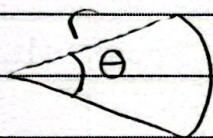
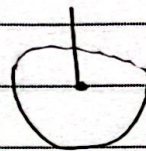
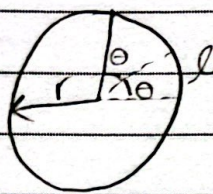


* Circular Motion :

 $r = \text{radius}$ 

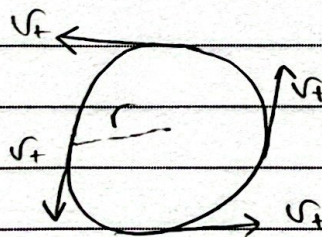
l arc length

طول القوس

$$l = r\theta \leftarrow \theta \text{ in radians (rad)}$$

$$30^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{6} \text{ rad} \quad \text{حيث } 3.14$$

$$\theta \times \frac{\pi}{180^\circ} \leftarrow \text{لما بدى تحول من درجات للراديان}$$

 v_t : tangential Velocity

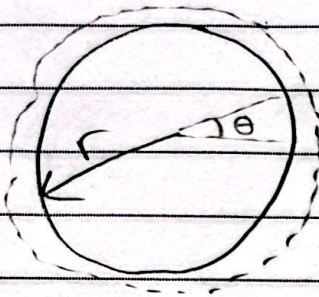
سرعة مماسية

* Uniform Circular motion $\rightarrow |v_t|$ is constant
ثابتة كمقدار
لكنها متغيرة كاتجاه

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

a_c = Centripetal acceleration
تسارع مركزي
ودائماً باتجاه وتجاه المركز

$$a_c = \frac{v^2}{r} = \frac{(\text{السرعة المماسية})^2}{\text{نصف قطر}}$$



دورة واحدة

$$1 \text{ revolution (rev)} = 2\pi \text{ rad}$$

$$\text{distance} \rightarrow l = r\theta$$

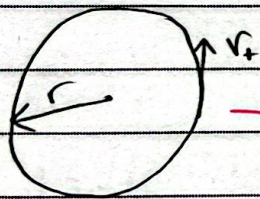
$$l = 2\pi r \leftarrow \text{لوكمل دورة كاملة}$$

⇒ Time to complete one revolution (T) Periodic time الزمن الدوري

$$T = \frac{\text{distance}}{\text{velocity}} = \frac{2\pi r}{v}$$

$$\frac{\pi r}{v} = \frac{1}{2} (2\pi r) \text{ نصف دورة يعني } * \text{ مثلاً لو دار } \frac{1}{2} \text{ دورة يعني}$$

$$\frac{4\pi r}{v} = 2 (2\pi r) \text{ لو دار } 2 \text{ دورة يعني}$$



$$\omega = \frac{2\pi \text{ rad}}{T} \text{ (angular velocity)}$$

السرعة الزاوية

$$\omega = \frac{2\pi}{T} \text{ كمقدار}$$

كاتجاه يا إما للداخل أو للخارج حسب الجسم كيف يتحرك

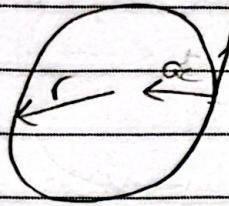
للداخل ← إذا كان يتحرك مع عقارب الساعة

للخارج ← إذا كان يتحرك عكس عقارب الساعة

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{\frac{2\pi r}{v}} = \frac{v}{r} \text{ تقاس بوحدة rad/s}$$

$$* a_c = \frac{v_t^2}{r} = \frac{(r\omega)^2}{r} = r^2 \omega^2 / r = r\omega^2 \rightsquigarrow a_c = r\omega^2$$

⇒



a_t : tangential acceleration

تكون تتغير اتجاه السرعة المماسية وعمودياً على التماسي المركزي
if the motion is uniform

v_t is constant

$$a_t = \left| \frac{dv_t}{dt} \right| = 0$$

none-uniform $\rightarrow r(t)$

إذا الحركة ثابتة

السرعة ثابتة

التسارع صفر

لما يبدى أحسب التسارع كما ذكرنا ثابتة نبسط السرعة ونضع عند t التي بنينا إياها

⇒ a_r : radial acceleration

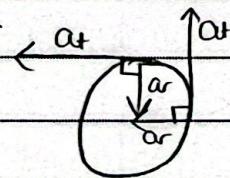


هو تسارعي مقدار قسماً بمقدار التماسي المركزي

$$|a_r| = |a_c| = \frac{v^2}{r}$$

اتجاههم عكس بعض $a_r = -a_c = -\frac{v^2}{r}$

$$|a^{total}| = \sqrt{|a_r|^2 + |a_t|^2}$$



Ex: A car leaves a stop sign and exhibits a constant acceleration of 0.3 m/s^2 parallel to the roadway. The car passes over a rise in the roadway such that the top of the rise is shaped like an arc of a circle of radius 500 m . At the moment the car is at the top of the rise, its velocity vector is horizontal and has a magnitude of 6 m/s . What are the magnitude and direction of the total acceleration vector for the car at this instant?

$$a^{total} = a_t + a_r \rightarrow a_r = -\frac{v^2}{r} = -\frac{36}{500} \text{ m/s}^2$$

$$a^{total} = \sqrt{(0.3)^2 + \left(-\frac{36}{500}\right)^2} = \dots \text{ m/s}^2$$

$$\tan \theta = \frac{a_r}{a_t}$$

$$\theta = \tan^{-1} \left(\frac{a_r}{a_t} \right)$$

$$= \tan^{-1} \left(\frac{-36/500}{0.3} \right)$$

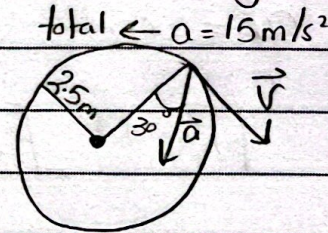
Ex: A tire 0.5 m in radius rotates at a constant rate of 200 rev/min. Find the speed and acceleration of a small stone lodged in the tread of the tire (on its outer edge).

$$200 \frac{\text{rev}}{\text{min}} \rightarrow \omega = 200 \frac{\text{rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} \approx 20 \text{ rad/s}$$

$$\omega = \frac{v}{r} \rightarrow v = r\omega = (0.5)(20) = 10 \text{ m/s}$$

$$a_c = \frac{v^2}{r} = \frac{100}{0.5} = 200 \text{ m/s}^2$$

Ex: Figure P4.40 represents the total acceleration of a particle moving clockwise in a circle of radius 2.5 m at a certain instant of time. For that instant, Find (a) the radial acceleration of the particle, (b) the speed of the particle, (c) its tangential acceleration?



$$a_t = |a| \sin 30$$

$$= 15 \times \frac{1}{2} = 7.5 \text{ m/s}^2$$

$$a_r = |a| \cos 30^\circ = 15 \times \frac{\sqrt{3}}{2} = 12.99 \text{ m/s}^2$$

$$a_r = \frac{v^2}{r}$$

$$v^2 = a_r r$$

$$v = \sqrt{2.5(12.99)} \text{ m/s}$$

* laws of motion (Newton's laws of motion) قوانين نيوتن للحركة

Particle $\rightarrow v, a, \Delta x$

action $\rightarrow$ Force

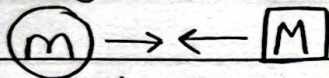
gravitational force قوة الجاذبية

electromagnetic force قوة الكهرومغناطيسية

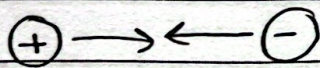
weak nuclear force القوة النووية الضعيفة

Strong nuclear force القوة النووية القوية

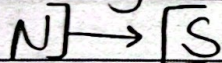
gravitational



electric



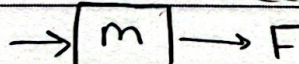
magnetic



$\Rightarrow$ masses : kilogram (kg) $[m] \leftarrow$ Property of an object

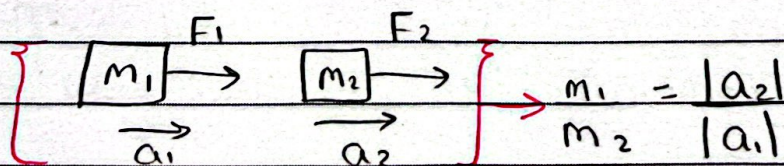
$\uparrow$ resistance to external action
"force"

$\rightarrow$ acceleration



$$m \propto \frac{1}{a}$$

الكتلة تتناسب عكسياً مع التسارع



* Force $\rightarrow$ Vector

F_1 F_2

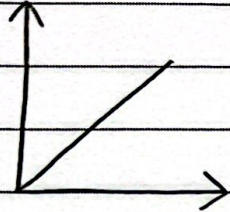
الحالة جمع

F_2

F_1

الحالة طرح

ومع اتجاه الأكبر

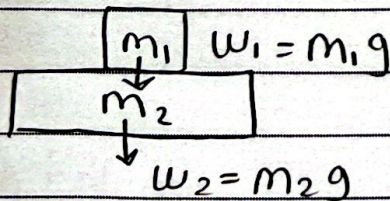
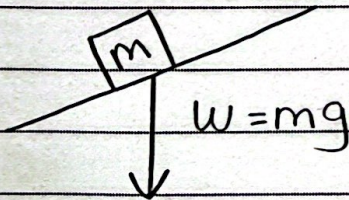
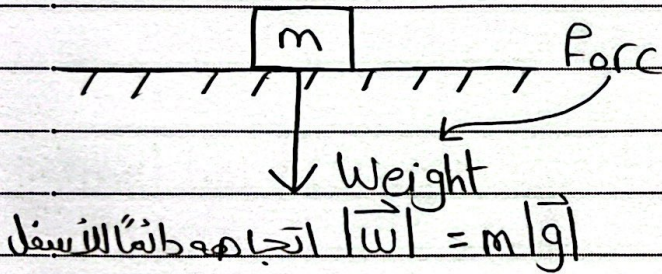


يمكن الزاوية 90°

يكون بالضعف

$$\sqrt{F_1^2 + F_2^2}$$

1) gravitational force : قوة الجاذبية



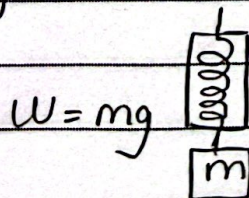
أو يعتبرها جسم واحد

$$W^{total} = m_1g + m_2g$$

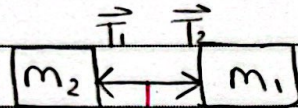
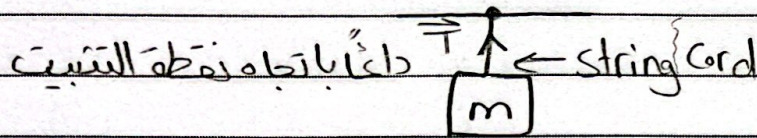
$$W^{total} = (m_1 + m_2)g$$

* to measure weight of a mass "m" ← لقياس الوزن لأي كتلة

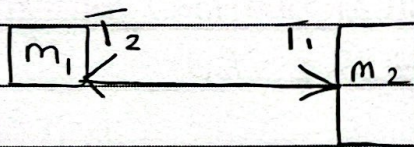
يستخدم ميزان زنبركي Spring Scale



2) Tension Force (T) قوة الشد

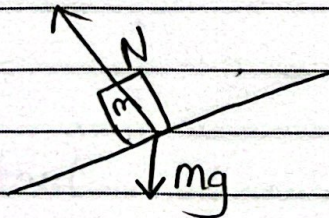
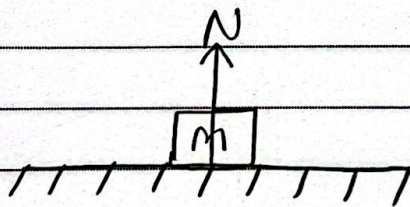


T is Property of string
خاصية الشد



$$|T_1| = |T_2|$$

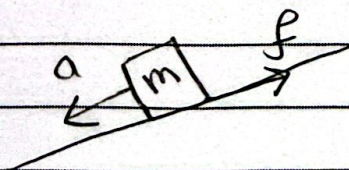
3) Normal Force (N) $\rightarrow$ Contact Force قوة التماس



4) Friction Force (f) قوة الاحتكاك $\rightarrow$ Property Surface

$$f \propto N$$

rough surfaces أسطح خشنة



$$\Rightarrow \vec{W} = m\vec{g}$$

$$= \text{Kg} \cdot \text{m/s}^2 = \text{Newton (N)}$$

يمكن تقيسها بـ KiloNewton أو مئة نيوتن

اسم تكون وحدة القياس Newton

* Newton's laws :

1) An Object at rest remains at rest الجسم الساكن يبقى ساكن
and an Object in motion Continues in motion with const velocity
والجسم المتحرك يبقى متحركاً في سرعة ثابتة

at rest

m

$$a = 0$$

$$\text{net force } \sum \vec{F} = 0$$

Resultant force

m $\rightarrow v \sim \text{const}$

$$a = 0$$

$$\sum \vec{F} = 0$$

N

m

mg

at rest

$$a = 0$$

$$\sum \vec{F} = 0$$

$$\sum \vec{F} = (N - mg) = 0$$

$$N = mg$$

- at rest or v constant

بعدد الحالة يبقى ساكناً أو

والقوى متساوية ومتعاكسة

at rest

mg

$$|\vec{F}_1| = |\vec{F}_2|$$

$$N = mg$$

2) Acceleration of an object proportional to the net force acting on it and inversely proportional to its mass

التسارع لأي جسم يتناسب طردي مع القوة الكلية المؤثرة عليه ويتناسب عكسي مع كتلته

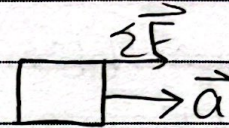
$$a = \frac{\sum \vec{F}}{m}$$

$$\sum \vec{F} = m\vec{a} \rightarrow \text{constant } v$$

Scalar

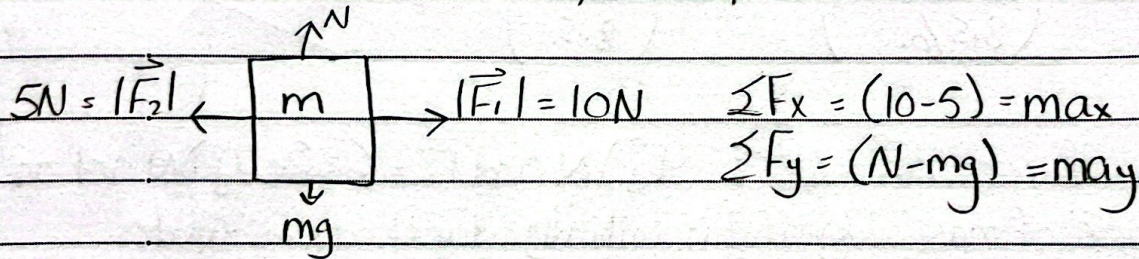
$$a = 0$$

$$\sum F = 0$$

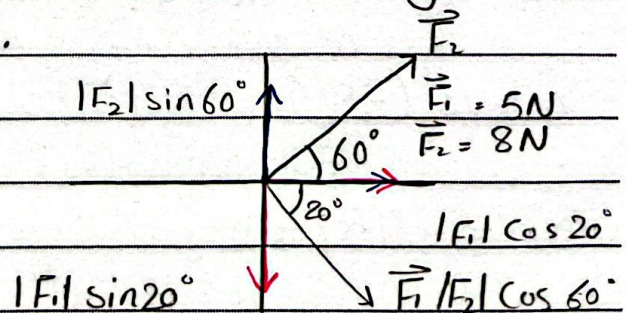


$$\sum \vec{F} = m\vec{a}$$

$$\sum F_x = ma_x, \quad \sum F_y = ma_y, \quad \sum F_z = ma_z$$



*Example 5.1: A hockey puck having a mass of 0.30 kg slides on the frictionless, horizontal surface of an ice rink. Two hockey sticks strike the puck simultaneously, exerting the forces on the puck shown in Figure 5.4. The force $\vec{F}_1$ has a magnitude of 5 N, and is directed at $\theta = 20^\circ$ below the x-axis. The force $\vec{F}_2$ has a magnitude of 8 N and its direction is $\theta = 60^\circ$ above the x-axis. Determine both the magnitude and the direction of the puck's acceleration.



$$\sum F_x = m a_x$$

$$|F_1| \cos 20 + |F_2| \cos 60 = 0.3 a_x$$

$$5(0.9) + 8(0.5) = 0.3 a_x \rightarrow 8.5 = a_x$$

$$\sum F_y = m a_y$$

$$|F_2| \sin 60 - |F_1| \sin 20 = 0.3 a_y \rightarrow 8\left(\frac{\sqrt{3}}{2}\right) - 5(0.3) = 0.3 a_y \rightarrow a_y = \frac{5.5}{0.3}$$

$$\sum F_z = m a_z$$

$$N - mg = 0 \rightarrow N = mg = 3M$$

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$\vec{a} = \frac{8.5}{0.3} \hat{i} + \frac{5.5}{0.3} \hat{j} \rightarrow |\vec{a}| = \sqrt{\left(\frac{8.5}{0.3}\right)^2 + \left(\frac{5.5}{0.3}\right)^2} = \dots m/s^2$$

$$\theta = \tan^{-1} \left(\frac{5.5/0.3}{8.5/0.3} \right) = \tan^{-1} \left(\frac{5.5}{8.5} \right)$$

EX: Two forces, $\vec{F}_1 = (-6\hat{i} - 4\hat{j})N$ and $\vec{F}_2 = (-3\hat{i} + 7\hat{j})N$ act on a particle of mass 2 kg that is initially at rest at coordinates $(-2m, +4m)$. (a) What are the components of the particle's velocity at $t=10s$? (b) In what direction is the particle moving at $t=10s$? (c) What displacement does the particle undergo during the first 10s? (d) What are the coordinates of the particle at $t=10s$?

$$1) \vec{v}_F = \vec{v}_i + a t$$

$$\vec{v}_F = 10 \vec{a}$$

$$\vec{v}_F = 10 \left(-\frac{9}{2} \hat{i} + \frac{3}{2} \hat{j} \right)$$

$$\vec{v}_F = -45\hat{i} + 15\hat{j}$$

$$\sum \vec{F} = m \vec{a}$$

$$\vec{F}_1 + \vec{F}_2 = 2\vec{a}$$

$$(-6\hat{i} - 4\hat{j}) + (-3\hat{i} + 7\hat{j}) = 2\vec{a}$$

$$\vec{a} = -\frac{9}{2} \hat{i} + \frac{3}{2} \hat{j}$$

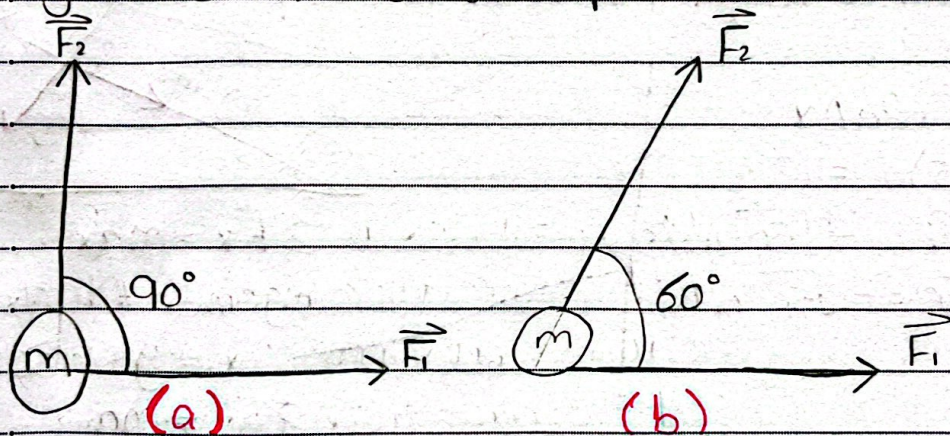
$$2) \tan \theta = \frac{15}{-45} \rightarrow \theta = \tan^{-1} -\frac{1}{3}$$

$$3) \Delta \vec{r} = \vec{v}_i t + \frac{1}{2} a t^2 \rightarrow \Delta \vec{r} = \frac{1}{2} * \left(-\frac{9}{2} \hat{i} + \frac{3}{2} \hat{j} \right) (10)^2 = -\frac{450}{2} \hat{i} + \frac{150}{2} \hat{j}$$

$$4) \Delta \vec{r} = \vec{r}_f - \vec{r}_i \rightarrow \vec{r}_f = \Delta \vec{r} + \vec{r}_i = (-225 \hat{i} + 75 \hat{j}) + (-2 \hat{i} + 4 \hat{j})$$

$$\vec{r}_f = -227 \hat{i} + 79 \hat{j}$$

Ex: Two forces $\vec{F}_1$ and $\vec{F}_2$ act on a 5kg object. Taking $F_1 = 20\text{N}$ and $F_2 = 15\text{N}$, find the accelerations of the object for the configurations of forces shown in parts (a) and (b) of Figure p.5.19.



(b) $\rightarrow$

Free body diagram for (b) showing forces: F_1 (horizontal right), F_2 (at 60°), mg (vertical down), and N (normal force, dashed line).

$$\sum F_x = \max$$

$$|F_2| \cos 60 + |F_1| = 5a_x$$

$$15 \cos 60 + 20 = 5a_x$$

$$a_x = \frac{27.5}{5} \text{ m/s}^2$$

Free body diagram for (a) showing forces: F_1 (horizontal right), F_2 (vertical up), mg (vertical down), and N (normal force, dashed line).

$$\sum F_y = \max$$

$$|F_2| \sin 60 = 5a_y$$

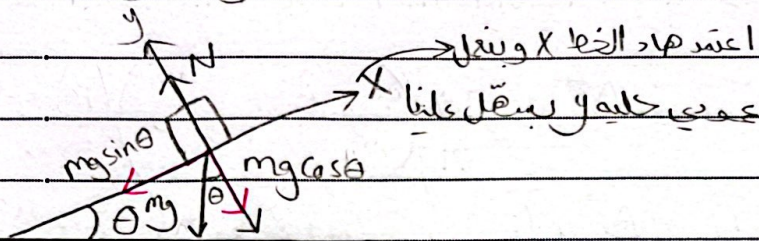
$$15 \sin 60 = 5a_y$$

$$a_y = \frac{3\sqrt{3}}{2} \text{ m/s}^2$$

Final acceleration vector:

$$\vec{a} = \frac{27.5}{5} \hat{i} + \frac{3\sqrt{3}}{2} \hat{j}$$

* Incline (تِلَا):



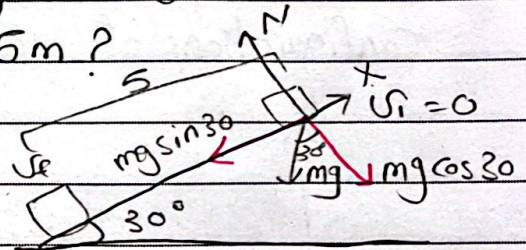
Ex: A car of mass 1000 kg is on icy driveway inclined at angle $\theta = 30^\circ$ with the horizontal, if the car starts from rest at the top of incline, find its velocity at the bottom of the incline knowing that the length of the incline is 5 m?

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$v_f^2 = 2(5)a = 10a$$

$$v_f = \pm \sqrt{50} \text{ m/s}$$

$$v_f = -\sqrt{50} \text{ m/s}$$



$$\sum F_x = \text{max}$$

$$mg \sin 30 = \text{max}$$

$$a_x = g \sin 30 = 5 \text{ m/s}^2$$

$$\sum F_y = \text{may}$$

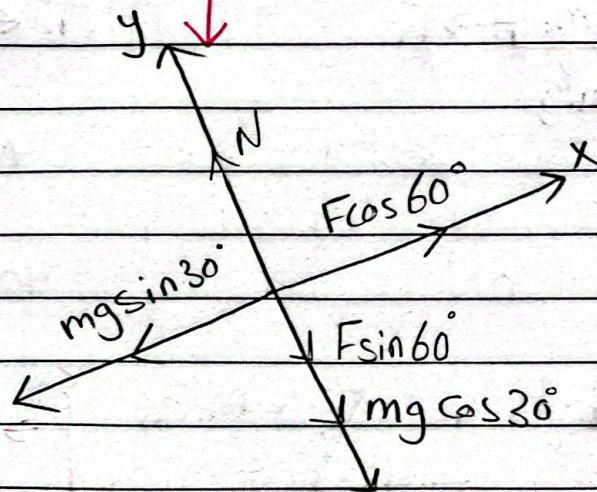
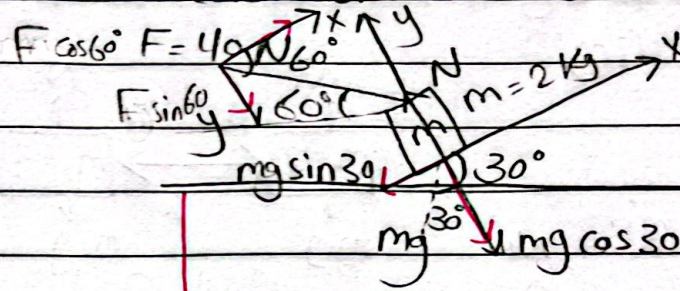
$$N - mg \cos 30 = 0$$

$$N = mg \cos 30$$

$$N = 1000 \times 10 \times \frac{\sqrt{3}}{2}$$

$$N = 5000\sqrt{3}$$

Ex: In the figure, find the acceleration of the block P



$$\sum F_x = m a_x$$

$$F \cos 60 - mg \sin 30 = m a_x$$

$$20 - 10 = 2 a_x$$

$$a_x = 5 \text{ m/s}^2$$

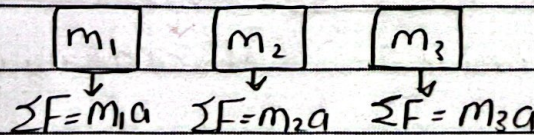
$$\sum F_y = m a_y$$

$$N - (mg \cos 30 + F \sin 60) = 0$$

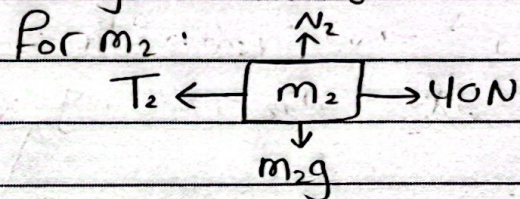
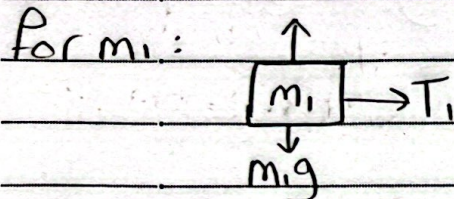
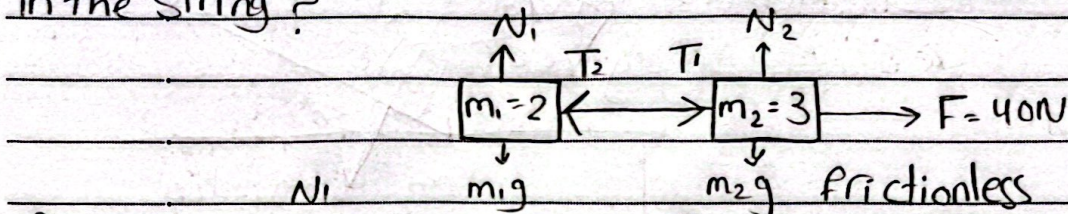
$$N = mg \cos 30 + F \sin 60$$

$$= 10\sqrt{3} + 20\sqrt{3} = 51 \text{ N}$$

* For systems with more than one mass:



Ex: In the figure, find the acceleration of m_1 and the tension in the string?



$$\sum F_y = 0$$

$$N_1 = m_1g$$

$$= 2 \times 10 = 20\text{N}$$

$$T_1 = m_1a_x$$

$$N_2 = m_2g$$

$$= 3 \times 10 = 30\text{N}$$

$$40 - T_2 = m_2a_x$$

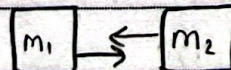
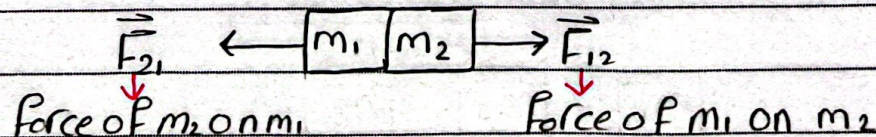
$$\begin{aligned} + T_1 &= m_1a_x \\ - 40 - T_2 &= m_2a_x \end{aligned} \quad \left. \begin{array}{l} \text{Same string} \\ |T_1| = |T_2| \end{array} \right\}$$

$$40 = m_1a_x + m_2a_x$$

$$40 = (m_1 + m_2)a_x$$

$$40 = 5a_x \rightarrow a_x = 8\text{m/s}^2$$

* Newton's Third law (Contact forces قوة التماس):



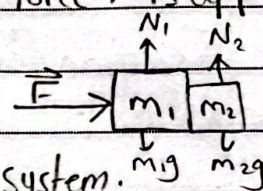
$$|\vec{F}_{12}| = |\vec{F}_{21}|$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

Newton's Third Law

Chapter 5

Ex: Two blocks of masses m_1 and m_2 , with $m_1 > m_2$ are placed in contact with each other on a frictionless, horizontal surface as in Figure 5.12a. A constant horizontal force $\vec{F}$ is applied to m_1 as shown.



A) Find the magnitude of the acceleration of the system.
 $m_1 = 3 \text{ kg}$, $m_2 = 1 \text{ kg}$, $|\vec{F}| = 20 \text{ N}$ عشان نفهم نحل بتقريب ارقام

$$\sum F_x = ma_x$$

$$20 = 4a_x$$

$$a_x = 5 \text{ m/s}^2$$

B) Determine the magnitude of the contact force between the two blocks



$$m_1: \sum F = m_1 a$$

$$m_2: \sum F = m_2 a$$

$$20 - F_{21} = 3a \quad \text{--- ①}$$

$$F_{12} = a \quad \text{--- ②}$$

$$20 - F_{21} = 3a$$

$$|F_{12}| = |F_{21}| \quad \text{كذلك}$$

$$+ F_{12} = a$$

$$20 = 4a$$

$$a = 5 \text{ m/s}^2$$

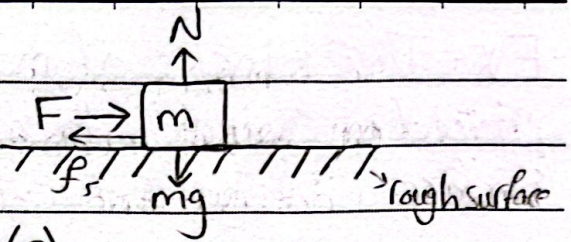
$$F_{12} = 1 \times 5 = 5 \text{ N}$$

$$F_{21} = -5 \text{ N}$$

Find the force from m_2 on m_1 ? $F_{21} = -5 \text{ N}$

magnitude ? 5 N

* Friction Forces :



- Static Friction force (f_s):

$$f_s \propto N$$

$$f_s = \mu_s N$$

μ_s : Coefficient of static friction

Surface Property

معامل الاحتكاك الساكن

خاصية السطح

"about to move" على وشك الحركة

لما ما تحرك (هنا القوة بآثار فيها حتى يهرب

الجسم على وشك الحركة)

- Kinetic Friction force (f_k):

$$f_k = \mu_k N$$

μ_k : Coefficient of kinetic friction

معامل الاحتكاك الحركي

* الخصائص المشتركة لـ μ_s, μ_k :

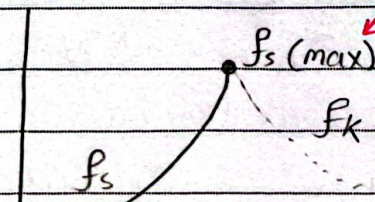
1) μ_s, μ_k are dimensionless بدون وحدات قياس

2) $0 < \mu_s, \mu_k < 1$

Frictionless

(Ideal)

about to move



3) $\mu_s > \mu_k$

4) Property of surface

Ex: The following is a simple method of measuring coefficients of friction. Suppose a block is placed on a rough surface inclined relative to the horizontal as shown in Figure 5.8. The incline angle is increased until the block starts to move. Show that you can obtain μ_s by measuring the critical angle θ_c at which this slipping just occurs

θ_c : Critical angle الزاوية الحرجة

$\sum F_x = 0$ لا تسارع في اتجاه x

$$\sum F_x = \max$$

$$mg \sin \theta - f_s = 0$$

$$mg \sin \theta = f_s$$

$$mg \sin \theta = \mu_s N \quad \text{--- ①}$$

$$\sum F_y = \max$$

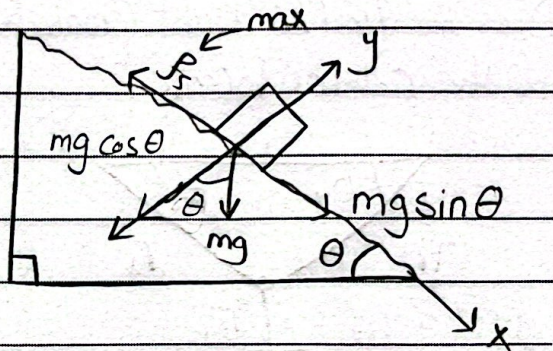
$$N - mg \cos \theta = 0$$

$$N = mg \cos \theta$$

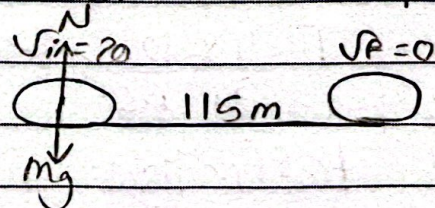
$$mg \sin \theta = \mu_s mg \cos \theta$$

$$(\mu_s)_{\max} = \tan \theta_c$$

$$\theta_c = \tan^{-1}(\mu_s)$$



Ex: A hockey puck on a frozen pond is given an initial speed of 20 m/s. If the puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and ice.



$$\sum F_x = \max$$

$$-f_k = ma$$

$$-\mu_k N = ma$$

$$-\mu_k mg = ma$$

$$\mu_k = -\frac{a}{g} = -\frac{-40}{23} = \frac{4}{23}$$

$$\sum F_y = 0$$

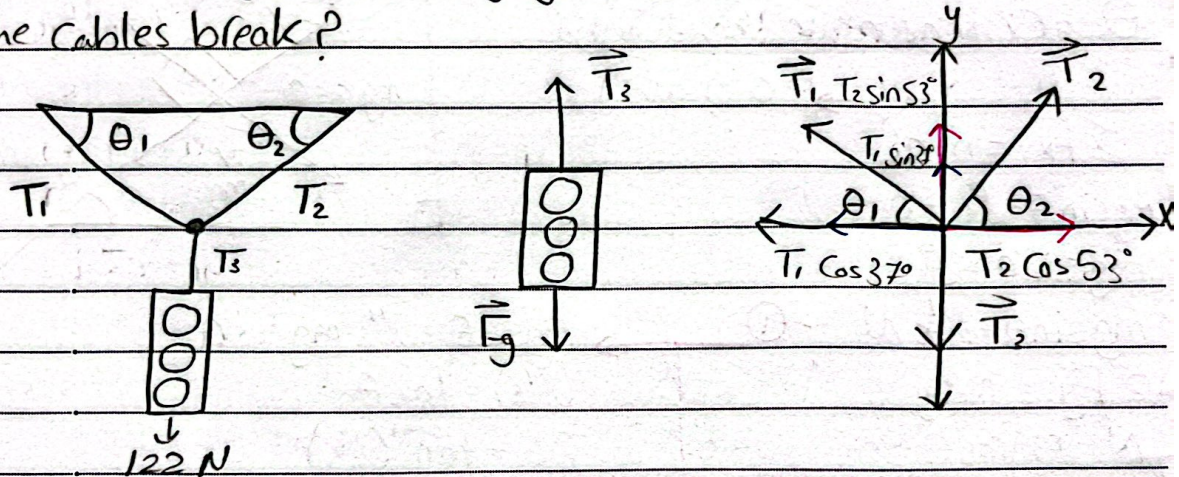
$$N = mg$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$0 = 400 + 230a$$

$$a = -\frac{40}{23} \text{ m/s}^2$$

Ex: A traffic light weighting 122 N hangs from a cable tied to two other cables fastened to a support as in Figure 5.10 a. The upper cables make angles of $\theta_1 = 37^\circ$ and $\theta_2 = 53^\circ$ with the horizontal. These upper cables are not as strong as the vertical cable and will break if the tension in them exceeds 100 N. Does the traffic light remain hanging in this situation, or will one of the cables break?



Stationary is law

$$\sum F = 0$$

$\sum F_x = 0$ (stationary)

$T_2 \cos 53^\circ = T_1 \cos 37^\circ$ equilibrium

$$T_2 (0.6) = T_1 (0.8)$$

$$T_2 = T_1 (0.8)$$

$$T_2 = \frac{4}{3} \times \frac{122}{1.6} > 100$$

$$T_2 = \frac{4}{3} T_1^{0.6} \text{ --- ①}$$

traffic light
will break

$$\sum F_y = 0$$

$$T_1 \sin 37^\circ + T_2 \sin 53^\circ = T_3$$

$$T_1 (0.6) + T_2 (0.8) = 122$$

$$T_1 (0.6) + \frac{4}{3} T_1 (0.8) = 122$$

$$0.6 T_1 + \frac{3.2}{3} T_1 = 122$$

$$T_1 = \frac{122 \times 3}{1.6} \text{ N}$$

Ex: When two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass as in Figure 5.14 a, the arrangement is called an Atwood machine. The device is sometimes used in the laboratory to determine the value of g . Determine the magnitude of the acceleration of the two objects and the tension in the lightweight string.

$$m_1 = 1 \text{ kg and } m_2 = 3 \text{ kg}$$

Find a ? and T ?

$$m_1:$$

$$\sum F = m_1 a$$

$$T_1 - m_1 g = m_1 a$$

$$T_1 - 10 = a \dots \textcircled{1}$$

$$m_2:$$

$$\sum F = m_2 a$$

$$m_2 g - T_2 = m_2 a$$

$$30 - T_2 = 3a \dots \textcircled{2}$$

$$T_1 - 10 = a$$

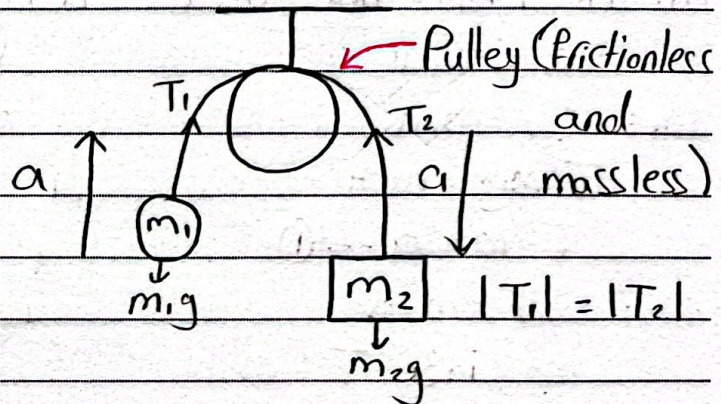
$$+ 30 - T_2 = 3a$$

$$20 = 4a$$

$$a = 5 \text{ m/s}^2$$

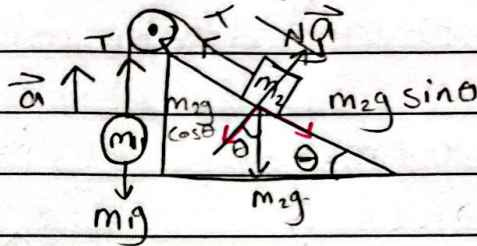
$$T_1 - 10 = 5$$

$$T_1 = 15 \text{ N}$$



Ex: A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass as in Figure 5.15a. The block lies on a frictionless incline of angle θ . Find the magnitude of the acceleration of the two objects and the tension in the

$m_1 = 1 \text{ kg}, m_2 = 4 \text{ kg}, \theta = 30^\circ$



$m_1: \sum F = m_1 a$

$T - m_1 g = m_1 a$

$T - 10 = a \dots (1)$

$m_2: \sum F = m_2 a$

$m_2 g \sin 30^\circ - T = m_2 a$

$40 \left(\frac{1}{2} \right) - T = 4a$

$20 - T = 4a \dots (2)$

$+ T - 10 = a$

$20 - T = 4a$

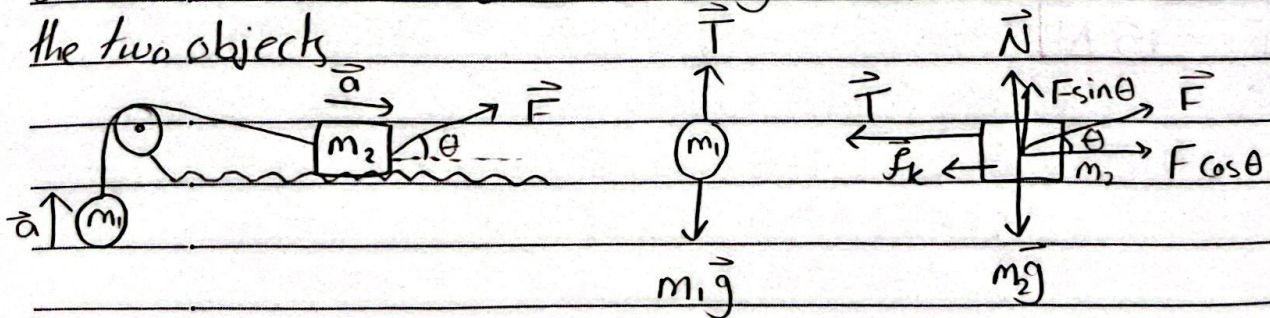
$10 = 5a$

$a = 2 \text{ m/s}^2$

$T - 10 = 2$

$T = 12 \text{ N}$

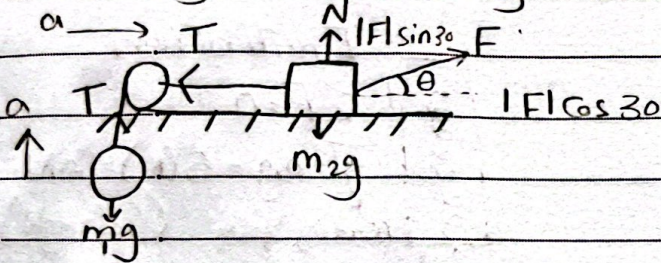
Ex: A block of mass m_2 on a rough, horizontal surface is connected to a ball of mass m_1 by a lightweight cord over a lightweight, frictionless pulley as shown in Figure 5.20a. A force of magnitude F at an angle θ with the horizontal is applied to the block as shown, and the block slides to the right. The coefficient of kinetic friction between the block and surface is μ_k . Determine the magnitude of the acceleration of the two objects.



law of motion

Chapter 5

$$m_1 = 1 \text{ kg}, m_2 = 4 \text{ kg}, |F| = 60 \text{ N}, \theta = 30^\circ, \mu_k = 0.1$$



For m_1 :

$$\sum F = m_1 a$$

$$T - m_1 g = m_1 a$$

$$T - 10 = a \dots \textcircled{1}$$

For m_2 :

$$\sum F = m_2 a$$

$$|F| \cos 30 - (T + f_k) = m_2 a$$

$$60 \left(\frac{\sqrt{3}}{2} \right) - T - \mu_k N = 4a$$

$$N + |F| \sin 30 = m_2 g$$

$$N = m_2 g - |F| \sin 30$$

$$N = 4(10) - 60 \left(\frac{1}{2} \right)$$

$$N = 10 \text{ N}$$

$$51 - T - (0.1)(10) = 4a$$

$$50 - T = 4a \dots \textcircled{2}$$

$$+ \quad T - 10 = a \dots \textcircled{1}$$

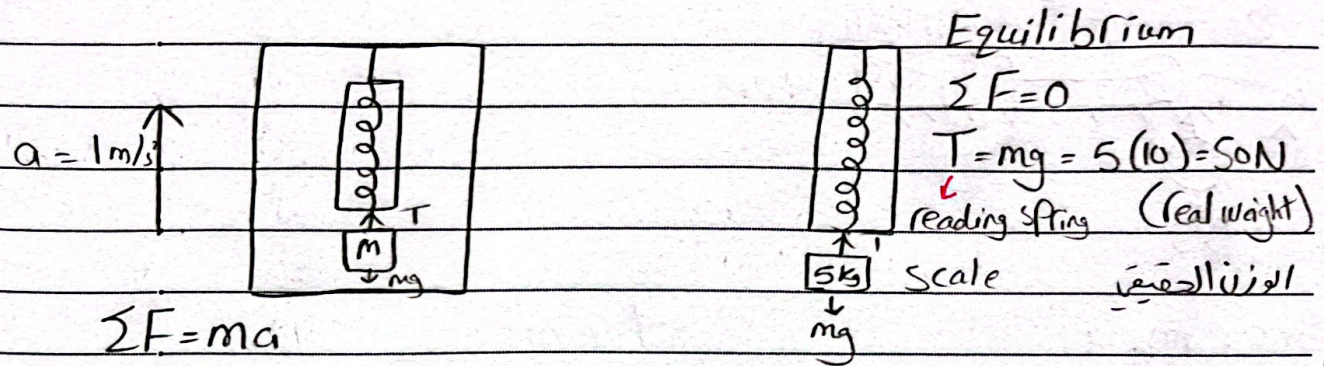
$$40 = 5a$$

$$a = 8 \text{ m/s}^2$$

$$T - 10 = a$$

$$T = 18 \text{ N}$$

*Elevators and scales:



$$\Sigma F = ma$$

$$T - mg = ma$$

$$T = mg + ma$$

الوزن بزيادة مقدار ma

$$T = 50 + 5 = 55 \text{ N}$$

الوزن الظاهري (غير الحقيقي) apparent weight

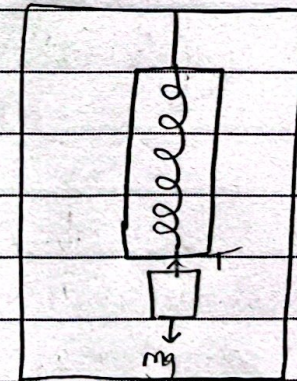
$$\Sigma F = ma$$

$$mg - T = ma$$

$$T = mg - ma$$

$$T = 50 - 5 = 45 \text{ N}$$

الوزن بقل مقدار ma



$$\Sigma F = 0$$

$$N - mg = ma$$

$$N = mg + ma$$

$$N = 600 + 120$$

$$N = 720 \text{ N}$$

Real weight ($a=0$)

$$N = mg$$

$$N = 60 \times 10$$

$$N = 600 \text{ N}$$

Diagram of a person standing on a scale in an elevator. The person has mass $m = 60 \text{ kg}$. The scale reads N . The forces shown are normal force N upwards and weight mg downwards. The elevator is accelerating upwards with $a = 2 \text{ m/s}^2$. The scale is labeled $L=3$ and $G=0$.

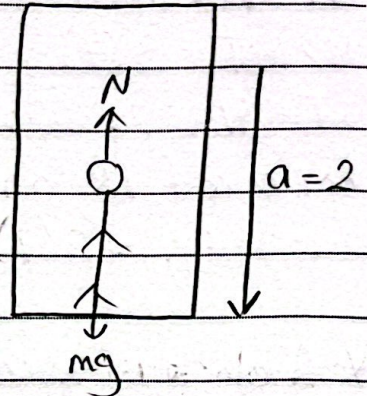
$$\sum F = ma$$

$$mg - N = ma$$

$$N = mg - ma$$

$$N = 600 - 120$$

$$N = 480 \text{ N}$$



* If the elevator is deaccelerating upward by 1 m/s^2 ?

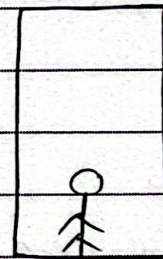
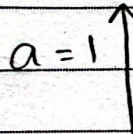
$$\sum F = ma$$

$$N - mg = -ma$$

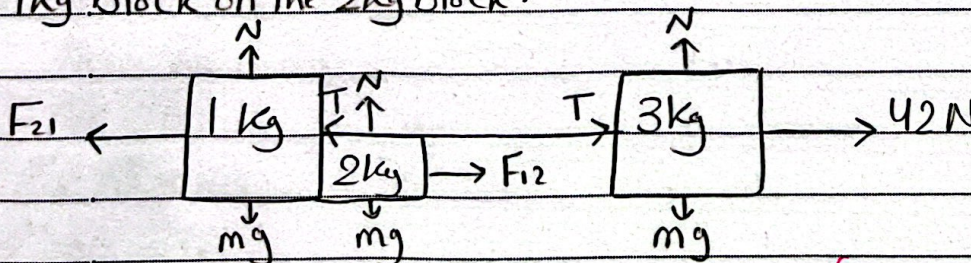
$$N = mg - ma$$

$$N = 600 - 120$$

$$N = 480 \text{ N}$$



* Assume the three blocks portrayed in Figure P5.29 move on a frictionless surface and a 42-N force acts as shown on the 3kg block. Determine (a) the acceleration given this system, (b) the tension in the cord connecting the 3kg and the 1kg blocks, (c) the force exerted by the 1kg block on the 2kg block.



$$\sum F = m_1 a$$

$$T - F_{21} = a \dots \textcircled{1}$$

$$\sum F = m_2 a$$

$$F_{12} = 2a \dots \textcircled{2}$$

$$\sum F = m_3 a$$

$$42 - T = 3a \dots \textcircled{3}$$

$$+ T - F_{21} = a \rightarrow a = 7$$

$$F_{12} = 2a$$

$$\sum F = (m_1 + m_2 + m_3) a$$

$$42 = 6a$$

$$a = 7 \text{ m/s}^2$$

law of motion

Chapter 5

$$42 - T = 3a$$

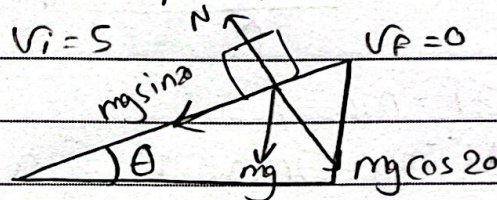
$$42 - T = 3(7)$$

$$T = 42 - 21 = 21 \text{ N}$$

Force of 1 kg on 2 kg (F_{12})

$$F_{12} = 2a = 2(7) = 14 \text{ N}$$

* A block is given an initial velocity of 5 m/s up a frictionless incline of angle $\theta = 20^\circ$. How far up the incline does the block slide before coming to rest?



$$\Sigma F = ma$$

$$-mg \sin 20 = ma$$

$$a = -g \sin 20$$

$$a = -10(0.9)$$

$$a = -9 \text{ m/s}^2$$

$$\Delta x = \frac{v_f^2 - v_i^2}{2a}$$

$$= \frac{0 - 25}{2(-9)}$$

$$\Delta x = \frac{25}{18} \text{ m}$$

$$v_i = 5$$

$$v_f = 0$$

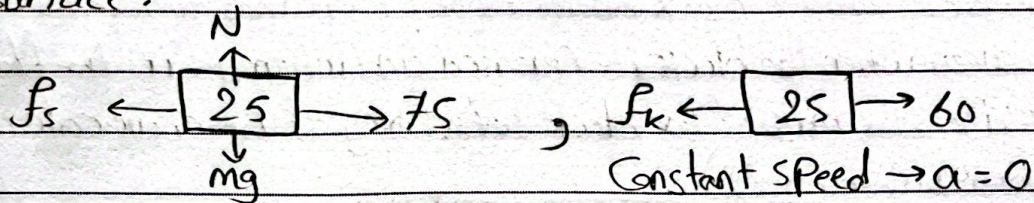
$$v_f^2 = v_i^2 + 2a\Delta x$$

$$0 = 25 + 2a\Delta x$$

$$\Delta x = -25 \dots \textcircled{1}$$

$$2a = ?$$

* A 25 kg block is initially at rest on a horizontal surface. A horizontal force of 75 N is required to set the block in motion, after which a horizontal force of 60 N is required to keep the block moving with constant speed. Find (a) the coefficient of static friction and (b) the coefficient of kinetic friction between the block and the surface.



$$f_s = 75$$

$$\mu_s N = 75$$

$$\mu_s mg = 75$$

$$\mu_s = \frac{75}{250}$$

Constant speed $\rightarrow a = 0$

$$60 - f_k = 0$$

$$f_k = 60$$

$$\mu_k N = 60$$

$$\mu_k mg = 60$$

$$\mu_k = \frac{60}{250} = \frac{6}{25}$$

$\Rightarrow$ If you required 60 N so the block has acceleration of 2 m/s^2

$$\Sigma F = ma$$

$$60 - f_k = ma$$

$$f_k = 60 - ma$$

$$f_k = 60 - 50 \quad \text{هذا الطريقه لو كانت}$$

$$f_k = 10 \quad \text{اذا تسارع}$$

$$\mu_k N = 10$$

$$\mu_k mg = 10$$

$$\mu_k = \frac{10}{250} = \frac{1}{25}$$

* A block of mass $m = 2 \text{ kg}$ is released from rest at $h = 0.5 \text{ m}$ above the surface of a table, at the top of a $\theta = 30^\circ$ incline as shown in Figure. The frictionless incline is fixed on a table of height $H = 2 \text{ m}$. (a) Determine the acceleration of the block as it slides down the incline. (b) What is the velocity of the block as it leaves the incline? (c) How far from the table will the block hit the floor? (d) What time interval elapses between when the block is released and when it hits the floor? (e) Does the mass of the block effect any of the above calculations? (الرسمة سؤال 101 Problem)

$$v_i = 0$$

$$h = 0.5 \text{ m}$$

$$\sin \theta = \frac{h}{d}$$

$$\sin 30^\circ = \frac{0.5}{d} \rightarrow d = 1 \text{ m}$$

$$\sum F = ma$$

$$mg \sin 30^\circ = ma \rightarrow a = 5 \text{ m/s}^2$$

$$v_f^2 = v_i^2 + 2a\Delta r$$

$$v_f^2 = 0 + 2(5)(1) = 10$$

$$v_f = \sqrt{10}$$

$$v_{ix} = v_i \cos 30^\circ$$

$$= \sqrt{10} \left(\frac{\sqrt{3}}{2} \right) = \frac{\sqrt{30}}{2} \text{ m/s}$$

$$v_{iy} = v_i \sin 30^\circ$$

$$= \sqrt{10} \left(\frac{1}{2} \right) = -\frac{\sqrt{10}}{2} \text{ m/s}$$

$$x = R = v_{ix} t$$

$$= \frac{\sqrt{30}}{2} t$$

$$\Delta y = v_{iy} t - \frac{1}{2} g t^2$$

$$0 - 2 = -\frac{\sqrt{10}}{2} t - 5 t^2 \rightarrow t \approx 0.4 \text{ s}$$

$$x = R = \frac{\sqrt{30}}{2} (0.4) = 0.2\sqrt{30} \text{ m}$$

$$\Delta t = 0.62 - 0.4 = 0.22 \text{ s}$$

$$t^{\text{total}} = 1.2 \text{ s}$$

Find t ? $v_i = 0$, $v_f = \sqrt{10}$, $a = 5$ mass has no effect

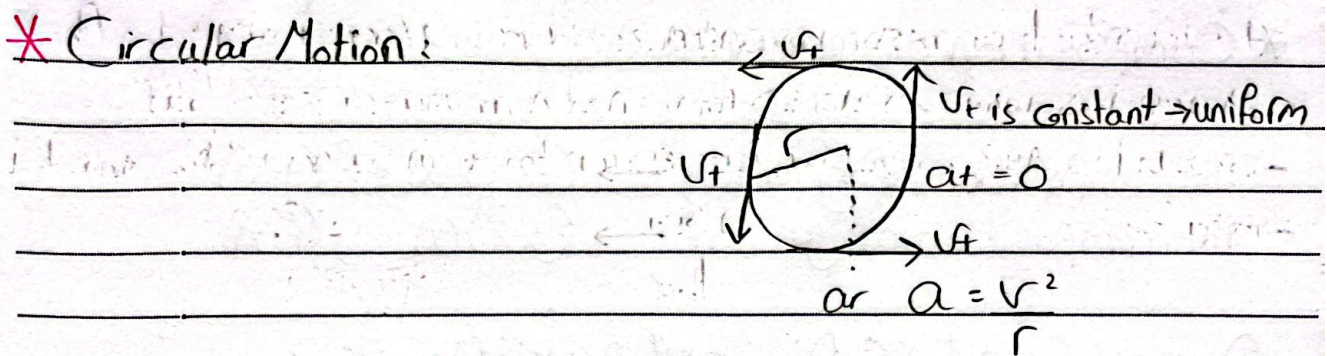
$$v_f = v_i + at$$

$$\sqrt{10} = 5t \rightarrow t = \frac{\sqrt{10}}{5} \approx 0.62 \text{ s}$$

Circular Motion

Chapter 6

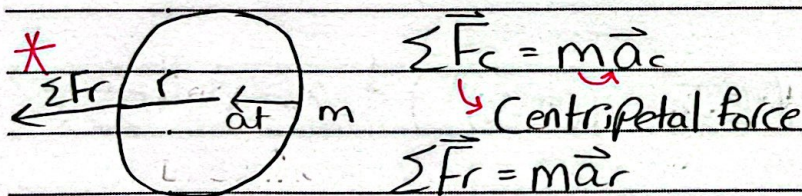
* Circular Motion:



Circular Motion

Uniform
 $\downarrow$
magnitude of v is
constant, v change
its direction $\rightarrow a_c$

None-uniform
 $\downarrow$
 v change both magnitude
and direction

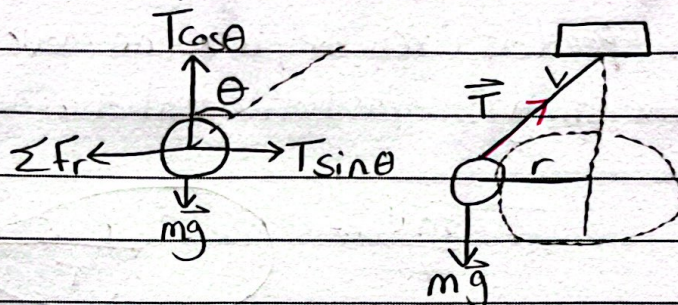


if v is constant $\rightarrow \Sigma F = 0$

Circular Motion

chapter 6

- * A small ball of mass m is suspended from a string of length L . The ball revolves with constant speed v in a horizontal circle of radius r as shown in Figure. Find an expression for v in terms of the geometry in Figure.



v is constant $\rightarrow \sum F_y = 0$

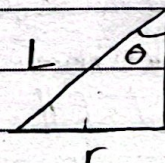
$$\begin{aligned} \therefore T \cos \theta &= mg \\ T \sin \theta &= \frac{mv^2}{r} \end{aligned} \rightarrow \frac{mv^2}{r} = \frac{mg \sin \theta}{\cos \theta}$$

$$\tan \theta = \frac{mv^2}{mg} = \frac{v^2}{rg} \rightarrow v = \sqrt{rg \tan \theta}$$

$$\sin \theta = \frac{r}{L}$$

$$r = L \sin \theta$$

$$v = \sqrt{L \sin \theta g \tan \theta}$$



* A Puck of mass 0.5 kg is attached to the end of a cord 1.5 m long. The Puck moves in a horizontal circle as shown in Figure 6.1. If the cord can withstand a maximum tension of 50 N , what is the maximum speed at which the Puck can move before the cord breaks? Assume the string remains horizontal during the motion.

$$T = \frac{mv^2}{r}$$

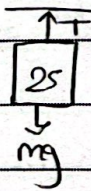

$$v^2 = \frac{rT}{m} \rightarrow v = \pm \sqrt{\frac{rT}{m}}$$

$$= + \sqrt{\frac{1.5(50)}{0.5}} = \sqrt{150} \text{ m/s}$$

* A light string can support a stationary hanging load of 25 kg before breaking. An Object of mass $m = 3 \text{ kg}$ attached to the string rotates on a frictionless, horizontal table in a circle of radius $r = 0.800 \text{ m}$, and the other end of the string is held fixed as in Figure. What range of speeds can the object have before the string breaks?

$$T = mg = 25 \times 10 = 250 \text{ N}$$

$$m = 3 \rightarrow N = mg = 30 \text{ N}$$

$$r = 0.8$$


$$T = \frac{mv^2}{r} \rightarrow v^2 = \frac{rT}{m} = \pm \sqrt{\frac{rT}{m}} = \pm \sqrt{\frac{200}{3}}$$

Circular Motion

Chapter 6

* A car initially travelling eastward turns north by traveling in a circular path at uniform speed as shown in Figure. The length of the arc ABC is 235 m, and the car completes the turn in 36 s. (a) What is the acceleration when the car is B located at an angle of 35° ? Express your answer in terms of the unit vectors $\hat{i}$ and $\hat{j}$. Determine (b) the car's average speed and (c) its average acceleration during the 36 s interval.

$$\text{arc length} = 235 \text{ m}$$

$$t = 36 \text{ s}$$

$$\frac{1}{4} (2\pi r) = 235$$

$$\frac{\pi r}{2} = 235$$

$$r = \frac{470}{\pi} = \frac{470}{3.14} \text{ m} \approx 150 \text{ m}$$

$$a_c = \frac{v^2}{r} \rightarrow v = \frac{d}{t} = \frac{235}{36} = 6.5 \text{ m/s}$$

$$a_c = \frac{v^2}{r} = \frac{42}{150} \approx 0.28 \text{ m/s}^2$$

$$\vec{a}_c = |a_c| \cos 35^\circ \hat{i}$$

$$\vec{a}_c = -0.28 \cos 35^\circ \hat{i} + 0.28 \sin 35^\circ \hat{j}$$

average speed and average acceleration in 36 s?

$$v_{\text{avg}} = \frac{\Delta r}{\Delta t} = \frac{r_f - r_i}{36}$$

$$r_i = r(-\hat{j}) = -150 \hat{j}$$

$$r_f = r(+\hat{i}) = 150 \hat{i}$$

$$v_{\text{avg}} = \frac{150 \hat{i} - (-150 \hat{j})}{36} = \frac{150}{36} \hat{i} + \frac{150}{36} \hat{j}$$

Circular Motion

Chapter 6

$$\text{average speed} = |\vec{v}_{\text{avg}}| = \sqrt{2} \left(\frac{150}{36} \right) \text{ m/s}$$

$$\text{Speed} = \sqrt{\left(\frac{150}{36} \right)^2 + \left(\frac{150}{36} \right)^2}$$

$$\vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{36} \quad a_{\text{avg}} = \frac{7\hat{j}}{36} - \frac{7\hat{i}}{36}$$

$$\vec{v}_i = |\vec{v}| \hat{i} = 7\hat{i}$$

$$\vec{v}_f = |\vec{v}| \hat{j} = 7\hat{j}$$

* A 1500 kg car moving on a flat, horizontal road negotiates a curve as shown in Figure. If the radius of the curve is 35 m and the coefficient of static friction between the tires and dry pavement is 0.523, find the maximum speed the car can have and still make the turn successfully

$$f_s = \frac{mv^2}{R}$$

$$\mu_s N = \frac{mv^2}{R}$$

$$\mu_s (mg) = \frac{mv^2}{R}$$

$$v = \sqrt{Rg\mu_s} = \sqrt{(35)(10)(0.523)} \text{ m}$$

$$\boxed{v_{\text{max}} = \sqrt{rg\mu_s}} \quad \text{Circle Flat}$$

* A civil engineer wishes to redesign the curved roadway in example 6.3 in such a way that a car will not have to rely on friction to round the curve without skidding. In other words, a car moving at the designated speed can negotiate the curve even when the road is covered with ice. Such a road is usually banked, which means that the roadway is tilted toward the inside of the curve as seen in the opening photograph for this chapter. Suppose the designated speed for the road is to be 13.4 m/s (30 mi/h) and the radius of the curve is 35 m . At what angle should the curve be banked?

$$V_{\text{max}} = 13.4 \text{ m/s}$$

$$R = 35 \text{ m}$$

$$\theta = ? \text{ banked angle}$$

$$N \sin \theta = \frac{mv^2}{R}$$

$$\div$$

$$N \cos \theta = mg$$

$$\tan \theta = \frac{v^2}{Rg}$$

$$\Rightarrow \tan \theta = \frac{v^2}{Rg} \rightarrow v = \sqrt{Rg \tan \theta}$$

$$\text{For flat circle } v = \sqrt{Rg}$$

$$\tan \theta_c = \mu_s$$

$$\tan \theta = \frac{(13.4)^2}{35(10)} \rightarrow \theta = \tan^{-1} \left(\frac{(13.4)^2}{350} \right)$$

$$\text{Flat, circle} \rightarrow v_{\text{max}} = \sqrt{Rg \mu_s}$$

$$\text{banked, circle} \rightarrow v_{\text{max}} = \sqrt{Rg \tan \theta}$$

* A child of mass m rides on a Ferris wheel as shown in figure. The child moves in a vertical circle of radius 10m at a constant speed of 3m/s .

A) Determine the force exerted by the seat on the child at the bottom of the ride. Express your answer in terms of the weight of the child.

B) Determine the force exerted by the seat on the child at the top of the ride

$$\text{at top: } N_{\text{top}} + mv^2/r = mg$$

$$N_{\text{top}} = mg - mv^2/r$$

$$\text{at bottom: } mg + mv^2/r = N_{\text{bot}}$$

$$N_{\text{bot}} = ma$$

$$N_{\text{bot}} = mg + mv^2/r \quad \text{if stationary}$$

$$N_{\text{top}} = mg - mv^2/r$$

For high speed

$$N_{\text{bot}} = 20(10) + \frac{20(3)^2}{10} = 218\text{N}$$

$$(N_{\text{top}} < 0)$$

$$N_{\text{top}} = 20(10) - \frac{20(3)^2}{10} = 182\text{N}$$

$$v = 3\text{m/s}, \quad v = 11\text{m/s}$$

$$N_{\text{top}} = mg - \frac{mv^2}{r}$$

$$= 20(10) - \frac{20(11)^2}{10} = -42\text{N}$$

* Consider a conical pendulum with a bob of mass $m = 80 \text{ kg}$ on a string of length $L = 10 \text{ m}$ that makes an angle of $\theta = 5^\circ$ with the vertical. Determine (a) the horizontal and vertical components of the force exerted by the string on the mass m ?

$$T_x = T \sin \theta = \frac{mv^2}{r}$$

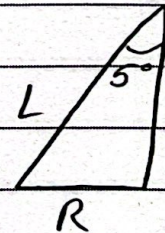
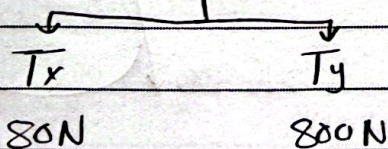
$$T_y = T \cos \theta = mg \leftarrow T_y = mg = 800 \text{ N}$$

$$\tan \theta = \frac{v^2}{rg} \rightarrow v = \sqrt{rg \tan \theta}$$

$$v = \sqrt{L \sin \theta g \tan \theta} = \sqrt{10(0.1)(10)(0.1)} = \sqrt{1} = 1 \text{ m/s}$$

$$T_x = T \sin \theta = \frac{mv^2}{r} = 80 \text{ N}$$

Force exerted by the string on mass



$$\sin 5 = \frac{R}{L}$$

$$R = L \sin 5$$

$$R = 1 \text{ m}$$

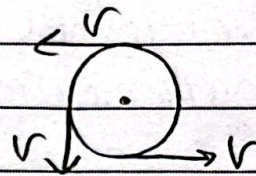
b) Find radial acceleration?

$$a_r = \frac{v^2}{r} = \frac{1}{1} = 1 \text{ m/s}^2$$

* Uniform speed or constant speed $\rightarrow a_t = 0$

$$a_t = 0 = \frac{dv}{dt}$$

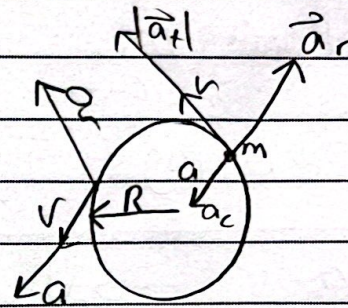
$$\vec{a}^{\text{total}} = \vec{a}_r + \vec{a}_t$$



* Non-uniform Circular motion



v changes both magnitude and direction



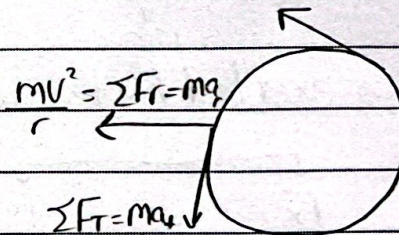
$v(t)$

a_t

$$\sum F_t = ma_t$$

$$\frac{dv}{dt} = a_t$$

$$a_t = \frac{\sum F_t}{m}$$



Ex: A small sphere of mass m is attached to the end of a cord of length R and set into motion in a vertical circle about a fixed point O as illustrated in Figure. Determine the tangential acceleration of the sphere and the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle θ with the vertical.

$$\frac{dv}{dt} \quad a_t \quad \Sigma F_T = ma$$

$$\Sigma F_T = mg \sin \theta = ma_t$$

$$a_t = g \sin \theta$$

$$T - mg \cos \theta = \frac{mv^2}{r}$$

$$T = mg \cos \theta + \frac{mv^2}{r}$$

at any angle except at top and bottom

$$a_t = \frac{dv}{dt} = 0$$

$$\text{at top: } mg + T_{\text{top}} = \frac{mv^2}{r}$$

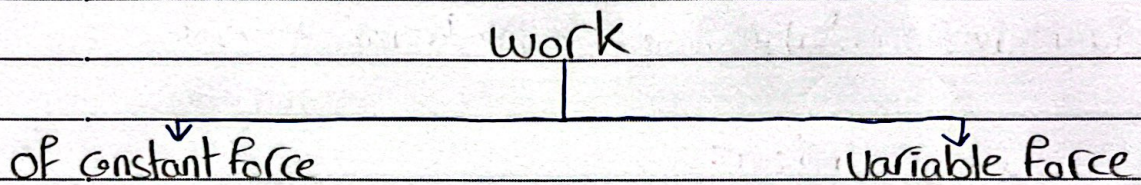
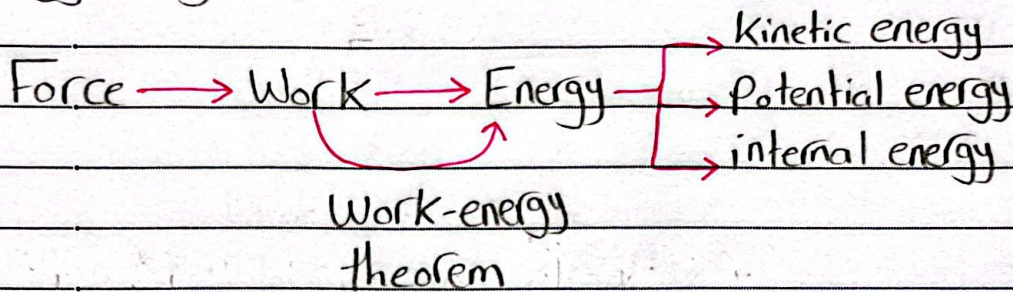
$$T_{\text{top}} = \frac{mv^2}{r} - mg$$

$$\text{at bottom: } mg + \frac{mv^2}{r} = T_{\text{bottom}}$$

$$T_{\text{bottom}} = \frac{mv^2}{r} + mg$$

$$\text{at any other point at angle } \theta \rightarrow T = \frac{mv^2}{r} + mg \cos \theta$$

* Energy of system :



* Dot Product : الضرب النقطي $\vec{A} \cdot \vec{B}$ (النتيجة عبارة عن رقم $\pm$)

$$\begin{aligned} \vec{A} &= A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \\ \vec{B} &= B_x \hat{i} + B_y \hat{j} + B_z \hat{k} \end{aligned} \quad \vec{A} \cdot \vec{B} = A_x B_x (\hat{i} \cdot \hat{i}) + A_x B_y (\hat{i} \cdot \hat{j}) + A_x B_z (\hat{i} \cdot \hat{k}) + A_y B_x (\hat{j} \cdot \hat{i}) + \dots + A_z B_z (\hat{k} \cdot \hat{k})$$

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1 \quad (\text{بسبب التشابهات مع الـ 1 في الجوانب})$$

$$\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = \dots = 0$$

إذا $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = \text{Scalar}$ رقم

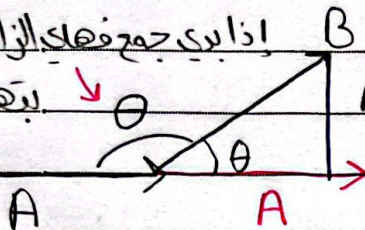
$\Rightarrow \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

لازم المتجهين $\vec{A}$ و $\vec{B}$ يشيران لنفس النقطة حتى نحصل على ضرب نقطي

Diagram showing vector $\vec{A}$ and vector $\vec{B}$ with angle θ between them. The projection of $\vec{B}$ onto $\vec{A}$ is $|\vec{B}| \cos \theta$. The dot product is $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$.

إذا برى جمع هذه الزاوية إلى

بهمين



$$A \cdot B = |A||B| \cos \theta$$

Ex: Let $\vec{A} = 2\hat{i} + \hat{j}$ and $\vec{B} = -\hat{i} - 4\hat{j}$ are two vectors, Find (1) the angle between $\vec{A}$ and $\vec{B}$, (2) the angle between $\vec{A}$ and x-axis, (3) the angle between $\vec{B}$ and y-axis.

$$1) \vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos \theta$$

$$\vec{A} \cdot \vec{B} = (2\hat{i} + \hat{j}) \cdot (-\hat{i} - 4\hat{j}) = (2 \times -1) + (1 \times -4) = -2 - 4 = -6$$

$$|\vec{A}| = \sqrt{(2)^2 + (1)^2} = \sqrt{5}$$

$$|\vec{B}| = \sqrt{(-1)^2 + (-4)^2} = \sqrt{17}$$

$$-6 = \sqrt{5} \times \sqrt{17} \times \cos \theta$$

$$\theta = \cos^{-1} \left(\frac{-6}{\sqrt{85}} \right)$$

$$2) \text{ x-axis} = \hat{i}$$

$$\vec{A} \cdot \hat{i} = |\vec{A}||\hat{i}| \cos \theta$$

$$\vec{A} \cdot \hat{i} = (2\hat{i} + \hat{j}) \cdot \hat{i} = 2 \times 1 = 2$$

$$|\vec{A}| = \sqrt{5}$$

$$|\hat{i}| = 1$$

$$2 = \sqrt{5} \cos \theta$$

$$\theta = \cos^{-1} \left(\frac{2}{\sqrt{5}} \right)$$

$$3) \vec{B} = -\hat{i} - 4\hat{j} \quad \text{y-axis}$$

$$\vec{B} \cdot \hat{j} = |\vec{B}||\hat{j}| \cos \theta$$

$$\vec{B} \cdot \hat{j} = (-\hat{i} - 4\hat{j}) \cdot \hat{j} = -4$$

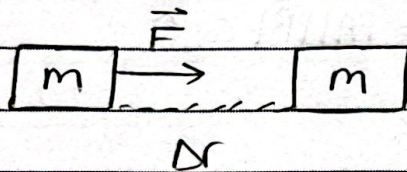
$$|\vec{B}| = \sqrt{17}$$

$$|\hat{j}| = 1$$

$$-4 = \sqrt{17} \cos \theta$$

$$\theta = \cos^{-1} \left(\frac{-4}{\sqrt{17}} \right)$$

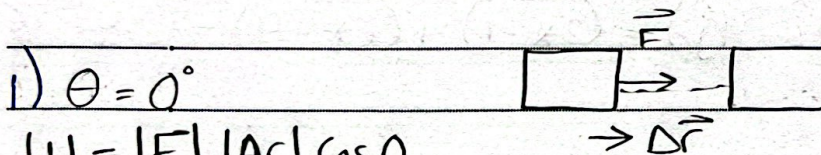
* Work of Constant force :



$$W = \vec{F} \cdot \Delta \vec{r} = |\vec{F}| |\Delta \vec{r}| \cos \theta$$

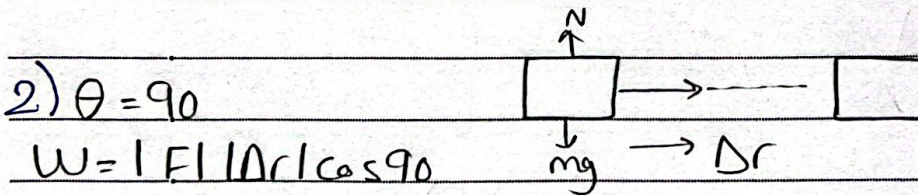
$$\left. \begin{array}{l} \\ \end{array} \right\} \rightarrow W^{\text{total}} = \sum \vec{F} \cdot \Delta \vec{r}$$

$W_{\text{Total}} = W_1 + W_2 + W_3 + W_4 = \text{Scalar (+, -) without direction}$



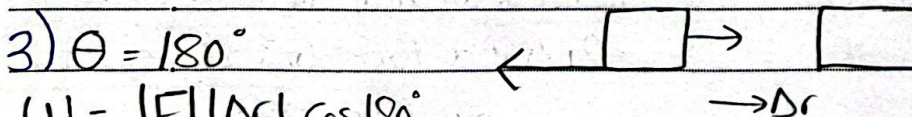
$$W = |\vec{F}| |\Delta \vec{r}| \cos 0$$

$$= |\vec{F}| |\Delta \vec{r}| \leftarrow \text{Maximum}$$



$$W = |\vec{F}| |\Delta \vec{r}| \cos 90^\circ$$

$$= 0$$



$$W = |\vec{F}| |\Delta \vec{r}| \cos 180^\circ$$

$$= -|\vec{F}| |\Delta \vec{r}| \leftarrow \text{Minimum work}$$

* For $\vec{A} = 3\hat{i} + \hat{j} - \hat{k}$, $\vec{B} = -\hat{i} + 2\hat{j} + 5\hat{k}$ and $\vec{C} = 2\hat{j} - 3\hat{k}$.
Find $\vec{C} \cdot (\vec{A} - \vec{B})$.

$$\vec{D} = \vec{A} - \vec{B}$$

$$\vec{D} = (3\hat{i} + \hat{j} - \hat{k}) - (-\hat{i} + 2\hat{j} + 5\hat{k})$$

$$\vec{D} = 4\hat{i} - \hat{j} - 6\hat{k}$$

$$\begin{aligned}\vec{C} \cdot \vec{D} &= (2\hat{j} - 3\hat{k}) \cdot (4\hat{i} - \hat{j} - 6\hat{k}) \\ &= (-2) + (18) = 16\end{aligned}$$

* A particle moving in the xy plane undergoes a displacement given by $\Delta\vec{r} = (2\hat{i} + 3\hat{j})\text{m}$ as a constant force $\vec{F} = (5\hat{i} + 2\hat{j})\text{N}$ acts on the particle. Calculate the work done by $\vec{F}$ on the particle.

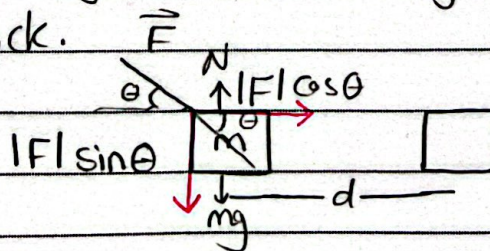
$$W = \vec{F} \cdot \Delta\vec{r}$$

$$= (5\hat{i} + 2\hat{j}) \cdot (2\hat{i} + 3\hat{j})$$

$$= (5 \times 2) + (2 \times 3)$$

$$= 10 + 6 = 16$$

* A block of mass $m = 2.5\text{ kg}$ is pushed a distance $d = 2.20\text{ m}$ along a frictionless, horizontal table by a constant applied force of magnitude $F = 16\text{ N}$ directed at an angle $\theta = 25^\circ$ below the horizontal as shown in Figure. Determine the work done on the block by (a) the applied force, (b) the normal force exerted by the table, (c) the gravitational force, (d) the net force on the block.



Energy of system

Chapter 7

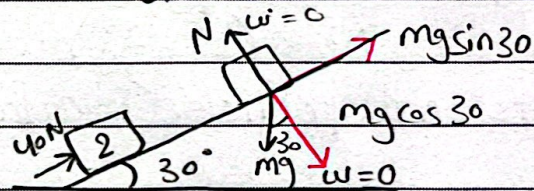
$$\begin{aligned}\vec{F} &= |F| \cos 25 \hat{i} + |F| \sin 25 \hat{j} \\ &= 16 \cos 25 \hat{i} + \underbrace{16 \sin 25 \hat{j}}_{W=0}\end{aligned}$$

$$W_F = |F| \cos \theta d = 16 \cos 25 (2.2) = 35.3 \cos 25 \text{ J}$$

Work of Normal + gravitational = 0

Ex: A 2kg block is pushed by 40N force a distance 2m on an incline of angle 30° with horizontal as shown in the figure, if $\mu_k = 0.1$ of incline, find the net work done on the block?

$$\begin{aligned}F_1 &= 40 \text{ N} \\ f_k &\end{aligned}$$



$$F_2 = mg \sin 30$$

$$\begin{aligned}W_F &= |F| |\Delta r| \cos 0 \\ &= 40 \times 2 = 80 \text{ J}\end{aligned}$$

$$\begin{aligned}W_2 &= |F_2| |\Delta r| \cos 180 \\ &= - (2)(10)(1)(2) = -20 \text{ J}\end{aligned}$$

$$\begin{aligned}W_{f_k} &= -f_k \Delta r = -\mu_k N \Delta r \\ &= -\mu_k mg \cos 30 \Delta r \\ &= -0.1 (2)(10)(\sqrt{3})(2) = -3.4 \text{ J}\end{aligned}$$

$$W_{\text{Total}} = 80 - 20 - 3.4 = 56.6 \text{ J}$$