# Retaining Structures

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SLOPE STABILITY AND RETAINING WALL

# Lateral Earth Pressure

Lateral Earth Pressure depend on:

- The unit weight of the soil (natural soil or backfill)
- The type and amount of wall movement, (deformation)
- The shear strength parameters of the retained soil, interlocking (friction, resistance, cohesion).
- The shear strength parameters of the foundation soil
- The drainage conditions in the backfill.

# Three type of lateral earth pressure:

*At-rest earth pressure*

*Active earth pressure*

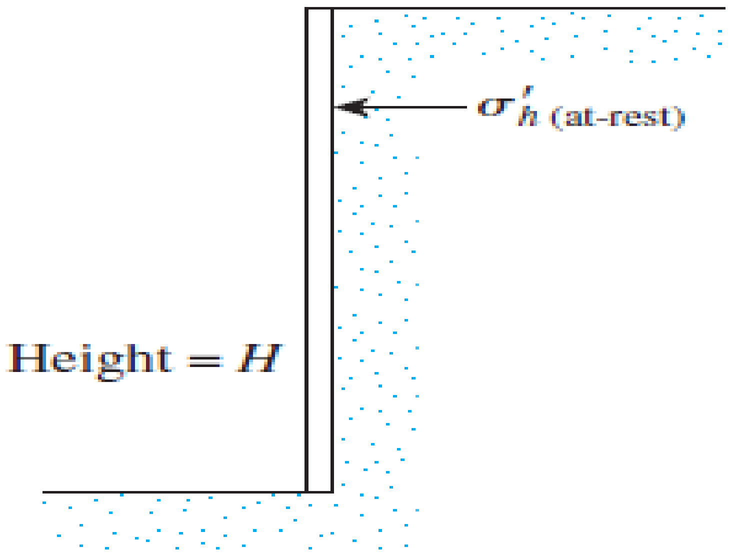
*Passive earth pressure.*

# At-rest earth pressure

The wall may be restrained from moving. The lateral earth pressure on the wall at any depth is called the *at-rest earth pressure* .

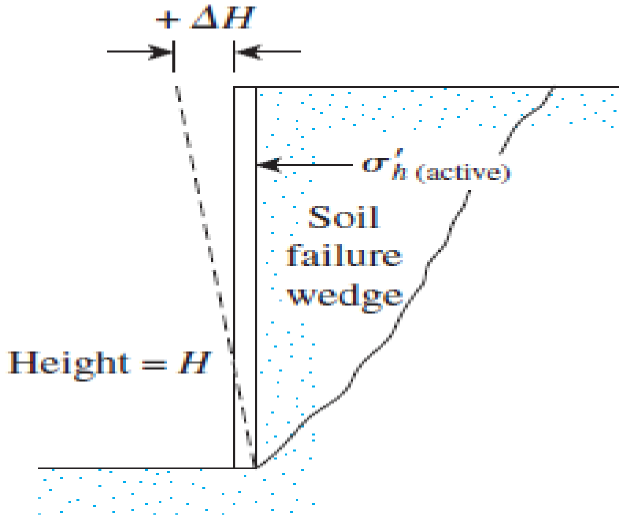
*The system is very strong*

*The deformation is very small*



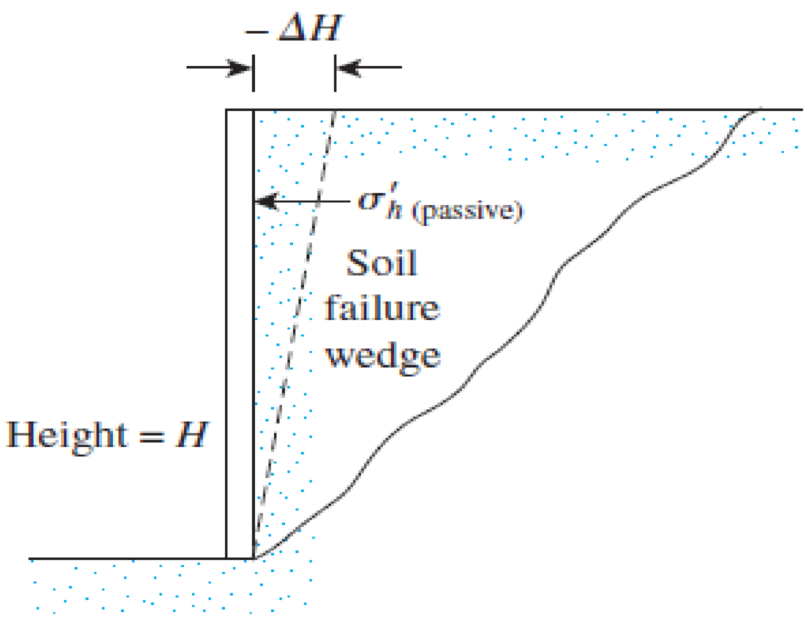
# Active earth pressure

The wall may tilt away from the soil that is retained (Figure 7.1b). With sufficient wall tilt, a triangular soil wedge behind the wall will fail. The lateral pressure for this condition is referred to as *active earth pressure.*



# passive earth pressure

The wall may be pushed into the soil that is retained. With sufficient wall movement, a soil wedge will fail. The lateral pressure for this condition is referred to as *passive earth pressure.*



# Lateral Earth pressure

Consider the following conditions:

Vertical wall of height *H*,

Retaining a soil having a unit weight of $\gamma$

Uniformly distributed load

The shear strength of the soil is:

$$s = c' + \sigma' \tan \phi'$$

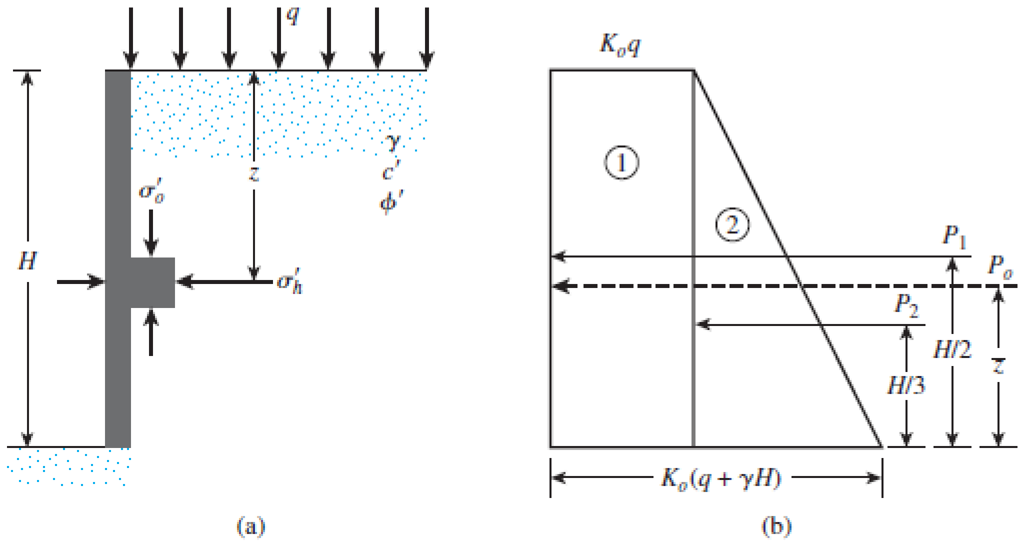
where

$c'$ = cohesion

$\phi'$ = effective angle of friction

$\sigma'$ = effective normal stress

# Lateral Earth Pressure At-rest



# Lateral Earth Pressure At-rest

At any depth *z* below the ground surface, the vertical subsurface stress is:

$$\sigma'_o = q + \gamma z$$

If the *wall is at rest and is not allowed to move at all,* either away from the soil mass or into the soil mass the lateral pressure at a depth *z* is:

$$\sigma_h = K_o \sigma'_o + u$$

where

$u$ = pore water pressure
$K_o$ = coefficient of at-rest earth pressure

# Lateral Earth Pressure At-rest

For normally consolidated soil, the relation for $K_o$ (Jaky, 1944) is

$$K_o \approx 1 - \sin\phi'$$

For overconsolidated soil, the at-rest earth pressure coefficient may be expressed as (Mayne and Kulhawy, 1982)

$$K_o = (1 - \sin\phi')\text{OCR}^{\sin\phi'}$$

where OCR = overconsolidation ratio.

The total force (Po), *per unit length* of the wall can be obtained

from the area of the pressure diagram is:

$$P_o = P_1 + P_2 = qK_oH + \tfrac{1}{2}\gamma H^2 K_o$$

where

$P_1$ = area of rectangle 1
$P_2$ = area of triangle 2

The location of the line of action of the resultant force, $P_o$, can be obtained by taking the moment about the bottom of the wall. Thus,

$$\bar{z} = \frac{P_1\left(\frac{H}{2}\right) + P_2\left(\frac{H}{3}\right)}{P_o}$$

# Lateral Pressure Theories

Rankine Theory ( Frictionless walls)

Coloumb Theory

# Lateral Pressure Theories

22/3/2020

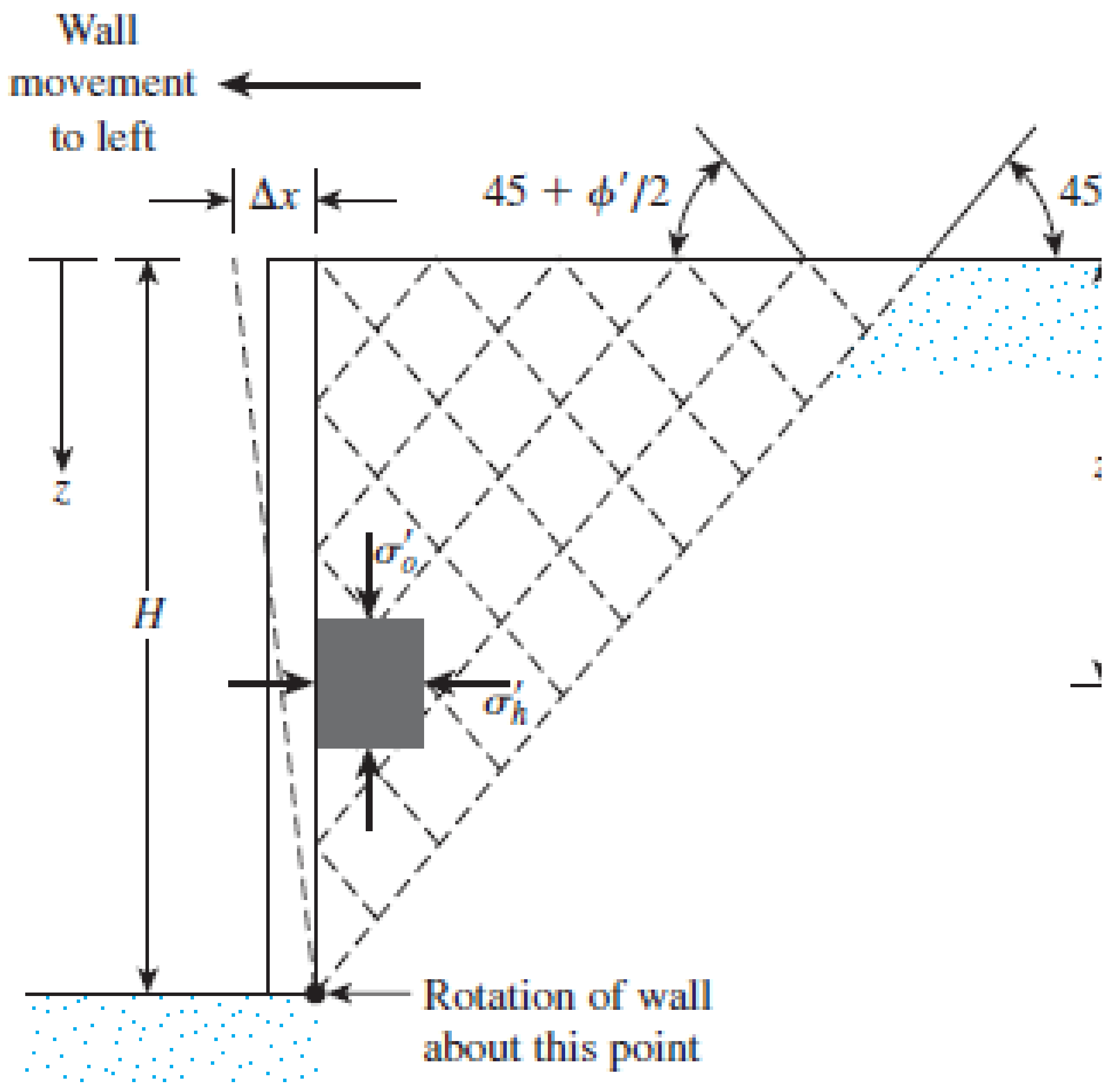
- Rankine Theory (Frictionless wall)
  - Active pressure
  - Passive pressure

- Coloumb Theory
  - Active pressure
  - Passive pressure

# Rankine Theory

Rankine Active Earth Pressure:

- if a wall tends to move away from the soil a distance $\Delta x$ as shown below.
- The soil pressure on the wall at any depth will decrease.
- With $\Delta x > 0$, $\sigma_h$ will be less than $\{(K_o\ \sigma_o)\}$ at rest pressure



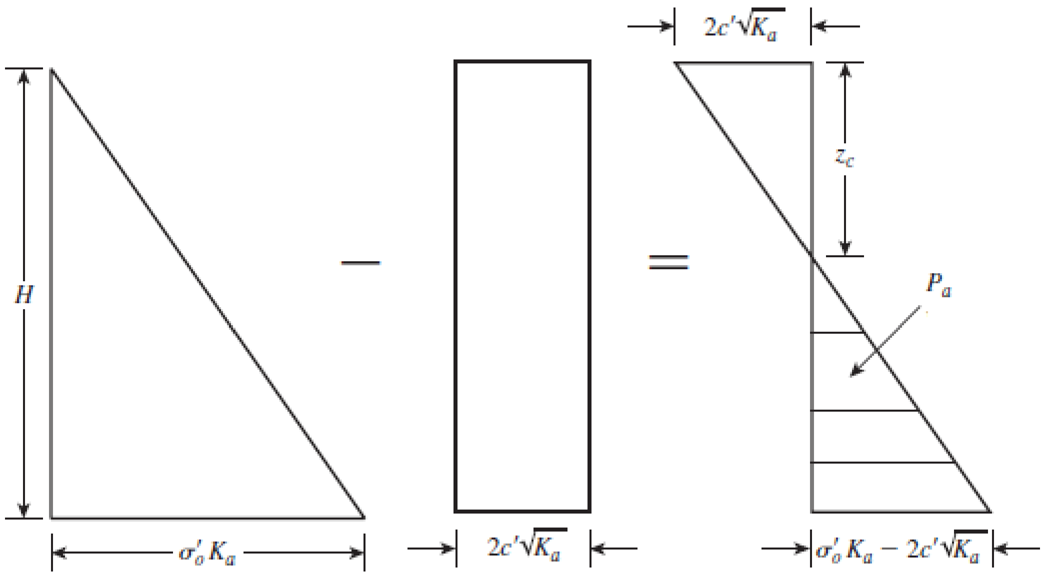
# Active lateral pressure:

$$\sigma'_a = \sigma'_o \tan^2\left(45 - \frac{\phi'}{2}\right) - 2c' \tan\left(45 - \frac{\phi'}{2}\right)$$
$$= \sigma'_o K_a - 2c'\sqrt{K_a}$$

← For c-ϕ soil

where $K_a = \tan^2(45 - \phi'/2)$ = Rankine active-pressure coefficient.

The variation of the active pressure with depth for the wall shown in previous slide is given here →



(c)

Where: $z_c = \frac{2c'}{\gamma\sqrt{K_a}}$

$z_c$ the depth is usually referred to as the *depth of tensile crack*

# Active lateral pressure:

- The total Rankine active force per unit length of the wall ($p_a$) **before** the tensile crack occurs is:

$$= \frac{1}{2}\gamma H^2 K_a - 2c' H \sqrt{K_a}$$

- **After** the tensile crack appears, the force per unit length on the wall will be caused only by the pressure distribution between depths z=$z_c$ and z=H, and its:

$$P_a = \frac{1}{2}(H - z_c)(\gamma H K_a - 2c'\sqrt{K_a})$$

# Active lateral pressure:

- For granular soil (c=0)

$$\sigma'_a = \sigma'_o \tan^2\left(45 - \frac{\phi'}{2}\right)$$

# How much the movement of wall in <u>active condition</u>:

- For granular soil backfills: $\Delta x = 0.001H$ to $0.004H$
- For cohesive soil backfills: $\Delta x = 0.01H$ to $0.04H$

# Example:

- A 6-m-high retaining wall is to support a soil with unit weight $\gamma = 17.4$ kN/m$^3$, soil friction angle $\phi' = 26°$, and cohesion $c' = 14.36$ kN/m$^2$. Determine the Rankine active force per unit length of the wall both before and after the tensile crack occurs, and determine the line of action of the resultant in both cases.

- **Reference:** Das, B.M. (2010). *Principles of Foundation Engineering*, 7th ed., CL Engineering
- **Note:** you can find this reference on CIVILITTEE web site in FOUNDATION ENGINEERING course.

# Solution

For $\phi' = 26°$,

$$K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2(45 - 13) = 0.39$$

$$\sqrt{K_a} = 0.625$$

$$\sigma'_a = \gamma H K_a - 2c'\sqrt{K_a}$$

At z = 0 $\sigma'_a = -2c'\sqrt{K_a} = -2(14.36)(0.625) = -17.95 \text{ kN/m}^2$

and at $z = 6$ m,

$$\sigma'_a = (17.4)(6)(0.39) - 2(14.36)(0.625)$$
$$= 40.72 - 17.95 = 22.77 \text{ kN/m}^2$$

Active Force before the Tensile Crack Appeared: Eq. (7.10)

$$P_a = \tfrac{1}{2}\gamma H^2 K_a - 2c'H\sqrt{K_a}$$
$$= \tfrac{1}{2}(6)(40.72) - (6)(17.95) = 122.16 - 107.7 = \mathbf{14.46 \text{ kN/m}}$$

The line of action of the resultant can be determined by taking the moment of the area of the pressure diagrams about the bottom of the wall, or

$$P_a\bar{z} = (122.16)\left(\frac{6}{3}\right) - (107.7)\left(\frac{6}{2}\right)$$

Thus,

$$\bar{z} = \frac{244.32 - 323.1}{14.46} = \mathbf{-5.45 \text{ m}}.$$

Active Force after the Tensile Crack Appeared: Eq. (7.9)

$$z_c = \frac{2c'}{\gamma\sqrt{K_a}} = \frac{2(14.36)}{(17.4)(0.625)} = 2.64 \text{ m}$$

Using Eq. (7.11) gives

$$P_a = \tfrac{1}{2}(H - z_c)(\gamma H K_a - 2c'\sqrt{K_a}) = \tfrac{1}{2}(6 - 2.64)(22.77) = \mathbf{38.25 \text{ kN/m}}$$

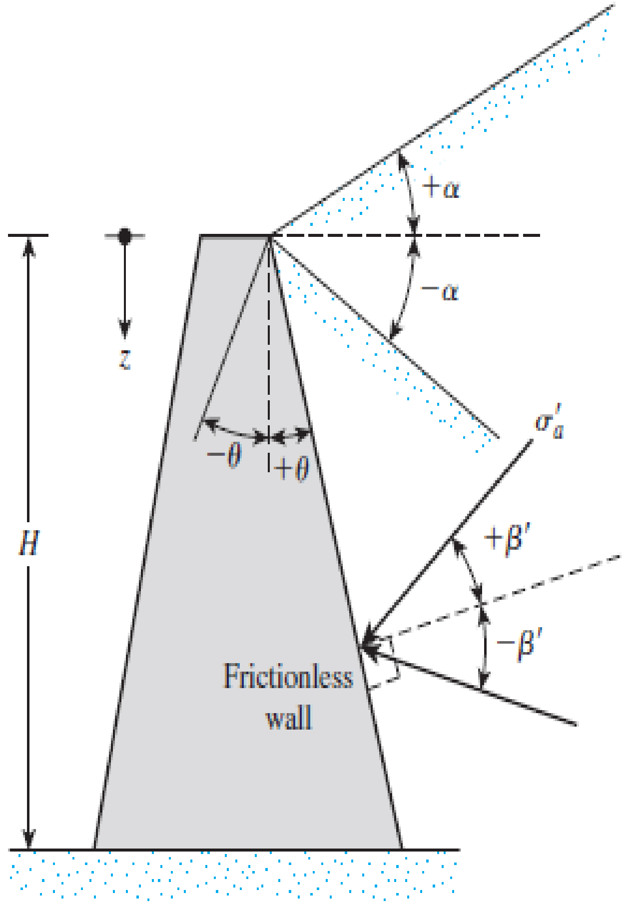
Figure 7.6c indicates that the force $P_a = 38.25$ kN/m is the area of the hatched triangle. Hence, the line of action of the resultant will be located at a height $\bar{z} = (H - z_c)/3$ above the bottom of the wall, or

$$\bar{z} = \frac{6 - 2.64}{3} = \mathbf{1.12 \text{ m}}$$

■

# A Generalized Case for Rankine Active Pressure

- **Granular Backfill**
- general cases of frictionless walls with inclined backs and inclined backfills.
- The granular backfill is inclined at an <u>angle α</u> with the horizontal.



# A Generalized Case for Rankine Active Pressure

- The lateral eath pressure at a depth *z* can be given as (Chu, 1991):

$$\sigma'_a = \frac{\gamma z \cos\alpha \sqrt{1 + \sin^2\phi' - 2\sin\phi' \cos\psi_a}}{\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}}$$

where $\psi_a = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right) - \alpha + 2\theta.$

# A Generalized Case for Rankine Active Pressure

- The pressure $\sigma`_a$ will be inclined at an angle $\beta$ with the plane drawn at right angle to the backface of the wall, and

$$\beta' = \tan^{-1}\left(\frac{\sin\phi' \sin\psi_a}{1 - \sin\phi' \cos\psi_a}\right)$$

The active force $P_a$ for unit length of the wall then can be calculated as

$$P_a = \frac{1}{2}\gamma H^2 K_a$$

where

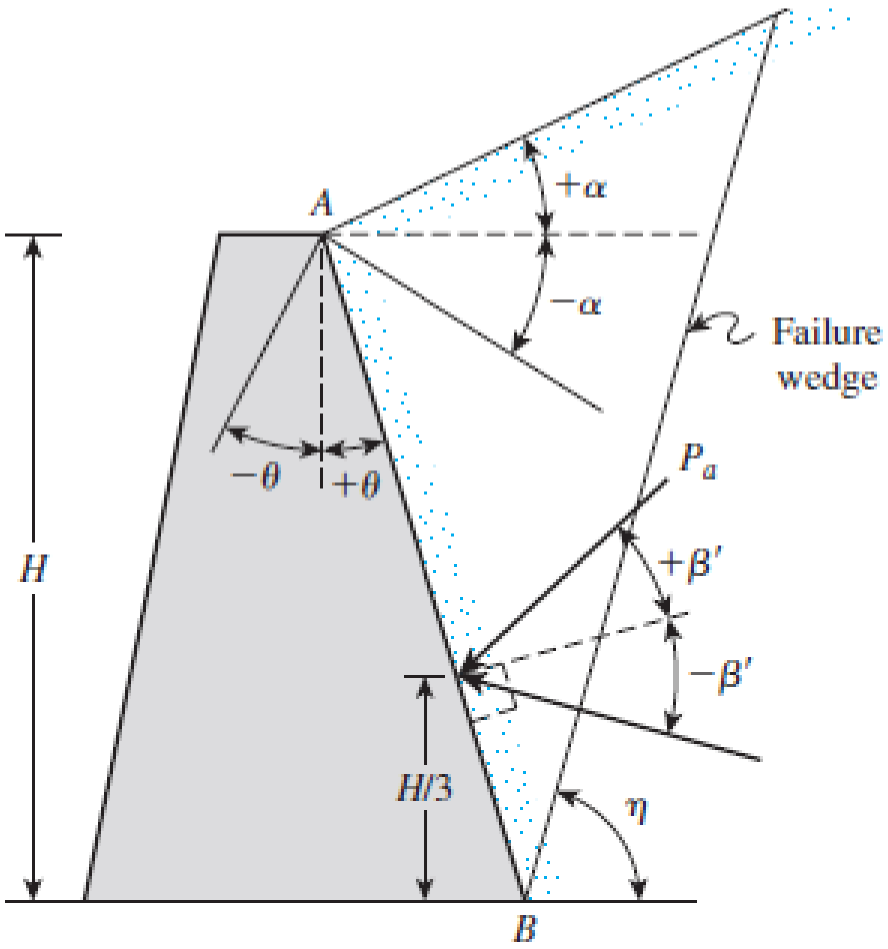
$$K_a = \frac{\cos(\alpha - \theta)\sqrt{1 + \sin^2\phi' - 2\sin\phi' \cos\psi_a}}{\cos^2\theta\left(\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}\right)}$$

= Rankine active earth-pressure coefficient for generalized case

# A Generalized Case for Rankine Active Pressure

- The location and direction of the resultant force $P_a$ is shown in Figure below. Also shown
- in this figure is the failure wedge, *ABC*. Note that *BC* will be inclined at an angle η

$$\eta = \frac{\pi}{4} + \frac{\phi'}{2} + \frac{\alpha}{2} - \frac{1}{2}\sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right)$$




$$\eta = \frac{\pi}{4} + \frac{\phi'}{2} + \frac{\alpha}{2} - \frac{1}{2}\sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right)$$

# Granular Backfill with Vertical Back Face

- As a special case, for a vertical backface of a wall (that is, $\theta = 0$ ), as shown in the figure:
- If the backfill of a frictionless retaining wall

is a *granular soil* and rises at an angle $\alpha$ with

respect to the horizontal

- **The *active earth-pressure coefficient* =**

$$K_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}}$$

Table 7.1 presents the values of $K_a$ (active earth pressure) for various values of $\alpha$ and $\phi$. (Next slide)

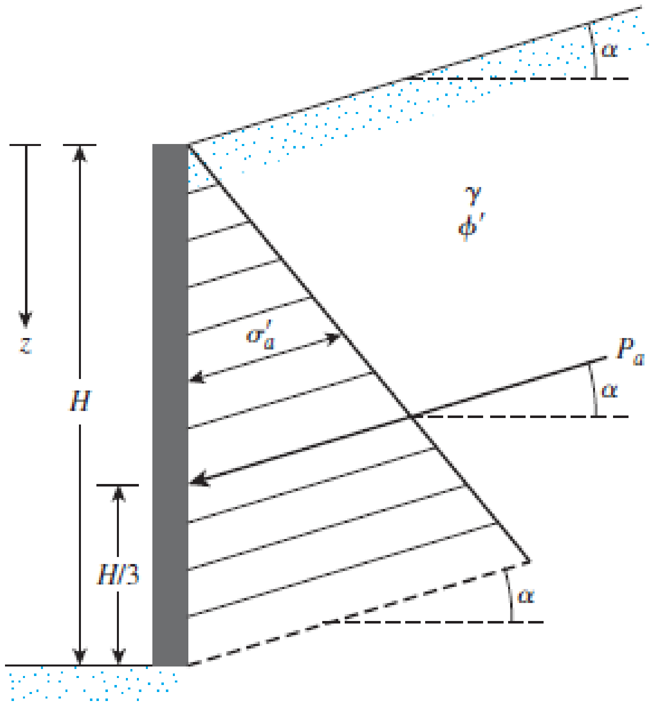


**Table 7.1** Values of $K_a$

| α (deg) ↓ | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | φ′ (deg) → | | | | | | |
| 0 | 0.3610 | 0.3470 | 0.3333 | 0.3201 | 0.3073 | 0.2948 | 0.2827 | 0.2710 | 0.2596 | 0.2486 | 0.2379 | 0.2275 | 0.21 |
| 1 | 0.3612 | 0.3471 | 0.3335 | 0.3202 | 0.3074 | 0.2949 | 0.2828 | 0.2711 | 0.2597 | 0.2487 | 0.2380 | 0.2276 | 0.21 |
| 2 | 0.3618 | 0.3476 | 0.3339 | 0.3207 | 0.3078 | 0.2953 | 0.2832 | 0.2714 | 0.2600 | 0.2489 | 0.2382 | 0.2278 | 0.21 |
| 3 | 0.3627 | 0.3485 | 0.3347 | 0.3214 | 0.3084 | 0.2959 | 0.2837 | 0.2719 | 0.2605 | 0.2494 | 0.2386 | 0.2282 | 0.21 |
| 4 | 0.3639 | 0.3496 | 0.3358 | 0.3224 | 0.3094 | 0.2967 | 0.2845 | 0.2726 | 0.2611 | 0.2500 | 0.2392 | 0.2287 | 0.21 |
| 5 | 0.3656 | 0.3512 | 0.3372 | 0.3237 | 0.3105 | 0.2978 | 0.2855 | 0.2736 | 0.2620 | 0.2508 | 0.2399 | 0.2294 | 0.21 |
| 6 | 0.3676 | 0.3531 | 0.3389 | 0.3253 | 0.3120 | 0.2992 | 0.2868 | 0.2747 | 0.2631 | 0.2518 | 0.2409 | 0.2303 | 0.22 |
| 7 | 0.3701 | 0.3553 | 0.3410 | 0.3272 | 0.3138 | 0.3008 | 0.2883 | 0.2761 | 0.2644 | 0.2530 | 0.2420 | 0.2313 | 0.22 |
| 8 | 0.3730 | 0.3580 | 0.3435 | 0.3294 | 0.3159 | 0.3027 | 0.2900 | 0.2778 | 0.2659 | 0.2544 | 0.2432 | 0.2325 | 0.22 |
| 9 | 0.3764 | 0.3611 | 0.3463 | 0.3320 | 0.3182 | 0.3049 | 0.2921 | 0.2796 | 0.2676 | 0.2560 | 0.2447 | 0.2338 | 0.22 |
| 10 | 0.3802 | 0.3646 | 0.3495 | 0.3350 | 0.3210 | 0.3074 | 0.2944 | 0.2818 | 0.2696 | 0.2578 | 0.2464 | 0.2354 | 0.22 |
| 11 | 0.3846 | 0.3686 | 0.3532 | 0.3383 | 0.3241 | 0.3103 | 0.2970 | 0.2841 | 0.2718 | 0.2598 | 0.2482 | 0.2371 | 0.22 |
| 12 | 0.3896 | 0.3731 | 0.3573 | 0.3421 | 0.3275 | 0.3134 | 0.2999 | 0.2868 | 0.2742 | 0.2621 | 0.2503 | 0.2390 | 0.22 |
| 13 | 0.3952 | 0.3782 | 0.3620 | 0.3464 | 0.3314 | 0.3170 | 0.3031 | 0.2898 | 0.2770 | 0.2646 | 0.2527 | 0.2412 | 0.23 |
| 14 | 0.4015 | 0.3839 | 0.3671 | 0.3511 | 0.3357 | 0.3209 | 0.3068 | 0.2931 | 0.2800 | 0.2674 | 0.2552 | 0.2435 | 0.23 |
| 15 | 0.4086 | 0.3903 | 0.3729 | 0.3564 | 0.3405 | 0.3253 | 0.3108 | 0.2968 | 0.2834 | 0.2705 | 0.2581 | 0.2461 | 0.23 |
| 16 | 0.4165 | 0.3975 | 0.3794 | 0.3622 | 0.3458 | 0.3302 | 0.3152 | 0.3008 | 0.2871 | 0.2739 | 0.2612 | 0.2490 | 0.23 |
| 17 | 0.4255 | 0.4056 | 0.3867 | 0.3688 | 0.3518 | 0.3356 | 0.3201 | 0.3053 | 0.2911 | 0.2776 | 0.2646 | 0.2521 | 0.24 |
| 18 | 0.4357 | 0.4146 | 0.3948 | 0.3761 | 0.3584 | 0.3415 | 0.3255 | 0.3102 | 0.2956 | 0.2817 | 0.2683 | 0.2555 | 0.24 |
| 19 | 0.4473 | 0.4249 | 0.4039 | 0.3842 | 0.3657 | 0.3481 | 0.3315 | 0.3156 | 0.3006 | 0.2862 | 0.2724 | 0.2593 | 0.24 |
| 20 | 0.4605 | 0.4365 | 0.4142 | 0.3934 | 0.3739 | 0.3555 | 0.3381 | 0.3216 | 0.3060 | 0.2911 | 0.2769 | 0.2634 | 0.25 |
| 21 | 0.4758 | 0.4498 | 0.4259 | 0.4037 | 0.3830 | 0.3637 | 0.3455 | 0.3283 | 0.3120 | 0.2965 | 0.2818 | 0.2678 | 0.25 |
| 22 | 0.4936 | 0.4651 | 0.4392 | 0.4154 | 0.3934 | 0.3729 | 0.3537 | 0.3356 | 0.3186 | 0.3025 | 0.2872 | 0.2727 | 0.25 |
| 23 | 0.5147 | 0.4829 | 0.4545 | 0.4287 | 0.4050 | 0.3832 | 0.3628 | 0.3438 | 0.3259 | 0.3091 | 0.2932 | 0.2781 | 0.26 |
| 24 | 0.5404 | 0.5041 | 0.4724 | 0.4440 | 0.4183 | 0.3948 | 0.3731 | 0.3529 | 0.3341 | 0.3164 | 0.2997 | 0.2840 | 0.26 |
| 25 | 0.5727 | 0.5299 | 0.4936 | 0.4619 | 0.4336 | 0.4081 | 0.3847 | 0.3631 | 0.3431 | 0.3245 | 0.3070 | 0.2905 | 0.27 |

# Granular Backfill with Vertical Back Face

# Granular Backfill with Vertical Back Face

- At any depth *z,* the *Rankine active pressure* may be expressed as:

$$\sigma'_a = \gamma z K_a$$

Also, the total force per unit length of the wall is:

$$P_a = \frac{1}{2}\gamma H^2 K_a$$

Note that, in this case, the direction of the resultant force $p_a$ is *inclined at an angle with the horizontal* and intersects the wall at a distance *H/3* from the base of the wall.

# Vertical Backface with (c`-ϕ`) Soil Backfill

- For a retaining wall with a *vertical back* $(\theta = 0)$ and *inclined backfill* of (c`-ϕ`) soil.

$$\sigma'_a = \gamma z K_a = \gamma z K'_a \cos\alpha$$

where

$$K'_a = \frac{1}{\cos^2\phi'}\left\{\begin{matrix} 2\cos^2\alpha + 2\left(\frac{c'}{\gamma z}\right)\cos\phi'\sin\phi' \\ -\sqrt{\left[4\cos^2\alpha(\cos^2\alpha - \cos^2\phi') + 4\left(\frac{c'}{\gamma z}\right)^2\cos^2\phi' + 8\left(\frac{c'}{\gamma z}\right)\cos^2\alpha\sin\phi'\cos\phi'\right]}\end{matrix}\right\} - 1$$

## Vertical Backface with (c`-ϕ`) Soil Backfill

- Some values of $k`_a$ are given in Table 7.2.
- For a problem of this type,
- the depth of tensile crack is given as:

$$z_c = \frac{2c'}{\gamma}\sqrt{\frac{1+\sin\phi'}{1-\sin\phi'}}$$

Table 7.2 Values of $K'_a$

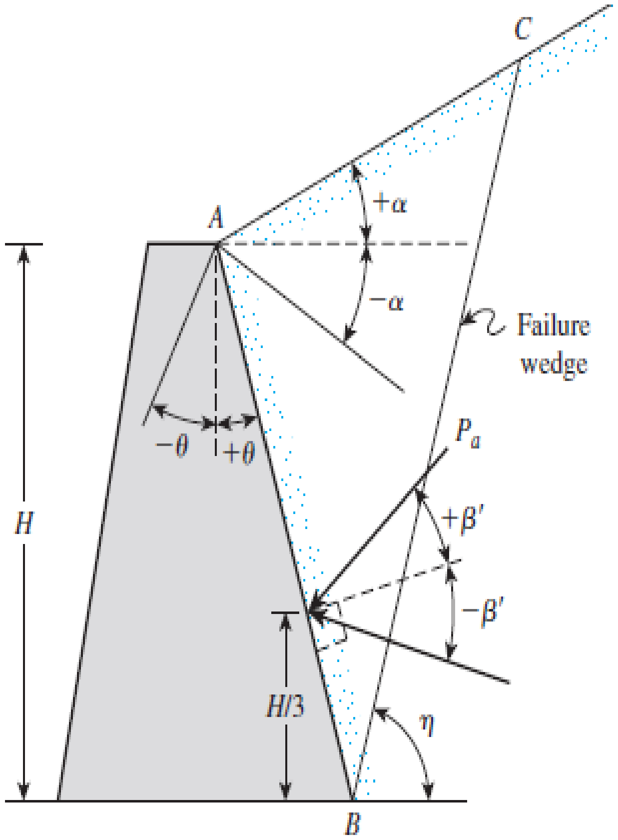
| $\phi'$ (deg) | $\alpha$ (deg) | $\frac{c'}{\gamma z}$ = 0.025 | 0.05 | 0.1 | 0.5 |
|---|---|---|---|---|---|
| 15 | 0 | 0.550 | 0.512 | 0.435 | −0.179 |
| | 5 | 0.566 | 0.525 | 0.445 | −0.184 |
| | 10 | 0.621 | 0.571 | 0.477 | −0.186 |
| | 15 | 0.776 | 0.683 | 0.546 | −0.196 |
| 20 | 0 | 0.455 | 0.420 | 0.350 | −0.210 |
| | 5 | 0.465 | 0.429 | 0.357 | −0.212 |
| | 10 | 0.497 | 0.456 | 0.377 | −0.218 |
| | 15 | 0.567 | 0.514 | 0.417 | −0.229 |
| 25 | 0 | 0.374 | 0.342 | 0.278 | −0.231 |
| | 5 | 0.381 | 0.348 | 0.283 | −0.233 |
| | 10 | 0.402 | 0.366 | 0.296 | −0.239 |
| | 15 | 0.443 | 0.401 | 0.321 | −0.250 |
| 30 | 0 | 0.305 | 0.276 | 0.218 | −0.244 |
| | 5 | 0.309 | 0.280 | 0.221 | −0.246 |
| | 10 | 0.323 | 0.292 | 0.230 | −0.252 |
| | 15 | 0.350 | 0.315 | 0.246 | −0.263 |

# Example:

Refer to the retaining wall in the figure The backfill is granular soil. Given:

| | |
|---|---|
| Wall: | $H = 10\,\text{ft}$ |
| | $\theta = +10°$ |
| Backfill: | $\alpha = 15°$ |
| | $\phi' = 35°$ |
| | $c' = 0$ |
| | $\gamma = 110\,\text{lb/ft}^3$ |

Determine the Rankine active force, $P_a$, and its location and direction.



# Solution

$$\psi_a = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right) - \alpha + 2\theta = \sin^{-1}\left(\frac{\sin 15}{\sin 35}\right) - 15 + (2)(10) = 31.82°$$

$$K_a = \frac{\cos(\alpha-\theta)\sqrt{1+\sin^2\phi' - 2\sin\phi'\cos\psi_a}}{\cos^2\theta\left(\cos\alpha+\sqrt{\sin^2\phi'-\sin^2\alpha}\right)}$$

$$= \frac{\cos(15-10)\sqrt{1+\sin^2 35 - (2)(\sin 35)(\sin 31.82)}}{\cos^2 10\left(\cos 15+\sqrt{\sin^2 35-\sin^2 15}\right)} = 0.59$$

$$P_a = ½\,\gamma H^2 K_a = (½)(110)(10)^2(0.59) \;\; = \mathbf{3245\ lb/ft}$$

$$\beta' = \tan^{-1}\left(\frac{\sin\phi'\sin\psi_a}{1-\sin\phi'\cos\psi_a}\right) = \tan^{-1}\left[\frac{(\sin 35)(\sin 31.82)}{1-(\sin 35)(\cos 31.82)}\right] = \mathbf{30.5°}$$

The force $P_a$ will act at a distance of $10/3 = 3.33$ ft above the bottom of the wall and will be inclined at an angle of $+30.5°$ to the normal drawn to the back face of the wall. ■

# Lateral Pressure Theories

24/3/2020

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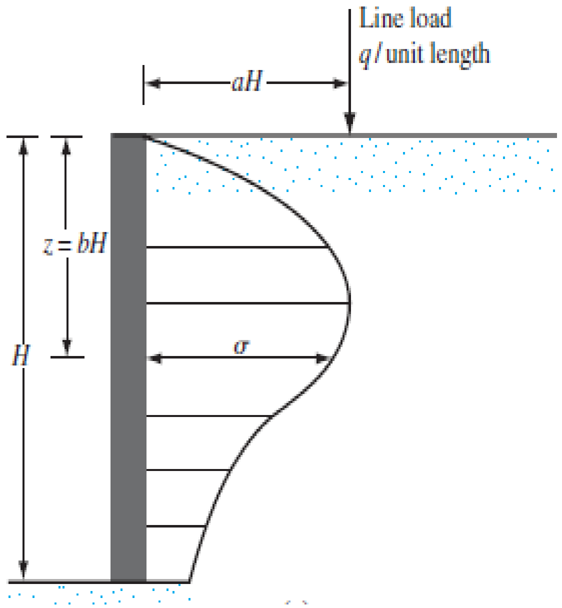
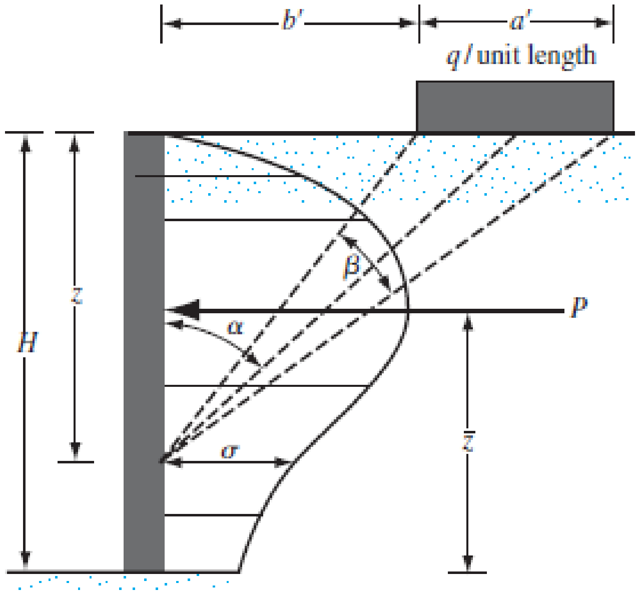
# Lateral Earth Pressure Due to Surcharge

The theory of elasticity is used to determine the lateral earth pressure on unyielding retaining structures caused by various types of surcharge loading, such as *line loading* and *strip loading*




**For line loading**

$$\sigma = \frac{2q}{\pi H}\frac{a^2 b}{(a^2 + b^2)^2}$$

**For surcharge load**

$$\sigma = \frac{2q}{\pi}(\beta - \sin\beta \cos 2\alpha)$$

- The total force per unit length (*P*) due to the *strip loading only:*

$$P = \frac{q}{90}\left[H\left(\theta_2 - \theta_1\right)\right]$$

$$\theta_1 = \tan^{-}\left(\frac{b'}{H}\right) \quad (\text{deg})$$

$$\theta_2 = \tan^{-1}\left(\frac{a' + b'}{H}\right) \quad (\text{deg})$$

# Lateral Earth Pressure Due to Surcharge

The location *z* of the resultant force, *P*, can be given as:

$$\bar{z} = H - \left[\frac{H^2(\theta_2 - \theta_1) + (R - Q) - 57.3a'H}{2H(\theta_2 - \theta_1)}\right]$$

$$R = (a' + b')^2(90 - \theta_2)$$
$$Q = b'^2(90 - \theta_1)$$

# Example:

## Example 7.8

Refer to Figure 7.14b. Here, $a' = 2$ m, $b' = 1$ m, $q = 40$ kN/m$^2$, and $H = 6$ m. Determine the total force on the wall (kN/m) caused by the strip loading only.

**Solution** From Eqs. (7.35) and (7.38),

$$\theta_1 = \tan^{-1}\left(\frac{1}{6}\right) = 9.46°$$

$$\theta_2 = \tan^{-1}\left(\frac{2 + 1}{6}\right) = 26.57°$$

From Eq. (7.34)

$$P = \frac{q}{90}[H(\theta_2 - \theta_1)] = \frac{40}{90}[6(26.57 - 9.46)] = \mathbf{45.63\ kN/m}$$

■

## Example 7.9

Refer to Example 7.8. Determine the location of the resultant $\bar{z}$.

**Solution**

From Eqs. (7.38) and (7.39),

$$R = (a' + b')^2(90 - \theta_2) = (2 + 1)^2(90 - 26.57) = 570.87$$

$$Q = b'^2(90 - \theta_1) = (1)^2(90 - 9.46) = 80.54$$

From Eq. (7.37),

$$\bar{z} = H - \left[\frac{H^2(\theta_2 - \theta_1) + (R - Q) - 57.3a'H}{2H(\theta_2 - \theta_1)}\right]$$

$$= 6 - \left[\frac{(6)^2(26.57 - 9.46) + (570.87 - 80.54) - (57.3)(2)(6)}{(2)(6)(26.57 - 9.46)}\right] = \mathbf{3.96\ m}$$

■

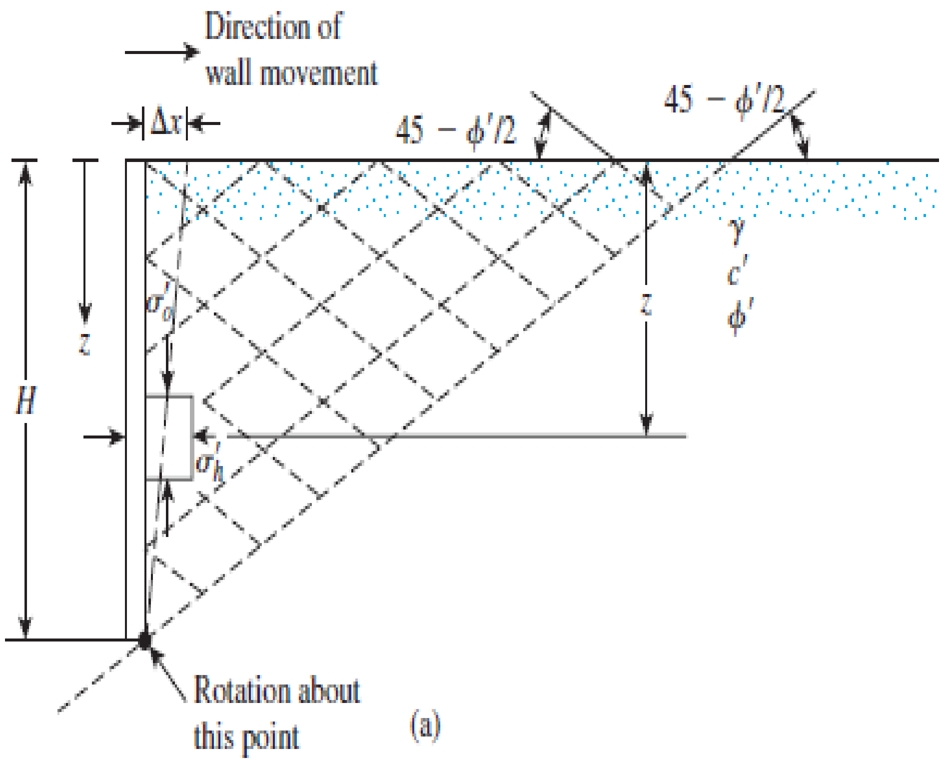
# Rankine Passive Earth Pressure

$$\sigma'_p = \sigma'_o \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

$K_p$ = Rankine passive earth pressure coefficient

$$= \tan^2\left(45 + \frac{\phi'}{2}\right)$$

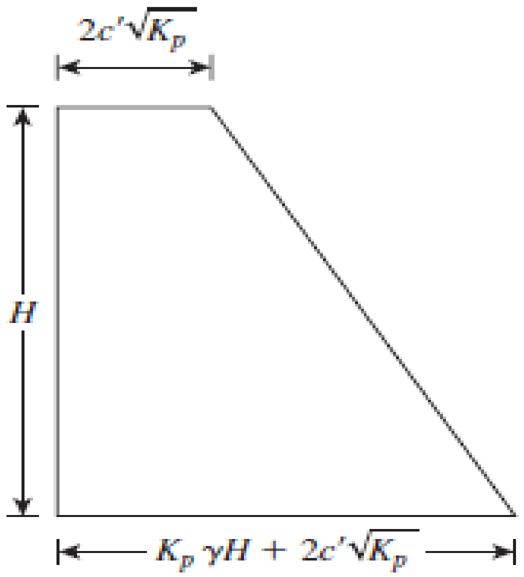
$$\sigma'_p = \sigma'_o K_p + 2c'\sqrt{K_p}$$



# Rankine Passive Earth Pressure

- The passive force per unit length of the wall can be determined from the area of the pressure diagram, or

$$P_p = \frac{1}{2}\gamma H^2 K_p + 2c' H\sqrt{K_p}$$




(c)

Rankine passive pressure

# Rankine Passive Earth Pressure

- **Rankine Passive Earth Pressure: Vertical Back face and Inclined Backfill**

## ➢ Granular Soil

For a frictionless vertical retaining wall with a *granular backfill (c`=0)* the Rankine passive pressure at any depth can be determined in a manner similar to that done in the case of active pressure.

$$\sigma'_p = \gamma z K_p$$

and the passive force is

$$P_p = \frac{1}{2}\gamma H^2 K_p$$

where

$$K_p = \cos\alpha \frac{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}}$$

$$K_p = \cos\alpha \frac{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}}$$

*Table 7.8* Passive Earth Pressure Coefficient $K_p$ [from Eq. (7.67)]

| | $\phi'$ (deg)→ | | | | | | |
|---|---|---|---|---|---|---|---|
| ↓$\alpha$ (deg) | 28 | 30 | 32 | 34 | 36 | 38 | 40 |
| 0 | 2.770 | 3.000 | 3.255 | 3.537 | 3.852 | 4.204 | 4.599 |
| 5 | 2.715 | 2.943 | 3.196 | 3.476 | 3.788 | 4.136 | 4.527 |
| 10 | 2.551 | 2.775 | 3.022 | 3.295 | 3.598 | 3.937 | 4.316 |
| 15 | 2.284 | 2.502 | 2.740 | 3.003 | 3.293 | 3.615 | 3.977 |
| 20 | 1.918 | 2.132 | 2.362 | 2.612 | 2.886 | 3.189 | 3.526 |
| 25 | 1.434 | 1.664 | 1.894 | 2.135 | 2.394 | 2.676 | 2.987 |

# Rankine Passive Earth Pressure

## ➢c`-ϕ` soil

If the backfill of the frictionless vertical retaining wall is a **c`-ϕ** soil (see Figure 7.10), then

$$\sigma'_a = \gamma z K_p = \gamma z K'_p \cos\alpha$$

where

$$K'_p = \frac{1}{\cos^2\phi'}\left\{\frac{2\cos^2\alpha + 2\left(\frac{c'}{\gamma z}\right)\cos\phi'\sin\phi'}{+\sqrt{4\cos^2\alpha(\cos^2\alpha - \cos^2\phi') + 4\left(\frac{c'}{\gamma z}\right)^2\cos^2\phi' + 8\left(\frac{c'}{\gamma z}\right)\cos^2\alpha\sin\phi'\cos\phi'}}\right\} - 1$$

The variation of $K'_p$ with $\phi', \alpha$, and $c'/\gamma z$ is given in Table 7.9 (Mazindrani and Ganjali, 1997).

**Table 7.9** Values of $K'_p$

| | | $c'/\gamma z$ | | | |
|---|---|---|---|---|---|
| $\phi'$ (deg) | $\alpha$ (deg) | 0.025 | 0.050 | 0.100 | 0.500 |
| 15 | 0 | 1.764 | 1.829 | 1.959 | 3.002 |
| | 5 | 1.716 | 1.783 | 1.917 | 2.971 |
| | 10 | 1.564 | 1.641 | 1.788 | 2.880 |
| | 15 | 1.251 | 1.370 | 1.561 | 2.732 |
| 20 | 0 | 2.111 | 2.182 | 2.325 | 3.468 |
| | 5 | 2.067 | 2.140 | 2.285 | 3.435 |
| | 10 | 1.932 | 2.010 | 2.162 | 3.339 |
| | 15 | 1.696 | 1.786 | 1.956 | 3.183 |
| 25 | 0 | 2.542 | 2.621 | 2.778 | 4.034 |
| | 5 | 2.499 | 2.578 | 2.737 | 3.999 |
| | 10 | 2.368 | 2.450 | 2.614 | 3.895 |
| | 15 | 2.147 | 2.236 | 2.409 | 3.726 |
| 30 | 0 | 3.087 | 3.173 | 3.346 | 4.732 |
| | 5 | 3.042 | 3.129 | 3.303 | 4.674 |
| | 10 | 2.907 | 2.996 | 3.174 | 4.579 |
| | 15 | 2.684 | 2.777 | 2.961 | 4.394 |

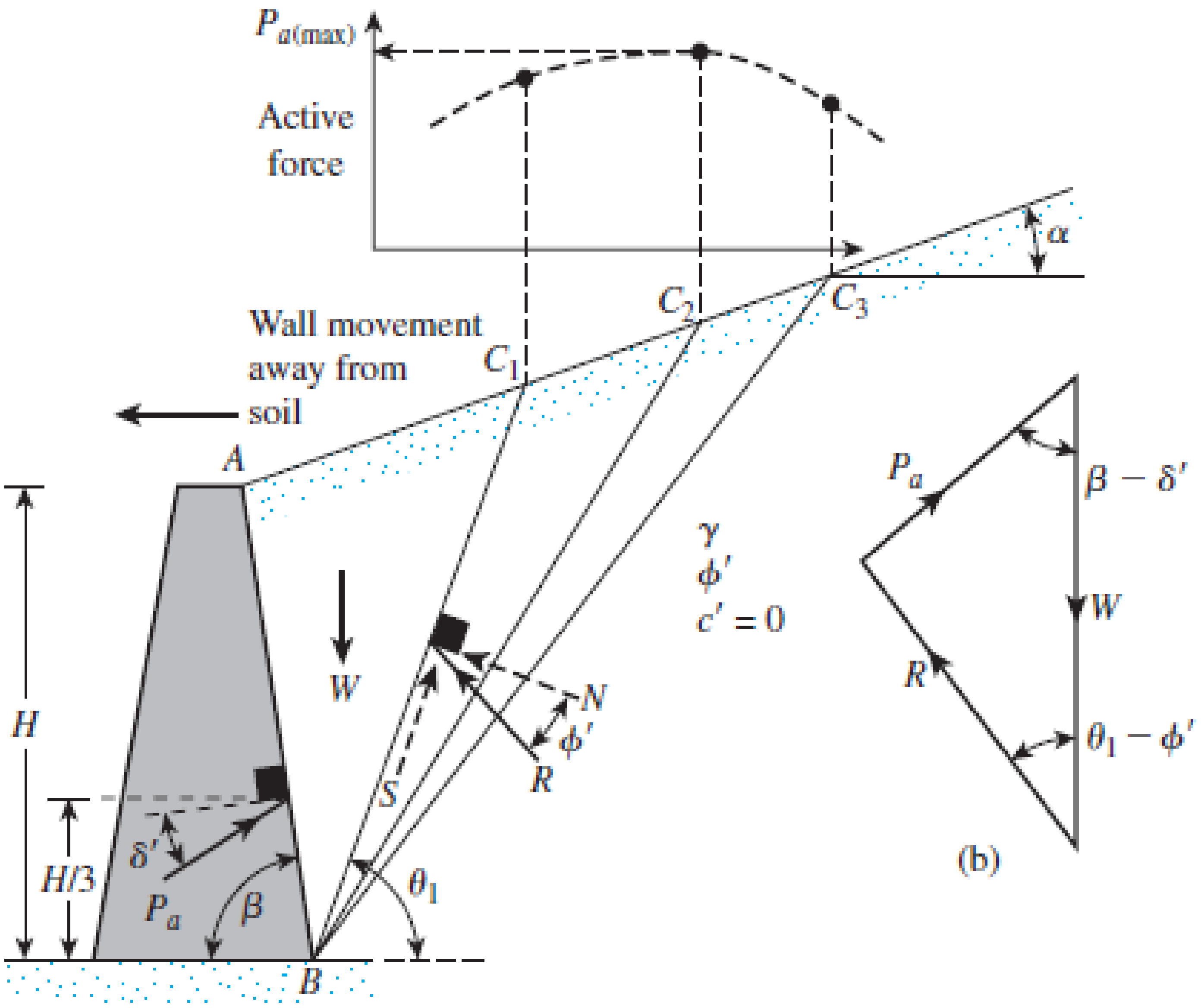
# Coulomb’s Active Earth Pressure

- Coulomb proposed a theory for calculating the lateral earth pressure on a retaining wall with granular soil backfill. This theory takes wall friction into consideration.

❖ Back face inclined at an angle $\beta$ with the horizontal, as shown in Figure.

❖ The backfill is a granular soil that slopes at an angle $\alpha$ with the horizontal.

❖ Let δ` be the angle of friction between the soil and the wall (i.e., the angle of wall friction).



# Coulomb's Active Earth Pressure

- Assumptions:
- Wall friction
- The failure surface in soil mass would be a plane BC1, BC2..

# Coulomb's Active Earth Pressure

- The maximum value of $p_a$ thus determined is Coulomb's active force which may be expressed as:

$$P_a = \tfrac{1}{2} K_a \gamma H^2$$

$K_a$ = Coulomb's active earth pressure coefficient

$$= \frac{\sin^2(\beta + \phi')}{\sin^2\beta \sin(\beta - \delta')\left[1 + \sqrt{\dfrac{\sin(\phi' + \delta')\sin(\phi' - \alpha)}{\sin(\beta - \delta')\sin(\alpha + \beta)}}\right]^2}$$

# Coulomb's Active Earth Pressure

- The values of the active earth pressure coefficient $k_a$ , for a vertical retaining wall ($\beta = 90^o$) with horizontal backfill ($\alpha = 0$) are given in Table 7.3.

**Table 7.3** Values of $K_a$ [Eq. (7.26)] for $\beta = 90°$ and $\alpha = 0°$

| | $\delta'$ (deg) | | | | | |
|---|---|---|---|---|---|---|
| $\phi'$ (deg) | 0 | 5 | 10 | 15 | 20 | 25 |
| 28 | 0.3610 | 0.3448 | 0.3330 | 0.3251 | 0.3203 | 0.3186 |
| 30 | 0.3333 | 0.3189 | 0.3085 | 0.3014 | 0.2973 | 0.2956 |
| 32 | 0.3073 | 0.2945 | 0.2853 | 0.2791 | 0.2755 | 0.2745 |
| 34 | 0.2827 | 0.2714 | 0.2633 | 0.2579 | 0.2549 | 0.2542 |
| 36 | 0.2596 | 0.2497 | 0.2426 | 0.2379 | 0.2354 | 0.2350 |
| 38 | 0.2379 | 0.2292 | 0.2230 | 0.2190 | 0.2169 | 0.2167 |
| 40 | 0.2174 | 0.2098 | 0.2045 | 0.2011 | 0.1994 | 0.1995 |
| 42 | 0.1982 | 0.1916 | 0.1870 | 0.1841 | 0.1828 | 0.1831 |

**Table 7.4** Values of $K_a$ [from Eq. (7.26)] for $\delta' = \frac{2}{3}\phi'$

| | | β (deg) | | | | | |
|---|---|---|---|---|---|---|---|
| α (deg) | φ′ (deg) | 90 | 85 | 80 | 75 | 70 | 65 |
| 0 | 28 | 0.3213 | 0.3588 | 0.4007 | 0.4481 | 0.5026 | 0.5662 |
| | 29 | 0.3091 | 0.3467 | 0.3886 | 0.4362 | 0.4908 | 0.5547 |
| | 30 | 0.2973 | 0.3349 | 0.3769 | 0.4245 | 0.4794 | 0.5435 |
| | 31 | 0.2860 | 0.3235 | 0.3655 | 0.4133 | 0.4682 | 0.5326 |
| | 32 | 0.2750 | 0.3125 | 0.3545 | 0.4023 | 0.4574 | 0.5220 |
| | 33 | 0.2645 | 0.3019 | 0.3439 | 0.3917 | 0.4469 | 0.5117 |
| | 34 | 0.2543 | 0.2916 | 0.3335 | 0.3813 | 0.4367 | 0.5017 |
| | 35 | 0.2444 | 0.2816 | 0.3235 | 0.3713 | 0.4267 | 0.4919 |
| | 36 | 0.2349 | 0.2719 | 0.3137 | 0.3615 | 0.4170 | 0.4824 |
| | 37 | 0.2257 | 0.2626 | 0.3042 | 0.3520 | 0.4075 | 0.4732 |
| | 38 | 0.2168 | 0.2535 | 0.2950 | 0.3427 | 0.3983 | 0.4641 |
| | 39 | 0.2082 | 0.2447 | 0.2861 | 0.3337 | 0.3894 | 0.4553 |
| | 40 | 0.1998 | 0.2361 | 0.2774 | 0.3249 | 0.3806 | 0.4468 |
| | 41 | 0.1918 | 0.2278 | 0.2689 | 0.3164 | 0.3721 | 0.4384 |
| | 42 | 0.1840 | 0.2197 | 0.2606 | 0.3080 | 0.3637 | 0.4302 |
| 5 | 28 | 0.3431 | 0.3845 | 0.4311 | 0.4843 | 0.5461 | 0.6190 |
| | 29 | 0.3295 | 0.3709 | 0.4175 | 0.4707 | 0.5325 | 0.6056 |
| | 30 | 0.3165 | 0.3578 | 0.4043 | 0.4575 | 0.5194 | 0.5926 |
| | 31 | 0.3039 | 0.3451 | 0.3916 | 0.4447 | 0.5067 | 0.5800 |
| | 32 | 0.2919 | 0.3329 | 0.3792 | 0.4324 | 0.4943 | 0.5677 |
| | 33 | 0.2803 | 0.3211 | 0.3673 | 0.4204 | 0.4823 | 0.5558 |
| | 34 | 0.2691 | 0.3097 | 0.3558 | 0.4088 | 0.4707 | 0.5443 |
| | 35 | 0.2583 | 0.2987 | 0.3446 | 0.3975 | 0.4594 | 0.5330 |
| | 36 | 0.2479 | 0.2881 | 0.3338 | 0.3866 | 0.4484 | 0.5221 |
| | 37 | 0.2379 | 0.2778 | 0.3233 | 0.3759 | 0.4377 | 0.5115 |
| | 38 | 0.2282 | 0.2679 | 0.3131 | 0.3656 | 0.4273 | 0.5012 |
| | 39 | 0.2188 | 0.2582 | 0.3033 | 0.3556 | 0.4172 | 0.4911 |
| | 40 | 0.2098 | 0.2489 | 0.2937 | 0.3458 | 0.4074 | 0.4813 |
| | 41 | 0.2011 | 0.2398 | 0.2844 | 0.3363 | 0.3978 | 0.4718 |
| | 42 | 0.1927 | 0.2311 | 0.2753 | 0.3271 | 0.3884 | 0.4625 |
| 10 | 28 | 0.3702 | 0.4164 | 0.4686 | 0.5287 | 0.5992 | 0.6834 |
| | 29 | 0.3548 | 0.4007 | 0.4528 | 0.5128 | 0.5831 | 0.6672 |
| | 30 | 0.3400 | 0.3857 | 0.4376 | 0.4974 | 0.5676 | 0.6516 |
| | 31 | 0.3259 | 0.3713 | 0.4230 | 0.4826 | 0.5526 | 0.6365 |
| | 32 | 0.3123 | 0.3575 | 0.4089 | 0.4683 | 0.5382 | 0.6219 |
| | 33 | 0.2993 | 0.3442 | 0.3953 | 0.4545 | 0.5242 | 0.6078 |
| | 34 | 0.2868 | 0.3314 | 0.3822 | 0.4412 | 0.5107 | 0.5942 |
| | 35 | 0.2748 | 0.3190 | 0.3696 | 0.4283 | 0.4976 | 0.5810 |
| | 36 | 0.2633 | 0.3072 | 0.3574 | 0.4158 | 0.4849 | 0.5682 |
| | 37 | 0.2522 | 0.2957 | 0.3456 | 0.4037 | 0.4726 | 0.5558 |
| | 38 | 0.2415 | 0.2846 | 0.3342 | 0.3920 | 0.4607 | 0.5437 |
| | 39 | 0.2313 | 0.2740 | 0.3231 | 0.3807 | 0.4491 | 0.5321 |
| | 40 | 0.2214 | 0.2636 | 0.3125 | 0.3697 | 0.4379 | 0.5207 |
| | 41 | 0.2119 | 0.2537 | 0.3021 | 0.3590 | 0.4270 | 0.5097 |
| | 42 | 0.2027 | 0.2441 | 0.2921 | 0.3487 | 0.4164 | 0.4990 |
| 15 | 28 | 0.4065 | 0.4585 | 0.5179 | 0.5868 | 0.6685 | 0.7670 |

*(continued)*

**Table 7.4** *(Continued)*

| α (deg) | φ′ (deg) | β (deg) 90 | 85 | 80 | 75 | 70 | 65 |
|---|---|---|---|---|---|---|---|
| | 29 | 0.3881 | 0.4397 | 0.4987 | 0.5672 | 0.6483 | 0.7463 |
| | 30 | 0.3707 | 0.4219 | 0.4804 | 0.5484 | 0.6291 | 0.7265 |
| | 31 | 0.3541 | 0.4049 | 0.4629 | 0.5305 | 0.6106 | 0.7076 |
| | 32 | 0.3384 | 0.3887 | 0.4462 | 0.5133 | 0.5930 | 0.6895 |
| | 33 | 0.3234 | 0.3732 | 0.4303 | 0.4969 | 0.5761 | 0.6721 |
| | 34 | 0.3091 | 0.3583 | 0.4150 | 0.4811 | 0.5598 | 0.6554 |
| | 35 | 0.2954 | 0.3442 | 0.4003 | 0.4659 | 0.5442 | 0.6393 |
| | 36 | 0.2823 | 0.3306 | 0.3862 | 0.4513 | 0.5291 | 0.6238 |
| | 37 | 0.2698 | 0.3175 | 0.3726 | 0.4373 | 0.5146 | 0.6089 |
| | 38 | 0.2578 | 0.3050 | 0.3595 | 0.4237 | 0.5006 | 0.5945 |
| | 39 | 0.2463 | 0.2929 | 0.3470 | 0.4106 | 0.4871 | 0.5805 |
| | 40 | 0.2353 | 0.2813 | 0.3348 | 0.3980 | 0.4740 | 0.5671 |
| | 41 | 0.2247 | 0.2702 | 0.3231 | 0.3858 | 0.4613 | 0.5541 |
| | 42 | 0.2146 | 0.2594 | 0.3118 | 0.3740 | 0.4491 | 0.5415 |
| 20 | 28 | 0.4602 | 0.5205 | 0.5900 | 0.6714 | 0.7689 | 0.8880 |
| | 29 | 0.4364 | 0.4958 | 0.5642 | 0.6445 | 0.7406 | 0.8581 |
| | 30 | 0.4142 | 0.4728 | 0.5403 | 0.6195 | 0.7144 | 0.8303 |
| | 31 | 0.3935 | 0.4513 | 0.5179 | 0.5961 | 0.6898 | 0.8043 |
| | 32 | 0.3742 | 0.4311 | 0.4968 | 0.5741 | 0.6666 | 0.7799 |
| | 33 | 0.3559 | 0.4121 | 0.4769 | 0.5532 | 0.6448 | 0.7569 |
| | 34 | 0.3388 | 0.3941 | 0.4581 | 0.5335 | 0.6241 | 0.7351 |
| | 35 | 0.3225 | 0.3771 | 0.4402 | 0.5148 | 0.6044 | 0.7144 |
| | 36 | 0.3071 | 0.3609 | 0.4233 | 0.4969 | 0.5856 | 0.6947 |
| | 37 | 0.2925 | 0.3455 | 0.4071 | 0.4799 | 0.5677 | 0.6759 |
| | 38 | 0.2787 | 0.3308 | 0.3916 | 0.4636 | 0.5506 | 0.6579 |
| | 39 | 0.2654 | 0.3168 | 0.3768 | 0.4480 | 0.5342 | 0.6407 |
| | 40 | 0.2529 | 0.3034 | 0.3626 | 0.4331 | 0.5185 | 0.6242 |
| | 41 | 0.2408 | 0.2906 | 0.3490 | 0.4187 | 0.5033 | 0.6083 |
| | 42 | 0.2294 | 0.2784 | 0.3360 | 0.4049 | 0.4888 | 0.5930 |

*Table 7.5* Values of $K_a$ [from Eq. (7.26)] for $\delta' = \phi'/2$

| | | β (deg) | | | | | |
|---|---|---|---|---|---|---|---|
| α (deg) | φ′ (deg) | 90 | 85 | 80 | 75 | 70 | 65 |
| 0 | 28 | 0.3264 | 0.3629 | 0.4034 | 0.4490 | 0.5011 | 0.5616 |
| | 29 | 0.3137 | 0.3502 | 0.3907 | 0.4363 | 0.4886 | 0.5492 |
| | 30 | 0.3014 | 0.3379 | 0.3784 | 0.4241 | 0.4764 | 0.5371 |
| | 31 | 0.2896 | 0.3260 | 0.3665 | 0.4121 | 0.4645 | 0.5253 |
| | 32 | 0.2782 | 0.3145 | 0.3549 | 0.4005 | 0.4529 | 0.5137 |
| | 33 | 0.2671 | 0.3033 | 0.3436 | 0.3892 | 0.4415 | 0.5025 |
| | 34 | 0.2564 | 0.2925 | 0.3327 | 0.3782 | 0.4305 | 0.4915 |
| | 35 | 0.2461 | 0.2820 | 0.3221 | 0.3675 | 0.4197 | 0.4807 |
| | 36 | 0.2362 | 0.2718 | 0.3118 | 0.3571 | 0.4092 | 0.4702 |

**Table 7.5** *(Continued)*

| α (deg) | ϕ′ (deg) | β (deg) 90 | 85 | 80 | 75 | 70 | 65 |
|---|---|---|---|---|---|---|---|
| | 37 | 0.2265 | 0.2620 | 0.3017 | 0.3469 | 0.3990 | 0.4599 |
| | 38 | 0.2172 | 0.2524 | 0.2920 | 0.3370 | 0.3890 | 0.4498 |
| | 39 | 0.2081 | 0.2431 | 0.2825 | 0.3273 | 0.3792 | 0.4400 |
| | 40 | 0.1994 | 0.2341 | 0.2732 | 0.3179 | 0.3696 | 0.4304 |
| | 41 | 0.1909 | 0.2253 | 0.2642 | 0.3087 | 0.3602 | 0.4209 |
| | 42 | 0.1828 | 0.2168 | 0.2554 | 0.2997 | 0.3511 | 0.4177 |
| 5 | 28 | 0.3477 | 0.3879 | 0.4327 | 0.4837 | 0.5425 | 0.6115 |
| | 29 | 0.3337 | 0.3737 | 0.4185 | 0.4694 | 0.5282 | 0.5972 |
| | 30 | 0.3202 | 0.3601 | 0.4048 | 0.4556 | 0.5144 | 0.5833 |
| | 31 | 0.3072 | 0.3470 | 0.3915 | 0.4422 | 0.5009 | 0.5698 |
| | 32 | 0.2946 | 0.3342 | 0.3787 | 0.4292 | 0.4878 | 0.5566 |
| | 33 | 0.2825 | 0.3219 | 0.3662 | 0.4166 | 0.4750 | 0.5437 |
| | 34 | 0.2709 | 0.3101 | 0.3541 | 0.4043 | 0.4626 | 0.5312 |
| | 35 | 0.2596 | 0.2986 | 0.3424 | 0.3924 | 0.4505 | 0.5190 |
| | 36 | 0.2488 | 0.2874 | 0.3310 | 0.3808 | 0.4387 | 0.5070 |
| | 37 | 0.2383 | 0.2767 | 0.3199 | 0.3695 | 0.4272 | 0.4954 |
| | 38 | 0.2282 | 0.2662 | 0.3092 | 0.3585 | 0.4160 | 0.4840 |
| | 39 | 0.2185 | 0.2561 | 0.2988 | 0.3478 | 0.4050 | 0.4729 |
| | 40 | 0.2090 | 0.2463 | 0.2887 | 0.3374 | 0.3944 | 0.4620 |
| | 41 | 0.1999 | 0.2368 | 0.2788 | 0.3273 | 0.3840 | 0.4514 |
| | 42 | 0.1911 | 0.2276 | 0.2693 | 0.3174 | 0.3738 | 0.4410 |
| 10 | 28 | 0.3743 | 0.4187 | 0.4688 | 0.5261 | 0.5928 | 0.6719 |
| | 29 | 0.3584 | 0.4026 | 0.4525 | 0.5096 | 0.5761 | 0.6549 |
| | 30 | 0.3432 | 0.3872 | 0.4368 | 0.4936 | 0.5599 | 0.6385 |
| | 31 | 0.3286 | 0.3723 | 0.4217 | 0.4782 | 0.5442 | 0.6225 |
| | 32 | 0.3145 | 0.3580 | 0.4071 | 0.4633 | 0.5290 | 0.6071 |
| | 33 | 0.3011 | 0.3442 | 0.3930 | 0.4489 | 0.5143 | 0.5920 |
| | 34 | 0.2881 | 0.3309 | 0.3793 | 0.4350 | 0.5000 | 0.5775 |
| | 35 | 0.2757 | 0.3181 | 0.3662 | 0.4215 | 0.4862 | 0.5633 |
| | 36 | 0.2637 | 0.3058 | 0.3534 | 0.4084 | 0.4727 | 0.5495 |
| | 37 | 0.2522 | 0.2938 | 0.3411 | 0.3957 | 0.4597 | 0.5361 |
| | 38 | 0.2412 | 0.2823 | 0.3292 | 0.3833 | 0.4470 | 0.5230 |
| | 39 | 0.2305 | 0.2712 | 0.3176 | 0.3714 | 0.4346 | 0.5103 |
| | 40 | 0.2202 | 0.2604 | 0.3064 | 0.3597 | 0.4226 | 0.4979 |
| | 41 | 0.2103 | 0.2500 | 0.2956 | 0.3484 | 0.4109 | 0.4858 |
| | 42 | 0.2007 | 0.2400 | 0.2850 | 0.3375 | 0.3995 | 0.4740 |
| 15 | 28 | 0.4095 | 0.4594 | 0.5159 | 0.5812 | 0.6579 | 0.7498 |
| | 29 | 0.3908 | 0.4402 | 0.4964 | 0.5611 | 0.6373 | 0.7284 |
| | 30 | 0.3730 | 0.4220 | 0.4777 | 0.5419 | 0.6175 | 0.7080 |
| | 31 | 0.3560 | 0.4046 | 0.4598 | 0.5235 | 0.5985 | 0.6884 |
| | 32 | 0.3398 | 0.3880 | 0.4427 | 0.5059 | 0.5803 | 0.6695 |
| | 33 | 0.3244 | 0.3721 | 0.4262 | 0.4889 | 0.5627 | 0.6513 |
| | 34 | 0.3097 | 0.3568 | 0.4105 | 0.4726 | 0.5458 | 0.6338 |
| | 35 | 0.2956 | 0.3422 | 0.3953 | 0.4569 | 0.5295 | 0.6168 |

*(continued)*

Table 7.5 (Continued)

| α (deg) | φ′ (deg) | β (deg) 90 | 85 | 80 | 75 | 70 | 65 |
|---|---|---|---|---|---|---|---|
| | 36 | 0.2821 | 0.3282 | 0.3807 | 0.4417 | 0.5138 | 0.6004 |
| | 37 | 0.2692 | 0.3147 | 0.3667 | 0.4271 | 0.4985 | 0.5846 |
| | 38 | 0.2569 | 0.3017 | 0.3531 | 0.4130 | 0.4838 | 0.5692 |
| | 39 | 0.2450 | 0.2893 | 0.3401 | 0.3993 | 0.4695 | 0.5543 |
| | 40 | 0.2336 | 0.2773 | 0.3275 | 0.3861 | 0.4557 | 0.5399 |
| | 41 | 0.2227 | 0.2657 | 0.3153 | 0.3733 | 0.4423 | 0.5258 |
| | 42 | 0.2122 | 0.2546 | 0.3035 | 0.3609 | 0.4293 | 0.5122 |
| 20 | 28 | 0.4614 | 0.5188 | 0.5844 | 0.6608 | 0.7514 | 0.8613 |
| | 29 | 0.4374 | 0.4940 | 0.5586 | 0.6339 | 0.7232 | 0.8313 |
| | 30 | 0.4150 | 0.4708 | 0.5345 | 0.6087 | 0.6968 | 0.8034 |
| | 31 | 0.3941 | 0.4491 | 0.5119 | 0.5851 | 0.6720 | 0.7772 |
| | 32 | 0.3744 | 0.4286 | 0.4906 | 0.5628 | 0.6486 | 0.7524 |
| | 33 | 0.3559 | 0.4093 | 0.4704 | 0.5417 | 0.6264 | 0.7289 |
| | 34 | 0.3384 | 0.3910 | 0.4513 | 0.5216 | 0.6052 | 0.7066 |
| | 35 | 0.3218 | 0.3736 | 0.4331 | 0.5025 | 0.5851 | 0.6853 |
| | 36 | 0.3061 | 0.3571 | 0.4157 | 0.4842 | 0.5658 | 0.6649 |
| | 37 | 0.2911 | 0.3413 | 0.3991 | 0.4668 | 0.5474 | 0.6453 |
| | 38 | 0.2769 | 0.3263 | 0.3833 | 0.4500 | 0.5297 | 0.6266 |
| | 39 | 0.2633 | 0.3120 | 0.3681 | 0.4340 | 0.5127 | 0.6085 |
| | 40 | 0.2504 | 0.2982 | 0.3535 | 0.4185 | 0.4963 | 0.5912 |
| | 41 | 0.2381 | 0.2851 | 0.3395 | 0.4037 | 0.4805 | 0.5744 |
| | 42 | 0.2263 | 0.2725 | 0.3261 | 0.3894 | 0.4653 | 0.5582 |

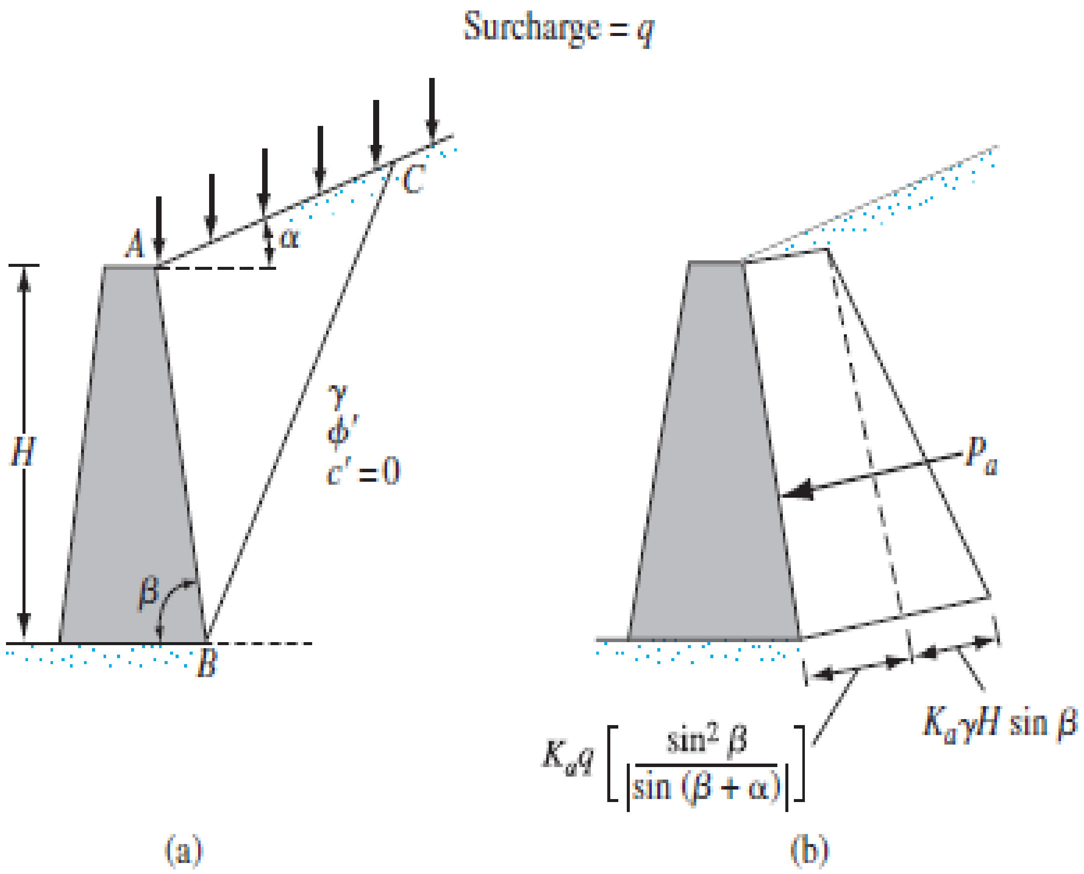
# Coulomb's active pressure with a surcharge on the backfill

If a uniform surcharge of intensity *q* is located above the backfill, as shown in Figure the active force, *Pa*, can be calculated as:

$$P_a = \frac{1}{2} K_a \gamma_{eq} H^2$$

↑
Eq. (7.25)

$$\gamma_{eq} = \gamma + \left[\frac{\sin\beta}{\sin(\beta + \alpha)}\right]\left(\frac{2q}{H}\right)$$



# Retaining Wall Stability

5/4/2020

# Retaining wall

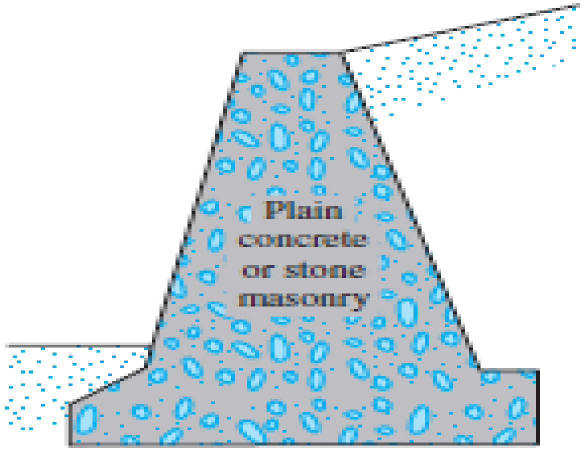
- In general, retaining walls can be divided into two major categories:

(a) conventional retaining walls

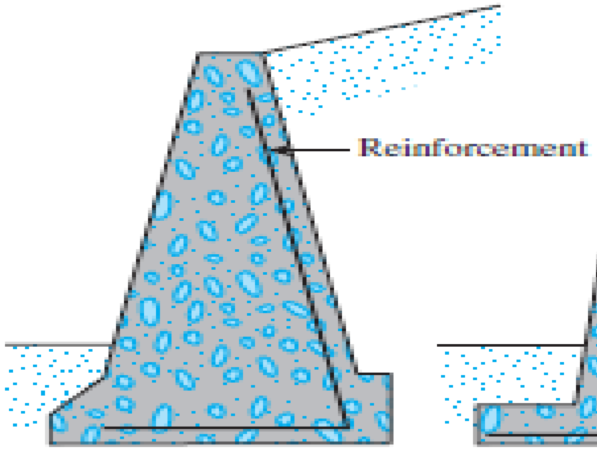
(b) mechanically stabilized earth walls (MSE)

# Retaining wall

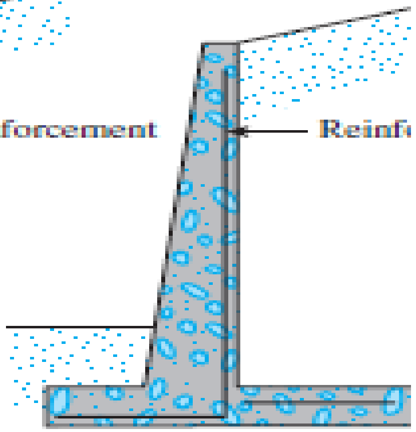
- Conventional retaining walls can generally be classified into four varieties:
- 1. Gravity retaining walls
- 2. Semigravity retaining walls
- 3. Cantilever retaining walls
- 4. Counterfort retaining walls

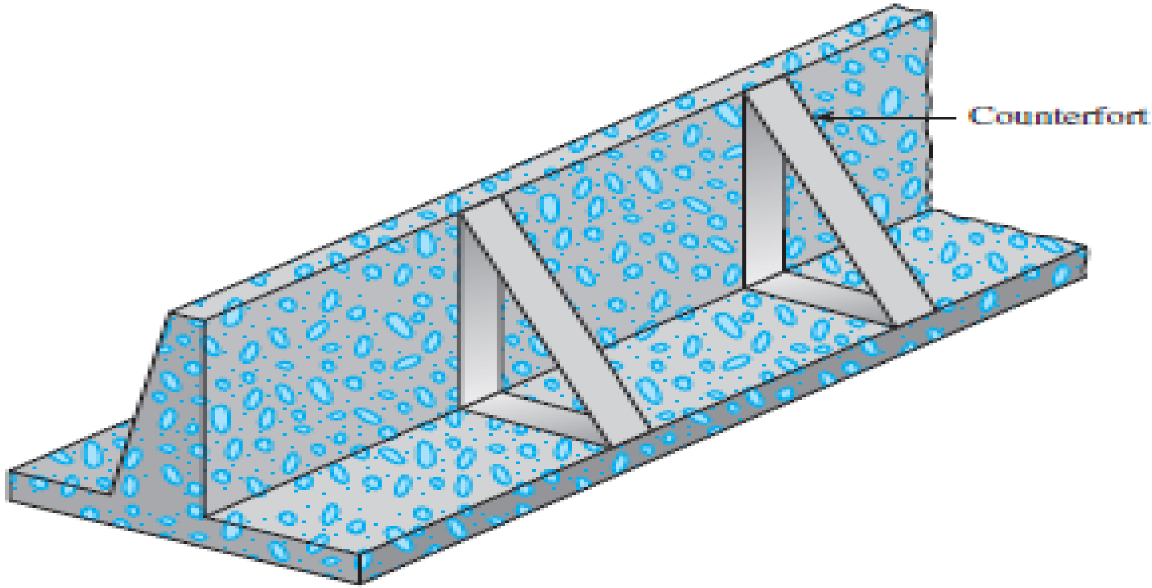


(a) Gravity wall



(b) Semigravity wall


(c) Cantilever wall



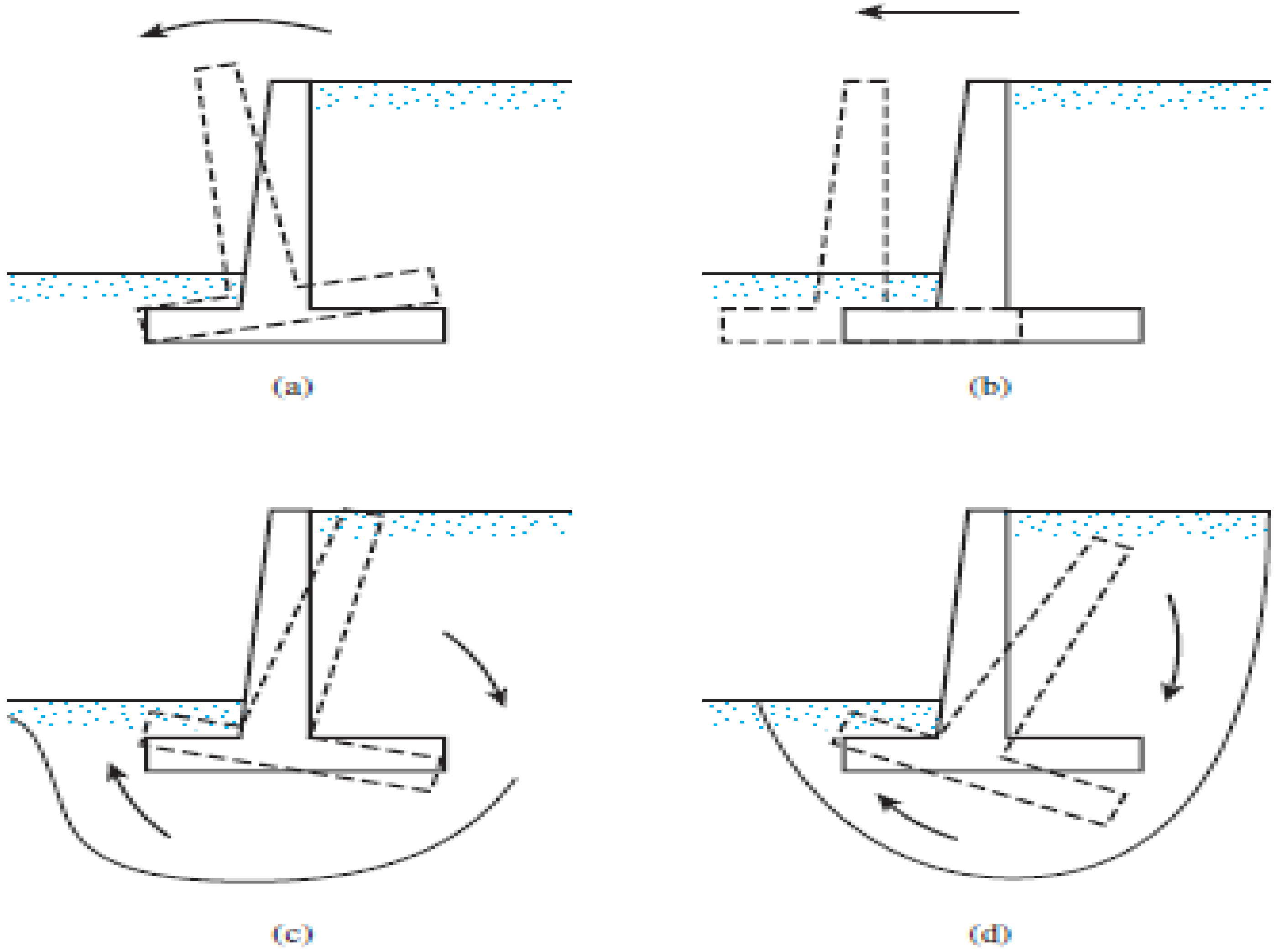
(d) Counterfort wall

# Stability of retaining wall:

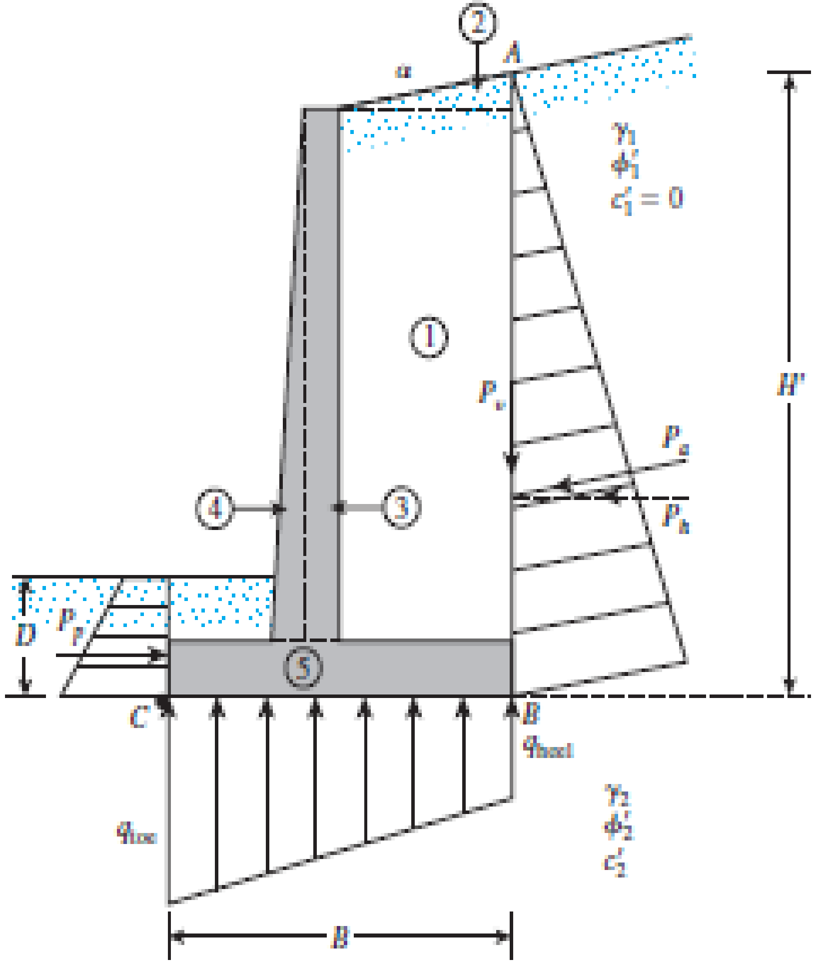
- The structure is examined for possible:
- ***Overturning***
- ***Sliding***
- ***Bearing capacity* failures.**

# Stability of Retaining Walls:

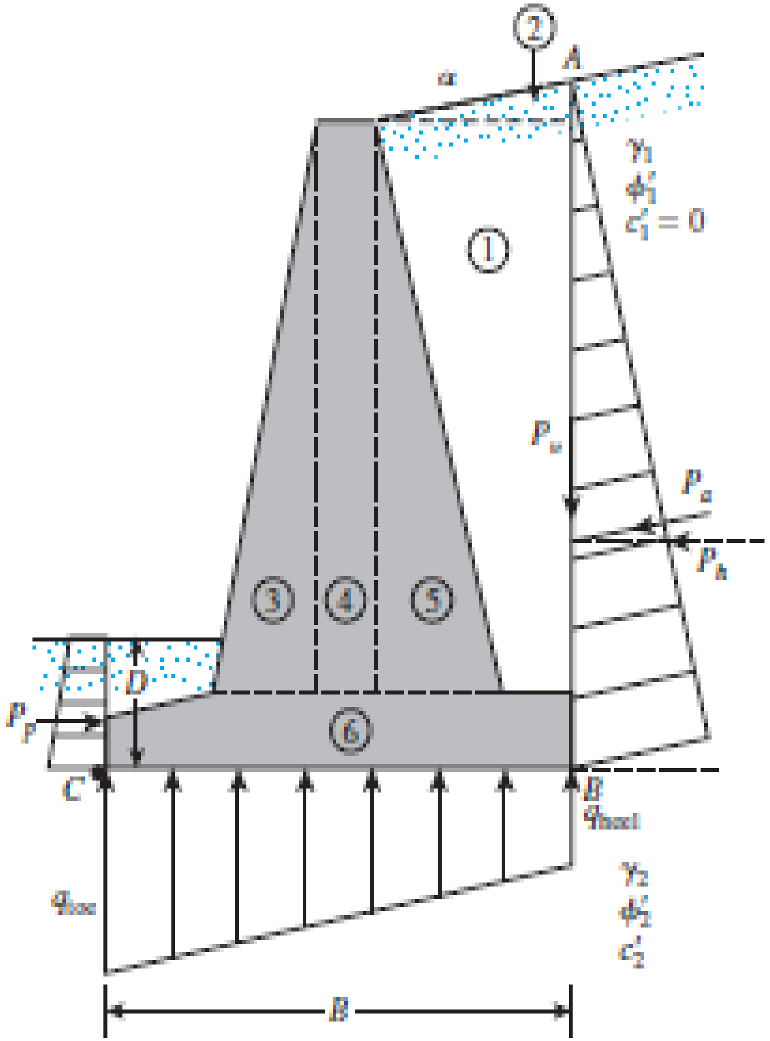
Failure of retaining wall:
(a) by overturning;
(b) by sliding;
(c) by bearing capacity failure;
(d) by deep-seated shear failure



# Check for Overturning



# Check for Overturning



# Check for Overturning

- The factor of safety against overturning about the toe—that is, about point *C* may be expressed as:

$$FS_{(overturning)} = \frac{\Sigma M_R}{\Sigma M_o}$$

where

$\Sigma M_o$ = sum of the moments of forces tending to overturn about point $C$
$\Sigma M_R$ = sum of the moments of forces tending to resist overturning about point $C$

# Check for Overturning

- The overturning moment is:

$$\Sigma M_o = P_h\left(\frac{H'}{3}\right)$$

where $P_h = P_a \cos\alpha$.

# Check for Overturning

To calculate the resisting moment, $\Sigma M_R$ (neglecting $p_p$):

- The weight of the soil above the heel and the weight of the concrete.
- The force $p_v$ also contributes to the resisting moment

($p_v$ is the vertical component of the active force)

$$P_v = P_a \sin\alpha$$

The moment of the force $P_v$ about $C$ is

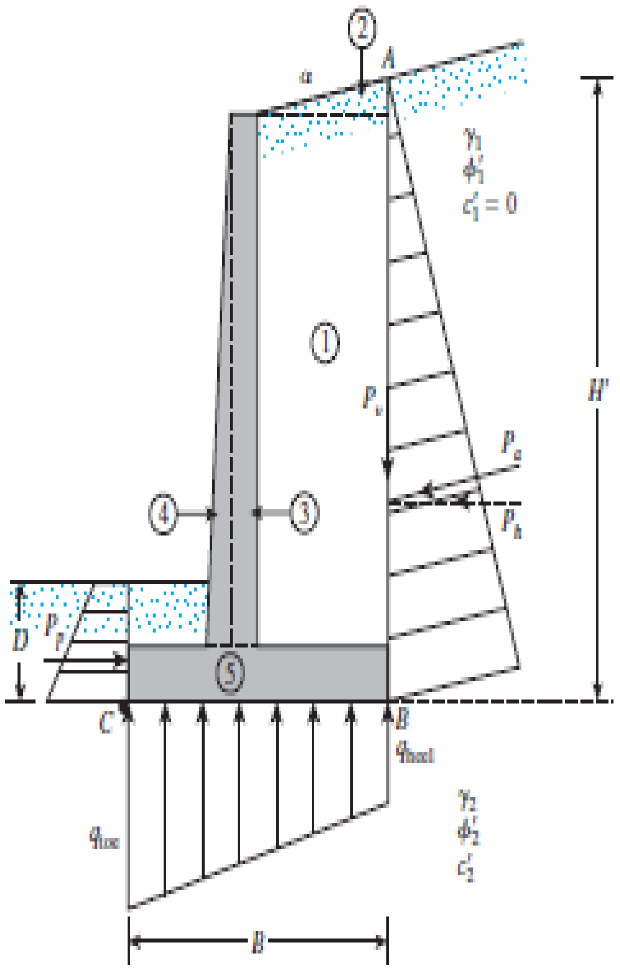
$$M_v = P_v B = P_a \sin\alpha B$$

# To calculate $\Sigma M_R$

**Table 8.1** Procedure for Calculating $\Sigma M_R$

| Section (1) | Area (2) | Weight/unit length of wall (3) | Moment arm measured from *C* (4) | Moment about *C* (5) |
|---|---|---|---|---|
| 1 | $A_1$ | $W_1 = \gamma_1 \times A_1$ | $X_1$ | $M_1$ |
| 2 | $A_2$ | $W_2 = \gamma_1 \times A_2$ | $X_2$ | $M_2$ |
| 3 | $A_3$ | $W_3 = \gamma_c \times A_3$ | $X_3$ | $M_3$ |
| 4 | $A_4$ | $W_4 = \gamma_c \times A_4$ | $X_4$ | $M_4$ |
| 5 | $A_5$ | $W_5 = \gamma_c \times A_5$ | $X_5$ | $M_5$ |
| 6 | $A_6$ | $W_6 = \gamma_c \times A_6$ | $X_6$ | $M_6$ |
| | | $P_v$ | $B$ | $M_v$ |
| | | $\Sigma V$ | | $\Sigma M_R$ |

(*Note:* $\gamma_l$ = unit weight of backfill
$\gamma_c$ = unit weight of concrete)



The factor of safety can be calculated as:

$$FS_{(overturning)} = \frac{M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_v}{P_a \cos\alpha (H'/3)}$$

The usual minimum desirable value of the factor of safety with respect to overturning is **2 to 3**.

# Check for Sliding along the Base

- The factor of safety against sliding may be expressed by the equation:

$$FS_{(sliding)} = \frac{\Sigma F_{R'}}{\Sigma F_d}$$

where

$\Sigma F_{R'}$ = sum of the horizontal resisting forces
$\Sigma F_d$ = sum of the horizontal driving forces

# Check for Sliding

- The shear strength of the soil immediately below the base slab may be represented as:

$$s = \sigma' \tan \delta' + c'_a$$

where

$\delta'$ = angle of friction between the soil and the base slab
$c'_a$ = adhesion between the soil and the base slab

- The maximum resisting force that can be derived from the soil per unit length of the wall along the bottom of the base slab is:

$$R' = s(\text{area of cross section}) = s(B \times 1) = B\sigma' \tan \delta' + Bc'_a$$

$$B\sigma' = \text{sum of the vertical force} = \Sigma V \text{ (see Table 8.1)}$$

$$R' = (\Sigma V) \tan \delta' + Bc'_a$$

# Check for Sliding

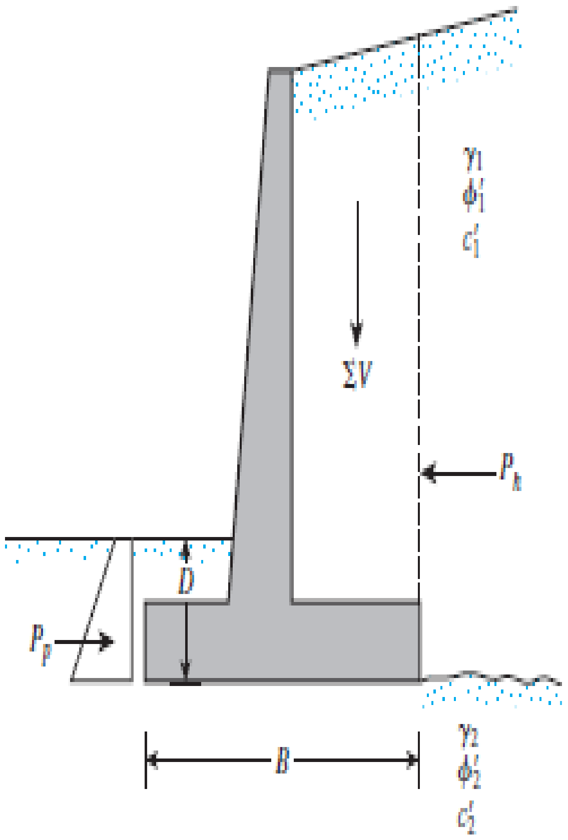
horizontal resisting force:

.

$$\sum F_{R'} = (\sum V)\tan\delta' + Bc'_a + P_p$$

The only horizontal force that will tend to cause the wall to slide (a *driving force*) is the horizontal component of the active force $p_a$:

$$\sum F_d = P_a \cos\alpha$$



$$\mathrm{FS}_{(\text{sliding})} = \frac{(\Sigma V)\tan\delta' + Bc'_a + P_p}{P_a\cos\alpha}$$

we can write $\delta' = k_1\phi'_2$ and $c'_a = k_2c'_2$.

In most cases, $k_1$ and $k_2$ are in the range from 1/2 to 2/3

$$\mathrm{FS}_{(\text{sliding})} = \frac{(\Sigma V)\tan(k_1\phi'_2) + Bk_2c'_2 + P_p}{P_a\cos\alpha}$$

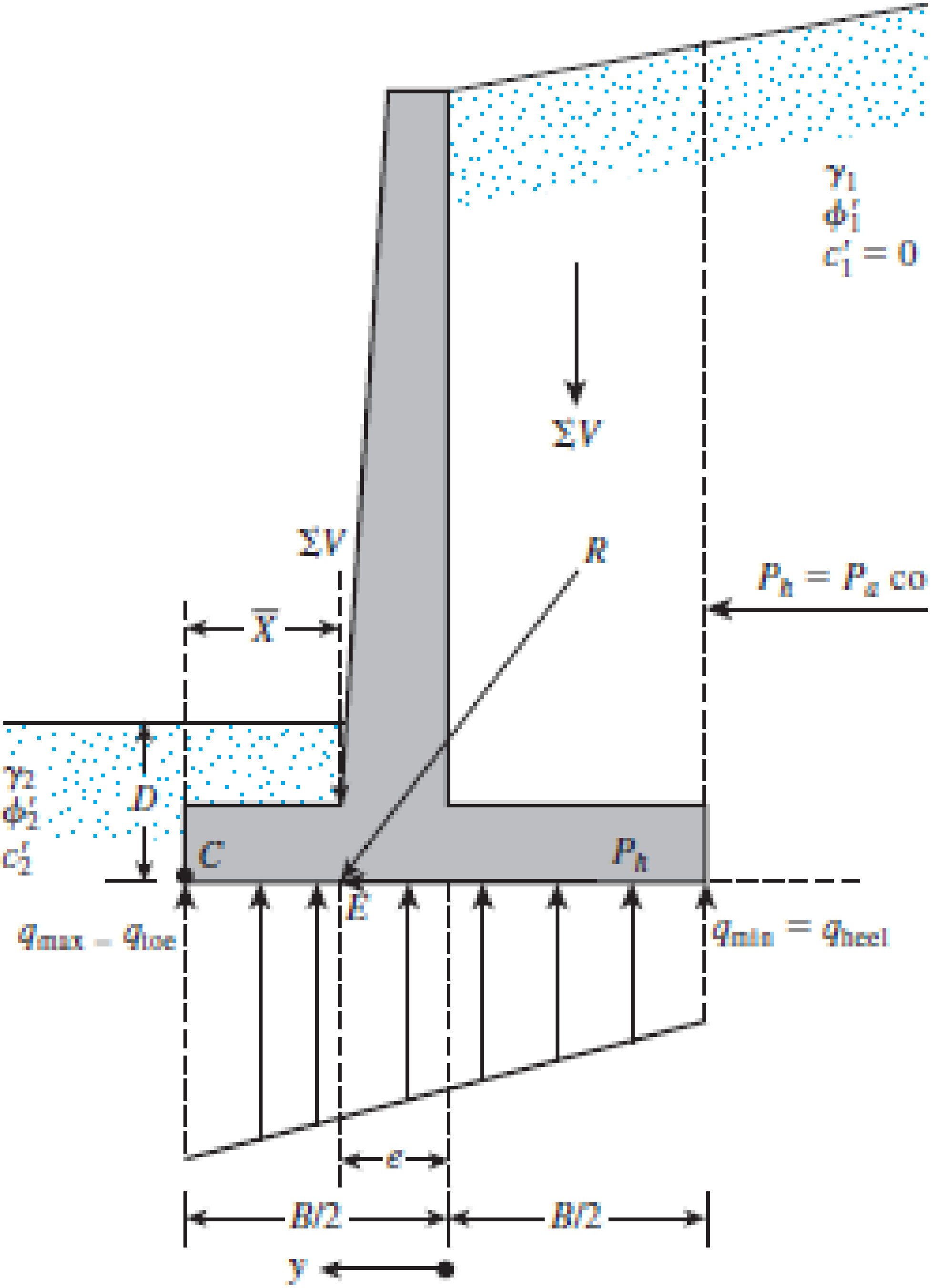
A minimum factor of safety of 1.5 against sliding is generally required

# Stability of retaining wall

**&**

**Examples**

**7/4/2020**




# Check for Bearing Capacity Failure

- The vertical pressure transmitted to the soil by the base slab of the retaining wall should be checked against the ultimate bearing capacity of the soil.

- $q_{toe}$ and $q_{heel}$ are the *maximum* and the *minimum* pressures occurring at the ends of the toe and heel sections, respectively.

For maximum and minimum pressures:

$$q_{max} = q_{toe} = \frac{\Sigma V}{B}\left(1 + \frac{6e}{B}\right)$$

$$q_{min} = q_{heel} = \frac{\Sigma V}{B}\left(1 - \frac{6e}{B}\right)$$

when the value of the eccentricity *e* becomes greater than B/6 ⟶ $q_{min}$ becomes negative.

The eccentricity due moment:

$$e = \frac{B}{2} - \frac{\Sigma M_R - \Sigma M_o}{\Sigma V}$$

$$FS_{(bearing\ capacity)} = \frac{q_u}{q_{max}} \longrightarrow \frac{\text{Ultimate bearing capacity}}{\text{Maximum pressure of soil}}$$

$$q_u = c_2' N_c F_{cd} F_{ci} + q N_q F_{qd} F_{qi} + \frac{1}{2}\gamma_2 B' N_\gamma F_{\gamma d} F_{\gamma i}$$

where

$$q = \gamma_2 D$$

$$B' = B - 2e$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi_2'}$$

$$F_{qd} = 1 + 2 \tan \phi_2' (1 - \sin \phi_2')^2 \frac{D}{B'}$$

$$F_{\gamma d} = 1$$

$$F_{ci} = F_{qi} = \left(1 - \frac{\psi^\circ}{90^\circ}\right)^2$$

$$F_{\gamma i} = \left(1 - \frac{\psi^\circ}{\phi_2'^\circ}\right)^2$$

$$\psi^\circ = \tan^{-1}\left(\frac{P_a \cos \alpha}{\Sigma V}\right)$$

**Table 3.3** Bearing Capacity Factors

| $\phi'$ | $N_c$ | $N_q$ | $N_\gamma$ | $\phi'$ | $N_c$ | $N_q$ | $N_\gamma$ |
|---|---|---|---|---|---|---|---|
| 0 | 5.14 | 1.00 | 0.00 | 26 | 22.25 | 11.85 | 12.54 |
| 1 | 5.38 | 1.09 | 0.07 | 27 | 23.94 | 13.20 | 14.47 |
| 2 | 5.63 | 1.20 | 0.15 | 28 | 25.80 | 14.72 | 16.72 |
| 3 | 5.90 | 1.31 | 0.24 | 29 | 27.86 | 16.44 | 19.34 |
| 4 | 6.19 | 1.43 | 0.34 | 30 | 30.14 | 18.40 | 22.40 |
| 5 | 6.49 | 1.57 | 0.45 | 31 | 32.67 | 20.63 | 25.99 |
| 6 | 6.81 | 1.72 | 0.57 | 32 | 35.49 | 23.18 | 30.22 |
| 7 | 7.16 | 1.88 | 0.71 | 33 | 38.64 | 26.09 | 35.19 |
| 8 | 7.53 | 2.06 | 0.86 | 34 | 42.16 | 29.44 | 41.06 |
| 9 | 7.92 | 2.25 | 1.03 | 35 | 46.12 | 33.30 | 48.03 |
| 10 | 8.35 | 2.47 | 1.22 | 36 | 50.59 | 37.75 | 56.31 |
| 11 | 8.80 | 2.71 | 1.44 | 37 | 55.63 | 42.92 | 66.19 |
| 12 | 9.28 | 2.97 | 1.69 | 38 | 61.35 | 48.93 | 78.03 |
| 13 | 9.81 | 3.26 | 1.97 | 39 | 67.87 | 55.96 | 92.25 |
| 14 | 10.37 | 3.59 | 2.29 | 40 | 75.31 | 64.20 | 109.41 |
| 15 | 10.98 | 3.94 | 2.65 | 41 | 83.86 | 73.90 | 130.22 |
| 16 | 11.63 | 4.34 | 3.06 | 42 | 93.71 | 85.38 | 155.55 |
| 17 | 12.34 | 4.77 | 3.53 | 43 | 105.11 | 99.02 | 186.54 |
| 18 | 13.10 | 5.26 | 4.07 | 44 | 118.37 | 115.31 | 224.64 |
| 19 | 13.93 | 5.80 | 4.68 | 45 | 133.88 | 134.88 | 271.76 |
| 20 | 14.83 | 6.40 | 5.39 | 46 | 152.10 | 158.51 | 330.35 |
| 21 | 15.82 | 7.07 | 6.20 | 47 | 173.64 | 187.21 | 403.67 |
| 22 | 16.88 | 7.82 | 7.13 | 48 | 199.26 | 222.31 | 496.01 |
| 23 | 18.05 | 8.66 | 8.20 | 49 | 229.93 | 265.51 | 613.16 |
| 24 | 19.32 | 9.60 | 9.44 | 50 | 266.89 | 319.07 | 762.89 |
| 25 | 20.72 | 10.66 | 10.88 | | | | |

# Example1:

The cross section of a cantilever retaining wall is shown in Figure 8.12. Calculate the factors of safety with respect to overturning, sliding, and bearing capacity.

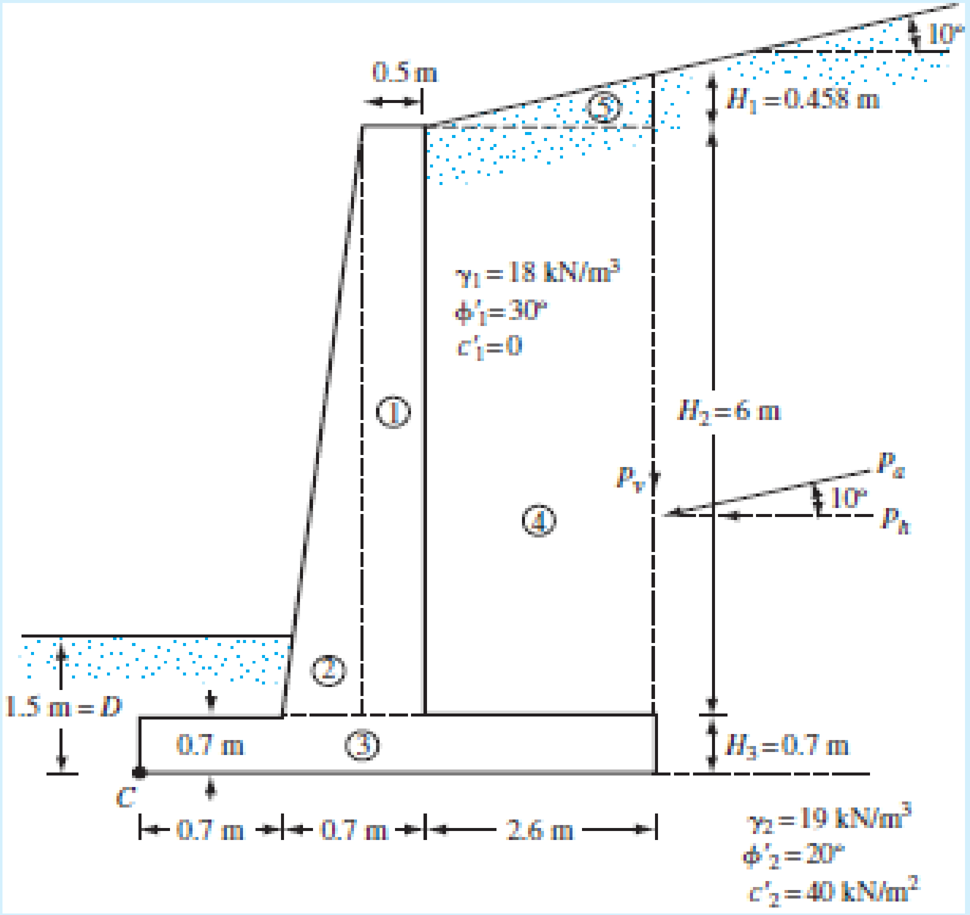



*Figure 8.12* Calculation of stability of a retaining wall

From the figure,

$$H' = H_1 + H_2 + H_3 = 2.6 \tan 10° + 6 + 0.7$$
$$= 0.458 + 6 + 0.7 = 7.158 \text{ m}$$

The Rankine active force per unit length of wall $= P_p = \frac{1}{2}\gamma_1 H'^2 K_a$. For $\phi_1' = 30°$ and $\alpha = 10°$, $K_a$ is equal to 0.3532. (See Table 7.1.) Thus,

$$P_a = \tfrac{1}{2}(18)(7.158)^2(0.3532) = 162.9 \text{ kN/m}$$

$$P_v = P_a \sin 10° = 162.9 (\sin 10°) = 28.29 \text{ kN/m}$$

and

$$P_h = P_a \cos 10° = 162.9 (\cos 10°) = 160.43 \text{ kN/m}$$

### Factor of Safety against Overturning

The following table can now be prepared for determining the resisting moment:

| Section no.[a] | Area (m²) | Weight/unit length (kN/m) | Moment arm from point $C$ (m) | Moment (kN-m/m) |
|---|---|---|---|---|
| 1 | $6 \times 0.5 = 3$ | 70.74 | 1.15 | 81.35 |
| 2 | $\frac{1}{2}(0.2)6 = 0.6$ | 14.15 | 0.833 | 11.79 |
| 3 | $4 \times 0.7 = 2.8$ | 66.02 | 2.0 | 132.04 |
| 4 | $6 \times 2.6 = 15.6$ | 280.80 | 2.7 | 758.16 |
| 5 | $\frac{1}{2}(2.6)(0.458) = 0.595$ | 10.71 | 3.13 | 33.52 |
| | | $P_v = 28.29$ | 4.0 | 113.16 |
| | | $\Sigma V = 470.71$ | | $1130.02 = \Sigma M_R$ |

[a]For section numbers, refer to Figure 8.12

$\gamma_{concrete} = 23.58 \text{ kN/m}^3$

The overturning moment

$$M_o = P_h\left(\frac{H'}{3}\right) = 160.43\left(\frac{7.158}{3}\right) = 382.79 \text{ kN-m/m}$$

and

$$FS_{(overturning)} = \frac{\Sigma M_R}{M_o} = \frac{1130.02}{382.79} = \mathbf{2.95 > 2, OK}$$

Factor of Safety against Sliding
From Eq. (8.11),

$$FS_{(\text{sliding})} = \frac{(\Sigma V)\tan(k_1\phi_2') + Bk_2c_2' + P_p}{P_a\cos\alpha}$$

Let $k_1 = k_2 = \frac{2}{3}$. Also,

$$P_p = \tfrac{1}{2}K_p\gamma_2 D^2 + 2c_2'\sqrt{K_p}D$$

$$K_p = \tan^2\left(45 + \frac{\phi_2'}{2}\right) = \tan^2(45 + 10) = 2.04$$

and

$$D = 1.5 \text{ m}$$

So

$$P_p = \tfrac{1}{2}(2.04)(19)(1.5)^2 + 2(40)(\sqrt{2.04})(1.5)$$
$$= 43.61 + 171.39 = 215 \text{ kN/m}$$

Hence,

$$FS_{(\text{sliding})} = \frac{(470.71)\tan\left(\frac{2 \times 20}{3}\right) + (4)\left(\frac{2}{3}\right)(40) + 215}{160.43}$$

$$= \frac{111.56 + 106.67 + 215}{160.43} = \mathbf{2.7 > 1.5, OK}$$

*Note*: For some designs, the depth $D$ in a passive pressure calculation may be taken to be *equal to the thickness of the base slab.*

Factor of Safety against Bearing Capacity Failure
Combining Eqs. (8.16), (8.17), and (8.18) yields

$$e = \frac{B}{2} - \frac{\Sigma M_R - \Sigma M_o}{\Sigma V} = \frac{4}{2} - \frac{1130.02 - 382.79}{470.71}$$
$$= 0.411 \text{ m} < \frac{B}{6} = \frac{4}{6} = 0.666 \text{ m}$$

Again, from Eqs. (8.20) and (8.21)

$$q_{\substack{\text{toe}\\\text{heel}}} = \frac{\Sigma V}{B}\left(1 \pm \frac{6e}{B}\right) = \frac{470.71}{4}\left(1 \pm \frac{6 \times 0.411}{4}\right) = 190.2 \text{ kN/m}^2 \text{ (toe)}$$
$$= 45.13 \text{ kN/m}^2 \text{ (heel)}$$

The ultimate bearing capacity of the soil can be determined from Eq. (8.22)

$$q_u = c_2' N_c F_{cd} F_{ci} + q N_q F_{qd} F_{qi} + \frac{1}{2}\gamma_2 B' N_\gamma F_{\gamma d} F_{\gamma i}$$

For $\phi_2' = 20°$ (see Table 3.3), $N_c = 14.83$, $N_q = 6.4$, and $N_\gamma = 5.39$. Also,

$$q = \gamma_2 D = (19)(1.5) = 28.5 \text{ kN/m}^2$$
$$B' = B - 2e = 4 - 2(0.411) = 3.178 \text{ m}$$
$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi_2'} = 1.148 - \frac{1 - 1.148}{(14.83)(\tan 20)} = 1.175$$
$$F_{qd} = 1 + 2\tan\phi_2'(1 - \sin\phi_2')^2\left(\frac{D}{B'}\right) = 1 + 0.315\left(\frac{1.5}{3.178}\right) = 1.148$$
$$F_{\gamma d} = 1$$
$$F_{ci} = F_{qi} = \left(1 - \frac{\psi°}{90°}\right)^2$$

and

$$\psi = \tan^{-1}\left(\frac{P_a\cos\alpha}{\Sigma V}\right) = \tan^{-1}\left(\frac{160.43}{470.71}\right) = 18.82°$$

So

So

$$F_{ci} = F_{qi} = \left(1 - \frac{18.82}{90}\right)^2 = 0.626$$

and

$$F_{\gamma i} = \left(1 - \frac{\psi}{\phi'_2}\right)^2 = \left(1 - \frac{18.82}{20}\right)^2 \approx 0$$

Hence,

$$q_u = (40)(14.83)(1.175)(0.626) + (28.5)(6.4)(1.148)(0.626) + \tfrac{1}{2}(19)(5.93)(3.178)(1)(0)$$
$$= 436.33 + 131.08 + 0 = 567.41 \text{ kN/m}^2$$

and

$$FS_{(\text{bearing capacity})} = \frac{q_u}{q_{\text{toe}}} = \frac{567.41}{190.2} = \mathbf{2.98}$$

*Note*: $FS_{(\text{bearing capacity})}$ is less than 3. Some reproportioning will be needed. ■

# Example2:

A gravity retaining wall is shown in Figure 8.13. Use $\delta' = 2/3\phi'_1$ and Coulomb's active earth pressure theory. Determine

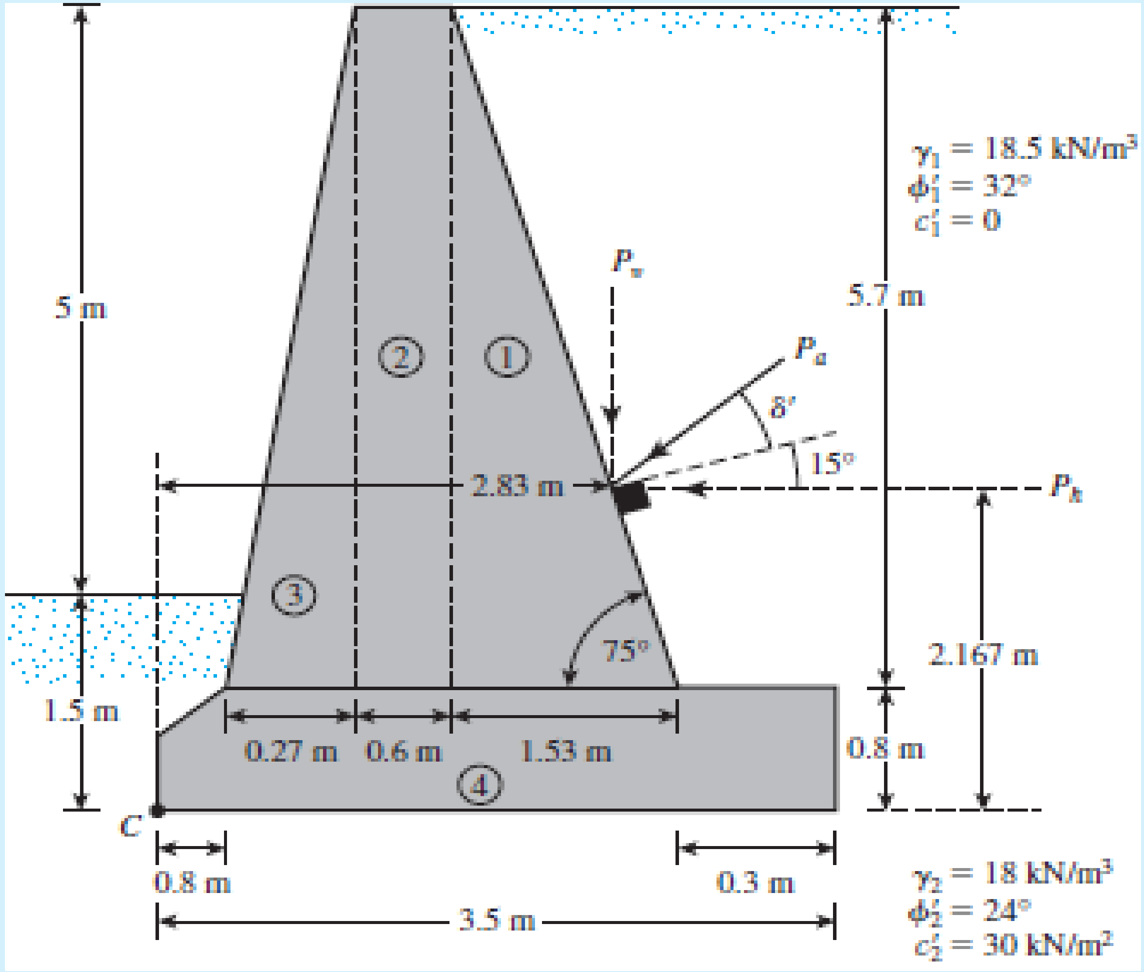



*Figure 8.13* Gravity retaining wall (not to scale)

a. The factor of safety against overturning
b. The factor of safety against sliding
c. The pressure on the soil at the toe and heel

**Solution**

The height

$$H' = 5 + 1.5 = 6.5 \text{ m}$$

Coulomb's active force is

$$P_a = \tfrac{1}{2}\gamma_1 H'^2 K_a$$

With $\alpha = 0°$, $\beta = 75°$, $\delta' = 2/3\phi'_1$, and $\phi'_1 = 32°$, $K_a = 0.4023$. (See Table 7.4.) So,

$$P_a = \tfrac{1}{2}(18.5)(6.5)^2(0.4023) = 157.22 \text{ kN/m}$$

$$P_h = P_a \cos\left(15 + \tfrac{2}{3}\phi'_1\right) = 157.22 \cos 36.33 = 126.65 \text{ kN/m}$$

and

$$P_v = P_a \sin\left(15 + \tfrac{2}{3}\phi'_1\right) = 157.22 \sin 36.33 = 93.14 \text{ kN/m}$$

Part a: Factor of Safety against Overturning

From Figure 8.13, one can prepare the following table:

| Area no. | Area ($m^2$) | Weight* (kN/m) | Moment arm from *C* (m) | Moment (kN-m/m) |
|---|---|---|---|---|
| 1 | $\tfrac{1}{2}(5.7)(1.53) = 4.36$ | 102.81 | 2.18 | 224.13 |
| 2 | $(0.6)(5.7) = 3.42$ | 80.64 | 1.37 | 110.48 |
| 3 | $\tfrac{1}{2}(0.27)(5.7) = 0.77$ | 18.16 | 0.98 | 17.80 |
| 4 | $\approx (3.5)(0.8) = 2.8$ | 66.02 | 1.75 | 115.54 |
| | | $P_v = 93.14$ | 2.83 | 263.59 |
| | | $\Sigma V = 360.77$ kN/m | | $\Sigma M_R = 731.54$ kN-m/m |

*$\gamma_{concrete} = 23.58 \text{ kN/m}^3$

Note that the weight of the soil above the back face of the wall is not taken into account in the preceding table. We have

$$\text{Overturning moment} = M_o = P_h\left(\frac{H'}{3}\right) = 126.65(2.167) = 274.45 \text{ kN-m/m}$$

Hence,

$$\text{FS}_{(\text{overturning})} = \frac{\Sigma M_R}{\Sigma M_o} = \frac{731.54}{274.45} = \mathbf{2.67 > 2, OK}$$

Part b: Factor of Safety against Sliding

We have

$$FS_{(sliding)} = \frac{(\Sigma V)\tan\left(\frac{2}{3}\phi_2'\right) + \frac{2}{3}c_2'B + P_p}{P_h}$$

$$P_p = \tfrac{1}{2}K_p\gamma_2 D^2 + 2c_2'\sqrt{K_p}D$$

and

$$K_p = \tan^2\left(45 + \frac{24}{2}\right) = 2.37$$

Hence,

$$P_p = \tfrac{1}{2}(2.37)(18)(1.5)^2 + 2(30)(1.54)(1.5) = 186.59 \text{ kN/m}$$

So

$$FS_{(sliding)} = \frac{360.77\tan\left(\frac{2}{3}\times 24\right) + \frac{2}{3}(30)(3.5) + 186.59}{126.65}$$

$$= \frac{103.45 + 70 + 186.59}{126.65} = \mathbf{2.84}$$

If $P_p$ is ignored, the factor of safety is **1.37**.

Part c: Pressure on Soil at Toe and Heel

From Eqs. (8.16), (8.17), and (8.18),

$$e = \frac{B}{2} - \frac{\Sigma M_R - \Sigma M_o}{\Sigma V} = \frac{3.5}{2} - \frac{731.54 - 274.45}{360.77} = 0.483 < \frac{B}{6} = 0.583$$

$$q_{toe} = \frac{\Sigma V}{B}\left[1 + \frac{6e}{B}\right] = \frac{360.77}{3.5}\left[1 + \frac{(6)(0.483)}{3.5}\right] = \mathbf{188.43 \text{ kN/m}^2}$$

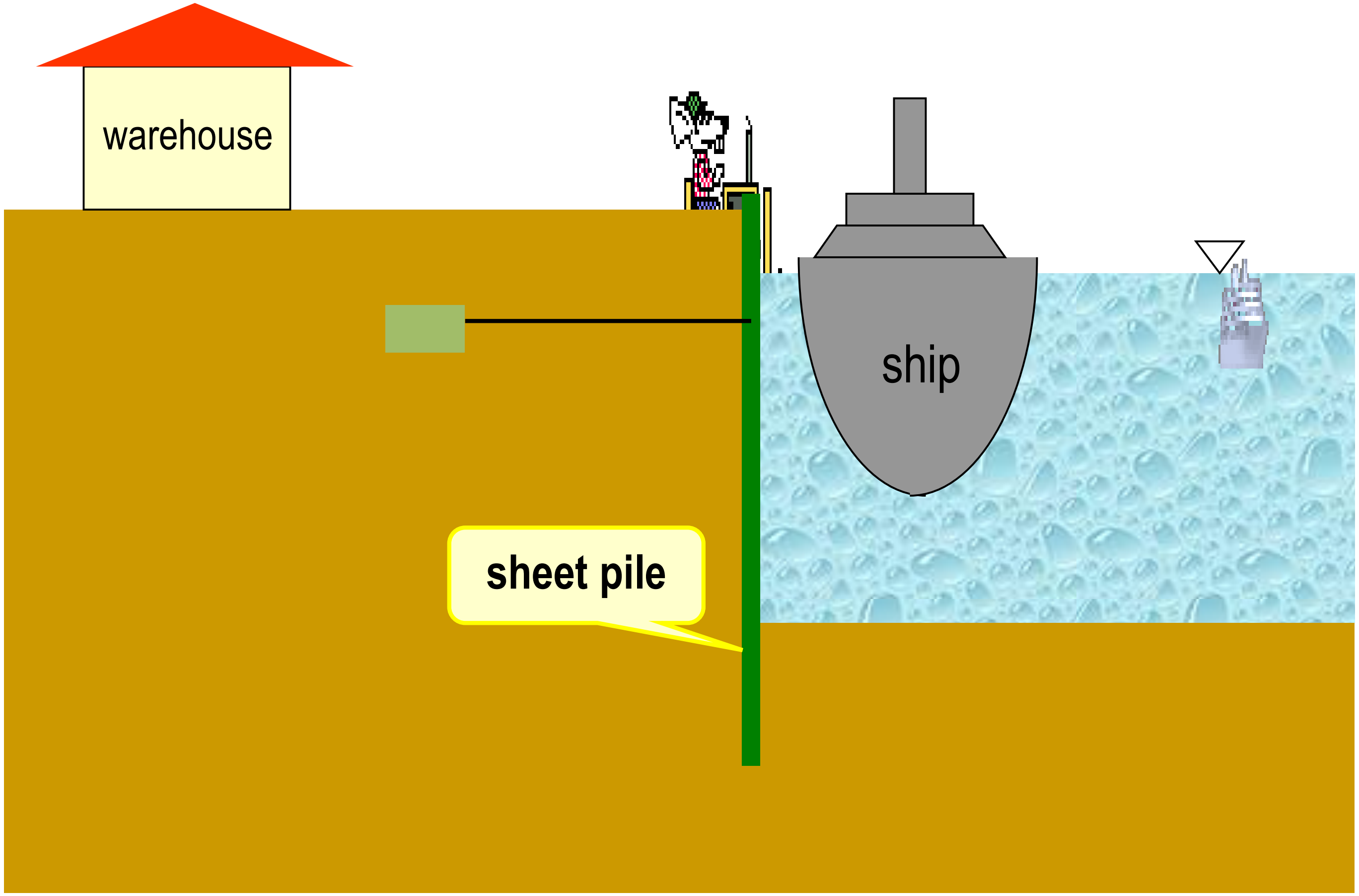
and

$$q_{heel} = \frac{V}{B}\left[1 - \frac{6e}{B}\right] = \frac{360.77}{3.5}\left[1 - \frac{(6)(0.483)}{3.5}\right] = \mathbf{17.73 \text{ kN/m}^2}$$ ■

# Sheet Pile Walls Design

# Sheet Piles

~ sheets of interlocking-steel or timber driven into the ground, forming a continuous sheet



# Sheet Piles

~ resist lateral earth pressures

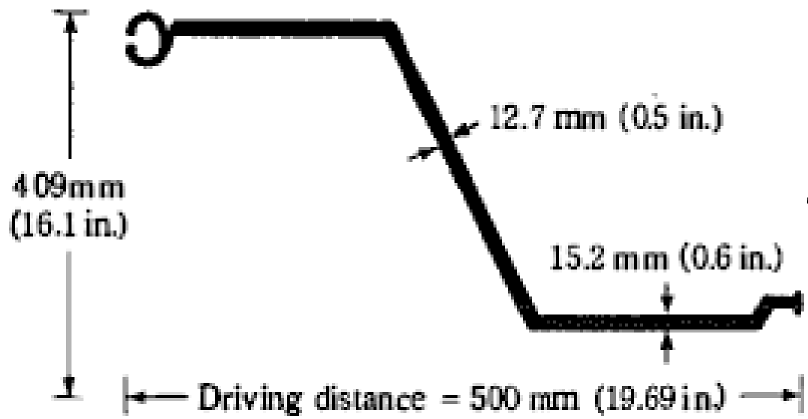
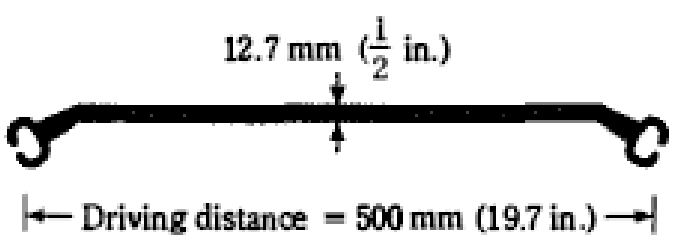
~ used in excavations, waterfront structures, ..

# Sheet Pile At Woolcock St SIMILAR TO THOSE AT Egyptians Gaza Border!

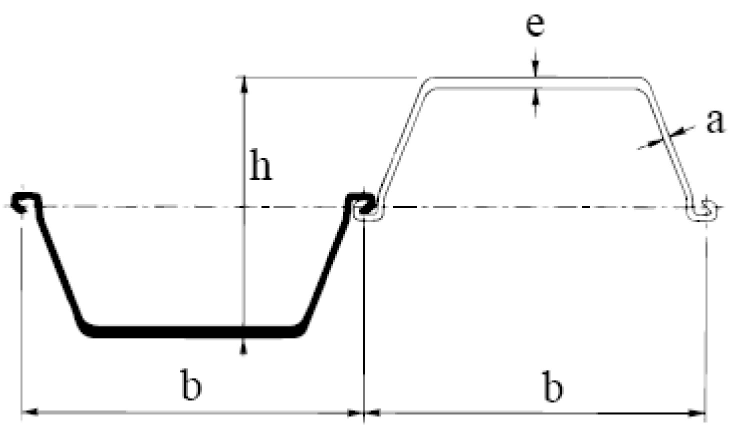
# Sheet Piles

~ used in temporary works

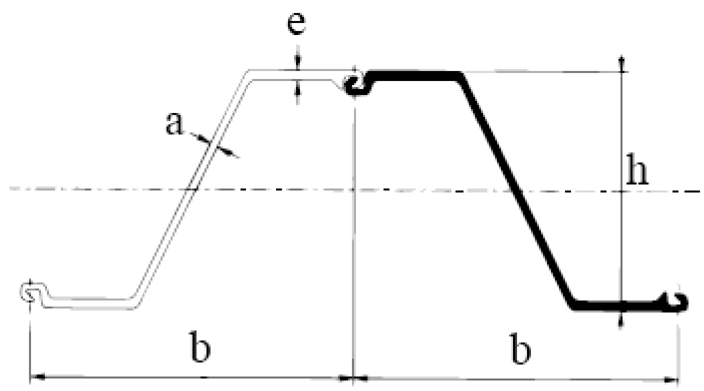
# Properties of Some Sheet-Pile Sections

| Section designation | Sketch of section | Section modulus | | Moment of inertia | |
|---|---|---|---|---|---|
| | | $m^3/m$ of wall | $in^3/ft$ of wall | $m^4/m$ of wall | $in^4/ft$ of wall |
| PZ-40 |   | $326.4 \times 10^{-5}$ | 60.7 | $670.5 \times 10^{-6}$ | 490.8 |
| PSA-31 |   | $10.8 \times 10^{-5}$ | 2.01 | $4.41 \times 10^{-6}$ | 3.23 |

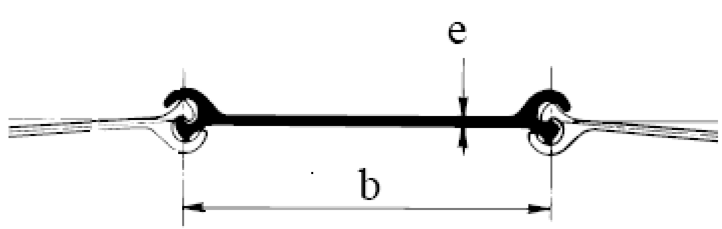
# Sheet Pile Configuration Sections



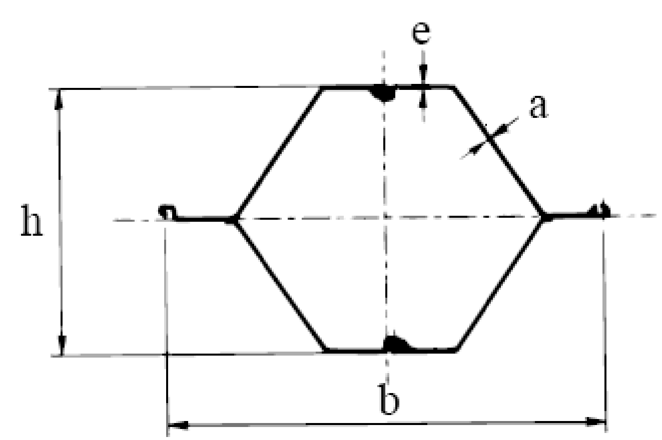

U Section




Z Section




Straight Section




Box Section

# Sheet pile wall

- Advantages

1. Conventional Wall System with Well established design procedure & performance characteristics

2. Wall system can be used for application in which wall can penetrates below ground water table

3. Wall system is suitable for temporary applications

# Sheet pile wall

**Disadvantages**

1. Requires specialized equipment
2. Driven sheet pile is noisy and it can be introduce vibration
3. Difficult to drive sheet in hard or dense or gravelly soil
4. Wall height is limited based on required structural sections
5. Wall system may undergo relatively movements which may be detrimental to nearby structure

# Construction Methods

- ❑ Construction methods generally can be divided into two categories
  - – Backfilled structure
  - – Dredge Structure

## Sequence of construction for backfilled structure

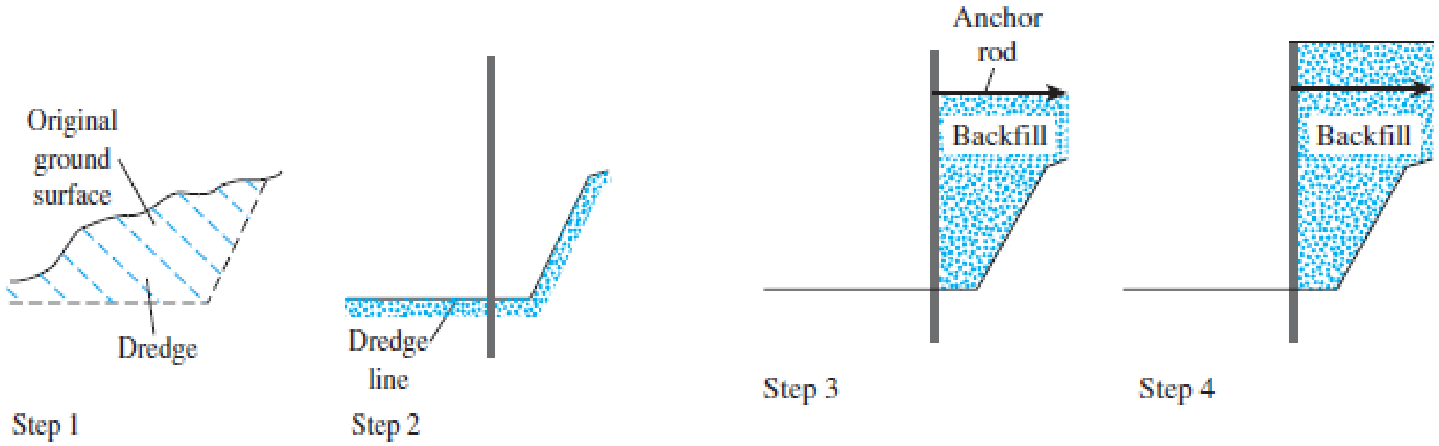
The sequence of construction for a *backfilled structure* is as follows (see Figure 9.5):

*Step 1*. Dredge the *in situ* soil in front and back of the proposed structure.

*Step 2*. Drive the sheet piles.

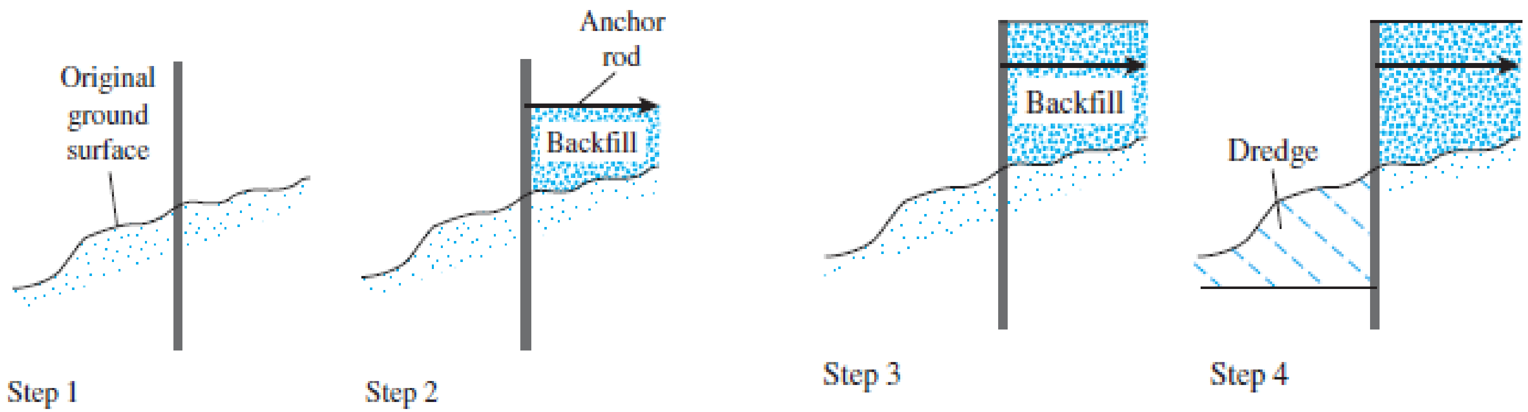
*Step 3*. Backfill up to the level of the anchor, and place the anchor system.

*Step 4*. Backfill up to the top of the wall.

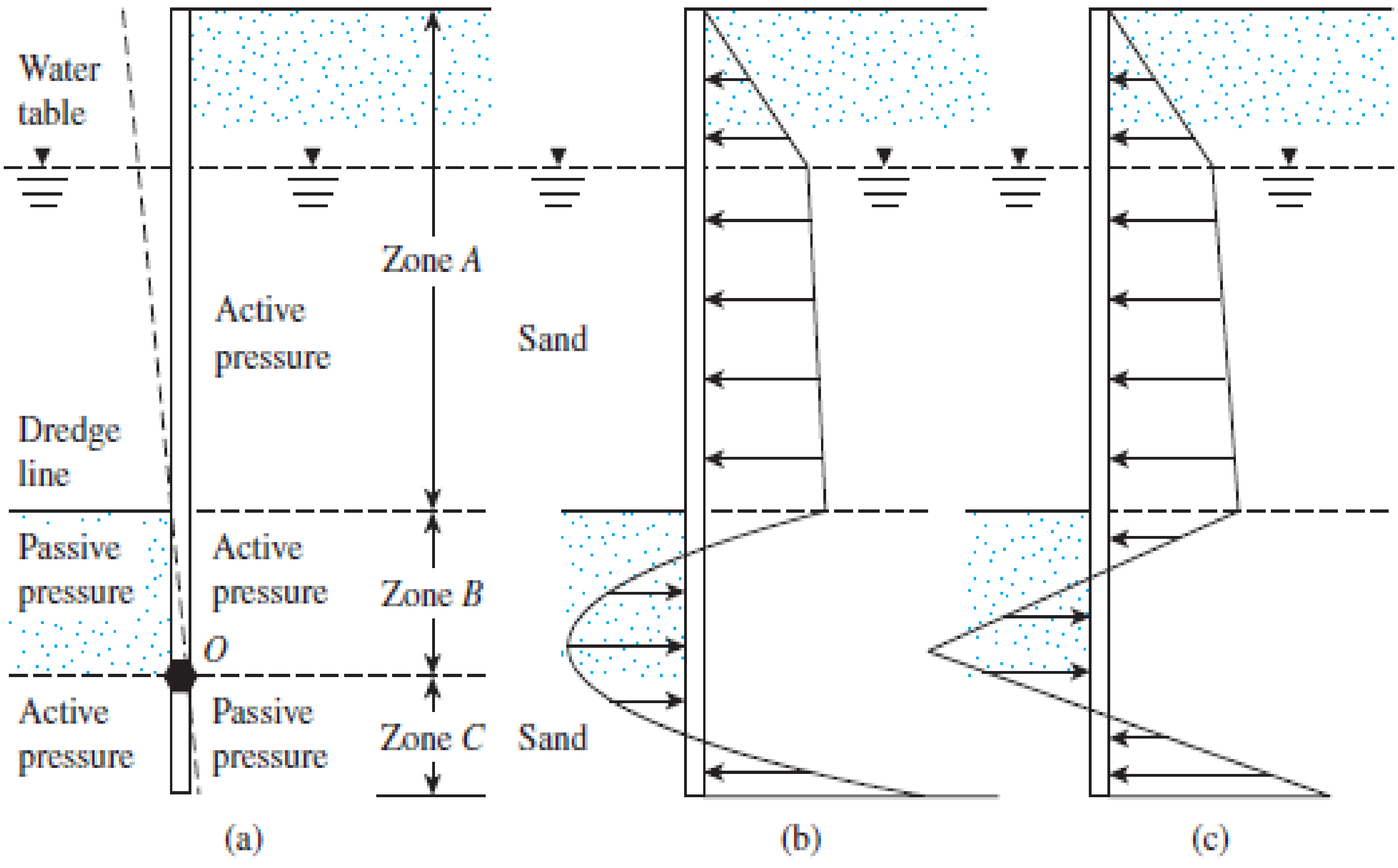


# Sequence of construction for a dredged structure

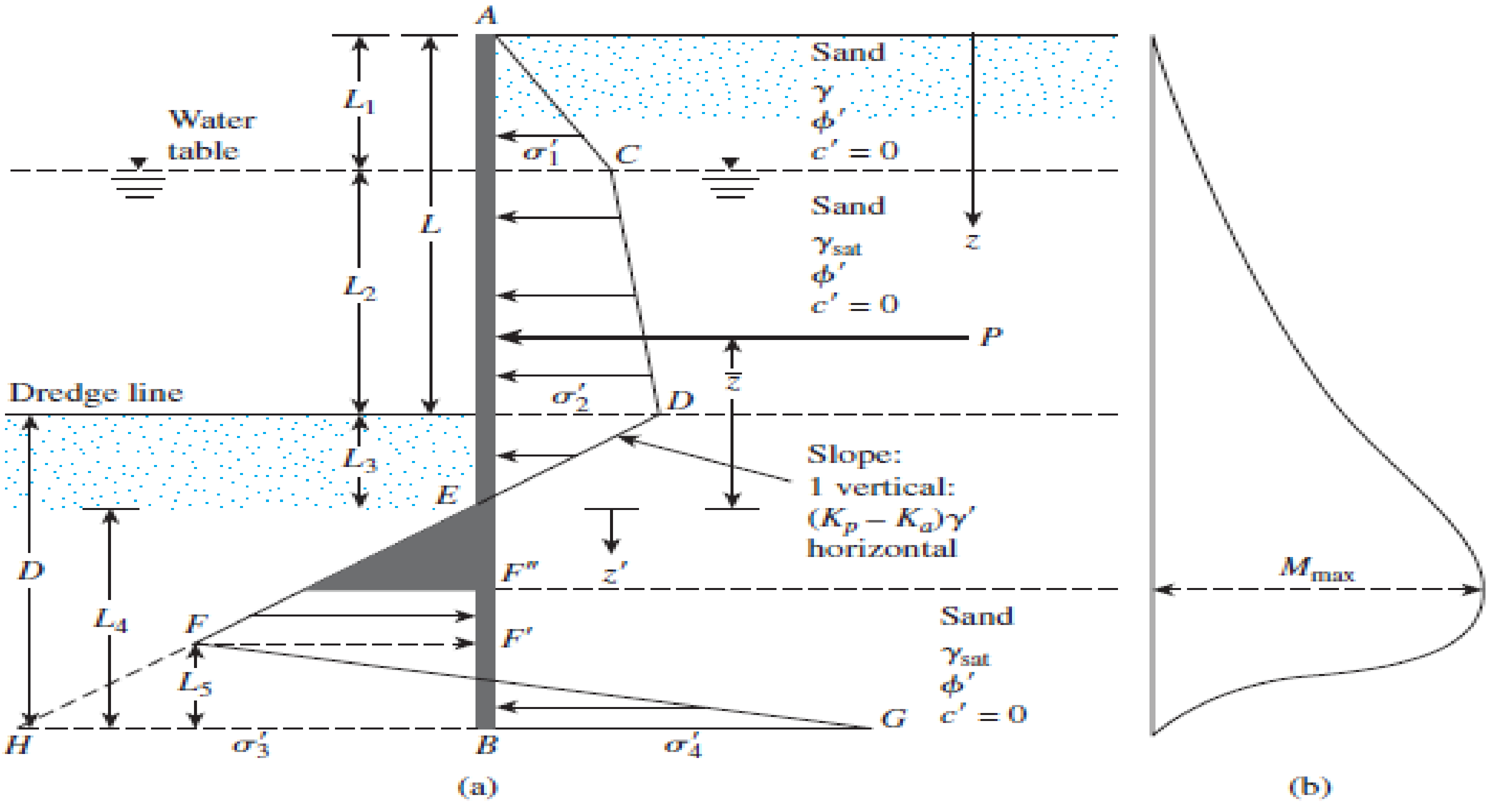
*Step 1*. Drive the sheet piles.
*Step 2*. Backfill up to the anchor level, and place the anchor system.
*Step 3*. Backfill up to the top of the wall.
*Step 4*. Dredge the front side of the wall.



# Cantilever Sheet Piling Penetrating Sandy Soils



# Cantilever Sheet Piling Penetrating Sandy Soils




Cantilever sheet pile penetrating sand: (a) variation of net pressure diagram; (b) variation of moment

# Cantilever Sheet Piling

- The mode of failure is by rotation about a point O near the lower end of the wall
- Consequently, passive resistance acts in front of the wall above O and behind the wall below O,
- this pressure distribution is an idealization as there is unlikely to be a complete change in passive resistance from the front to the back of the wall at point O
- To allow for over-excavation it is recommended that the soil level in front of the wall should be reduced by 10% of the retained height, subject to a maximum of 0.5 m.
- A minimum surcharge pressure of 10 kN/m2 should be assumed to act on the soil surface behind the wall.

# Simplified Cantilever Sheet Piling analysis

- The net passive resistance below point O is represented by a concentrated force R acting at a point C, slightly below O, at depth d below the lower soil surface.
- Determining the depth d by equating moments about C
  - Factor of safety FS being applied to the restoring moment, i.e. the available passive resistance in front of the wall is divided by FS
- The value of d is then increased arbitrarily by 20% to allow for the simplification involved in the method, i.e. the required depth of embedment is 1.2d
- Evaluate R by equating horizontal forces
- Check if net passive resistance available over the additional 20% embedded depth is equal to or greater than R.
- Determine the location of zero shear, and maximum moment
- Determine the section modulus by dividing the maximum moment allowable flexural stress of sheet pile
- Select the section type (see table 9.1 for example)

# General Note

- Please note this methods is also applicable for layered soil and for pure clayey soil
- In case of layered soil (sand above clay) its advisable to evaluate the long term condition (i.e. effective stress method)
- In case of clayey soil its advisable to check both condition
  - Long term condition (effective stress approach), and
  - End of construction condition (total stress approach)

## Step-by-Step Procedure for Obtaining the Pressure Diagram

Based on the preceding theory, a step-by-step procedure for obtaining the pressure diagram
for a cantilever sheet pile wall penetrating a granular soil is as follows:

*Step 1.* Calculate $K_a$ and $K_p$.

*Step 2.* Calculate $\sigma_1'$ [Eq. (9.1)] and $\sigma_2'$ [Eq. (9.2)]. (*Note:* $L_1$ and $L_2$ will be given.)

*Step 3.* Calculate $L_3$ [Eq. (9.6)].

*Step 4.* Calculate $P$.

*Step 5.* Calculate $\bar{z}$ (i.e., the center of pressure for the area *ACDE*) by taking the moment about *E*.

*Step 6.* Calculate $\sigma_5'$ [Eq. (9.11)].

*Step 7.* Calculate $A_1$, $A_2$, $A_3$, and $A_4$ [Eqs. (9.17) through (9.20)].

*Step 8.* Solve Eq. (9.16) by trial and error to determine $L_4$.

*Step 9.* Calculate $\sigma_4'$ [Eq. (9.10)].

*Step 10.* Calculate $\sigma_3'$ [Eq. (9.7)].

*Step 11.* Obtain $L_5$ from Eq. (9.15).

*Step 12.* Draw a pressure distribution diagram like the one shown in Figure 9.8a.

*Step 13.* Obtain the theoretical depth [see Eq. (9.12)] of penetration as $L_3 + L_4$. The actual depth of penetration is increased by about 20 to 30%.

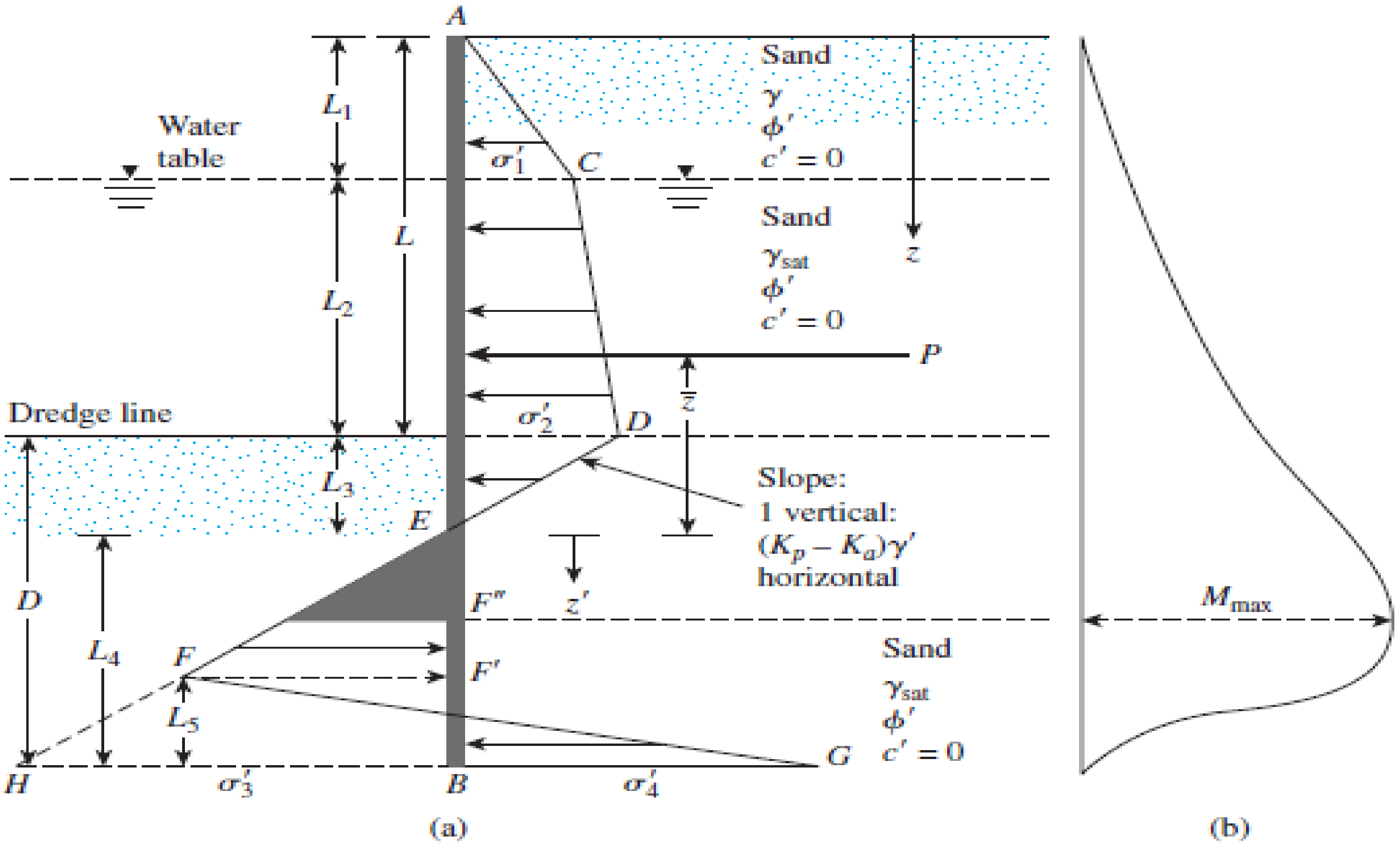
# Sheet pile walls 2

# Cantilever Sheet Piling Penetrating Sandy Soils

**Step-by-Step Procedure for Obtaining the Pressure Diagram**
Based on the preceding theory, a step-by-step procedure for obtaining the pressure diagram for a cantilever sheet pile wall penetrating a granular soil is as follows:

*Step 1.* Calculate $K_a$ and $K_p$.
*Step 2.* Calculate $\sigma'_1$ [Eq. (9.1)] and $\sigma'_2$ [Eq. (9.2)]. (*Note:* $L_1$ and $L_2$ will be given.)
*Step 3.* Calculate $L_3$ [Eq. (9.6)].
*Step 4.* Calculate $P$.
*Step 5.* Calculate $\bar{z}$ (i.e., the center of pressure for the area $ACDE$) by taking the moment about $E$.
*Step 6.* Calculate $\sigma'_5$ [Eq. (9.11)].
*Step 7.* Calculate $A_1$, $A_2$, $A_3$, and $A_4$ [Eqs. (9.17) through (9.20)].
*Step 8.* Solve Eq. (9.16) by trial and error to determine $L_4$.
*Step 9.* Calculate $\sigma'_4$ [Eq. (9.10)].
*Step 10.* Calculate $\sigma'_3$ [Eq. (9.7)].
*Step 11.* Obtain $L_5$ from Eq. (9.15).
*Step 12.* Draw a pressure distribution diagram like the one shown in Figure 9.8a.
*Step 13.* Obtain the theoretical depth [see Eq. (9.12)] of penetration as $L_3 + L_4$. The actual depth of penetration is increased by about 20 to 30%.

# Cantilever Sheet Piling Penetrating Sandy Soils




Cantilever sheet pile penetrating sand: (a) variation of net pressure diagram; (b) variation of moment

Let the effective angle of friction of the sand be $\phi'$. The intensity of the active pressure at a depth $z = L_1$ is

$$\sigma'_1 = \gamma L_1 K_a \tag{9.1}$$

where

$K_a$ = Rankine active pressure coefficient = $\tan^2(45 - \phi'/2)$
$\gamma$ = unit weight of soil above the water table

Similarly, the active pressure at a depth $z = L_1 + L_2$ (i.e., at the level of the dredge line) is

$$\sigma'_2 = (\gamma L_1 + \gamma' L_2) K_a \tag{9.2}$$

where $\gamma'$ = effective unit weight of soil = $\gamma_{sat} - \gamma_w$.

Also, the passive pressure at depth $z$ is

$$\sigma'_p = \gamma'(z - L_1 - L_2) K_p \tag{9.4}$$

where $K_p$ = Rankine passive pressure coefficient = $\tan^2(45 + \phi'/2)$ .

Combining Eqs. (9.3) and (9.4) yields the net lateral pressure, namely,

$$\sigma' = \sigma'_a - \sigma'_p = (\gamma L_1 + \gamma' L_2)K_a - \gamma'(z - L_1 - L_2)(K_p - K_a)$$

$$= \sigma'_2 - \gamma'(z - L)(K_p - K_a) \tag{9.5}$$

where $L = L_1 + L_2$.

The net pressure, $\sigma'$ equals zero at a depth $L_3$ below the dredge line, so

$$\sigma'_2 - \gamma'(z - L)(K_p - K_a) = 0$$

or

$$(z - L) = L_3 = \frac{\sigma'_2}{\gamma'(K_p - K_a)} \tag{9.6}$$

Equation (9.6) indicates that the slope of the net pressure distribution line *DEF* is 1 vertical to $(K_p - K_a)\gamma'$ horizontal, so, in the pressure diagram,

$$\overline{HB} = \sigma'_3 = L_4(K_p - K_a)\gamma' \tag{9.7}$$

At the bottom of the sheet pile, passive pressure, $\sigma'_p$, acts from the right toward the left side, and active pressure acts from the left toward the right side of the sheet pile, so, at $z = L + D$,

$$\sigma'_p = (\gamma L_1 + \gamma' L_2 + \gamma' D)K_p \tag{9.8}$$

At the same depth,

$$\sigma'_a = \gamma' D K_a \tag{9.9}$$

Hence, the net lateral pressure at the bottom of the sheet pile is

$$\begin{aligned}\sigma'_p - \sigma'_a = \sigma'_4 &= (\gamma L_1 + \gamma' L_2)K_p + \gamma' D(K_p - K_a)\\ &= (\gamma L_1 + \gamma' L_2)K_p + \gamma' L_3(K_p - K_a) + \gamma' L_4(K_p - K_a)\\ &= \sigma'_5 + \gamma' L_4(K_p - K_a)\end{aligned} \tag{9.10}$$

where

$$\sigma'_5 = (\gamma L_1 + \gamma' L_2)K_p + \gamma' L_3(K_p - K_a) \tag{9.11}$$

$$D = L_3 + L_4 \tag{9.12}$$

For the stability of the wall, the principles of statics can now be applied:

$$\Sigma \text{ horizontal forces per unit length of wall} = 0$$

and

$$\Sigma \text{ moment of the forces per unit length of wall about point } B = 0$$

For the summation of the horizontal forces, we have

$$\text{Area of the pressure diagram } ACDE - \text{area of } EFHB + \text{area of } FHBG = 0$$

or

$$P - \tfrac{1}{2}\sigma_3' L_4 + \tfrac{1}{2} L_5 (\sigma_3' + \sigma_4') = 0 \tag{9.13}$$

where $P$ = area of the pressure diagram $ACDE$.

Summing the moment of all the forces about point $B$ yields

$$P(L_4 + \bar{z}) - \left(\frac{1}{2} L_4 \sigma_3'\right)\left(\frac{L_4}{3}\right) + \frac{1}{2} L_5 (\sigma_3' + \sigma_4')\left(\frac{L_5}{3}\right) = 0 \tag{9.14}$$

From Eq. (9.13),

$$L_5 = \frac{\sigma_3' L_4 - 2P}{\sigma_3' + \sigma_4'} \tag{9.15}$$

Combining Eqs. (9.7), (9.10), (9.14), and (9.15) and simplifying them further, we obtain the following fourth-degree equation in terms of $L_4$:

$$L_4^4 + A_1 L_4^3 - A_2 L_4^2 - A_3 L_4 - A_4 = 0 \tag{9.16}$$

In this equation,

$$A_1 = \frac{\sigma_5'}{\gamma'(K_p - K_a)} \tag{9.17}$$

$$A_2 = \frac{8P}{\gamma'(K_p - K_a)} \tag{9.18}$$

$$A_3 = \frac{6P[2\bar{z}\gamma'(K_p - K_a) + \sigma_5']}{\gamma'^2(K_p - K_a)^2} \tag{9.19}$$

$$A_4 = \frac{P(6\bar{z}\sigma_5' + 4P)}{\gamma'^2(K_p - K_a)^2} \tag{9.20}$$

# Calculation of Maximum Bending Moment

- The maximum moment will occur between points $E$ and $F$`. Obtaining the maximum moment $(M_{max})$ per unit length of the wall requires determining the point of zero shear. For a new axis $z$` (with origin at point $E$) for zero shear,

$$P = \frac{1}{2}(z')^2(K_p - K_a)\gamma'$$

or

$$z' = \sqrt{\frac{2P}{(K_p - K_a)\gamma'}} \qquad (9.21)$$

The magnitude of the maximum moment can be obtained as

$$M_{max} = P(\bar{z} + z') - \left[\frac{1}{2}\gamma' z'^2 (K_p - K_a)\right]\left(\frac{1}{3}\right)z' \qquad (9.22)$$

The necessary profile of the sheet piling is then sized according to the allowable flexural stress of the sheet pile material, or

$$S = \frac{M_{\text{max}}}{\sigma_{\text{all}}} \tag{9.23}$$

where

$S$ = section modulus of the sheet pile required per unit length of the structure
$\sigma_{\text{all}}$ = allowable flexural stress of the sheet pile

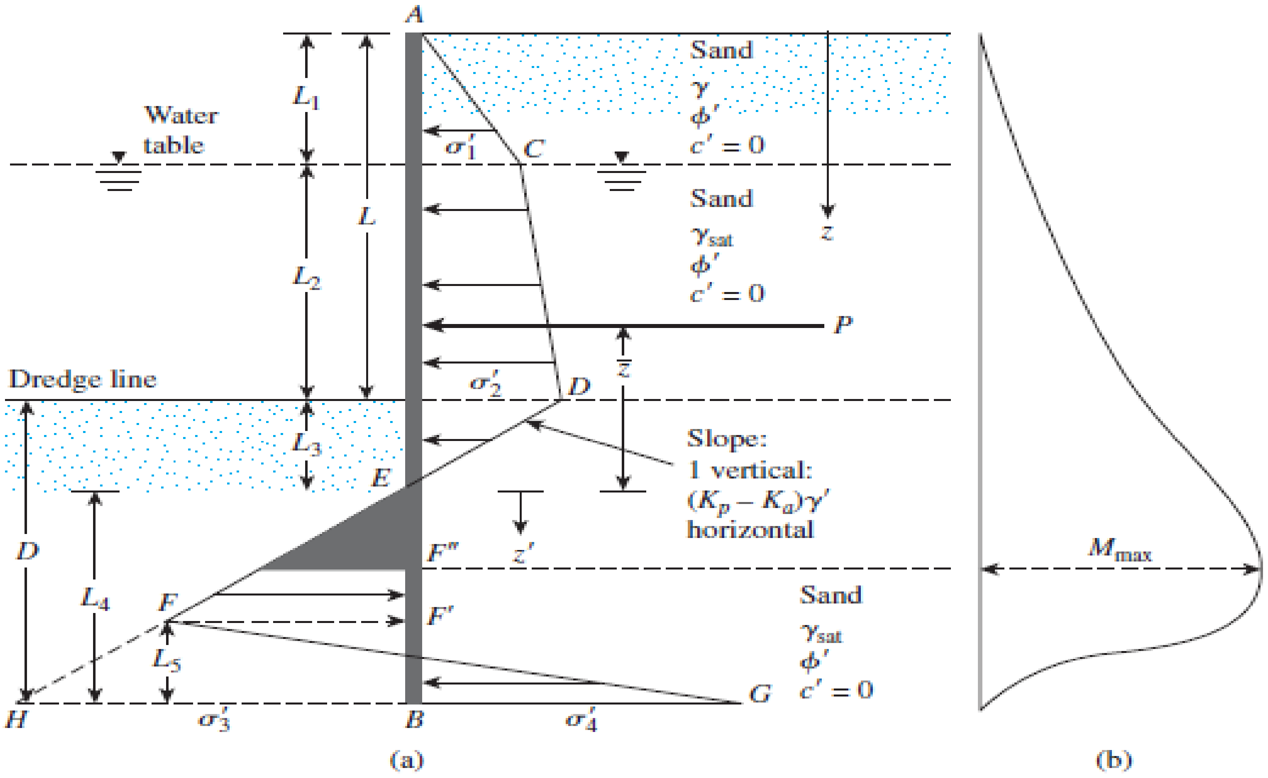
Figure 9.9 shows a cantilever sheet pile wall penetrating a granular soil. Here, $L_1 = 2$ m, $L_2 = 3$ m, $\gamma = 15.9$ kN/m$^3$, $\gamma_{sat} = 19.33$ kN/m$^3$, and $\phi' = 32°$.

a. What is the theoretical depth of embedment, $D$?
b. For a 30% increase in $D$, what should be the total length of the sheet piles?
c. What should be the minimum section modulus of the sheet piles? Use $\sigma_{all} =$ 172 MN/m$^2$.

## Solution

### Part a

Using Figure 9.8a for the pressure distribution diagram, one can now prepare the following table for a step-by-step calculation.




Cantilever sheet pile penetrating sand: (a) variation of net pressure diagram; (b) variation of moment

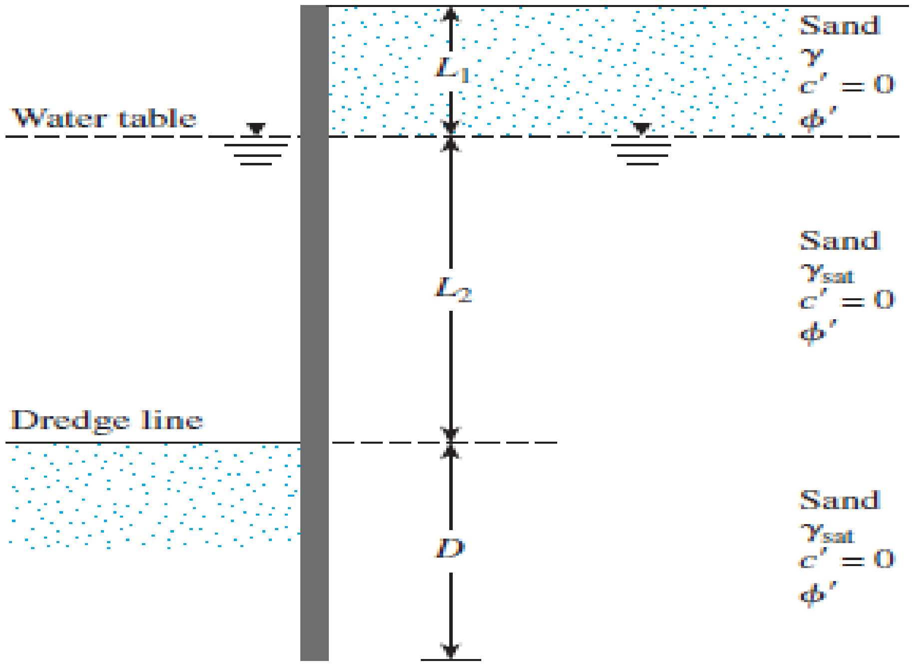



**Figure 9.9** Cantilever sheet-pile wall

| Quantity required | Eq. no. | Equation and calculation |
|---|---|---|
| $K_a$ | — | $\tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2\left(45 - \frac{32}{2}\right) = 0.307$ |
| $K_p$ | — | $\tan^2\left(45 + \frac{\phi'}{2}\right) = \tan^2\left(45 + \frac{32}{2}\right) = 3.25$ |
| $\sigma_1'$ | 9.1 | $\gamma L_1 K_a = (15.9)(2)(0.307) = 9.763\ \text{kN/m}^2$ |
| $\sigma_2'$ | 9.2 | $(\gamma L_1 + \gamma' L_2)K_a = [(15.9)(2) + (19.33 - 9.81)(3)](0.307) = 18.53\ \text{kN/m}^2$ |
| $L_3$ | 9.6 | $\frac{\sigma_2'}{\gamma'(K_p - K_a)} = \frac{18.53}{(19.33 - 9.81)(3.25 - 0.307)} = 0.66\ \text{m}$ |
| $P$ | — | $\frac{1}{2}\sigma_1' L_1 + \sigma_1' L_2 + \frac{1}{2}(\sigma_2' - \sigma_1')L_2 + \frac{1}{2}\sigma_2' L_3$ <br> $= (\frac{1}{2})(9.763)(2) + (9.763)(3) + (\frac{1}{2})(18.53 - 9.763)(3) + (\frac{1}{2})(18.53)(0.66)$ <br> $= 9.763 + 29.289 + 13.151 + 6.115 = 58.32\ \text{kN/m}$ |

| | | |
|---|---|---|
| $\bar{z}$ | — | $\dfrac{\Sigma M_E}{P} = \dfrac{1}{58.32}\left[\begin{array}{l} 9.763(0.66 + 3 + \frac{2}{3}) + 29.289(0.66 + \frac{3}{2}) \\ + 13.151(0.66 + \frac{3}{3}) + 6.115(0.66 \times \frac{2}{3}) \end{array}\right] = 2.23\text{ m}$ |
| $\sigma_5'$ | 9.11 | $(\gamma L_1 + \gamma' L_2)K_p + \gamma' L_3(K_p - K_a) = [(15.9)(2) + (19.33 - 9.81)(3)](3.25) + (19.33 - 9.81)(0.66)(3.25 - 0.307) = 214.66\text{ kN/m}^2$ |
| $A_1$ | 9.17 | $\dfrac{\sigma_5'}{\gamma'(K_p - K_a)} = \dfrac{214.66}{(19.33 - 9.81)(3.25 - 0.307)} = 7.66$ |
| $A_2$ | 9.18 | $\dfrac{8P}{\gamma'(K_p - K_a)} = \dfrac{(8)(58.32)}{(19.33 - 9.81)(3.25 - 0.307)} = 16.65$ |
| $A_3$ | 9.19 | $\dfrac{6P[2\bar{z}\gamma'(K_p - K_a) + \sigma_5']}{\gamma'^2(K_p - K_a)^2} = \dfrac{(6)(58.32)[(2)(2.23)(19.33 - 9.81)(3.25 - 0.307) + 214.66]}{(19.33 - 9.81)^2(3.25 - 0.307)^2} = 151.93$ |
| $A_4$ | 9.20 | $\dfrac{P(6\bar{z}\sigma_5' + 4P)}{\gamma'^2(K_p - K_a)^2} = \dfrac{58.32[(6)(2.23)(214.66) + (4)(58.32)]}{(19.33 - 9.81)^2(3.25 - 0.307)^2} = 230.72$ |
| $L_4$ | 9.16 | $L_4^4 + A_1 L_4^3 - A_2 L_4^2 - A_3 L_4 - A_4 = 0$<br>$L_4^4 + 7.66L_4^3 - 16.65L_4^2 - 151.93L_4 - 230.72 = 0;\ L_4 \approx 4.8\text{ m}$ |

Thus,

$$D_{\text{theory}} = L_3 + L_4 = 0.66 + 4.8 = \mathbf{5.46\ m}$$

Part b

The total length of the sheet piles is

$$L_1 + L_2 + 1.3(L_3 + L_4) = 2 + 3 + 1.3(5.46) = \mathbf{12.1\ m}$$

Part c

Finally, we have the following table.

| Quantity required | Eq. no. | Equation and calculation |
|---|---|---|
| $z'$ | 9.21 | $\sqrt{\dfrac{2P}{(K_p - K_a)\gamma'}} = \sqrt{\dfrac{(2)(58.32)}{(3.25 - 0.307)(19.33 - 9.81)}} = 2.04\ \text{m}$ |
| $M_{\max}$ | 9.22 | $P(\bar{z} + z') - \left[\dfrac{1}{2}\gamma' z'^2 (K_p - K_a)\right]\dfrac{z'}{3} = (58.32)(2.23 + 2.04)$ $-\left[\left(\dfrac{1}{2}\right)(19.33 - 9.81)(2.04)^2(3.25 - 0.307)\right]\dfrac{2.04}{3}$ $= 209.39\ \text{kN}\cdot\text{m/m}$ |
| $S$ | 9.29 | $\dfrac{M_{\max}}{\sigma_{\text{all}}} = \dfrac{209.39\ \text{kN}\cdot\text{m}}{172 \times 10^3\ \text{kN/m}^2} = \mathbf{1.217 \times 10^{-3}\ m^3/m\ of\ wall}$ ■ |

# Special Cases for Cantilever Walls Penetrating a Sandy Soil

- **Sheet Pile Wall with the Absence of Water Table**
- the net pressure diagram on the cantilever sheet-pile wall will be as shown in Figure 9.10

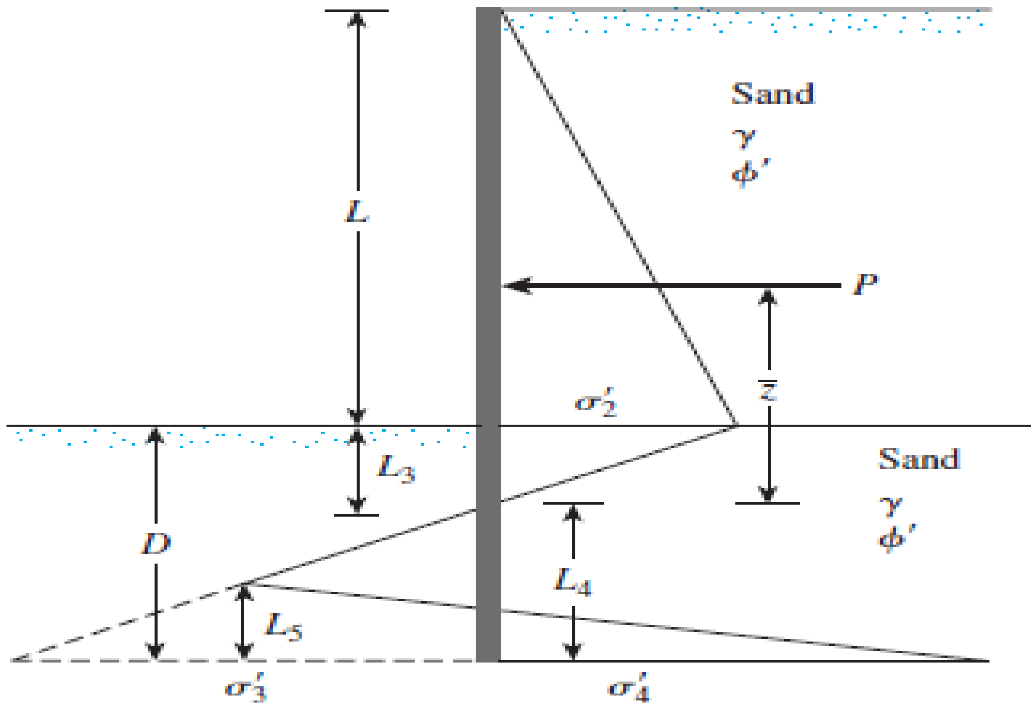



*Figure 9.10* Sheet piling penetrating a sandy soil in the absence of the water table

$$\sigma'_2 = \gamma L K_a \quad (9.24)$$

$$\sigma'_3 = L_4(K_p - K_a)\gamma \quad (9.25)$$

$$\sigma'_4 = \sigma'_5 + \gamma L_4(K_p - K_a) \quad (9.26)$$

$$\sigma'_5 = \gamma L K_p + \gamma L_3(K_p - K_a) \quad (9.27)$$

$$L_3 = \frac{\sigma'_2}{\gamma(K_p - K_a)} = \frac{LK_a}{(K_p - K_a)} \quad (9.28)$$

$$P = \tfrac{1}{2}\sigma'_2 L + \tfrac{1}{2}\sigma'_2 L_3 \quad (9.29)$$

$$\overline{z} = L_3 + \frac{L}{3} = \frac{LK_a}{K_p - K_a} + \frac{L}{3} = \frac{L(2K_a + K_p)}{3(K_p - K_a)} \quad (9.30)$$

and Eq. (9.16) transforms to

$$L_4^4 + A'_1 L_4^3 - A'_2 L_4^2 - A'_3 L_4 - A'_4 = 0 \quad (9.31)$$

where

$$A'_1 = \frac{\sigma'_5}{\gamma(K_p - K_a)} \quad (9.32)$$

$$A'_2 = \frac{8P}{\gamma(K_p - K_a)} \quad (9.33)$$

$$A'_3 = \frac{6P[2\overline{z}\gamma(K_p - K_a) + \sigma'_5]}{\gamma^2(K_p - K_a)^2} \quad (9.34)$$

$$A'_4 = \frac{P(6\overline{z}\sigma'_5 + 4P)}{\gamma^2(K_p - K_a)^2} \quad (9.35)$$

# Free Cantilever Sheet Piling

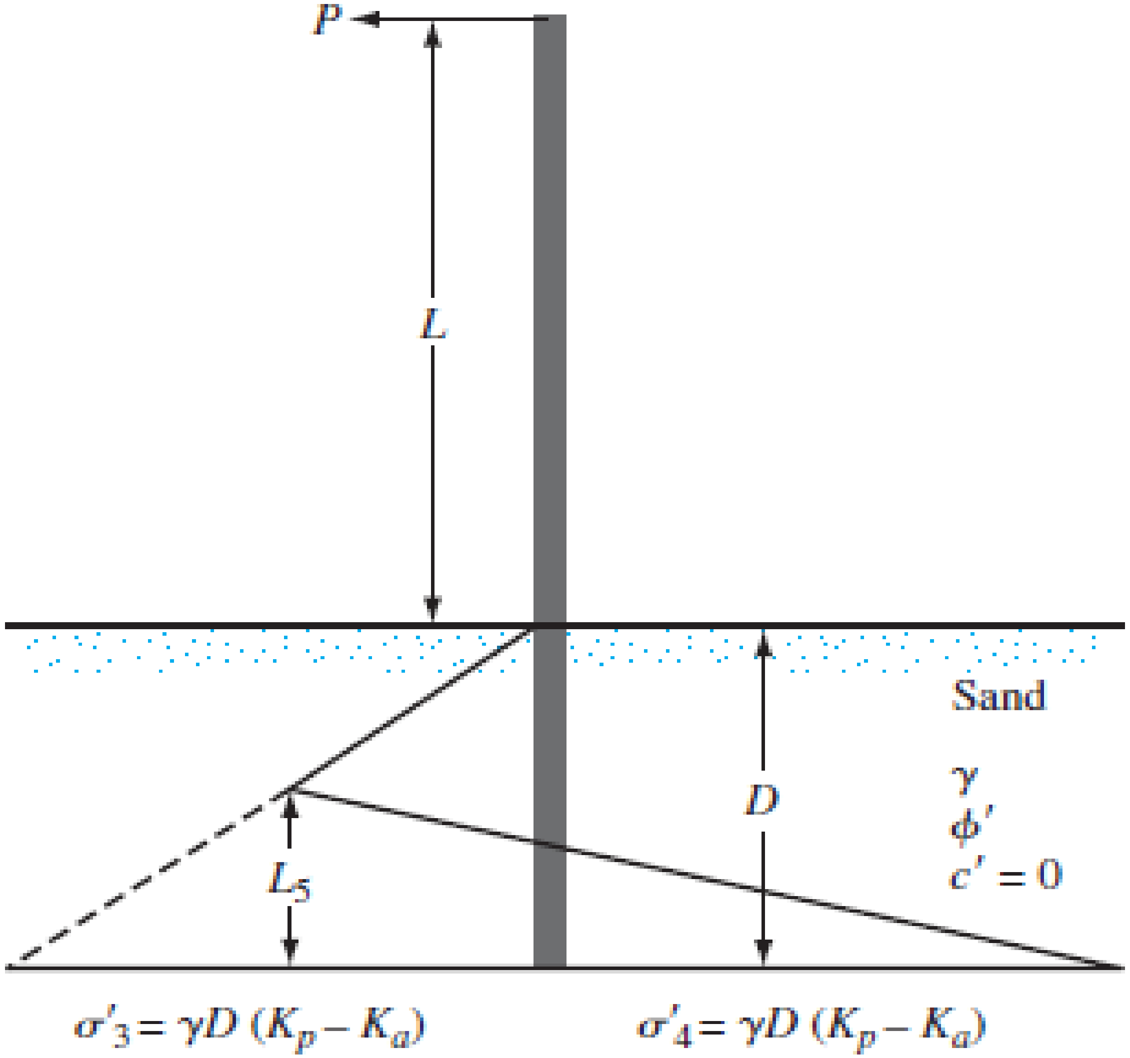



*Figure 9.11* Free cantilever sheet piling penetrating a layer of sand

Figure 9.11 shows a free cantilever sheet-pile wall penetrating a sandy soil and subjected to a line load of *P* per unit length of the wall. For this case,

$$D^4 - \left[\frac{8P}{\gamma(K_p - K_a)}\right]D^2 - \left[\frac{12PL}{\gamma(K_p - K_a)}\right]D - \left[\frac{2P}{\gamma(K_p - K_a)}\right]^2 = 0 \quad (9.36)$$

$$L_5 = \frac{\gamma(K_p - K_a)D^2 - 2P}{2D(K_p - K_a)\gamma} \quad (9.37)$$

$$M_{max} = P(L + z') - \frac{\gamma z'^3(K_p - K_a)}{6} \quad (9.38)$$

and

$$z' = \sqrt{\frac{2P}{\gamma'(K_p - K_a)}} \quad (9.39)$$

**Example**

Redo parts a and b of Example 9.1, assuming the absence of the water table. Use $\gamma = 15.9\ \text{kN/m}^3$ and $\phi' = 32°$. *Note:* $L = 5$ m.

## Solution

Part a

| Quantity required | Eq. no. | Equation and calculation |
| --- | --- | --- |
| $K_a$ | — | $\tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2\left(45 - \frac{32}{2}\right) = 0.307$ |
| $K_p$ | — | $\tan^2\left(45 + \frac{\phi'}{2}\right) = \tan^2\left(45 + \frac{32}{2}\right) = 3.25$ |
| $\sigma_2'$ | 9.24 | $\gamma L K_a = (15.9)(5)(0.307) = 24.41\ \text{kN/m}^2$ |
| $L_3$ | 9.28 | $\frac{LK_a}{K_p - K_a} = \frac{(5)(0.307)}{3.25 - 0.307} = 0.521\ \text{m}$ |
| $\sigma_5'$ | 9.27 | $\gamma L K_p + \gamma L_3(K_p - K_a) = (15.9)(5)(3.25) + (15.9)(0.521)(3.25 - 0.307) = 282.76\ \text{kN/m}^2$ |
| $P$ | 9.29 | $\frac{1}{2}\sigma_2' L + \frac{1}{2}\sigma_2' L_3 = \frac{1}{2}\sigma_2'(L + L_3) = \left(\frac{1}{2}\right)(24.41)(5 + 0.521) = 67.38\ \text{kN/m}$ |
| $\bar{z}$ | 9.30 | $\frac{L(2K_a - K_p)}{3(K_p - K_a)} = \frac{5[(2)(0.307) + 3.25]}{3(3.25 - 0.307)} = 2.188\ \text{m}$ |

| | | |
|---|---|---|
| $A_1'$ | 9.32 | $\dfrac{\sigma_5'}{\gamma(K_p - K_a)} = \dfrac{282.76}{(15.9)(3.25 - 0.307)} = 6.04$ |
| $A_2'$ | 9.33 | $\dfrac{8P}{\gamma(K_p - K_a)} = \dfrac{(8)(67.38)}{(15.9)(3.25 - 0.307)} = 11.52$ |
| $A_3'$ | 9.34 | $\dfrac{6P[2\bar{z}\gamma(K_p - K_a) + \sigma_5']}{\gamma^2(K_p - K_a)^2}$ $= \dfrac{(6)(67.38)[(2)(2.188)(15.9)(3.25 - 0.307) + 282.76]}{(15.9)^2(3.25 - 0.307)^2} = 90.01$ |
| $A_4'$ | 9.35 | $\dfrac{P(6\bar{z}\sigma_5' + 4P)}{\gamma^2(K_p - K_a)^2} = \dfrac{(67.38)[(6)(2.188)(282.76) + (4)(67.38)]}{(15.9)^2(3.25 - 0.307)^2} = 122.52$ |
| $L_4$ | 9.31 | $L_4^4 + A_1'L_4^3 - A_2'L_4^2 - A_3'L_4 - A_4' = 0$ <br> $L_4^4 + 6.04L_4^3 - 11.52L_4^2 - 90.01L_4 - 122.52 = 0;\ L_4 \approx 4.1$ m |

$$D_{theory} = L_3 + L_4 = 0.521 + 4.1 \approx \mathbf{4.7\ m}$$

Part b

Total length, $L + 1.3(D_{theory}) = 5 + 1.3(4.7) = \mathbf{11.11\ m}$

# Mechanically Stabilized Earth Walls (MSE Walls)

13/4/2020

# Components of MSE walls

- Backfill
- Reinforced elements (Geogrids, Metallic strips, geotextile)
- Wall facing (wall skin, panels)

# Design of wall

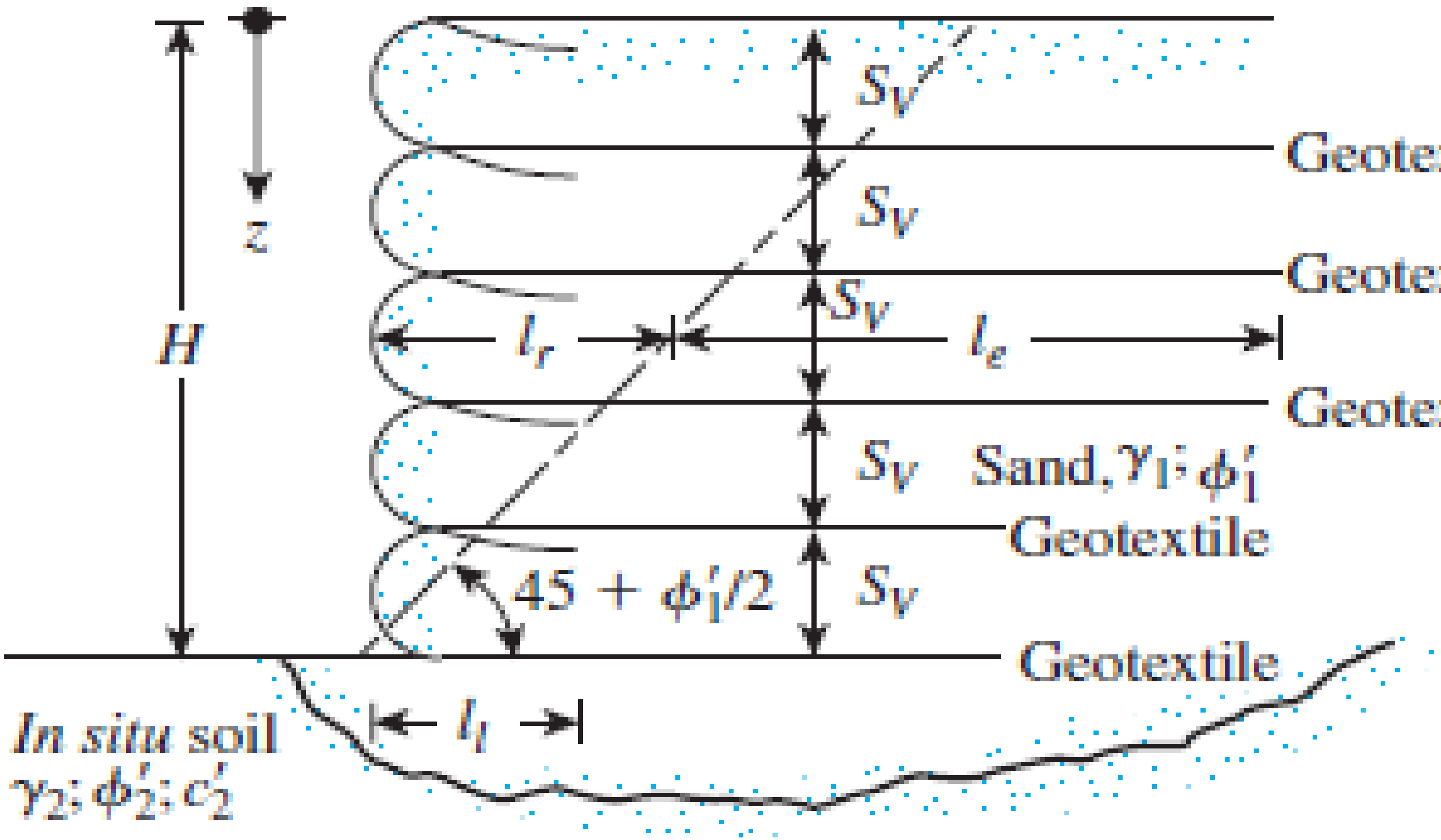
- External stabilitychecks
  - Sliding
  - Overturning
  - Bearing capacity

- Internal Stability
  - Breakage of reinforcement
  - Pullout Mechanism

# Retaining Walls with Geotextile Reinforcement

Figure 8.35 A completed geotextile-reinforced retaining wall in DeBeque Canyon, Colorado (*Courtesy of Jonathan T. H. Wu, University of Colorado at Denver, Denver, Colorado*)

# Retaining Walls with Geotextile Reinforcement



# Design procedures:

- *Step 1.* Determine the active pressure distribution on the wall from the formula

$$\sigma'_a = K_a \sigma'_o = K_a \gamma_1 z$$

where

$K_a$ = Rankine active pressure coefficient = $\tan^2(45 - \phi'_1/2)$
$\gamma_1$ = unit weight of the granular backfill
$\phi'_1$ = friction angle of the granular backfill

- The recommended values of the reduction factor are as follows (Koerner, 2005)

$$T_{all} = \frac{T_{ult}}{RF_{id} \times RF_{cr} \times RF_{cbd}}$$

where

$T_{ult}$ = ultimate tensile strength
$RF_{id}$ = reduction factor for installation damage
$RF_{cr}$ = reduction factor for creep
$RF_{cbd}$ = reduction factor for chemical and biological degradation

The recommended values of the reduction factor are as follows (Koerner, 2005)

| | |
|---|---|
| $RF_{id}$ | 1.1–2.0 |
| $RF_{cr}$ | 2–4 |
| $RF_{cbd}$ | 1–1.5 |

*Step 3.* Determine the vertical spacing of the layers at any depth *z* from the formula

$$S_V = \frac{T_{all}}{\sigma'_a \mathrm{FS}_{(B)}} = \frac{T_{all}}{(\gamma_1 z K_a)[\mathrm{FS}_{(B)}]}$$

The magnitude of $FS_{(B)}$ is generally 1.3 to 1.5.

$FS_{(B)}$ : Factor of safety against tie breakage.

*Step 4.* Determine the length of each layer of geotextile from the formula

$$L = l_r + l_e$$

where

$$l_r = \frac{H - z}{\tan\left(45 + \frac{\phi'_1}{2}\right)}$$

and

$$l_e = \frac{S_V \sigma'_a [\mathrm{FS}_{(P)}]}{2\sigma'_o \tan \phi'_F}$$

in which

$$\sigma'_a = \gamma_1 z K_a$$

$$\sigma'_o = \gamma_1 z$$

$$\mathrm{FS}_{(P)} = 1.3 \text{ to } 1.5$$

$$\phi'_F = \text{friction angle at geotextile–soil interface} \approx \tfrac{2}{3}\phi'_1$$

- *Step 5.* Determine the lap length, $l_l$ from

$$l_l = \frac{S_V \sigma'_a \mathrm{FS}_{(P)}}{4\sigma'_o \tan\phi'_F}$$

The minimum lap length should be 1 m.

*Step 6.* Check the factors of safety against overturning, sliding, and bearing capacity

# Example:

A geotextile-reinforced retaining wall 5 m high is shown in Figure 8.36. For the granular backfill, $\gamma_1 = 15.7\ \mathrm{kN/m^3}$ and $\phi'_1 = 36°$. For the geotextile, $T_{ult} = 52.5\ \mathrm{kN/m}$. For the design of the wall, determine $S_V$, $L$, and $l_l$. Use $RF_{id} = 1.2$, $RF_{cr} = 2.5$, and $RF_{cbd} = 1.25$.

**Solution**

We have

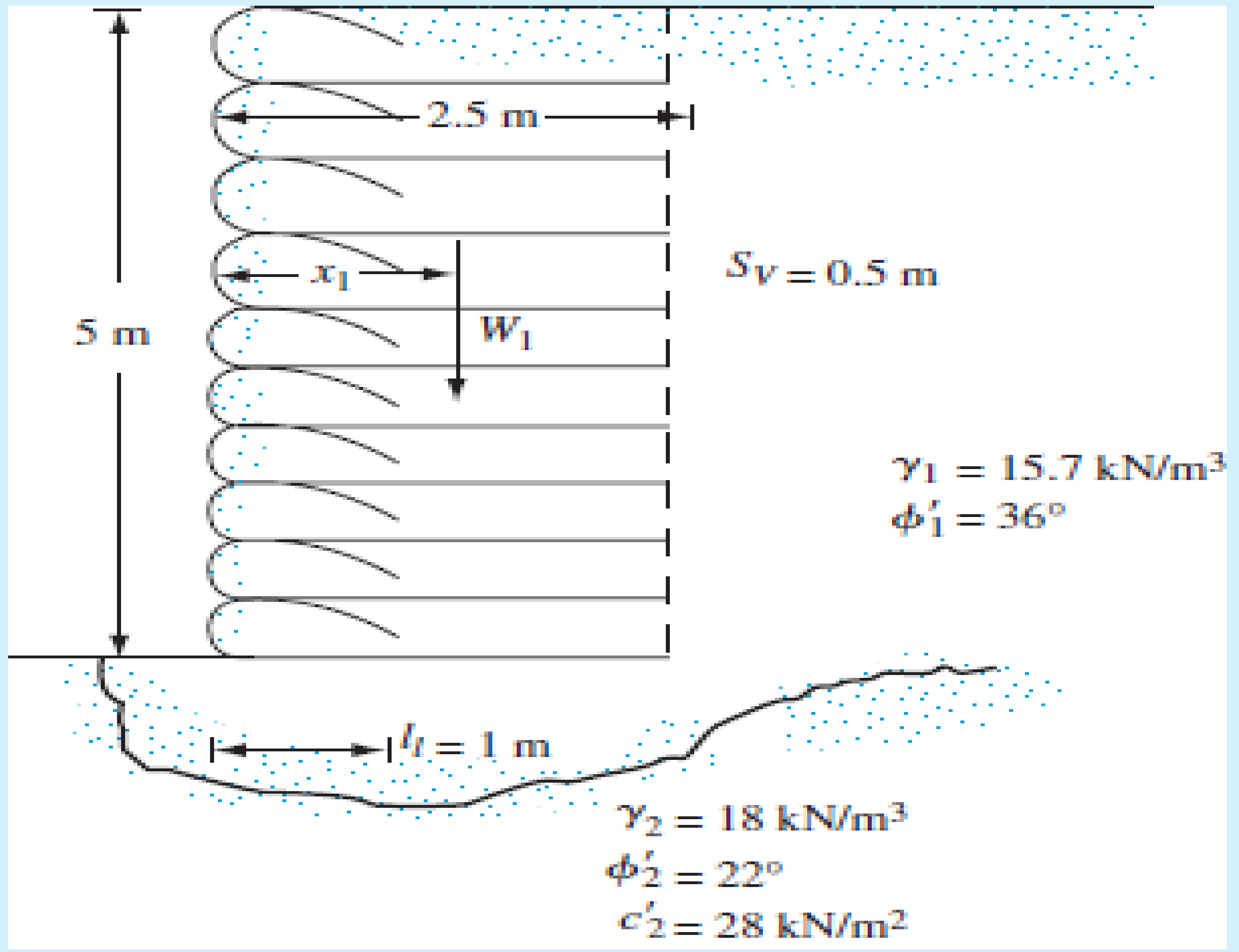
$$K_a = \tan^2\left(45 - \frac{\phi'_1}{2}\right) = 0.26$$

Determination of $S_V$

To find $S_V$, we make a few trials. From Eq. (8.57),

$$S_V = \frac{T_{all}}{(\gamma_1 z K_a)[FS_{(B)}]}$$

# Example:



2.5 m
$S_V = 0.5$ m
$x_1$
$W_1$
5 m
$\gamma_1 = 15.7$ kN/m³
$\phi'_1 = 36°$
$l_l = 1$ m
$\gamma_2 = 18$ kN/m³
$\phi'_2 = 22°$
$c'_2 = 28$ kN/m²

# Example:

$$T_{all} = \frac{T_{uef}}{RF_{id} \times RF_{cr} \times RF_{cbd}} = \frac{52.5}{1.2 \times 2.5 \times 1.25} = 14 \text{ kN/m}$$

With $FS_{(B)} = 1.5$ at $z = 2$ m,

$$S_V = \frac{14}{(15.7)(2)(0.26)(1.5)} = 1.14 \text{ m}$$

At $z = 4$ m,

$$S_V = \frac{14}{(15.7)(4)(0.26)(1.5)} = 0.57 \text{ m}$$

At $z = 5$ m,

$$S_V = \frac{14}{(15.7)(5)(0.26)(1.5)} = 0.46 \text{ m}$$

**So, use $S_V = 0.5$ m for $z = 0$ to $z = 5$ m** (See Figure 8.36.)

Determination of $L$

From Eqs. (8.58), (8.59), and (8.60),

$$L = \frac{(H - z)}{\tan\left(45 + \frac{\phi'_1}{2}\right)} + \frac{S_V K_a[FS_{(P)}]}{2 \tan \phi'_F}$$

# Example:

For $FS_{(P)} = 1.5$, $\tan\phi'_F = \tan[(\frac{2}{3})(36)] = 0.445$, and it follows that

$$L = (0.51)(H - z) + 0.438S_V$$

$H = 5$ m, $S_V = 0.5$ m

At $z = 0.5$ m: $L = (0.51)(5 - 0.5) + (0.438)(0.5) = 2.514$ m
At $z = 2.5$ m: $L = (0.51)(5 - 2.5) + (0.438)(0.5) = 1.494$ m

**So, use $L = 2.5$ m throughout.**

Determination of $l_l$
From Eq. (8.61),

$$l_l = \frac{S_V \sigma'_a [FS_{(P)}]}{4\sigma'_o \tan\phi'_F}$$

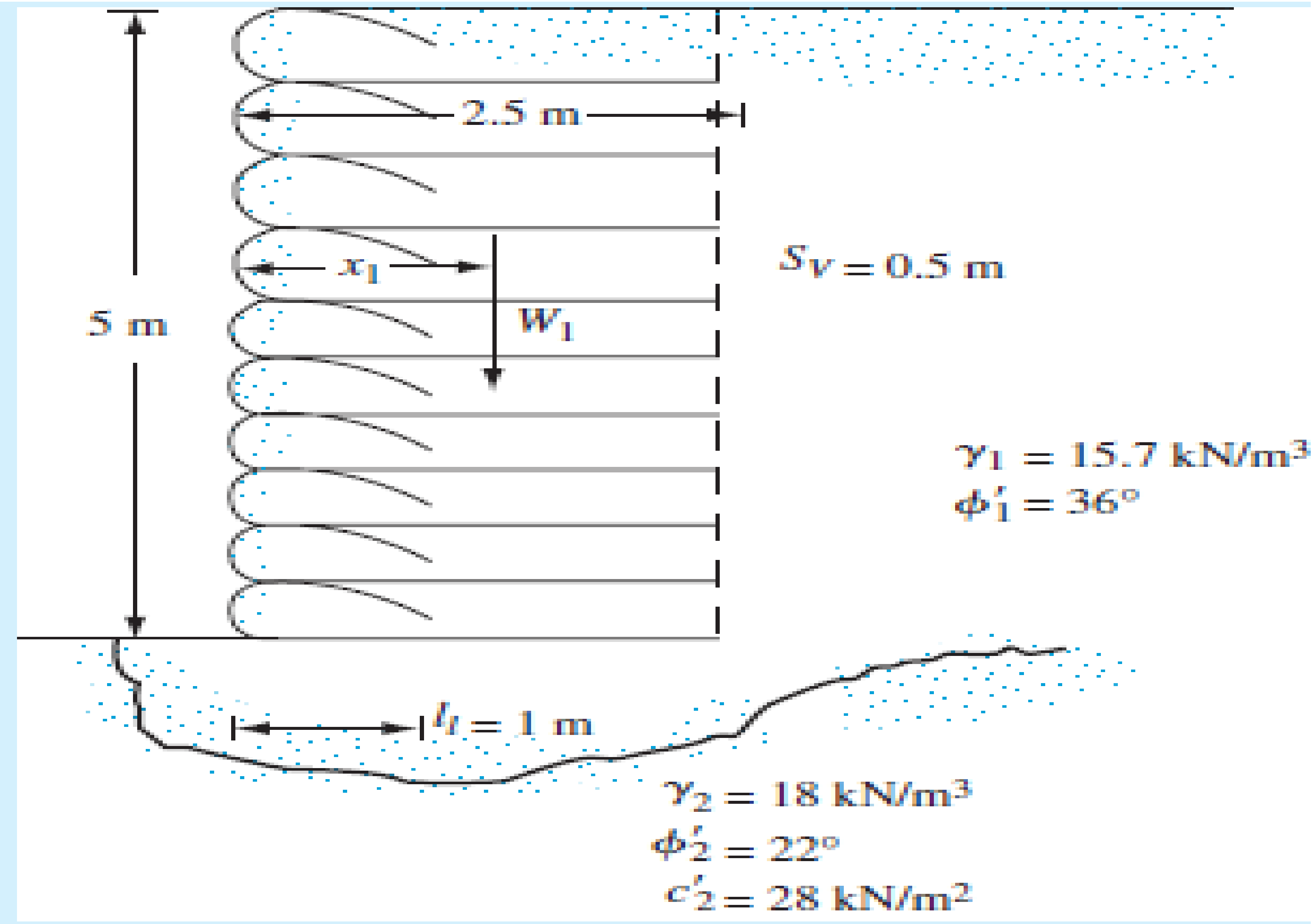
$\sigma'_a = \gamma_1 z K_a$, $FS_{(P)} = 1.5$; with $\sigma'_o = \gamma_1 z$, $\phi'_F = \frac{2}{3}\phi'_1$. So

$$l_l = \frac{S_V K_a [FS_{(P)}]}{4 \tan \phi'_F} = \frac{S_V (0.26)(1.5)}{4\tan[(\frac{2}{3})(36)]} = 0.219S_V$$

$$l_l = 0.219S_V = (0.219)(0.5) = 0.11 \text{ m} \le 1 \text{ m}$$

So, use $\boldsymbol{l_l} = \mathbf{1}$ **m.**

For the same example, calculate the factor of safety against overturning, sliding, and bearing capacity failure.



## Factor of Safety Against Overturning

From Eq. (8.50), $FS_{(overturning)} = \frac{W_1 x_1}{(P_a)\left(\frac{H}{3}\right)}$

$$W_1 = (5)(2.5)(15.7) = 196.25 \text{ kN/m}$$

$$x_1 = \frac{2.5}{2} = 1.25 \text{ m}$$

$$P_a = \frac{1}{2}\gamma H^2 K_a = \left(\frac{1}{2}\right)(15.7)(5)^2(0.26) = 51.03 \text{ kN/m}$$

Hence,

$$FS_{(overturning)} = \frac{(196.25)(1.25)}{51.03(5/3)} = 2.88 < 3$$

(increase length of geotextile layers to 3 m)

Factor of Safety Against Sliding

From Eq. (8.51),

$$FS_{(\text{sliding})} = \frac{W_1 \tan\left(\frac{2}{3}\phi_1'\right)}{P_a} = \frac{(196.25)\left[\tan\left(\frac{2}{3} \times 36\right)\right]}{51.03} = \mathbf{1.71 > 1.5 - O.K.}$$

Factor of Safety Against Bearing Capacity Failure

From Eq. (8.52), $q_u = c_2' N_c + \frac{1}{2}\gamma_2 L_2 N_\gamma$

Given: $\gamma_2 = 18$ kN/m$^3$, $L_2 = 2.5$ m, $c_2' = 28$ kN/m$^2$, and $\phi_2' = 22°$. From Table 3.3, $N_c = 16.88$, and $N_\gamma = 7.13$.

$$q_u = (28)(16.88) + \left(\frac{1}{2}\right)(18)(2.5)(7.13) \approx 633 \text{ kN/m}^2$$

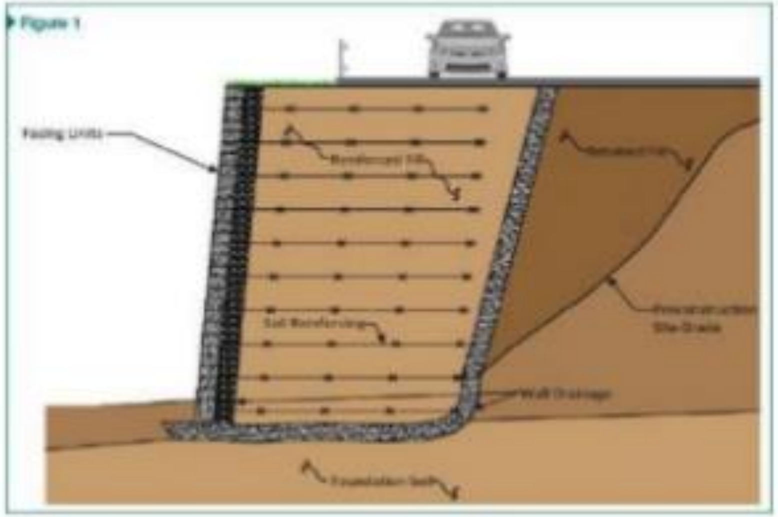
From Eq. (8.54),

$$FS_{(\text{bearing capacity})} = \frac{q_u}{\sigma'_{o(H)}} = \frac{633}{\gamma_1 H} = \frac{633}{(15.7)(5)} = \mathbf{8.06 > 3 - O.K.}$$

■

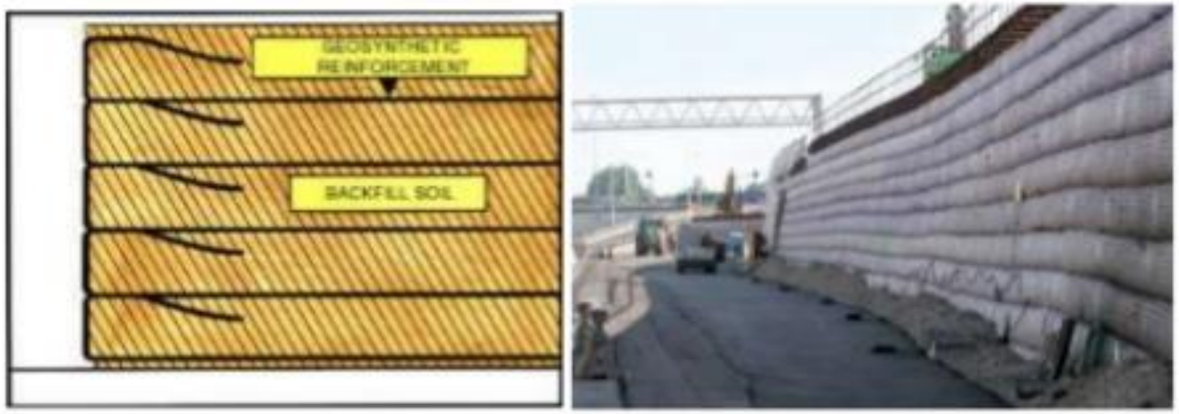
# MSE Walls

# MSE Geotextile wall




Δ *MSE retaining wall*

Since our main topic is about geotextile MSE wall, I will now only focus on the main component of geotextile MSE wall which is geotextile.

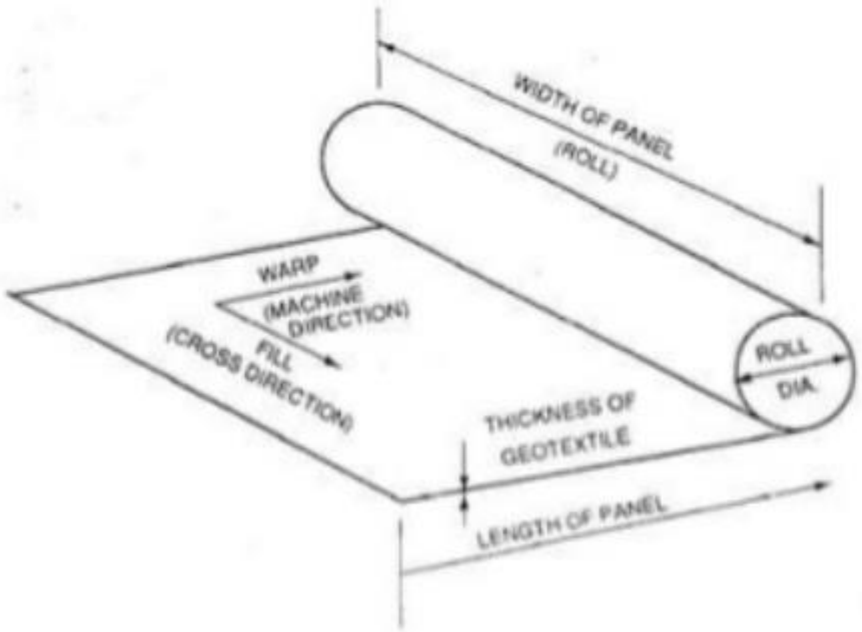


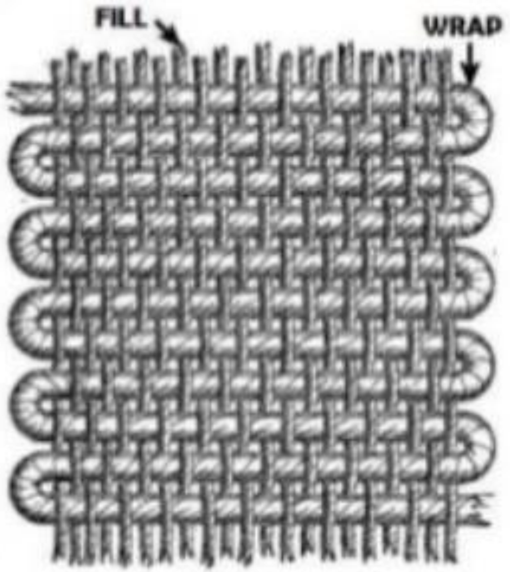

Δ *MSE wall with geotextile as soil reinforcement*

Type of Geotextile:

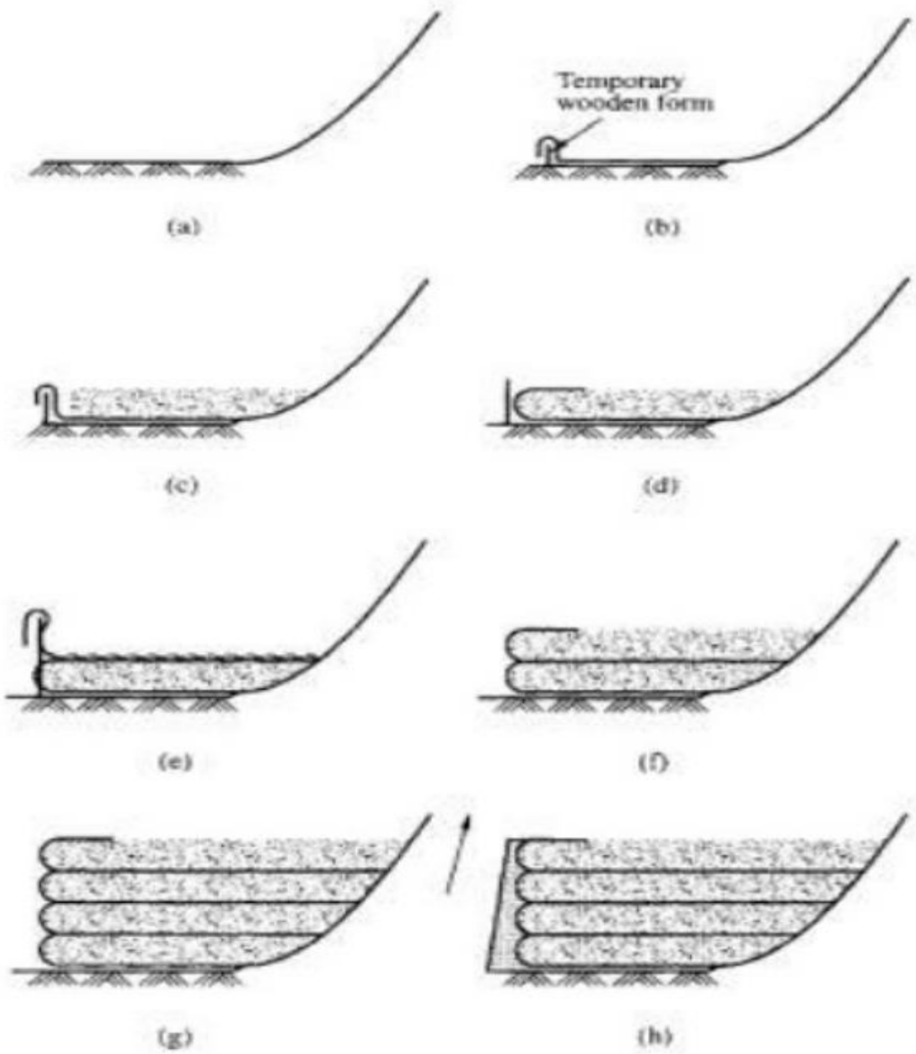
Δ *Woven Geotextile*

Δ *Non-woven Geotextile*


WIDTH OF PANEL
(ROLL)
WARP
(MACHINE
DIRECTION)
FILL
(CROSS DIRECTION)
ROLL
DIA
THICKNESS OF
GEOTEXTILE
LENGTH OF PANEL


FILL
WRAP


# Construction process of geotextile MSE retaining wall



# Construction process of geotextile MSE retaining wall

1. Start with an adequate working surface and staging area
2. Lay a geotextile sheet of proper width on the ground surface with 4 to 7 ft at the wall face draped over a temporary wooden form.
3. Backfill over this sheet with soil.
4. Construction equipment must work from the soil backfill and be kept off the unprotected geotextile. The spreading equipment should be a wide-tracked bulldozer that exerts little pressure against the ground on which it rests. Rolling equipment likewise should be of relatively light weight.
5. When the first layer has been folded over the process should be repeated for the second layer with the temporary facing form being extended from the original ground surface or the wall being stepped back about 6 inches so that the form can be supported from the first layer. In the latter case, the support stakes must penetrate the fabric.
6. This process is continued until the wall reaches its intended height.
7. For protection against ultraviolet light and safety against vandalism the faces of such walls must be protected. Both shotcrete and gunite have been used for this purpose.

Please watch the video in the link below:

- https://www.youtube.com/watch?v=W8JlgcwmRf8

Example:

| | |
|---|---|
| H (m) = | 6 |
| $\phi'_1$ (deg) = | 36 |
| $FS_{(P)}$ = | 1.4 |
| *Cr* = | 1 |
| Tult (kN/m) = | 50 |
| Tall (kN/m) = | 13.1 |
| $\phi'_F$ (deg) = | 24 |
| $\gamma_1$ (kN/m$^3$) = | 18 |
| q (kPa) = | 10 |
| Ka = | 0.26 |

Examples : (1)

| Layer No. | z (m) | Sv (m) | σ'a (total) (kPa) | σ'o (kPa) | Le (m) | Min. Le (m) | Lr (m) | Calculated L (m) | Recommended L (m) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.65 | 0.65 | 5.64 | 21.7 | 0.27 | 1.0 | 2.73 | 3.73 | 4.0 |
| 2 | 1.30 | 0.65 | 8.68 | 33.4 | 0.27 | 1.0 | 2.39 | 3.39 | 4.0 |
| 3 | 1.80 | 0.50 | 11.02 | 42.4 | 0.20 | 1.0 | 2.14 | 3.14 | 4.0 |
| 4 | 2.30 | 0.50 | 13.36 | 51.4 | 0.20 | 1.0 | 1.89 | 2.89 | 3.0 |
| 5 | 2.80 | 0.50 | 15.70 | 60.4 | 0.20 | 1.0 | 1.63 | 2.63 | 3.0 |
| 6 | 3.30 | 0.50 | 18.04 | 69.4 | 0.20 | 1.0 | 1.38 | 2.38 | 3.0 |
| 7 | 3.60 | 0.30 | 19.45 | 74.8 | 0.12 | 1.0 | 1.22 | 2.22 | 3.0 |
| 8 | 3.90 | 0.30 | 20.85 | 80.2 | 0.12 | 1.0 | 1.07 | 2.07 | 3.0 |
| 9 | 4.20 | 0.30 | 22.26 | 85.6 | 0.12 | 1.0 | 0.92 | 1.92 | 2.5 |
| 10 | 4.50 | 0.30 | 23.66 | 91.0 | 0.12 | 1.0 | 0.76 | 1.76 | 2.5 |
| 11 | 4.80 | 0.30 | 25.06 | 96.4 | 0.12 | 1.0 | 0.61 | 1.61 | 2.5 |
| 12 | 5.10 | 0.30 | 26.47 | 101.8 | 0.12 | 1.0 | 0.46 | 1.46 | 2.5 |
| 13 | 5.40 | 0.30 | 27.87 | 107.2 | 0.12 | 1.0 | 0.31 | 1.31 | 2.5 |
| 14 | 5.70 | 0.30 | 29.28 | 112.6 | 0.12 | 1.0 | 0.15 | 1.15 | 2.5 |
| 15 | 6.00 | 0.30 | 30.68 | 118.0 | 0.12 | 1.0 | 0.00 | 1.00 | 2.5 |

# MSE Geotextile wall