Matrix:- المصفوفات

$4x_1 + 6x_2 + 9x_3 = 6$
$6x_1 - 2x_3 = 20$
$5x_1 - 8x_2 + x_3 = 10$

$$A = \begin{bmatrix} 4 & 6 & 9 \\ 6 & 0 & -2 \\ 5 & -8 & 1 \end{bmatrix}$$

العناصر داخل المصفوفة
augmanred matrix → element

$$\hat{A} = \left[\begin{array}{ccc:c} 4 & 6 & 9 & 6 \\ 6 & 0 & -2 & 20 \\ 5 & -8 & 1 & 10 \end{array}\right]$$ +الجواب

m → rows صفوف
n → colums → اعمدة
m x n ⇒ size

Find the elements of the matrix A??

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

* انتبه: A = B ← جميع الelement متساوية

$$B = \begin{bmatrix} 4 & 0 \\ 5 & 3 \end{bmatrix}$$

sol

$$A = \begin{bmatrix} 4 & 0 \\ 5 & 3 \end{bmatrix}$$

Find the size of the matrix :

1) $\begin{bmatrix} 0.3 & 1 & -3 \\ 2 & 4 & 3 \end{bmatrix} \rightarrow$ اعمدة X صفوف
2×3

2) $\begin{bmatrix} 5 \\ 1 \end{bmatrix} \rightarrow 2\times1$ → one colum

3) $[1 \;\; 2 \;\; 5] \rightarrow 1\times3$ → one rawe

squar matrix $\Leftarrow m = n$

~~الجـ~~ الجمع الـ matrix

→ عند الجمع يجب ان يكون نفس الحجم
same size

EX8 $A = \begin{bmatrix} -4 & 6 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 5 & -1 & 0 \\ 3 & 1 & 0 \end{bmatrix}$

Size → 2×3 $\qquad 2\times3$

$A + B = ??$

$-4+5 \qquad 6+-1 \qquad 3+0$

$= \begin{bmatrix} 1 & 5 & 3 \\ 3 & 2 & 2 \end{bmatrix}$

$0+3 \qquad 1+1 \qquad 2+0$

* الضرب ب ثابت :

$A = \begin{bmatrix} 2.7 & -1.8 \\ 0 & 0.9 \\ 9 & -4.5 \end{bmatrix}$

$2.7 * -1 \qquad -1.8 * -1$

$-1A = \begin{bmatrix} -2.7 & 1.8 \\ 0 & -0.9 \\ -9 & 4.5 \end{bmatrix}$

$0 * -1 \qquad 0.9 * -1$

$9 * -1 \qquad -4.5 * -1$

* ضرب الـ matrix :

$A \rightarrow [m \times n]$ (صف، عمود)

$B \rightarrow [n \times p]$ (صف، عمود) → $n = n$

$A = \begin{bmatrix} 3 & 5 & -1 \\ 4 & 0 & 2 \\ -6 & -3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -2 & 3 & 1 \\ 5 & 0 & 7 & 8 \\ 9 & -4 & 1 & 1 \end{bmatrix}$

size ~~A*B~~ → 3×3 equal 3×4

$= 3\times4$

الصف ١ * العمود ١

الصف الاول * العمود الاول

$\Rightarrow a_{11} = 3\times2 + 5\times5 + -1\times9$

$= 22$

$a_{12} = 3\times-2 + 5\times0 + -1\times\cancel{3}\ \overset{-4}{\underline{\ }}\ -2$

الصف الاول
العمود الثاني

$a_{13} = 3\times3 + 5(7) + -1(1) = 43$

$\cancel{a_{21}} = a_{14} \rightarrow 3(1) + 5(8) + -1(1) = 42$

$a_{21} = 4(2) + 0(5) + 2(9) = 26$

$a_{22} = 4(-2) + 0(0) + 2(-4) = -16$

$a_{23} = \cancel{4}\ 4(3) + 0(7) + 2(1) = 14$

$a_{24} = 4(1) + 0(8) + 2(1) = 6$

$a_{31} = -6(2) + -3(5) + 2\cancel{8}(9) = -9$

$a_{32} = -6(-2) + -3(0) + 2(-4) = 4$

$a_{33} = -6(3) + -3(7) + 2(1) = -37$

$a_{34} = -6(1) + -3(8) + 2(1) = -28$

$$\begin{bmatrix} 22 & -2 & 43 & 42 \\ 26 & -16 & 14 & 6 \\ -9 & 4 & -37 & -28 \end{bmatrix} \rightarrow 3\times4$$

ex:

$4\times3+2(5)$

$$\begin{bmatrix} 4 & 2 \\ 1 & 8 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \cancel{12 - 4}\begin{bmatrix} 22 \\ 40 \end{bmatrix}$$

$\underbrace{2\times2 \quad 2\times1}$ $= 2\times1$

$1\times3+8\times5$

ex:

$3(1)+6(2)+\cancel{1}(4)$

$$\begin{bmatrix} 3 & 6 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 19 \end{bmatrix}$$

$\underbrace{1\times3 \quad 3\times1}$ $= 1\times1$

ex:

$$\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 1 & 8 \end{bmatrix} \rightarrow \text{غير معرف}$$

$\underbrace{3\times1 \quad 2\times2}_{b}$

not equal

$$\underset{3\times1}{\begin{bmatrix}1\\2\\4\end{bmatrix}}\ \underset{1\times3}{\begin{bmatrix}3 & 6 & 1\end{bmatrix}} = \begin{bmatrix}3 & 6 & 1\\6 & 12 & 2\\12 & 24 & 4\end{bmatrix}$$

3x1 6x1 1x1

3x2 2x6

$3\times\underbrace{1\ \ \ 1}\times3$

equal

ex:
$$\underset{2\times2}{\begin{bmatrix}1 & 1\\100 & 100\end{bmatrix}}\ \underset{2\times2}{\begin{bmatrix}-1 & 1\\1 & -1\end{bmatrix}} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$a_{11} = 1\times-1 + 1(1) = 0$ $\quad a_{11}$

$a_{12}\ \ 100(-1) + 100(1) = 0$ $\quad$ الصف الاول × العمود الاول

$a_{21} = \cancel{100(1)} +$

$a_{21} = 100(-1) + 100(1) = 0$ $\quad a_{21}$

$a_{22} = 100(1) + 100(-1) = 0$ $\quad$ الصف الثاني × العمود الاول

• $A(BC) = (AB)C \neq (BC)A$ مش نغير مكانات

$(A+B)C = \cancel{CA+CB} \to AC+BC \neq CA+CB$ عكس

a

$C(A+B) \to CA+CB \neq AC+BC$

* ننتبه على الترتيب

$AB = 0 \not\Rightarrow BA = 0$ $\quad B=0\ \ A=0$ مش لازم

Trans Position:- تبديل الصفوف والعمود

$A = \ \ a_{11}\ \ a_{12}\ \ a_{13}$

EX
$$\begin{bmatrix}5 & -8 & 1\\4 & 0 & 0\end{bmatrix}^T = \begin{bmatrix}5 & 4\\-8 & 0\\1 & 0\end{bmatrix}$$

size ⇒ 2 x 3 $\quad$ 3 x 2

EX
$$\underset{1\times3}{\begin{bmatrix}6 & 2 & 3\end{bmatrix}} = \underset{3\times1}{\begin{bmatrix}6\\2\\3\end{bmatrix}}$$

EX 3

$$\underset{3\times3}{\begin{bmatrix}3 & 0 & 7\\8 & -1 & 5\\1 & -9 & 5\end{bmatrix}}^T = \underset{3\times3}{\begin{bmatrix}3 & 8 & 1\\0 & -1 & -9\\7 & 5 & 5\end{bmatrix}}$$

EX:

$$\begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}^T \rightarrow [6\ 2\ 3]$$

3×1 1×3

$* (A^T)^T = A$ $* (A+B)^T = A^T + B^T$

$* (CA)^T = CA^T$ $* (AB)^T = B^T \times A^T$

* symmetric and skew symmetric:-

$A^T = A \Rightarrow$ symmetric

$A^T = -A \Rightarrow$ skew-symmetric

EX:

$$\begin{bmatrix} 20 & 120 & 200 \\ 120 & 10 & 150 \\ 200 & 150 & 30 \end{bmatrix} \rightarrow \begin{bmatrix} 20 & 120 & 200 \\ 120 & 10 & 150 \\ 200 & 150 & 30 \end{bmatrix}$$

3×3

symmetric

$$A = \begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & 2 \\ -3 & -2 & 0 \end{bmatrix}$$

تعرف لاحقاً ← skew-symmetrix

Triangular Matrix:-

1) $\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$ → upper triangular

2) $\begin{bmatrix} 1 & 4 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 6 \end{bmatrix}$ → upper triangular

3) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 7 & 6 & 8 \end{bmatrix}$ → lower triangular

1) Diagonal matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

ارقام مختلفة

2) scaller matrix

ارقام متشابه

$$= \begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix}$$

3) unit identity matrix:

$AI = IA = A$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Linear system of equation:

$a_{11}x_1 + \dots + a_{mn}x_n = b_1$

$a_{21}x_1 + \dots + a_{mn}x_n = b_2$

$\vdots$

$a_{m1}x_1 + \dots + a_{mn}x_n = b_m$

* $a_{11} \dots a_{mm} \rightarrow$ cofficient of the system
* if all $b = 0 \rightarrow$ homogeneous system

* Gauss — الطريقة الاولى حل المعادلات الخطية باستخدام الmatrix

$a_{11}x_1 + a_{12}x_2 = b_1$

$a_{21}x_1 + a_{22}x_2 = b_2$

Pivot

$$\hat{A} = \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} a_{11} & a_{12} & b_1 \\ 0 & a'_{22} & b'_2 \end{array}\right]$$

$0 + a'_{22}x_2 = b'_2$

$$x_2 = \frac{b'_2}{a'_{22}}$$

EX: solve the linear system:

$2x_1 + 5x_2 = 2$

$-4x_1 + 3x_2 = -30$

باستخدام طريقة الـ Gauss:

Pivot

$$A = \begin{bmatrix} 2 & 5 & 2 \\ -4 & 3 & -30 \end{bmatrix}$$

* نبحث عن العدد اللي تحت الـ Pivot

ما هو العدد اللي نضربه بالـ Pivot

والجمع العدد اللي تحت الـ Pivot = 0

→ العدد اللي تحت الـ Pivot + كم نضرب بالـ Pivot = 0

Pivot

$(2)(2) + -4 = 0$

$2\,Row_1 + Row_2$

طريقة الحساب

$$\begin{array}{ccc} 2\times 2 & 5\times 2 & 2\times 2 \\ \begin{bmatrix} 4+ & 10+ & 2+ \\ -4 & 3 & -30 \end{bmatrix} \\ 0 & 13 & -28 \end{array}$$

$$\begin{bmatrix} 2 & 5 & 2 \\ 0 & 13 & -28 \end{bmatrix}$$

$0x_1 + 13x_2 = -28$

$\frac{13}{13}\boxed{x_2 = -2}$

$2x_1 + 5x_2 = 2$

$2x_1 + 5(-2) = 2$

$x_1 = 6$

ex: solve the ~~len~~ linear system:

$x_1 - x_2 + x_3 = 0$

$-x_1 + x_2 - x_3 = 0$

$10x_2 + 25x_3 = 90$

$20x_1 + 10x_2 = 80$

sol.

Pivot

$$\bar{A} = \begin{bmatrix} 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & 10 & 25 & 90 \\ 20 & 10 & 0 & 80 \end{bmatrix} \begin{array}{l} \\ (1)Row_1 + Row_2 \\ \\ -20\,Row_1 + Row_4 \end{array}$$

* كل الـ element اللي تحت الـ (Pivot) = 0

$$= \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 30 & -20 & 80 \end{bmatrix}$$

$(1)(1)+-1$, $1(1)+-1$, $(1)(-1)+1$

$-20(1)+20$, $-20(-1)+20$, $-20(1)+0$, $-20(0)+80$

ما عدنا كلياتي نعمل لانو اكو كم اللي تحت الـ Pivot = 0

نحط الصف اللي كلو 0 الى الاسفل اخر شي

Pivot

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 30 & -20 & 80 \\ 0 & 0 & 0 & 0 \end{bmatrix} -3\,Row_2 + Row_3$$

$-0.2(3)+0.6$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 0 & -95 & -190 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 - x_2 + x_3 = 0$

$10x_2 + 25x_3 = 90$

$-95x_3 = -190 \rightarrow x_3 = 2$

$10x_2 + 25(2) = 90$

$x_2 = 4$

$x_1 - 4 + 2 = 0$

$x_1 = 2$

Ex: $3x_1 + 2x_2 + 2x_3 - 5x_4 = 8$

$0.6x_1 + 1.5x_2 + 1.5x_3 - 5.4x_4 = 2.7$

$1.2x_1 - 0.3x_2 - 0.3x_3 + 2.4x_4 = 2.1$

Sol

$$\begin{array}{l} \quad \text{Pivot} \\ \hat{A} = \begin{bmatrix} 3 & 2 & 2 & -5 & 8 \\ 0.6 & 1.5 & 1.5 & -5.4 & 2.7 \\ 1.2 & -0.3 & -0.3 & 2.4 & 2.1 \end{bmatrix} \end{array} \begin{array}{l} \\ -0.2\,Row_1 + Row_2 \\ \cancel{-0.4\,Row_1 + Row_3} \\ -0.4\,Row_1 + Row_3 \end{array}$$

Pivot

$$= \begin{bmatrix} 3 & 2 & 2 & -5 & 8 \\ 0 & 1.1 & 1.1 & -4.4 & 1.1 \\ \cancel{1.2} & \cancel{-0.3} & \cancel{-0.3} & \cancel{2.4} & \cancel{2.1} \\ 0 & -1.1 & -1.1 & 4.4 & -1.1 \end{bmatrix} \quad Row_2 + Row_3$$

$$\begin{bmatrix} 3 & 2 & 2 & -5 & 8 \\ 0 & 1.1 & 1.1 & -4.4 & 1.1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$3x_1 + 2x_2 + 2x_3 - 5x_4 = 8$

$1.1x_2 + 1.1x_3 - 4.4x_4 = 1.1$

بما ان عدد المجاهيل اكبر من عدد المعادلات

يمكن وضع اي عدد في x_4 و x_2 لمطالع الاجابة وراح تطلع

عدد لانهائي من الاجوبة

= infinitly many solution

Ex: $3x_1 + 2x_2 + x_3 = 3$

$2x_1 + x_2 + x_3 = 0$

$6x_1 + 2x_2 + 4x_3 = 6$

sol.

$$\begin{array}{l} \quad \text{Pivot} \\ \hat{A} = \begin{bmatrix} 3 & 2 & 1 & 3 \\ 2 & 1 & 1 & 0 \\ 6 & 2 & 4 & 6 \end{bmatrix} \end{array} \begin{array}{l} \\ -2/3(Row_1) + Row_2 \\ -2\,Row_1 + Row_3 \end{array}$$

$-\frac{2}{3}\,Row_1 + Row_2$ | $-2\,Row_1 + Row_3$

$\frac{3x}{3} = \frac{2}{3} \rightarrow \cancel{0.66}\ \frac{2}{3}$

Pivot

$$\left[\begin{array}{cccc} 3 & 2 & 1 & 3 \\ 0 & -1/3 & +1/3 & -2 \\ 0 & -2 & 2 & 0 \end{array}\right] \quad -6\,Row_2 + Row_3$$

$$\frac{-1/3(X)}{-1/3} = \frac{-2}{-1/3} \quad \boxed{X=6} \qquad -6\,Row_2 + Row_3$$

$$\left[\begin{array}{cccc} 3 & 2 & 1 & 3 \\ 0 & -1/3 & 1/3 & -2 \\ 0 & 0 & 0 & 12 \end{array}\right]$$

$3X_1 + 2X_2 + X_3 + \cancel{3X_4} = 3$

$-1/3X_2 + 1/3X_3 = -2$

$0 \neq 12$

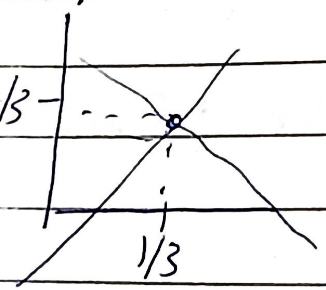
no solution

حالات الاولوية

one solution/unique solution

$X_1 + X_2 = 1 \quad X_1 = 1/3$

$2X_1 - X_2 = 0 \quad X_2 = 2/3$

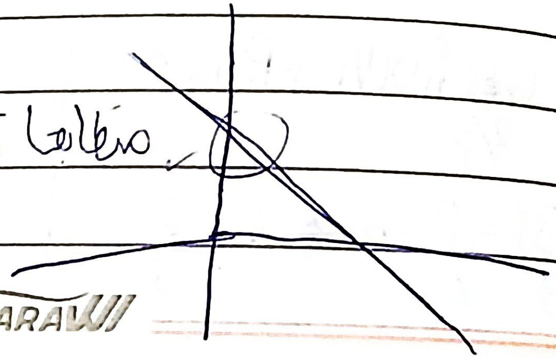


infinitly many solu.

$X_1 + X_2 = 1$

$2X_1 + 2X_2 = 2$

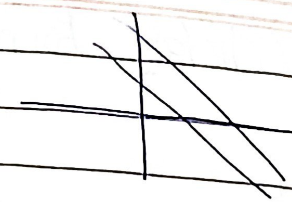
متطابقة



3) No - solution

$X_1 + X_2 = 1$

$X_1 + X_2 = 0$



→ A linear system is called [over determined] if equation more than unkown

$m > n$ → سدرالعامل

b

سدرالطلاك

2) determined $m = n$

3) under determined $m < n$

(#)

* A system is called : 1) consistent

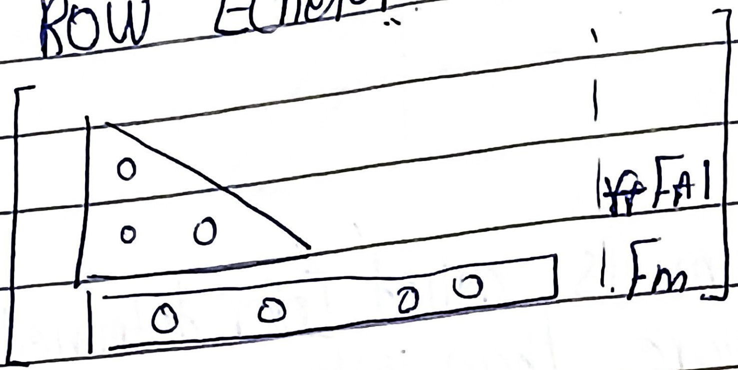
→ one solution

→ infinitely solution

2) in consistent

→ no solution

Row Echelor Form



1) unique solution : $r = n \rightarrow$ كدا الحل وحيد

كدا الصفوف اللي هاتصفرن F_{r+1} to F_m = zero

2) infintely many solution : $r < n$

F_{r+1} to F_m = zero

3) No solution $r < m \rightarrow$ كدا الحطاكن

F_{r+1} to F_m is non zero

١٢٥
١٢٥
٢٤٥

٧٥

Rank of a matrix   رتبة المصفوفة

1) جاوس

2) عدد الصفوف التي لم تصفر = rank

Ex: $A = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$ $2Row_1 + Row_2$, $-7Row_1 + Row_3$

$\begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 42 & 28 & 58 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $Row_3 + 1/2\, Row_2$

rank A = 2

Pivot

Ex: $\begin{bmatrix} 2 & 5 & 2 \\ -4 & 3 & -30 \end{bmatrix}$ $2Row_1 + Row_2$

vector 1

$\begin{bmatrix} 2 & 5 & 2 \\ 0 & 13 & -26 \end{bmatrix}$ rank = 2

vector 2

(independent)

n = 2   Vector = Rnk

rank A = 2   2 = 2

rank (AB) = 2 → unique solution

→ Rank = vector → independent
Rank < vector → dependent
إذا لم يكن هناك أي صف صفري بعد عملية جاوس
(independent)

2) A and A^T have the same rank

3) → unique solution → rank(A) = rank($\bar{A}$) = n
لـ عدد المجاهيل

→ infinitely many solution → rank(A) = rank($\bar{A}$) < n

→ no solution → rank(A) ≠ rank($\bar{A}$)

* Determinants:-

$$D = \det A = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & & \vdots \\ a_{m1} & & a_{m} \end{vmatrix}$$

- second-order Determinants:-

$$D = \det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = (a_{11}a_{22}) - (a_{21}a_{12})$$

- Third order Det

← اختار صفاو عمود الاول

نختار لوحة

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\rightarrow a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

EX:

$$D = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix} = 1\begin{bmatrix} 6 & 4 \\ 0 & 2 \end{bmatrix} - 3\begin{bmatrix} 2 & 4 \\ -1 & 2 \end{bmatrix} + 0\cancel{\begin{bmatrix} 2 & 6 \\ -1 & 0 \end{bmatrix}}$$

$$= 1((6)(2) - (4)(0)) - 3((2)(2) - (4)(-1))$$

$$= 12 - (12 + 12) = 12 - 24 = -12$$

⟹ minors and cofactors:-

$$C_{jk} = (-1)^{j+k} m_{jk}$$

$$D = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Minor:

$$m_{11} = \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} \quad m_{12} = \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} \cdots\cdots \quad m_{33} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

cofactor:

$$C_{11} = (-1)^{1+1} m_{11} = +1\, m_{11} = m_{11}$$

$$C_{12} = (-1)^{1+2} m_{12} = -m_{12} \;\cdot\cdot\; C_{33} = (-1)^{3+3} m_{33} = +m_{33}$$

الشكل العام لل cofactor التسهيل →

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

EX: $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$

Find Cofactor matrix

$\rightarrow \text{cof}A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$

1. $m_{11} = \begin{bmatrix} -1 & 1 \\ 3 & 4 \end{bmatrix} = -4 - 3 = -7$
2. $m_{12} = \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix} = 12 - (-1) = 12 + 1 = 13$
3. $m_{13} = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$ ~~$(4)(1) - 8$~~ $(4) - 6 = 8$ $= -2$
4. $m_{21} = -4 - 2 - 6 = 2$
5. $m_{22} = -2$
6. $m_{23} = -2$
7. $m_{31} = 3$
8. $m_{32} = -7$
9. $m_{33} = 2$

$\rightarrow A = \begin{bmatrix} -7 & -13 & 8 \\ 2 & -2 & 2 \\ 3 & 7 & -2 \end{bmatrix}$

$\Rightarrow$ Adjoint (Adjugate) matrix:

$\text{adj} A = (\text{cof} A)^T$

$A(\text{adj}A) = \text{adj}(A)(\det A) I$

* 1) Determinant (tringular matrix / diagonal matrix / scaler / I)

فقط نضرب قطر المصفوفة

2)

EX:- $\begin{bmatrix} -3 & 0 & 0 \\ 6 & 4 & 0 \\ -1 & 2 & 5 \end{bmatrix} = -3 * 4 * 5 = -60$

نضرب فقط القطر

← اذا بدلنا صف مع صف نضرب ال det بـ -1.

←

$\rightarrow 2 * \begin{bmatrix} -3 & 0 & 0 \\ 6 & 4 & 0 \\ -1 & 2 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} -6 & 0 & 0 \\ 12 & 8 & 0 \\ -2 & 4 & 10 \end{bmatrix} = -6 * 8 * 10 = -480$

للتعديل det

$(\text{عدد الصفوف})^2 * -60 = -480$

$(2)^2 * -60 = -480$

$\begin{bmatrix} -3 & 0 & 0 \\ 6 & 4 & 0 \\ -1 & 2 & 5 \end{bmatrix}^T = \begin{bmatrix} -3 & 6 & -1 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{bmatrix} \rightarrow \det = -3 * 4 * 5 = -6$

← اذا عملية ال (T) لا تؤثر على قيمة ال det

← اذا كان عندي صف او عمود كامل يساوي صفر فان ال det = 0

←

Gauss (3 بدل الـ Pivot ونعوض اللي تحته)

2. نجيب الـ det

EX ⇒ $D = \begin{vmatrix} 2 & 0 & -4 & 6 \\ 4 & 5 & 1 & 0 \\ 0 & 2 & 6 & -1 \\ -3 & 8 & 9 & 1 \end{vmatrix}$ $\begin{matrix} \rightarrow -2\,Row_1 + Row_2 \\ \\ 1.5\,Row_1 + Row_4 \end{matrix}$

Pivot

$\frac{2(X)}{2} = \frac{-3}{2}$

$X = -3/2$

$\begin{vmatrix} 2 & 0 & -4 & 6 \\ 0 & 5 & 9 & -12 \\ 0 & 2 & 6 & -1 \\ 0 & 8 & 3 & 10 \end{vmatrix}$ $\begin{matrix} -0.4\,Row_2 + Row_3 \\ Row_4 - 1.6\,Row_2 \end{matrix}$ Pivot

$\begin{vmatrix} 2 & 0 & -4 & 6 \\ 0 & 5 & 9 & -12 \\ 0 & 0 & 2.4 & 3.8 \\ 0 & 0 & -11.4 & 29.2 \end{vmatrix}$ Pivot

$\frac{5X}{5} = \frac{2}{5} \rightarrow X = 0.4$

$4.75\,Row_3 + Row_4$

$2.4X = -11.4$

$X = 4.75$

$\begin{vmatrix} 2 & 0 & -4 & 6 \\ 0 & 5 & 9 & -12 \\ 0 & 0 & 2.4 & 3.8 \\ 0 & 0 & 0 & 47.25 \end{vmatrix} \rightarrow \det = 2 * 5 * 2.4 = 1134$

⟹ Cramer's Rule

1) $a_{11}X_1 + a_{12}X_2 = b_1$

2) $a_{21}X_1 + a_{22}X_2 = b_2$

→ $X_1 = D_1 / D$ (det)

$X_2 = D_2 / D$

$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

$D_1 = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}$

$D_2 = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}$

Ex: $4X_1 + 3X_2 = 12$

$2X_1 + 5X_2 = -8$

$X_1 = D_1/D = 84/14 = 6$

$X_2 = D_2/D = 56/6 = -4$

$D = \begin{vmatrix} 4 & 3 \\ 2 & 5 \end{vmatrix} = 4(5) - 3(2) = 14$

$D_1 = \begin{vmatrix} 12 & 3 \\ -8 & 5 \end{vmatrix} = 12(5) - (3(-8)) = 84$

$D_2 = \begin{bmatrix} 4 & 12 \\ 2 & -8 \end{bmatrix} = 4(-8) - (12(2)) = -56$

If the system is homogenous (all b=0) and $D \neq 0$:-

then the solution

$X_1 = 0, X_2 = 0, X_3 = 0$

⇒ Inverse of a matrix:
It has be squar matrix?

- $A A^{-1} = A A^{-1} = I$ $AB \neq BA$
→ A has a matrix then A is called a non-singular (invertible) matrix
→ if A has inverse then the an inverse is uniqule
→ A has an inverse if rank = n
→ A has an inverse if det ≠ 0

$$\Rightarrow A^{-1} = 1/\det A \, (C)^T$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ is } A^{-1} = 1/\det \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{22} \end{bmatrix}$$

$$[\mathrm{Cof}(A)]^T = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{22} \end{bmatrix}$$

Ex:

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$ Find A^{-1}:

$$A^{-1} = \frac{1}{\det A} * [\mathrm{Cof}]^T$$

$$D = (3)(4) - (1)(2) = 10$$

$$[\mathrm{Cof}]^T = \begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{10} \begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 0.4 & -0.1 \\ -0.2 & 0.3 \end{bmatrix}$$

Ex:

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$ Find the inverse of:

$$A^{-1} = 1/\det * [\mathrm{Cof}]^T$$

$C_{11} = -7$ $C_{21} = 2$ $C_{31} = 3$
$C_{12} = -13$ $C_{22} = -2$ $C_{32} = 7$
$C_{13} = 8$ $C_{23} = 2$ $C_{33} = -2$

$$D \Rightarrow (-1)\begin{vmatrix} -1 & 1 \\ 3 & 4 \end{vmatrix} - (1)\begin{vmatrix} 3 & 1 \\ -1 & 4 \end{vmatrix} + (2)\begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix}$$

$$= (-1)(-4-3) - (1)(12-1) + (2)(9-(+1))$$

$$= 10$$

$$A^{-1} = 1/10 \begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix} = \begin{bmatrix} -0.7 & 0.2 & 0.3 \\ -1.3 & -0.2 & 0.7 \\ 0.8 & 0.2 & -0.2 \end{bmatrix}$$

Ex:

$$\begin{bmatrix} -0.5 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ diagonal matrix A^T?? →

$$= \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$1/0.5$ $1/4$ $1/1$

* Solution system for linear equation: using matrix inver

EX

$$x_1 + x_2 - x_3 = 3$$
$$-x_1 + x_2 + x_3 = -1$$
$$x_1 - x_2 + x_3 = 5$$

$$AX = b \quad .1$$
$$A^{-1}AX = A^{-1}b \quad .2$$
$$X = A^{-1}b \quad 3$$

$$A^{-1} = 1/\det [C]^T \rightarrow D = \begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 4$$

$$C = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 2 \\ 2 & 0 & 2 \end{bmatrix} \rightarrow [C]^T = \begin{bmatrix} 2 & 0 & 2 \\ 2 & 2 & 0 \\ 0 & 2 & 2 \end{bmatrix} * \frac{1}{4}$$

$$A^{-1} = \begin{vmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{vmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = A^{-1} b = \underset{3 \times 3}{\begin{vmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{vmatrix}} \underset{3 \times 1}{\begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix} \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$x_1 = 4 \quad x_2 = 1 \quad x_3 = 2$$