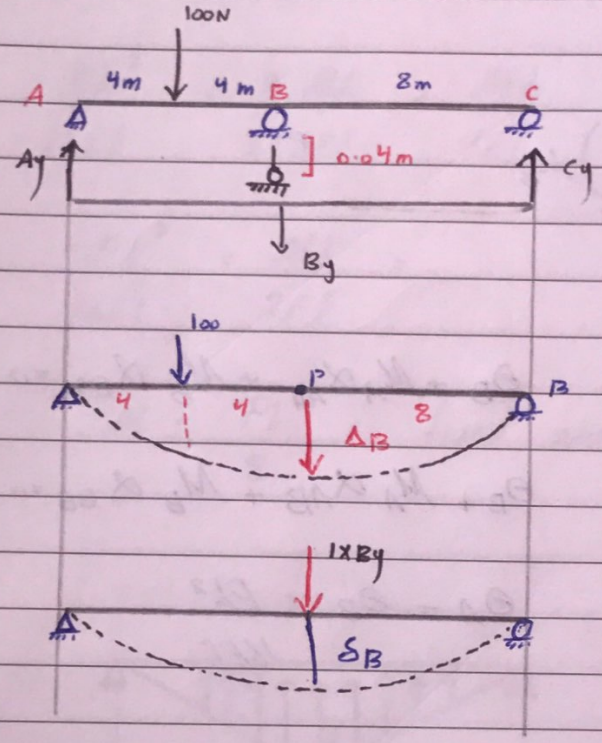
(n)

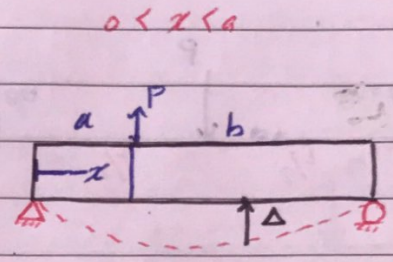
$$\Delta = \sum NnL/EA \qquad \delta = \sum nnL/EA$$

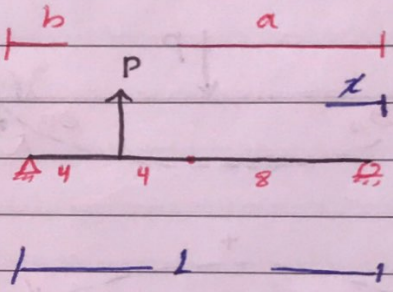
$$\Delta + Bx\delta = 0$$

4/3/2019

Find support Reaction if support sinks 40mm







$$\frac{Pbx(L^2-b^2-x^2)}{6LEI} \Rightarrow \frac{(100)(4)(8)}{(16)(16)EI}(16^2-4^2-8^2) = \frac{5866.67}{EI}$$

$$\delta_B = \frac{(1)(8)(3\times16^2-(4)(8)^2)}{48EI} = \frac{86.33}{EI}$$

$$\Delta_B + By\,\delta_B = 0.04$$

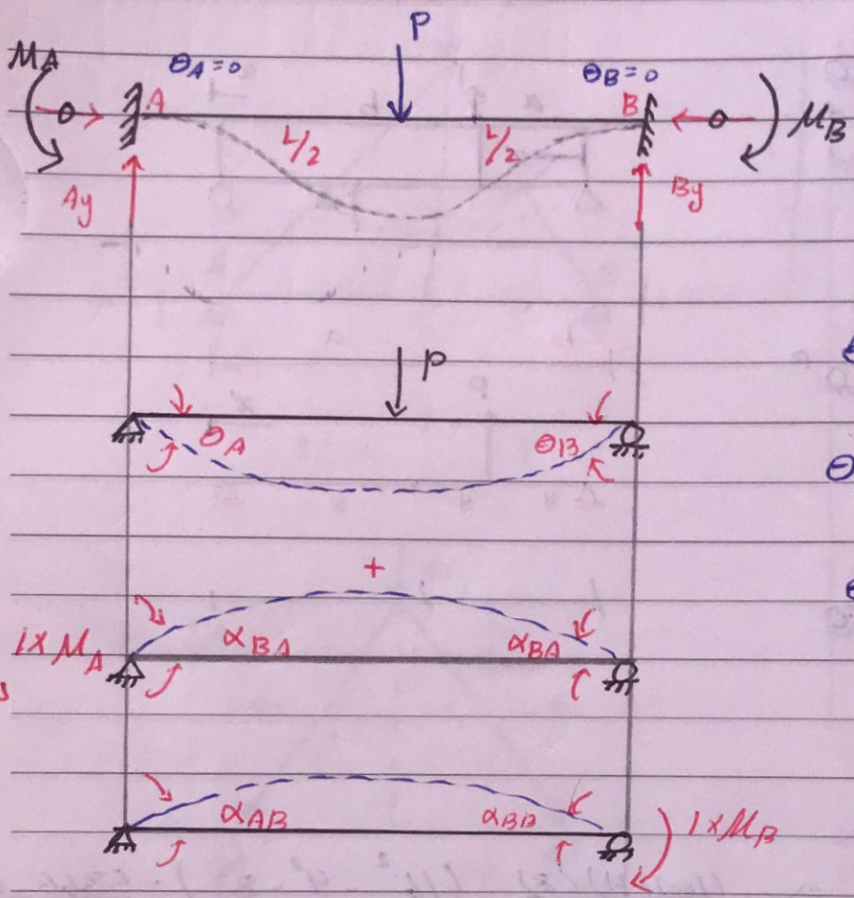
$$\frac{5866.67}{EI} + \frac{(By)(85.22)}{EI} = 0.04$$

$$By = -\frac{68.8}{EI}$$

Five Apple

Find support Reaction Δ δ $\quad \theta$ α

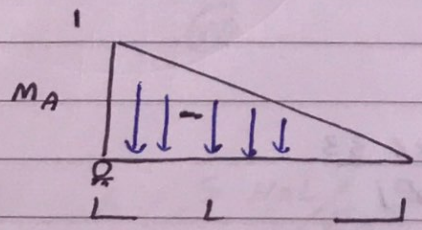
$\alpha_{AB} = \alpha_{BA}$



$$\theta_B + M_A \alpha_{AA} + M_B \alpha_{BA} = 0$$

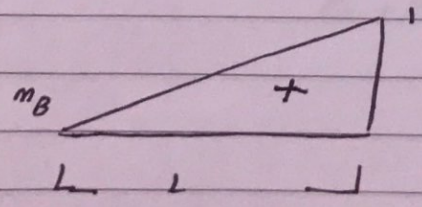
$$\theta_B + M_A \alpha_{AB} + M_B \alpha_{BB} = 0$$

$$\theta_A = \theta_B = \frac{PL^2}{16EI}$$



$$\alpha_{AA} = \frac{\int m_A m_A \, dx}{EI} = \frac{L}{3EI}$$

$$\alpha_{BB} = \frac{\int m_B m_B \, dx}{EI} = \frac{L}{3EI}$$



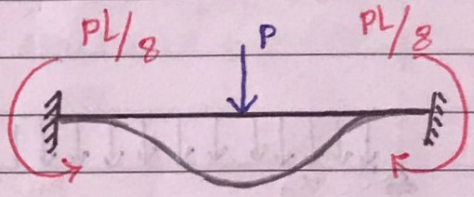
$$\alpha_{AB} = \alpha_{BA} = \int \frac{M_A M_B \, dx}{EI}$$

$$= \frac{L}{6EI}$$

Five Apple

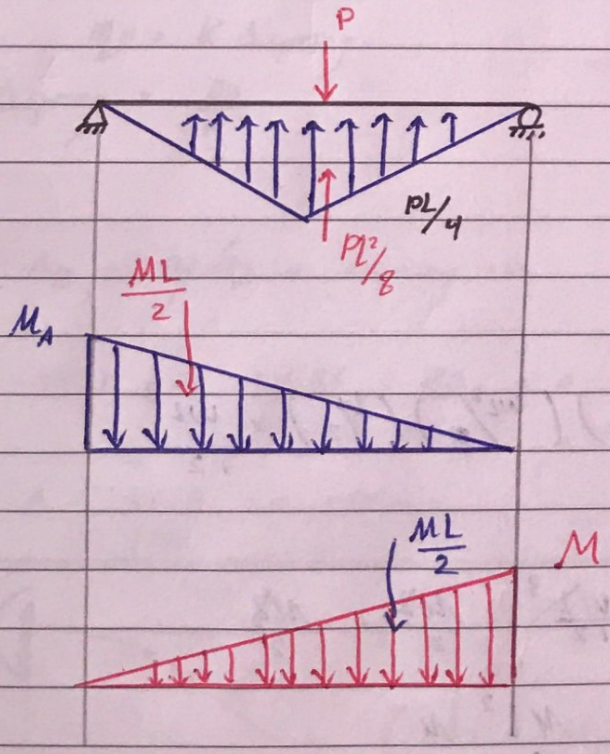
$M_A = M_B = M$

$$\frac{PL^2}{16EI} = \frac{2ML}{2 \times 3EI} + \frac{ML}{6EI} = \frac{3ML}{6EI}$$



$$\frac{PL^2}{16EI} = \frac{ML}{2EI}$$

$M = \frac{PL}{8}$ fixed and moment



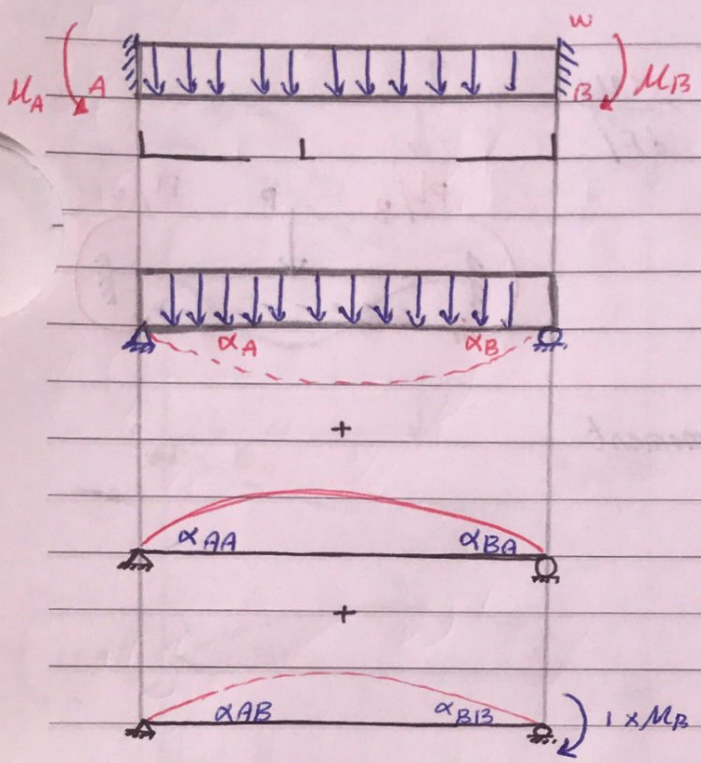
$$\frac{PL^2}{8} - \frac{ML}{2} - \frac{ML}{2} = 0$$

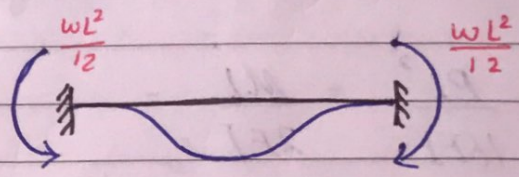
$$\frac{PL^2}{8} = ML$$

$$M = \frac{PL}{8}$$

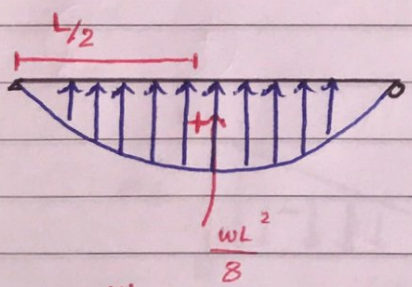
Five Apple

Fixed and Moment

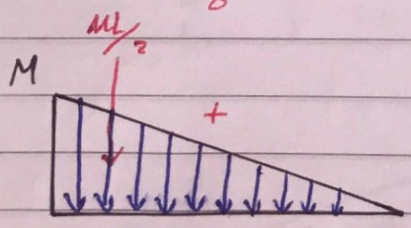




$M_A = M_B = M$

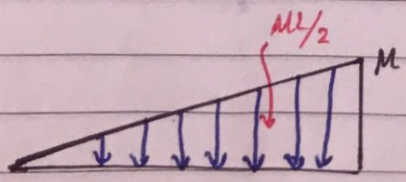


$$(4/3)\left(\frac{wL^2}{8}\right)\left(\frac{L}{2}\right) = \frac{wL^3}{12}$$

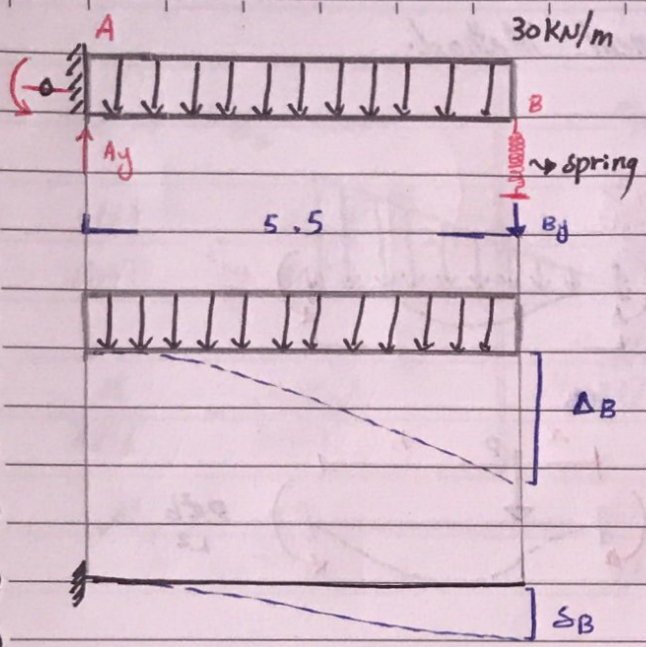


$$\frac{wL^3}{12} = \frac{ML}{2} + \frac{ML}{2}$$

$$\frac{ML^2}{12} = M$$



Five Apple



$K = 1750\ kN/m$

$E = 200\ Gpa$

$I = 120 \times 10^6\ mm^4$

$$\Delta_B = \frac{(30)(5.5)^4}{8EI} = \frac{-3431}{EI}$$

$$\delta_B = \frac{(1)(5.5)^3}{3EI} = \frac{-55.458}{EI}$$

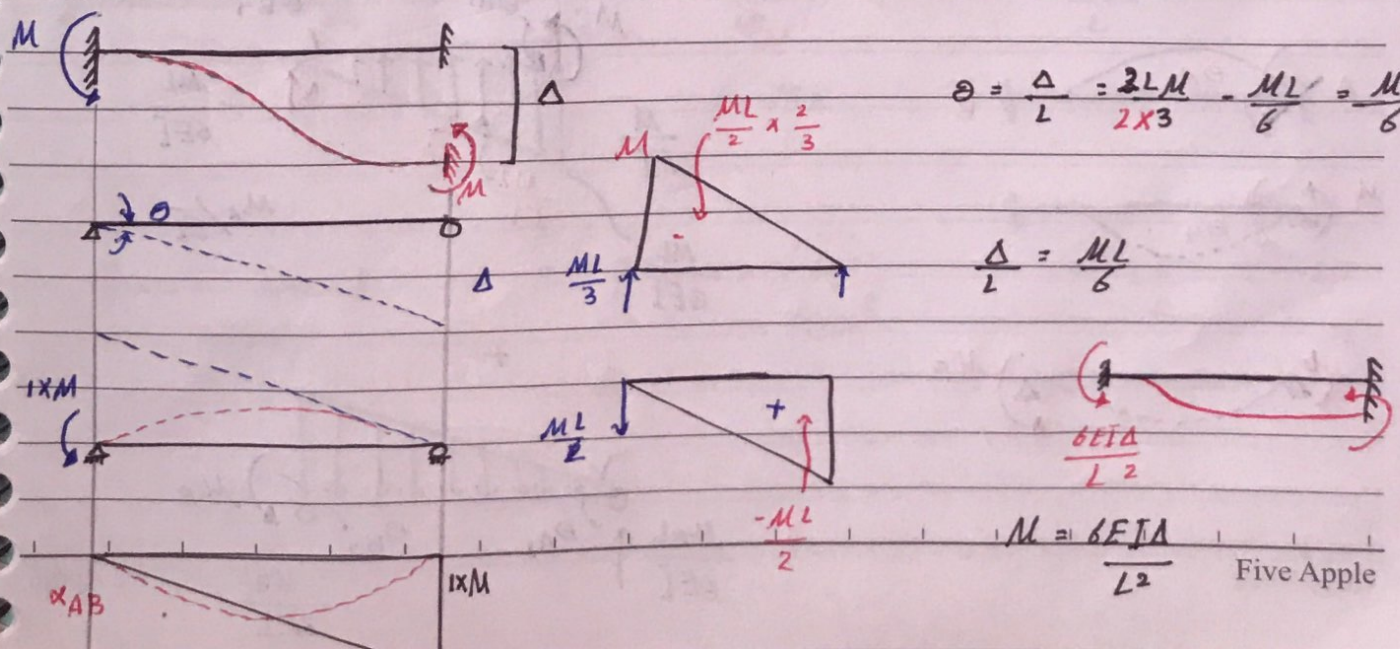
$$By = K\,\Delta_{spring}$$

$$\Delta_{spring} = \frac{By}{K}$$

$$\Delta_B + By\,\delta_B + \Delta_{spring} = 0$$

$$-3431 + By\,\frac{55.46}{EI} - \frac{By}{1750} = 0 \quad \Rightarrow By = -49.5$$

$$\Delta = \frac{49.5}{1750} = 0.028m$$



$$\theta = \frac{\Delta}{L} = \frac{2LM}{2\times3} = \frac{ML}{6} = \frac{ML}{6}$$

$$\frac{\Delta}{L} = \frac{ML}{6}$$

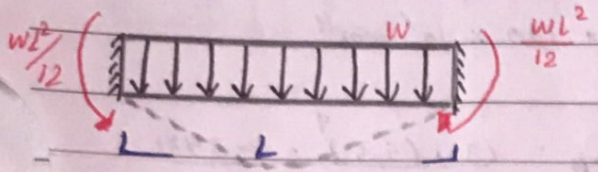
$$M = \frac{6EI\Delta}{L^2}$$

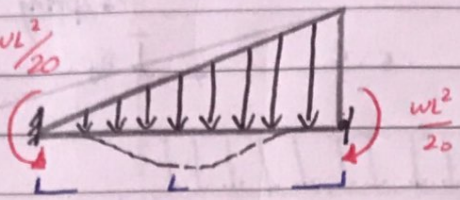
Five Apple

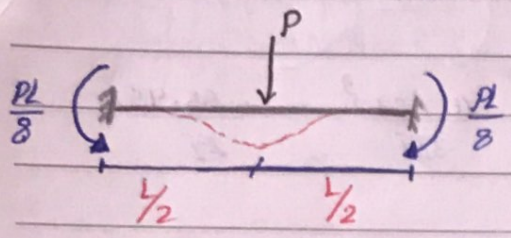
6/2/2019

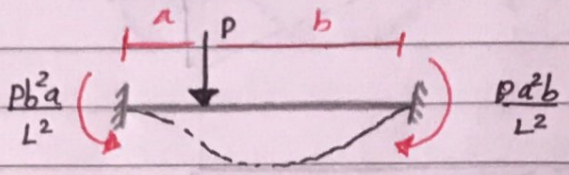
11 Moment Distribution Displacment Method.

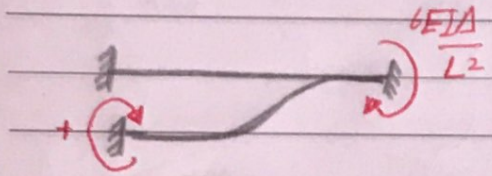
① Fixed End moment

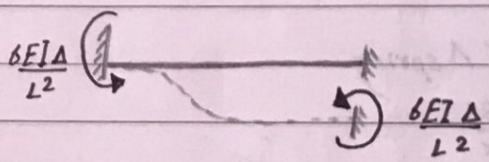




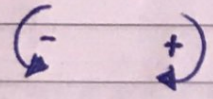




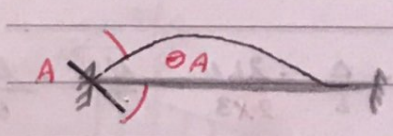


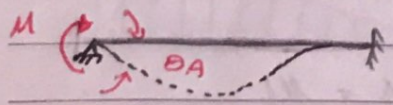


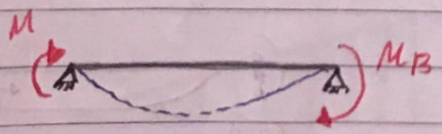
sign Coaversion

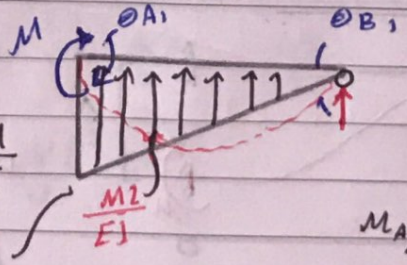


- conryover moment
- stiffness $M \rightarrow M/2$

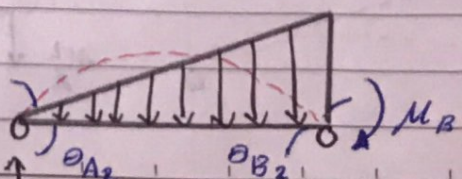








+



Five Apple

$\theta_{B_1} + \theta_{B_2} = 0$

$\frac{ML}{6EI} - \frac{M_B L}{3EI} = 0$

$\frac{M}{6EI} - \frac{2M_B L}{6EI}$

$M_B = \frac{M}{2}$

$\theta_{A_1} + \theta_{A_2} = \theta$

$\frac{ML}{3EI} - \frac{M_B L}{6EI} = \theta$

$\frac{4ML}{4 \times 3EI} - \frac{ML}{12EI} = \theta$

$\frac{3ML}{12EI} = \theta \qquad \frac{ML}{4EI}$

$M = \frac{4EI}{L}\theta$

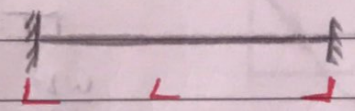
K

stiffness

$F = K\Delta$

$M = K\Delta$

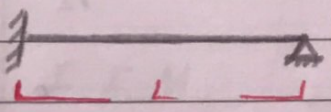
EI



$K = \frac{4EI}{L}$

Relative stiff

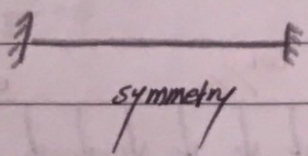
$\frac{I}{L}$



$K = \frac{3EI}{L} = 3 \times \frac{4EI}{4L}$

$= 0.75K$

$\frac{0.75}{L}$



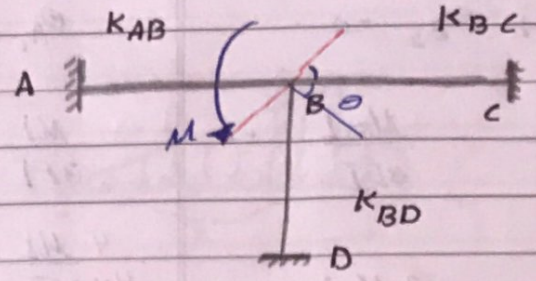
$K = \frac{2EI}{L} = 2 \times \frac{4EI}{4L}$

$K = 0.5K$

$\frac{0.5I}{L}$

- Distribution factor

$$M_{BC} = \frac{K_{BC} M}{\Sigma K}$$



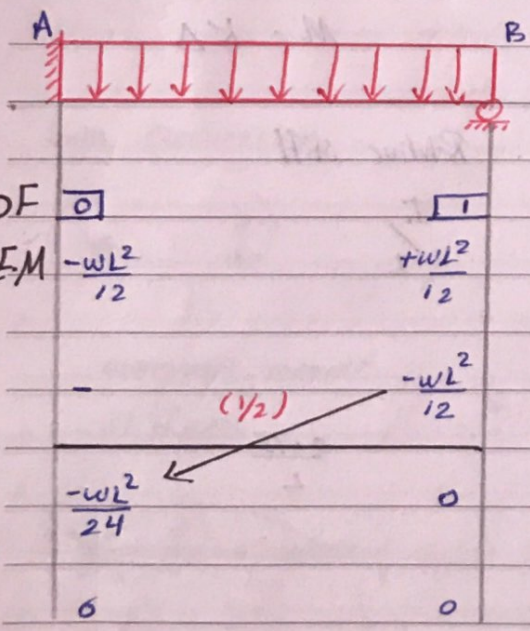
$$M_{BA} + M_{BC} + M_{BD} = M$$

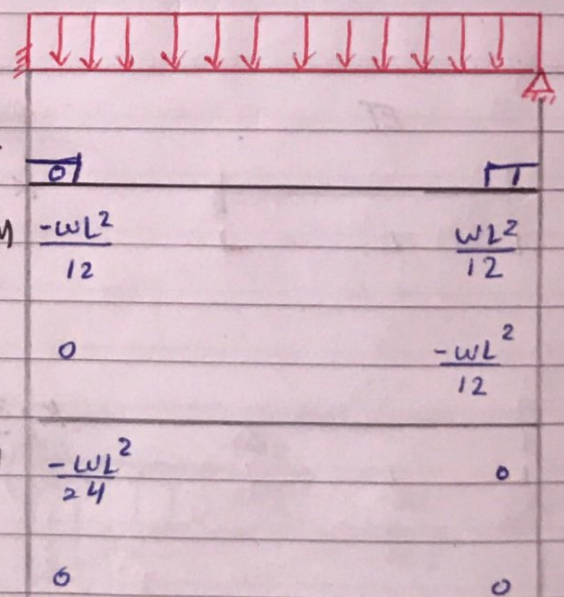
$$K_{AB}\theta + K_{BC}\theta + K_{BD}\theta = M$$

$$(K_{AB} + K_{BC} + K_{BD})\theta = M$$

$$\theta = M/\Sigma K$$

$$M_{BA} = \frac{K_{AB} M}{\Sigma K}$$





fixed → 0 , Roller pin → 1

* distribution factor :- $K_i/\Sigma K$

A ——— ⇒ distribution factor of fixed equal (0)

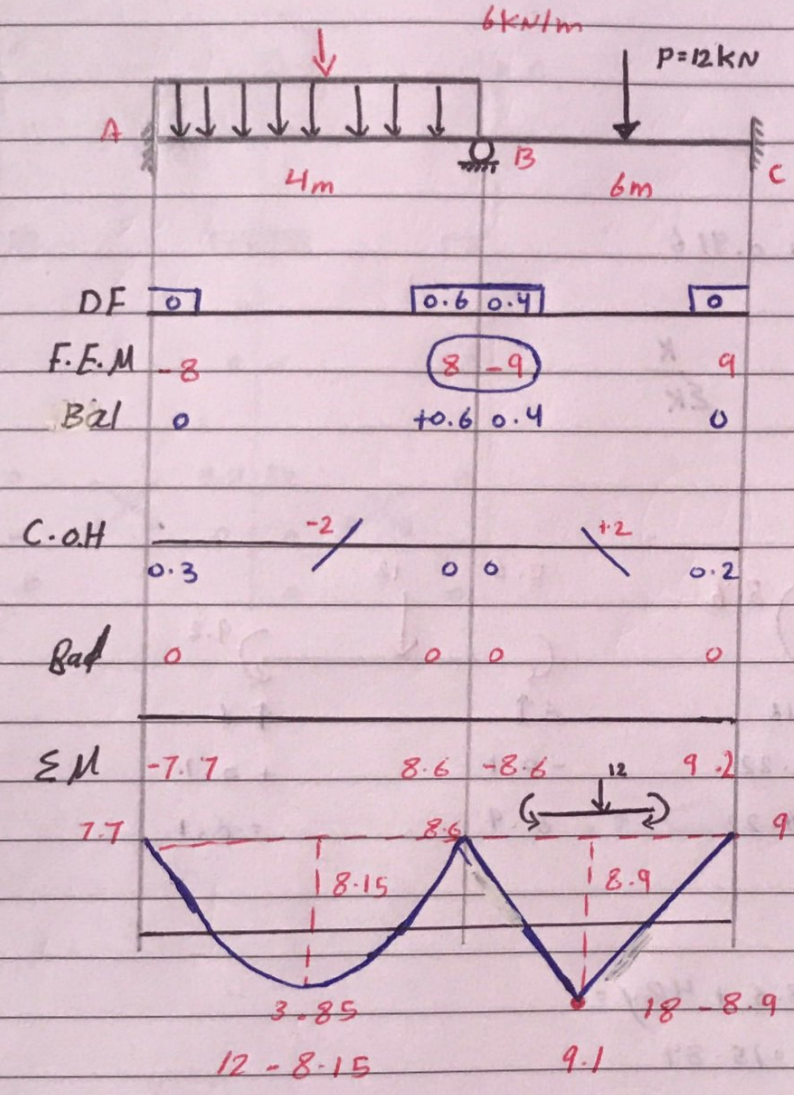
——— ⇒ distribution factor of hinge and Roller = 1

Σ $-3L^2/24 = WL^2/8$

$\frac{WL^2}{8}$

EI = constant

Draw S.D M.D



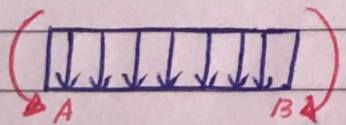
$\frac{8.6 + 7.7}{2} = 8.15$

$\frac{WL^2}{8} = \frac{(6)(4)^2}{8} = 12$

$\frac{8.6 + 9.2}{2} = 8.9$

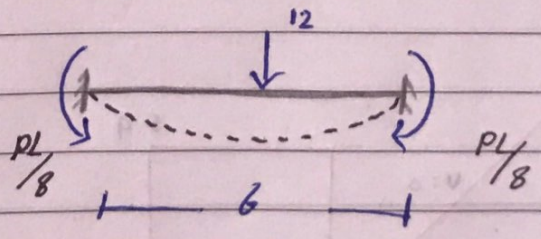
$\frac{(12)(6)}{4} = 18$

- F.E.M

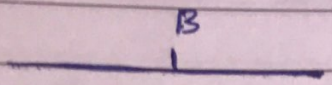


$\frac{(6)(4)^2}{12} = 8$

$\frac{WL^2}{12} = 8$



$\frac{(12)(6)}{8} = 9$

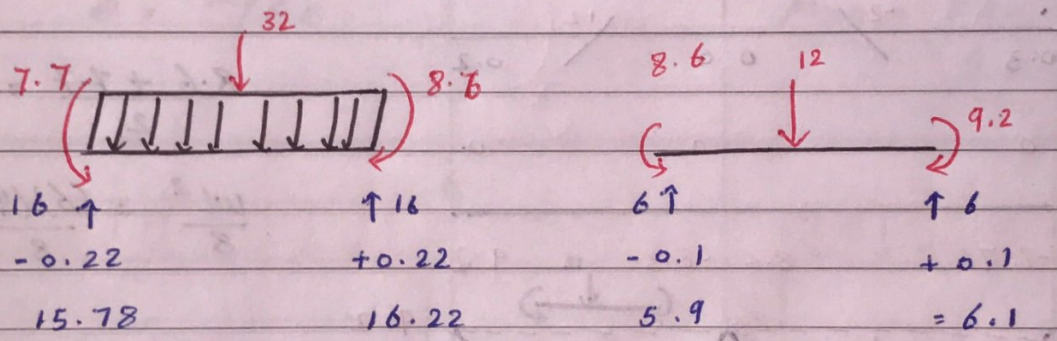


$\frac{1}{4}$ $\frac{1}{6}$

0.25 : 0.167 : 0.416

$\frac{0.25}{0.416}$: $\frac{0.167}{0.416}$ $\frac{K}{\Sigma K}$

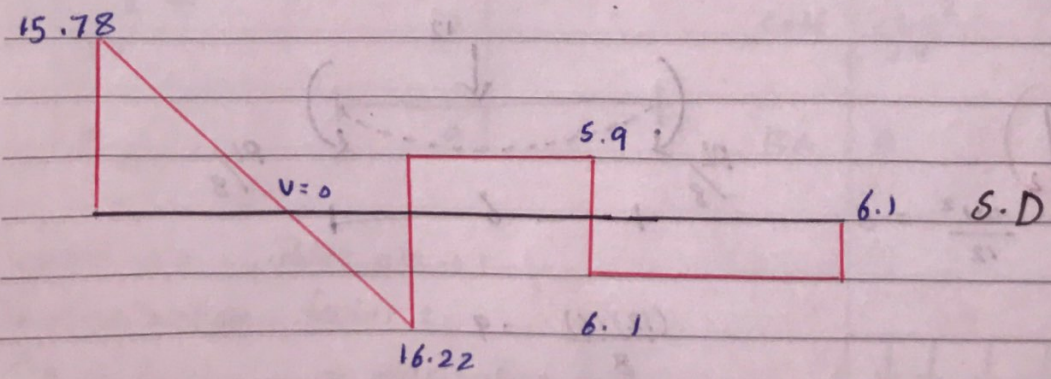
0.6 : 0.4



$\Sigma M_A = 0$

$(7.7) - (32)(2) - 8.6 + 4By = 0$

$By = 16.22$, $Ay = 15.87$

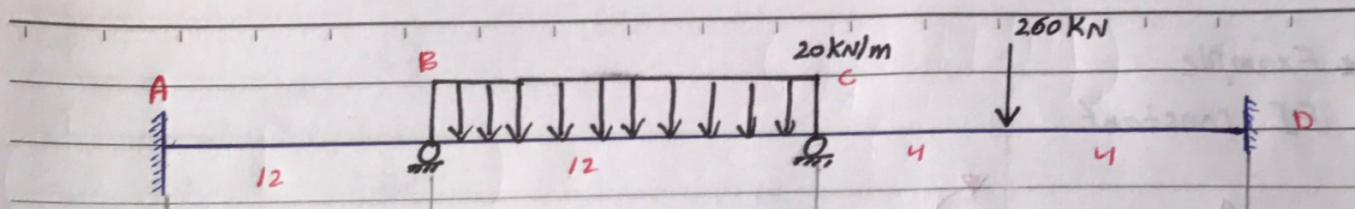


Five Apple

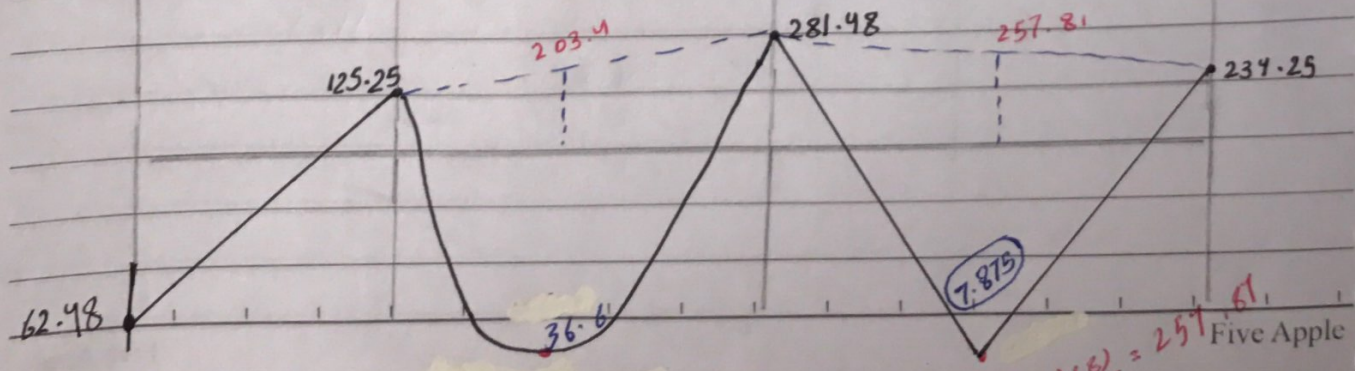
Example c/n

EI : constant

11/3/2019



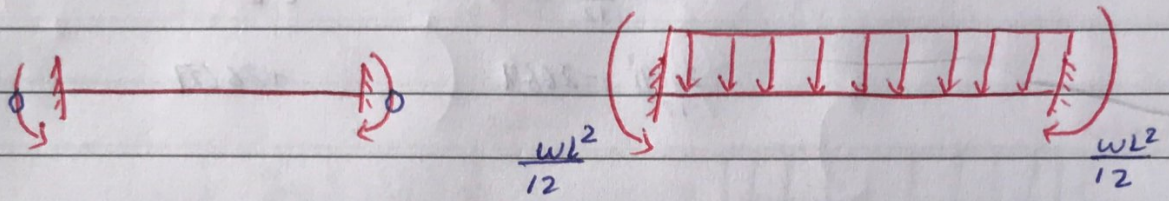
	A	B		C		D
DF	0	0.5	0.5	0.4	0.6	0
F.E.M	0	0	-240	+240	-250	250
				96	144	
				-100	-150	
				-4	-6	
Bal	0	+120	+120	4	6	0
	60	0	2	60	0	3
Bal	0	-1	-1	-24	-36	0
C.O	-0.5	0	-12	-0.5	0	-18
Bal	0	+6	+6	0.2	0.3	0
C.O	3	0	0.1	3	0	0.15
Bal	0	-0.05	-0.05	-1.2	-1.8	0
C.O	-0.025	0	-0.6	-0.025	0	-0.09
Bal	0	+0.3	+0.3	+0.01	0.015	0
Σ	62.475	125.25	-125.25	281.48	-281.5	234.25



$DF \rightarrow \frac{k}{\Sigma k}$

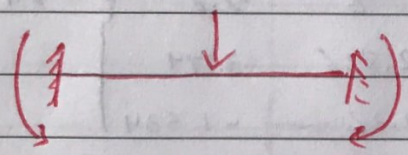
$DF_{BA} = 0.5$, $DF_{BC} = 0.5$, $DF_{CB} = 0.4$ $DF_{CD} = 0.6$

$DF_{AB} = 0$



$\frac{WL^2}{12}$ $\frac{WL^2}{12}$

$\frac{20(12)^2}{12} = -240$ $+240$



$\frac{PL}{8}$ $\frac{PL}{8}$

$\frac{250(8)}{8} = -250$ $+250$

$\therefore M_{BC} = -240$

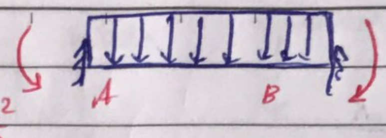
$M_{CB} = 240$

$M_{CD} = -250$

$M_{DC} = 250$

Examplea 2,,

EI constant



$M_{AB} = -6$

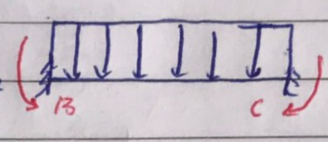
$M_{BA} = 6$

R.S => BA BC

$\frac{1}{6}$ $\quad$ $1/4 \times 0.75$

$\frac{2 \times (6)^2}{12} = -6$ $\quad$ $+6$

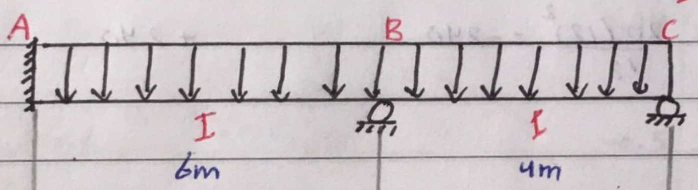
DF $\frac{K}{\Sigma K}$: 0.471 $\quad$ 0.529



$M_{BC} = -2.67$

$M_{CB} = 2.67$

$\frac{2 \cdot (4)^2}{12} = -2.67$ $\quad$ $+2.67$



6	-2.6	2.1
2.826	3.174	
-1.36	-1.534	

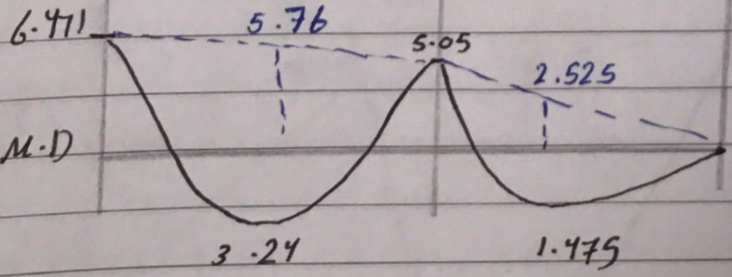
	A	BA	BC	C
DF	0	0.471	0.529	1
F.M.E	-6	6	-2.67	2.67
			-1.335 ←	-2.67
F.M.E	-6	+6	-3.9	0
Bal	0	-0.942	-1.058	0
C.O.M	0.471	0	0	-0.529

لا مايكون عالطرف fixed ← لازم
تحوله لـ fixed ← لكن ال moment ازا
طلعناه [2.67] نأخذ عكس الاشارة
-2.67 عشان نأخذ (zero) ونضعها نصها
بالاشارة على الجهة الأخرى ونكمل الحل عادي !!

Final Moment

Σ	-6.471	5.05	-5.05	0

M.D



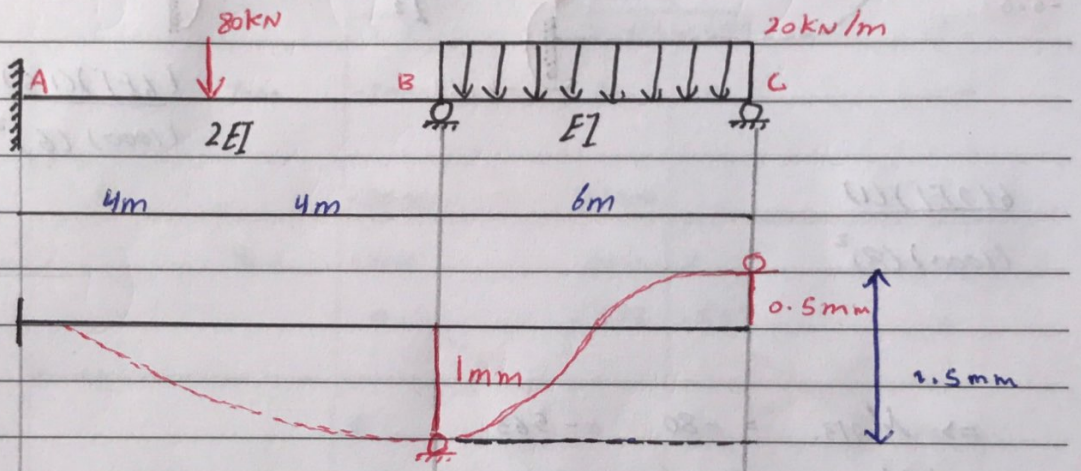
Example "3"

$K = \frac{4EI}{L}$

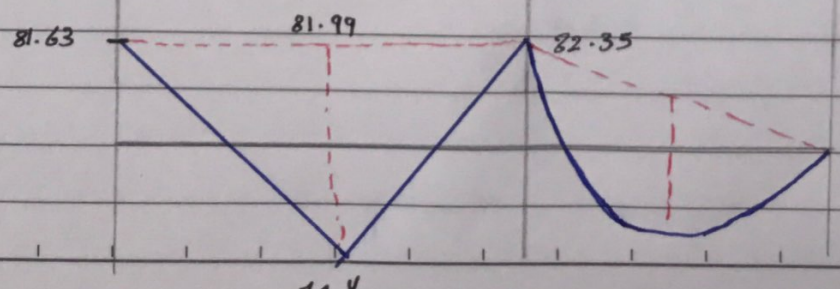
$EI = 30{,}000$

B sinks 1mm C rises 0.5mm Draw M.D

	B		
	BA	BC	
	$\frac{2}{8}$	$(1/6)(0.75)$	
DF =	0.67	0.33	

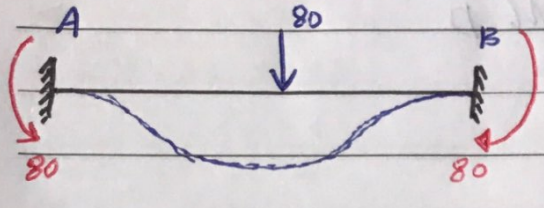


	0	0.67	0.33	0
F.E.M	-85.63	74.37	-52.5	67.5
			-33.75 ←	-67.5
F.EM	-85.63	74.37	-86.25	0
Bal	0	+8	+13.9	0
C.O	4	0	0	-
Bal	0	0	0	0
	-81.63	82.35	-82.32	0

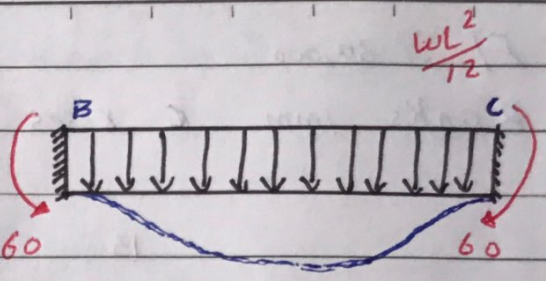


Five Apple

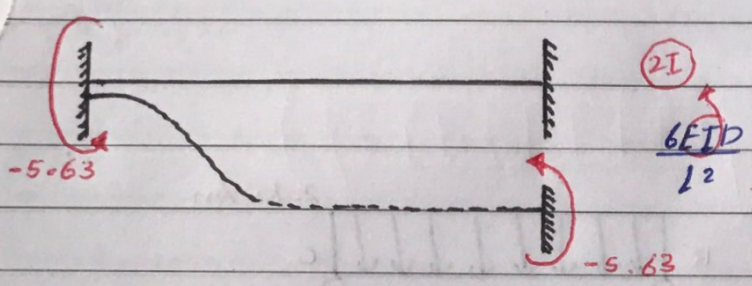
F.E.M



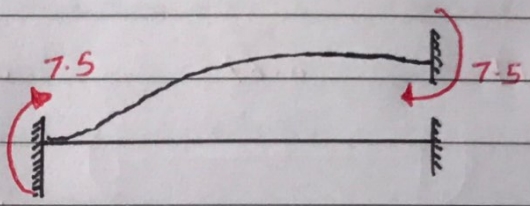
$\frac{PL}{8}$



$\frac{wL^2}{12}$



$\frac{6EI\Delta}{l^2}$



$\frac{(6EI)(1.5)}{(1000)(6)^2}$

$\frac{6(2EI)(1)}{(1000)(8)^2}$

$\Rightarrow M_{AB} = -80 \quad -563$

$\Rightarrow M_{BA} = +80 \quad -563$

$\Rightarrow M_{BC} = -60 \quad +75$

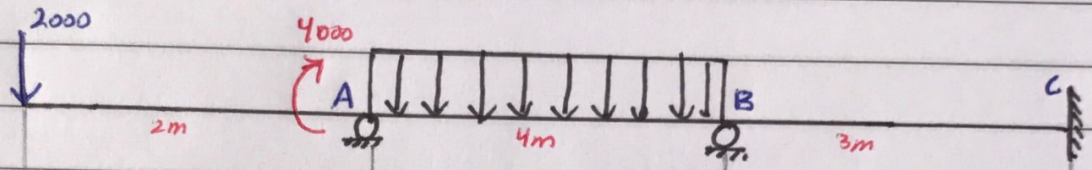
$\Rightarrow M_{CB} = 60 \quad +7.5$

Example "4"

$I_{AB} = 300 \times 10^6$

$I_{BC} = 240 \times 10^6$

	BA	BC
Rs	$\frac{(0.75)(300)}{4}$	$\frac{240}{3}$
DF	0.413	0.587



		1	0.413	0.587	0
F.E.M	4000	-2000	2000	0	0
		-2000	½ → -1000		
F.E.M	4000	-4000	1000	0	0
		0	-413	-587	0
		0	0	0	-293.5
		0	0		
Σ	4000	-4000	587	-587	-293.5