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Cartesian Coordinate system

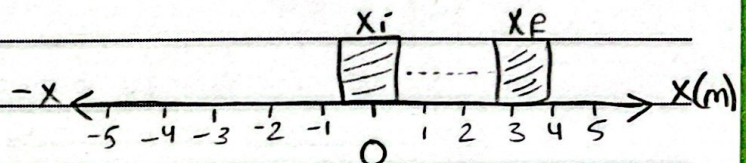
y

-x

x

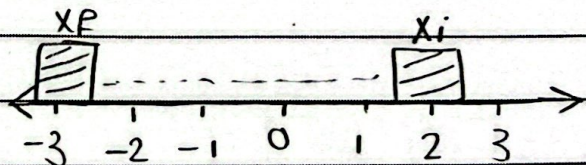
-y

* Motion along x-axis:

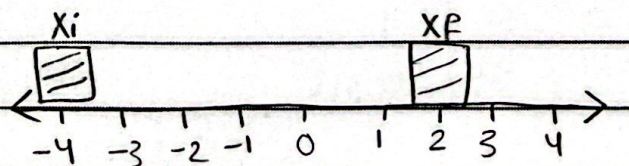


$$\begin{aligned} \Delta x &= x_f - x_i \\ &= 3 - 0 = 3\text{m} \end{aligned}$$

Displacement $\left\{ \begin{array}{l} x_i = 0\text{m} \\ x_f = 3\text{m} \end{array} \right.$



$$\begin{aligned} \Delta x &= x_f - x_i \\ &= -3 - 2 = -5 \end{aligned}$$



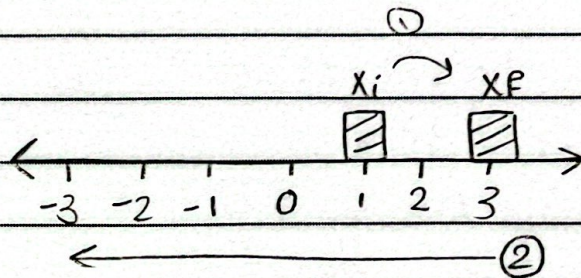
$$\begin{aligned} \Delta x &= x_f - x_i \\ &= 2 - (-4) = 6 \end{aligned}$$

⇒ Displacement depends on initial and Final Position Only

$$\left. \begin{array}{l} X_i = -4 \\ X_f = -2 \end{array} \right\} \rightarrow \Delta x = X_f - X_i = -2 - (-4) = 2 \text{ m}$$

Displacement: $\Delta x = X_f - X_i$

* Distance (d):



* Displacement: Δx

Distance: d

Distance: d

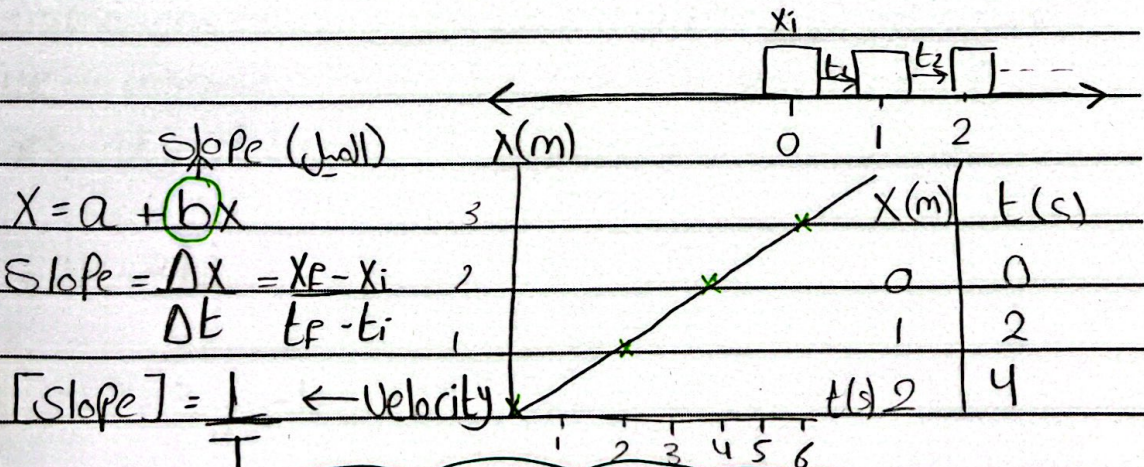
بناخذ المطلوعين موجب

$$d_1 = |\Delta x| = |X_f - X_i| = |3 - 1| = 2$$

$$d_2 = |\Delta x| = |X_f - X_i| = |-3 - 1| = 6$$

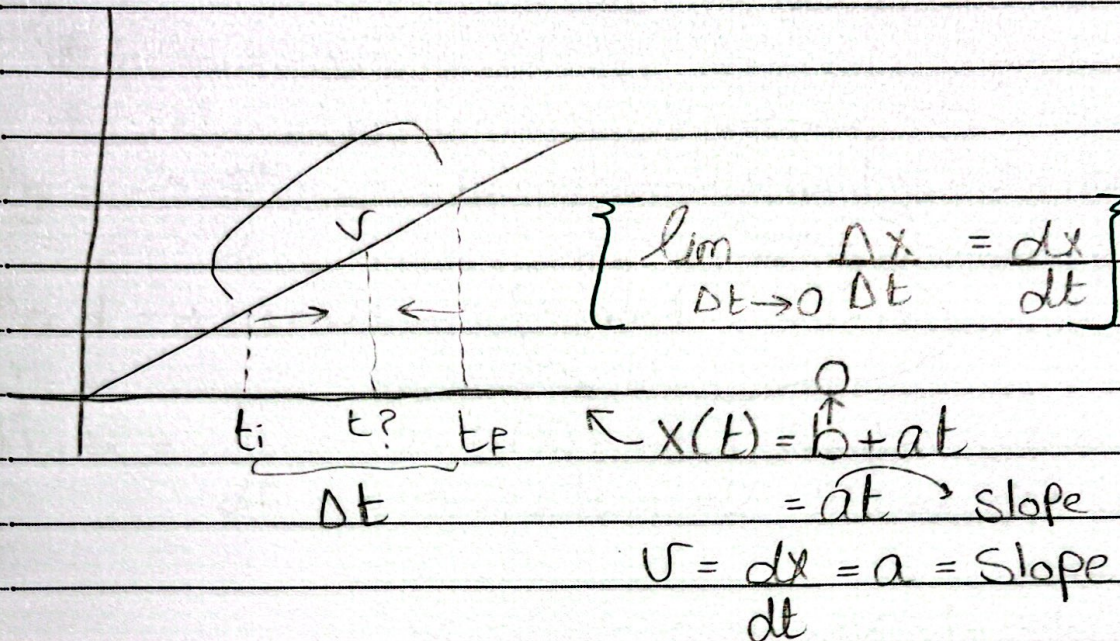
$$d_{\text{total}} = d_1 + d_2 = 2 + 6 = 8 \text{ m}$$

$$\Delta x_{\text{total}} = \Delta x_1 + \Delta x_2 = 2 + (-6) = -4 \text{ m}$$



Distance: $|\Delta x| = |X_f - X_i|$

* Average Velocity: $V_{avg} = \frac{\Delta x}{\Delta t}$ ← ~~average~~



Ex: a particle is moving along x-axis according to $x(t) = 2t - 3t^2$, where x is the particle position in meters and t is the time in seconds:

- 1) the position of the particle at $t = 1s$?
- 2) the displacement of the particle between $t = 0$ and $t = 2s$?
- 3) the velocity and speed between $t = 1s$ and $t = 3s$?
- 4) the velocity at $t = 4s$?

$$x(t) = 2t - 3t^2$$

$$1) x|_{t=1} = 2 - 3 = -1$$

$$2) t_i = 0, t_f = 2 \longrightarrow \Delta x = x_f - x_i = -8 - 0 = -8m$$

$$x_f|_{t_f=2} = -8, x_i|_{t_i=0} = 0$$

$$3) V_{avg} = \frac{x_f - x_i}{t_f - t_i} = \frac{x_f - x_i}{3 - 1}$$

$$x_f | t_f = 3 = -21 \text{ m}$$

$$x_i | t_i = 1 = -1 \text{ m}$$

$$V_{avg} = \frac{-21 - (-1)}{2} = -10 \text{ m/s} \rightarrow \text{Velocity - أو + سكون}$$

$$\text{speed} = +10 \text{ m/s} \rightarrow \text{Speed دائماً موجب}$$

سرعة لحظية

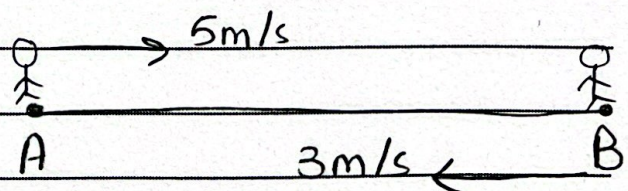
$$4) v | t=4s = \frac{dx}{dt} = 2 - 3(2)t \rightarrow \text{لما يطلب السرعة اللحظية بنسبة معينة}$$

$$= 2 - 6t | t=4 = 2 - 24 = -22 \text{ m/s}$$

$$\text{Speed} = \text{Velocity} \times \text{في طرأ سالت في سرعة فقط}$$

Q) A person walks first at a constant speed of (5 m/s) along a straight line from Point A to Point B and then back along the line from B to A at a constant speed of (3 m/s).

a) What is her average speed over the entire trip?



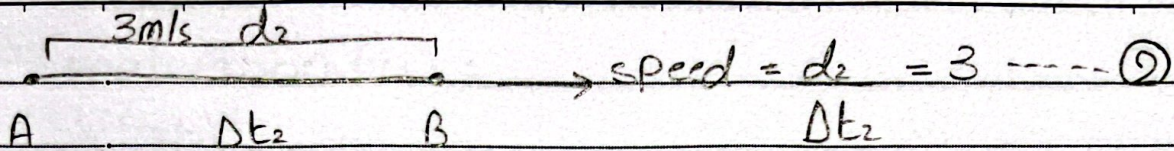
$$\text{average speed} \neq 0 = \frac{d_{total}}{\Delta t_{total}}$$

The diagram shows a horizontal line segment from point A to point B. Above the segment, an arrow points from A to B with the label '5 m/s'. Below the segment, the distance is labeled 'd1' and the time is labeled 'Δt1'. To the right of the diagram, the equation 'Speed = d1 / Δt1 = 5' is written, followed by a circled number 1.

$$\text{Speed} = \frac{d_1}{\Delta t_1} = 5 \text{ --- ①}$$

Motion in one dimension

Chapter 2



From ① : $5 = \frac{d_1}{\Delta t_1} \rightarrow \Delta t_1 = \frac{d_1}{5}$

From ② : $3 = \frac{d_2}{\Delta t_2} \rightarrow \Delta t_2 = \frac{d_2}{3}$

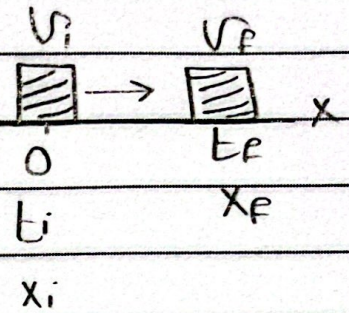
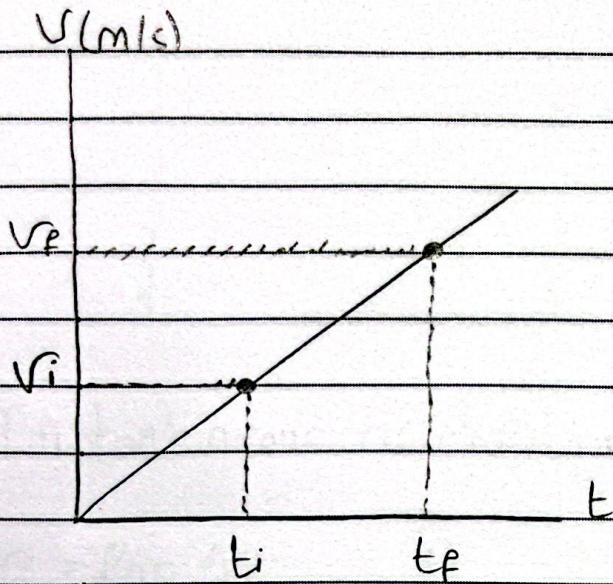
$$\text{Speed} = \frac{d^{\text{total}}}{\Delta t^{\text{total}}} = \frac{d_1 + d_2}{\Delta t_1 + \Delta t_2}, \text{ but } d_1 = d_2 = \underline{\underline{d}}$$

$$\text{Speed} = \frac{2d}{\Delta t_1 + \Delta t_2} = \frac{2d}{\frac{d}{5} + \frac{d}{3}} = \frac{2d}{d(\frac{1}{5} + \frac{1}{3})} = \frac{2d}{d(\frac{8}{15})} = \frac{2d \cdot 15}{8d} = \frac{30}{8} \text{ m/s}$$

b) What is her average velocity over the entire trip?

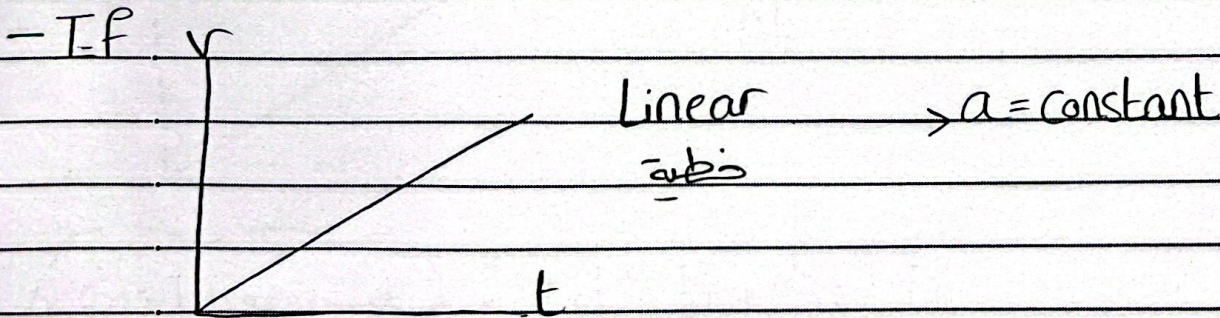
$$v_{\text{avg}} = \frac{x_f - x_i}{\Delta t} = 0 \leftarrow [\Delta x = 0] \text{ "رجع لوضعك الأول"}$$

* Acceleration (a):



$$[\text{Slope}] = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{1}{T^2} \text{ [m/s}^2\text{]}$$

$$\text{Acceleration} = \frac{\Delta v}{\Delta t}$$



-TF. V

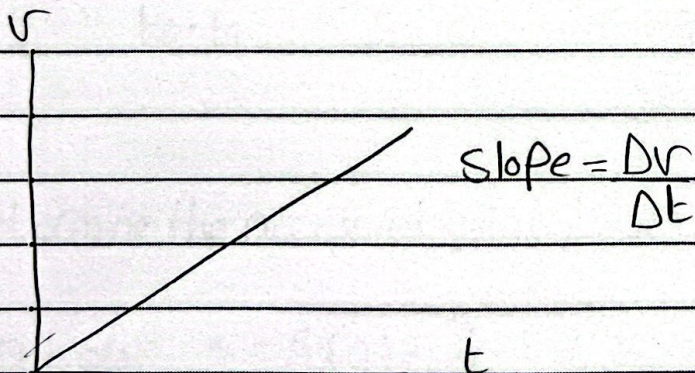
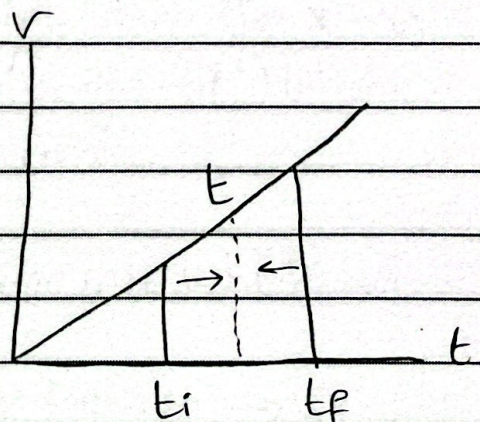
Constant
Velocity

$$a = \frac{v_f - v_i}{t_f - t_i} = 0$$

* Instantaneous acceleration (a):

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$$

$$a = \frac{dv}{dt} \leftarrow v(t)$$



* الخلاصة: إذا كانت

العلاقة خطية بين السرعة

والزمن فالمتسارع الخطي

يساوي a_{avg} وهو

constant

$$v(t) = b + st$$

intercept $\leftarrow$

$$a = \frac{dv}{dt} = s = \frac{\Delta v}{\Delta t} = a_{avg}$$

$$a = a_{avg} \text{ (linear)}$$

* $v(t) \rightarrow a = \frac{dv}{dt}$ but $v = \frac{dx}{dt}$

$$a = \frac{d}{dt} \left(\frac{dx}{dt} \right) = \frac{d^2x}{dt^2}$$

* إذا كانت العلاقة للسرعة كدالة لـ t (Velocity as a function of t)
معروفة فالنسبة $\frac{dx}{dt}$ هي مشتقة السرعة بالنسبة إلى الزمن
(السرعة إذا كانت دالة لـ x بمشتقة مرتين)

Ex : The velocity of a particle moving along the x-axis varies according to the expression ($v_x = 40 - 5t^2$) where v_x is in meters per second and t is in seconds.

A) Find the average acceleration in the time interval $t=0$ to $t=2$?

$$a_{avg} = \frac{v_f - v_i}{t_f - t_i} = \frac{20 - 40}{2 - 0} = -10 \text{ m/s}^2 \left\{ \begin{array}{l} \text{at } t_i = 0 \rightarrow v_i = 40 \text{ m/s} \\ \text{at } t_f = 2 \rightarrow v_f = 20 \text{ m/s} \end{array} \right.$$

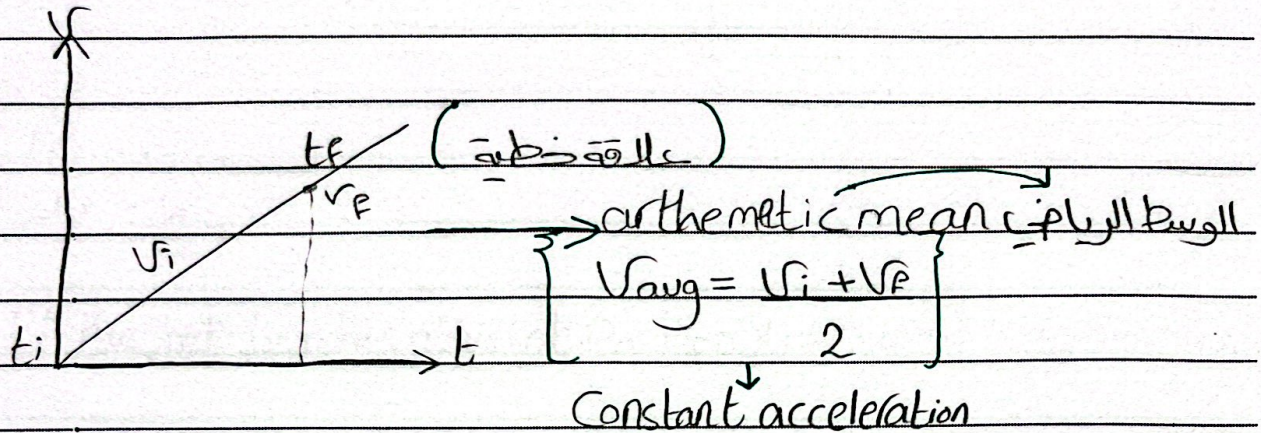
deceleration (تباطؤ)

B) Determine the acceleration at $t=2$ s?

$$a = \frac{dv}{dt} = -5(2)t = -10t \Big|_{t=2} = -10(2) = -20 \text{ m/s}^2$$

* Analysis Model : Particle Under Constant Acceleration: تحليل نموذج

⇒ If an object is moving along x-axis with constant acceleration



* Kinematic equation of motion:

If velocity is constant $\left\{ \begin{array}{l} 1) v_f = v_i + at \leftarrow \text{الزيادة في السرعة في ظل تسارع ثابت}$
 $\downarrow$
 $a = 0$
 $\downarrow$
 $v_f = v_i$
 $\Delta x = v_i t$
 $\downarrow$
 السكون
 $\uparrow$
 $\text{constant acceleration}$
 $2) v_f^2 = v_i^2 + 2a\Delta x \leftarrow$
 $3) \Delta x = v_i t + \frac{1}{2} at^2 \leftarrow$

* If an Object starts from rest $\rightarrow v_i = 0$

Ex: A jet lands on an aircraft carrier at a speed of 140 mi/h $\downarrow$ 63 m/s

1) What is its acceleration (assumed constant) if it stops in 2 s due to an arresting cable that snags the jet and brings it to a stop?

Motion in one dimension

Chapter 2

assumed constant $\rightarrow$ بافتراض ثابت

$$a = ? , v_i = 63 , v_f = 0 , \Delta t = 2$$

$$v_f = v_i + at$$

$$0 = 63 + 2a$$

$$-63 = 2a$$

$$a = \frac{-63}{2} \text{ m/s}^2$$

2) If the jet touches down at position $x_i = 0$, what is its final position?

$$x_f \rightarrow \Delta x = x_f - x_i^0 = x_f = ?$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$0 = (63)^2 + 2(-63)x_f$$

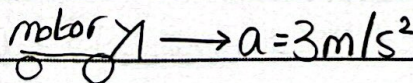
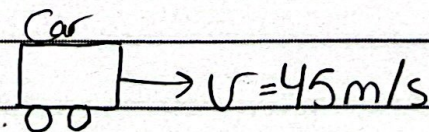
$$0 = (63)^2 - 63x_f$$

$$63x_f = (63)^2$$

$$x_f = 63 \text{ m}$$

Ex: A car traveling at a constant speed of 45 m/s passes a trooper on a motorcycle hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch the car, accelerating at a constant rate of 3 m/s². How long does it take the trooper to overtake the car?

$$\Rightarrow \Delta x_{\text{car}} = \Delta x_{\text{motor}}$$
$$(v_i t + \frac{1}{2} a t^2)_{\text{car}} = (v_i t + \frac{1}{2} a t^2)_{\text{motor}}$$
$$(v_i t)_{\text{car}} = (\frac{1}{2} a t^2)_{\text{motor}}$$



One-second later $\left\{ \begin{array}{l} \rightarrow t+1 \text{ (car)} \\ \rightarrow t-1 \text{ (motor)} \end{array} \right.$

$$v_i(t+1) = \frac{1}{2}at^2$$

$$45(t+1) = \frac{1}{2}(3)t^2$$

$$\frac{2}{3} * 45t + 45 = \frac{3t^2}{2} * \frac{2}{3}$$

$$30t + 30 = t^2$$

$$t^2 - 30t - 30 = 0$$

$$a=1, b=-30, c=-30$$

$$t = \frac{30 \pm \sqrt{(-30)^2 - 4(1)(-30)}}{2}$$

$$\left\{ \begin{array}{l} \text{* تنكيز: } at^2 + bt + c = 0 \\ t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{array} \right.$$

$$= \frac{30 \pm \sqrt{1020}}{2} \quad \left\{ \begin{array}{l} \rightarrow \frac{30 + \sqrt{1020}}{2} \rightarrow t = \frac{30 + \sqrt{1020}}{2} = 31s \\ \rightarrow \frac{30 - \sqrt{1020}}{2} \quad X \end{array} \right.$$

***Ex:** An Object moves along the x-axis according to the equation $x = 3t^2 - 2t + 3$, where x is in meters and t is in seconds.

A) Determine the average speed between $t = 2s$ and $t = 3s$?

$$V_{avg} = \frac{x_f - x_i}{t_f - t_i} = \frac{24 - 11}{3 - 2} = 13 \text{ m/s} \quad \left\{ \begin{array}{l} t_i = 2 \rightarrow x_i = 11 \\ t_f = 3 \rightarrow x_f = 24 \end{array} \right.$$

B) Determine the instantaneous speed at $t=2\text{ s}$ and $t=3\text{ s}$?

$$v = \frac{dx}{dt} = 6t - 2 \rightarrow v|_{t=2} = 10 \text{ m/s}$$

$$v|_{t=3} = 16 \text{ m/s}$$

C) Determine the average acceleration between $t=2\text{ s}$ and $t=3\text{ s}$?

$$a_{\text{avg}} = \frac{v_f - v_i}{t_f - t_i} = \frac{16 - 10}{3 - 2} = 6 \text{ m/s}^2$$

$$v(t) = \frac{dx}{dt} = 6t - 2$$

$$v_i|_{t_i=2} = 10 \text{ m/s}$$

$$v_f|_{t_f=3} = 16 \text{ m/s}$$

D) Determine the instantaneous acceleration at $t=2\text{ s}$ and $t=3\text{ s}$?

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \rightarrow x(t) = 3t^2 - 2t + 3$$

$$\frac{dx}{dt} = 6t - 2$$

$$a = \frac{d^2x}{dt^2} = 6 \text{ m/s}^2$$

E) At what time is the object at rest?

$$v(t) = \frac{dx}{dt} = 6t - 2 = 0$$

$$6t = 2$$

$$t = \frac{1}{3} \text{ s}$$

Ex: A Particle moves along the x-axis. Its position is given by the equation $x = 2 + 3t - 4t^2$, with x in meters and t in seconds.

A) Determine its position when it changes direction?

$$v \neq 0$$

$$v(t) = \frac{dx}{dt} = 3 - 8t$$

$$3 - 8t = 0$$

$$3 = 8t \rightarrow t = \frac{3}{8} \text{ s}$$

$$x = 2 + 3\left(\frac{3}{8}\right) - 4\left(\frac{3}{8}\right)^2$$

$$= \frac{2}{1} + \frac{9}{8} - \frac{36}{64} = 2.6 \text{ m}$$

B) Determine its velocity when it returns to the position at $t=0$

$$v_f = v_i + at \dots\dots\dots ①$$

$$v(t) = \frac{dx}{dt} = 3 - 8t \Big|_{t=0} = 3 \text{ m/s}$$

$$v_i = 3 \text{ m/s} \checkmark$$

$$a = -8 \text{ m/s}^2 \checkmark$$

$$a = \frac{dv}{dt} = -8 \text{ m/s}^2$$

when return to initial position $\rightarrow \Delta x = 0$

$$\Delta x = v_i t + \frac{1}{2} at^2 = 0 \rightarrow v_i t = -\frac{1}{2} at^2$$

$$v_i = -\frac{1}{2} at \rightarrow t = \frac{-2v_i}{a} = \frac{2(3)}{8} = \frac{3}{4} \text{ s}$$

$$v_f = v_i + at \rightarrow v_f = 3 + (-8)\left(\frac{3}{4}\right) = -3 \text{ m/s}$$

* Motion in 1-D along y-axis: $\rightarrow$ Free Fall $\uparrow \downarrow$

acceleration due gravity $\sim g = 9.81 \text{ m/s}^2 \cong 10 \text{ m/s}^2$

$\rightarrow$ along - x - axis

$$v_f = v_i + at$$

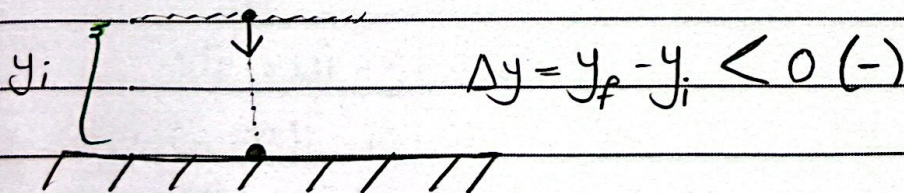
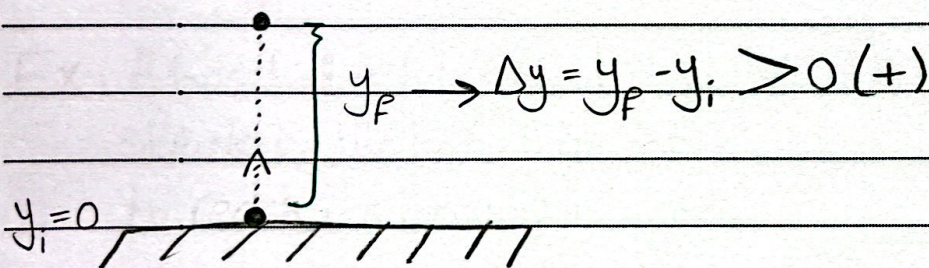
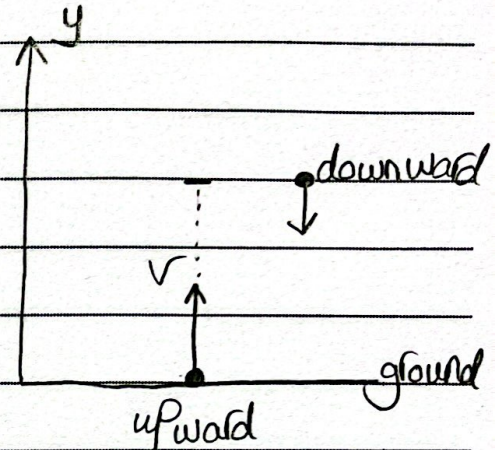
$$v_f^2 = v_i^2 + 2a\Delta x$$

$$\Delta x = v_i t + \frac{1}{2}at^2$$

$$a \rightarrow g$$

$$\Delta x \rightarrow \Delta y$$

$\rightarrow$ Constant acceleration



$$\left[\begin{array}{l} v_f = v_i - gt \\ v_f^2 = v_i^2 - 2g\Delta y \\ \Delta y = v_i t - \frac{1}{2}gt^2 \end{array} \right]$$

Ex: Rock is thrown downward from the top of a 40 m-tall tower with an initial speed of 12 m/s. Assuming negligible air resistance, what is the speed of the rock just before hitting the ground?

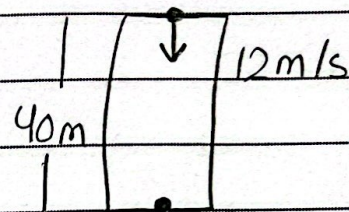
$$v_f^2 = v_i^2 - 2g\Delta y$$

$$v_f^2 = (-12)^2 - 2(10)(-40)$$

$$v_f^2 = 144 + 800$$

$$v_f = \pm\sqrt{944}$$

$$\text{Speed} = v_f = \sqrt{944} \text{ m/s}, \text{ Velocity } v_f = -\sqrt{944} \text{ m/s}$$



Ex: A baseball is hit so that it travels straight upward after being struck by the bat. A fan observes that it takes 3 s for the ball to reach its maximum height.

A) Find the ball's initial velocity?

$$v_f = v_i - gt_{\max}$$

$$0 = v_i - 10(3)$$

$$v_i = +30 \text{ m/s}$$

B) Find the height it reaches?

$$v_f^2 = v_i^2 - 2g\Delta y$$

$$0 = (30)^2 - 2(10)\Delta y$$

$$\Delta y = 45 \text{ m}$$

Ex: A ball is thrown directly downward with an initial speed of 8 m/s from a height of 30 m. After what time interval does it strike the ground?

$$\Delta y = v_i t - \frac{1}{2} g t^2$$

$$-30 = -8t - 5t^2$$

$$5t^2 + 8t - 30 = 0$$

$$a = 5, b = 8, c = -30$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-8 \pm \sqrt{(8)^2 - 4(5)(-30)}}{10} = \frac{-8 \pm \sqrt{664}}{10}$$

Ex: The height of a helicopter above the ground is given by $h = 3t^3$, where h is in meters and t is in seconds. At $t = 2$ the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground?

$$h = 3t^3, \quad t = 2s$$

$$\text{at } t = 2s \rightarrow h = 3(2)^3 = 24m$$

$$v = \frac{dh}{dt} = 9t^2 \quad \Big|_{t=2} = 9(2)^2 = 36 \text{ m/s}$$

$$t_{\max} \rightarrow v_f = v_i - g t_{\max}$$

$$0 = 36 - 10 t_{\max}$$

$$10 t_{\max} = 36$$

$$t_{\max} = 3.6s$$

} time needed the mailbag

} to return to launch point

$$t = 2 t_{\max}$$

$$t = 2(3.6) = 7.2s$$

Ex: A ball is thrown upward from the ground with an initial speed of 25 m/s , at the same instant, another ball is dropped from a building 15 m high. After how long will the balls be at the same height above the ground?

$$(y_f)_1 = (y_f)_2$$

$$\Delta y_1 = y_f - y_i = v_i t - \frac{1}{2} g t^2$$

$$y_{f1} = 25t - 5t^2 \dots \dots \textcircled{1}$$

$$y_{f1} = y_{f2}$$

$$25t - 5t^2 = 15 - 5t^2$$

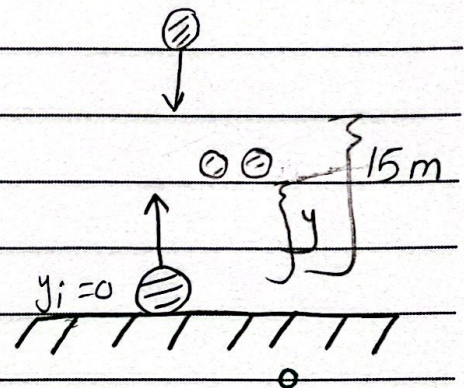
$$25t = 15$$

$$t = \frac{15}{25} = \frac{3}{5} \text{ s}$$

$$\Delta y_2 = y_f - y_i = v_i t - \frac{1}{2} g t^2$$

$$y_f - 15 = -5t^2$$

$$y_{f2} = 15 - 5t^2 \dots \dots \textcircled{2}$$



⇒ After how long the two balls will pass each other?

(تذكر من صيغة السؤال وفيه اعداد طوب)

Physical quantities

Vector

(magnitude, direction)

المقدار و الاتجاه

Ex: displacement, velocity
acceleration, force

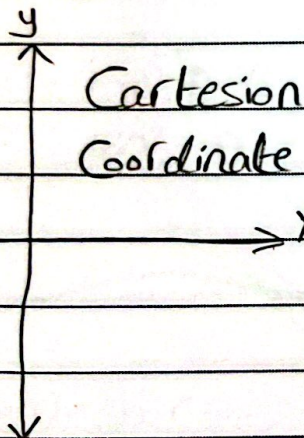
Scalar

(magnitude)

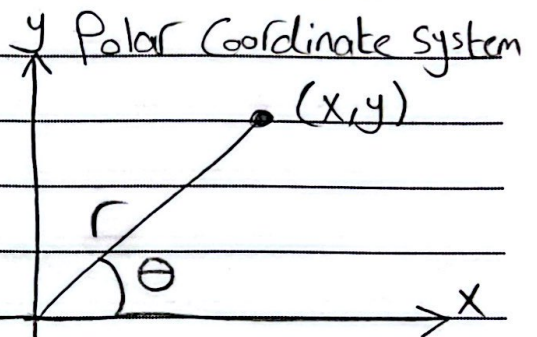
المقدار فقط

Ex: distance, density
work, Energy

* Coordinate system:



Cartesian
Coordinate



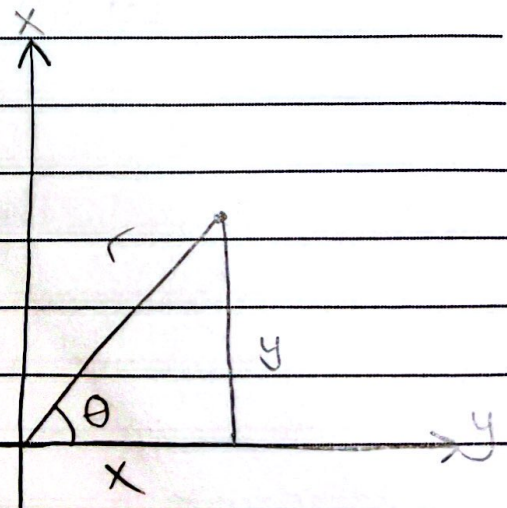
Polar Coordinate system

$$(x, y) \longrightarrow (r, \theta)$$

$$r^2 = x^2 + y^2$$

OR

$$r = \sqrt{x^2 + y^2}$$



$$\Rightarrow \sin \theta = \frac{y}{r} \Rightarrow y = r \sin \theta$$

$$\cos \theta = \frac{x}{r} \Rightarrow x = r \cos \theta$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{y}{x} \Rightarrow \theta = \tan^{-1} \left(\frac{y}{x} \right)$$

Vectors

Chapter 3

Ex: The Cartesian coordinates of a point in the xy plane are $(x, y) = (-3.5, -2.5)$ m as shown in Figure 3.3. Find the Polar coordinates of this point.

$$(x, y) = (-3.5, -2.5)$$

Find its Polar coordinate

$$r = \sqrt{(-3.5)^2 + (-2.5)^2} = 4.3 \text{ m}$$

(r, θ) يعني يعني

$$\tan \theta = \frac{-2.5}{-3.5} = 0.71$$

$$\theta' = \tan^{-1} 0.71 = 35.5^\circ$$

$$\theta = 180^\circ + \theta' = 180 + 35.5^\circ = 215.5^\circ$$

$r \rightarrow$ Vector $\Rightarrow$ notation

in your book $\left\{ \begin{array}{l} \vec{A} \text{ (vector)} \\ A \text{ (magnitude)} \end{array} \right\} \rightarrow \left\{ \begin{array}{l} \vec{A} \text{ (vector)} \\ |\vec{A}| \text{ (magnitude)} \end{array} \right\} \rightarrow$ في كتابي يعني يعني

Tail $\leftarrow \vec{A}_1, |\vec{A}_1| = 5 \text{ unit}$

$\rightarrow$ Tip or head

$\vec{A}_2, |\vec{A}_2| = 3 \text{ unit}$

(المقدار يعني $|\vec{A}|$ magnitude)

* Graphical Representation of Vectors :

التساوي

1) equality

$$\begin{matrix} \vec{A}, |\vec{A}| \\ \vec{B}, |\vec{B}| \end{matrix} \rightarrow \vec{A} = \vec{B}$$

بشرط أن يكونا في نفس الاتجاه

$\vec{A}$ and $\vec{B}$ in the same direction

$$\vec{A} = \vec{B} \quad \vec{A} \quad \vec{B}$$

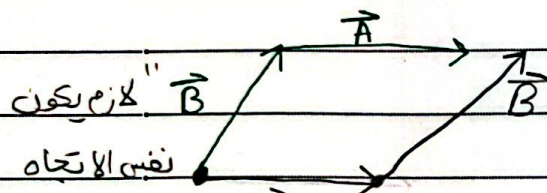
وقد قيمة A تساوي قيمة B

$$|\vec{A}| = |\vec{B}|$$

الجمع

2) addition

$$\vec{R} = \vec{A} + \vec{B}$$



"لازم يكون نفس الاتجاه"

واختيار من هذا الشكل

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

الجمع التجميعي

$$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$$

الطرح

3) Subtraction $\rightarrow \vec{R} = \vec{A} - \vec{B}$?

$$\vec{R} = \vec{A} + \vec{B}$$

negative vector

$$\vec{B}, |\vec{B}| = 5$$

$$\vec{R} = \vec{A} - \vec{B} \quad \vec{A} \quad -\vec{B}$$

$$-\vec{B}, |\vec{B}| = 5$$

(نفس المقدار، عكس الاتجاه)

* Resultant :

1) $\theta = 0^\circ$

$\vec{A}, |\vec{A}|$
 $\vec{B}, |\vec{B}|$
 $\theta = 0^\circ$
 $\vec{R} = \vec{A} + \vec{B}$

حاصل جمع
 بنفس الاتجاه
 $|\vec{R}| = |\vec{A}| + |\vec{B}|$

2) $\theta = 180^\circ$

$\vec{A}, |\vec{A}|$
 $\vec{B}, |\vec{B}|$
 $\vec{R}, |\vec{R}|$
 $|\vec{R}| = |\vec{A}| - |\vec{B}|$

حاصل طرح
 اتجاه مع الاتجاه الأكبر
 $|\vec{R}| = |\vec{A}| - |\vec{B}|$

3) $\theta = 90^\circ$

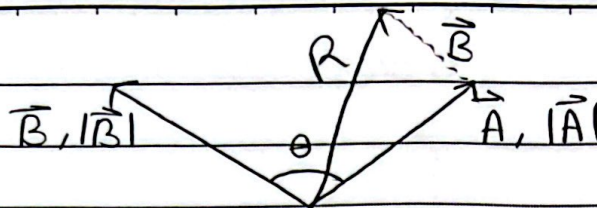
$\vec{B}, |\vec{B}|$
 $\vec{R} = \vec{A} + \vec{B}$
 $\vec{A}, |\vec{A}|$

$|\vec{R}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2}$

direction θ ? $\tan \theta = \frac{|\vec{B}|}{|\vec{A}|}$

$\theta = \tan^{-1} \left(\frac{|\vec{B}|}{|\vec{A}|} \right)$

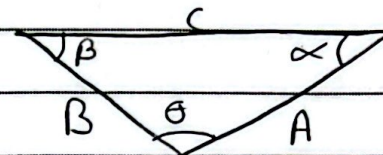
4) $90^\circ < \theta \leq 120^\circ$



$$\vec{R} = \vec{A} + \vec{B}$$

$$|R| = \sqrt{|A|^2 + |B|^2 - 2|A||B|\cos\theta} \quad (\text{cosine law})$$

* Sine law: $\frac{\sin\theta}{C} = \frac{\sin\alpha}{B} = \frac{\sin\beta}{A}$



Ex: A car travels 20 km due north and then 35 km in a direction 60° west of north as shown as in Figure 5.11. Find the magnitude and direction of the car's resultant displacements.

$$|\vec{R}| = \sqrt{(20)^2 + (35)^2 - 2(20)(35)\cos 120} = 48 \text{ km}$$

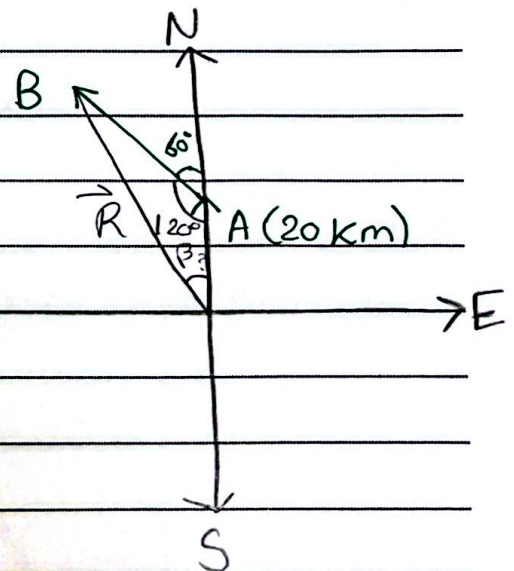
$$\frac{\sin\beta}{35} = \frac{\sin 120}{48}$$

$$\sin\beta = \frac{35}{48} \sin 120 \cong 0.63$$

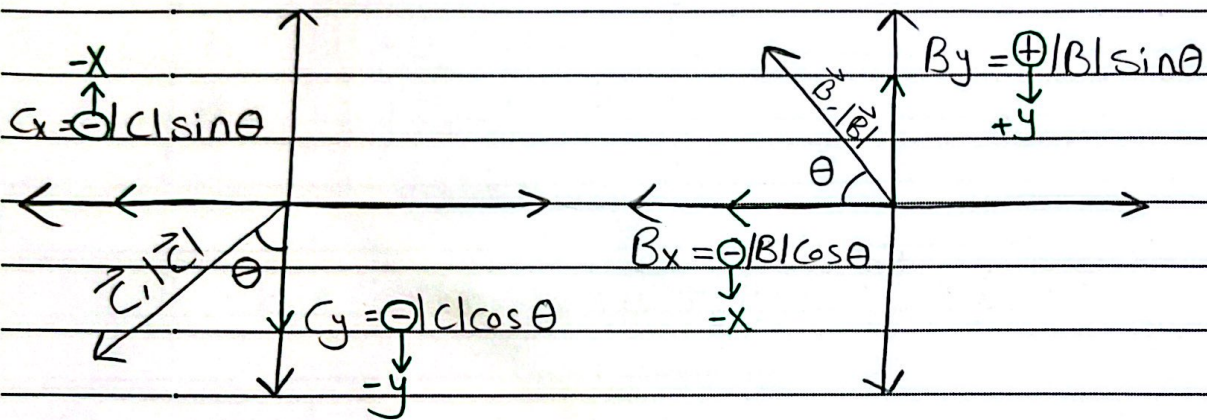
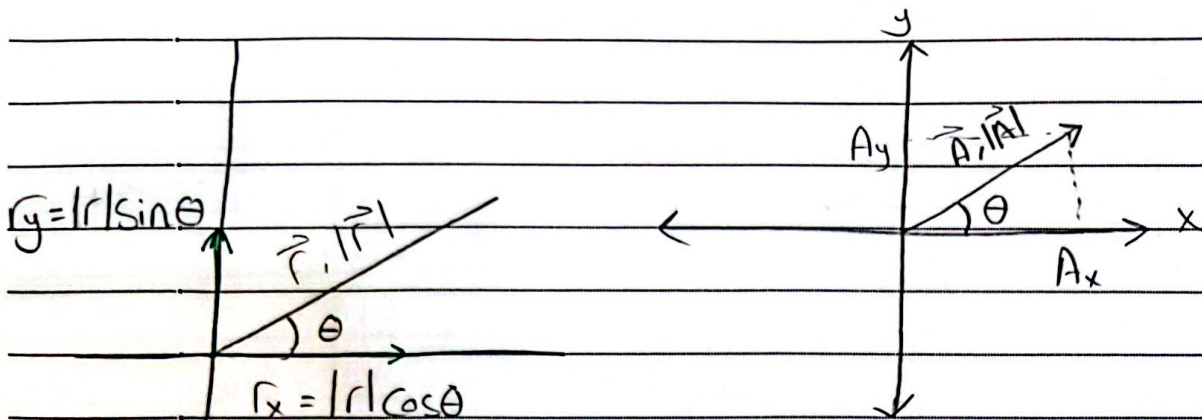
$$\beta = \sin^{-1}(0.63) \cong 39^\circ$$

with respect X-axis

$$\text{direction} = 39^\circ + 90^\circ = 129^\circ$$

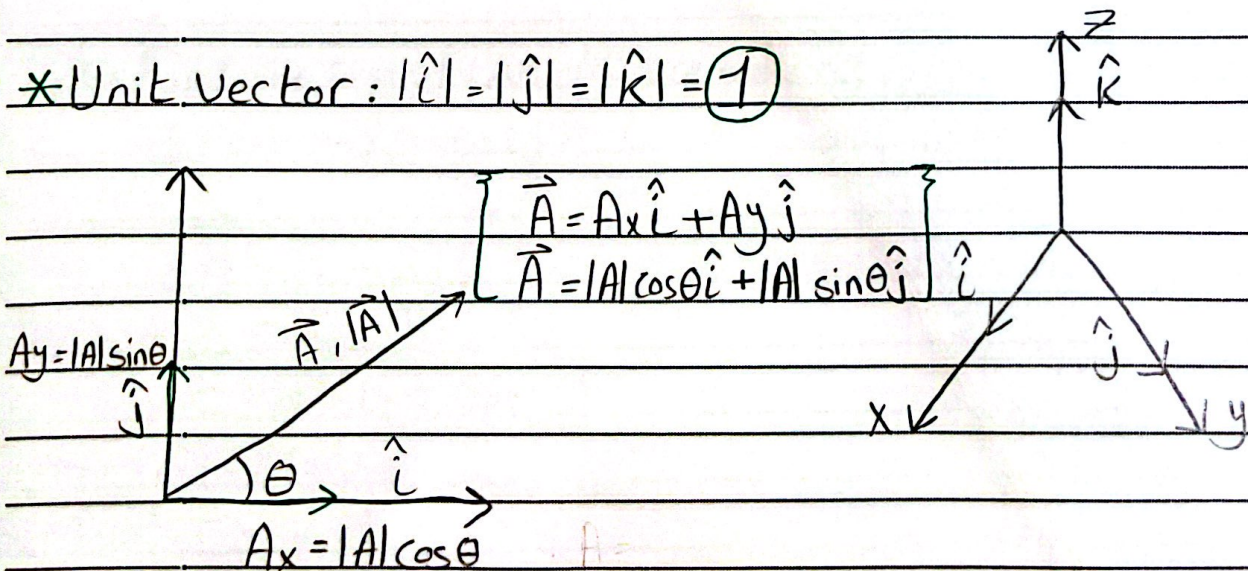


* Components of Vectors : مركبات المتجهات



* الی باسم مع الزاویة بیاخذ $\cos \theta$ والی فی بیاخذ $\sin \theta$
اوجب والسالب حسب المحاور $(-y, +y, -x, +x)$

* Unit Vector : $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$

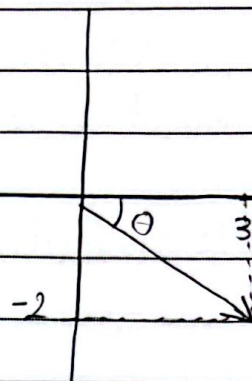


* Consider: $\vec{A} = 3\hat{i} - 2\hat{j} + 0\hat{k}$

$$\begin{pmatrix} A_x = 3 \\ A_y = -2 \\ A_z = 0 \end{pmatrix}$$

$$\Rightarrow |\vec{A}| = \sqrt{A_x^2 + A_y^2} \\ = \sqrt{(3)^2 + (-2)^2} \\ = \sqrt{13}$$

$$\tan \theta = \frac{-2}{3} \rightarrow \theta = \tan^{-1}\left(\frac{-2}{3}\right)$$



* Let $\left\{ \begin{array}{l} \vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \\ \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k} \end{array} \right\}$

$$\left\{ \vec{A} + \vec{B} = \vec{C} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k} \right\}$$

$\Rightarrow a$ is constant

$$\vec{C} = a\vec{A} = aA_x\hat{i} + aA_y\hat{j} + aA_z\hat{k}$$

$$\Rightarrow \vec{R} = \vec{A} + \vec{B}$$

Ex: Find the sum of two displacement vectors $\vec{A}$ and $\vec{B}$ lying in the xy Plane and given by $\vec{A} = (2\hat{i} + 2\hat{j})$ and $\vec{B} = (2\hat{i} - 4\hat{j})$.

$$\vec{A} = 2\hat{i} + 2\hat{j}$$

$$+ \vec{B} = 2\hat{i} - 4\hat{j}$$

$$\vec{C} = \vec{A} + \vec{B} = 4\hat{i} - 2\hat{j}$$

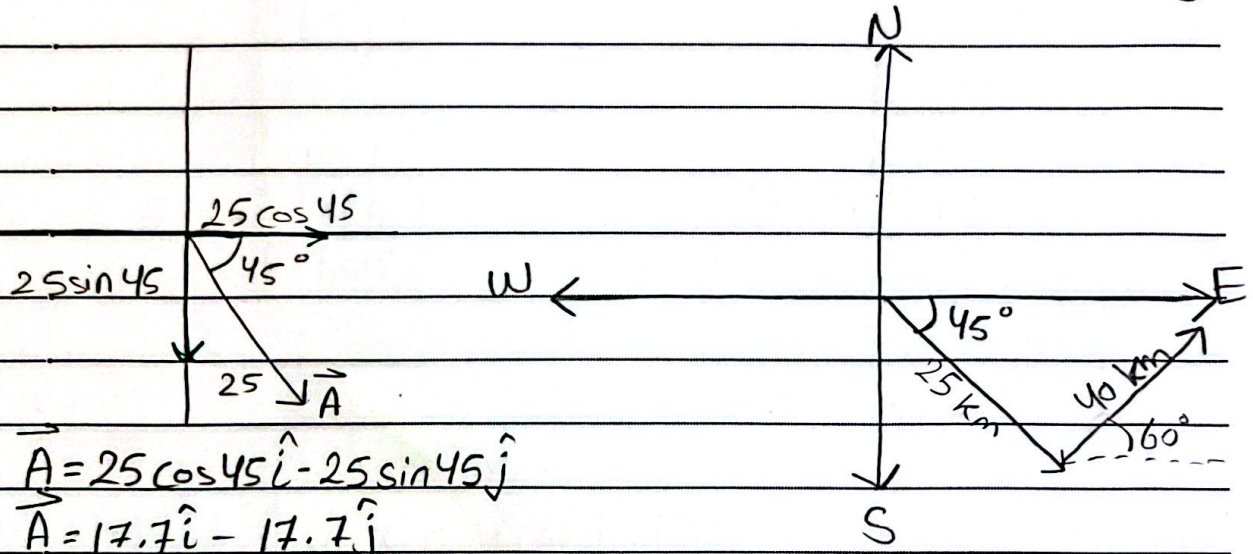
$$\text{direction } \theta = \tan^{-1}\left(\frac{-2}{4}\right)$$

$$\Rightarrow |\vec{C}| = \sqrt{(4)^2 + (-2)^2} = \sqrt{20}$$

$$\tan \theta = \frac{C_y}{C_x} = \frac{-2}{4}$$

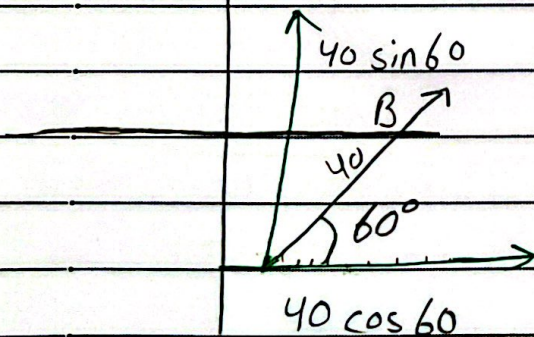
Ex: A hiker begins a trip by first walking 25 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40 km in a direction 60° north of east, at which point she discovers a forest ranger's tower.

A) Determine the components of the hiker's displacement for each day.



$$\vec{A} = 25 \cos 45^\circ \hat{i} - 25 \sin 45^\circ \hat{j}$$

$$\vec{A} = 17.7 \hat{i} - 17.7 \hat{j}$$



$$\vec{B} = 40 \cos 60^\circ \hat{i} + 40 \sin 60^\circ \hat{j}$$

$$\vec{B} = 20 \hat{i} + 34.6 \hat{j}$$

$$\Rightarrow \vec{R} = \vec{A} + \vec{B}$$

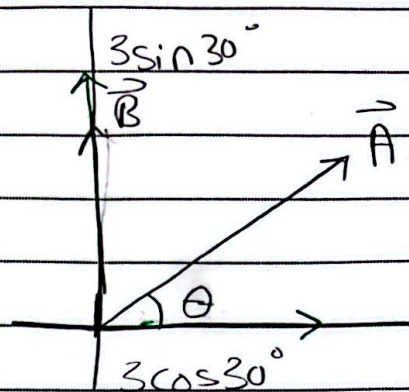
$$\vec{R} = 37.7 \hat{i} + 16.3 \hat{j}$$

$$\text{Total distance: } |\vec{R}| = \sqrt{(37.7)^2 + (16.3)^2}$$

Vectors

Chapter 3

Ex: The displacement vectors $\vec{A}$ and $\vec{B}$ shown in Figure P3.11 both have magnitudes of 3m. The direction of vector $\vec{A}$ is $\theta = 30^\circ$. Find graphically:



$$1) \vec{A} + \vec{B} = 1.5\sqrt{3}\hat{i} + 4.5\hat{j}$$

$$2) \vec{A} - \vec{B} = 1.5\sqrt{3}\hat{i} - 1.5\hat{j}$$

$$3) \vec{B} - \vec{A} = -1.5\sqrt{3}\hat{i} + 1.5\hat{j}$$

$$4) \vec{A} - 2\vec{B} = 1.5\sqrt{3}\hat{i} - 4.5\hat{j}$$

$$|\vec{A} - 2\vec{B}| = \sqrt{(1.5\sqrt{3})^2 + (-4.5)^2}$$

$$\text{direction: } \theta = \tan^{-1}\left(\frac{-4.5}{1.5\sqrt{3}}\right)$$

$$\begin{aligned}\vec{A} &= A_x\hat{i} + A_y\hat{j} \\ &= 3\cos 30^\circ\hat{i} + 3\sin 30^\circ\hat{j} \\ &= 1.5\sqrt{3}\hat{i} + 1.5\hat{j} \\ \vec{B} &= 3\hat{j} + 0\hat{i}\end{aligned}$$

Ex: A vector has an x component of -25 units and a y component of 40 units. Find the magnitude and direction of this vector.

$$\vec{A} = -25\hat{i} + 40\hat{j}$$

$$|\vec{A}| = \sqrt{(-25)^2 + (40)^2} = \sqrt{2225}$$

$$\tan \theta = \frac{40}{-25} \rightarrow \theta = \tan^{-1}\left(\frac{-40}{25}\right)$$

Ex: Consider the three displacement vectors $\vec{A} = (3\hat{i} - 3\hat{j})$ m

$\vec{B} = (\hat{i} - 4\hat{j})$ and $\vec{C} = (-2\hat{i} + 5\hat{j})$. Use the component method to determine:

A) the magnitude and direction of $\vec{D} = \vec{A} + \vec{B} + \vec{C}$

$$\vec{A} = 3\hat{i} - 3\hat{j}$$

$$\vec{B} = \hat{i} - 4\hat{j}$$

$$\vec{C} = -2\hat{i} + 5\hat{j}$$

$$\vec{D} = \vec{A} + \vec{B} + \vec{C} = 2\hat{i} - 2\hat{j}$$

$$|\vec{D}| = \sqrt{(2)^2 + (-2)^2} = \sqrt{8}$$

$$\theta = \tan^{-1}\left(\frac{-2}{2}\right)$$

B) $-\vec{A} = -3\hat{i} + 3\hat{j}$

$$-\vec{B} = -\hat{i} + 4\hat{j}$$

$$\vec{C} = -2\hat{i} + 5\hat{j}$$

$$\vec{E} = -\vec{A} - \vec{B} + \vec{C} = -6\hat{i} + 12\hat{j}$$

$$|\vec{E}| = \sqrt{(-6)^2 + (12)^2} =$$

$$\theta = \tan^{-1}\left(\frac{12}{-6}\right) = \tan^{-1}(-2)$$

Ex: Vector $\vec{A}$ has x and y components of -8.7 cm and 15 cm respectively, vector $\vec{B}$ has x and y components of 13.2 and -6.6 cm, respectively, IF $\vec{A} - \vec{B} + 3\vec{C} = 0$, what are the components $\vec{C}$?

$$\vec{A} = -8.7\hat{i} + 15\hat{j}$$

$$-\vec{B} = 13.2\hat{i} - 6.6\hat{j}$$

$$\vec{A} - \vec{B} = -21.9\hat{i} + 21.6\hat{j}$$

$$\vec{A} - \vec{B} + 3\vec{C} = 0$$

$$\vec{A} - \vec{B} = -3\vec{C} \quad \text{y-component}$$

$$\vec{C} = \frac{21.9}{3}\hat{i} - \frac{21.6}{3}\hat{j}$$

Ex: (a) taking $\vec{A} = (6\hat{i} - 8\hat{j})$ units, $\vec{B} = (-8\hat{i} + 3\hat{j})$ units, and $\vec{C} = (26\hat{i} + 19\hat{j})$ units, determine a and b such that $a\vec{A} + b\vec{B} + \vec{C} = \vec{0}$.

(b) Find the constants a and b .

$$\Rightarrow a\vec{A} + b\vec{B} + \vec{C} = \vec{0}$$

$$a(6\hat{i} - 8\hat{j}) + b(-8\hat{i} + 3\hat{j}) = -\vec{C}$$

$$[6a\hat{i} - 8a\hat{j}] + [-8b\hat{i} + 3b\hat{j}] = -26\hat{i} - 19\hat{j}$$

$$(6a - 8b)\hat{i} + (3b - 8a)\hat{j} = -26\hat{i} - 19\hat{j}$$

$$6a - 8b = -26 \quad \text{--- (1)}$$

$$3b - 8a = -19 \quad \text{--- (2)}$$

$$(6a - 8b = -26) * -1$$

$$8b - 6a = 26$$

$$4b - 3a = 13$$

$$b = \frac{13 + 3a}{4}$$

$$(3b - 8a = -19) * -1$$

$$8a - 3b = 19$$

$$8a - 3\left(\frac{13 + 3a}{4}\right) = 19$$

$$8a - \frac{39}{4} - \frac{9a}{4} = 19$$

$$\left(\frac{8a - 9a}{4} = 19 + \frac{39}{4}\right) * 4$$

$$a = \frac{76}{23} + \frac{39}{23} = \frac{115}{23} = 5$$

$$a = 5 \rightarrow b =$$

$$\begin{aligned} v &\rightarrow \vec{v} \\ a &\rightarrow \vec{a} \\ \Delta x &\rightarrow \vec{\Delta x} \end{aligned}$$

applications

projectile motion

circular motion

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

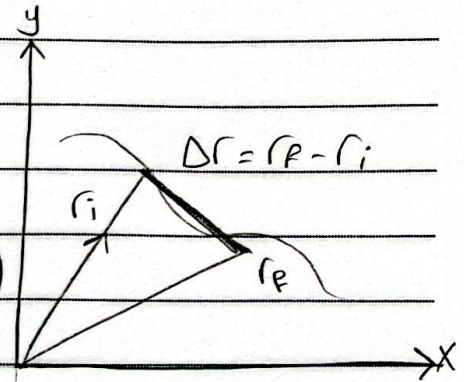
$$\Rightarrow \vec{r} = x\hat{i} + y\hat{j}$$

$$\Delta \vec{r} = (x_f\hat{i} + y_f\hat{j}) - (x_i\hat{i} + y_i\hat{j})$$

$$= x_f\hat{i} + y_f\hat{j} - x_i\hat{i} - y_i\hat{j}$$

$$= (x_f - x_i)\hat{i} + (y_f - y_i)\hat{j}$$

$$= \Delta x\hat{i} + \Delta y\hat{j}$$

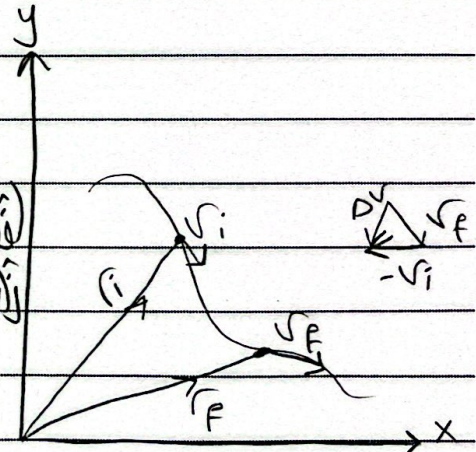


$$\Delta \vec{v} = \vec{v}_f - \vec{v}_i$$

$$= (v_{fx}\hat{i} + v_{fy}\hat{j}) - (v_{ix}\hat{i} + v_{iy}\hat{j})$$

$$= (v_{fx} - v_{ix})\hat{i} + (v_{fy} - v_{iy})\hat{j}$$

$$= \Delta v_x\hat{i} + \Delta v_y\hat{j}$$



* average velocity: $\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$

Chapter 2:

$$v_{avg} = \frac{\Delta x}{\Delta t}$$

$$\vec{v}_{avg} = \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}$$

$$= (v_{avg})_x\hat{i} + (v_{avg})_y\hat{j}$$

* instantaneous Velocity:

$$\begin{aligned}\vec{v} &= \frac{d\vec{r}}{dt} = \frac{d}{dt} (x\hat{i} + y\hat{j}) \\ &= \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} \\ &= v_x \hat{i} + v_y \hat{j}\end{aligned}$$

Chapter 2:

$$v = \frac{dx}{dt}$$

* average acceleration:

$$\begin{aligned}\vec{a}_{avg} &= \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta v_x \hat{i} + \Delta v_y \hat{j}}{\Delta t} \\ &= \frac{\Delta v_x}{\Delta t} \hat{i} + \frac{\Delta v_y}{\Delta t} \hat{j} \\ &= (a_{avg})_x \hat{i} + (a_{avg})_y \hat{j}\end{aligned}$$

* instantaneous acceleration:

$$\begin{aligned}\vec{a} &= \frac{d\vec{v}}{dt} \\ &= \frac{d}{dt} (v_x \hat{i} + v_y \hat{j}) \\ &= \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} \\ \vec{a} &= a_x \hat{i} + a_y \hat{j}\end{aligned}$$

$$a = \frac{d^2 r}{dt^2} = \frac{d}{dt} \left(\frac{dr}{dt} \right) = \frac{d}{dt} \left(\frac{d}{dt} (x\hat{i} + y\hat{j}) \right)$$

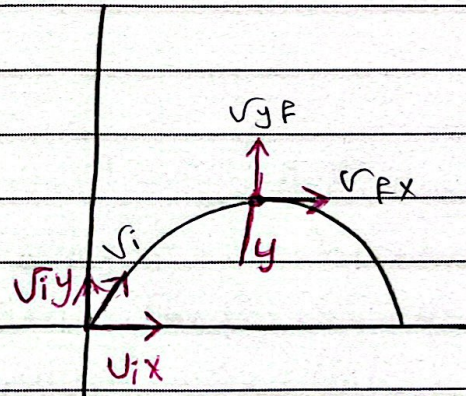
$$= \frac{d}{dt} (v_x\hat{i} + v_y\hat{j}) = a_x\hat{i} + a_y\hat{j}$$

* الفكرة لو كانت مستقيمة أدنى أو ثابته ما بهتت ما عادي الحل بنفسه

$$v, a, \Delta r \begin{cases} \hat{i} \\ \hat{j} \end{cases}$$

⇒ applications: Projectile motion:

"موضوع كامل زحني عنه"



2-D:

$$\vec{v}_F = \vec{v}_i + \vec{a}t$$

$$v_{Fx} = v_{ix} + a_x t, \quad v_{Fy} = v_{iy} + a_y t$$

1-D:

$$v_F = v_i + at$$

$$\Delta x = v_i t + \frac{1}{2} a t^2$$

$$v_F^2 = v_i^2 + 2a\Delta x$$

$$\Delta \vec{r} = \vec{v}_i t + \frac{1}{2} \vec{a} t^2$$

$$\Delta x = v_{ix} t + \frac{1}{2} a_x t^2$$

$\hat{i}$

$$\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$$

$\hat{j}$

$$\Delta \vec{r} = \Delta x \hat{i} + \Delta y \hat{j}$$

$$v_f^2 = v_i^2 + 2\vec{a}\Delta\vec{r} \leftarrow \text{Scalar (رقم)}$$

موجب أو سالب

$$\vec{v}_f = v_{fx} \hat{i} + v_{fy} \hat{j}$$

$$|\vec{v}_f| = \sqrt{(v_{fx})^2 + (v_{fy})^2}$$

Ex: A particle moves in the xy plane, starting from the origin at $t=0$ with an initial velocity having an x component of 20 m/s and a y component of -15 m/s . The particle experiences an acceleration in the x direction, given by $a_x = 4 \text{ m/s}^2$.

A) Determine the total velocity vector at anytime.

$$\vec{v}_i = 20\hat{i} - 15\hat{j}$$

$$\vec{a} = 4\hat{i} + 0\hat{j}$$

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

$$\vec{v}_f = (20\hat{i} - 15\hat{j}) + (4\hat{i} + 0\hat{j})t$$

$$= 20\hat{i} - 15\hat{j} + 4t\hat{i} = (20 + 4t)\hat{i} - 15\hat{j}$$

B) Calculate the velocity and speed of the particle at $t=5$ and the angle the velocity vector makes with the x-axis.

$$\vec{v}_P = (20 + 4t)\hat{i} - 15\hat{j} \big|_{t=5}$$

$$= 40\hat{i} - 15\hat{j}$$

OR

$$\begin{aligned} v_{Px} &= v_{ix} + a_x t \\ &= 20 + 4(5) \\ &= 40\hat{i} \end{aligned} \quad \left\{ \begin{aligned} v_{Py} &= v_{iy} + a_y t \\ &= -15 + 0t \\ &= -15\hat{j} \end{aligned} \right.$$

$$\vec{v}_P = v_{Px}\hat{i} + v_{Py}\hat{j}$$

$$= 40\hat{i} - 15\hat{j}$$

$$\text{direction: } \theta = \tan^{-1}(-15)$$

$$\text{speed} = |\vec{v}_P| = \sqrt{(40)^2 + (-15)^2}$$

C) Determine the x and y coordinates of the particle at any time and its position vector at this time.

$$\begin{aligned} \Delta x, \Delta y &\leftarrow \Delta \vec{r} = \vec{v}_i t + \frac{1}{2} \vec{a} t^2 \\ &= (20\hat{i} - 15\hat{j})(5) + \frac{1}{2} (4\hat{i})(5)^2 \\ &= 100\hat{i} - 75\hat{j} + 50\hat{i} \\ &= 150\hat{i} - 75\hat{j} \end{aligned}$$

$$x\text{-coordinate} = \Delta x = 150 \text{ m}$$

$$y\text{-coordinate} = -75 \text{ m}$$

$$\text{direction of } \Delta \vec{r}: \theta = \tan^{-1}(-75)$$

$$\text{distance} = |\Delta \vec{r}| = \sqrt{(150)^2 + (-75)^2}$$

Ex: A particle initially located at the origin has an acceleration of $\vec{a} = 3\hat{j}$ and an initial velocity of $\vec{v}_i = 5\hat{i}$.

A) Find the vector position of the particle at any time t .

B) Find the velocity of the particle at any time t .

C) Find the coordinates of the particle at $t=2$.

D) Find the speed of the particle at $t=2$.

$$\left. \begin{array}{l} a_y = 3\hat{j} \\ a_x = 0 \\ v_{ix} = 5\hat{i} \\ v_{iy} = 0 \end{array} \right\} \begin{array}{l} \text{at } t=2 \\ \vec{v}_f = \vec{v}_i + \vec{a}t \\ \vec{v}_f = (5\hat{i}) + (3\hat{j})(2) \\ = 5\hat{i} + 6\hat{j} \end{array}$$

$$\text{direction : } \theta = \tan^{-1}(6)$$

$$\text{Speed} = |\vec{v}_f| = \sqrt{5^2 + 6^2}$$

$$\Delta \vec{r} = \vec{v}_i t + \frac{1}{2} \vec{a} t^2$$

$$= 5\hat{i}t + \frac{1}{2}(3\hat{j})t^2$$

$$\Delta \vec{r} = 10\hat{i} + 6\hat{j}$$

$$\text{direction : } \theta = \tan^{-1}(6)$$

$$|\Delta \vec{r}| = \sqrt{(10)^2 + (6)^2}$$

Ex: The vector position of a particle varies in time according to the expression $\vec{r} = 3\hat{i} - 6t^2\hat{j}$, where $\vec{r}$ is in meters and t is in seconds.

A) Find velocity between $t=0$ and $t=1$?

$$V_{avg} = \frac{\Delta r}{\Delta t} = \frac{r_f - r_i}{t_f - t_i}$$

$$r_i |_{t_i=0} = 3\hat{i} - 6(0)^2\hat{j} = 3\hat{i}$$

$$r_f |_{t_f=1} = 3\hat{i} - 6(1)^2\hat{j} = 3\hat{i} - 6\hat{j}$$

$$V_{avg} = \frac{3\hat{i} - 6\hat{j} - 3\hat{i}}{1-0} = -6\hat{j}$$

B) Find the speed at $t=3$?

$$v = \frac{dr}{dt} = -12t\hat{j} \quad \Big|_{t=3} = -12(3)\hat{j} = -36\hat{j} \text{ m/s}$$

$$\text{Speed} = |v| = 36 \text{ m/s}$$

C) Find the acceleration between $t=1$ and $t=3$?

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

$$v_i |_{t_i=1} = -12(1)\hat{j} = -12\hat{j}$$

$$v_f |_{t_f=3} = -12(3)\hat{j} = -36\hat{j}$$

$$a_{avg} = \frac{-36\hat{j} - (-12\hat{j})}{3-1} = \frac{-24\hat{j}}{2} = -12\hat{j} \text{ m/s}^2$$

D) Find acceleration at $t=4$?

$$a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$$

$$r(t) = 3\hat{i} - 6t^2\hat{j}$$

$$\frac{dr}{dt} = -12t\hat{j}$$

$$a = \frac{d^2r}{dt^2} = -12\hat{j} \big|_{t=4} \text{ m/s}^2$$

Ex: A fish swimming in a horizontal plane has velocity $\vec{v}_i = 4\hat{i} + 1\hat{j}$ at a point in the ocean where the position relative to a certain rock is $\vec{r}_i = 10\hat{i} - 4\hat{j}$. After the fish swims with constant acceleration for 20 s, its velocity is $\vec{v} = 20\hat{i} - 5\hat{j}$.

A) What are the components of the acceleration of the fish?

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

$$(20\hat{i} - 5\hat{j}) = (4\hat{i} + 1\hat{j}) + a(20)$$

$$20\hat{i} - 5\hat{j} - 4\hat{i} - 1\hat{j} = 20a$$

$$16\hat{i} - 6\hat{j} = 20a$$

$$\vec{a} = \frac{16\hat{i}}{20} - \frac{6\hat{j}}{20} = \frac{4}{5}\hat{i} - \frac{3}{10}\hat{j} \text{ m/s}^2$$

B) What is the direction of its acceleration with respect to unit vector $\hat{i}$?

$$\tan \theta \rightarrow \theta = \tan^{-1} \left(\frac{-3}{\frac{4}{5}} \right) = \tan^{-1} \left(\frac{-15}{40} \right)$$

C) If the fish maintains constant acceleration, where is it at $t=25$ and in what direction is it moving?

$$a = \frac{4}{5} \hat{i} - \frac{3}{10} \hat{j}$$

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i = \vec{v}_i t + \frac{1}{2} a t^2$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2} a t^2$$

$$= (10\hat{i} - 4\hat{j}) + (4\hat{i} + \hat{j})(25) + \frac{1}{2} (4\hat{i} - 3\hat{j})(25)^2$$

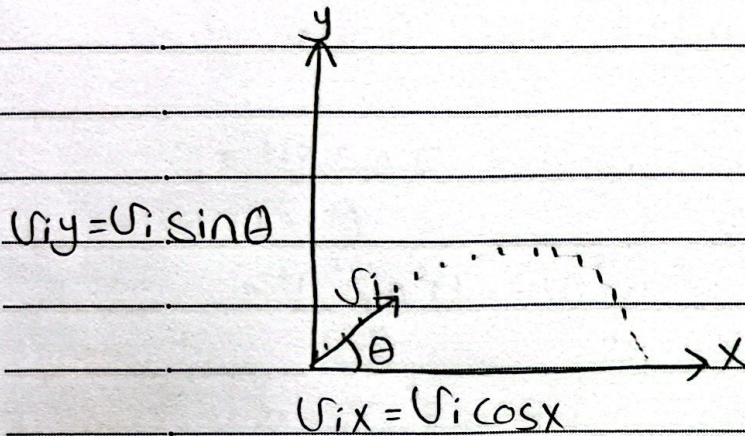
$$= 10\hat{i} - 4\hat{j} + 100\hat{i} + 25\hat{j} + 300\hat{i} - \frac{3 \times 625}{2} \hat{j}$$

$$\vec{r}_f = 410\hat{i} + (21 - 3 \times 625) \hat{j}$$

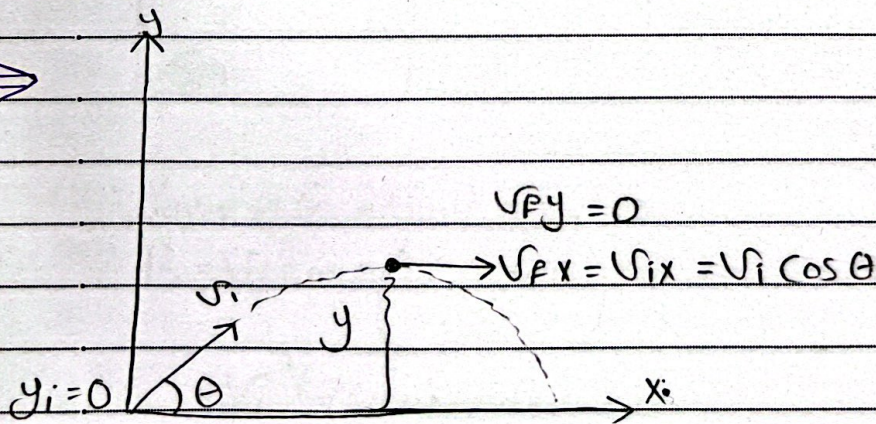
$$\Rightarrow \text{distance} = |\Delta \vec{r}|$$

* Projectile motion:

1) an object is fired with angle θ above horizontal ($\vec{a} = -g\hat{j}$)



X-axis	Y-axis $\uparrow \downarrow$
$u_{ix} = u_i \cos \theta$	$u_{iy} = u_i \sin \theta$
$a = 0$ (constant velocity)	$v_{fy} = u_{iy} - gt$
$v_{fx} = u_{ix} = u_i \cos \theta$	$v_{fy}^2 = u_{iy}^2 - 2g\Delta y$
$x = u_{ix}t = u_i \cos \theta t$	$\Delta y = u_{iy}t - \frac{1}{2}gt^2$



at max: $v_{fy} = u_{iy} - gt$

$0 = u_i \sin \theta - gt_{max}$

$t_{max} = \frac{u_i \sin \theta}{g}$

$$\Delta y = y_f - y_i = v_{iy}t - \frac{1}{2}gt^2$$

$$y_f = v_i \sin \theta t - \frac{1}{2}gt^2$$

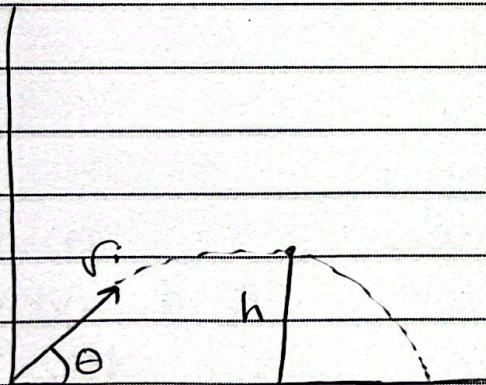
$$y_{\max} = h = v_i \sin \theta t_{\max} - \frac{1}{2}gt_{\max}^2$$

$$h = v_i \sin \theta \left(\frac{v_i \sin \theta}{g} \right) - \frac{1}{2}g \left(\frac{v_i \sin \theta}{g} \right)^2$$

$$= \frac{v_i^2 \sin^2 \theta}{g} - \frac{1}{2}g \left(\frac{v_i^2 \sin^2 \theta}{g^2} \right)$$

$$= \frac{v_i^2 \sin^2 \theta}{g} - \frac{v_i^2 \sin^2 \theta}{2g}$$

$$h = \frac{v_i^2 \sin^2 \theta}{2g}$$



$$x_{\max} = \text{Range (R)}$$

$$t_{\max} = \frac{v_i \sin \theta}{g}$$

$$t_{\text{flying}} = 2t_{\max} = \frac{2v_i \sin \theta}{g}$$

$$x_{\max} = R = v_{ix} t_{\text{flying}}$$

$$R = v_i \cos \theta \left(\frac{2v_i \sin \theta}{g} \right)$$

$$= \frac{v_i^2 (2 \sin \theta \cos \theta)}{g}$$

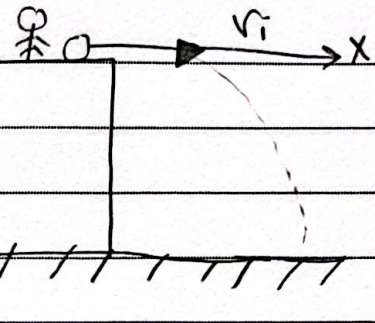
$$R = \frac{v_i^2 \sin 2\theta}{g}$$

2) a projectile is fired horizontally

$$\theta = 0$$

x

y



$$v_{ix} = v_i \cos \theta = v_i \cos 0 = v_i$$

$$v_{iy} = v_i \sin \theta = v_i \sin 0 = 0$$

$$x = v_{ix} t = v_i t$$

$$v_{fy} = v_{iy} - gt$$

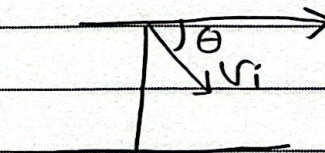
$$v_{fy}^2 = v_{iy}^2 - 2g\Delta y$$

$$\Delta y = v_{iy} t - \frac{1}{2} g t^2$$

3) A Projectile is fired with angle θ below horizontal

x

y



$$v_{ix} = v_i \cos \theta$$

$$v_{iy} = -v_i \sin \theta$$

$$v_{fx} = v_{ix} = v_i \cos \theta$$

$$v_{fy} = v_{iy} - gt$$

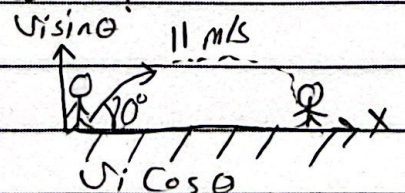
$$x = v_{ix} t = v_i \cos \theta t$$

$$\Delta y = v_{iy} t - \frac{1}{2} g t^2$$

Ex: A long jumper leaves the ground at an angle of 20° above the horizontal and at a speed of 11 m/s.

A) How far does he jump in the horizontal direction?

$$R = X_{\max} = \frac{v_i^2 \sin 2\theta}{g} = \frac{(11)^2 \sin(40)}{10} \text{ m}$$



B) What the maximum height reached?

$$h = \frac{v_i^2 \sin^2 \theta}{2g} = \frac{(11)^2 (\sin 20)^2}{2 \times 10} \text{ m}$$

Ex: A ski jumper leaves the ski track moving in the horizontal direction with a speed of 25 m/s as shown in Figure 4.14. The landing incline below her falls off with a slope of 35° . Where does she land on the incline?

d?

horizontally $\theta = 0$

$$v_{ix} = v_i \cos \theta = v_i = 25 \text{ m/s}$$

$$v_{iy} = v_i \sin \theta = 0$$

$$\Rightarrow \cos 35 = \frac{x}{d}$$

$$\Rightarrow x = v_{ix} t = v_i t$$

$$\cos 35 = \frac{v_i t}{d} \quad \text{--- (1)}$$

$$\sin 35 = \frac{y}{d}$$

$$\Delta y = y_F - y_i = v_{iy} t - \frac{1}{2} g t^2$$

$$0 - y_i = -\frac{1}{2} g t^2$$

$$t y_i = +\frac{1}{2} g t^2$$

$$y_i = \frac{1}{2} g t^2$$

$$\sin 35 = \frac{\frac{1}{2} g t^2}{d}$$

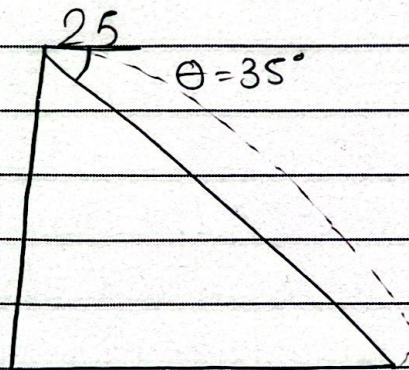
$$\cos 35 = \frac{v_i t}{d}$$

$$\tan 35 = \frac{g t}{2 v_i}$$

$$t = \frac{2 v_i \tan 35}{g}$$

$$= \frac{2(25) \tan 35}{10}$$

$$\sin 35 = \frac{\frac{1}{2} g t^2}{d} \Rightarrow d = \frac{\frac{1}{2} (10) (3.5)^2}{\sin 35} = 3.5 \text{ s}$$



Ex: A Projectile is fired in such a way that its horizontal range is equal to three times its maximum height. What is the angle of projection?

$$R = 3h$$

$$\frac{v_i^2 \sin 2\theta}{g} = 3 \frac{v_i^2 \sin^2 \theta}{2g} \Rightarrow \sin 2\theta = \frac{3}{2} \sin^2 \theta$$

$$2 \sin \theta \cos \theta = \frac{3}{2} \sin^2 \theta \Rightarrow \frac{2 \cos \theta}{\sin \theta} = \frac{3}{2} \Rightarrow \frac{\cos \theta}{\sin \theta} = \frac{3}{4}$$

$$\tan \theta = \frac{4}{3} \Rightarrow \theta = \tan^{-1} \left(\frac{4}{3} \right) \approx 53^\circ$$

Ex: To start an avalanche on a mountain slope, an artillery shell is fired with an initial velocity of 300 m/s at 55° above the horizontal. It explodes on the mountainside 42 s after firing. What are the x and y coordinates of the shell where it explodes, relative to its firing point?

$$v_{ix} = v_i \cos 55 = 300 \cos 55$$

$$v_{iy} = v_i \sin 55 = 300 \sin 55$$

$$v_x = v_{ix} t = (300 \cos 55)(42) = \dots$$

$$\Delta y = y_f - y_i = y_f = v_{iy} t - \frac{1}{2} g t^2 = (300 \sin 55)(42) - \frac{1}{2} (10)(42)^2$$

$$= \dots$$

