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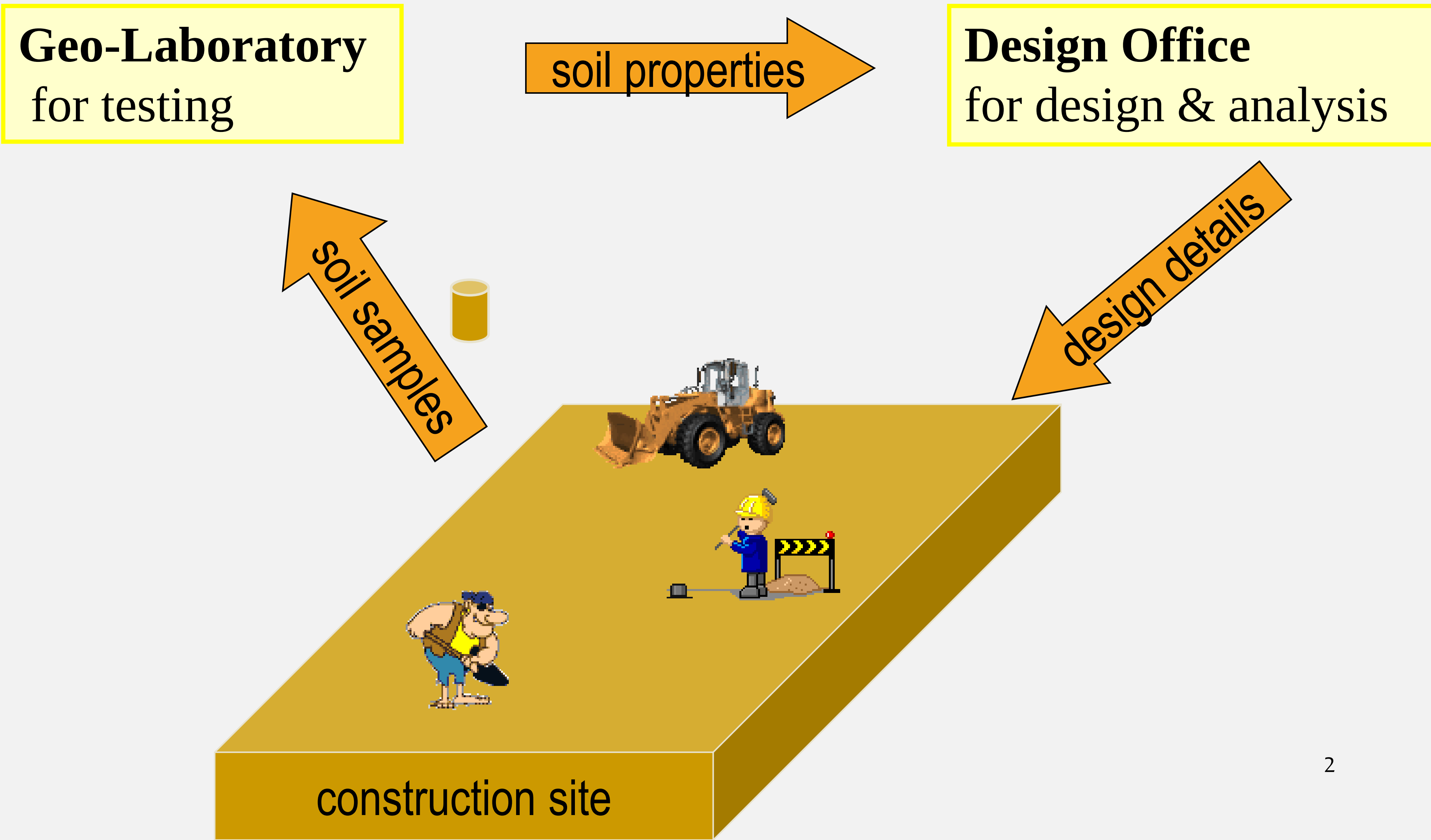
INTRODUCTION TO GEOTECHNICAL ENGINEERING

TYPICAL GEOTECHNICAL PROJECT



GEOTECHNICAL ENGINEERING AND SOIL MECHANICS

Soils are the oldest and most complex engineering materials.

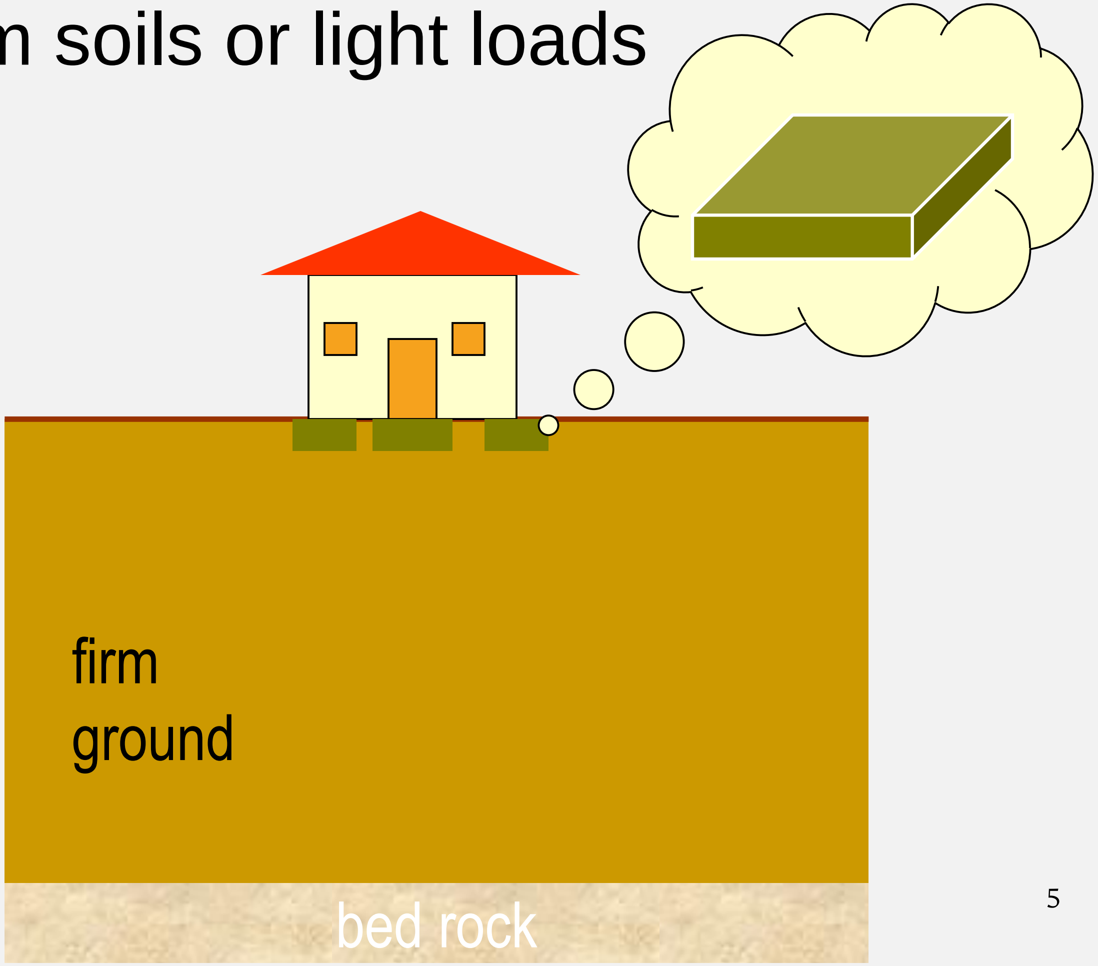
Our ancestors used soils as a construction material for flood protection and shelters

Soil mechanics is a subset of geotechnical engineering, which involves the application of soil mechanics, geology, and hydraulics to the analysis and design of geotechnical systems such as dams, embankments, tunnels, canals and waterways, foundations for bridges, roads, and buildings,

Geotechnical Applications

SHALLOW FOUNDATIONS

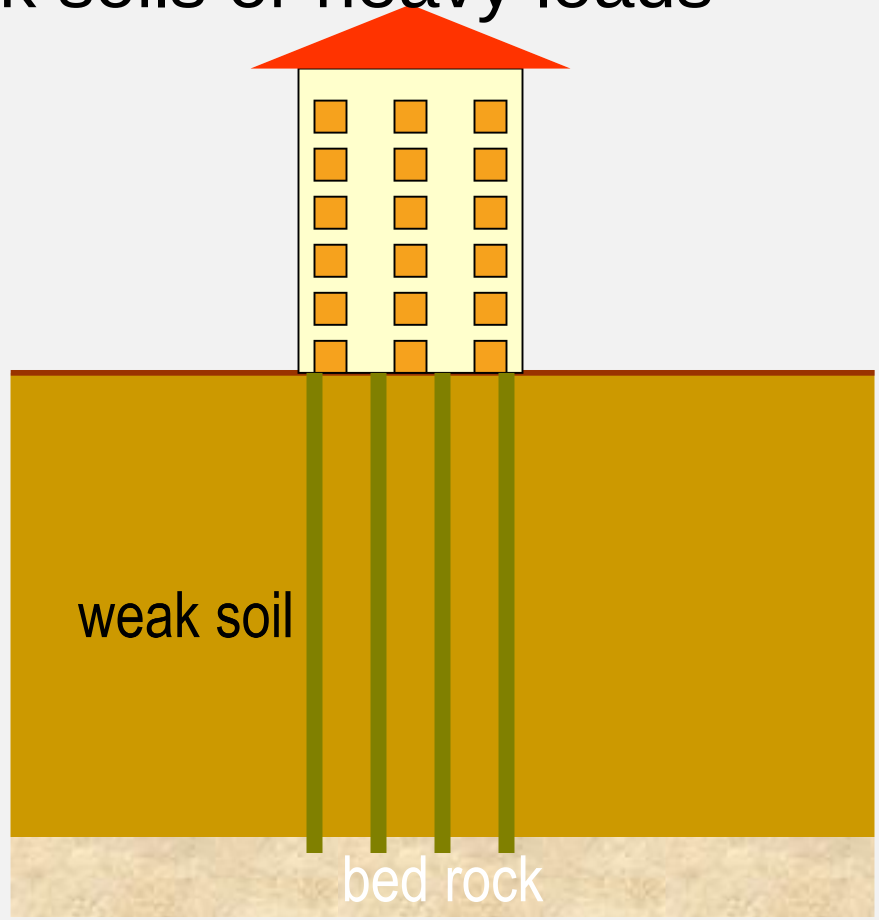
- for transferring building loads to underlying ground
- mostly for firm soils or light loads



SHALLOW FOUNDATIONS

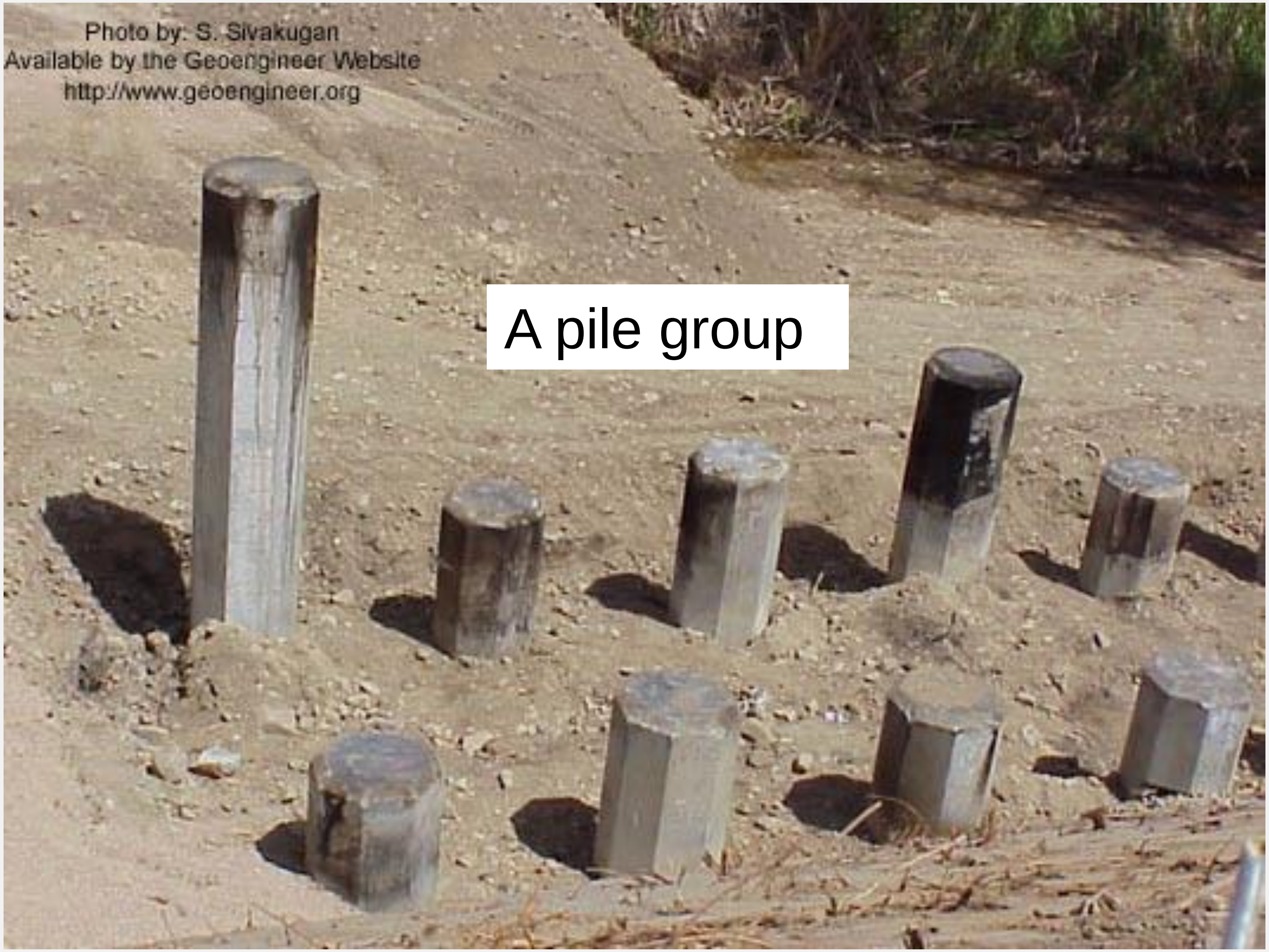
DEEP FOUNDATIONS

- for transferring building loads to underlying ground
- mostly for weak soils or heavy loads



PILE DRIVING RIG – ROSS RIVER DAM

PILE DRIVING RIG – ROSS RIVER DAM



DEEP FOUNDATIONS

Driven timber piles, Pacific Highway

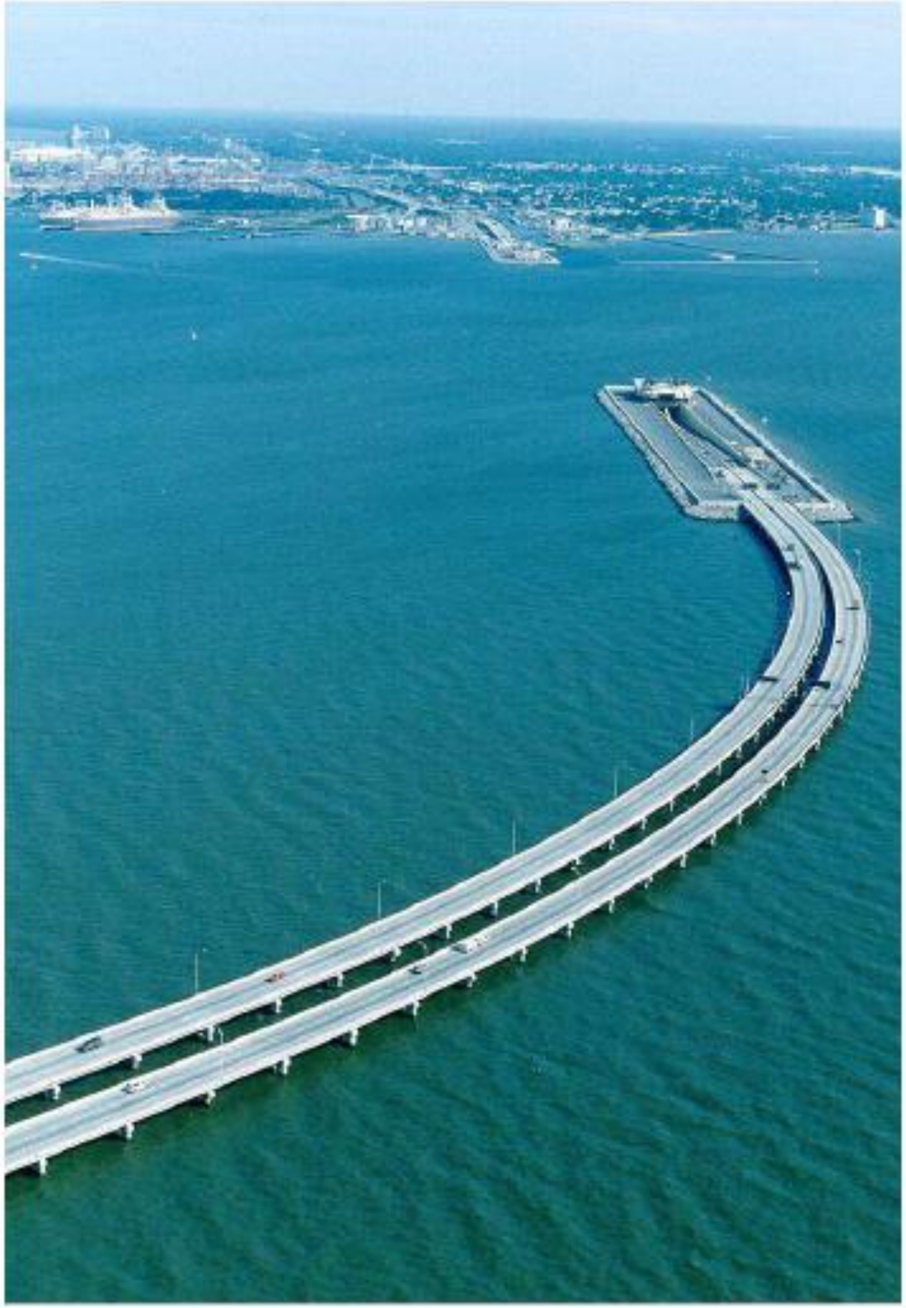
PIER FOUNDATIONS FOR BRIDGES

Millau Viaduct in France (2005)

- Cable-stayed bridge
- Supported on 7 piers, 342 m apart
- Longest pier (336) in the world

PIER FOUNDATIONS FOR BRIDGES

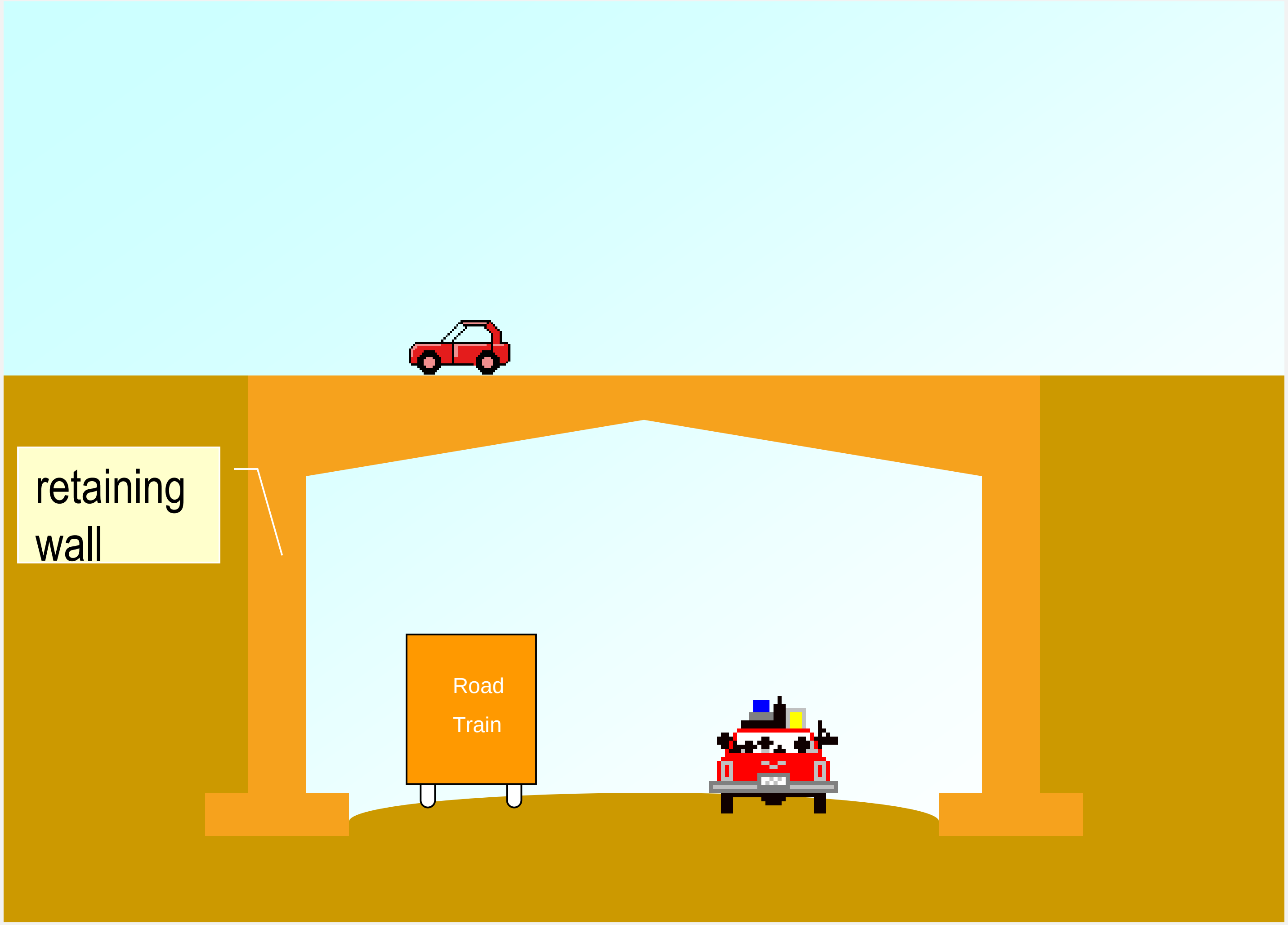
Millau Viaduct in France (2005)



The new link between Copenhagen, (Denmark) and Malmo (Sweden) includes the causeway and its tunnel seen in this photograph, plus one of the world's longest cable-stayed bridges (not seen here).

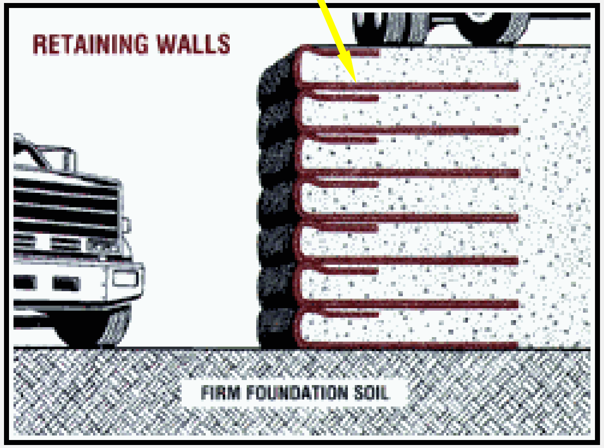
RETAINING WALLS

- for retaining soils from spreading laterally



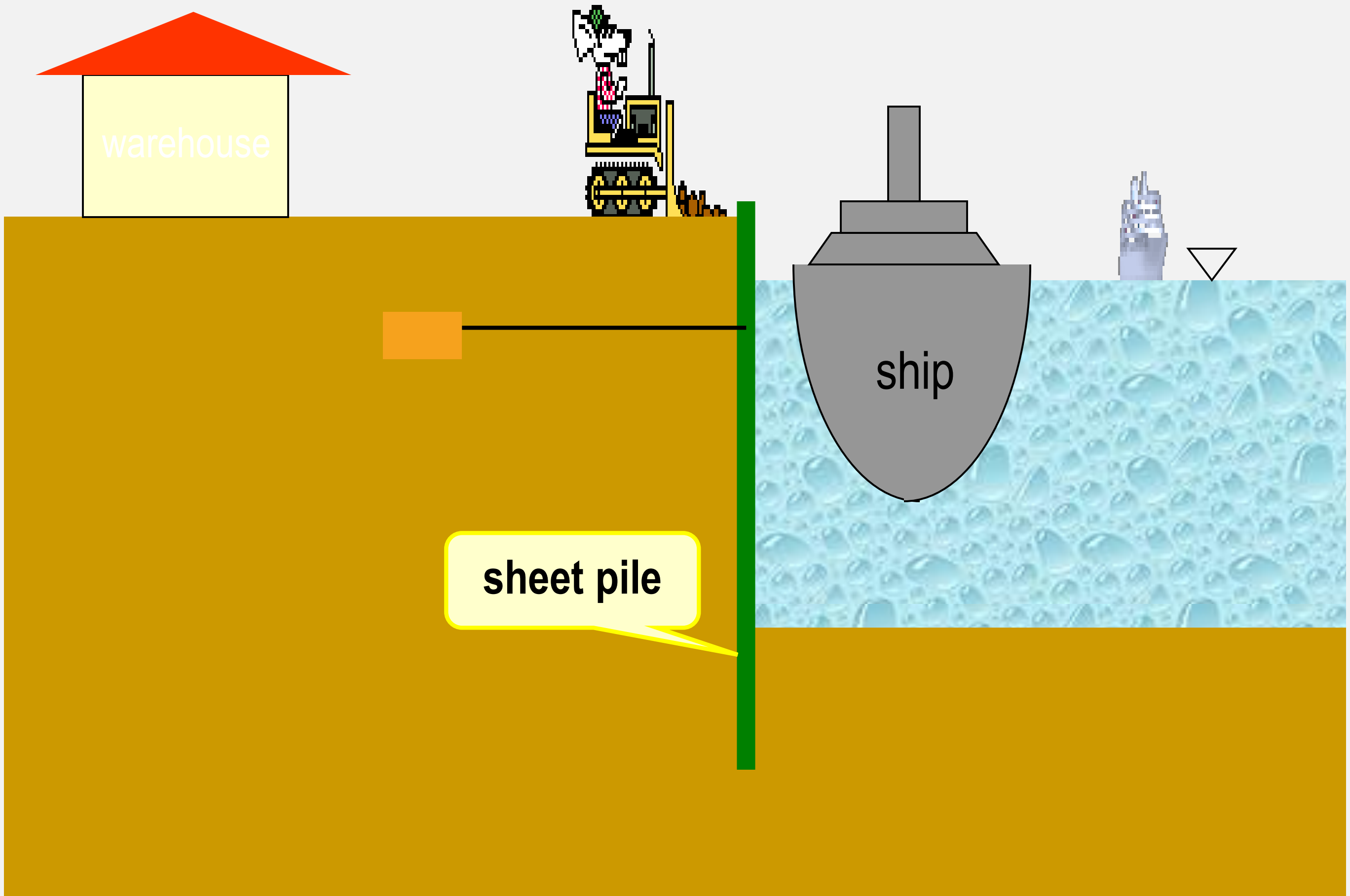
REINFORCED EARTH WALLS

~ using geofabrics to strengthen the soil



SHEET PILES

~ sheets of interlocking-steel or timber driven into the ground, forming a continuous sheet



SHEET PILES

~ resist lateral earth pressures

~ used in excavations, waterfront structures, ..

SHEET PILE AT WOOLCOCK ST

SHEET PILES

~ used in temporary works

COFFERDAM

~ sheet pile walls enclosing an area, to prevent water seeping in

COFFERDAM

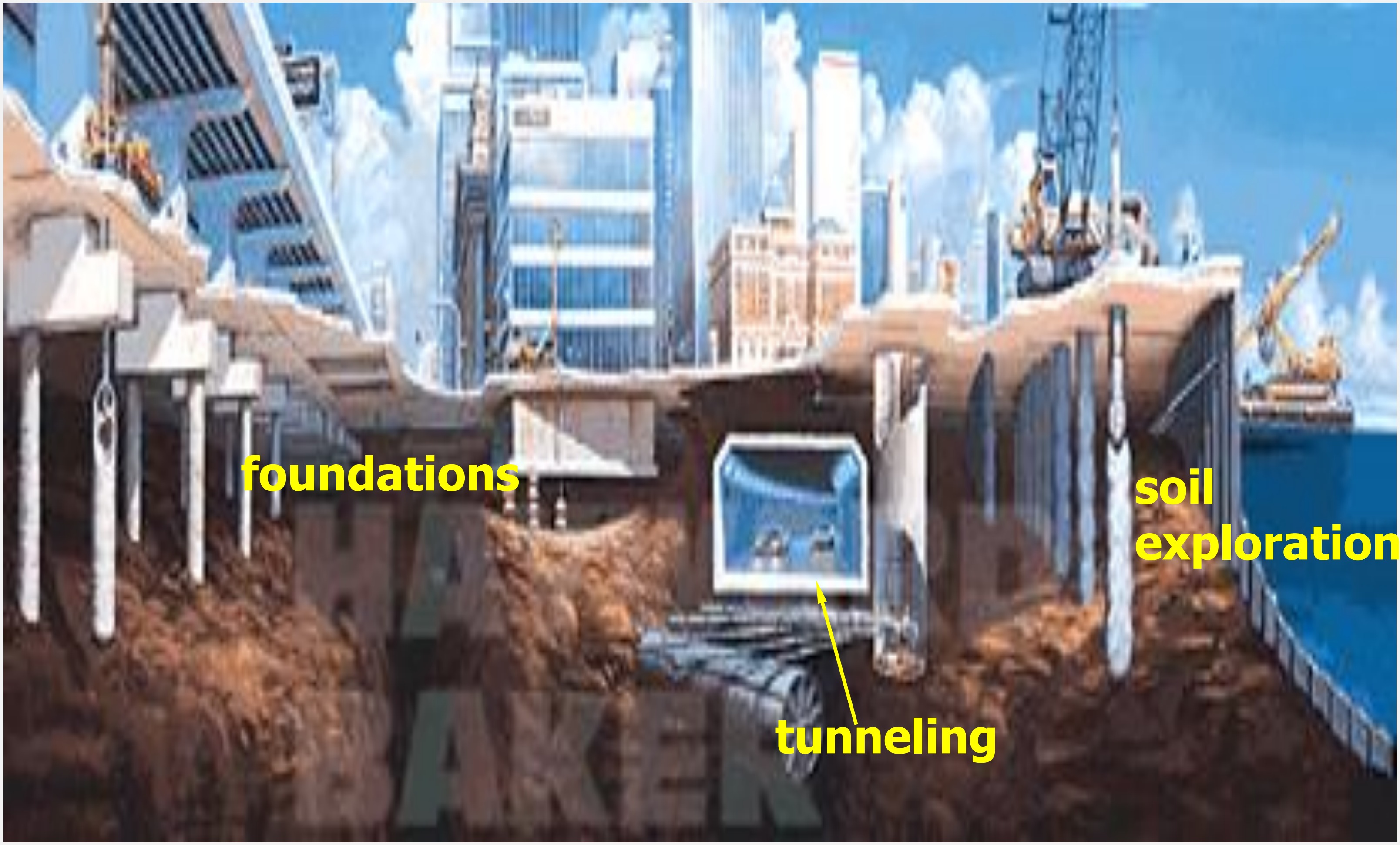
~ sheet pile walls enclosing an area, to prevent water seeping in

SHORING

propping and supporting the exposed walls to resist lateral earth pressures

EXCAVATIONS

SOME CIVIL ENGINEERING MARVELS ....



... buried right under your feet.

GREAT CONTRIBUTORS TO THE DEVELOPMENTS IN GEOTECHNICAL ENGINEERING

Karl Terzaghi
1883-1963

C.A.Coulomb
1736-1806

M. Rankine
1820-1872

GEOTECHNICAL ENGINEERING LANDMARKS

LEANING TOWER OF PISA

Our blunders become monuments!

FORMATION OF SOIL

THE BASIC CHARACTERISTICS OF SOILS ENGINEERING DEFINITION OF SOIL

- Soil: is an uncemented or weakly cemented aggregate of solid grains formed due to weathering of rocks and decayed organic matter, soil is considered to be particulate system.
- If the grains are connected together by strong and permanent cohesive forces, the material called "rock" or "stone".
- Soils tend to be complex and heterogeneous materials with a wide range of mechanical behavior.

THE BASIC CHARACTERISTICS OF SOILS ENGINEERING DEFINITION OF SOIL

- The mineral grains that form the solid phase of a soil aggregate are the product of rock weathering.
- The physical properties of soil are dictated by the size, shape, and chemical composition of the grains, and hence the rock from which is derived.
- Rocks are compact, semi-hard to hard mass composed of one or several minerals.

SOIL FORMATION

- Soils are formed from the:
- **Physical weathering**
- **Chemical weathering**

PHYSICAL WEATHERING

- 1-Physical (mechanical) weathering

1) Temperature changes: may cause the rock to disintegrate either because of

- (A) fatigue due to cyclic stresses of compression and tension.
- (B) differential thermal expansions of the different minerals within the rock, or both.

2) Alteration freezing and thawing: may widen the crack in the rock by

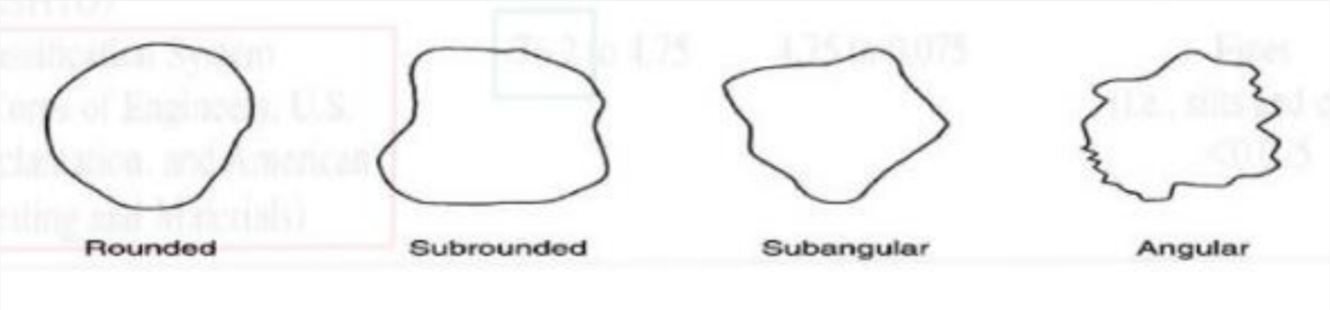
- expansion of water in the cracks, or may create new cracks in the sound
- rock if the pores or voids of the rock are filled with water.

3) Splitting action of plant roots: roots may penetrate rock pores or

- existing cracks to further increase their size.
- 4) Erosion by the action of wind, water, or glaciers (abrasive movement)

PHYSICAL (MECHANICAL) WEATHERING

- The resultant soil particles due to physical weathering retain the same compositionas the parent rock.
- •The shape of particles may be:
- Rounded, subrounded, angular, or subangular



2- CHEMICAL WEATHERING

1) **Oxidation**: is an agent in the decomposition process whereby oxygen ions combine with some minerals in the rock which subsequently decomposed in a manner similar to the rusting of steel.

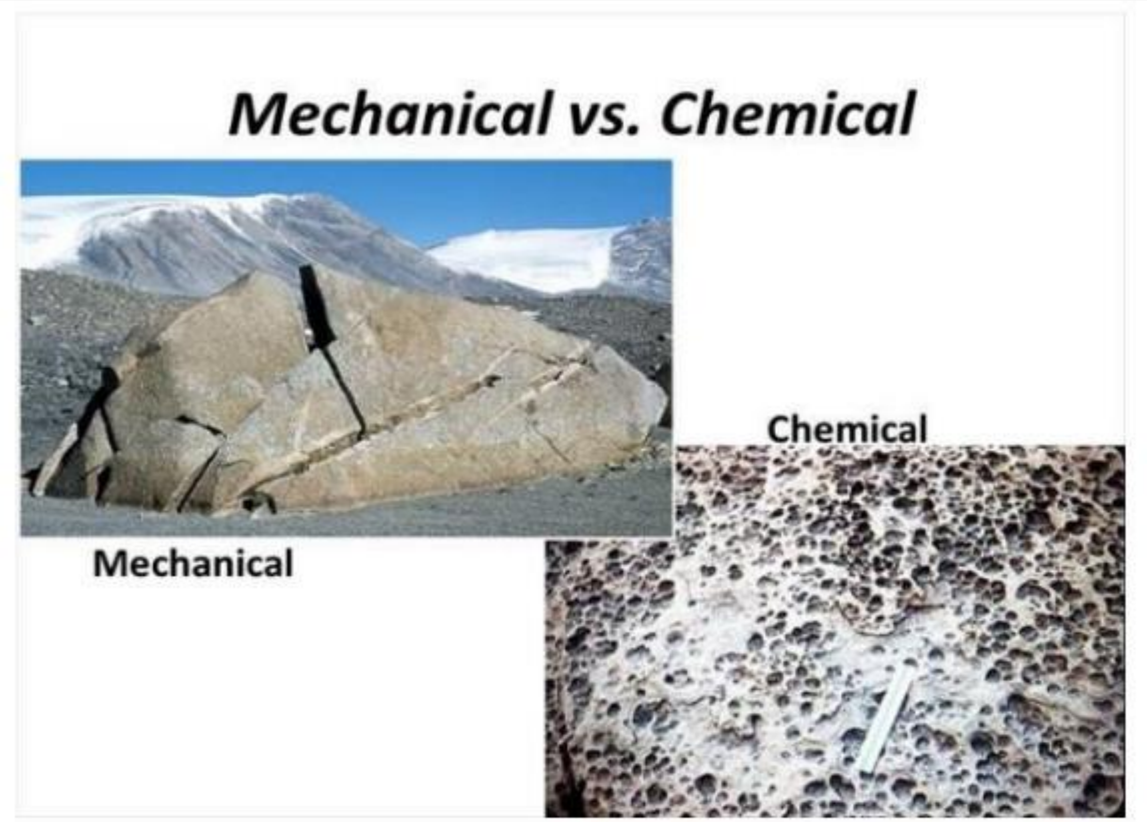
2) **Carbonation**: is a form of decomposition where carbon dioxide and water form carbonic acid which decomposes many minerals containing iron, calcium, sodium, etc.

3) **Hydration**: is the process of chemical addition of water to the minerals that subsequently convert into new minerals.

4) **Vegetation**: decaying vegetation may be a factor in the production of organic acids, carbon dioxide, and oxygen, which mixed with water penetrating through rock, may extract certain chemical elements from the rock.

CHEMICAL WEATHERING

- The resultant soil particles due to chemical weathering have a various composition about the parent rock
- .



FORMATION OF SOIL

- residual soils: Soils that remain at the site of weathering are called.
- These soils retain many of the elements that comprise the parent rock.

- Alluvial soils, also called fluvial soils, are soils that were transported by rivers and streams.
- The composition of these soils depends on the environment under which they were transported and is often different from the parent rock.

ROCKS TYPES

- Rocks can be divided into three basic types:
- **Igneous rocks**

Formed by solidification of molten Magma ejected from deep within earth's mantle.

- **Sedimentary rocks**

Sedimentary rocks are widely spread over the surface of earth.

Weathering reduces the exposed rock mass to fragmented particles which can be more easily transported more easily by wind, water, and ice.

ROCKS TYPES

- The process through which sediments are converted into sedimentary rocks is called DIAGENSIS. It includes the following phases:
- 1. Cementation Water percolating through the voids (or pores) between the particles of sediment carries mineral matter which coats the grain and acts as cement that bind them together.
- 2. Compaction The weight of top layers compacts sediments and expels water out.
- 3. Crystallization Sometimes grains of sediments are joined together due to crystallization of some of their constituents due to pressure.

ROCKS TYPES

- **Metamorphic rocks**

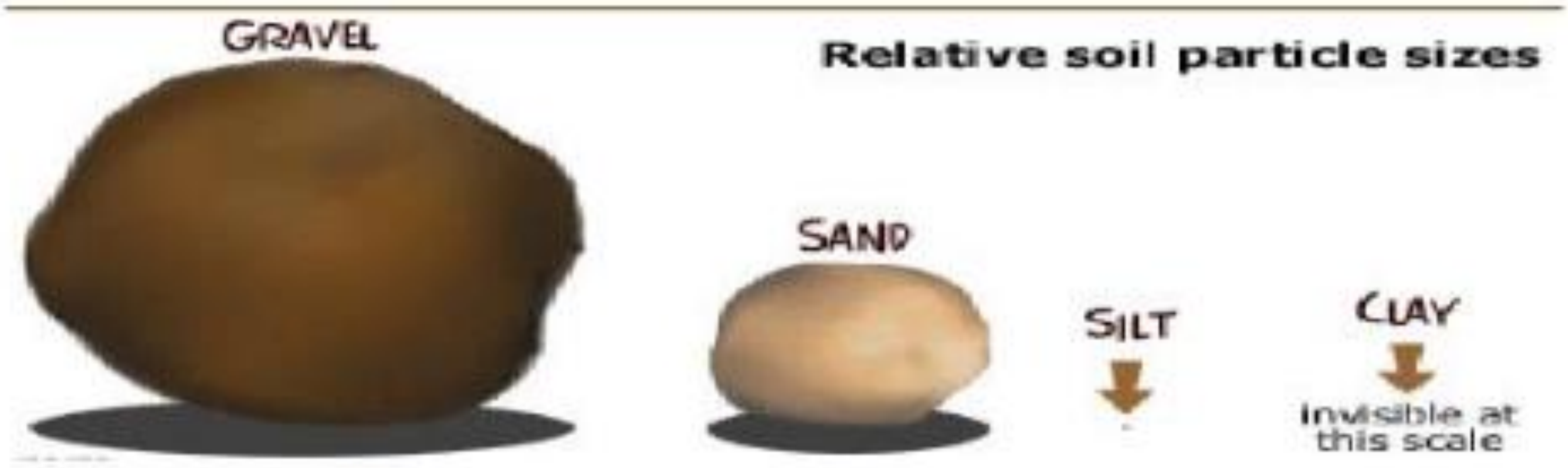
Metamorphic rocks are formed if a rock is subjected to increase in temperature, pressure, or both, to such degree that a new TEXTURE or possibly a new MINERAL composition is produced.

- The original rock may be igneous, sedimentary or metamorphic.

SOIL TYPES

- **Soil Types**
- Common descriptive terms such as:
- gravels, sands, silts, and clays are used to identify specific textures in soils.
- Texture: refers to the appearance or feel of a soil.

- Sands and gravels are grouped together as coarse-grained soils.
- Coarse-grained soils feel gritty and hard
- Clays and silts are fi ne-grained soils..
- Fine-grained soils feel smooth.


GRAVEL
Relative soil particle sizes
SAND
SILT
CLAY
invisible at
this scale

MAIN CLASSES OF SOIL

Main Classes of Soils

Soil Type	Type of Grains	Predominant Size	Intergrain Bonding
Gravels	quartz, feldspars, rounded and/or angular	2mm < D < 76mm	Frictional
Sands	same as above	.075mm < D < 2mm	Frictional & chemical
Silts	primarily quartz flake-like grains	2μm < D < 2mm	Frictional, chemical, electrical
Clays	micah, kaolinite, bentonite, etc. very small plate-like grains	D < 2μm	Chemical & electrical

CLASSES OF SOIL DEPENDING ON SIZE PARTICLES

Table 2.3 Particle-Size Classifications

Coarse grained soils — Fine grained soils

Name of organization	Grain size (mm)			
	Gravel	Sand	Silt	Clay
Massachusetts Institute of Technology (MIT)	>2	2 to 0.06	0.06 to 0.002	<0.002
U.S. Department of Agriculture (USDA)	>2	2 to 0.05	0.05 to 0.002	<0.002
American Association of State Highway and Transportation Officials (AASHTO)	76.2 to 2	2 to 0.075	0.075 to 0.002	<0.002
Unified Soil Classification System (U.S. Army Corps of Engineers, U.S. Bureau of Reclamation, and American Society for Testing and Materials)	76.2 to 4.75	4.75 to 0.075	Fines (i.e., silts and clays) <0.075	

PARTICLE FORCES AND BEHAVIOR

- • Behavior of individual soil particles and their interaction with other particles influenced by:

1. weight of the particle forces (Fg): weight force is the result of gravitational forces. It's a function of volume of particles.
2. particle surfaces forces (Fs): surface forces are of an electrical nature. It's a function of surface area.

- soil particles are assumed to be sphere particles of diameter (D).
- The ratio Fg/Fs is directly proportional to D. which explain the nature of forces between coarse grained soils and fine grained soils.

Clay Mineralogy

CLAY MINERALOGY

- Why we study clay minerals?
- When we can called a material " Clay"?

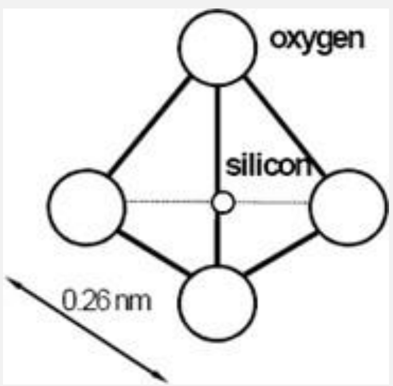
1. crystalline particles of very small size (**<0.002mm**)
2. Develop *plasticity* when mixed with limited amount of water.
3. *Member of Phyllosilicate* family.

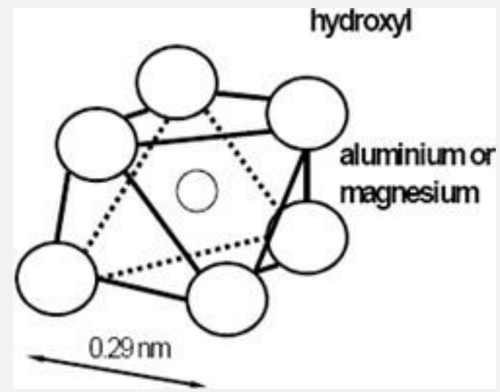
BASIC STRUCTURAL UNITS

structural units

Tetrahedral sheet

Octahedral sheet





Dioctahedral sheet

Aluminum

trioctahedral sheet

Magnesium

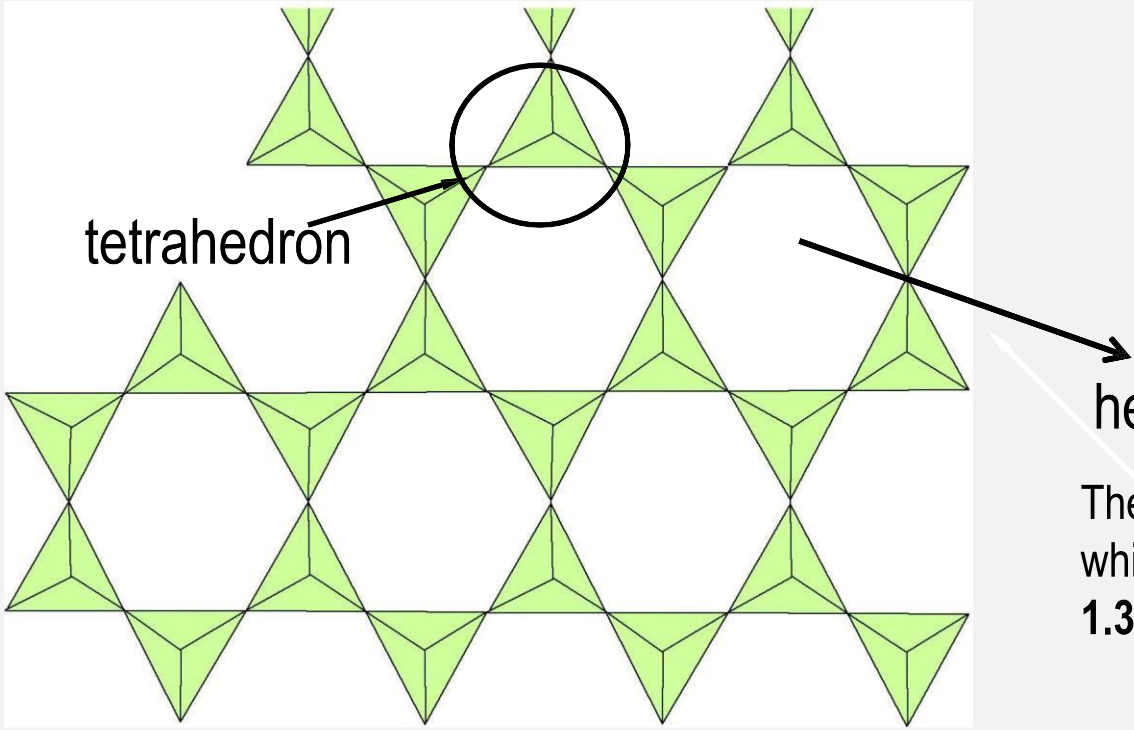
Si

G

B

TETRAHEDRAL SHEET

Several tetrahedrons joined together form a tetrahedral sheet.

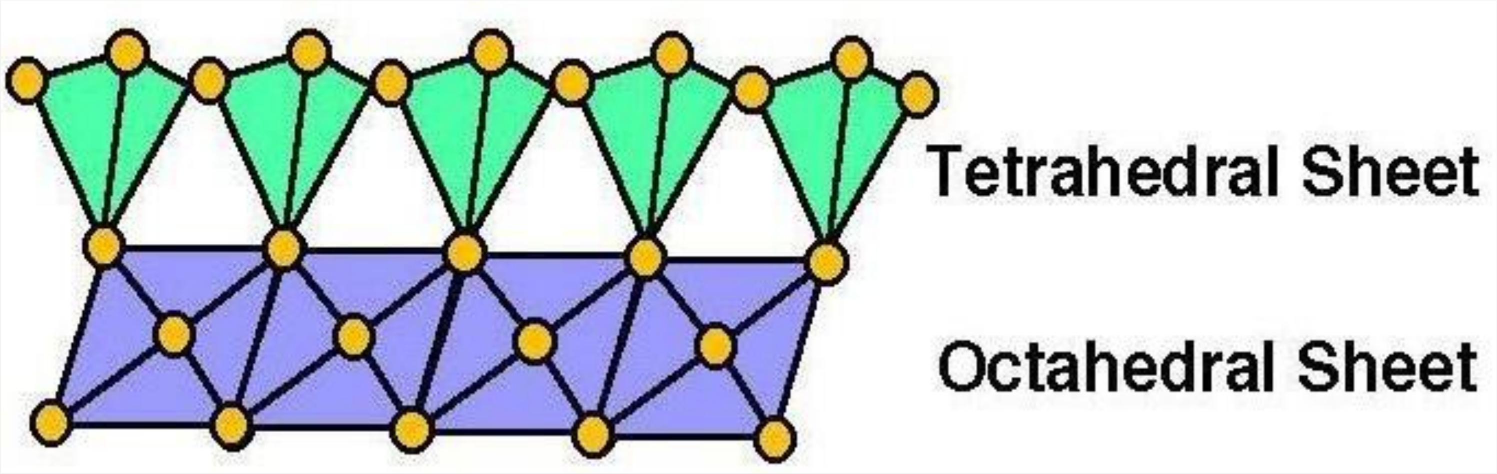


The radius of circle which fits into hole is **1.32 A°**

DIFFERENT CLAY MINERALS

Different combinations of tetrahedral and octahedral sheets form different clay minerals:

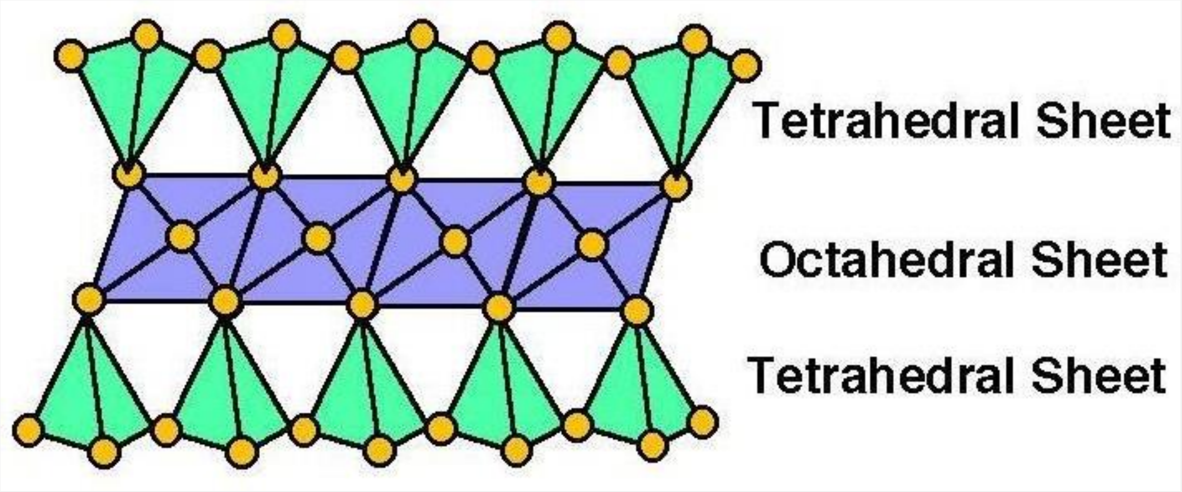
<u>1:1 Clay Mineral</u> (e.g., kaolinite, halloysite):



DIFFERENT CLAY MINERALS

Different combinations of tetrahedral and octahedral sheets form different clay minerals:

<u>2:1 Clay Mineral</u> (e.g., montmorillonite, illite)

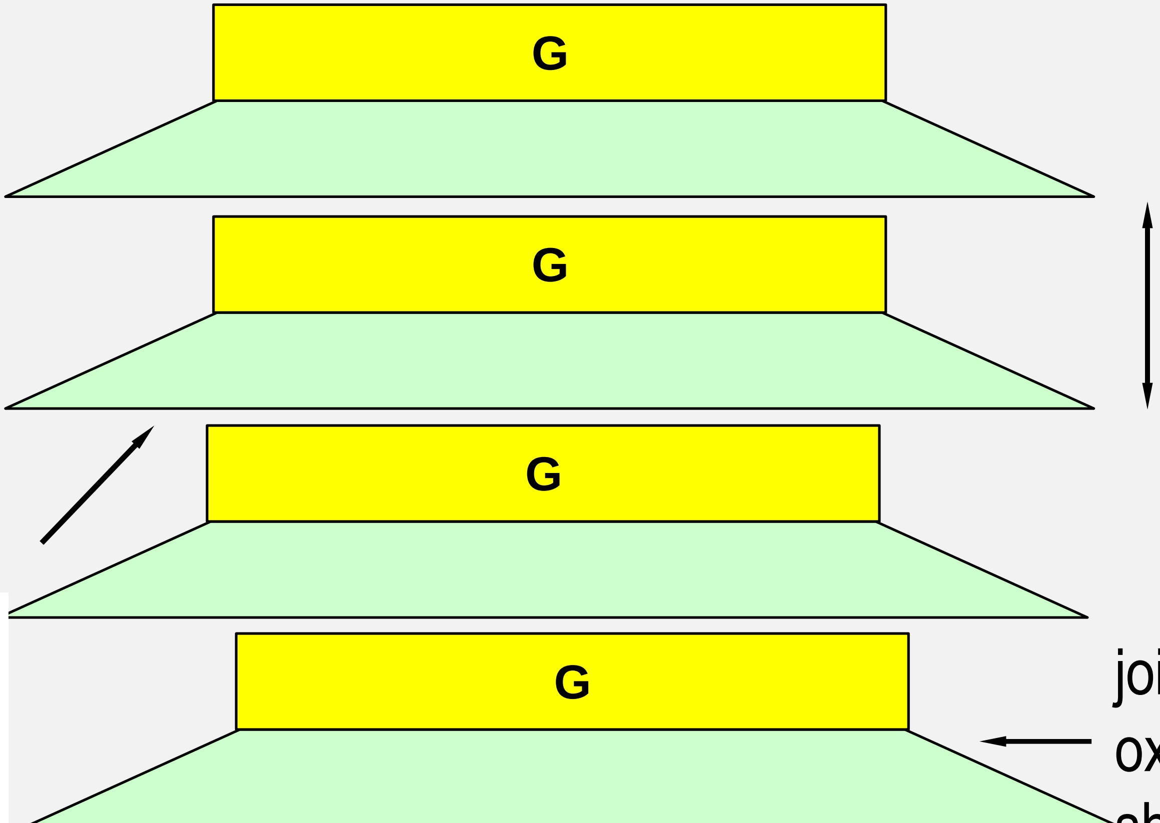


DIFFERENT CLAY MINERALS

- There are many different types of clay minerals, the most important clay minerals are:

1. *Kaolinite*
2. *Illite*
3. *Montmorillonite*

KAOLINITE
1:1 DIOCTAHEDRAL

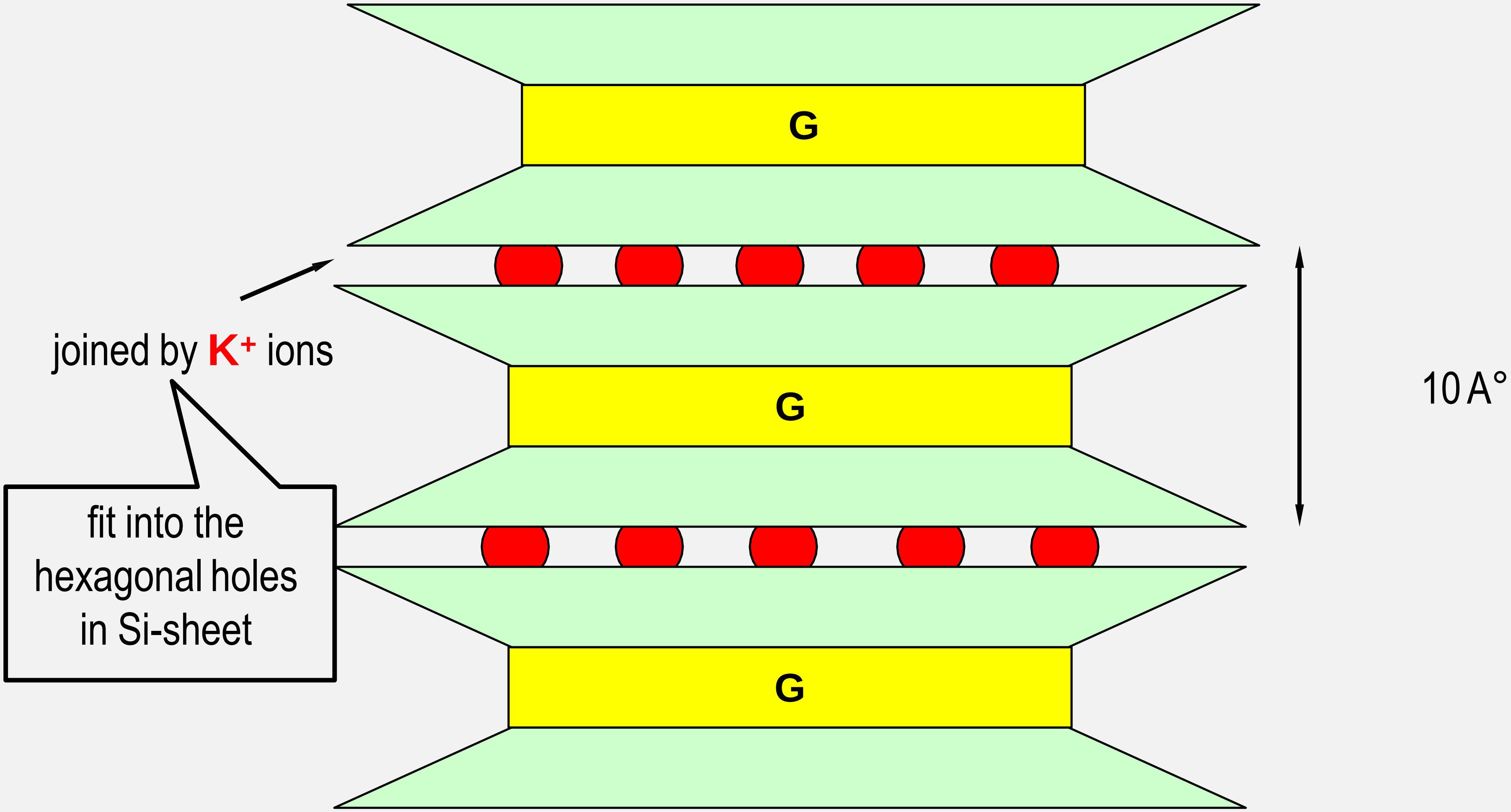


joined by strong H-bond ∴ no easy separation

joined by oxygen sharing

ILLITE

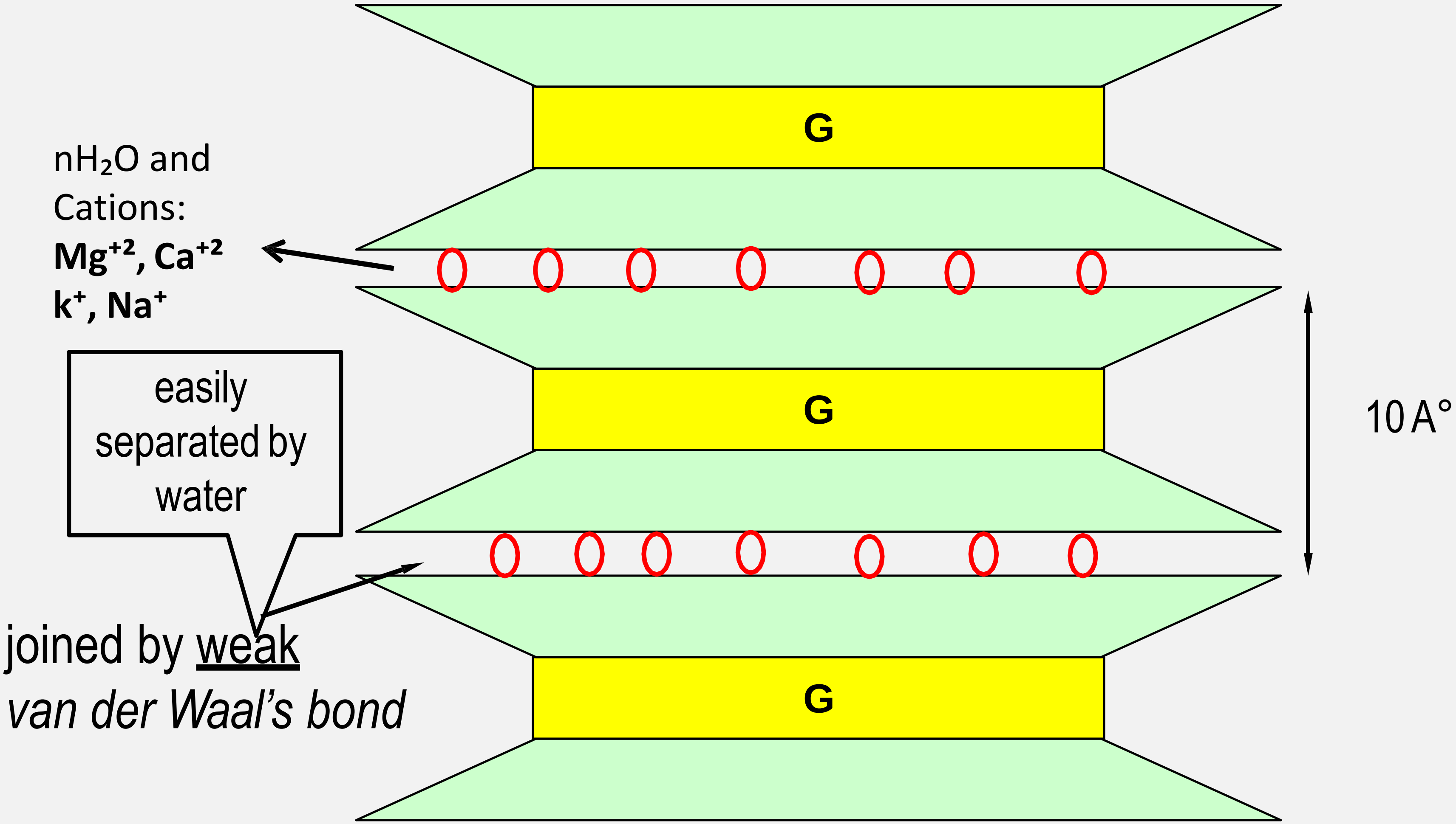
2:1 DIOCTAHEDRAL



MONTMORILLONITE

2:1 Dioctahedral

also called **smectite**; expands on contact with water.



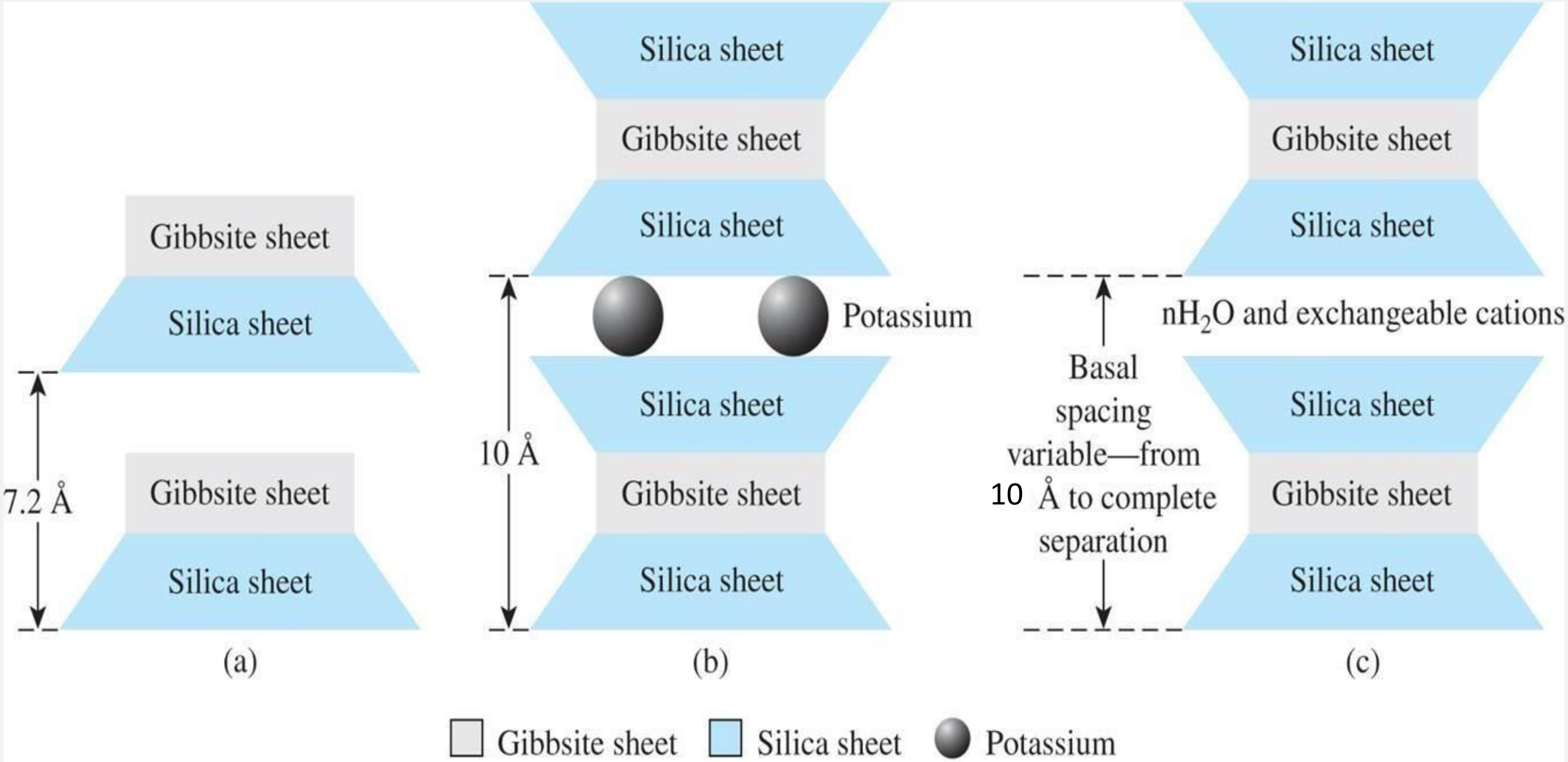


Figure 2.12 Diagram of the structures of (a) kaolinite; (b) illite; (c) montmorillonite

CLAY PARTICLE

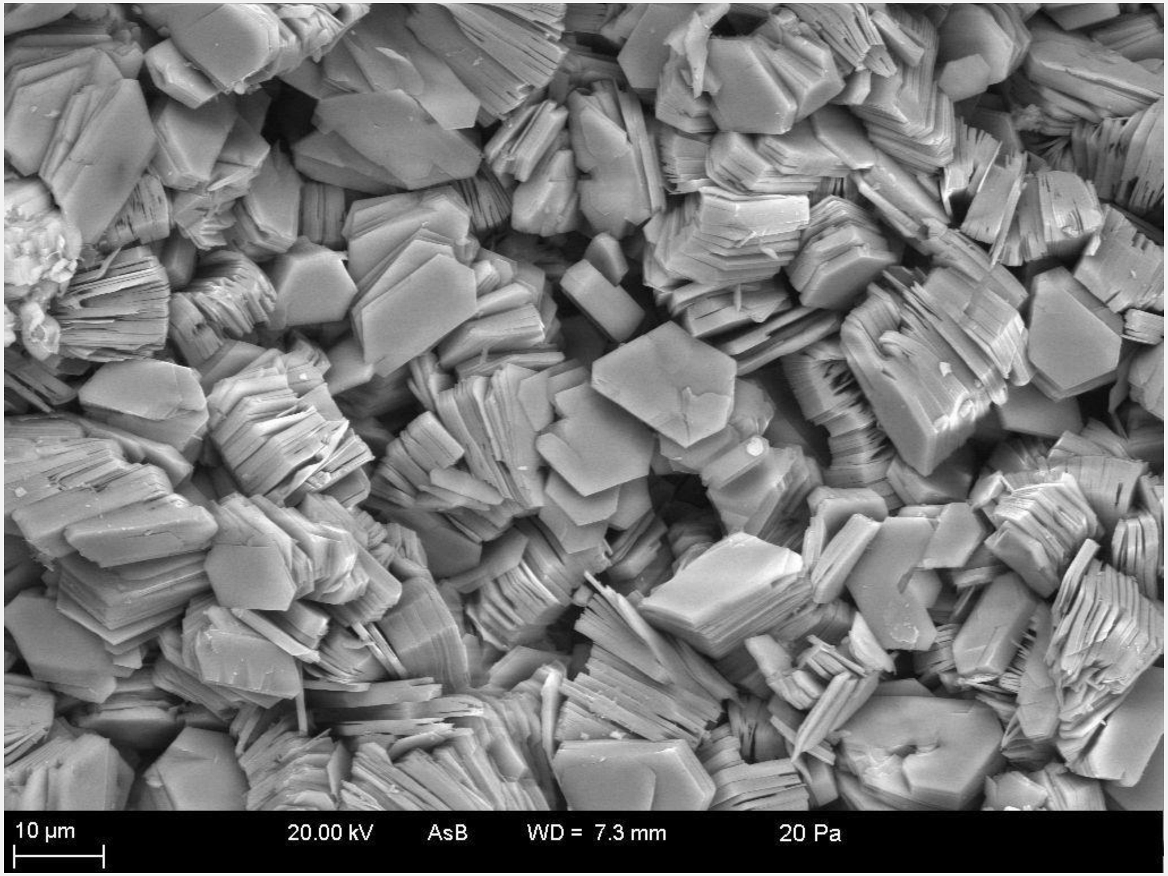
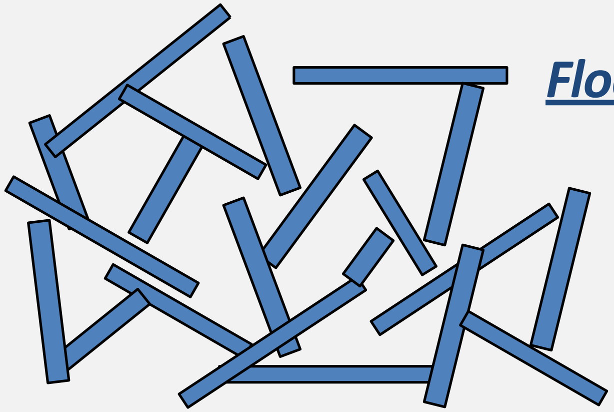


Plate-like or Flaky Shape

CLAY STRUCTURE

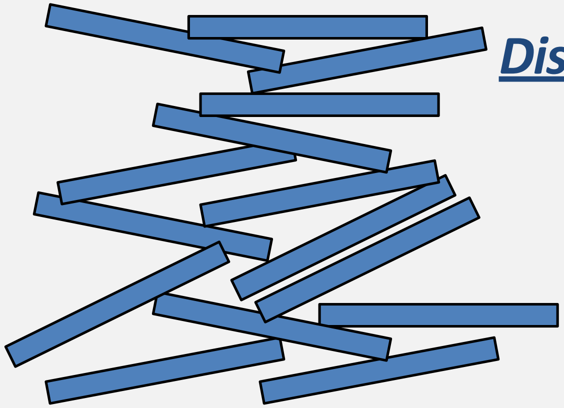
- Clay structure: is the arrangement of clay mineral particles with respect to each other.
- The arrangement is due to the net forces acting between particles. The net forces are:

1. Attraction: the structure is Flocculated.
2. Repulsion: the structure is Dispersed.



Flocculated

edge-to-face contact or
edge-to-edge contact



Dispersed

face-to-face contact

SPECIFIC SURFACE

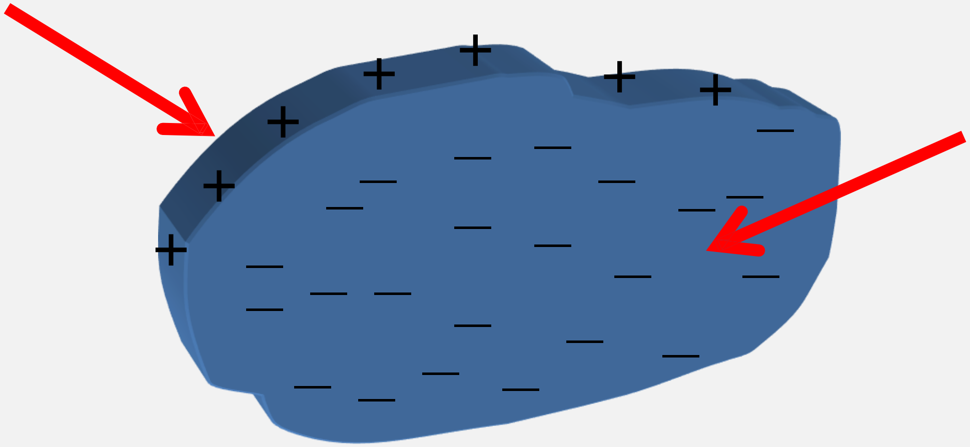
- Specific surface is the surface area per unit mass (m^2/g)
- Clay minerals have a large specific surface
- Clay properties are highly affected by specific surface

Mineral	Specific surface (m^2/g)
Kaolinite	10-20
Illite	80-100
Montmorillonite	800

CHARGE OF CLAY PARTICLES

- The surface of clay mineral particles is ***negatively*** charged, due to these reasons:

1. Isomorphous substitution
2. Dissociation of hydroxyl ions.
3. Unsatisfied charges due to *broken bonds* at the edges of particles.
4. Adsorption of anions.

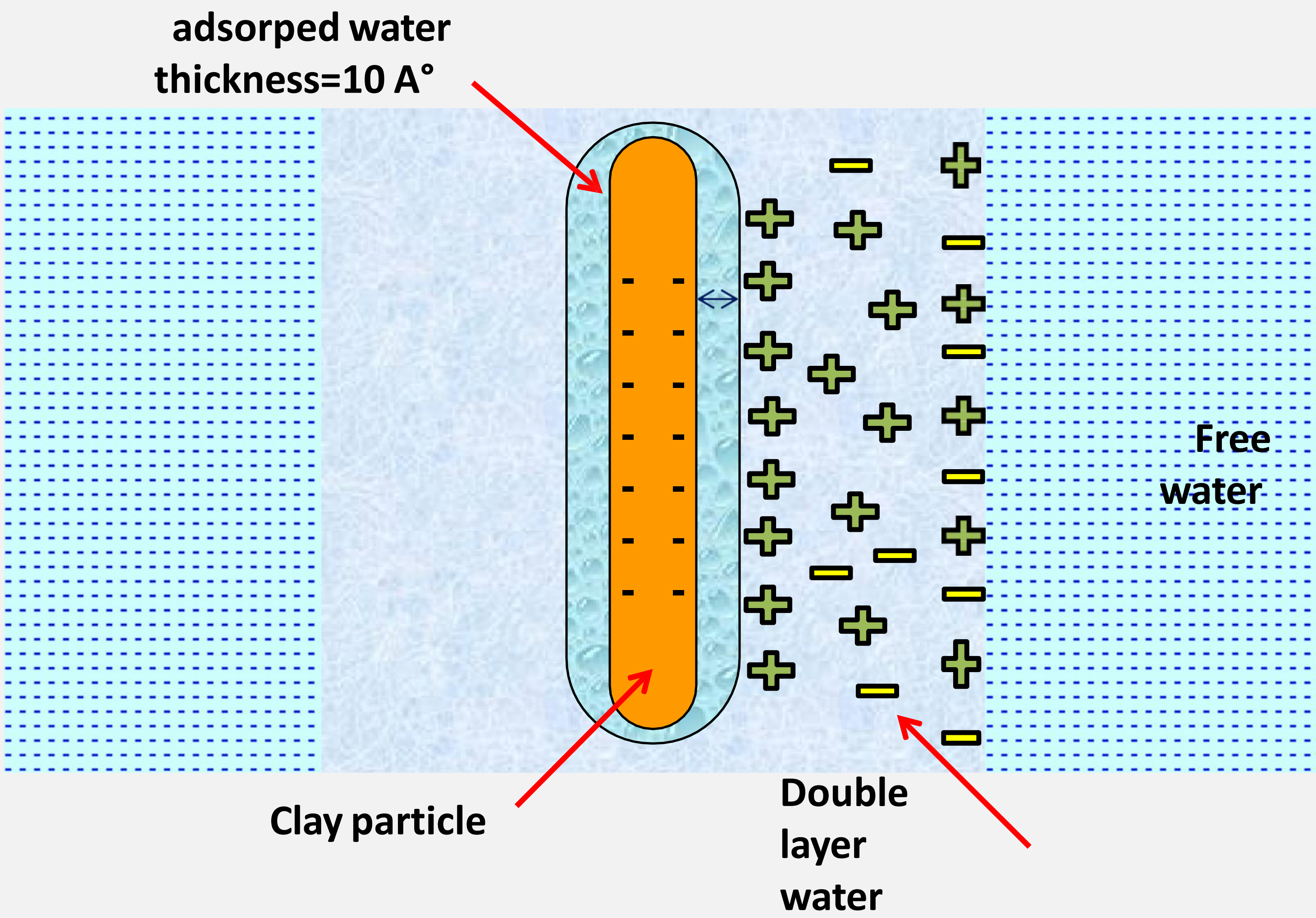


ISOMORPHOUS SUBSTITUTION

- **Isomorphous substitution**: is substitution of aluminum or silicon by atom of lower valance, which leads to a net negative charge on clay mineral particles.
- the arrangement of negative charge on clay minerals:
  - **Montmorillonite> Illite> Kaolinite**
- Isomorphous substitution in clay minerals as shown:

Type of I.S	Kaolinite	Illite	Montmorillonite
Type of I.S	Al^{+3} for Si^{+4}	Al^{+3} for Si^{+4}	Mg^{+2} for Al^{+3}
Amount of I.S	1:400	1:7	1:6
Location of I.S	Silica sheet	Silica sheet	Gibbsite

CLAY PARTICLES IN WATER



CLAY PARTICLES IN WATER

- Adsorbed water: water nearest to the clay particle which strongly held by hydrogen bond (because water molecules are dipolar) and appears to have high viscosity.

- D.D.L: describes the negatively charged particle surface and the dispersed layer of cations.

- Free water: water which is not affected by the charge of clay particles, and have lower viscosity.

Phase Relations

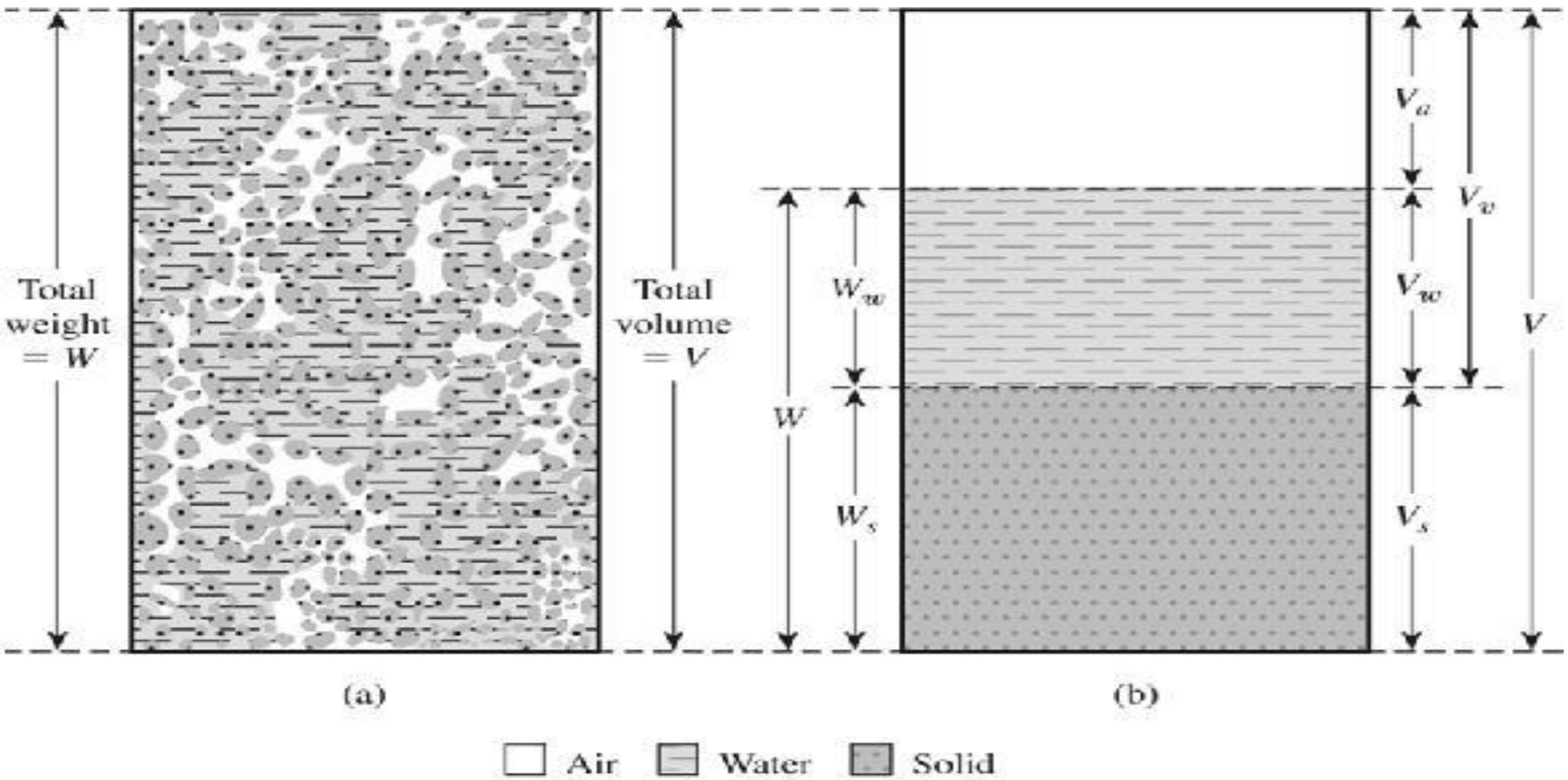


Figure 3.1 (a) Soil element in natural state; (b) three phases of the soil element

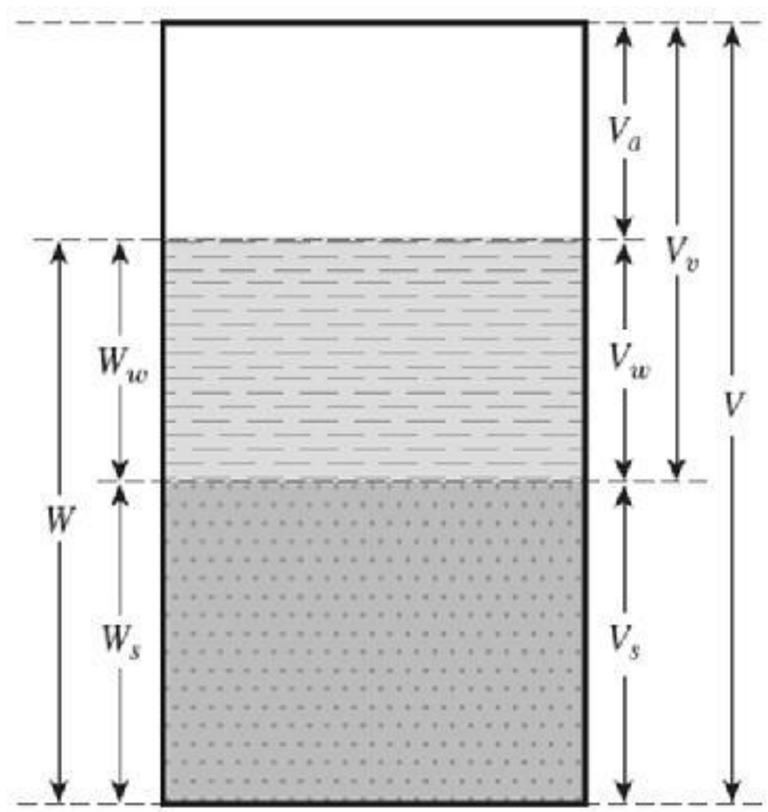
Phase Relations

Phase	Volume	Mass	Weight
Air	V_A	0	0
Water	V_W	M_W	W_W
Solid	V_S	M_S	W_S

Table 1 Distribution by Volume, Mass, and Weight.

Objectives: To compute the masses (or weights) and volumes of the <u>three</u> different phases.

Notation
W = Weight
V = volume
s = soil grains
w= water
a = air
v = voids



Phase Diagram

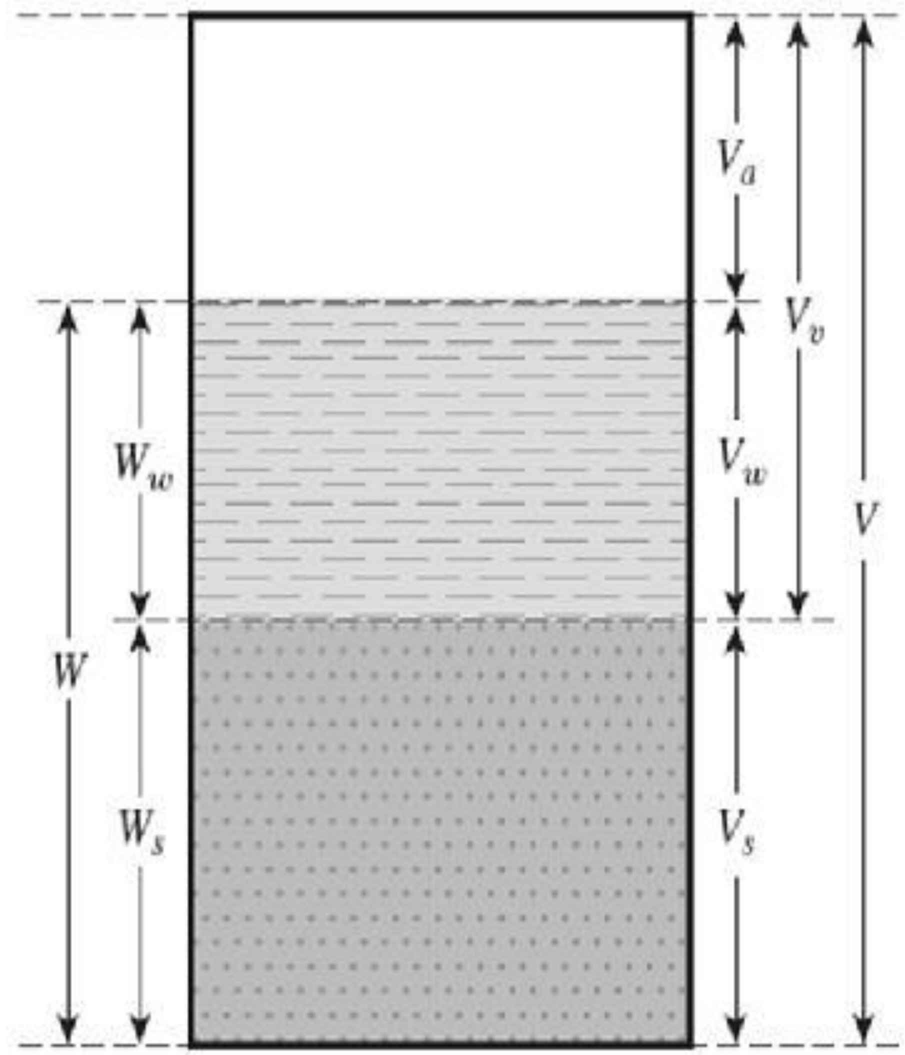
Definitions

Water content (w) is a measure of the water present in the soil.

$$\boxed{w = \frac{W_W}{W_S} \times 100\%}$$

Expressed as percentage.

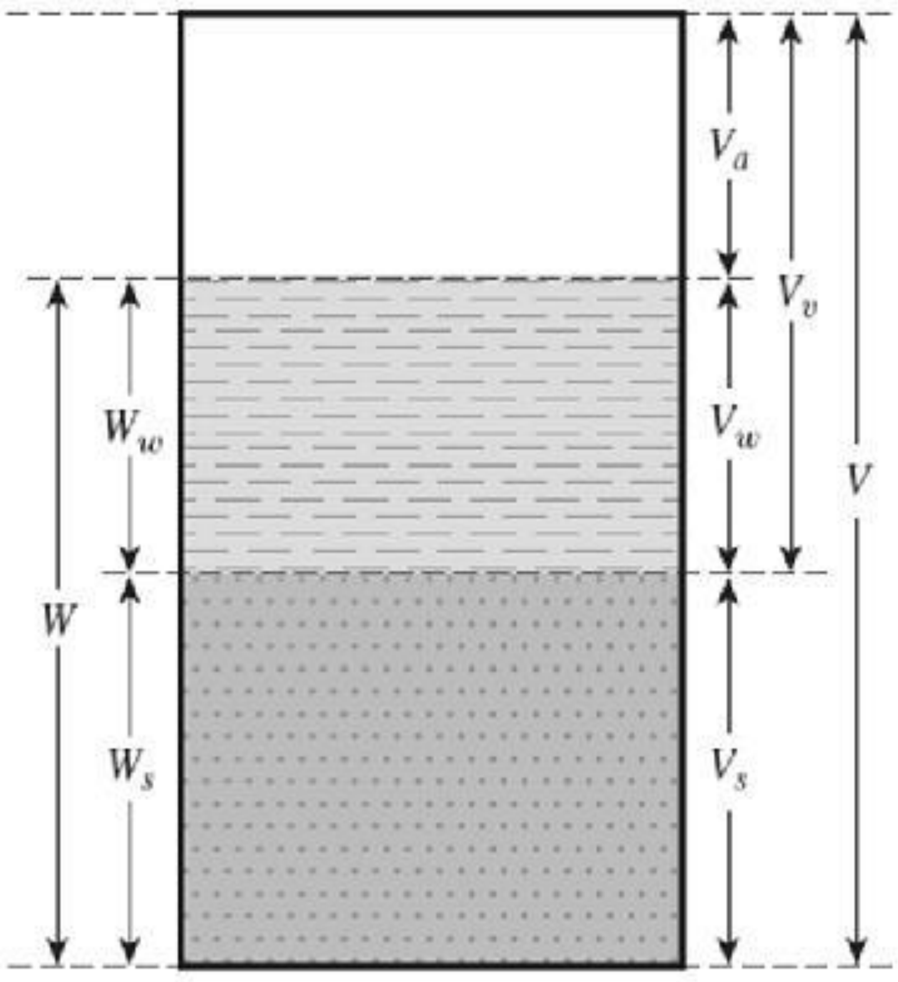
Range = 0 – 100%.



Definitions

Void ratio (e) is a measure of the void volume.

$$\boxed{e = \frac{V_V}{V_S}}$$



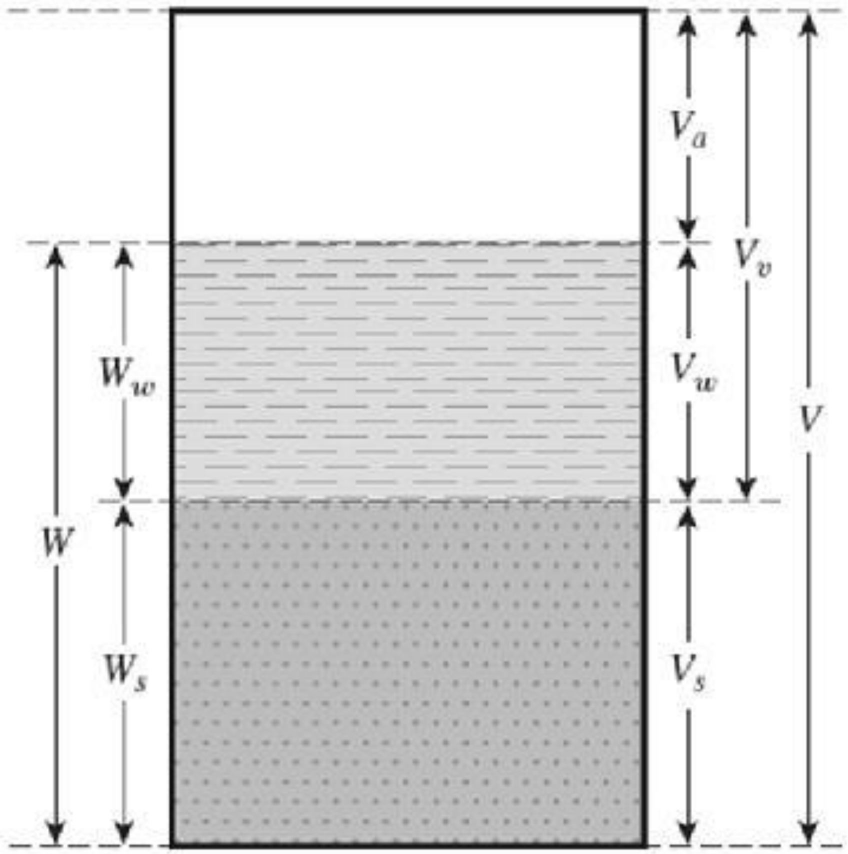
Phase Diagram

Definitions

Porosity (n) is also a measure of the void volume, expressed as a percentage.

$$\boxed{n = \frac{V_V}{V} \times 100\%}$$

Theoretical range:
0 – 100%



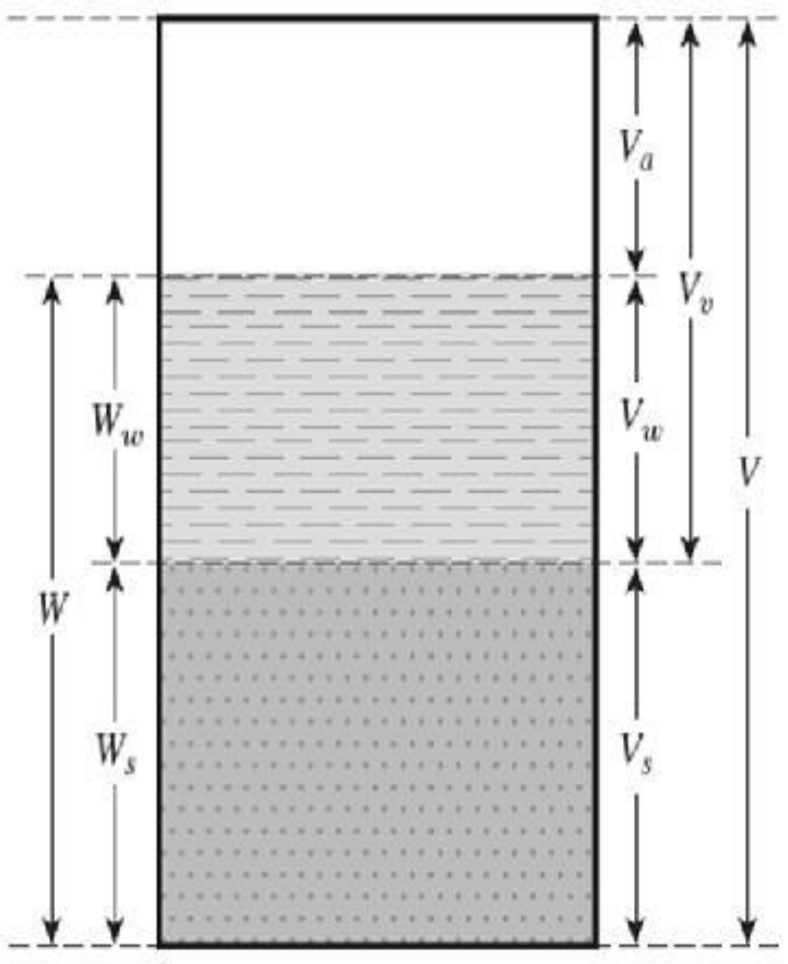
Definitions

Degree of saturation (S) is the percentage of the void volume filled by water.

$$\boxed{S = \frac{V_W}{V_V} \times 100\%}$$

Range: 0 – 100%

Dry - Saturated



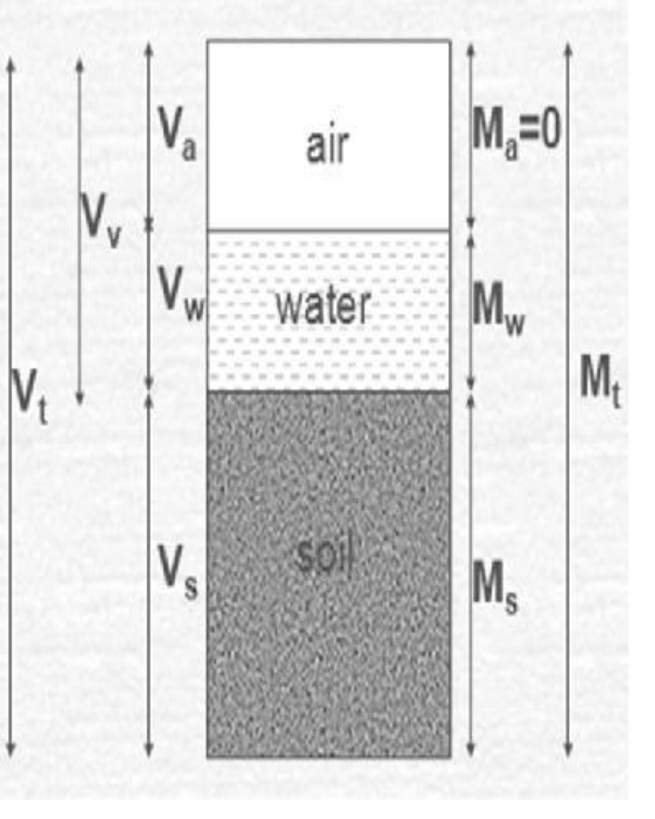
Phase Diagram

Definitions

Soil density (ρ) (bulk density) : is the density of the soil in the current state.

$$\boxed{\rho = \frac{M}{V}}$$

Units:
t/m^3,
g/ml,
kg/m^3



Phase Diagram

Definitions

Saturated density (ρ_{sat}) is the density of the soil when the voids are filled with water.

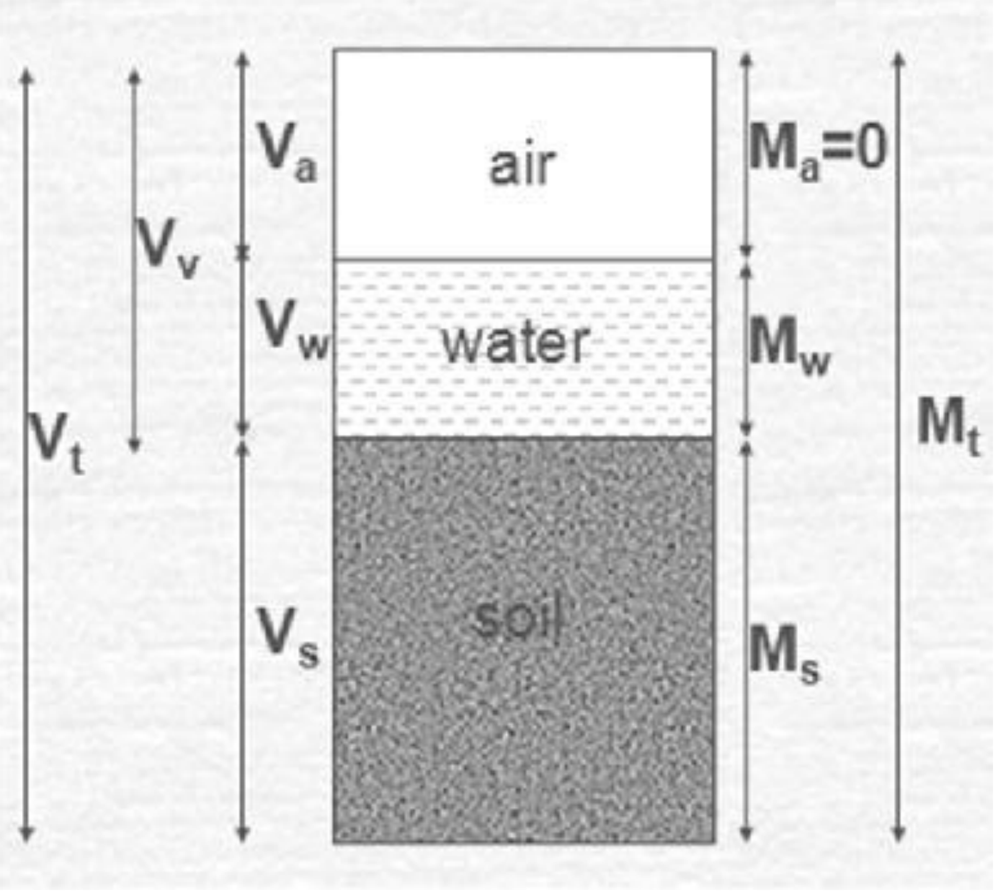
Submerged density (ρ') is the effective density of the soil when it is submerged.

$$\rho' = \rho_{sat} - \rho_w$$

Definitions

Dry density (ρ_d) is the density of the soil in dry state.

Units: t/m^3, g/ml, kg/m^3



Phase Diagram

Definitions

Bulk, saturated, dry and submerged *unit weights (γ)* are defined in a similar manner.

Here, use weight (kN) instead of mass (kg).

$\gamma = \rho * g$

N/m^3 kg/m^3 m/s^2

$$\boxed{\gamma = \frac{M\,g}{V}}$$

Because the force is usually required it is often convenient in calculations to use the unit weight, g (weight per unit volume).

$$\boxed{\gamma = \frac{W}{V}}$$

Definitions

Specific gravity *of the soil grains (G_s)* the ratio of the unit weight of a given material to the unit weight of water.

$$G_s = \frac{\rho_s}{\rho_w} = \frac{\gamma_s}{\gamma_w}$$

We can use G_s to calculate the density or unit weight of the solid particles:

$$\rho_s = G_s \, \rho_w$$
$$\gamma_s = G_s \, \gamma_w$$

Phase Relations

Consider a fraction of the soil where $\mathbf{V_s = 1}$.

The other volumes can be obtained from the previous definitions.

The weight can be obtained from:

Weight = Density x Volume x gravity accelerating

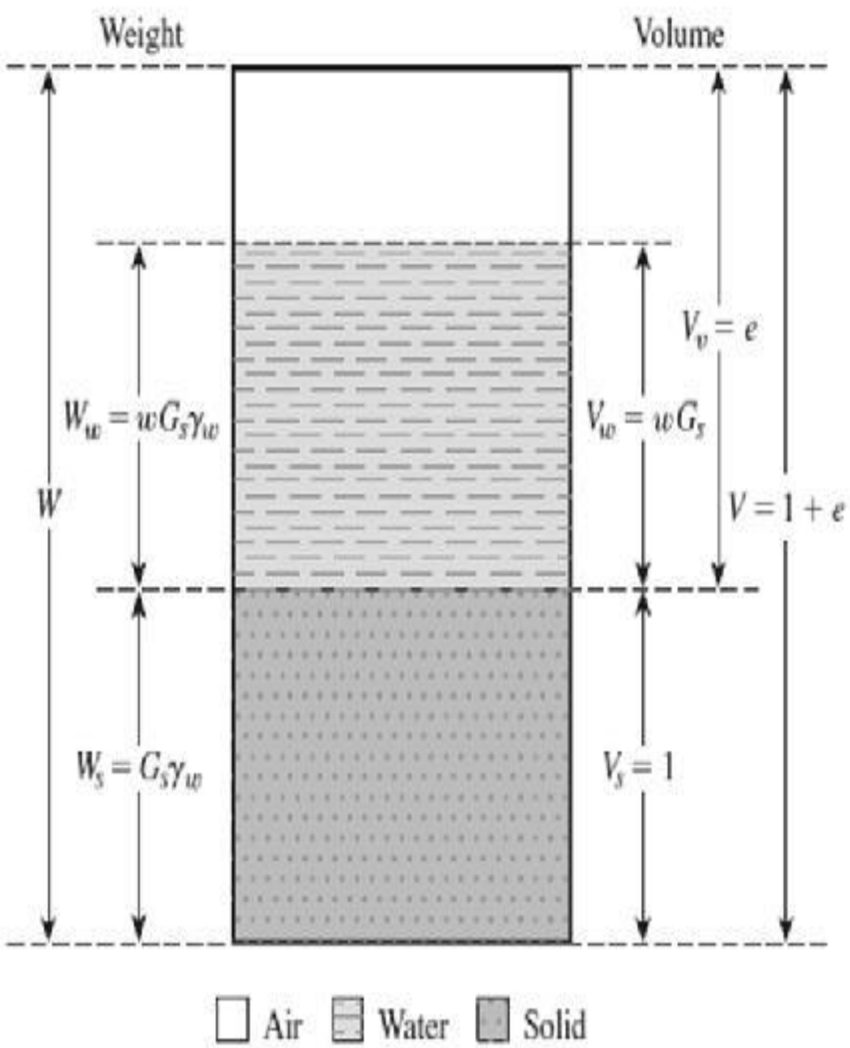


Figure 3.2 Three separate phases of a soil element with volume of soil solids equal to one

Phase Relations

Consider a fraction of the soil where $\mathbf{V_s = 1}$.

Phase	Volume	Mass	Weight
Air	$e\,(1 - S)$	0	0
Water	$e\,S$	$e\,S\,\rho_w$	$e\,S\,\gamma_w$
Solid	1	$G_s\,\rho_w$	$G_s\,\gamma_w$

Table 2 Distribution by Volume, Mass and Weight in Soil

Note that Table 2 assumes a solid volume $V_s = 1\ m^3$, All terms in the table should be multiplied by V_s if this is not the case.

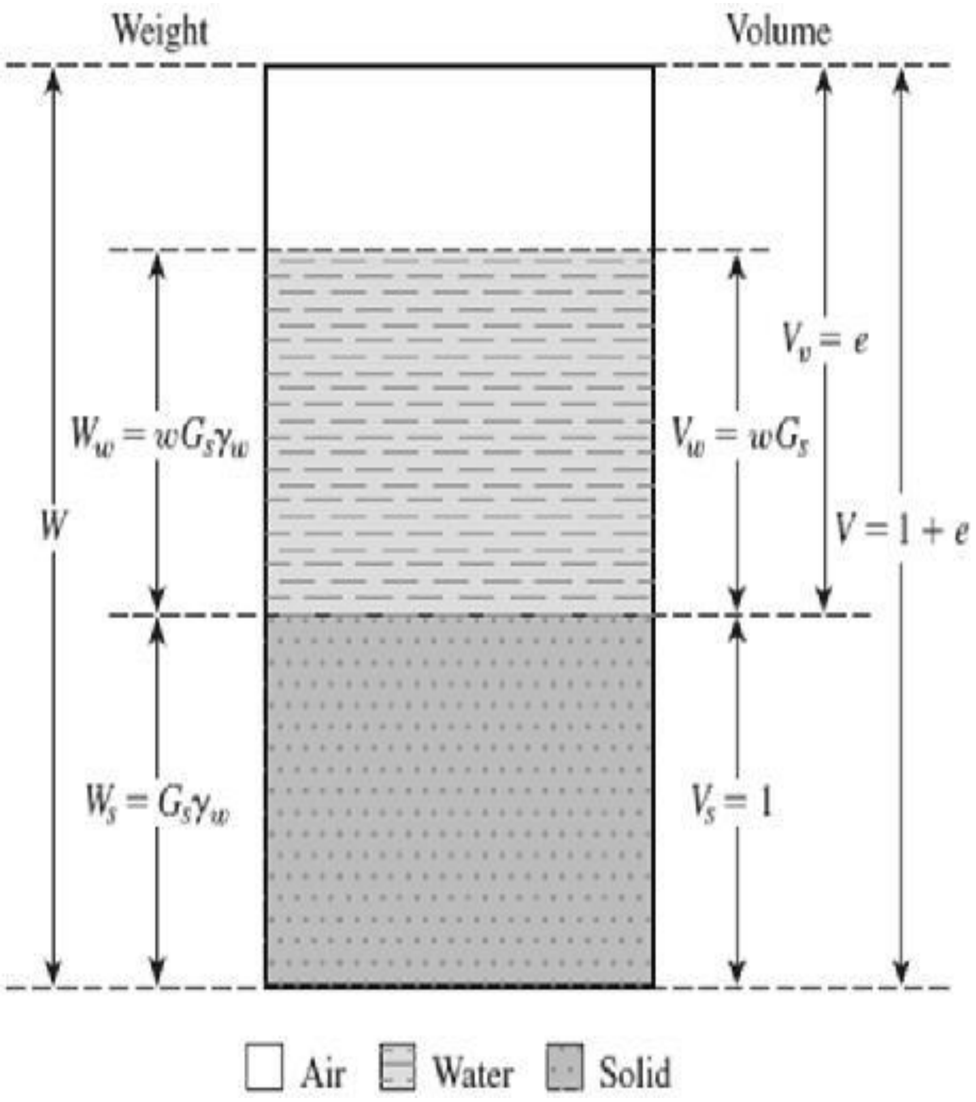
Phase Relations

From the previous definitions,

$$\boxed{w = \frac{W_W}{W_S} = \frac{Se}{G_S}}$$

$$\boxed{n = \frac{V_V}{V} = \frac{e}{1+e}}$$

$$e = \frac{V_v}{V_s} = \frac{V_v}{(1-n)V} = \frac{n}{1-n}$$

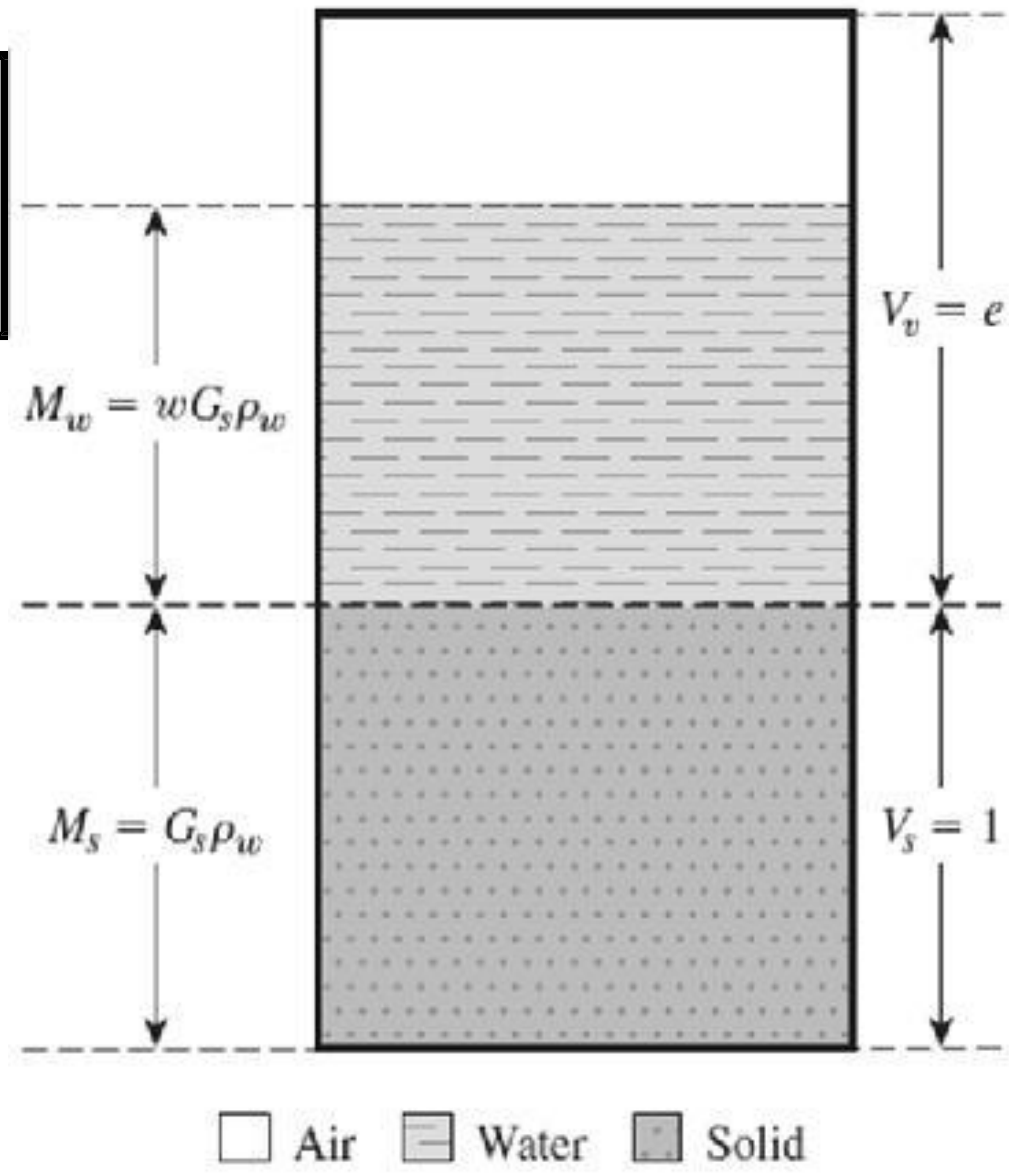


Phase Relations

$$\boxed{\rho_m = \frac{M}{V} = \frac{G_S + Se}{1+e}\rho_W}$$

$$\boxed{\rho_{sat} = \frac{M}{V} = \frac{G_S + e}{1+e}\rho_W}$$

$$\boxed{\rho_d = \frac{M_S}{V} = \frac{G_S}{1+e}\rho_W}$$



Unit Weights

Several unit weights are used in Soil Mechanics. These are the bulk, saturated, dry, and submerged unit weights.

The bulk unit weight is simply defined as the weight per unit volume

$$\boxed{\gamma_{bulk} = \frac{W}{V}}$$

When all the voids are filled with water the bulk unit weight is identical to the saturated unit weight, γ_{sat}, and when all the voids are filled with air the bulk unit weight is identical with the dry unit weight, γ_{dry}.

$$\gamma_{bulk} = \frac{W}{V} = \frac{\gamma_w G_s + \gamma_w e S}{1+e} = \frac{\gamma_w (G_s + eS)}{1+e}$$

$$\gamma_{sat} = \frac{\gamma_w (G_s + e)}{1+e} \qquad S = 1$$

$$\gamma_{dry} = \frac{\gamma_w G_s}{1+e} \qquad S = 0$$

Important Notes

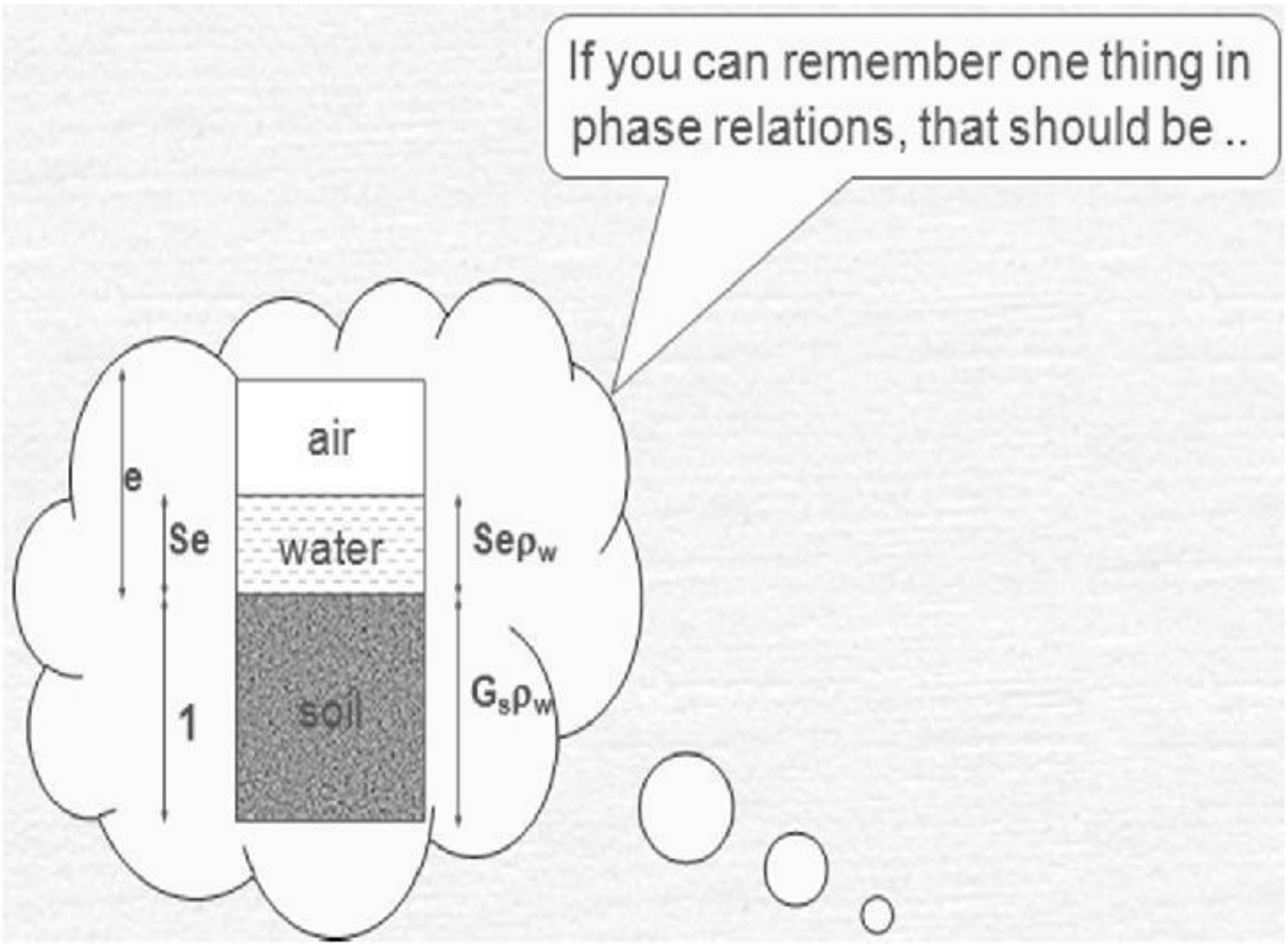
- Try not to *memorise* the equations. ***Understand*** the definitions, and develop the relations from the phase diagram with $V_S = 1$;

- Assume G_S (2.6-2.8) when not given;

- Do not mix densities and unit weights;

- Soil grains are incompressible. Their mass and volume remain the same at any void ratio.

A Suggestion..



Relationship among *unit weight, porosity, and moisture content:*

Consider a soil with a total volume equal to one V=1

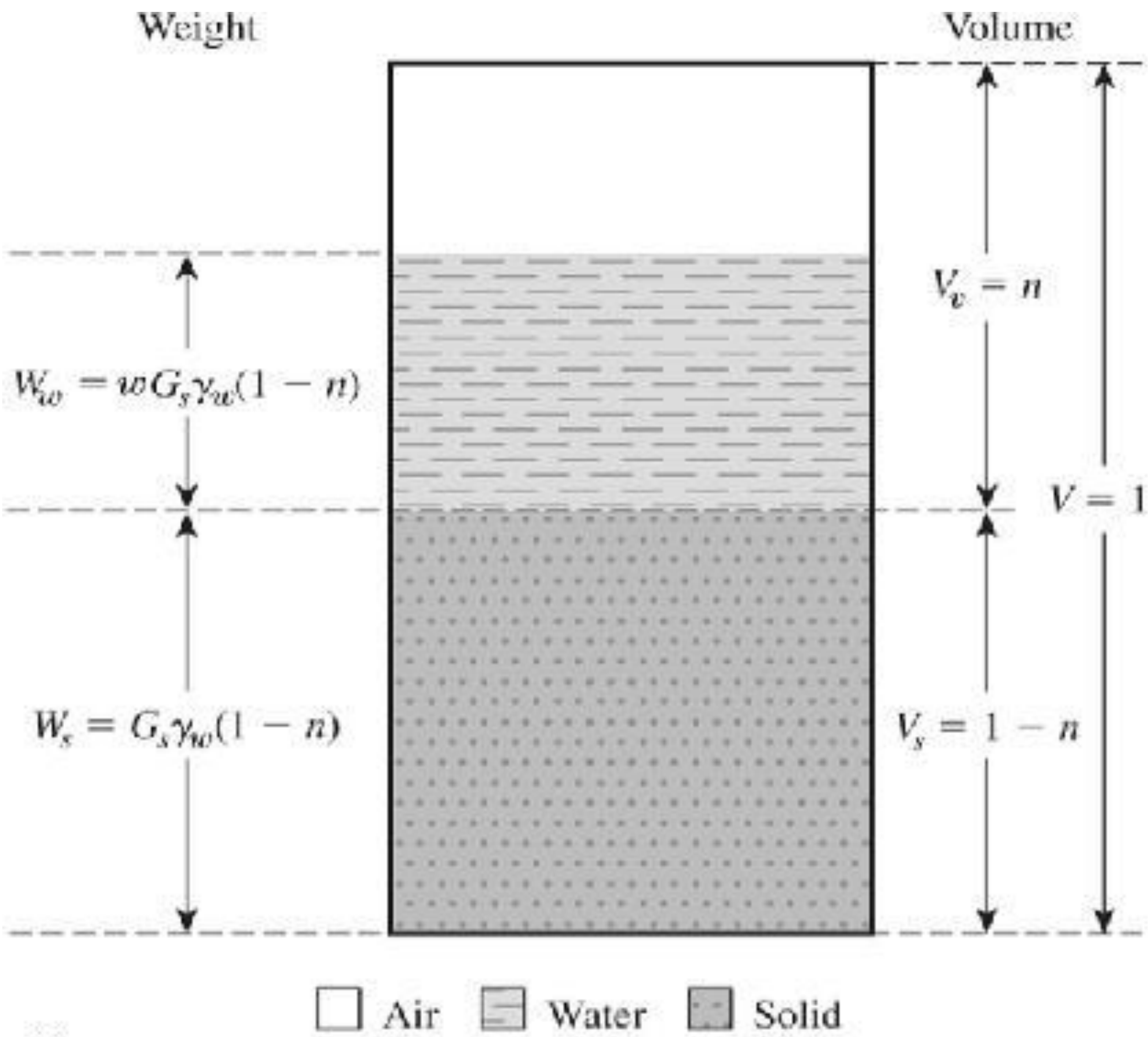


Figure 3.5
Soil element with total volume equal to one

$$n = \frac{V_V}{V}$$

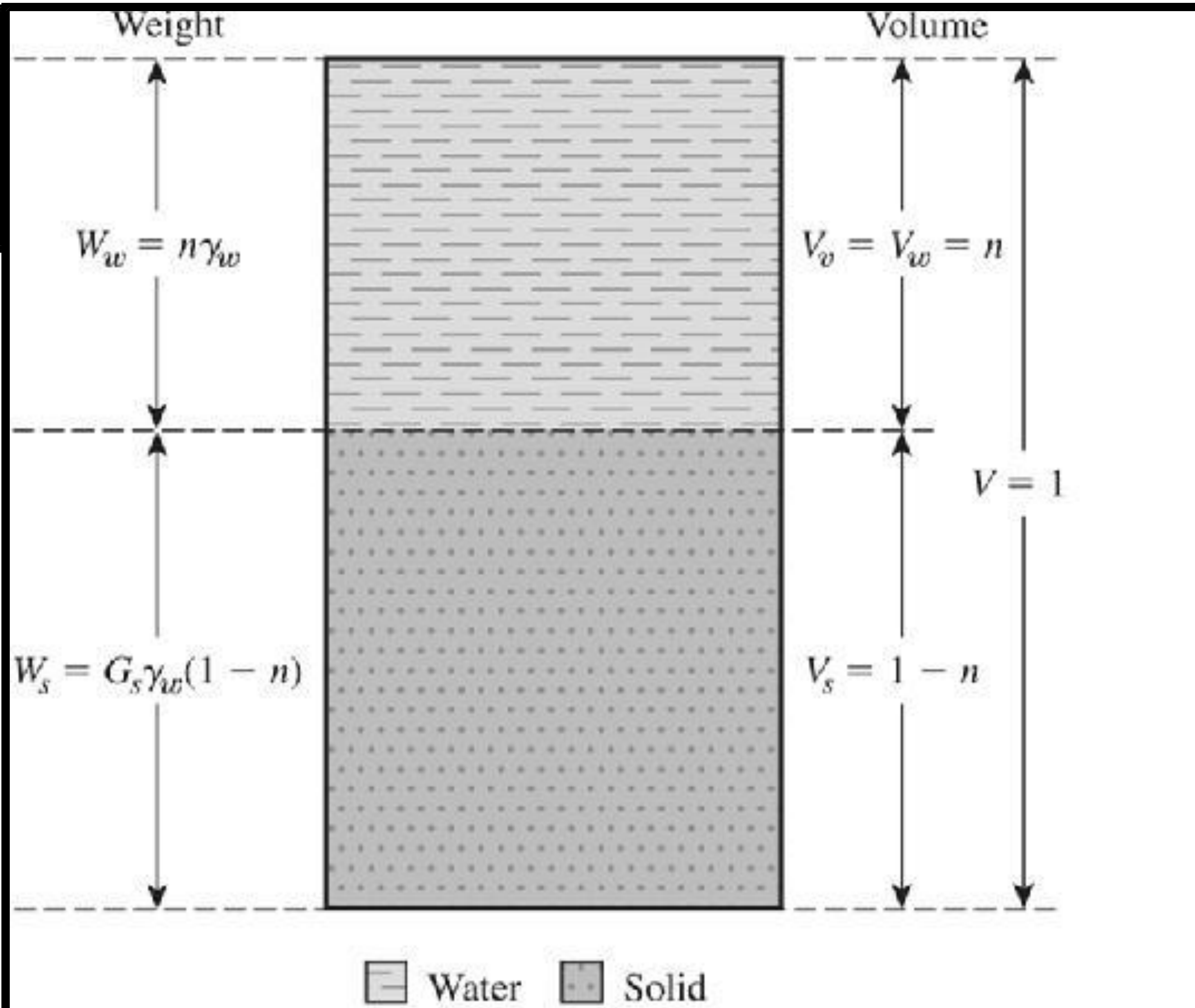


Figure 3.6 Saturated soil element with total volume equal to one

Various Unit-Weight Relationships

Table 3.1 Various Forms of Relationships for γ, γ_d, and γ_{sat}

Moist unit weight (γ)		Dry unit weight (γ_d)		Saturated unit weight (γ_{sat})	
Given	Relationship	Given	Relationship	Given	Relationship
w, G_s, e	$\dfrac{(1+w)G_s\gamma_w}{1+e}$	γ, w	$\dfrac{\gamma}{1+w}$	G_s, e	$\dfrac{(G_s+e)\gamma_w}{1+e}$
S, G_s, e	$\dfrac{(G_s+Se)\gamma_w}{1+e}$	G_s, e	$\dfrac{G_s\gamma_w}{1+e}$	G_s, n	$[(1-n)G_s+n]\gamma_w$
w, G_s, S	$\dfrac{(1+w)G_s\gamma_w}{1+\dfrac{wG_s}{S}}$	G_s, n	$G_s\gamma_w(1-n)$	G_s, w_{sat}	$\left(\dfrac{1+w_{sat}}{1+w_{sat}G_s}\right)G_s\gamma_w$
w, G_s, n	$G_s\gamma_w(1-n)(1+w)$	G_s, w, S	$\dfrac{G_s\gamma_w}{1+\left(\dfrac{wG_s}{S}\right)}$	e, w_{sat}	$\left(\dfrac{e}{w_{sat}}\right)\left(\dfrac{1+w_{sat}}{1+e}\right)\gamma_w$
S, G_s, n	$G_s\gamma_w(1-n)+nS\gamma_w$	e, w, S	$\dfrac{eS\gamma_w}{(1+e)w}$	n, w_{sat}	$n\left(\dfrac{1+w_{sat}}{w_{sat}}\right)\gamma_w$
		γ_{sat}, e	$\gamma_{sat}-\dfrac{e\gamma_w}{1+e}$	γ_d, e	$\gamma_d+\left(\dfrac{e}{1+e}\right)\gamma_w$
		γ_{sat}, n	$\gamma_{sat}-n\gamma_w$	γ_d, n	$\gamma_d+n\gamma_w$
		γ_{sat}, G_s	$\dfrac{(\gamma_{sat}-\gamma_w)G_s}{(G_s-1)}$	γ_d, S	$\left(1-\dfrac{1}{G_s}\right)\gamma_d+\gamma_w$
				γ_d, w_{sat}	$\gamma_d(1+w_{sat})$

Table 3.2 Void Ratio, Moisture Content, and Dry Unit Weight for Some Typical Soils in a Natural State

Type of soil	Void ratio, e	Natural moisture content in a saturated state (%)	Dry unit weight, γ_d (kN/m^3)
Loose uniform sand	0.8	30	14.5
Dense uniform sand	0.45	16	18
Loose angular-grained silty sand	0.65	25	16
Dense angular-grained silty sand	0.4	15	19
Stiff clay	0.6	21	17
Soft clay	0.9–1.4	30–50	11.5–14.5
Loess	0.9	25	13.5
Soft organic clay	2.5–3.2	90–120	6–8
Glacial till	0.3	10	21

Relative density

The term *relative density is commonly used to indicate the in situ denseness or looseness* of granular soil.
It is defined as:

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}}$$

Table 3.3 Qualitative Description of Granular Soil Deposits

Relative density (%)	Description of soil deposit
0–15	Very loose
15–50	Loose
50–70	Medium
70–85	Dense
85–100	Very dense

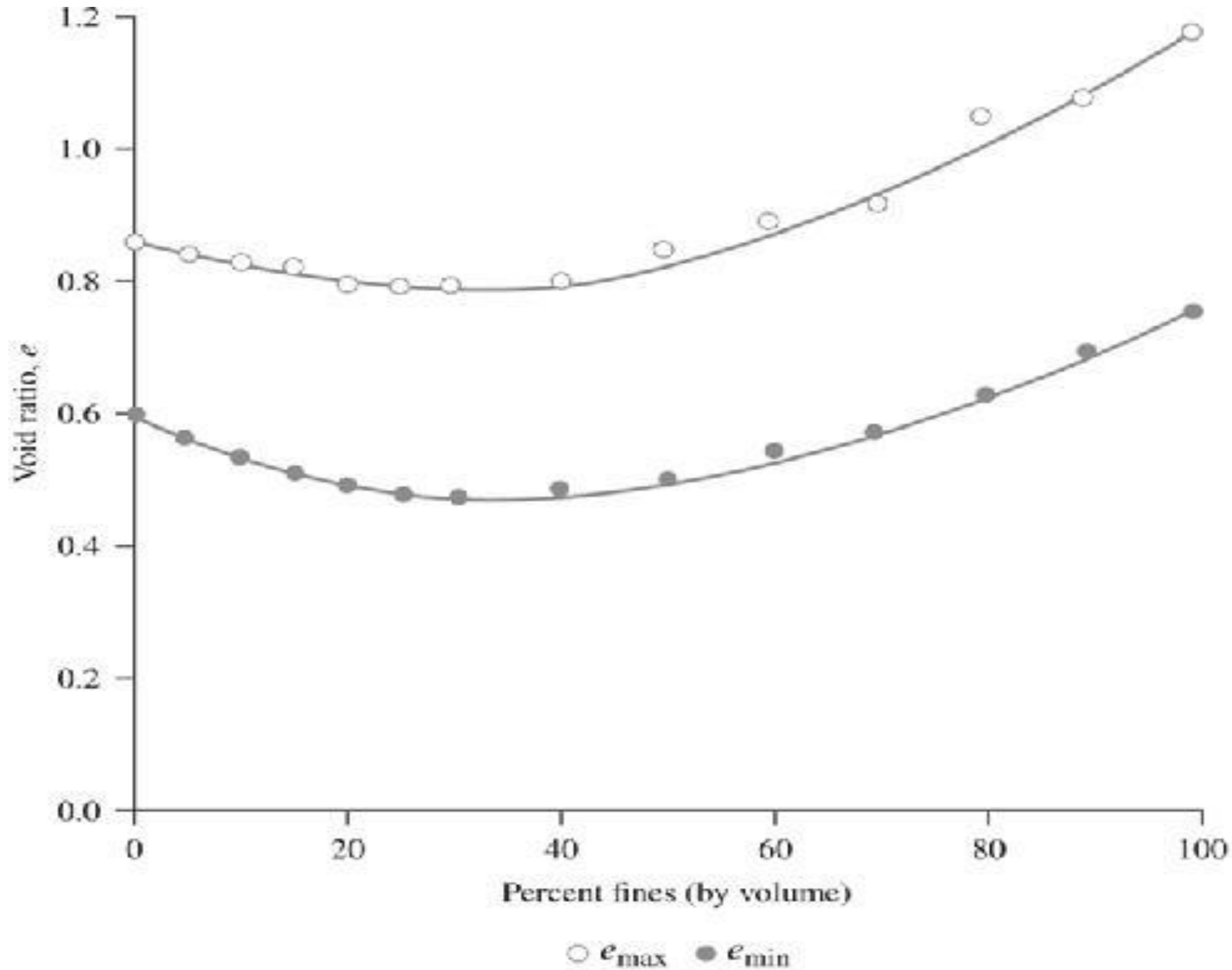


Figure 3.8 Variation of e_{max} and e_{min} (for Nevada 50/80 sand) with percentage of non-plastic fines (*Redrawn from Lade et al, 1998. Copyright ASTM INTERNATIONAL. Reprinted with permission.*)

Phase Relations Examples

- In its natural condition a soil sample has a mass of 2290 g and a volume of 1.15x10-3 m3 . After being completely dried in an oven the mass of the sample is 2035 g. The value of G_s for the soil is 2.68. Determine:
- 1. the bulk density, $=M_t/V_t$, $=2.290$ kg/ ($1.15\times10^{-3}m^3$) $=1991.3kg/ m^3$
- 2. unit weight= density *g = 1991.3*9.806/1000 $=19.527kN/m^3$
- 3. water content, $=M_w/M_s$ = (2290-2035)*100%/2035 =12.53%
- 4. void ratio, $=V_v/V_s$, $Gs=M_s/V_s$,vs=2.035/(2.68*1000) =7.593x10-4m3, e= ($1.15\times10^{-3}m^3$- $7.593\times10^{-4}m^3$)/ 7.593×10^{-4}m3= 0.515

- 5- porosity,n= v_v*100%/v_t = ($1.15x10^{-3}m^3$-$7.593x10^{-4}m^3$)*100%/ ($1.15x10^{-3}m^3$) =**33.97%**
- •6-degree of saturation =V_W/V_V, 1000=M_w/V_w, V_w= (2.290-2.035)/1000 = $2.55x10^{-4}m^3$, S = $2.55x10^{-4}m^3$*100% /($1.15x10^{-3}m^3$-$7.593x10^{-4}m^3$)=65.3%

- 7 -air content. $A=V_a*100\%/V_t$, $V_v=Vw+Va$
- •V_a = (1.15x10-3m3-7.593x10$_{-4}$m^3)-(2.55x10-4m^3)= 1.357x10^{-4}m3
- •A=1.357x10-4m^3 *100%/ 1.15x10^{-3}m^3 =11.8%

EXAMPLE 2

The mass of a moist soil sample having a volume of 0.0057 m^3 is 10.5 kg. The moisture content (w) and the specific gravity of soil solids (G_s) were determined to be 13% and 2.68, respectively. Determine

- **a.** Moist density, ρ (kg/m^3)
- **b.** Dry density, ρ_d (kg/m^3)
- **c.** Void ratio, e
- **d.** Porosity, n
- **e.** Degree of saturation, S (%)

Solution

a. From Eq. (3.13),

$$\rho = \frac{M}{V} = \frac{10.5}{0.0057} = \mathbf{1842\ kg/m^3}$$

b. From Eqs. (3.21) and (3.22),

$$\rho_d = \frac{\rho}{1 + w} = \frac{1842}{1 + \frac{13}{\ }} = \mathbf{1630\ kg/m^3}$$

c. From Eq. (3.22),

$$e = \frac{G_s \gamma_w}{\rho_d} - 1 = \frac{(2.68)(1000)}{1630} - 1 = \mathbf{0.64}$$

d. From Eq. (3.7),

$$n = \frac{e}{1 + e} = \frac{0.64}{1 + 0.64} = \mathbf{0.39}$$

e. From Eq. (3.18),

$$S(\%) = \frac{wG_s}{e} \times 100 = \frac{(0.13)(2.68)}{0.64} \times 100 = \mathbf{54.4\%}$$

EXAMPLE 3

The saturated unit weight, γ_{sat}, of a soil is 19.5 kN/m^3, and the specific gravity of soil solids is 2.65.

a. Derive an expression for γ_d in terms of γ_{sat}, γ_w, and G_s.
b. Using the expression derived in part (a), determine the dry unit weight of the soil.

Solution

a. From Eq. (3.19),

$$\gamma_{sat} = \frac{G_s\gamma_w + e\gamma_w}{1+e}$$

$$\gamma_{sat} - \gamma_w = \frac{G_s\gamma_w + e\gamma_w}{1+e} - \gamma_w = \frac{G_s\gamma_w + e\gamma_w - \gamma_w - e\gamma_w}{1+e} = \frac{\gamma_w(G_s - 1)}{1+e}$$

$$\gamma_{sat} - \gamma_w = \frac{\gamma_w(G_s - 1)G_s}{(1+e)G_s} = \frac{\gamma_d(G_s - 1)}{G_s}$$

or

$$\gamma_d = \frac{(\gamma_{sat} - \gamma_w)G_s}{G_s - 1}$$

b. Given that $\gamma_{sat} = 19.5$ kN/m^3 and $G_s = 2.65$,

$$\gamma_d = \frac{(\gamma_{sat} - \gamma_w)G_s}{G_s - 1} = \frac{(19.5 - 9.81)(2.65)}{2.65 - 1} = 15.56 \text{ kN/m}^3$$ ∎

EXAMPLE 4

In its natural state, a moist soil has a volume of 0.33 ft³ and weighs 39.93 lb. The oven-dried weight of the soil is 34.54 lb. If $G_s = 2.67$, calculate

a. Moisture content (%)
b. Moist unit weight (lb/ft³)
c. Dry unit weight (lb/ft³)
d. Void ratio
e. Porosity
f. Degree of saturation (%)

Solution

a. From Eq. (3.8),

$$w = \frac{W_w}{W_s} = \frac{39.93 - 34.54}{34.54}(100) = \mathbf{15.6\%}$$

b. From Eq. (3.9),

$$\gamma = \frac{W}{V} = \frac{39.93}{0.33} = \mathbf{121\ lb/ft^3}$$

c. From Eq. (3.11),

$$\gamma_d = \frac{W_s}{V} = \frac{34.54}{0.33} = \mathbf{104.7\ lb/ft^3}$$

d. The volume of solids is

$$V_s = \frac{W_s}{G_s\gamma_w} = \frac{34.54}{(2.67)(62.4)} = 0.207\ \text{ft}^3$$

Thus,

$$V_v = V - V_s = 0.33 - 0.207 = 0.123\ \text{ft}^3$$

The volume of water is

$$V_w = \frac{W_w}{\gamma_w} = \frac{39.93 - 34.54}{62.4} = 0.086\ \text{ft}^3$$

Now, refer to Figure 3.7. From Eq. (3.3),

$$e = \frac{V_v}{V_s} = \frac{0.123}{0.207} = \mathbf{0.59}$$

EXAMPLE 4

$$n = \frac{V_v}{V} = \frac{0.123}{0.33} = \mathbf{0.37}$$

$$S = \frac{V_w}{V_v} = \frac{0.086}{0.123} = 0.699 = \mathbf{69.9\%}$$

EXAMPLE 5

For a saturated soil, given $w = 40\%$ and $G_s = 2.71$, determine the saturated and dry unit weights in lb/ft^3 and kN/m^3.

Solution

For saturated soil, from Eq. (3.20),

$$e = wG_s = (0.4)(2.71) = 1.084$$

From Eq. (3.19),

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1 + e} = \frac{(2.71 + 1.084)62.4}{1 + 1.084} = \mathbf{113.6\ lb/ft^3}$$

Also,

$$\gamma_{sat} = (113.6)\left(\frac{9.81}{62.4}\right) = \mathbf{17.86\ kN/m^3}$$

From Eq. (3.16),

$$\gamma_d = \frac{G_s\gamma_w}{1 + e} = \frac{(2.71)(62.4)}{1 + 1.084} = \mathbf{81.1\ lb/ft^3}$$

Also,

$$\gamma_d = (81.1)\left(\frac{9.81}{62.4}\right) = \mathbf{12.75\ kN/m^3}$$

EXAMPLE 6

The mass of a moist soil sample collected from the field is 465 grams, and its oven dry mass is 405.76 grams. The specific gravity of the soil solids was determined in the laboratory to be 2.68. If the void ratio of the soil in the natural state is 0.83, find the following:

a. The moist density of the soil in the field (kg/m^3)
b. The dry density of the soil in the field (kg/m^3)
c. The mass of water, in kilograms, to be added per cubic meter of soil in the field for saturation

Solution
Part a

$$w = \frac{M_w}{M_s} = \frac{465 - 405.76}{405.76} = \frac{59.24}{405.76} = 14.6\%$$

From Eq. (3.21),

$$\rho = \frac{G_s\rho_w + wG_s\rho_w}{1 + e} = \frac{G_s\rho_w(1 + w)}{1 + e} = \frac{(2.68)(1000)(1.146)}{1.83}$$

$$= \mathbf{1678.3\ kg/m^3}$$

Part b
From Eq. (3.22),

$$\rho_d = \frac{G_s\rho_w}{1 + e} = \frac{(2.68)(1000)}{1.83} = \mathbf{1468.48\ kg/m^3}$$

Part c
Mass of water to be added $= \rho_{sat} - \rho$
From Eq. (3.23),

$$\rho_{sat} = \frac{G_s\rho_w + e\rho_w}{1 + e} = \frac{\rho_w(G_s + e)}{1 + e} = \frac{(1000)(2.68 + 0.83)}{1.83} = 1918\ kg/m^3$$

So the mass of water to be added = 1918−1678.3 = **239.7 kg/m^3**.

EXAMPLE 7

For a given sandy soil, $e_{max} = 0.82$ and $e_{min} = 0.42$. Let $G_s = 2.66$. In the field, the soil is compacted to a moist density of 1720 kg/m^3 at a moisture content of 9%. Determine the relative density of compaction.

Solution

From Eq. (3.21),

$$\rho = \frac{(1 + w)G_s\rho_w}{1 + e}$$

or

$$e = \frac{G_s\rho_w(1 + w)}{\rho} - 1 = \frac{(2.66)(1000)(1 + 0.09)}{1720} - 1 = 0.686$$

From Eq. (3.30),

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}} = \frac{0.82 - 0.686}{0.82 - 0.42} = \mathbf{0.335 = 33.5\%}$$

PLASTICITY AND STRUCTURE OF SOIL

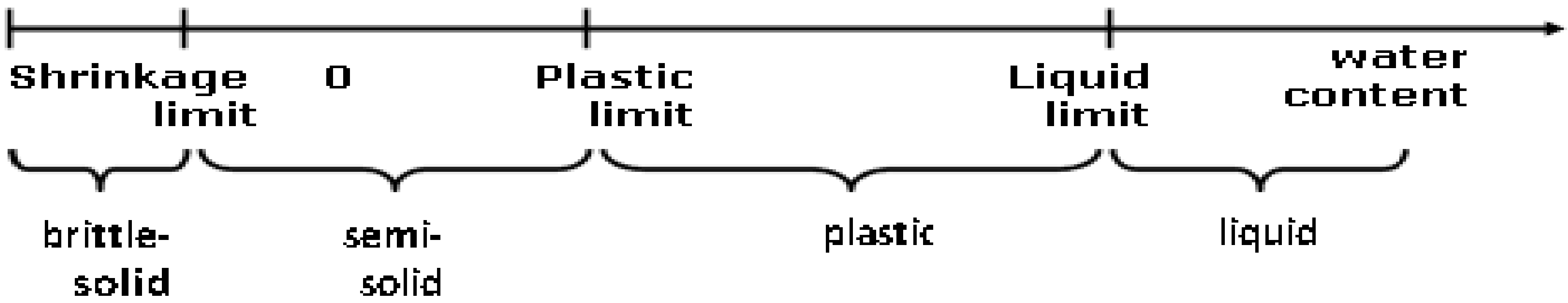
- **Plasticity:**

The ability of a soil to undergo unrecoverable deformation at a constant volume without cracking or crumbling of the soil, due to the presences of clay minerals or organic material.

Consistency: the physical state of a fine-grained soil at a particular water content.

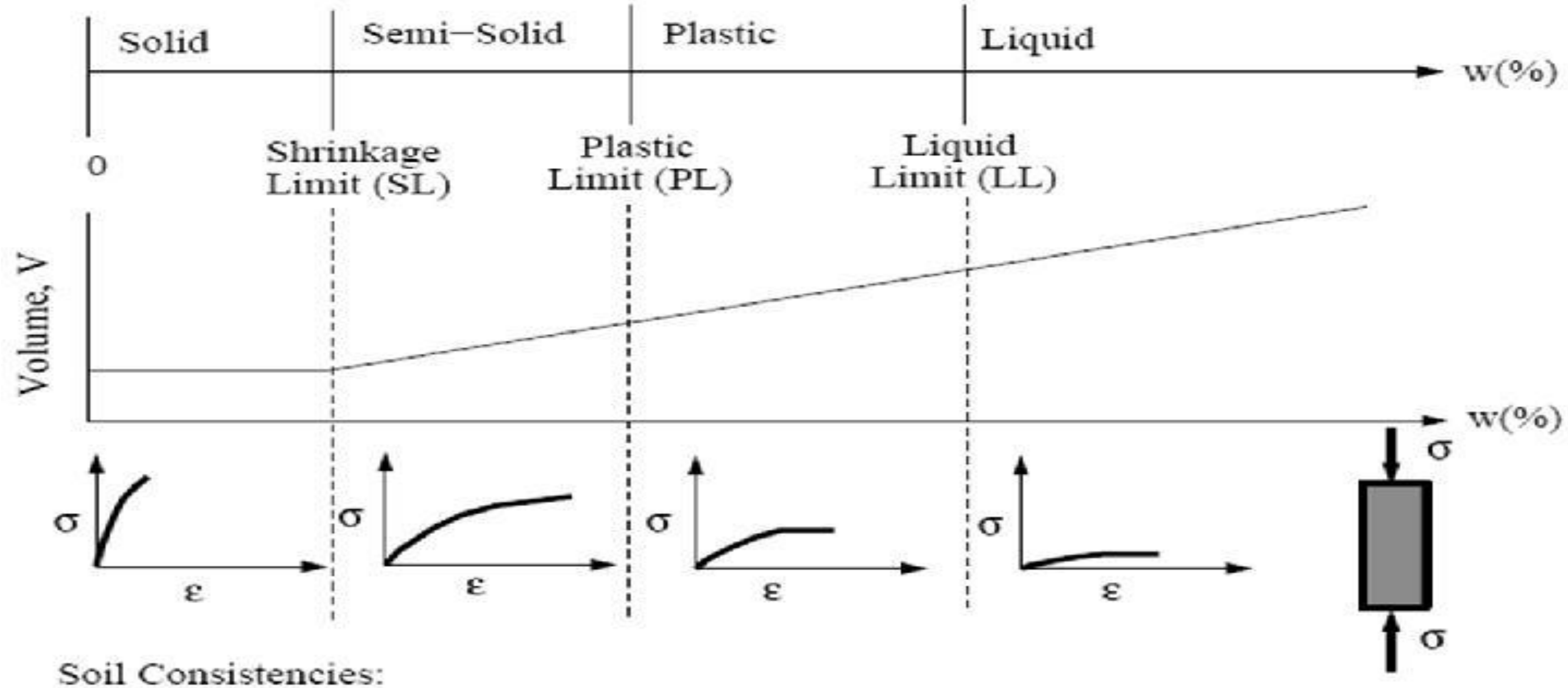
Atterberg Limits

- Border line **water contents**, separating the different **states** of a fine grained soil



Atterberg

- **Albert Atterberg was a Swedish chemist and agricultural scientist.**
- **Conducted studies to identify the specific minerals that give a clayey soil its plastic nature**
- **In each state the consistency and behavior of a soil is different and thus so are its engineering properties.**
- **The boundary between each state can be defined based on a change in the soil's behavior.**



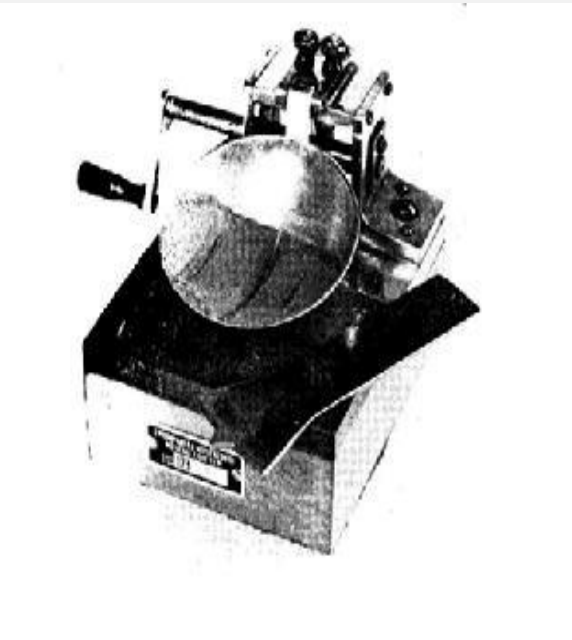
Soil Consistencies:

Solid: soil is hard and brittle
Semi–Solid: soil has combined brittle/ductile behavior (like stiff cheese)
Plastic: soil has very ductile, malleable behavior
Liquid: soil behaves like a thick or thin viscous fluid

Liquid limit

•The liquid limit (LL) is the water content where a soil changes from liquid to plastic behavior

•Determined using a Casagrande cup (lab) or cone penetrometer (field)

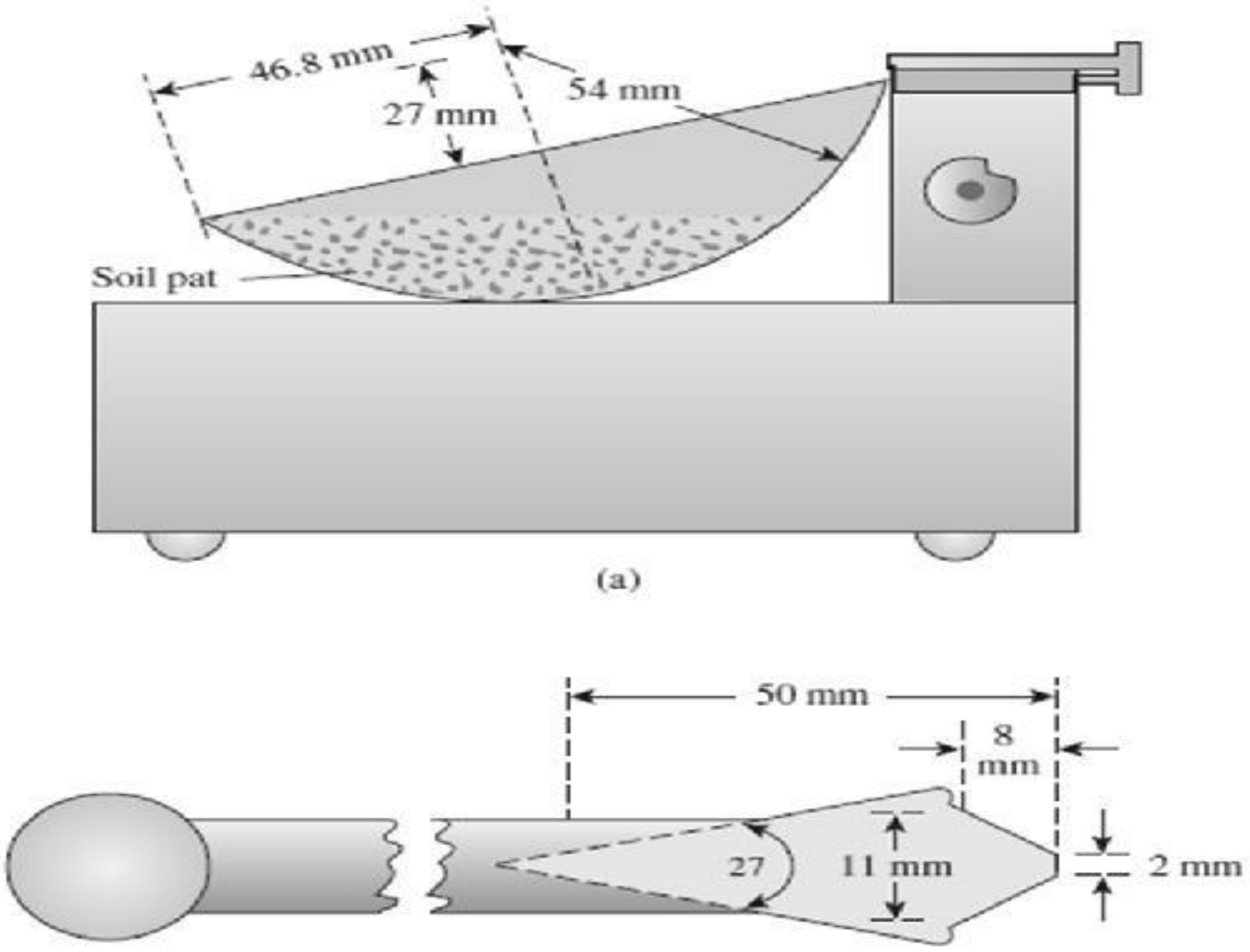


- Soil Classification
- Seepage through earth structures
- Shear Strength

Best Teacher in The Harvard University

ARTHUR CASAGRANDE

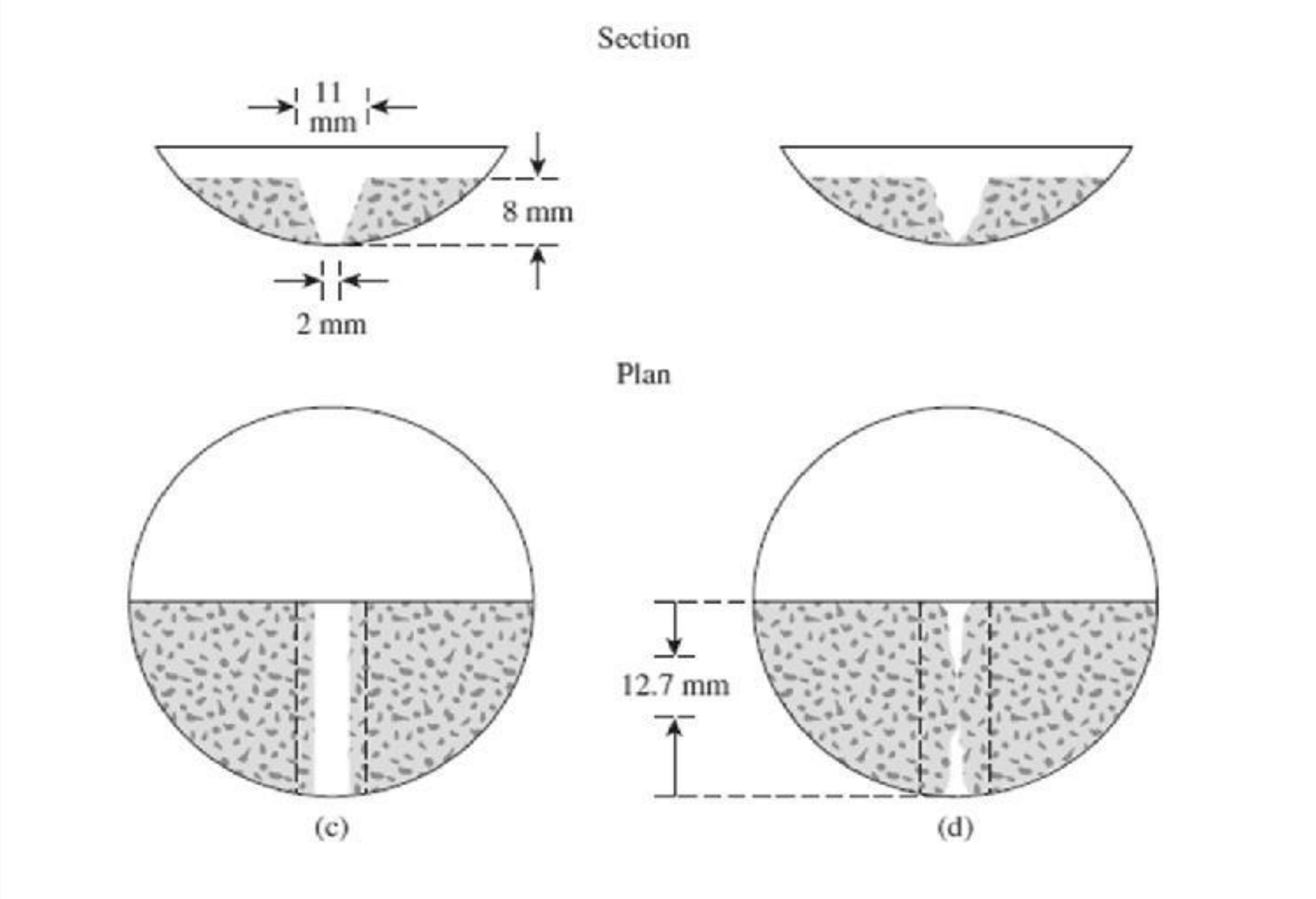
(1902 - 1981)



(a)

(b)

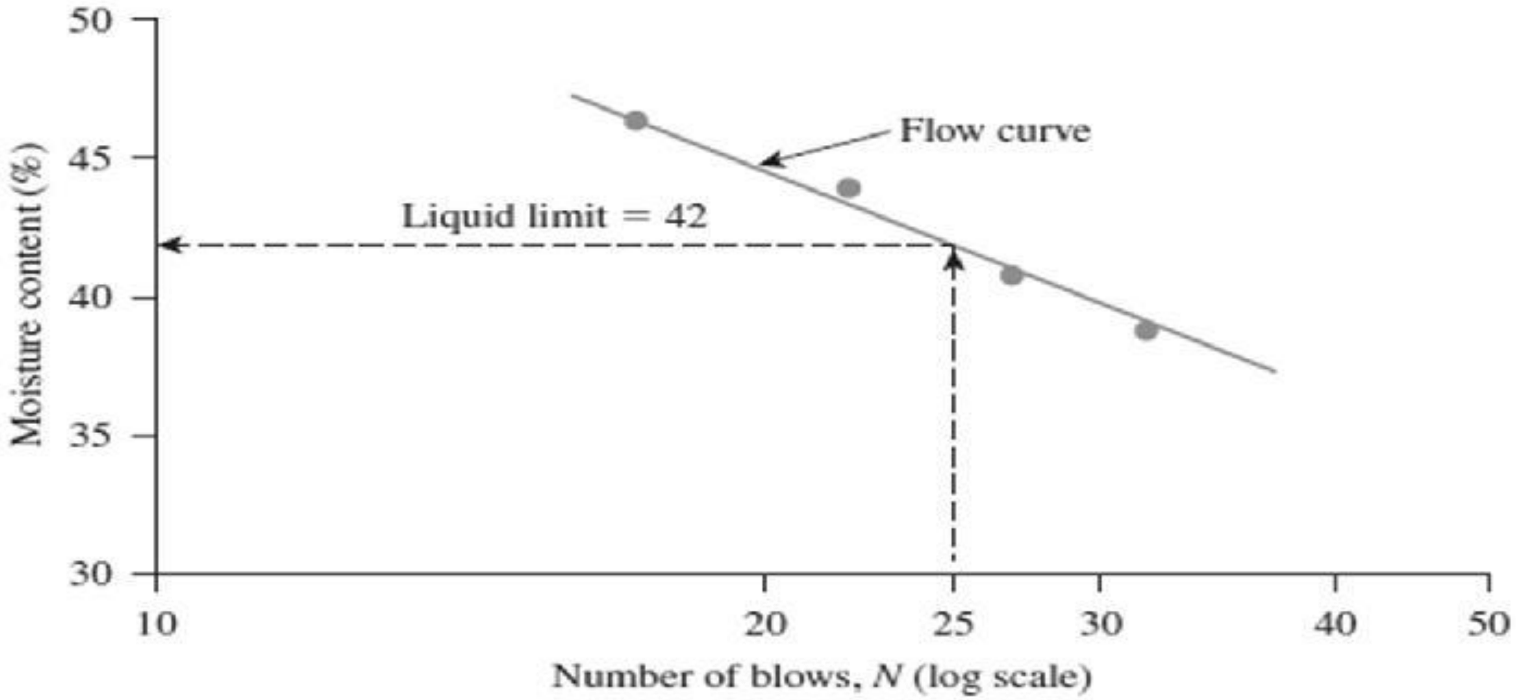
The moisture content, in percent, required to close a distance of 12.7 mm (0.5 in.) along the bottom of the groove (see Figures c and d (after 25 blows is defined as the *liquid limit.)*



liquid limit (LL) is defined as:

The moisture content, in percent, required to close a distance of *12.7 mm (0.5 in.)* along the bottom of the groove after 25 blows.

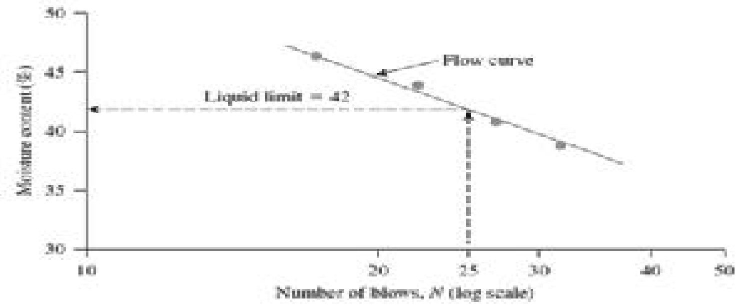
- ***#of blows should be within 35-15blows*.**



flow index $\longrightarrow$ $I_F = \dfrac{w_1 - w_2}{\log\left(\dfrac{N_2}{N_1}\right)}$

flow line **can be written in a general form as:**

$$w = -I_F \log N + C$$



the U.S. Army Corps of Engineers proposed an empirical equation for estimating the LL:

$$LL = w_N \left(\frac{N}{25} \right)^{\tan \beta}$$

** Note: good results for N between ((30-20blows.*

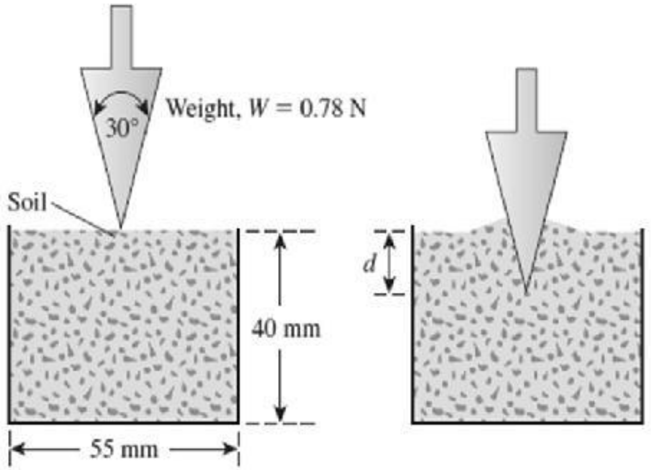
$$\tan \beta = 0.121$$

** Note: diff. β for diff. soil types*

Fall Cone method (British Standard—BS.(1377

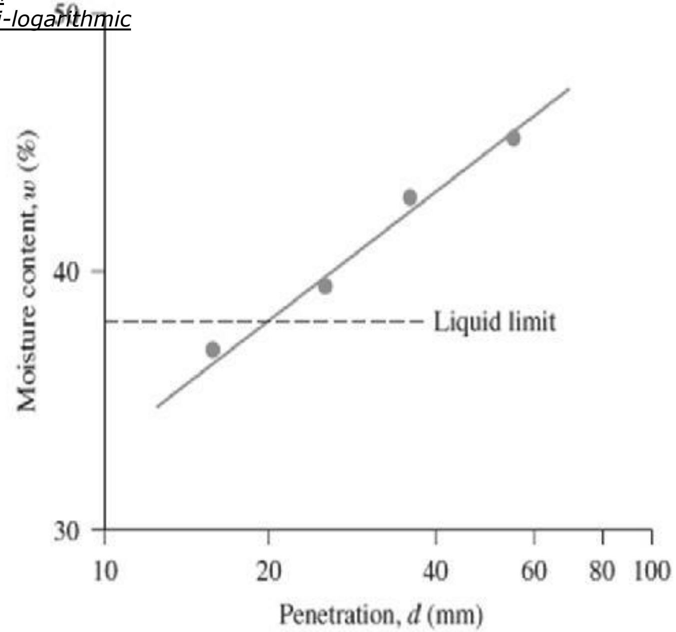
LL= moisture content at which a standard cone of apex angle 30 and weight of 0.78 N will penetrate a distance *d = 20 mm in 5 seconds when allowed to drop* from a position of point contact with the soil surface.

14



Note:

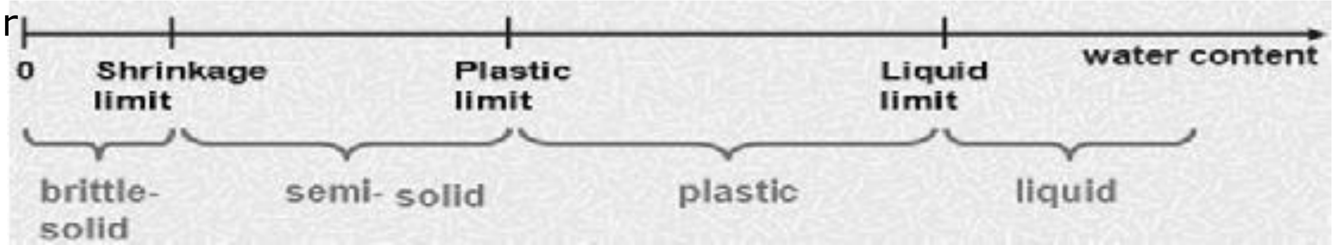
A semi-logarithmic



$$\textit{flow index} \longrightarrow \quad I_{FC} = \frac{w_2(\%) - w_1(\%)}{\log d_2 - \log d_1}$$

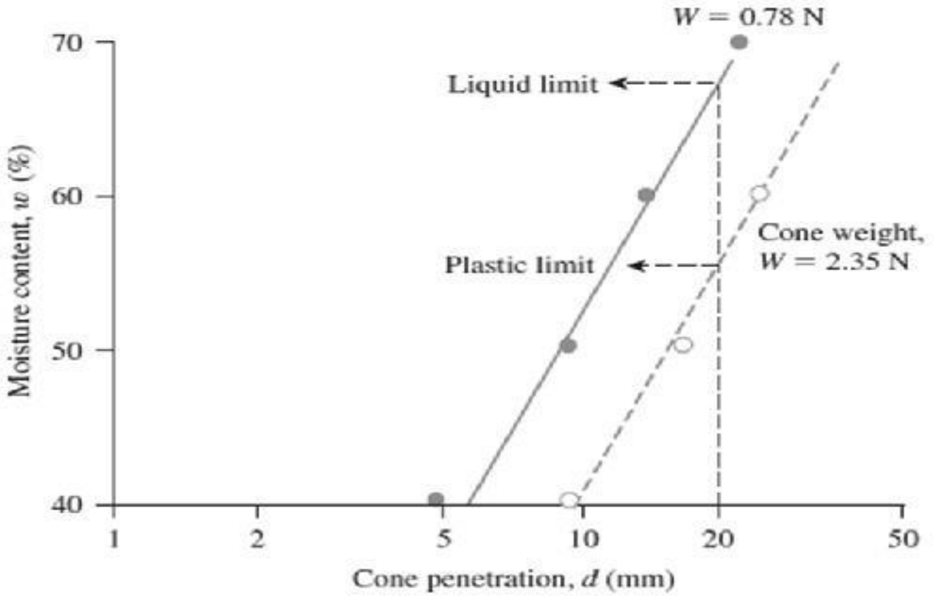
Plastic limit

•The plastic limit (PL) is the water content (w%) where soil starts to exhibit plastic behavior.
•The moisture content in percent, at which the soil crumbles, when rolled into threads of 4.2 mm (in.) in diameter



Fall Cone method (British Standard.(

PL= moisture content at which a standard cone of apex angle 30 and weight of 2.35 N will penetrate a distance *d =20 mm*.



Mineral	Liquid limit, *LL*	Plastic limit, *PL*
Kaolinite	35–100	20–40
Illite	60–120	35–60
Montmorillonite	100–900	50–100
Halloysite (hydrated)	50–70	40–60
Halloysite (dehydrated)	40–55	30–45
Attapulgite	150–250	100–125
Allophane	200–250	120–150

Shrinkage limit

- The shrinkage limit (SL) is the water content where further loss of moisture will not result in any more volume reduction

- The shrinkage limit is much less the liquid limit commonly used than and the plastic limit.

Plasticity Index

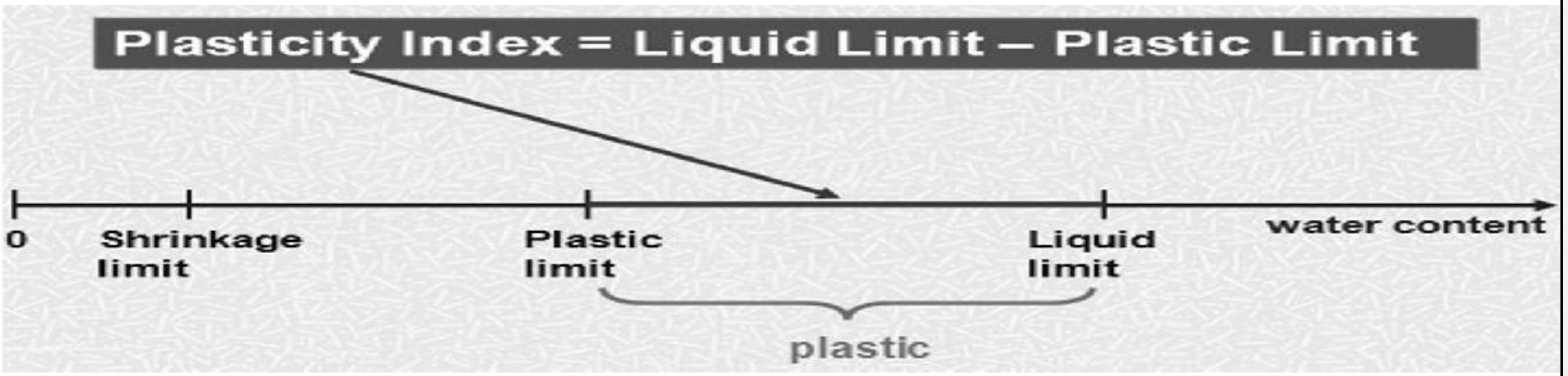
- The PI is the difference between the liquid limit and the plastic limit (PI = LL-PL)

$$PI = LL - PL$$

- The plasticity index is Important in classifying fine-grained soils.
- **Comments**:
  - High PI tend to be clay
  - Low PI tend to be silt
  - PI of 0 tend to have little or no silt or clay.

Plasticity Index (PI)

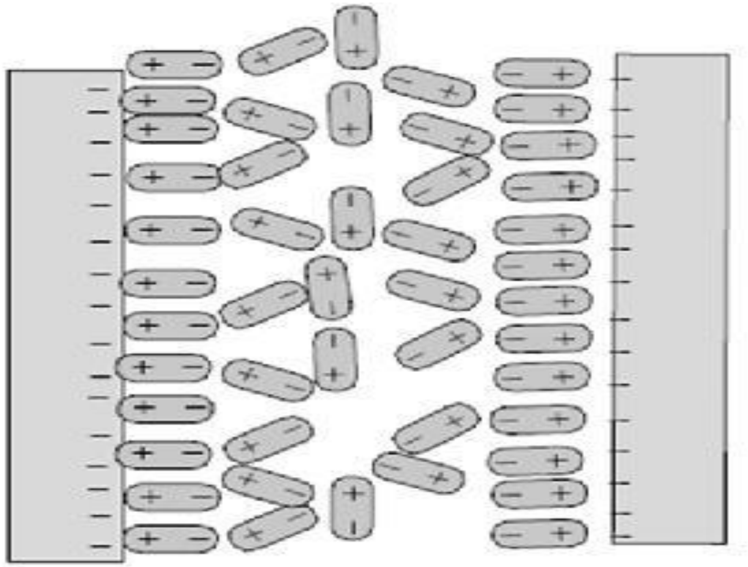
Range of water content over which the soil remains plastic



Plasticity Index (PI(

<Clay soils with high SSA's and charged particles will be able to hold a large amount of water between grains due to their charge field and the polar nature of water molecules.

Clay soils with high SSA's and charged surfaces are able to bind/assimilate water molecules and the overall soil will still behave as a plastic solid. Such soils will have high plasticity indexes.



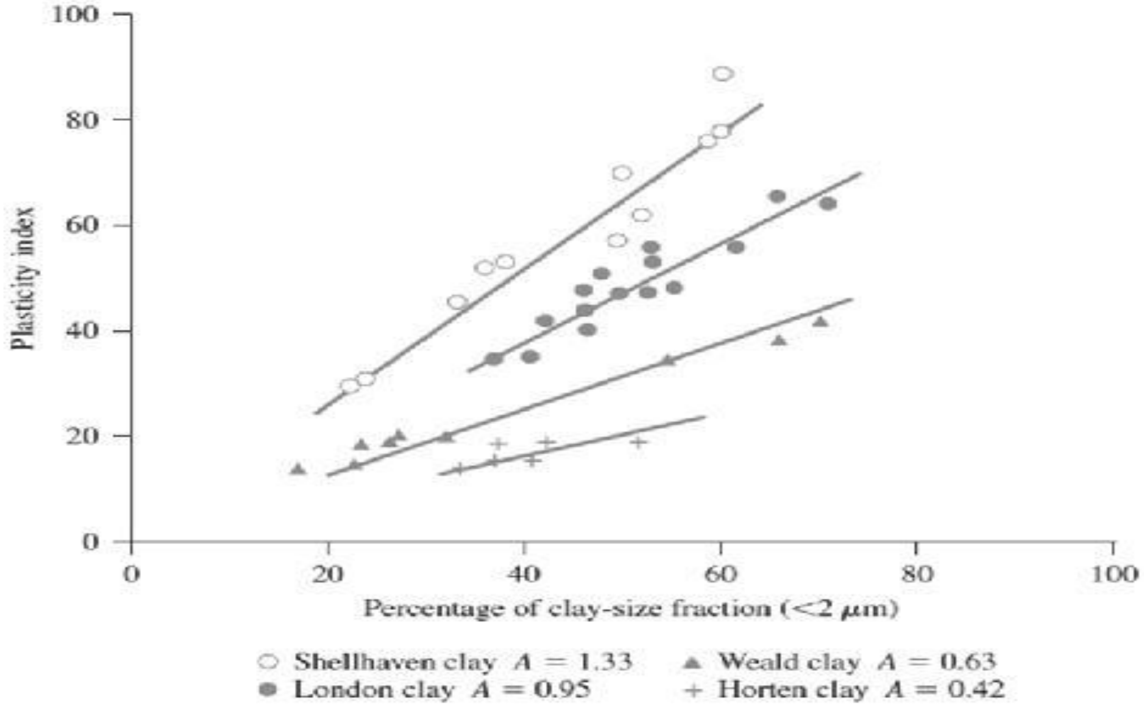
Soils with comparatively lower SSA's will not be able to bind/assimilate water molecules and thus will have much smaller PI values.

PI	Description
0	Nonplastic
1–5	Slightly plastic
5–10	Low plasticity
10–20	Medium plasticity
20–40	High plasticity
>40	Very high plasticity

Activity

The PI of a soil is a measure of the ***activity A*** *of the soil grains*.

Skempton observation:



Based on the results, Skempton defined the ***activity*** *as the* ***slope*** *of the line correlating* ***PI*** *and* ***ht renif %an 2 mm****.*

$$A = \frac{PI}{(\%\text{of clay-size fraction, by weight})}$$

$$A = \frac{PI}{\%\text{of clay-size fraction} - C'}$$

25

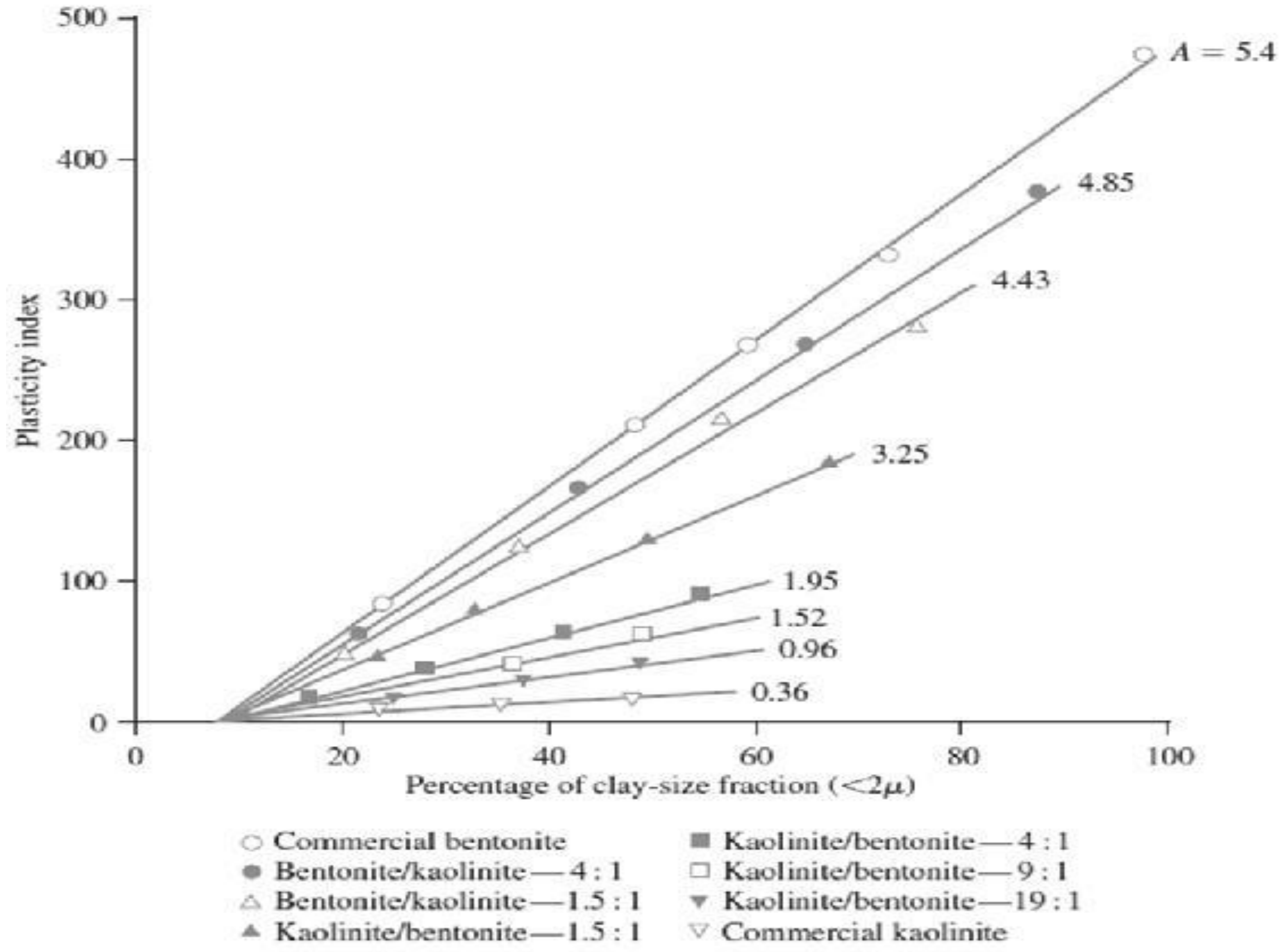
Note C′ ~ 9

- *Fundamentals of effective stress*
- *Pore pressures in clays*
- *Bearing capacity*
- *Slope stability*

Best Teacher in The Imperial College in The University of London

ALEC WESTLEY SKEMPTON

(1914 – 2001)



Plasticity index
Percentage of clay-size fraction (<2μ)
A = 5.4
4.85
4.43
3.25
1.95
1.52
0.96
0.36
Commercial bentonite
Bentonite/kaolinite — 4 : 1
Bentonite/kaolinite — 1.5 : 1
Kaolinite/bentonite — 1.5 : 1
Kaolinite/bentonite — 4 : 1
Kaolinite/bentonite — 9 : 1
Kaolinite/bentonite — 19 : 1
Commercial kaolinite

The ***activity A*** of a fine−grained soil can be useful in identifying the type of clay contained in a soil.

For example:

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Mineral	Activity, *A*
Kaolinite	0.3–0.5
Illite	0.5–1.2
Montmorillonite	1.5–7.0
Halloysite (hydrated)	0.1–0.2
Halloysite (dehydrated)	0.4–0.6
Attapulgite	0.4–1.3
Allophane	0.4–1.3

Liquidity Index and Consistency Index

The relative consistency of a cohesive soil in the natural state can be defined by a ratio called the *liquidity index*

•Liquidity index is a measure of sensitivity…

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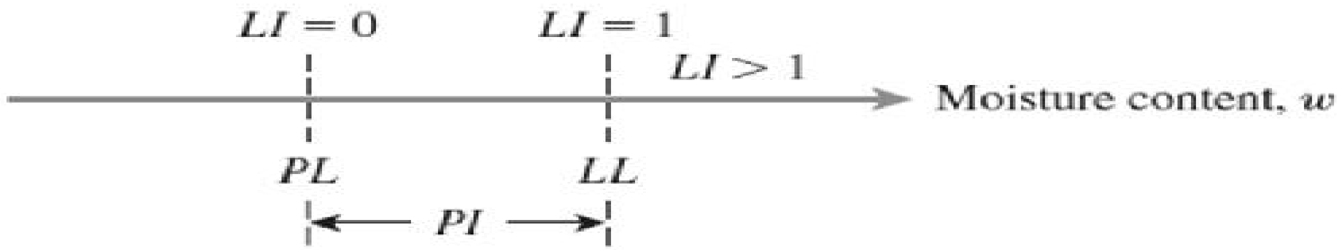
$$LI = \frac{w - PL}{LL - PL}$$

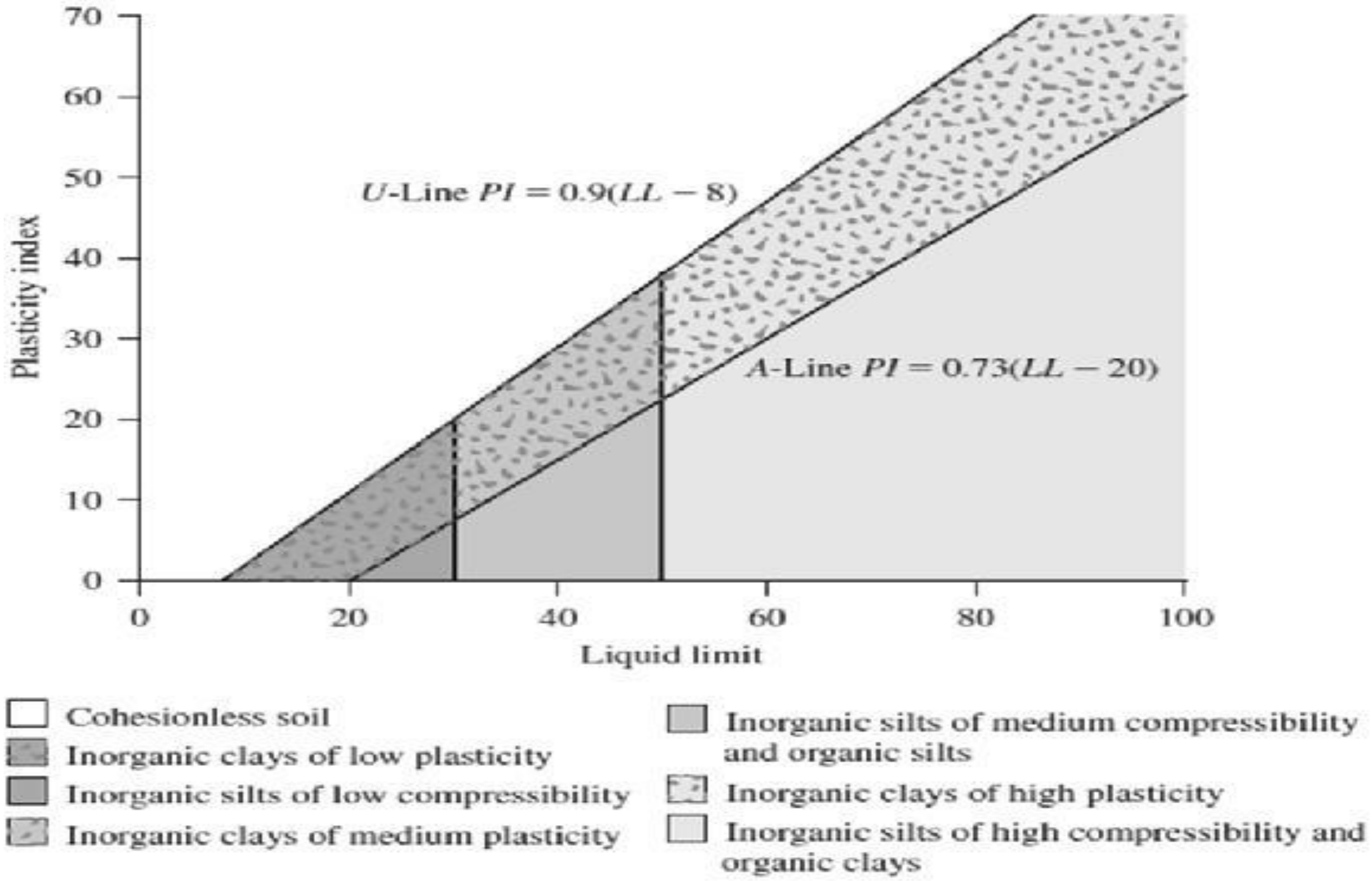


Liquidity Index and Consistency Index

Consistency index (CI), may be defined as:

$$CI = \frac{LL - w}{LL - PI}$$

Plasticity Chart



INDEX PROPERTIES AND SOIL CLASSIFICATION

Table 2.3 Particle-Size Classifications

Name of organization	Grain size (mm)			
	Gravel	**Sand**	**Silt**	**Clay**
Massachusetts Institute of Technology (MIT)	>2	2 to 0.06	0.06 to 0.002	<0.002
U.S. Department of Agriculture (USDA)	>2	2 to 0.05	0.05 to 0.002	<0.002
American Association of State Highway and Transportation Officials (AASHTO)	76.2 to 2	2 to 0.075	0.075 to 0.002	<0.002
Unified Soil Classification System (U.S. Army Corps of Engineers, U.S. Bureau of Reclamation, and American Society for Testing and Materials)	76.2 to 4.75	4.75 to 0.075	Fines (i.e., silts and clays) <0.075	

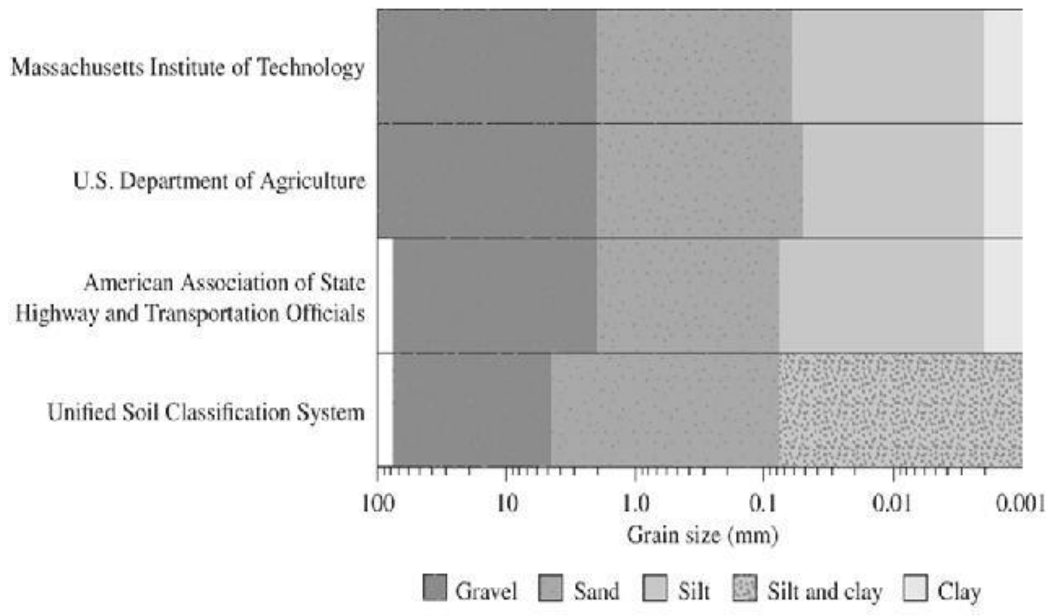


Figure 2.7 Soil-separate-size limits by various systems

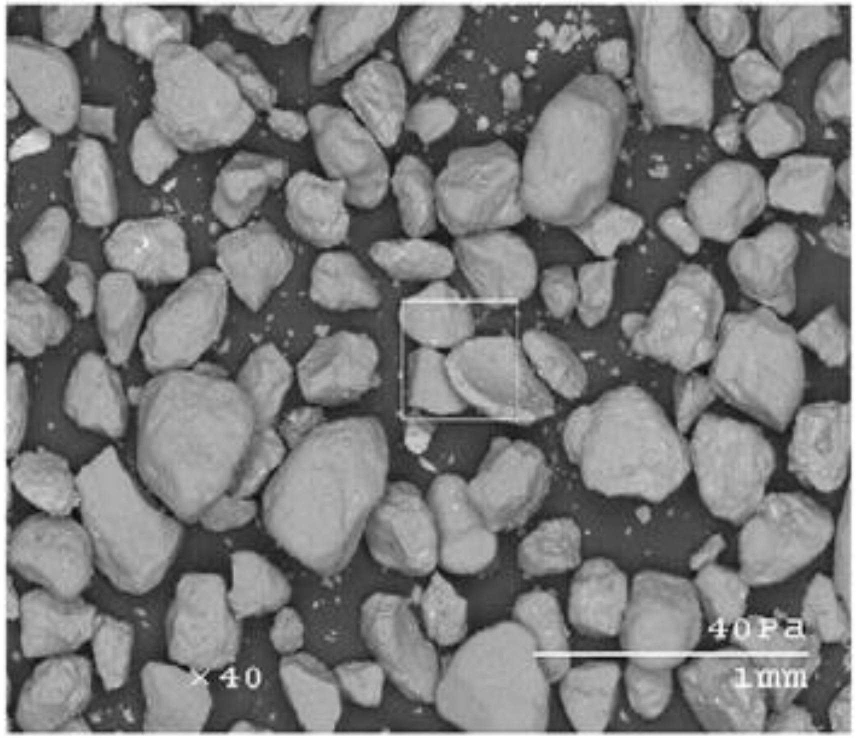


Figure 2.8 Scanning electron micrograph of some sand grains (*Courtesy of David J. White, Iowa State University, Ames, Iowa*)

Specific Gravity

Specific gravity is defined as the ratio of the unit weight of a given material to the unit weight of water.

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$$G_s = \frac{\rho_s}{\rho_w} = \frac{\gamma_s}{\gamma_w}$$

Table 2.4 Specific Gravity of Common Minerals

Mineral	Specific gravity, G_s
Quartz	2.65
Kaolinite	2.6
Illite	2.8
Montmorillonite	2.65–2.80
Halloysite	2.0–2.55
Potassium feldspar	2.57
Sodium and calcium feldspar	2.62–2.76
Chlorite	2.6–2.9
Biotite	2.8–3.2
Muscovite	2.76–3.1
Hornblende	3.0–3.47
Limonite	3.6–4.0
Olivine	3.27–3.7

Soil-Particle Size

Table 2.3 Particle-Size Classifications

Name of organization	Grain size (mm)			
	Gravel	Sand	Silt	Clay
Massachusetts Institute of Technology (MIT)	>2	2 to 0.06	0.06 to 0.002	<0.002
U.S. Department of Agriculture (USDA)	>2	2 to 0.05	0.05 to 0.002	<0.002
American Association of State Highway and Transportation Officials (AASHTO)	76.2 to 2	2 to 0.075	0.075 to 0.002	<0.002
Unified Soil Classification System (U.S. Army Corps of Engineers, U.S. Bureau of Reclamation, and American Society for Testing and Materials)	76.2 to 4.75	4.75 to 0.075	Fines (i.e., silts and clays) <0.075	

Note: Sieve openings of 4.75 mm are found on a U.S. No. 4 sieve; 2-mm openings on a U.S. No. 10 sieve; 0.075-mm openings on a U.S. No. 200 sieve. See Table 2.5.

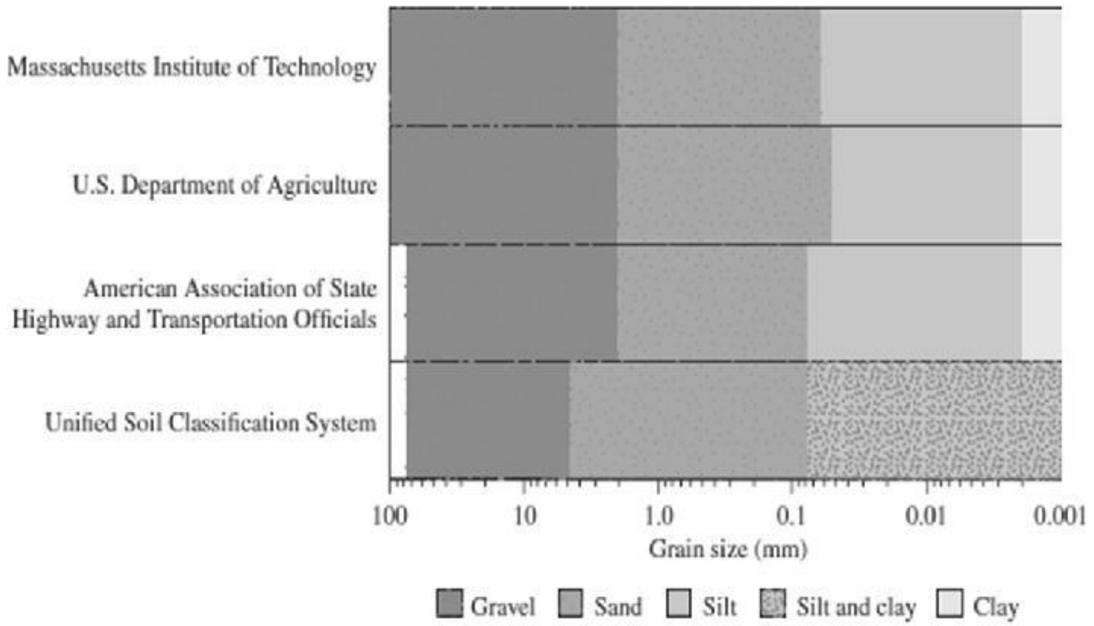

Figure 2.7 Soil-separate-size limits by various systems

•***Coarse-grained soils***

•These include sands, gravels and larger particles.

•For these soils the grains are well defined and may be seen by the naked eye

•The individual particles may vary from perfectly round to highly angular.

• ***Fine-grained soils***

• These include the silts and clays

• Silts : These can be visually differentiated from clays because they exhibit the property of dilatancy. If a moist sample is shaken in the hand water will appear on the surface. If the sample is then squeezed in the fingers the water will disappear.

• Clays : Clays exhibit plasticity, they may be readily remoulded when moist, and if left to dry can attain high strengths

Organic

•These may be of either clay or silt sized particles.

•Contain significant amounts of vegetable matter.

•The soils as a result are usually dark grey or black and have a noticeable odour from decaying matter.

•Generally only a surface phenonomen but layers of peat may be found at depth.

•These are very poor soils for most engineering purposes.

Mechanical Analysis of Soil

Mechanical analysis: is the determination of the size range of particles present in a soil, expressed as a percentage of the total dry weight.

Two methods generally are used to find the particle-size distribution of soil:

- *sieve analysis—for particle sizes larger than 0.075* mm in diameter, and
- *hydrometer analysis—for particle sizes smaller than 0.075 mm in* diameter.

Table 2.5 U.S. Standard Sieve Sizes

Sieve no.	Opening (mm)	Sieve no.	Opening (mm)
4	4.75	35	0.500
5	4.00	40	0.425
6	3.35	50	0.355
7	2.80	60	0.250
8	2.36	70	0.212
10	2.00	80	0.180
12	1.70	100	0.150
14	1.40	120	0.125
16	1.18	140	0.106
18	1.00	170	0.090
20	0.850	200	0.075
25	0.710	270	0.053
30	0.600		

- **Sieving procedure**

- (1) Write down the weight of each sieve as well as the bottom pan to be used in the analysis.
- (2) Record the weight of the given dry soil sample.
- (3) Make sure that all the sieves are clean, and assemble them in the ascending order of sieve numbers (#4 sieve at top and #200 sieve at bottom). Place the pan below #200 sieve. Carefully pour the soil sample into the top sieve and place the cap over it.
- (4) Place the sieve stack in the mechanical shaker and shake for 10 minutes.
- (5) Remove the stack from the shaker and carefully weigh and record the weight of each sieve with its retained soil. In addition, remember to weigh and record the weight of the bottom pan with its retained fine soil.

Data Analysis:

(1) Obtain the mass of soil retained on each sieve by subtracting the weight of the empty sieve from the mass of the sieve + retained soil, and record this mass as the weight retained on the data sheet. The sum of these retained masses should be approximately equals the initial mass of the soil sample. A loss of more than two percent is unsatisfactory.

(2) Calculate the percent retained on each sieve by dividing the weight retained on each sieve by the original sample mass.

(3) Calculate the percent passing (or percent finer) by starting with 100 percent and subtracting the percent retained on each sieve as a cumulative procedure.


Set of Sieves
Soil Specimen
Balance
PAN


PAN


Sieve Shaker
PAN


0.824

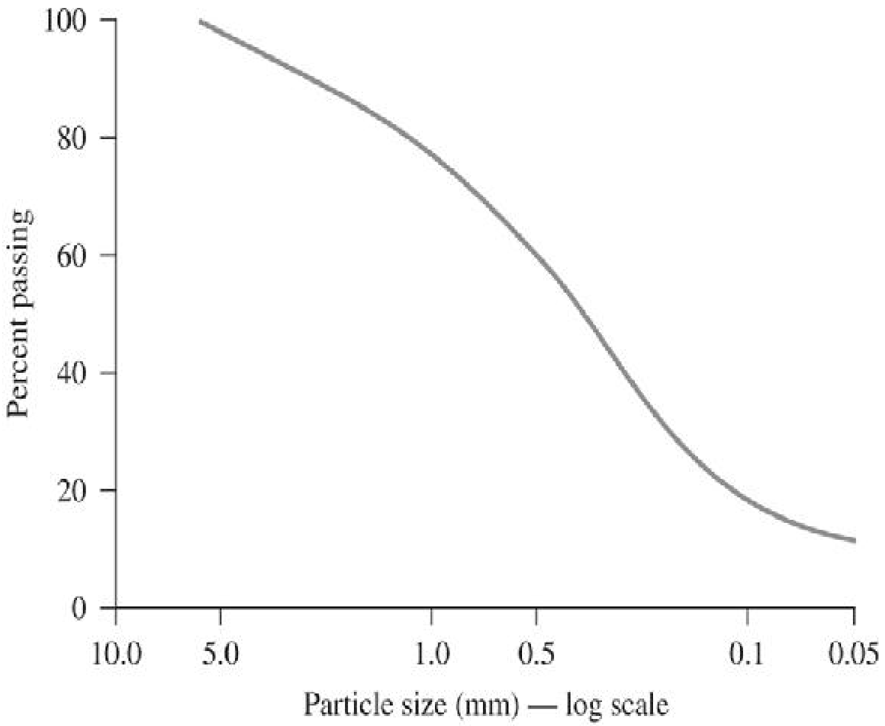

Figure 2.22 Particle-size distribution curve

Hydrometer (Fine)

- To determine the grain size distribution of material passing the 75μm sieve.
- The soil is mixed with water and a dispersing agent and allowed to settle to the bottom of a measuring cylinder.
- As the soil particles settle out of suspension the specific gravity of the mixture reduces. The hydrometer is used to record the variation of specific gravity with time.
- By making use of Stoke's Law, which relates the velocity of a free-falling sphere to its diameter, the test data is reduced to provide particle diameters and the % by weight of the sample finer than a particular particle size.

$$v = \frac{\rho_s - \rho_w}{18\eta} D^2 \Rightarrow D\,(\text{mm}) = K\sqrt{\frac{L\,(\text{cm})}{t\,(\text{min})}} \Rightarrow K = \sqrt{\frac{30\eta}{(G_s - 1)}}$$

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Figure 2.23
ASTM 152H hydrometer
(*Courtesy of ELE International*)

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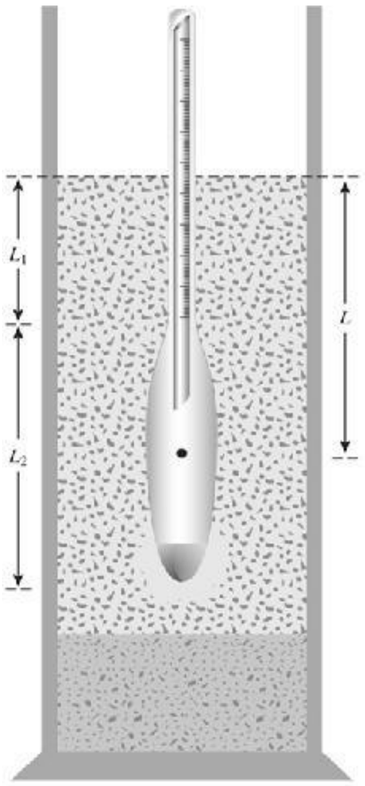

Figure 2.24 Definition of L in hydrometer test

Particle-Size Distribution Curve

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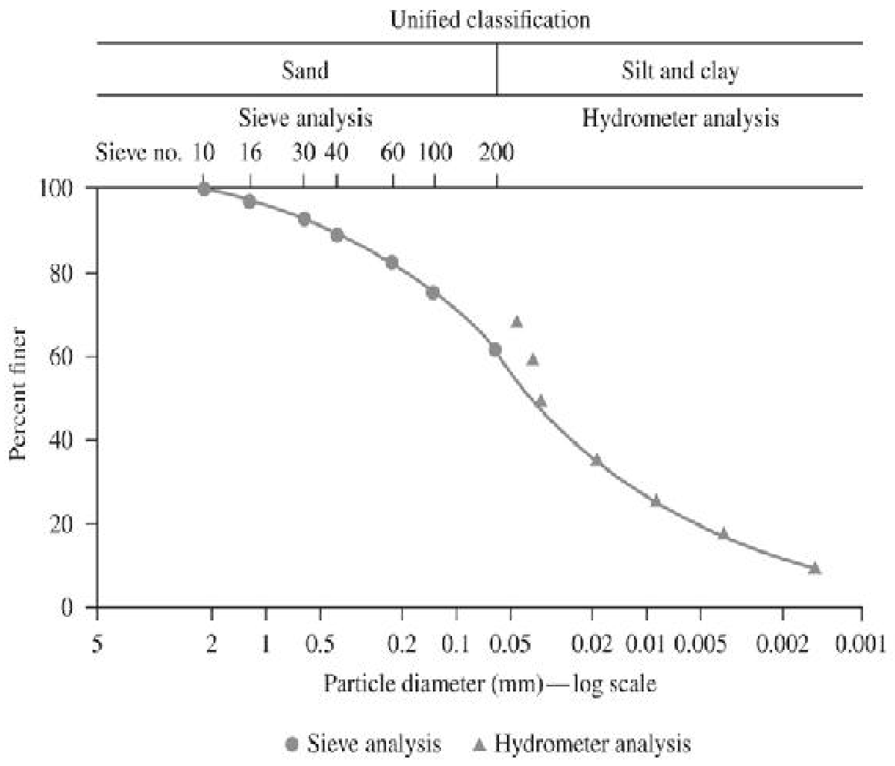

Figure 2.25 Particle-size distribution curve—sieve analysis and hydrometer analysis

A particle-size distribution curve can be used to determine the following four parameters:

•*Effective size (D noitubirtsid ezis-elcitrap eht ni retemaid eht si retemarap sihT :(10* curve corresponding to 10% finer. The effective size of a granular soil is a good measure to estimate the hydraulic conductivity and drainage through soil.

•*Uniformity coefficient: (Cu)*

•*Coefficient of gradation: (Cc)*

•*Sorting coefficient: (So)*

$$C_u = \frac{D_{60}}{D_{10}}$$

$$C_c = \frac{D_{30}^2}{D_{60} \times D_{10}}$$

$$S_0 = \sqrt{\frac{D_{75}}{D_{25}}}$$

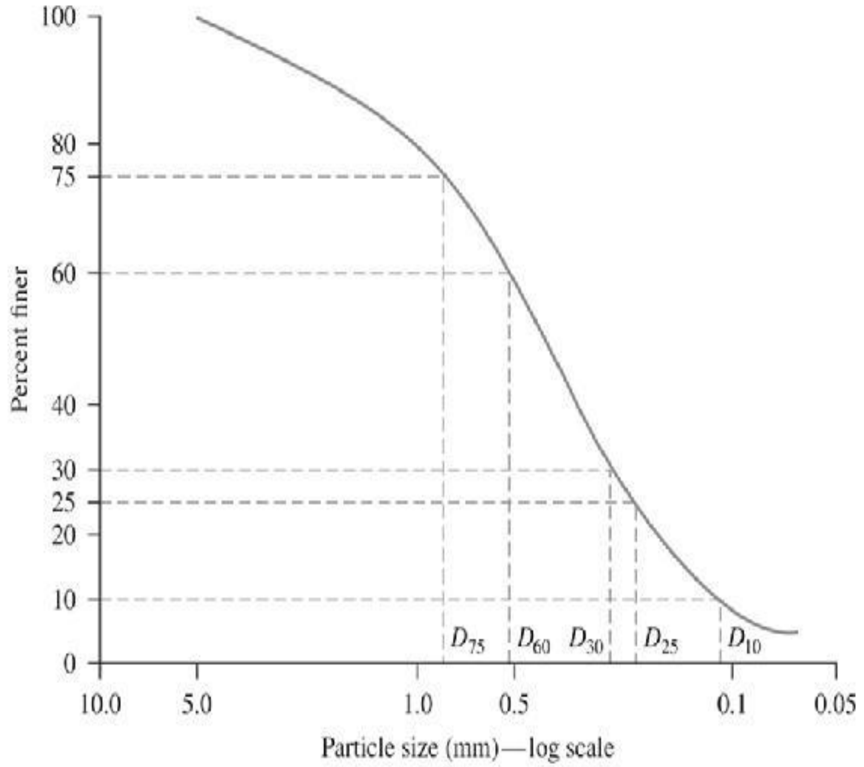

Figure 2.26 Definition of D_{75}, D_{60}, D_{30}, D_{25}, and D_{10}

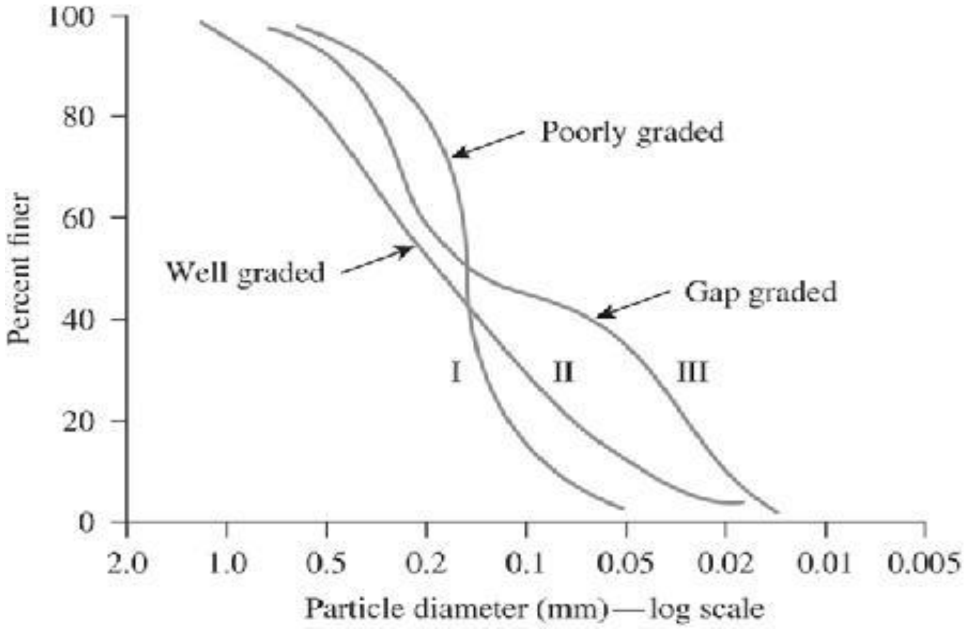

Figure 2.27 Different types of particle-size distribution curves

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The particle-size distribution curve is shown in Figure 2.28.

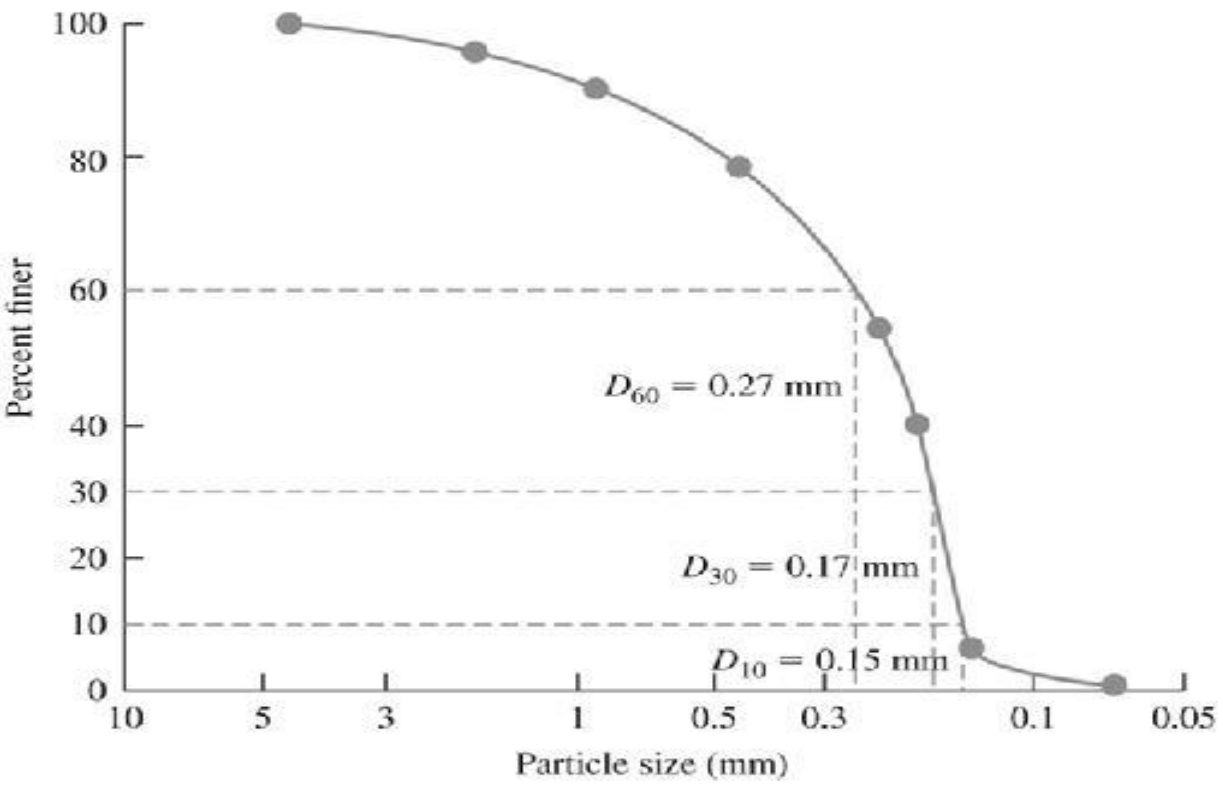

Figure 2.28 Particle-size distribution curve

particle shape

generally can be divided into three major categories:

- Bulky
- Flaky
- Needle shaped

Bulky particles are formed mostly by mechanical weathering of rock and minerals.

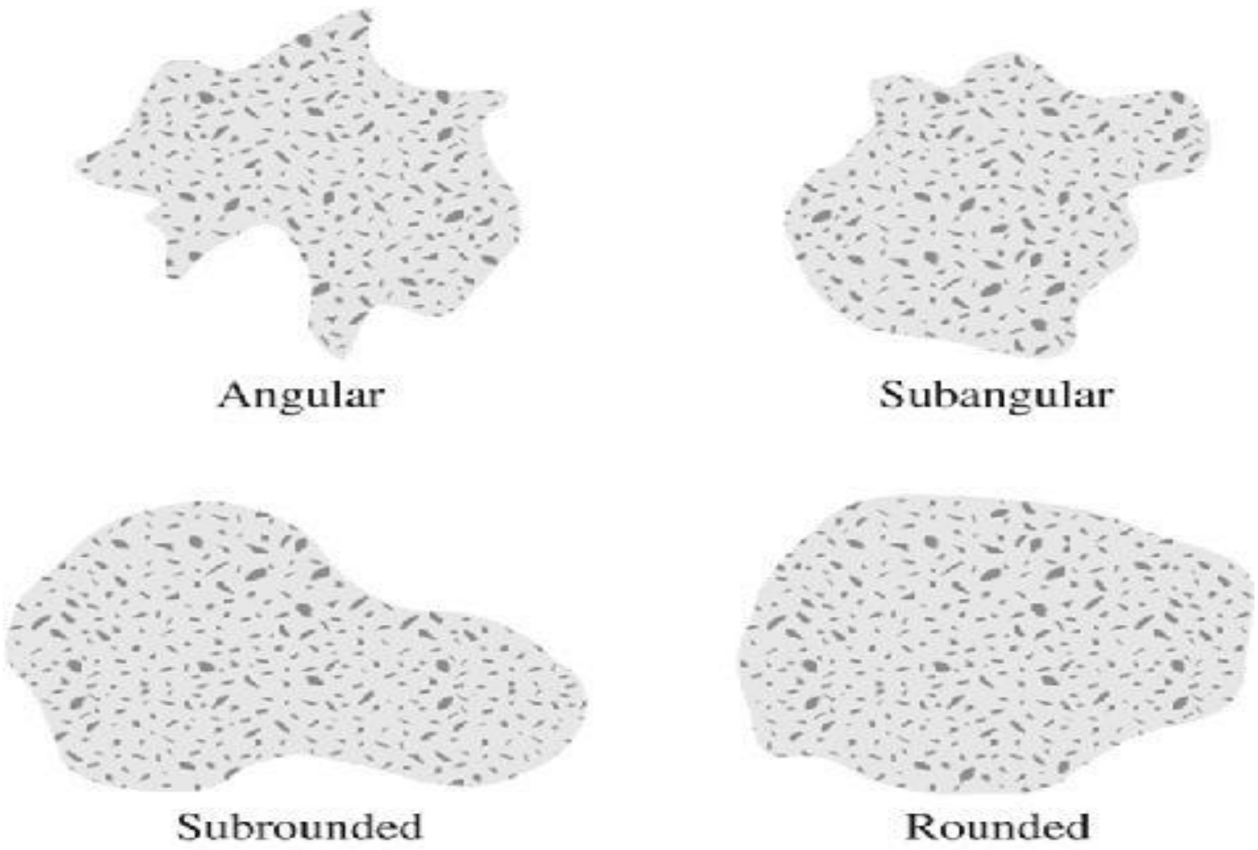

Figure 2.29 Shape of bulky particles

Grain Size Distribution

Examples

Example 2.1

Following are the results of a sieve analysis. Make the necessary calculations and draw a particle-size distribution curve.

U.S. sieve no.	Mass of soil retained on each sieve (g)
4	0
10	40
20	60
40	89
60	140
80	122
100	210
200	56
Pan	12

Solution

The following table can now be prepared.

U.S. sieve (1)	Opening (mm) (2)	Mass retained on each sieve (g) (3)	Cumulative mass retained above each sieve (g) (4)	Percent finer[a] (5)
4	4.75	0	0	100
10	2.00	40	$0 + 40 = 40$	94.5
20	0.850	60	$40 + 60 = 100$	86.3
40	0.425	89	$100 + 89 = 189$	74.1
60	0.250	140	$189 + 140 = 329$	54.9
80	0.180	122	$329 + 122 = 451$	38.1
100	0.150	210	$451 + 210 = 661$	9.3
200	0.075	56	$661 + 56 = 717$	1.7
Pan	–	12	$717 + 12 = 729 = \Sigma M$	0

[a] $\dfrac{\sum M - \text{col. }4}{\sum M} \times 100 = \dfrac{729 - \text{col. }4}{729} \times 100$

The particle-size distribution curve is shown in Figure 2.28.

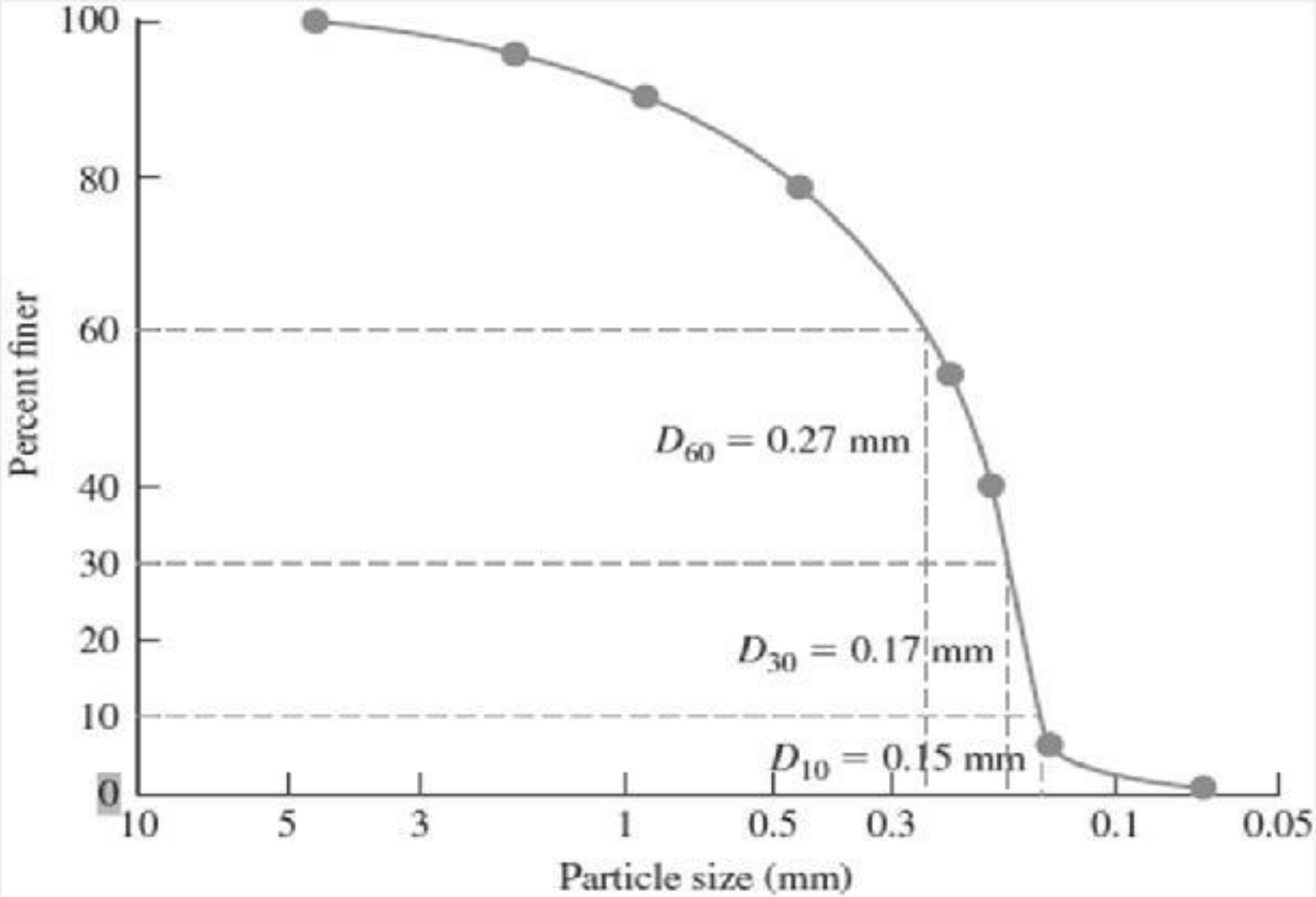

Figure 2.28 Particle-size distribution curve

Example 2.2

For the particle-size distribution curve shown in Figure 2.28 determine

a. D_{10}, D_{30}, and D_{60}
b. Uniformity coefficient, C_u
c. Coefficient of gradation, C_z

Solution

Part a

From Figure 2.28,

$$D_{10} = \mathbf{0.15\ mm}$$

$$D_{30} = \mathbf{0.17\ mm}$$

$$D_{60} = \mathbf{0.27\ mm}$$

Part b

$$C_u = \frac{D_{60}}{D_{10}} = \frac{0.27}{0.15} = \mathbf{1.8}$$

$$C_z = \frac{D_{30}^2}{D_{60} \times D_{10}} = \frac{(0.17)^2}{(0.27)(0.15)} = \mathbf{0.71}$$

Example 2.3

For the particle-size distribution curve shown in Figure 2.28, determine the percentages of gravel, sand, silt, and clay-size particles present. Use the Unified Soil Classification System.

Solution

From Figure 2.28, we can prepare the following table.

Size (mm)	Percent finer	
76.2	100	
		$100 - 100 =$ **0% gravel**
4.75	100	
		$100 - 1.7 =$ **98.3% sand**
0.075	1.7	
		$1.7 - 0 =$ **1.7% silt and clay**
–	0	

Table 2.3 Particle-Size Classifications

Name of organization	Grain size (mm)			
	Gravel	Sand	Silt	Clay
Massachusetts Institute of Technology (MIT)	>2	2 to 0.06	0.06 to 0.002	<0.002
U.S. Department of Agriculture (USDA)	>2	2 to 0.05	0.05 to 0.002	<0.002
American Association of State Highway and Transportation Officials (AASHTO)	76.2 to 2	2 to 0.075	0.075 to 0.002	<0.002
Unified Soil Classification System (U.S. Army Corps of Engineers, U.S. Bureau of Reclamation, and American Society for Testing and Materials)	76.2 to 4.75	4.75 to 0.075	Fines (i.e., silts and clays) <0.075	

CLASSIFICATION OF SOIL

Soil Classification

- Different Soils with similar properties may be classified into groups and sub-groups according to their engineering behavior.

- Classification systems provide a common language to concisely express the general characteristics of soils, which are infinitely varied, without detailed descriptions.

Textural Classification

- *Texture of soil refers to its surface appearance. Soil texture is influenced* by the size of the individual particles present in it.

- Table 2.3 divided soils into gravel, sand, silt, and clay categories on the basis of particle size. In most cases, natural soils are mixtures of particles from several size groups.

Textural Classification

In the textural classification system, the soils are named after their principal components, such as sandy clay, silty clay, and so forth.

Table 2.3 Particle-Size Classifications

Name of organization	Grain size (mm) Gravel	Sand	Silt	Clay
Massachusetts Institute of Technology (MIT)	>2	2 to 0.06	0.06 to 0.002	<0.002
U.S. Department of Agriculture (USDA)	>2	2 to 0.05	0.05 to 0.002	<0.002
American Association of State Highway and Transportation Officials (AASHTO)	76.2 to 2	2 to 0.075	0.075 to 0.002	<0.002
Unified Soil Classification System (U.S. Army Corps of Engineers, U.S. Bureau of Reclamation, and American Society for Testing and Materials)	76.2 to 4.75	4.75 to 0.075	Fines (i.e., silts and clays) <0.075	

Note: Sieve openings of 4.75 mm are found on a U.S. No. 4 sieve; 2-mm openings on a U.S. No. 10 sieve; 0.075-mm openings on a U.S. No. 200 sieve. See Table 2.5.

This chart is based only on the fraction of soil that passes through the No. 10 sieve

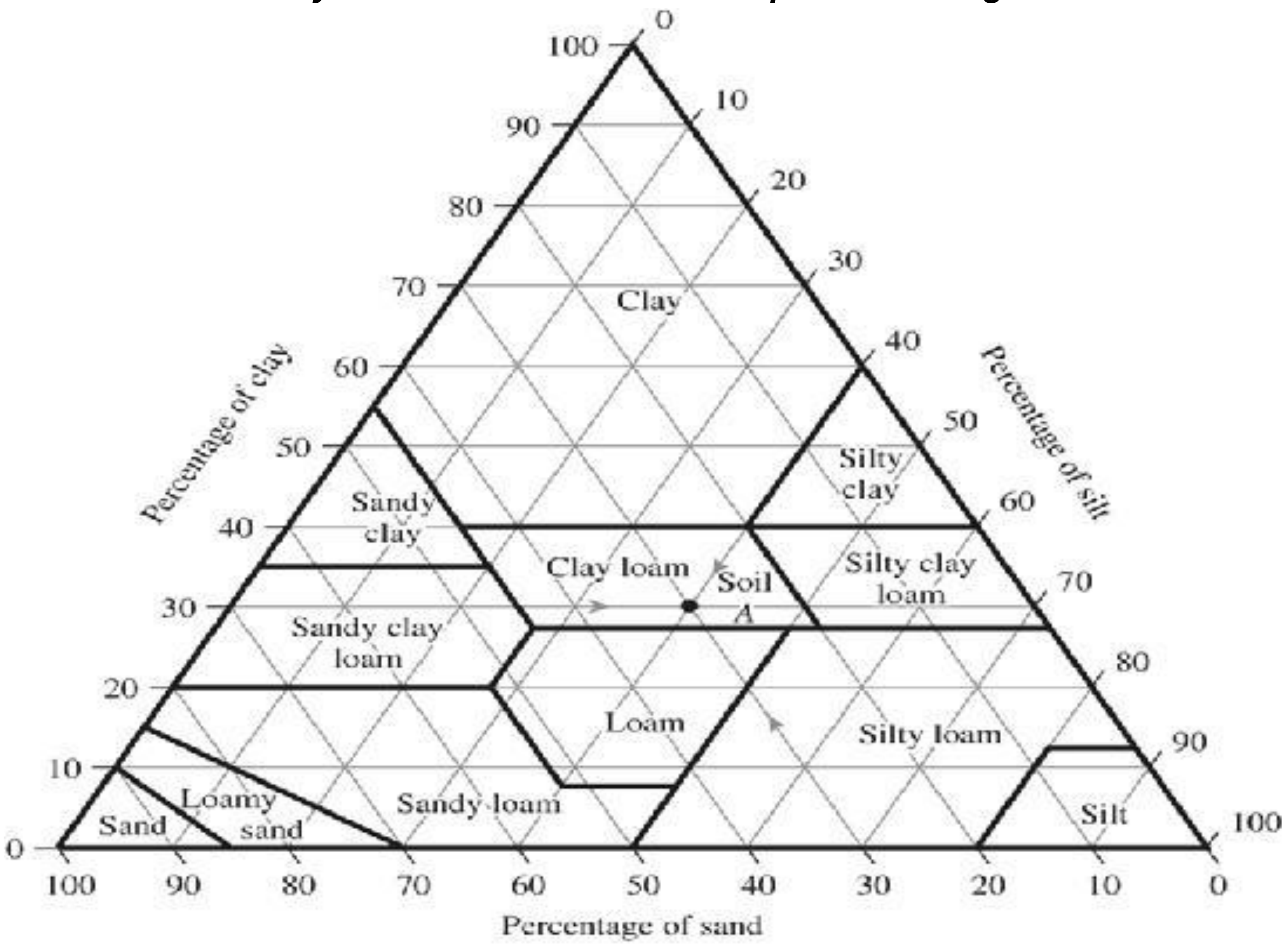

Figure 5.1 U.S. Department of Agriculture textural classification

Example.

If the particle-size distribution of soil *A shows* 30% *sand,* 40% *silt, and* 30% *clay-size particles, determine its textural* classification?

Solution.

This soil falls into the zone of clay loam

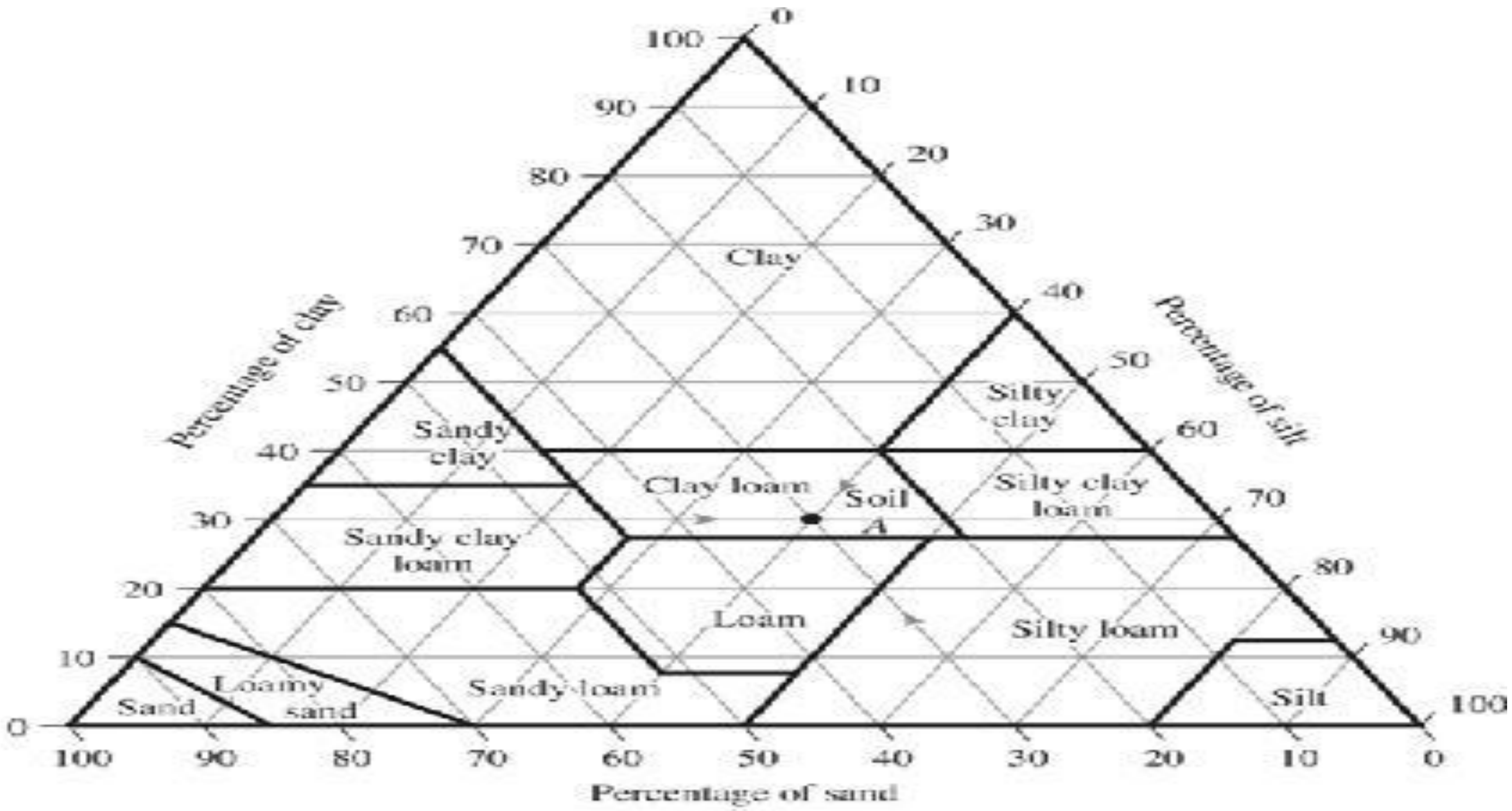

Figure 5.1 U.S. Department of Agriculture textural classification

Example:

if a soil *has a particle size* distribution of 20% gravel, 10% sand, 30% silt, and 40% clay, classify the soil according to textural classification system;

Solution:

the modified textural compositions is needed

$$Sand\ size: \frac{10 \times 100}{100 - 20} = 12.5\%$$

$$Silt\ size: \frac{30 \times 100}{100 - 20} = 37.5\%$$

$$Clay\ size: \frac{40 \times 100}{100 - 20} = 50.0\%$$

Clay or gravely Clay

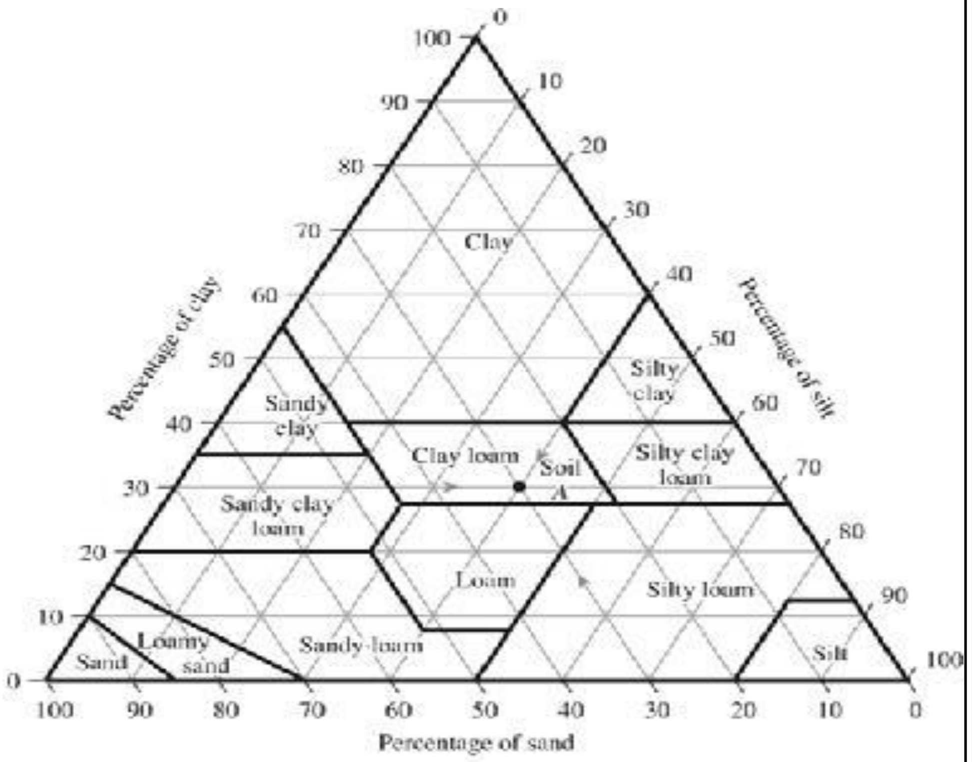

Figure 5.1 U.S. Department of Agriculture textural classification

AASHTO CLASSIFICATION SYSTEM

•Soil is classified into seven major groups: A-1 through A-.7

•Soils classified under groups A-1, A-2, and A-3 are granular materials of which 35% or less of the particles pass through the No. 200 sieve.

•Soils of which more than 35% pass through the No. 200 sieve are classified under groups A-4, A-5, A-6, and A-7 .These soils are mostly silt and clay-type materials.

Table 5.1 Classification of Highway Subgrade Materials

General classification	Granular materials (35% or less of total sample passing No. 200)						
	A-1			A-2			
Group classification	A-1-a	A-1-b	A-3	A-2-4	A-2-5	A-2-6	A-2-7
Sieve analysis (percentage passing)							
No. 10	50 max.						
No. 40	30 max.	50 max.	51 min.				
No. 200	15 max.	25 max.	10 max.	35 max.	35 max.	35 max.	35 max.
Characteristics of fraction passing No. 40							
Liquid limit				40 max.	41 min.	40 max.	41 min.
Plasticity index	6 max.		NP	10 max.	10 max.	11 min.	11 min.
Usual types of significant constituent materials	Stone fragments, gravel, and sand		Fine sand	Silty or clayey gravel and sand			
General subgrade rating	Excellent to good						

General classification	Silt-clay materials (more than 35% of total sample passing No. 200)			
Group classification	A-4	A-5	A-6	A-7 A-7-5[a] A-7-6[b]
Sieve analysis (percentage passing)				
No. 10				
No. 40				
No. 200	36 min.	36 min.	36 min.	36 min.
Characteristics of fraction passing No. 40				
Liquid limit	40 max.	41 min.	40 max.	41 min.
Plasticity index	10 max.	10 max.	11 min.	11 min.
Usual types of significant constituent materials	Silty soils		Clayey soils	
General subgrade rating	Fair to poor			

[a] For A-7-5, $PI \leq LL - 30$
[b] For A-7-6, $PI > LL - 30$

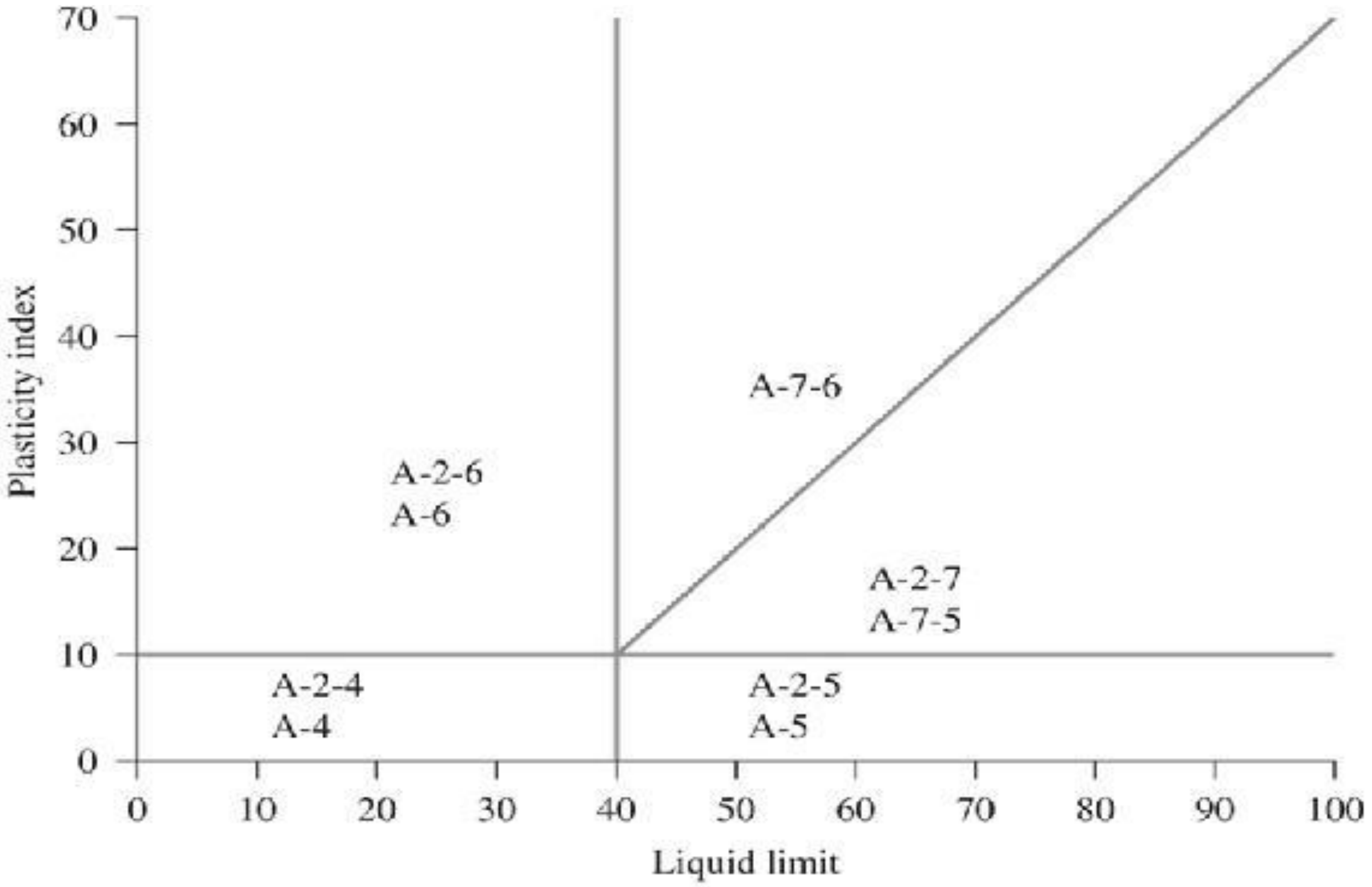

Figure 5.2 Range of liquid limit and plasticity index for soils in groups A-2, A-4, A-5, A-6, and A-7

Unified Soil Classification System

This system classifies soils into two broad categories:

•Coarse-grained soils that are gravelly and sandy in nature with less than 50% passing through the No. 200 sieve.

.The group symbols start with a prefix of G or S. G stands for gravel or gravelly soil, and S for sand or sandy soil.

•Fine-grained soils are with 50% or more passing through the No. 200 sieve.

.The group symbols start with prefixes of M, which stands for inorganic silt, C for inorganic clay, or O for organic silts and clays.

.The symbol Pt is used for peat, muck, and other highly organic soils.

Other symbols used for the classification are :

- W—well graded
- **P—poorly graded**
- L—low plasticity (liquid limit less than 50)
- H—high plasticity (liquid limit more than 50)

Table 5.2 Unified Soil Classification System (Based on Material Passing 76.2-mm Sieve)

Criteria for assigning group symbols				Group symbol
Coarse-grained soils More than 50% of retained on No. 200 sieve	**Gravels** More than 50% of coarse fraction retained on No. 4 sieve	Clean Gravels Less than 5% fines[a]	$C_u \geq 4$ and $1 \leq C_c \leq 3$[c]	GW
			$C_u < 4$ and/or $1 > C_c > 3$[c]	GP
		Gravels with Fines More than 12% fines[a,d]	$PI < 4$ or plots below "A" line (Figure 5.3)	GM
			$PI > 7$ and plots on or above "A" line (Figure 5.3)	GC
	Sands 50% or more of coarse fraction passes No. 4 sieve	Clean Sands Less than 5% fines[b]	$C_u \geq 6$ and $1 \leq C_c \leq 3$[c]	SW
			$C_u < 6$ and/or $1 > C_c > 3$[c]	SP
		Sands with Fines More than 12% fines[b,d]	$PI < 4$ or plots below "A" line (Figure 5.3)	SM
			$PI > 7$ and plots on or above "A" line (Figure 5.3)	SC
Fine-grained soils 50% or more passes No. 200 sieve	**Silts and clays** Liquid limit less than 50	Inorganic	$PI > 7$ and plots on or above "A" line (Figure 5.3)[e]	CL
			$PI < 4$ or plots below "A" line (Figure 5.3)[e]	ML
		Organic	$\dfrac{\text{Liquid limit — oven dried}}{\text{Liquid limit — not dried}} < 0.75$; see Figure 5.3; OL zone	OL
	Silts and clays Liquid limit 50 or more	Inorganic	PI plots on or above "A" line (Figure 5.3)	CH
			PI plots below "A" line (Figure 5.3)	MH
		Organic	$\dfrac{\text{Liquid limit — oven dried}}{\text{Liquid limit — not dried}} < 0.75$; see Figure 5.3; OH zone	OH
Highly Organic Soils	Primarily organic matter, dark in color, and organic odor			Pt

[a] Gravels with 5 to 12% fine require dual symbols: GW-GM, GW-GC, GP-GM, GP-GC.

[b] Sands with 5 to 12% fines require dual symbols: SW-SM, SW-SC, SP-SM, SP-SC.

[c] $C_u = \dfrac{D_{60}}{D_{10}}$; $C_c = \dfrac{(D_{30})^2}{D_{60} \times D_{10}}$

[d] If $4 \leq PI \leq 7$ and plots in the hatched area in Figure 5.3, use dual symbol GC-GM or SC-SM.

[e] If $4 \leq PI \leq 7$ and plots in the hatched area in Figure 5.3, use dual symbol CL-ML.

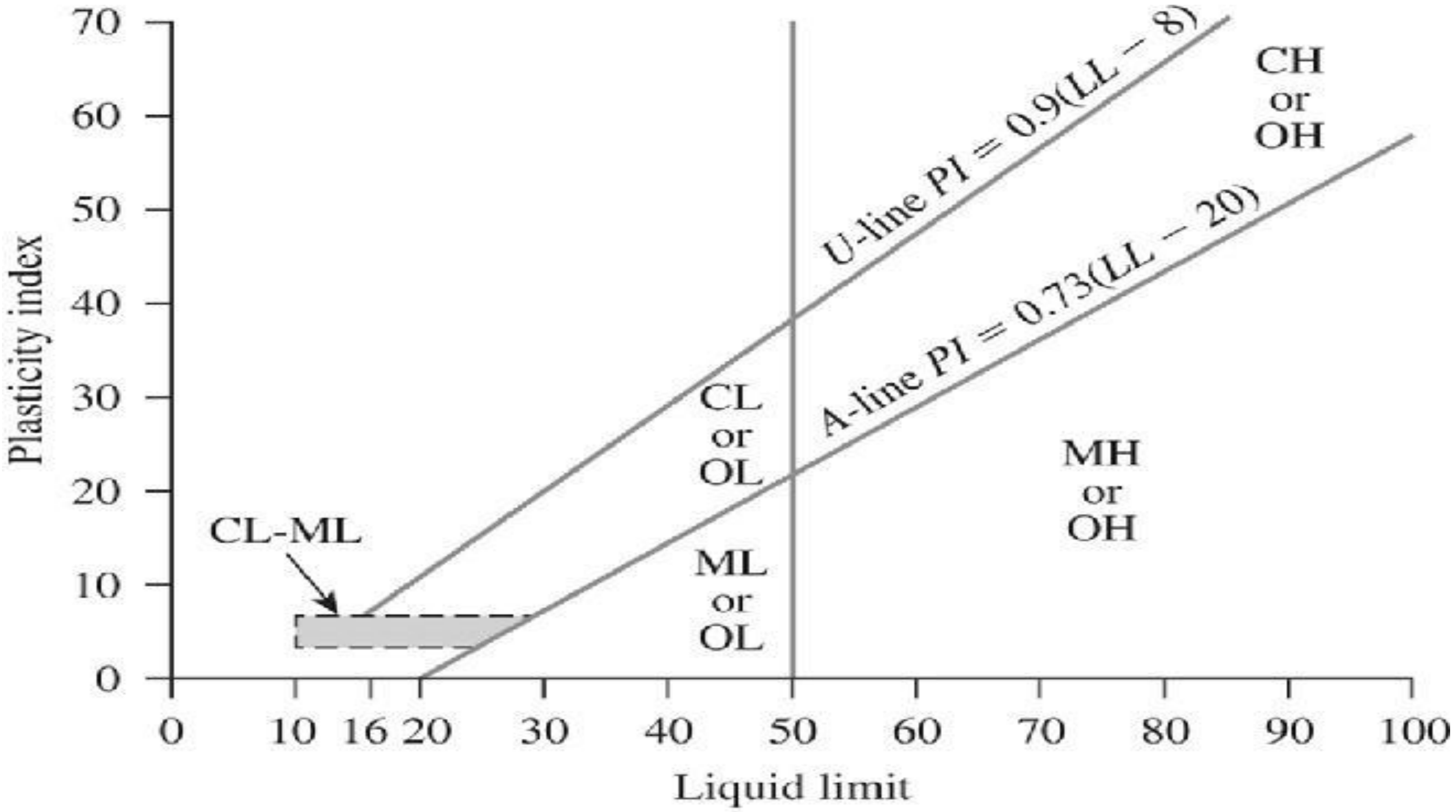

Figure 5.3 Plasticity chart

Group symbol		Group name
GW	<15% sand	Well-graded gravel
	≥15% sand	Well-graded gravel with sand
GP	<15% sand	Poorly graded gravel
	≥15% sand	Poorly graded gravel with sand
GW-GM	<15% sand	Well-graded sand with silt
	≥15% sand	Well-graded gravel with silt and sand
GW-GC	<15% sand	Well-graded gravel with clay (or silty clay)
	≥15% sand	Well-graded gravel with clay and sand (or silty clay and sand)
GP-GM	<15% sand	Poorly graded gravel with silt
	≥15% sand	Poorly graded gravel with silt and sand
GP-GC	<15% sand	Poorly graded gravel with clay (or silty clay)
	≥15% sand	Poorly graded gravel with clay and sand (or silty clay and sand)
GM	<15% sand	Silty gravel
	≥15% sand	Silty gravel with sand
GC	<15% sand	Clayey gravel
	≥15% sand	Clayey gravel with sand
GC-GM	<15% sand	Silty clayey gravel
	≥15% sand	Silty clayey gravel with sand
SW	<15% gravel	Well-graded sand
	≥15% gravel	Well-graded sand with gravel
SP	<15% gravel	Poorly graded sand
	≥15% gravel	Poorly graded sand with gravel
SW-SM	<15% gravel	Well-graded sand with silt
	≥15% gravel	Well-graded sand with silt and gravel
SW-SC	<15% gravel	Well-graded sand with clay (or silty clay)
	≥15% gravel	Well-graded sand with clay and gravel (or silty clay and gravel)
SP-SM	<15% gravel	Poorly graded sand with silt
	≥15% gravel	Poorly graded sand with silt and gravel
SP-SC	<15% gravel	Poorly graded sand with clay (or silty clay)
	≥15% gravel	Poorly graded sand with clay and gravel (or silty clay and gravel)
SM	<15% gravel	Silty sand
	≥15% gravel	Silty sand with gravel
SC	<15% gravel	Clayey sand
	≥15% gravel	Clayey sand with gravel
SC-SM	<15% gravel	Silty clayey sand
	≥15% gravel	Silty clayey sand with gravel

Figure 5.4 Flowchart group names for gravelly and sandy soil (*Source:* From "Annual Book of ASTM Standards, 04.08." Copyright 2007 American Society for Testing and Materials.)

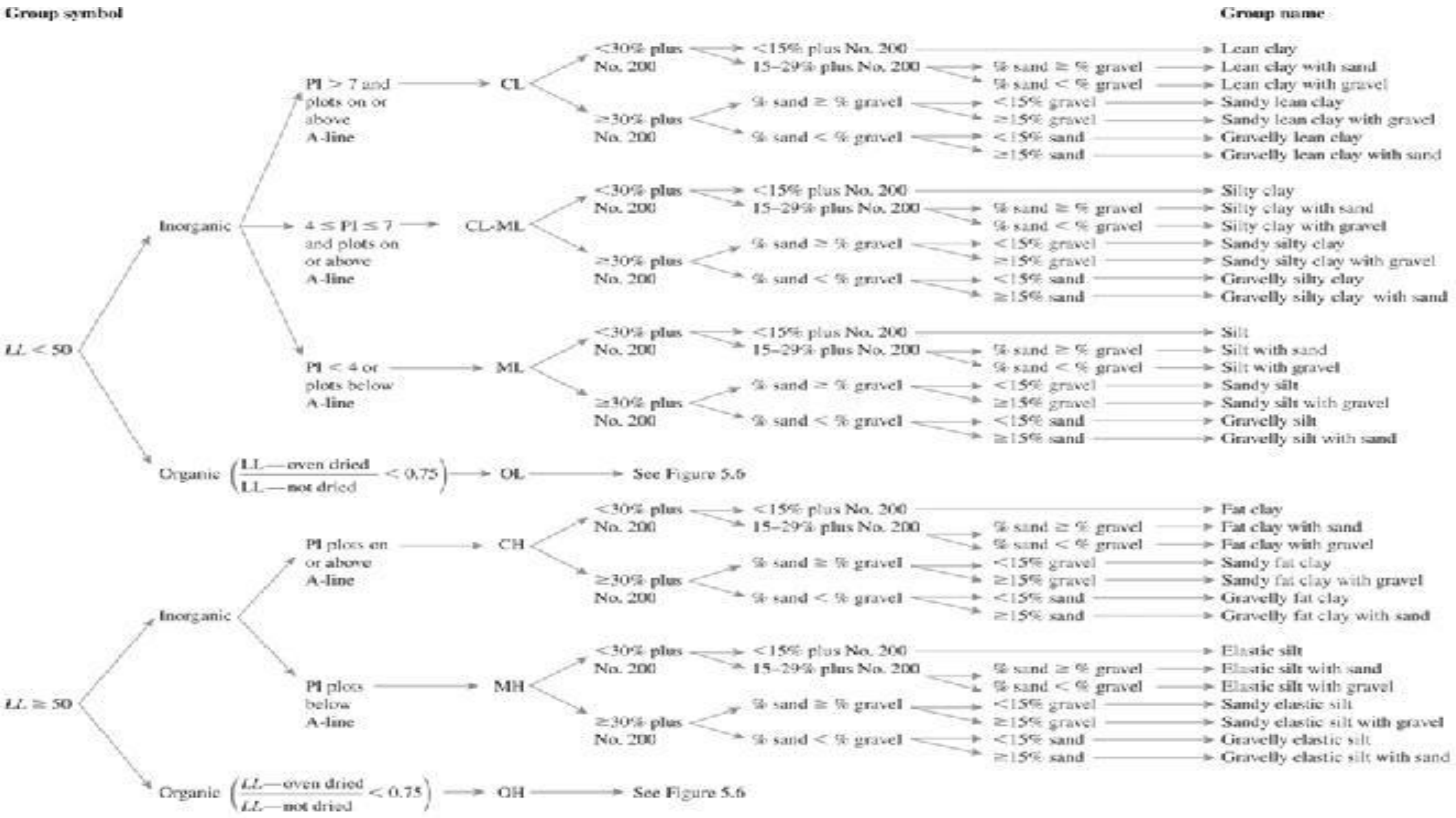

Figure 5.6 Flowchart group names for inorganic silty and clayey soils (*Source:* From "Annual Book of ASTM Standards, 04.08." Copyright 2007 American Society for Testing and Materials.)

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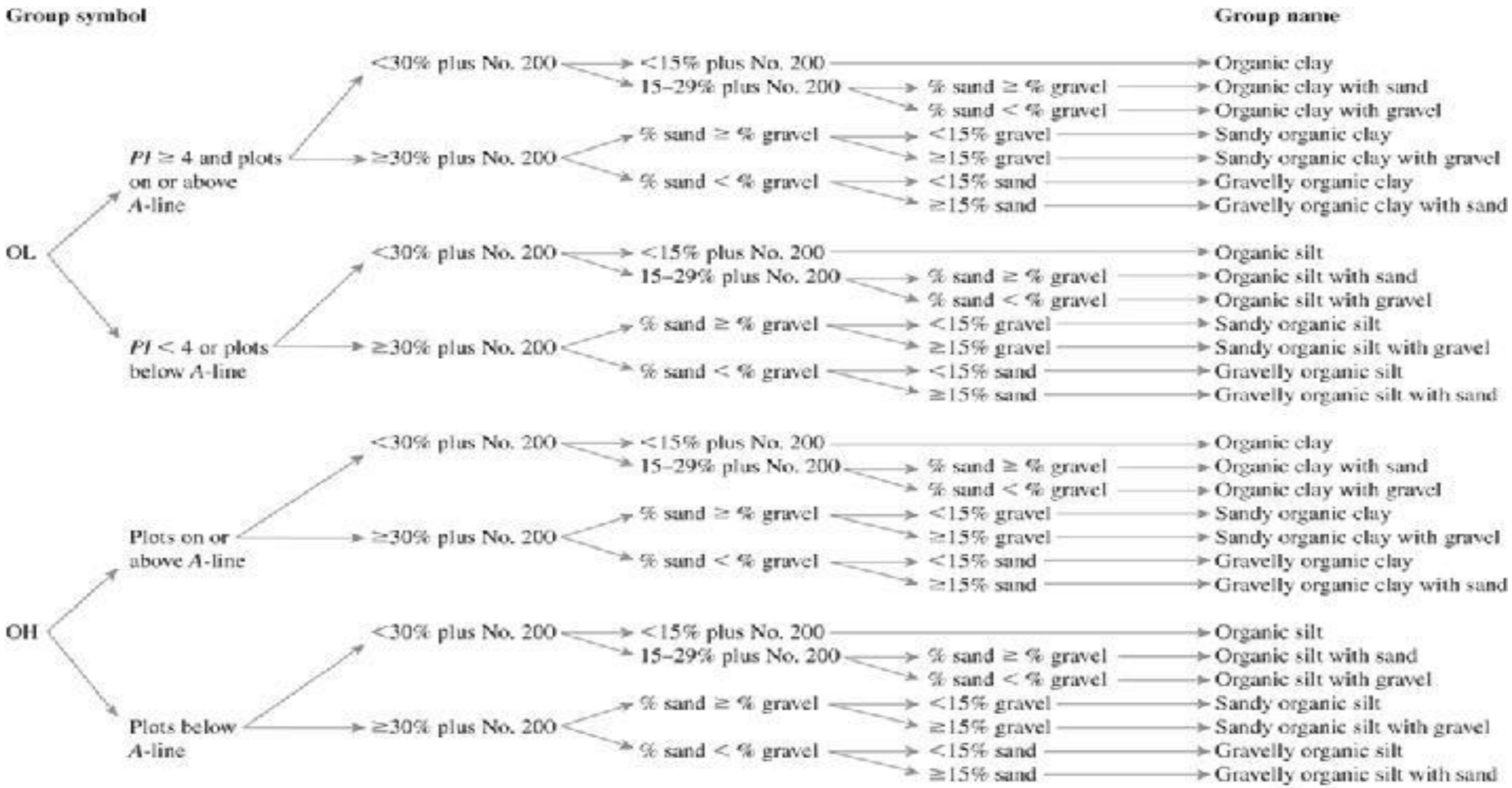

Figure 5.6 Flowchart group names for organic silty and clayey soils (*Source:* From "Annual Book of ASTM Standards, 04.08." Copyright 2007 American Society for Testing and Materials.)

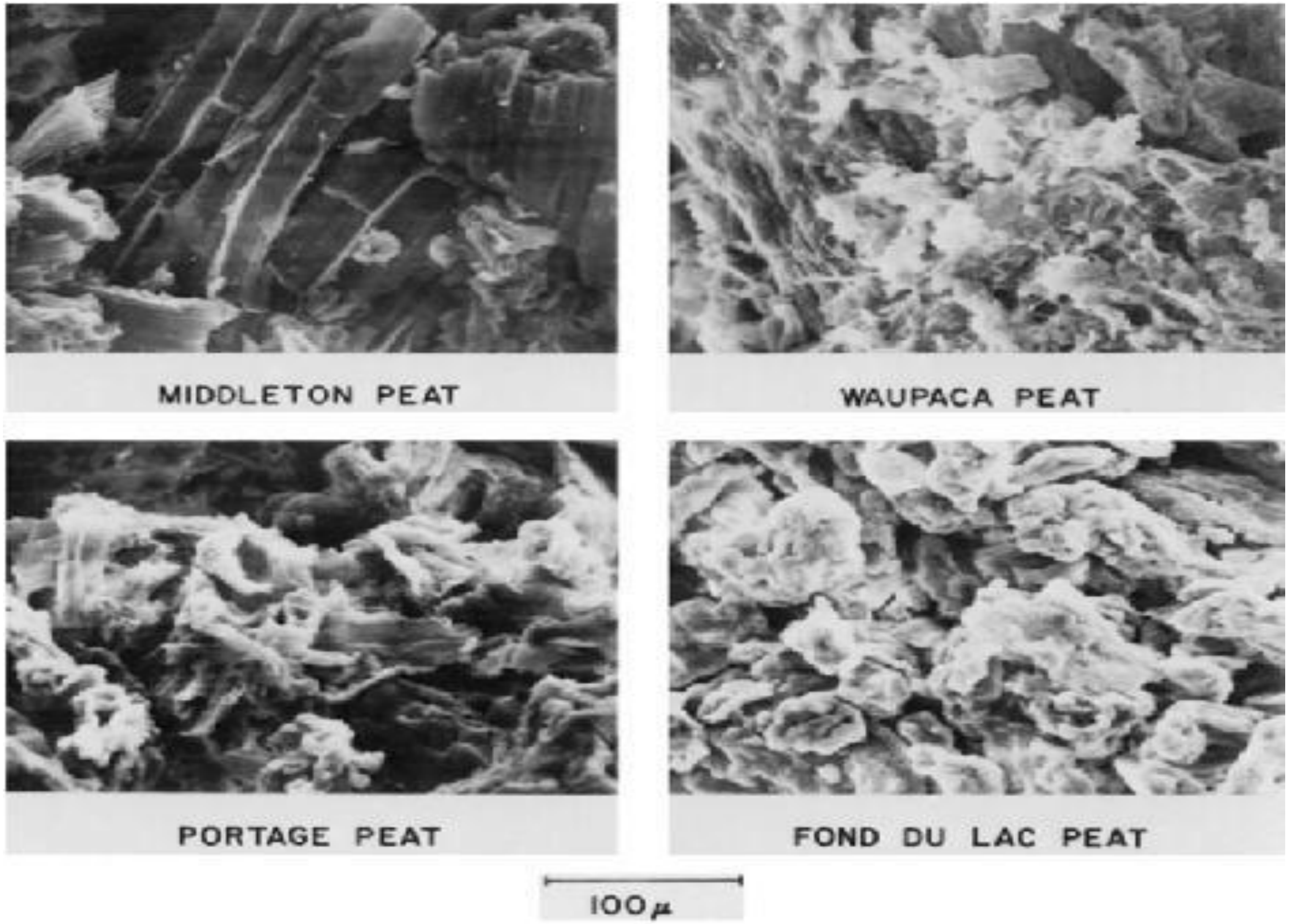

Figure 5.8 Scanning electron micrographs for four peat samples (*After Dhowian and Edil, 1980. Copyright ASTM INTERNATIONAL. Reprinted with permission.*)

Table 5.3 Properties of the Peats Shown in Figure 5.8

Source of peat	Moisture content (%)	Unit weight (kN/m^3)	Specific gravity, G_s	Ash content (%)
Middleton	510	9.1	1.41	12.0
Waupaca County	460	9.6	1.68	15.0
Portage	600	9.6	1.72	19.5
Fond du Lac County	240	10.2	1.94	39.8

Table 5.4 Comparison of the AASHTO System with the Unified System*

Soil group in AASHTO system	Comparable soil groups in Unified system		
	Most probable	Possible	Possible but improbable
A-1-a	GW, GP	SW, SP	GM, SM
A-1-b	SW, SP, GM, SM	GP	—
A-3	SP	—	SW, GP
A-2-4	GM, SM	GC, SC	GW, GP, SW, SP
A-2-5	GM, SM	—	GW, GP, SW, SP
A-2-6	GC, SC	GM, SM	GW, GP, SW, SP
A-2-7	GM, GC, SM, SC	—	GW, GP, SW, SP
A-4	ML, OL	CL, SM, SC	GM, GC
A-5	OH, MH, ML, OL	—	SM, GM
A-6	CL	ML, OL, SC	GC, GM, SM
A-7-5	OH, MH	ML, OL, CH	GM, SM, GC, SC
A-7-6	CH, CL	ML, OL, SC	OH, MH, GC, GM, SM

Table 5.5 Comparison of the Unified System with the AASHTO System*

Soil group in Unified system	Comparable soil groups in AASHTO system		
	Most probable	Possible	Possible but improbable
GW	A-1-a	—	A-2-4, A-2-5, A-2-6, A-2-7
GP	A-1-a	A-1-b	A-3, A-2-4, A-2-5, A-2-6, A-2-7
GM	A-1-b, A-2-4, A-2-5, A-2-7	A-2-6	A-4, A-5, A-6, A-7-5, A-7-6, A-1-a
GC	A-2-6, A-2-7	A-2-4	A-4, A-6, A-7-6, A-7-5
SW	A-1-b	A-1-a	A-3, A-2-4, A-2-5, A-2-6, A-2-7
SP	A-3, A-1-b	A-1-a	A-2-4, A-2-5, A-2-6, A-2-7
SM	A-1-b, A-2-4, A-2-5, A-2-7	A-2-6, A-4	A-5, A-6, A-7-5, A-7-6, A-1-a
SC	A-2-6, A-2-7	A-2-4, A-6, A-4, A-7-6	A-7-5
ML	A-4, A-5	A-6, A-7-5, A-7-6	—
CL	A-6, A-7-6	A-4	—
OL	A-4, A-5	A-6, A-7-5, A-7-6	—
MH	A-7-5, A-5	—	A-7-6
CH	A-7-6	A-7-5	—
OH	A-7-5, A-5	—	A-7-6
Pt	—	—	—

SOIL CLASSIFICATION

Examples

EXAMPLE 1:

The results of the particle-size analysis of a soil are as follows:

- Percent passing the No. 10 sieve = 42
- Percent passing the No. 40 sieve = 35
- Percent passing the No. 200 sieve = 20

The liquid limit and plasticity index of the minus No. 40 fraction of the soil are 25 and 20, respectively. Classify the soil by the AASHTO system.

Solution

Since 20% (i.e., less than 35%) of soil is passing No. 200 sieve, it is a granular soil. Hence it can be A-1, A-2, or A-3. Refer to Table 5.1. Starting from the left of the table, the soil falls under A-1-b (see the table below).

Parameter	Specifications in Table 5.1	Parameters of the given soil
Percent passing sieve		
No. 10	—	
No. 40	50 max	35
No. 200	25 max	20
Plasticity index (PI)	6 max	$PI = LL - PL = 25 - 20 = 5$

EXAMPLE 2:

Ninety-five percent of a soil passes through the No. 200 sieve and has a liquid limit of 60 and plasticity index of 40. Classify the soil by the AASHTO system.

Solution

Ninety-five percent of the soil (which is ≥ 36%) is passing through No. 200 sieve. So it is a silty-clay material. Now refer to Table 5.1. Starting from the left of the table, it falls under A-7-6 (see the table below).

Parameter	Specifications in Table 5.1	Parameters of the given soil
Percent passing No. 200 sieve	36 min.	95
Liquid limit (*LL*)	41 min.	60
Plasticity index (*PI*)	11 min.	40
PI	$> LL - 30$	$PI = 40 > LL - 30 = 60 - 30 = 30$

$$
\begin{aligned}
GI &= (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10) \\
&= (95 - 35)[0.2 + 0.005(60 - 40)] + (0.01)(95 - 15)(40 - 10) \\
&= 42
\end{aligned}
$$

So, the classification is **A-7-6(42)**.

EXAMPLE 3:

The results of the particle-size analysis of a soil are as follows:

Percent passing through the No. 10 sieve = 100
Percent passing through the No. 40 sieve = 80
Percent passing through the No. 200 sieve = 58

The liquid limit and plasticity index of the minus No. 40 fraction of the soil are 30 and 10, respectively. Classify the soil by the Unified classification system.

Solution

Refer to Table 5.2. Since 58% of the soil passes through the No. 200 sieve, it is a fine-grained soil. Referring to the plasticity chart in Figure 5.3, for $LL = 30$ and $PI = 10$, it can be classified (group symbol) as CL.

In order to determine the group name, we refer to Figure 5.5 and Figure 5.7, which is taken from Figure 5.5. The percent passing No. 200 sieve is more than 30%. Percent of gravel = 0; percent of sand = (100 − 58) − (0) = 42. Hence, percent sand > percent gravel. Also, percent gravel is less than 15%. Hence the group name is sandy lean clay.

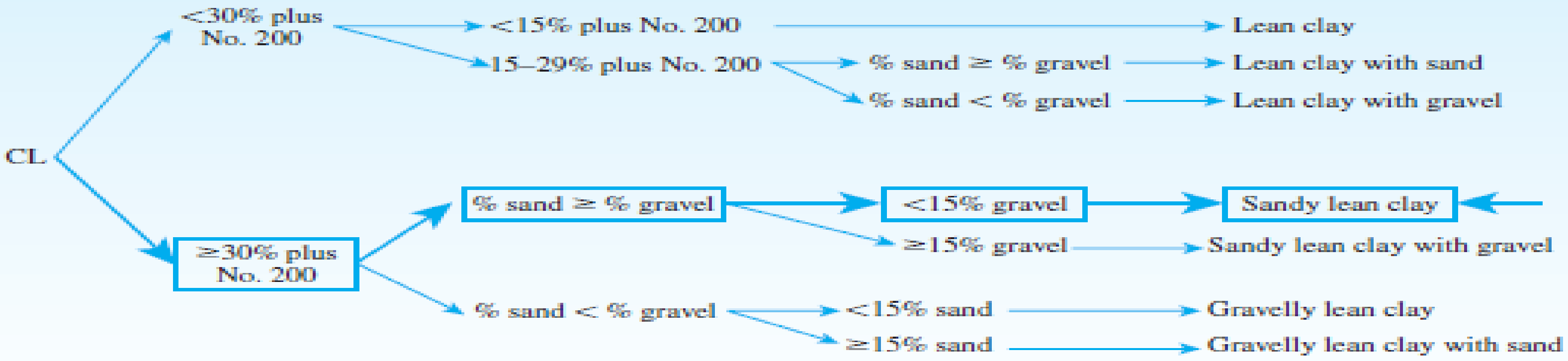

Figure 5.7 Determination of group name for the soil in Example 5.4

EXAMPLE 4:

Figure 5.9 gives the grain-size distribution of two soils. The liquid and plastic limits of minus No. 40 sieve fraction of the soil are as follows:

	Soil *A*	Soil *B*
Liquid limit	30	26
Plastic limit	22	20

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Determine the group symbols and group names according to the Unified Soil Classification System.

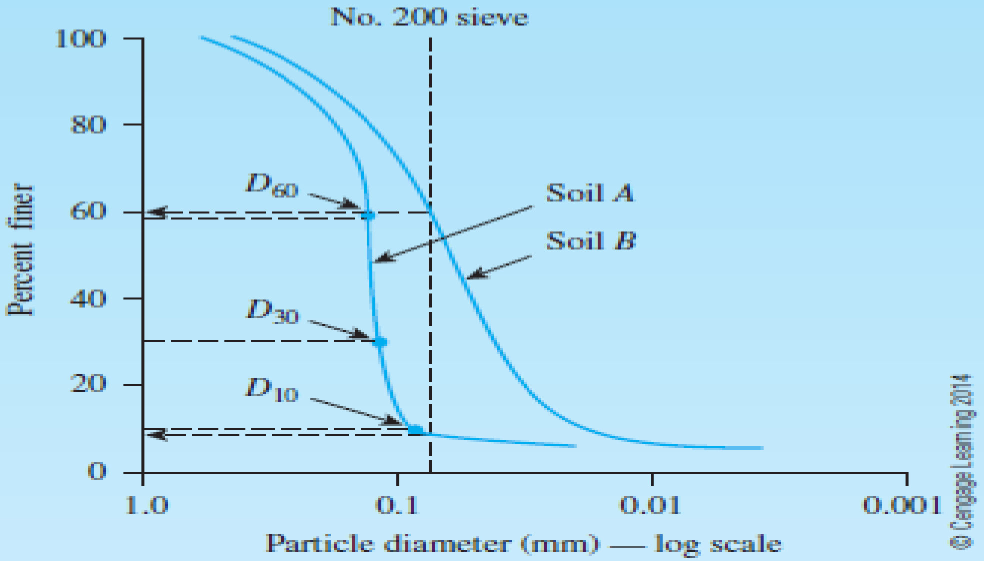


Figure 5.9 Particle-size distribution of two soils

Solution

Soil *A*

The grain-size distribution curve (Figure 5.9) indicates that percent passing No. 200 sieve is 8. According to Table 5.2, it is a coarse-grained soil. Also, from Figure 5.9, the percent retained on No. 4 sieve is zero. Hence, it is a sandy soil.

From Figure 5.9, $D_{10} = 0.085$ mm, $D_{30} = 0.12$ m, and $D_{60} = 0.135$ mm. Thus,

$$C_u = \frac{D_{60}}{D_{10}} = \frac{0.135}{0.085} = 1.59 < 6$$

$$C_c = \frac{D_{30}^2}{D_{60} \times D_{10}} = \frac{(0.12)^2}{(0.135)(0.085)} = 1.25 > 1$$

With $LL = 30$ and $PI = 30 - 22 = 8$ (which is greater than 7), it plots above the *A*-line in Figure 5.3. Hence, the group symbol is **SP-SC**.

In order to determine the group name, we refer to Figure 5.4 and Figure 5.10.

$$\text{Percentage of gravel} = 0 \text{ (which is} < 15\%\text{)}$$

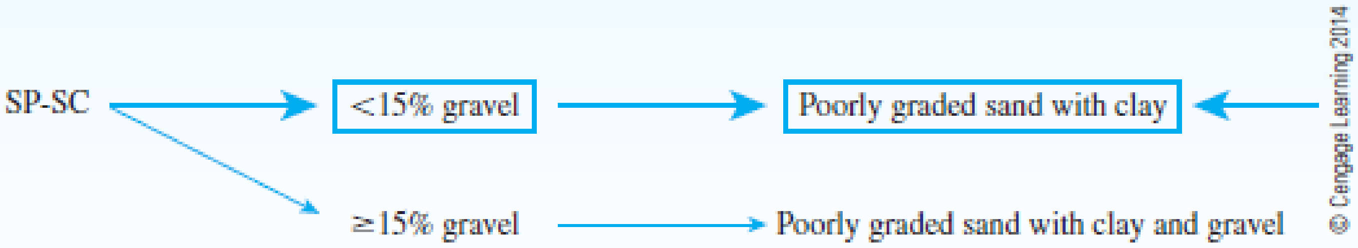


Figure 5.10 Determination of group name for soil *A* in Example 5.6

So, the group name is **poorly graded sand with clay.**

Soil *B*

The grain-size distribution curve in Figure 5.9 shows that percent passing No. 200 sieve is 61 (>50%); hence, it is a fine-grained soil. Given: $LL = 26$ and $PI = 26 - 20 = 6$. In Figure 5.3, the PI plots in the hatched area. So, from Table 5.2, the group symbol is **CL-ML.**

For group name (assuming that the soil is inorganic), we go to Figure 5.5 and obtain Plus No. 200 sieve $= 100 - 61 = 39$ (which is greater than 30).

$$\text{Percentage of gravel} = 0; \text{ percentage of sand} = 100 - 61 = 39$$

Thus, because the percentage of sand is greater than the percentage of gravel, the soil is **sandy silty clay** as shown in Figure 5.11.

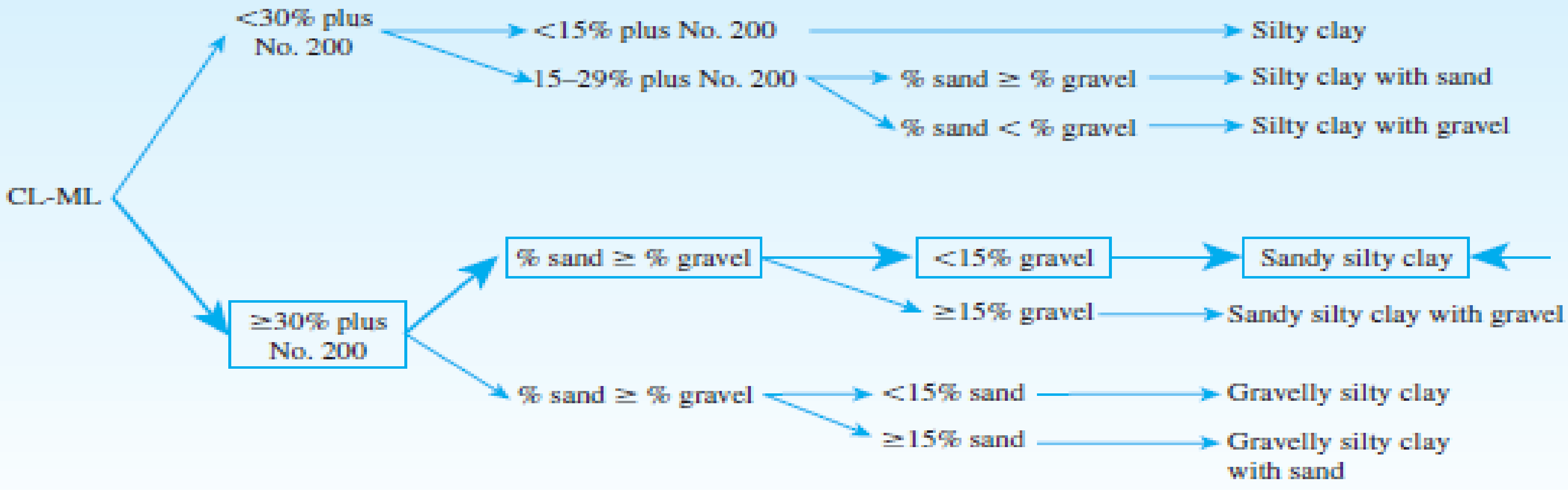


Figure 5.11 Determination of group name for soil *B* in Example 5.6

SOIL COMPACTION

INTRODUCTION:

•In the construction of highway embankments, earth dams, and many other engineering structures, loose soils must be compacted to increase their unit weights.

•Compaction increases the strength characteristics of soils, which increase the bearing capacity of foundations constructed over them.

•Compaction also decreases the amount of undesirable settlement of structures and increases the stability of slopes of embankments.

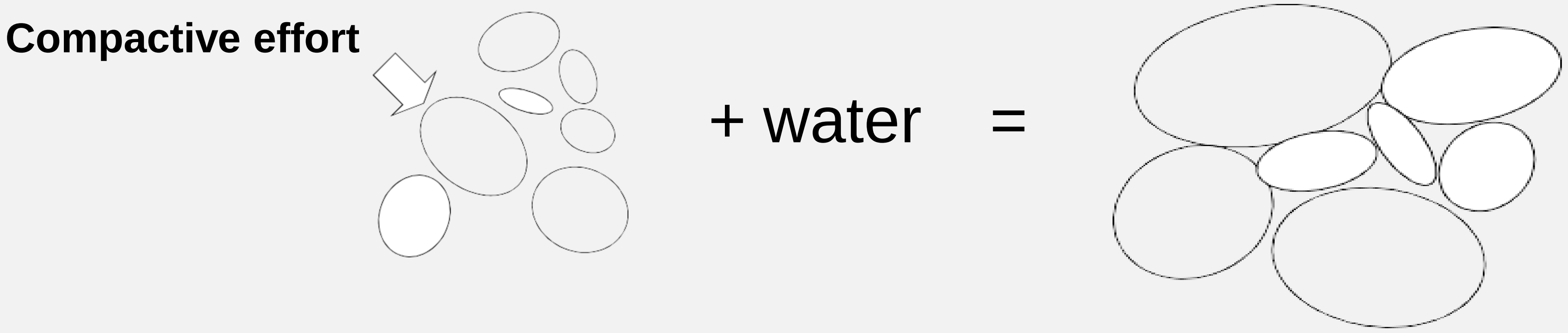
COMPACTION—GENERAL PRINCIPLES:

What is compaction؟

•Is the densification of soil by removal of air, which requires mechanical energy.

•A simple ground improvement technique, where the soil is densified through external compactive effort.

4



Compaction

- Compaction – expelling air from the void space
  - Consolidation – extrusion of water
- Effects of compaction
  - Increase soil's shear strength
  - Decrease in future settlement of the soil
  - Decrease in soil permeability
- How to quantify – use dry unit weight of soil

$$\gamma_d = \frac{\gamma}{1+w}$$

$\gamma =$ wet unit weigh t

$w =$ moisture content

Compaction

- What does water do for compaction؟
  - Lubricant)softening agent)
  - Too much water → lesser density
  - Optimum moisture content
    (= maximum dry unit weight) → best compaction

- How to use maximum dry unit weight؟
  - Target unit weight at the job site
    - Need to know how much the soil can be compacted.

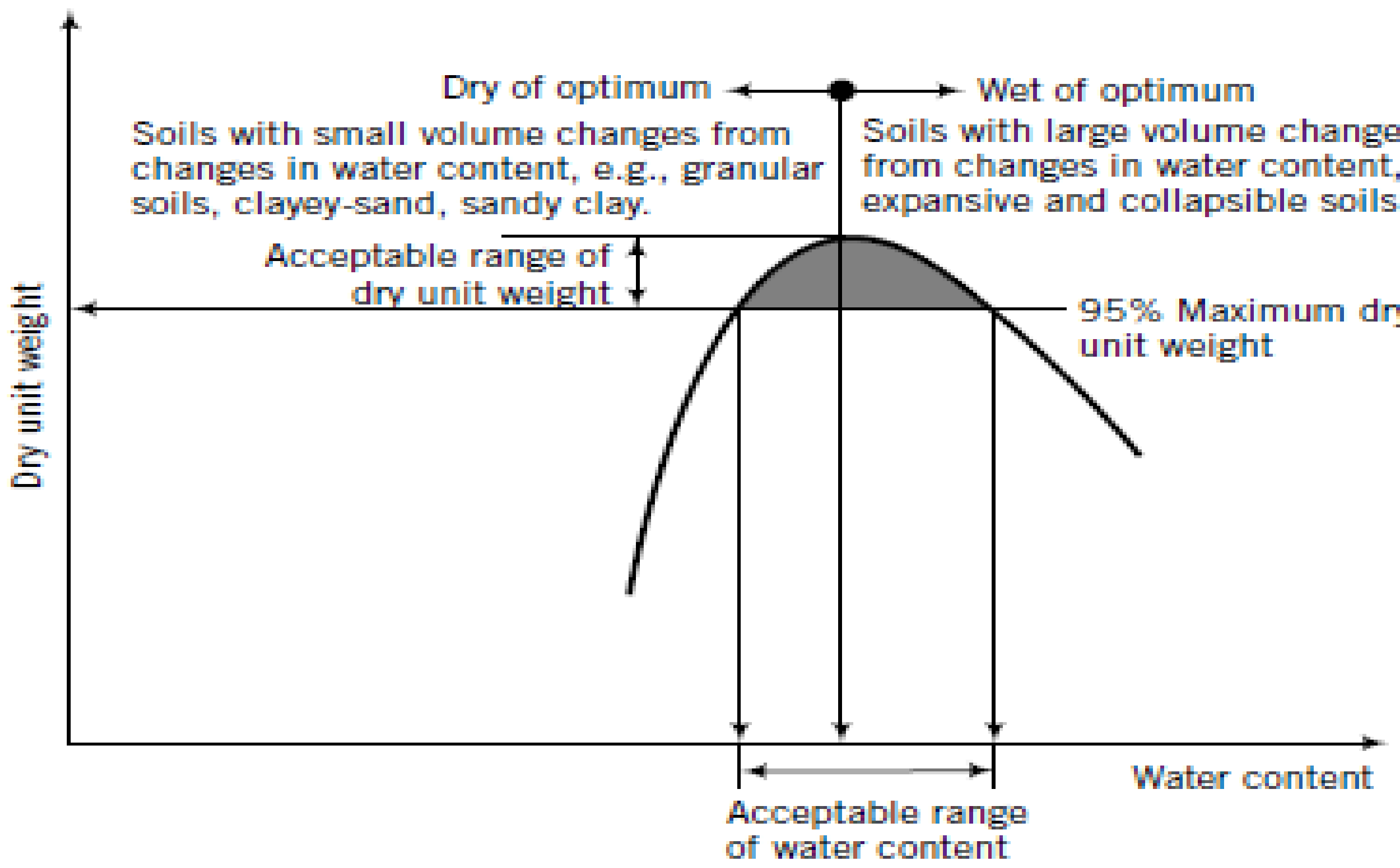
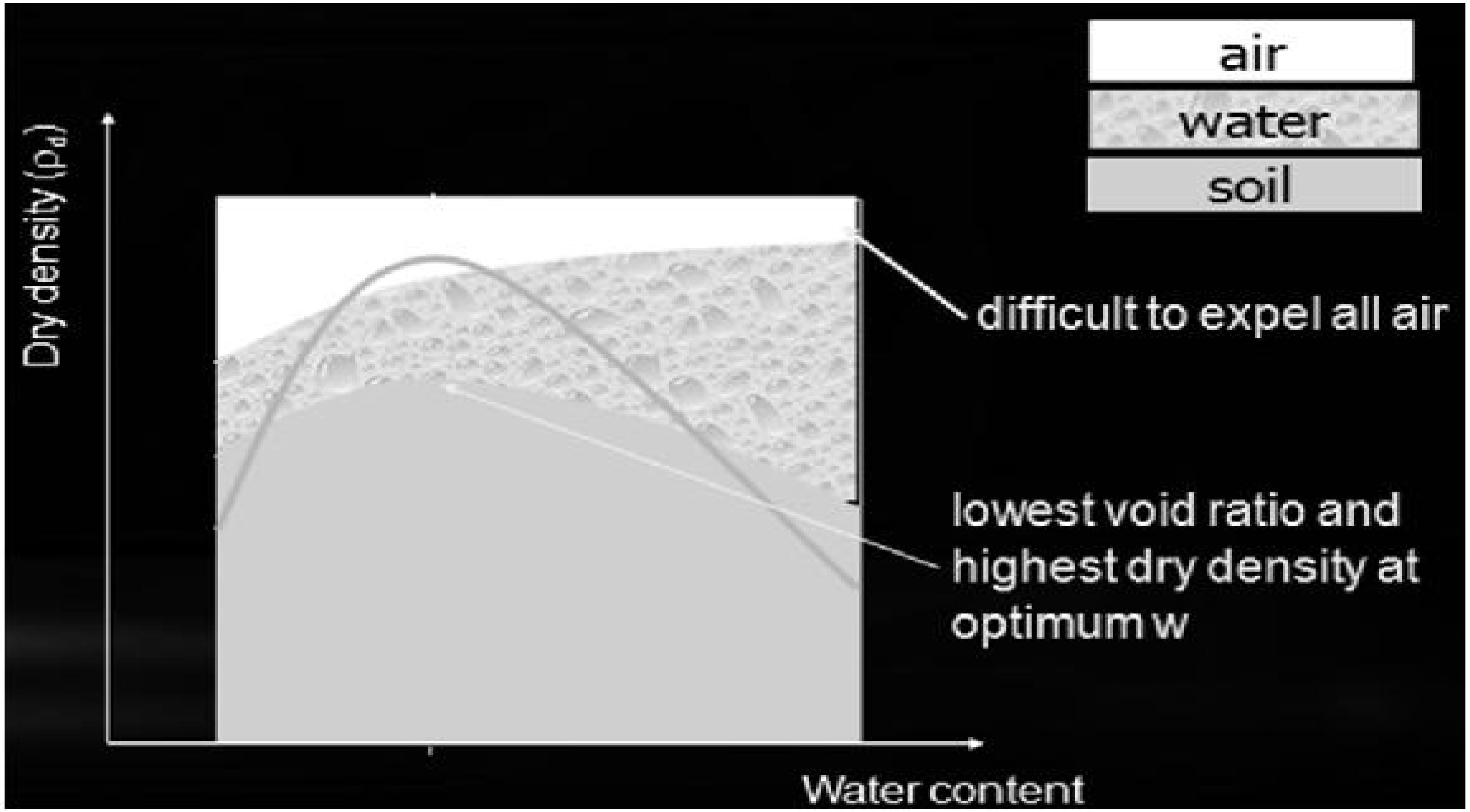


FIGURE 5.6 Illustration of compaction specification of soils in the field.

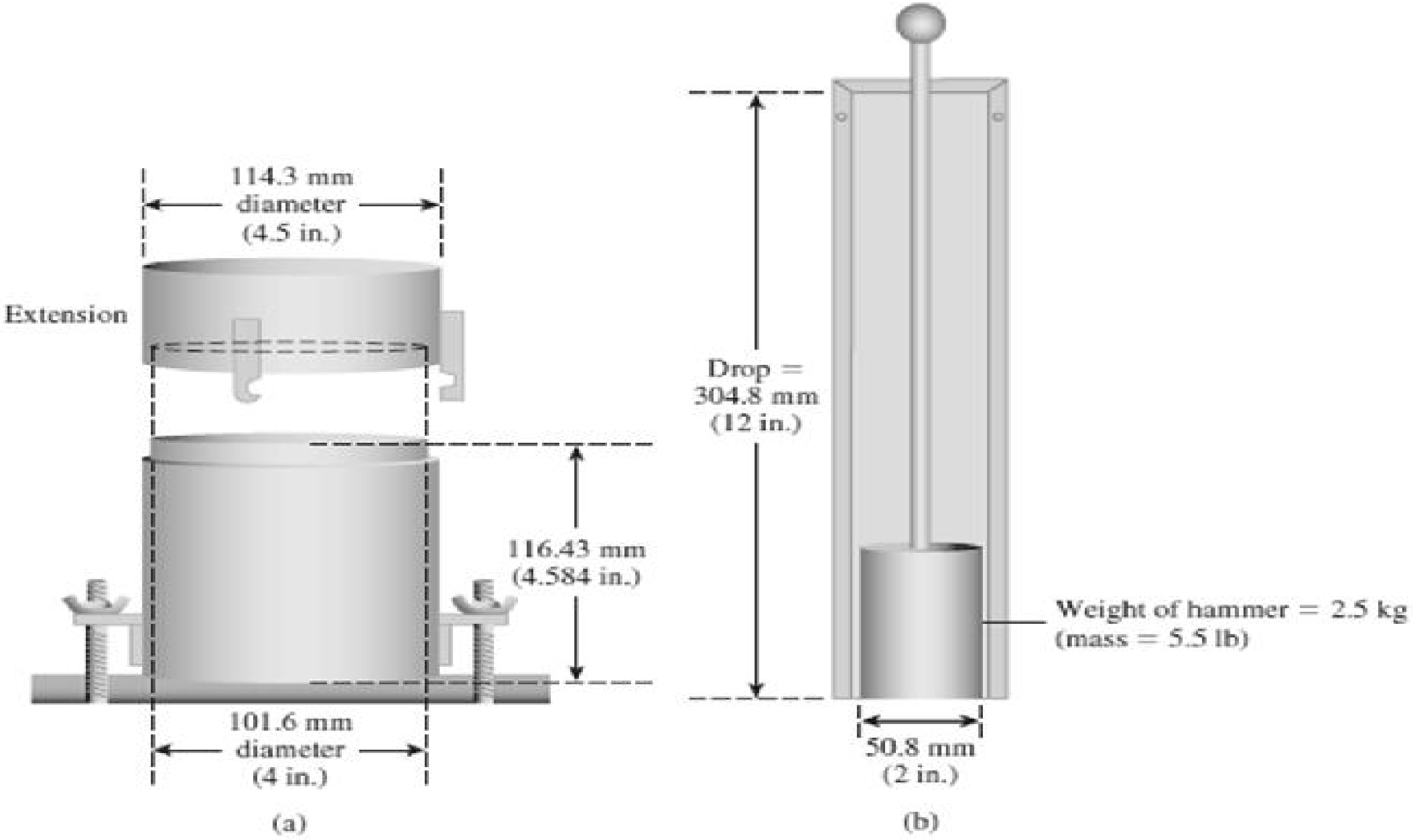
Compaction Curve

What happens to the relative quantities of the three phases with addition of water?

8



• **Standard Proctor Test:**



•For each test, the moist unit weight of compaction, γ, can be calculate as:

$$\gamma = \frac{W}{V_{(m)}}$$

•For each test, the moisture content of the compacted soil is determined in the laboratory. With the known moisture content, the dry unit weight can be calculated as:

$$\gamma_d = \frac{\gamma}{1 + \frac{w\,(\%)}{100}}$$

For a given *moisture content w and degree of saturation S, the dry unit weight of* compaction can be calculated as follows.

$$\gamma_d = \frac{G_s \gamma_w}{1 + e} \quad \xrightarrow{Se = G_s w} \quad \gamma_d = \frac{G_s \gamma_w}{1 + \frac{G_s w}{S}}$$

-

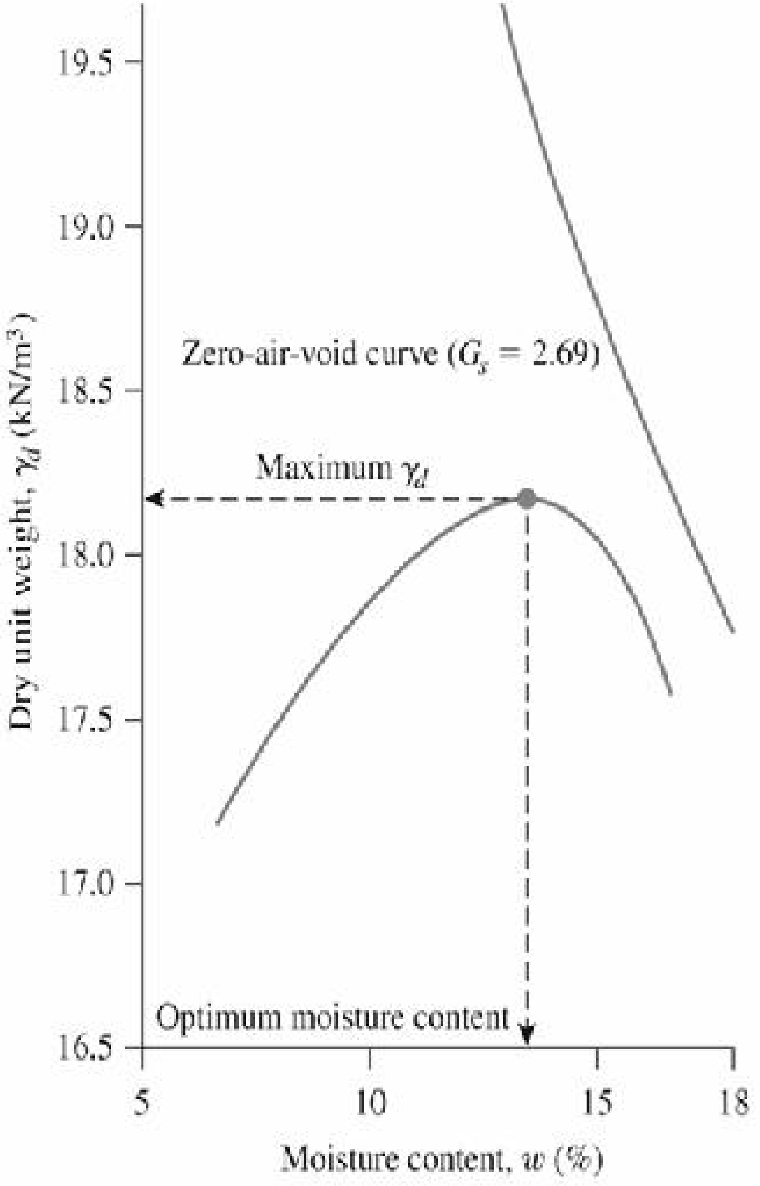


Figure 6.3 Standard Proctor compaction test results for a silty clay

Theoretical maximum dry unit weight is obtained when no air is in the void spaces.

S = 100 %

$$\gamma_{zav} = \frac{G_s \gamma_w}{1 + wG_s} = \frac{\gamma_w}{w + \dfrac{1}{G_s}}$$

Factors Affecting Compaction:

- Moisture content
- Soil Type
- Compaction Effort

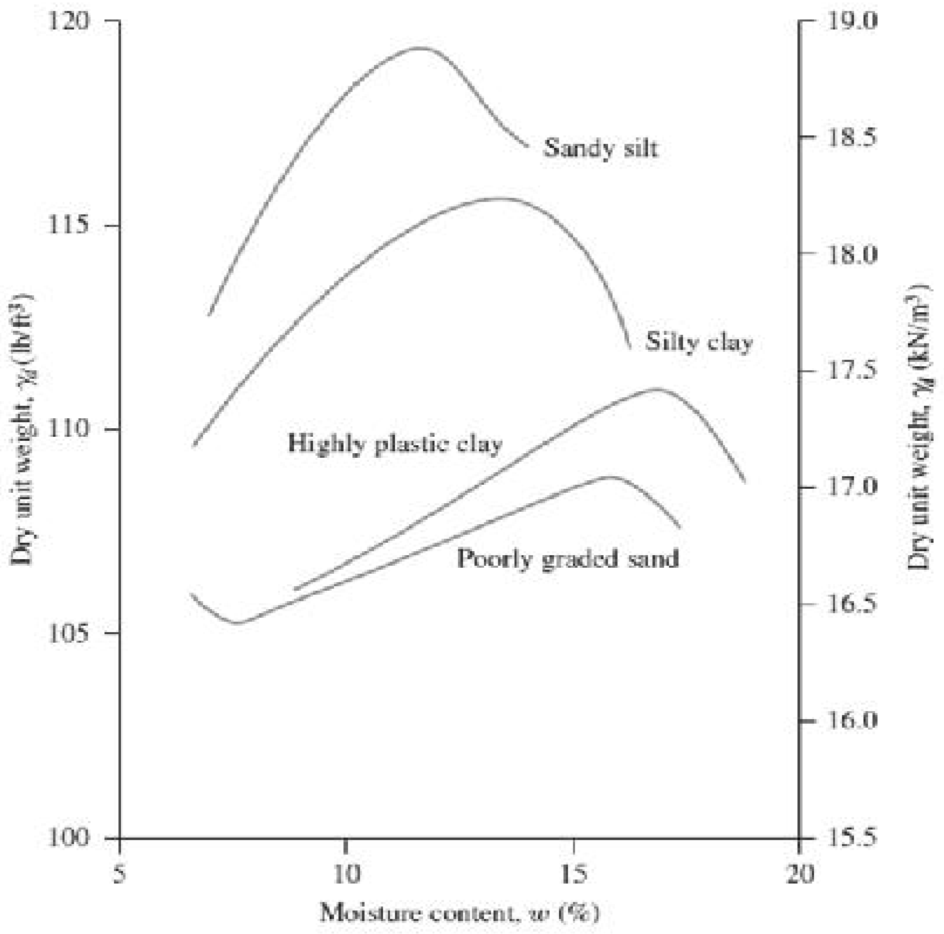
Effect of Soil Type:

•grain-size distribution

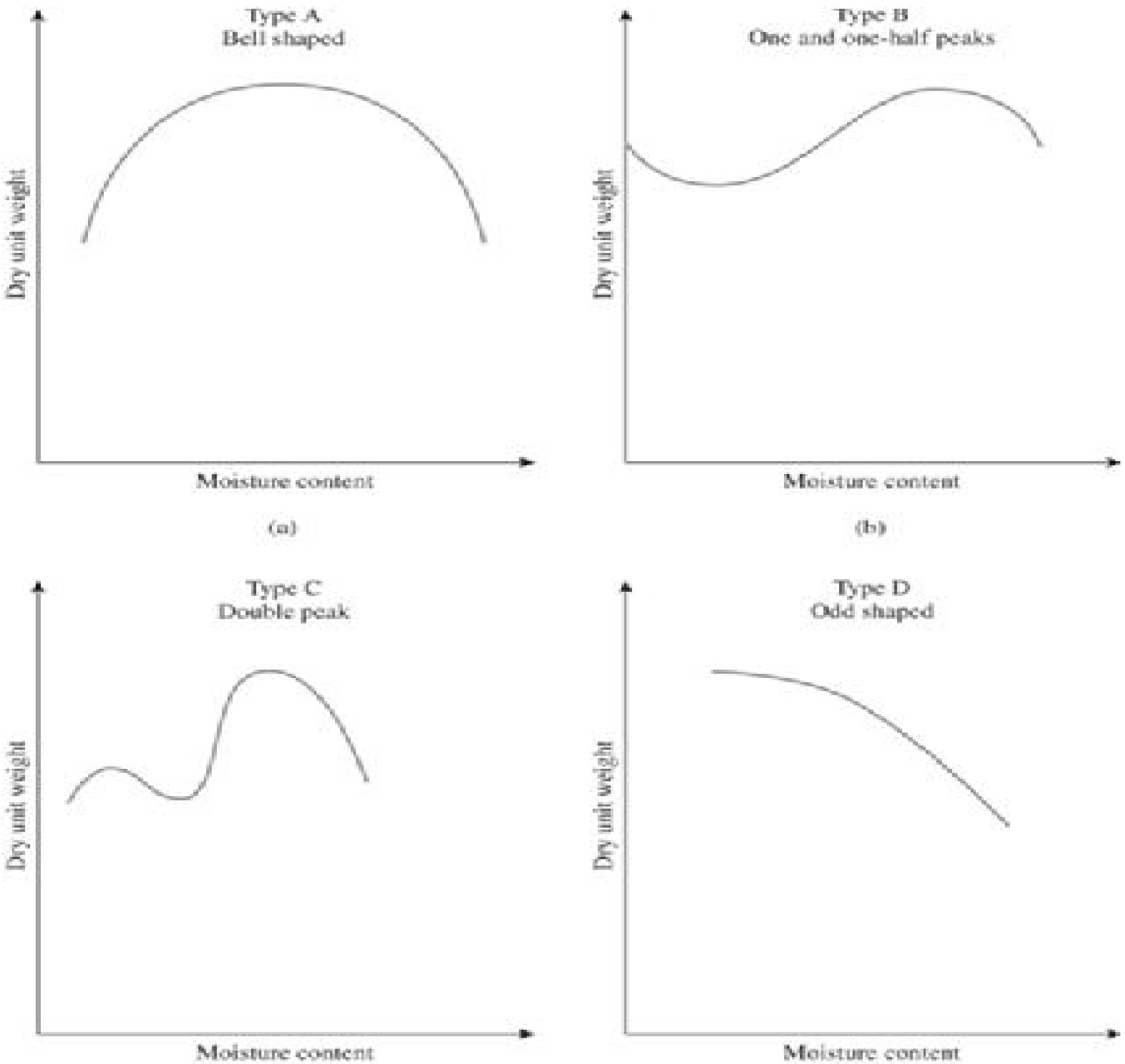
•shape of the soil grains

•specific gravity of soil solids

•amount and type of clay minerals present



Type of compaction curve (Figure 6.5)	Description of curve	Liquid limit
A	Bell shaped	Between 30 to 70
B	1-1/2 peak	Less than 30
C	Double peak	Less than 30 and those greater than 70
D	Odd shaped	Greater than 70



Effect of Compaction Effort:

$$E = \frac{\begin{pmatrix}\text{Number}\\ \text{of blows}\\ \text{per layer}\end{pmatrix} \times \begin{pmatrix}\text{Number}\\ \text{of}\\ \text{layers}\end{pmatrix} \times \begin{pmatrix}\text{Weight}\\ \text{of}\\ \text{hammer}\end{pmatrix} \times \begin{pmatrix}\text{Height of}\\ \text{drop of}\\ \text{hammer}\end{pmatrix}}{\text{Volume of mold}}$$

$$E = \frac{(25)(3)\left(\frac{2.5 \times 9.81}{1000}\ \text{kN}\right)(0.305\ \text{m})}{944 \times 10^{-6}\ \text{m}^3}$$

$$= 594\ \text{kN-m/m}^3 \approx 600\ \text{kN-m/m}^3$$

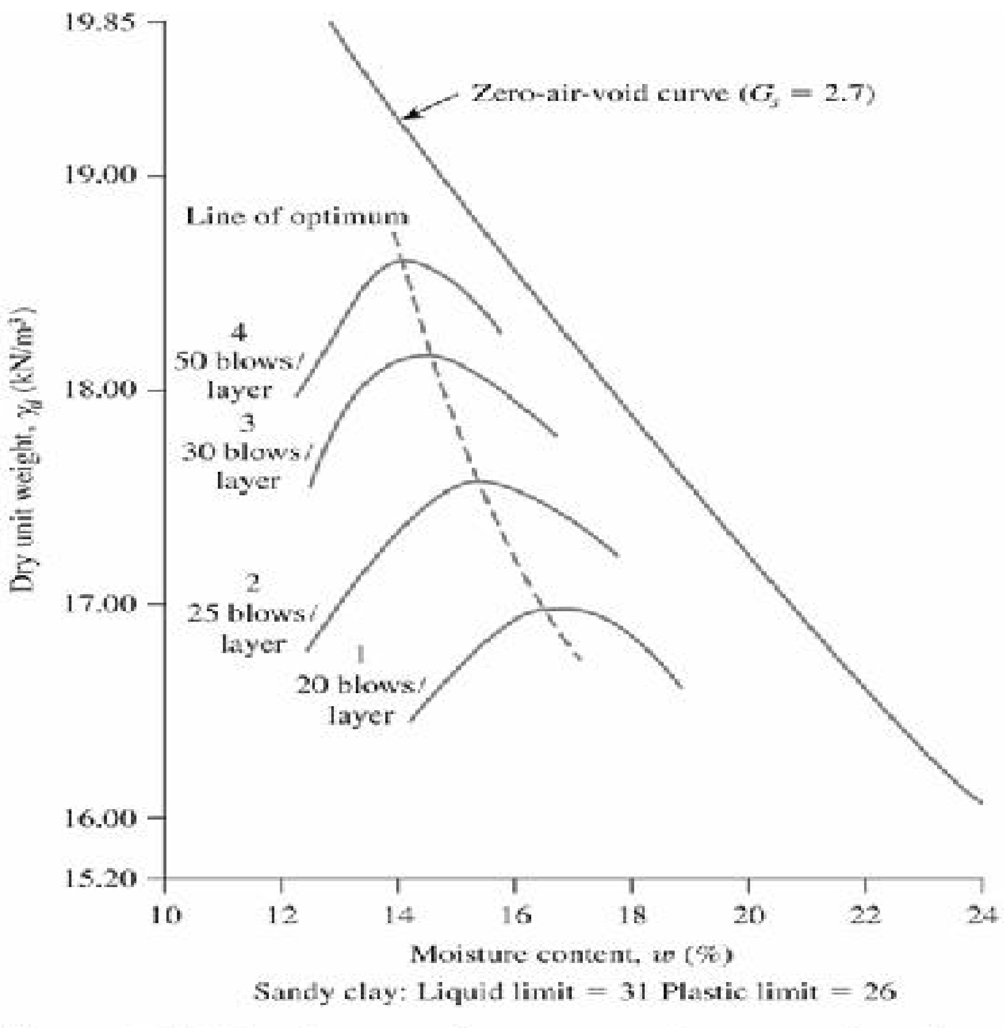
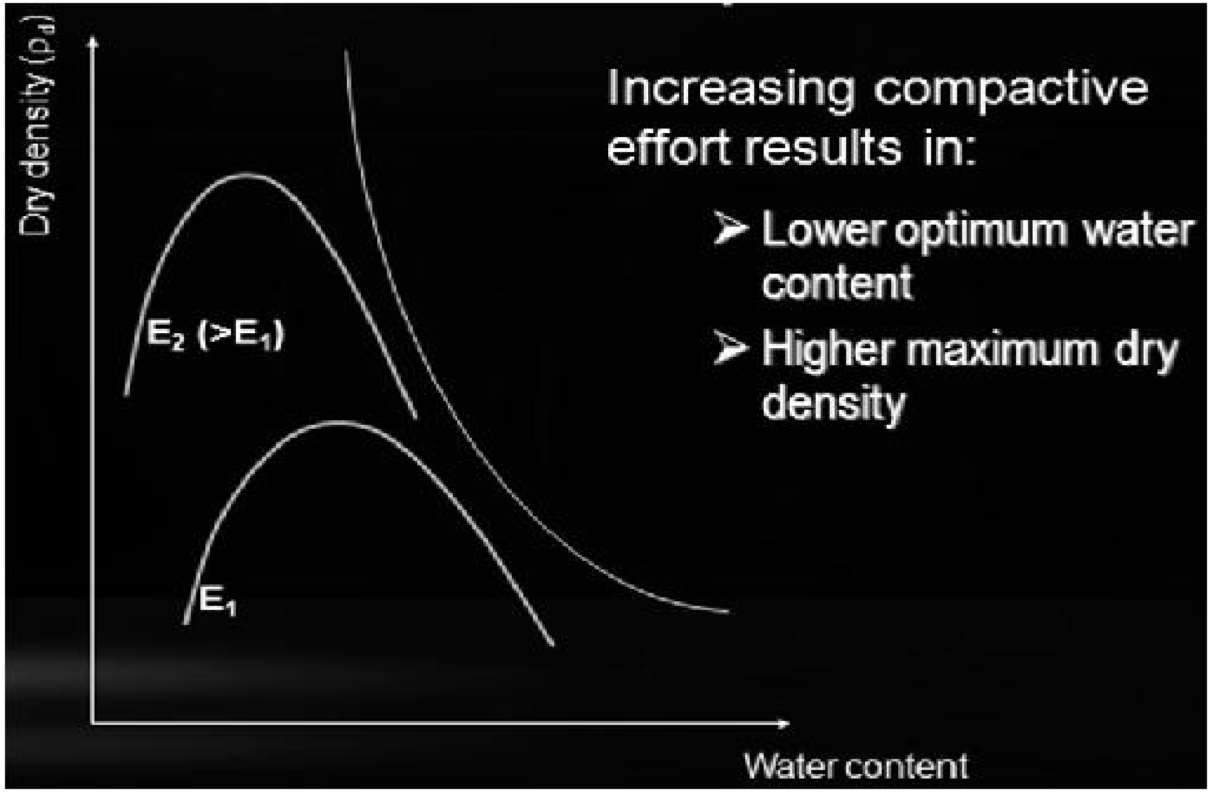


Figure 6.6 Effect of compaction energy on the compaction of a sandy clay

From the preceding observation we can say that:

•As the compaction effort is increased, the maximum dry unit weight of compaction is also increased.
•As the compaction effort is increased, the optimum moisture content is decreased to some extent.



Laboratory Compaction Test

-To obtain the compaction curve and define the optimum water content and maximum dry density for a specific compactive effort.

Standard Proctor:

•3 layers
•25 blows per layer
•2.5 kg hammer
•305 mm drop
•E= 594kN-m/m3

hammer

944 ml compaction mould

Modified Proctor:

• 5 layers
• 25 blows per layer
•4.54 kg hammer
• 450 mm drop
•E= 2700kN-m/m3

Figure 6.7 Comparison between standard Proctor hammer (left) and modified Proctor hammer (right) (*Courtesy of Braja M. Das, Henderson, Nevada*)

Table 6.1 Summary of Standard and Modified Proctor Compaction Test Specifications (ASTM D-698 and D-1557)

	Description	Method A	Method B	Method C
Physical data for the tests	Material	Passing No. 4 sieve	Passing 9.5 mm sieve	Passing 19 mm sieve
	Use	Used if 20% or less by weight of material is retained on No. 4 (4.75 mm) sieve	Used if more than 20% by weight of material is retained on No. 4 (4.75 mm) sieve and 20% or less by weight of material is retained on 9.5 mm sieve	Used if more than 20% by weight of material is retained on 9.5 mm sieve and less than 30% by weight of material is retained on 19 mm sieve
	Mold volume	944 cm^3	944 cm^3	2124 cm^3
	Mold diameter	101.6 mm	101.6 mm	152.4 mm
	Mold height	116.4 mm	116.4 mm	116.4 mm
Standard Proctor test	Weight of hammer	24.4 N	24.4 N	24.4 N
	Height of drop	305 mm	305 mm	305 mm
	Number of soil layers	3	3	3
	Number of blows/layer	25	25	56
Modified Proctor test	Weight of hammer	44.5 N	44.5 N	44.5 N
	Height of drop	457 mm	457 mm	457 mm
	Number of soil layers	5	5	5
	Number of blows/layer	25	25	56

• **Structure of Compacted Clay Soil:**

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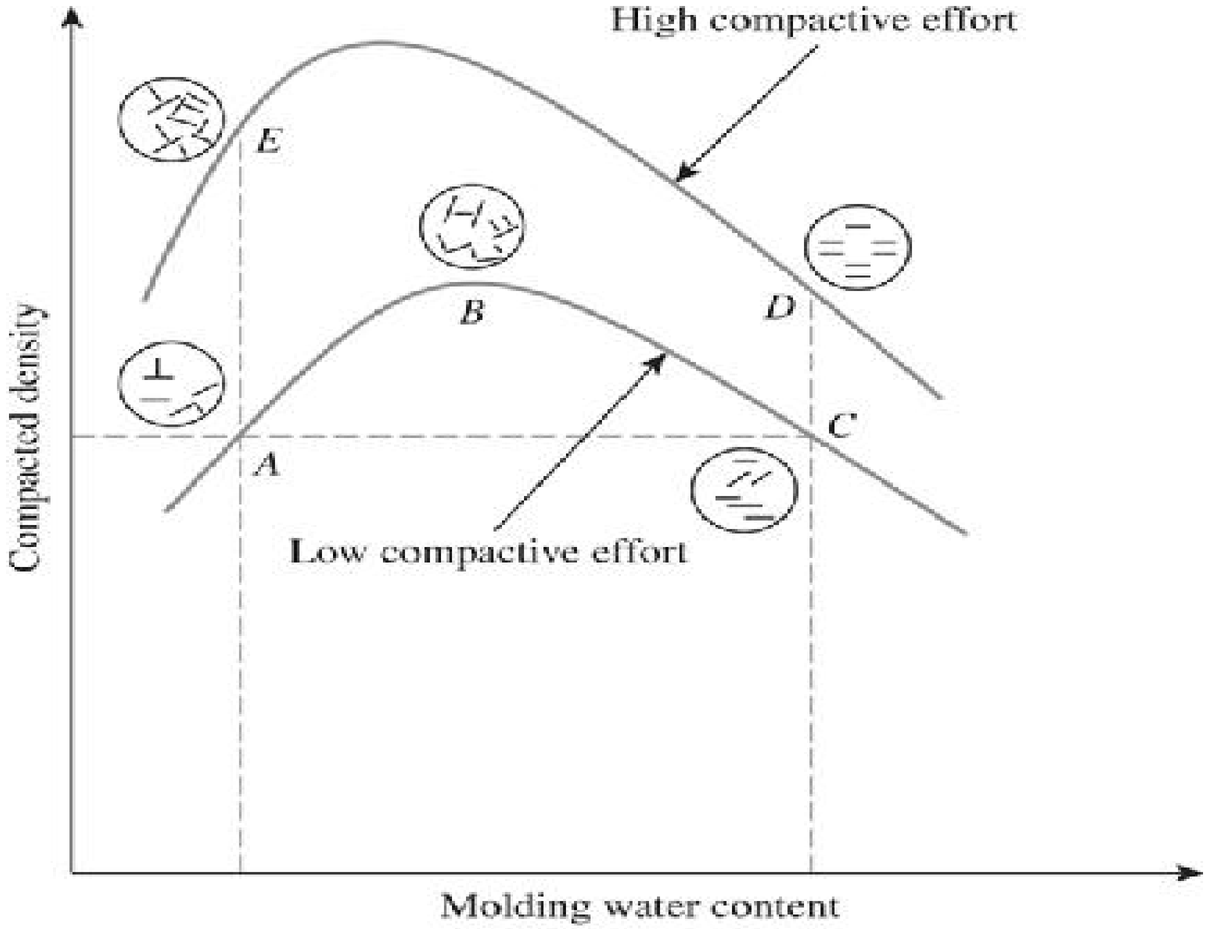


Figure 6.9
Effect of compaction on structure of clay soils (*Redrawn after Lambe, 1958a. With permission from ASCE.*)

Remarks on the previous curve:

At point *A, clay particles possess a flocculent structure. This* type of structure results because, at low moisture content, the diffuse double layers of ions surrounding the clay particles cannot be fully developed; hence, the inter-particle repulsion is reduced. This reduced repulsion results in a more random particle orientation and a lower dry unit weight.

When the moisture content of compaction is increased, as shown by point *B, the* diffuse double layers around the particles expand, which increases the repulsion between the clay particles and gives a lower degree of flocculation and a higher dry unit weight.

A continued increase in moisture content from *B to C expands the double layers more. This expansion* results in a continued increase of repulsion between the particles and thus a still greater degree of particle orientation and a more or less dispersed structure.

How is Soil Compacted in the Field؟

- Smooth-wheel rollers (or smooth- drum rollers).
- Pneumatic rubber-tired rollers.
- Sheepsfoot rollers.
- Vibratory rollers.
- Trucks &tractors.
- Hand tampers &rammers.
- Dynamic compaction.

Smooth-Wheel Rollers

Smooth-Wheel Rollers

- Suitable for finishing operations of fills with sandy & clayey soils.

- Provides 100% coverage under wheels.

- Ground contact pressures as high as 310 – 380kPa.

- Compacts effectively only to 200-300 mm; therefore, soil must be placed in *shallow* layers (lifts).

- Not suitable for producing high unit weights of compaction when used on thicker layers.

Pneumatic Rubber-Tired Rollers

Photo courtesy of Caterpiller Corporation
& UC Davis GeoPhoto Album

Photo courtesy of Professor
Sivakugan, James Cook University,
Australia

Pneumatic Rubber-Tired Rollers

•Are heavily loaded wagons with several rows of closely-spaced tires.

•Compaction is achieved by a combination of pressure & kneading action.

•Ground contact pressures as high as 600 – 700kPa.

•Produce 70% to %80coverage.

•Can be used for sandy & clayey soils.

Sheepsfoot Rollers

Sheepsfoot Rollers

•Are drums with a large number of projections.

•Area of projections ranges from 25 to 85 sq. cm.

•Provides kneading action.

•Ground contact pressures as high as 6900 –1380 kPa.

•Most effective in compacting clayey soils.

•Initial passes compact the lower portion of a lift; middle & top portions are compacted at a later stage.

•When the fill has been densified to some degree, the roller “walks out,” & entire weight is supported on pads resting on top of the fill.

Sheepsfoot Rollers

Vibratory padded drum roller (similar to a sheepsfoot roller) compacting clay.

Vibratory Rollers

Vibratory Rollers

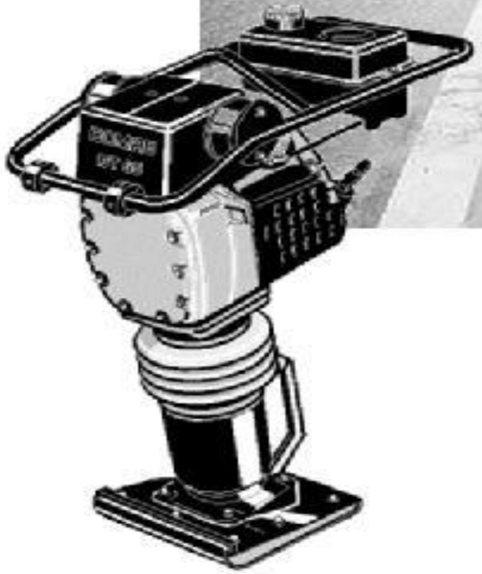
- Vibration is more effective for compacting sands and gravels than static pressure.

- Water conditioning is not as important for compacting sands and gravels as it is for compacting clays.

- The total force applied by a vibratory roller is equal to the weight of the roller plus the dynamic vibratory force.

- Can be attached to to smooth-wheel, pneumatic rubber- tired, or sheepsfoot rollers.

Hand Tampers



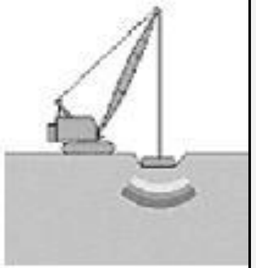
Hand Tampers

- Effective for compaction of granular soils (sands & gravels).

- Practically only used to compact such soils over a *limited area*.

Dynamic Compaction

Pounding the ground by a heavy weight

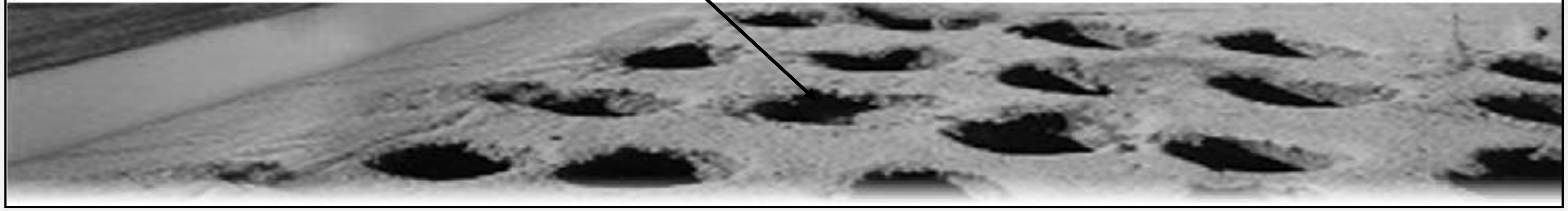


Suitable for granular soils, landfills and karst terrain with sinkholes.

)Slide courtesy of Professor Sivakugan, James Cook University, Australia(

Pounder (Tamper)

Crater created by the impact
)to be backfilled)



Dynamic Compaction

Vibroflotation

Suitable for *granular* soils

Practiced in several forms:
- •Vibro -compaction
- •Stone columns
- •Vibro -replacement

Vibroflot (vibrating unit)
Length = 2 - 3 m
Diameter = 0.3 -0.5
Mass = 2tonnes

(Slide courtesy of Professor Sivakugan, James Cook University, Australia)

(Lowered into the ground &vibrated)

Factors Affecting Field Compaction:

- Soil type
- Moisture content
- Thickness of lift
- Intensity of pressure applied
- Area over which the pressure is applied

Specifications for Field Compaction:

In most specifications for earthwork, the contractor is instructed to achieve a compacted field dry unit weight of 90 to 95% of the maximum dry unit weight determined in the laboratory by either the standard or modified Proctor test.

- Relative Compaction

$$R(\%) = \frac{\gamma_{d(\text{field})}}{\gamma_{d(\text{max—lab})}} \times 100$$

$$D_r = \left[\frac{\gamma_{d(\text{field})} - \gamma_{d(\text{min})}}{\gamma_{d(\text{max})} - \gamma_{d(\text{min})}}\right]\left[\frac{\gamma_{d(\text{max})}}{\gamma_{d(\text{field})}}\right]$$

$$R = \frac{R_0}{1 - D_r(1 - R_0)}$$

$$R_0 = \frac{\gamma_{d(\text{min})}}{\gamma_{d(\text{max})}}$$

Correlation between *R and Dr for granular soils:* $\longrightarrow$ $R = 80 + 0.2D_r$

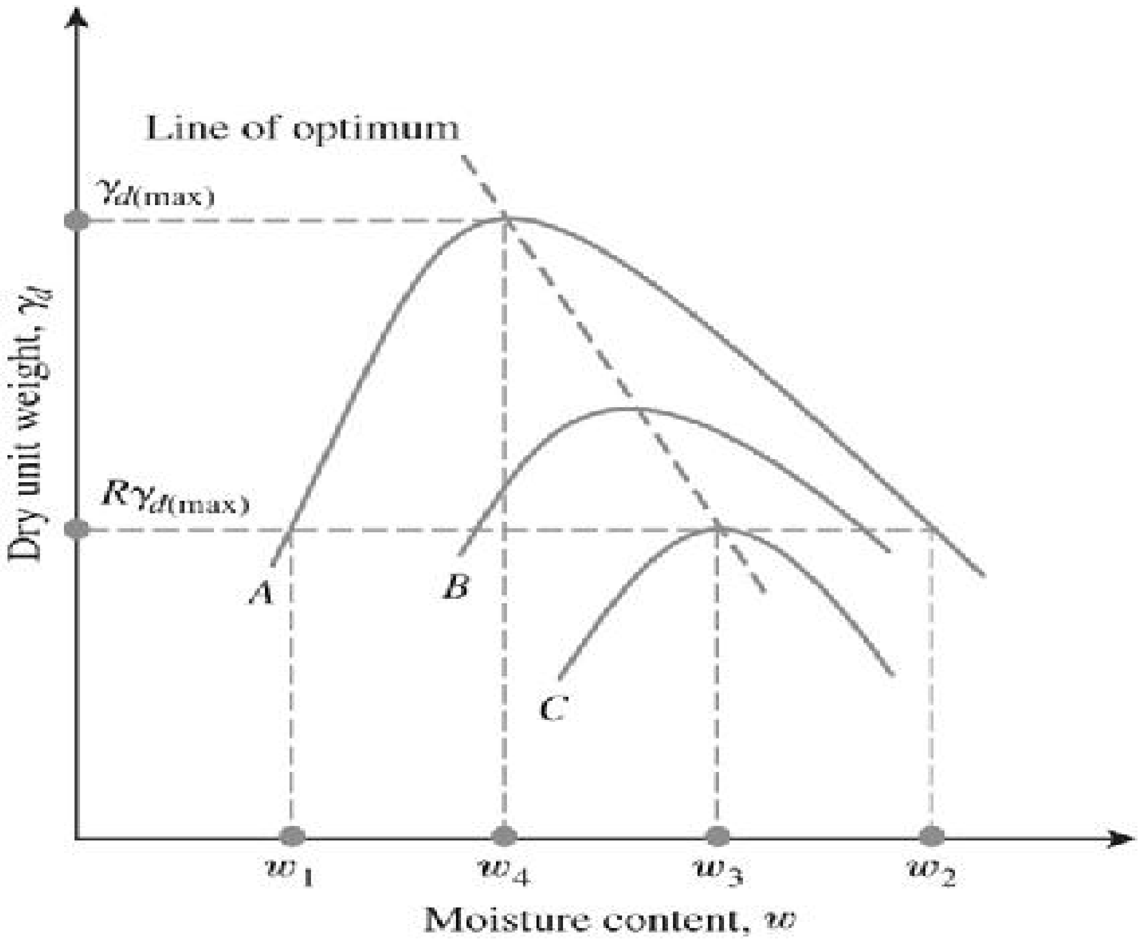


Figure 6.21 Most economical compaction condition

Table 6.2 Requirements to Achieve R = 95 to 100% (based on standard Proctor maximum dry unit-weight)[a]

Equipment type	Applicability	Requirements for compaction of 95 to 100% standard Proctor maximum dry unit weight					Possible variations in equipment
		Compacted lift thickness	Passes or coverages	Dimensions and weight of equipment			
				Soil type	*Foot contact area*	*Foot contact pressures*	
Sheepsfoot rollers	For fine-grained soils or dirty coarse-grained soils with more than 20% passing the No. 200 sieve. Not suitable for clean, coarse-grained soils. Particularly appropriate for compaction of impervious zone for earth dam or linings where bonding of lifts is important.	150 mm	4 to 6 passes for fine-grained soil	Fine-grained soil, $PI > 30$	30 to 80 cm^2	1700 to 3400 kN/m^2	For earth dam, highway, and airfield work, a drum of 1.5 m diameter, loaded to 45 to 90 kN per linear meter of drum is generally utilized. For smaller projects, a 1 m diameter drum, loaded to 22 to 50 kN per linear meter of drum, is used. Foot contact pressure should be regulated to avoid shearing the soil on the third or fourth pass.
				Fine-grained soil, $PI < 30$	45 to 90 cm^2	1400 to 2800 kN/m^2	
			6 to 8 passes for coarse-grained soil	Coarse-grained soil	65 to 90 cm^2	1000 to 1800 kN/m^2	
				Efficient compaction of soils on the wet side of the optimum requires less contact pressure than that required by the same soils at lower moisture contents.			
Rubber-tired rollers	For clean, coarse-grained soils with 4 to 8% passing the No. 200 sieve.	250 mm	3 to 5 coverages	Tire inflation pressures of 400 to 550 kN/m^2 for clean granular material or base course and subgrade compaction. Wheel load, 80 to 110 kN.			A wide variety of rubber-tired compaction equipment is available. For cohesive soils, light-wheel loads, such as provided by wobble-wheel equipment, may be substituted for heavy-wheel loads if lift thickness is decreased. For cohesionless soils, large-size tires are desirable to avoid shear and rutting.
	For fine-grained soils or well-graded, dirty, coarse-grained soils with more than 8% passing the No. 200 sieve.	150 to 200 mm	4 to 6 coverages	Tire inflation pressures in excess of 450 kN/m^2 for fine-grained soils of high plasticity. For uniform clean sands or silty fine sands, use larger-size tires with pressures of 280 to 350 kN/m^2			

Smooth-wheel rollers	Appropriate for subgrade or base course compaction of well-graded sand-gravel mixtures.	200 to 300 mm	4 coverages	Tandem-type rollers for base-course or sub-grade compaction, 90 to 135 kN weight, 53 to 88 kN per linear meter of width of rear roller.	Three-wheel rollers are obtainable in a wide range of sizes. Two-wheel tandem rollers are available in the 9 to 180 kN weight range. Three-axle tandem rollers are generally used in the 90 to 180 kN weight range. Very heavy rollers are used for proof rolling of subgrade or base course.
	May be used for fine-grained soils other than in earth dams. Not suitable for clean, well-graded sands or silty, uniform sands.	150 to 200 mm	6 coverages	Three-wheel roller for compaction of fine-grained soil; weights from 45 to 55 kN for materials of low plasticity to 90 kN for materials of high plasticity.	
Vibrating baseplate compactors	For coarse-grained soils with less than about 12% passing the No. 200 sieve. Best suited for materials with 4 to 8% passing the No. 200 sieve, placed thoroughly wet.	200 to 250 mm	3 coverages	Single pads or plates should weigh no less than 0.9 kN. May be used in tandem where working space is available. For clean, coarse-grained soil, vibration frequency should be no less than 1600 cycles per minute.	Vibrating pads or plates are available, hand-propelled or self-propelled, single or in gangs, with width of coverage from 0.45 to 4.5 m. Various types of vibrating-drum equipment should be considered for compaction in large areas.
Crawler tractor	Best suited for coarse-grained soils with less than 4 to 8% passing the No. 200 sieve, placed thoroughly wet.	250 to 300 mm	3 to 4 coverages	No smaller than D8 tractor with blade, 153 kN weight, for high compaction.	Tractor weights up to 265 kN
Power tamper or rammer	For difficult access, trench backfill. Suitable for all inorganic soils.	100 to 150 mm for silt or clay; 150 mm for coarse-grained soils	2 coverages	130 N minimum weight. Considerable range is tolerable, depending on materials and conditions.	Weights up to 1.1 kN, foot diameter, 100 to 250 mm

* After U.S. Navy (1971). Published by U.S. Government Printing Office

• Determination of Field Unit Weight of Compaction:

-The field unit weight of compaction an be determined by one of the following methods:

- Sand cone method

- Rubber balloon method

- Nuclear method

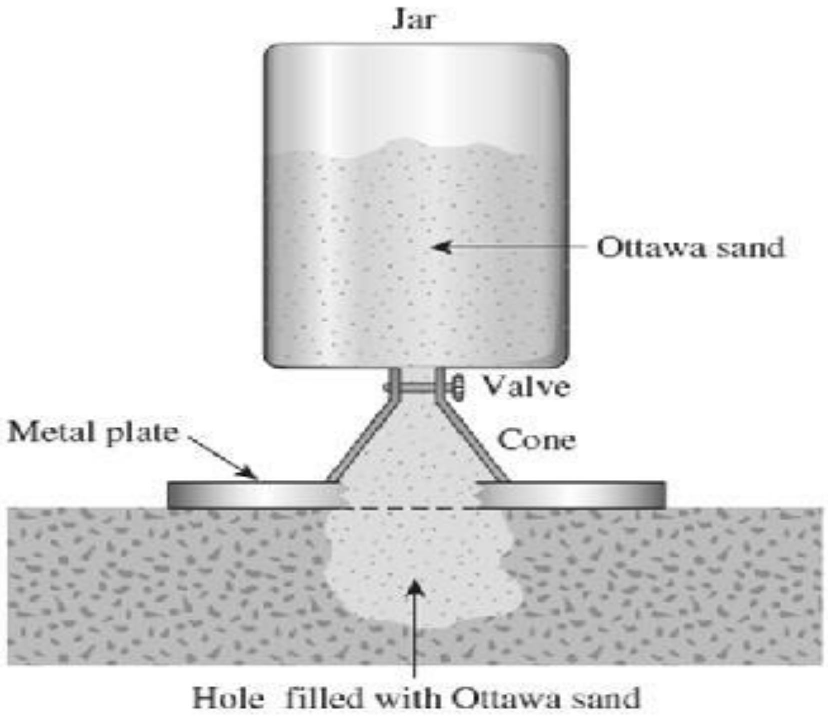
Sand Cone Method (ASTM Designation D-1556)

Figure 6.22
Glass jar filled with Ottawa sand with sand cone attached (*Courtesy of Braja M. Das, Henderson, Nevada*)

Sand Cone Method (ASTM Designation D-1556):

If the weight of the moist soil excavated from the hole (*W2*) is *determined and the moisture content of* the excavated soil is known, the dry weight (*W3*) of the soil can be obtained as:

$$W_3 = \frac{W_2}{1 + \frac{w(\%)}{100}}$$



$$W_5 = W_1 - W_4$$

where W_5 = weight of sand to fill the hole and cone.

The volume of the excavated hole can then be determined as

$$V = \frac{W_5 - W_c}{\gamma_{d(\text{sand})}}$$

where W_c = weight of sand to fill the cone only

$\gamma_{d(\text{sand})}$ = dry unit weight of Ottawa sand used

Rubber balloon method:

Figure 6.24 Calibrated vessel used with rubber balloon (not shown) (*Courtesy of John Hester, Carterville, Illinois*)

Nuclear method:

Figure 6.25 Nuclear density meter (*Courtesy of Braja M. Das, Henderson, Nevada*)

Example 1

The laboratory test results of a standard Proctor test are given in the following table.

Volume of mold (cm^3)	Weight of moist soil in mold (N)	Moisture content, w (%)
944	16.81	10
944	17.84	12
944	18.41	14
944	18.33	16
944	17.84	18
944	17.35	20

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a. Determine the maximum dry unit weight of compaction and the optimum moisture content.

b. Calculate and plot γ_d versus the moisture content for degree of saturation, S = 80, 90, and 100% (i.e., γ_{zav}). Given: $G_s = 2.7$.

Solution

Part a

The following table can be prepared.

Volume of mold V_m (cm^3)	Weight of soil, W (N)	Moist unit weight, γ (kN/m^3)[a]	Moisture content, w (%)	Dry unit weight, γ_d (kN/m^3)[b]
944	16.81	17.81	10	16.19
944	17.84	18.90	12	16.87
944	18.41	19.50	14	17.11
944	18.33	19.42	16	16.74
944	17.84	18.90	18	16.02
944	17.35	18.38	20	15.32

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[a] $\gamma = \dfrac{W}{V_m}$

[b] $\gamma_d = \dfrac{\gamma}{1 + \dfrac{w\%}{100}}$

The plot of γ_d versus w is shown at the bottom of Figure 6.8. From the plot, we see that the maximum dry unit weight $\gamma_{d(\max)} = \mathbf{17.15\ kN/m^3}$ and the optimum moisture content is **14.4%**.

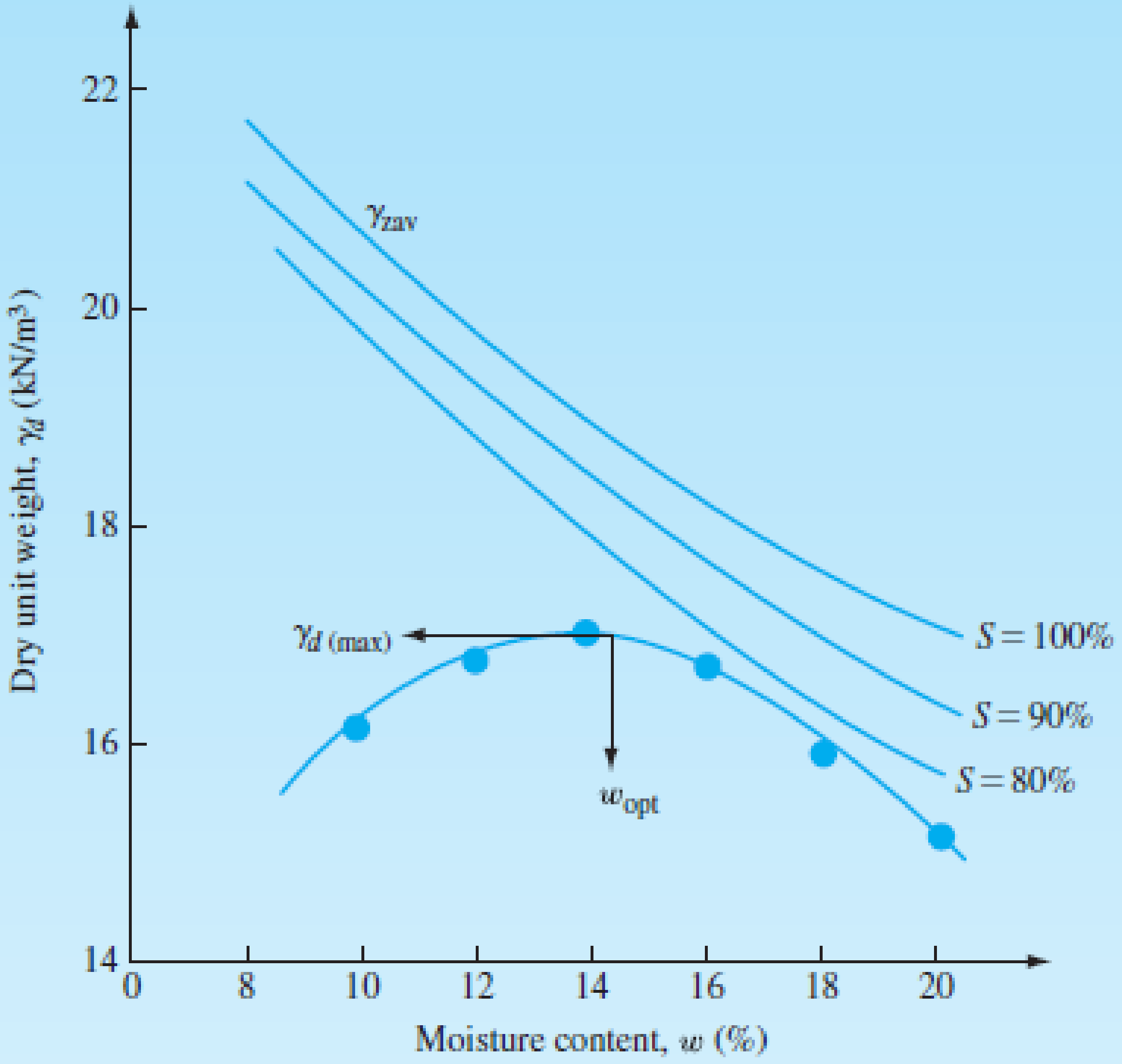

Figure 6.8 Moisture content–unit weight curves

Part b

From Eq. (6.3),

$$\gamma_d = \frac{G_s \gamma_w}{1 + \dfrac{G_s w}{S}}$$

The following table can be prepared.

		γ_d (kN/m³)		
G_s	w (%)	S = 80%	S = 90%	S = 100%
2.7	8	20.84	21.37	21.79
2.7	10	19.81	20.37	20.86
2.7	12	18.85	19.48	20.01
2.7	14	17.99	18.65	19.23
2.7	16	17.20	17.89	18.50
2.7	18	16.48	17.20	17.83
2.7	20	15.82	16.55	17.20

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The plot of γ_d versus w for the various degrees of saturation is also shown in Figure 6.8.

Example 2

Laboratory compaction test results for a clayey silt are given in the following table.

Moisture content (%)	Dry unit weight (kN/m³)
6	14.80
8	17.45
9	18.52
11	18.9
12	18.5
14	16.9

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Following are the results of a field unit-weight determination test performed on the same soil by means of the sand cone method:

- Calibrated dry density of Ottawa sand = 1570 kg/m^3
- Calibrated mass of Ottawa sand to fill the cone = 0.545 kg
- Mass of jar + cone + sand (before use) = 7.59 kg
- Mass of jar + cone + sand (after use) = 4.78 kg
- Mass of moist soil from hole = 3.007 kg
- Moisture content of moist soil = 10.2%

Determine:

a. Dry unit weight of compaction in the field
b. Relative compaction in the field

Solution

Part a

In the field,

$$\text{Mass of sand used to fill the hole and cone} = 7.59 \text{ kg} - 4.78 \text{ kg} = 2.81 \text{ kg}$$

$$\text{Mass of sand used to fill the hole} = 2.81 \text{ kg} - 0.545 \text{ kg} = 2.265 \text{ kg}$$

$$\text{Volume of the hole } (V) = \frac{2.265 \text{ kg}}{\text{Dry density of Ottawa sand}}$$

$$= \frac{2.265 \text{ kg}}{1570 \text{ kg/m}^3} = 0.0014426 \text{ m}^3$$

$$\text{Moist density of compacted soil} = \frac{\text{Mass of moist soil}}{\text{Volume of hole}}$$

$$= \frac{3.007}{0.0014426} = 2.084.4 \text{ kg/m}^3$$

$$\text{Moist unit weight of compacted soil} = \frac{(2084.4)(9.81)}{1000} = 20.45 \text{ kN/m}^3$$

Hence,

$$\gamma_d = \frac{\gamma}{1 + \dfrac{w(\%)}{100}} = \frac{20.45}{1 + \dfrac{10.2}{100}} = \mathbf{18.56 \text{ kN/m}^3}$$

Part b

The results of the laboratory compaction test are plotted in Figure 6.27. From the plot, we see that $\gamma_{d(\max)} = 19\ \text{kN/m}^3$. Thus, from Eq. (6.19),

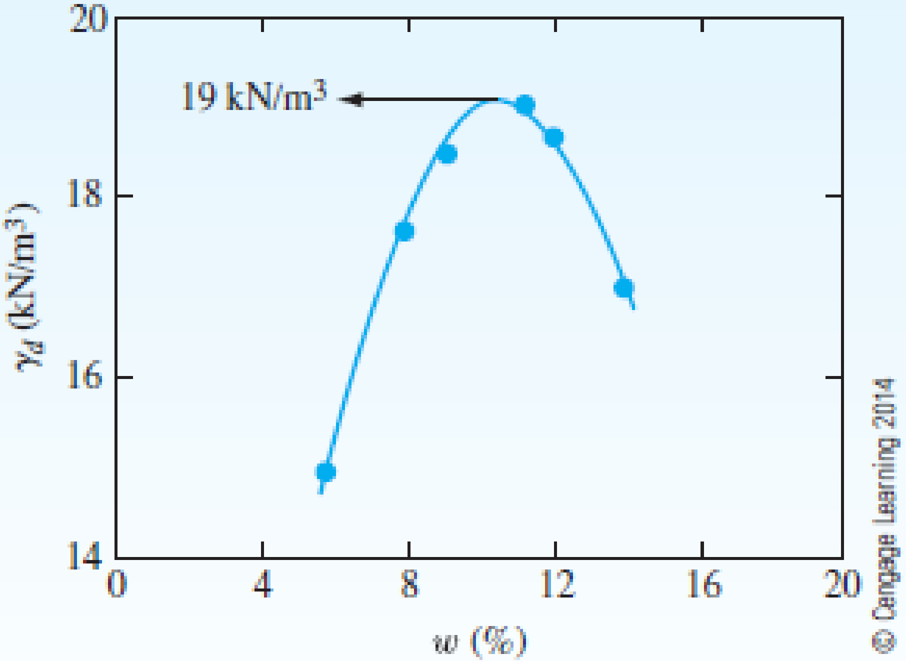


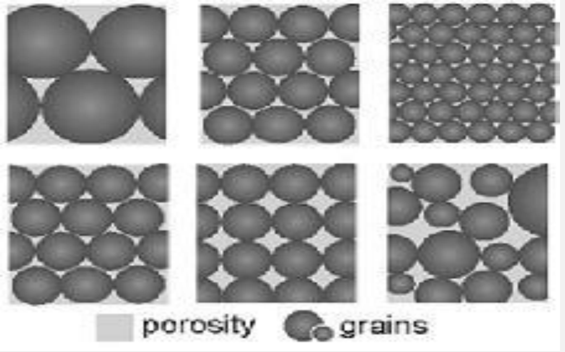
Figure 6.27 Plot of laboratory-compaction test results

$$R = \frac{\gamma_{d(\text{field})}}{\gamma_{d(\max)}} = \frac{18.56}{19.0} = \mathbf{97.7\%}$$

8- PERMEABILITY

<u>INTRODUCTION:</u>

Soils are permeable due to the *existence of interconnected voids through which water can flow* from points of high energy to points of low energy.



The study of the flow of water through permeable soil media is necessary for.

1. estimating ciluardyh suoirav rednu egapees dnuorgrednu fo ytitnauq eht .snoitidnoc
2. for investigating problems involving the pumping of water for underground construction.
3. for making stability analyses of earth dams and earth-retaining structures that are subject to seepage forces.

PERMEABILITY

- The property of soils that allows water to pass through them at some rate

- The property is a product of the granular nature of the soil, although it can be affected by other factors (such as water bonding in clays)

- Different soil has different permeabilities.

Bernoulli's Equation:

•According to Bernoulli's equation, the total head at a point in water under motion:

$$h = \underbrace{\frac{u}{\gamma_w}}_{\substack{\uparrow \\ \text{Pressure} \\ \text{head}}} + \underbrace{\frac{v^2}{2g}}_{\substack{\uparrow \\ \text{Velocity} \\ \text{head}}} + \underbrace{Z}_{\substack{\uparrow \\ \text{Elevation} \\ \text{head}}}$$

•Velocity head can be neglected

$$h = \frac{u}{\gamma_w} + Z$$

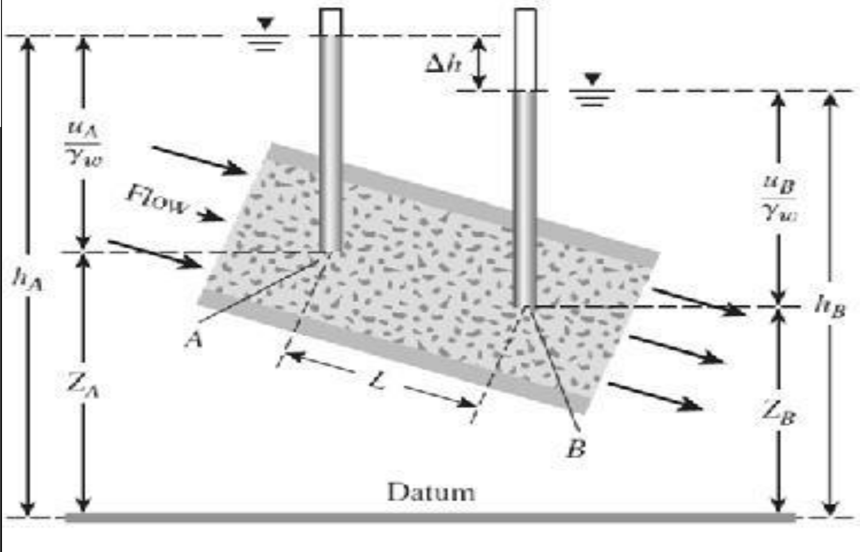
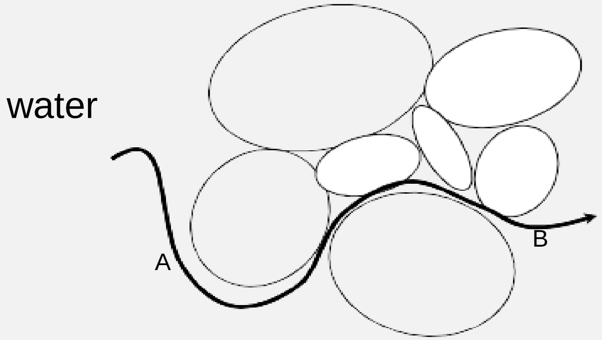


Figure 7.1 Pressure, elevation, and total heads for flow of water through soil

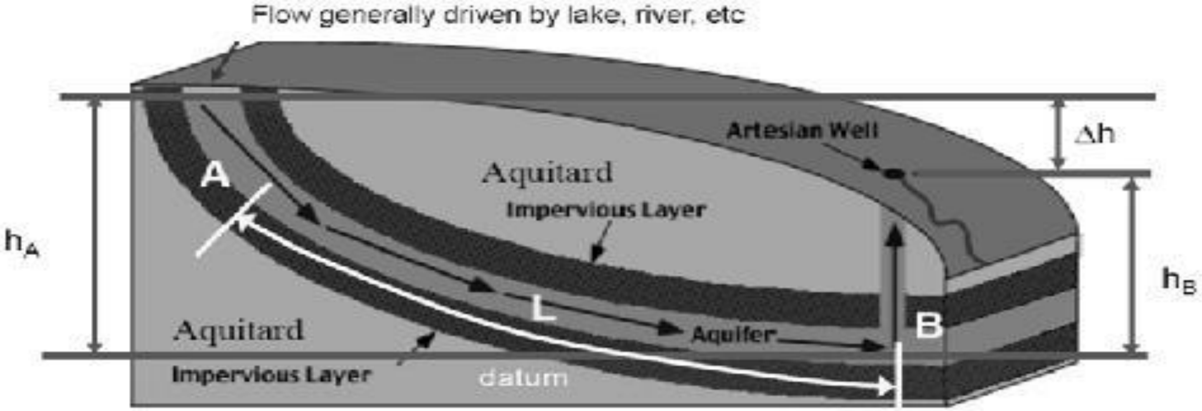
Some Notes:

If flow is from A to B, **total head** is higher at A than at B.

Energy is dissipated in overcoming the soil resistance and hence is the head loss.



Hydraulic Gradient:



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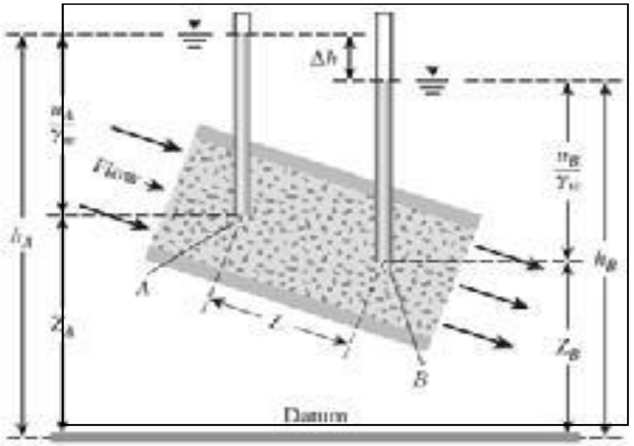


Figure 7.1 Pressure, elevation, and total heads for flow of water through soil

•The loss of head between two points, *A and B, can be given by:*

$$\Delta h = h_A - h_B = \left(\frac{u_A}{\gamma_w} + Z_A\right) - \left(\frac{u_B}{\gamma_w} + Z_B\right)$$

•The head loss, Δ*h, can be expressed in a nondimensional form as:*

$$i = \frac{\Delta h}{L}$$

itneidarG ciluardyh eht si ,

•The variation of the velocity (v) with the hydraulic gradient (*i) is as shown* in Figure 7.2 . This figure is divided into three zones:

1. Laminar flow zone (Zone I)
2. Transition zone (Zone II)
3. Turbulent flow zone (Zone III)

-In most soils, the flow of water through the void spaces can be considered laminar; thus,

$$v \propto i$$

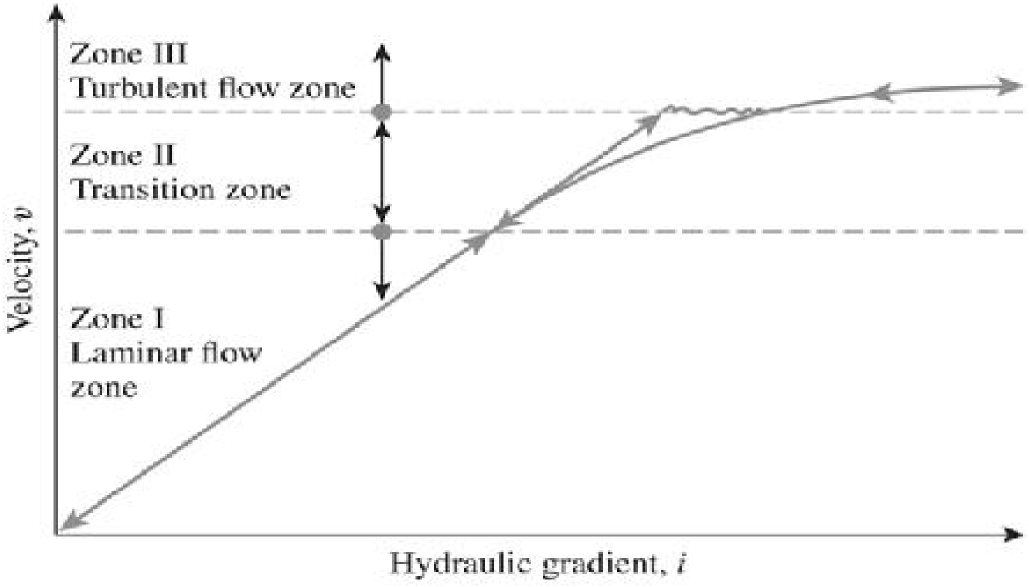


Figure 7.2 Nature of variation of v with hydraulic gradient, i

Darcy’s Law:

)Henry Philibert Gaspard Darcy, (1803-1858)

Velocity (**v**tneidarg ciluardyh eht ot lanoitroporp si wolf fo (
(**i**) - Darcy (1856)

$$v = ki$$

v is the discharge velocity of water based on the gross cross-sectional area of the soil.

Permeability

- or hydraulic conductivity
- unit of velocity (cm/s)

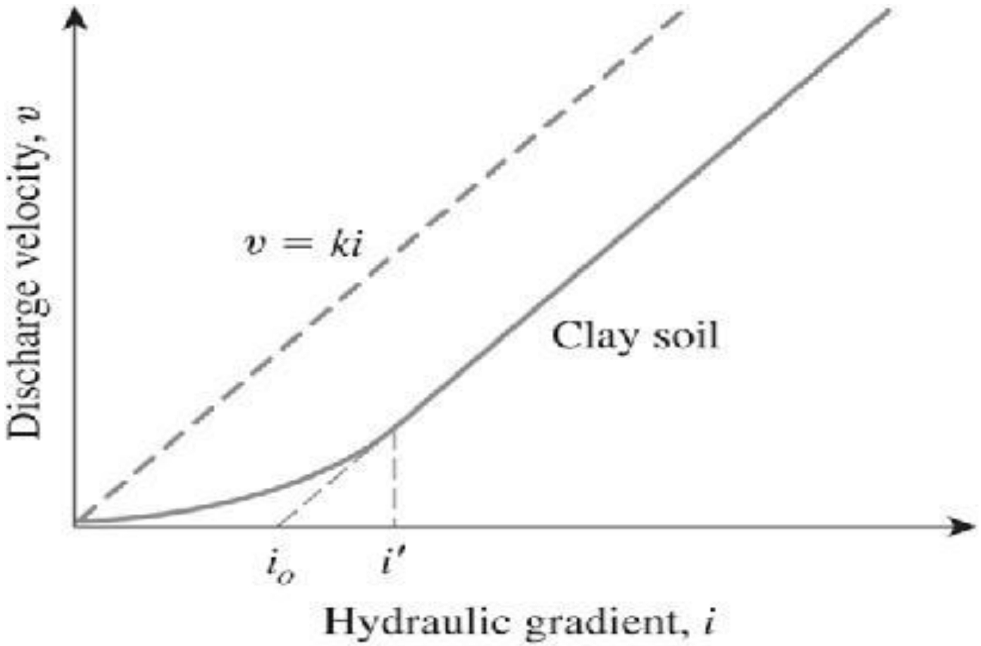


Figure 7.4
Variation of discharge velocity with hydraulic gradient in clay

$$v = k(i - i_0) \qquad (\text{for } i \geq i')$$

$$v = ki^m \qquad (\text{for } i < i')$$

Hansbo (1960)

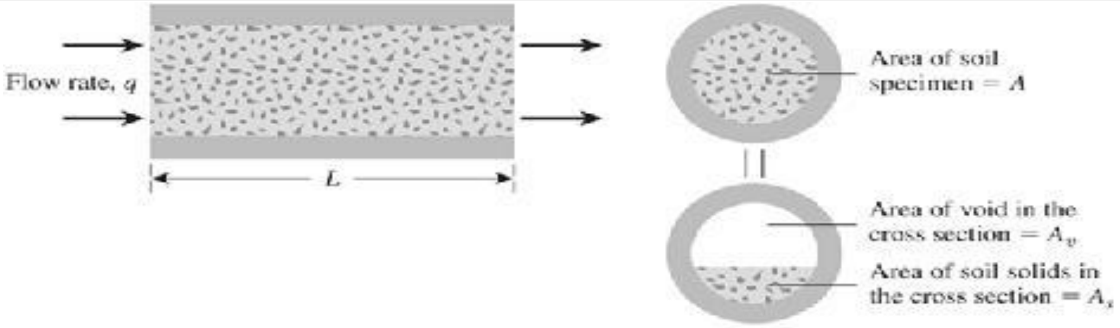


Figure 7.3 Derivation of Eq. (7.10)

$$q = vA = A_v v_s \tag{7.7}$$

where v_s = seepage velocity
A_v = area of void in the cross section of the specimen

However,

$$A = A_v + A_s \tag{7.8}$$

where A_s = area of soil solids in the cross section of the specimen.

Combining Eqs. (7.7) and (7.8) gives

$$q = v(A_v + A_s) = A_v v_s$$

or

$$v_s = \frac{v(A_v + A_s)}{A_v} = \frac{v(A_v + A_s)L}{A_v L} = \frac{v(V_v + V_s)}{V_v} \tag{7.9}$$

where V_v = volume of voids in the specimen
V_s = volume of soil solids in the specimen

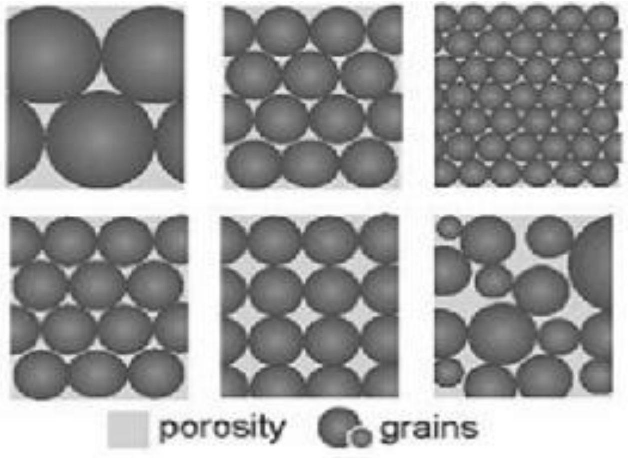
Equation (7.9) can be rewritten as

$$v_s = v\left[\frac{1 + \left(\dfrac{V_v}{V_s}\right)}{\dfrac{V_v}{V_s}}\right] = v\left(\frac{1+e}{e}\right) = \frac{v}{n} \tag{7.10}$$

Hydraulic Conductivity:

Coefficient permeability = hydraulic conductivity

Depend on:



- Fluid viscosity
- Pore-size distribution
- Grain size distribution
- Void ratio
- Roughness of mineral particles
- Degree of soil saturation
    - Permeability of saturated soil is lower and increases rapidly with degree of saturation

In clayey soils, structure plays an important role in hydraulic conductivity.

Factors affect the permeability of clays are:

- The ionic concentration
- The thickness of layers of water held to the clay particles.

Table 7.1 Typical Values of Hydraulic Conductivity of Saturated Soils

Soil type	k (cm/sec)
Clean gravel	100−1.0
Coarse sand	1.0−0.01
Fine sand	0.01−0.001
Silty clay	0.001−0.00001
Clay	<0.000001

- The hydraulic conductivity of a soil is also related to the properties of the fluid flowing through it by the equation

$$k = \frac{\gamma_w}{\eta} \overline{K}$$

where γ_w = unit weight of water
η = viscosity of water
$\overline{K}$ = absolute permeability

From which;

$$\frac{k_{T_1}}{k_{T_2}} = \left(\frac{\eta_{T_2}}{\eta_{T_1}}\right)\left(\frac{\gamma_{w(T_1)}}{\gamma_{w(T_2)}}\right)$$

assume that $\gamma_{w(T_1)} \simeq \gamma_{w(T_2)}$ $\Longrightarrow$

$$k_{20C} = \left(\frac{\eta_{TC}}{\eta_{20C}}\right) k_{TC}$$

Table 7.2 Variation of $\eta_{T^\circ C}/\eta_{20^\circ C}$

Temperature, T (°C)	$\eta_{T^\circ C}/\eta_{20^\circ C}$	Temperature, T (°C)	$\eta_{T^\circ C}/\eta_{20^\circ C}$
15	1.135	23	0.931
16	1.106	24	0.910
17	1.077	25	0.889
18	1.051	26	0.869
19	1.025	27	0.850
20	1.000	28	0.832
21	0.976	29	0.814
22	0.953	30	0.797

• **Laboratory Determination of Hydraulic Conductivity**

1. Constant Head

2. Falling Head

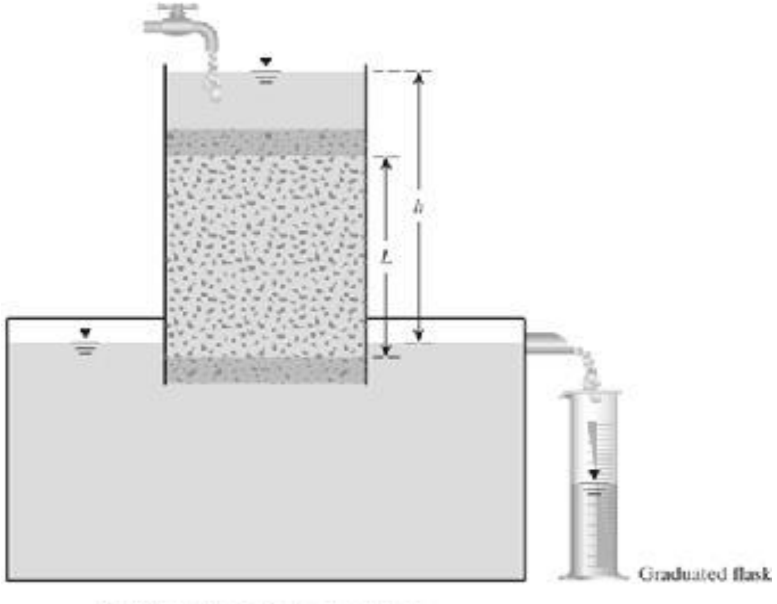


Figure 7.5 Constant-head permeability test

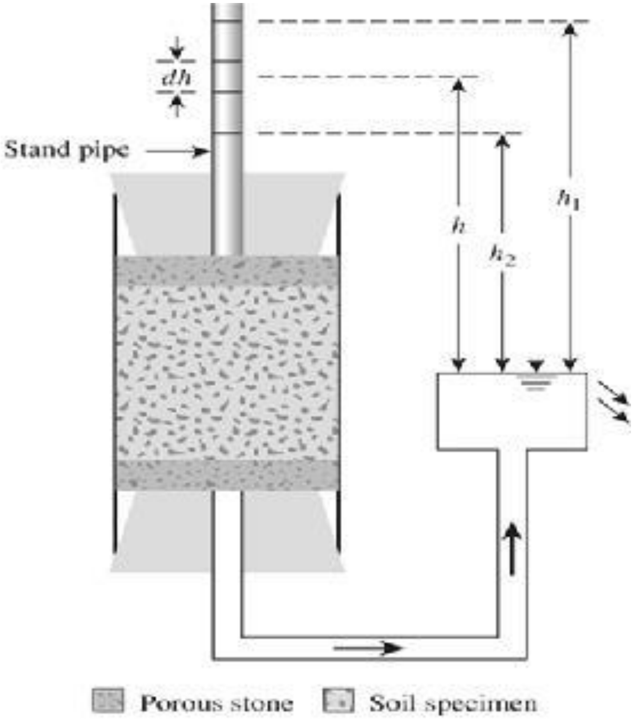
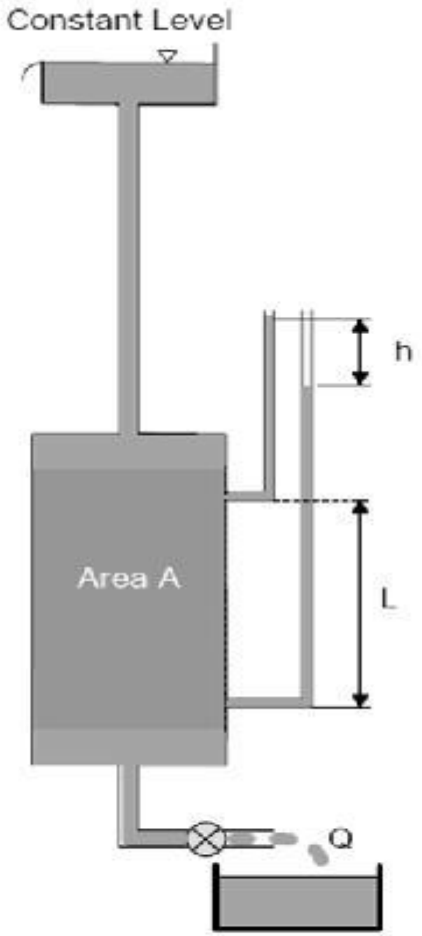


Figure 7.6 Falling-head permeability test

1. Constant Head

17



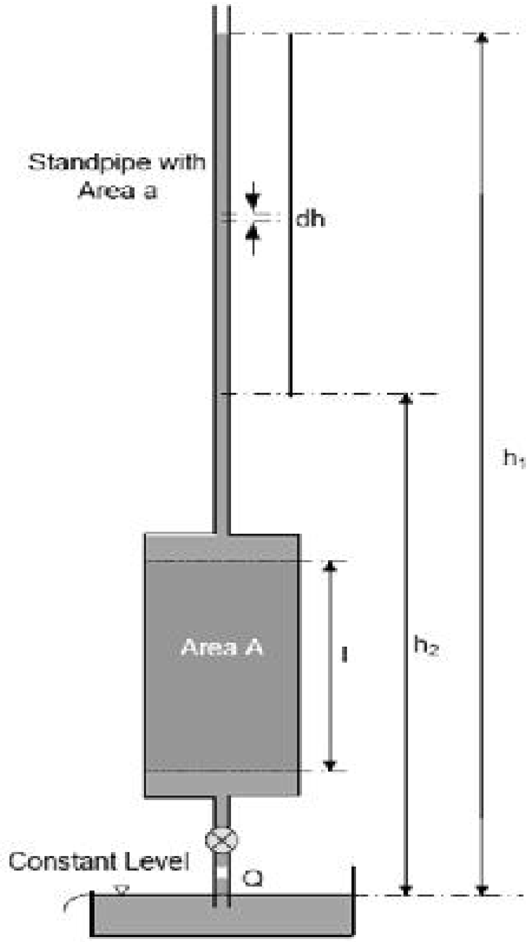
for coarse soil

$$Q = A.v.t$$
$$= A.(k.i).t$$
$$= A.(k\frac{h}{L}).t$$
$$k = \frac{QL}{Aht} \qquad [m/s]$$

Q = volume of water collected
A = area of cross section of the soil sample
T = duration of collection of water

2. Falling Head

18



for fine soil

$$q = k\frac{h}{L}.A = -a\frac{dh}{dt}$$

$$dt = \frac{aL}{Ak}\left(-\frac{dh}{h}\right)$$

$$\int_0^{t_1} dt = -\frac{aL}{Ak}\int_{h_2}^{h_1}\frac{dh}{h}$$

$$t = \frac{aL}{Ak}\ln\frac{h_1}{h_2}$$

$$\boxed{k = \frac{aL}{At}\ln\frac{h_1}{h_2}} \quad \boxed{= 2.33\frac{aL}{At}\log\frac{h_1}{h_2}}$$

q = rate of flow
a = cross sectional area of standpipe
A = cross section of the soil sample

- **Relationships for Hydraulic Conductivity —**
- **Granular Soil:**
- 1.Hazen (1930) For fairly uniform sand (small uniformity coefficient

$$k\ (\text{cm/sec}) = cD_{10}^2$$

where c = a constant that varies from 1.0 to 1.5
D_{10} = the effective size, in mm

***Important**

2. Kozeny-Carman (Kozeny, 1927; Carman, 1938, 1956) (for Sandy soil)

$$k = \frac{1}{C_s S_s^2 T^2} \frac{\gamma_w}{\eta} \frac{e^3}{1+e}$$

C_s = shape factor, which is a function of the shape of flow channels
S_s = specific surface area per unit volume of particles
T = tortuosity of flow channels
γ_w = unit weight of water
η = viscosity of permeant
e = void ratio

3.Carrier (2003)

$$k = 1.99 \times 10^4 \left[\frac{100\%}{\Sigma \frac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}}}\right]^2 \left(\frac{1}{SF}\right)^2 \left(\frac{e^3}{1+e}\right)$$

f_i = fraction of particles between two sieve sizes, in percent
(*Note:* larger sieve, l; smaller sieve, s)
SF = shape factor

Note: $k \propto \frac{e^3}{1+e}$ **Important**

U.S. Navy ,1971 Hydraulic Conductivity—Granular Soil-

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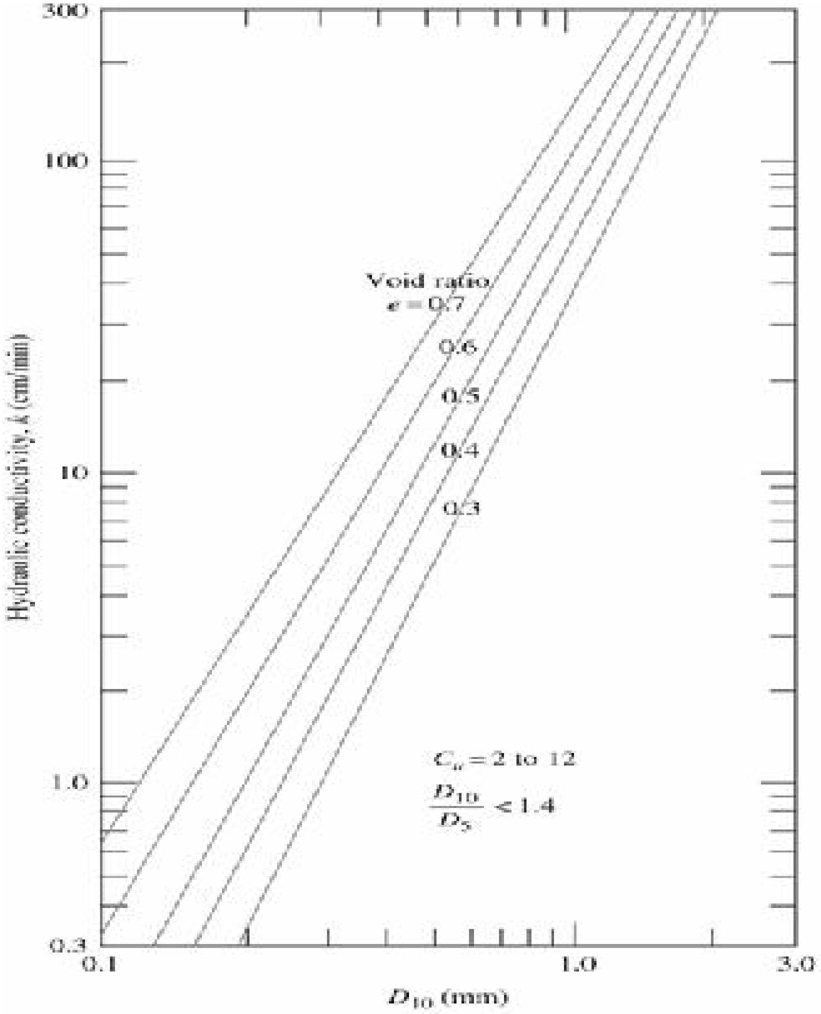

Figure 7.9 Hydraulic conductivity of granular soils (*After U.S. Department of Navy, 1971*)

• Relationships for Hydraulic Conductivity—Cohesive Soil:

1. Taylor (1948)

$$\log k = \log k_o - \frac{e_o - e}{C_k}$$

k_o = *in situ* hydraulic conductivity at a void ratio e_o
k = hydraulic conductivity at a void ratio e
C_k = hydraulic conductivity change index

*Good for $e_o > 2.5$. *In this equation,*
**The value of C_k *may be taken to be about* $0.5\ e_o$

2. Samarasinghe, *et al. (1982), fo*r normally consolidated Clay:

$$k = C\left(\frac{e^n}{1 + e}\right)$$

C and n are constants to be determined experimentally.

3. Mesri and Olson (1971)

$$\log k = A' \log e + B'$$

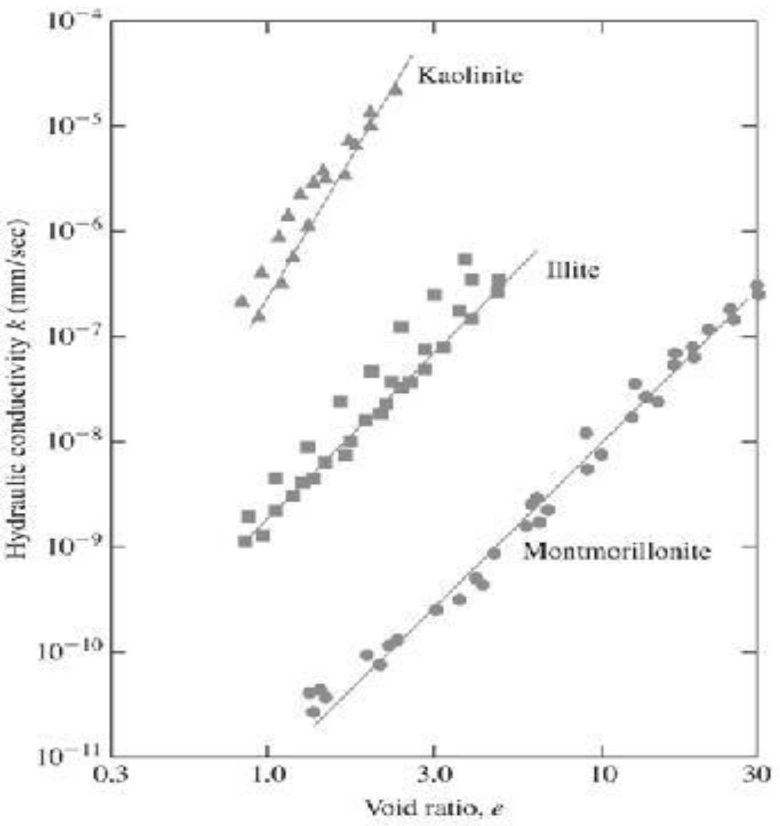

Figure 7.12 Variation of hydraulic conductivity of sodium clay minerals (*Based on Mesri and Olson, 1971*)

4. Tavenas, *et al.* (*1983*)

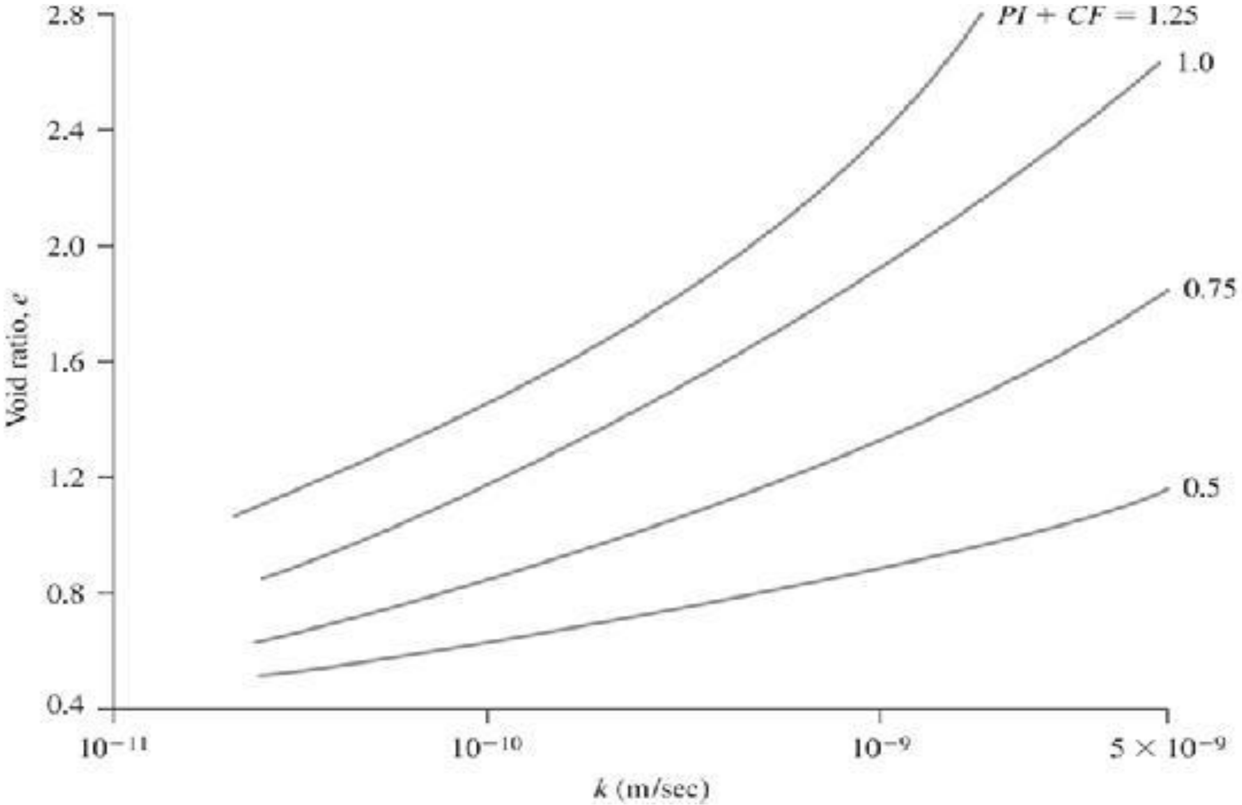

Figure 7.13 Variation of void ratio with hydraulic conductivity of clayey soils (*Based on Tavenas, et al, 1983*)

- *PI, the plasticity index, and*
- *CF, the* clay-size fraction in the soil, *.mrof (lamiced)*

• **Directional Variation of Permeability**:

- soils are not isotropic with respect to permeability.

- In a given soil deposit, the magnitude of *k changes with respect to the direction of flow*.

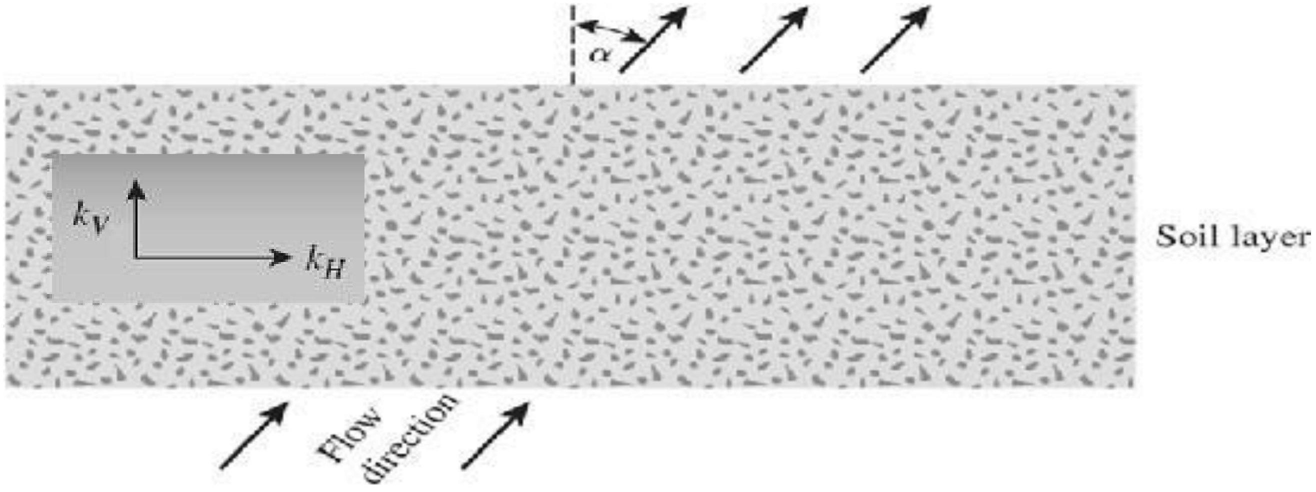

Figure 7.14 Directional variation of permeability

•Fukushima and Ishii (1986) related to k_V *and* k_H *for compacted Masa-do soil (weathered granite)*

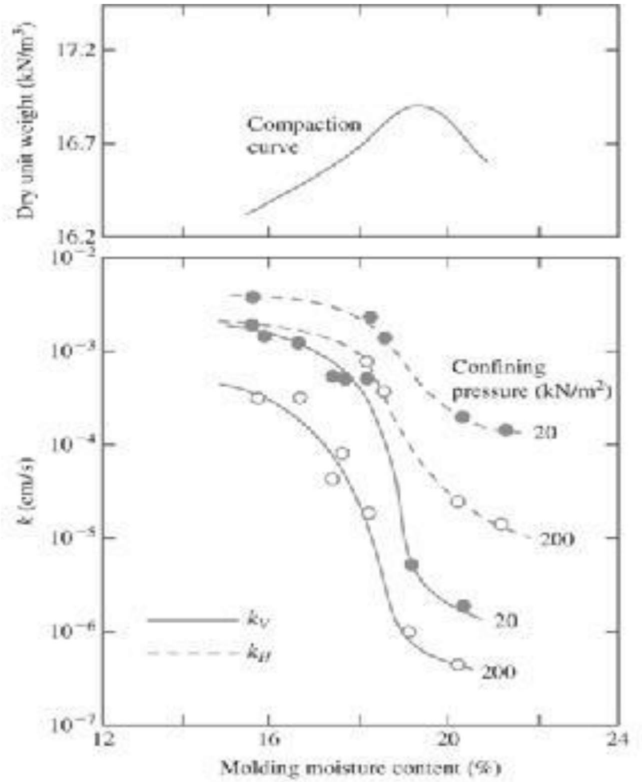

Figure 7.15 Variation of k_V and k_H for Masa-do soil compacted in the laboratory (*Based on the results of Fukushima and Ishii, 1986*)

Note that, for any given molding moisture content and confining pressure, k_H *is larger than* k_V.

Table 7.3 k_H/k_V for Fine-Grained Soils—Summary of Several Studies

Soil type	k_H/k_V	Reference
Organic silt with peat	1.2 to 1.7	Tsien (1955)
Plastic marine clay	1.2	Lumb and Holt (1968)
Soft clay	1.5	Basett and Brodie (1961)
Varved clay	1.5 to 1.7	Chan and Kenney (1973)
Varved clay	1.5	Kenney and Chan (1973)
Varved clay	3 to 15	Wu, *et al*. (1978)
Varved clay	4 to 40	Casagrande and Poulos (1969)

• **Equivalent Hydraulic Conductivity in Stratified Soil:**

➢ *horizontal direction*

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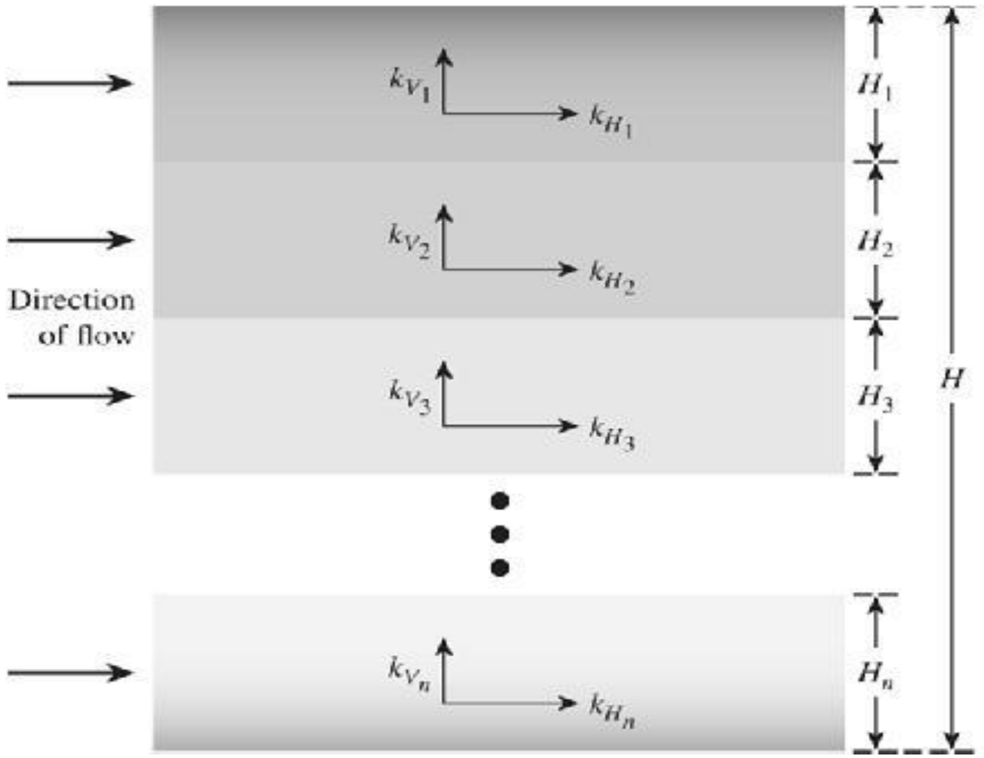


Figure 7.16 Equivalent hydraulic conductivity determination—horizontal flow in stratified soil

Derivation in the horizontal direction:

$$q = v \cdot 1 \cdot H$$
$$= v_1 \cdot 1 \cdot H_1 + v_2 \cdot 1 \cdot H_2 + v_3 \cdot 1 \cdot H_3 + \cdots + v_n \cdot 1 \cdot H_n$$

where v = average discharge velocity
$v_1, v_2, v_3, \ldots, v_n$ = discharge velocities of

$$v = k_{H(\mathrm{eq})} i_{\mathrm{eq}}; \quad v_1 = k_{H_1} i_1; \quad v_2 = k_{H_2} i_2; \quad \ldots \quad v_n = k_{H_n} i_n$$

$$k_{H(\mathrm{eq})} = \frac{1}{H}(k_{H_1}H_1 + k_{H_2}H_2 + k_{H_3}H_3 + \cdots + k_{H_n}H_n)$$

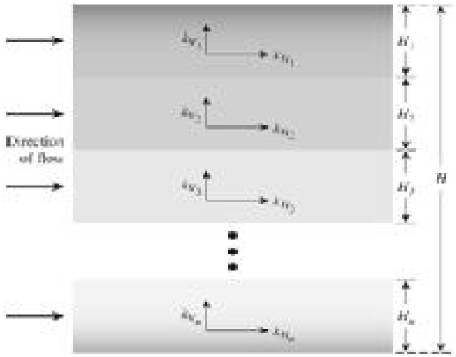


Figure 7.16 Equivalent hydraulic conductivity determination—horizontal flow in stratified soil

• **Equivalent Hydraulic Conductivity in Stratified Soil:**

- *Vertical direction*

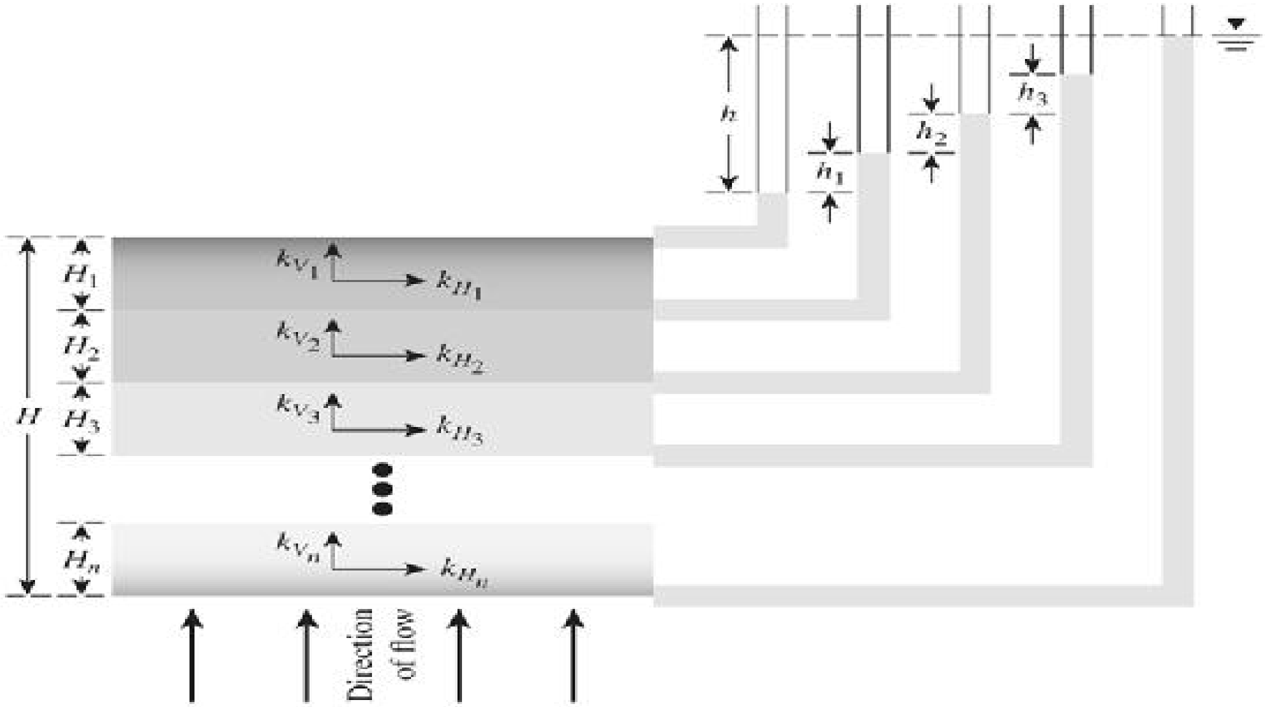


Figure 7.17 Equivalent hydraulic conductivity determination—vertical flow in stratified soil

*Derivation in the vertical direction:

the velocity of flow through all the layers is the same. However, the total head loss, *h, is* equal to the sum of the head losses in all layers. Thus,

Using Darcy's law

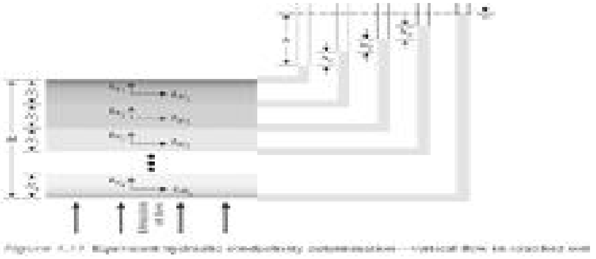
$$v = v_1 = v_2 = v_3 = \cdots = v_n$$

$$h = h_1 + h_2 + h_3 + \cdots + h_n$$

$$k_{V(\mathrm{eq})}\left(\frac{h}{H}\right) = k_{V_1}i_1 = k_{V_2}i_2 = k_{V_3}i_3 = \cdots = k_{V_n}i_n$$

$$h = H_1 i_1 + H_2 i_2 + H_3 i_3 + \cdots + H_n i_n$$

$$k_{V(\mathrm{eq})} = \frac{H}{\left(\frac{H_1}{k_{V_1}}\right) + \left(\frac{H_2}{k_{V_2}}\right) + \left(\frac{H_3}{k_{V_3}}\right) + \cdots + \left(\frac{H_n}{k_{V_n}}\right)}$$



- **Permeability Test in the Field by Pumping from Wells:**

In the field, the average hydraulic conductivity of a soil deposit in the direction of flow can be determined by performing pumping tests from wells.

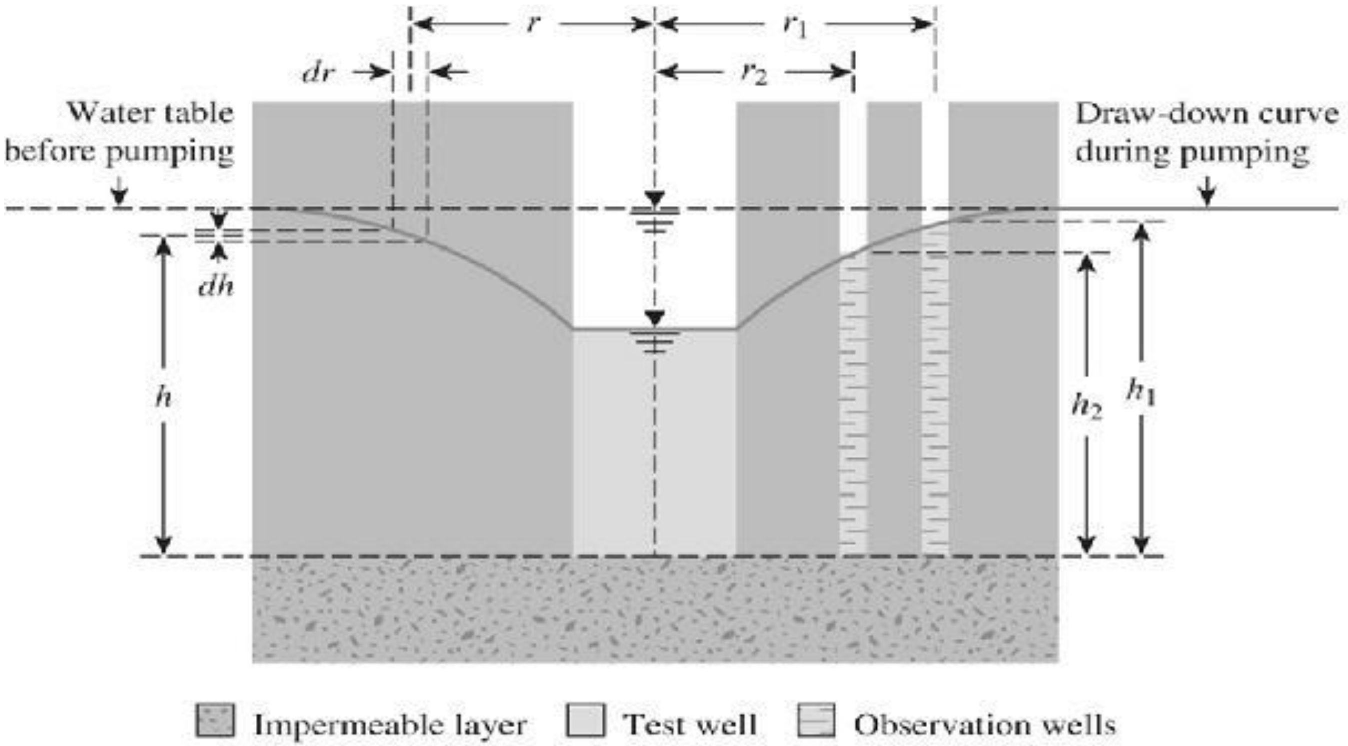


Figure 7.21 Pumping test from a well in an unconfined permeable layer underlain by an impermeable stratum.

Average hydraulic conductivity of a top permeable layer (**unconfined aquifer)**

- Continuous observations of the water level in the test well and in the observation, wells are made after the start of pumping, until a steady state is reached.
- The steady state is established when the water level in the test and observation wells becomes constant

$$q = k\left(\frac{dh}{dr}\right)2\pi rh$$

$$\int_{r_2}^{r_1}\frac{dr}{r} = \left(\frac{2\pi k}{q}\right)\int_{h_2}^{h_1} h\,dh$$

$$k = \frac{2.303q\log_{10}\left(\frac{r_1}{r_2}\right)}{\pi(h_1^2 - h_2^2)}$$

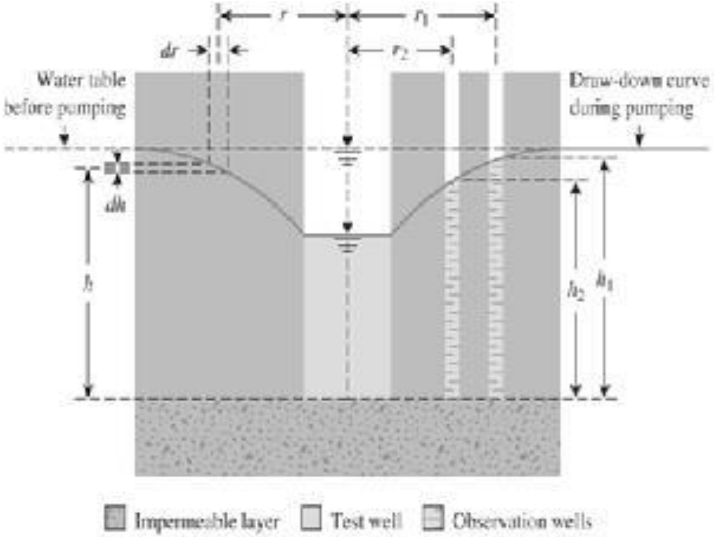


Figure 7.21 Pumping test from a well in an unconfined permeable layer underlain by an impermeable stratum.

Average hydraulic conductivity of a **confined aquifer**

- Conducting a pumping test from a well with a perforated casing that penetrates the full depth of the aquifer
- By observing the piezometric level in a number of observation wells at various radial distances
- Pumping is continued at a uniform rate *q until a* steady state is reached

$$q = k\left(\frac{dh}{dr}\right)2\pi rH$$

$$\int_{r_2}^{r_1}\frac{dr}{r} = \int_{h_2}^{h_1}\frac{2\pi kH}{q}\,dh$$

$$k = \frac{q\log_{10}\left(\frac{r_1}{r_2}\right)}{2.727H(h_1 - h_2)}$$

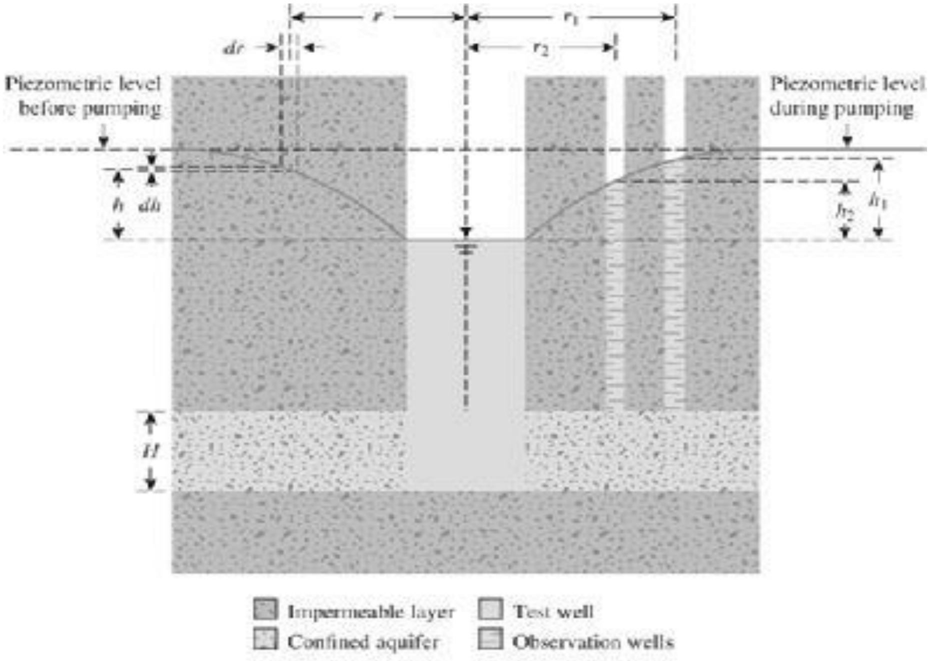


Figure 7.22 Pumping test from a well penetrating the full depth in a confined aquifer

In Situ Hydraulic Conductivity of Compacted Clay Soils:

Daniel (1989) provided an excellent review of nine methods to estimate the *in situ hydraulic* conductivity of compacted clay layers;

DANIEL, D. E. (1989). "*In Situ Hydraulic Conductivity Tests for Compacted Clay*", *Journal of Geotechnical Engineering, ASCE, Vol. 115, No. 9, 1205–1226.*

1. Boutwell Permeameter:

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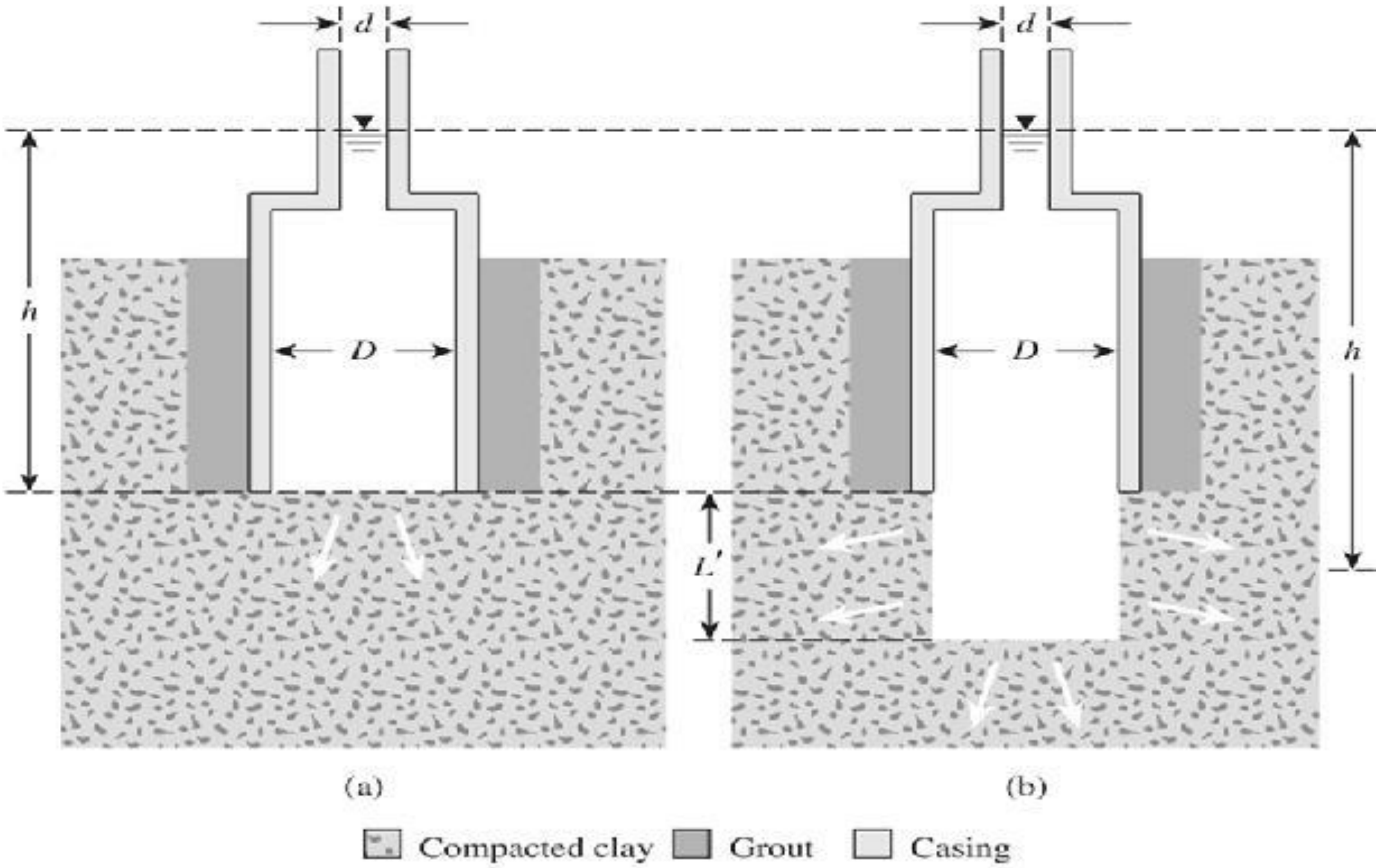

Figure 7.23 Permeability test with Boutwell permeameter

Constant-Head Borehole Permeameter:

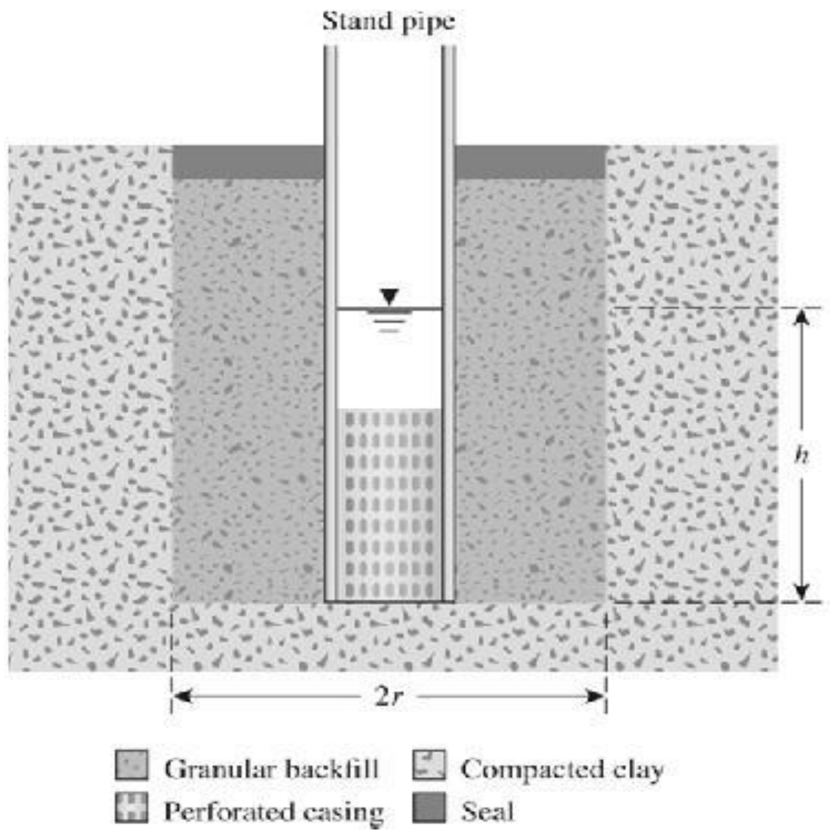

Figure 7.25 Borehole test with constant water level

Porous Probes:

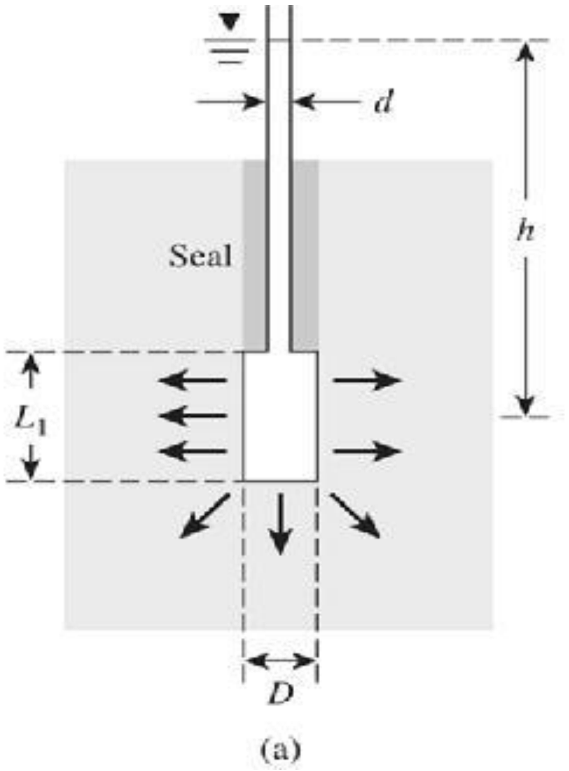

Figure 7.26 Porous probe: (a) test with permeable base; (b) test with impermeable base

PERMEABILITY

Examples

Example I

The grain-size distribution curve for a sand is shown in Figure 7.13. Estimate the hydraulic conductivity using Eq. (7.30). Given: The void ratio of the sand is 0.6. Use $SF = 7$.

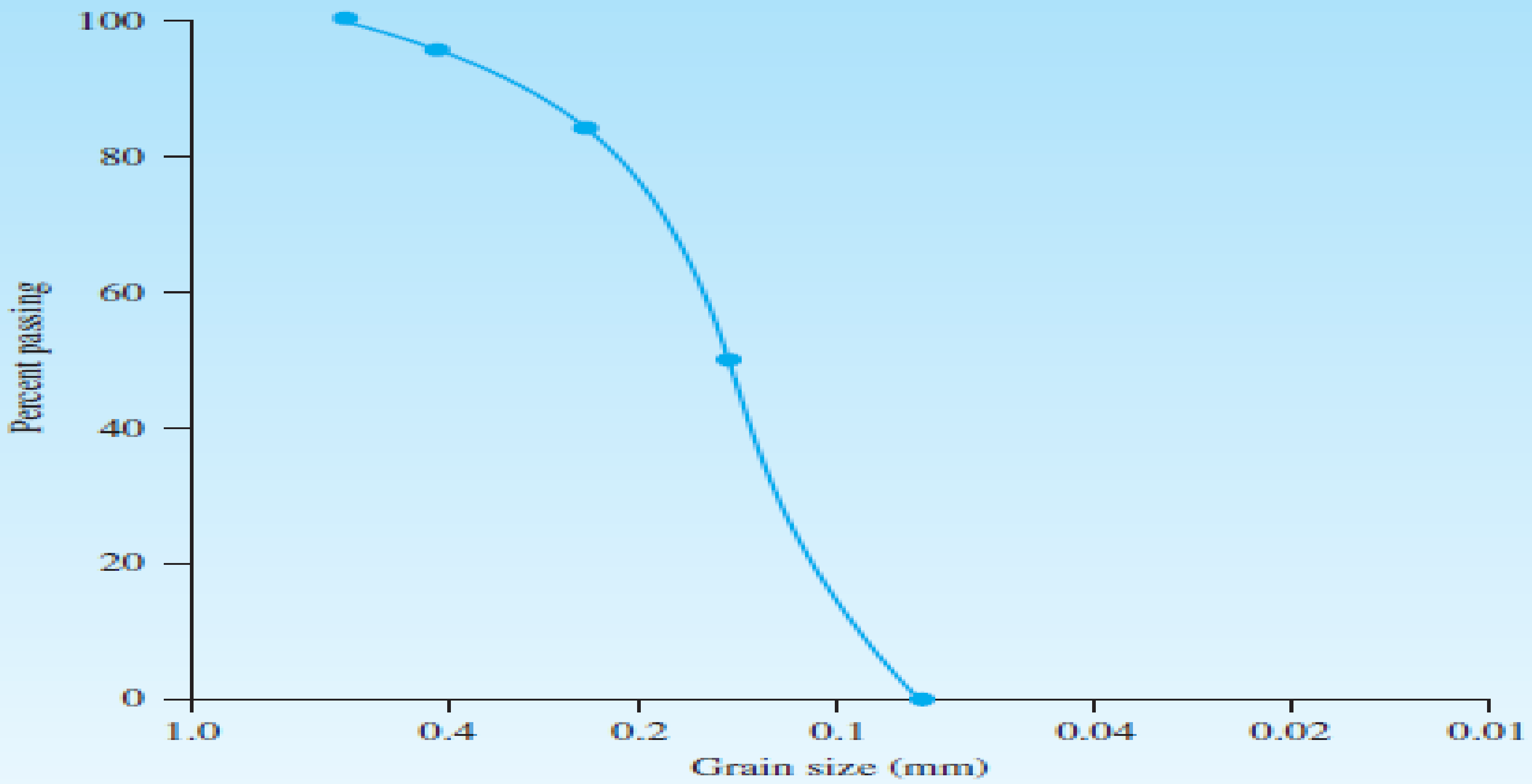

Figure 7.13

Solution

From Figure 7.13, the following table can be prepared.

Sieve no.	Sieve opening (cm)	Percent passing	Fraction of particles between two consecutive sieves (%)
30	0.06	100	4
40	0.0425	96	12
60	0.02	84	34
100	0.015	50	50
200	0.0075	0	

For fraction between Nos. 30 and 40 sieves;

$$\frac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}} = \frac{4}{(0.06)^{0.404} \times (0.0425)^{0.595}} = 81.62$$

For fraction between Nos. 40 and 60 sieves;

$$\frac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}} = \frac{12}{(0.0425)^{0.404} \times (0.02)^{0.595}} = 440.76$$

Similarly, for fraction between Nos. 60 and 100 sieves;

$$\frac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}} = \frac{34}{(0.02)^{0.404} \times (0.015)^{0.595}} = 2009.5$$

And, for between Nos. 100 and 200 sieves;

$$\frac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}} = \frac{50}{(0.015)^{0.404} \times (0.0075)^{0.595}} = 5013.8$$

$$\frac{100\%}{\Sigma \dfrac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}}} = \frac{100}{81.62 + 440.76 + 2009.5 + 5013.8} \approx 0.0133$$

From Eq. (7.30),

$$k = (1.99 \times 10^4)(0.0133)^2\left(\frac{1}{7}\right)^2\left(\frac{0.6^3}{1 + 0.6}\right) = \mathbf{0.0097\ cm/s}$$

Example 2

Refer to the constant-head permeability test arrangement shown in Figure 7.5. A test gives these values:

- $L = 30$ cm
- A = area of the specimen = 177 cm^2
- Constant-head difference, $h = 50$ cm
- Water collected in a period of 5 min = 350 cm^3

Calculate the hydraulic conductivity in cm/sec.

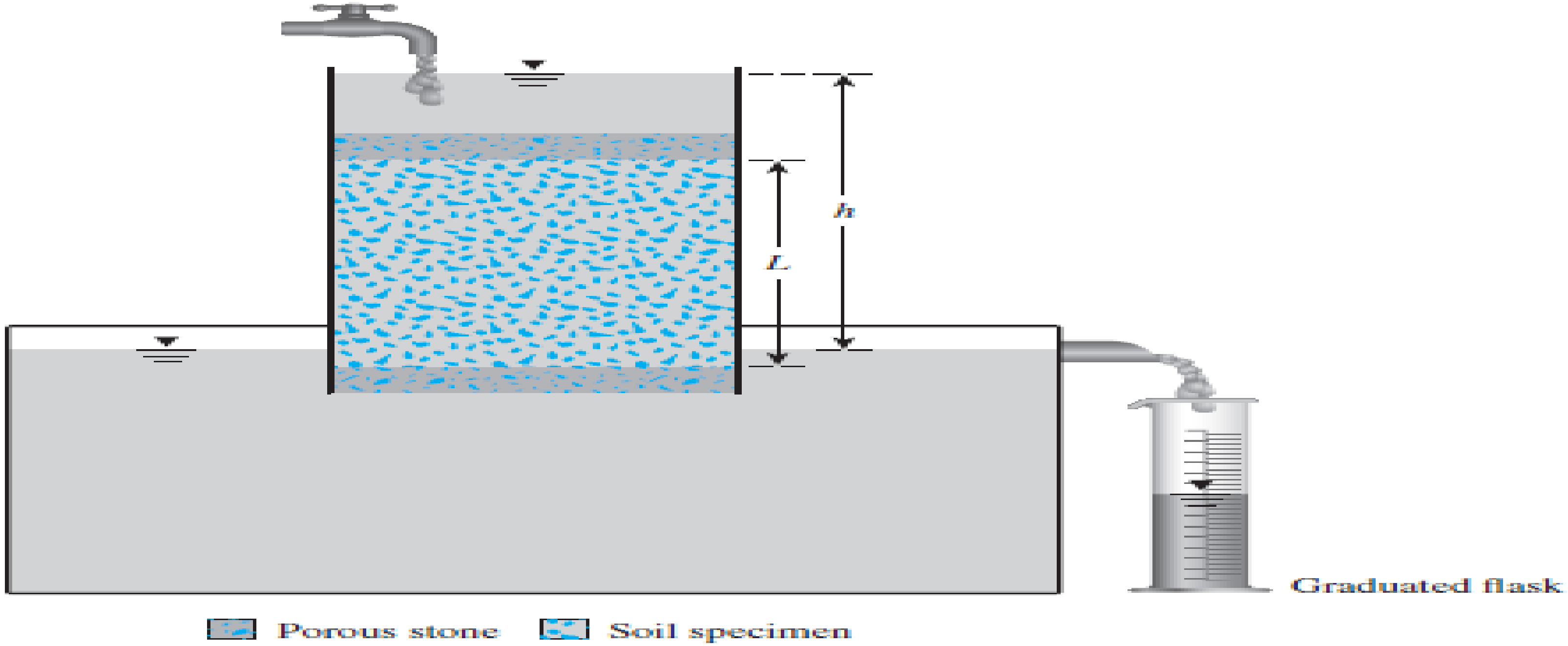

Figure 7.5 Constant-head permeability test

$$k = \frac{QL}{Aht}$$

Given $Q = 350\ \text{cm}^3$, $L = 30$ cm, $A = 177\ \text{cm}^2$, $h = 50$ cm, and $t = 5$ min, we have

$$k = \frac{(350)(30)}{(177)(50)(5)(60)} = \mathbf{3.95 \times 10^{-3}\ cm/sec}$$

Example 3

For a normally consolidated clay soil, the following values are given:

Void ratio	k (cm/sec)
1.1	0.302×10^{-7}
0.9	0.12×10^{-7}

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Estimate the hydraulic conductivity of the clay at a void ratio of 0.75.

$$k = C\left(\frac{e^n}{1+e}\right)$$

$$\frac{k_1}{k_2} = \frac{\left(\dfrac{e_1^n}{1+e_1}\right)}{\left(\dfrac{e_2^n}{1+e_2}\right)}$$

$$\frac{0.302 \times 10^{-7}}{0.12 \times 10^{-7}} = \frac{\dfrac{(1.1)^n}{1+1.1}}{\dfrac{(0.9)^n}{1+0.9}}$$

$$2.517 = \left(\frac{1.9}{2.1}\right)\left(\frac{1.1}{0.9}\right)^n$$

$$2.782 = (1.222)^n$$

$$n = \frac{\log\ (2.782)}{\log\ (1.222)} = \frac{0.444}{0.087} = 5.1$$

so

$$k = C\left(\frac{e^{5.1}}{1 + e}\right)$$

To find C,

$$0.302 \times 10^{-7} = C\left[\frac{(1.1)^{5.1}}{1 + 1.1}\right] = \left(\frac{1.626}{2.1}\right)C$$

$$C = \frac{(0.302 \times 10^{-7})(2.1)}{1.626} = 0.39 \times 10^{-7}$$

Hence,

$$k = (0.39 \times 10^{-7}\ \text{cm/sec})\left(\frac{e^{n}}{1 + e}\right)$$

At a void ratio of 0.75,

$$k = (0.39 \times 10^{-7})\left(\frac{0.75^{5.1}}{1 + 0.75}\right) = \mathbf{0.514 \times 10^{-8}\ cm/sec}$$

Example 4

A layered soil is shown in Figure 7.24. Given:

- $H_1 = 1.5$ m $k_1 = 10^{-4}$ cm/sec
- $H_2 = 3$ m $k_2 = 3.2 \times 10^{-2}$ cm/sec
- $H_3 = 2$ m $k_3 = 4.1 \times 10^{-5}$ cm/sec

Estimate the ratio of equivalent hydraulic conductivity,

$$\frac{k_{H(eq)}}{k_{V(eq)}}$$

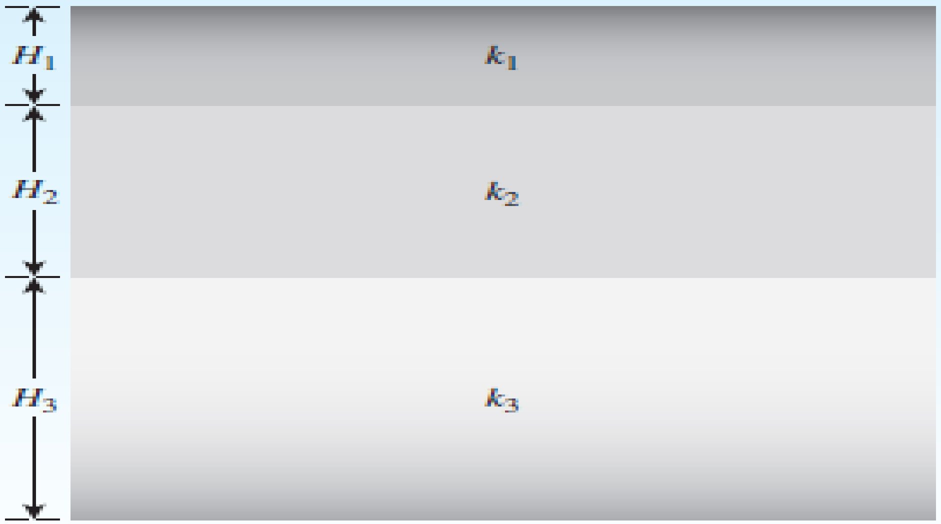

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Figure 7.24 A layered soil profile

$$k_{H(\text{eq})} = \frac{1}{H}\left(k_{H_1}H_1 + k_{H_2}H_2 + k_{H_3}H_3\right)$$

$$= \frac{1}{(1.5 + 3 + 2)}\left[(10^{-4})(1.5) + (3.2 \times 10^{-2})(3) + (4.1 \times 10^{-5})(2)\right]$$

$$= 148.05 \times 10^{-4} \text{ cm/sec}$$

$$k_{v(\text{eq})} = \frac{H}{\left(\frac{H_1}{k_{V_1}}\right) + \left(\frac{H_2}{k_{V_2}}\right) + \left(\frac{H_3}{k_{V_3}}\right)}$$

$$= \frac{1.5 + 3 + 2}{\left(\frac{1.5}{10^{-4}}\right) + \left(\frac{3}{3.2 \times 10^{-2}}\right) + \left(\frac{2}{4.1 \times 10^{-5}}\right)}$$

$$= 1.018 \times 10^{-4} \text{ cm/sec}$$

Hence,

$$\frac{k_{H(\text{eq})}}{k_{V(\text{eq})}} = \frac{148.05 \times 10^{-4}}{1.018 \times 10^{-4}} = \mathbf{145.4}$$

SEEPAGE

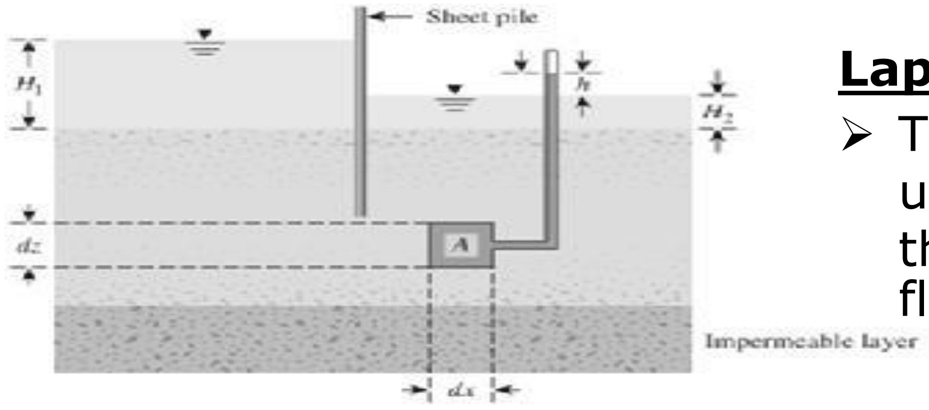
- In the preceding chapter, we considered some simple cases for which direct application of Darcy's law was required to calculate the flow of water through soil.

- In many instances, the flow of water through soil is not in one direction only, nor is it uniform over the entire area perpendicular to the flow.

- In such cases, the groundwater flow is generally calculated by the use of graphs referred to as *flow nets. The concept of the flow* net is based on *Laplace's equation of continuity, which governs the steady flow condition* for a given point in the soil mass.

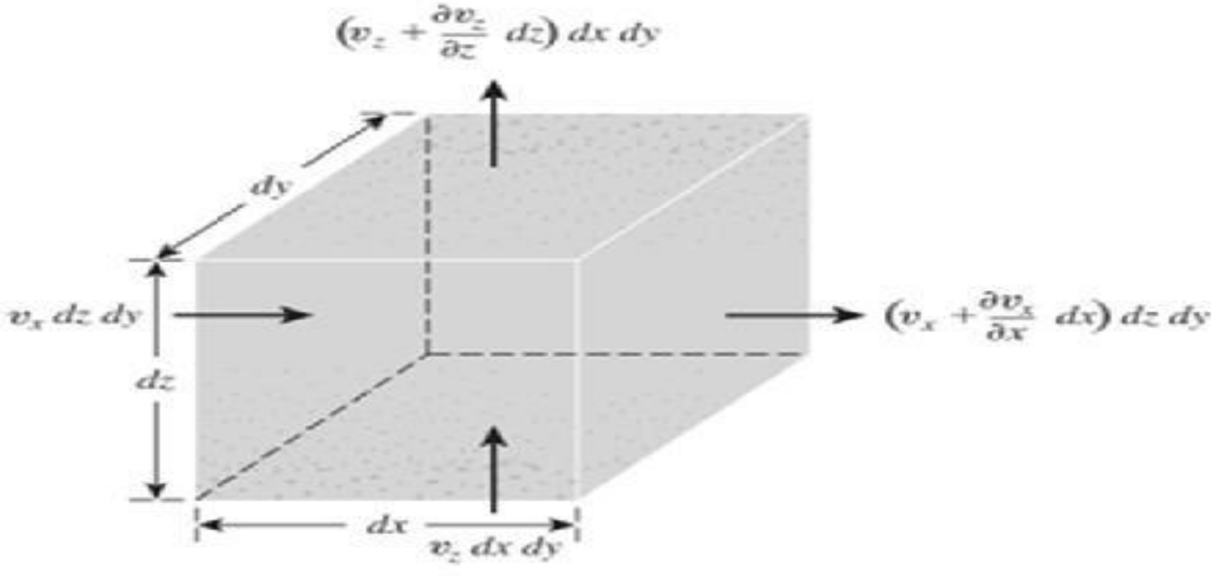
Laplace's Equation of Continuity

- The steady state flow of water from the upstream to the downstream side through the permeable layer is a two dimensional flow.



(a)

- The rate of flow of water into the elemental block in the horizontal direction is equal to v*xdz.dy, and in the vertical direction it is vzdx.dy*.



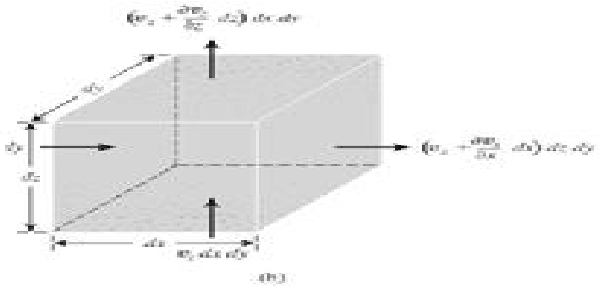
(b)

Laplace’s Equation of Continuity

Derivation…..

The rates of outflow from the block in the horizontal and vertical directions are, respectively,

$$\left(v_x + \frac{\partial v_x}{\partial x} dx\right) dz\, dy, \quad \text{and} \left(v_z + \frac{\partial v_z}{\partial z} dz\right) dx\, dy$$



…Assuming that water is incompressible and that no volume change in the soil mass occurs, we know that the total rate of inflow should equal the total rate of outflow. Thus,

$$\left[\left(v_x + \frac{\partial v_x}{\partial x} dx\right) dz\, dy + \left(v_z + \frac{\partial v_z}{\partial z} dz\right) dx\, dy\right] - [v_x\, dz\, dy + v_z\, dx\, dy] = 0$$

or $$\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0$$ $\Rightarrow$ With Darcy's law. $$v_x = k_x i_x = k_x \frac{\partial h}{\partial x}$$

and $$v_z = k_z i_z = k_z \frac{\partial h}{\partial z}$$

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

Assuming the soil is isotropic with respect to the hydraulic conductivity), ***(kx=kz)***, **then**;

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

*<u>***continuity equation for two-dimensional flow</u>*

Laplace's Equation of Continuity

Derivation…..(continue…(

$$\partial^2 h/\partial x^2 + \partial^2 h/\partial y^2 + \partial^2 h/\partial z^2 = 0$$

Laplace Equation which governs seepage in homogeneous, isotropic soil deposits.

*)***general form)*

❑ Possible Methods for Solving the Laplace Equation.

1. Analytical, closed form or series solutions of the PDE.
   - quite mathematical, and not very general.

2. Numerical solution methods
   - typically, the *finite element method or the finite difference method*.
   - very powerful and easy to apply
   - can deal with heterogeneity, anisotropy, 2D, 3D

3. Graphical Techniques − *Flow−net Methods*
   - commonly used in engineering practice to solve 2D flow problems.
   - the ideas behind this method are now explained.

<u>Flow Nets</u>

Solutions of Laplace equation consist of two families of orthogonal curves in the (x,z) plane.

These families of curves make a ***<u>flow net</u>***.

<u>flow net:</u> is the combination of a number of *flow lines* and *equipotential* lines.

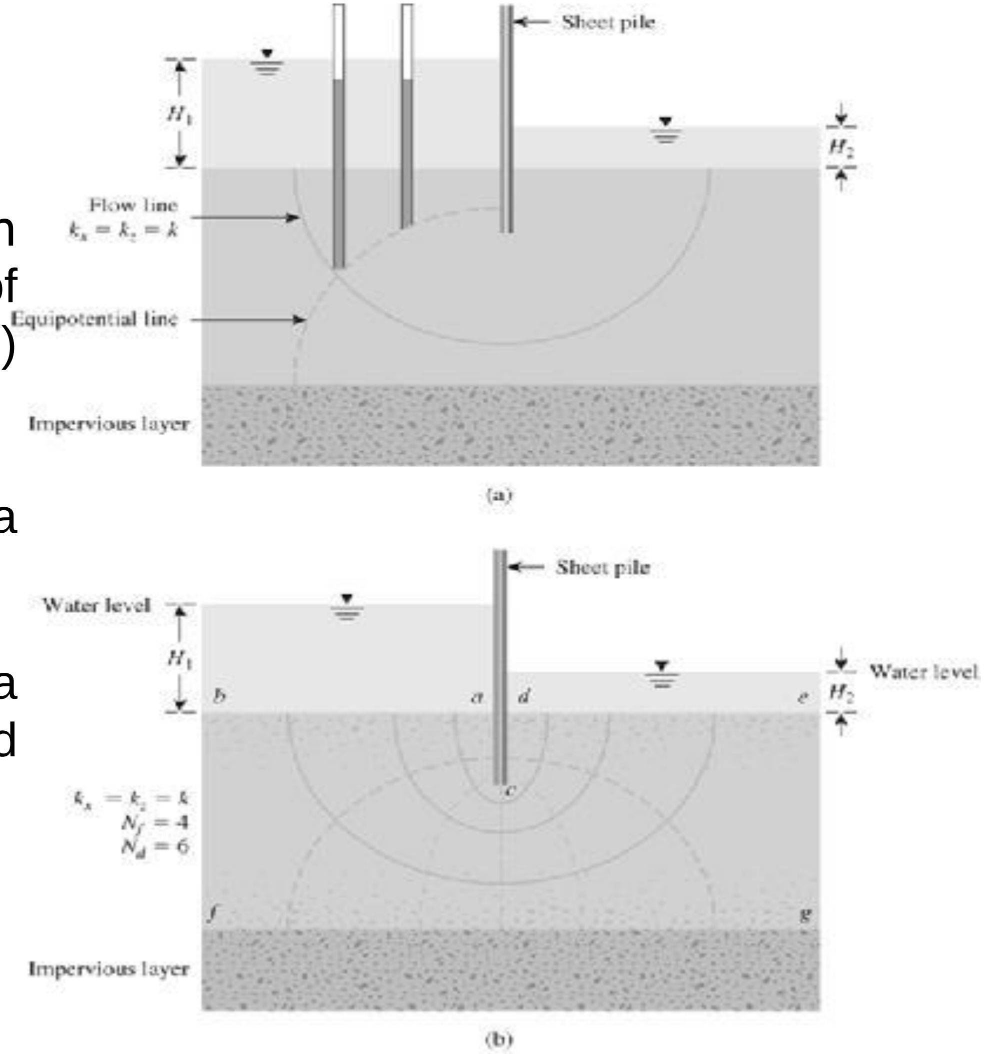


Figure 8.3 (a) Definition of flow lines and equipotential lines; (b) completed flow net

•
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• *Flow line: is a line along* which a water particle will travel from upstream to the downstream side in the permeable soil medium.

• *Equipotential line: is a line along which the potential head at all points is* equal.

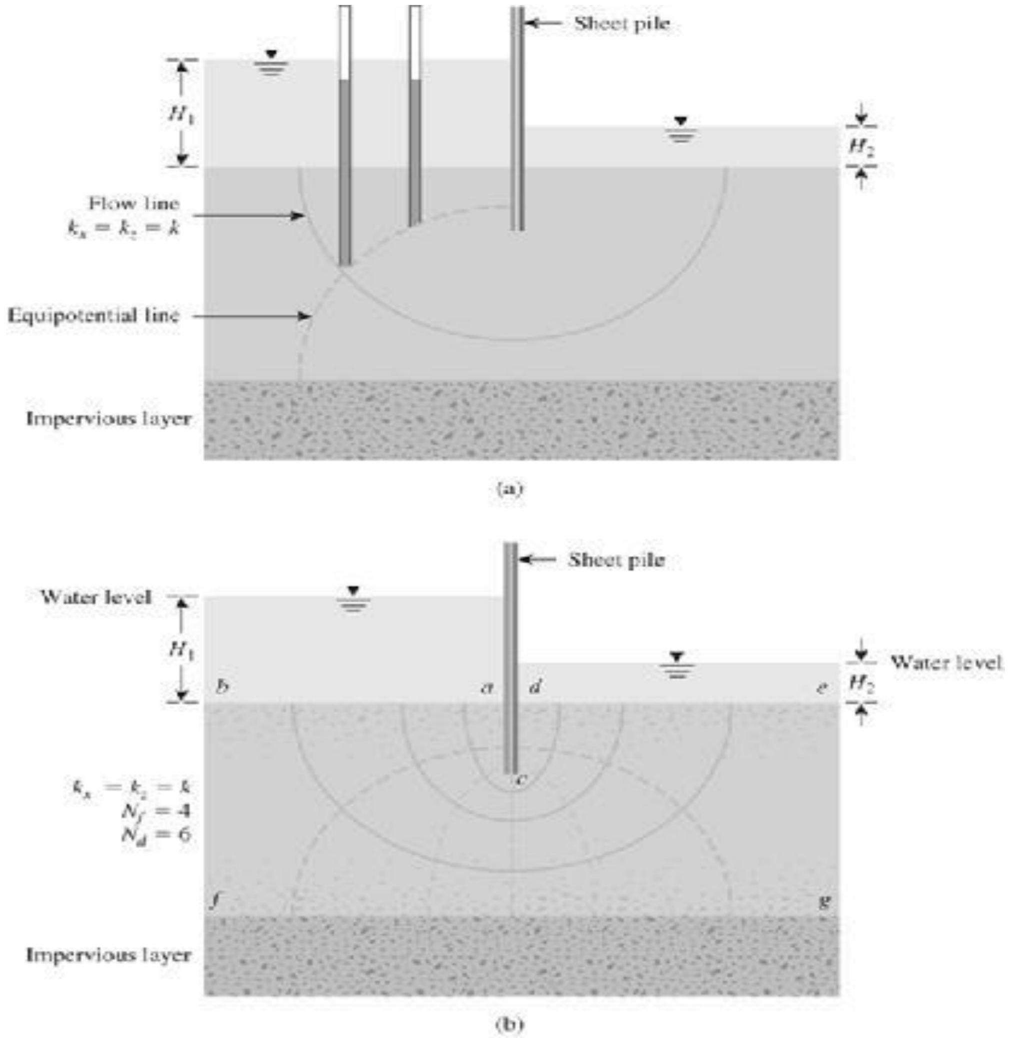


Figure 8.3 (a) Definition of flow lines and equipotential lines; (b) completed flow net

Flow net

To complete the graphic construction
of a flow net, one must draw the flow and equipotential lines
in such a way that

1. The equipotential lines intersect the flow lines at right angles.
2. The flow elements formed are approximate squares.

Example of flow net in isotropic permeable layer is given in
Figure 8.4. In these figures, N_f is the number
of flow channels in the flow net, and N_d is the number of
potential drops

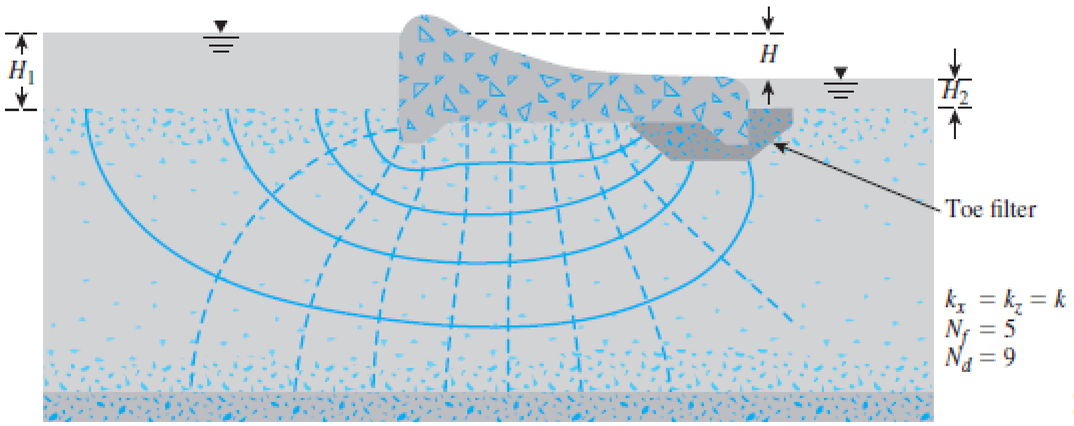


Figure 8.4 Flow net under a dam with toe filter

Drawing a flow net takes several trials. While constructing the flow net, keep the boundary conditions in mind. For the flow net shown in Figure 8.3b, the following four boundary conditions apply:

Condition 1: The upstream and downstream surfaces of the permeable layer (lines *ab* and *de*) are equipotential lines.

Condition 2: Because *ab* and *de* are equipotential lines, all the flow lines intersect them at right angles.

Condition 3: The boundary of the impervious layer—that is, line *fg*—is a flow line, and so is the surface of the impervious sheet pile, line *acd*.

Condition 4: The equipotential lines intersect *acd* and *fg* at right angles.

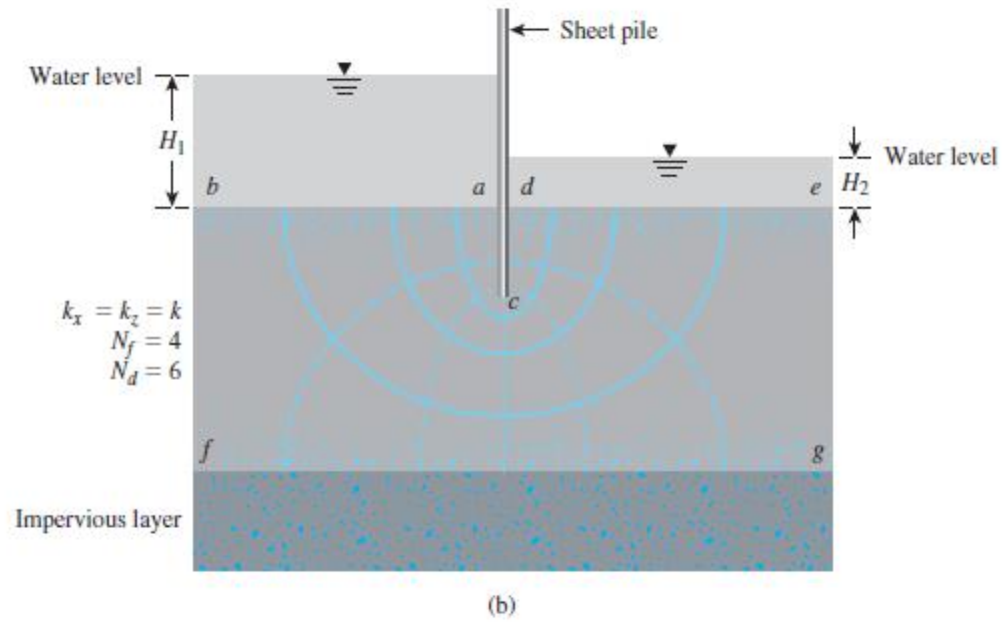
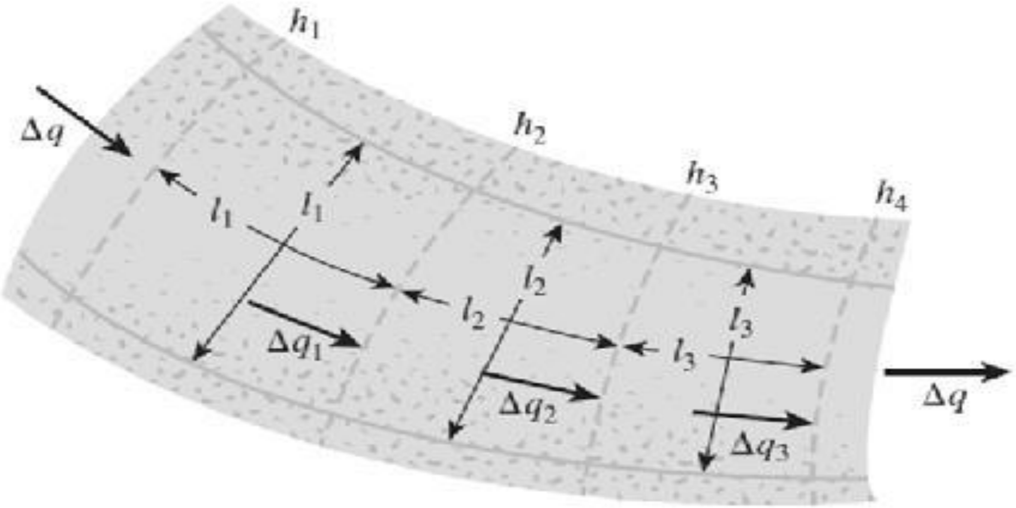


Figure 8.3 (a) Definition of flow lines and equipotential lines; (b) completed flow net

• Seepage Calculation from a Flow Net:

Consider the adjacent flow channel….



• The rate of seepage through the flow channel per unit length can be calculated as follows.

Because there is no flow across the flow lines,

$$\Delta q_1 = \Delta q_2 = \Delta q_3 = \cdots = \Delta q$$

From Darcy's law, the flow rate is equal to kiA. Thus,

$$\Delta q = k\left(\frac{h_1 - h_2}{l_1}\right)l_1 = k\left(\frac{h_2 - h_3}{l_2}\right)l_2 = k\left(\frac{h_3 - h_4}{l_3}\right)l_3 = \cdots$$

if the flow elements are drawn as approximate squares, the ***potential drop*** in the piezometric level between any two adjacent equipotential lines is the same. *Thus,*

$$h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = \cdots = \frac{H}{N_d}$$

$$\Delta q = k\frac{H}{N_d}$$

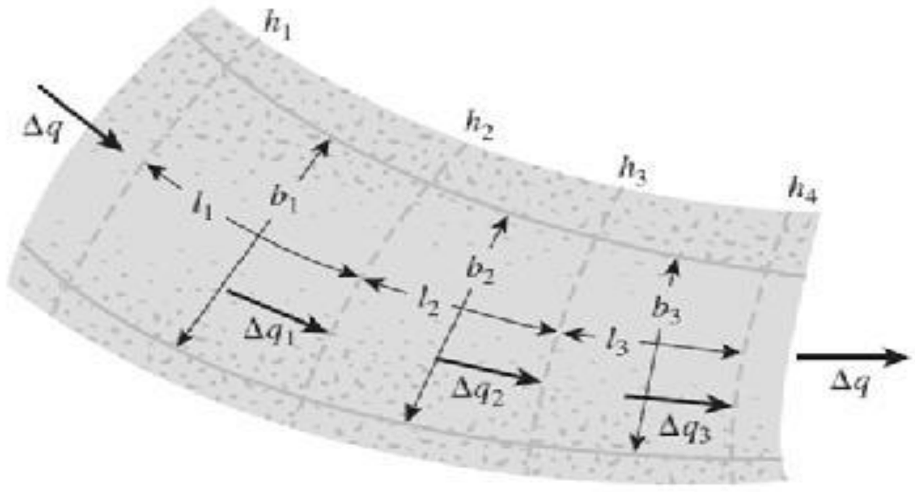
where H = head difference between the upstream and downstream sides

N_d = number of potential drops

•If the number of flow channels in a flow net is equal to *Nf* , ***the total rate of flow*** through all the channels per unit length can be given by;

$$q = k\frac{HN_f}{N_d}$$

If $b_1/l_1 = b_2/l_2 = b_3/l_3 = \cdots = n$ (i.e., the elements are not square)



The rate of flow through the channel

$$\Delta q = k\left(\frac{h_1 - h_2}{l_1}\right)b_1 = k\left(\frac{h_2 - h_3}{l_2}\right)b_2 = k\left(\frac{h_3 - h_4}{l_3}\right)b_3 = \cdots$$

$$b_1/l_1 = b_2/l_2 = b_3/l_3 = \cdots = n$$

$$\Delta q = kH\left(\frac{n}{N_d}\right)$$

$$q = kH\left(\frac{N_f}{N_d}\right)n$$

Example….

[19]

$$\Delta q_1 + \Delta q_2 = \frac{k}{N_d}H + \frac{k}{N_d}H = \frac{2kH}{N_d} \qquad \Delta q_3 = \frac{k}{N_d}H(0.38)$$

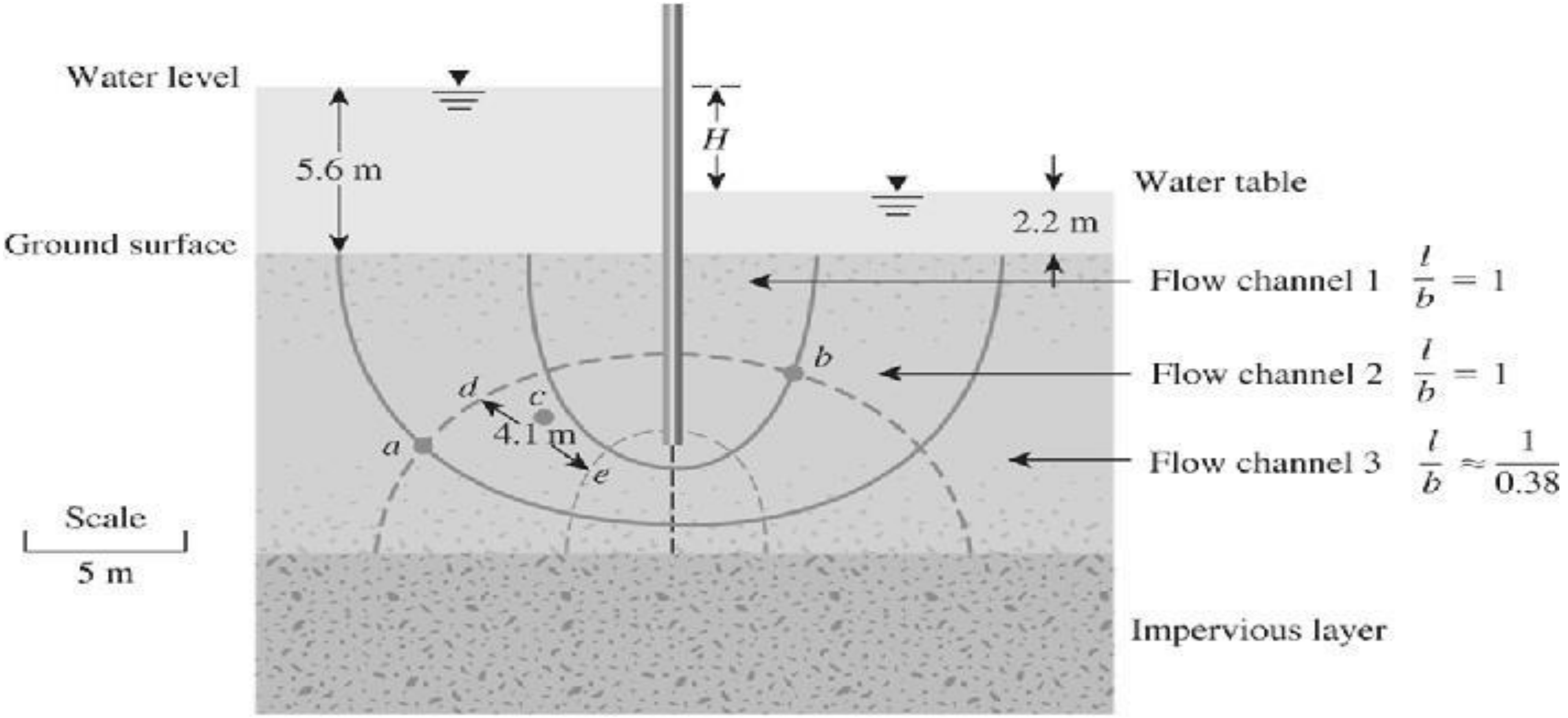


Figure 8.7 Flow net for seepage around a single row of sheet piles

$$q = \Delta q_1 + \Delta q_2 + \Delta q_3 = 2.38\frac{kH}{N_d}$$

20

Example 8.2

A flow net for flow around a single row of sheet piles in a permeable soil layer is shown in Figure 8.7. Given that $k_x = k_z = k = 5 \times 10^{-3}$ cm/sec, determine

- **a.** How high (above the ground surface) the water will rise if piezometers are placed at points a and b.
- **b.** The total rate of seepage through the permeable layer per unit length
- **c.** The approximate average hydraulic gradient at c.

Solution

Part a

From Figure 8.7, we have $N_d = 6$, $H_1 = 5.6$ m, and $H_2 = 2.2$ m. So the head loss of each potential drop is

$$\Delta H = \frac{H_1 - H_2}{N_d} = \frac{5.6 - 2.2}{6} = 0.567\,\text{m}$$

At point a, we have gone through one potential drop. So the water in the piezometer will rise to an elevation of

$$(5.6 - 0.567) = \mathbf{5.033\ m\ above\ the\ ground\ surface}$$

At point b, we have five potential drops. So the water in the piezometer will rise to an elevation of

$$[5.6 - (5)(0.567)] = \mathbf{2.765\ m\ above\ the\ ground\ surface}$$

Part b

From Eq. (8.25),

$$q = 2.38\frac{k(H_1 - H_2)}{N_d} = \frac{(2.38)(5 \times 10^{-5}\,\text{m/sec})(5.6 - 2.2)}{6}$$

$$= \mathbf{6.74 \times 10^{-5}\,m^3/sec/m}$$

Part c

The average hydraulic gradient at c can be given as

$$i = \frac{\text{head loss}}{\text{average length of flow between } d \text{ and } e} = \frac{\Delta H}{\Delta L} = \frac{0.567\,\text{m}}{4.1\,\text{m}} = \mathbf{0.138}$$

SEEPAGE

Part 11

<u>Flow Nets in Anisotropic Soil</u>:

- To account for soil anisotropy with respect to hydraulic conductivity, we must modify the flow net construction.

- The differential equation of continuity for a two-dimensional flow

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

- For anisotropic soils, ($k_x \neq k_z$) *In this case, the equation represents two families* of curves that *do not meet at 90.*

$$\frac{\partial^2 h}{(k_z/k_x)\, \partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad \Longrightarrow \quad \text{Substituting } x' = \sqrt{k_z/k_x}\, x$$

$$\frac{\partial^2 h}{\partial x'^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

- <u>New transformed coordinate</u>
  <u>***(x replaced by x′)***</u>

Procedure to construct the flow net in Anisotropic Soil:

Step 1: Adopt a vertical scale (that is, z axis) for drawing the cross section.

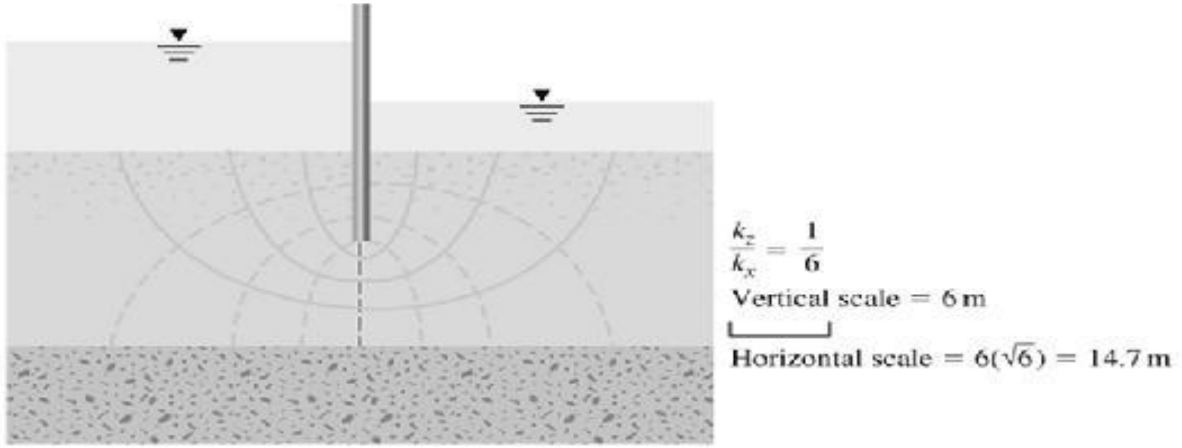
Step 2: Adopt a horizontal scale (that is, x axis) such that horizontal scale $= \sqrt{k_z/k_x} \times$ vertical scale.

Step 3: With scales adopted as in Steps 1 and 2, plot the vertical section through the permeable layer parallel to the direction of flow.

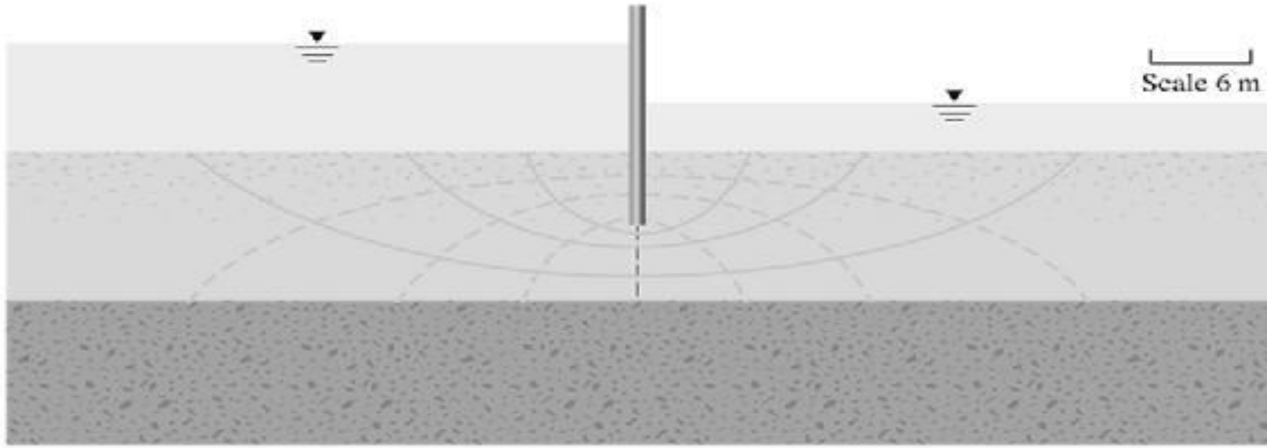
Step 4: Draw the flow net for the permeable layer on the section obtained from Step 3, with flow lines intersecting equipotential lines at right angles and the elements as approximate squares.

The rate of seepage per unit length can be calculated by

$$q = \sqrt{k_x k_z}\frac{H N_f}{N_d}$$



(a)



(b)

Figure 8.8
A flow element in anisotropic soil: (a) in transformed section; (b) in true section

Example 8.3

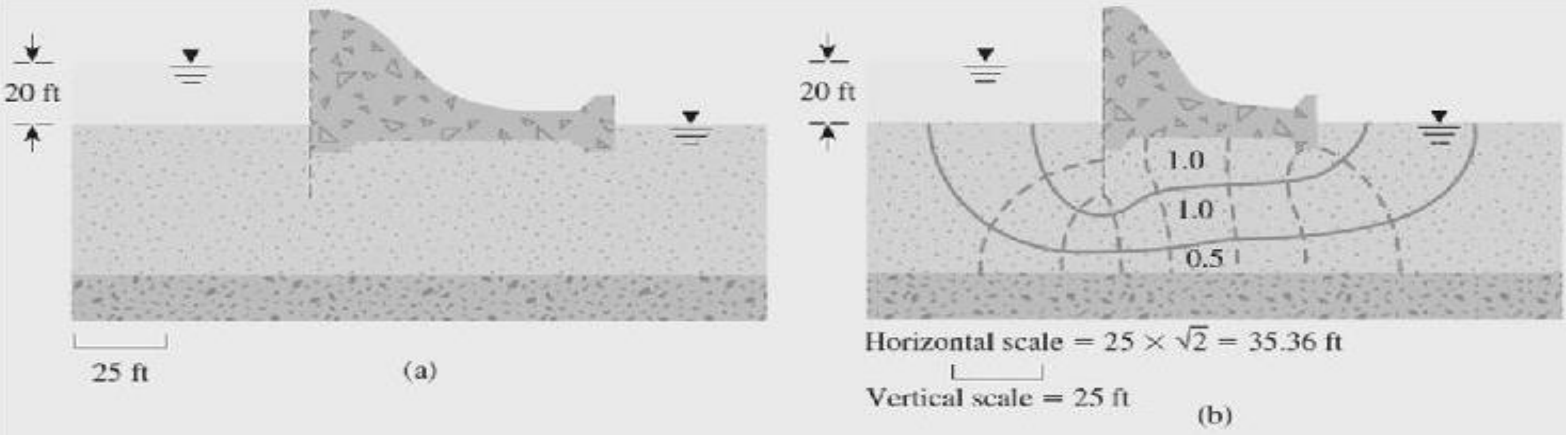
A dam section is shown in Figure 8.9a. The hydraulic conductivity of the permeable layer in the vertical and horizontal directions are 2×10^{-2} mm/s and 4×10^{-2} mm/s, respectively. Draw a flow net and calculate the seepage loss of the dam in ft^3/day/ft

Solution

From the given data,

$$k_z = 2 \times 10^{-2} \text{ mm/s} = 5.67 \text{ ft/day}$$

$$k_x = 4 \times 10^{-2} \text{ mm/s} = 11.34 \text{ ft/day}$$



and $h = 20$ ft. For drawing the flow net,

$$\text{Horizontal scale} = \sqrt{\frac{2 \times 10^{-2}}{4 \times 10^{-2}}}(\text{vertical scale})$$
$$= \frac{1}{\sqrt{2}}(\text{vertical scale})$$

On the basis of this, the dam section is replotted, and the flow net drawn as in Figure 8.9b. The rate of seepage is given by $q = \sqrt{k_x k_z}\, H(N_f/N_d)$. From Figure 8.9b, $N_d = 8$ and $N_f = 2.5$ (the lowermost flow channel has a width-to-length ratio of 0.5). So,

$$q = \sqrt{(5.67)(11.34)}\,(20)(2.5/8) = 50.12\ \text{ft}^3/\text{day/ft}$$

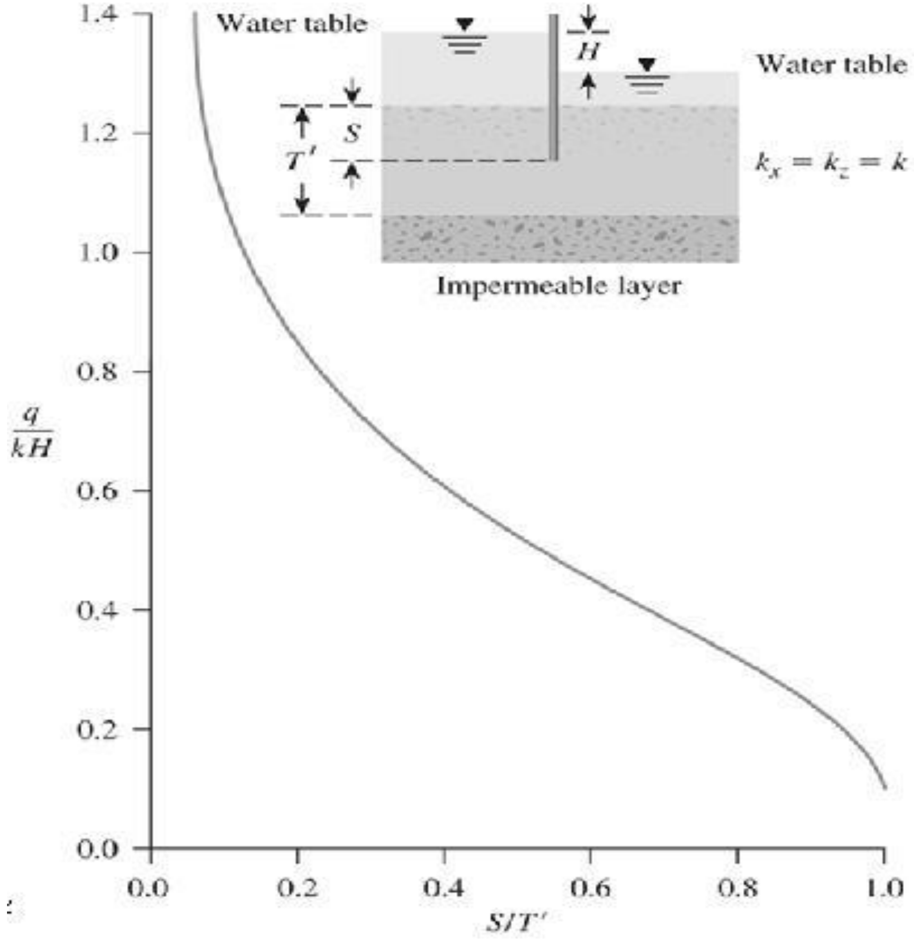
■

Mathematical Solution for Seepage:

The seepage under several simple hydraulic structures can be solved mathematically.

Harr (1962) has analyzed many such conditions.

•A nondimensional plot for the rate of seepage around a single row of sheet piles.



S: the depth of penetration of the sheet pile

T′: is *the thickness of the permeable soil layer.*

Figure 8.10 Plot of q/kH against S/T' for flow around a single row of sheet piles (*After Harr, 1962*)

• A nondimensional plot for the rate of seepage under a dam.

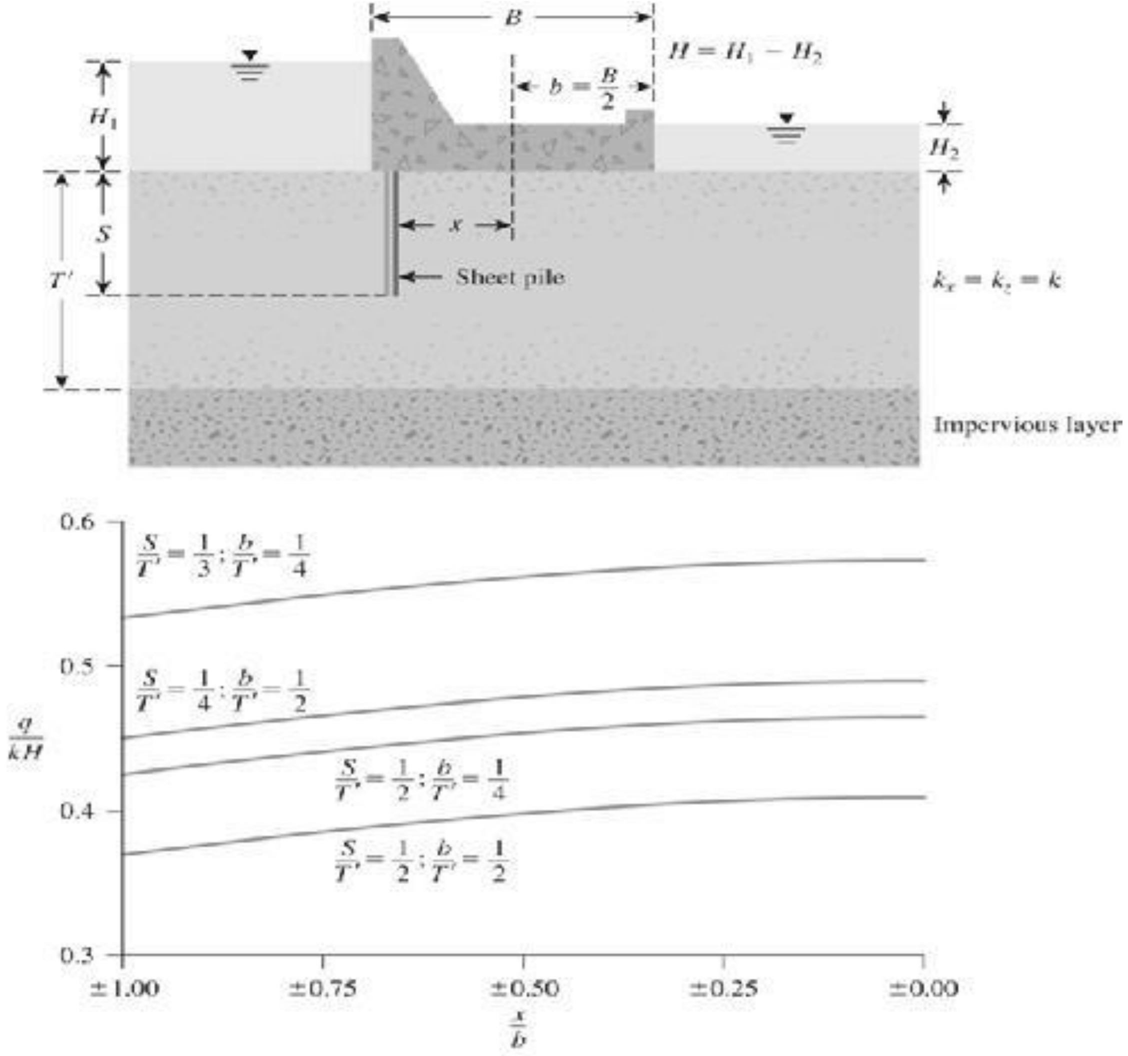


Figure 8.11 Seepage under a dam (*After Harr, 1962*)

Example 8.4

Refer to Figure 8.11. Given; the width of the dam, $B = 6$ m; length of the dam, $L = 120$ m; $S = 3$ m; $T' = 6$ m; $x = 2.4$ m; and $H_1 - H_2 = 5$ m. If the hydraulic conductivity of the permeable layer is 0.008 cm/sec, estimate the seepage under the dam (Q) in m^3/day/m.

Solution

Given that $B = 6$ m, $T' = 6$ m, and $S = 3$ m, so $b = B/2 = 3$ m.

$$\frac{b}{T'} = \frac{3}{6} = 0.5$$

$$\frac{S}{T'} = \frac{3}{6} = 0.5$$

$$\frac{x}{b} = \frac{2.4}{3} = 0.8$$

From Figure 8.11, for $b/T' = 0.5$, $S/T' = 0.5$, and $x/b = 0.8$, the value of $q/kH \approx 0.378$. Thus,

$$\begin{aligned} Q = q\,L = 0.378\,k\,HL &= (0.378)(0.008 \times 10^{-2} \times 60 \times 60 \times 24 \text{ m/day})(5)(120) \\ &= \mathbf{1567.64\ m^3/day} \end{aligned}$$

■

Uplift Pressure Under Hydraulic Structures:

$$H/N_d = 7/7 = 1 \text{ m}$$

The uplift pressure at
a (left corner of the base)
$= (\text{Pressure head at } a) \times (\gamma_w)$
$= [(7 + 2) - 1]\gamma_w = 8\gamma_w$

$$b = [9 - (2)(1)]\gamma_w = 7\gamma_w$$

$$f = [9 - (6)(1)]\gamma_w = 3\gamma_w$$

*The uplift **force** per unit length can be calculated by finding the area of the pressure diagram.*

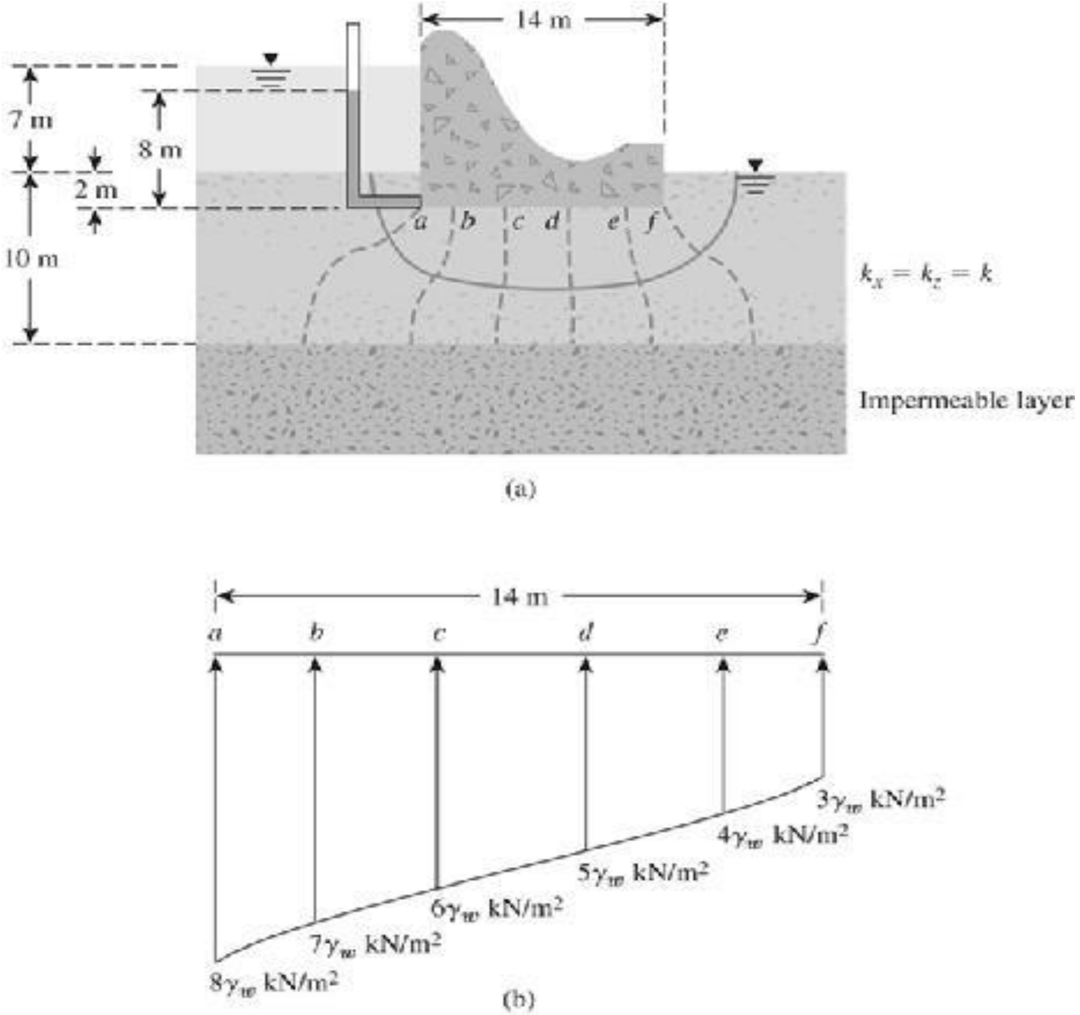


Figure 8.12 (a) A weir; (b) uplift force under a hydraulic structure

Seepage Through an Earth Dam on an Impervious Base:

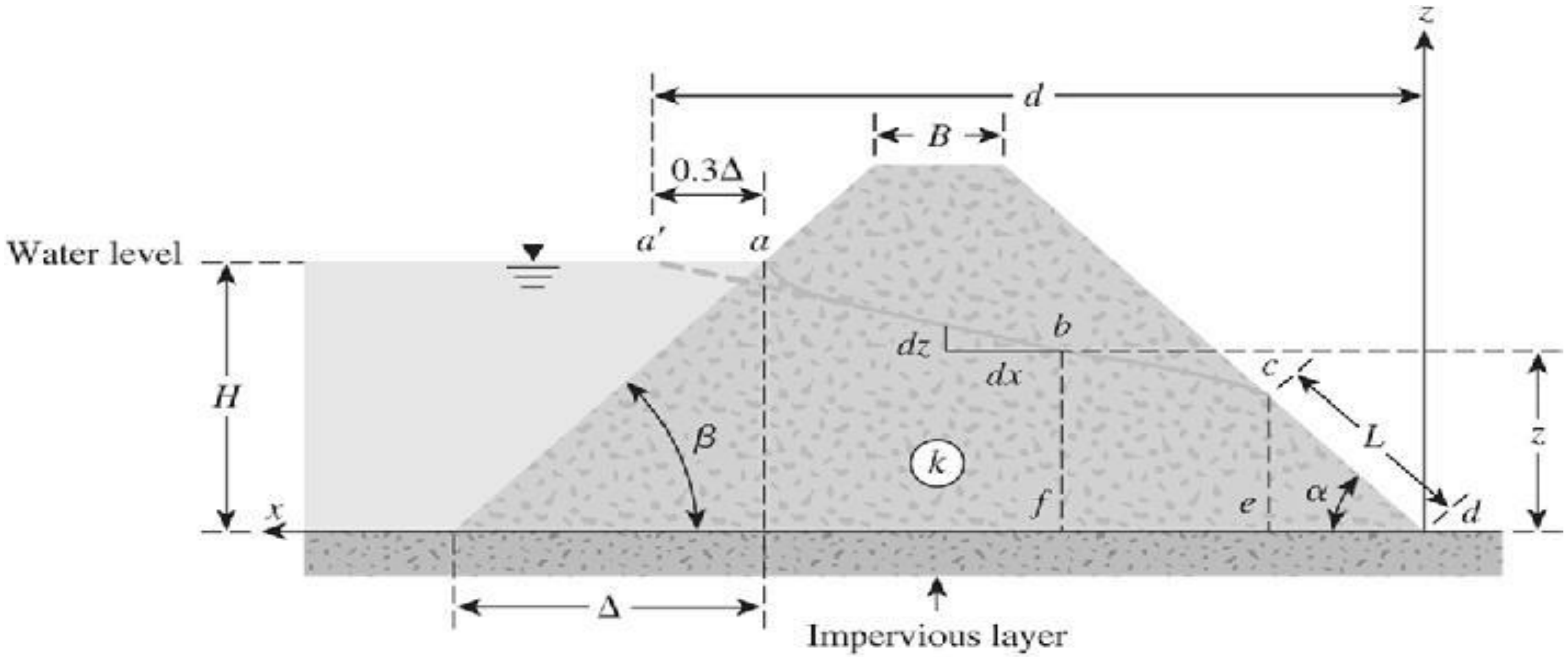


Figure 8.13 Flow through an earth dam constructed over an impervious base

- *a'bc is parabolic.*
- *The slope of the free surface can be assumed to be equal to the* hydraulic gradient.

- It also is assumed that hydraulic gradient is constant with depth (Dupuit, ;(1863

...

$$i \simeq \frac{dz}{dx}$$

- Considering the triangle *cde, we can give the rate of seepage per unit length of the dam;*

$$q = kiA$$

$$i = \frac{dz}{dx} = \tan \alpha$$

$$A = (\overline{ce})(1) = L \sin \alpha$$

$$q = k(\tan \alpha)(L \sin \alpha) = kL \tan \alpha \sin \alpha \qquad (8.30)$$

(at right angles to the cross section)

…

Again, the rate of seepage (per unit length of the dam) through the section *bf* is

$$q = kiA = k\left(\frac{dz}{dx}\right)(z \times 1) = kz\frac{dz}{dx} \qquad (8.31)$$

For continuous flow,

$$q_{\text{Eq. (8.30)}} = q_{\text{Eq. (8.31)}}$$

or

$$kz\frac{dz}{dx} = kL \tan\alpha \sin\alpha$$

or

$$\int_{z=L\sin\alpha}^{z=H} kz\,dz = \int_{x=L\cos\alpha}^{x=d} (kL \tan\alpha \sin\alpha)\,dx$$

$$\tfrac{1}{2}(H^2 - L^2\sin^2\alpha) = L\tan\alpha\sin\alpha(d - L\cos\alpha)$$

$$\frac{H^2}{2} - \frac{L^2\sin^2\alpha}{2} = Ld\left(\frac{\sin^2\alpha}{\cos\alpha}\right) - L^2\sin^2\alpha$$

$$\frac{H^2 \cos\alpha}{2\sin^2\alpha} - \frac{L^2\cos\alpha}{2} = Ld - L^2\cos\alpha$$

or
$$L^2\cos\alpha - 2Ld + \frac{H^2\cos\alpha}{\sin^2\alpha} = 0$$

$$L = \frac{d}{\cos\alpha} - \sqrt{\frac{d^2}{\cos^2\alpha} - \frac{H^2}{\sin^2\alpha}}$$

This solution is known as: Schaffernak's solution (1917) with Casagrande's correction

Following is a step-by-step procedure to obtain the seepage rate q (per unit length of the dam):

Step 1: Obtain α.
Step 2: Calculate Δ (see Figure 8.13) and then 0.3Δ.
Step 3: Calculate d.
Step 4: With known values of α and d, calculate L from Eq. (8.32).
Step 5: With known value of L, calculate q from Eq. (8.30).

Example 8.5

Refer to the earth dam shown in Figure 8.13. Given that $\beta = 45°$, $\alpha = 30°$, $B = 10$ ft, $H = 20$ ft, height of dam $= 25$ ft, and $k = 2 \times 10^{-4}$ ft/min, calculate the seepage rate, q, in ft^3/day/ft length.

Solution

We know that $\beta = 45°$ and $\alpha = 30°$. Thus,

$$\Delta = \frac{H}{\tan\beta} = \frac{20}{\tan 45°} = 20\,\text{ft} \quad 0.3\Delta = (0.3)(20) = 6\,\text{ft}$$

$$d = 0.3\Delta + \frac{(25 - 20)}{\tan\beta} + B + \frac{25}{\tan\alpha}$$

$$= 6 + \frac{(25 - 20)}{\tan 45°} + 10 + \frac{25}{\tan 30} = 64.3\,\text{ft}$$

From Eq. (8.32),

$$L = \frac{d}{\cos\alpha} - \sqrt{\frac{d^2}{\cos^2\alpha} - \frac{H^2}{\sin^2\alpha}}$$

$$= \frac{64.3}{\cos 30} - \sqrt{\left(\frac{64.3}{\cos 30}\right)^2 - \left(\frac{20}{\sin 30}\right)^2} = 11.7\ \text{ft}$$

From Eq. (8.30)

$$q = kL\tan\alpha\sin\alpha = (2 \times 10^{-4})(11.7)(\tan 30)(\sin 30)$$

$$= 6.754 \times 10^{-4}\ \text{ft}^3/\text{min/ft} = \mathbf{0.973\ ft^3/day/ft}$$

L. Casagrande's Solution for Seepage Through an Earth Dam:

Casagrande (1932) showed that, when the downstream slope ngle in Figure 8.13 becomes (a 30), deviations from Dupuit's assumption (*i ≈dz/dx).* become more noticeable. Thus, L. Casagrande (1932) suggested that:

$$i = \frac{dz}{ds} = \sin \alpha$$

where $ds = \sqrt{dx^2 + dz^2}$

$$q = kiA = k \sin \alpha (L \sin \alpha) = kL \sin^2 \alpha$$

$$q = kiA = k\left(\frac{dz}{ds}\right)(1 \times z)$$

$$\int_{L \sin \alpha}^{H} z\, dz = \int_{L}^{s} L \sin^2 \alpha\, ds$$

where s = length of curve $a'bc$

$$\frac{1}{2}(H^2 - L^2 \sin^2 \alpha) = L \sin^2 \alpha (s - L)$$

$$L = s - \sqrt{s^2 - \frac{H^2}{\sin^2 \alpha}}$$

With about 4 to 5% error, we can write

$$s = \sqrt{d^2 + H^2}$$

$$L = \sqrt{d^2 + H^2} - \sqrt{d^2 - H^2 \cot^2 \alpha}$$

Once the magnitude of L is known, the rate of seepage can be calculated

$$q = kL \sin^2 \alpha$$

to avoid the approximation a graphical solution was provided by Gilboy (1934). Note,

$$m = \frac{L \sin \alpha}{H}$$

In order to use the graph,

Step 1: Determine d/H.

Step 2: For a given d/H and α, determine m.

Step 3: Calculate $L = \dfrac{mH}{\sin \alpha}$.

Step 4: Calculate $kL \sin^2 \alpha$.

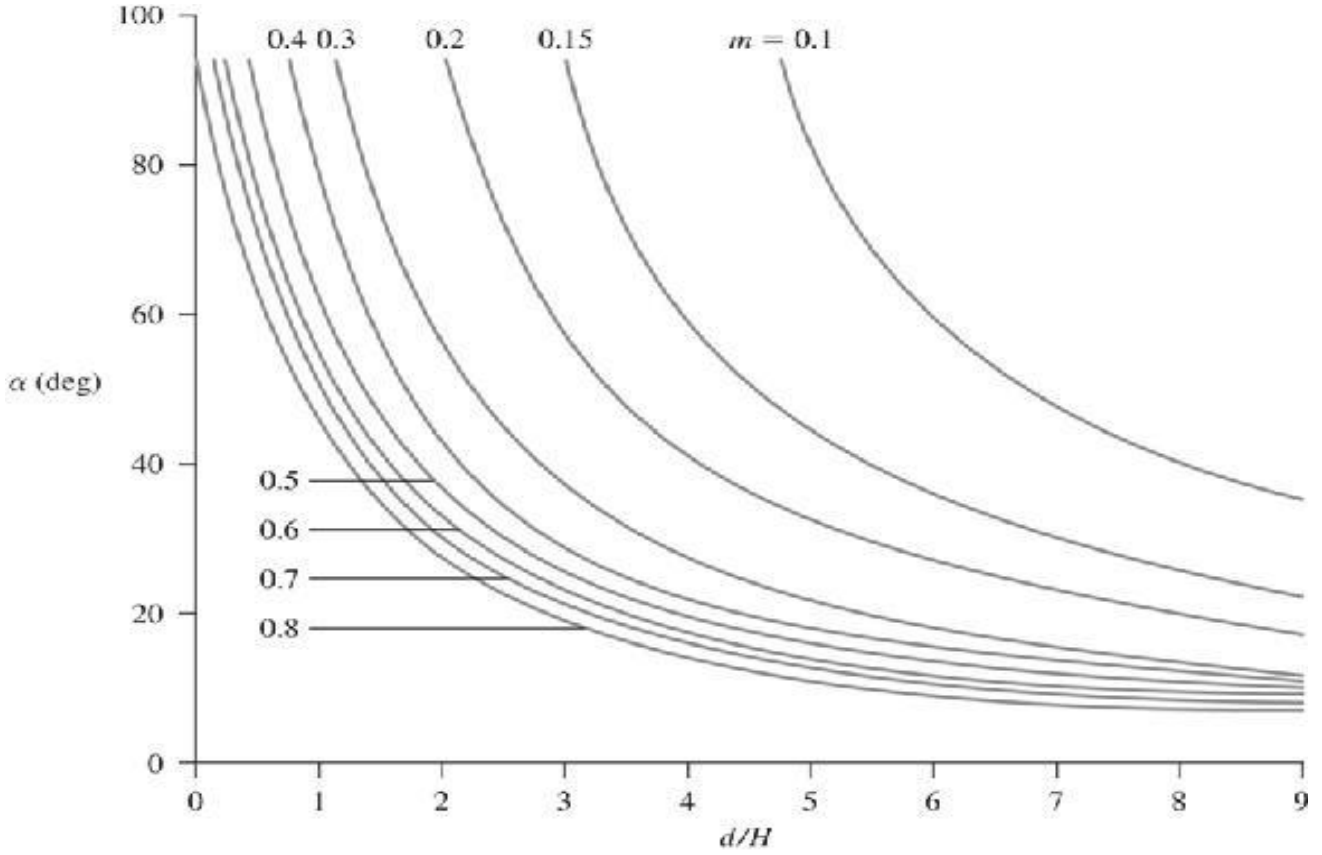


Figure 8.14 Chart for solution by L. Casagrande's method based on Gilboy's solution

Filter Design:

For proper selectionof the filter material,

Condition 1: The size of the voids in the filter material should be small enough to hold the larger particles of the protected material in place.

Condition 2: The filter material should have a high hydraulic conductivity to prevent buildup of large seepage forces and hydrostatic pressures in the filters.

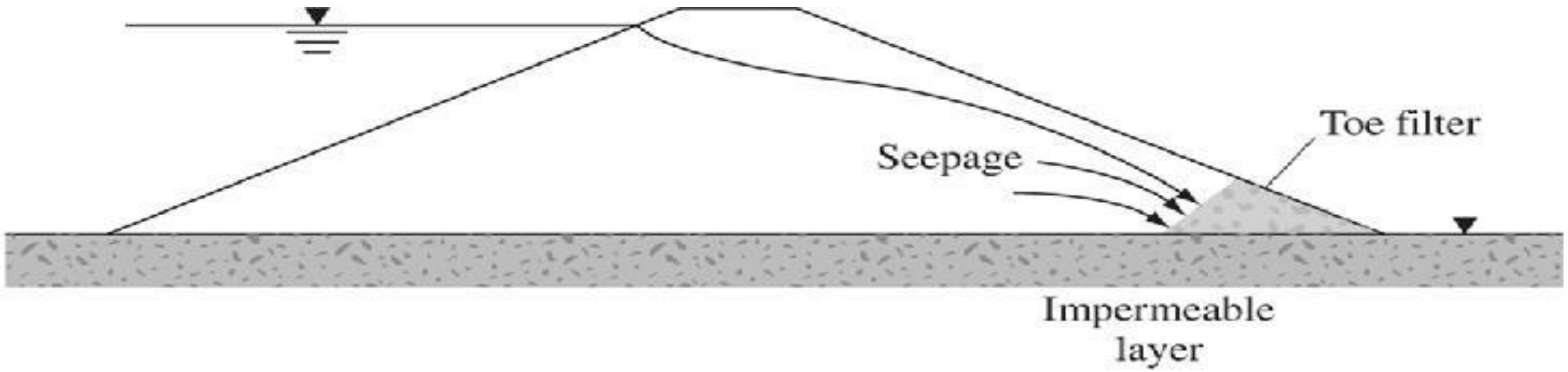


Figure 8.15 Steady-state seepage in an earth dam with a toe filter

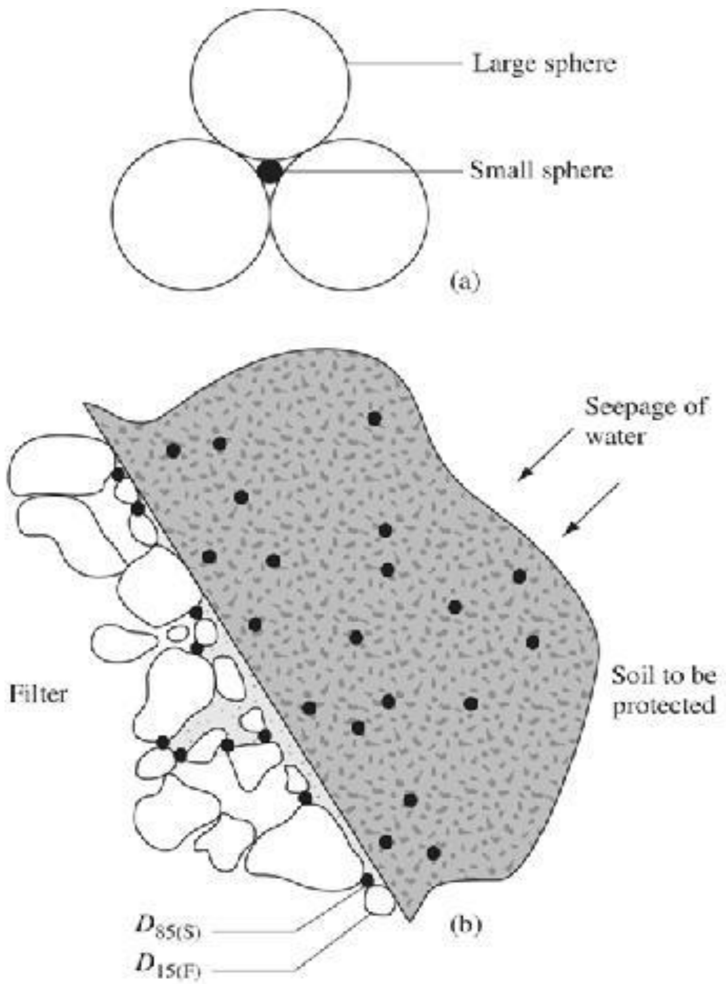


Figure 8.16 (a) Large spheres with diameters of 6.5 times the diameter of the small sphere; (b) boundary between a filter and the soil to be protected

$$\frac{D_{15(F)}}{D_{85(S)}} \leq 4 \text{ to } 5 \qquad \text{(to satisfy Condition 1)}$$

$$\frac{D_{15(F)}}{D_{15(S)}} \geq 4 \text{ to } 5 \qquad \text{(to satisfy Condition 2)}$$

where $D_{15(F)}$ = diameter through which 15% of filter material will pass
$D_{15(S)}$ = diameter through which 15% of soil to be protected will pass
$D_{85(S)}$ = diameter through which 85% of soil to be protected will pass

Let the grain-size distribution of this soil be given by curve *a* in Figure 8.17.
We can now determine $5D_{85(S)}$ and $5D_{15(S)}$ and plot them as shown in Figure 8.17.

The acceptable grain-size distribution of the filter material will have to lie in the shaded zone. (*Note:* The shape of curves *b* and *c* are approximately the same as curve *a*.)

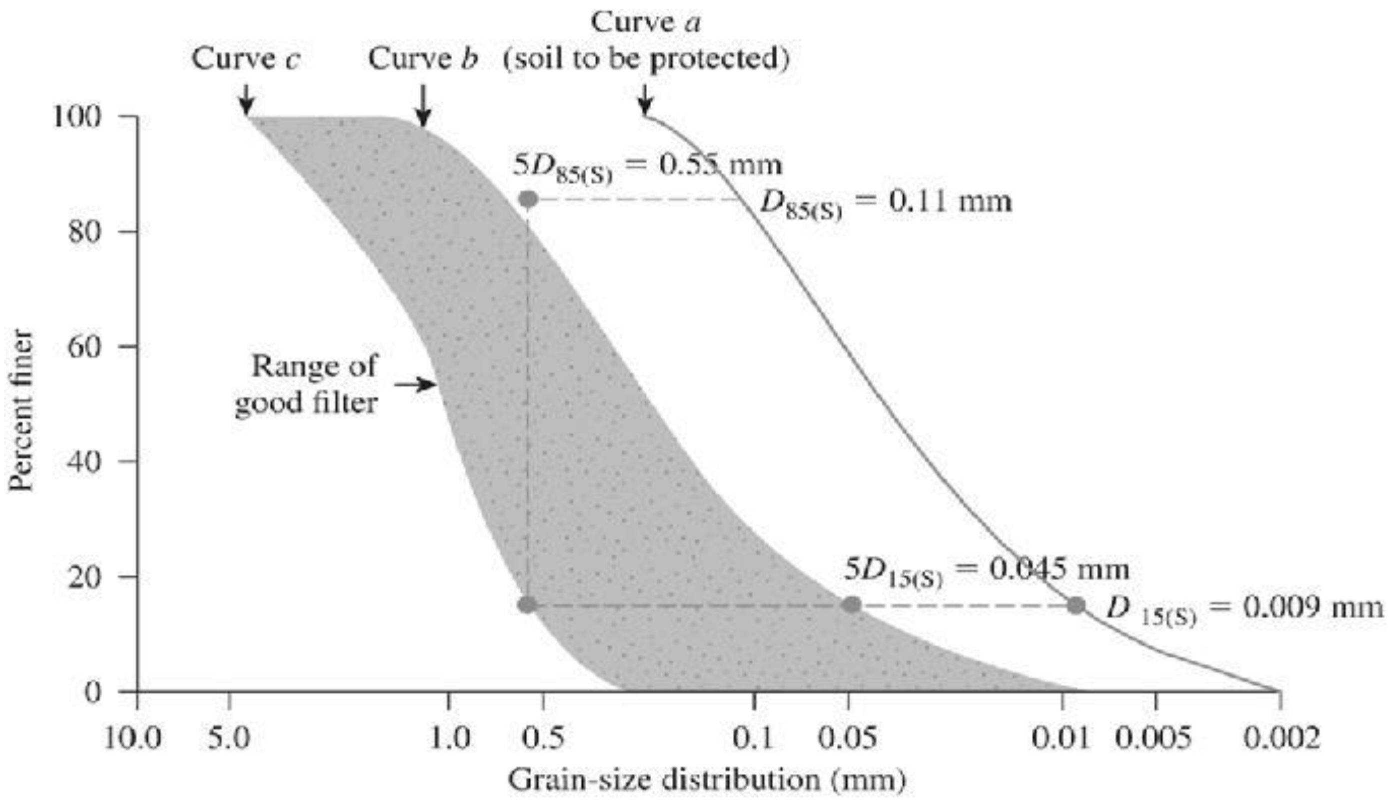


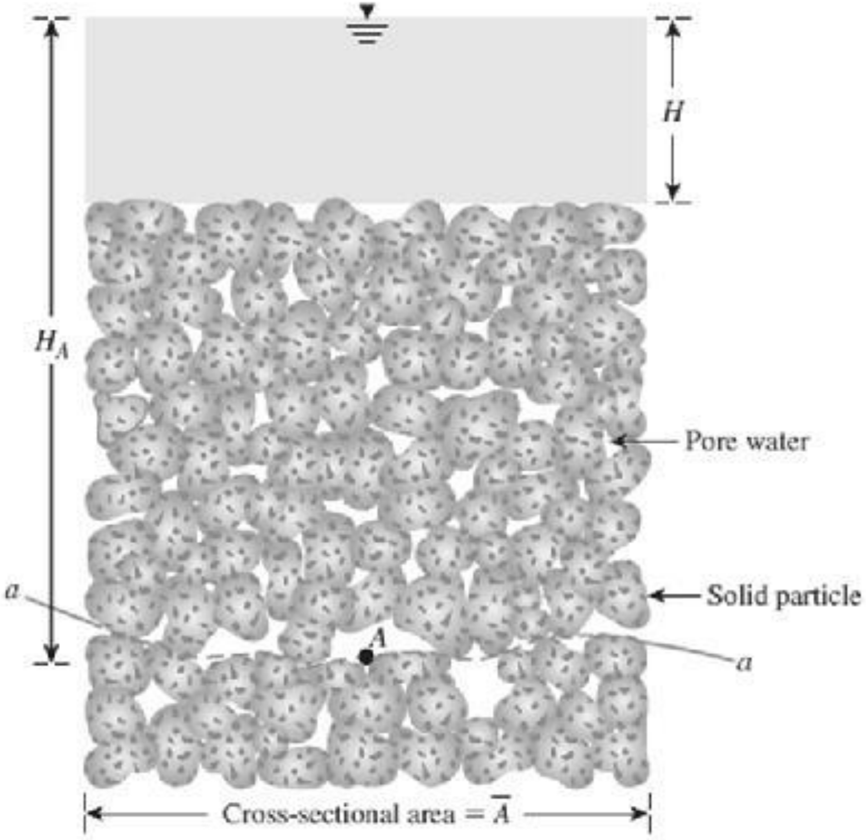
Figure 8.17 Determination of grain-size distribution of filter using Eqs. (8.41) and (8.42)

In situ Stress

• Stresses in Saturated Soil without Seepage

$$\sigma = H\gamma_w + (H_A - H)\gamma_{sat}$$

σ = total stress at the elevation of point A
γ_w = unit weight of water
γ_{sat} = saturated unit weight of the soil
H = height of water table from the top of the soil column
H_A = distance between point A and the water table



(a)

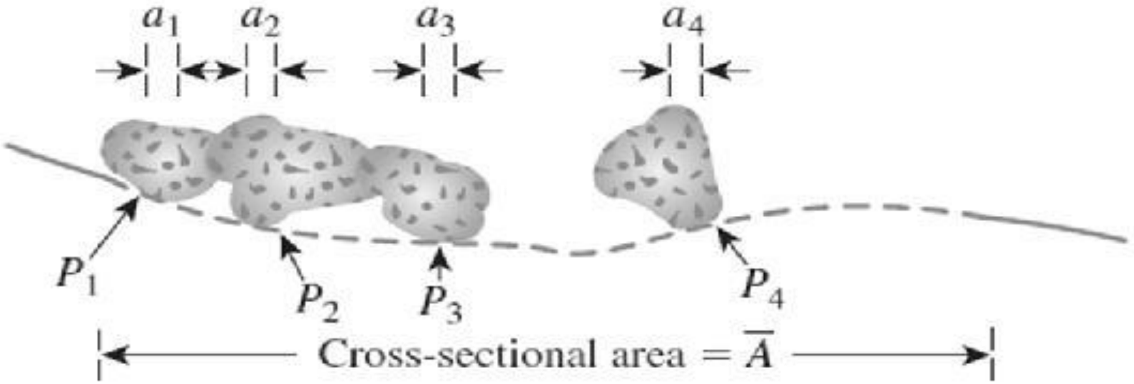
The total stress, σ, given by the following equation can be divided into two parts:

$$\sigma = H\gamma_w + (H_A - H)\gamma_{sat}$$

1. A portion is carried by water in the continuous void spaces. This portion acts with equal intensity in all directions.

2. The rest of the total stress is carried by the soil solids at their points of contact. The sum of the vertical components of the forces developed at the points of contact of the solid particles per unit cross-sectional area of the soil mass is called the *effective stress.*

The value of $a`_s$ is small and can be neglected in practical problems



$$\sigma' = \frac{P_{1(v)} + P_{2(v)} + P_{3(v)} + \cdots + P_{n(v)}}{\overline{A}}$$

$$\sigma = \sigma' + \frac{u(\overline{A} - a_s)}{\overline{A}} = \sigma' + u(1 - a'_s)$$

The value of ($a`_s$) is extremely small and can be neglected

$$\sigma = \sigma' + u$$

$$\sigma = \sigma' + u$$

$$\begin{aligned}\sigma' &= [H\gamma_w + (H_A - H)\gamma_{sat}] - H_A\gamma_w \\ &= (H_A - H)(\gamma_{sat} - \gamma_w) \\ &= (\text{Height of the soil column}) \times \gamma'\end{aligned}$$

where $\gamma' = \gamma_{sat} - \gamma_w$ equals the submerged unit weight of soil.

- ***effective stress: is** approximately the force per unit area carried by the soil skeleton.*
- The effective stress in a soil mass controls its volume change and strength.
- Increasing the effective stress induces soil to move into a denser state of packing.

- The effective stress principle is probably the most important concept in geotechnical engineering.

- The compressibility and shearing resistance of a soil depend to a great extent on the effective stress.

- The concept of effective stress is significant in solving geotechnical engineering problems, such as:

    1. the lateral earth pressure on retaining structures
    2. The load-bearing capacity
    3. settlement of foundations
    4. the stability of earth slopes

•**Submerged soil in a tank with no seepage**.

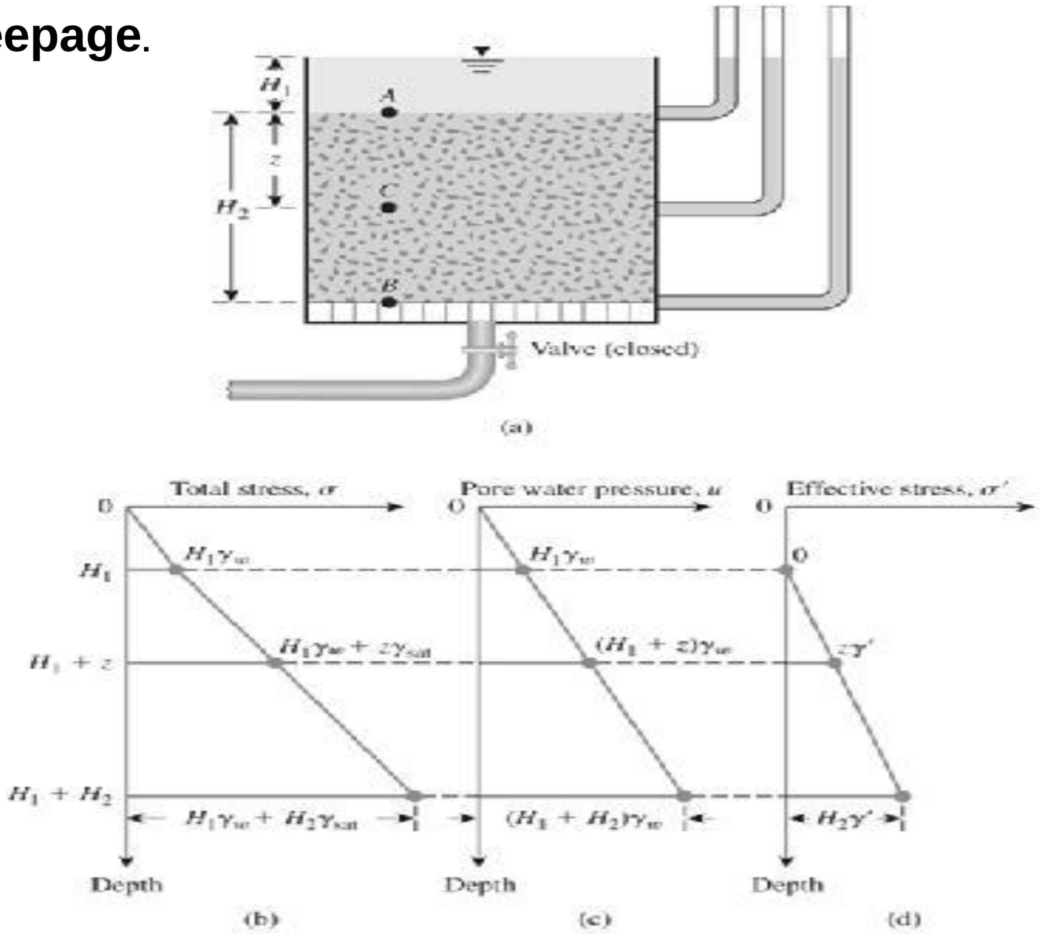


Figure 9.2 (a) Layer of soil in a tank where there is no seepage; Variation of (b) total stress, (c) pore water pressure, and (d) effective stress with depth for a submerged soil layer without seepage

•For fine-grained soils, intergranular contact may not physically be there, because the clay particles are surrounded by tightly held water film. In a more general sense, we can be write:

$$\sigma = \sigma_{ig} + u(1 - a_s') - A' + R'$$

σ_{ig} = intergranular stress
A' = electrical attractive force per unit cross-sectional area of soil
R' = electrical repulsive force per unit cross-sectional area of soil

For granular soils, silts, and clays of low plasticity, the magnitudes of A′ and R′ are small. Hence, for all practical purposes,

$$\sigma_{ig} = \sigma' \approx \sigma - u$$

However, if $A' - R'$ is large, then $\sigma_{ig} \neq \sigma'$.

Example 9.1

A soil profile is shown in Figure 9.3. Calculate the total stress, pore water pressure, and effective stress at points *A*, *B*, and *C*.

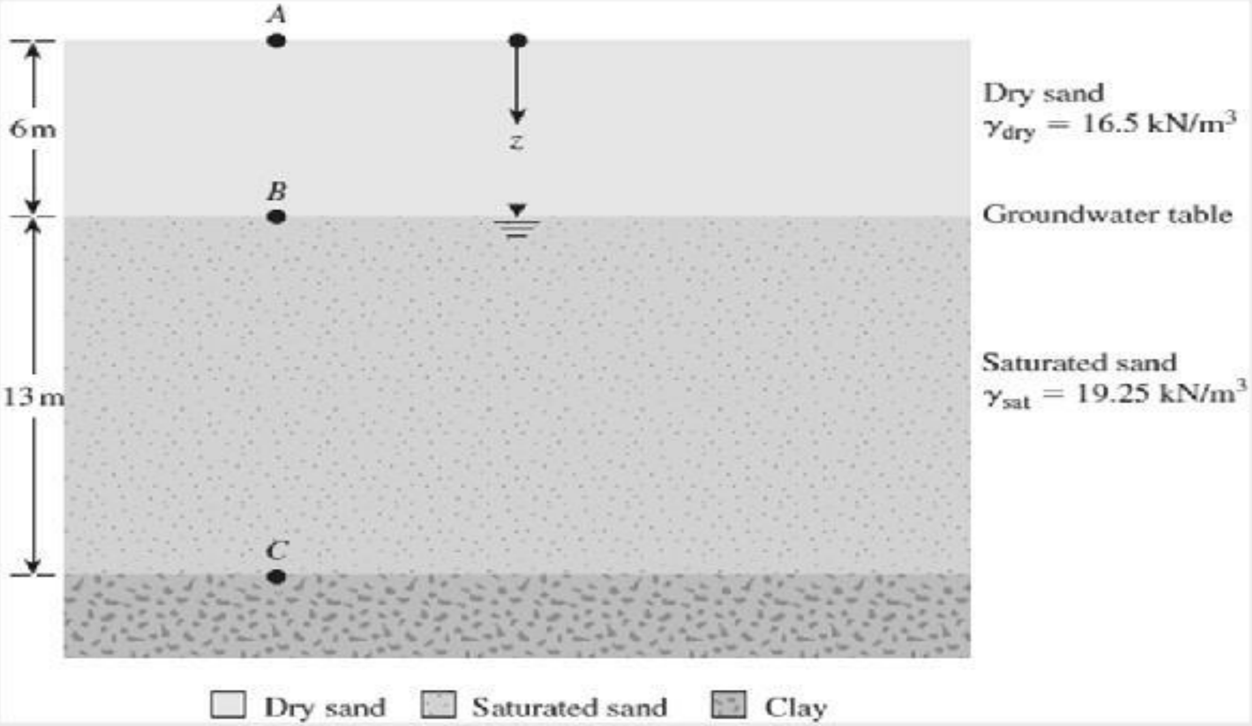


Figure 9.3 Soil profile

Solution

At Point A,

Total stress: $\sigma_A = \mathbf{0}$
Pore water pressure: $u_A = \mathbf{0}$
Effective stress: $\sigma'_A = \mathbf{0}$

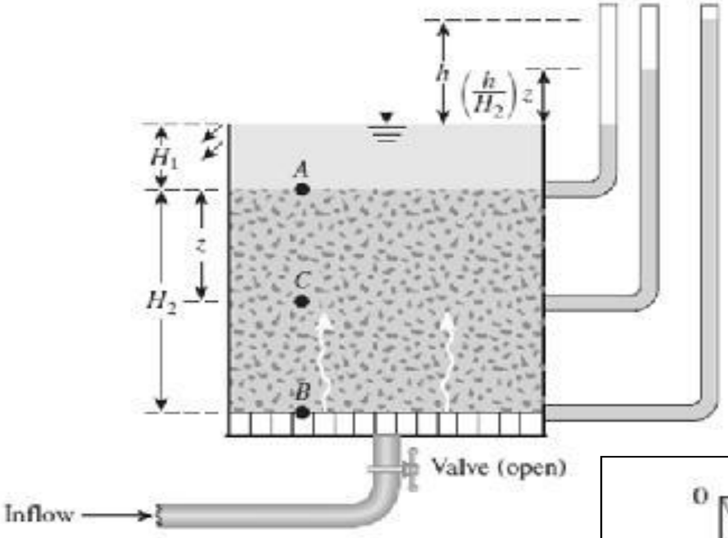
At Point B,

$$\sigma_B = 6\gamma_{\text{dry(sand)}} = 6 \times 16.5 = \mathbf{99\ kN/m^2}$$
$$u_B = \mathbf{0\ kN/m^2}$$
$$\sigma'_B = 99 - 0 = \mathbf{99\ kN/m^2}$$

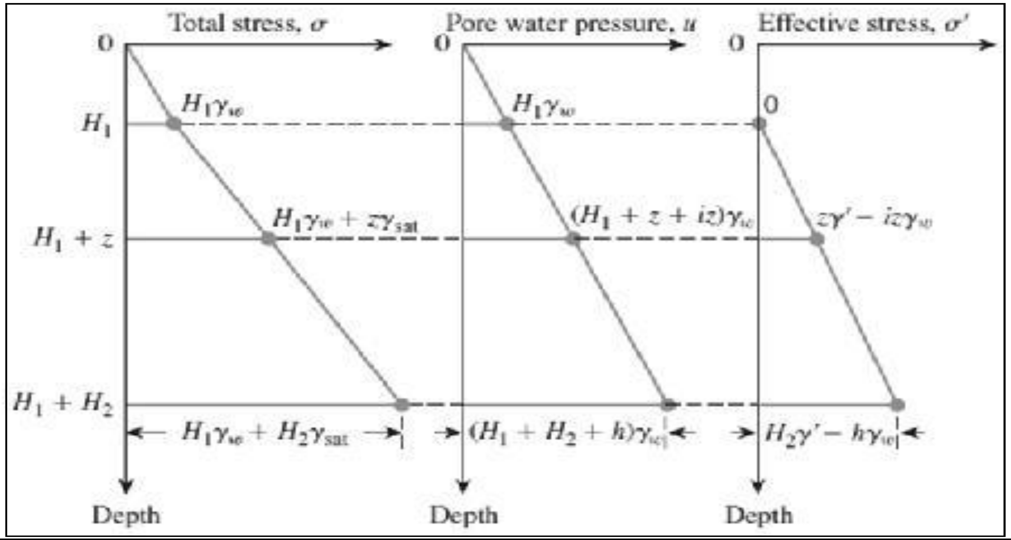
At Point C,

$$\sigma_C = 6\gamma_{\text{dry(sand)}} + 13\gamma_{\text{sat(clay)}}$$
$$= 6 \times 16.5 + 13 \times 19.25$$
$$= 99 + 250.25 = \mathbf{349.25\ kN/m^2}$$
$$u_C = 13\gamma_w = 13 \times 9.81 = \mathbf{127.53\ kN/m^2}$$
$$\sigma'_C = 349.25 - 127.53 = \mathbf{221.72\ kN/m^2}$$

• Stresses in Saturated Soil with Upward Seepage



➢The total stress at any point in the soil mass is due solely to the weight of soil and water above it.



At A,

- Total stress: $\sigma_A = H_1\gamma_w$
- Pore water pressure: $u_A = H_1\gamma_w$
- Effective stress: $\sigma'_A = \sigma_A - u_A = 0$

At B,

- Total stress: $\sigma_B = H_1\gamma_w + H_2\gamma_{sat}$
- Pore water pressure: $u_B = (H_1 + H_2 + h)\gamma_w$
- Effective stress: $\sigma'_B = \sigma_B - u_B$

$$= H_2(\gamma_{sat} - \gamma_w) - h\gamma_w$$
$$= H_2\gamma' - h\gamma_w$$

Similarly, the effective stress at a point C located at a depth z below the top of the soil surface can be calculated as follows:

At C,

- Total stress: $\sigma_C = H_1\gamma_w + z\gamma_{sat}$
- Pore water pressure: $u_C = \left(H_1 + z + \frac{h}{H_2}z\right)\gamma_w$
- Effective stress: $\sigma'_C = \sigma_C - u_C$

$$= z(\gamma_{sat} - \gamma_w) - \frac{h}{H_2}z\gamma_w$$
$$= z\gamma' - \frac{h}{H_2}z\gamma_w$$

Note that h/H_2 is the hydraulic gradient i caused by the flow, and therefore,

$$\sigma'_C = z\gamma' - iz\gamma_w$$

- If the rate of seepage and thereby the hydraulic gradient gradually are increased, a limiting condition will be reached, at which point:

$$\sigma'_C = z\gamma' - i_{cr} z\gamma_w = 0$$

- where i_{cr} *critical hydraulic gradient (for zero effective stress).* Under such a situation, soil stability is lost. This situation generally is referred to **as *boiling, or a quick*** *condition.*

$$i_{cr} = \frac{\gamma'}{\gamma_w}$$

Example 9.2

A 20-ft thick layer of stiff saturated clay is underlain by a layer of sand (Figure 9.5). The sand is under artesian pressure. Calculate the maximum depth of cut H that can be made in the clay.

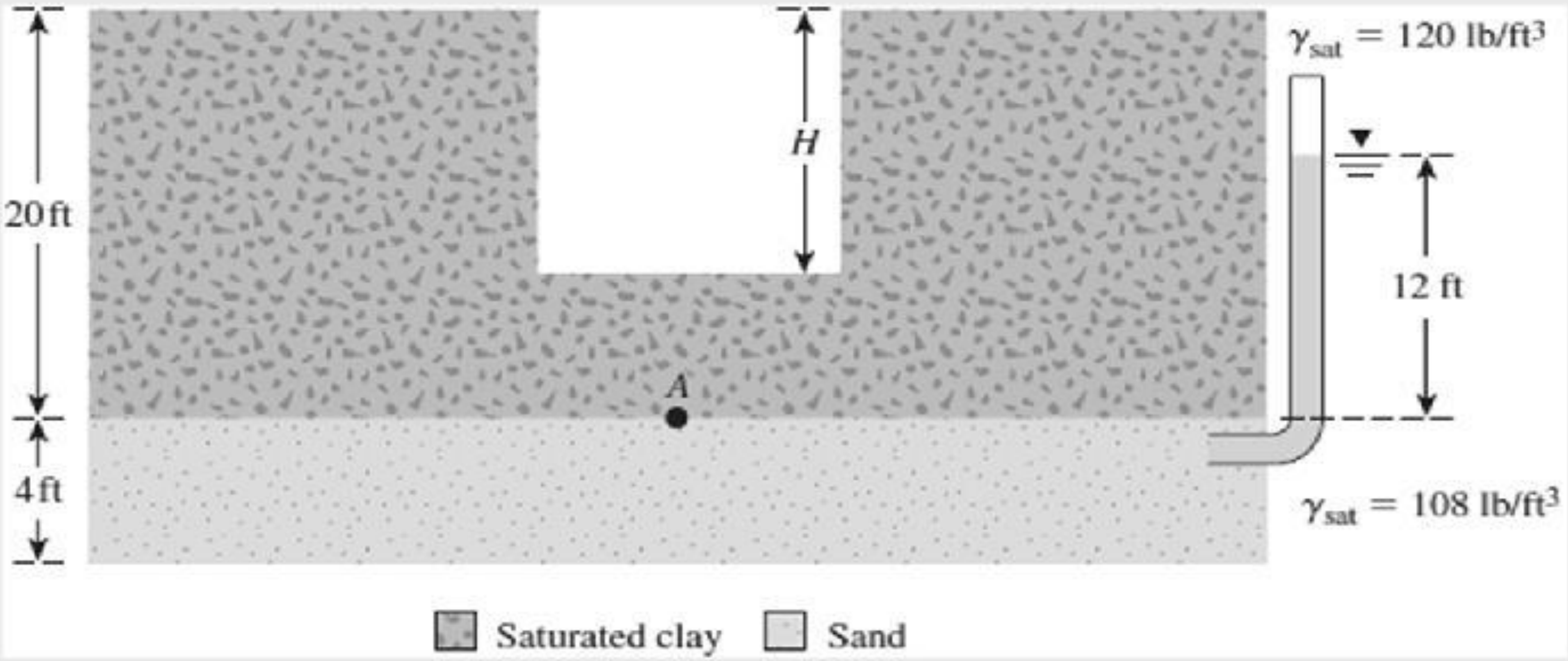


Figure 9.5

Solution

Due to excavation, there will be unloading of the overburden pressure. Let the depth of the cut be H, at which point the bottom will heave. Let us consider the stability of point A at that time:

$$\sigma_A = (20 - H)\gamma_{\mathrm{sat(clay)}}$$
$$u_A = 12\gamma_w$$

For heave to occur, σ'_A should be 0. So

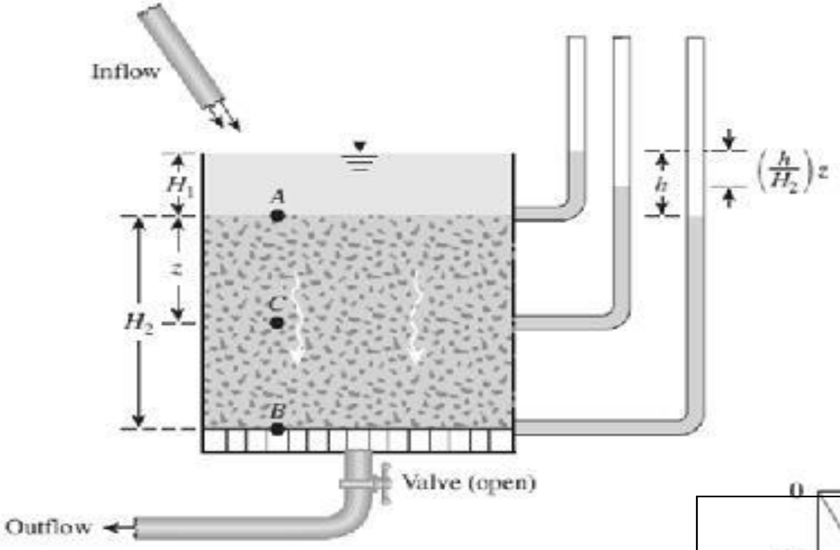
$$\sigma_A - u_A = (20 - H)\gamma_{\mathrm{sat(clay)}} - 12\gamma_w$$

or

$$(20 - H)120 - (12)62.4 = 0$$

$$H = \frac{(20)120 - (12)62.4}{120} = \mathbf{13.76\ ft}$$

• **Stresses in Saturated Soil with Downward Seepage**

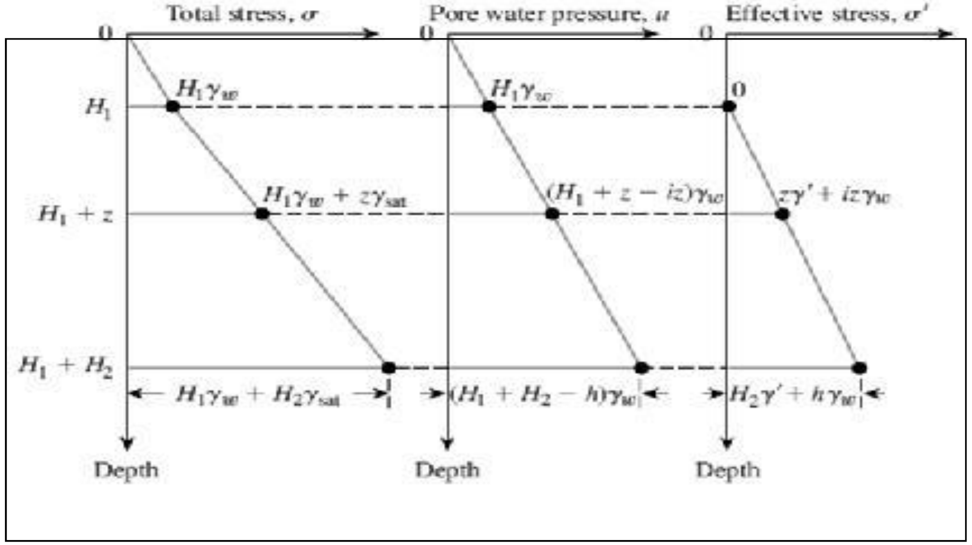


$$\sigma_C = H_1\gamma_w + z\gamma_{sat}$$

$$u_C = (H_1 + z - iz)\gamma_w$$

$$\sigma'_C = (H_1\gamma_w + z\gamma_{sat}) - (H_1 + z - iz)\gamma_w$$

$$= z\gamma' + iz\gamma_w$$



- **Seepage Force**

The effect of seepage is to increase or decrease the effective stress at a point in a layer of soil. Expressing the seepage force per unit volume of soil is convenient.

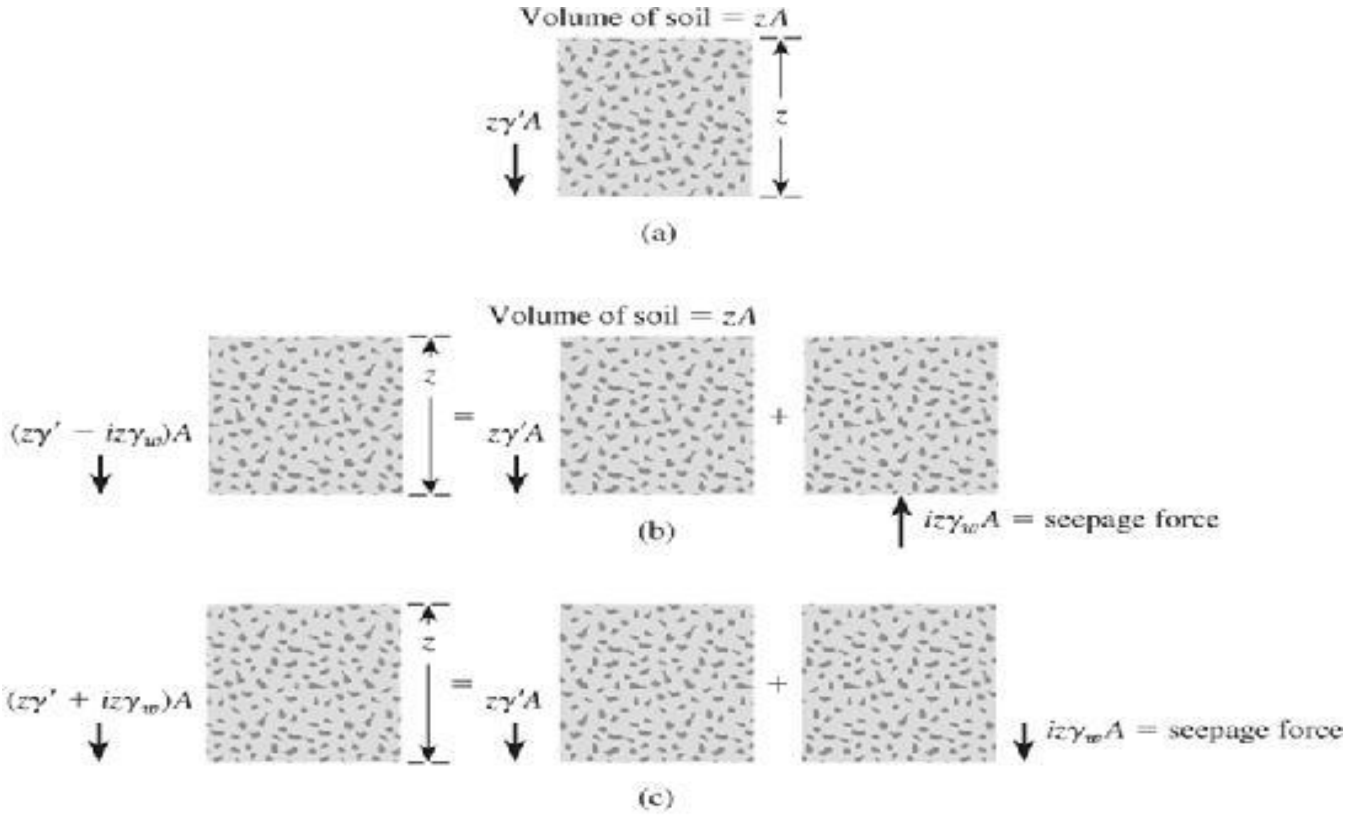


Figure 9.7 Force due to (a) no seepage; (b) upward seepage; (c) downward seepage on a volume of soil

Example 9.3

Consider the upward flow of water through a layer of sand in a tank as shown in Figure 9.8. For the sand, the following are given: void ratio (e) = 0.52 and specific gravity of solids = 2.67.

a. Calculate the total stress, pore water pressure, and effective stress at points A and B.
b. What is the upward seepage force per unit volume of soil?

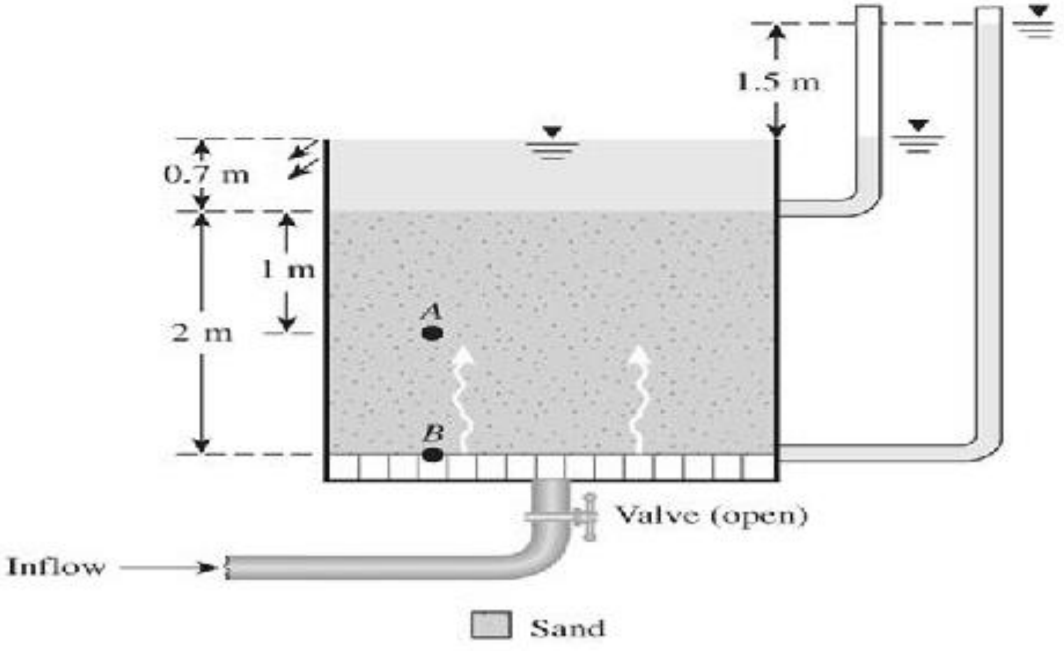


Figure 9.8 Upward flow of water through a layer of sand in a tank

Solution

Part a

The saturated unit weight of sand is calculated as follows:

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1 + e} = \frac{(2.67 + 0.52)9.81}{1 + 0.52} = 20.59 \text{ kN/m}^3$$

Now, the following table can be prepared:

Point	Total stress, σ (kN/m²)	Pore water pressure, u (kN/m²)	Effective stress, $\sigma' = \sigma - u$ (kN/m²)
A	$0.7\gamma_w + 1\gamma_{sat} = (0.7)(9.81) + (1)(20.59) = \mathbf{27.46}$	$\left[(1 + 0.7) + \left(\frac{1.5}{2}\right)(1)\right]\gamma_w = (2.45)(9.81) = \mathbf{24.03}$	**3.43**
B	$0.7\gamma_w + 2\gamma_{sat} = (0.7)(9.81) + (2)(20.59) = \mathbf{48.05}$	$(2 + 0.7 + 1.5)\gamma_w = (4.2)(9.81) = \mathbf{41.2}$	**6.85**

Part b

Hydraulic gradient (i) = 1.5/2 = 0.75. Thus, the seepage force per unit volume can be calculated as

$$i\gamma_w = (0.75)(9.81) = \mathbf{7.36 \text{ kN/m}^3}$$

■

• **Heaving in Soil Due to Flow Around Sheet Piles**

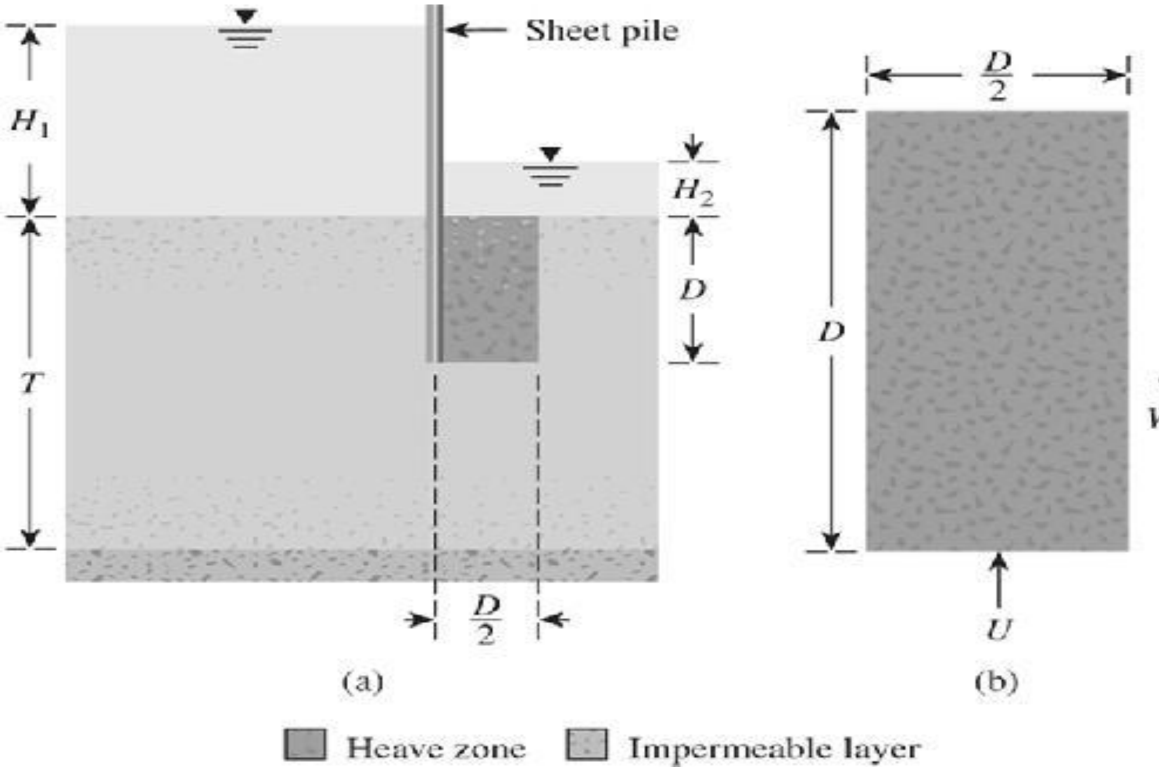


Figure 9.9
(a) Check for heaving on the downstream side for a row of sheet piles driven into a permeable layer; (b) enlargement of heave zone

•Terzaghi (1922) concluded that heaving generally occurs within a distance of ***D/2*** *from the sheet piles (when D equals* depth of embedment of sheet piles into the permeable layer).

The factor of safety against heaving can be given by

$$FS = \frac{W'}{U} \tag{9.14}$$

where FS = factor of safety

W' = submerged weight of soil in the heave zone per unit length of sheet pile $= D(D/2)(\gamma_{sat} - \gamma_w) = (\frac{1}{2})D^2\gamma'$

U = uplifting force caused by seepage on the same volume of soil

From Eq. (9.13),

$$U = (\text{Soil volume}) \times (i_{av}\gamma_w) = \tfrac{1}{2}D^2 i_{av}\gamma_w$$

where i_{av} = average hydraulic gradient at the bottom of the block of soil

Substituting the values of W' and U in Eq. (9.14), we can write

$$\boxed{FS = \frac{\gamma'}{i_{av}\gamma_w}} \tag{9.15}$$

For the case of *flow around a sheet pile* in a *homogeneous soil*, as shown in Figure 9.9, it can be demonstrated that

$$\frac{U}{0.5\gamma_w D(H_1 - H_2)} = C_o$$

where C_o is a function of D/T (see Table 9.1). Hence, from Eq. (9.14),

$$\boxed{FS = \frac{W'}{U} = \frac{0.5D^2\gamma'}{0.5C_o\gamma_w D(H_1 - H_2)} = \frac{D\gamma'}{C_o\gamma_w(H_1 - H_2)}}$$

Table 9.1 Variation of C_o with D/T

D/T	C_o
0.1	0.385
0.2	0.365
0.3	0.359
0.4	0.353
0.5	0.347
0.6	0.339
0.7	0.327
0.8	0.309
0.9	0.274

• Effective Stress in Partially Saturated Soil

In partially saturated soil, water in the void spaces is not continuous, and it is a three-phase system—that is, solid, pore water, and pore air.

❑Bishop, *et al. (1960) gave the following equation for effective stress* in partially saturated soils…

$$\sigma' = \sigma - u_a + \chi(u_a - u_w)$$

where σ' = effective stress
σ = total stress
u_a = pore air pressure
u_w = pore water pressure

χ represents the fraction of a unit cross-sectional area of the soil occupied by water.
For dry soil $\chi = 0$, and for saturated soil $\chi = 1$.

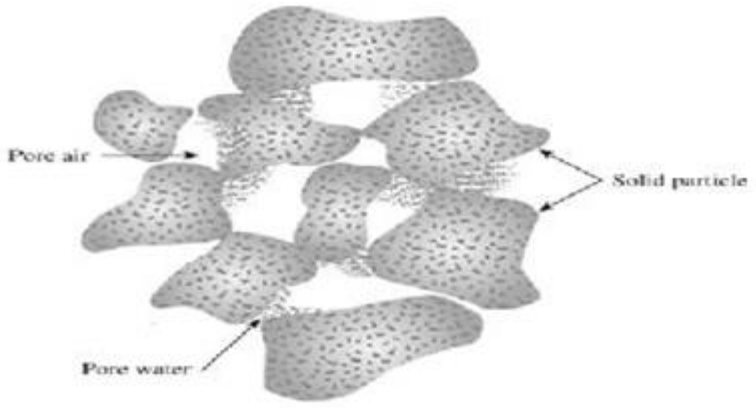


Figure 9.13 Partially saturated soil

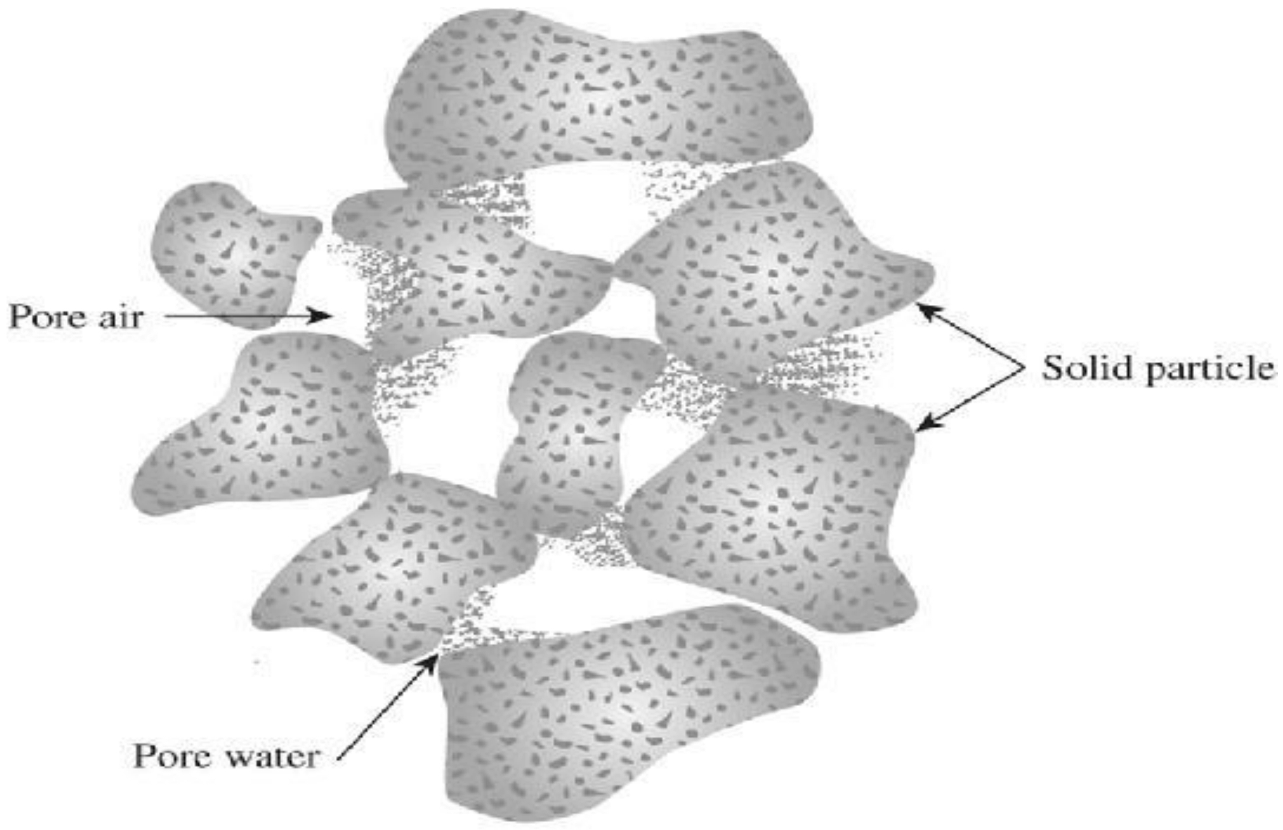


Figure 9.13 Partially saturated soil

•Bishop, *et al. have pointed out that the intermediate values of x will depend primarily* on the degree of saturation *S.*

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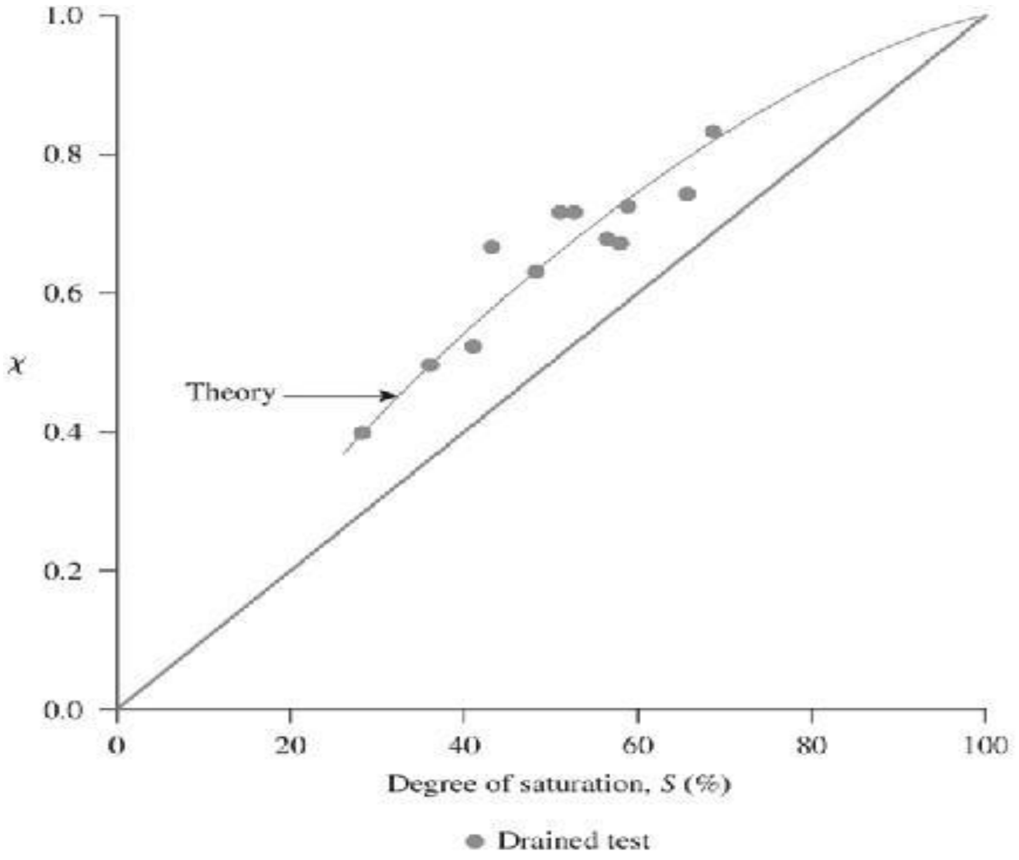


Figure 9.14 Relationship between the parameter χ and the degree of saturation for Bearhead silt (*After Bishop et al, 1960. With permission from ASCE.*)

• Capillary Rise in Soils

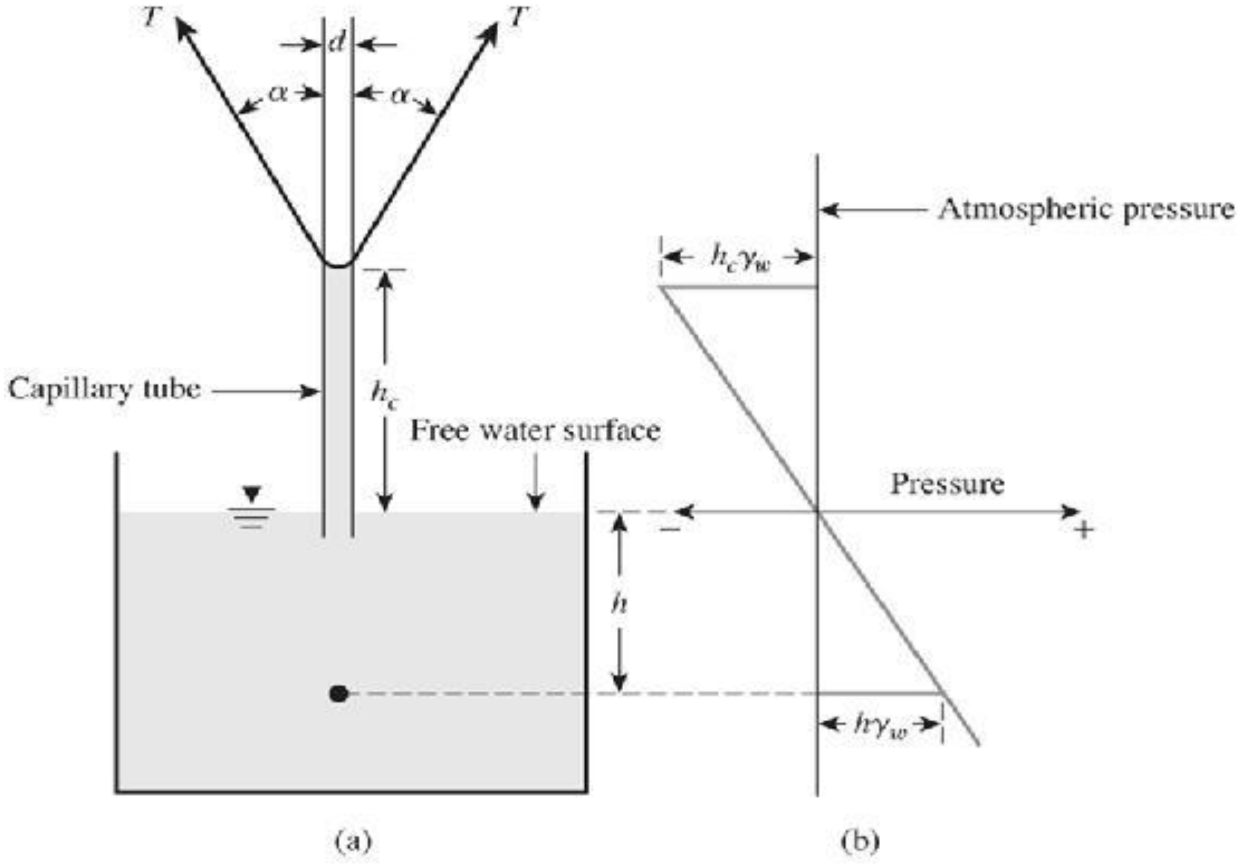


Figure 9.15 (a) Rise of water in the capillary tube; (b) pressure within the height of rise in the capillary tube (atmospheric pressure taken as datum)

- The height of rise of water in the capillary tube can be given by summing the forces in the vertical direction, or

$$\left(\frac{\pi}{4}d^2\right)h_c\gamma_w = \pi d T \cos\alpha$$

$$h_c = \frac{4T\cos\alpha}{d\gamma_w} \qquad (9.21)$$

where T = surface tension (force/length)
α = angle of contact
d = diameter of capillary tube
γ_w = unit weight of water

For pure water and clean glass, $\alpha = 0$. Thus, Eq. (9.21) becomes

$$h_c = \frac{4T}{d\gamma_w} \qquad (9.22)$$

For water, $T = 72$ mN/m. From Eq. (9.22), we see that the height of capillary rise

$$h_c \propto \frac{1}{d} \qquad (9.23)$$

Thus, the smaller the capillary tube diameter, the larger the capillary rise.

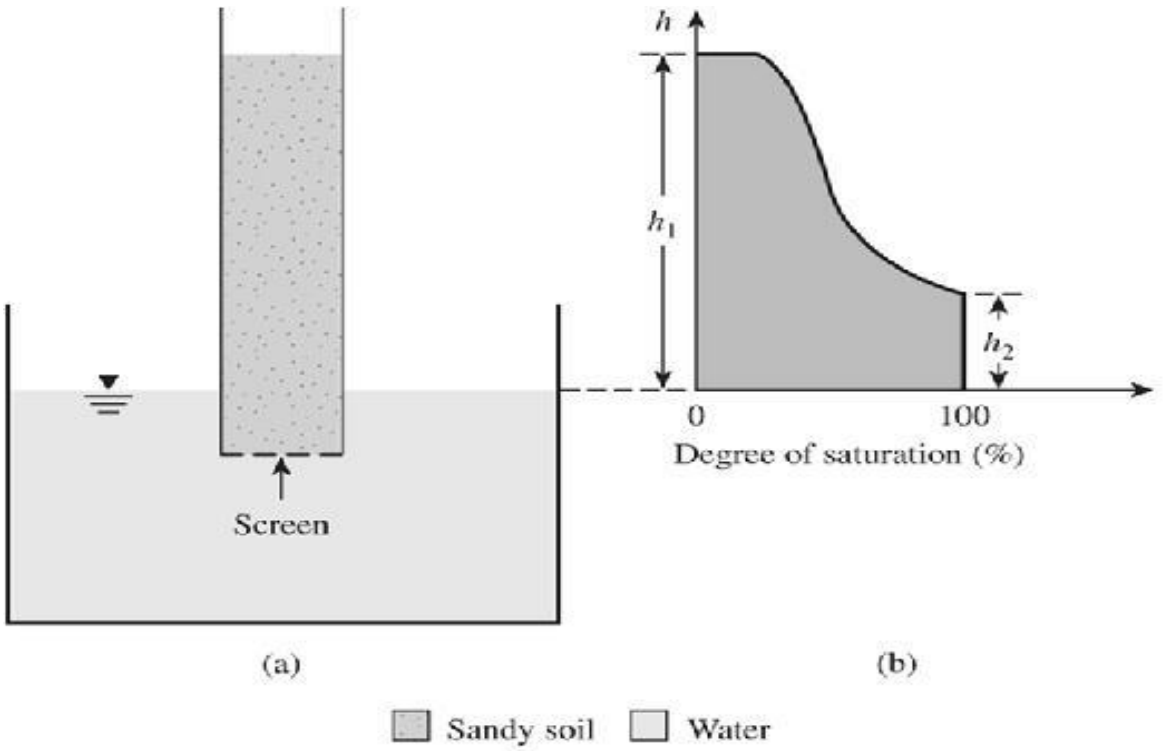


Figure 9.16 Capillary effect in sandy soil: (a) a soil column in contact with water; (b) variation of degree of saturation in the soil column

Table 9.2 Approximate Range of Capillary Rise in Soils

Soil type	Range of capillary rise (m)
Coarse sand	0.1–0.2
Fine sand	0.3–1.2
Silt	0.75–7.5
Clay	7.5–23

- **Effective Stress in the Zone of Capillary Rise**

The pore water pressure *u at a point in a layer of soil fully saturated by capillary* rise is equal to ***-γh.***

(h = height of the point under consideration measured from the groundwater table)

- If partial saturation is caused by capillary action, *pore water pressure u* can be approximated as:

$$u = -\left(\frac{S}{100}\right)\gamma_w h$$

- where *S = degree of saturation, in percent.*

Example 9.5

A soil profile is shown in Figure 9.17. Given: $H_1 = 6$ ft, $H_2 = 3$ ft, $H_3 = 6$ ft. Plot the variation of σ, u, and σ' with depth.

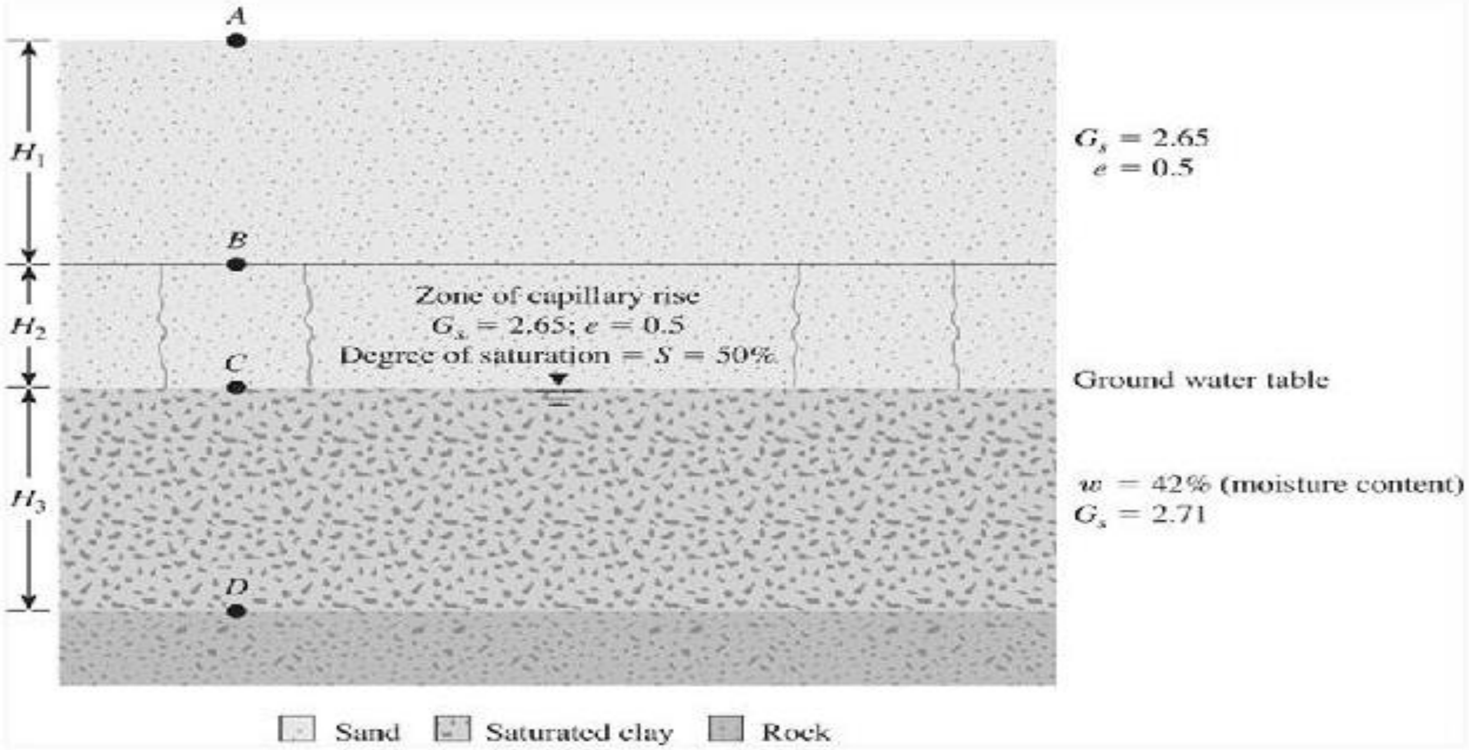


Figure 9.17

Solution

Determination of Unit Weight

Dry sand:

$$\gamma_{d(\text{sand})} = \frac{G_s \gamma_w}{1 + e} = \frac{(2.65)(62.4)}{1 + 0.5} = 110.24 \text{ lb/ft}^3$$

Moist sand:

$$\gamma_{\text{sand}} = \frac{(G_s + Se)\gamma_w}{1 + e} = \frac{[2.65 + (0.5)(0.5)]62.4}{1 + 0.5} = 120.64 \text{ lb/ft}^3$$

Saturated clay:

$$e = \frac{G_s w}{S} = \frac{(2.71)(0.42)}{1.0} = 1.1382$$

$$\gamma_{\text{sat(clay)}} = \frac{(G_s + e)\gamma_w}{1 + e} = \frac{(2.71 + 1.1382)62.4}{1 + 1.1382} = 112.3 \text{ lb/ft}^3$$

Calculation of Stress

At the ground surface (i.e., point A):

$$\sigma = 0$$
$$u = 0$$
$$\sigma' = \sigma - u = 0$$

At depth H_1 (i.e., point B):

$$\sigma = \gamma_d(\text{sand})(6) = (110.24)(6) = \mathbf{661.44\ lb/ft^2}$$

$$u = \mathbf{0}\ \text{(immediately above)}$$

$$u = -(S\gamma_w H_2) = -(0.5)(62.4)(3) = \mathbf{-93.6\ lb/ft^2}\ \text{(immediately below)}$$

$$\sigma' = 661.44 - 0 = \mathbf{661.44\ lb/ft^2}\ \text{(immediately above)}$$

$$\sigma' = 661.44 - (-93.6) = \mathbf{755.04\ lb/ft^2}\ \text{(immediately below)}$$

At depth $H_1 + H_2$ (i.e., at point C):

$$\sigma = (110.24)(6) + (120.64)(3) = \mathbf{1023.36\ lb/ft^2}$$

$$u = \mathbf{0}$$

$$\sigma' = 1023.36 - 0 = \mathbf{1023.36\ lb/ft^2}$$

At depth $H_1 + H_2 + H_3$ (i.e., at point D):

$$\sigma = 1023.36 + (112.3)(6) = \mathbf{1697.17\ lb/ft^2}$$

$$u = 6\gamma_w = (6)(62.4) = \mathbf{374.4\ lb/ft^2}$$

$$\sigma' = 1697.17 - 374.4 = \mathbf{1322.77\ lb/ft^2}$$

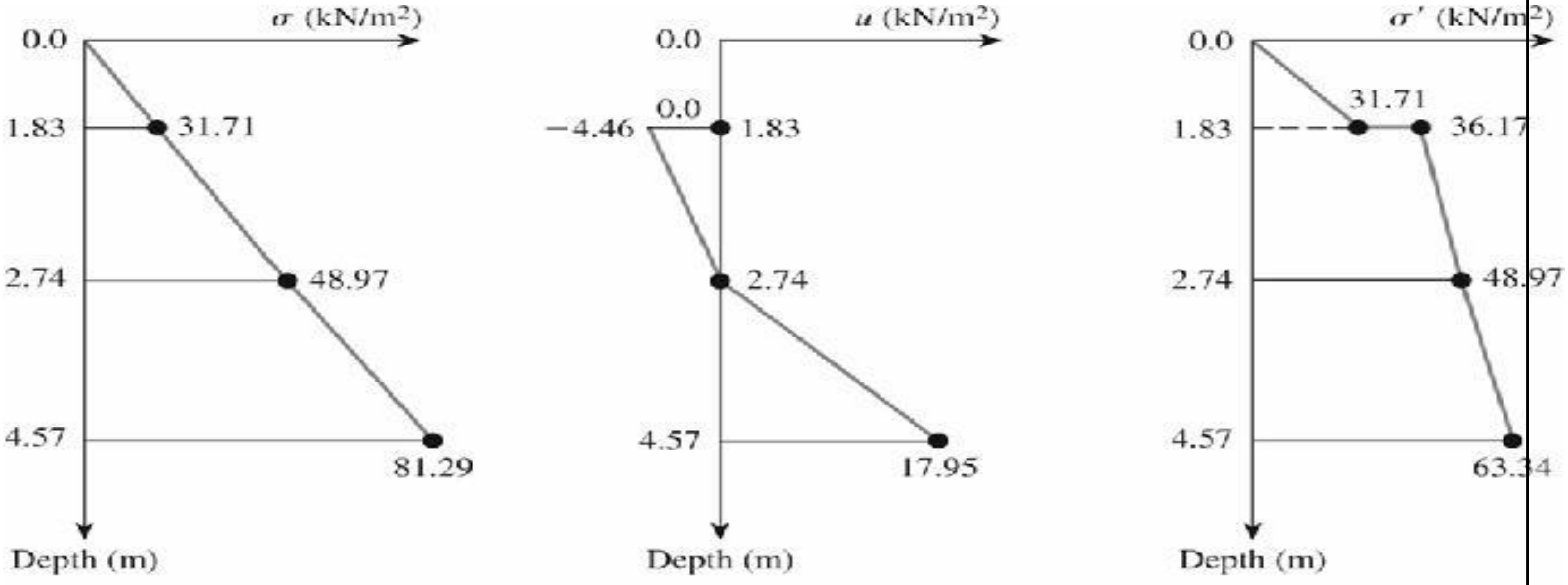

σ (kN/m²)
0.0
1.83
31.71
2.74
48.97
4.57
81.29
Depth (m)
u (kN/m²)
0.0
0.0
−4.46
1.83
2.74
4.57
17.95
Depth (m)
σ′ (kN/m²)
0.0
31.71
1.83
36.17
2.74
48.97
4.57
63.34
Depth (m)

Figure 9.18

Stresses in a soil mass

• Normal and Shear Stresses on a Plane

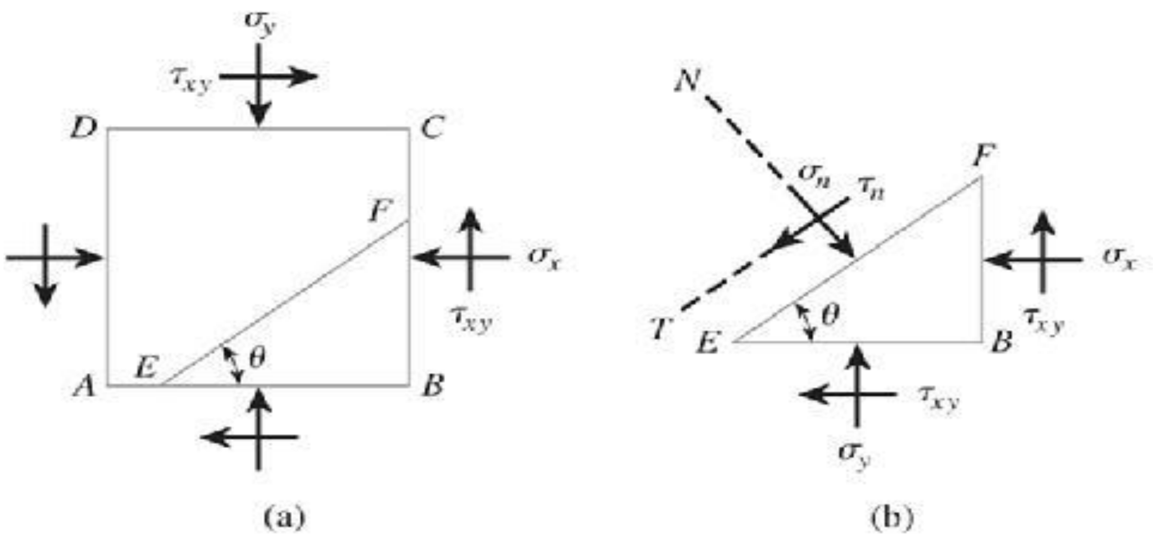


Figure 10.1 (a) A soil element with normal and shear stresses acting on it; (b) free body diagram of *EFB* as shown in (a)

σ_n : normal stress
τ_n: shear stress

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_y - \sigma_x}$$

Major principal stress:

$$\sigma_n = \sigma_1 = \frac{\sigma_y + \sigma_x}{2} + \sqrt{\left[\frac{(\sigma_y - \sigma_x)}{2}\right]^2 + \tau_{xy}^2}$$

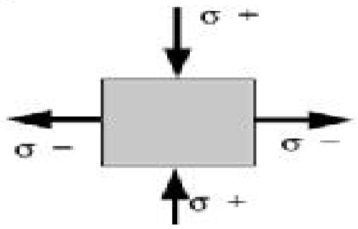
Minor principal stress:

$$\sigma_n = \sigma_3 = \frac{\sigma_y + \sigma_x}{2} - \sqrt{\left[\frac{(\sigma_y - \sigma_x)}{2}\right]^2 + \tau_{xy}^2}$$

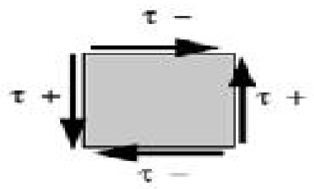
- The normal stress and shear stress that act on any plane can also be determined by plotting a Mohr's circle.

Mohr's Circle Sign Conventions:

- Compressive normal stresses are positive

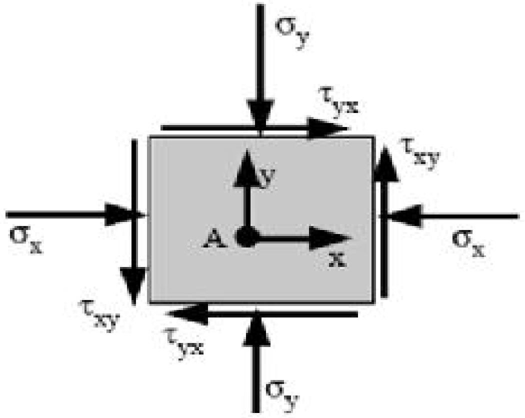


- Shear stresses are positive, if when they act on two opposing faces, they tend to produce a counterclockwise rotation.



The question that Mohr's Circle helps to answer is:

Given the stresses on any two perpendicular planes passing through a point A, what are the stresses on all other planes passing through the same point?



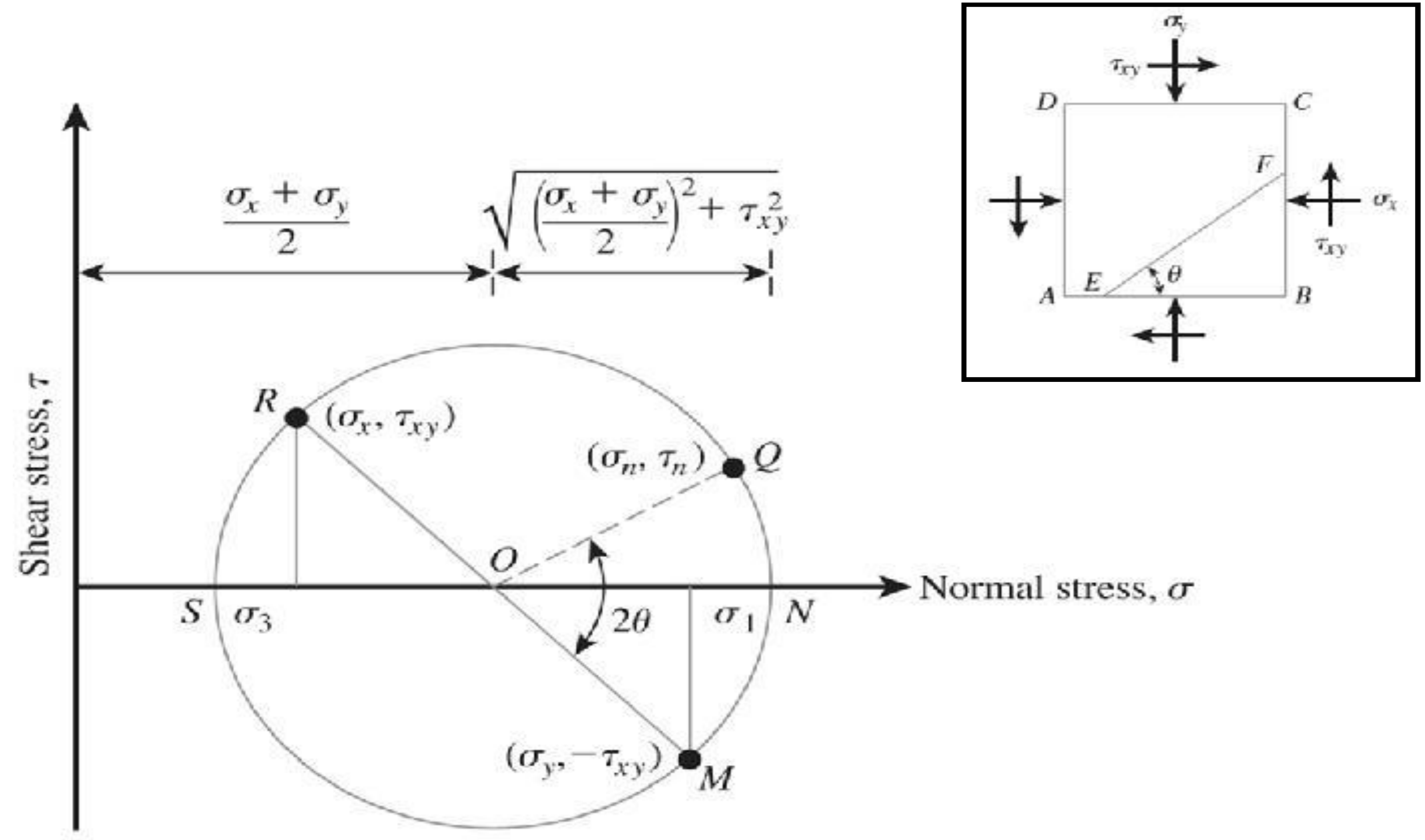


Figure 10.2 Principles of the Mohr's circle

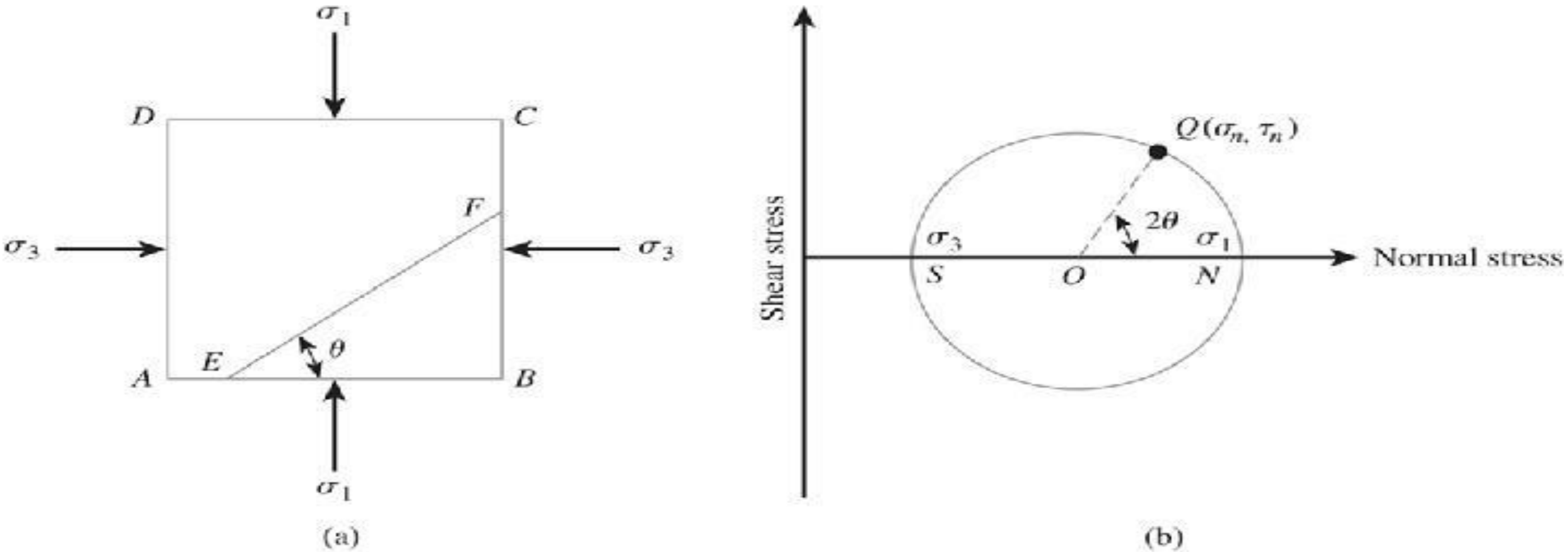


Figure 10.3 (a) Soil element with *AB* and *AD* as major and minor principal planes; (b) Mohr's circle for soil element shown in (a)

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$$

$$\tau_n = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$$

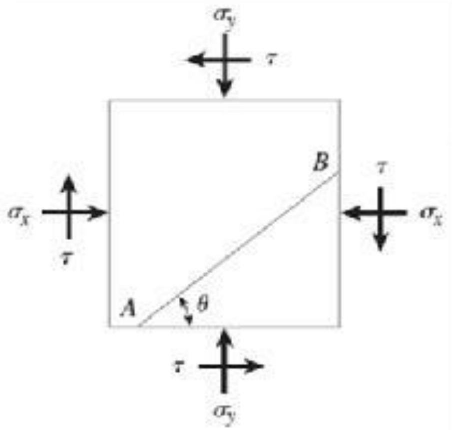
*)*special case only when the stresses acting on the planes are principal stresses).*

Example 10.1

A soil element is shown in Figure 10.4. The magnitudes of stresses are σ_x = 2000 lb/ft², τ = 800 lb/ft², σ_y = 2500 lb/ft², and θ = 20°. Determine

a. Magnitudes of the principal stresses

b. Normal and shear stresses on plane *AB*. Use Eqs. (10.3), (10.4), (10.6), and (10.7).



Solution

Part a

From Eqs. (10.6) and (10.7),

$$\left.\begin{matrix}\sigma_3\\ \sigma_1\end{matrix}\right\} = \frac{\sigma_y + \sigma_x}{2} \pm \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2}$$

$$= \frac{2500 + 2000}{2} \pm \sqrt{\left[\frac{2500 - 2000}{2}\right]^2 + (-800)^2}$$

$$\sigma_1 = \mathbf{3088.15\ lb/ft^2}$$

$$\sigma_3 = \mathbf{1411.85\ lb/ft^2}$$

Part b

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau \sin 2\theta$$

$$= \frac{2500 + 2000}{2} + \frac{2500 - 2000}{2}\cos(2 \times 20) + (-800)\sin(2 \times 20)$$

$$= \mathbf{1927.28\ lb/ft^2}$$

❑ The Pole Method of Finding Stresses Along a Plane

According to the pole method, we draw a line from a known point on the Mohr's circle parallel to the plane on which the state of stress acts. The point of intersection of this line with the Mohr's circle is called the pole.

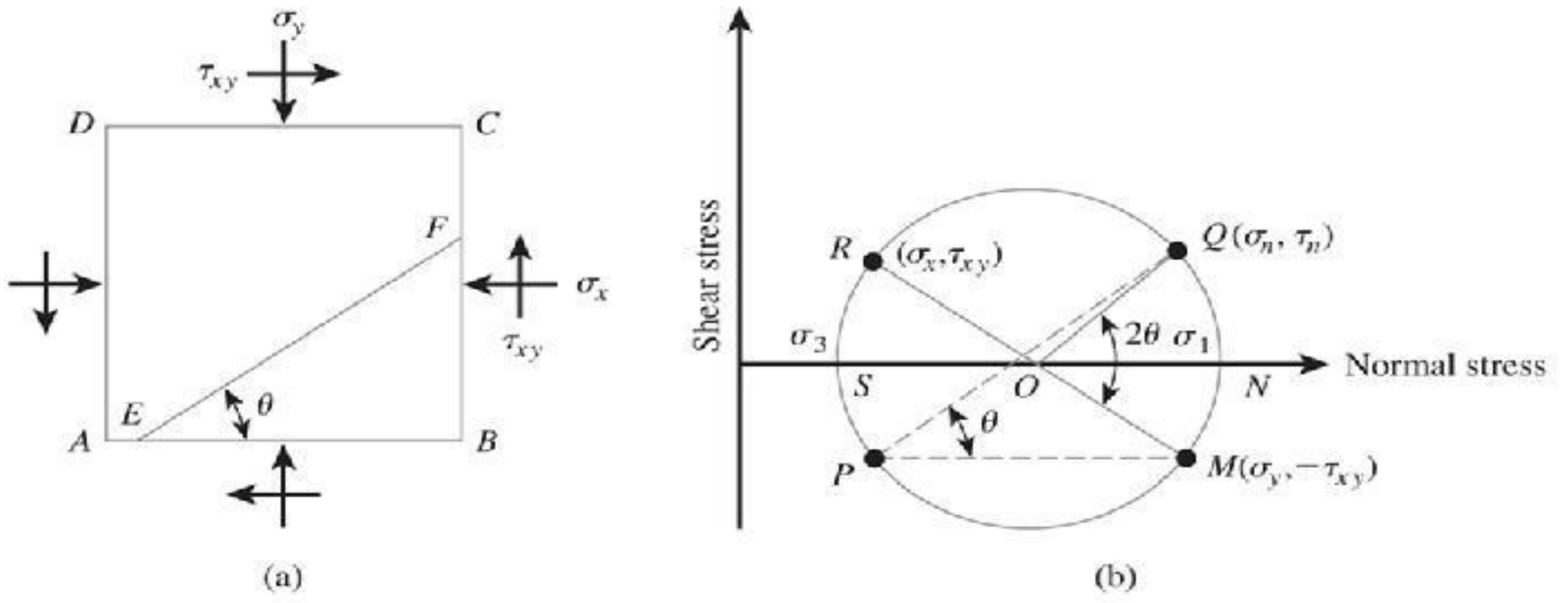


Figure 10.5 (a) Soil element with normal and shear stresses acting on it; (b) use of pole method to find the stresses along a plane

❑ The Pole Method of Finding Stresses Along a Plane

To find the stresses on a plane of any orientation:

1. draw a line through the pole P parallel to the plane;

2. the point where this line intersects the Mohr's circle gives the stresses (σ, ζ) on the plane of interest.

Example 10.2

For the stressed soil element shown in Figure 10.6a, determine

a. Major principal stress
b. Minor principal stress
c. Normal and shear stresses on the plane *DE*

Use the pole method.

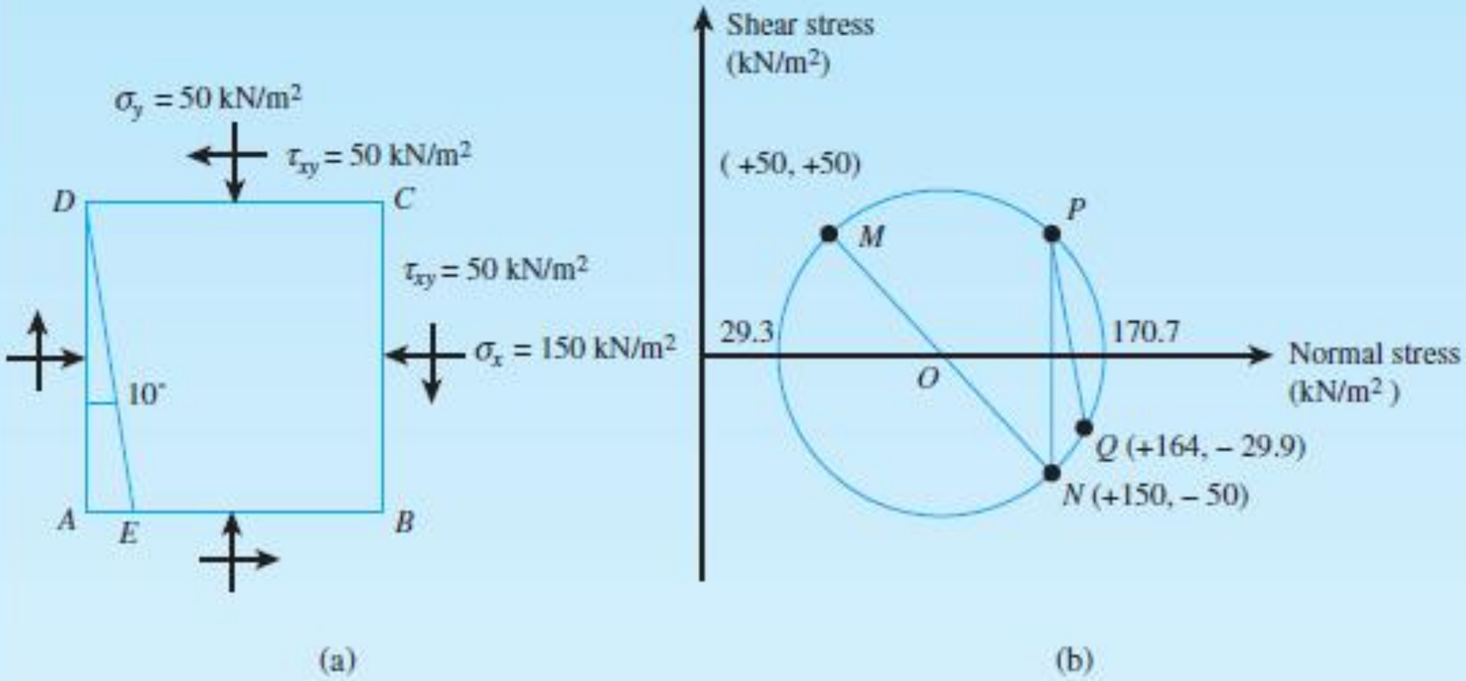


Figure 10.6 (a) Stressed soil element; (b) Mohr's circle for the soil element

Solution

On plane *AD:*

$$\text{Normal stress} = +150\ \text{kN/m}^2$$
$$\text{Shear stress} = -50\ \text{kN/m}^2$$

On plane *AB:*

$$\text{Normal stress} = +50\ \text{kN/m}^2$$
$$\text{Shear stress} = +50\ \text{kN/m}^2$$

The Mohr's circle is plotted in Figure 10.6b. From the plot,

a. Major principal stress = **170.7 kN/m²**
b. Minor principal stress = **29.3 kN/m²**
c. *NP* is the line drawn parallel to the plane *CB.*

P is the pole. *PQ* is drawn parallel to *DE* (Figure 10.6a). The coordinates of point *Q* give the stress on the plane *DE.* Thus,

$$\text{Normal stress} = \mathbf{164\ kN/m^2}$$
$$\text{Shear stress} = \mathbf{-29.9\ kN/m^2}$$

• Stresses Caused by a Point Load

Boussinesq (1883) solved the problem of stresses produced at any point in a homogeneous, elastic, and isotropic medium as the result of a point load applied on the surface of an infinitely large half-space.

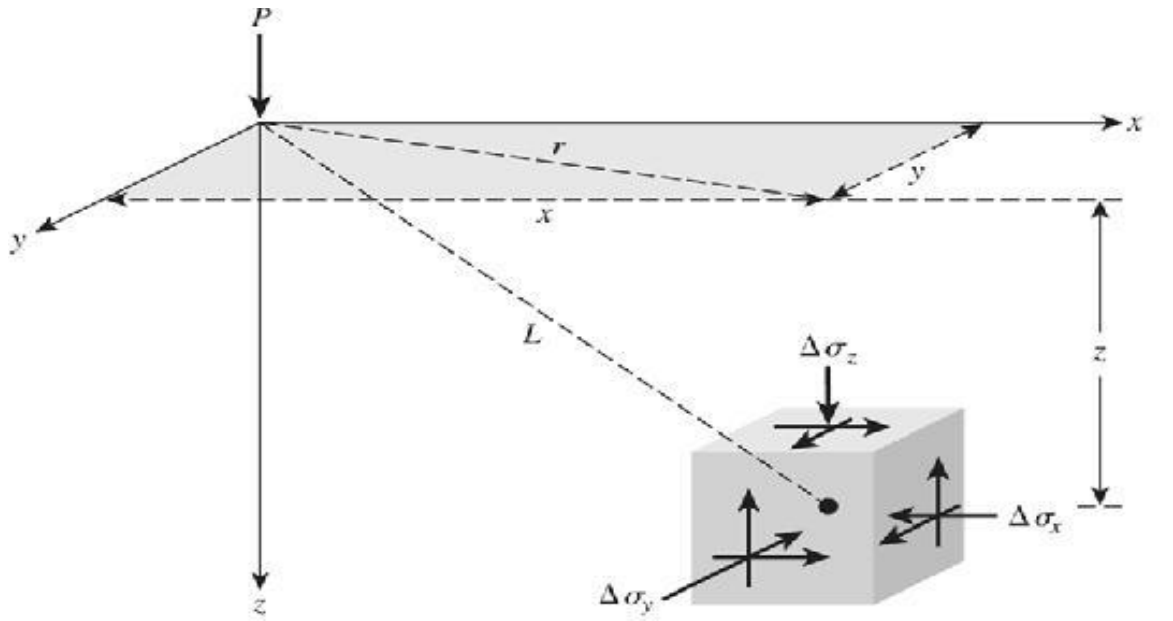


Figure 10.7 Stresses in an elastic medium caused by a point load

$$\Delta\sigma_z = \frac{3P}{2\pi}\frac{z^3}{L^5} = \frac{3P}{2\pi}\frac{z^3}{(r^2 + z^2)^{5/2}}$$

$$\Delta\sigma_z = \frac{P}{z^2}\left\{\frac{3}{2\pi}\frac{1}{[(r/z)^2 + 1]^{5/2}}\right\} = \frac{P}{z^2}I_1$$

$$\boxed{I_1 = \frac{3}{2\pi}\frac{1}{[(r/z)^2 + 1]^{5/2}}}$$

Table 10.1 Variation of I_1 for Various Values of r/z [Eq. (10.14)]

r/z	I_1	r/z	I_1	r/z	I_1
0	0.4775	0.36	0.3521	1.80	0.0129
0.02	0.4770	0.38	0.3408	2.00	0.0085
0.04	0.4765	0.40	0.3294	2.20	0.0058
0.06	0.4723	0.45	0.3011	2.40	0.0040
0.08	0.4699	0.50	0.2733	2.60	0.0029
0.10	0.4657	0.55	0.2466	2.80	0.0021
0.12	0.4607	0.60	0.2214	3.00	0.0015
0.14	0.4548	0.65	0.1978	3.20	0.0011
0.16	0.4482	0.70	0.1762	3.40	0.00085
0.18	0.4409	0.75	0.1565	3.60	0.00066
0.20	0.4329	0.80	0.1386	3.80	0.00051
0.22	0.4242	0.85	0.1226	4.00	0.00040
0.24	0.4151	0.90	0.1083	4.20	0.00032
0.26	0.4050	0.95	0.0956	4.40	0.00026
0.28	0.3954	1.00	0.0844	4.60	0.00021
0.30	0.3849	1.20	0.0513	4.80	0.00017
0.32	0.3742	1.40	0.0317	5.00	0.00014
0.34	0.3632	1.60	0.0200		

• **Vertical Stress Caused by a Vertical Line Load**

$$\Delta\sigma_z = \frac{2qz^3}{\pi(x^2 + z^2)^2}$$

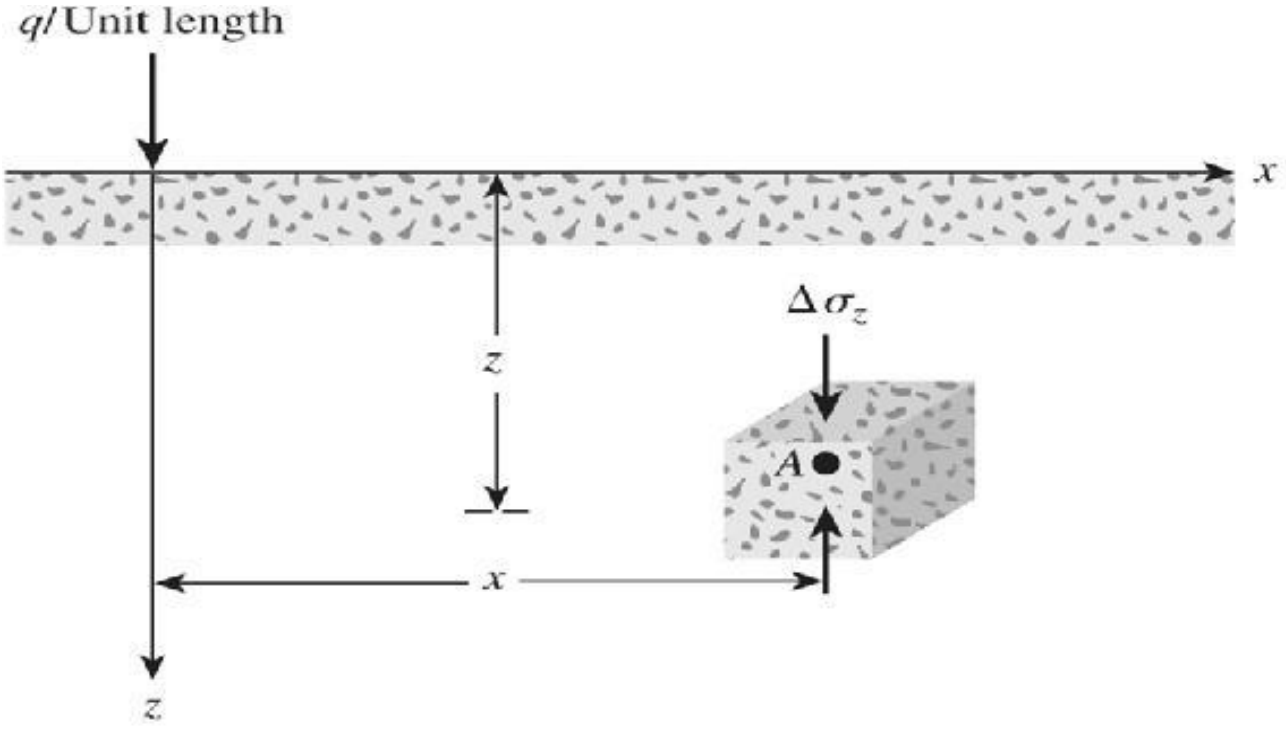


Figure 10.8 Line load over the surface of a semi-infinite soil mass

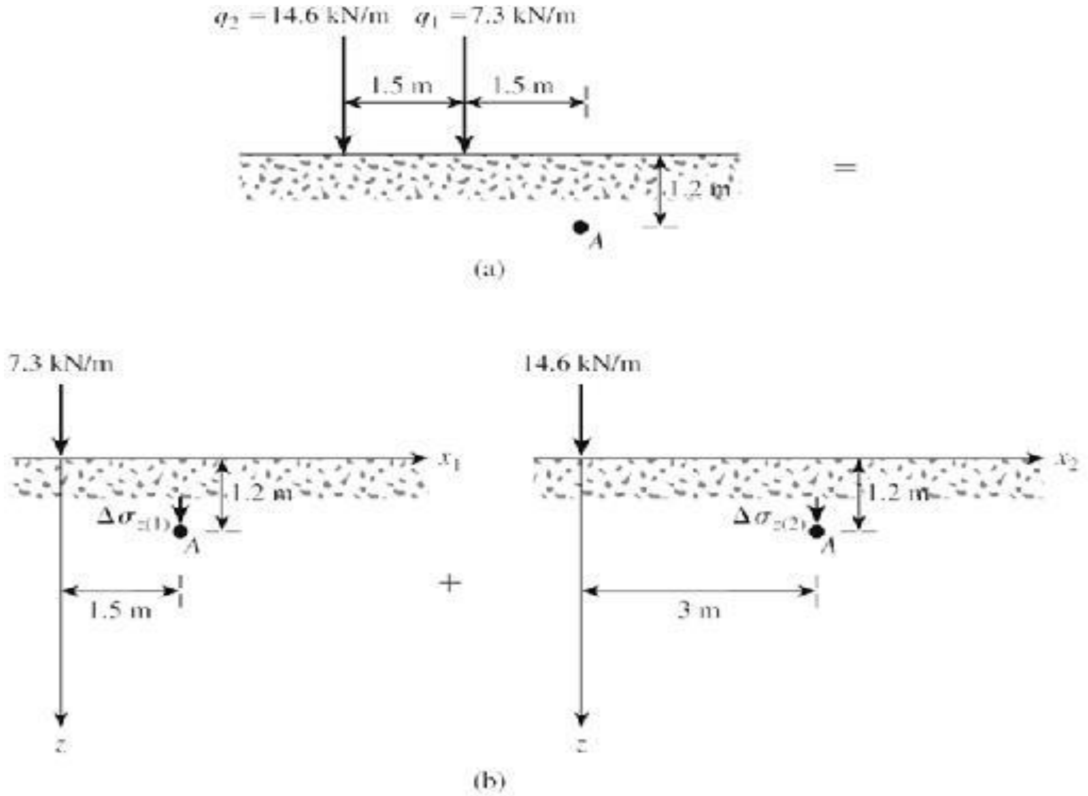


Figure 10.9 (a) Two line loads on the ground surface; (b) use of superposition principl to obtain stress at point A ■

• Vertical Stress Caused by a Horizontal Line Load

$$\Delta\sigma_z = \frac{2q}{\pi}\frac{xz^2}{(x^2 + z^2)^2}$$

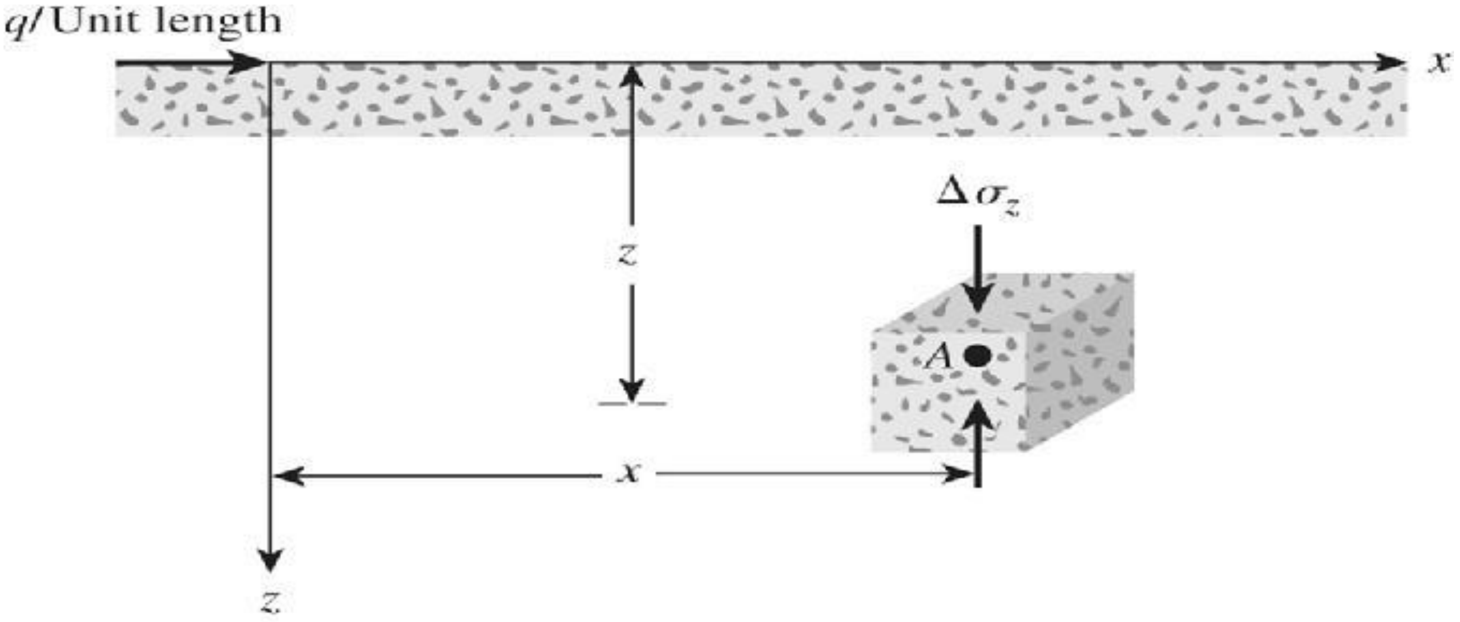


Figure 10.10 Horizontal line load over the surface of a semi-infinite soil mass

Table 10.3 Variation of $\Delta\sigma_z/(q/z)$ with x/z

x/z	$\Delta\sigma_z/(q/z)$	x/z	$\Delta\sigma_z/(q/z)$
0	0	0.7	0.201
0.1	0.062	0.8	0.189
0.2	0.118	0.9	0.175
0.3	0.161	1.0	0.159
0.4	0.189	1.5	0.090
0.5	0.204	2.0	0.051
0.6	0.207	3.0	0.019

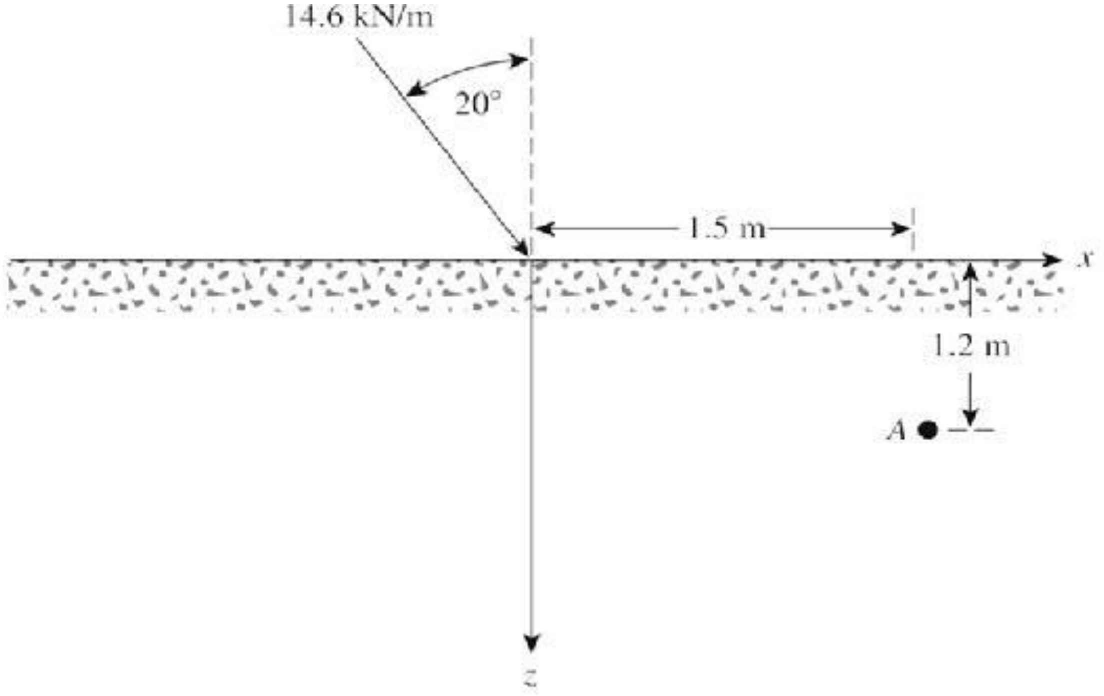


Figure 10.11

Vertical Stress Caused by a Vertical Strip Load (Finite Width and Infinite Length)

use table 10:4

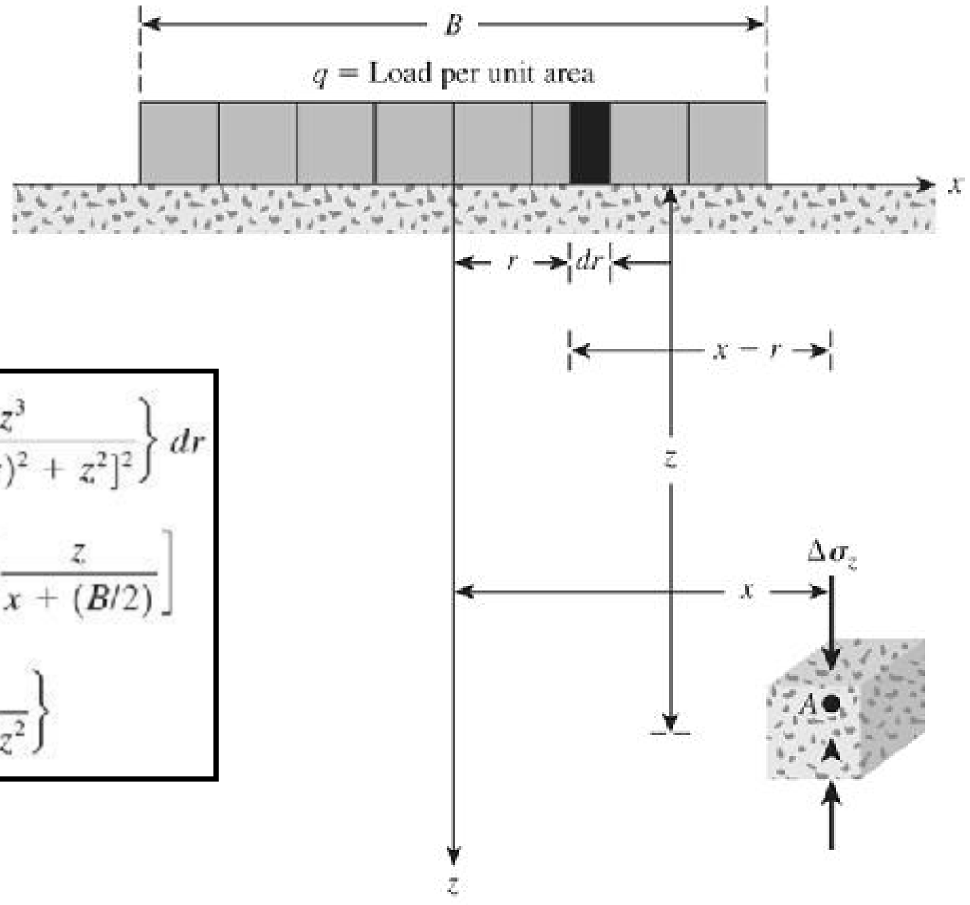


Figure 10.12 Vertical stress caused by a flexible strip load

$$\Delta\sigma_z = \int d\sigma_z = \int_{-B/2}^{+B/2}\left(\frac{2q}{\pi}\right)\left\{\frac{z^3}{[(x-r)^2+z^2]^2}\right\}dr$$

$$= \frac{q}{\pi}\left\{\tan^{-1}\left[\frac{z}{x-(B/2)}\right] - \tan^{-1}\left[\frac{z}{x+(B/2)}\right] - \frac{Bz[x^2-z^2-(B^2/4)]}{[x^2+z^2-(B^2/4)]^2+B^2z^2}\right\}$$

Table 10.4 Variation of $\Delta\sigma_z/q$ with $2z/B$ and $2x/B$ [Eq. (10.19)]

	$2x/B$										
$2z/B$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.000
0.10	1.000	1.000	0.999	0.999	0.999	0.998	0.997	0.993	0.980	0.909	0.500
0.20	0.997	0.997	0.996	0.995	0.992	0.988	0.979	0.959	0.909	0.775	0.500
0.30	0.990	0.989	0.987	0.984	0.978	0.967	0.947	0.908	0.833	0.697	0.499
0.40	0.977	0.976	0.973	0.966	0.955	0.937	0.906	0.855	0.773	0.651	0.498
0.50	0.959	0.958	0.953	0.943	0.927	0.902	0.864	0.808	0.727	0.620	0.497
0.60	0.937	0.935	0.928	0.915	0.896	0.866	0.825	0.767	0.691	0.598	0.495
0.70	0.910	0.908	0.899	0.885	0.863	0.831	0.788	0.732	0.662	0.581	0.492
0.80	0.881	0.878	0.869	0.853	0.829	0.797	0.755	0.701	0.638	0.566	0.489
0.90	0.850	0.847	0.837	0.821	0.797	0.765	0.724	0.675	0.617	0.552	0.485
1.00	0.818	0.815	0.805	0.789	0.766	0.735	0.696	0.650	0.598	0.540	0.480
1.10	0.787	0.783	0.774	0.758	0.735	0.706	0.670	0.628	0.580	0.529	0.474
1.20	0.755	0.752	0.743	0.728	0.707	0.679	0.646	0.607	0.564	0.517	0.468
1.30	0.725	0.722	0.714	0.699	0.679	0.654	0.623	0.588	0.548	0.506	0.462
1.40	0.696	0.693	0.685	0.672	0.653	0.630	0.602	0.569	0.534	0.495	0.455
1.50	0.668	0.666	0.658	0.646	0.629	0.607	0.581	0.552	0.519	0.484	0.448
1.60	0.642	0.639	0.633	0.621	0.605	0.586	0.562	0.535	0.506	0.474	0.440
1.70	0.617	0.615	0.608	0.598	0.583	0.565	0.544	0.519	0.492	0.463	0.433
1.80	0.593	0.591	0.585	0.576	0.563	0.546	0.526	0.504	0.479	0.453	0.425
1.90	0.571	0.569	0.564	0.555	0.543	0.528	0.510	0.489	0.467	0.443	0.417
2.00	0.550	0.548	0.543	0.535	0.524	0.510	0.494	0.475	0.455	0.433	0.409
2.10	0.530	0.529	0.524	0.517	0.507	0.494	0.479	0.462	0.443	0.423	0.401
2.20	0.511	0.510	0.506	0.499	0.490	0.479	0.465	0.449	0.432	0.413	0.393
2.30	0.494	0.493	0.489	0.483	0.474	0.464	0.451	0.437	0.421	0.404	0.385
2.40	0.477	0.476	0.473	0.467	0.460	0.450	0.438	0.425	0.410	0.395	0.378
2.50	0.462	0.461	0.458	0.452	0.445	0.436	0.426	0.414	0.400	0.386	0.370
2.60	0.447	0.446	0.443	0.439	0.432	0.424	0.414	0.403	0.390	0.377	0.363
2.70	0.433	0.432	0.430	0.425	0.419	0.412	0.403	0.393	0.381	0.369	0.355
2.80	0.420	0.419	0.417	0.413	0.407	0.400	0.392	0.383	0.372	0.360	0.348
2.90	0.408	0.407	0.405	0.401	0.396	0.389	0.382	0.373	0.363	0.352	0.341
3.00	0.396	0.395	0.393	0.390	0.385	0.379	0.372	0.364	0.355	0.345	0.334
3.10	0.385	0.384	0.382	0.379	0.375	0.369	0.363	0.355	0.347	0.337	0.327
3.20	0.374	0.373	0.372	0.369	0.365	0.360	0.354	0.347	0.339	0.330	0.321
3.30	0.364	0.363	0.362	0.359	0.355	0.351	0.345	0.339	0.331	0.323	0.315
3.40	0.354	0.354	0.352	0.350	0.346	0.342	0.337	0.331	0.324	0.316	0.308
3.50	0.345	0.345	0.343	0.341	0.338	0.334	0.329	0.323	0.317	0.310	0.302
3.60	0.337	0.336	0.335	0.333	0.330	0.326	0.321	0.316	0.310	0.304	0.297
3.70	0.328	0.328	0.327	0.325	0.322	0.318	0.314	0.309	0.304	0.298	0.291
3.80	0.320	0.320	0.319	0.317	0.315	0.311	0.307	0.303	0.297	0.292	0.285
3.90	0.313	0.313	0.312	0.310	0.307	0.304	0.301	0.296	0.291	0.286	0.280
4.00	0.306	0.305	0.304	0.303	0.301	0.298	0.294	0.290	0.285	0.280	0.275
4.10	0.299	0.299	0.298	0.296	0.294	0.291	0.288	0.284	0.280	0.275	0.270
4.20	0.292	0.292	0.291	0.290	0.288	0.285	0.282	0.278	0.274	0.270	0.265
4.30	0.286	0.286	0.285	0.283	0.282	0.279	0.276	0.273	0.269	0.265	0.260
4.40	0.280	0.280	0.279	0.278	0.276	0.274	0.271	0.268	0.264	0.260	0.256
4.50	0.274	0.274	0.273	0.272	0.270	0.268	0.266	0.263	0.259	0.255	0.251
4.60	0.268	0.268	0.268	0.266	0.265	0.263	0.260	0.258	0.254	0.251	0.247
4.70	0.263	0.263	0.262	0.261	0.260	0.258	0.255	0.253	0.250	0.246	0.243
4.80	0.258	0.258	0.257	0.256	0.255	0.253	0.251	0.248	0.245	0.242	0.239
4.90	0.253	0.253	0.252	0.251	0.250	0.248	0.246	0.244	0.241	0.238	0.235
5.00	0.248	0.248	0.247	0.246	0.245	0.244	0.242	0.239	0.237	0.234	0.231

Table 10.4 (continued)

	2x/B									
2z/B	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.10	0.091	0.020	0.007	0.003	0.002	0.001	0.001	0.000	0.000	0.000
0.20	0.225	0.091	0.040	0.020	0.011	0.007	0.004	0.003	0.002	0.002
0.30	0.301	0.165	0.090	0.052	0.031	0.020	0.013	0.009	0.007	0.005
0.40	0.346	0.224	0.141	0.090	0.059	0.040	0.027	0.020	0.014	0.011
0.50	0.373	0.267	0.185	0.128	0.089	0.063	0.046	0.034	0.025	0.019
0.60	0.391	0.298	0.222	0.163	0.120	0.088	0.066	0.050	0.038	0.030
0.70	0.403	0.321	0.250	0.193	0.148	0.113	0.087	0.068	0.053	0.042
0.80	0.411	0.338	0.273	0.218	0.173	0.137	0.108	0.086	0.069	0.056
0.90	0.416	0.351	0.291	0.239	0.195	0.158	0.128	0.104	0.085	0.070
1.00	0.419	0.360	0.305	0.256	0.214	0.177	0.147	0.122	0.101	0.084
1.10	0.420	0.366	0.316	0.271	0.230	0.194	0.164	0.138	0.116	0.098
1.20	0.419	0.371	0.325	0.282	0.243	0.209	0.178	0.152	0.130	0.111
1.30	0.417	0.373	0.331	0.291	0.254	0.221	0.191	0.166	0.143	0.123
1.40	0.414	0.374	0.335	0.298	0.263	0.232	0.203	0.177	0.155	0.135
1.50	0.411	0.374	0.338	0.303	0.271	0.240	0.213	0.188	0.165	0.146
1.60	0.407	0.373	0.339	0.307	0.276	0.248	0.221	0.197	0.175	0.155
1.70	0.402	0.370	0.339	0.309	0.281	0.254	0.228	0.205	0.183	0.164
1.80	0.396	0.368	0.339	0.311	0.284	0.258	0.234	0.212	0.191	0.172
1.90	0.391	0.364	0.338	0.312	0.286	0.262	0.239	0.217	0.197	0.179
2.00	0.385	0.360	0.336	0.311	0.288	0.265	0.243	0.222	0.203	0.185
2.10	0.379	0.356	0.333	0.311	0.288	0.267	0.246	0.226	0.208	0.190
2.20	0.373	0.352	0.330	0.309	0.288	0.268	0.248	0.229	0.212	0.195
2.30	0.366	0.347	0.327	0.307	0.288	0.268	0.250	0.232	0.215	0.199
2.40	0.360	0.342	0.323	0.305	0.287	0.268	0.251	0.234	0.217	0.202
2.50	0.354	0.337	0.320	0.302	0.285	0.268	0.251	0.235	0.220	0.205
2.60	0.347	0.332	0.316	0.299	0.283	0.267	0.251	0.236	0.221	0.207
2.70	0.341	0.327	0.312	0.296	0.281	0.266	0.251	0.236	0.222	0.208
2.80	0.335	0.321	0.307	0.293	0.279	0.265	0.250	0.236	0.223	0.210
2.90	0.329	0.316	0.303	0.290	0.276	0.263	0.249	0.236	0.223	0.211
3.00	0.323	0.311	0.299	0.286	0.274	0.261	0.248	0.236	0.223	0.211
3.10	0.317	0.306	0.294	0.283	0.271	0.259	0.247	0.235	0.223	0.212
3.20	0.311	0.301	0.290	0.279	0.268	0.256	0.245	0.234	0.223	0.212
3.30	0.305	0.296	0.286	0.275	0.265	0.254	0.243	0.232	0.222	0.211
3.40	0.300	0.291	0.281	0.271	0.261	0.251	0.241	0.231	0.221	0.211
3.50	0.294	0.286	0.277	0.268	0.258	0.249	0.239	0.229	0.220	0.210
3.60	0.289	0.281	0.273	0.264	0.255	0.246	0.237	0.228	0.218	0.209
3.70	0.284	0.276	0.268	0.260	0.252	0.243	0.235	0.226	0.217	0.208
3.80	0.279	0.272	0.264	0.256	0.249	0.240	0.232	0.224	0.216	0.207
3.90	0.274	0.267	0.260	0.253	0.245	0.238	0.230	0.222	0.214	0.206
4.00	0.269	0.263	0.256	0.249	0.242	0.235	0.227	0.220	0.212	0.205
4.10	0.264	0.258	0.252	0.246	0.239	0.232	0.225	0.218	0.211	0.203
4.20	0.260	0.254	0.248	0.242	0.236	0.229	0.222	0.216	0.209	0.202
4.30	0.255	0.250	0.244	0.239	0.233	0.226	0.220	0.213	0.207	0.200
4.40	0.251	0.246	0.241	0.235	0.229	0.224	0.217	0.211	0.205	0.199
4.50	0.247	0.242	0.237	0.232	0.226	0.221	0.215	0.209	0.203	0.197
4.60	0.243	0.238	0.234	0.229	0.223	0.218	0.212	0.207	0.201	0.195
4.70	0.239	0.235	0.230	0.225	0.220	0.215	0.210	0.205	0.199	0.194
4.80	0.235	0.231	0.227	0.222	0.217	0.213	0.208	0.202	0.197	0.192
4.90	0.231	0.227	0.223	0.219	0.215	0.210	0.205	0.200	0.195	0.190
5.00	0.227	0.224	0.220	0.216	0.212	0.207	0.203	0.198	0.193	0.188

• Vertical Stress Due to Embankment Loading

$$\Delta\sigma_z = \frac{q_o}{\pi}\left[\left(\frac{B_1 + B_2}{B_2}\right)(\alpha_1 + \alpha_2) - \frac{B_1}{B_2}(\alpha_2)\right]$$

$$\Delta\sigma_z = q_o I_2$$

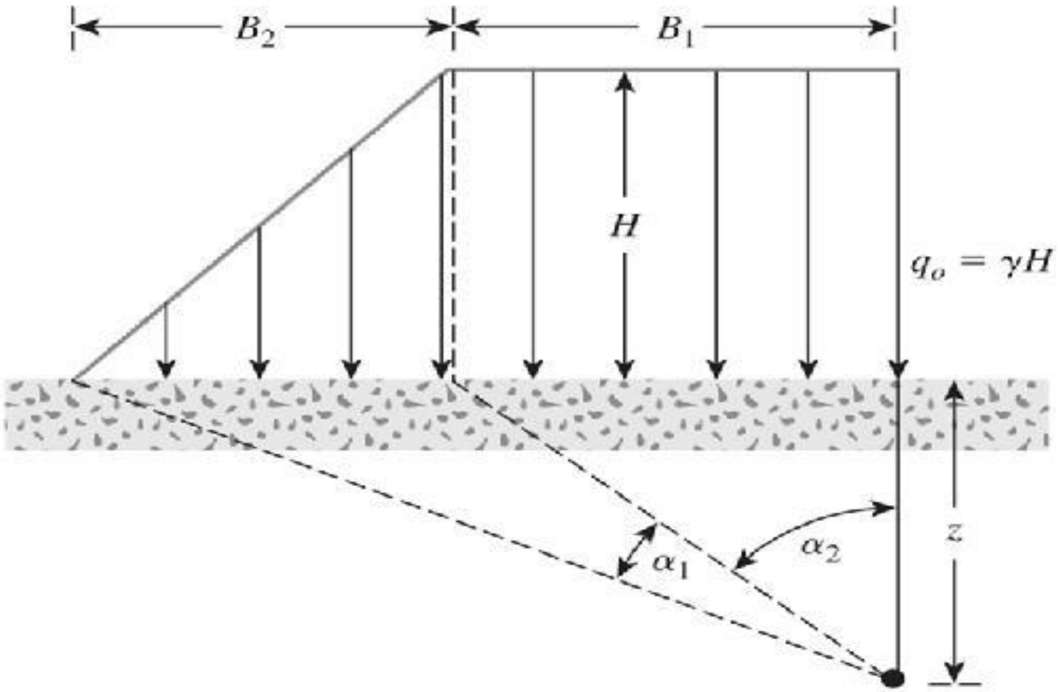


Figure 10.14 Embankment loading

$$\Delta\sigma_z = q_o I_2$$

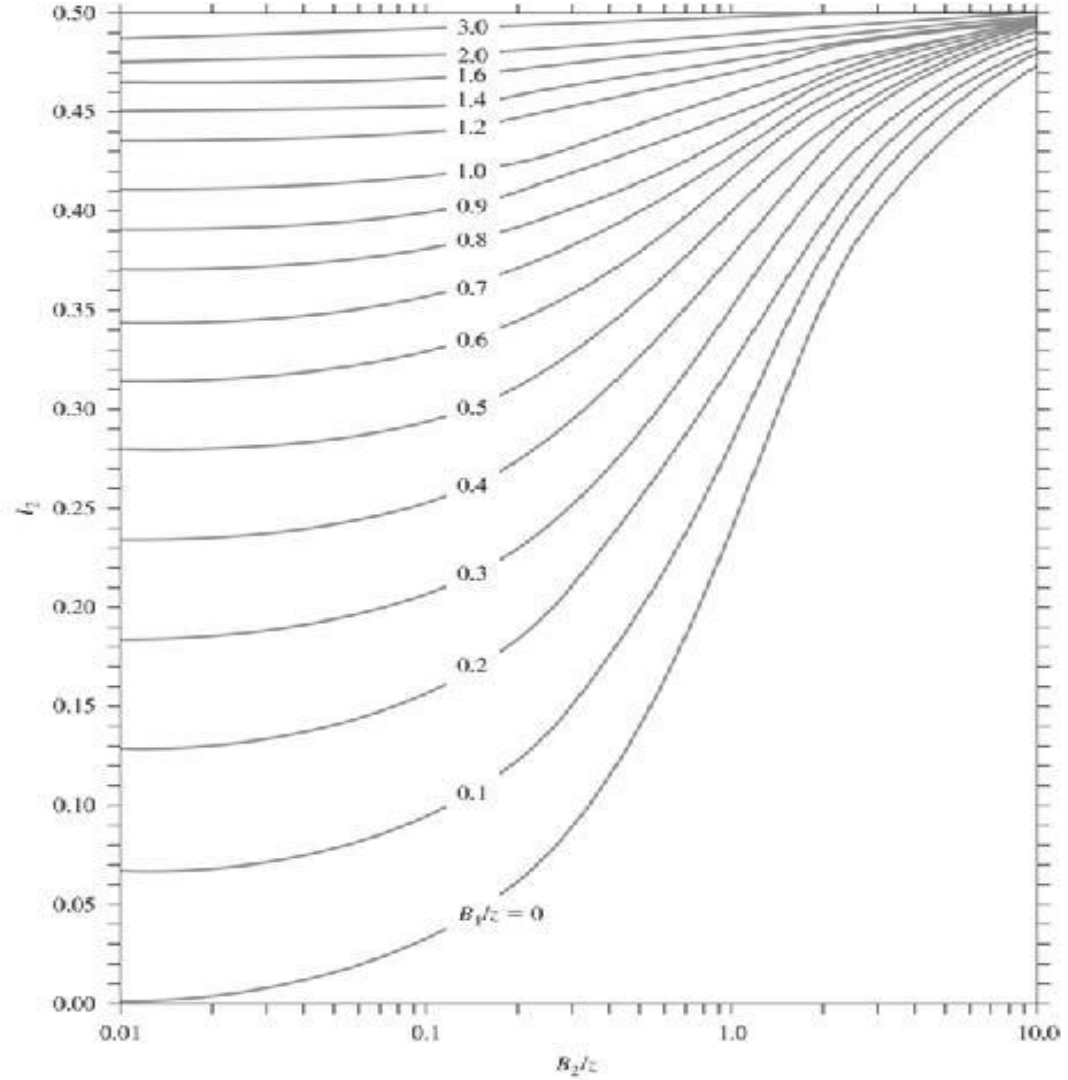
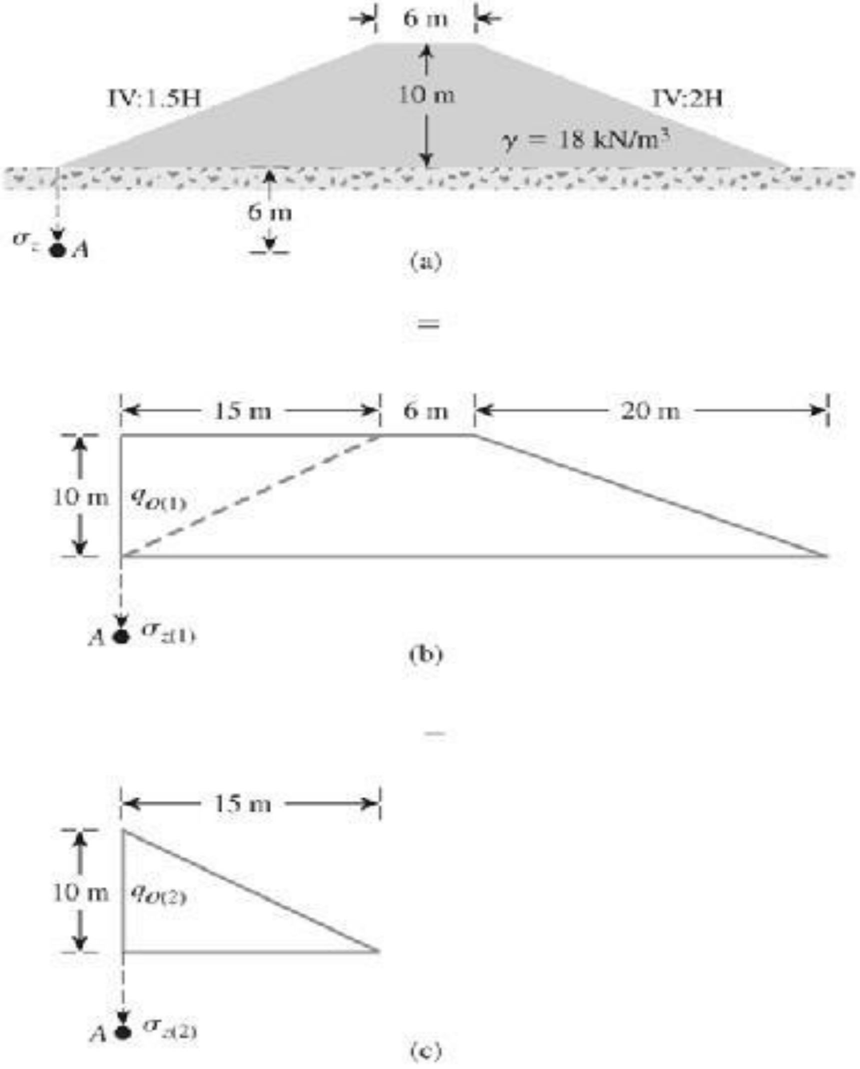


Figure 10.15 Osterberg's chart for determination of vertical stress due to embankment loading



6 m
IV:1.5H
10 m
IV:2H
γ = 18 kN/m³
6 m
σz
A
(a)
=
15 m
6 m
20 m
10 m
qO(1)
A
σz(1)
(b)
−
15 m
10 m
qO(2)
A
σz(2)
(c)

Figure 10.16

• Vertical Stress Below the Center of a Uniformly Loaded Circular Area

$$\Delta\sigma_z = q\left\{1 - \frac{1}{[(R/z)^2 + 1]^{3/2}}\right\}$$

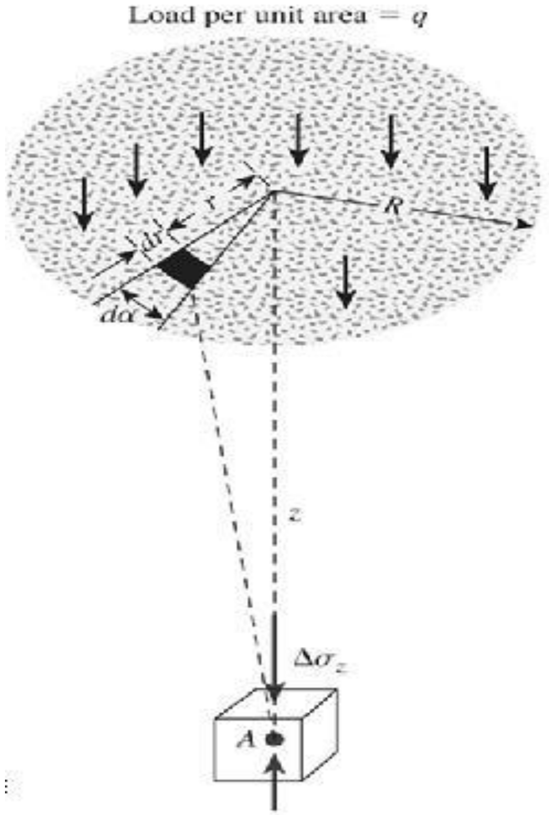


Figure 10.17
Vertical stress below the center of a uniformly loaded flexible circular area

Table 10.5 Variation of $\Delta\sigma_z/q$ with z/R [Eq. (10.25)]

z/R	$\Delta\sigma_z/q$	z/R	$\Delta\sigma_z/q$
0	1	1.0	0.6465
0.02	0.9999	1.5	0.4240
0.05	0.9998	2.0	0.2845
0.10	0.9990	2.5	0.1996
0.2	0.9925	3.0	0.1436
0.4	0.9488	4.0	0.0869
0.5	0.9106	5.0	0.0571
0.8	0.7562		

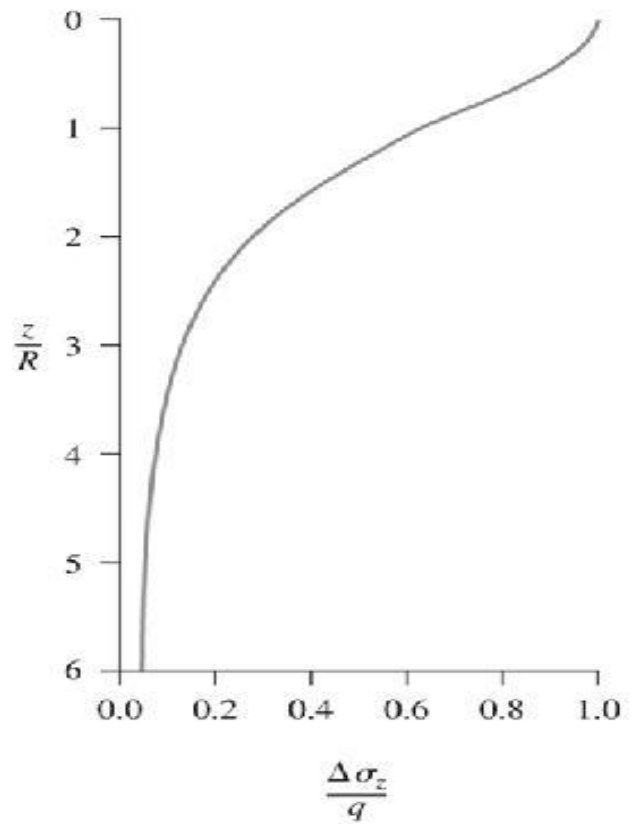


Figure 10.18 Stress under the center of a uniformly loaded flexible circular area

• Vertical Stress at Any Point Below a Uniformly Loaded Circular Area

$$\Delta\sigma_z = q(A' + B')$$ where A' and B' are functions of z/R and r/R.

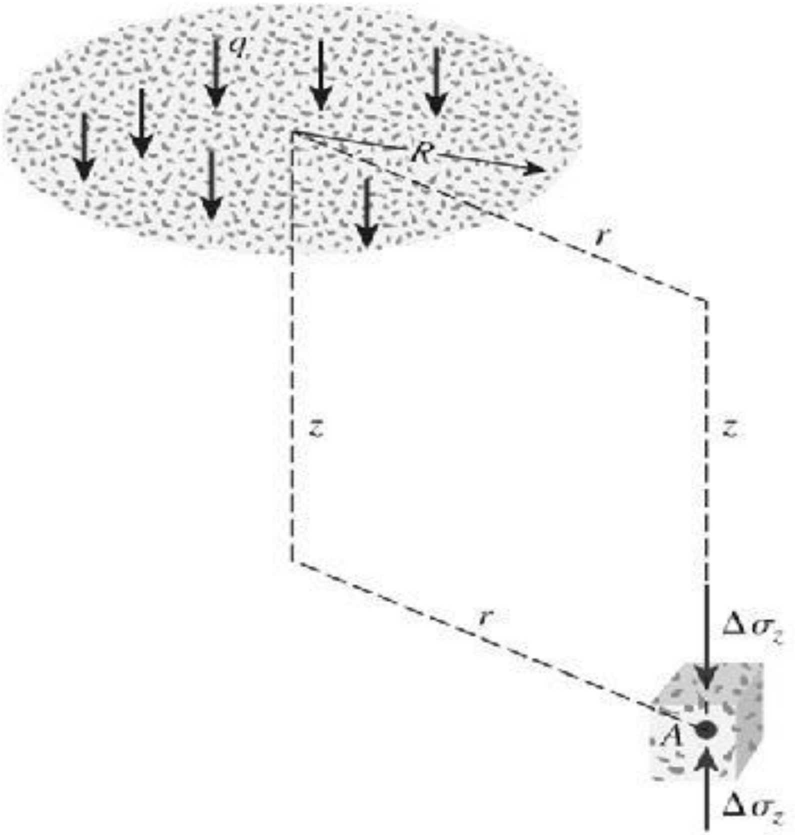


Figure 10.19 Vertical stress at any point below a uniformly loaded circular area

Table 10.6 Variation of A' with z/R and r/R*

	r/R								
z/R	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2
0	1.0	1.0	1.0	1.0	1.0	0.5	0	0	0
0.1	0.90050	0.89748	0.88679	0.86126	0.78797	0.43015	0.09645	0.02787	0.00856
0.2	0.80388	0.79824	0.77884	0.73483	0.63014	0.38269	0.15433	0.05251	0.01680
0.3	0.71265	0.70518	0.68316	0.62690	0.52081	0.34375	0.17964	0.07199	0.02440
0.4	0.62861	0.62015	0.59241	0.53767	0.44329	0.31048	0.18709	0.08593	0.03118
0.5	0.55279	0.54403	0.51622	0.46448	0.38390	0.28156	0.18556	0.09499	0.03701
0.6	0.48550	0.47691	0.45078	0.40427	0.33676	0.25588	0.17952	0.10010	
0.7	0.42654	0.41874	0.39491	0.35428	0.29833	0.21727	0.17124	0.10228	0.04558
0.8	0.37531	0.36832	0.34729	0.31243	0.26581	0.21297	0.16206	0.10236	
0.9	0.33104	0.32492	0.30669	0.27707	0.23832	0.19488	0.15253	0.10094	
1	0.29289	0.28763	0.27005	0.24697	0.21468	0.17868	0.14329	0.09849	0.05185
1.2	0.23178	0.22795	0.21662	0.19890	0.17626	0.15101	0.12570	0.09192	0.05260
1.5	0.16795	0.16552	0.15877	0.14804	0.13436	0.11892	0.10296	0.08048	0.05116
2	0.10557	0.10453	0.10140	0.09647	0.09011	0.08269	0.07471	0.06275	0.04496
2.5	0.07152	0.07098	0.06947	0.06698	0.06373	0.05974	0.05555	0.04880	0.03787
3	0.05132	0.05101	0.05022	0.04886	0.04707	0.04487	0.04241	0.03839	0.03150
4	0.02986	0.02976	0.02907	0.02802	0.02832	0.02749	0.02651	0.02490	0.02193
5	0.01942	0.01938				0.01835			0.01573
6	0.01361					0.01307			0.01168
7	0.01005					0.00976			0.00894
8	0.00772					0.00755			0.00703
9	0.00612					0.00600			0.00566
10								0.00477	0.00465

Source: From Ahlvin, R. G., and H. H. Ulery. Tabulated Values for Determining the Complete Pattern of Stresses, Strains, and Deflections Beneath a Uniform Circular Load on a Homogeneous Half Space. In Highway Research Bulletin 342, Highway Research Board, National Research Council, Washington, D.C., 1962, Tables 1 and 2, p. 3. Reproduced with permission of the Transportation Research Board.

Table 10.6 *(continued)*

3	4	5	6	7	8	10	12	14
0	0	0	0	0	0	0	0	0
0.00211	0.00084	0.00042						
0.00419	0.00167	0.00083	0.00048	0.00030	0.00020			
0.00622	0.00250							
0.01013	0.00407	0.00209	0.00118	0.00071	0.00053	0.00025	0.00014	0.00009
0.01742	0.00761	0.00393	0.00226	0.00143	0.00097	0.00050	0.00029	0.00018
0.01935	0.00871	0.00459	0.00269	0.00171	0.00115			
0.02142	0.01013	0.00548	0.00325	0.00210	0.00141	0.00073	0.00043	0.00027
0.02221	0.01160	0.00659	0.00399	0.00264	0.00180	0.00094	0.00056	0.00036
0.02143	0.01221	0.00732	0.00463	0.00308	0.00214	0.00115	0.00068	0.00043
0.01980	0.01220	0.00770	0.00505	0.00346	0.00242	0.00132	0.00079	0.00051
0.01592	0.01109	0.00768	0.00536	0.00384	0.00282	0.00160	0.00099	0.00065
0.01249	0.00949	0.00708	0.00527	0.00394	0.00298	0.00179	0.00113	0.00075
0.00983	0.00795	0.00628	0.00492	0.00384	0.00299	0.00188	0.00124	0.00084
0.00784	0.00661	0.00548	0.00445	0.00360	0.00291	0.00193	0.00130	0.00091
0.00635	0.00554	0.00472	0.00398	0.00332	0.00276	0.00189	0.00134	0.00094
0.00520	0.00466	0.00409	0.00353	0.00301	0.00256	0.00184	0.00133	0.00096
0.00438	0.00397	0.00352	0.00326	0.00273	0.00241			

Table 10.7 Variation of B' with z/R and r/R*

									r/R
z/R	**0**	**0.2**	**0.4**	**0.6**	**0.8**	**1**	**1.2**	**1.5**	**2**
0	0	0	0	0	0	0	0	0	0
0.1	0.09852	0.10140	0.11138	0.13424	0.18796	0.05388	−0.07899	−0.02672	−0.00845
0.2	0.18857	0.19306	0.20772	0.23524	0.25983	0.08513	−0.07759	−0.04448	−0.01593
0.3	0.26362	0.26787	0.28018	0.29483	0.27257	0.10757	−0.04316	−0.04999	−0.02166
0.4	0.32016	0.32259	0.32748	0.32273	0.26925	0.12404	−0.00766	−0.04535	−0.02522
0.5	0.35777	0.35752	0.35323	0.33106	0.26236	0.13591	0.02165	−0.03455	−0.02651
0.6	0.37831	0.37531	0.36308	0.32822	0.25411	0.14440	0.04457	−0.02101	
0.7	0.38487	0.37962	0.36072	0.31929	0.24638	0.14986	0.06209	−0.00702	−0.02329
0.8	0.38091	0.37408	0.35133	0.30699	0.23779	0.15292	0.07530	0.00614	
0.9	0.36962	0.36275	0.33734	0.29299	0.22891	0.15404	0.08507	0.01795	
1	0.35355	0.34553	0.32075	0.27819	0.21978	0.15355	0.09210	0.02814	−0.01005
1.2	0.31485	0.30730	0.28481	0.24836	0.20113	0.14915	0.10002	0.04378	0.00023
1.5	0.25602	0.25025	0.23338	0.20694	0.17368	0.13732	0.10193	0.05745	0.01385
2	0.17889	0.18144	0.16644	0.15198	0.13375	0.11331	0.09254	0.06371	0.02836
2.5	0.12807	0.12633	0.12126	0.11327	0.10298	0.09130	0.07869	0.06022	0.03429
3	0.09487	0.09394	0.09099	0.08635	0.08033	0.07325	0.06551	0.05354	0.03511
4	0.05707	0.05666	0.05562	0.05383	0.05145	0.04773	0.04532	0.03995	0.03066
5	0.03772	0.03760				0.03384			0.02474
6	0.02666					0.02468			0.01968
7	0.01980					0.01868			0.01577
8	0.01526					0.01459			0.01279
9	0.01212					0.01170			0.01054
10								0.00924	0.00879

See Example …10:8

Table 10.7 *(continued)*

3	4	5	6	7	8	10	12	14
0	0	0	0	0	0	0	0	0
−0.00210	−0.00084	−0.00042						
−0.00412	−0.00166	−0.00083	−0.00024	−0.00015	−0.00010			
−0.00599	−0.00245							
−0.00991	−0.00388	−0.00199	−0.00116	−0.00073	−0.00049	−0.00025	−0.00014	−0.00009
−0.01115	−0.00608	−0.00344	−0.00210	−0.00135	−0.00092	−0.00048	−0.00028	−0.00018
−0.00995	−0.00632	−0.00378	−0.00236	−0.00156	−0.00107			
−0.00669	−0.00600	−0.00401	−0.00265	−0.00181	−0.00126	−0.00068	−0.00040	−0.00026
0.00028	−0.00410	−0.00371	−0.00278	−0.00202	−0.00148	−0.00084	−0.00050	−0.00033
0.00661	−0.00130	−0.00271	−0.00250	−0.00201	−0.00156	−0.00094	−0.00059	−0.00039
0.01112	0.00157	−0.00134	−0.00192	−0.00179	−0.00151	−0.00099	−0.00065	−0.00046
0.01515	0.00595	0.00155	−0.00029	−0.00094	−0.00109	−0.00094	−0.00068	−0.00050
0.01522	0.00810	0.00371	0.00132	0.00013	−0.00043	−0.00070	−0.00061	−0.00049
0.01380	0.00867	0.00496	0.00254	0.00110	0.00028	−0.00037	−0.00047	−0.00045
0.01204	0.00842	0.00547	0.00332	0.00185	0.00093	−0.00002	−0.00029	−0.00037
0.01034	0.00779	0.00554	0.00372	0.00236	0.00141	0.00035	−0.00008	−0.00025
0.00888	0.00705	0.00533	0.00386	0.00265	0.00178	0.00066	0.00012	−0.00012
0.00764	0.00631	0.00501	0.00382	0.00281	0.00199			

• **Vertical Stress Caused by a Rectangularly Loaded Area**

$$\Delta\sigma_z = \int d\sigma_z = \int_{y=0}^{B}\int_{x=0}^{L} \frac{3qz^3(dx\,dy)}{2\pi(x^2+y^2+z^2)^{5/2}} = qI_3$$

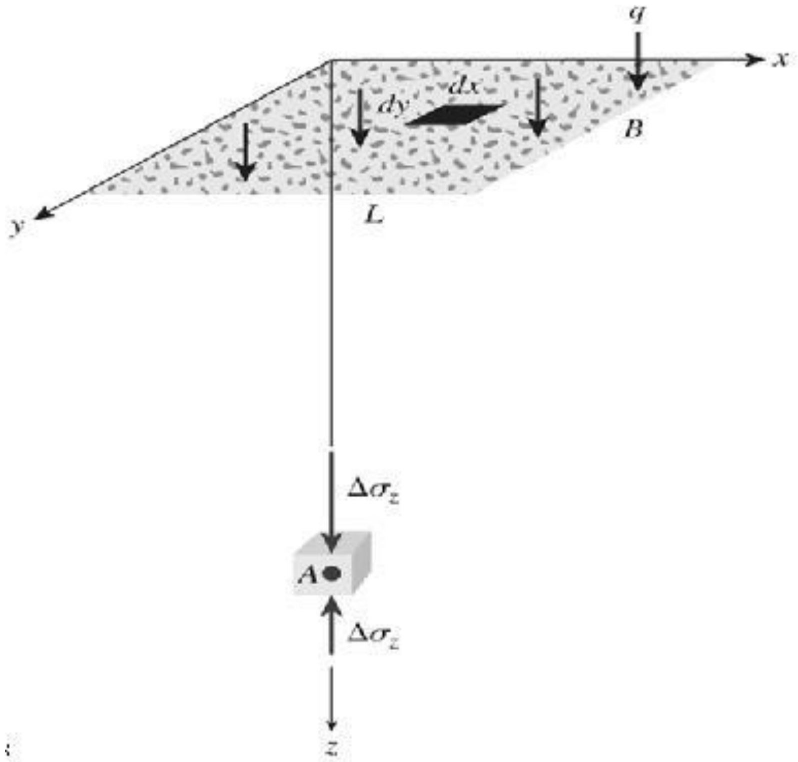


Figure 10.20
Vertical stress below the corner of a uniformly loaded flexible rectangular area

$$m = \frac{B}{z} \qquad n = \frac{L}{z}$$

Table 10.8 Variation of I_3 with m and n [Eq. (10.30)]

	m									
n	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
0.1	0.0047	0.0092	0.0132	0.0168	0.0198	0.0222	0.0242	0.0258	0.0270	0.0279
0.2	0.0092	0.0179	0.0259	0.0328	0.0387	0.0435	0.0474	0.0504	0.0528	0.0547
0.3	0.0132	0.0259	0.0374	0.0474	0.0559	0.0629	0.0686	0.0731	0.0766	0.0794
0.4	0.0168	0.0328	0.0474	0.0602	0.0711	0.0801	0.0873	0.0931	0.0977	0.1013
0.5	0.0198	0.0387	0.0559	0.0711	0.0840	0.0947	0.1034	0.1104	0.1158	0.1202
0.6	0.0222	0.0435	0.0629	0.0801	0.0947	0.1069	0.1168	0.1247	0.1311	0.1361
0.7	0.0242	0.0474	0.0686	0.0873	0.1034	0.1169	0.1277	0.1365	0.1436	0.1491
0.8	0.0258	0.0504	0.0731	0.0931	0.1104	0.1247	0.1365	0.1461	0.1537	0.1598
0.9	0.0270	0.0528	0.0766	0.0977	0.1158	0.1311	0.1436	0.1537	0.1619	0.1684
1.0	0.0279	0.0547	0.0794	0.1013	0.1202	0.1361	0.1491	0.1598	0.1684	0.1752
1.2	0.0293	0.0573	0.0832	0.1063	0.1263	0.1431	0.1570	0.1684	0.1777	0.1851
1.4	0.0301	0.0589	0.0856	0.1094	0.1300	0.1475	0.1620	0.1739	0.1836	0.1914
1.6	0.0306	0.0599	0.0871	0.1114	0.1324	0.1503	0.1652	0.1774	0.1874	0.1955
1.8	0.0309	0.0606	0.0880	0.1126	0.1340	0.1521	0.1672	0.1797	0.1899	0.1981
2.0	0.0311	0.0610	0.0887	0.1134	0.1350	0.1533	0.1686	0.1812	0.1915	0.1999
2.5	0.0314	0.0616	0.0895	0.1145	0.1363	0.1548	0.1704	0.1832	0.1938	0.2024
3.0	0.0315	0.0618	0.0898	0.1150	0.1368	0.1555	0.1711	0.1841	0.1947	0.2034
4.0	0.0316	0.0619	0.0901	0.1153	0.1372	0.1560	0.1717	0.1847	0.1954	0.2042
5.0	0.0316	0.0620	0.0901	0.1154	0.1374	0.1561	0.1719	0.1849	0.1956	0.2044
6.0	0.0316	0.0620	0.0902	0.1154	0.1374	0.1562	0.1719	0.1850	0.1957	0.2045

Table 10.8 (*continued*)

1.2	1.4	1.6	1.8	2.0	2.5	3.0	4.0	5.0	6.0
0.0293	0.0301	0.0306	0.0309	0.0311	0.0314	0.0315	0.0316	0.0316	0.0316
0.0573	0.0589	0.0599	0.0606	0.0610	0.0616	0.0618	0.0619	0.0620	0.0620
0.0832	0.0856	0.0871	0.0880	0.0887	0.0895	0.0898	0.0901	0.0901	0.0902
0.1063	0.1094	0.1114	0.1126	0.1134	0.1145	0.1150	0.1153	0.1154	0.1154
0.1263	0.1300	0.1324	0.1340	0.1350	0.1363	0.1368	0.1372	0.1374	0.1374
0.1431	0.1475	0.1503	0.1521	0.1533	0.1548	0.1555	0.1560	0.1561	0.1562
0.1570	0.1620	0.1652	0.1672	0.1686	0.1704	0.1711	0.1717	0.1719	0.1719
0.1684	0.1739	0.1774	0.1797	0.1812	0.1832	0.1841	0.1847	0.1849	0.1850
0.1777	0.1836	0.1874	0.1899	0.1915	0.1938	0.1947	0.1954	0.1956	0.1957
0.1851	0.1914	0.1955	0.1981	0.1999	0.2024	0.2034	0.2042	0.2044	0.2045
0.1958	0.2028	0.2073	0.2103	0.2124	0.2151	0.2163	0.2172	0.2175	0.2176
0.2028	0.2102	0.2151	0.2184	0.2206	0.2236	0.2250	0.2260	0.2263	0.2264
0.2073	0.2151	0.2203	0.2237	0.2261	0.2294	0.2309	0.2320	0.2323	0.2325
0.2103	0.2183	0.2237	0.2274	0.2299	0.2333	0.2350	0.2362	0.2366	0.2367
0.2124	0.2206	0.2261	0.2299	0.2325	0.2361	0.2378	0.2391	0.2395	0.2397
0.2151	0.2236	0.2294	0.2333	0.2361	0.2401	0.2420	0.2434	0.2439	0.2441
0.2163	0.2250	0.2309	0.2350	0.2378	0.2420	0.2439	0.2455	0.2461	0.2463
0.2172	0.2260	0.2320	0.2362	0.2391	0.2434	0.2455	0.2472	0.2479	0.2481
0.2175	0.2263	0.2324	0.2366	0.2395	0.2439	0.2460	0.2479	0.2486	0.2489
0.2176	0.2264	0.2325	0.2367	0.2397	0.2441	0.2463	0.2482	0.2489	0.2492

$$m = \frac{B}{z} \qquad n = \frac{L}{z}$$

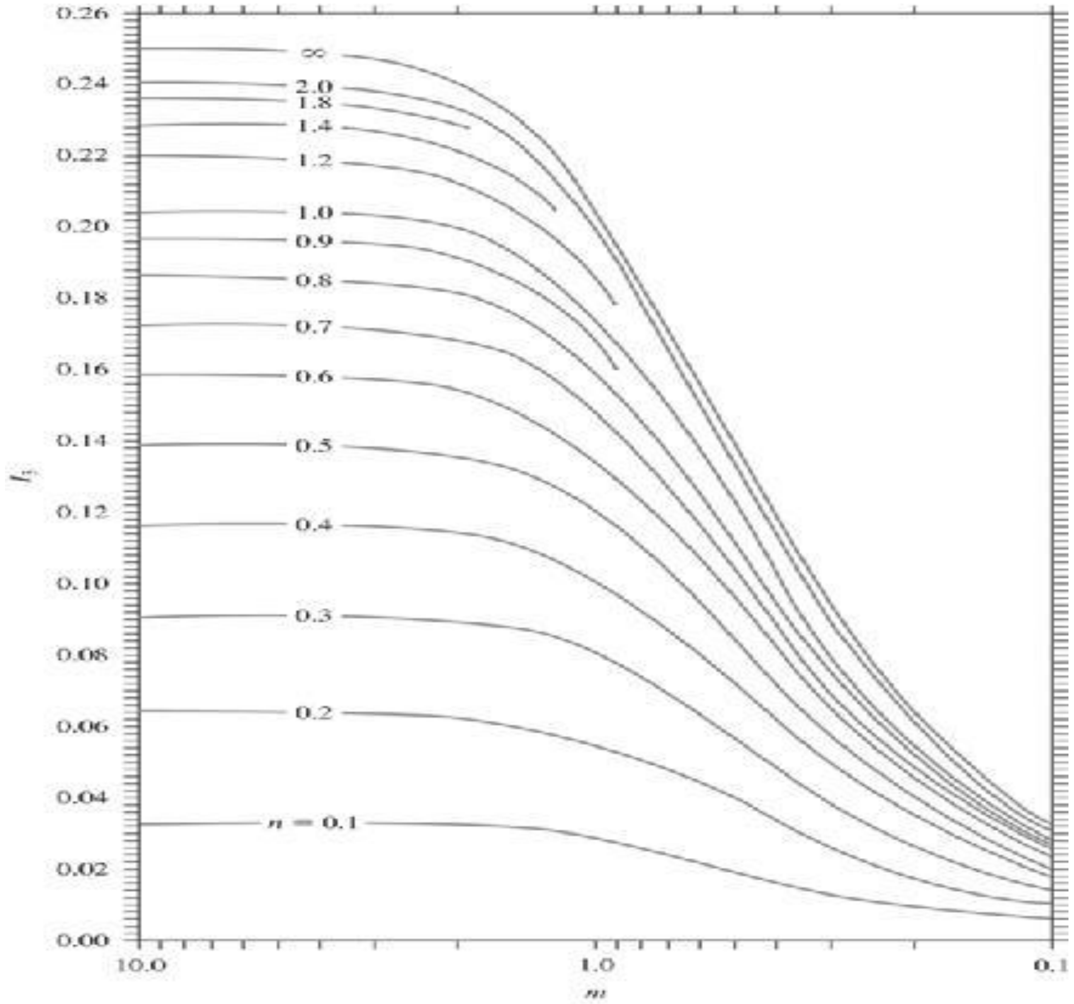


Figure 10.21 Variation of I_3 with m and n

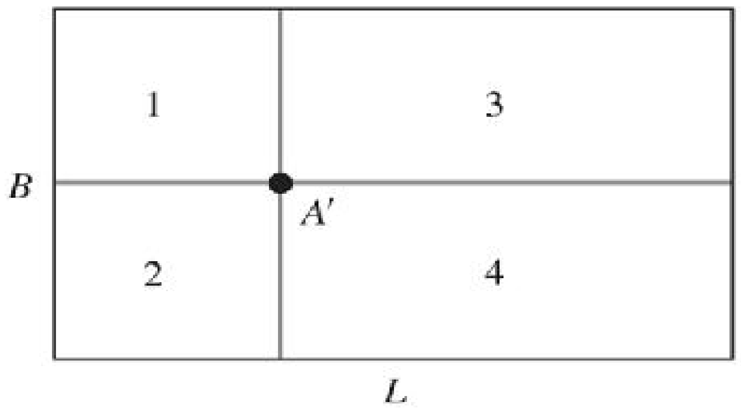


Figure 10.22 Increase of stress at any point below a rectangularly loaded flexible area

•Vertical stress below the center of a uniformly loaded flexible rectangular area

$$\Delta\sigma_z = qI_4$$

$$m_1 = \frac{L}{B} \qquad n_1 = \frac{z}{b}$$

$$b = \frac{B}{2}$$

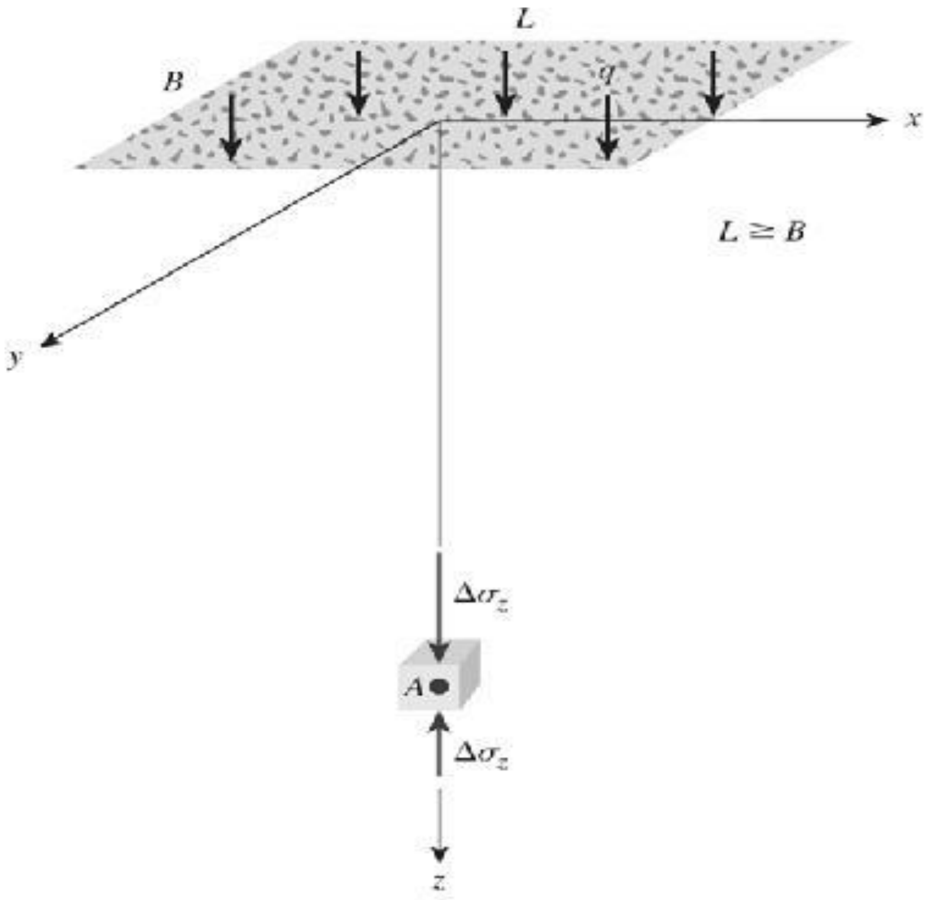


Figure 10.23 Vertical stress below the center of a uniformly loaded flexible rectangular area

Table 10.9 Variation of I_4 with m_1 and n_1 [Eq. (10.35)]

	m_1									
n_1	1	2	3	4	5	6	7	8	9	10
0.20	0.994	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
0.40	0.960	0.976	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.977
0.60	0.892	0.932	0.936	0.936	0.937	0.937	0.937	0.937	0.937	0.937
0.80	0.800	0.870	0.878	0.880	0.881	0.881	0.881	0.881	0.881	0.881
1.00	0.701	0.800	0.814	0.817	0.818	0.818	0.818	0.818	0.818	0.818
1.20	0.606	0.727	0.748	0.753	0.754	0.755	0.755	0.755	0.755	0.755
1.40	0.522	0.658	0.685	0.692	0.694	0.695	0.695	0.696	0.696	0.696
1.60	0.449	0.593	0.627	0.636	0.639	0.640	0.641	0.641	0.641	0.642
1.80	0.388	0.534	0.573	0.585	0.590	0.591	0.592	0.592	0.593	0.593
2.00	0.336	0.481	0.525	0.540	0.545	0.547	0.548	0.549	0.549	0.549
3.00	0.179	0.293	0.348	0.373	0.384	0.389	0.392	0.393	0.394	0.395
4.00	0.108	0.190	0.241	0.269	0.285	0.293	0.298	0.301	0.302	0.303
5.00	0.072	0.131	0.174	0.202	0.219	0.229	0.236	0.240	0.242	0.244
6.00	0.051	0.095	0.130	0.155	0.172	0.184	0.192	0.197	0.200	0.202
7.00	0.038	0.072	0.100	0.122	0.139	0.150	0.158	0.164	0.168	0.171
8.00	0.029	0.056	0.079	0.098	0.113	0.125	0.133	0.139	0.144	0.147
9.00	0.023	0.045	0.064	0.081	0.094	0.105	0.113	0.119	0.124	0.128
10.00	0.019	0.037	0.053	0.067	0.079	0.089	0.097	0.103	0.108	0.112

The plan of a uniformly loaded rectangular area is shown in Figure 10.30a. Determine the vertical stress increase $\Delta\sigma_z$ below point A' at a depth of $z = 4$ m.

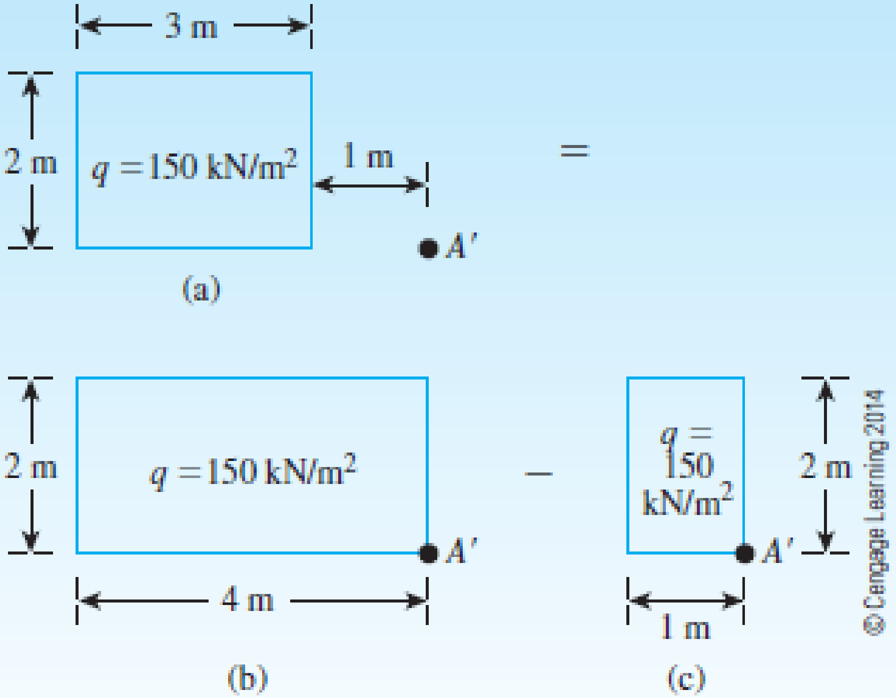


Figure 10.30

Solution

The stress increase $\Delta\sigma_z$ can be written as

$$\Delta\sigma_z = \Delta\sigma_{z(1)} - \Delta\sigma_{z(2)}$$

where

$\Delta\sigma_{z(1)}$ = stress increase due to the loaded area shown in Figure 10.30b
$\Delta\sigma_{z(2)}$ = stress increase due to the loaded area shown in Figure 10.30c

For the loaded area shown in Figure 10.30b:

$$m = \frac{B}{z} = \frac{2}{4} = 0.5$$

$$n = \frac{L}{z} = \frac{4}{4} = 1$$

From Figure 10.21 for $m = 0.5$ and $n = 1$, the value of $I_3 = 0.1225$. So

$$\Delta\sigma_{z(1)} = qI_3 = (150)(0.1202) = 18.38 \text{ kN/m}^2$$

Similarly, for the loaded area shown in Figure 10.30c:

$$m = \frac{B}{z} = \frac{1}{4} = 0.25$$

$$n = \frac{L}{z} = \frac{2}{4} = 0.5$$

Thus, $I_3 = 0.0473$. Hence

$$\Delta\sigma_{z(2)} = (150)(0.0473) = 7.1 \text{ kN/m}^2$$

So

$$\Delta\sigma_z = \Delta\sigma_{z(1)} - \Delta\sigma_{z(2)} = 18.38 - 7.1 = \mathbf{11.28\ kN/m^2}$$

• **Stress Isobars**

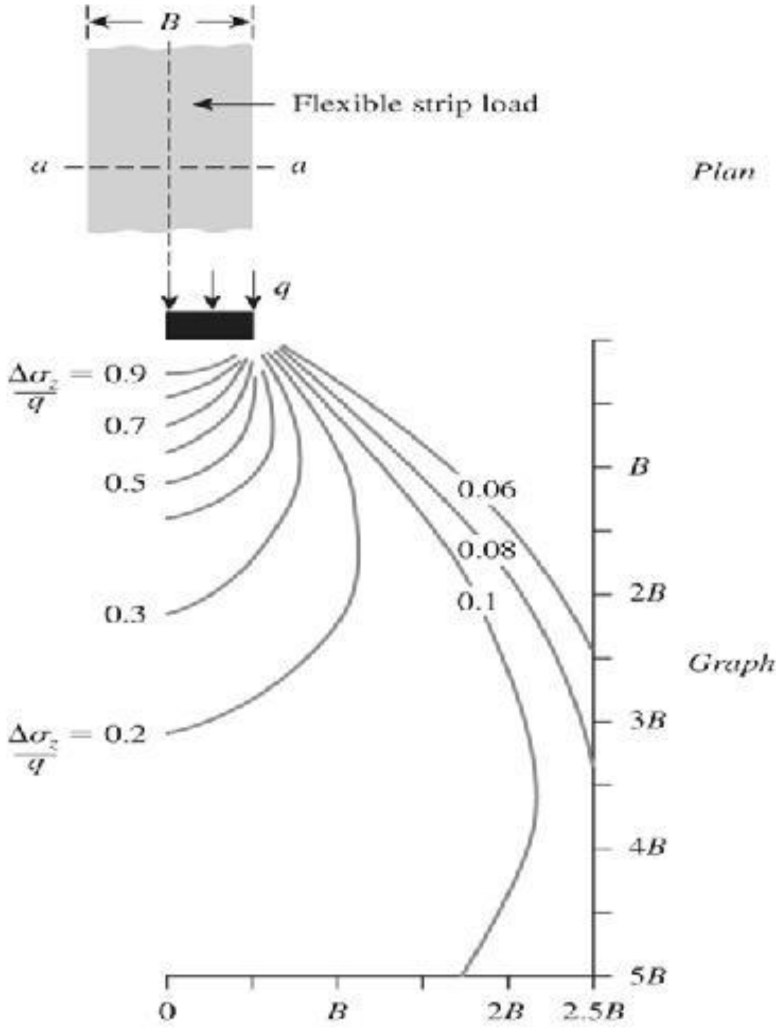


Figure 10.25 Vertical pressure isobars under a flexible strip load (*Note:* Isobars are for line *a–a* as shown on the plan)

• **Stress Isobars**

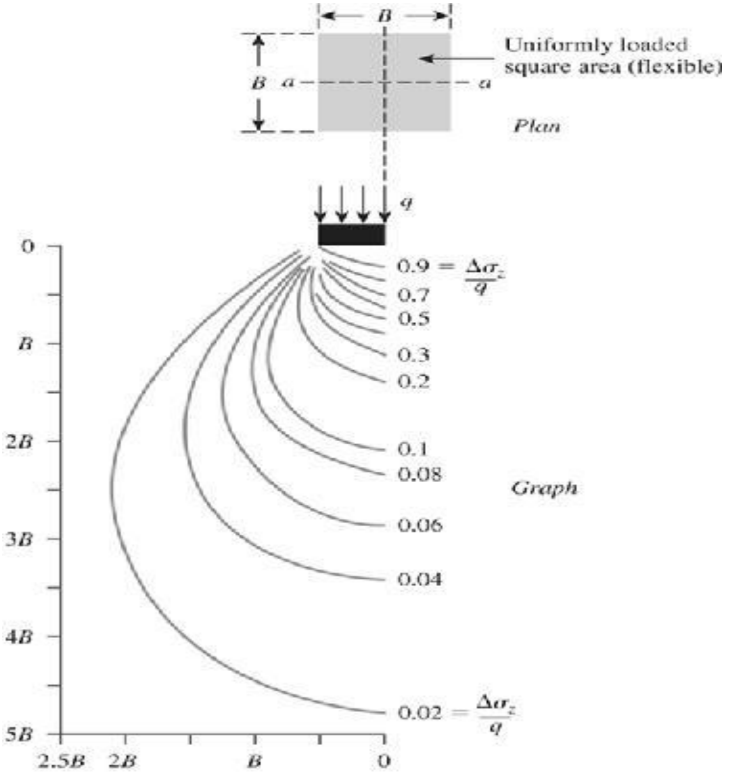


Figure 10.26 Vertical pressure isobars under a uniformly loaded square area (*Note:* Isobars are for line a–a as shown on the plan)

• Influence Chart for Vertical Pressure

$$\frac{R}{z} = \sqrt{\left(1 - \frac{\Delta\sigma_z}{q}\right)^{-2/3} - 1}$$

Table 10.10 Values of R/z for Various Pressure Ratios [Eq. (10.39)]

$\Delta\sigma_z/q$	R/z	$\Delta\sigma_z/q$	R/z
0	0	0.55	0.8384
0.05	0.1865	0.60	0.9176
0.10	0.2698	0.65	1.0067
0.15	0.3383	0.70	1.1097
0.20	0.4005	0.75	1.2328
0.25	0.4598	0.80	1.3871
0.30	0.5181	0.85	1.5943
0.35	0.5768	0.90	1.9084
0.40	0.6370	0.95	2.5232
0.45	0.6997	1.00	∞
0.50	0.7664		

•Using the values of *R/z obtained for various pressure ratios,* **Newmark (1942)** presented an influence chart that can be used to determine the vertical pressure at any point below a uniformly loaded flexible area of **any shape.**

$$\Delta\sigma_z = (IV)qM$$

IV=1/N

N=No. of elements

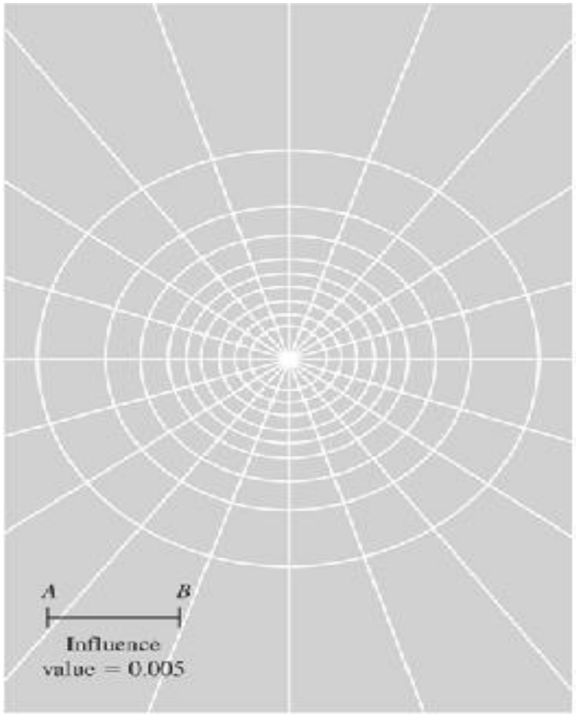


Figure 10.27
Influence chart for vertical pressure based on Boussinesq's theory (*Bulletin No. 338. Influence Charts for Computation of Stresses in Elastic. Foundations, by Nathan M. Newmark. University of Illinois, 1942.*)

•**The procedure for obtaining vertical pressure at any point below a loaded area is as follows:**

1. Determine the depth *z below the uniformly loaded area at which the stress increase* is required.

2. Plot the plan of the loaded area with a scale of *z equal to the unit length of* the chart (AB).

3. Place the plan (plotted in step 2) on the influence chart in such a way that the point below which the stress is to be determined is located at the center of the chart.

4. Count the number of elements (*M) of the chart enclosed by the plan of the loaded area.*

The increase in the pressure at the point under consideration is given by

$$\Delta\sigma_z = (IV)qM$$

where IV = influence value

q = pressure on the loaded area

Example 10.10

The cross section and plan of a column footing are shown in Figure 10.28. Find the increase in vertical stress produced by the column footing at point *A*.

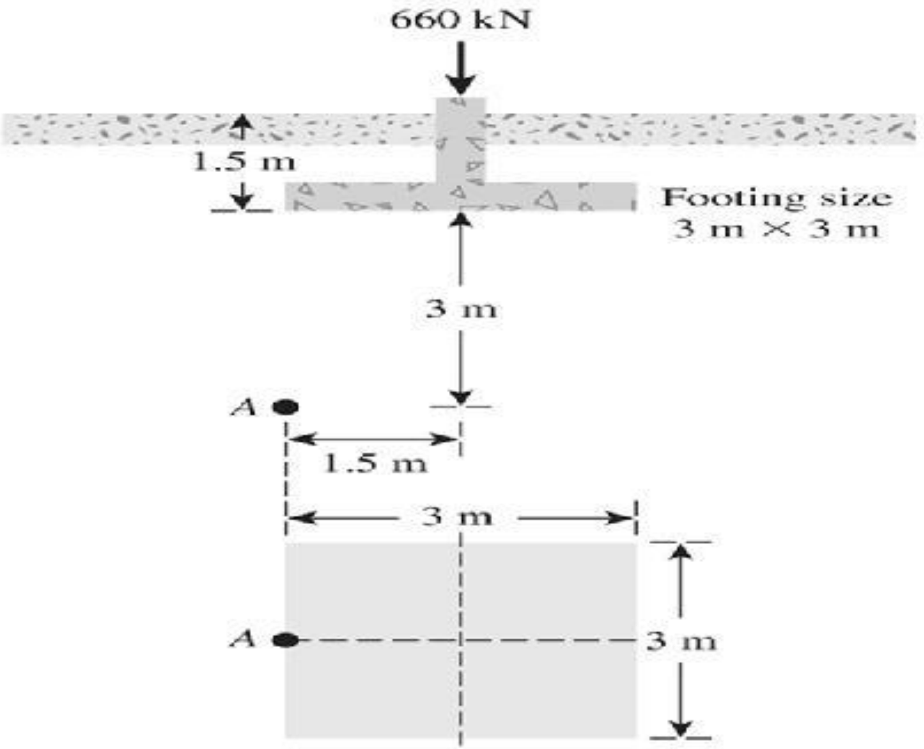


Figure 10.28
Cross section and plan of a column footing

4

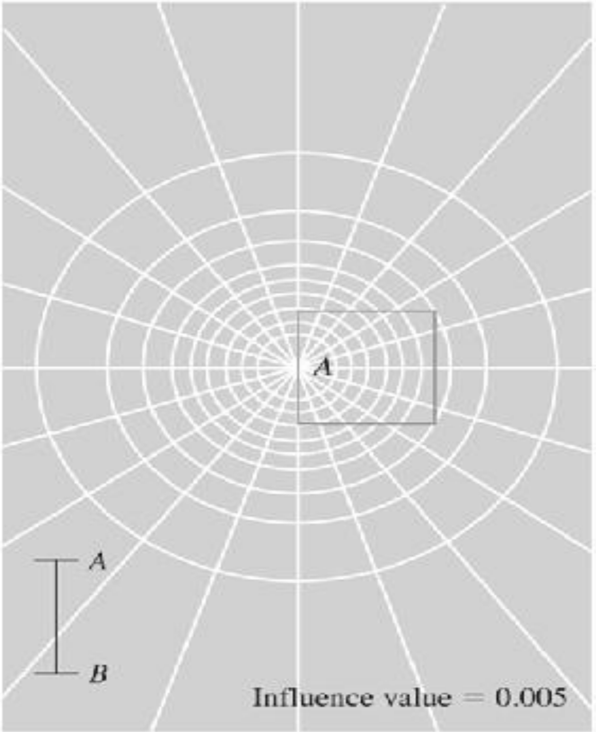


Figure 10.29 Determination of stress at a point by use of Newmark's influence chart

$$\Delta\sigma_z = (IV)qM = 0.005\left(\frac{660}{3 \times 3}\right)48.5 = \mathbf{17.78\ kN/m^2}$$

Compressibility of soil

A stress increase caused by the construction of foundations or other loads compresses soil layers.

The compression is caused by : (a)deformation of soil particles,

•relocations of soil particles, and

•Expulsion (removal) of water or air from the void spaces.

A stress increase caused by the construction of foundations or other loads compresses soil layers.

The compression is caused by :

(a)deformation of soil particles,

(b) relocations of soil particles, and

(c) Expulsion (removal) of water or air from the void spaces.

The Soil settlement caused by loads may be divided into **three broad** categories:

1. Elastic settlement (or immediate settlement), which is caused by the elastic deformation of dry soil and of moist and saturated soils without any change in the moisture content. Elastic settlement calculations generally are based on equations derived from the theory of elasticity.

2. *Primary consolidation settlement, which is the result of a volume change in saturated* cohesive soils because of expulsion of the water that occupies the void spaces.

3. *Secondary consolidation settlement, which is observed in saturated cohesive soils* and is the result of the plastic adjustment of soil fabrics. It is an additional form of compression that occurs at constant effective stress.

$$S_T = S_c + S_s + S_e$$

ELASTIC SETTLEMENT

• Contact Pressure and Settlement Profile

A perfectly flexible : If the foundation is subjected to a uniformly distributed load, the contact pressure will be uniform and the foundation will experience a sagging profile. foundation resting on an elastic material such as saturated clay. (Fig 11.1a).

A perfectly rigid : the ground surface subjected to a uniformly distributed load, the foundation will undergo a uniform settlement and the contact pressure will be redistributed. The contact pressure and foundation settlement profile will be as shown in Figure 11.1b:

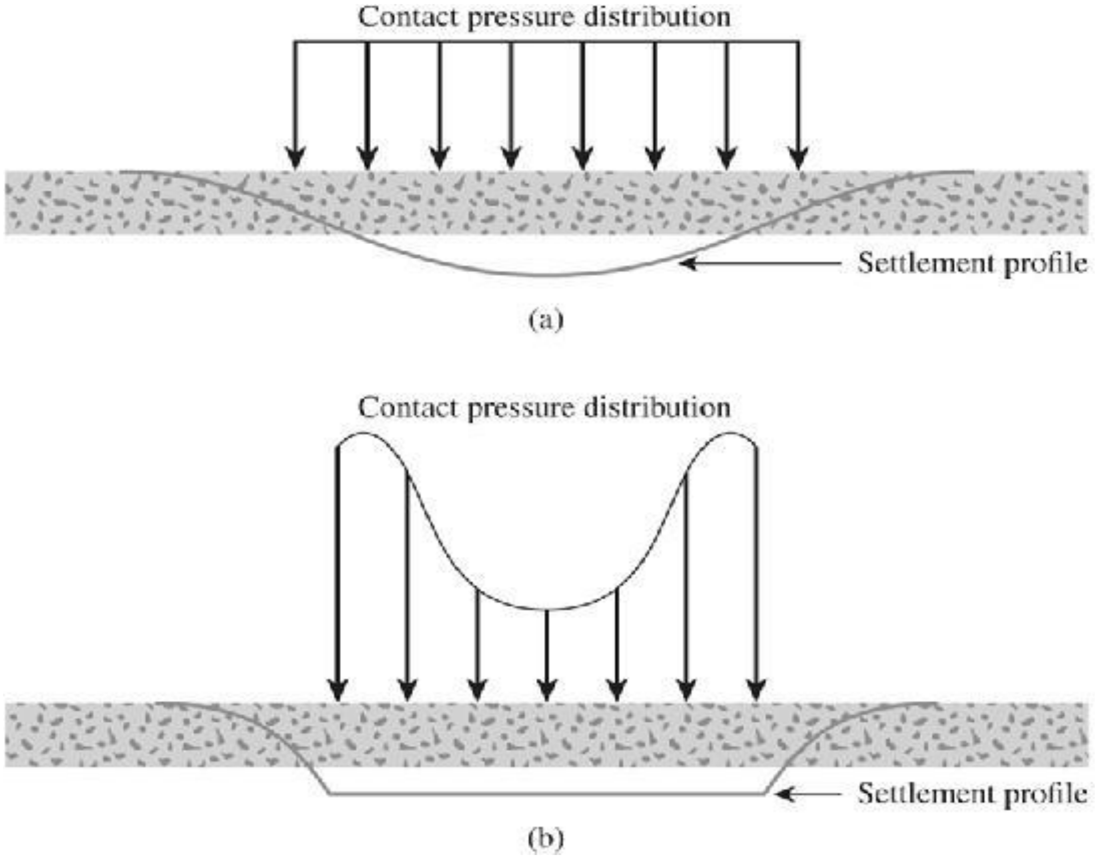


Figure 11.1 Elastic settlement profile and contact pressure in clay: (a) flexible foundation; (b) rigid foundation

In the case of cohesionless sand:

1. the modulus of elasticity increases with depth.
2. there is a lack of lateral confinement on the edge of the foundation at the ground surface. The sand at the edge of a flexible foundation is pushed outward, and the deflection curve of the foundation takes a concave downward shape.

The distributions of contact pressure and the settlement profiles of a flexible and a rigid foundation resting on sand and subjected to uniform loading are shown in Figures 11.2a and 11.2b, respectively.

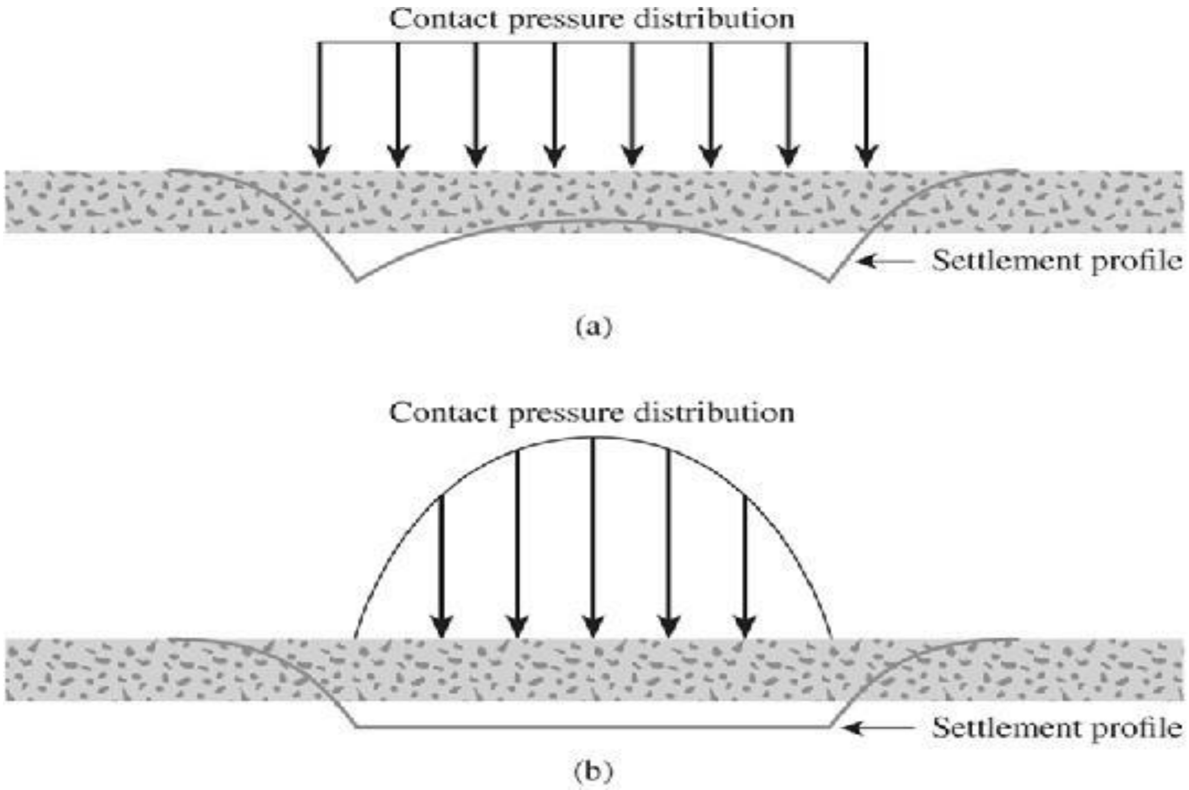


Figure 11.2 Elastic settlement profile and contact pressure in sand: (a) flexible foundation; (b) rigid foundation

Relations for Elastic Settlement Calculation

If the foundation is perfectly flexible, the settlement may be expressed as:

expressed as:

$$S_e = \Delta\sigma(\alpha B') \frac{1 - \mu_s^2}{E_s} I_s I_f$$

where $\Delta\sigma$ = net applied pressure on the foundation

μ_s = Poisson's ratio of soil

E_s = average modulus of elasticity of the soil under the foundation measured from $z = 0$ to about $z = 4B$

$B' = B/2$ for center of foundation

$= B$ for corner of foundation

I_s = shape factor (Steinbrenner, 1934)

$$= F_1 + \frac{1 - 2\mu_s}{1 - \mu_s} F_2$$

$$F_1 = \frac{1}{\pi}(A_0 + A_1)$$

$$F_2 = \frac{n'}{2\pi} \tan^{-1} A_2$$

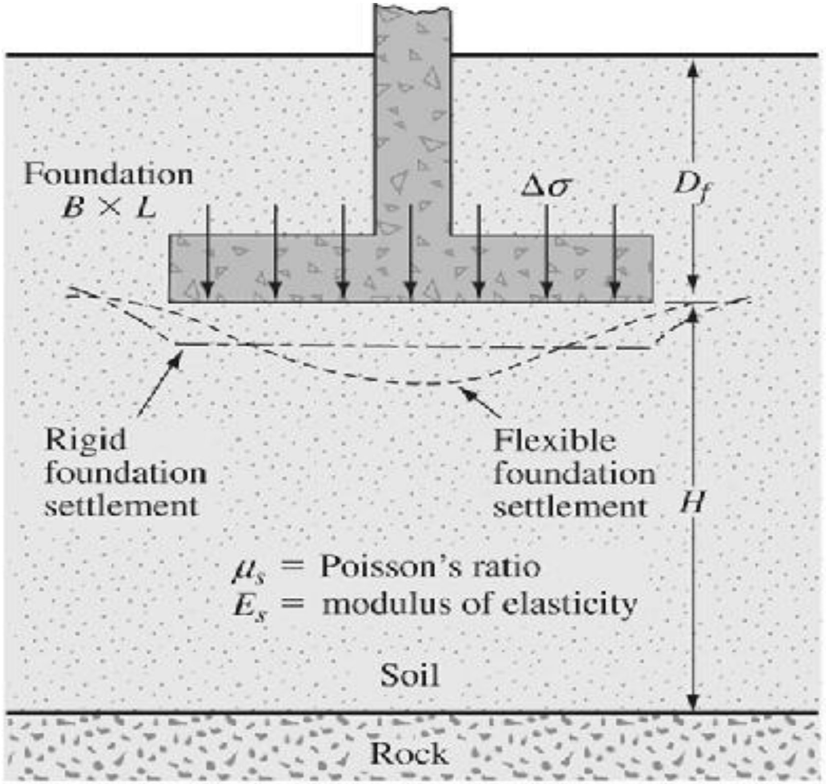


Figure 11.3 Elastic settlement of flexible and rigid foundations

- For calculation of settlement at the *center* of the foundation:

$$\alpha = 4 \qquad m' = \frac{L}{B} \qquad n' = \frac{H}{\left(\frac{B}{2}\right)}$$

- For calculation of settlement at a *corner* of the foundation:

$$\alpha = 1 \qquad m' = \frac{L}{B} \qquad n' = \frac{H}{B}$$

The variations of F_1 and F_2 with m' and n' given in Tables 11.1 and 11.2. Also the variation of I_f with D_f/B and μ_s is given in Table 11.3. Note that *when* $D_f = 0$, *the value of* $I_f = 1$ *in all cases*.

Table 11.1 Variation of F_1 with m' and n'

	m'									
n'	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	3.5	4.0
0.25	0.014	0.013	0.012	0.011	0.011	0.011	0.010	0.010	0.010	0.010
0.50	0.049	0.046	0.044	0.042	0.041	0.040	0.038	0.038	0.037	0.037
0.75	0.095	0.090	0.087	0.084	0.082	0.080	0.077	0.076	0.074	0.074
1.00	0.142	0.138	0.134	0.130	0.127	0.125	0.121	0.118	0.116	0.115
1.25	0.186	0.183	0.179	0.176	0.173	0.170	0.165	0.161	0.158	0.157
1.50	0.224	0.224	0.222	0.219	0.216	0.213	0.207	0.203	0.199	0.197
1.75	0.257	0.259	0.259	0.258	0.255	0.253	0.247	0.242	0.238	0.235
2.00	0.285	0.290	0.292	0.292	0.291	0.289	0.284	0.279	0.275	0.271
2.25	0.309	0.317	0.321	0.323	0.323	0.322	0.317	0.313	0.308	0.305
2.50	0.330	0.341	0.347	0.350	0.351	0.351	0.348	0.344	0.340	0.336
2.75	0.348	0.361	0.369	0.374	0.377	0.378	0.377	0.373	0.369	0.365
3.00	0.363	0.379	0.389	0.396	0.400	0.402	0.402	0.400	0.396	0.392
3.25	0.376	0.394	0.406	0.415	0.420	0.423	0.426	0.424	0.421	0.418
3.50	0.388	0.408	0.422	0.431	0.438	0.442	0.447	0.447	0.444	0.441
3.75	0.399	0.420	0.436	0.447	0.454	0.460	0.467	0.458	0.466	0.464
4.00	0.408	0.431	0.448	0.460	0.469	0.476	0.484	0.487	0.486	0.484
4.25	0.417	0.440	0.458	0.472	0.481	0.484	0.495	0.514	0.515	0.515
4.50	0.424	0.450	0.469	0.484	0.495	0.503	0.516	0.521	0.522	0.522
4.75	0.431	0.458	0.478	0.494	0.506	0.515	0.530	0.536	0.539	0.539
5.00	0.437	0.465	0.487	0.503	0.516	0.526	0.543	0.551	0.554	0.554
5.25	0.443	0.472	0.494	0.512	0.526	0.537	0.555	0.564	0.568	0.569
5.50	0.448	0.478	0.501	0.520	0.534	0.546	0.566	0.576	0.581	0.584
5.75	0.453	0.483	0.508	0.527	0.542	0.555	0.576	0.588	0.594	0.597
6.00	0.457	0.489	0.514	0.534	0.550	0.563	0.585	0.598	0.606	0.609
6.25	0.461	0.493	0.519	0.540	0.557	0.570	0.594	0.609	0.617	0.621
6.50	0.465	0.498	0.524	0.546	0.563	0.577	0.603	0.618	0.627	0.632
6.75	0.468	0.502	0.529	0.551	0.569	0.584	0.610	0.627	0.637	0.643
7.00	0.471	0.506	0.533	0.556	0.575	0.590	0.618	0.635	0.646	0.653
7.25	0.474	0.509	0.538	0.561	0.580	0.596	0.625	0.643	0.655	0.662
7.50	0.477	0.513	0.541	0.565	0.585	0.601	0.631	0.650	0.663	0.671
7.75	0.480	0.516	0.545	0.569	0.589	0.606	0.637	0.658	0.671	0.680
8.00	0.482	0.519	0.549	0.573	0.594	0.611	0.643	0.664	0.678	0.688
8.25	0.485	0.522	0.552	0.577	0.598	0.615	0.648	0.670	0.685	0.695
8.50	0.487	0.524	0.555	0.580	0.601	0.619	0.653	0.676	0.692	0.703
8.75	0.489	0.527	0.558	0.583	0.605	0.623	0.658	0.682	0.698	0.710
9.00	0.491	0.529	0.560	0.587	0.609	0.627	0.663	0.687	0.705	0.716
9.25	0.493	0.531	0.563	0.589	0.612	0.631	0.667	0.693	0.710	0.723
9.50	0.495	0.533	0.565	0.592	0.615	0.634	0.671	0.697	0.716	0.719
9.75	0.496	0.536	0.568	0.595	0.618	0.638	0.675	0.702	0.721	0.735
10.00	0.498	0.537	0.570	0.597	0.621	0.641	0.679	0.707	0.726	0.740
20.00	0.529	0.575	0.614	0.647	0.677	0.702	0.756	0.797	0.830	0.858
50.00	0.548	0.598	0.640	0.678	0.711	0.740	0.803	0.853	0.895	0.931
100.00	0.555	0.605	0.649	0.688	0.722	0.753	0.819	0.872	0.918	0.956

Table 11.1 (*continued*)

	m'									
n'	4.5	5.0	6.0	7.0	8.0	9.0	10.0	25.0	50.0	100.0
0.25	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
0.50	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036
0.75	0.073	0.073	0.072	0.072	0.072	0.072	0.071	0.071	0.071	0.071
1.00	0.114	0.113	0.112	0.112	0.112	0.111	0.111	0.110	0.110	0.110
1.25	0.155	0.154	0.153	0.152	0.152	0.151	0.151	0.150	0.150	0.150
1.50	0.195	0.194	0.192	0.191	0.190	0.190	0.189	0.188	0.188	0.188
1.75	0.233	0.232	0.229	0.228	0.227	0.226	0.225	0.223	0.223	0.223
2.00	0.269	0.267	0.264	0.262	0.261	0.260	0.259	0.257	0.256	0.256
2.25	0.302	0.300	0.296	0.294	0.293	0.291	0.291	0.287	0.287	0.287
2.50	0.333	0.331	0.327	0.324	0.322	0.321	0.320	0.316	0.315	0.315
2.75	0.362	0.359	0.355	0.352	0.350	0.348	0.347	0.343	0.342	0.342
3.00	0.389	0.386	0.382	0.378	0.376	0.374	0.373	0.368	0.367	0.367
3.25	0.415	0.412	0.407	0.403	0.401	0.399	0.397	0.391	0.390	0.390
3.50	0.438	0.435	0.430	0.427	0.424	0.421	0.420	0.413	0.412	0.411
3.75	0.461	0.458	0.453	0.449	0.446	0.443	0.441	0.433	0.432	0.432
4.00	0.482	0.479	0.474	0.470	0.466	0.464	0.462	0.453	0.451	0.451
4.25	0.516	0.496	0.484	0.473	0.471	0.471	0.470	0.468	0.462	0.460
4.50	0.520	0.517	0.513	0.508	0.505	0.502	0.499	0.489	0.487	0.487
4.75	0.537	0.535	0.530	0.526	0.523	0.519	0.517	0.506	0.504	0.503
5.00	0.554	0.552	0.548	0.543	0.540	0.536	0.534	0.522	0.519	0.519
5.25	0.569	0.568	0.564	0.560	0.556	0.553	0.550	0.537	0.534	0.534
5.50	0.584	0.583	0.579	0.575	0.571	0.568	0.585	0.551	0.549	0.548
5.75	0.597	0.597	0.594	0.590	0.586	0.583	0.580	0.565	0.583	0.562
6.00	0.611	0.610	0.608	0.604	0.601	0.598	0.595	0.579	0.576	0.575
6.25	0.623	0.623	0.621	0.618	0.615	0.611	0.608	0.592	0.589	0.588
6.50	0.635	0.635	0.634	0.631	0.628	0.625	0.622	0.605	0.601	0.600
6.75	0.646	0.647	0.646	0.644	0.641	0.637	0.634	0.617	0.613	0.612
7.00	0.656	0.658	0.658	0.656	0.653	0.650	0.647	0.628	0.624	0.623
7.25	0.666	0.669	0.669	0.668	0.665	0.662	0.659	0.640	0.635	0.634
7.50	0.676	0.679	0.680	0.679	0.676	0.673	0.670	0.651	0.646	0.645
7.75	0.685	0.688	0.690	0.689	0.687	0.684	0.681	0.661	0.656	0.655
8.00	0.694	0.697	0.700	0.700	0.698	0.695	0.692	0.672	0.666	0.665
8.25	0.702	0.706	0.710	0.710	0.708	0.705	0.703	0.682	0.676	0.675
8.50	0.710	0.714	0.719	0.719	0.718	0.715	0.713	0.692	0.686	0.684
8.75	0.717	0.722	0.727	0.728	0.727	0.725	0.723	0.701	0.695	0.693
9.00	0.725	0.730	0.736	0.737	0.736	0.735	0.732	0.710	0.704	0.702
9.25	0.731	0.737	0.744	0.746	0.745	0.744	0.742	0.719	0.713	0.711
9.50	0.738	0.744	0.752	0.754	0.754	0.753	0.751	0.728	0.721	0.719
9.75	0.744	0.751	0.759	0.762	0.762	0.761	0.759	0.737	0.729	0.727
10.00	0.750	0.758	0.766	0.770	0.770	0.770	0.768	0.745	0.738	0.735
20.00	0.878	0.896	0.925	0.945	0.959	0.969	0.977	0.982	0.965	0.957
50.00	0.962	0.989	1.034	1.070	1.100	1.125	1.146	1.265	1.279	1.261
100.00	0.990	1.020	1.072	1.114	1.150	1.182	1.209	1.408	1.489	1.499

Table 11.2 Variation of F_2 with m' and n'

n'	m' 1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	3.5	4.0
0.25	0.049	0.050	0.051	0.051	0.051	0.052	0.052	0.052	0.052	0.052
0.50	0.074	0.077	0.080	0.081	0.083	0.084	0.086	0.086	0.0878	0.087
0.75	0.083	0.089	0.093	0.097	0.099	0.101	0.104	0.106	0.107	0.108
1.00	0.083	0.091	0.098	0.102	0.106	0.109	0.114	0.117	0.119	0.120
1.25	0.080	0.089	0.096	0.102	0.107	0.111	0.118	0.122	0.125	0.127
1.50	0.075	0.084	0.093	0.099	0.105	0.110	0.118	0.124	0.128	0.130
1.75	0.069	0.079	0.088	0.095	0.101	0.107	0.117	0.123	0.128	0.131
2.00	0.064	0.074	0.083	0.090	0.097	0.102	0.114	0.121	0.127	0.131
2.25	0.059	0.069	0.077	0.085	0.092	0.098	0.110	0.119	0.125	0.130
2.50	0.055	0.064	0.073	0.080	0.087	0.093	0.106	0.115	0.122	0.127
2.75	0.051	0.060	0.068	0.076	0.082	0.089	0.102	0.111	0.119	0.125
3.00	0.048	0.056	0.064	0.071	0.078	0.084	0.097	0.108	0.116	0.122
3.25	0.045	0.053	0.060	0.067	0.074	0.080	0.093	0.104	0.112	0.119
3.50	0.042	0.050	0.057	0.064	0.070	0.076	0.089	0.100	0.109	0.116
3.75	0.040	0.047	0.054	0.060	0.067	0.073	0.086	0.096	0.105	0.113
4.00	0.037	0.044	0.051	0.057	0.063	0.069	0.082	0.093	0.102	0.110
4.25	0.036	0.042	0.049	0.055	0.061	0.066	0.079	0.090	0.099	0.107
4.50	0.034	0.040	0.046	0.052	0.058	0.063	0.076	0.086	0.096	0.104
4.75	0.032	0.038	0.044	0.050	0.055	0.061	0.073	0.083	0.093	0.101
5.00	0.031	0.036	0.042	0.048	0.053	0.058	0.070	0.080	0.090	0.098
5.25	0.029	0.035	0.040	0.046	0.051	0.056	0.067	0.078	0.087	0.095
5.50	0.028	0.033	0.039	0.044	0.049	0.054	0.065	0.075	0.084	0.092
5.75	0.027	0.032	0.037	0.042	0.047	0.052	0.063	0.073	0.082	0.090
6.00	0.026	0.031	0.036	0.040	0.045	0.050	0.060	0.070	0.079	0.087
6.25	0.025	0.030	0.034	0.039	0.044	0.048	0.058	0.068	0.077	0.085
6.50	0.024	0.029	0.033	0.038	0.042	0.046	0.056	0.066	0.075	0.083
6.75	0.023	0.028	0.032	0.036	0.041	0.045	0.055	0.064	0.073	0.080
7.00	0.022	0.027	0.031	0.035	0.039	0.043	0.053	0.062	0.071	0.078
7.25	0.022	0.026	0.030	0.034	0.038	0.042	0.051	0.060	0.069	0.076
7.50	0.021	0.025	0.029	0.033	0.037	0.041	0.050	0.059	0.067	0.074
7.75	0.020	0.024	0.028	0.032	0.036	0.039	0.048	0.057	0.065	0.072
8.00	0.020	0.023	0.027	0.031	0.035	0.038	0.047	0.055	0.063	0.071
8.25	0.019	0.023	0.026	0.030	0.034	0.037	0.046	0.054	0.062	0.069
8.50	0.018	0.022	0.026	0.029	0.033	0.036	0.045	0.053	0.060	0.067
8.75	0.018	0.021	0.025	0.028	0.032	0.035	0.043	0.051	0.059	0.066
9.00	0.017	0.021	0.024	0.028	0.031	0.034	0.042	0.050	0.057	0.064
9.25	0.017	0.020	0.024	0.027	0.030	0.033	0.041	0.049	0.056	0.063
9.50	0.017	0.020	0.023	0.026	0.029	0.033	0.040	0.048	0.055	0.061
9.75	0.016	0.019	0.023	0.026	0.029	0.032	0.039	0.047	0.054	0.060
10.00	0.016	0.019	0.022	0.025	0.028	0.031	0.038	0.046	0.052	0.059
20.00	0.008	0.010	0.011	0.013	0.014	0.016	0.020	0.024	0.027	0.031
50.00	0.003	0.004	0.004	0.005	0.006	0.006	0.008	0.010	0.011	0.013
100.00	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.005	0.006	0.006

Table 11.2 (*continued*)

	m'									
n'	4.5	5.0	6.0	7.0	8.0	9.0	10.0	25.0	50.0	100.0
0.25	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053
0.50	0.087	0.087	0.088	0.088	0.088	0.088	0.088	0.088	0.088	0.088
0.75	0.109	0.109	0.109	0.110	0.110	0.110	0.110	0.111	0.111	0.111
1.00	0.121	0.122	0.123	0.123	0.124	0.124	0.124	0.125	0.125	0.125
1.25	0.128	0.130	0.131	0.132	0.132	0.133	0.133	0.134	0.134	0.134
1.50	0.132	0.134	0.136	0.137	0.138	0.138	0.139	0.140	0.140	0.140
1.75	0.134	0.136	0.138	0.140	0.141	0.142	0.142	0.144	0.144	0.145
2.00	0.134	0.136	0.139	0.141	0.143	0.144	0.145	0.147	0.147	0.148
2.25	0.133	0.136	0.140	0.142	0.144	0.145	0.146	0.149	0.150	0.150
2.50	0.132	0.135	0.139	0.142	0.144	0.146	0.147	0.151	0.151	0.151
2.75	0.130	0.133	0.138	0.142	0.144	0.146	0.147	0.152	0.152	0.153
3.00	0.127	0.131	0.137	0.141	0.144	0.145	0.147	0.152	0.153	0.154
3.25	0.125	0.129	0.135	0.140	0.143	0.145	0.147	0.153	0.154	0.154
3.50	0.122	0.126	0.133	0.138	0.142	0.144	0.146	0.153	0.155	0.155
3.75	0.119	0.124	0.131	0.137	0.141	0.143	0.145	0.154	0.155	0.155
4.00	0.116	0.121	0.129	0.135	0.139	0.142	0.145	0.154	0.155	0.156
4.25	0.113	0.119	0.127	0.133	0.138	0.141	0.144	0.154	0.156	0.156
4.50	0.110	0.116	0.125	0.131	0.136	0.140	0.143	0.154	0.156	0.156
4.75	0.107	0.113	0.123	0.130	0.135	0.139	0.142	0.154	0.156	0.157
5.00	0.105	0.111	0.120	0.128	0.133	0.137	0.140	0.154	0.156	0.157
5.25	0.102	0.108	0.118	0.126	0.131	0.136	0.139	0.154	0.156	0.157
5.50	0.099	0.106	0.116	0.124	0.130	0.134	0.138	0.154	0.156	0.157
5.75	0.097	0.103	0.113	0.122	0.128	0.133	0.136	0.154	0.157	0.157
6.00	0.094	0.101	0.111	0.120	0.126	0.131	0.135	0.153	0.157	0.157
6.25	0.092	0.098	0.109	0.118	0.124	0.129	0.134	0.153	0.157	0.158
6.50	0.090	0.096	0.107	0.116	0.122	0.128	0.132	0.153	0.157	0.158
6.75	0.087	0.094	0.105	0.114	0.121	0.126	0.131	0.153	0.157	0.158
7.00	0.085	0.092	0.103	0.112	0.119	0.125	0.129	0.152	0.157	0.158
7.25	0.083	0.090	0.101	0.110	0.117	0.123	0.128	0.152	0.157	0.158
7.50	0.081	0.088	0.099	0.108	0.115	0.121	0.126	0.152	0.156	0.158
7.75	0.079	0.086	0.097	0.106	0.114	0.120	0.125	0.151	0.156	0.158
8.00	0.077	0.084	0.095	0.104	0.112	0.118	0.124	0.151	0.156	0.158
8.25	0.076	0.082	0.093	0.102	0.110	0.117	0.122	0.150	0.156	0.158
8.50	0.074	0.080	0.091	0.101	0.108	0.115	0.121	0.150	0.156	0.158
8.75	0.072	0.078	0.089	0.099	0.107	0.114	0.119	0.150	0.156	0.158
9.00	0.071	0.077	0.088	0.097	0.105	0.112	0.118	0.149	0.156	0.158
9.25	0.069	0.075	0.086	0.096	0.104	0.110	0.116	0.149	0.156	0.158
9.50	0.068	0.074	0.085	0.094	0.102	0.109	0.115	0.148	0.156	0.158
9.75	0.066	0.072	0.083	0.092	0.100	0.107	0.113	0.148	0.156	0.158
10.00	0.065	0.071	0.082	0.091	0.099	0.106	0.112	0.147	0.156	0.158
20.00	0.035	0.039	0.046	0.053	0.059	0.065	0.071	0.124	0.148	0.156
50.00	0.014	0.016	0.019	0.022	0.025	0.028	0.031	0.071	0.113	0.142
100.00	0.007	0.008	0.010	0.011	0.013	0.014	0.016	0.039	0.071	0.113

Table 11.3 Variation of I_f with L/B and D_f/B

		I_f		
L/B	D_f/B	$\mu_s = 0.3$	$\mu_s = 0.4$	$\mu_s = 0.5$
1	0.5	0.77	0.82	0.85
	0.75	0.69	0.74	0.77
	1	0.65	0.69	0.72
2	0.5	0.82	0.86	0.89
	0.75	0.75	0.79	0.83
	1	0.71	0.75	0.79
5	0.5	0.87	0.91	0.93
	0.75	0.81	0.86	0.89
	1	0.78	0.82	0.85

The elastic settlement of a *rigid foundation* can be estimated as

$$S_{e(\text{rigid})} \approx 0.93 S_{e(\text{flexible, center})}$$

Due to the nonhomogeneous nature of soil deposits, the magnitude of E_s may vary with depth. For that reason, Bowles (1987) recommended using a weighted average value of E_s in Eq. (11.1) or

$$E_s = \frac{\Sigma E_{s(i)} \Delta z}{\bar{z}}$$

Where $E_{s(i)}$ = soil modulus of elasticity within a depth Δz
$\bar{z} = H$ or $5B$, whichever is smaller

Table 11.4 Representative Values of the Modulus of Elasticity of Soil

Soil type	E_s **(kN/m^2)**
Soft clay	1,800–3,500
Hard clay	6,000–14,000
Loose sand	10,000–28,000
Dense sand	35,000–70,000

Table 11.5 Representative Values of Poisson's Ratio

Type of soil	Poisson's ratio, μ_s
Loose sand	0.2–0.4
Medium sand	0.25–0.4
Dense sand	0.3–0.45
Silty sand	0.2–0.4
Soft clay	0.15–0.25
Medium clay	0.2–0.5

Example 11.1

A rigid shallow foundation 1 m × 2 m is shown in Figure 11.4. Calculate the elastic settlement at the center of the foundation.

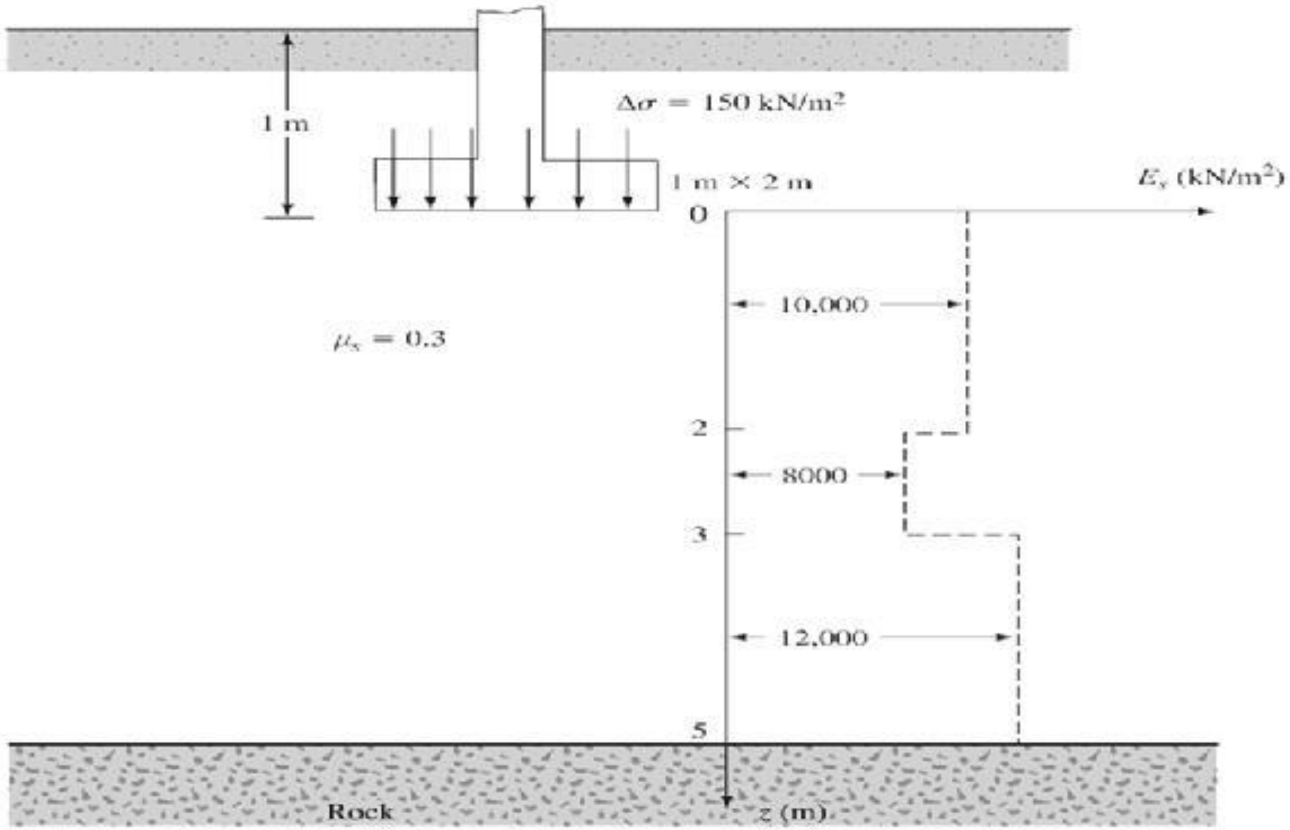


Figure 11.4

Solution

Given: $B = 1$ m and $L = 2$ m. Note that $\bar{z} = 5$ m $= 5B$. From Eq. (11.10),

$$E_s = \frac{\Sigma E_{s(i)}\Delta z}{\bar{z}}$$

$$= \frac{(10{,}000)(2) + (8{,}000)(1) + (12{,}000)(2)}{5} = 10{,}400\,\text{kN/m}^2$$

For the *center of the foundation*,

$$\alpha = 4$$

$$m' = \frac{L}{B} = \frac{2}{1} = 2$$

$$n' = \frac{H}{\left(\frac{B}{2}\right)} = \frac{5}{\left(\frac{1}{2}\right)} = 10$$

From Tables 11.1 and 11.2, $F_1 = 0.641$ and $F_2 = 0.031$. From Eq. (11.2).

$$I_s = F_1 + \frac{2-\mu_s}{1-\mu_s}F_2$$

$$= 0.641 + \frac{2-0.3}{1-0.3}(0.031) = 0.716$$

Again, $\frac{D_f}{B} = \frac{1}{1} = 1$, $\frac{L}{B} = 2$, $\mu_s = 0.3$. From Table 11.3, $I_f = 0.71$. Hence,

$$S_{e(\text{flexible})} = \Delta\sigma(\alpha B')\frac{1-\mu_s^2}{E_s}I_s I_f$$

$$= (150)\left(4 \times \frac{1}{2}\right)\left(\frac{1-0.3^2}{10{,}400}\right)(0.716)(0.71) = 0.0133\,\text{m} = 13.3\,\text{mm}$$

Since the foundation is rigid, from Eq. (11.9),

$$S_e(\text{rigid}) = (0.93)(13.3) = 12.4\text{ mm}$$ ■

• **<u>CONSOLIDATION SETTLEMENT</u>** **<u>Fundamentals of Consolidation</u>**

1. **Sandy soils:**

When a saturated soil layer is subjected to a stress increase, the pore water pressure is increased suddenly. In sandy soils that are highly permeable, the drainage caused by the increase in the pore water pressure is completed immediately. Pore water drainage is accompanied by a reduction in the volume of the soil mass, which results in settlement. Because of rapid drainage of the pore water in sandy soils, elastic settlement and consolidation occur simultaneously.

• CONSOLIDATION SETTLEMENT Fundamentals of Consolidation

2. Clayey soils:

When a saturated compressible clay layer is subjected to a stress increase, elastic settlement occurs immediately. Because the hydraulic conductivity of clay is significantly smaller than that of sand , the excess pore water pressure generated by loading gradually dissipates over a long period . Thus, the associated volume change (that is, the consolidation) in the clay may continue long after the elastic settlement. The settlement caused by consolidation in clay may be several times greater than the elastic settlement.

The time-dependent deformation of saturated clayey soil best can be understood by considering a simple model that consists of a cylinder with a spring at its center.

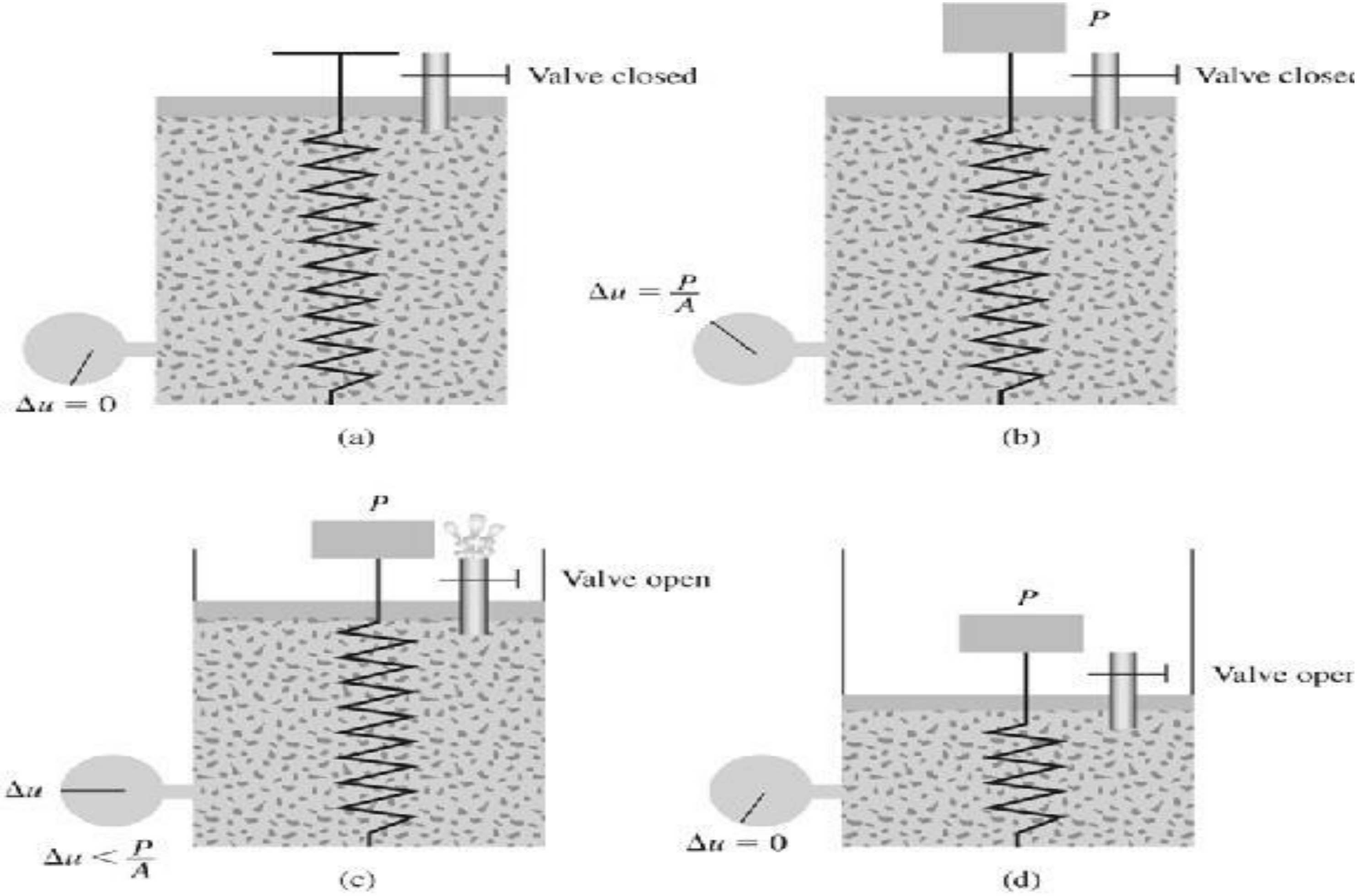


Figure 11.5 Spring-cylinder model

$$\Delta\sigma = \Delta\sigma' + \Delta u$$

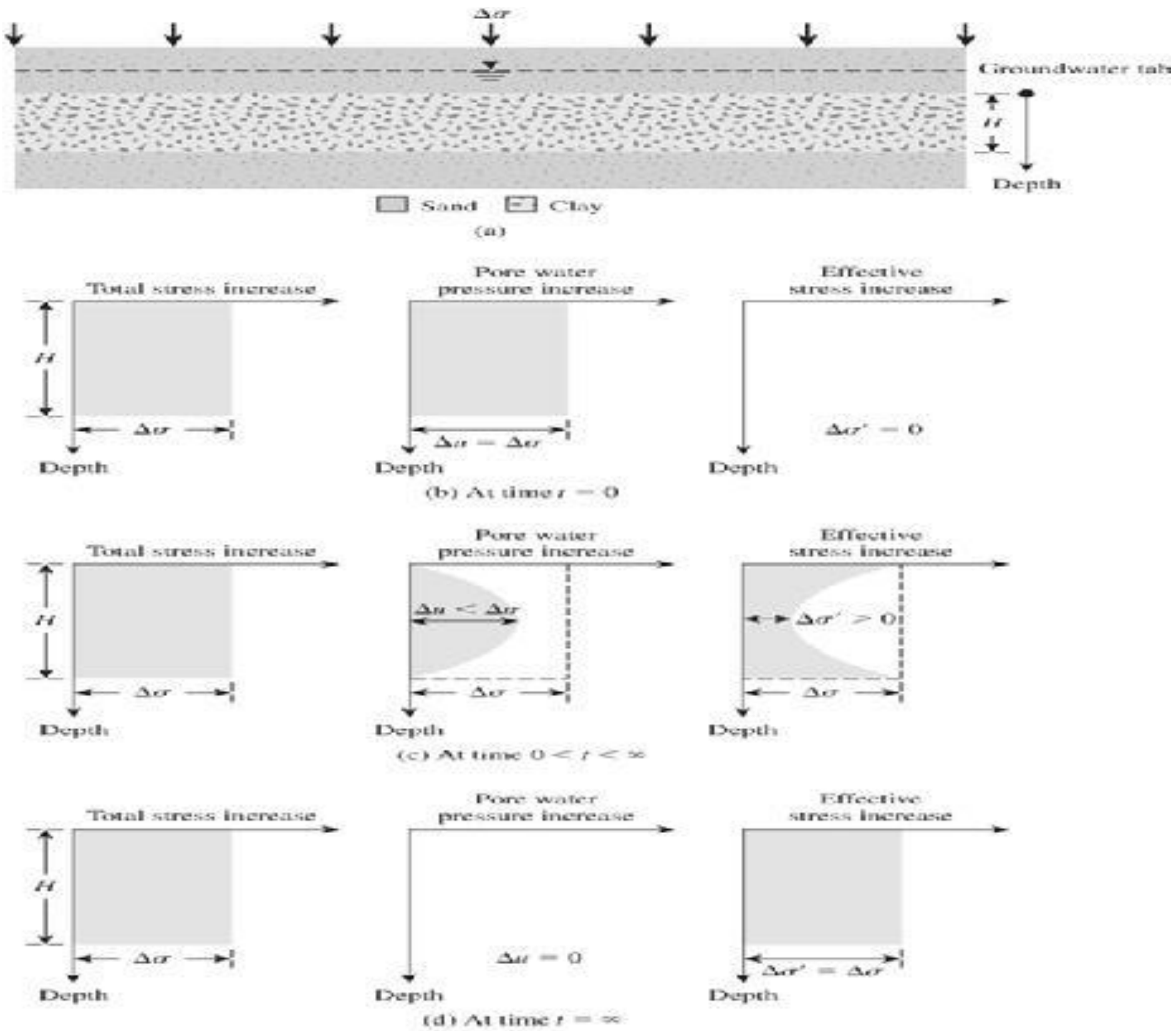


Figure 11.6 Variation of total stress, pore water pressure, and effective stress in a clay layer drained at top and bottom as the result of an added stress, $\Delta\sigma$

Compressibility of soil

Part II

❑ CONSOLIDATION SETTLEMENT

Fundamentals of Consolidation

1. **Sandy soils:**

When a saturated soil layer is subjected to a stress increase, the pore water pressure is increased suddenly. In sandy soils that are highly permeable, the drainage caused by the increase in the pore water pressure is completed immediately. Pore water drainage is accompanied by a reduction in the volume of the soil mass, which results in settlement. Because of rapid drainage of the pore water in sandy soils, elastic settlement and consolidation occur together.

❑ CONSOLIDATION SETTLEMENT

Fundamentals of Consolidation

2. Clayey soils:

When a saturated compressible clay layer is subjected to a stress increase, elastic settlement occurs immediately. Because the hydraulic conductivity of clay is significantly smaller than that of sand , the excess pore water pressure generated by loading gradually dissipates over a long period . Thus, the associated volume change (that is, the consolidation) in the clay may continue long after the elastic settlement. The settlement caused by consolidation in clay may be several times greater than the elastic settlement.

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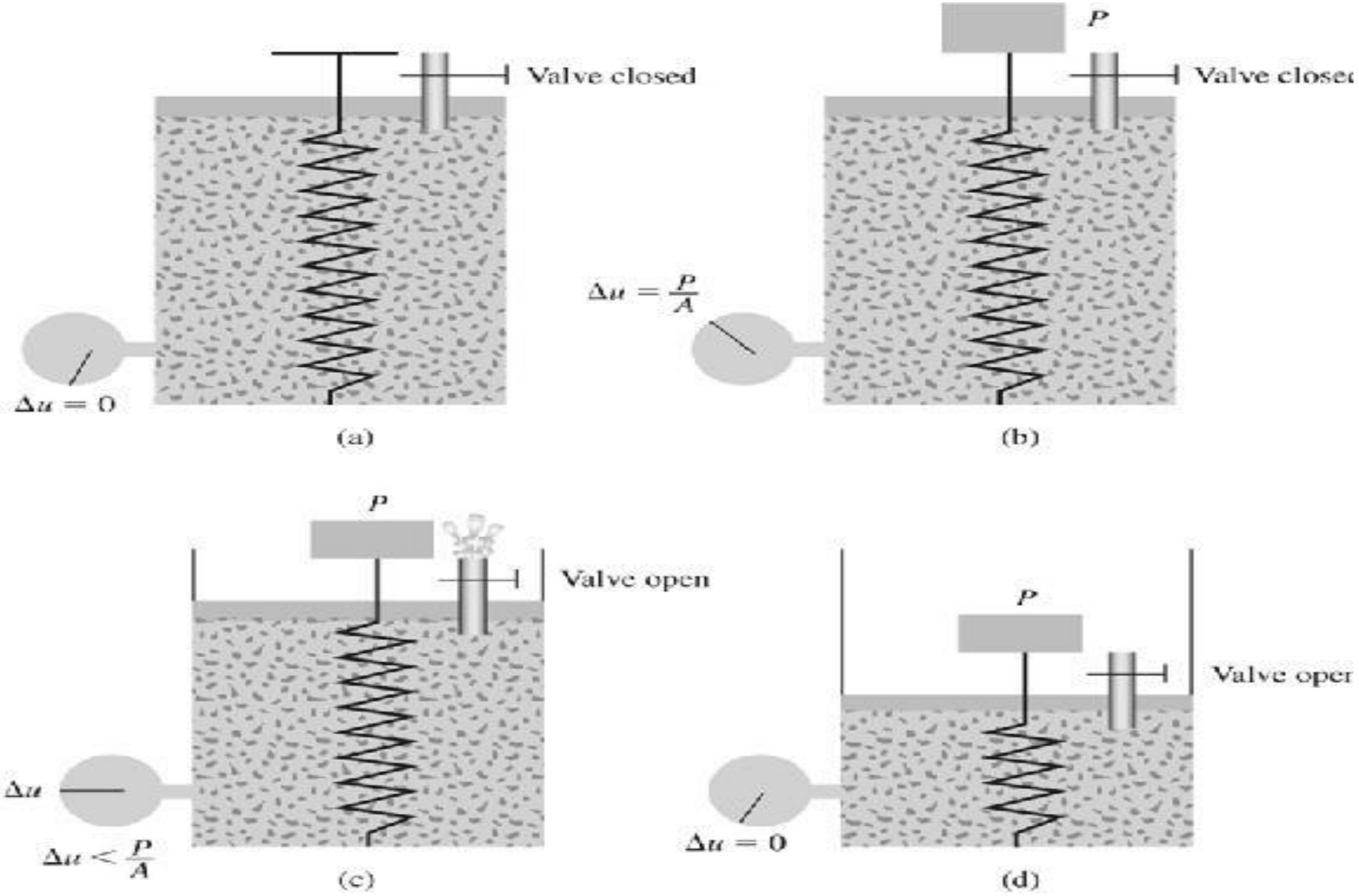


Figure 11.5 Spring-cylinder model

$$\Delta\sigma = \Delta\sigma' + \Delta u$$

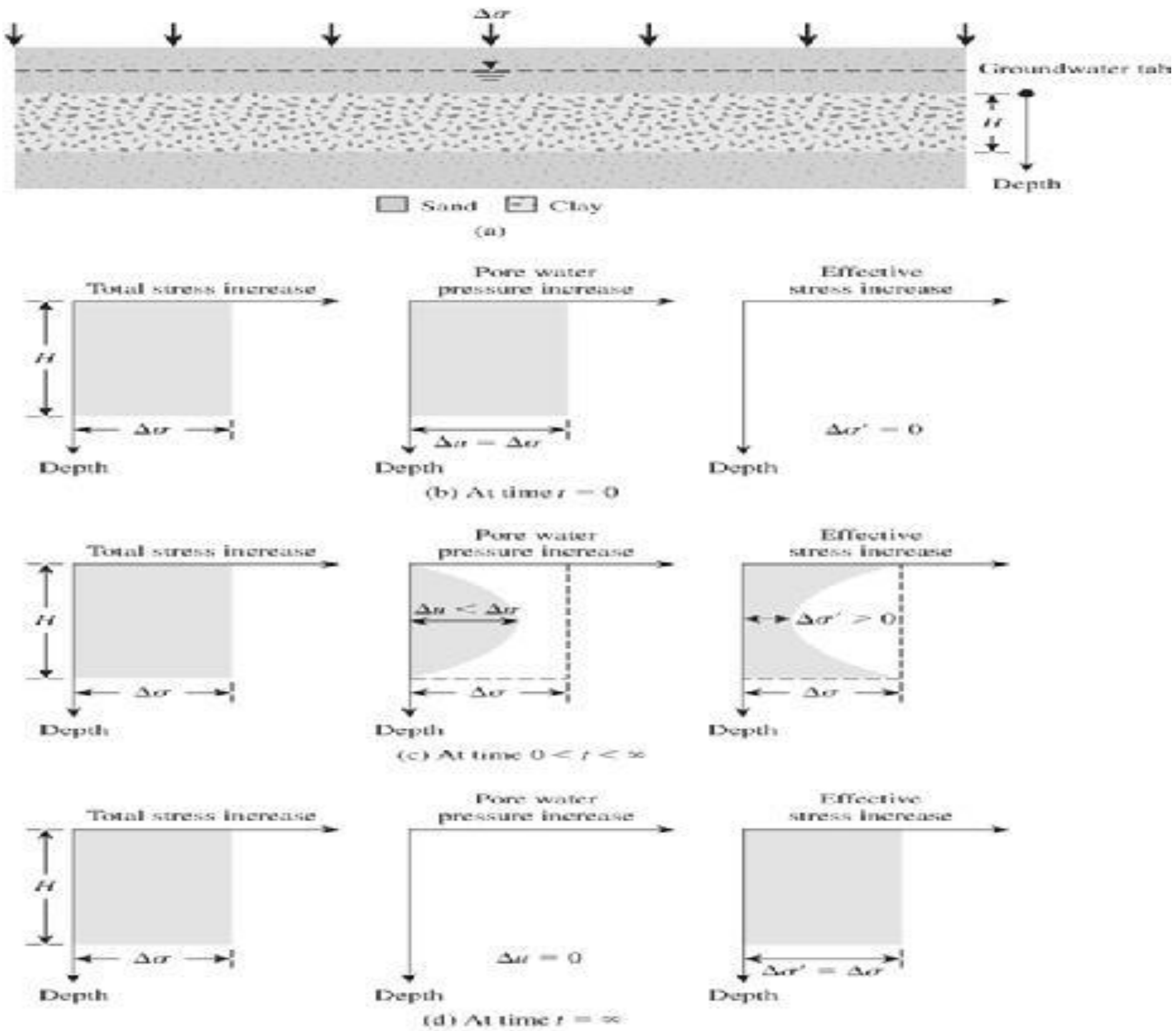


Figure 11.6 Variation of total stress, pore water pressure, and effective stress in a clay layer drained at top and bottom as the result of an added stress, $\Delta\sigma$

❑ <u>One-Dimensional Laboratory Consolidation Test</u>

- The one-dimensional consolidation testing procedure was first suggested by Terzaghi. This test is performed in a consolidometer (sometimes referred to as an *oedometer).*

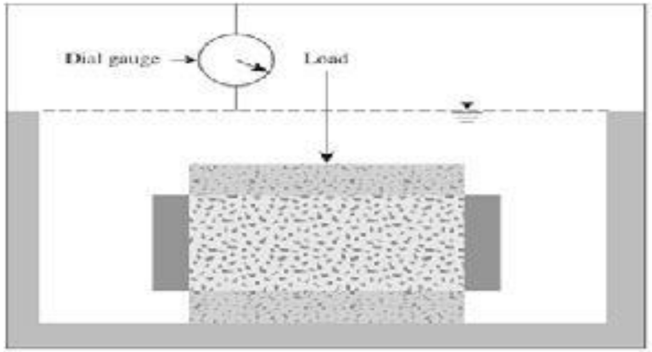
• *<u>The test simple procedure:</u>*

• The soil specimen is placed inside a metal ring with two porous stones, one at the top of the specimen and another at the bottom.

• The specimens are usually 64 mm (2.5 in.) in diameter and 25 mm. (1 in.) thick.

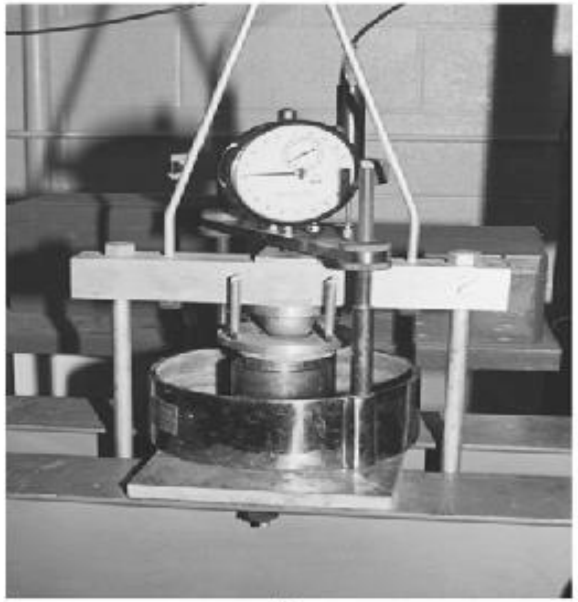
• The load on the specimen is applied through a lever arm, and compression is measured by a micrometer dial gauge.

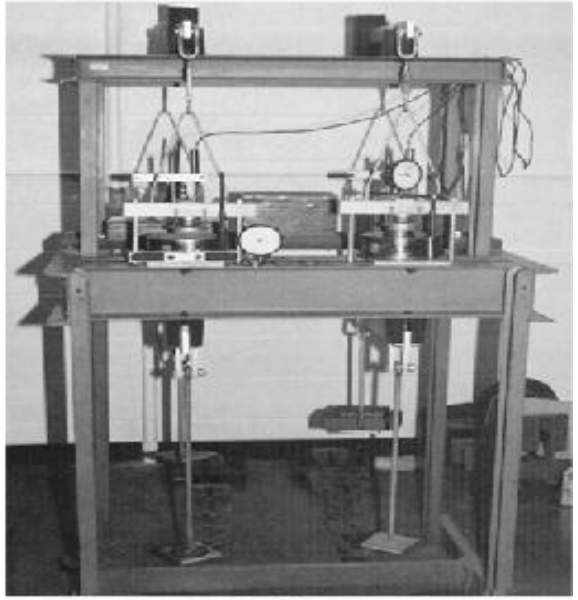
• The specimen is kept under water during the test.



(a)

Figure 11.7
(a) Schematic diagram of a consolidometer;
(b) photograph of a consolidometer; (c) a consolidation test in progress (right-hand side) (*Courtesy of Braja M. Das, Henderson, Nevada*)


(b)


(c)

-The general shape of the plot of deformation against time for a given load increment during consolidation test.

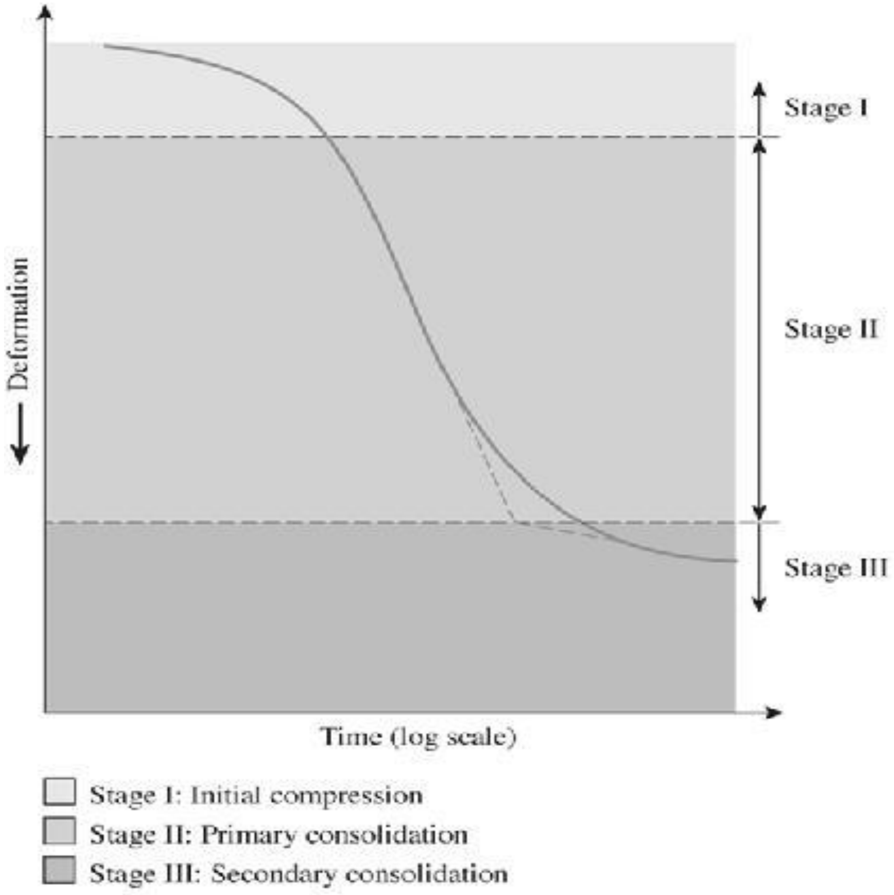
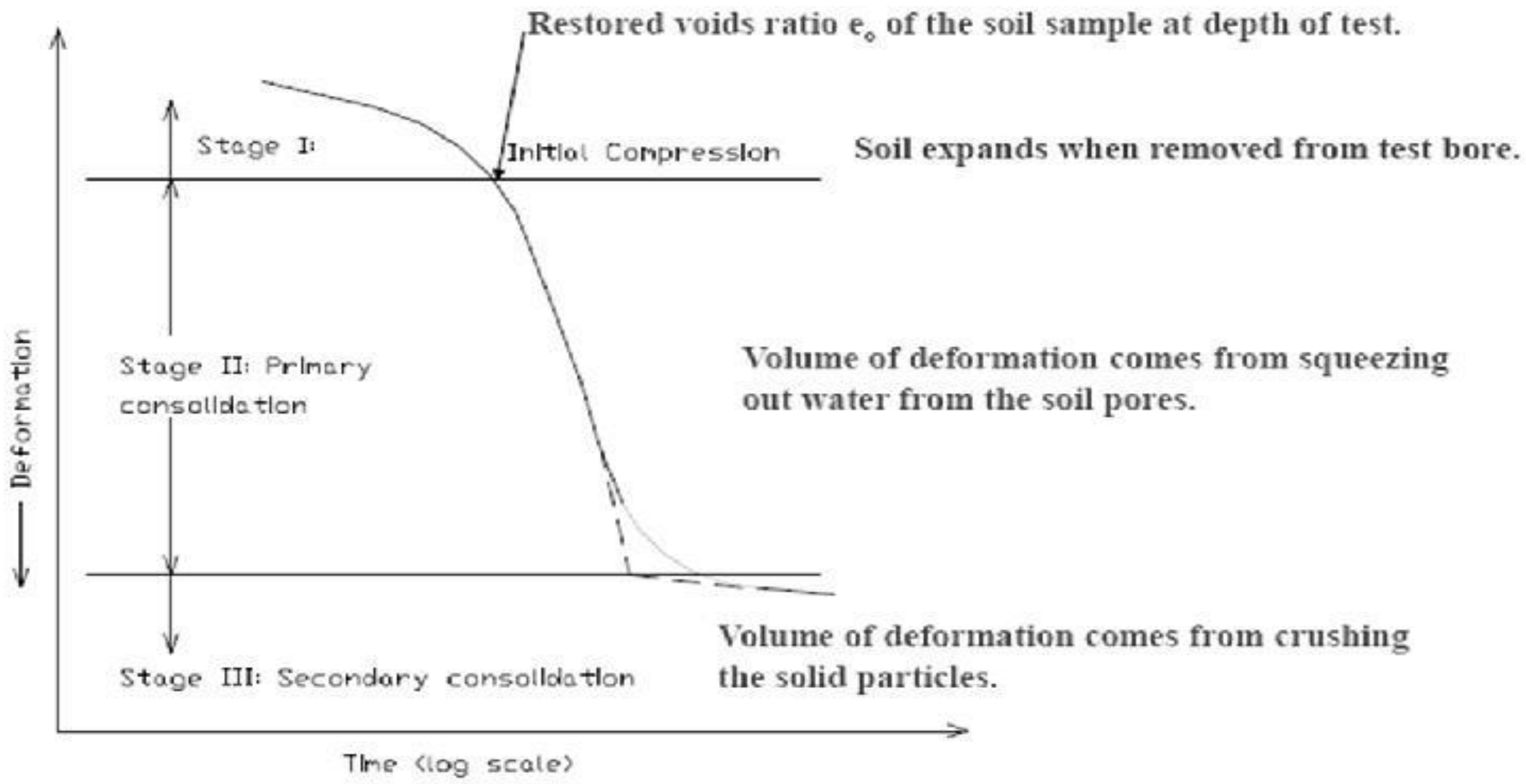


Figure 11.8
Time–deformation plot during consolidation for a given load increment



Time-deformation plot during consolidation for a given load increment

THE THREE STAGES DURING CONSOLIDATION TEST:

Stage I: Initial compression, which is caused mostly by preloading.

Stage II: Primary consolidation, during which excess pore water pressure gradually is transferred into effective stress because of the expulsion of pore water.

Stage III: Secondary consolidation, which occurs after complete dissipation of the excess porewater pressure, when some deformation of the specimen takes place because of the plastic readjustment of soil fabric.

• Void Ratio–Pressure Plots

•*step-by-step procedure for studying the change in the void ratio of the specimen with pressure:*

Step 1: Calculate the height of solids, H_s, in the soil specimen using the equation

$$H_s = \frac{W_s}{AG_s\gamma_w} = \frac{M_s}{AG_s\rho_w}$$

Step 2: Calculate the initial height of voids as

$$H_v = H - H_s$$

Step 3: Calculate the initial void ratio, e_O, of the specimen,

$$e_O = \frac{V_v}{V_s} = \frac{H_v}{H_s}\frac{A}{A} = \frac{H_v}{H_s}$$

Step 4: For the first incremental loading, σ_1 (total load/unit area of specimen), which causes a deformation ΔH_1, calculate the change in the void ratio as

$$\Delta e_1 = \frac{\Delta H_1}{H_s}$$

Step 5: Calculate the new void ratio after consolidation caused by the pressure increment as

$$e_1 = e_O - \Delta e_1$$

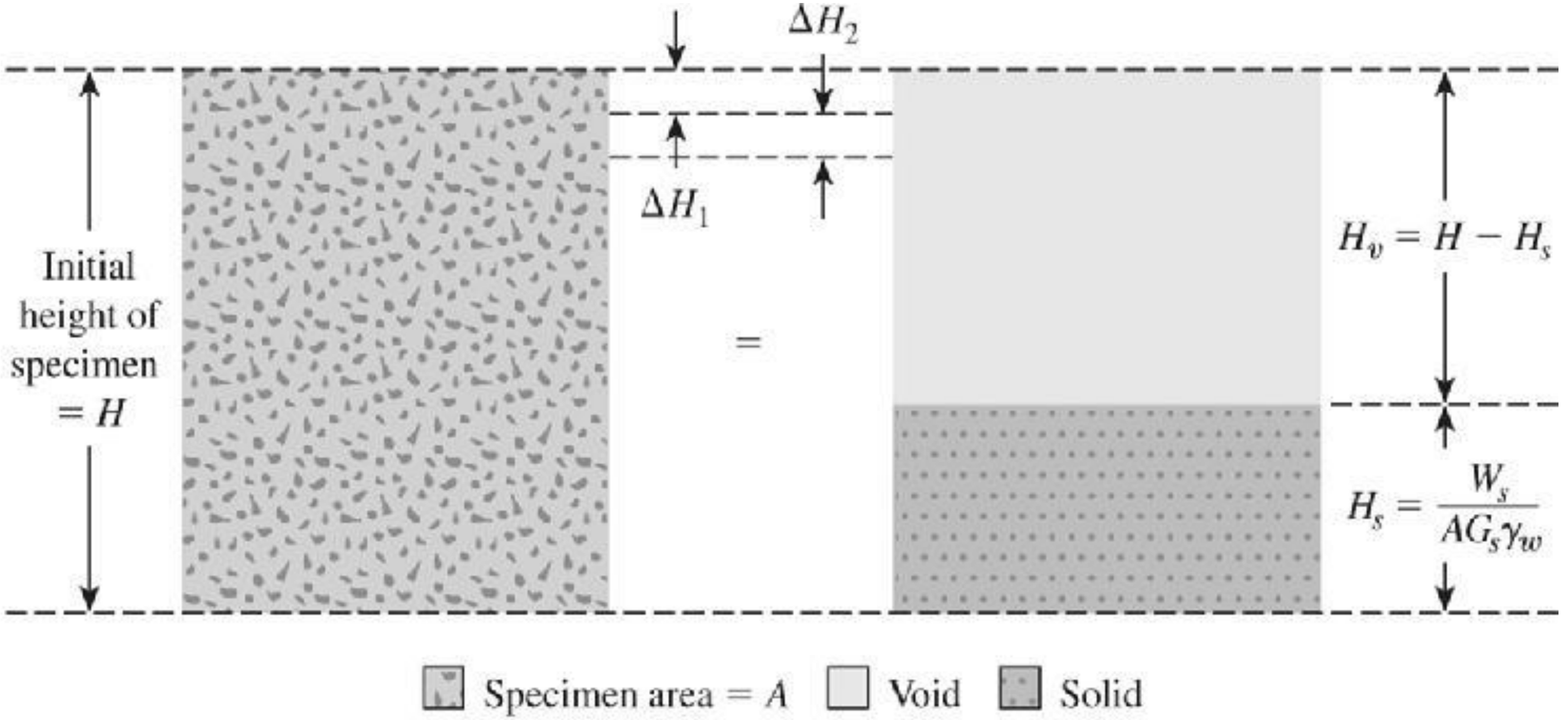


Figure 11.9 Change of height of specimen in one-dimensional consolidation test

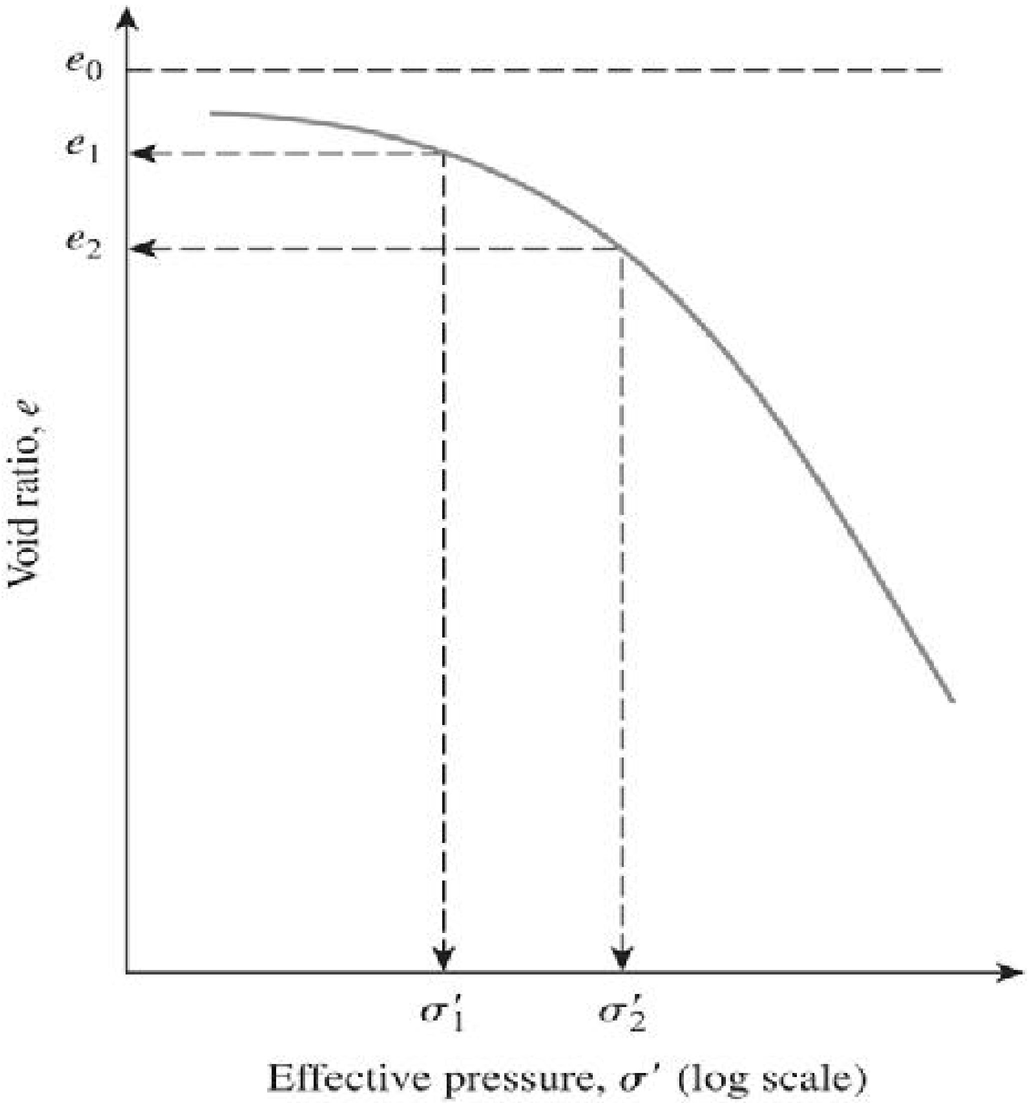


Figure 11.10 Typical plot of e against log σ'

• Normally Consolidated and Overconsolidated Clays

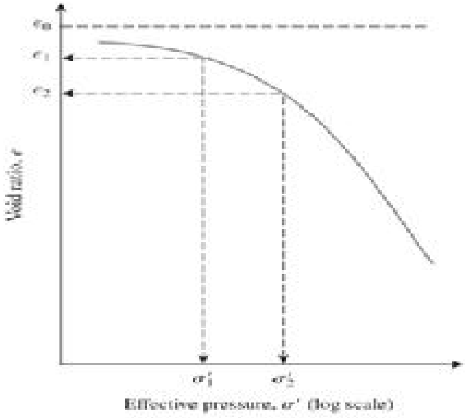


Figure 11.10 Typical plot of e against log σ'

•A soil in the field at some depth has been subjected to a certain maximum effective past pressure in its geologic history.

•This maximum effective past pressure may be equal to or less than the existing effective overburden pressure at the time of sampling.

•The reduction of effective pressure in the field may be caused by natural geologic processes or human processes.

•During the soil sampling, the existing effective overburden pressure is also released, which results in some expansion.

•When this specimen is subjected to a consolidation test, a small amount of compression (that is, a small change in void ratio) will occur when the effective pressure applied is less than the maximum effective overburden pressure in the field to which the soil has been subjected in the past.

•When the effective pressure on the specimen becomes greater than the maximum effective past pressure, the change in the void ratio is much larger, and the *e–log σ' relationship is practically* linear with a steeper slope.

•<u>This leads us to the two basic definitions of clay based on stress history: Normally Consolidated, and Overconsolidated</u>.

1. *Normally consolidated, whose present effective overburden pressure is the maximum* pressure that the soil was subjected to in the past.

2. *Overconsolidated, whose present effective overburden pressure is less than that* which the soil experienced in the past. The maximum effective past pressure is called the *preconsolidation pressure.*

3. *Under consolidated,* A soil deposit that has not consolidated under the present overburden pressure (effective stress) is called Under Consolidated Soil. These soils are susceptible to larger deformation and cause distress in buildings built on these deposits .

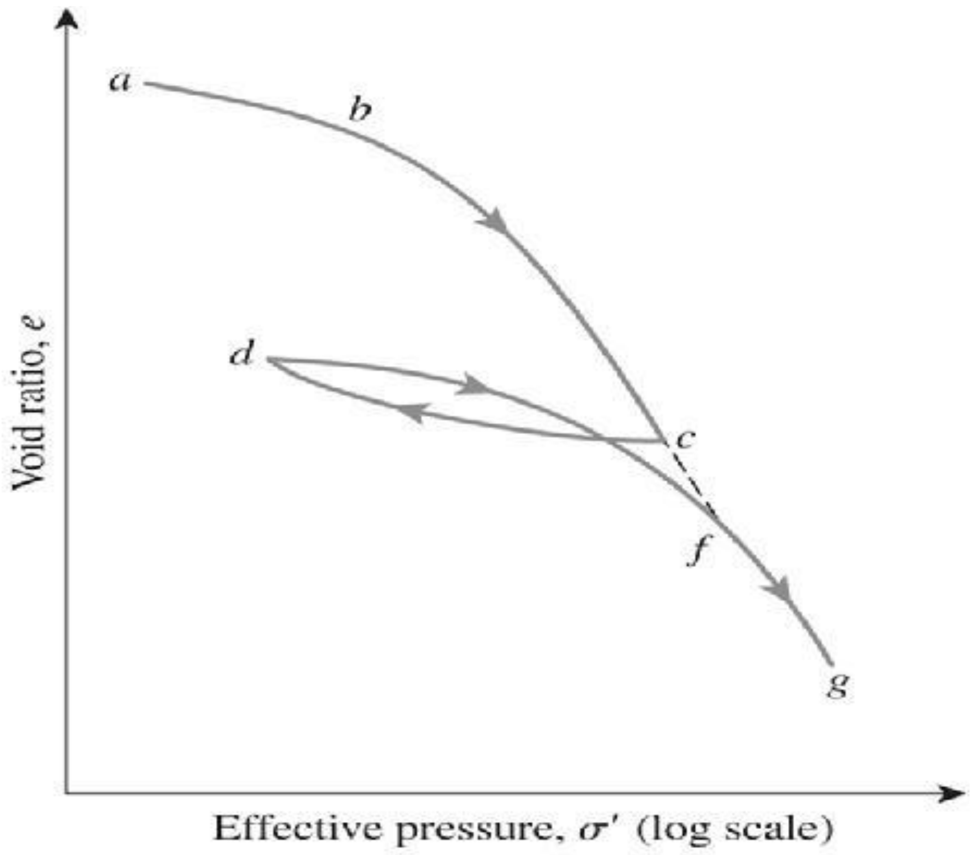


Figure 11.12 Plot of e against log σ' showing loading, unloading, and reloading branches

*Graphic construction to determine preconsolidation pressure ’σc *from the laboratory e–log ’σplot*
Casagrande (1936).

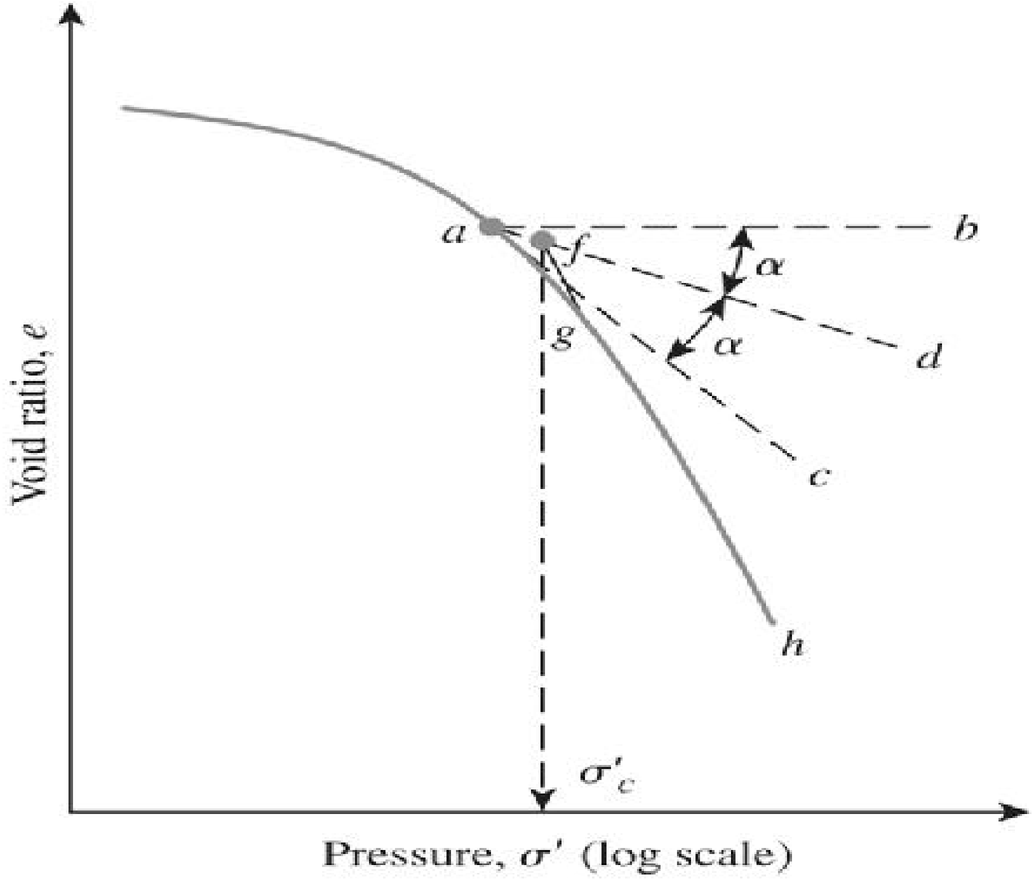


Figure 11.13 Graphic procedure for determining preconsolidation pressure

Compressibility of soil

PART III

The overconsolidation ratio (OCR) for a soil can now be defined as:

$$OCR = \frac{\sigma'_c}{\sigma'}$$

where σ'_c = preconsolidation pressure of a specimen
σ' = present effective vertical pressure

Empirical relationships are available to predict the preconsolidation pressure:

- Nagaraj and Murty (1985):

$$\log \sigma'_c = \frac{1.112 - \left(\frac{e_o}{e_L}\right) 0.0463 \sigma'_o}{0.188}$$

where e_o = *in situ* void ratio
e_L = void ratio at liquid limit = $\left[\frac{LL(\%)}{100}\right] G_s$
G_s = specific gravity of soil solids
σ'_o = *in situ* effective overburden pressure
(*Note*: σ'_c and σ'_o are in kN/m^2)

- Stas and Kulhawy (1984):

$$\frac{\sigma'_c}{p_a} = 10^{[1.11 - 1.62(LI)]}$$

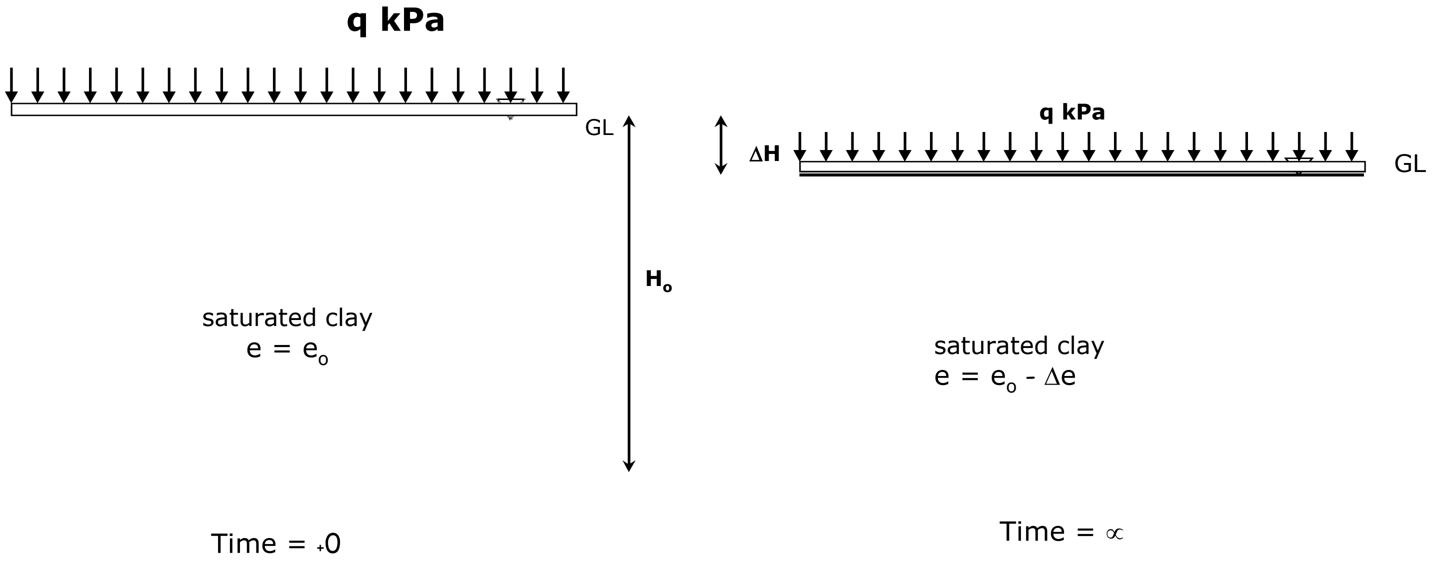
where p_a = atmospheric pressure ($\approx$100 kN/m^2)
LI = liquidity index

• Effect of Disturbance on Void Ratio–Pressure Relationship

A soil specimen will be remolded when it is subjected to some degree of disturbance. This remolding will result in some deviation of the *e–log σ' plot as observed in the laboratory* from the actual behavior in the field. The field *e–log σ' plot can be reconstructed* from the laboratory test results in the manner described by (Terzaghi and Peck, 1967).

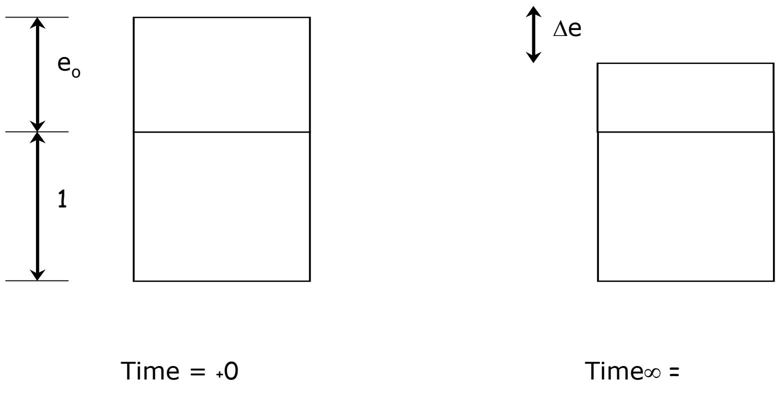
• Calculation of Settlement from One-Dimensional Primary Consolidation

average vertical strain
=



• Calculation of Settlement from One-Dimensional Primary Consolidation (continue…)

Consider an element where $V_s = 1$ initially.



so the average vertical strain=

• Calculation of Settlement from One-Dimensional Primary Consolidation (continue…)

Equating the two expressions for average vertical strain,

consolidation settlement

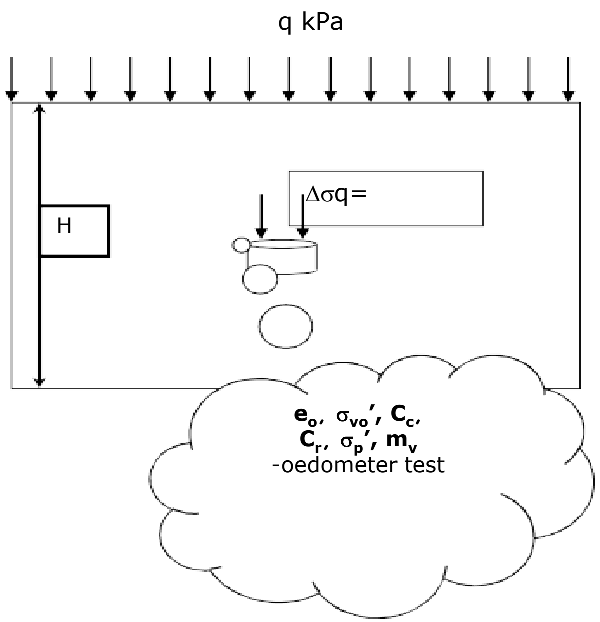
change in void ratio

initial thickness of clay layer

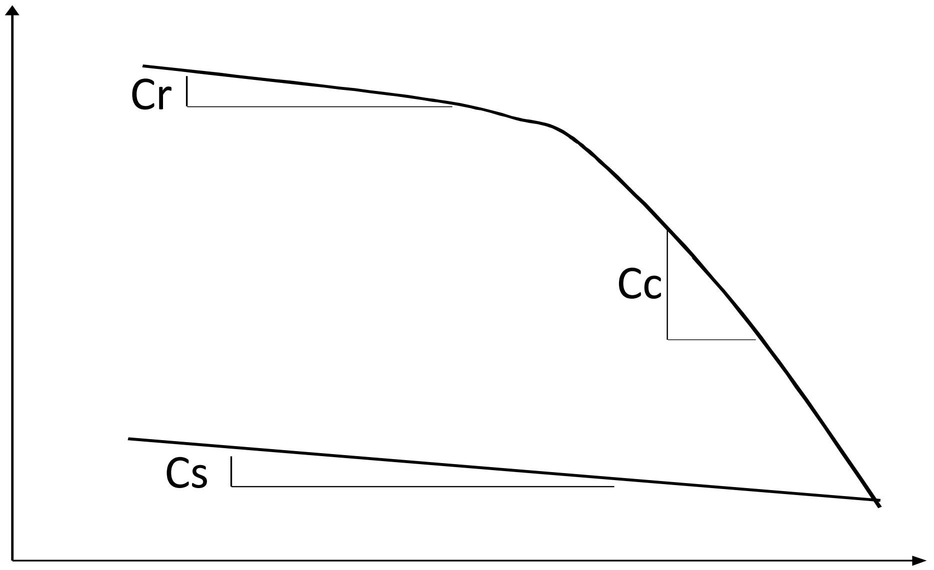
initial void ratio

•**Note That $\Delta H = S_c$**

(a) using e-log $\sigma_v$tolp ’

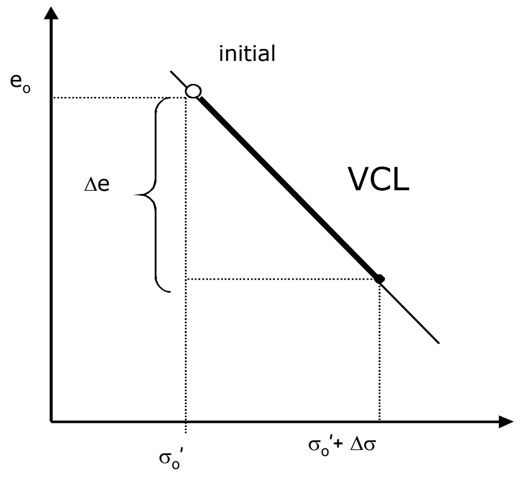


C_c ~ compression index
C_r ~ recompression index (or swelling index $\mathbf{C_s}$)



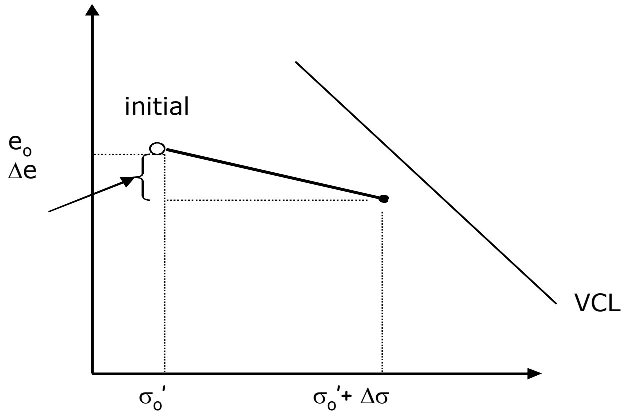
If the clay is normally consolidated,

the entire loading path is along the VCL.



If the clay is overconsolidated, and remains so by the end of consolidation,

$$\sigma'_O + \Delta\sigma' \le \sigma'_c,$$

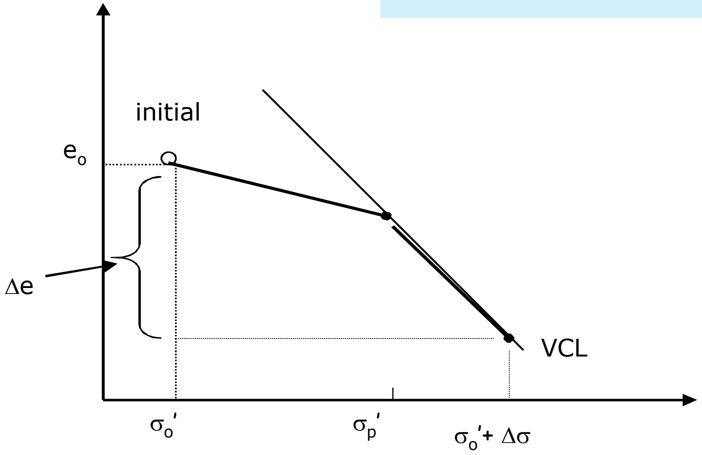


$$S_c = \frac{C_s H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$

If an overconsolidated clay becomes normally consolidated by the end of consolidation,

$$\sigma'_O + \Delta\sigma' > \sigma'_c:$$

$$S_c = \frac{C_s H}{1 + e_o} \log \frac{\sigma'_c}{\sigma'_o} + \frac{C_c H}{1 + e_o} \log \left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c} \right)$$



• Compression Index (Cc):

•The compression index for the calculation of field settlement caused by consolidation can be determined by graphic construction (as discussed previously) after one obtains the laboratory test results for void ratio and pressure.

Skempton (1944) suggested the following empirical expression for the compression index for undisturbed clays:

$$C_c = 0.009(LL - 10)$$

Rendon-Herrero (1983) gave the relationship for the compression index in the form

$$C_c = 0.141 G_s^{1.2} \left(\frac{1 + e_o}{G_s} \right)^{2.38}$$

Table 11.6 Correlations for Compression Index, C_c*

Equation	Reference	Region of applicability
$C_c = 0.007(LL - 7)$	Skempton (1944)	Remolded clays
$C_c = 0.01 w_N$		Chicago clays
$C_c = 1.15(e_O - 0.27)$	Nishida (1956)	All clays
$C_c = 0.30(e_O - 0.27)$	Hough (1957)	Inorganic cohesive soil: silt, silty clay, clay
$C_c = 0.0115 w_N$		Organic soils, peats, organic silt, and clay
$C_c = 0.0046(LL - 9)$		Brazilian clays
$C_c = 0.75(e_O - 0.5)$		Soils with low plasticity
$C_c = 0.208 e_O + 0.0083$		Chicago clays
$C_c = 0.156 e_O + 0.0107$		All clays

Table 11.7 Compression and Swell of Natural Soils

Soil	Liquid limit	Plastic limit	Compression index, C_c	Swell index, C_s
Boston blue clay	41	20	0.35	0.07
Chicago clay	60	20	0.4	0.07
Ft. Gordon clay, Georgia	51	26	0.12	—
New Orleans clay	80	25	0.3	0.05
Montana clay	60	28	0.21	0.05

Swell Index (Cs):

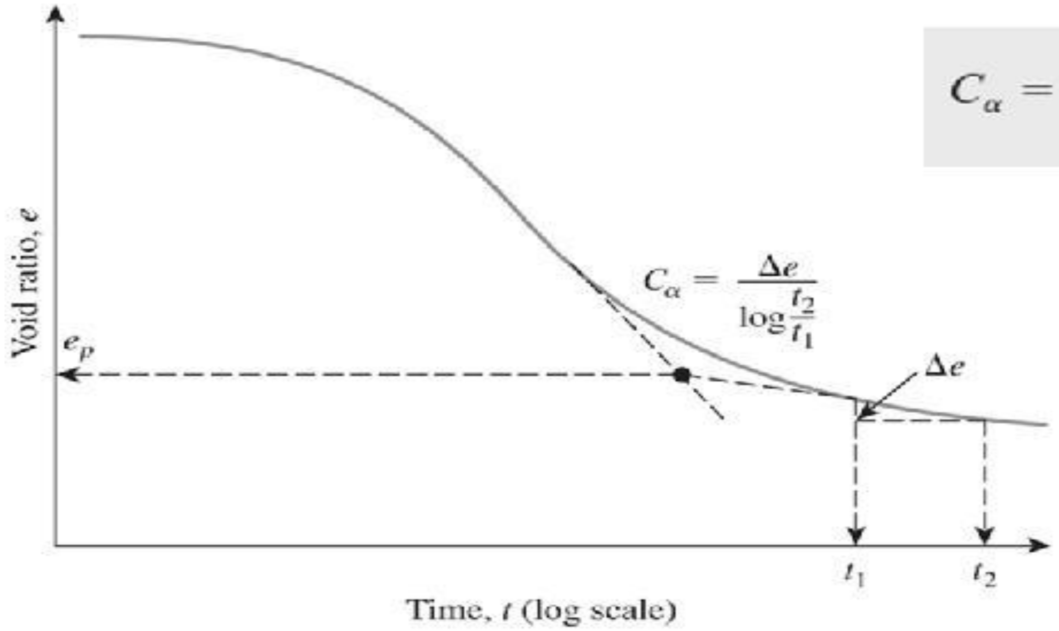
The swell index is appreciably smaller in magnitude than the compression index and generally can be determined from laboratory tests. In most cases,

$$C_s \simeq \frac{1}{5} \text{ to } \frac{1}{10} C_c$$

The swell index was expressed by Nagaraj and Murty (1985) as

$$C_s = 0.0463\left[\frac{LL(\%)}{100}\right]G_s$$

• **Secondary Consolidation Settlement:**



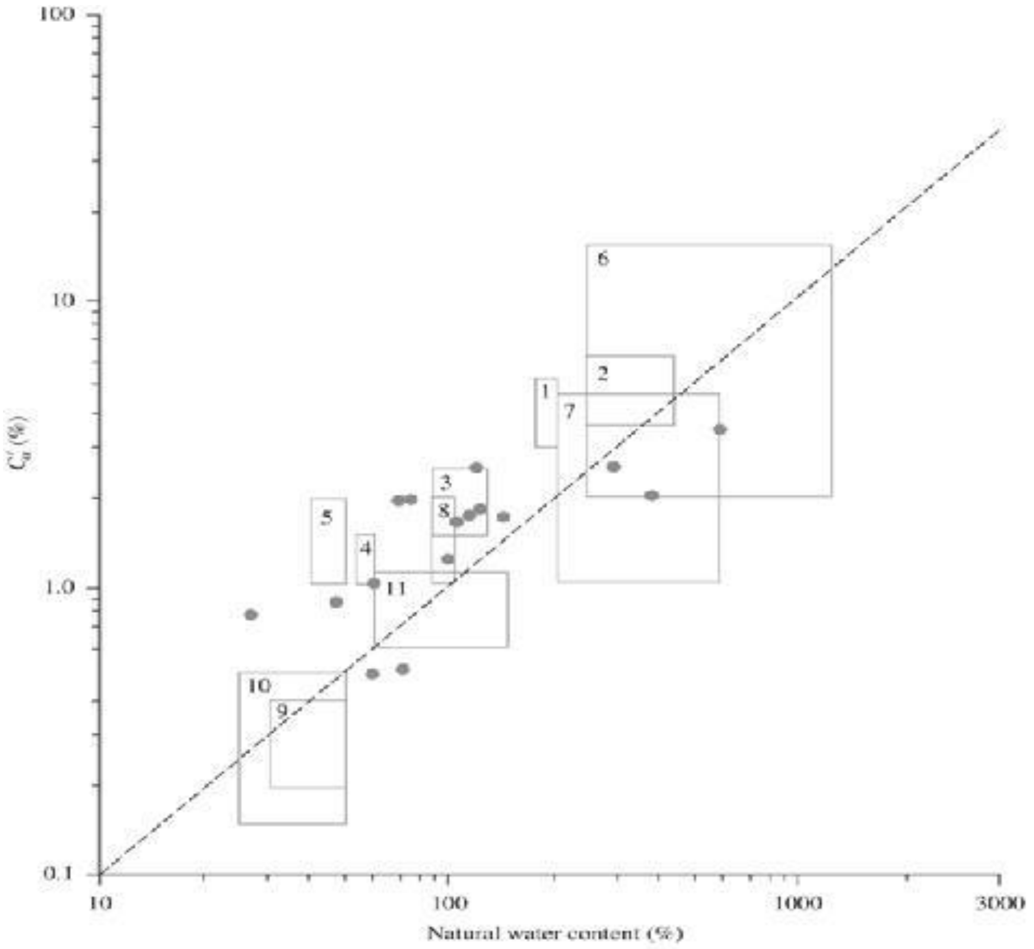
$$C_\alpha = \frac{\Delta e}{\log t_2 - \log t_1} = \frac{\Delta e}{\log(t_2/t_1)}$$

Figure 11.20 Variation of e with log t under a given load increment and definition of secondary consolidation index

$$S_s = C'_\alpha H \log\left(\frac{t_2}{t_1}\right)$$

$$C'_\alpha = \frac{C_\alpha}{1 + e_p}$$

e_p = void ratio at the end of primary consolidation



1—Whangamarino clay
2—Mexico City clay
3—Calcareous organic silt
4—Leda clay
5—Norwegian plastic clay
6—Amorphous and fibrous peat
7—Canadian muskeg
8—Organic marine deposits
9—Boston blue clay
10—Chicago blue clay
11—Organic silty clay
●—Organic silt, etc.

Figure 11.21 C'_α for natural soil deposits (*After Mesri, 1973. With permission from ASCE.*)

• **Time Rate of Consolidation:**

Terzaghi (1925) proposed the first theory to consider the rate of one-dimensional consolidation for saturated clay soils.

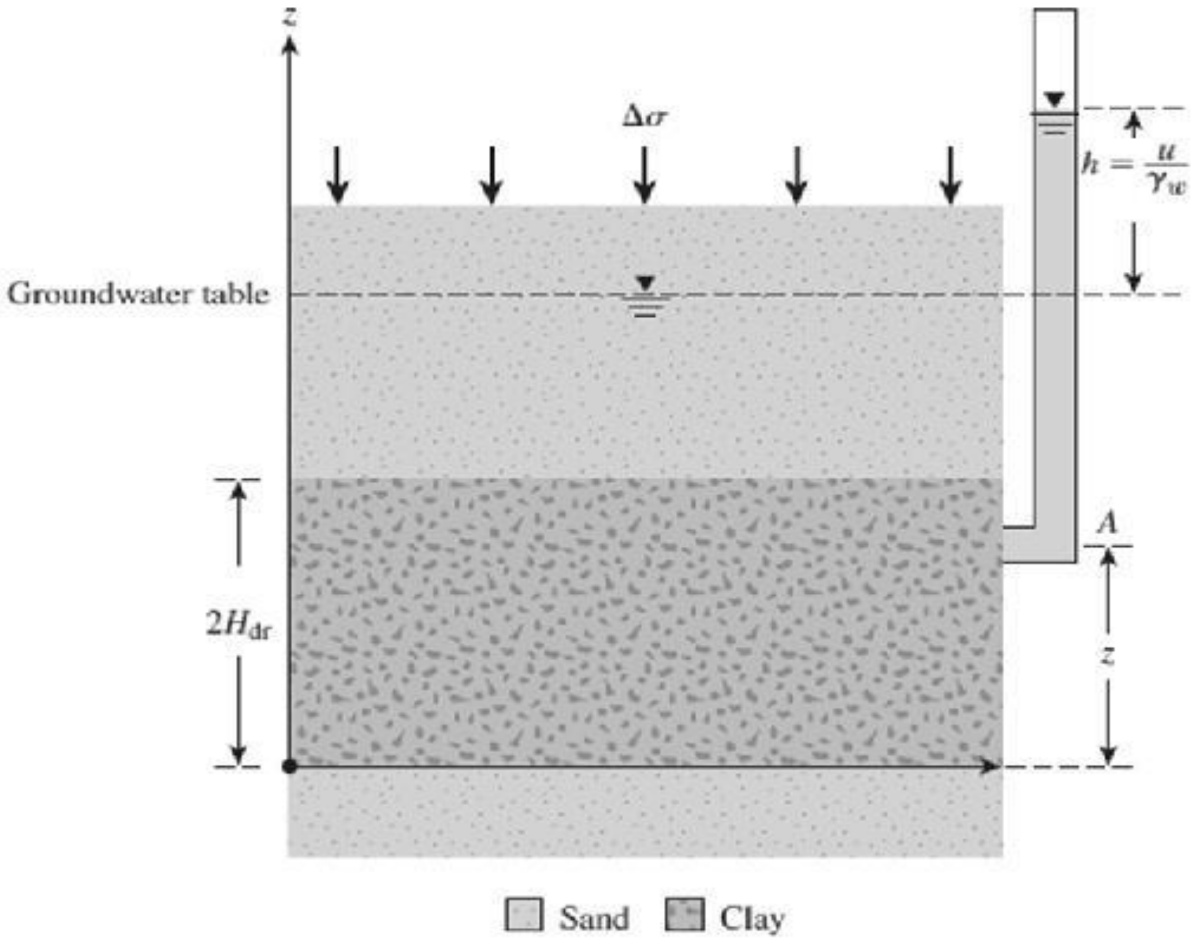
Mathematical derivation assumptions:

- The clay–water system is homogeneous.
- Saturation is complete.
- Compressibility of water is negligible.
- Compressibility of soil grains is negligible.
- The flow of water is in one direction only.
- Darcy's law is valid.

• Time Rate of Consolidation:

Consider a layer of clay of thickness $2H_{dr}$

(H_{dr} :length of maximum drainage path) located between two highly permeable sand layers and is subjected to an increased pressure.



Rate of outflow of water − Rate of inflow of water = Rate of volume change

Thus, $$\left(v_z + \frac{\partial v_z}{\partial z} dz\right) dx\, dy - v_z\, dx\, dy = \frac{\partial V}{\partial t}$$

where V = volume of the soil element
v_z = velocity of flow in z direction

$$\frac{\partial v_z}{\partial z} dx\, dy\, dz = \frac{\partial V}{\partial t} \quad \text{(a)}$$

Using Darcy's law, we have

$$v_z = ki = -k\frac{\partial h}{\partial z} = -\frac{k}{\gamma_w}\frac{\partial u}{\partial z} \quad \text{(b)}$$

u = excess pore water pressure caused by the increase of stress.

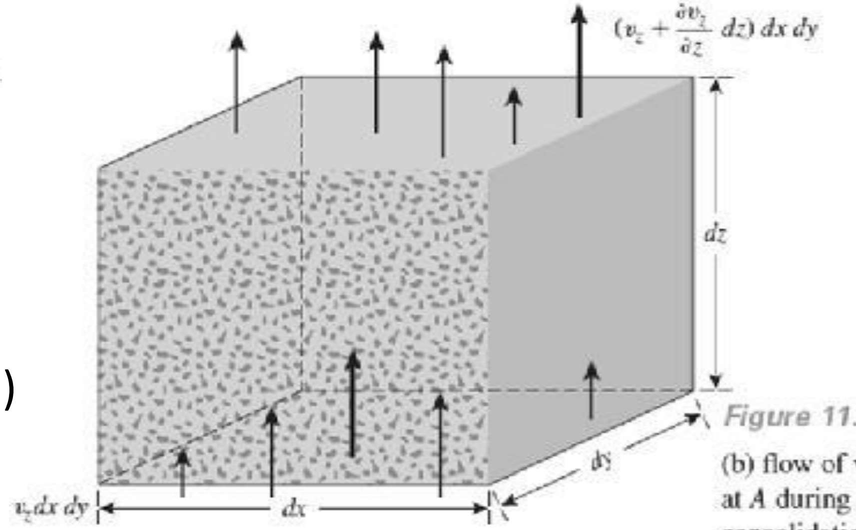


Figure 11.22
(b) flow of water at A during consolidation

From (a) & (b) $$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{dx\, dy\, dz}\frac{\partial V}{\partial t}$$

•During consolidation, the rate of change in the volume of the soil element is equal to the rate of change in the volume of voids. Thus,

$$\frac{\partial V}{\partial t} = \frac{\partial V_v}{\partial t} = \frac{\partial (V_s + eV_s)}{\partial t} = \frac{\partial V_s}{\partial t} + V_s\frac{\partial e}{\partial t} + e\frac{\partial V_s}{\partial t} \qquad (c)$$

V_s = volume of soil solids
V_v = volume of voids

•Assuming that soil solids are incompressible:

$$\frac{\partial V_s}{\partial t} = 0 \quad \text{and} \quad V_s = \frac{V}{1 + e_o} = \frac{dx\,dy\,dz}{1 + e_o}$$ *Sub. Into eq. (c)*

$$\frac{\partial V}{\partial t} = \frac{dx\,dy\,dz}{1 + e_o}\frac{\partial e}{\partial t}$$)d) *Combining eqs. (c) and (b…(*

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{1 + e_o}\frac{\partial e}{\partial t}$$

The change in the void ratio is caused by the increase of effective stress (i.e., a decrease of excess pore water pressure). Assuming that they are related linearly, we have:

$$\partial e = a_v \partial(\Delta\sigma') = -a_v \partial u$$

where $\partial(\Delta\sigma')$ = change in effective pressure

a_v = coefficient of compressibility (a_v can be considered constant for a narrow range of pressure increase)

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = -\frac{a_v}{1 + e_o}\frac{\partial u}{\partial t} = -m_v\frac{\partial u}{\partial t}$$

By combining eqs. (d) and (e)

m_v = coefficient of volume compressibility $= a_v/(1 + e_o)$

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

*Basic differential equation of Terzaghi's consolidation theory (important)

where

c_v = coefficient of consolidation $= k/(\gamma_w m_v)$ Thus, $c_v = \dfrac{k}{\gamma_w m_v} = \dfrac{k}{\gamma_w\left(\dfrac{a_v}{1 + e_o}\right)}$

•Solution of Terzaghi’s basic differential equation of consolidation theory:

•Boundary conditions:

$z = 0, \quad u = 0 \qquad z = 2H_{dr}, \quad u = 0$

$t = 0, \quad u = u_O$

The solution yields

$$u = \sum_{m=0}^{m=\infty} \left[\frac{2u_o}{M} \sin\left(\frac{Mz}{H_{dr}} \right) \right] e^{-M^2 T_v}$$

where m = an integer
$M = (\pi/2)(2m + 1)$
u_O = initial excess pore water pressure

$$T_v = \frac{c_v t}{H_{dr}^2} = \text{time factor}$$

•***Important***

Consolidation progresses by the dissipation of excess pore water pressure, thus, degree of consolidation at a distance *z at any time t is:*

$$U_z = \frac{u_o - u_z}{u_o} = 1 - \frac{u_z}{u_o}$$

•***Important***

*To obtain the variation of degree of consolidation at any depth *z. the following figure can also be provided.*

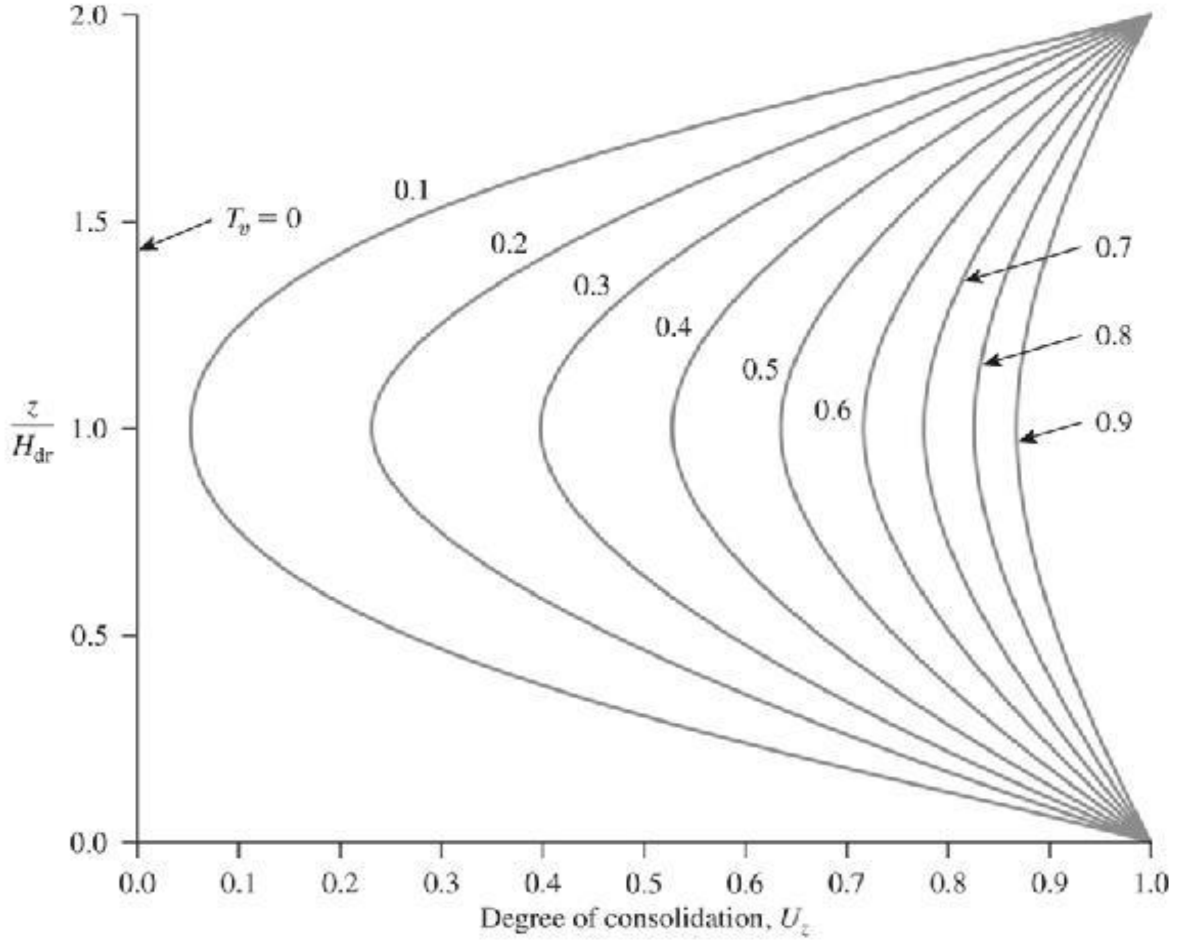


Figure 11.23
Variation of U_z with T_v and z/H_{dr}

$$U = \frac{S_{c(t)}}{S_c} = 1 - \frac{\left(\frac{1}{2H_{dr}}\right)\int_0^{2H_{dr}} u_z\,dz}{u_o}$$

Average degree of consolidation

(*important)

-The values of the time factor and their corresponding average degrees of consolidation may be approximated by the following simple relationship:

For $U = 0$ to 60%, $T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2$

For $U > 60\%$, $T_v = 1.781 - 0.933\log(100 - U\%)$

*Also, see the next figure and table .(**Important…)**

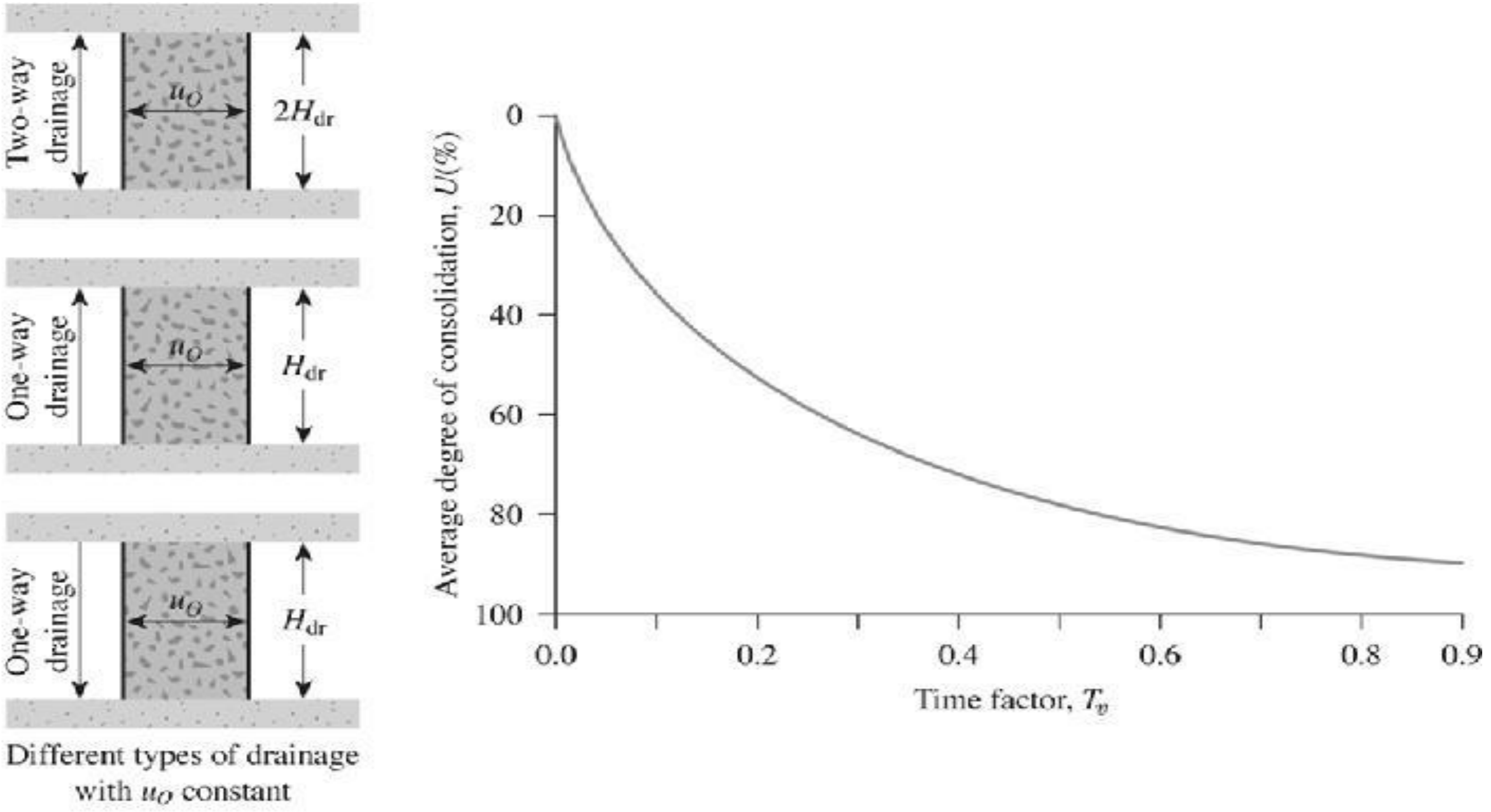


Figure 11.24 Variation of average degree of consolidation with time factor, T_v (u_O constant with depth)

Important…

Important...

Table 11.8 Variation of T_v with U

U (%)	T_v	U (%)	T_v	U (%)	T_v	U (%)	T_v
0	0	26	0.0531	52	0.212	78	0.529
1	0.00008	27	0.0572	53	0.221	79	0.547
2	0.0003	28	0.0615	54	0.230	80	0.567
3	0.00071	29	0.0660	55	0.239	81	0.588
4	0.00126	30	0.0707	56	0.248	82	0.610
5	0.00196	31	0.0754	57	0.257	83	0.633
6	0.00283	32	0.0803	58	0.267	84	0.658
7	0.00385	33	0.0855	59	0.276	85	0.684
8	0.00502	34	0.0907	60	0.286	86	0.712
9	0.00636	35	0.0962	61	0.297	87	0.742
10	0.00785	36	0.102	62	0.307	88	0.774
11	0.0095	37	0.107	63	0.318	89	0.809
12	0.0113	38	0.113	64	0.329	90	0.848
13	0.0133	39	0.119	65	0.304	91	0.891
14	0.0154	40	0.126	66	0.352	92	0.938
15	0.0177	41	0.132	67	0.364	93	0.993
16	0.0201	42	0.138	68	0.377	94	1.055
17	0.0227	43	0.145	69	0.390	95	1.129
18	0.0254	44	0.152	70	0.403	96	1.219
19	0.0283	45	0.159	71	0.417	97	1.336
20	0.0314	46	0.166	72	0.431	98	1.500
21	0.0346	47	0.173	73	0.446	99	1.781
22	0.0380	48	0.181	74	0.461	100	∞
23	0.0415	49	0.188	75	0.477		
24	0.0452	50	0.197	76	0.493		
25	0.0491	51	0.204	77	0.511		

Compressibility of soil

PART IV

*To obtain the variation of degree of consolidation at any depth *z. the following figure can also be provided.*

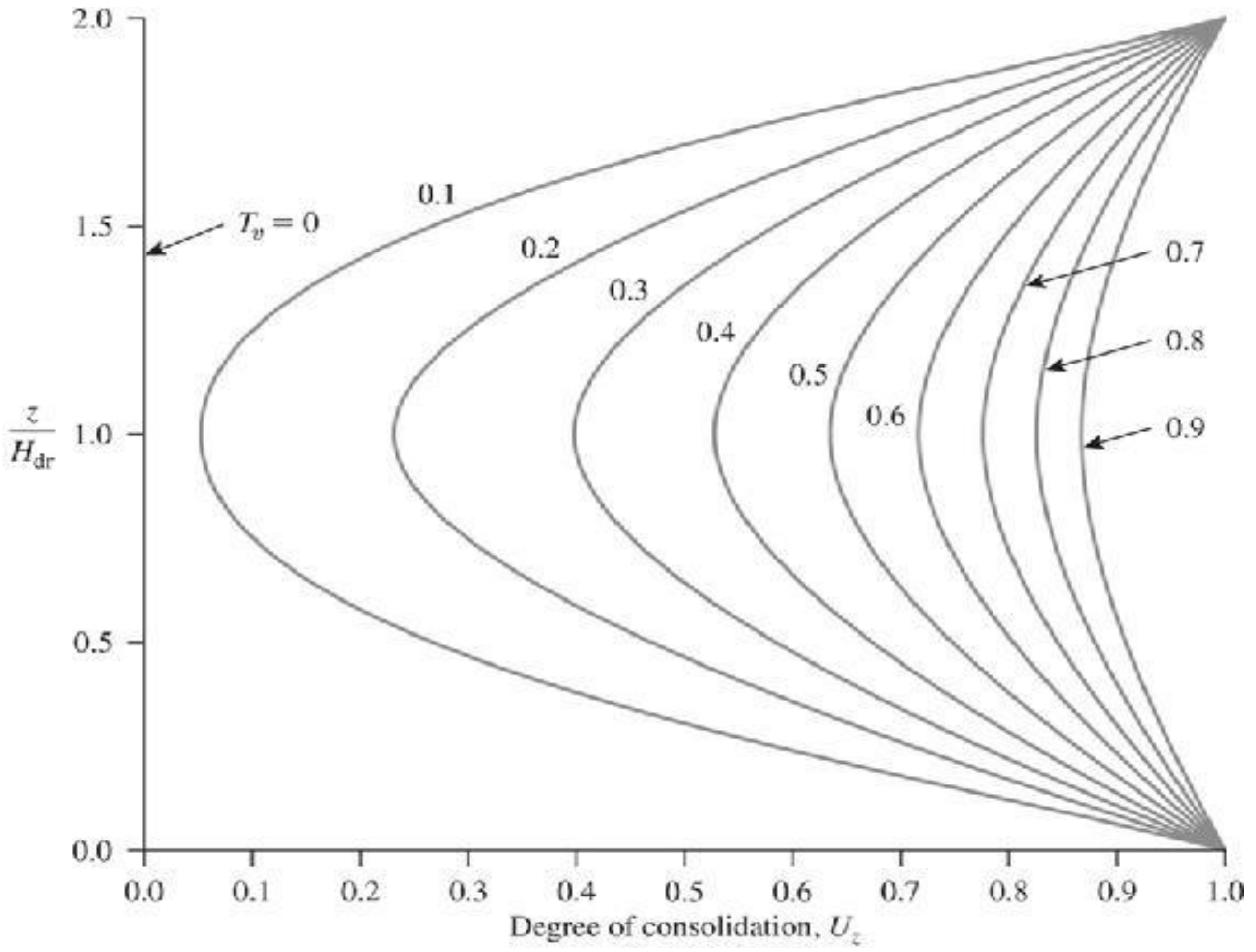


Figure 11.23
Variation of U_z with T_v and z/H_{dr}

$$U = \frac{S_{c(t)}}{S_c} = 1 - \frac{\left(\frac{1}{2H_{dr}}\right)\int_0^{2H_{dr}} u_z\,dz}{u_o}$$

<u>Average degree of consolidation</u>

<u>(*important)</u>

-The values of the time factor and their corresponding average degrees of consolidation may be approximated by the following simple relationship:

$$\text{For } U = 0 \text{ to } 60\%, \quad T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2$$

$$\text{For } U > 60\%, \quad T_v = 1.781 - 0.933\log(100 - U\%)$$

*Also, see the next figure and table .(**Important…)**

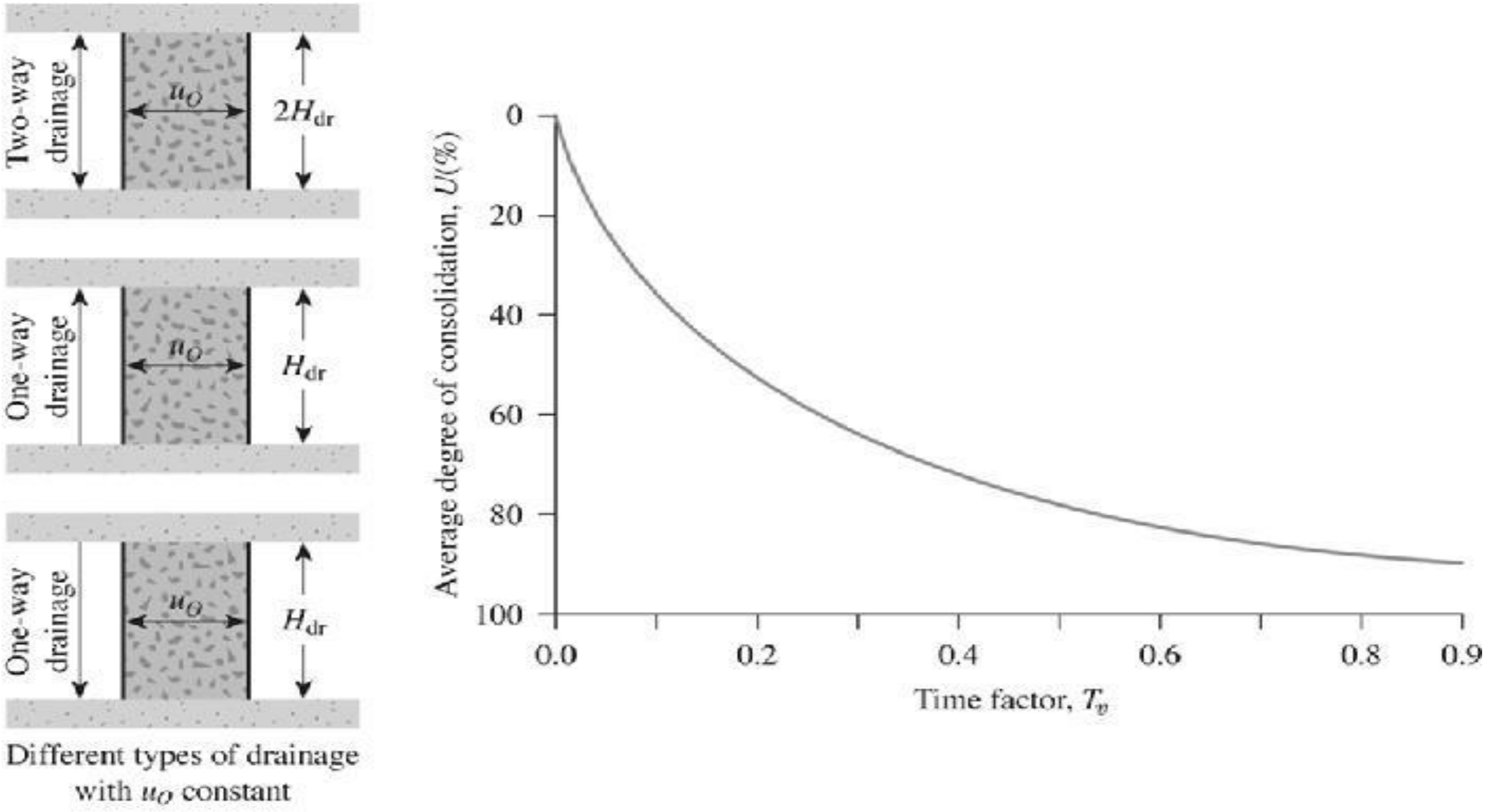


Figure 11.24 Variation of average degree of consolidation with time factor, T_v (u_O constant with depth)

***Important…**

Table 11.8 Variation of T_v with U

U (%)	T_v	U (%)	T_v	U (%)	T_v	U (%)	T_v
0	0	26	0.0531	52	0.212	78	0.529
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23	0.0415	49	0.188	75	0.477		
24	0.0452	50	0.197	76	0.493		
25	0.0491	51	0.204	77	0.511		

Example 11.7

The time required for 50% consolidation of a 25-mm-thick clay layer (drained at both top and bottom) in the laboratory is 2 min. 20 sec. How long (in days) will it take for a 3-m-thick clay layer of the same clay in the field under the same pressure increment to reach 50% consolidation? In the field, there is a rock layer at the bottom of the clay.

Solution

$$T_{50} = \frac{c_v t_{\text{lab}}}{H^2_{\text{dr(lab)}}} = \frac{c_v t_{\text{field}}}{H^2_{\text{dr(field)}}}$$

or

$$\frac{t_{\text{lab}}}{H^2_{\text{dr(lab)}}} = \frac{t_{\text{field}}}{H^2_{\text{dr(field)}}}$$

$$\frac{140\,\text{sec}}{\left(\dfrac{0.025\,\text{m}}{2}\right)^2} = \frac{t_{\text{field}}}{(3\,\text{m})^2}$$

$$t_{\text{field}} = 8{,}064{,}000\,\text{sec} = \mathbf{93.33\ days}$$

Example 11.9

A 3-m-thick layer (double drainage) of saturated clay under a surcharge loading underwent 90% primary consolidation in 75 days. Find the coefficient of consolidation of clay for the pressure range.

Solution

$$T_{90} = \frac{c_v t_{90}}{H_{dr}^2}$$

Because the clay layer has two-way drainage, $H_{dr} = 3 \text{ m}/2 = 1.5$ m. Also, $T_{90} = 0.848$ (see Table 11.8). So,

$$0.848 = \frac{c_v(75 \times 24 \times 60 \times 60)}{(1.5 \times 100)^2}$$

$$c_v = \frac{0.848 \times 2.25 \times 10^4}{75 \times 24 \times 60 \times 60} = \mathbf{0.00294\ cm^2/sec}$$

■

Example 11.10

For a normally consolidated laboratory clay specimen drained on both sides, the following are given:

- $\sigma'_O = 3000 \text{ lb/ft}^2$, $e = e_O = 1.1$
- $\sigma'_O + \Delta\sigma' = 6000 \text{ lb/ft}^2$, $e = 0.9$
- Thickness of clay specimen = 1 in.
- Time for 50% consolidation = 2 min

a. Determine the hydraulic conductivity (ft/min) of the clay for the loading range.

b. How long (in days) will it take for a 6-ft clay layer in the field (drained on one side) to reach 60% consolidation?

Solution

Part a

The coefficient of compressibility is

$$m_v = \frac{a_v}{1 + e_{av}} = \frac{\left(\dfrac{\Delta e}{\Delta\sigma'}\right)}{1 + e_{av}}$$

$$\Delta e = 1.1 - 0.9 = 0.2$$

$$\Delta\sigma' = 6000 - 3000 = 3000 \text{ lb/ft}^2$$

$$e_{av} = \frac{1.1 + 0.9}{2} = 1.0$$

So

$$m_v = \frac{\frac{0.2}{3000}}{1 + 1.0} = 3.33 \times 10^{-5} \text{ft}^2/\text{lb}$$

From Table 11.8, for $U = 50\%$, $T_v = 0.197$; thus,

$$c_v = \frac{(0.197)\left(\frac{1}{2 \times 12}\right)^2}{2} = 1.71 \times 10^{-4} \text{ft}^2/\text{min}$$

$$k = c_v m_v \gamma_w = (1.71 \times 10^{-4} \text{ft}^2/\text{min})(3.33 \times 10^{-5} \text{ft}^2/\text{lb})(62.4 \text{ lb/ft}^3)$$

$$= \mathbf{3.55 \times 10^{-7} \text{ ft/min}}$$

Part b

$$T_{60} = \frac{c_v t_{60}}{H_{dr}^2}$$

$$t_{60} = \frac{T_{60} H_{dr}^2}{c_v}$$

From Table 11.8, for $U = 60\%$, $T_v = 0.286$,

$$t_{60} = \frac{(0.286)(6)^2}{1.71 \times 10^{-4}} = 60{,}211 \text{ min} = \mathbf{41.8 \text{ days}}$$

❑ Coefficient of Consolidation:

- ❖ The coefficient of consolidation c_v *generally decreases as the liquid limit of soil increases.* The range of variation of c_v *for a given liquid limit of soil is wide.*

- ❖ For a given load increment on a specimen, two graphical methods commonly are used for determining c_v *from laboratory one-dimensional consolidation tests.*

- ➢ *The first is the logarithm- of-time method proposed by Casagrande and Fadum (1940),*

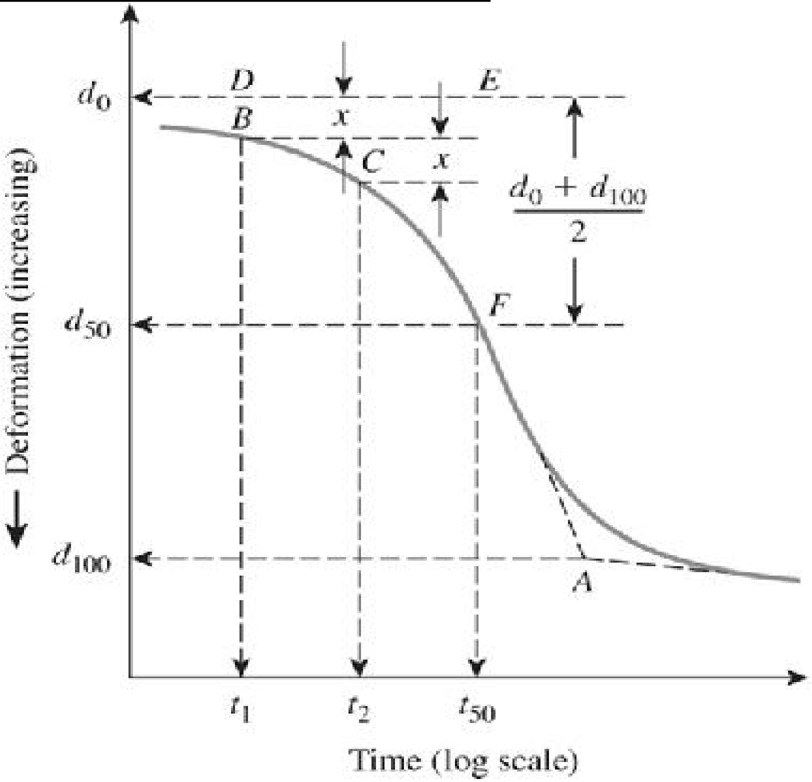
- ➢ *The other is the square-root-of-time method given by Taylor (1942).*

- ❖ *More recently, at least two other methods* were proposed. They are
- ➢ the *hyperbola method (Sridharan and Prakash, 1985) and*
- ➢ *the early stage log-t method (Robinson and Allam, 1996).*

Only the first two methods will be presented here…

• Coefficient of Consolidation

)logarithm- of-time method)



For 50% average degree of consolidation, $T_v = 0.197$

$$c_v = \frac{0.197 H_{dr}{}^2}{t_{50}} \qquad T_{50} = \frac{c_v t_{50}}{H_{dr}^2}$$

Figure 11.25
Logarithm-of-time method for determining coefficient of consolidation

• Coefficient of Consolidation

)square-root-of-time method)

For 90% consolidation,
$T_{90} = 0.848$

$$T_{90} = 0.848 = \frac{c_v t_{90}}{H_{dr}^2}$$

$$c_v = \frac{0.848 H_{dr}^2}{t_{90}}$$

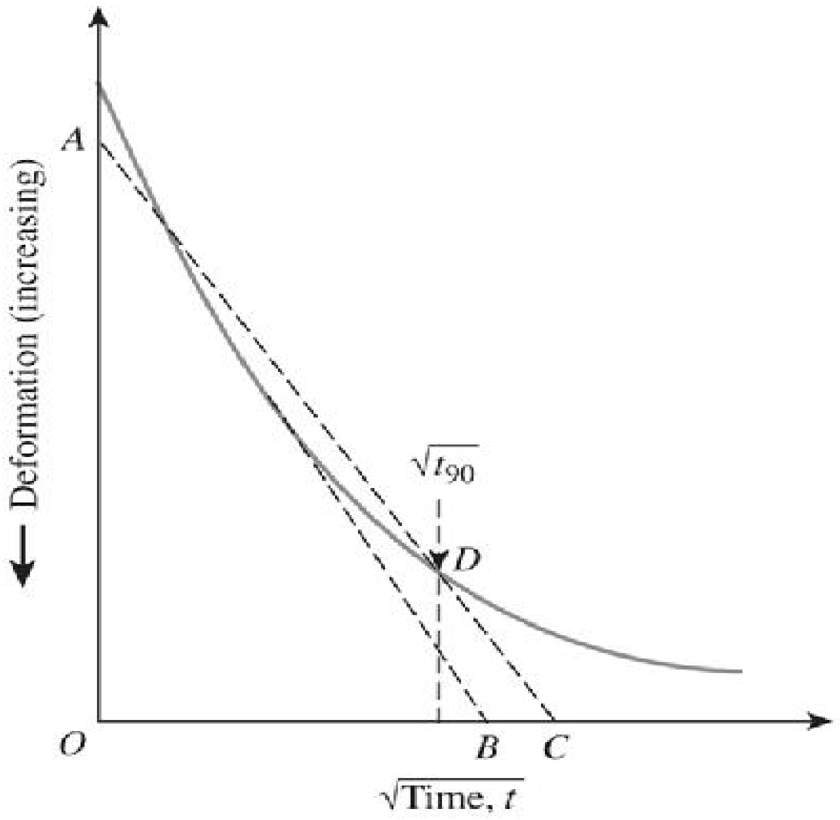


Figure 11.26 Square-root-of-time fitting method

Example 11.11

During a laboratory consolidation test, the time and dial gauge readings obtained from an increase of pressure on the specimen from 50 kN/m^2 to 100 kN/m^2 are given here.

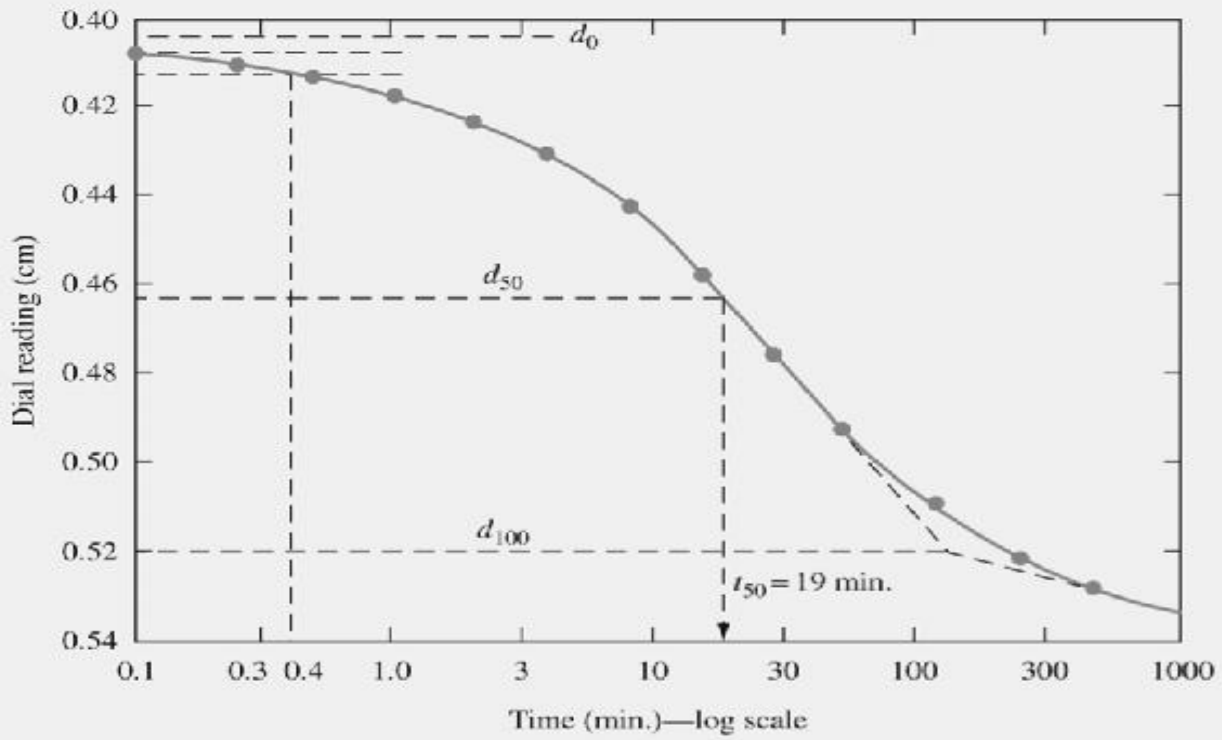
Time (min)	Dial gauge reading (cm × 10^4)	Time (min)	Dial gauge reading (cm × 10^4)
0	3975	16.0	4572
0.1	4082	30.0	4737
0.25	4102	60.0	4923
0.5	4128	120.0	5080
1.0	4166	240.0	5207
2.0	4224	480.0	5283
4.0	4298	960.0	5334
8.0	4420	1440.0	5364

Using the logarithm-of-time method, determine c_v. The average height of the specimen during consolidation was 2.24 cm, and it was drained at the top and bottom.

Solution

The semi-logarithmic plot of dial reading versus time is shown in Figure 11.29. For this, $t_1 = 0.1$ min, $t_2 = 0.4$ min to determine d_0. Following the procedure outlined in Figure 11.25, $t_{50} \approx 19$ min. From Eq. (11.66)

$$C_v = \frac{0.197 H_{dr}^2}{t_{50}} = \frac{0.197\left(\frac{2.24}{2}\right)^2}{19} = 0.013 \text{cm}^2/\text{min} = \mathbf{2.17 \times 10^{-4} cm^2/sec}$$



0.40
0.42
0.44
0.46
0.48
0.50
0.52
0.54
Dial reading (cm)
d_0
d_{50}
d_{100}
$t_{50} = 19$ min.
0.1
0.3
0.4
1.0
3
10
30
100
300
1000
Time (min.)—log scale

Figure 11.29

• Calculation of Consolidation Settlement Under a Foundation

Assuming that the pressure increase varies parabolically, using Simpson's rule, we can estimate the value of ∆:sa 'σ

$$\Delta\sigma'_{av} = \frac{\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b}{6}$$

where $\Delta\sigma'_t$, $\Delta\sigma'_m$, and $\Delta\sigma'_b$ represent the increase in the effective pressure at the top, middle, and bottom of the layer, respectively.

Also, we can use the 2:1 method or other method stated in Chapter10 for estimating the average stress increase.

For the estimation of settlement; we can divide the clay layer into sub- layers and estimate the consolidation settlement for the hole layer by summing up each sub-layer settlement to get more accurate estimation for the settlement.

Example 11.12

Calculate the settlement of the 10-ft-thick clay layer (Figure 11.30) that will result from the load carried by a 5-ft-square footing. The clay is normally consolidated. Use the weighted average method [Eq. (11.70)] to calculate the average increase of effective pressure in the clay layer.

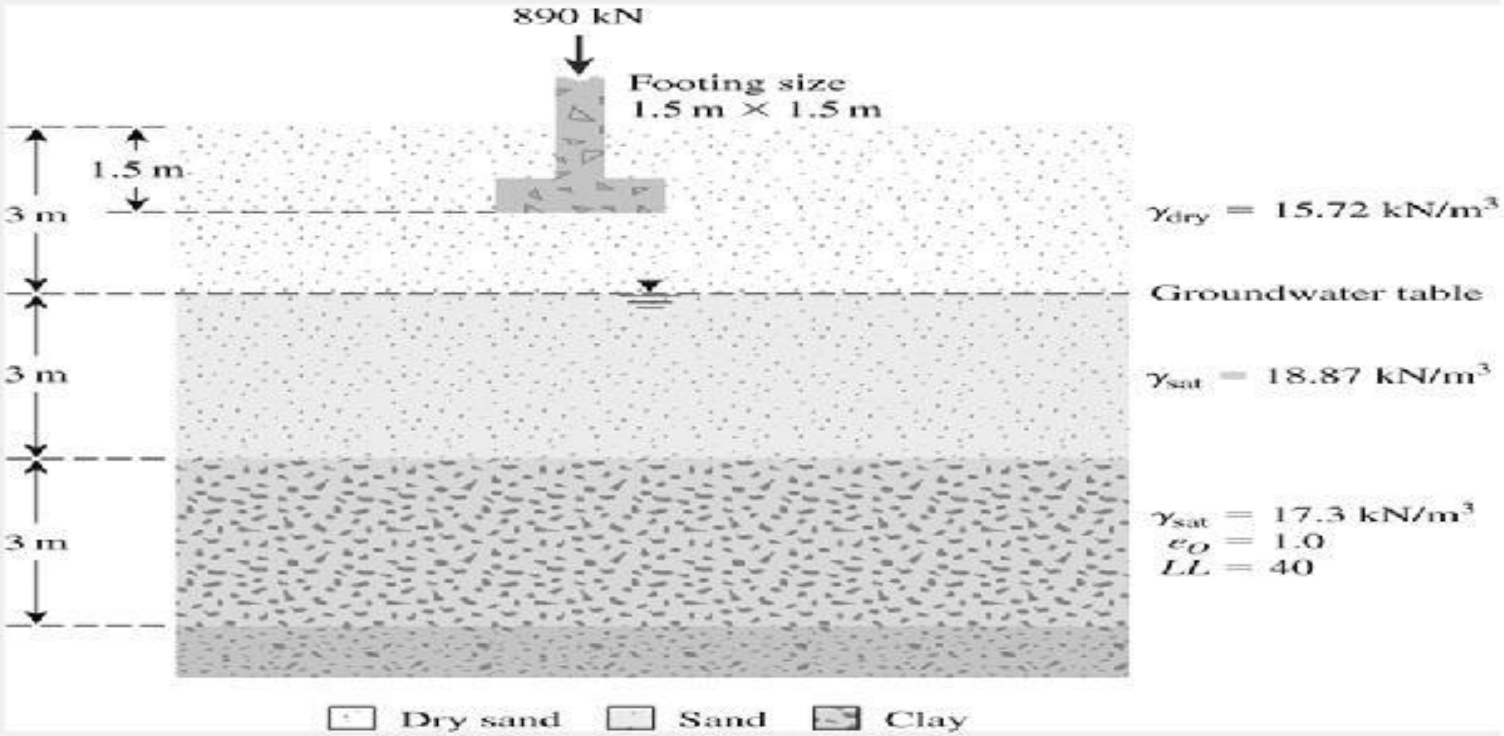


Figure 11.30

Solution

For normally consolidated clay, from Eq. (11.31),

$$S_e = \frac{C_c H}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o}$$

where

$$C_c = 0.009(LL - 10) = 0.009(40 - 10) = 0.27$$

$$H = 10 \times 12 = 120 \text{ in.}$$

$$e_o = 1.0$$

$$\sigma'_O = 10 \text{ ft} \times \gamma_{dry(sand)} + 10 \text{ ft}[\gamma_{sat(sand)} - 62.4] + \frac{10}{2}[\gamma_{sat(clay)} - 62.4]$$

$$= 10 \times 100 + 10(120 - 62.4) + 5(110 - 62.4)$$

$$= 1814 \text{ lb/ft}^2$$

From Eq. (11.70), $$\Delta\sigma'_{av} = \frac{\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b}{6}$$

$\Delta\sigma'_t$, $\Delta\sigma'_m$, and $\Delta\sigma'_b$ below the center of the footing can be obtained from Eq. (10.34).

Now we can prepare the following table (*Note: L/B* = 5/5 = 1):

m_1	z (ft)	$b = B/2$ (ft)	$n_1 = z/b$	q (kip/ft²)	I_4	$\Delta\sigma' = qI_4$ (kip/ft²)
1	15	2.5	6	$\frac{200}{5 \times 5} = 8$	0.051	$0.408 = \Delta\sigma'_t$
1	20	2.5	8	8	0.029	$0.232 = \Delta\sigma'_m$
1	25	2.5	10	8	0.019	$0.152 = \Delta\sigma'_b$

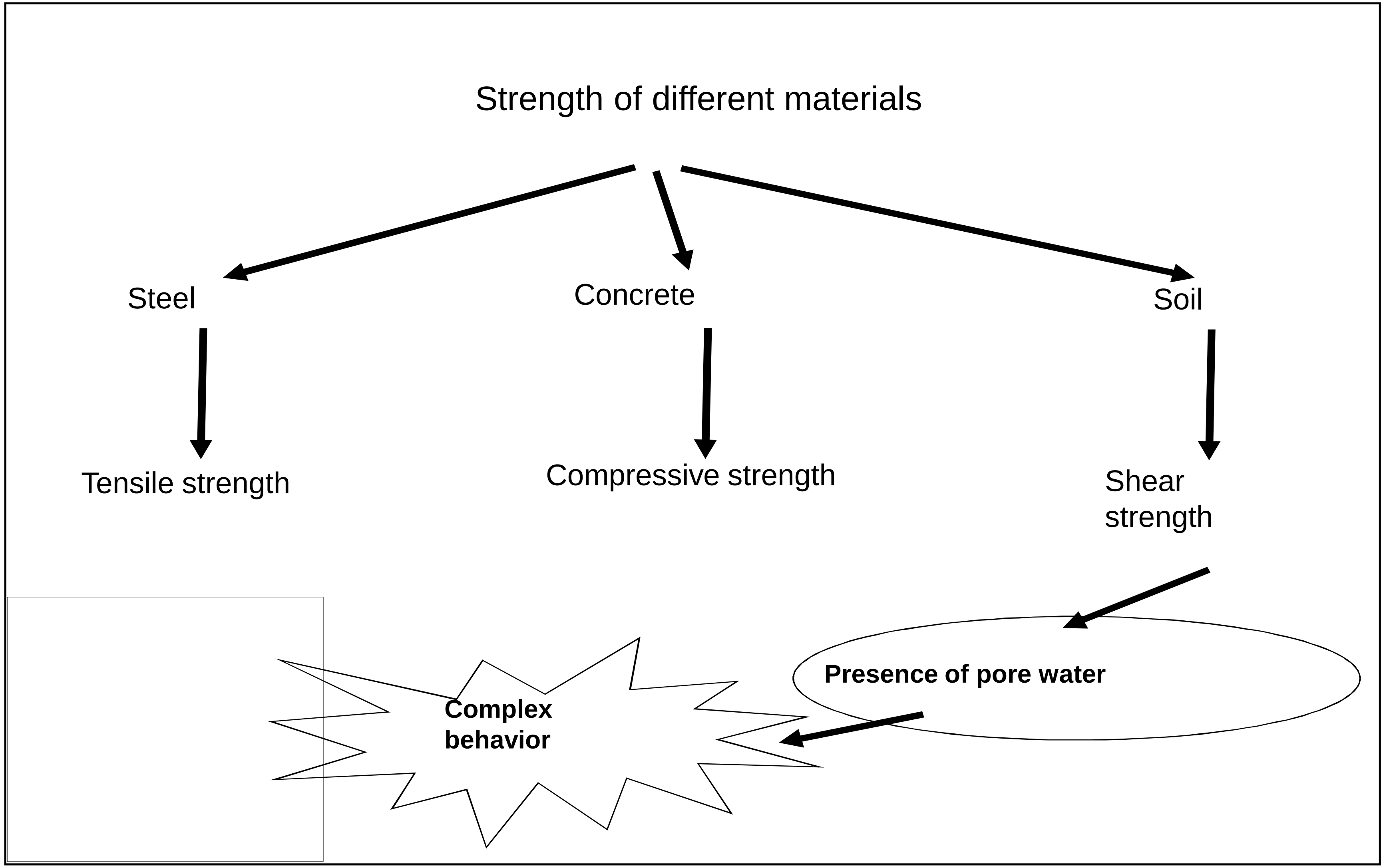
So,

$$\Delta\sigma'_{av} = \frac{0.408 + (4)(0.232) + 0.152}{6} = 0.248 \text{ kip/ft}^2 = 248 \text{ lb/ft}^2$$

Hence,

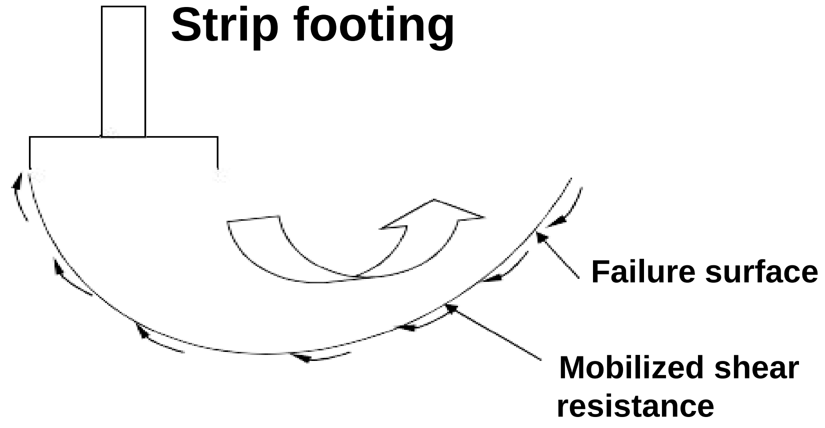
$$S_c = \frac{(0.27)(120)}{1 + 1} \log \frac{1814 + 248}{1814} \approx \mathbf{0.9 \text{ in.}}$$ ■

Shear Strength of soil


Strength of different materials
Steel
Concrete
Soil
Tensile strength
Compressive strength
Shear strength
Presence of pore water
Complex behavior

Shear failure of soils

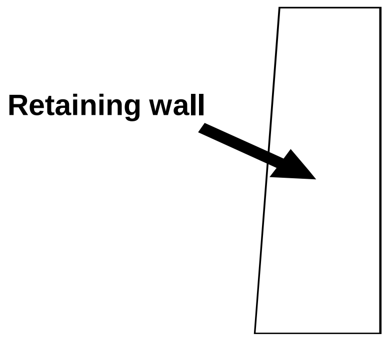
Soils generally fail in <u>shear</u>



- At failure, shear stress along the failure surface (mobilized shear resistance) reaches the shear strength.

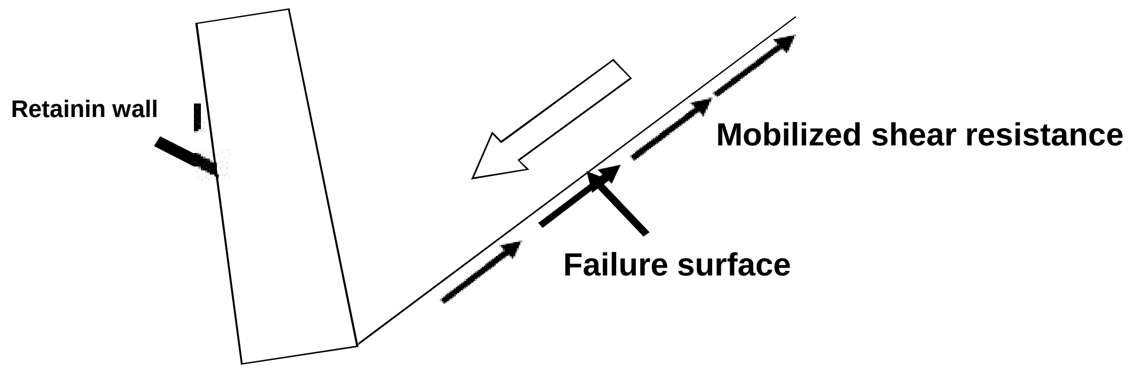
Shear failure of soils

Soils generally fail in shear



Shear failure of soils

Soils generally fail in <u>shear</u>



At failure, shear stress along the failure surface (mobilized shear resistance) reaches the shear strength.

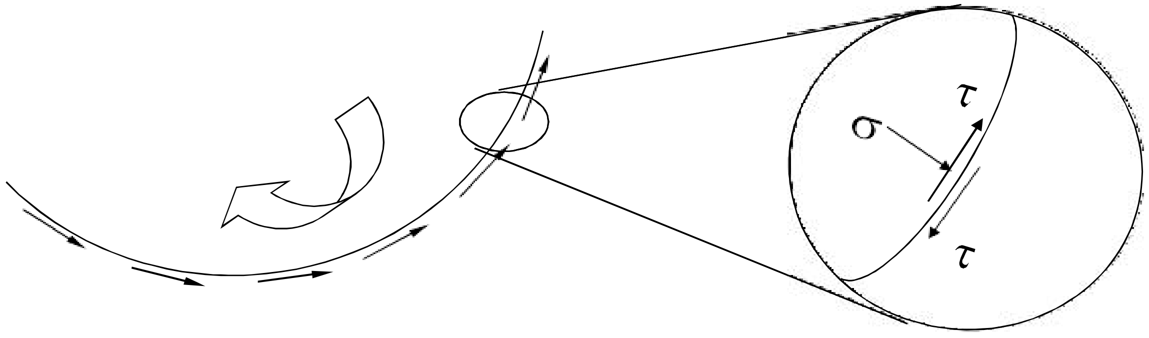
Shear failure mechanism

<u>failure surface</u>

- ❑ The soil grains slide over each other along the failure surface.

- ❑ No crushing of individual grains.

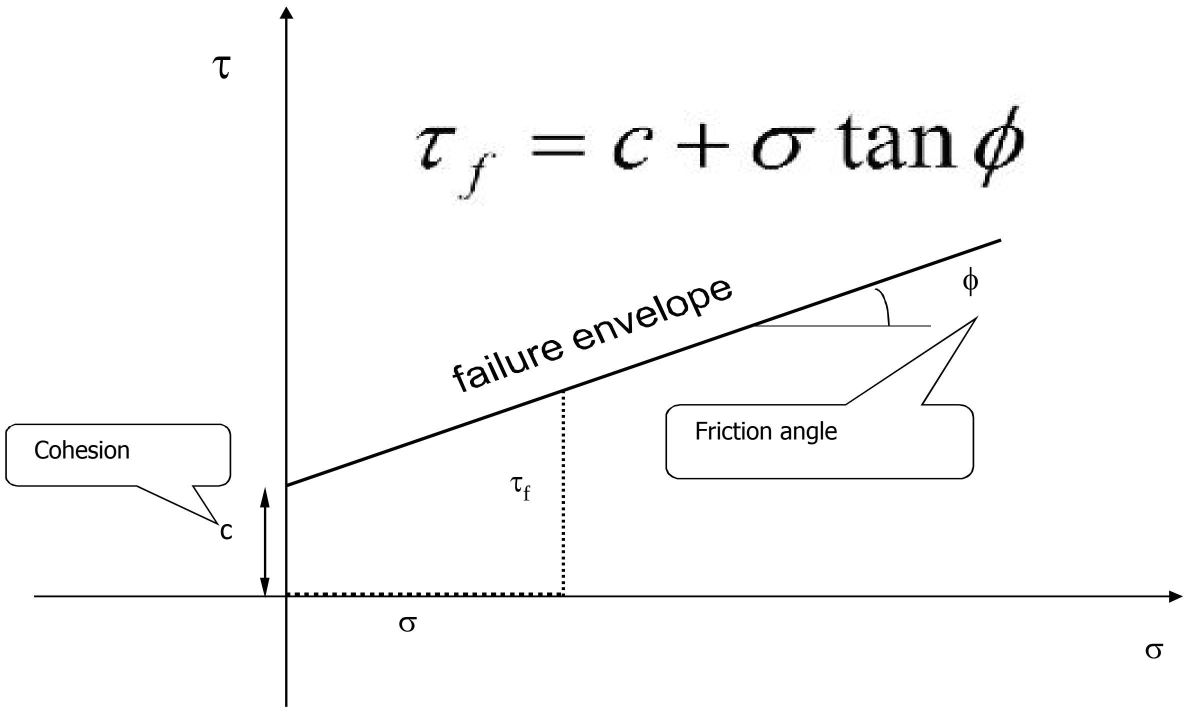
Shear failure mechanism



At failure, shear stress along the failure surface (τ) reaches the shear strength (τ_f)

Mohr-Coulomb Failure Criterion
(in terms of total stresses)

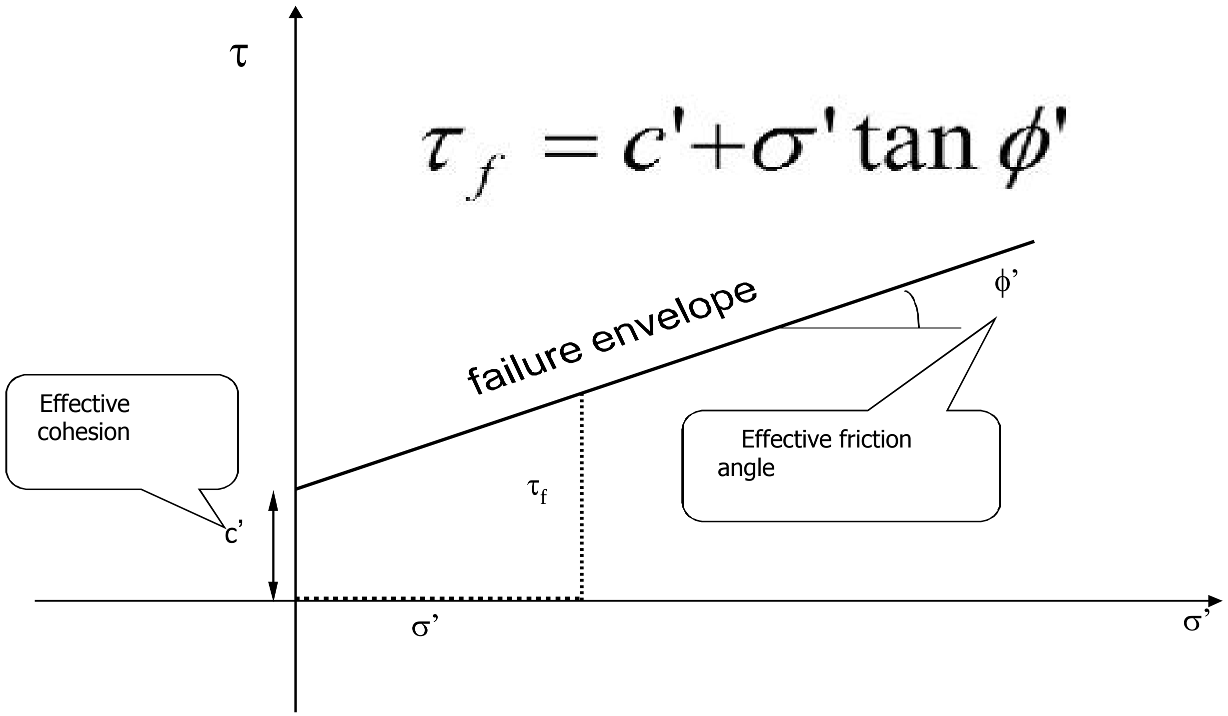
$$\tau_f = c + \sigma \tan\phi$$



τ_f lamron rednu ,eruliaf tuohtiw ekat nac lios eht sserts raehs mumixam eht si
fo ssertsσ.

Mohr-Coulomb Failure Criterion
)in terms of effective stresses)

$$\tau_f = c' + \sigma' \tan \phi'$$



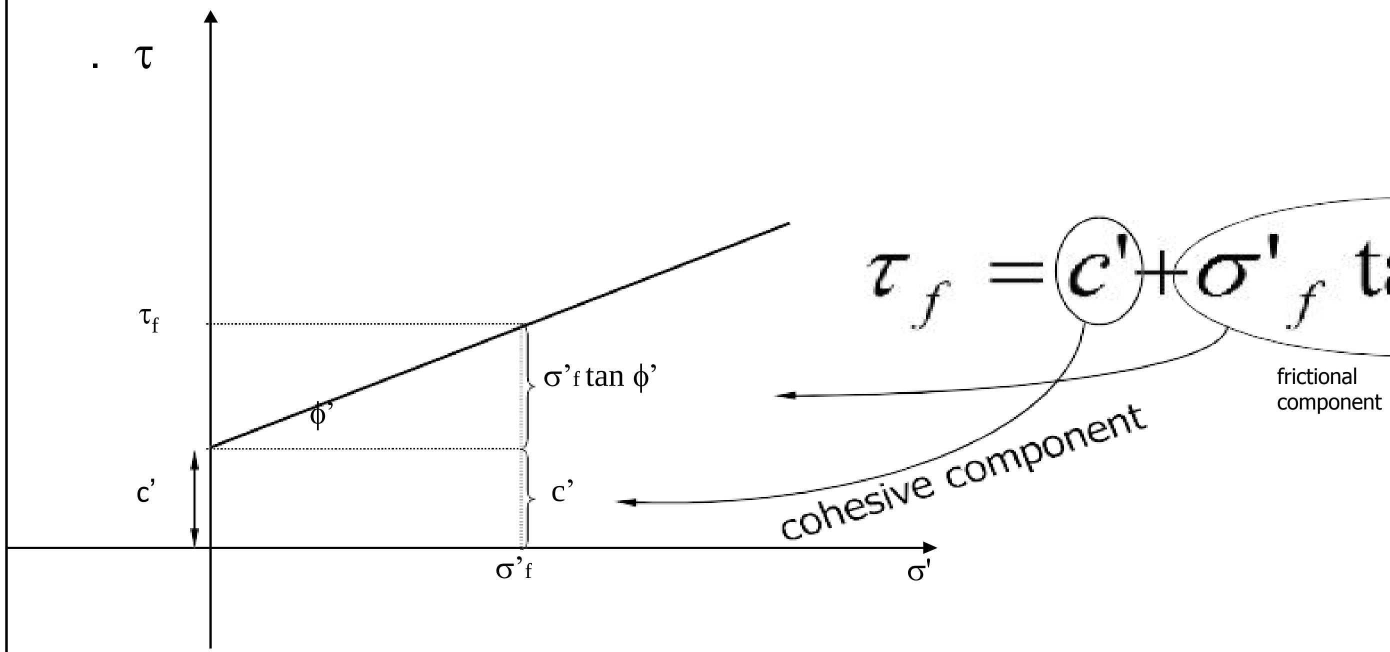
$$\sigma' = \sigma - u$$

u = pore water pressure

τ_f lamron rednu ,eruliaf tuohtiw ekat nac lios eht sserts raehs mumixam eht si
fo sserts evitceffeσ.'

Mohr-Coulomb Failure Criterion

Shear strength consists of two components: cohesive and frictional

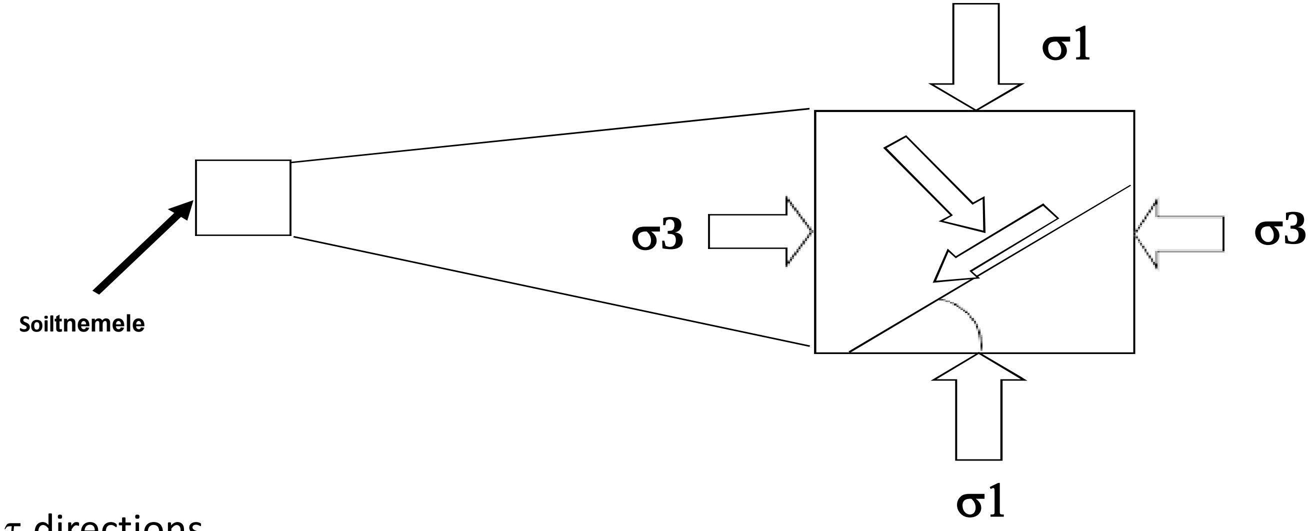


$$\tau_f = c' + \sigma'_f \tan \phi'$$

c and ϕ are measures of shear strength .

Higher the values, higher the shear strength.

Mohr Circle of stress



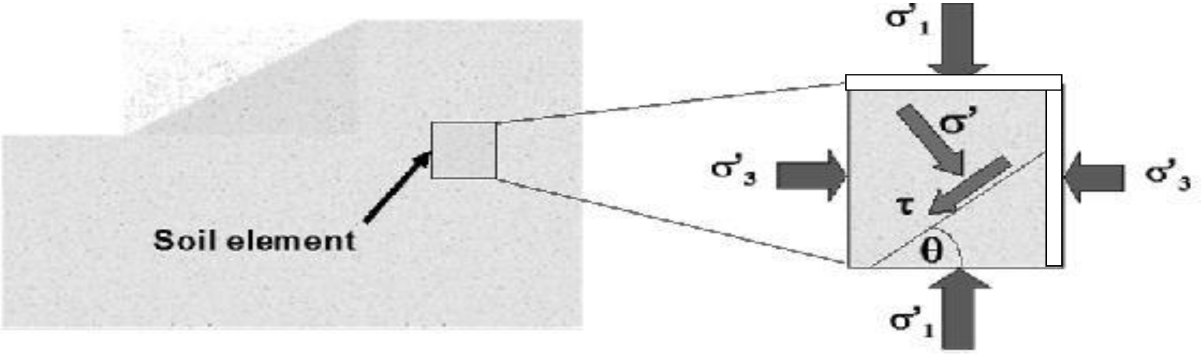
Resolving ni secrof σ and τ directions,

$$\tau = \frac{\sigma_1' - \sigma_3'}{2} Sin2\theta$$

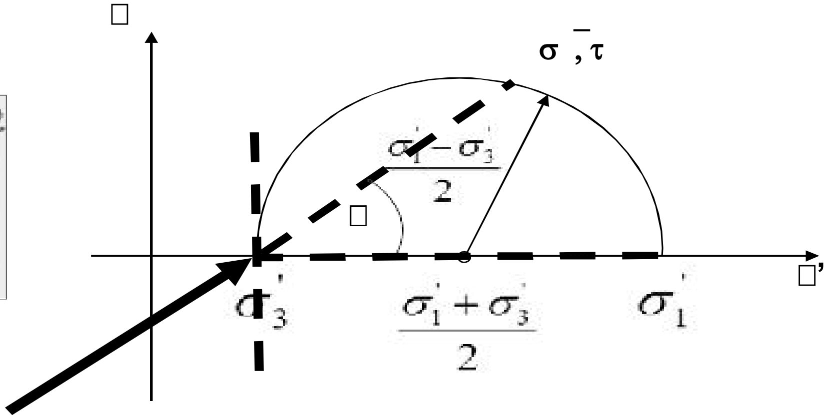
$$\sigma' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} Cos2\theta$$

$$\tau^2 + \left(\sigma' - \frac{\sigma_1' + \sigma_3'}{2}\right)^2 = \left(\frac{\sigma_1' - \sigma_3'}{2}\right)^2$$

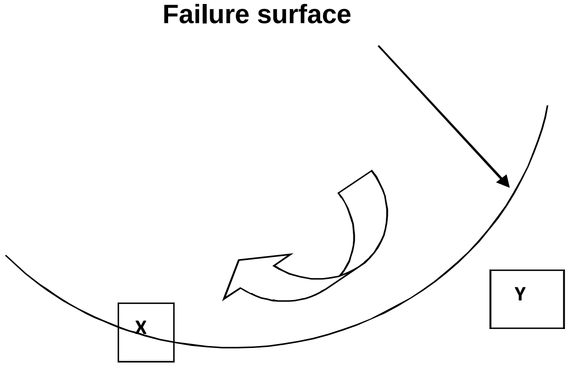
Mohr Circle of stress



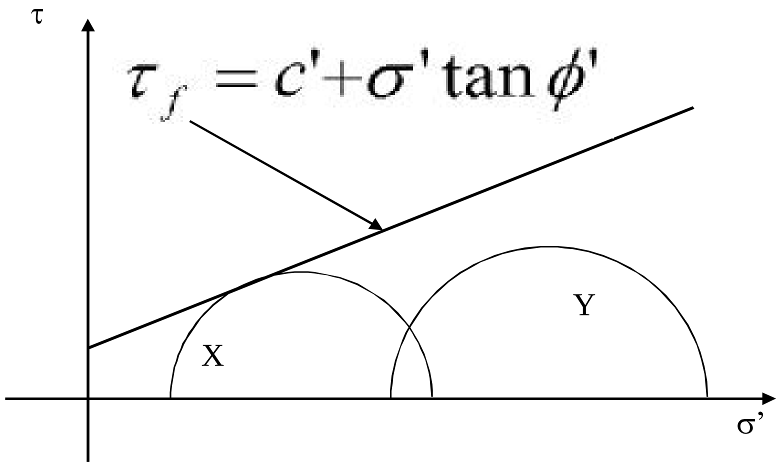
$$\tau^2 + \left(\sigma' - \frac{\sigma_1' + \sigma_3'}{2} \right)^2 = \left(\frac{\sigma_1' - \sigma_3'}{2} \right)^2$$



Mohr Circles & Failure Envelope



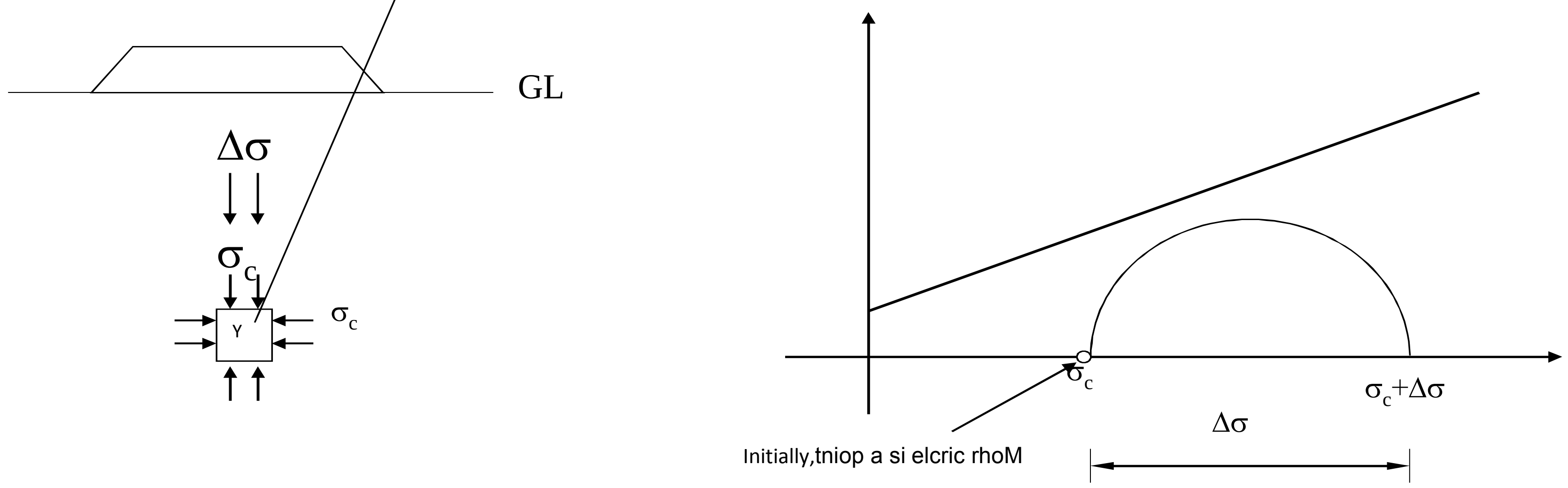
Soil elements at different locations



Y elbats ~
X eruliaf ~

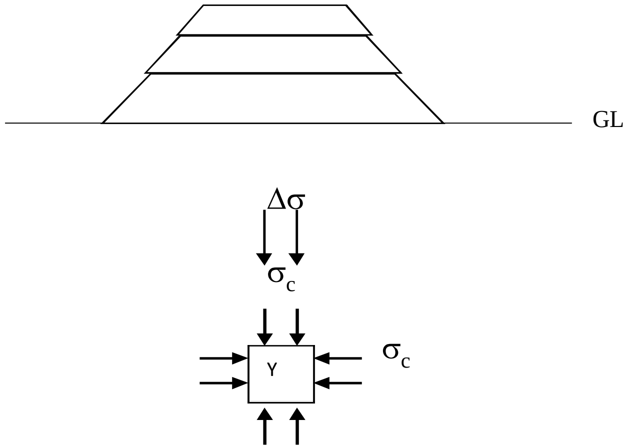
Mohr Circles & Failure Envelope

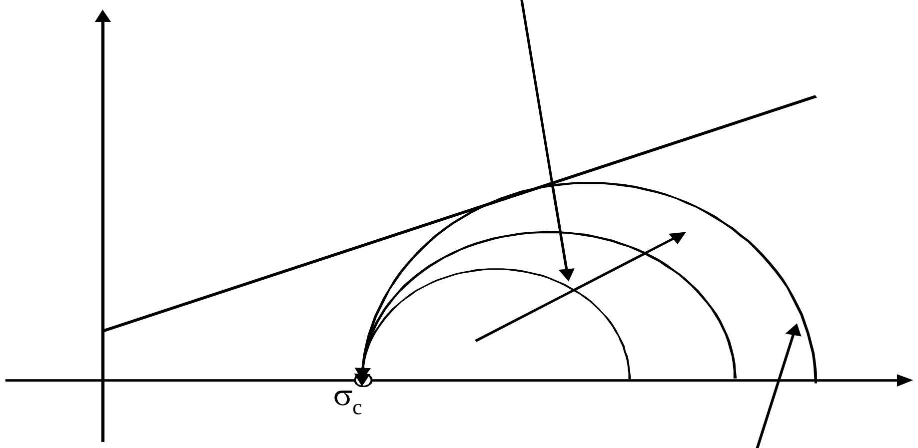
The soil element does not fail
if the Mohr circle is contained
within the envelope



Mohr Circles & Failure Envelope

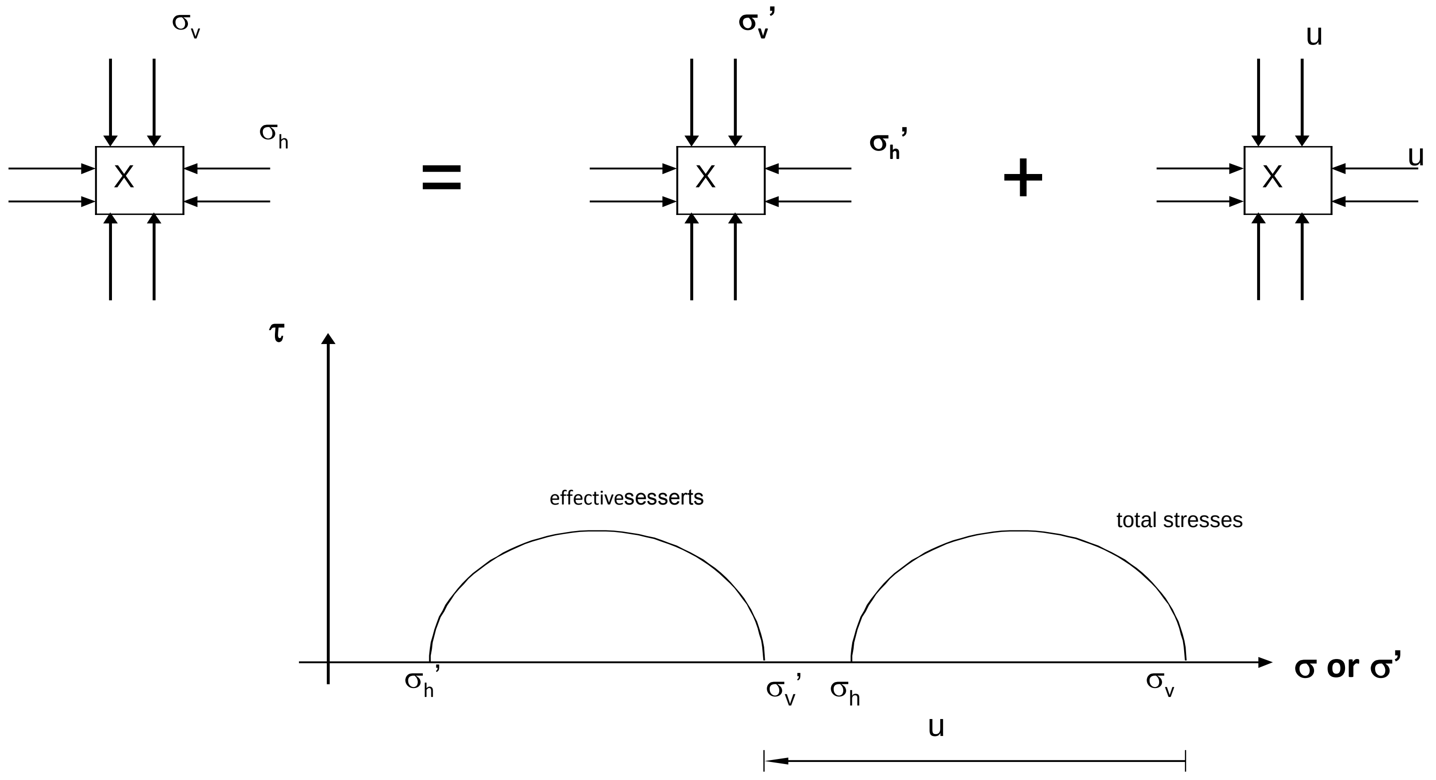
As loading progresses, Mohr circle becomes larger…



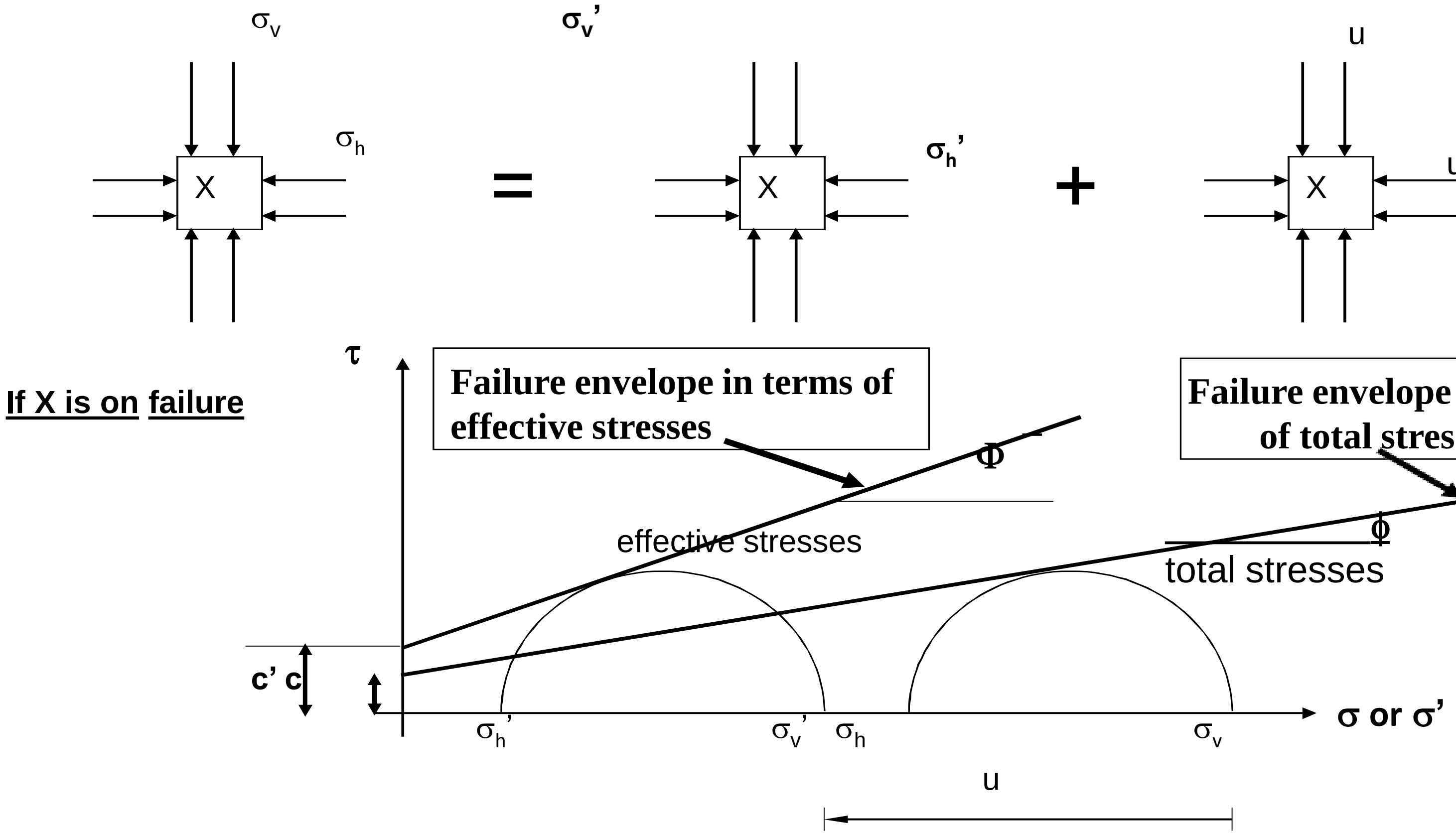


..and finally failure occurs rhoM nehw epolevne eht sehcuot elcric

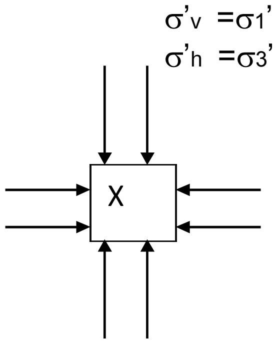
Mohr circles in terms of total and effective stresses



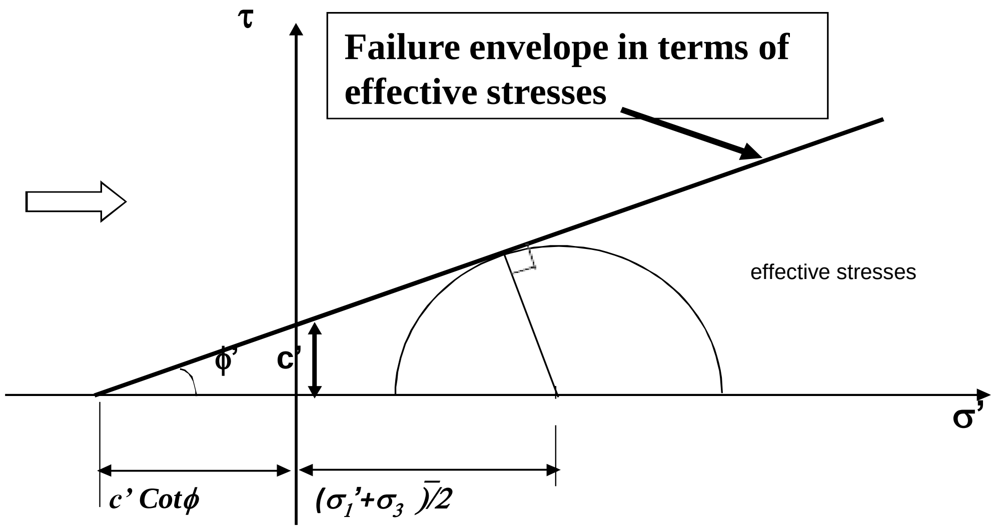
Failure envelopes in terms of total & effective stresses



Mohr Coulomb failure criterion with Mohr circle of stress



X is on failure



Therefore,

$$\left[c'\,Cot\,\phi' + \left(\frac{\sigma_1' + \sigma_3'}{2}\right)\right] Sin\,\phi' = \left(\frac{\sigma_1' - \sigma_3'}{2}\right)$$

Mohr Coulomb failure criterion with Mohr circle of stress

$$\sigma'_1 = \sigma'_3 \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

**Very important

Determination of shear strength parameters of soils c, ϕ or c', ϕ'

Laboratory tests on specimens taken from representative undisturbed samples

Most common laboratory tests to determine the shear strength parameters are,

1. Direct shear test
2. Triaxial shear test

Other laboratory tests include, Direct simple shear test, torsional ring shear test, plane strain triaxial test, laboratory vane shear test, laboratory fall cone test

Field tests

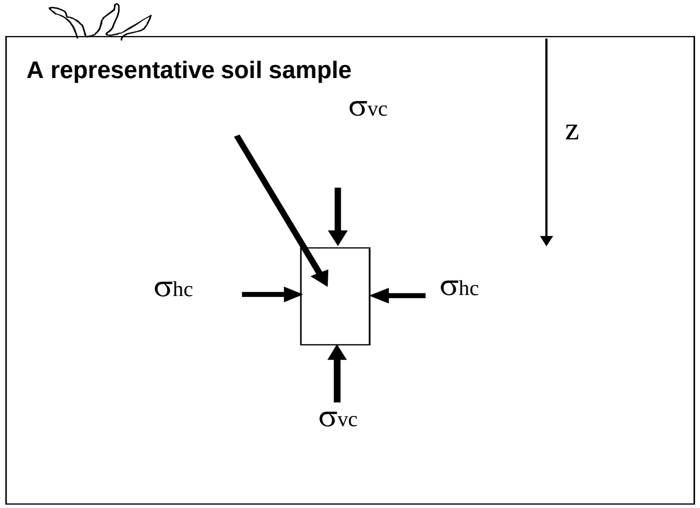
1. Vane shear test
2. Torvane
3. Pocket penetrometer
4. Fall cone
5. Pressuremeter
6. Static cone penetrometer
7. Standard penetration test

Laboratory tests

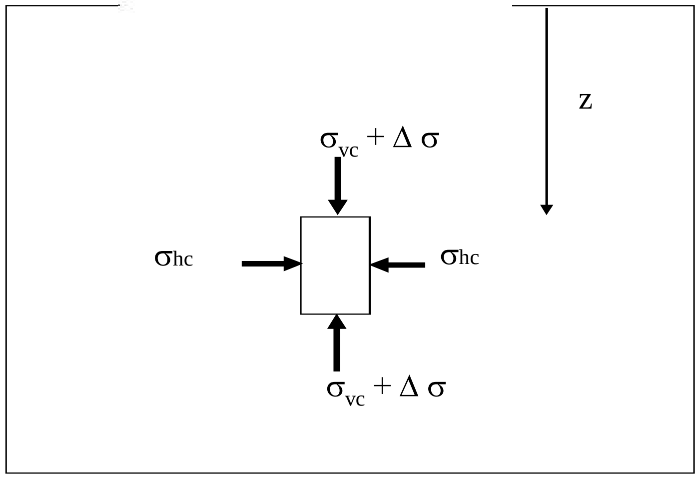
How to take undisturbed samples

Laboratory tests

Field conditions



Beforenoitcurtsnoc



After and during construction

Laboratory tests
Simulating field conditions in the laboratory

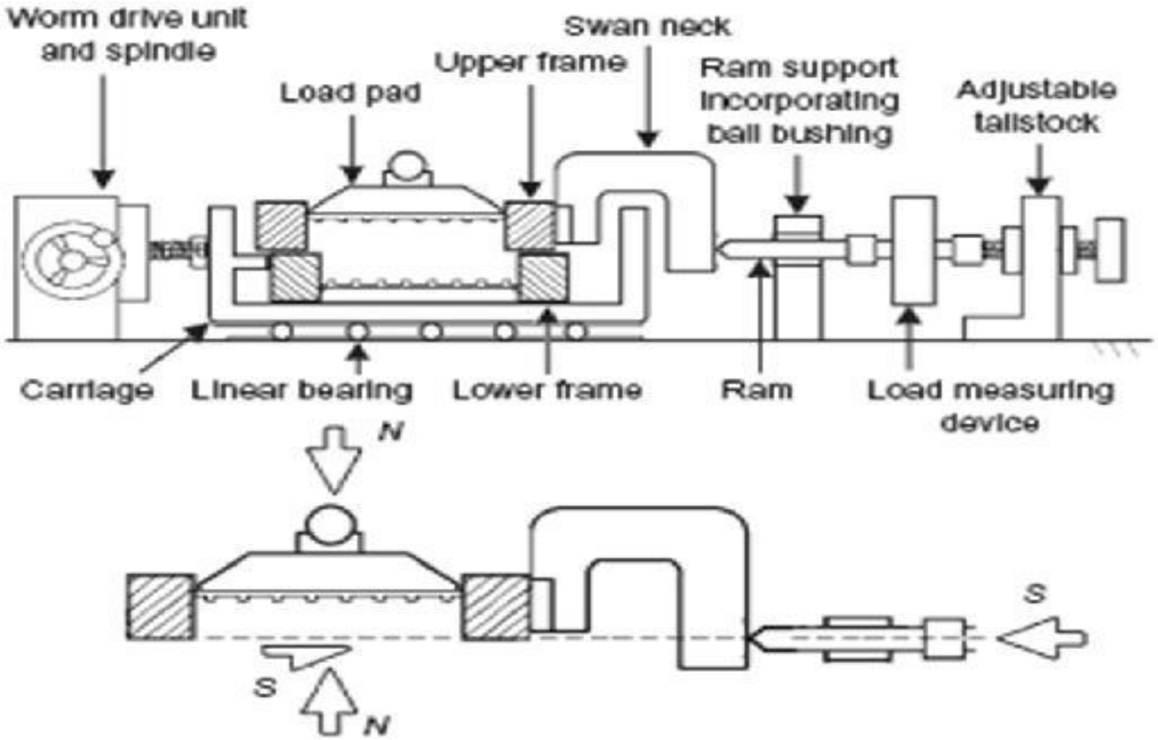
Representative soil sample taken from the site

Step 1
Set the specimen in the apparatus and apply the initial stress condition

Step 2
Apply the corresponding field stress conditions

Direct shear test

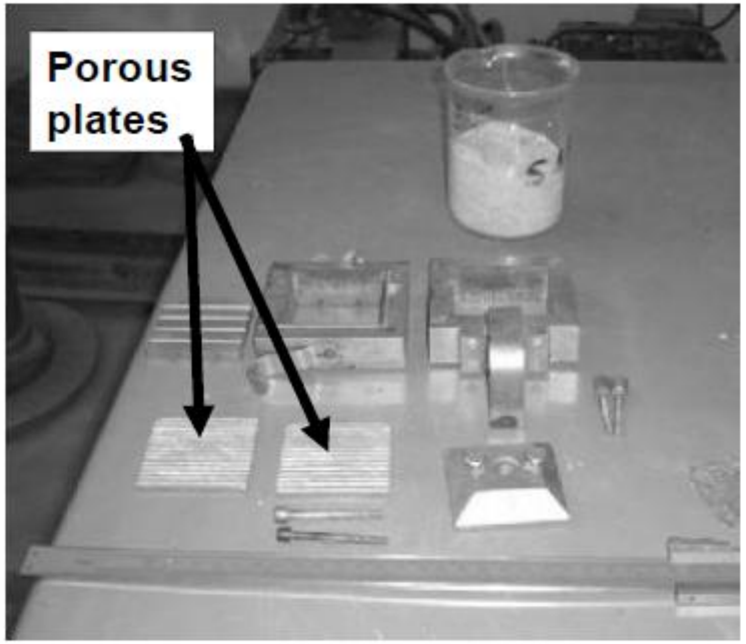
Schematic diagram of the direct shear apparatus



Direct shear test

Direct shear test is most suitable for consolidated drained tests specially on granular soils (e.g.: sand) or stiff clays

Preparation of a sand specimen



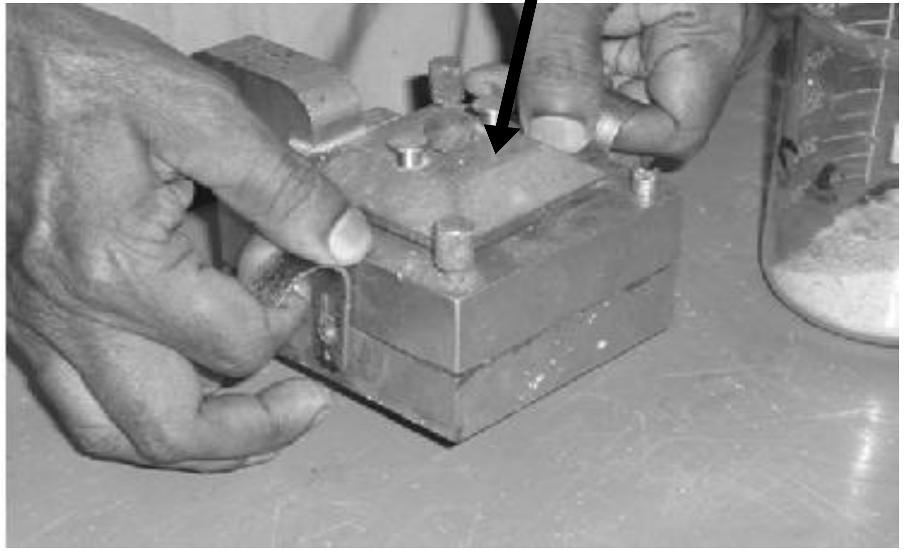
Components of the shear box

Preparation of a sand specimen

Direct shear test

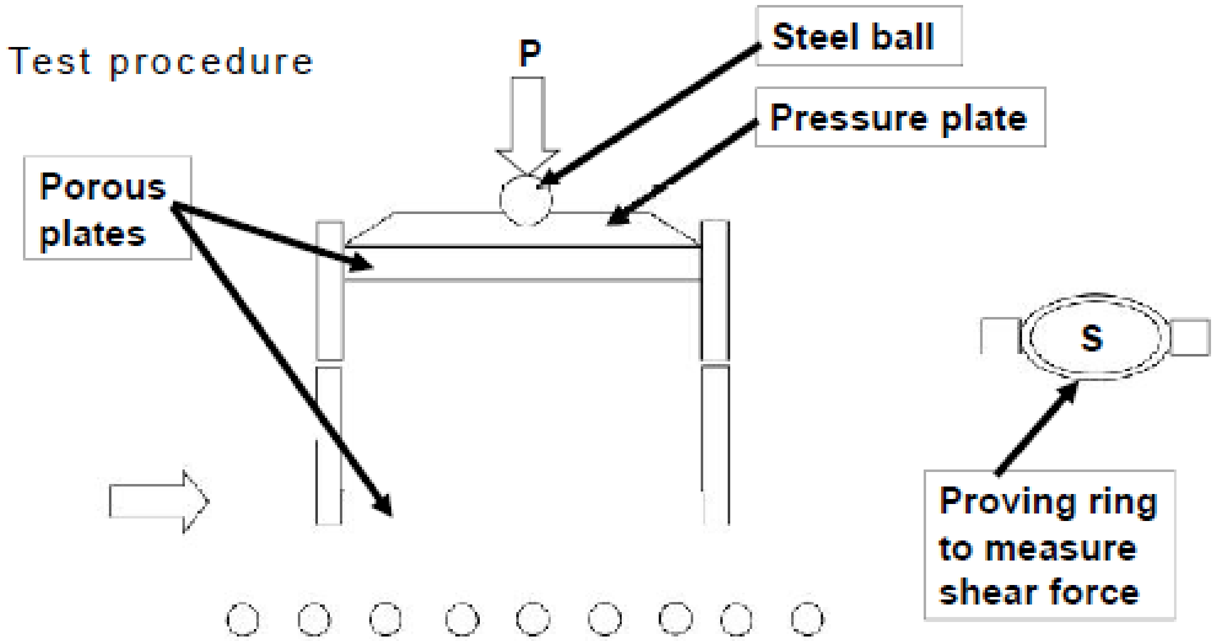
Preparation of a sand specimen

Leveling the top surface of specimen



Specimen preparation completed

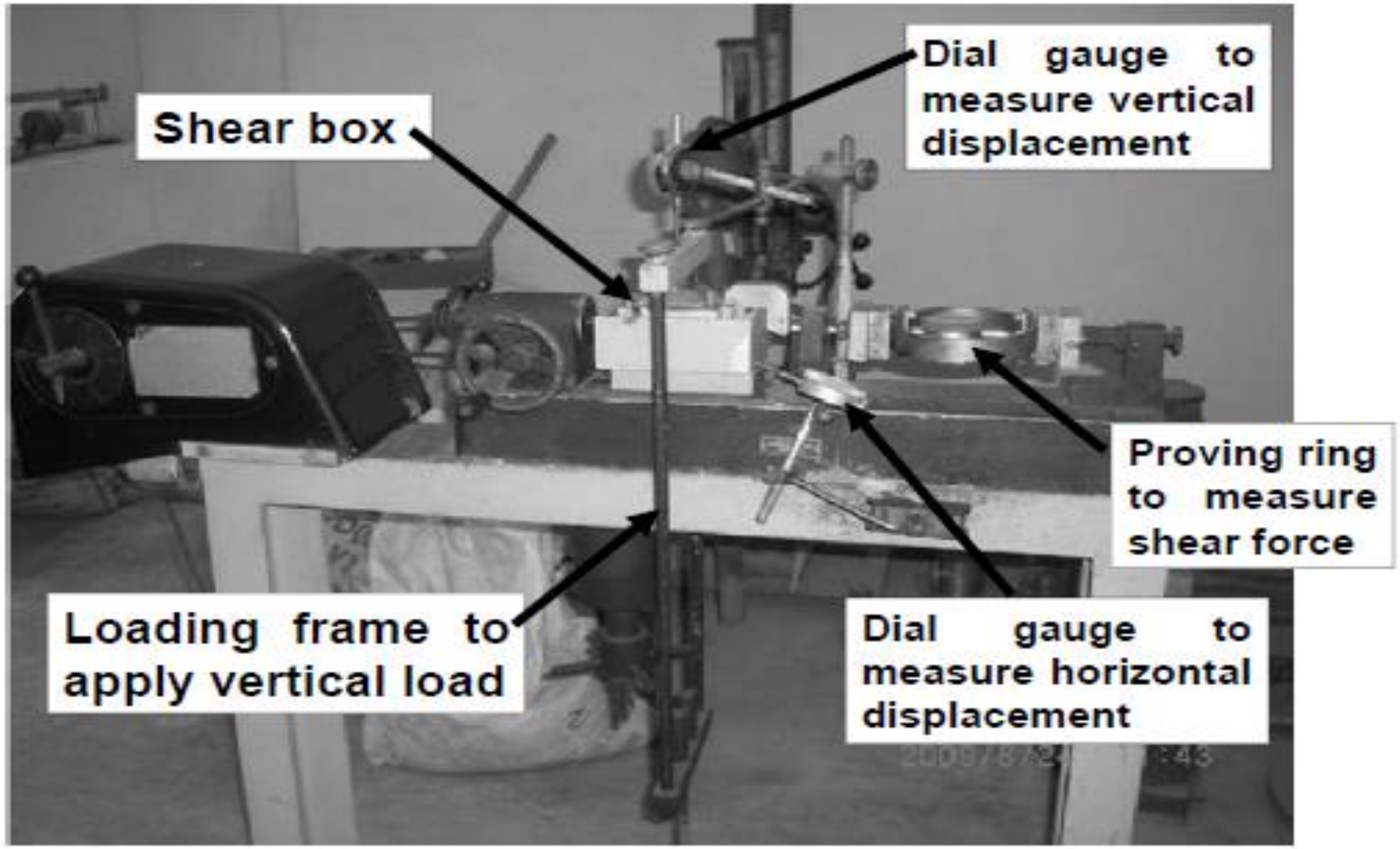
Direct shear test



Step 1: Apply a vertical load to the specimen and wait for consolidation

Step 2: Lower box is subjected to a horizontal displacement at a constant rate

Direct shear test



$$\tau = \text{Shear stress} = \frac{\text{Shear resistance developed at the sliding surface (S)}}{\text{Area of cross section of the sample}}$$

Direct shear test

Analysis of test results

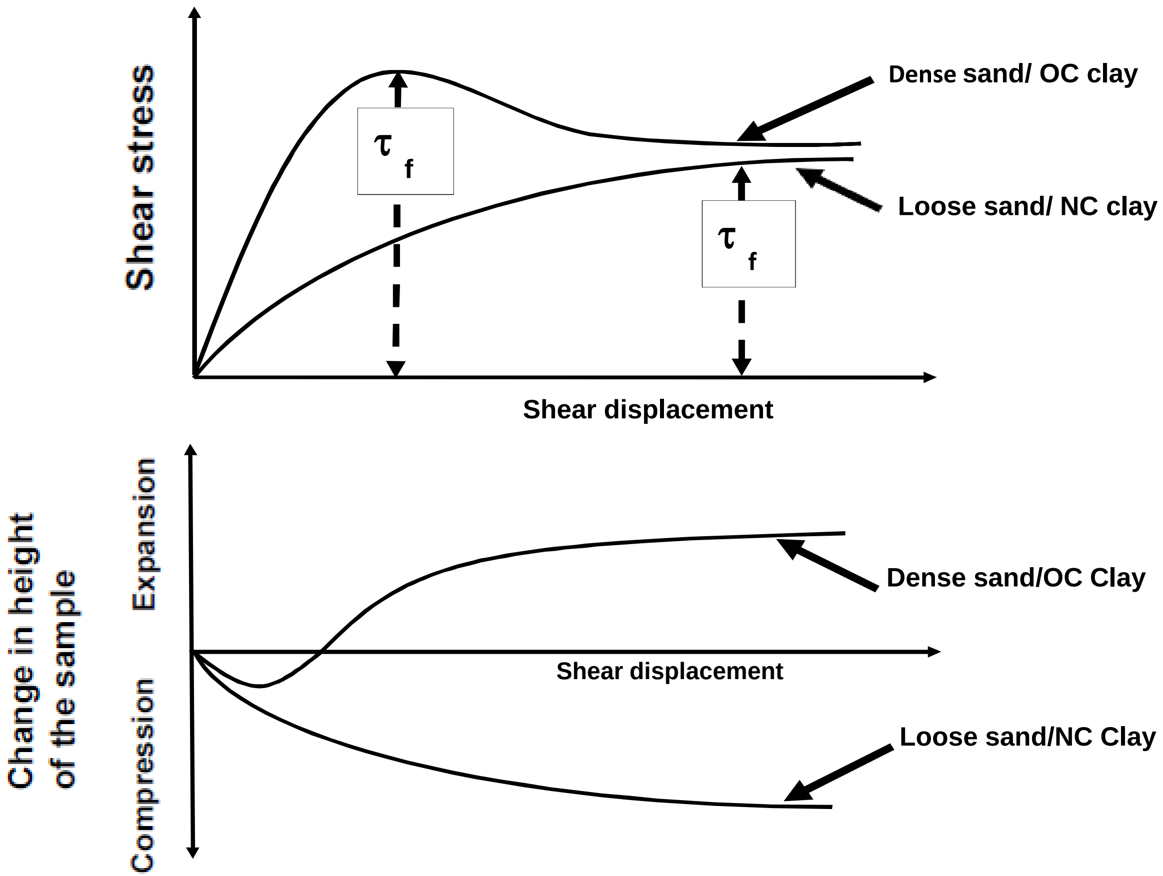
$$\sigma = \text{Normal stress} = \frac{\text{Normal force (P)}}{\text{Area of cross section of the sample}}$$

$$\tau = \text{Shear stress} = \frac{\text{Shear resistance developed at the sliding surface (S)}}{\text{Area of cross section of the sample}}$$

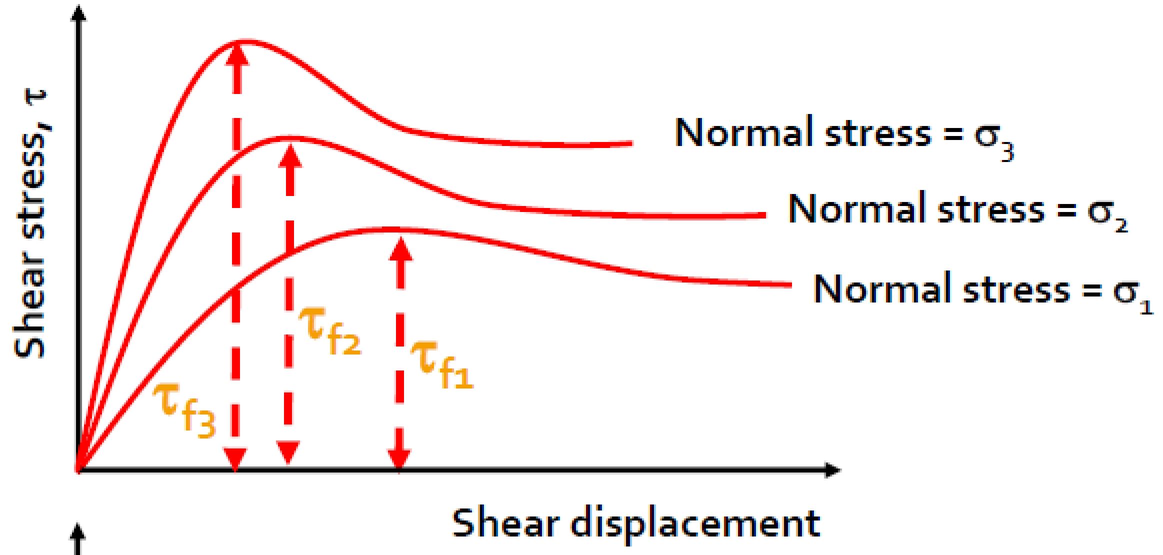
Note: Cross-sectional area of the sample changes with the horizontal displacement

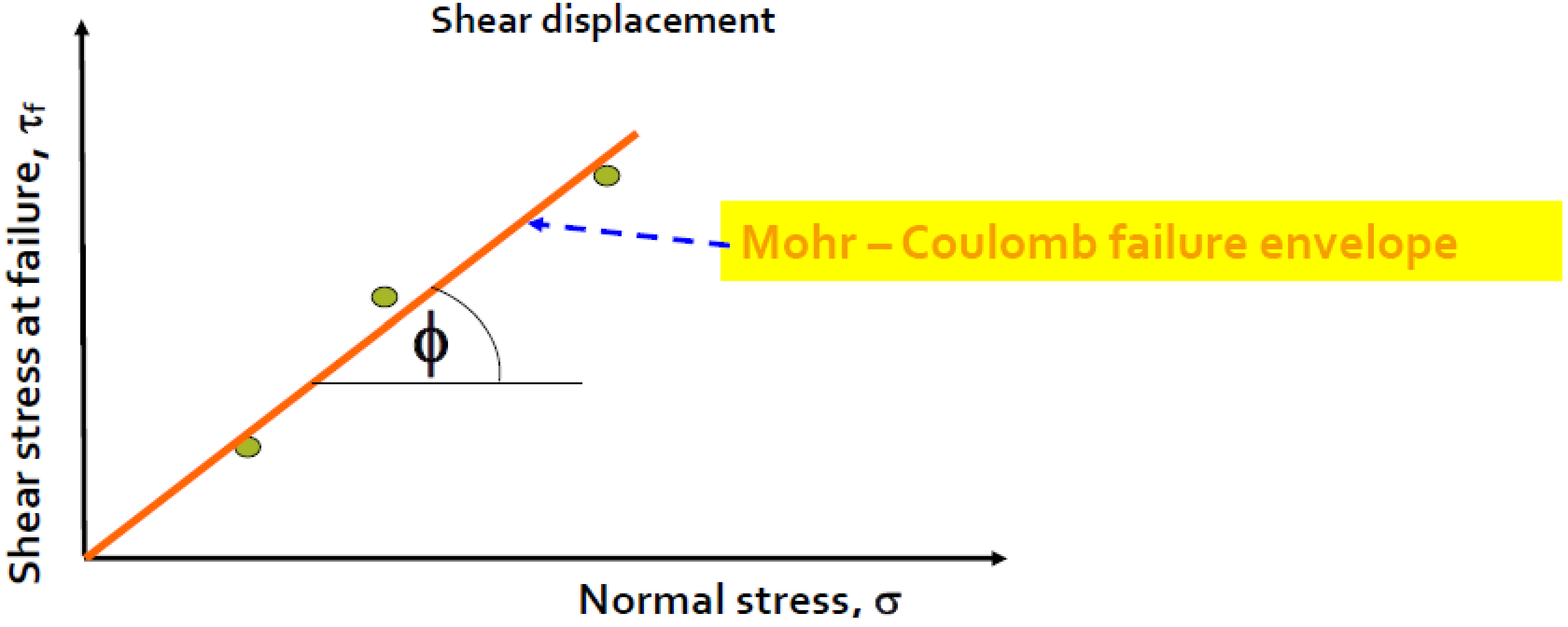
Direct shear tests on sands

Stress-strain relationship



How to determine strength parameters c and ϕ





Direct shear tests on sands

Sand is cohesionless hence c = 0

Direct shear tests are drained and pore water pressures are dissipated , Hence u = 0

Therefore,
ϕ = 'ϕ and c` = c = 0

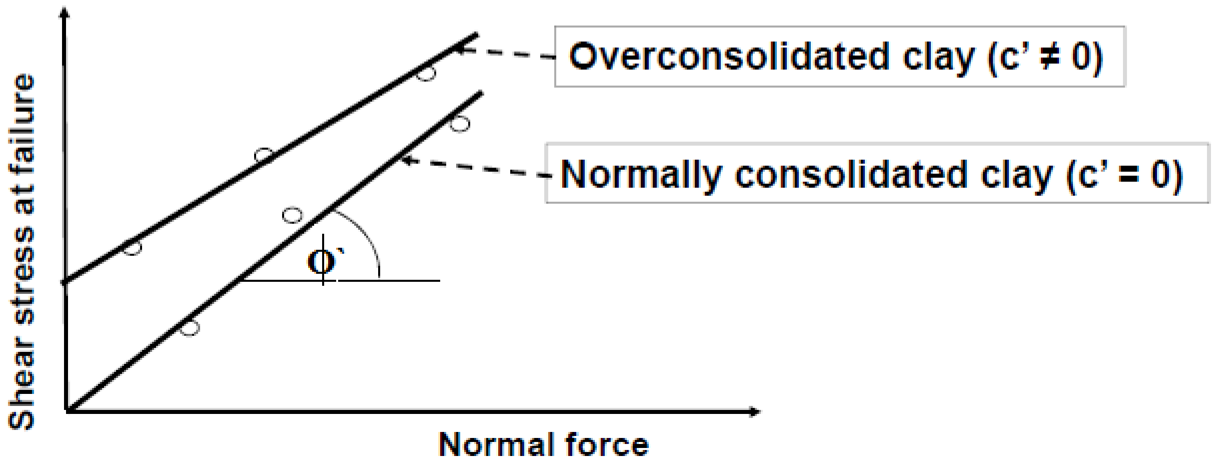
Some important facts on strength parameters c and ϕ of sand

Direct shear tests on clays

Failure envelopes for clay from drained direct shear tests

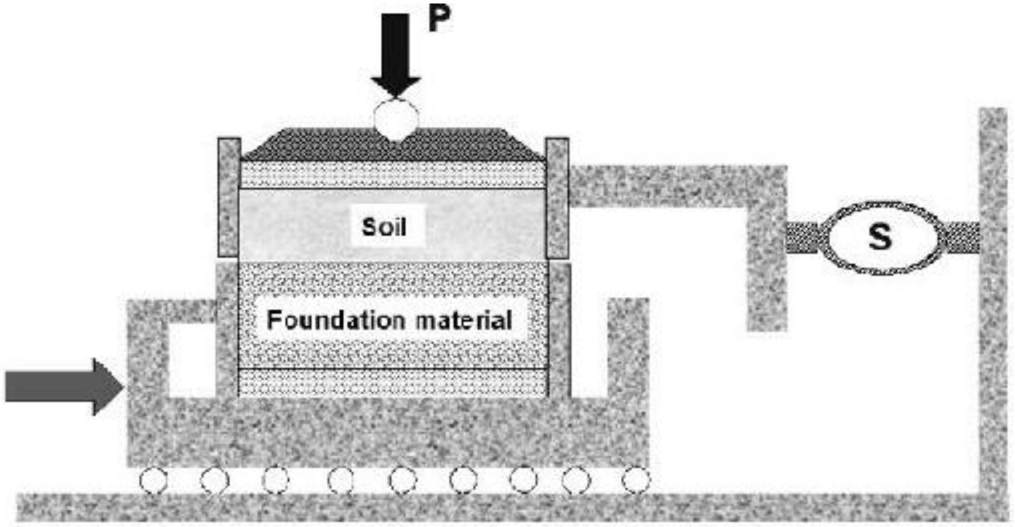
In case of clay, horizontal displacement should be applied at a very slow rate to allow dissipation of pore water pressure (therefore, one test would take several days to finish)

Failure envelopes for clay from drained direct shear tests



Interface tests on direct shear apparatus

- In many foundation design problems and retaining wall problems, it is required to determine the angle of internal friction between soil and the structural material (concrete, steel or wood)



$$\tau_f = c_a + \sigma' \tan \delta$$

Where,
c_a = adhesion,
ϕ =angle of internal friction

Advantages of direct shear apparatus

- Due to the smaller thickness of the sample, rapid drainage can be achieved
- Can be used to determine interface strength parameters
- Clay samples can be oriented along the plane of weakness or an identified failure plane

Disadvantages of direct shear apparatus

- Failure occurs along a predetermined failure plane
- Area of the sliding surface changes as the test progresses
- Non-uniform distribution of shear stress along the failure surface