سلايدات

خرسانة مسبقة الإجهاد

د. عبد الله الغصون

PRESTRESSED CONCRETE
-INTRODUCTION-

The Hashemite University
Department of Civil Engineering
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Spring 2025

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COURSE INFO

Instructor:

➤ Abdullah Alghossoon, PhD, Assistant professor and the director of BESSC (office 3035)

➤ Doctor of philosophy in structural engineering, Purdue University (3rd in Civil Engineering)

➤ Area of interest :

1. High-strength steel.
2. Steel-to-concrete composite structures

➤ Current activities:

➤ Working with the AISC TC5 Committee on CFT design equations

➤ Experimental Research in BESSC (1 Ph.D and 2 M.Sc.)

➤ Publications: Google scholar: Abdullah alghossoon

➤ Practical experience: Structural Design Engineer

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COURSE INFO

Course Description:

- This course covers the fundamentals of prestressed concrete, including prestressing methods, material properties, and structural behavior. Key topics include flexural and shear design, anchorage zone detailing, prestress losses, crack control, and deflection estimation. Students will learn analysis and design techniques for prestressed beams, ensuring structural efficiency and durability in real-world applications.

- Grading:

First Exam	(20 Points)	TBA
Second	(20 Points)	TBA
Project	(20 Points)	Last week of the semester
Extra	(10 Points)	Last week of the semester
Final Exam	(40 Points)	Will be announced by the registrar

- General Notes

- **No make up exams**.

- You are allowed a maximum of 15% absence, which equates to 6 lectures

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MAIN TOPICS

❖Main Topics:

1. Units and Significant Figures
2. Introduction and Overview
3. Basic concept of prestressing
4. Materials
5. Prestressing losses
6. Flexural stresses and service load consideration
7. Ultimate strength design
8. Shear design of prestressed beams
9. Deflection calculations
10. Anchorage zone design

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UNITS AND SIGNIFICANT FIGURES

❖System of Units

- Every mechanical quantity or an algebraic combination of mechanical quantities has units.

- Who cares, as long as I got the correct number

- Incorrect units are often related to incorrect answers (but not necessarily vice versa). Typically, a unit-related error is very large.

- Tracing incorrect units might help with finding errors – at certain step, there can be a mismatch between units – lack of homogeneity (guesswork in choosing equations, adding or missing equation's elements).

❖Note: Incorrect units are considered an error, despite having a correct answer.

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UNITS AND SIGNIFICANT FIGURES

❖Three of the kinetic units referred to as basic units that may be defined arbitrarily are length, time, and mass. However, force is considered as a derived unit.

SI Unit (System International) =Metric = International

Basic units: L (meter), t (second), m (kilogram).

Derived units: $F\left(\text{newton=}N = \dfrac{kg.m}{s^2} \right)$

Imperial Unit = US Unit (only difference in volume measurement)= British

Basic units: f (foot), t (second), lb (pound).

Derived units: $F\left(\text{pound force=}Ibf = \dfrac{slug.ft}{s^2} \right)$

One slug is a mass equal to 32.2 $\underline{lb}$ (14.6 $\underline{kg}$) and weight 32.2 Ibf or 143 N

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UNITS AND SIGNIFICANT FIGURES

❖System of Units (SI System):

The SI system (International System of Units) is the modern metric system of measurement and the dominant system of international commerce and trade. SI units are gradually replacing Imperial (old English unit) and USCS units (The United States Customary System Units are the measuring units used in the U.S. consisting of the mile, inch, gallon, second and pound).

The SI system includes:

• SI base and derived units described in terms of acceptable SI units.

• SI Prefixes.

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SI UNITS AND ENGLISH UNITS

Quantity	Symbol	SI Units	English Units	To Convert from English to SI units Multiply by
Length	L	m	ft	0.3048
Mass	M	kg	Ibm	0.4536
Time	T	s	sec	1
Area	A	m^2	ft^2	0.0929
Volume	V	m^3	ft^3	0.02832
Velocity	V	m/s	ft/sec	0.3048
Acceleration	a	m/s^2	ft/sec^2	0.3048
Angular velocity	ω	Rad/s	$Rad/sec, rpm$	1, 9.55
Force, Weight	F, W	N	Ibf	4.448
Density	Y	kg/m^3	Ibm/ft^3	16.02
Specific weight	g	N/m^3	Ibf/ft^3	157.1
Pressure, Stress	σ	$MPa, N/mm^2$	ksi	6.895
Work, Energy	E,U	$J, N.m$	$ft.Ibf$	1.356
Moment, Torque	M	$N.m$	$ft.Ibf$	1.356

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SI PREFIXES

Prefix	Symbol	Abbreviation
1E+18	exa	E
1E+15	peta	P
1E+12	tera	T
1E+09	giga	G
1E+06	mega	M
1E+03	kilo	k
1E+02	hecto	h
1E+01	deka	da
1E-01	deci	d
1E-02	centi	c
1E-03	milli	m
1E-06	micro	µ
1E-09	nano	n
1E-12	pico	p

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UNIT CONVERSION

❖Example

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SIGNIFICANT FIGURES

❖Significant figures, also known as significant digits or sig figs, are a fundamental concept in mathematics and science that play a crucial role in expressing the precision and accuracy of numerical values. They represent the meaningful digits in a measured or calculated quantity and indicate the reliability of the measurement or calculation.

❖In scientific and engineering calculations, it is essential to understand and apply significant figures correctly to ensure the accuracy of results and maintain consistency in data reporting. By adhering to the rules of significant figures, scientists and engineers can convey the appropriate level of precision in their measurements, calculations, and experimental findings.

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SIGNIFICANT FIGURES

❖Rules for deciding the number of significant figures in a measured quantity:

(1) All nonzero digits are significant: 1.234 g has 4 significant figures, 1.2 g has 2 significant figures.

(2) Zeroes between nonzero digits are significant: 1002 kg has 4 significant figures, 3.07 mL has 3 significant figures.

(3) Leading zeros to the left of the first nonzero digits are not significant; such zeroes merely indicate the position of the decimal point: 0.001 has only 1 significant figure, 0.012 g has 2 significant figures.

(4) Trailing zeroes that are also to the right of a decimal point in a number are significant: 0.0230 mL has 3 significant figures, 0.20 g has 2 significant figures.

(5) When a number ends in zeroes that are not to the right of a decimal point, the zeroes are not necessarily significant: 190 miles may be 2 or 3 significant figures, 50,600 calories may be 3, 4, or 5 significant figures.

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UNITS AND SIGNIFICANT FIGURES

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UNITS AND SIGNIFICANT FIGURES

❖ Rules for deciding the number of significant figures in a measured quantity:

a.	508.0 m	4 significant figures
b.	8304000.0 mm	7 significant figures
c.	1.0200×10^5 kg	5 significant figures
d.	807000 kg	3 significant figures
e.	1.5×10^5 N	2 significant figures
f.	Round 36.4**7**3 to 4 significant figures	36.47
g.	Round 0.4**7**3 to 2 significant figures	0.47
h.	Round 36.0**4**7 to 4 significant figures	36.05
i.	Round **3**6.470 to 1 significant figures	40

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STRUCTURAL SYSTEMS IN CIVIL ENGINEERING

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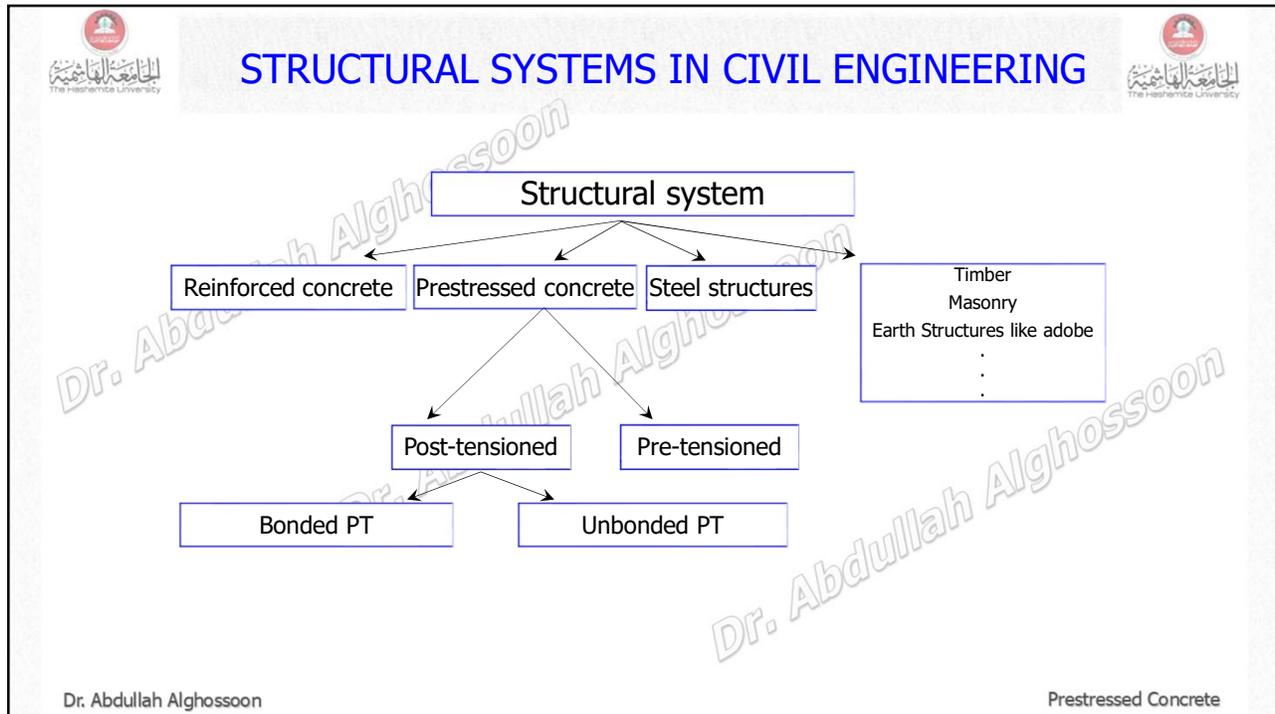
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WHAT IS PRESTRESSING

- **What is prestressing:**
  Generating stresses opposite to that from structure's design loads.

- **Mechanism:**
  Applying tensile force on a group of high strength steel wires passing through the concrete member with a specific path.

  Steel wires profile allow controlling the stresses at the extreme fibers of the concrete member (Eccentricity)

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WHAT IS PRESTRESSING

Before applying prestressing force
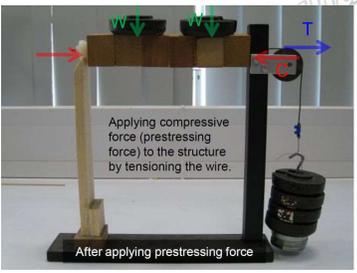
Applying compressive force (prestressing force) to the structure by tensioning the wire.

After applying prestressing force
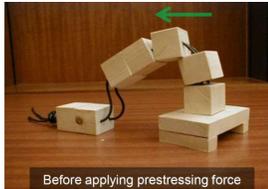
Before applying prestressing force
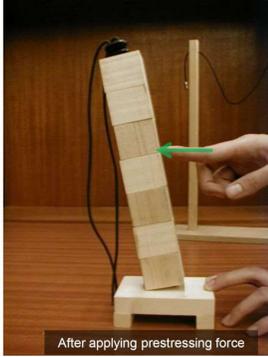
After applying prestressing force
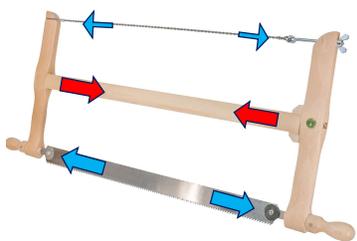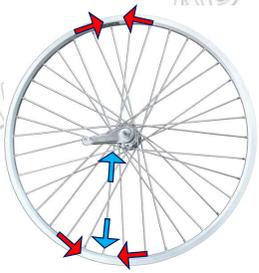
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REAL-LIFE APPLICATIONS

- Everyday example of prestressing:
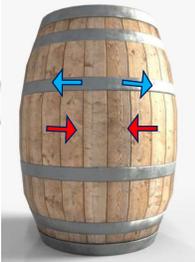 The concept of prestressing has been used widely in many applications other than prestressed concrete structures

Prestressed concrete beam

Multi-strand system with smart strand and steel strands

Multi-strand system with steel strands only

Frame saw

Bicycle spokes

Rupert's drop

Liquid barrel

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Bruce Buckley. "Creative Solutions Make Needed Chesapeake Bridge a Reality". December 18, 2015,
https://www.enr.com/mid-atlantic

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PS IN CIVIL ENGINEERING

Segmental bridge

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PS IN CIVIL ENGINEERING

Span – Longitudinal Tendons

Cross Section – Transverse Tensons

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TYPICAL CONSTRUCTION STAGES OF BRIDGES

Balanced Cantilever Construction

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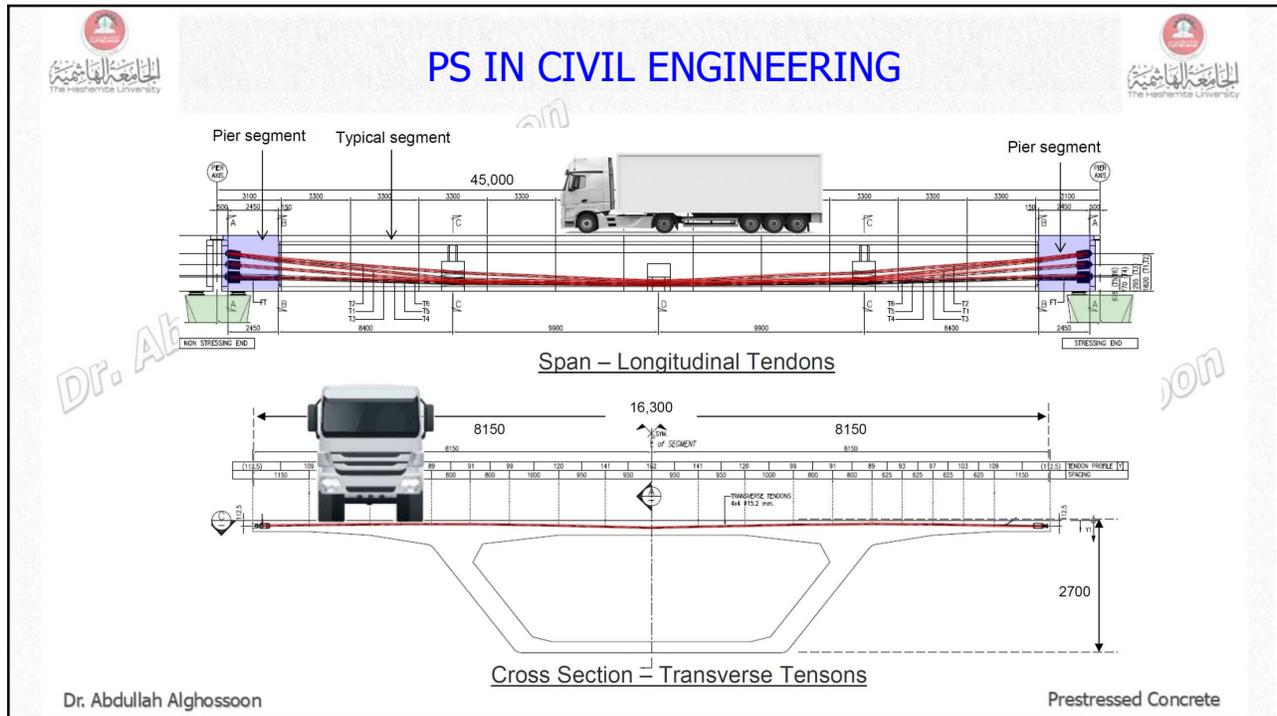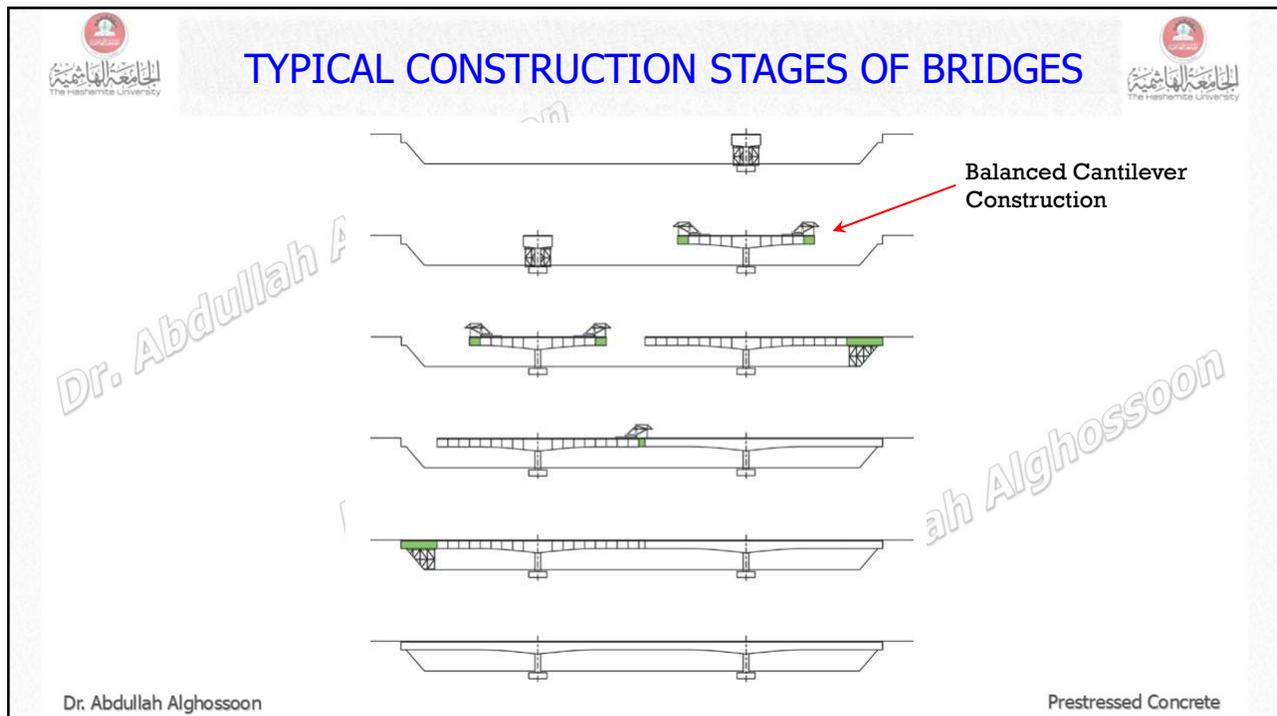
TYPICAL CONSTRUCTION STAGES OF BRIDGES

Span by Span construction
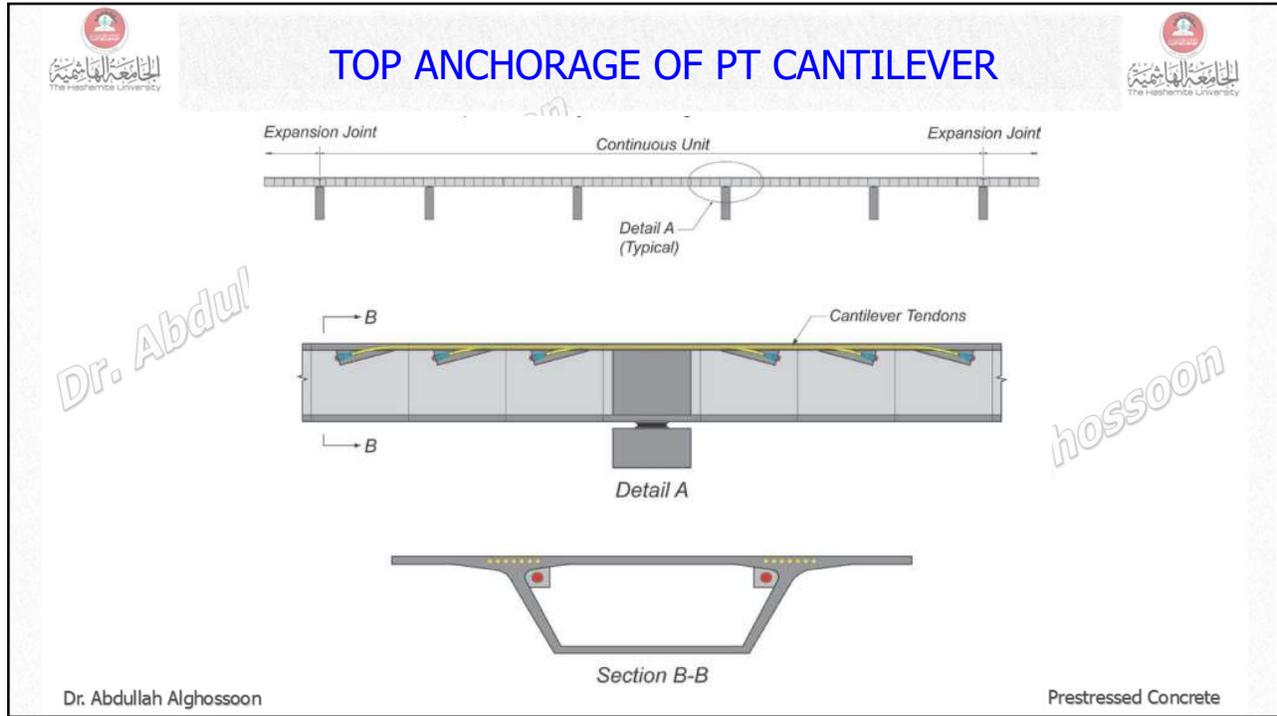

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TYPICAL SEGMENT

Top Slab Shear Key

Cantilever Tendons

Cantilever Tendons Anchor ("Face Anchored")

Top Temporary PT Bars

Bottom Continuity Anchor Blister

Cantilever Tendons Anchor Blister

Web Shear Keys

Bottom Slab Shear Key

Bottom PT Bars

Bottom Continuity Tendons

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TOP ANCHORAGE OF PT CANTILEVER

Detail A

Section B-B

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BOTTOM ANCHORAGE OF PT CANTILEVER

Detail A

Section B-B

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HAMMERHEAD PIERS

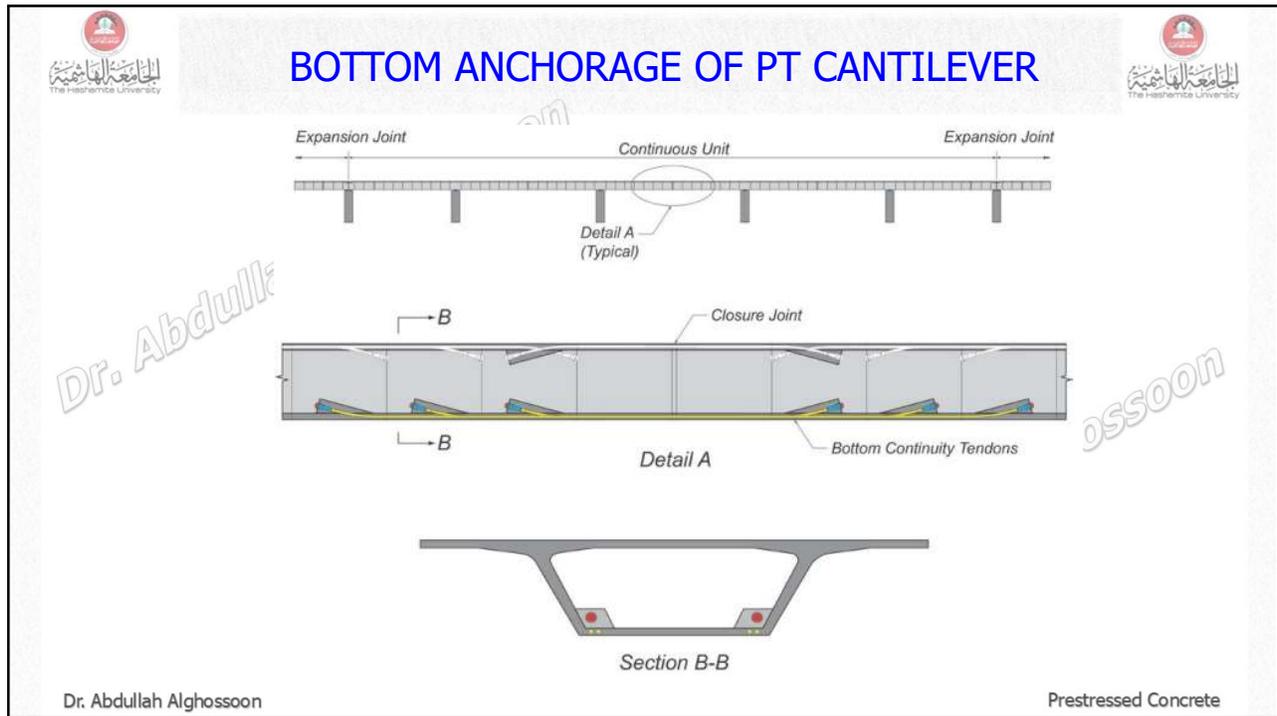
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CANTILEVER PIERS

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STRADDLE BENT

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PS IN CIVIL ENGINEERING

Pre-tensioned solid slab

Hollow core slab

Post-tensioned continuous solid slab
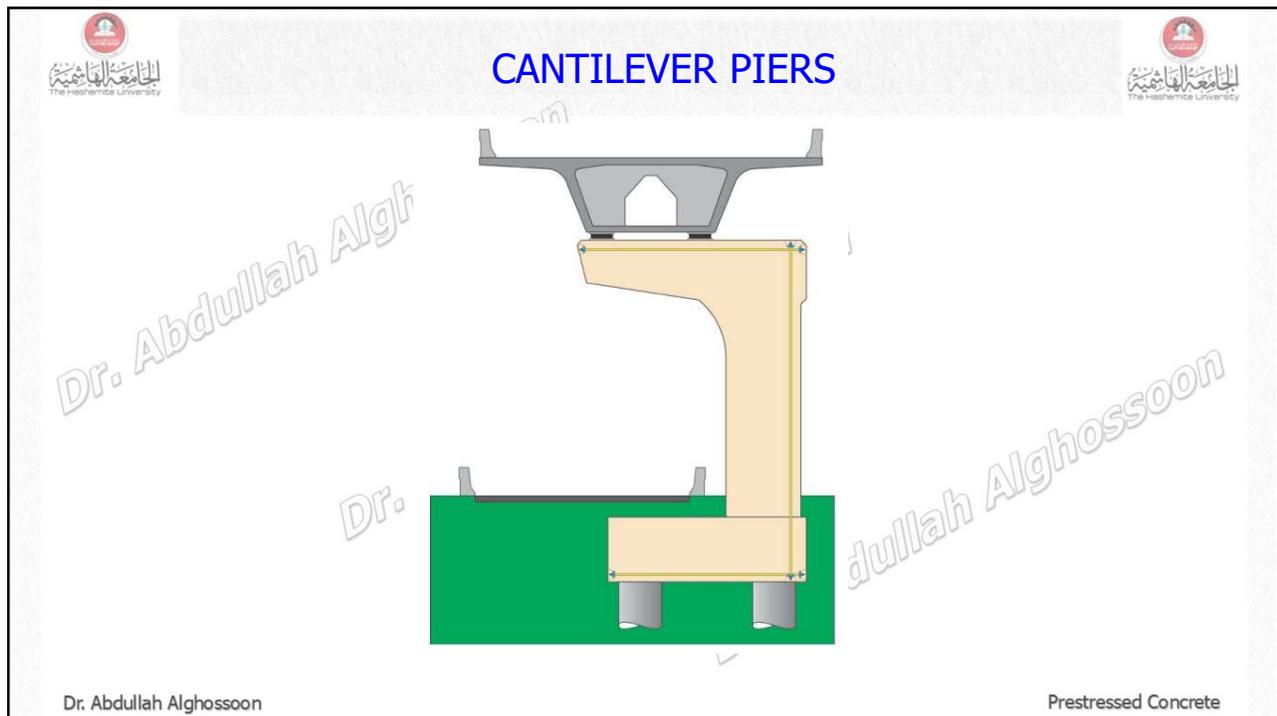
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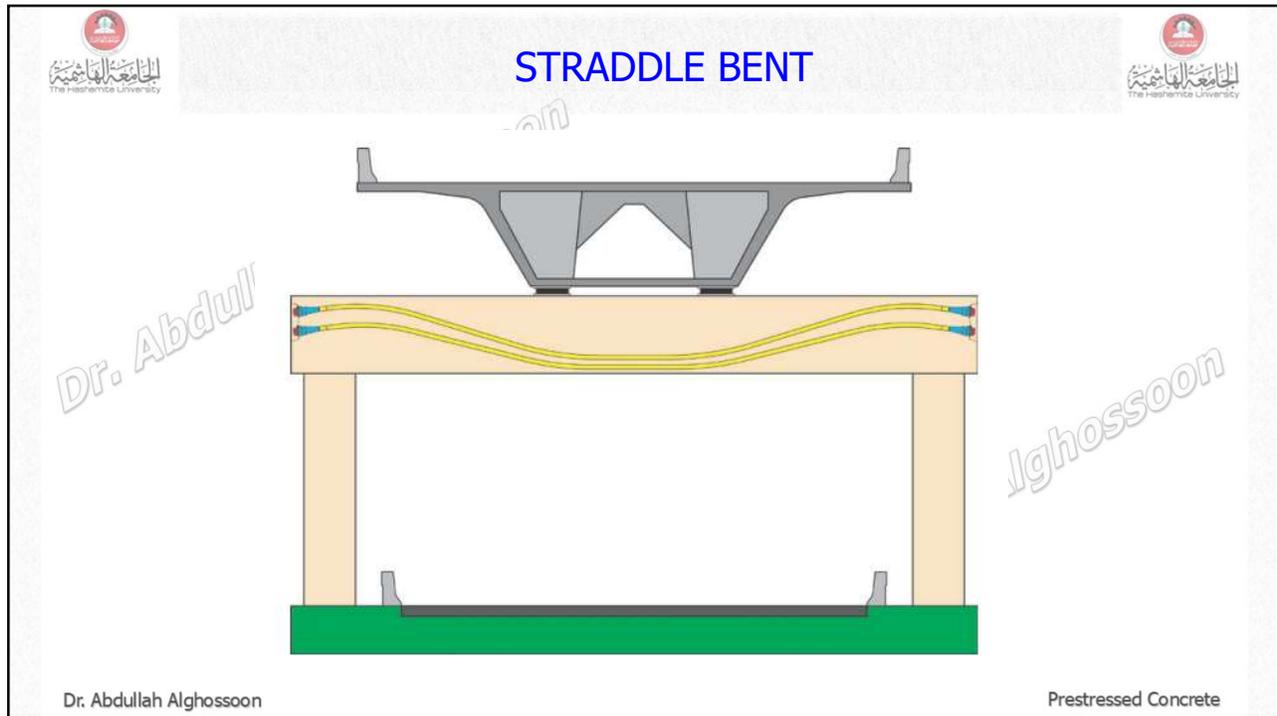
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HISTORY OF PRESTRESSING

- **Reinforced Concrete (1854):**
  Aspdin, J., (England) invented Portland cement in 1824, 30 years later, Monier, J., (France) introduce steel wire in concrete in 1857 (what we know now as reinforced concrete).

- **First application (1886):**
  Early attempt by P.H Jackson in the US
  (He is cited as the first engineer to patent prestressed concrete structures)

- **First application in Europe (1888):**
  By C.E.W Doehring in Germany
  (His model cited as the first prestressed structural system in Europe)

- **Significant advancement (late in 19th century):**
  By Thomas A. Lee
  (He used high-strength steel strands in precast concrete or masonry elements)
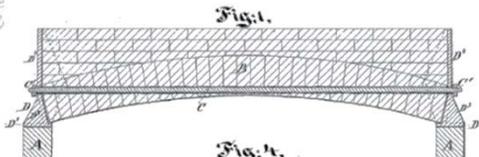
  These attempts were not successful because of the huge amount of steel and the prestressing loss with time (nearly 10% of the initial stresses)

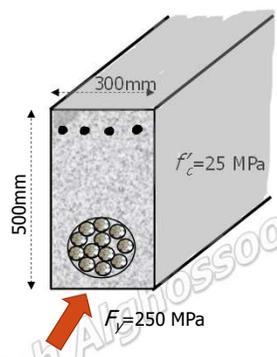Dr. Abdullah Alghossoon   Prestressed Concrete

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HISTORY OF PRESTRESSING

- **Early Efforts and Innovations:**

  Stainer, C. R., (USA, 1908)
  Recognized losses due to shrinkage and creep, and suggested retightening the rods to recover lost prestress Bolton and Lund (1910)

  Emperger, F., (Austria)
  Developed a method of winding and pre- tensioning high tensile steel wires around concrete pipes

  Hewett, W. H., (USA)
  Introduced hoop-stressed horizontal reinforcement around walls of concrete tanks through the use of turnbuckles.

  Dill, R. H., (USA)
  Used high strength unbonded steel rods. The rods were tensioned and anchored after hardening of the concrete.

  Early engineers struggled with issues like shrinkage and creep, often failing to account for time-dependent material properties (huge amount of steel and the prestressing loss with time)

300mm

500mm

f'_c=25 MPa

F_y=250 MPa

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HISTORY OF PRESTRESSING

- **The first applicable prestressing system(1939):**
  Proposed by Eugene Fryssinet of France.

  Fryssinet proposed method to overcome prestress losses by using high-strength steel with ultimate strength as high as 1725 MPa and yield stress over 1240 MPa. He also developed conical wedges for end anchorages for post-tensioning and developed double-acting jacks.

- **Further development :**

1938 Hoyer, E., (Germany)
  Developed 'long line' pre-tensioning method.

1940 Magnel, G., (Belgium)
  Developed an anchoring system for post-tensioning, using flat wedges.

Fig 1.1 Portrait of Eugene Freyssinet
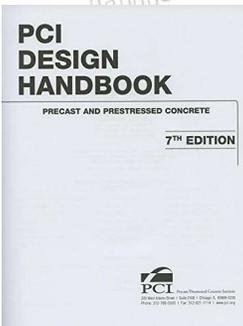
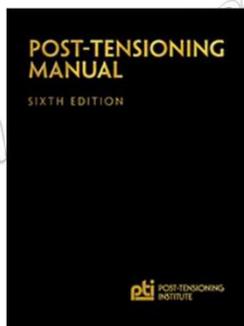Dr. Abdullah Alghossoon — Prestressed Concrete

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HISTORY OF PRESTRESSING

- During the Second World War, applications of prestressed and precast concrete increased rapidly.
- The International Federation for Prestressing (FIP), a professional organization in Europe was established in 1952. The Precast/Prestressed Concrete Institute (PCI) was established in USA in 1954.

PCI DESIGN HANDBOOK
PRECAST AND PRESTRESSED CONCRETE
7TH EDITION

POST-TENSIONING MANUAL
SIXTH EDITION

An ACI Standard
Building Code Requirements for Structural Concrete (ACI 318-19)
Commentary on Building Code Requirements for Structural Concrete (ACI 318R-19)
ACI 318-19
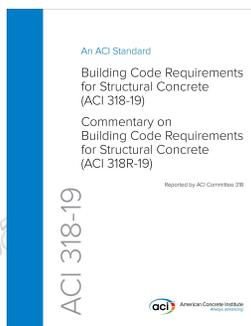
The design prestressed concrete in the US is governed by the ACI code and the Pre-stressed Concrete Institution (PCI)

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HISTORY OF PRESTRESSING

- Prestressed concrete in Jordan:
  Freyssinate (Zahran Street, Amman).
  ASSAS (Alqstal, Amman).
  Philadelphia (Amman)/ Omar Shekhalard
  Modular concrete Co.
  Assas for concrete products Co.
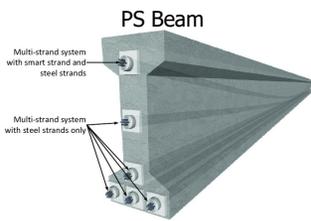 .
  .

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PS Vs. RC STRUCTURES

PS Beam

RC Beam

Multi-strand system with smart strand and steel strands

Multi-strand system with steel strands only

Application	Minimum f_c', psi
General	2500
Foundations for structures assigned to SDC A, B, or C	2500
Foundations for Residential and Utility use and occupancy classification with stud bearing wall construction two stories or less assigned to SDC D, E, or F	2500
Foundations for structures assigned to SDC D, E, or F other than Residential and Utility use and occupancy classification with stud bearing wall construction two stories or less	3000
Special moment frames Special structural walls with Grade 60 or 80 reinforcement	3000
Special structural walls with Grade 100 reinforcement	5000
Precast-nonprestressed driven piles Drilled shafts	4000
Precast-prestressed driven piles	5000

Prestressed members	RC members
Induced with tension (Active action)	No tension induced (Passive action)
High strength steel tendons	Mild steel deformed bars
Initial and final concrete compressive strength are considered	Only concrete compressive strength is considered
Typical range of concrete strength is more than 35 MPa	Min concrete strength in the ACI starts at 17 MPa

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PRESTRESSING STEEL

Tendons   Strand (7-wire strand)   Strands roll   Bars

The main difference between reinforced concrete steel and stressed concrete is the fact that the steel is :

ACTIVE in the stressed concrete with yield strength of 1860 MPa

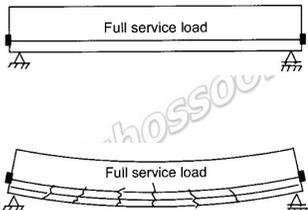**PASSIVE** in the stressed concrete with yield strength of 414 MPa

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ADVANTAGES OF PRESTRESSING

Criterion	Prestressed member	Reinforced concrete
member effectiveness	Utilizing the entire section	Only uncracked potion
Cracking resistance	Can be eliminated at service load	Inevitable
Corrosion resistance	Higher control	More suitable to corrosion
Span to Depth ratio (slabs)	≈ 45 (longer spans)	≈ 28
Aesthetically aspect	Slender section	Heavy and bulky
Amount of steel	Requires less reinforcement	Requires more reinforcement
Deflection	Less (controlled) deflection	More deflection
Load/shock resistance	More	Less
Cost	More economical in large spans and repetitive members	More economical in short span and small buildings
Quality control	More	Less

Full service load

Full service load

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CHALLENGES OF PRESTRESSING

Criterion	Prestressed member	Reinforced concrete
Equipment	Requires specialized and expensive equipment	Doesn't need specialized equipment
Labor	Requires highly skilled labor	Doesn't need highly skilled labor
Delivery	Length of PS segments might be controlled with the truck length	Usually cast in-situ
Design	Requires special attention and involving more steps	Easier to design
Failure	Usually catastrophic	Less dangerous

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OBJECTIVES OF PRESTRESSING

1. Control tensile stresses and eliminate concrete cracking.

2. Control deflection at service load

3. Allow using of high-strength materials

4. Improve performance of concrete (Corrosion resistance, shear strength)

5. Extend the range of concrete applications.

6. Controlling member slenderness for a more aesthetic appearance.

7. More economical in some situations.

8. Rehabilitation of existing structures.

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TYPES OF PRESTRESSING

Pre-tensioned

Forces transmitted through the bond between steel and concrete

Post-tensioned

Forces transmitted through the end anchorage

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TYPES OF PRESTRESSING

Internal prestressing
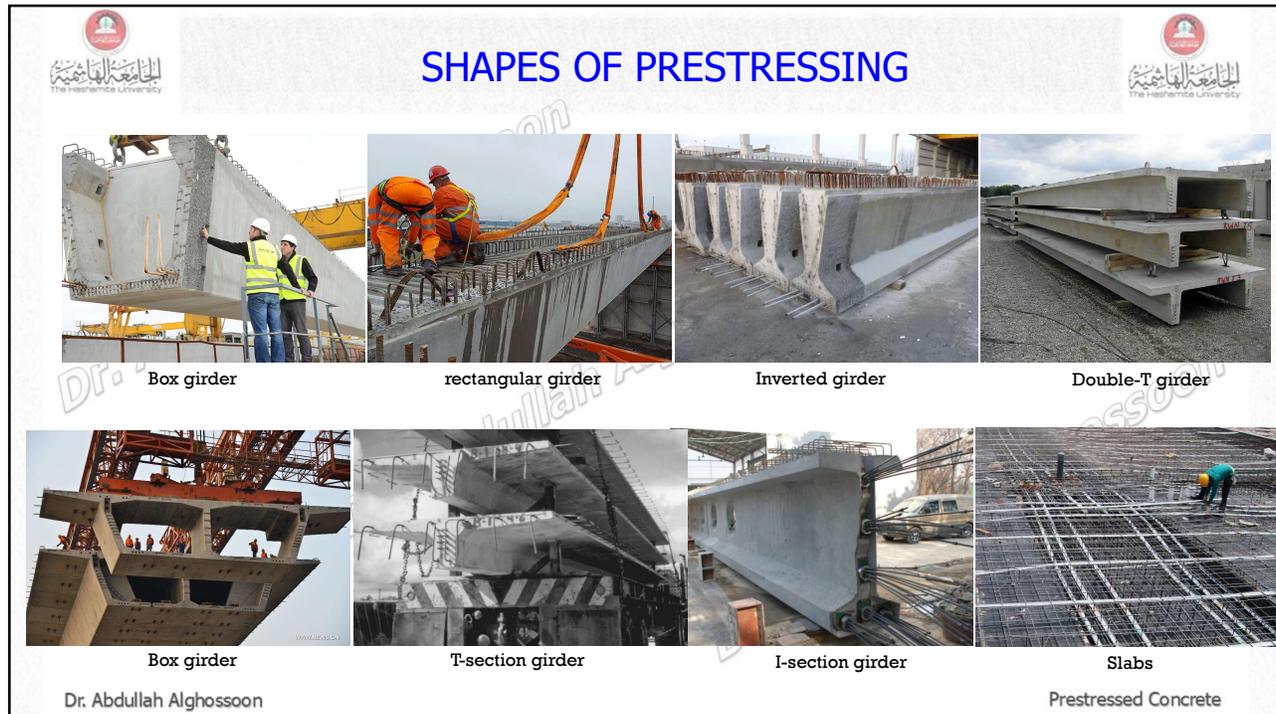

tendons are not visible inside the concrete block

External prestressing

visible tendons lie outside or inside the concrete block

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SHAPES OF PRESTRESSING

Box girder

rectangular girder

Inverted girder

Double-T girder

Box girder

T-section girder

I-section girder

Slabs

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PROS AND CONS OF PRE-TENSIONING

Advantages of Pre-tensioning

The relative advantages of pre-tensioning as compared to post-tensioning are as follows:

1. Pre-tensioning is suitable for precast members produced in bulk.
2. In pre-tensioning large anchorage device is not present.

Disadvantages of Pre-tensioning

The relative disadvantages are as follows.
1. A prestressing bed is required for the pre-tensioning operation.
2. There is a waiting period in the prestressing bed, before the concrete attains
3. sufficient strength.
4. There should be good bond between concrete and steel over the transmission length.

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CONCEPT OF PRE-TENSIONING

Fixed abetment

1. Steel tendons are first stressed (tensioned) and fixed to the abutment.
2. Steel reinforcements are placed (stirrups and longitudinal steel).
3. Concrete is then poured into the formwork around the **stressed** tendons.
4. Wires are cut after the concrete hardening and reach a specific compressive strength $(f_{ci}^{'})$.

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PRE-TENSIONING SYSTEM

Prestressing in the lab environment, Purdue University

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SEQUENCE OF PRE-TENSIONING IN FACTORY

Machine for Prestressing

Setting reinforcing bars

Casting bed

Setting reinforcement

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SEQUENCE OF PRE-TENSIONING IN FACTORY

Formwork

Casting of concrete and removal of form

Formwork

Casting concrete

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SEQUENCE OF PRE-TENSIONING IN-SITU

Prestressing steel extended along between the fixed abutments

Fixed abutment

Tensioning strands

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SEQUENCE OF PRE-TENSIONING IN-SITU

Setting of reinforcing bars/stirrups

Casting concrete and removing formwork

Cutting strands and transferring forces to the concrete member

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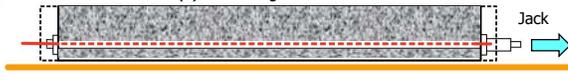
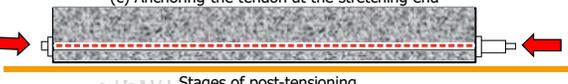Prestressed Concrete

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CONCEPT OF POST-TENSIONING

(a) Casting of concrete

(b) Tensioning of tendons

(c) Anchoring the tendon at the stretching end

Stages of post-tensioning

1. Tendon are going through ducts in a certain profile prior to concrete pouring.

2. After concrete hardening with sufficient strength, the tendons are stressed using a hydraulic jack and anchored at the end of the member.

3. Unless un-bonded tendons are being used, the ducts are grouted.

4. The shape of the duct profile is designed to control the flexural stresses along the member (usually parabolic).

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SEQUENCE OF POST-TENSIONING

Setting ducts and reinforcement steel

Formwork and casting concrete

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SEQUENCE OF POST-TENSIONING

Curing concrete until reaching the intended strength

Tensioning process

Grouting

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PROS AND CONS OF POST-TENSIONING

- Advantages of Post-tensioning

The relative advantages of post-tensioning as compared to pre-tensioning are as follows:

1. Post-tensioning is suitable for heavy cast-in-place members.
2. The waiting period in the casting bed is less.
3. The transfer of prestress is independent of transmission length.

- Disadvantage of Post-tensioning

The relative disadvantage of post-tensioning as compared to pre-tensioning is the requirement of anchorage device and grouting equipment.

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POST-TENSIONING

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POST-TENSIONING

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POST-TENSIONING COMPONENTS

External post-tensioning

Internal post-tensioning

Hydraulic machine

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POST-TENSIONING COMPONENTS

anchor block

jaws

guide

sheath

tendon

Jaws / Wedges

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ANCHORAGE SYSTEM

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ANCHORAGE SYSTEM FOR THIN PLATES

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ANCHORAGE SYSTEM

The **strain** of the unboned tendon is **averaged over the length between the anchorages**, and thus is much smaller than the strain increase in the tendon taken at the section of the maximum moment (**lower moment strength & larger cracks**).
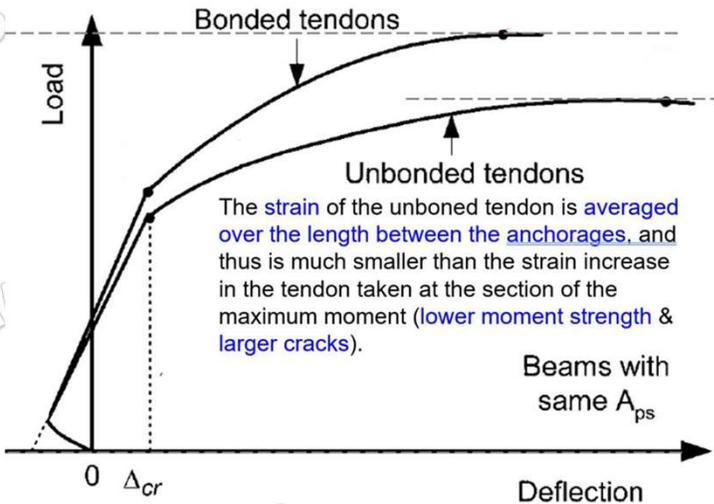
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POST-TENSIONING ACCESSORIES

1- Placing of anchor head and wedges

2- Positioning of the jack

3- Stressing

4- Removing Jack

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POST-TENSIONING PROJECTS IN JORDAN

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POST-TENSIONING PROJECTS IN JORDAN

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TYPES OF POST-TENSIONING

- Bounded construction:

Procedure where ducts are grouted with a paste-type grout.

- Unbounded

Procedure where ducts are left un-grouted (i.e., external prestressed beam, greased mono-strands wrapped with flexible plastic thin hose).

Used for economic reasons such as the ability to replace the strands in case of rapture, repositioning the end anchorages, less labor cost and saving material (gout)

- The main difference between these procedures is how we are going to compute the stresses at ultimate in prestressing steel (f_{ps})

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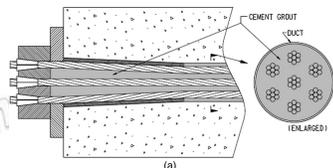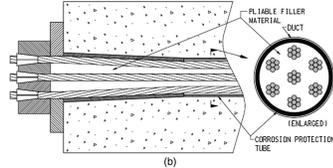
PRESTRESSED CONCRETE
-DESIGN PHILOSOPHY & ANALYSIS METHODS-

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DESIGN PHILOSOPHY

- Structural Design: is the art of designing structures that can safely resist the expected forces with minimal amount of material and overall structure cost.

- Basic principle of structure design:

Load: What the structure must hold	Resistance: How strong the structure is	Serviceability: Functionality and comfort

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DESIGN PHILOSOPHY

❖ Design philosophy of Prestressed Concrete:

➢ Generating compressive stresses using high-strength steel tendons in a concrete member before loads are applied to balance/mitigate the tensile stresses imposed in the member during <u>service</u> loading combinations.

➢ Compressive stresses are induced in prestressed concrete either by pre-tensioning or post-tensioning the steel reinforcement. The definition of the latter terms depends on the state of concrete at the time of prestressing.

➢ The steel wires of a pre-stressed concrete member do not retain all the preliminary pre-stress due to the nature of the material, friction, adjustment of anchorage tools, elastic shorting, etc. Hence losses of prestressing must be accounted for during the design.

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OBJECTIVES FOR STRUCTURAL DESIGN

▪ Sizing the structural components to avoid failure.

What is failure?

▪ Meeting the architectural/client demands (Aesthetics).

▪ Ensure a doable structural system considering the available resources and surviving the construction process.

▪ Ensure an economical design structure, considering:
1. Time of construction.
2. Fabrication and erection cost
3. Materials

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Type of Structural Problems

Analysis

▶ Checking the adequacy of existing prestressed concrete member in terms of maximum stresses and deformation when all information are available such as: member geometry, material properties and loading conditions.

1- Basic Method.

2- C-line Method.

3- Load Balancing Method.

Design

▶ Adopting some initial assumptions based some rue of thumbs, code specifications/constraints and engineering sense or experience for calculating the unknow variables. The design assumptions are then refined for optimizing the design.

(Iterative process)

▶ **Serviceability limit state**:

Allowable stress, crack width and deflection limitation.

▶ **Ultimate limit state:**

Control structural collapse

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Types of Structural loading

▪ The main purpose of the design codes is to ensure that engineers are using consistent loads.

▪ The ASCE 7, IBC and AASHTO are examples of the common codes used in the industry.

▪ These code outline the minimum design loads for buildings and structures, but it is up to the designer to change these values whenever is needed.

ASCE STANDARD ASCE/SEI 7-16
Minimum Design Loads and Associated Criteria for Buildings and Other Structures

IBC

AASHTO

LRFD BRIDGE DESIGN SPECIFICATIONS
9th Edition | 2020

2021
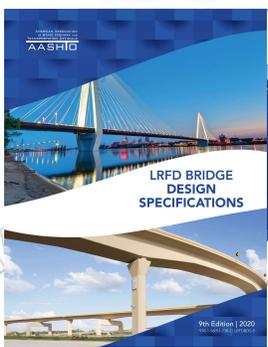
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LIMIT STATES

Working stress design

- Referred to as Allowable Stress Design (ASD).
- Calculated stresses can't exceed service load stresses.
- Structure is meant to be linear elastic.
- Taken as a fraction of the characteristic strength.
- Load types are treated the same way.
- Design criteria (Actual Cal ≤ Allowable limit).

$$\left(\sum Q \leq R_{\text{allowable}} \leq \frac{R_n}{\Omega}\right)$$

Ultimate strength design

- Referred to as Load and Resistance Factor Design (LRFD).
- Calculated stresses can't exceed the nominal strength limitations.
- Different load types are multiplied by different factors reflecting the uncertainties in their magnitudes and occurrence.
- Capacity reduction factor is applied to account for material uncertainties and analysis approximations.
- Design criteria $\left(\sum \gamma_i Q_i \leq \phi R_n\right)$

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LIMIT STATES

Working stress design

2.4 COMBINING NOMINAL LOADS USING ALLOWABLE STRESS DESIGN

2.4.1 Basic Combinations

Loads listed herein shall be considered to act in the following combinations; whichever produces the most unfavorable effect in the building, foundation, or structural member being considered. Effects of one or more loads not acting shall be considered.

1. D
2. $D + L$
3. $D + (L_r$ or S or $R)$
4. $D + 0.75L + 0.75(L_r$ or S or $R)$
5. $D + (0.6W$ or $0.7E)$
6a. $D + 0.75L + 0.75(0.6W) + 0.75(L_r$ or S or $R)$
6b. $D + 0.75L + 0.75(0.7E) + 0.75S$
7. $0.6D + 0.6W$
8. $0.6D + 0.7E$

ASCE STANDARD ASCE/SEI 7-10

Minimum Design Loads for Buildings and Other Structures

This document uses both the International System of Units (SI) and customary units

ASCE

Ultimate strength design
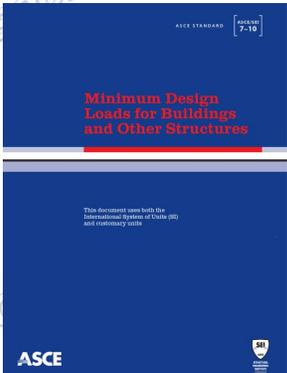
2.3 COMBINING FACTORED LOADS USING STRENGTH DESIGN

2.3.1 Applicability

The load combinations and load factors given in Section 2.3.2 shall be used only in those cases in which they are specifically authorized by the applicable material design standard.

2.3.2 Basic Combinations

Structures, components, and foundations shall be designed so that their design strength equals or exceeds the effects of the factored loads in the following combinations:

1. $1.4D$
2. $1.2D + 1.6L + 0.5(L_r$ or S or $R)$
3. $1.2D + 1.6(L_r$ or S or $R) + (L$ or $0.5W)$
4. $1.2D + 1.0W + L + 0.5(L_r$ or S or $R)$
5. $1.2D + 1.0E + L + 0.2S$
6. $0.9D + 1.0W$
7. $0.9D + 1.0E$

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LOADS

❖Dead load:

Self-weight and any superimposed loads such as slab topping, partitions, and false ceiling.
1. Unit weight of normal concrete is about 24 kN/m3 (including steel reinforcement).
2. Unit weight of lightweight concrete is about 19 kN/m3 (including steel reinforcement).
3. Unit weight of other materials in construction can be found in the design codes.

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LOADS

❖Live load:

Occupancy loads and vehicle loads (design codes plus the engineer's judgment)

≈750 kg/m2   ≈ 500 kg/m2   250 kg/m2

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LOADS

- HL-93 is the standard vehicular load model used in AASHTO LRDF for designing highway bridges.

Load Type	Description	Best for
Truck Load	Three-axle vehicle (HL-93 Truck)	General bridge design, maximum point load effects
Tandem Load	Two-axle vehicle with fixed spacing	Short-span bridges, high shear demands
Lane Load	Lane Load	Lane Load

- In Jordan, AASHTO LRFD live loads is multiplied by **1.35** to encounter the unexpected heavy live loads running on these bridges

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LOADS

- Truck load:

Moving of heavy vehicle loads applied on bridges
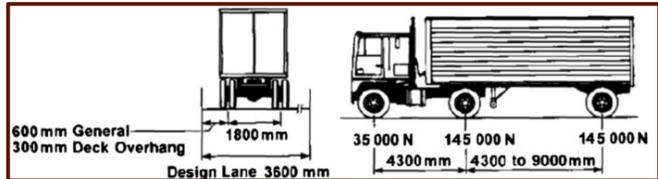
- Tandem Load

A pair of closely spaced axles, simulating vehicles like delivery trucks or small construction vehicles.
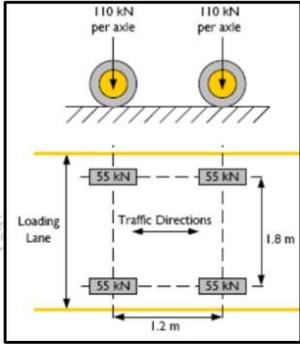
- Lane Load

A uniformly distributed load applied over a 12-foot-wide traffic lane, representing the presence of multiple vehicles.

The AASHTO LRFD lane load consists of:
1. 0.64 kips/ft (9.34 kN/m) distributed load
2. Additional concentrated load of 18 kips (80 kN) placed at the worst location

600 mm General / 300 mm Deck Overhang — 1800 mm — Design Lane 3600 mm — 35 000 N — 145 000 N — 145 000 N — 4300 mm — 4300 to 9000 mm

110 kN per axle — 110 kN per axle — 55 kN — Traffic Directions — 55 kN — Loading Lane — 55 kN — 55 kN — 1.8 m — 1.2 m

80 kN for moment / 115 kN for shear — 9.34 kN/m

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LOADS

❖Loading Scenarios:
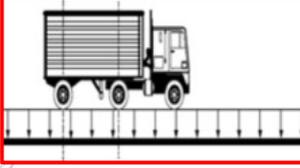
Truck load + Lane load
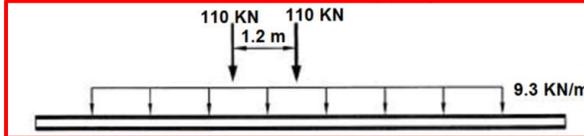
Tandem load + Lane load

Used for SS Elements

110 KN   110 KN
1.2 m

9.3 KN/m
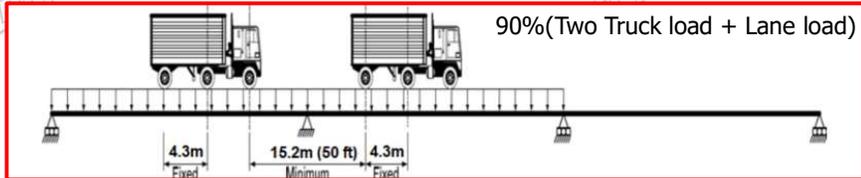
90%(Two Truck load + Lane load)

4.3m   15.2m (50 ft)   4.3m
Fixed   Minimum   Fixed

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LOADS

❖ Other types of loading:

Environmental loads, earthquake loads, Earth pressure loads, Impact load, etc.

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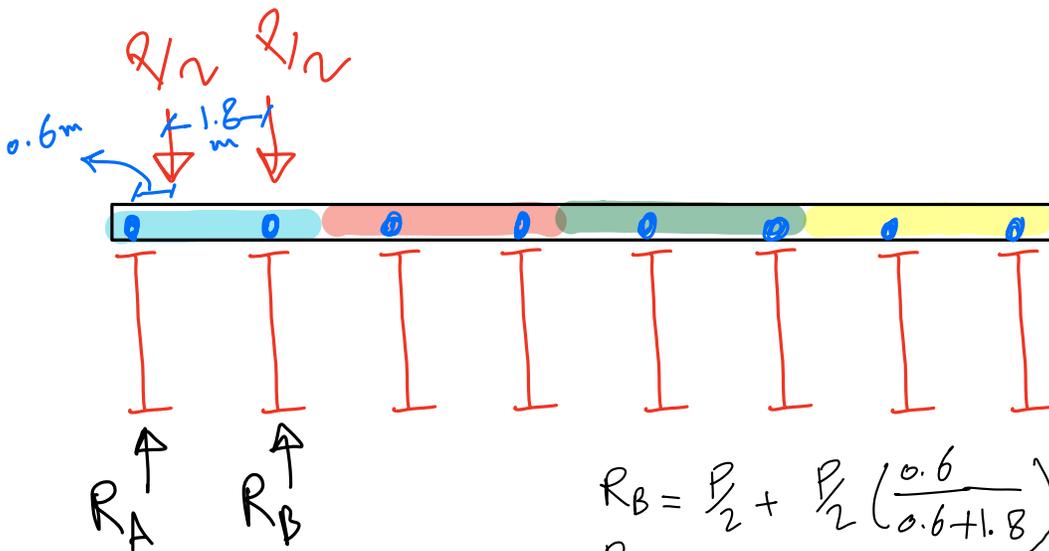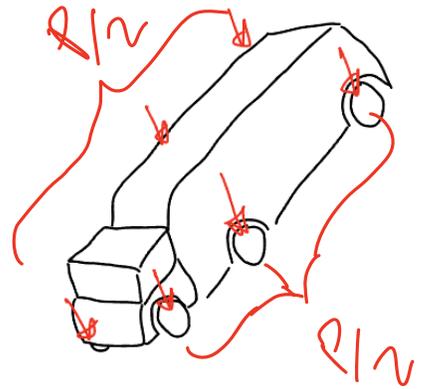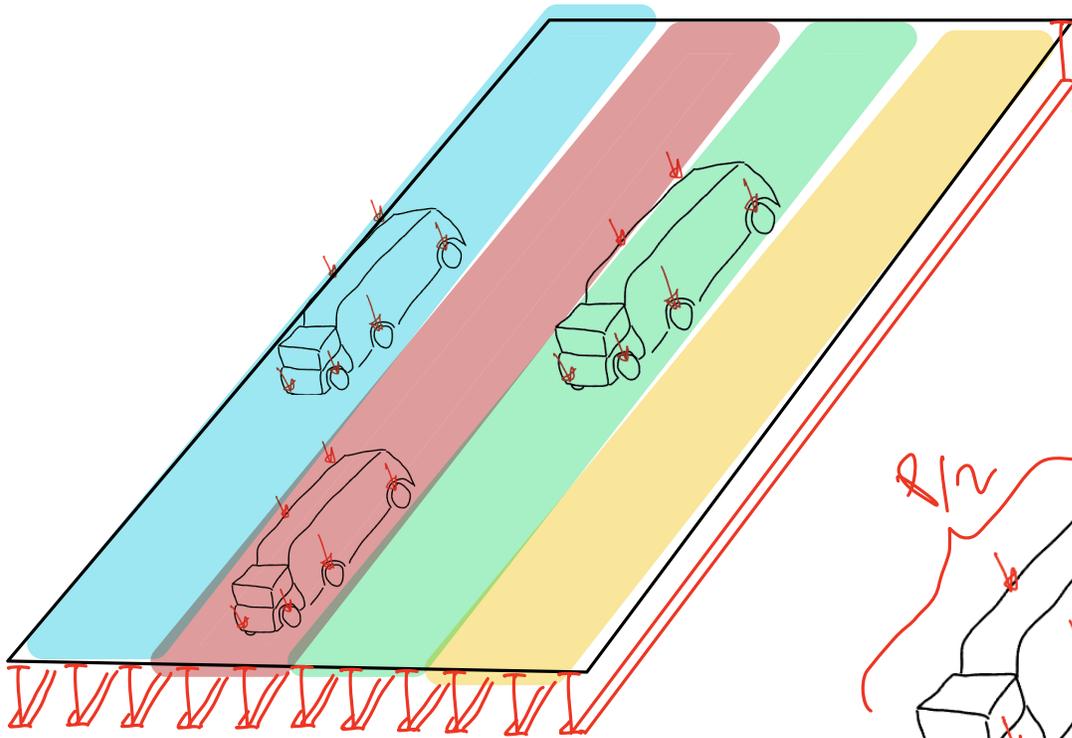
$$R_B = \frac{P}{2} + \frac{P}{2}\left(\frac{0.6}{0.6+1.8}\right) = 0.625P$$

$$R_A = \frac{P}{2}\left(\frac{1.8}{0.6+1.8}\right) = 0.375P$$

$$\times 1.2$$
MPF
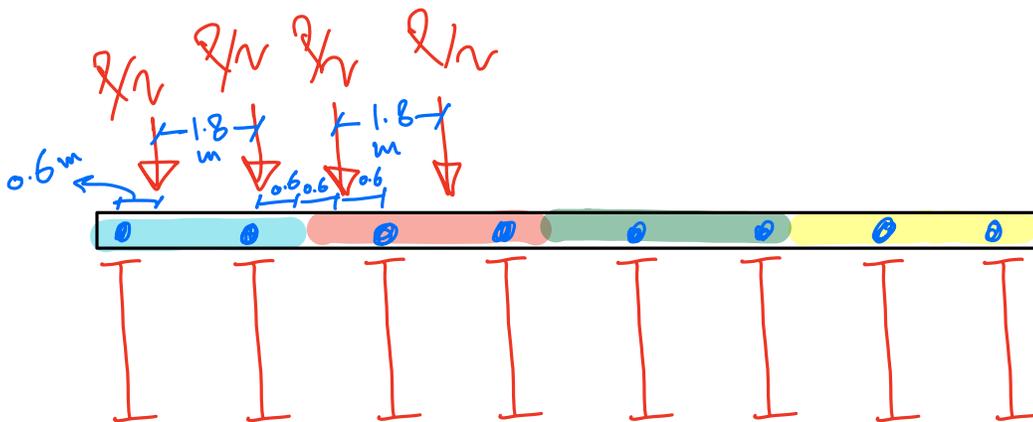
$$\uparrow R_B \qquad R_B = \frac{P}{2} + \frac{P}{2}\left(\frac{0.6}{0.6+1.8}\right) + \frac{P}{2}\left(\frac{0.6}{1.8}\right) = 0.79P \quad \times 1$$

MPF

0.75(35kN) 0.75(145kN) 0.75(145kN)

Case I

0.79(35kN) 0.79(145kN) 0.79(145kN)

Case II
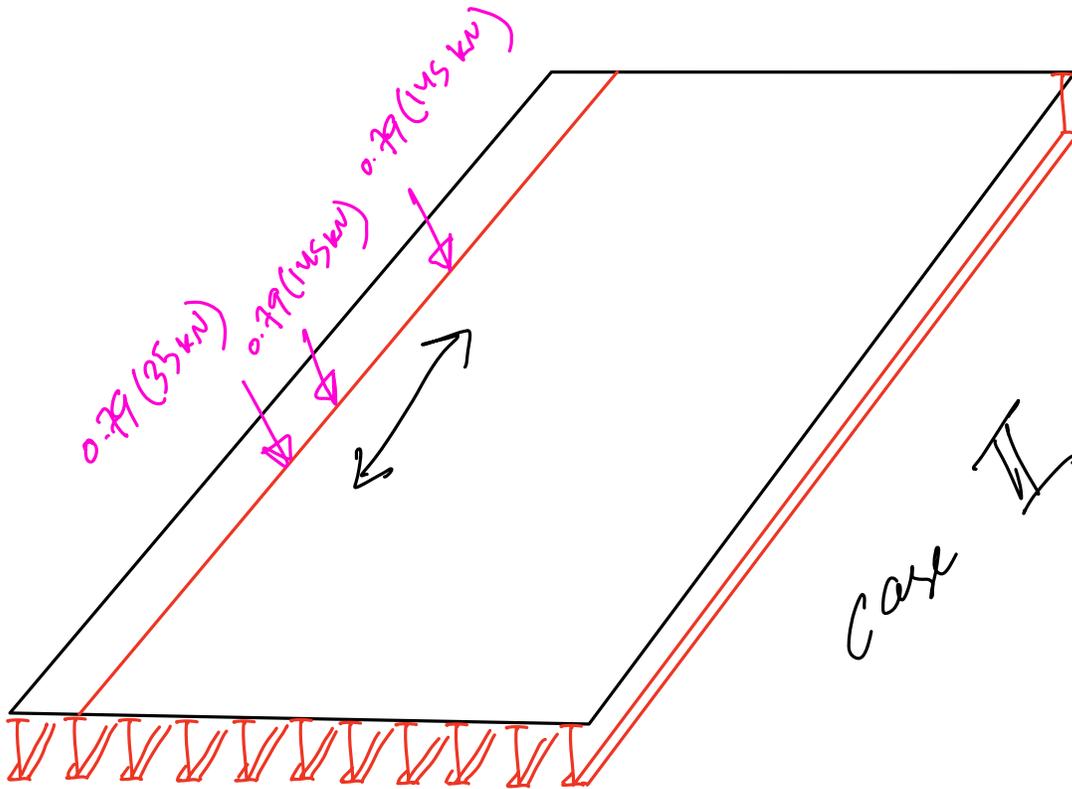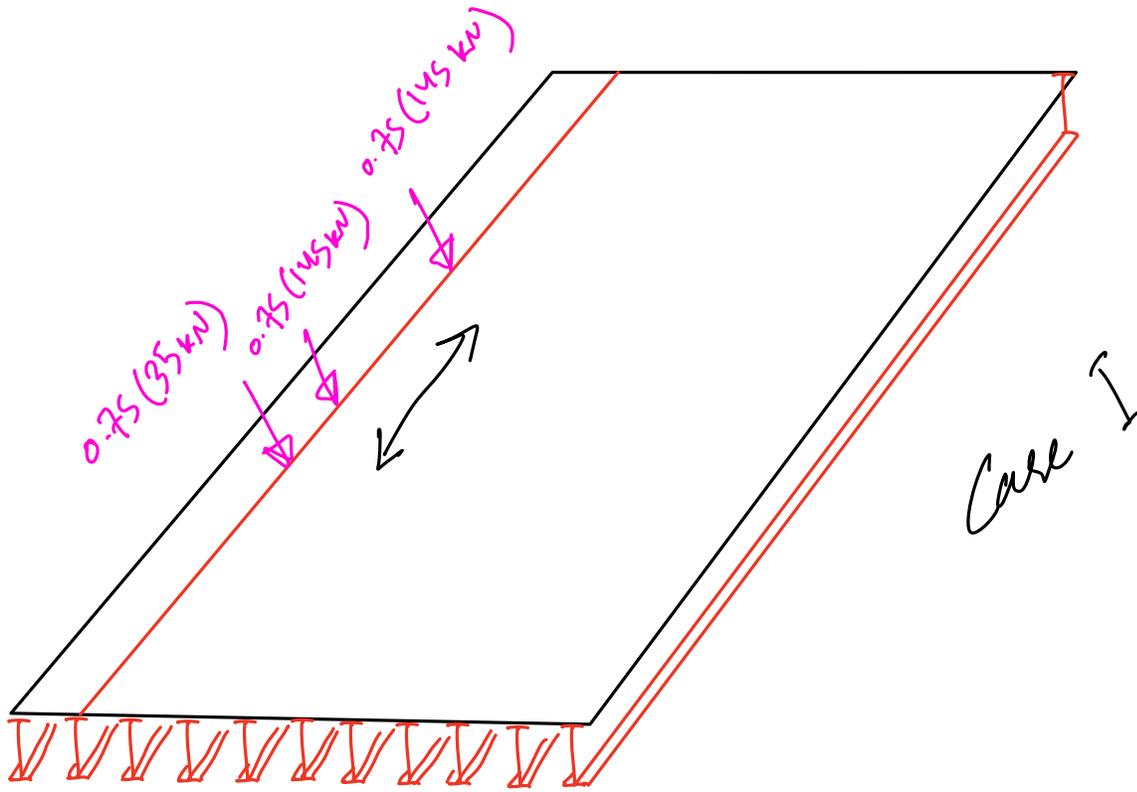
METHOD FOR ANALYSIS

❖ Analysis Stages
- Stresses are typically checked at significant stages.
- The number of stages varies with the complexity and type of prestressing.
- Stresses are usually calculated for the service level loads.

❖ Assumptions:
- Plane Sections Remain Plane
- Small Deformations
- Material is Linearly Elastic
- No Transverse Normal Stress
- Homogeneous and Isotropic Material
- Shear Deformation is Negligible (for Euler-Bernoulli Theory)

❖ Limitations of Euler-Bernoulli Beam Theory
- Not accurate for deep beams where shear deformation is significant.
- Does not consider material nonlinearity (plastic behavior, cracking in concrete).
- Ignores large deflections, making it unsuitable for beams with large deformations (e.g., flexible structures).
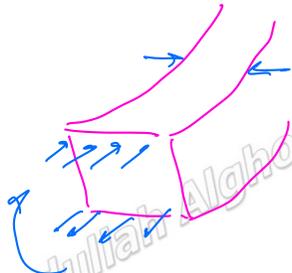
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BEAM THEORY

Plane section before bending remain plane after bending (i.e., small deflections)

$$\ddot{y} = \frac{1}{R} = k$$

$R = \infty$

$R = R_1$

$R = R_2$

$R = R_3$

$M = 0$

$M = M_1$

$M = M_2$

$M = M_3$

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BEAM THEORY

Plane section before bending remain plane after bending (i.e., small deflections)

$$M = \frac{1}{R} EI = \kappa EI$$

$$\kappa = \frac{M}{EI}$$

EI (Flexural rigidity)

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BEAM THEORY

Beam before loading

Beam after loading

$$\frac{x}{R} = \frac{\Delta x/2}{y}$$

$$\frac{x}{R} = \frac{\Delta x}{y}$$

$$\frac{\Delta x}{x} = \frac{y}{R} = y\kappa$$

$$\frac{\Delta x}{x} = \varepsilon = y\kappa = \frac{\sigma}{E} = y\frac{M}{EI}$$

$$\sigma = \frac{My}{I}$$

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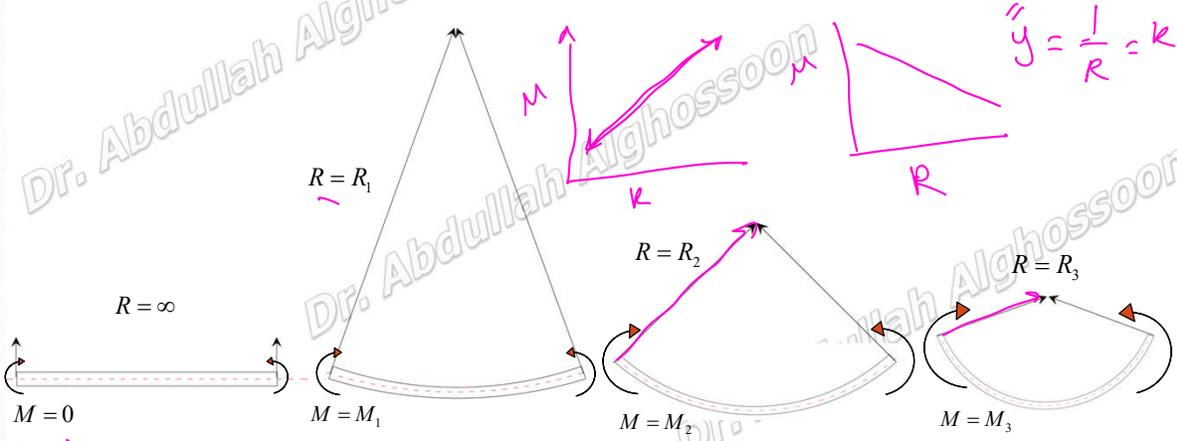
9

METHOD FOR ANALYSIS

$$\sigma = \frac{\pm My}{I} \qquad \sigma = \frac{P}{A}$$

❖Basic Concept Method:

  ▪ Concrete fiber stresses are directly computed from the external forces applied to the concrete by longitudinal prestressing and the external transverse load.

❖C-line Method:

  ▪ Beam is analyzed as if it were a plain concrete elastic beam using the basic principles of statics.

❖Load Balancing Method:

  ▪ A useful method for analyzing continuous prestressed beams (Developed by Lin).

  ▪ This method utilizes the vertical force of the draped or harped prestressing tendons to counteract or balance the imposed gravity loading.

  ▪ It is only applicable to non-straight prestressing tendons

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METHOD FOR ANALYSIS

❖C-line Method (Center of pressure):

  ▪ Prestressing forces is considered as an external compressive force, wit a constant tensile T in the tendon throughout the span

$$M = 0.85 f'_c \, a \, b \left(d - \frac{a}{2} \right)$$

$$100 \times 10^6 \; N.mm = 0.85 \, 25 \; a \; 300 \left(450 - \frac{a}{2} \right)$$

$$a = 36.3 \quad 8\%$$
$$a = 43.9 \quad 9\%$$
$$a = 76.16 \quad 16\%$$

$M = 0$
$M = 100 \, kN.m$
$\Sigma F = 0$
$\Sigma M = 0$

$T = C$
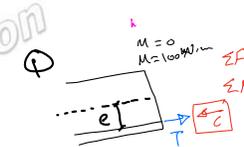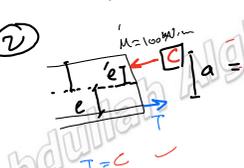
$a = \acute{e} + e$

$T = C$

$$M = Ta = Ca$$
$$\sigma = \frac{My}{I} = \frac{C\acute{e} \, y_b}{I}$$

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$$\sigma = \frac{-C}{A} \pm \frac{C\acute{e} \, y_{b/t}}{I}$$

$$15 \, Mpa : \quad \sigma_t = \frac{-C}{A} - \frac{C\acute{e} \, y_t}{I} \rightarrow \sigma_c$$

$$\sigma_b = \frac{-C}{A} + \frac{C\acute{e} \, y_b}{I} \rightarrow \sigma_T$$

METHOD FOR ANALYSIS

❖Load Balancing Method:

1. Section remains uncracked.

2. Stress-strain relationship is linear for both materials.

3. Use superposition to sum stress effect of each load

4. Prestressing is just another load.
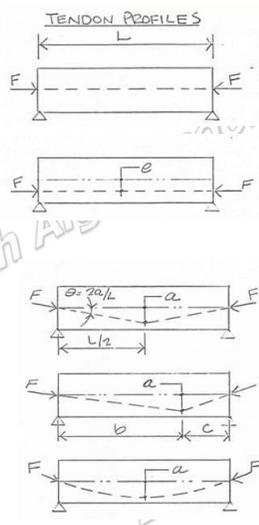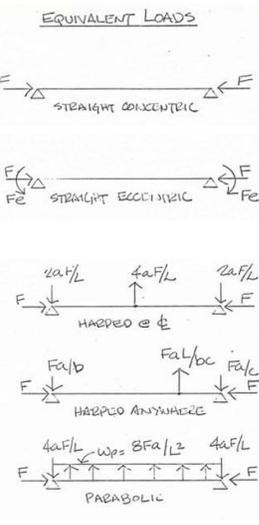
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BASIC CONCEPT

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STAGE OF LOADING

EXAMPLE

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Extreme fiber stresses along the length of SS beam (Immediately after transfer forces)

Extreme fiber stresses along the length of SS beam (After allowances of all losses)

Extreme fiber stresses along the length of SS beam (Immediately after transfer forces)

Extreme fiber stresses along the length of SS beam (After allowances of all losses)
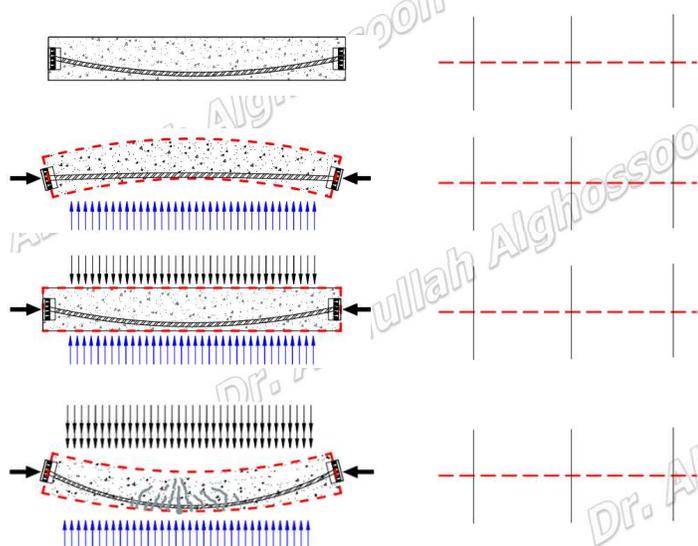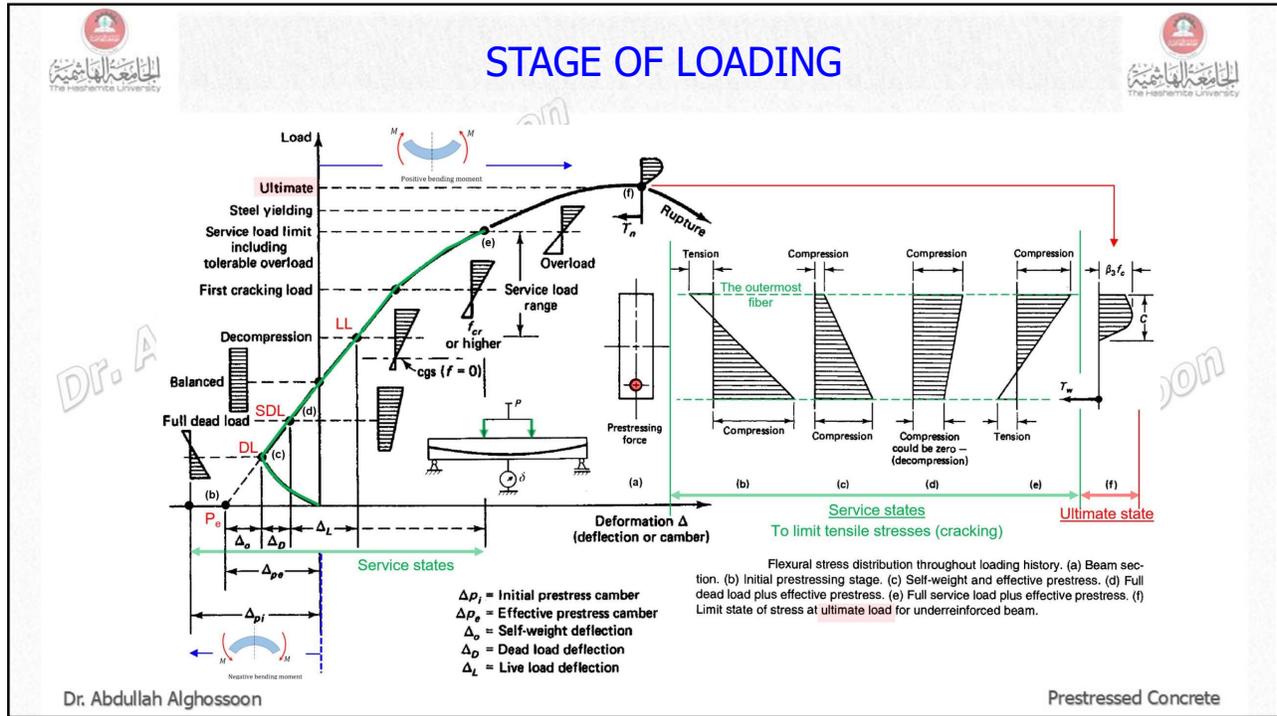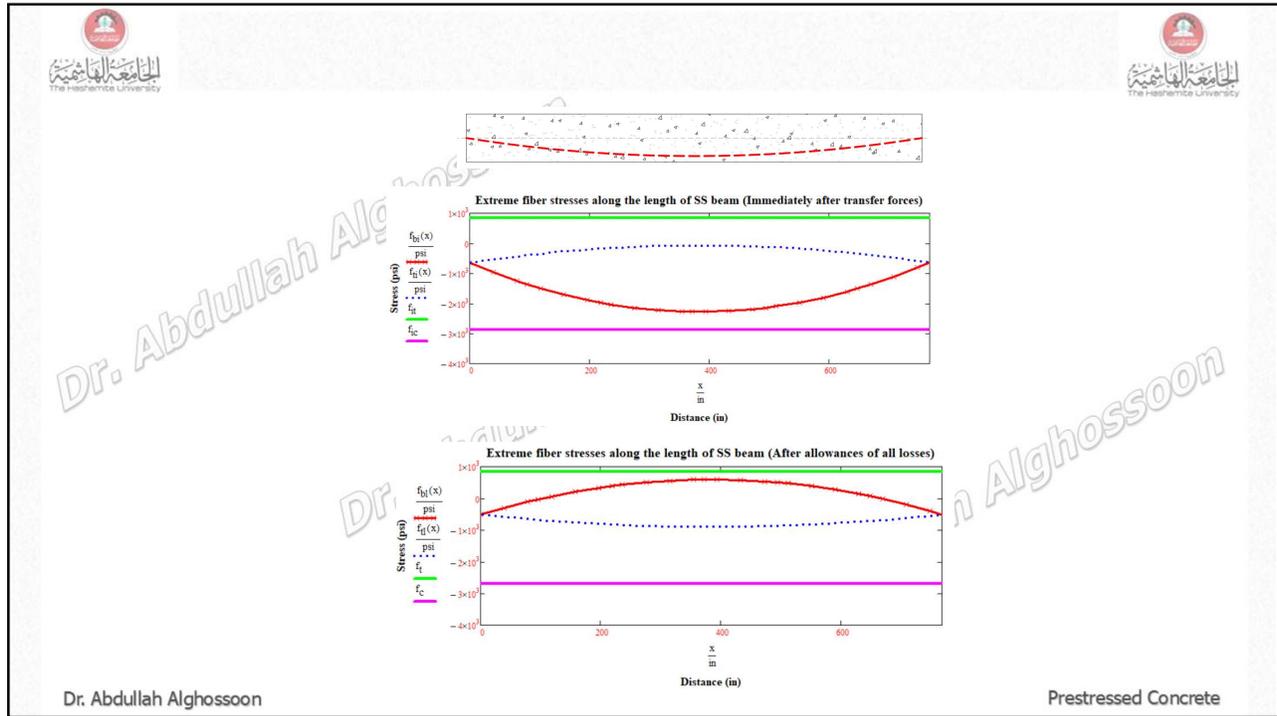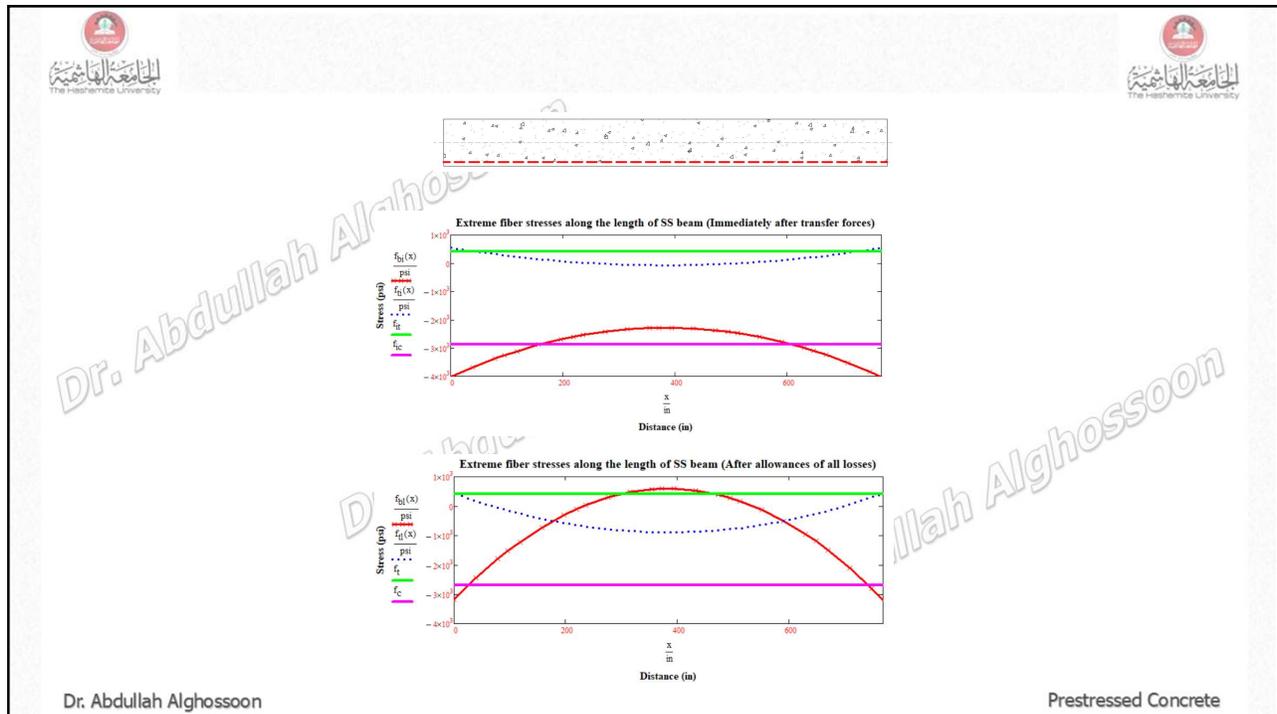
PRESTRESSED CONCRETE
-MATERIALS-

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Spring 2025

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MATERIALS (CONCRETE)

- Concrete is a composite material obtained by mixing: Portland cement, coarse aggregate, fine aggregate, water, chemical admixtures.

- According to the ASTM C150 there are five types of Portland cement:

    I.   general purpose
    II. general purpose with moderate exposure to sulfate the attack
    III. used when high early strength is required
    IV. used when low heat of hydration is desire
    V.   used when high sulfate resistance is needed
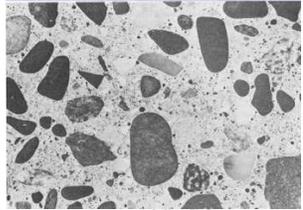
Petrographic section of hardened concrete

There are two types of concrete:
- ❖Normal weight concrete: 145~155 *pcf* (5% more to account for reinforcement)
- ❖Lightweight concrete: 90~120 *pcf*

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MATERIALS (CONCRETE)

- In prestressed concrete the technical level is higher and involve specifying compressive strength at time of tensioning f'_{ci} as well as at 28 days f'_c. Moreover, it is required in other time dependent properties such as creep, shrinkage and long term deflection.

- Compressive strength: Usually in the range 4.0 ksi to 12.0 ksi. However, 20.0 ksi can be achieved (this is almost half of the conventional steel strength). ASTM C-39 (Compressive Strength of Cylindrical Concrete Specimens).

- For concrete under compression due to axial load, the ultimate strain is restricted to 0.003.

- Concrete is assumed elastic up to 30%-45% of the compressive strength.

$$E_c = 33w_c^{1.5}\sqrt{f'_c} \quad \text{for } 90 < w_c < 155\,\text{lb/ft}^3$$

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CONCRETE BEHAVIOR
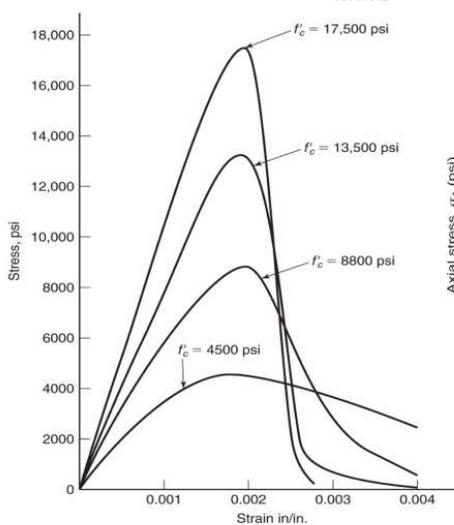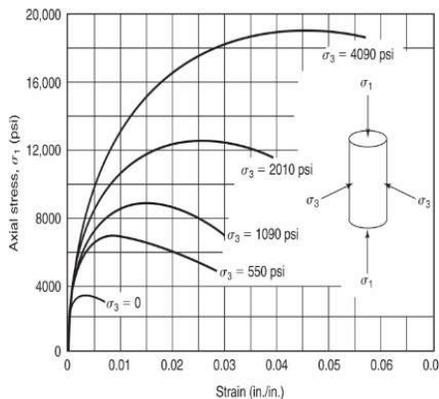
Fig. 3-15 Axial stress–strain curves from triaxial compression tests on concrete cylinders; unconfined compressive strength f'c = 3600 psi. (From [3-3].)

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CONCRETE STRESS-STRAIN CURVE

US
$$E_c = 33w_c^{1.5}\sqrt{f_c'} \quad \text{for } 90 < w_c < 155\,\text{lb/ft}^3$$

SI
$$E_c = 4700\sqrt{f_c'} \quad \text{for } f_c' \le 41.4\,MPa$$
$$E_c = 3320\sqrt{f_c'} + 6895 \quad \text{for } f_c' > 41.4\,MPa$$

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CONCRETE MODULUS OF ELASTICITY

$$E_c = 33w_c^{1.5}\sqrt{f_c'} \quad \text{for } 90 < w_c < 155\,\text{lb/ft}^3$$

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3
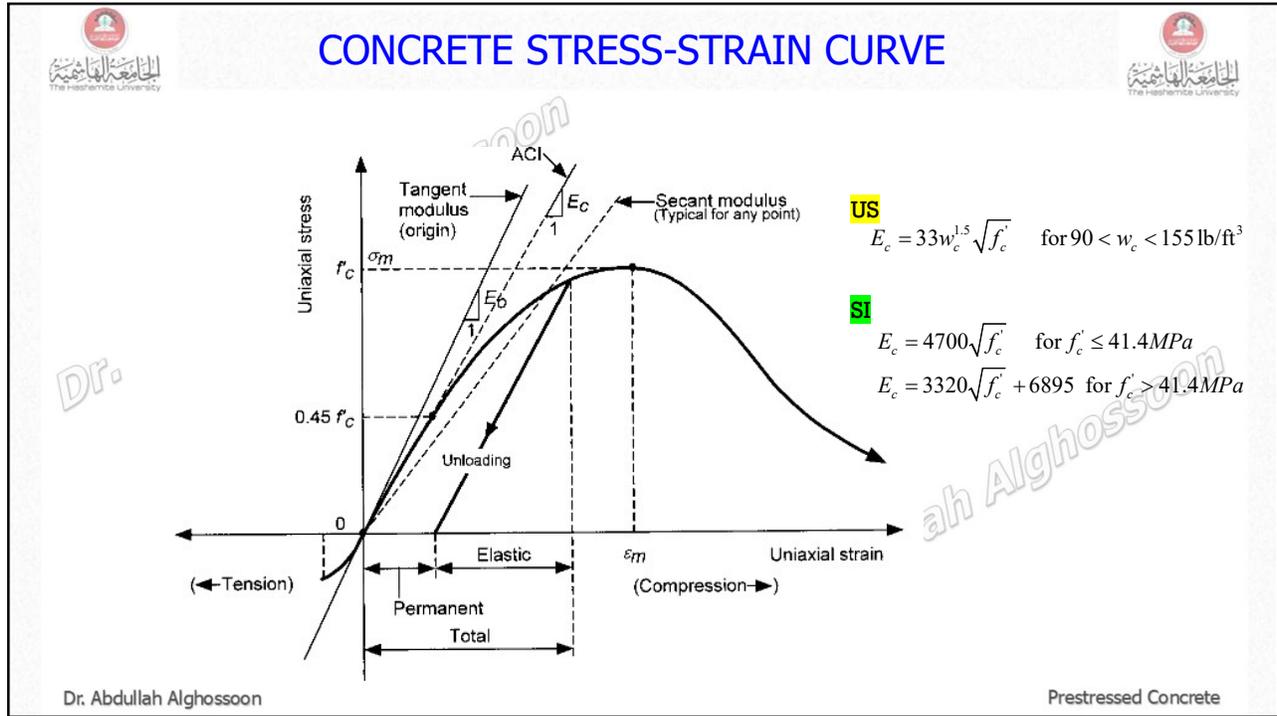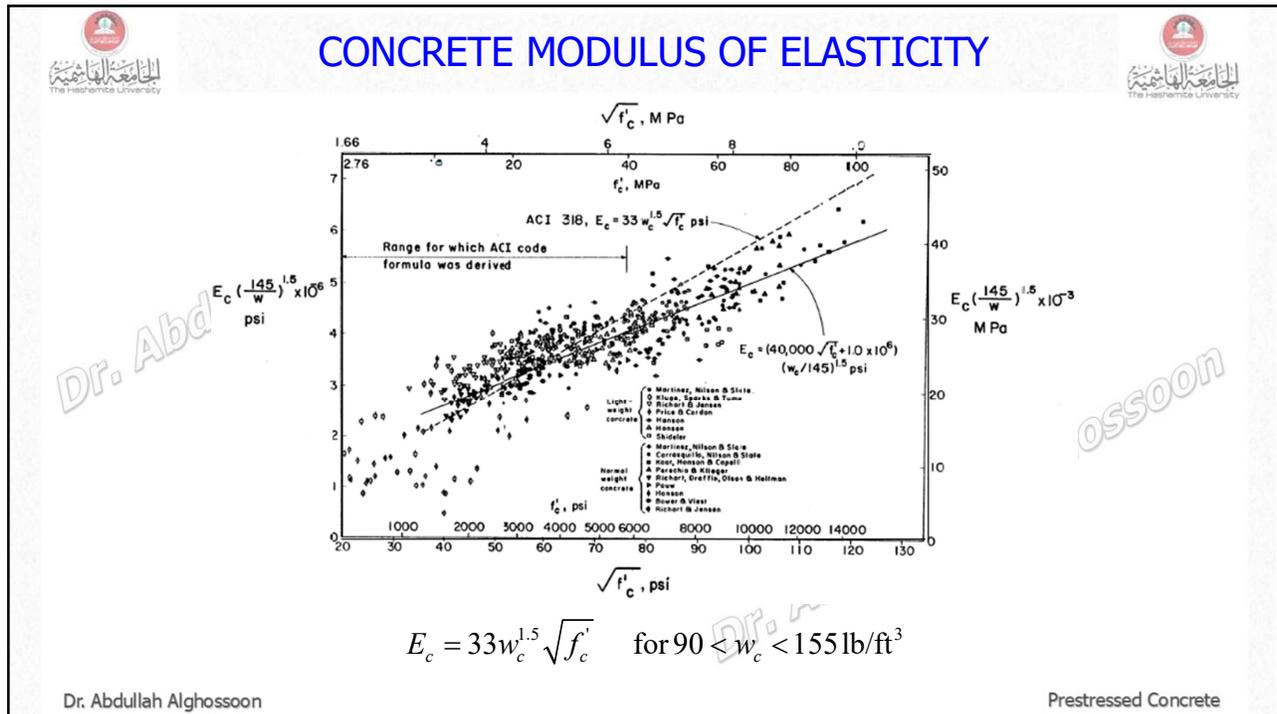
CONCRETE BEHAVIOR

- Cracks developed in concrete from day 1 due to shrinkage.

- Concrete is assumed elastic up to 30%-45% of the compressive strength and cracks remains stable (no progressive damage)

- Nonlinear behavior becomes prominent in the range of 50%-70% of the compressive strength due to the development of the unstable micro-cracks.

- The rate of cracks progression increases significantly beyond 70% of the compressive strength, concrete strain accordingly.

- At higher level of loading, concrete fails under sustained loading even below its maximum compressive strength.
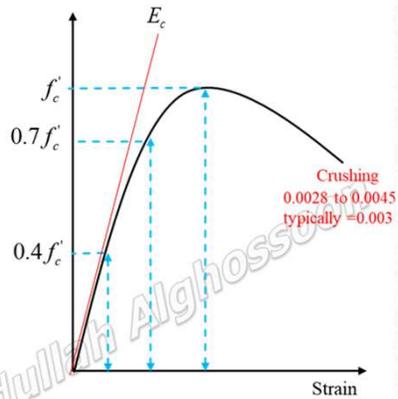
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TENSILE STRENGTH OF CONCRETE

- Tensile strength: calculated as a fraction of the compressive strength $0.1 f_c' \leq f_{ct} \leq 0.2 f_c'$

- For member subjected to bending we use modulus of rapture which is a bit higher than the tensile splitting strength ASTM C-78
$$f_r = 7.5\sqrt{f_c'}$$

- For light weight concrete:
$$f_r = 1.09 f_{ct} \leq 7.5\sqrt{f_c'}$$
    if f_{ct} is not spicified use 0.75 for lightweight concrete

    and 0.85 for sand-lightweight concrete

(a) Loading and cracks

(b) Bi-axial stress

$$f_{ts} = 2P_s/(\pi Dh_c)$$

- Shear strength is more difficult to be calculated and varies significantly (20% to 85% of the compressive strength)

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UNCERTAINTIES IN CONCRETE PROPERTIES

This is how accurate the equations we use!

Tensile strength

Shear strength strength

Fig. 6-13
Effect of reinforcement ratio, ρ_w, on shear capacity, V_c, of beams constructed with normal-weight concrete and without stirrups [6-6].

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CONCRETE STRESSES IN FLEXURE

Property	Concrete type	Observed range	Design value (US)	Design value (SI)
Direct tensile strength, f'_{tc}	Normal weight	$3\sqrt{f'_c}$ to $5\sqrt{f'_c}$	$3\sqrt{f'_c}$ to $1/3\sqrt{\gamma f'_c}$	$0.25\sqrt{f'_c}$ to $0.007\sqrt{\gamma f'_c}$
	Lightweight	$2\sqrt{f'_c}$ to $3.5\sqrt{f'_c}$	$2\sqrt{f'_c}$ to $1/3\sqrt{\gamma f'_c}$	$0.17\sqrt{f'_c}$ to $0.007\sqrt{\gamma f'_c}$
Split cylinder tensile test, f'_{ts}	Normal weight	$6\sqrt{f'_c}$ to $7\sqrt{f'_c}$	$6\sqrt{f'_c}$ to $0.6\sqrt{\gamma f'_c}$	$0.5\sqrt{f'_c}$ to $0.0014\sqrt{\gamma f'_c}$
	Lightweight	$4\sqrt{f'_c}$ to $5\sqrt{f'_c}$	$4\sqrt{f'_c}$ to $0.5\sqrt{\gamma f'_c}$	$0.33\sqrt{f'_c}$ to $0.00115\sqrt{\gamma f'_c}$
Modulus of rupture, f_r	Normal weight	$7.5\sqrt{f'_c}$ to $12\sqrt{f'_c}$	$7.5\sqrt{f'_c}$	$0.62\sqrt{f'_c}$
	Lightweight	$5\sqrt{f'_c}$ to $9\sqrt{f'_c}$	$0.75\left(7.5\sqrt{f'_c}\right)$ for LW sand $0.85\left(7.5\sqrt{f'_c}\right)$ for normal sand	$0.47\sqrt{f'_c}$ for LW sand $0.53\sqrt{f'_c}$ for normal sand
Modulus of elasticity, E	Normal weight	$(27\ to\ 35\ )\gamma^{1.5}\sqrt{f'_c}$	$35\,\gamma^{1.5}\sqrt{f'_c}$ or $57000\sqrt{f'_c}$	$0.043\,\gamma^{1.5}\sqrt{f'_c}$ or $4730\sqrt{f'_c}$
	Lightweight	$(25\ to\ 30\ )\gamma^{1.5}\sqrt{f'_c}$	$33\,\gamma^{1.5}\sqrt{f'_c}$	$0.043\,\gamma^{1.5}\sqrt{f'_c}$
Poisson's ratio, v	Normal weight	0.15 to 0.2	0.2	0.2
	Lightweight			

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CONCRETE CREEP

- Creep is the permanent deformation of the concrete material caused by sustained loading over the concrete structures

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CONCRETE CREEP

$$\frac{\varepsilon_{CR}}{\varepsilon_{EL}} = \phi(70y,t_0) = K_{CR} = C_t$$

$$\frac{\varepsilon_{CR}}{\varepsilon_{EL}} = \phi(70y,t_0)$$

TABLE 3-2   Creep Coefficient, $\phi(70y, t_0)$ for Normal-Weight Concrete after 70 Years of Loading

Age at Loading, t_0 (days)	Dry Atmospheric Conditions (RH = 50%)		Humid Atmospheric Conditions (RH = 80%)	
	Effective Thickness, h_e (Eq. 3-32)			
	6 in.	24 in.	6 in.	24 in.
1	4.8	3.9	3.4	3.0
7	3.3	2.7	2.4	2.1
28	2.6	2.1	1.8	1.6
90	2.1	1.7	1.5	1.3
365	1.6	1.3	1.1	1.0

$$\frac{\varepsilon_{CR}}{\varepsilon_{EL}} = K_{CR}$$

$K_{CR} = 2$ for pre-tensioned concrete

$\quad\quad = 1.6$ for post-tensioned concrete

K_{CR} reduced by 20% for LWC

$$\frac{\varepsilon_{CR}}{\varepsilon_{EL}} = C_t$$

$$C_t = \frac{t^{0.6}}{t^{0.6}+10}C_u$$

$$C_u = 2.35 \times \lambda_{rh} \times \lambda_{vs} \times \lambda_{to} \quad ....(2 \sim 4, \text{typically } 2.35)$$

$$\lambda_{rh} = 1.27 - 0.0067 \times RH$$

$$\lambda_{vs} = 0.67\left[1 + 1.13^{-0.54V/S}\right]$$

$$\lambda_{to} = \begin{cases} 1.25 \times t_o^{-0.118} & \text{moist-cured concrete} \\ 1.13 \times t_o^{-0.094} & \text{steam-cured concrete} \end{cases}$$

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CONCRETE SHRINKAGE

➤ Shrinkage is a reduction of the concrete volume due to loss of water during hardening.

➤ Types of Shrinkage:
  ➤ Drying shrinkage occurs as moisture diffuses out of the concrete.
  ➤ Autogenous shrinkage due to internal chemical reaction.
  ➤ Carbonation shrinkage occurs when the concrete is exposed to air containing carbon dioxide (significant for HR of 50%).

➤ We can observe that 80%-90% of the time-dependent strain occurs within the first two years.

➤ A value of 0.0005 strain can be considered upper bound and can be controlled using the proper admixture.

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Effect of concrete shrinkage
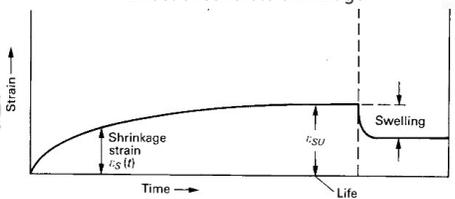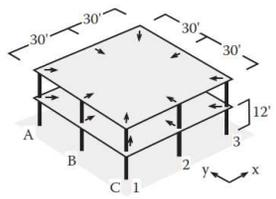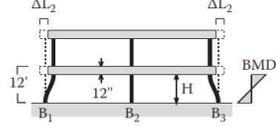
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CONCRETE SHRINKAGE

➤ Shrinkage depends on:
  ▪ Relative humidity (Significant if RH <40%)
  ▪ Volume to the surface ratio (exposed area and the shape of the cross-section).
  ▪ Amount of cement (Hydrated cement paste is responsible for shrinkage)
  ▪ Water to cement ratio.
  ▪ Room temperature.

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Effect of RH on shrinkage
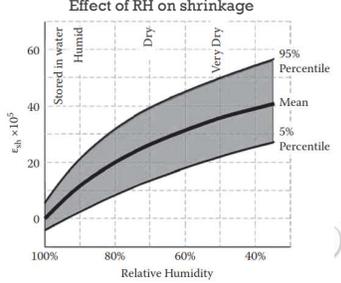
Effect of W/C ratio on shrinkage

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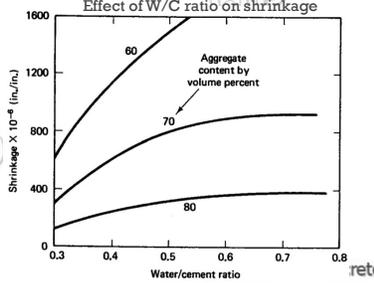
CONCRETE SHRINKAGE

Branson expression (Time dependent method):

$$\varepsilon_{SH}(t) = \begin{cases} \dfrac{t}{35+t}\varepsilon_{SH,u}\,k_{sh}\,\gamma_{vs} & for\ moist-cured \\[2mm] \dfrac{t}{55+t}\varepsilon_{SH,u}\,k_{sh}\,\gamma_{vs} & for\ steam-cured \end{cases}$$

$$\gamma_{vs} = 1.2^{-0.12V/S} \quad,\quad k_{sh} = \begin{cases} 1.4-0.01H & for\ 40\% < H \le 80\% \\ 3.0-0.03H & for\ 80\% < H \le 100\% \end{cases}$$

Where:
$\varepsilon_{SH,u} = 780\times10^{-6}\ in/in$ (measured for $V/S = 1.5$)
$t : in\ days$
$RH = H : relative\ humidity\ (i.e., 70)$

OR as per the PCI expression (K_{SH} method):

$$K_{SH} : \begin{cases} pre\text{-}tensioned\ member = 1 \\ post\text{-}tensioned\ member = use\ Table\ 3.6\ in\ the\ Textbook \end{cases}$$

$$\varepsilon_{SH}(t) = 8.2\times10^{-6}\,K_{SH}\left(1-0.06\dfrac{V}{S}\right)(100-RH)$$

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MATERIALS (STEEL)

Non-prestressing reinforcement

Hot-Rolled Deformed Bars

ASTM A 615
sizes 3 to 18 in Grade 60
sizes 3 to 6 in Grade 40
sizes 6 to 18 in Grade 75

ASTM A 706
(Bridges/seismic applications)
sizes 4 to 18 in Grade 60/80

Prestressing reinforcement
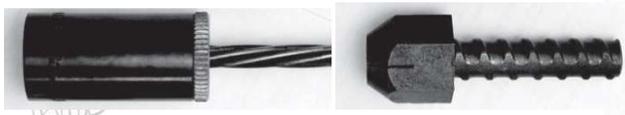
Seven-wire strand (ASTM A 416)   HS bar (ASTM A 722)

produced through a cold-working process and stress relieved by applying tension force to the steel and heating to specific temperature.

ASTM A 416
sizes 0.25 to 0.6 in Grade 250
sizes 0.375 to 0.6 in Grade 270

ASTM A 722
sizes 0.75 to 1.375in Grade 145
sizes 0.75 to 1.375 in Grade 160

Modulus of elasticity is the same for all types of steel $\approx$ 29000 ksi (200000 MPa)
Due to helical shape of the 7-wire strands we use effective modulus of elasticity $\approx$ **27000 ksi**.

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MATERIALS (STEEL)

Prestressing strand (270 ksi) (1860 MPa)

F_{py} =1650 MPa (1% yielding, ϕ12.7 mm)

Stress-relieved wire (235 ksi) (1620 MPa)

High-strength prestressing bars (160 ksi) (1103 MPa)

≈ 4 Times

Grade 60 reinforcing steel (60 ksi) with definite yieldplateau

Grade 60 steel with no definite yield

F_y = 420 MPa

Assuming same elastic modulus

Typical stress-strain curves of reinforcing and prestressing steels
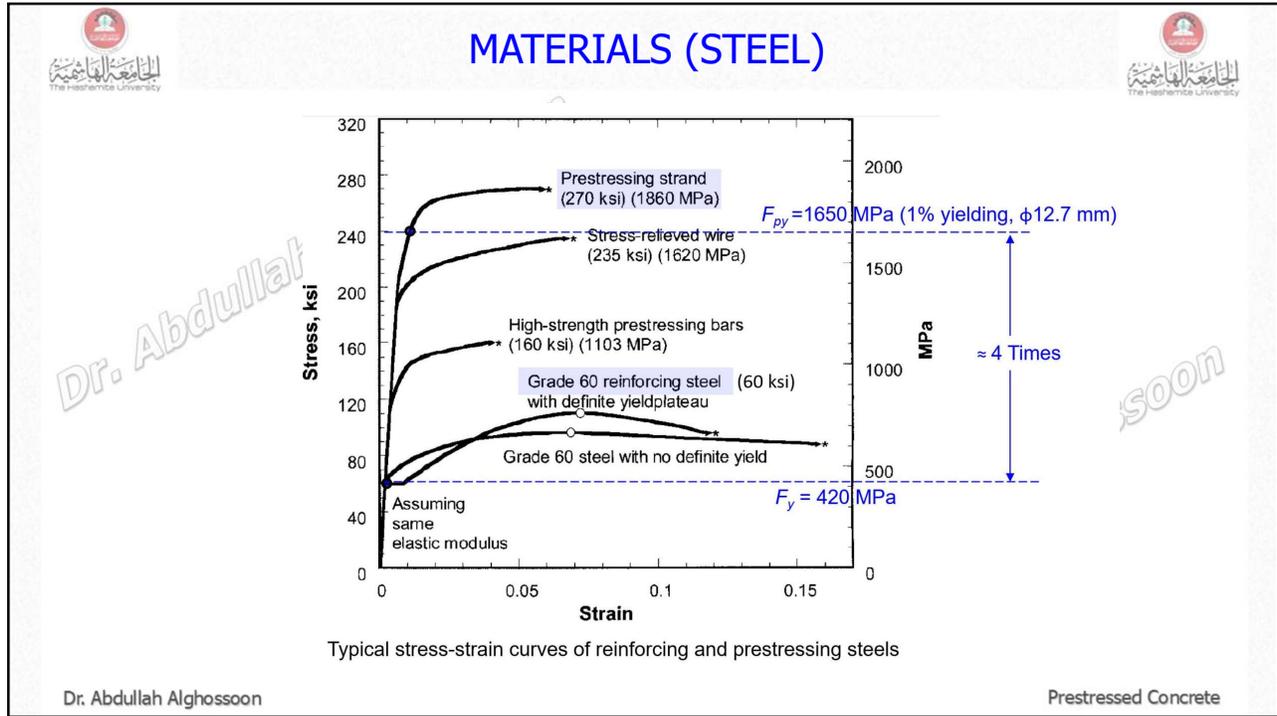
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MATERIALS (STEEL)

Material Test Certificate (MTC)

Customer	: ACROW MISR FORMWORK
	--
	AMMAN , JO
Description	: UNCOATED 7 STEEL WIRE STRAND FOR PRESTRESSED CONCRETE
Size	: 15.2
Standard	: SASO ASTM A416M:2022 / ASTM A416 (A416M) - 24 GRADE 270 (1860) - (W/OIL)

CertificateNo.	: 100000017577
Delivery No	: 71009918
Sale Order No.	: 31006852
LC No.	:
Quantity	: 7
Net Weight	: 19.835

Mechanical Properties

Coil.No	Strand Diameter	Diff. bet. C.W & O.W Dia.	Working sectional area	Maximum Force (Fm)	Load @ 1% Ext. Rt 1.0	Ft 1.0 / Fm	Elongation	Mass	Modulus	Strand Pitch	Heat Number
Unit	mm	mm	mm2	KN	KN	%	%	g/m	KN/mm2	Xd	-
Nom Value	-	-	140.00	-	-	-	-	-	-	-	-
Min.Std	15.05	0.10	-	261.00	234.60	90.00	3.50	-	185.00	12.00	-
Max.Std	15.85	-	-	-	-	-	-	-	205.00	16.00	-
2441318084	15.20	0.20	140.02	273.07	252.70	92.54	5.69	1099.16	199.32	15.09	K450004707
2441318087	15.24	0.20	140.37	274.92	252.76	91.94	5.85	1101.91	197.87	15.42	K450004707
2441318088	15.24	0.20	140.37	274.92	252.76	91.94	5.85	1101.91	197.87	15.42	K450004707
2441318090	15.24	0.20	140.41	275.89	254.81	92.36	5.96	1102.22	197.12	15.42	K450004707
2441318091	15.24	0.20	140.41	275.89	254.81	92.36	5.96	1102.22	197.12	15.42	K450004707
2441318092	15.24	0.20	140.41	275.89	254.81	92.36	5.96	1102.22	197.12	15.42	K450004707
2441318093	15.24	0.20	140.41	275.89	254.81	92.36	5.96	1102.22	197.12	15.42	K450004707

Typical Chemical Composition

Heat No	C	Si	Mn	P	S	Cr
K450004707	0.840	0.190	0.790	0.0140	0.0140	0.170

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MATERIALS (STEEL)

Heat No.	K450004707			Print Date	04/02/2025
Standard	ASTM A416-24 GRADE 270 (1860)			Diameter	15.2 mm
Coil No.	2441318084				

Test Data										
No.	So(A) mm^2	Fm kN	Ft kN	E GPa	Rm MPa	Agt %	Gage Length (mm)			
24413-18081	140.02	273.07	252.70	199.32	1950.21	5.69	600			

All Test Signatures
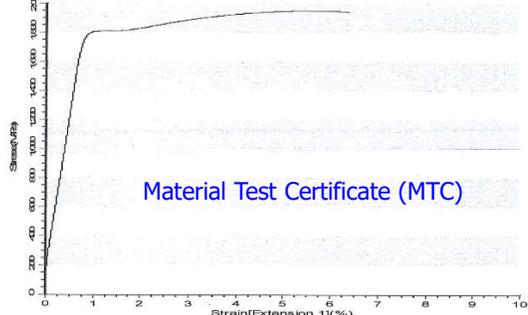
Material Test Certificate (MTC)

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MATERIALS (STEEL)

Prestressing wires

Prestressing strands

Prestressing bars

Type	Size (Diameter)		Shape
	mm	in.	
Plain round wire	2.0 ~ 9.0	0.06 ~ 0.360	
Indented wire	5.0 ~ 7.0	0.200 ~ 0.276	
Sumi - Twist	7.3 ~ 13.0	0.276 ~ 0.512	
Two-wire strand	2.9 x 2	0.114 x 2	
Seven-wire strand	6.2 ~ 15.2	0.250 ~ 0.600	
Nineteen-wire strand	17.8 ~ 21.8	0.700 ~ 0.860	
Round bar	9.2 ~ 32.0	0.362 ~ 1.260	
Threaded bar	23.0 ~ 32.0	0.906 ~ 1.260	
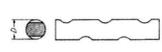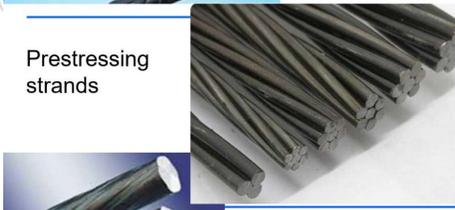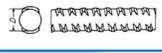
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MATERIALS (STEEL)

Typical characteristics of prestressing steel

Prestressing wires and strands

Prestressing steel	ASTM type or grade	Nominal diameter		Nominal area		Minimum tensile strength, f_{pu}	
		in.	mm	in^2	mm^2	ksi	Mpa
Stress-relieved wires (ASTM A421)	WA,†BA‡	0.192	4.88	0.0289	18.7	250	1725
	WA	0.196	4.98	0.0302	19.4	250	1725
	BA	0.196	4.98	0.0302	19.4	240	1655
	WA, BA	0.25	6.35	0.0491	31.6	240	1655
	WA	0.276	7.01	0.0598	38.7	235	1622
Stress-relieved 7-wire strands (ASTM A416)	Grade 250 (ksi)	0.25	6.35	0.036	23.22	250	1725
		0.313	7.94	0.058	37.42	250	1725
		0.375	9.53	0.080	51.61	250	1725
		0.438	11.11	0.108	69.68	250	1725
		0.500	12.54	0.144	92.90	250	1725
		0.600	15.24	0.216	139.35	250	1725
	Grade 270 (ksi)	0.375	9.53	0.085	54.84	270	1860
		0.438	11.11	0.115	74.19	270	1860
		0.500	12.54	0.153	98.71	270	1860
		0.563	14.29	0.192	123.87	270	1860
		0.600	15.24	0.216	139.35	270	1860

Prestressing bars

Type	ASTM grade	Nominal diameter		Nominal area†		Minimum tensile strength f_{pu}	
		in	mm	in^2	mm^2	ksi	MPa
Smooth alloyed steel bars (ASTM A722)	145 (ksi)	0.750	19.05	0.442	283.9	145	1000
		0.875	22.22	0.601	387.1	145	1000
		1.0	25.40	0.785	503.2	145	1000
		1.125	28.57	0.994	638.7	145	1000
		1.250	31.75	1.227	793.5	145	1000
		1.375	34.92	1.485	954.8	145	1000
	160 (ksi)	0.750	19.05	0.442	283.9	160	1104
		0.875	22.22	0.601	387.1	160	1104
		1.0	25.40	0.785	503.2	160	1104
		1.125	28.57	0.994	638.7	160	1104
		1.250	31.75	1.227	793.5	160	1104
		1.375	34.92	1.485	954.8	160	1104
Deformed bars‡	...	0.625	15.87	0.280	180.6	157	1083
		1.0	25.4	0.852	548.4	150	1035
		1.0	25.4	0.852	548.4	160	1104
		1.25	31.75	1.295	835.5	150	1035
		1.25	31.75	1.295	835.5	160	1104
		1.50	34.92	1.630	1051.6	150	1035

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PRESTRESSED CONCRETE
-PRESTRESS LOSSES-

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Department of Civil Engineering
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Fall 2024

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PRESTRESS LOSSES

▶ Prestressing forces decrease with time, starting at the time we cut off/release steel tendons. As mentioned earlier, this was the main hinder of adopting prestressing techniques for many years. Quantifying reasonably accurate values of prestressing losses is crucial in the design process.

▶ Loss of prestressing is the reduction of tensile stress in prestressing tendons due to:

1) Elastic shortening of the prestressed concrete member
2) Relaxation of tendons stress
3) Time-dependent deformations in the concrete such as shrinkage and creep Anchorage set
4) Friction.

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PRESTRESS LOSSES

▶ Underestimating prestress losses affect **service loading conditions** such as cracking, deflection and cambering. On the other hand, overestimating prestressing forces can be almost as detrimental as underestimation at service loads.

▶ Prestress losses has no effect on the ultimate flexural strength unless tendons are unbounded or in case of significant loss of stresses (less than 50% of the ultimate strength)

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LIMITATIONS ON TENSILE STRESSES IN THE PRESTRESSINGING STEEL

▶ Prestressing forces/stresses are denoted based on the time we are checking these forces/stresses, and usually expressed in terms of the mechanical steel properties such as the yield and ultimate strength.

20.3.2.5.1 The tensile stress in prestressed reinforcement shall not exceed the limits in Table 20.3.2.5.1.

Table 20.3.2.5.1—Maximum permissible tensile stresses in prestressed reinforcement

Stage	Location	Maximum tensile stress	
During stressing	At jacking end	Least of:	$0.94f_{py}$
			$0.80f_{pu}$
			Maximum jacking force recommended by the supplier of anchorage device
Immediately after force transfer	At post-tensioning anchorage devices and couplers	$0.70f_{pu}$	

Table R20.3.2.3.1—Ratio of f_{py}/f_{pu} associated with reinforcement type

Prestressing reinforcement type		f_{py}/f_{pu}
High-strength prestressing bars	ASTM A722 Type I (Plain)	≥ 0.85
	ASTM A722 Type II (Deformed)	≥ 0.80
Stress-relieved strand and wire	ASTM A416 ASTM A421	≥ 0.85
Low-relaxation strand and wire	ASTM A416 ASTM A421	≥ 0.90

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PRESTRESSING FORCE

▶ Prestressing forces are checked in three stages:

1. Jacking stage (while tensioning)
2. Initial stage (Immediately after transferring stresses i.e., 7 days from casting concrete)
3. Final stage (When the long-term effects such as concert creep and shrinkage take place i.e., 5 years)

Jacking Force	Initial Force	Effective force
❑ Before casting concrete for pre-tensioning. ❑ After the concert reaches the specified strength (i.e., 80% of its ultimate strength) for post-tensioning	❑ After instantaneous losses take place	❑ After long-term losses take place
f_{pJ} : expressed in terms of ultimate strength (f_{pu})	f_{pi}	f_{pe}

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PRESTRESS LOSSES

ACI 318-19

Table 3.3   Types of Prestress Loss

Type of prestress loss	Stage of occurrence		Tendon stress loss	
	Pretensioned members	Post-tensioned members	During time interval (t_i, t_j)	Total or during life
Elastic shortening of concrete (ES)	At transfer	At sequential jacking	. . .	Δf_{pES}
Relaxation of tendons (R)	Before and after transfer	After transfer	$\Delta f_{pR}(t_i, t_j)$	Δf_{pR}
Creep of concrete (CR)	After transfer	After transfer	$\Delta f_{pC}(t_i, t_j)$	Δf_{pCR}
Shrinkage of concrete (SH)	After transfer	After transfer	$\Delta f_{pS}(t_i, t_j)$	Δf_{pSH}
Friction (F)	. . .	At jacking	. . .	Δf_{pF}
Anchorage seating loss (A)	. . .	At transfer	. . .	Δf_{pA}
Total	Life	Life	$\Delta f_{pT}(t_i, t_j)$	Δf_{pT}

20.3.2.6 *Prestress losses*

20.3.2.6.1 Prestress losses shall be considered in the calculation of the effective tensile stress in the prestressed reinforcement, f_{se}, and shall include (a) through (f):

(a) Prestressed reinforcement seating at transfer
(b) Elastic shortening of concrete
(c) Creep of concrete
(d) Shrinkage of concrete
(e) Relaxation of prestressed reinforcement
(f) Friction loss due to intended or unintended curvature in post-tensioning tendons

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SOURCE OF PRESTRESS LOSSES

Short term (instantaneous)

▶ Elastic shortening of concrete Δf_{pES}

Concrete member is shortening due to the applied compressive force, so are the attached tendons.

▶ Anchorage set Δf_{pAS}

Wedges set-in a certain distance during the locking process (Only in post-tensioned members)

▶ Friction Δf_{pf}

The friction between tendons and duct due to change of duct profile (Only in post-tensioned member)

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SOURCE OF PRESTRESS LOSSES

Long term (time-dependent)

▶ Relaxation of the stressed tendons Δf_{pR}

Loss of tension force with time (depends on initial stress and temperature)

▶ Shrinkage (concrete) Δf_{pS}

Loss of free water in concrete induces shortening in concrete

▶ Creep (concrete) Δf_{pC}

Shortening of concrete strain due to sustained compressive stresses

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$$\sigma_{SH} = \varepsilon_{SH} * Es$$

$$\sigma_{SH} = \varepsilon_{SH} * fs/\varepsilon_s$$

$$\sigma_{SH} = \varepsilon_{SH} * fcs*n*/ \varepsilon_s$$

$$\sigma_{SH} = \varepsilon_{SH} * fcs*(Es/Ec)*/ \varepsilon_s$$

$$\sigma_{SH} = \varepsilon_{SH} / \varepsilon_s * fcs*(Es/Ec)$$

$$\sigma_{SH} = Ct* fcs*(Es/Ec)$$

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CONCRETE CREEP (ΔF_{pC})

$$C_t = \frac{t^{0.6}}{10+t^{0.6}} C_u$$

$$\Delta f_{pCR} = C_t \frac{E_{ps}}{E_c} f_{cs}$$

The ACI-ASCE committee expression has similar format:

$$\Delta f_{pCR} = K_{CR} \frac{E_{ps}}{E_c} \left(f_{CS}^- - f_{CSd}^- \right)$$

C_u = Creep coefficient ranges between 2 and 4

f_{CS}^- = stress in the concrete at the level of the centroid of the prestressing tendons immediately after transfer.

f_{CSd}^- = stress in the concrete at the level of the centroid of the prestressing tendons due to superimpoed dead load after prestressing.

t : is time in days

n : modular ratio

$$K_{CR} : \begin{pmatrix} 2 \text{ for pretensioned member} \\ 1.6 \text{ for post-tensioned member} \end{pmatrix} \text{ Reduced by 20\% for lightweight concrete}$$
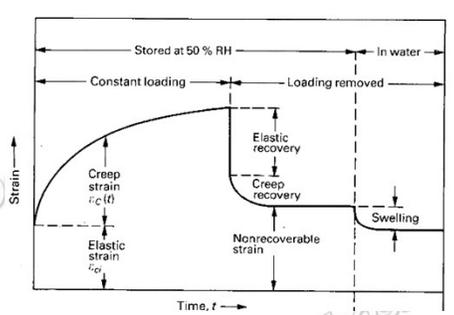
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CONCRETE SHRINKAGE (ΔF_{pS})

Branson expression (Time dependent method):

$$\Delta f_{pSH=} = \begin{cases} \dfrac{t}{35+t}\,\varepsilon_{SH,u}\,E_{ps}\,k_{sh} & \text{for moist–cured} \\[2mm] \dfrac{t}{55+t}\,\varepsilon_{SH,u}\,E_{ps}\,k_{sh} & \text{for steam–cured} \end{cases} \quad , \quad k_{sh} = \begin{cases} 1.4-0.01H & \text{for } 40\% < H \le 80\% \\[2mm] 3.0-0.03H & \text{for } 80\% < H \le 100\% \end{cases}$$

OR as per the PCI expression (K_{SH} method):

$$\Delta f_{pSH=} = \left\{ 8.2\times10^{-6}\,K_{SH}\times E_{ps}\left(1-0.06\dfrac{V}{S}\right)(100-RH) \right.$$

Where:

$\varepsilon_{SH,u} = 780\times10^{-6} \text{ to } 820\times10^{-6} \text{ in/in}$

$t : in\ days$

$RH = H : relative\ humidity\ (i.e., 70)$

$K_{SH} : from\ Table\ 3.6\ in\ the\ Textbook\ for\ post–tensioned$

$K_{SH} : 1\ for\ pre–tensioned$

Table 3.6 Values of K_{SH} for Post-Tensioned Members

Time from end of moist curing to application of prestress, days	1	3	5	7	10	20	30	60
K_{sh}	0.92	0.85	0.80	0.77	0.73	0.64	0.58	0.45

Source: Prestressed Concrete Institute.

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CONCRETE CREEP (ΔF_{pC}) AND SHRINKAGE (ΔF_{pS})

$$\Delta f_{pSH=} = \begin{cases} \dfrac{t}{35+t}\,\varepsilon_{SH,u}\,E_{ps}\,k_{sh} & \text{for moist–cured} \\[2mm] \dfrac{t}{55+t}\,\varepsilon_{SH,u}\,E_{ps}\,k_{sh} & \text{for steam–cured} \end{cases}$$

$$k_{sh} = \begin{cases} 1.4-0.01H & \text{for } 40\% < H \le 80\% \\[2mm] 3.0-0.03H & \text{for } 80\% < H \le 100\% \end{cases}$$

It is recommended to consider the information provided by the manufacturer.

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PRESTRESS RELAXATION (ΔF_{pR})

Relaxation of the stressed tendons vs time

$$\Delta f_{pR} = \begin{cases} f_{pi}\dfrac{\log t}{10}\dfrac{f_{pi}}{f_{py}} - 0.55 & \text{for stress relieved wires} \\[2ex] f_{pi}\dfrac{\log t}{45}\dfrac{f_{pi}}{f_{py}} - 0.55 & \text{for low relaxation wires} \end{cases}$$

Effect of temp on relaxation

Effect of initial stress on relaxation

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PRESTRESSING STEEL RELAXATION (R)

$$\Delta f_{pR} = f_{pi}'\left(\frac{\log t}{10}\right)\left(\frac{f_{pi}}{f_{py}} - 0.55\right) \quad \text{for stress relieved steel}$$

$$\Delta f_{pR} = f_{pi}'\left(\frac{\log t}{45}\right)\left(\frac{f_{pi}}{f_{py}} - 0.55\right) \quad \text{for low relaxation steel}$$

Where t: in hours

f_{pi}' : Reduced transferred stresses due to immediate and long term losses

▶ The loss in the prestressing force is attributed to the constant elongation with time. This reduction in force is expressed in terms of initial prestress to the yield stress (f_{pi} / f_{py})

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PRESTRESSING STEEL RELAXATION

❖ ACI-ASCE Method for calculating prestress relaxation loss:
This method combines the effect of elastic shortening, creep and shrinkage to evaluate the prestress steel relaxation based on the following equation:

$$\Delta f_{pR} = [K_{re} - J\Delta(f_{pES} + f_{pCR} + f_{pSR})] \times C$$

Table 3.4 Values of C

f_{pi}/f_{pu}	Stress-relieved strand or wire	Stress-relieved bar or low-relaxation strand or wire
0.80		1.28
0.79		1.22
0.78		1.16
0.77		1.11
0.76		1.05
0.75	1.45	1.00
0.74	1.36	0.95
0.73	1.27	0.90

Table 3.5 Values of K_{RE} and J

Type of tendon[a]	K_{RE}	J
270 Grade stress-relieved strand or wire	20,000	0.15
250 Grade stress-relieved strand or wire	18,500	0.14
240 or 235 Grade stress-relieved wire	17,600	0.13
270 Grade low-relaxation strand	5,000	0.040
250 Grade low-relaxation wire	4,630	0.037
240 or 235 Grade low-relaxation wire	4,400	0.035
145 or 160 Grade stress-relieved bar	6,000	0.05

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LOSSES DUE TO FRICTION (F)

► Friction loss takes place in post-tensioned concrete between in the tendons and the surrounding concrete ducts.

► The magnitude of the friction loss depends on the global and local tendon alignment:

Global alignment: Curvature effect (pre-determined).

Local alignment: wobble effect (refined during shop drawing preparations).

► Friction losses vary linearly along the member and has a maximum value at the far end (if jacking from one end).

Curvature and wobble effect   Idealized   Curvature and wobble effect
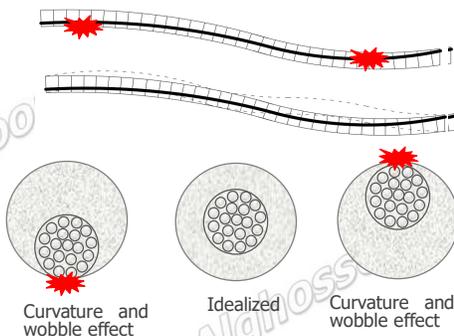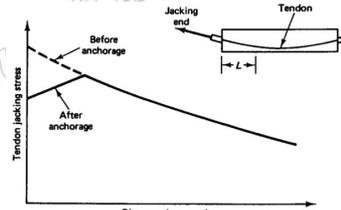
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LOSSES DUE TO FRICTION (F)

Curvature effect $\quad F_2 = F_1 e^{-\alpha\mu}$

Wobble effect $\quad\quad F_2 = F_1 e^{-KL}$

Curvature + Wobble = $F_2 = F_1 e^{-(\mu\alpha + KL)}$

$\Delta f_{pF} = F_1 - F_2 = F_1(1 - e^{-(\mu\alpha + KL)})$

For small difference in prestressing force between the start and the end of curved portion:

$1 - e^{-(\mu\alpha + KL)} = -(\mu\alpha + KL)$, Hence;

$\Delta f_{pF} = -F_1(\mu\alpha + KL)$

Where:

L: is the length of the curve

α : is approximately twice the solpe of the tendons at either end of the segment ($\approx \dfrac{8y}{x}$ radian)
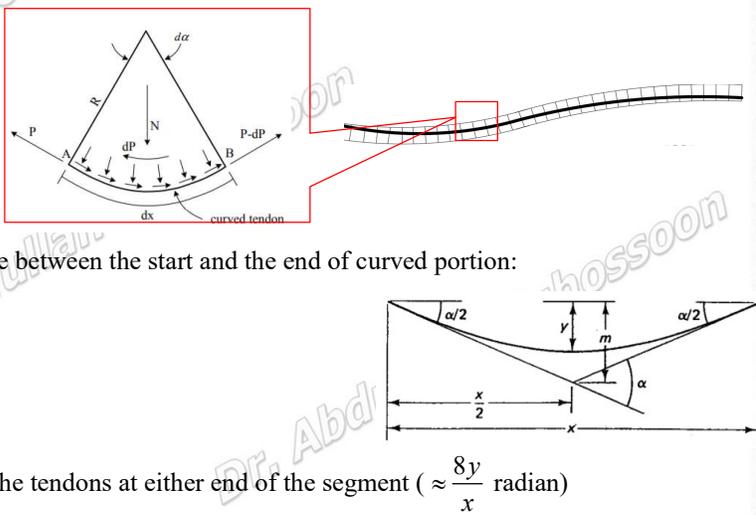
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LOSSES DUE TO FRICTION (F)

Be aware of Units

$$\Delta f_{pF} = -F_1(\mu\alpha + KL)$$

Multiply by 3.28 for metric units

Table 3.7 Wobble and Curvature Friction Coefficients

Type of tendon	Wobble coefficient, K per foot	Curvature coefficient, μ
Tendons in flexible metal sheathing		
Wire tendons	0.0010–0.0015	0.15–0.25
7-wire strand	0.0005–0.0020	0.15–0.25
High-strength bars	0.0001–0.0006	0.08–0.30
Tendons in rigid metal duct		
7-wire strand	0.0002	0.15–0.25
Mastic-coated tendons		
Wire tendons and 7-wire strand	0.0010–0.0020	0.05–0.15
Pregreased tendons		
Wire tendons and 7-wire strand	0.0003–0.0020	0.05–0.15

Source: Prestressed Concrete Institute.

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ANCHORAGE SEATING LOSSES

▶ This type of losses occurs due to the seating (slip) of wedges in the anchors when the jacking force is transferred to the anchorage.

▶ The range of anchorage-seating loss is between 6.0 mm to 10.0 mm. for the two-piece wedges.

▶ This type of losses can be compensated by overstressing based on the data provided by the manufacturer

$$\Delta f_{pA} = \frac{\Delta_A}{L} E_{ps}$$

Where:

Δ_A : The magnitude of the slip

L : The length of tendons

E_{ps} : The prestressing steel modulus

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LOSSES OF PRESTRESS

Jacking Force

Initial Force
(Prestressing force immediately after transfer)

Effective Force
(Final of effective prestressing force)

P_j — Immediate losses ($\approx 10\%$) → P_i — Time-dependent losses ($\approx 10\%$) → P_e — Total losses ($\approx 20\%$)

$\left(P_j \cong 0.75 - 0.80 P_u \right)$

Limited from the ultimate strength of prestressing steel

Losses	Pretensioned Members	Post-Tensioned Members
Immediate Losses		
1) Elastic Shortening (Concrete)	✓	✓
2) Anchorage (Steel)		✓
3) Friction (Concrete & Steel)		✓
Time-Dependent Losses		
1) Creep (Concrete)	✓	✓
2) Shrinkage (Concrete)	✓	✓
3) Steel Relaxation (Steel)	✓	✓

For single tendon, there is no loss as the tendon force is the same as jacking force.

For multi tendons, there is no loss only for the last stressed tendon as the tendon force is the same as jacking force, while other previous tendons are affected to the elastic shortening losses.

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EXAMPLE 1 (ELASTIC SHORTENING)

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EXAMPLE 1 (ELASTIC SHORTENING)
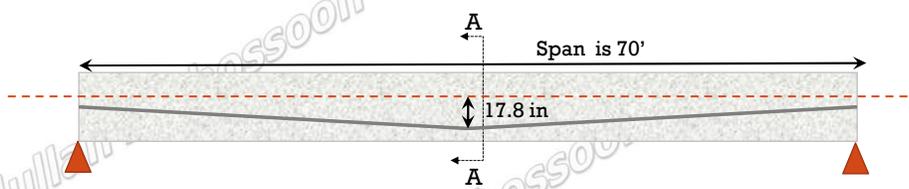
Loading:

W.D=491 plf (Concentrated)
W.SD=250 plf(Concentrated)

Material Properties:

Fpu=270.0 ksi (Stress relieved 7-wire strands)
Fpy=0.85 Fpu =229.5 ksi
Fpi=0.7 Fpu= 189.0 ksi
Aps = 12 x 0.153 in

Eps = 28000 ksi
F'c= 5 ksi
F'ci= 3.5 ksi

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SEQUENTIAL TENSIONING

Modular ratio

Total number steps

Prestressing force at j+1 level

Eccentricity of strands at the level j+1

$$(\Delta f_{PES})_J = n\Sigma_{j=J}^{N-1}\left[\frac{P_{i(j+1)}}{A_c} + \frac{P_{i(j+1)}.e_{(j+1)}}{I_c}.e_j\right]$$

Average elastic shortening losses

Area of PS section

Step No (prestressing)

Moment of inertia of PS section

Eccentricity of strands at the level of interest

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EXAMPLE 2 SEQUENTIAL TENSIONING

Find elastic shortening in the **post-tensioned** beam shown in the figure below when tendons are tensioned sequentially.

Given:

1- Four 7-wire strands with ultimate strength 270 ksi and 0.6 in dia.

2- Jacking force per strand is 225 kN.

3- Modular ratio = 6

250mm

250mm

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CONT..

$$(\Delta f_{PES})_J = n\Sigma_{j=J}^{N-1}\left[\frac{P_{i(j+1)}}{A_c} + \frac{P_{i(j+1)}.e_{(j+1)}}{I_c}.e_j\right]$$

225 kN

6

0.25 x 0.25
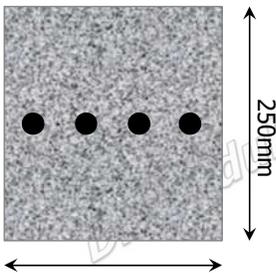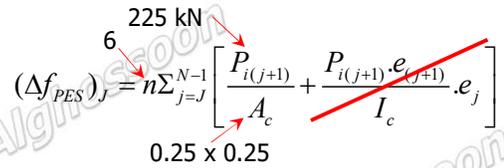
Jacking	Cable loss			
	1	2	3	4
1	0	-	-	-
2	$\frac{225kN}{(0.25m)^2}x6 = 21.6 MPa$	0	-	-
3	$21.6 MPa$	$21.6 MPa$	0	-
4	$21.6 MPa$	$21.6 MPa$	$21.6 MPa$	0
Total	$64.8 MPa$	$43.2 MPa$	$21.6 MPa$	0

250mm

250mm
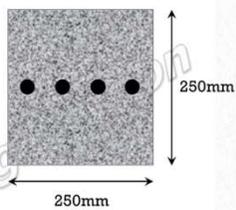
$$Average\ losses = \frac{64.8 MPa + 43.2 MPa + 21.6 MPa + 0 MPa}{4} = 32.4\ MPa$$

$This\ can\ also\ be\ calculated\ as\ the\ total\ loss\ in\ the\ first\ strands\ x\ 1/2$

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WHAT IF IT WAS PRETENSIONED

250mm

250mm
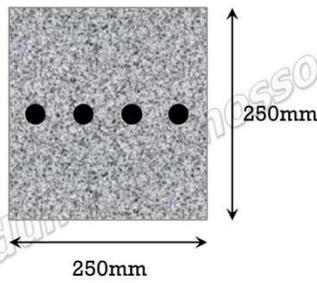
$$\Delta f_{PES} = 6\left(\frac{4*225kN}{(0.25m)^2}\right) = 86.4\ MPa \gg 32.4\ MPa$$

$Elastic\ shortening\ is\ more\ significant\ in\ case\ of\ pretensioning$

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EXAMPLE 3 (SEQUENTIAL TENSIONING)

❖ Find elastic shortening in the **pre-tensioned** beam shown in the figure below.

Given:

1- Six 7-wire strands with ultimate strength 270 ksi and 0.6 in dia.

2- Jacking force per strand is 225 kN.

3- Modular ratio = 6

125mm
125mm
50mm

600mm

300mm
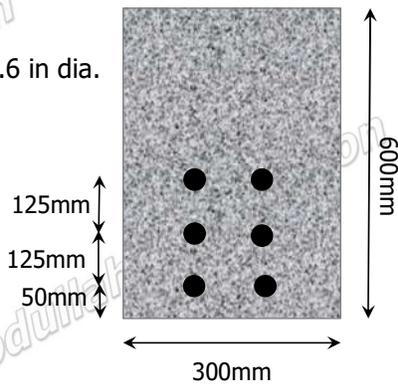
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CONT..

Because all strands are released together

Location of the equivalent PS force from N.A

Location of calculated stress

Tendons	Cable loss		
	F_{CS} (Stress at the concrete level)	$F_{PES} = nF_{CS}$	% loss
1	$\dfrac{225kNx6}{300x600}\left(1 + \dfrac{125x0}{30000}\right) = 7.5MPa$	$45MPa$	$\dfrac{45MPa}{225kN/140mm^2} = 2.8\%$
2	$\dfrac{225kNx6}{300x600}\left(1 + \dfrac{125x125}{30000}\right) = 11.41MPa$	$68.6MPa$	$\dfrac{68.6MPa}{225kN/140mm^2} = 4.27\%$
3	$\dfrac{225kNx6}{300x600}\left(1 + \dfrac{125x250}{30000}\right) = 15.3MPa$	$91.9MPa$	$\dfrac{91.9MPa}{225kN/140mm^2} = 5.72\%$
Total		$\dfrac{240.4MPa}{3} = 68.5\ MPa$	$\dfrac{68.5MPa}{225kN/140mm^2} = 4.26\%$

Compare the results with f_{pu}=F x A_{ps}

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WHAT IF IT WAS POST-TENSIONED

$$(\Delta f_{PES})_J = n\Sigma_{j=J}^{N-1}\left[\frac{P_{i(j+1)}}{A_c} + \frac{P_{i(j+1)}.e_{(j+1)}}{I_c}.e_j\right]$$

600mm

1

125mm

2

125mm

3

50mm

300mm

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CONT..

$$(\Delta f_{PES})_J = n\Sigma_{j=J}^{N-1}\left[\frac{P_{i(j+1)}}{A_c} + \frac{P_{i(j+1)}.e_{(j+1)}}{I_c}.e_j\right] = \frac{P_{i(j+1)}}{A_c}\left[1 + \frac{e_{(j+1)}}{r^2}.e_j\right]$$

Two tendons at once

Tendons	Cable loss			
	F_{CS} (Stress at the concrete level)			% loss
	1 (0mm)	2 (125mm)	3 (250mm)	
1 (0mm)	0	–	–	
2 (125mm)	$\frac{225kNx2}{300x600}\left(1+\frac{125x0}{30000}\right)=2.5MPa$	0	–	$\frac{20.2MPa}{225kN/140mm^2}=1.26\%$
3 (250mm)	$\frac{225kNx2}{300x600}\left(1+\frac{250x0}{30000}\right)=2.5MPa$	$\frac{225kNx2}{300x600}\left(1+\frac{250x125}{30000}\right)=5.1MPa$	0	
Total	0+2.5+2.5=5 MPa	0 + 5.1 = 5.1 MPa	0	
Avg	n(5+5.1)/3=20.2 MPa			

Modular ratio

Number of steps

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RECOMMENDED LUMP-SUM LOSSES

- The magnitude of losses can't be determined exactly as it depends on many interrelated factors.

- Empirical methods can be used for estimating losses in the preliminary member design under standard conditions (Some info is not available and the design may be changed many times over)

PCI

Similarly, the Precast/Prestressed Concrete Institute (PCI) published building code requirements for prestressed concrete (Structural Engineers Association of Northern California 1959) specifying the following lump-sum values for estimating prestress losses, excluding friction prestressing steel loss: pretensioning = 25,000 to 35,000 psi (172 to 241 MPa); and post-tensioning = 15,000 to 25,000 psi (103 to 172 MPa).

ACI-ASCE

Requirements for evaluating prestress loss first appeared in ACI 318-63, in new Chapter 29 on prestressed concrete. ACI 318-63 commentary recommended the following lump-sum losses, excluding friction and seating losses, based on the ACI-ASCE Committee 323 (1958) report: pretensioning = 35,000 psi (241 MPa); and post-tensioning = 25,000 psi (172 MPa).

FHWA

The Federal Highway Administration (FHWA), known as the Bureau of Public Roads (1954) at the time, published design criteria for prestressed concrete bridges in which losses due to creep, relaxation, shrinkage, and elastic deformation were specified as

$$\text{pretensioned concrete} = 6000 + 16 f_{cps} + 0.04 f_{pt} \ (\text{psi})$$
$$\text{pretensioned concrete} = 41.4 + 16 f_{cps} + 0.04 f_{pt} \ (\text{MPa}) \quad (3.2a)$$

$$\text{post-tensioned concrete} = 3000 + 11 f_{cps} + 0.04 f_{pt} \ (\text{psi})$$
$$\text{post-tensioned concrete} = 20.7 + 11 f_{cps} + 0.04 f_{pt} \ (\text{MPa}) \quad (3.2b)$$

For typical designs, the value of f_{cps} was approximately 1300 psi (8.96 MPa) and the value of f_{pt} was a maximum of 200,000 psi (1380 MPa). Using these typical values for f_{cps} and f_{pt}, the criteria stated (Eq. (3.2a) and Eq. (3.2b)) for losses equates to 34,800 psi (240 MPa) for pretensioned concrete and 25,300 psi (174 MPa) for post-tensioned concrete.

AASHTO

Table 3.1 AASHTO Lump-Sum Losses

Type of prestressing steel	Total loss	
	f_c' = 4,000 psi (27.6 N/mm²)	f_c' = 5,000 psi (34.5 N/mm²)
Pretensioning strand		45,000 psi (310 N/mm²)
Post-tensioning[a] wire or strand	32,000 psi (221 N/mm²)	33,000 psi (228 N/mm²)
Bars	22,000 psi (152 N/mm²)	23,000 psi (159 N/mm²)

[a]Losses due to friction are excluded. Such losses should be computed according to Section 6.5 of the AASHTO specifications.

Table 3.2 Approximate Prestress Loss Values for Post-Tensioning

Post-tensioning tendon material	Prestress loss, psi	
	Slabs	Beams and joists
Stress-relieved 270-K strand and stress-relieved 240-K wire	30,000 (207 N/mm²)	35,000 (241 N/mm²)
Bar	20,000 (138 N/mm²)	25,000 (172 N/mm²)
Low-relaxation 270-K strand	15,000 (103 N/mm²)	20,000 (138 N/mm²)

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LUMP-SUM LOSSES EXAMPLE

Measured losses from literature

$y = 0.0x + 19.5\%$

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LOSSES OF PRESTRESSING

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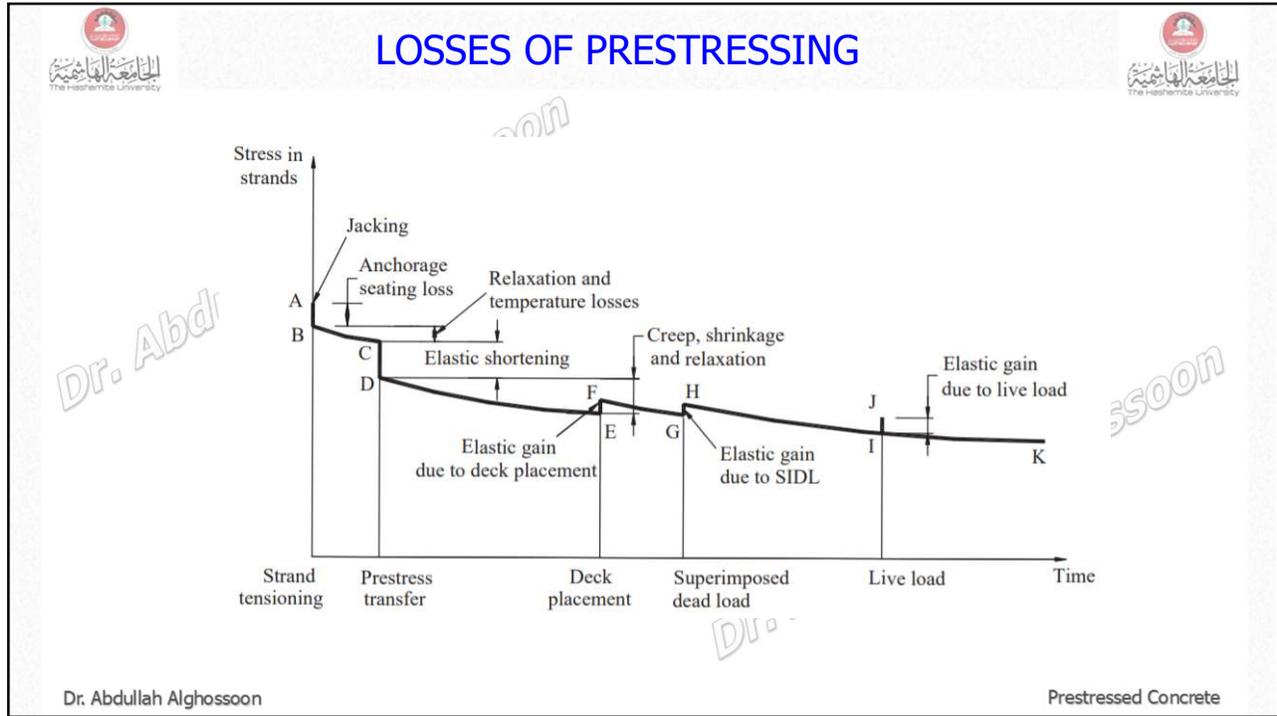
STAGES AT WHICH STRESSES ARE ESTIMATED

- At transfer of prestress force by checking stresses in the tendon and concrete and compare these to acceptable ACI 318-19 limits.
  - Initial Prestress
  - Self-weight application

- At service load level, the stresses in the concrete are also checked and compared to acceptable ACI 318-19 limits, and prevent excessive deflections and cracking.
  - Superimposed dead load
  - Decompression in steel
  - Service load limit

- Strength must be checked to ensure that the member can sustain overloads, M_n and V_n in accordance to ACI 318-19
  - Ultimate load state
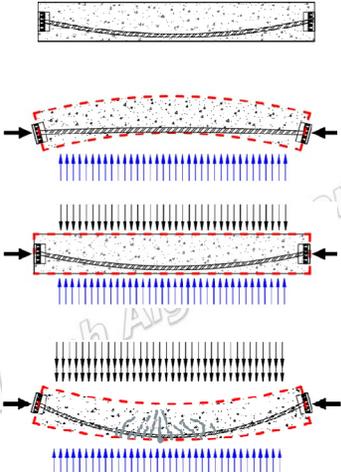
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PROPORTIONING OF PC MEMBERS

❖ Select cross section
  - Pretensioned – standard sections
  - Post-tensioned - L_n/h criteria

❖ Select prestressing
  - Service load: stress control/deflection for initial amount, then check strength
    - Or
  - Strength: M_n satisfied, then check service load conditions

❖ Check Shear Strength

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CONCRETE STRESSES IN FLEXURE

Immediately after prestress transfer

- Extreme fiber stress in compression $0.6f_{ci}'$
- Extreme fiber stress in tension except at ends of SS beam $3\sqrt{f_{ci}'}$
- Extreme fiber stress in tension at ends of SS beam. $6\sqrt{f_{ci}'}$

After allowances for all prestress losses

- Extreme fiber stress in compression due to prestress plus sustained loads (dead and live load). $0.45f_{ci}'$
- Extreme fiber stress in compression due to prestress plus total load if live load is transient. $0.6f_c'$
- Extreme fiber stress in tension in precompression tensile zone. $6\sqrt{f_c'}$
- Extreme fiber stress in tension in precompression tensile zone when long-term deflections comply with the ACI definition requirement in one-way system. $12\sqrt{f_c'}$

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SERVICEABILITY TENSION REQUIREMENTS ACI318-19

Table R24.5.2.1—Serviceability design requirements

	Prestressed		
	Class U	**Class T**	**Class C**
Assumed behavior	Uncracked	Transition between uncracked and cracked	Cracked
Section properties for stress calculation at service loads	Gross section 24.5.2.2	Gross section 24.5.2.2	Cracked section 24.5.2.3
Allowable stress at transfer	24.5.3	24.5.3	24.5.3
Allowable compressive stress based on uncracked section properties	24.5.4	24.5.4	No requirement
Tensile stress at service loads 24.5.2.1	$\leq 7.5\sqrt{f_c'}$	$7.5\sqrt{f_c'} < f_t \leq 12\sqrt{f_c'}$	No requirement
Deflection calculation basis	24.2.3.8, 24.2.4.2 Gross section	24.2.3.9, 24.2.4.2 Cracked section, bilinear	24.2.3.9, 24.2.4.2 Cracked section, bilinear
Crack control	No requirement	No requirement	24.3
Computation of Δf_{ps} or f_s for crack control	—	—	Cracked section analysis
Side skin reinforcement	No requirement	No requirement	9.7.2.3

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FLEXURAL DESIGN

❖ What do we need:
    To satisfy the stresses and deflection requirement at service load and member strength ta ultimate loading.

❖ How can we do that:
    By controlling some geometric and material properties such as:
    1-Cross-section dimensions.
    2-Prestress force.
    3-PS steel eccentricity.

❖ Basic flexure theory assumptions:
    1. Plane section before bending remain plane after bending (i.e. small deflections)
    2. Material is elastic
    3. Effect of transformed section is neglected
    4. Section is uncracked
    5. No variation of PS force along the beam
    6. Effect of small curvature is neglected

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STAGE OF STRESSES
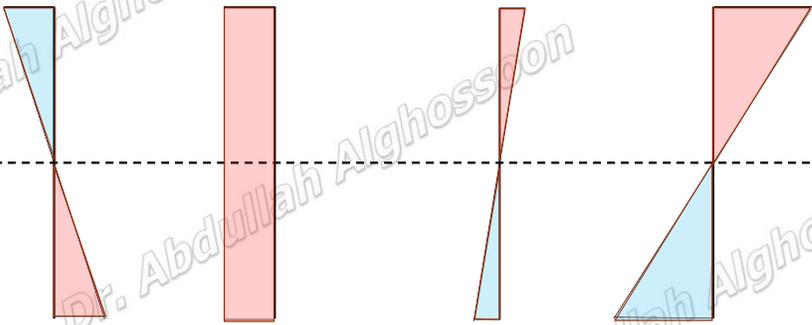
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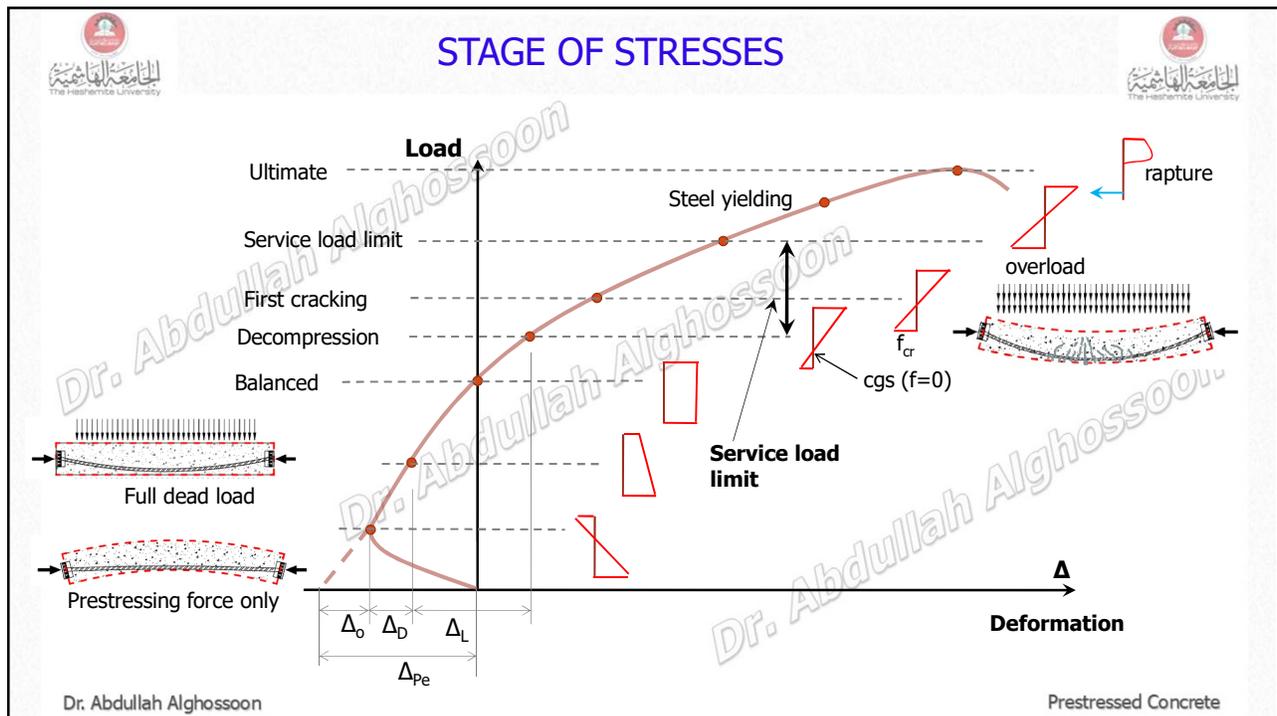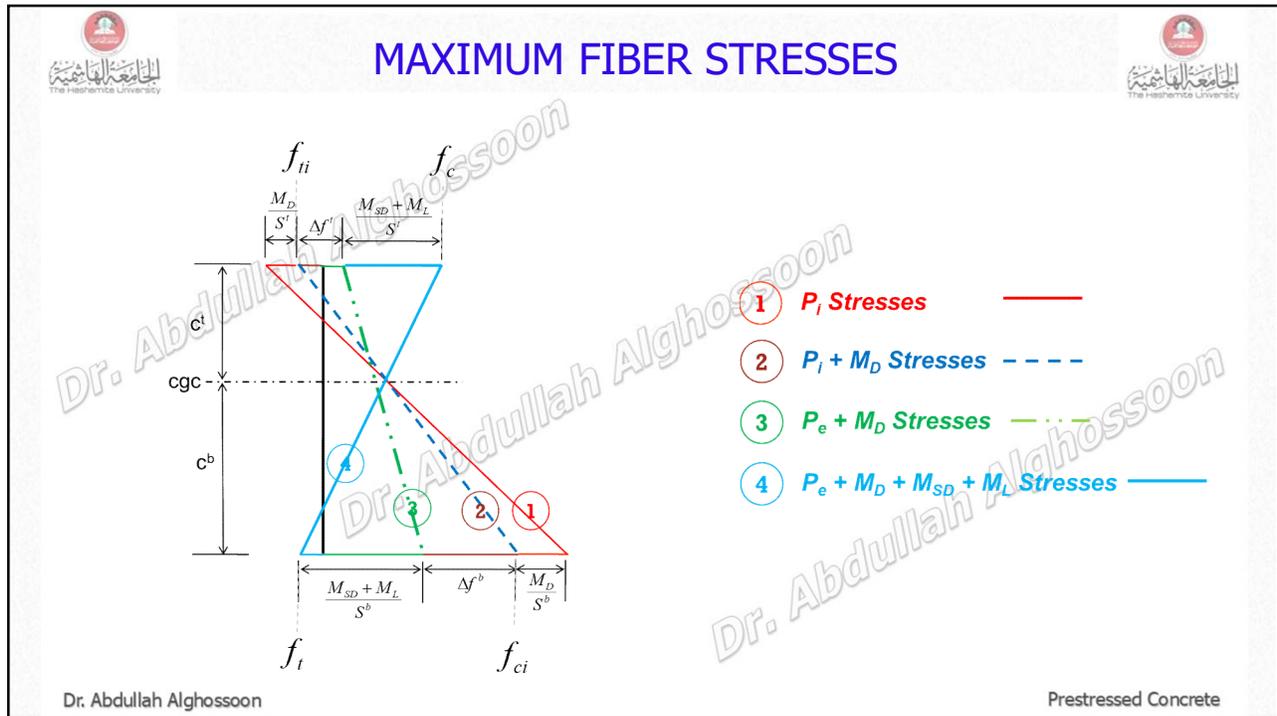
SELECTION OF GEOMETRIC PROPERTIES

❖ The minimum section moduli S^t & S^b is selected based on the allowable stress limits at initial and service loadings conditions. This involve losses, additional loading, and different allowable stress limits :

$$f_{ti} = 3\sqrt{f_{ci}'} \; OR$$

$$f_{ti} = 6\sqrt{f_{ci}'} \; \text{for SS at support}$$

$$f_{ci} = 0.6 f_{ci}'$$

$$f_c = 0.45 f_c' \; OR$$

$$f_c = 0.6 f_c'$$

$$f_t = 6\sqrt{f_c'} \; OR$$

$$f_t = 12\sqrt{f_c'} \quad \text{if Long-term deflection is met}$$

MPa or psi ?

(a) At Transfer

(B) At Service

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SELECTION OF GEOMETRIC PROPERTIES

❖ Stresses at transfer:

$$f^t = \frac{-P_i}{A_c}\left(1 - \frac{ec^t}{r^2}\right) - \frac{M_D}{S^t} \leq f_{ti} \;\; .....(1)$$

$$f^b = \frac{-P_i}{A_c}\left(1 + \frac{ec^b}{r^2}\right) + \frac{M_D}{S^b} \leq f_{ci}' \;\; ....(2)$$

P_i = initial prestressing force

❖ Effective prestressing after losses:

$$f^t = \frac{-P_e}{A_c}\left(1 - \frac{ec^t}{r^2}\right) - \frac{M_T}{S^t} \leq f_c' \;\; ....(3)$$

$$f^b = \frac{-P_e}{A_c}\left(1 + \frac{ec^b}{r^2}\right) + \frac{M_T}{S^b} \leq f_t \;\; ....(4)$$

P_e = effective prestressing force after losses

*$M_T = M_D + M_{SD} + M_L$

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SELECTION OF GEOMETRIC PROPERTIES

❖ Decompression stage is when the stress at the cgs is equal to zero. The change in the concrete stress due to decompression is:

$$f_{decomp} = \frac{P_e}{A_c}\left(1 + \frac{e^2}{r^2}\right)$$

❖ For variable tendon eccentricity: assume the effective prestress:

$$P_e = \gamma P_i$$

Where γ is the residual prestress ratio (i.e., 80%)

❖ In this case the losses will be: $\quad P_i - P_e = (1-\gamma)P_i \ldots\ldots\ldots(5)$

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MEMBERS WITH VARIABLE TENDON ECCENTRICITY
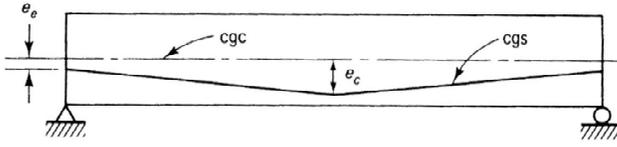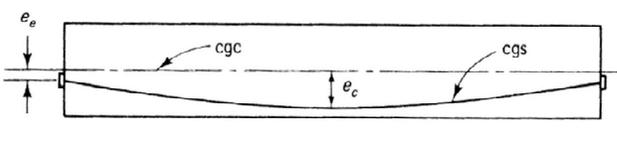
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SELECTION OF GEOMETRIC PROPERTIES

-By solving Equations (1) and (3) and using equation (5) we get:

$$S^t \geq \frac{(1-\gamma)M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c}$$

-By solving Equations (2) and (4) and using equation (5) we get:

$$S^b \geq \frac{(1-\gamma)M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}}$$

-In addition:

$$c^b + c^t = h, \quad Hence; \quad \frac{c^t}{c^b} = \frac{S^b}{S^t} = \frac{\gamma f_{ti} - f_c}{f_t - \gamma f_{ci}}$$

$$\frac{c^t + c^b}{h} = 1 \quad ; \frac{c^b}{h} = \frac{S^t}{S^t + S^b} \quad ; \frac{c^t}{h} = \frac{S^b}{S^t + S^b}$$

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REQUIRED ECCENTRICITY

❖ In addition to the cross-section modulus, maximum eccentricity can be determined using concrete centroidal stress under initial conditions.

$$\frac{-P_i}{A_c} + \frac{P_i e}{S_t} - \frac{M_D}{S_t} = f_{ti} \quad but \quad \frac{-P_i}{A_c} = f_{cgc} \quad Hence;$$

$$\frac{P_i e}{S_t} = (f_{ti} - f_{cgc}) + \frac{M_D}{S_t} \quad or \quad \boxed{e = (f_{ti} - f_{cgc})\frac{S^t}{P_i} + \frac{M_D}{P_i}}$$

We don't know f_{cgc} ?

$$\frac{f_{ti} - f_{cgc}}{c_t} = \frac{f_{ti} - f_{ci}}{h}$$

$$f_{cgc} = f_{ti} - \frac{c_t}{h}(f_{ti} - f_{ci})$$

Usually negative

This value is negative
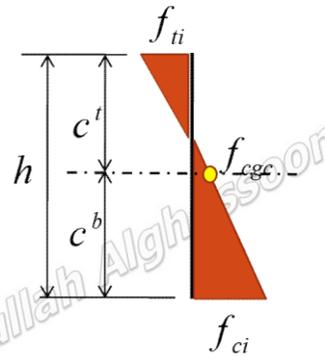
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MEMBERS WITH CONSTANT TENDON ECCENTRICITY

❖ Beams with constant tendons along the span are very common in moderate PS spans due to the economic fabrication and erection costs. However, the stress limits will most likely be exceeded at several point in the span, especially at supports du to the absence of external stresses (M_D, M_{SD}, M_L).

Allowable limits are violated at the support location

Stresses are within the allowable limits at the location of moment

Extreme fiber stresses along the length of SS beam (Immediately after transfer forces)

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MEMBERS WITH CONSTANT TENDON ECCENTRICITY

❖ Certain alternatives are available for reducing excessive stresses at supports, as follows:

cables

1- Debonding

Cable sheathed without grouting

2- Raised Tendons

L/4

3- Supplementary non-prestressed steel

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MEMBERS WITH CONSTANT TENDON ECCENTRICITY

❖ Smaller eccentricity is used in beams with constant tendons along the span compared with draped or harped tendons. We will use the same system of equations but without the effect of external stresses at load transfer stage.

Stresses at transfer:	Effective losses after losses:	Total service load:
$f^t = \dfrac{-P_i}{A_c}\left(1 - \dfrac{ec^t}{r^2}\right) \leq f_{ti}$	$f^t = \dfrac{-P_e}{A_c}\left(1 - \dfrac{ec^t}{r^2}\right) \leq f_t$	$f^t = \dfrac{-P_e}{A_c}\left(1 - \dfrac{ec^t}{r^2}\right) - \dfrac{M_T}{S^t} \leq f_c'$
$f^b = \dfrac{-P_i}{A_c}\left(1 + \dfrac{ec^b}{r^2}\right) \leq f_{ci}'$	$f^b = \dfrac{-P_e}{A_c}\left(1 + \dfrac{ec^b}{r^2}\right) \leq f_c'$	$f^b = \dfrac{-P_e}{A_c}\left(1 + \dfrac{ec^b}{r^2}\right) + \dfrac{M_T}{S^b} \leq f_t$

$$S^t \geq \frac{M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c} \qquad S^b \geq \frac{M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}} \qquad e = \left(f_{ti} - f_{cgc}\right)\frac{S^t}{P_i}$$

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PROPER SELECTION OF BEAM SECTION AND PROPERTIES

❖ Prestressed section are not fully standardized. It is up to the designer to select the proper section in the particular project.

❖ Tendon's profile is designed to control the amount of prestressing force, the larger the eccentricity the smaller is the needed prestressing force.

❖ For large eccentricity, a large concrete area at the top is needed; Hence a T-section of wide-flange I-section become suitable.

T-section girder

Double-T girder

I-section girder

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PROPER SELECTION OF BEAM SECTION AND PROPERTIES

❖End section (i.e., end or SS beam) is usually solid to avoid large eccentricity, increasing shear capacity and huge bearing stresses from the anchorage.

❖Double-T is more stable and easier to handle than single T section and usually used in parking garages.

❖I-section is used in a typical floor beams with composite slab at the top.

❖Hollow-core sections are used as easily erected floor slab.

❖Large Hollow-box girder is used in large span bridge known as *segmental bridge deck system.* These girder have high torsional strength and strength/weight ratio.

Hollow core floor system

Box girder

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TYPICAL SPAN-DEPTH RATIO

Type	Span/Depth Ratio
I-Beam and single T-beam	24 - 36
Double T-beams	30 - 40
Bridge Girders	25 - 30
One-way Solid Slabs	35 - 50
One-way Hollow core Slabs	40 - 50
Two-way Solid Flat Plates	40 - 50

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CONCRETE PROTECTION AND TENDON SPACING

❖ACI 7.7 imposes the minimum cover distance for PS concrete member.

❖For post-tensioned members, the cover requirements apply to the ducts and metal and fitting.

❖If the member is designed for a service load tension in excess of $0.5\sqrt{f_c'}$ cracks in concrete are likely, and the cover requirements must be increased by 50%.

❖At the mid-span and any elsewhere than at the ends, spacing between bars and strands is the larger of d_b and 25 mm.

❖At the ends of the pretensioned members, spacing is increased for proper bond, S ≥:

    ❖$4d_b$ for wires

    ❖$3d_b$ for strands

❖Elsewhere, bundling of no more than four tendons or bars is permitted.

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CONCRETE PROTECTION AND TENDON SPACING

CODE

25.2.4 For pretensioned strands at ends of a member, minimum center-to-center spacing s shall be the greater of the value in Table 25.2.4, and $[(4/3)d_{agg} + d_b]$.

7.7.6.3 Prestressed reinforcement

7.7.6.3.1 Spacing of slab tendons required by 7.6.4.2 and the distance between face of beam or wall to the nearest slab tendon shall not exceed 6 ft.

7.7.6.3.2 If spacing of slab tendons exceeds 4.5 ft, additional deformed shrinkage and temperature reinforcement conforming to 24.4.3 shall be provided parallel to the tendons, except 24.4.3.4 need not be satisfied. In calculating the area of additional reinforcement, it shall be permitted to take the gross concrete area in 24.4.3.2 as the slab area

Table 25.2.4—Minimum center-to-center spacing of pretensioned strands at ends of members

f_{ci}', psi	Nominal strand diameter, in.	Minimum s
< 4000	All	$4d_b$
≥ 4000	< 0.5 in.	$4d_b$
	0.5 in.	1-3/4 in.
	0.6 in.	2 in.

25.2.5 For pretensioned wire at ends of a member, minimum center-to-center spacing, s, shall be the greater of $5d_b$ and $[(4/3)d_{agg} + d_b]$.

25.2.6 Reduced vertical spacing including bundling of prestressed reinforcement shall be permitted in the middle portion of a span.

25.6.2 Post-tensioning ducts

25.6.2.1 Bundling of post-tensioning ducts shall be permitted if shown that concrete can be satisfactorily placed and if provision is made to prevent the prestressed reinforcement from breaking through the duct.

R25.6.2 Post-tensioning ducts

R25.6.2.1 Where ducts for prestressing reinforcement in a beam are arranged closely together vertically, provisions should be made to prevent the prestressed reinforcement from breaking through the duct when tensioned. Horizontal arrangement of ducts should allow proper placement of concrete. A clear spacing of one and one-third times the nominal maximum size of the coarse aggregate, but not less than 1 in., has proven satisfactory.

Where concentration of tendons or ducts tends to create a weakened plane in the concrete cover, reinforcement should be provided to control cracking.

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FLEXURAL CRACK CONTROL

Table R24.5.2.1—Serviceability design requirements

	Prestressed		
	Class U	**Class T**	**Class C**
Assumed behavior	Uncracked	Transition between uncracked and cracked	Cracked
Section properties for stress calculation at service loads	Gross section 24.5.2.2	Gross section 24.5.2.2	Cracked section 24.5.2.3
Allowable stress at transfer	24.5.3	24.5.3	24.5.3
Allowable compressive stress based on uncracked section properties	24.5.4	24.5.4	No requirement
Tensile stress at service loads 24.5.2.1	$\leq 7.5\sqrt{f'_c}$	$7.5\sqrt{f'_c} < f_t \leq 12\sqrt{f'_c}$	No requirement
Deflection calculation basis	24.2.3.8, 24.2.4.2 Gross section	24.2.3.9, 24.2.4.2 Cracked section, bilinear	24.2.3.9, 24.2.4.2 Cracked section, bilinear
Crack control	No requirement	No requirement	24.3
Computation of Δf_{ps} or f_s for crack control	—	—	Cracked section analysis
Side skin reinforcement	No requirement	No requirement	9.7.2.3

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FLEXURAL CRACK CONTROL

❖ Flexural tensile cracks may be limited or eliminated completely by prestressing. However, partial prestressing has gained increasing popularity due to technical and economical reasons resulting in need for crack width control.

❖ No special provisions are included in the ACI code for PS concrete. The provisions for regular RC members are applicable.

For post-tensioned members designed as cracked members, it will usually be advantageous to provide crack control by the use of deformed reinforcement, for which the provisions in Table 24.3.2 for deformed bars or wires may be used. Bonded reinforcement required by other provisions of the Code may also be used as crack control reinforcement.

R7.7.2.3 Editions of ACI 318 prior to 2019 excluded the provisions of 7.7.2.3 for prestressed concrete. However, Class T and C slabs prestressed with unbonded tendons rely solely on deformed reinforcement for crack control. Consequently, the requirements of 7.7.2.3 have been extended to apply to Class T and C slabs prestressed with unbonded tendons.

Table 24.3.2—Maximum spacing of bonded reinforcement in nonprestressed and Class C prestressed one-way slabs and beams

Reinforcement type		Maximum spacing s
Deformed bars or wires	Lesser of:	$15\left(\dfrac{40{,}000}{f_s}\right) - 2.5c_c$
		$12\left(\dfrac{40{,}000}{f_s}\right)$
Bonded prestressed reinforcement	Lesser of:	$\left(\dfrac{2}{3}\right)\left[15\left(\dfrac{40{,}000}{\Delta f_{ps}}\right) - 2.5c_c\right]$
		$\left(\dfrac{2}{3}\right)\left[12\left(\dfrac{40{,}000}{\Delta f_{ps}}\right)\right]$
Combined deformed bars or wires and bonded prestressed reinforcement	Lesser of:	$\left(\dfrac{5}{6}\right)\left[15\left(\dfrac{40{,}000}{\Delta f_{ps}}\right) - 2.5c_c\right]$
		$\left(\dfrac{5}{6}\right)\left[12\left(\dfrac{40{,}000}{\Delta f_{ps}}\right)\right]$

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ENVELOPES FOR TENDON PLACEMENT

❖There is an envelope within which the prestressing force can be applied with causing no tensile stresses or allowable stress
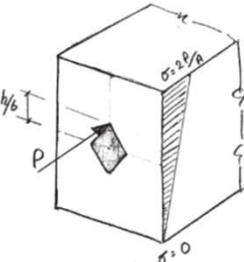
b) Force P at limit of kern of rectangular section
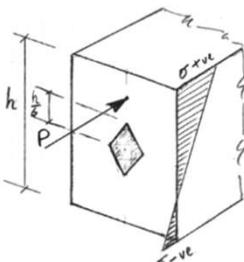
c) Force P outside kern of rectangular section

$$0 = \frac{-P_i}{A_c}\left(1 - \frac{e.c^t}{r^2}\right)$$

$$e = k_b = \frac{r^2}{c^t} = \frac{S^t}{A_c}$$

$$Similarly\ ,$$

$$e = k_t = \frac{r^2}{c^b} = \frac{S^b}{A_c}$$

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ENVELOPES FOR TENDON PLACEMENT

❖ In a similar manner, kern points can be established to the right and the left.

❖ To design the tendon along the span to develop no tension or limited tension, a draped or harped tendon should follow the shape of the bending moment diagram.

❖ Draped tendons are used for uniformly distributed loading

❖ Harped tendons are used for concentrated loading.

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ENVELOPES FOR NO TENSION

❖ Lower cgs envelope:
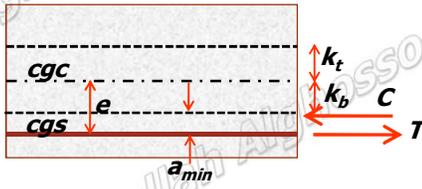
$$a_{\min} = \frac{M_D}{P_i}$$

$$e_{\max} = e_b = k_b + a_{\min}$$

❖ Upper cgs envelope:
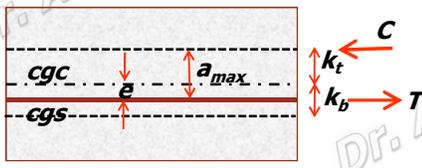
$$a_{\max} = \frac{M_T}{P_e}$$

$$e_{\min} = e_t = a_{\max} - k_t$$

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ENVELOPES FOR NO TENSION
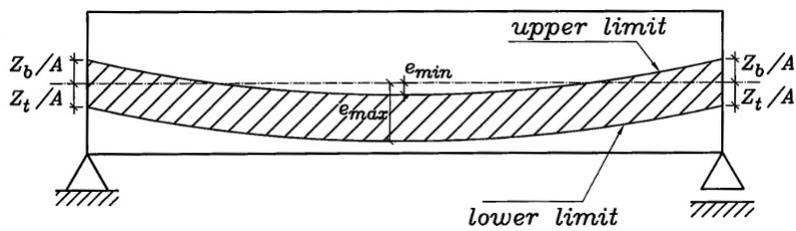
$$a_{\min} = \frac{M_D}{P_i}$$

$$e_{\max} = e_b = k_b + a_{\min}$$

$$a_{\max} = \frac{M_T}{P_e}$$

$$e_{\min} = e_t = a_{\max} - k_t$$

❖ We have to increase the section dimensions if:

$e_{max} < e_{min}$

$(e_{max}\ or\ e_{max}) > \frac{h}{2} - cover$

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ENVELOPES FOR NO TENSION

Waste of material and prestressing force
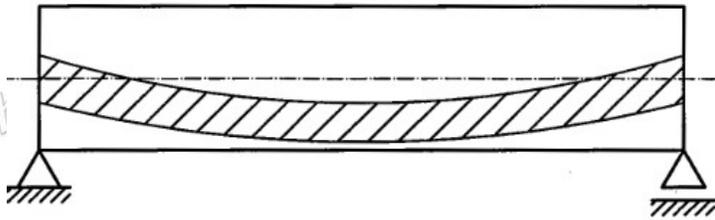
More economical design
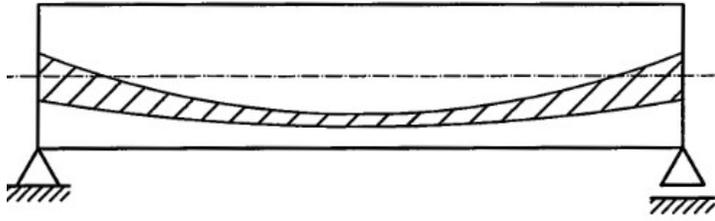
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ENVELOPES FOR LIMITING TENSION

❖ Additional eccentricity at the bottom:

Permitted Tension **No Tension**

k_t

e'_t

a_{max}

k_b

a_{min} e'_b
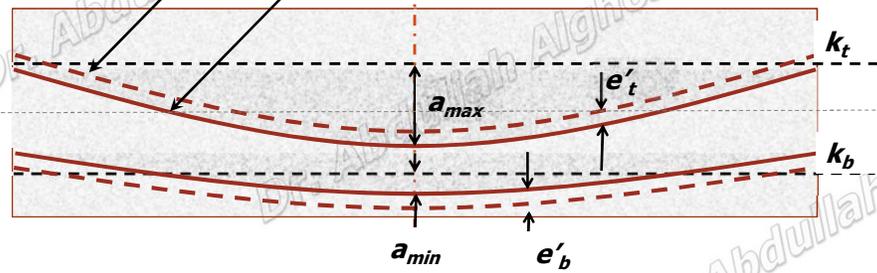
$$f_{ti} = \frac{P_i e'_b}{S^t}$$

$$e'_b = \frac{f_{ti} S^t}{P_i}$$

$$f_t = \frac{P_e e'_t}{S^b}$$

$$e'_t = \frac{f_t S^b}{P_e}$$

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EXAMPLE

Design a simply supported pretensioned double T-beam for a parking garage with harped tendon and with a span of 60 ft (18.3 m) using the ACI 318 Building Code allowable stresses. The beam has to carry a superimposed service live load of 1,100 plf (16.1 kN/m) and superimposed dead load of 100 plf (1.5 kN/m), and has no concrete topping. Assume the beam is made of normal-weight concrete with $f'_c = 5,000$ psi (34.5 MPa) and that the concrete strength f'_{ci} at transfer is 75 percent of the cylinder strength.

Assume also that the time-dependent losses of the initial prestress are 18 percent of the initial prestress, and that $f_{pu} = 270,000$ psi (1,862 MPa) for stress-relieved tendons, use $f_t = 12\sqrt{f'_c}$

Assume self weight = 1000 Ib/ft

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SOLUTION FROM THE PCI TABLES

Based on our calculations for the required top and bottom section modulus. Section 12DT32 satisfy these requirements. PCI Tables demonstrate draped profile of twenty ½″ diameters strand can take 140 psi superimposed service load (ML and MSD).

How much can we reduce the number of strand without violating the limiting stresses at transfer and at service loading condition.

Distance from the center of prestressing steel to the bottom fibers

Strand Pattern Designation

2 08 - D 1
- No. of strand (20)
- S = straight   D = depressed
- No. of depression points
- Diameter of strand in 16ths

Safe loads shown include dead load of 10 psf for untopped members and 15 psf for topped members. Remainder is live load. Long-time cambers include superimposed dead load but do not include live load.

Key
193 – Safe superimposed service load, psf
0.8 – Estimated camber at erection, in.
1.2 – Estimated long-time camber, in.

DOUBLE TEE
12'-0" x 32"
Normal Weight Concrete

12'-0"
3'-0"   6'-0"   3'-0"
7¾"
2"
3" Chamfer
32"
4¾"

$f'_c = 5,000$ psi
$f_{pu} = 270,000$ psi

Dead load

Section Properties	Untopped	Topped
A =	690 in.²	—
I =	64,620 in.⁴	82,413 in.⁴
y_b =	22.75 in.	25.50 in.
y_t =	9.25 in.	8.75 in.
S_b =	2,840 in.³	3,232 in.³
S_t =	6,986 in.³	9,696 in.³
wt =	719 plf	1,019 plf
DL =	60 psf	85 psf
V/S=	1.70 in.	

12DT32

Table of safe superimposed service load (psf) and cambers (in.) — No Topping

Strand Pattern	y_s(end) in. y_s(center) in.	40	42	44	46	48	50	52	54	56	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94	96
128-S	7.00 / 7.00	193 0.8 1.2	171 0.9 1.2	151 0.9 1.3	135 1.0 1.4	120 1.0 1.4	107 1.1 1.5	96 1.1 1.5	85 1.1 1.5	76 1.4	61 1.0 1.4	54 1.0 1.3	48 0.9 1.2	42 0.8 1.1	37 0.7 0.9	33 0.6 0.7	28 0.4 0.5												
148-S	8.00 / 8.00	196 1.0 1.4	174 1.1 1.5	156 1.1 1.6	139 1.2 1.7	125 1.2 1.7	112 1.3 1.7	101 1.3 1.7	90 1.7	8?	73 1.3 1.7	65 1.2 1.6	59 1.1 1.5	52 1.0 1.4	47 0.9 1.2	42 0.8 1.0	37 0.6 0.7	33 0.4 0.5	29										
168-S	9.00 / 9.00			194 1.2 1.6	174 1.2 1.7	156 1.3 1.8	140 1.3 1.8	126 1.4 1.9	114 1.4 1.9	103 1.4 2.0	84 1.4 1.9	75 1.4 1.9	68 1.4 1.8	61 1.3 1.7	55 1.3 1.6	49 1.2 1.4	44 1.0 1.2	40 0.9 0.9	35 0.7 0.6	31 0.5 0.2	27								
188-S	10.00 / 10.00				189 1.3 1.8	170 1.4 1.8	153 1.4 1.9	138 1.5 2.0	125 1.5 2.0	113 1.5 2.1	102 2.1	93 2.1	84 1.6 2.0	76 1.6 1.9	69 1.5 1.8	62 1.5 1.8	56 1.4 1.6	51 1.3 1.4	46 1.2 1.2	40 1.1 0.9	35 0.9 0.6	31 0.2	26						
208-D1	15.50 / 4.25										140 2.3 3.2	128 2.4 3.2	117 2.5 3.3	107 2.5 3.3	97 2.5 3.2	88 2.4 3.1	80 2.5 3.0	73 2.4 2.8	66 2.2 2.5	59 2.1 2.3	53 1.9 2.0	49 1.7 1.7	44 1.5 1.4	40 1.0 1.0	37 0.6 0.6	33 0.0 0.0	30	26	
228-D1	16.41 / 4.50														107 2.7 3.7	98 2.7 3.7	89 2.8 3.6	81 2.7 3.5	74 2.7 3.4	67 2.6 3.2	61 2.6 3.0	55 2.4 2.7	45 2.2 2.3	41 2.0 2.0	37 1.8 1.6	33 1.5 1.2	30 1.2 0.8	28 0.9 0.3	

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CHECKING PCI TABLE VALUES

(1) 10 psf is included based on the tables

$(0.75 f'_c)$ $(6\sqrt{f'_c})$

GEOMETRIC PROPERTIES		
L (Span)	in	720.00
Depth H	in	32

LOADING (MOMENT)		
M_D	lb.in	4537944
M_{SD}	lb.in	2484000
M_L	lb.in	5940000

MATERIALS PROPERTIES		
f'_c	psi	5000
f'_{ci}	psi	4300
f_{pu}	psi	270000
f_{pi}	psi	189000
f_{pe}	psi	154980
Density	psi	0.087
Area/Strand	in²	0.153

STRESS LIMITS		
f_{ti}	psi	196.72
f_{ci}	psi	3010.00
f_t	psi	848.53
f_c	psi	3000.00

(5) PCI have different limits $(0.7 f'_{ci})$ and $(\sqrt{f'_c})$

(3) P_i is based on $0.75(0.9 f_{pu})$ as per PCI

CALCULATED PROPERTIES			
A_c	in²	690	
c_t	in	9	
c_b	in	23	
I_c	in⁴	6.46E+04	
r^2	in²	94	
S_t	in³	6.99E+03	
S_b	in³	2.84E+03	1.29E+02
k_t	in	4	
k_b	in	10	

FORCE AND ECCENTRICITY		
f_{cgc}	psi	-730.22
P_i	lb	503,852
M_T	lb.in	12,961,944
e_c	in	18.5
N		20
A_{ps}	in²	3

CALCULATED STRESSES			
P_i	lb	578,340	
f_{ti}	psi	43.8	SAFE
f_{ci}	psi	-3007.3	SAFE
P_e	lb	474,239	
f_t	psi	-1286.9	SAFE
f_c	psi	787.3	SAFE

(4) Eccentricity is not given directley

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PARTIALLY PRESTRESSED CONCRETE

Typical of load-deflection relationship curve concrete structures (Naaman 1982)

In a combination of partially prestressed and non-prestressed reinforcements, both reinforcements contribute to the resistance of the structure. The advantage is to have a better control of camber and deflection, and to increase ductility.

1) Fully Prestressed
A_{ps}
min A_s
1.1) $\sigma \leq \sigma_{allowable}$
1.2) $M_u \leq \phi M_n$

2) Partially Prestressed
A_{ps}
A_s
2.1) $M_u \leq \phi M_n$

3) Reinforced Concrete
A_s
3.1) $M_u \leq \phi M_n$

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FLEXURAL DESIGN OF COMPOSITE BEAMS

Precast unit mostly pretensioned + cast-in-place unit (slab).
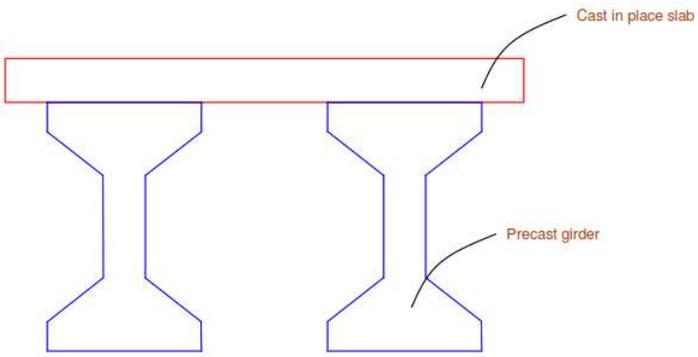
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FLEXURAL DESIGN OF COMPOSITE BEAMS

Typical cross-sections of composite beams.

❖ Double tee
❖ Single tee
❖ Bridge girders
❖ Box girders
❖ Hollow slabs
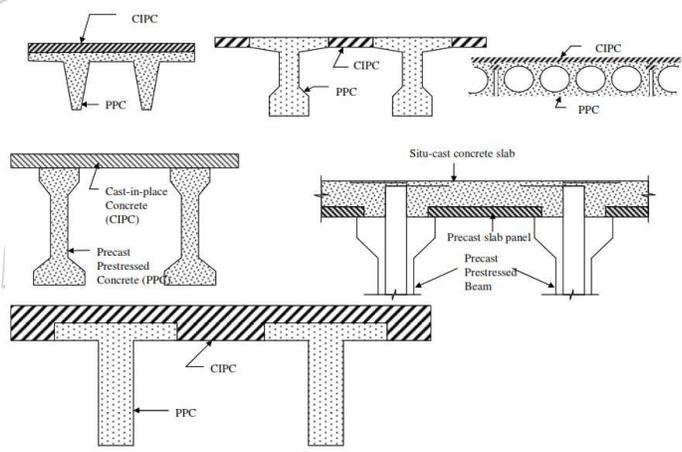
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SHORED AND UNSHORED SYSTEM

❖ The issue of shoring and non-shoring slab system has been a matter of debate

Shored slab system

❖ Beams are supported until the concrete slab has hardened, and then both act as a single structural element.

❖ Less materials (concrete, strands) are required compared with unshored slab system.

❖ Not applicable in some projects such as bridges with great height, heavy traffic locations

Unshored slab system

❖ In unshored construction, the beams support itself and designed with enough strength and stiffness to support the weight of the fresh concrete and any construction loads.

❖ cost is typically less than shored (in US), most structural engineers base their design on unshored construction

❖ The question is, are the cost saving in material greater than the cost of shoring

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FLEXURAL DESIGN OF COMPOSITE BEAMS

Shoring System

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FLEXURAL DESIGN OF COMPOSITE BEAMS

Advantages:

1. Stiffer, more resistance to deterioration.
2. Stronger.
3. Reduction or elimination of formworks (scaffolding).
4. Total construction time is reduced.
5. Excellent quality control.

Particular Design Aspects:
1. The loading stages and their relation to whether the beam responds as a composite or non-composite beam.
2. The transformed effective flange width (section properties).
3. The horizontal shear at the interface between the precast beam and the cast-in-place Slab.
4. Differential shrinkage

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FLEXURAL DESIGN OF COMPOSITE BEAMS

Advantages of Composite Construction

- ❖ There is a significant reduction in the construction time
- ❖ Low strength concrete can be used for in situ concrete
- ❖ No formwork is required when the beams are placed closely.
- ❖ An increase in the cross-section area of the section results in higher stiffness of the beam. As a result, beam bending and shear capacity increases.
- ❖ Due to the higher stiffness, it reduces the deflection of prestressed composite beams

Disadvantages of Composite Construction

- ❖ Designers need to pay more attention to the design procedures as they are a bit complex than the usual method. The correct idealization of the actional condition should be done during the design.
- ❖ Due to the composite slab, there will restrain resulting in an increase of prestress losses. Further, there could be a restrain moment due to this action.
- ❖ Different shrinkages in the section could induce additional stresses.

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STRESS DISTRIBUTION IN UNSHORED COMPOSITE BEAMS

STRESS DISTRIBUTION IN SHORED COMPOSITE BEAMS
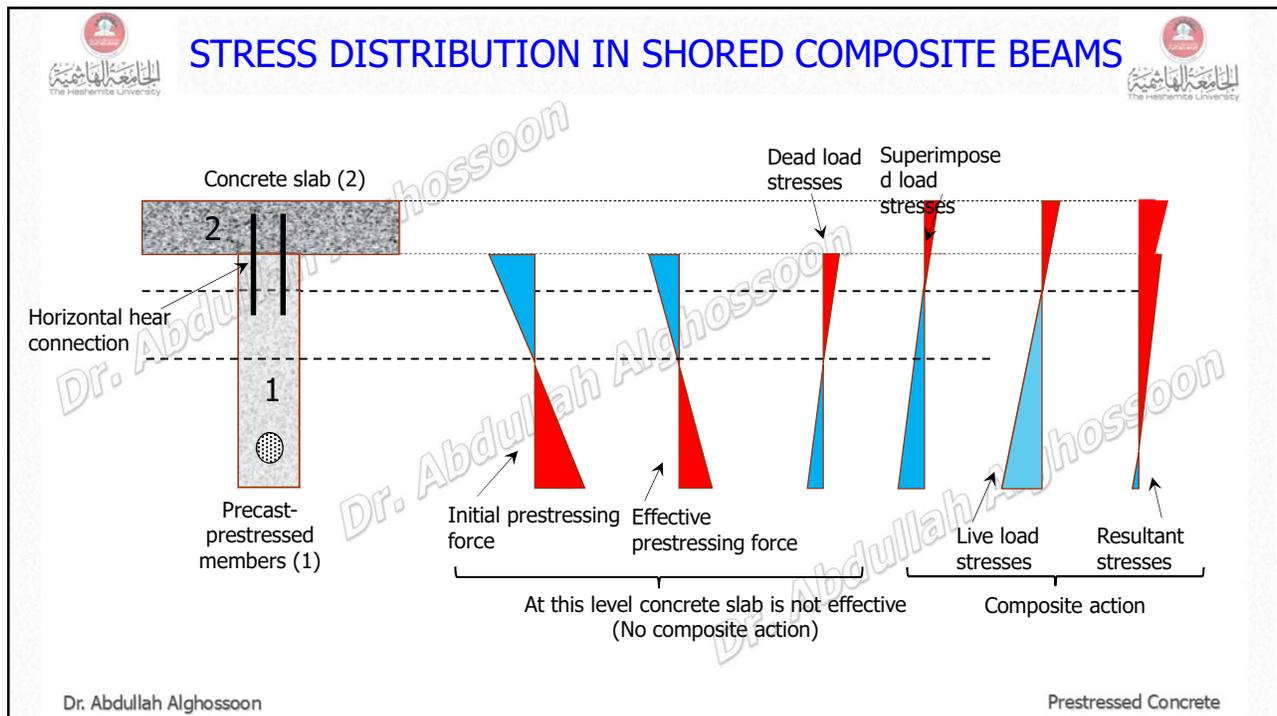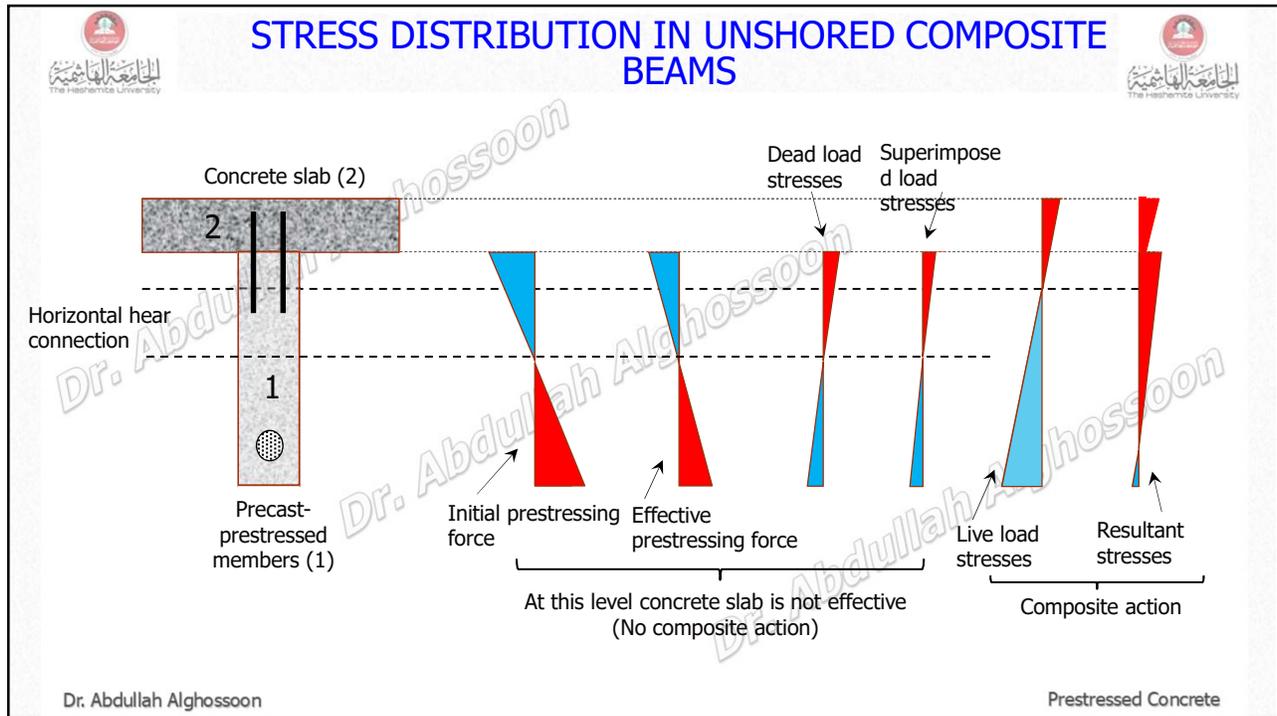
EFFECTIVE FLANGE WIDTH OF COMPOSITE SECTION

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EFFECTIVE FLANGE WIDTH OF COMPOSITE SECTION

❖ b_{eff} is the lesser of:

Code	Edge Beam	Intermediate Beam
ACI	$b_w + 6h_f$	$b_w + 16h_f$
	$b_w + 1/2L_c$	$b_w + L_c$
	$b_w + L/12$	$L/4$
AASHTO	$b_w + 6h_f$	$b_w + 12h_f$
	$b_w + 1/2L_c$	$b_w + L_c$
	$b_w + L/12$	$L/4$

* If the modulus of elasticity of the top slab E_{ct} and of the precast beam E_{ci} then the effective flange width b_{eff} must be modified by the modular ration 'n'

$$b_m = \frac{E_{ct}}{E_c} b_{eff}$$

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STRESS CALCULATIONS IN UNSHORED SLAB SYSTEM

1- Stresses before casting concrete:

$$f^t = \frac{-P_i}{A_c}\left(1 - \frac{ec^t}{r^2}\right) - \frac{M_D}{S^t}$$

$$f^b = \frac{-P_i}{A_c}\left(1 + \frac{ec^b}{r^2}\right) + \frac{M_D}{S^b}$$

P_i =initial prestressing force

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STRESS CALCULATIONS IN UNSHORED SLAB SYSTEM

2- Stresses after casting concrete before hardening:

$$f^t = \frac{-P_e}{A_c}\left(1 - \frac{ec^t}{r^2}\right) - \frac{M_D + M_{SD}}{S^t}$$

$$f^b = \frac{-P_e}{A_c}\left(1 + \frac{ec^b}{r^2}\right) + \frac{M_D + M_{SD}}{S^b}$$

P_e =effective prestressing force
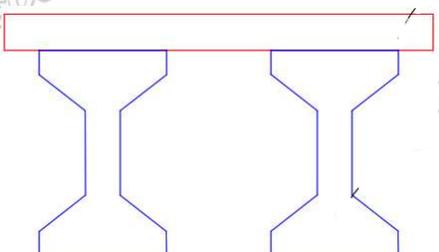
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STRESS CALCULATIONS IN UNSHORED SLAB SYSTEM

3- Stresses after concrete hardening:

Live load and any load after concrete hardening

$$f^t = \frac{-P_e}{A_c}\left(1 - \frac{ec^t}{r^2}\right) - \frac{M_D + M_{SD}}{S^t} - \frac{M_{CSD} + M_L}{S_c^t}$$

$$f^b = \frac{-P_e}{A_c}\left(1 + \frac{ec^b}{r^2}\right) + \frac{M_D + M_{SD}}{S^b} + \frac{M_{CSD} + M_L}{S_c^b}$$

P_e = effective prestressing force

M_{CSD} = the additional composite superimposed dead
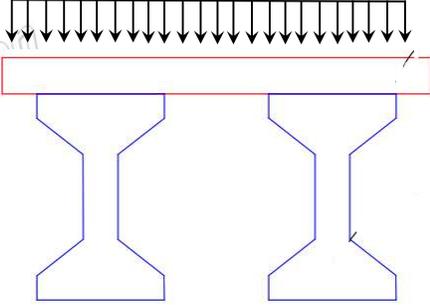
S_c^b and S_c^t are the section moduli of the composite section at the level of the bottom and top fibers, respectively, of the precast section.

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STRESS CALCULATIONS IN UNSHORED SLAB SYSTEM

4- Stresses at the top and bottom fibers of the situ-case slab:

Live load and any load after concrete hardening

$$f^{ts} = -\frac{M_{CSD} + M_L}{S^{tcs}}$$

$$f^{bs} = \frac{M_{CSD} + M_L}{S^{bcs}}$$
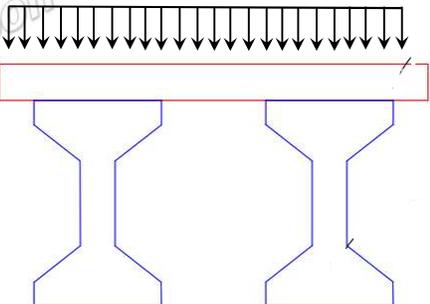
S^{bcs} and S^{tcs} are the section moduli of the composite section at the level of the bottom and top fibers, respectively, of the situ-cast slab

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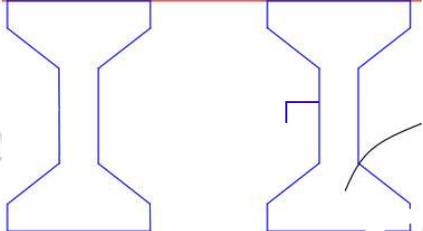Prestressed Concrete

STRESS CALCULATIONS IN SHORED SLAB SYSTEM

1- Stresses before casting concrete

$$f^t = \frac{-P_i}{A_c}\left(1 - \frac{ec^t}{r^2}\right) - \frac{M_D}{S^t}$$

$$f^b = \frac{-P_i}{A_c}\left(1 + \frac{ec^b}{r^2}\right) + \frac{M_D}{S^b}$$

P_e = effective prestressing force
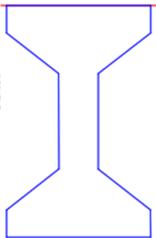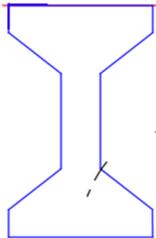
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STRESS CALCULATIONS IN SHORED SLAB SYSTEM

2- After the top slab is situ cast and full composite action is developed when the concrete hardens.

Live load and any load after concrete hardening

$$f^t = \frac{-P_e}{A_c}\left(1 - \frac{ec^t}{r^2}\right) - \frac{M_D}{S^t} - \frac{M_{SD} + M_{CSD} + M_L}{S^t_c}$$

$$f^b = \frac{-P_e}{A_c}\left(1 + \frac{ec^b}{r^2}\right) + \frac{M_D}{S^b} + \frac{M_{SD} + M_{CSD} + M_L}{S^b_c}$$

P_e = effective prestressing force

M_{CSD} = the additional composite superimposed dead load after erection

S^b_c and S^t_c are the section moduli of the composite section at the level of the bottom and top fibers, respectively, of the precast section.

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EXAMPLE

A two-lane, simply supported bridge has a 64 ft (19.5 m), center to center, of bearings. The width of the bridge is such that the exterior beams are 28 ft (8.53 m) center to center. The spacing of the interior beams is at 7 ft (2.13 m) center to center.

64 ft

Diaphragm

7 ft

28 ft

Assume:
Live load = $(4.7 \; kN/m^2)$
Superimposed Dead load = $(0.47 kN/m^2)$
Concrete weight = $145 \; lb/ft^3 (24 \; kN/m^3)$
f'c=45 MPa
f'ci=35MPa
Modular ratio = 0.65

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External loads

| GEOMETRIC PROPERTIES		
L (Span)	m	19.5
Depth H	mm	815
Flange width b_{f1}	mm	290
thickness h_{f1}	mm	145
Web b_w	mm	120
Clear web h_w	mm	525
Flange width b_{f2}	mm	290
Thickness h_{f2}	mm	145
CS Thickness hs	mm	120
Effective width bf	mm	1326

Selected based on the recommended L/D ratio

Usually 0.2 t0 0.3 of H

Pay attention to the different material elasticity

LOADING (MOMENT)		
$M_{Selfweight}$	kN.m	168
$M_{SD(Fresh\; concrete)}$	kN.m	291.6
$M_{SD(After\; Hardening)}$	kN.m	47.5
M_{live}	kN.m	475.3

System Type
Composite-Shored

Type of composite system

CALCULATED PROPERTIES			Composite Properties	
A_G	mm2	147,100	159,120	
c_t	mm	408	284.6	
c_b	mm	408	650.4	
I_c	mm4	1.10E+10	2.79E+10	
r^2	mm2	75,000	91,206	
S_t	mm3	2.71E+07	1.70E+08	9.81E+07
S_b	mm3	2.71E+07	4.29E+07	
k_t	mm	184	140	
k_b	mm	184	321	
S_t required		3.01E+07		
S_b required		3.61E+07		

Required section elastic modulus

Girder geometric properties

Composite geometric properties

FORCE AND ECCENTRICITY		
f_{tpc}	MPa	-9.76
P_i	kN	1,436
M_T	kN.m	982
e_{max1}	mm	357
e_{max}	mm	329
e_c	mm	307.6
A_{cs}	mm2	964

Required eccentricity

MATERIALS PROPERTIES		
f'$_c$ MPa	45	
f'$_{ci}$ MPa	35	
f$_{pu}$ MPa	1862	
f$_{pi}$ MPa	1489.6	
f$_{pe}$ MPa	1191.68	0.6
Ec MPa	31528.56	
E$_{cs}$/E$_c$	0.65	
Density kN/m3	24	
Area/Strand mm2	99.0	12.7
Concrete cover mm	50.8	

Difference between slab and beam material

STRESS LIMITS		
f$_{ti}$ MPa	1.48	
f$_{ci}$ MPa	21.00	
f$_t$ MPa	6.71	
f$_c$ MPa	27.00	

CALCULATED STRESSES			
P_i kN		1,475	
f$_{Top(Instantaneous)}$ MPa		0.53	SAFE
f$_{Bot(Instantaneous)}$ MPa		-20.58	SAFE
P_e kN		1179.76	
f$_{Top(Composite)}$ MPa		-5.62	SAFE
f$_{Bot(Composite)}$ MPa		3.74	SAFE
f$_{Top(Concrete\; Slab}$ MPa		-5.93	SAFE

Using girder geometry

All other loads are resisted by the composite section

Checking stresses on the concrete slab

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Slide 1

Notice the difference between the shored and unshored systems in terms of the number of strands, stresses and concrete that area

- Selected based on the recommended L/D ratio
- Usually 0.2 to 0.3 of H
- Pay attention to the different material elasticity
- Required section elastic modulus
- Girder geometric properties
- Composite geometric properties
- Required eccentricity
- More strands required
- Type of composite system
- External loads
- Difference between slab and beam material
- Using girder geometry
- All other loads are resisted by the composite section
- Checking stresses on the concrete slab

GEOMETRIC PROPERTIES

L (Span)	m	19.5
Depth H	mm	815
Flange width b_{f1}	mm	400
thickness h_{f1}	mm	200
Web b_w	mm	120
Clear web h_w	mm	415
Flange width b_{f2}	mm	400
Thickness h_{f2}	mm	200
CS Thickness hs	mm	120
Effective width bf	mm	1326

LOADING (MOMENT)

$M_{Selfweight}$	kN.m	239
$M_{SD(Fresh\ concrete)}$	kN.m	410.7
$M_{SD(After\ Hardening)}$	kN.m	47.5
M_{live}	kN.m	475.3

System Type
Composite-Unshored

MATERIALS PROPERTIES

f'_c MPa		45
f'_{ci} MPa		35
f_{pu} MPa		1862
f_{pj} MPa		1489.6
f_{pe} MPa		1191.68
Ec MPa		31528.56
E_{cs}/E_c		0.65
Density kN/m3		24
Area/Strand mm2		99.0
Concrete cover mm		50.8

STRESS LIMITS

f_{ti} MPa		1.48
f_{ci} MPa		21.00
f_t MPa		6.71
f_c MPa		27.00

CALCULATED PROPERTIES / **Composite Properties**

A_c	mm2	209,800	159,120
c_t	mm	408	325.9
c_b	mm	408	609.1
I_c	mm4	1.64E+10	3.63E+10
r^2	mm2	78,060	98,517
S_t	mm3	4.02E+07	1.77E+08
S_b	mm3	4.02E+07	5.97E+07
k_t	mm	192	162
k_b	mm	192	302
S_t required	3.48E+07		
S_b required	4.17E+07		

FORCE AND ECCENTRICITY

f_{cgc}	MPa	-9.76
P_i	kN	2,048
M_T	kN.m	1,173
e_{max1}	mm	357
e_{max}	mm	337
e_o	mm	307.5
N		13

CALCULATED STRESSES

P_i kN		1,917
$f_{Top(Instantaneous)}$ MPa		-0.42
$f_{Bot(Instantaneous)}$ MPa		-17.85	SAFE
P_e kN		1533.69
$f_{Top(Composite)}$ MPa		-14.71	SAFE
$f_{Bot(Composite)}$ MPa		5.89	SAFE
$f_{Top(Concrete\ Slab)}$ MPa		-3.05	SAFE

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Slide 2

STAGES AT WHICH STRESSES ARE ESTIMATED

- At transfer of prestressing force by checking stresses in the tendon and concrete and comparing these to acceptable ACI 318-19 limits.
  - Initial Prestress
  - Self-weight application

- At the service load level, the stresses in the concrete are also checked and compared to acceptable ACI 318-19 limits, and prevent excessive deflections and cracking.
  - Superimposed dead load
  - Decompression in steel
  - Service load limit

- Strength must be checked to ensure that the member can sustain overloads, M_n and V_n in accordance to ACI 318-19
  - Ultimate load state

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PRESTRESSED CONCRETE
-ULTIMATE FLEXURAL STRENGTH DESIGN-

The Hashemite University
Department of Civil Engineering
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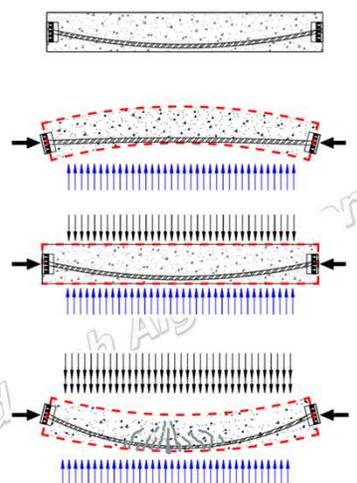Abdullahm_ab@hu.edu.jo
Spring 2025

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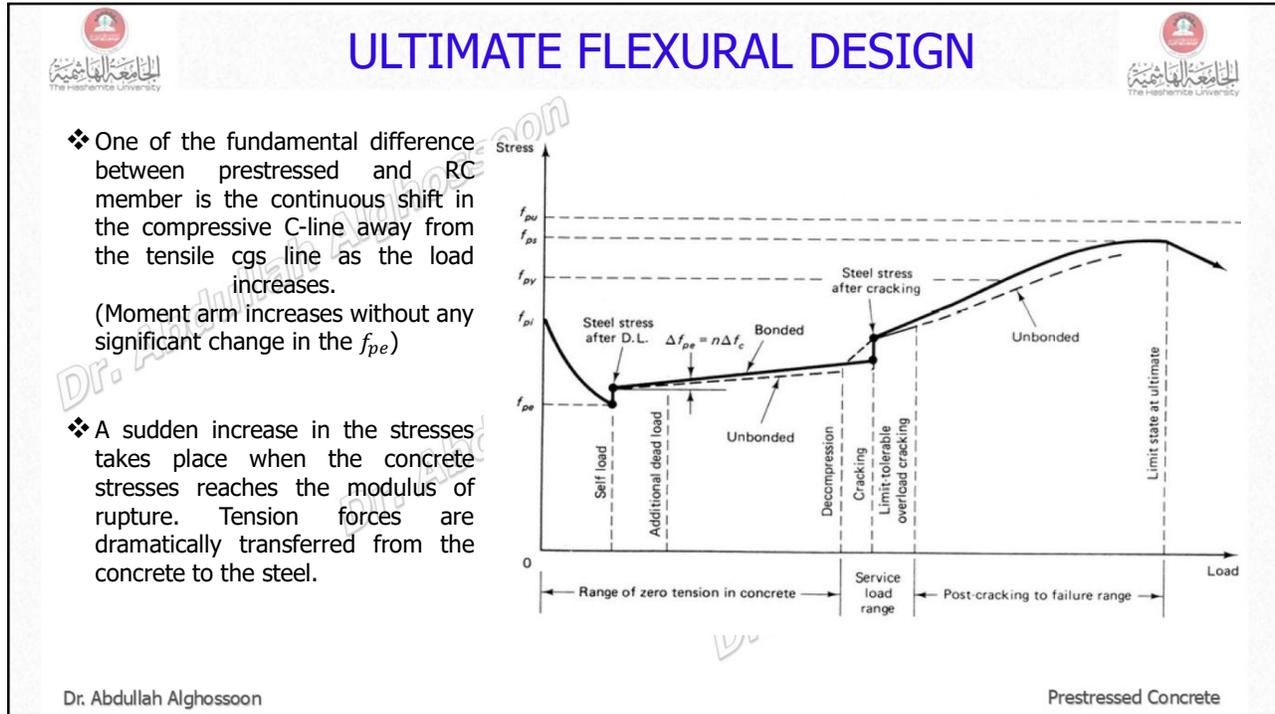# STAGES AT WHICH STRESSES ARE ESTIMATED

- At transfer of prestressing force by checking stresses in the tendon and concrete and comparing these to acceptable ACI 318-19 limits.
  - Initial Prestress
  - Self-weight application

- At the service load level, the stresses in the concrete are also checked and compared to acceptable ACI 318-19 limits, and prevent excessive deflections and cracking.
  - Superimposed dead load
  - Decompression in steel
  - Service load limit

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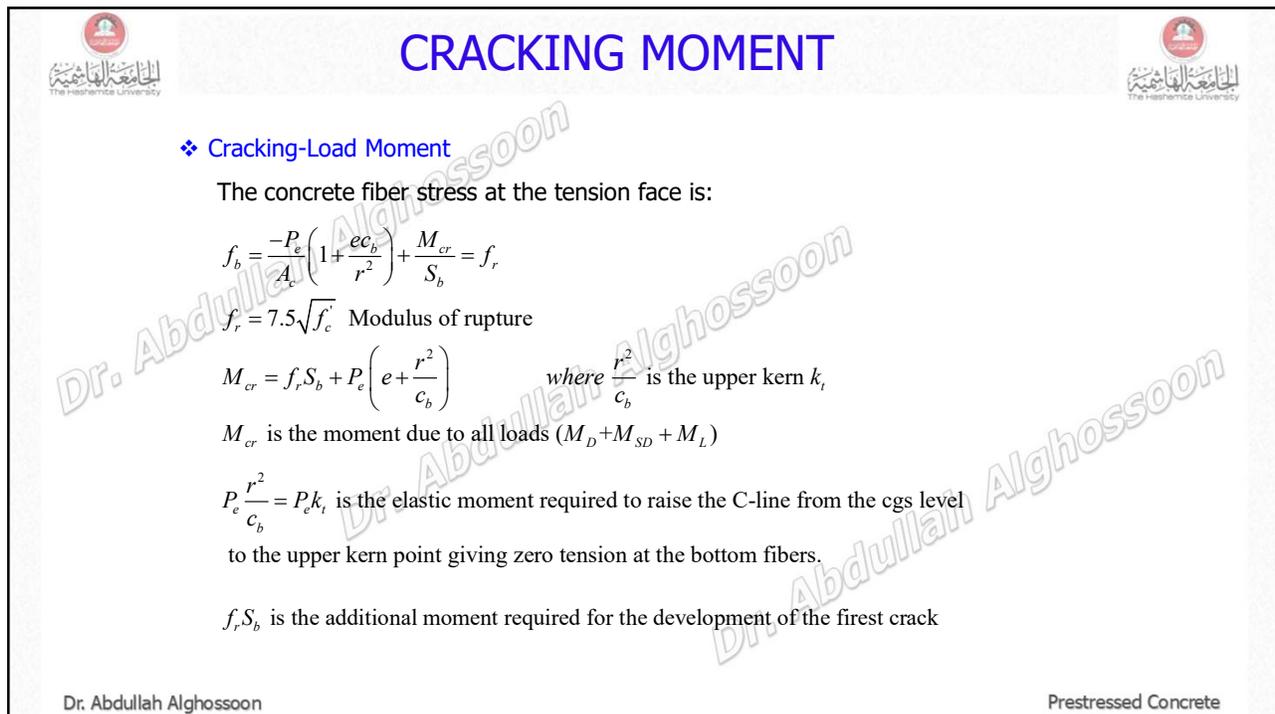 - Ultimate load state

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ULTIMATE FLEXURAL DESIGN

❖ One of the fundamental difference between prestressed and RC member is the continuous shift in the compressive C-line away from the tensile cgs line as the load increases.
(Moment arm increases without any significant change in the f_{pe})

❖ A sudden increase in the stresses takes place when the concrete stresses reaches the modulus of rupture. Tension forces are dramatically transferred from the concrete to the steel.

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CRACKING MOMENT

❖ Cracking-Load Moment

The concrete fiber stress at the tension face is:

$$f_b = \frac{-P_e}{A_c}\left(1+\frac{ec_b}{r^2}\right)+\frac{M_{cr}}{S_b} = f_r$$

$f_r = 7.5\sqrt{f_c'}$ Modulus of rupture

$$M_{cr} = f_r S_b + P_e\left(e+\frac{r^2}{c_b}\right) \qquad where \; \frac{r^2}{c_b} \; \text{is the upper kern} \; k_t$$

M_{cr} is the moment due to all loads $(M_D + M_{SD} + M_L)$

$P_e\dfrac{r^2}{c_b} = P_e k_t$ is the elastic moment required to raise the C-line from the cgs level

to the upper kern point giving zero tension at the bottom fibers.

$f_r S_b$ is the additional moment required for the development of the first crack

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ULTIMATE FLEXURAL DESIGN

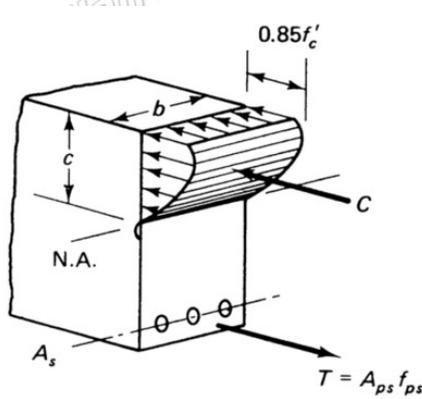Calculate the cracking moment M_{cr} in the I-beam of the last example

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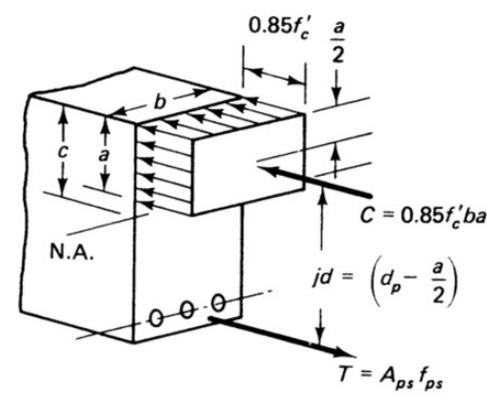# EQUIVALENT STRESS BLOCK

❖ Nominal Moment Strength of Rectangular Section
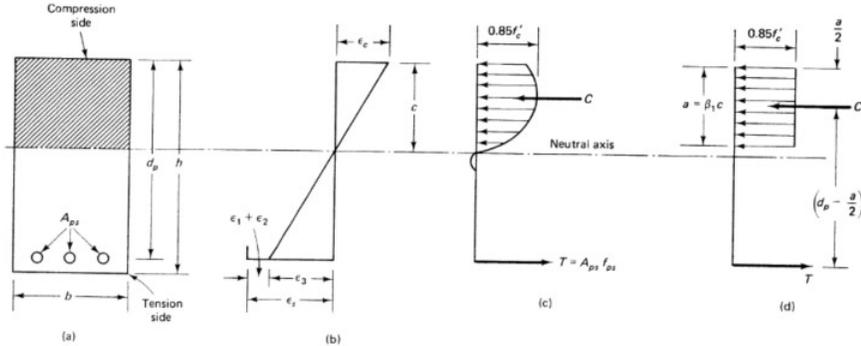

Whitney block

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EQUIVALENT STRESS BLOCK

$$A_{ps}f_{ps} = 0.85 f_c' ba$$

$$a = \beta_1 c = \frac{A_{ps}f_{ps}}{0.85 f_c' b}$$

$$M_n = A_{ps}f_{ps}\left(d - \frac{a}{2}\right)$$

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NOMINAL FLEXURAL STRENGTH

$$A_{ps}f_{ps} = 0.85 f_c' ba \qquad a = \beta_1 c = \frac{A_{ps}f_{ps}}{0.85 f_c' b} \qquad M_n = A_{ps}f_{ps}\left(d - \frac{a}{2}\right)$$

$$\rho_p = \frac{A_{ps}}{bd} \qquad \omega_p = \rho_p \frac{f_{ps}}{f_c'}$$

$$M_n = \rho_p f_{ps} b d_p^2 (1 - 0.59\omega_p)$$

❖ The contribution of the mild steel can be accounted for in the stress block as follows:

$$a = \beta_1 c = \frac{A_{ps}f_{ps} + A_y f_y}{0.85 f_c' b}$$

❖ The strain at the level of mild steel: $\varepsilon_3 = \varepsilon = \left(\frac{d - c}{c}\right)$
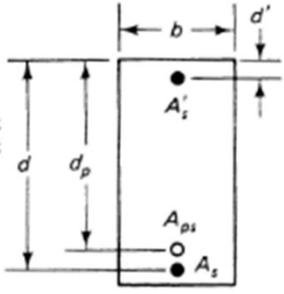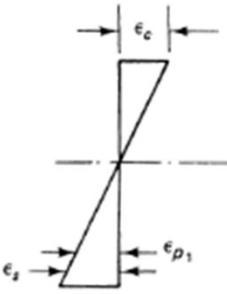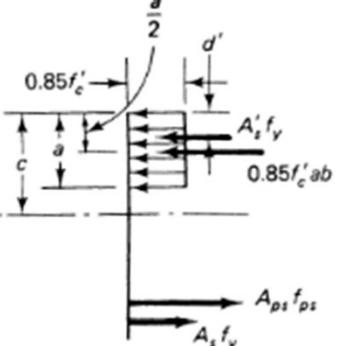
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NOMINAL FLEXURAL STRENGTH (RECTANGULAR)

(a) Beam section. (b) Strain. (c) Stresses and forces

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NOMINAL FLEXURAL STRENGTH

❖ The moment capacity of a rectangular section with mild tension steel and no compression steel is given by:

$$M_n = \rho_p f_{ps} b d_p^2 (1 - 0.59\omega_p) + \rho f_y b d^2 (1 - 0.59\omega)$$

$$\omega = \rho\left(\frac{f_y}{f_c'}\right)$$

❖ The moment capacity of a rectangular section with mild tension steel and compression steel is given by:

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_y f_y\left(d - \frac{a}{2}\right) + A_s' f_y\left(\frac{a}{2} - d'\right)$$

$$a = \beta_1 c = \frac{A_{ps} f_{ps} + A_y f_y - A_s' f_y}{0.85 f_c' b}$$

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NOMINAL FLEXURAL STRENGTH (T-SECTION)

(a) Beam section. (b) Strain. (c) Web stresses and forces. (d) Flange stress and force

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NOMINAL FLEXURAL STRENGTH (T-SECTION)

$$T_p + T_s = T_{pw} + T_{pf}$$

Where:

T_p = total prestressing force = $A_p f_{ps}$

T_s = ultimate force in the nonprestressed steel = $A_s f_y$

T_{pw} = part of the total force in the tension reinforcement

required to develop the web = $A_{pw} f_{ps} = 0.85 f_c' b_w a = C_w$

A_{pw} = total reinforcement area corresponding to the force T_{pw}

T_{pf} = part of the total force in the tension reinforcement

required to develop the flange = $C_f = 0.85 f_c' (b - b_w) h_f$

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NOMINAL FLEXURAL STRENGTH (T-SECTION)

$$C_w = 0.85 f_c' b_w a$$

$$T_{pw} = T_p + T_s - T_{pf} = A_{ps} f_{ps} + A_s f_y - 0.85 f_c'(b - b_w) h_f$$

$$T_{pw} + T_{pf} = C_w + C_f$$

$$T_{pw} = C_w + C_f - T_{pf}$$

$$a = \frac{T_{pw}}{0.85 f_c' b_w}$$

$$a = \frac{A_{ps} f_{ps} + A_s f_y - 0.85 f_c'(b - b_w) h_f}{0.85 f_c' b_w}$$

❖ For a flanged section where the neutral axis falls outside the flange and $a > h_f$ the moment capacity can be calculated as follows:

$$M_n = A_{pw} f_{ps} \left(d_p - \frac{a}{2} \right) + A_s f_y \left(d - d_p \right) + 0.85 f_c' \left(b - b_w \right) h_f \left(d_p - \frac{h_f}{2} \right)$$

$$M_u = \phi M_n$$

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STRENGTH REDUCTION FACTOR

ACI	φ
1- Beams and slabs in flexure	0.9
2- Columns with ties	0.65
3- Columns with spirals	0.70
4- Columns carrying small axial loads	0.65 – 0.9 or 0.70 – 0.9
5- Beams in shear or torsion	0.75

AASHTO	φ
1- Factory produced members	1.0
2- Post-tensioned cast in place	0.95
3- Shear and torsion	0.9

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NOMINAL FAILURE STRESSES OF PS STEEL

❖ For a flanged section where the neutral axis falls outside the flange and a > hf the moment capacity can be calculated as follows:

$$M_n = A_{pw}f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y\left(d - d_p\right) + 0.85 f_c'\left(b - b_w\right)h_f\left(d_p - \frac{h_f}{2}\right)$$

❖ How to find f_{ps} :

if $f_{pe} < 0.5 f_{pu}$ use strain compatability

if $f_{pe} \geq 0.5 f_{pu}$ use ACI approximate method:

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NOMINAL FAILURE STRESSES OF PS STEEL
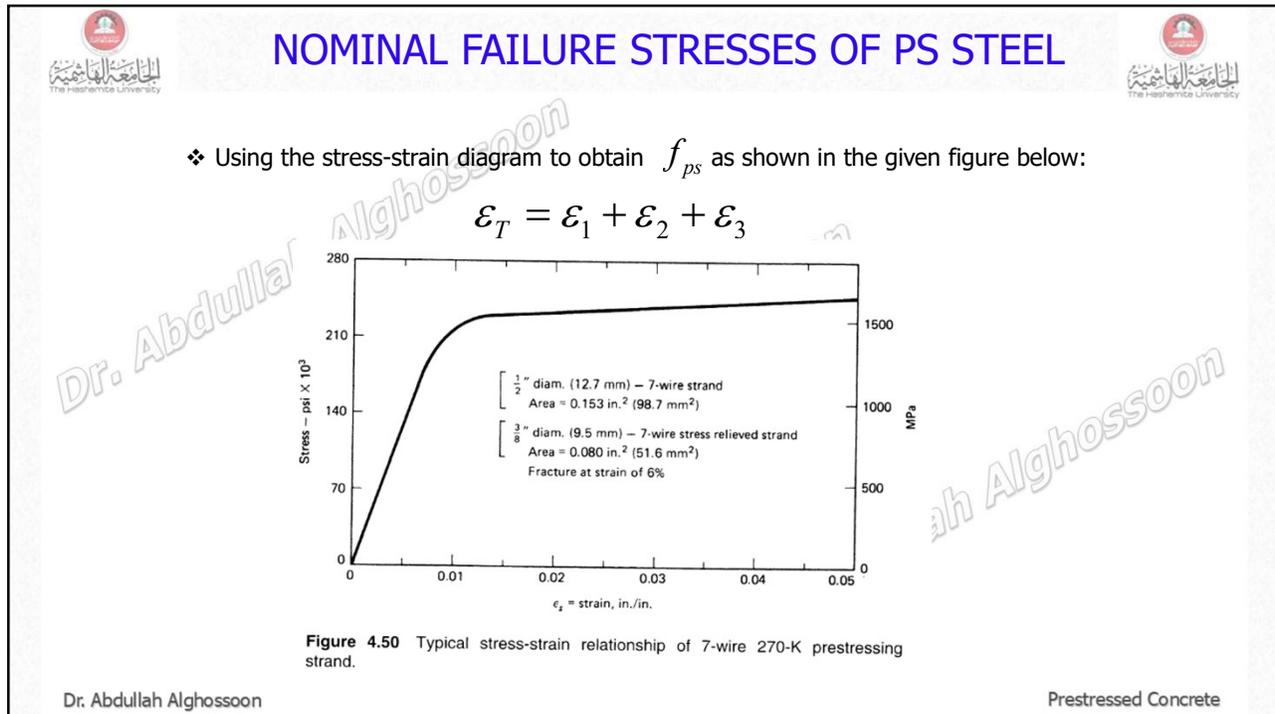
if $f_{pe} < 0.5 f_{pu}$ use strain compatability:

Stage I: P_e alone after all losses: $\varepsilon_1 = \varepsilon_{pe} = \dfrac{f_{pe}}{E_{ps}}$

Stage II: Intermediate step, concrete decompression at the PS steel level: $\varepsilon_2 = \varepsilon_{decomp} = \dfrac{P_e}{A_c E_c}\left(1 + \dfrac{e^2}{r^2}\right)$

Stage III: Overload to failure, N.A. at 'c' from top: $\varepsilon_3 = \dfrac{d-c}{c}\left(\varepsilon_{cu}\right)$

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NOMINAL FAILURE STRESSES OF PS STEEL

❖ Using the stress-strain diagram to obtain f_{ps} as shown in the given figure below:

$$\varepsilon_T = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

Figure 4.50 Typical stress-strain relationship of 7-wire 270-K prestressing strand.

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NOMINAL FAILURE STRESSES OF PS STEEL

if $f_{pe} \geq 0.5 f_{pu}$ use ACI approximate method:

* bonded tendons:

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\left[\rho_p \frac{f_{pu}}{f_c'} + \frac{d}{d_p}\left(\omega - \omega'\right)\right]\right)$$

$$\rho_p \frac{f_{pu}}{f_c'} + \frac{d}{d_p}\left(\omega - \omega'\right) \geq 0.17 \quad ; \quad d' \leq 0.15 d_p$$

$$\Rightarrow \omega' = \rho' \frac{f_y}{f_c'}$$

$$\Rightarrow \omega = \rho \frac{f_y}{f_c'}$$

$$\Rightarrow \rho_p = \frac{A_p}{b.d_p}$$

$\gamma_p = 0.55$ if $\dfrac{f_{py}}{f_{pu}} > 0.80 \Rightarrow$ High stress Bars

$\gamma_p = 0.40$ if $\dfrac{f_{py}}{f_{pu}} > 0.85 \Rightarrow$ Stress Relieved Strand

$\gamma_p = 0.28$ if $\dfrac{f_{py}}{f_{pu}} > 0.90 \Rightarrow$ Low-relaxation Strands

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NOMINAL FAILURE STRESSES OF PS STEEL

*unbonded tendons (MPa)

(a) $L/h \le 35 \Rightarrow f_{ps} = f_{pe} + 69 + f_c'/100\rho_p$

$$f_{ps} < f_{py}$$

$$f_{ps} < f_{pe} + 414 MPa$$

(b) $L/h > 35 \Rightarrow f_{ps} = f_{pe} + 69 + f_c'/300\rho_p$

$$f_{ps} < f_{py}$$

$$f_{ps} < f_{pe} + 207 MPa$$

*unbonded tendons (psi)

(a) $L/h \le 35 \Rightarrow f_{ps} = f_{pe} + 10,000 + f_c'/100\rho_p$ (psi)

$$f_{ps} < f_{py} (psi)$$

$$f_{ps} < f_{pe} + 60,000 (psi)$$

(b) $L/h > 35 \Rightarrow f_{ps} = f_{pe} + 10,000 + f_c'/300\rho_p$ (psi)

$$f_{ps} < f_{py} (psi)$$

$$f_{ps} < f_{pe} + 30,000 (psi)$$

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SUMMARY OF FLEXURAL STRENGTH

Reinforced Concrete

$(f_s = f_y$ at ultimate state$)$

$$a = \beta_1 c = \frac{A_s f_y}{0.85 f_c' b}$$

$$M_u \le \phi M_n = \phi \left[A_s f_y \left(d - \frac{a}{2} \right) \right]$$

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$(f_s = f_y$ at ultimate state as small amount of provided rebar$)$

$$a = \beta_1 c = \frac{\left(A_s f_y + A_{ps} f_{ps} \right)}{0.85 f_c' b}$$

$$M_u \le \phi M_n = \phi \left[A_s f_y \left(d - \frac{a}{2} \right) + A_{ps} f_{ps} \left(d_p - \frac{a}{2} \right) \right]$$

$(f_{ps}$ at failure is not readily as $f_y)$

A_{ps}

Unboned system (few large cracks)

$$f_{pe} = \frac{F_{ps}(0.8)}{A_{ps}}$$

$$\frac{L}{d} \le 35, \quad f_{ps} = f_{pe} + 700 + \frac{f_c'}{100\rho_p} \le \begin{cases} f_{py} \\ f_{pe} + 4,000 \end{cases}$$

$$\rho_p = \frac{n A_{ps}}{b d_p}$$

$$\frac{L}{d} > 35, \quad f_{ps} = f_{pe} + 700 + \frac{f_c'}{300\rho_p} \le \begin{cases} f_{py} \\ f_{pe} + 2,000 \end{cases}$$

Boned system (many small cracks)

$$f_{ps} = f_{pu} \left(1 - \frac{\gamma_p}{\beta_1} \left[\rho_p \frac{f_{pu}}{f_c'} + \frac{d}{d_p} \left(\omega - \omega' \right) \right] \right)$$

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MINIMUM AREA OF STEEL

∗ The total amount of prestressed and nonprestressed reinforcement required by the ACI should not be less than that required to develop a actored moment $Mu = \phi Mn$, such that

1) $M_u \geq 1.2M_{cr}$

Also, a minimum area of bonded non-prestressed reinforcement in beams

2) $As_{min} = 0.004A_t$

A_t is the section area between extreme tension fiber and cgc line of gross section.
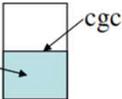
∗ In flat plates if tension stress exceeds use mild steel such that $A_s = N_c / 0.5f_y$

Where N_c : is concrete tension force due to $D + L$ loads.

∗ In slabs at negative moment $As_{min} = 0.0075hl$ in each direction

 $l =$ span length parallel to reinforcement used

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MAXIMUM AREA OF STEEL

∗ Rectangular sections with PS only:

$$\omega_p = \rho_p \frac{f_{ps}}{f_c'} \leq 0.32\beta_1$$

∗ Rectangular sections with PS and tensile and compressive reinforcement:

$$\omega_p + \frac{d}{d_p}(\omega - \omega') \leq 0.36\beta_1$$

∗ Flanged sections:

$$\omega_{pw} + \frac{d}{d_p}(\omega_w - \omega_w') = \frac{0.85a}{d_p} \leq 0.36\beta_1$$

$$\beta_1 = 0.85 - 0.00005(f_c' - 4000) \geq 0.65$$

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DESIGN STEPS

1- Find $M_u = 1.2DL + 1.6LL$

2- $M_n = \dfrac{M_u}{\phi}$

3- Trial depth ≈0.75 R.C. beams (0.6 in/ft or 1:20 m)

4- Assume $jd \approx 0.8h$ and $f_{ps} \approx 0.9\,f_{pu}$.

5- $M_n = T \times jd = C \times jd = A_{ps} \times f_{ps} \times jd = A_{ps} \times 0.9 f_u \times 0.8h = \dfrac{M_u}{\phi}$

6- $A_{ps} = \dfrac{M_u}{\phi(0.9 f_{pu})(0.8h)} = \dfrac{M_u}{\phi\,0.72 f_{pu} h}$

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DESIGN STEPS

For flanged section:

Assume $a = h_f$

$C = 0.85 f_c' b_f h_f$

$T = A_{ps}\,0.9 f_{pu} \dfrac{M_u}{0.8h}$

$C = T$

$0.85 f_c' A_c' = \dfrac{M_u}{0.8h}$ ➡ $A_c' = \dfrac{M_u}{0.68 h f_c'}$

$A_c' = b_f h_f$ choose b_f and get h_f

b_w will be selected based on shear design

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DESIGN STEPS

Determine if the section is rectangular or flanged by locating the N.A. if 'a' is less than 'h' then rectangular, otherwise it is flanged.

$$Rectangular: \quad a = \frac{A_{ps}f_{ps} + A_s f_y + A'_s f_y}{0.85 f'_c b}$$

$$Flanged: \quad a = \frac{A_{pw}f_{ps}}{0.85 f'_c b_w} \; ;$$

$$where \quad A_{pw}f_{ps} = A_{ps}f_{ps} + A_s f_y - 0.85 f'_c (b - b_w) h_f$$

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EXAMPLE 1

Design the bonded beam in *example 4.2* by the ultimate-load theory using non-prestressed reinforcement to partially carry part of the factored loads. Use strain compatibility to evaluate f_{ps}

Given the modified section in the figure given below.

I_c = 70,688 in⁴
r^2 = 187.5 in²
A_c = 377 in²
c_t = 21.16 in.
S^t = 3340 in³
c_b = 18.84 in.
S_b = 3750 in.³
w_D = 393 plf
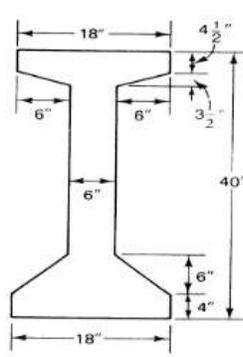
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EXAMPLE 1

Using

$$f_{pu} = 270,000 \, psi$$

$$f_{py} = 0.85 f_{pu} \text{ for stress relived strands}$$

$$f_y = 60,000 \, psi$$

$$f_c' = 5,000 \, psi$$

Use 7-wire ½-in diameter tendons. The nonprestressed partial mild steel is to be placed with a 1½-in clear cover, and no compression steel is to be accounted for, no wind or earthquake is taken into consideration.

EXAMPLE 1

From example 4.2
Superimposed service live load = 1100 plf
Superimposed service dead load = 100 plf
Assumed own weight = 393 plf
Beam Span = 65 ft

1- Determine the factored moment
The factored moment is given by

$$W_u = 1.2(W_D + W_{SD}) + 1.6W_L$$

$$= 1.2(100 + 393) + 1.6(1100) = 2352 \, plf$$

$$M_u = \frac{w_u l^2}{8} = \frac{2352 \times (65)^2 \times 12}{8} = 14,905,800 \, in. - lb$$

EXAMPLE 1

The required nominal moment strength is

$$M_n = \frac{M_u}{\phi} = \frac{14,905,800}{0.9} = 16,562,000 in.-lb$$

2- Choice of preliminary section

Assuming a depth of 0.6in./ft of span, we can have a trial section depth

$$h = 0.6 \times 65 = 39 in. \quad \text{use } h = 40 in.$$

$$A_c' = \frac{M_n}{0.68 f_c' h} = \frac{16562000}{0.68 \times 5000 \times 40} = 121.8 in.^2$$

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EXAMPLE 1

Assume a flange width of 18 in. Then the average flange thickness

$$h_f = 121.8/18 \cong 7 in$$

Use a web width, $b_w = 6 in$

To be subsequently verified for shear requirements.

The geometrical properties of the trial section are very close to the assumed dimensions for the depth h and the top flange width b. Hence, use the suggested section.

Minimum area of non-prestressed steel = 0.004A

$$= 0.004(18 \times 4 + 6(\frac{6+18}{2}) + 6 \times 8.84) = 0.788 \, in^2$$

Use 4 #4 ($A_s = 0.8 \, in^2$)

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EXAMPLE 1

3- Calculate the prestressing steel

$$A_{ps} = \frac{M_n}{0.72 f_{pu} h} = \frac{16562000}{0.72 \times 270,000 \times 40} = 2.13 in.^2$$

And the number of ½-in. stress-relieved 7-wire strands

$$= 2.13/0.153 = 13.9$$

Try thirteen (½-in) tendons, $A_{ps} = 13 \times 0.153 = 1.99\ in^2$.

From example 4.2, $f_{pe} \cong 155,000$ psi

$$p_e = 13 \times 0.153 \times 155,000 = 308,295 lb$$

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EXAMPLE 1

4- Calculate the stress f_{ps} in the prestressing tendons at nominal strength using the strain compatibility approach.

$$i - \varepsilon_1 = \varepsilon_{pe} = \frac{f_{pe}}{E_{ps}} = \frac{155,000}{28 \times 10^6} = 0.0055$$

ii- The increase in prestressing steel strain as the concrete is decompressed by the increased external load is given as

$$\varepsilon_2 = \varepsilon_{decomp} = \frac{P_e}{A_c E_c}\left(1 + \frac{e^2}{r^2}\right) = \frac{308,295}{377 \times 4.03 \times 10^6}(1 + 1.2) = 0.0004$$

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EXAMPLE 1

iii - Assume that the stress $f_{ps} \cong 205{,}000$ psi as a first trial.

Suppose the neutral axis inside the flange is verified on the basis of

$$h_f = 4\tfrac{1}{2} + \frac{\frac{18+6}{2} \times 3.5}{18} = 6.83 \ in.$$

Then, from Equation 4.42a

$$a = \frac{A_{ps}f_{ps} + A_s f_y}{0.85 f_c' b} = \frac{1.99 \times 205{,}000 + 0.80 \times 60{,}000}{0.85 \times 5000 \times 18} = 5.33 \ in < h_f$$

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EXAMPLE 1

Hence, the equivalent compressive block is inside the flange and the section has to be treated as rectangular.

Accordingly, for 5,000 psi concrete

$$\beta_1 = 0.85 - 0.05 = 0.8$$

$$c = \frac{a}{\beta_1} = \frac{5.33}{0.8} = 6.67 \ in.$$

$$d = 40 - (1.5 + \tfrac{1}{2} in. \ for \ stirrups + \tfrac{2}{8} in. \ for \ bars) \cong 37.8 \ in.$$

$$\varepsilon_c = 0.003$$

The increment of strain due to the ultimate is

$$\varepsilon_3 = \varepsilon_c \left(\frac{d-c}{c} \right) = 0.003 \left(\frac{37.8 - 6.67}{6.67} \right) = 0.014 \ in./in. \gg 0.005 \ in./in. \quad O.K.$$

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EXAMPLE 1

And the total strain is

$$\varepsilon_{ps} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0.0055 + 0.0004 + 0.014 = 0.0199 \ in./in.$$

From the stress-strain diagram f_{ps} corresponding to $\varepsilon_{ps} = 0.0199$

is 232,000 psi

Second trial for f_{ps} value, $f_{ps} = 230,000 \ psi$

$$a = \frac{A_{ps}f_{ps} + A_s f_y}{0.85 f_c' b} = \frac{1.99 \times 230,000 + 0.80 \times 60,000}{0.85 \times 5000 \times 18} = 6.61 in < h_f$$

Consider section as rectangular beam.

$$c = \frac{a}{\beta_1} = \frac{6.61}{0.8} = 8.26 \, in.$$

$$\varepsilon_3 = \varepsilon_c \left(\frac{d-c}{c}\right) = 0.003 \left(\frac{37.8 - 8.26}{8.26}\right) = 0.0107 \, in./in$$

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EXAMPLE 1

Then the total strain is

$$\varepsilon_{ps} = 0.0055 + 0.0004 + 0.0107 = 0.0166 \, in./in$$

From figure 4.50, $f_{ps} = 230,000 \, psi$
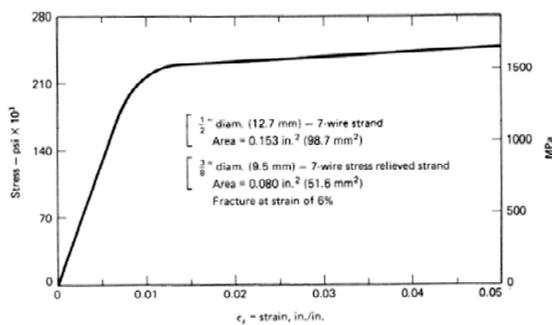
Figure 4.50 Typical stress-strain relationship of 7-wire 270-K prestressing strand.

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EXAMPLE 1

5- Available moment strength

$$M_n = 1.99 \times 230{,}000\left(36.16 - \frac{6.61}{2}\right) + 0.80 \times 60{,}000\left(37.8 - \frac{6.61}{2}\right)$$

$$= 15{,}037{,}370 + 1{,}655{,}760 = 16{,}693{,}490 \, in.-lb$$

$$> required \, M_n = 16{,}562{,}000 \, in.-lb, \, O.K.$$

The maximum steel index, from equation 4.57b, is

$$\omega_p + \frac{d}{d_p}(\omega - \omega') \le 0.36\beta_1 < 0.29 \qquad for \, \beta_1 = 0.80$$

The actual total reinforcement index is

$$\omega_T = \frac{1.99 \times 230{,}000}{18 \times 36.16 \times 5{,}000} + \frac{37.8}{36.16}\left(\frac{0.80 \times 60{,}000}{18 \times 37.8 \times 5{,}000}\right) = 0.0155 < 0.29 \quad O.K$$

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EXAMPLE 1

Alternatively, the ACI Code limit strain provisions as given in fig 4.45 do not prescribe a maximum percentage of reinforcement. They require that a check be made of the strain ε_T at the level of the extreme tensile reinforcement,

$$\varepsilon_t = 0.003 \times \left(\frac{37.8 - 6.67}{6.67}\right) = 0.014 > 0.005$$
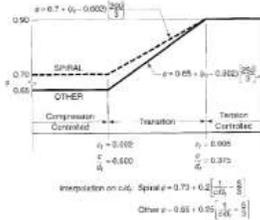

Figure 4.45 Strain limit zones and variation of strength reduction factor φ with the net Tensile Strain εₜ.

So, the beam is tension controlled, and the reduction factor $\phi = 0.90$

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EXAMPLE 2

❖ Design 18m span simply supported pretensioned beam with M_T = 435 kN.m (this includes self weight moment of M_D = 55 kN.m). Assume the ultimate total moment M_u = 650 kN.m (this includes M_D = 66 kN.m). The prestress at transfer is f_{pi} = 1035 MPa and at service f_{pe} = 860 MPa. Based on the grade of concrete M35, the allowable compressive stresses are 12.5 MPa at transfer, 11.0 MPa at service, and no tension stresses are allowed at any stage.

❖ The properties of the prestressing strands are given below:
  - ❖ Type of prestressing tendon : 7-wire strand
  - ❖ Ultimate strength = f_{pu} = 1860 MPa
  - ❖ Nominal diameter = 12.8 mm
  - ❖ Nominal area = 99.3 mm^2

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EXAMPLE 2 (PRELIMINARY SECTION)

Choose a preliminary section:

$$h = (50\,mm\,/\,m) \times 18\,m = 900\,mm$$

$$Flange\ area = A_c' = \frac{M_n}{0.68f_c'h} = \frac{723 \times 10^6}{0.68 \times 35 \times 900} = 33,754\,mm^2$$

$$Assume\ Flange\ width = b = 350\,mm$$

$$Flange\ thickness = h_f = \frac{33,754}{350} = 96\,mm \approx 100\,mm$$

Assume web thickness $t_w = 100\,mm$; *this should be verified for shear requirements.*

$$A_{ps} = \frac{M_n}{0.72f_{pu}h} = \frac{723 \times 10^6}{0.72 \times 1860 \times 900} = 600\,mm^2$$

$\therefore$ *Try Seven 12.8mm diameter 7-wire strands=695 mm^2*

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EXAMPLE 2 (SELECTED SECTION PROPERTIES)

Section Properties:

$A_c = 140,000\,mm^2$

$I_c = 1.412 \times 10^{10}\,mm^4$

$r^2 = 100,833\,mm^2$

$S = 31.37 \times 10^6\,mm^3$

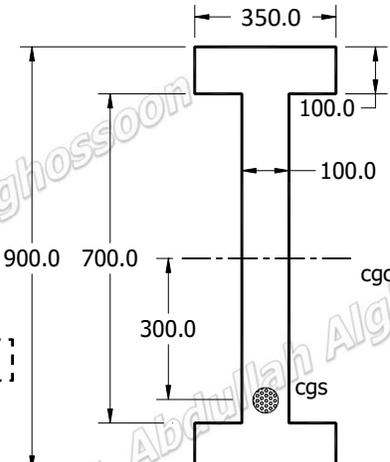for zero tension stresses,
kern points are:

$k_t = k_b = \dfrac{r^2}{c_b} = \dfrac{100,833}{450} = 224\,mm$

$e_{max} = k_b + \dfrac{M_D}{P_i}$ $\boxed{f_{pi} * A_{ps} \text{ or using } A_c * f_{cgc}}$

$e_{max} = 234 + \dfrac{136 \times 10^3}{695 \times 1035} = 423\,mm$

Select $e_c = 300\,mm$

350.0
100.0
100.0
900.0 700.0
cgc
300.0
cgs

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EXAMPLE 2 (CHECK SECTION AT SERVICE)

$\gamma = \dfrac{860}{1035} = 0.83;\quad 1-\gamma = 0.17$

$S^t = \dfrac{(1-\gamma)M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c}$

$S^t = \dfrac{0.17 \times 136 + 380}{0 + 11} = 36.65 \times 10^6\,mm^3$

$S^b = \dfrac{(1-\gamma)M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}}$

$S^b = \dfrac{0.17 \times 136 + 380}{0 + 0.83 \times 12.5} = 38.85 \times 10^6\,mm^3$

S values less than $S = 31.37 \times 10^6\,mm^3$

∴ Revise the beam section

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EXAMPLE 2 (REVISE SECTION)

Section Properties:

$A_c = 195,000\,mm^2$

$I_c = 1.876 \times 10^{10}\,mm^4$

$r^2 = 96,218\,mm^2$

$S^t = S^b = 41.69 \times 10^6\,mm^3$

$c_t = c_b = 450\,mm$

$k_t = k_b = \dfrac{r^2}{c_b} = 214\,mm$

$M_D = 0.195 \times 24 \times \dfrac{18^2}{8} = 190\,kN.m$

$M_T = 380 + 190 = 570\,kN.m$

$M_u = 584 + 1.2(190) = 812\,kN.m$
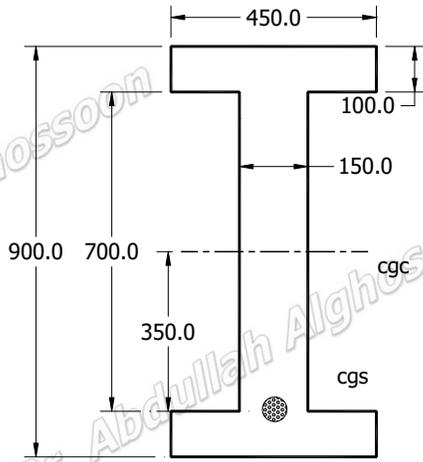
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EXAMPLE 2 (STRAIN COMPATIBILITY METHOD)

$$\bar{f}_{ci} = f_{ti} - \frac{c_t}{h}(f_{ti} - f_{ci})$$

$$\bar{f}_{ci} = 0 - 0.50(0 + 12.5) = -6.25\,MPa$$

$$P_i = A_c\bar{f}_{ci} = 1,219\,kN$$

$$e_c = (f_{ti} - \bar{f}_{ci})\frac{S^t}{P_i} + \frac{M_D}{P_i}$$

$$e_c = (0 + 6.25)\frac{41.69 \times 10^6}{1,219 \times 10^3} + \frac{190 \times 10^6}{1,219 \times 10^3}$$

$$e_c = 370\,mm \quad USE \quad e_c = 350\,mm$$

$$A_{ps} = \frac{1,219 \times 10^3}{1035} = 1,178\,mm^2$$

USE Twelve 12.8mm diameter 7-wire strands

$$A_{ps} = 1,192\,mm^2$$

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EXAMPLE 2 (CHECK FLEXURAL CAPACITY FM$_N$)

Strain due to effective prestressing (ε_1):

$$\varepsilon_1 = \varepsilon_{pe} = \frac{f_{pe}}{E_{ps}} = \frac{860}{193 \times 10^3} = 0.0045$$

$$P_e = 12 \times 99.3 \times 860 = 1,025 \, kN$$

Strain due to decompression (ε_2):

$$\varepsilon_2 = \varepsilon_{decomp} = \frac{P_e}{A_c E_c} \left(1 + \frac{e^2}{r^2} \right)$$

$$\varepsilon_2 = \frac{1,025,000}{195,000 \times 27,800} \left(1 + \frac{350^2}{96,218} \right) = 0.00043$$

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EXAMPLE 2 (STRAIN COMPATIBILITY METHOD)

Trial 1: assume $f_{ps} = 1400 \, MPa$

$$A = \frac{A_{ps} f_{ps}}{0.85 f_c'} = \frac{1192 \times 1400}{0.85 \times 35 \times 450} = 56,094 \, mm^2 > A_f$$

$\Rightarrow$ *Flanged section*

$$56,094 = (450 - 150)(100) + 150a \Rightarrow a = 174 \, mm$$

$$\beta_1 = 0.85 - \frac{0.05}{7}(35 - 30) = 0.814$$

$$c = \frac{a}{\beta_1} = 214 \, mm; \quad d_p = 450 + 350 = 800 \, mm$$

$$\therefore \varepsilon_3 = \left(\frac{d_p - c}{c} \right) \varepsilon_c = \left(\frac{800 - 214}{214} \right)(0.003) = 0.0082 > 0.005$$

$\therefore$ *Ductile behavior*

$$\varepsilon_{ps} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0.0131$$

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EXAMPLE 2 (STRAIN COMPATIBILITY METHOD)

CHECK ULTIMATE STRENGTH CAPACITY

Ultimate loading

γ_D		1.2
γ_L		1.6
$M_{u,D}$	kN.m	227
$M_{u,SD}$	kN.m	0
$M_{u,L}$	kN.m	608
$M_{u,T}$	kN.m	835
$V_{u,d/2}$	kN	177 120.9
$V_{i,d/2}$	kN	129
$V_{d,d/2}$	kN	40
$M_{d,d/2}$	kN.m	73
$V_{u,No\ need}$	kN	78 5200
v_h	Mpa	0.00

Failure stress in PS steel

assumed f_{ps}	MPa	**1,400**
Calc. f_{ps}	MPa	1,619
β_1		0.814
A_{comp}	mm²	56,075
Flanged Section		

Strain calculation

ε_e	mm/mm	0.00445
ε_{decomp}	mm/mm	0.00043
ε_3	mm/mm	0.00824
ε_T	mm/mm	0.01312
a	mm	174
c	mm	213
ω		0.185 **under-reinforced**

Tension reinforcement

d_p	mm	800
d	mm	810
A_s	mm²	0
f_y	MPa	414
16		4 804.2

Compression reinforcement

d'	mm	50.0
A_s'	mm²	0
f_y'	MPa	414

Moment strength

M_n	kN.m	1,223	
ϕM_n	kN.m	1,100	OK
M_{cr}	kN.m	154	OK
$A_{s,min}$	mm²	429	OK

Shear strength

S_{max}	mm	600.0
S_{req}	mm	#VALUE!
$S_{selected}$	mm	300.0
A_v	mm²	157.1
f_{yv}	Mpa	414
ϕV_n	kN	355.9

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EXAMPLE 2 (STRAIN COMPATIBILITY METHOD)

From stress-strain diagram f_{ps} corresponding to $\varepsilon_{ps} = 0.0131$ is $f_{ps} = 1515\ MPa$ Not O.k.

Figure 4.50 Typical stress-strain relationship of 7-wire 270-K prestressing strand.

- ½″ diam. (12.7 mm) — 7-wire strand
  Area = 0.153 in.² (98.7 mm²)
- ⅜″ diam. (9.5 mm) — 7-wire stress relieved strand
  Area = 0.080 in.² (51.6 mm²)
  Fracture at strain of 6%

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EXAMPLE 2

Trial 2: Assume $f_{ps} = 1490\,MPa$

$a = 198\,mm$; $c = 243\,mm$; $\varepsilon_3 = 0.0067$;

Thus: $\varepsilon_{ps} = 0.0118$

From stress-strain diagram f_{ps} corresponding

to $\varepsilon_{ps} = 0.0118$ is $f_{ps} = 1480\,MPa$ close enough O.k.

$$M_n = 0.85 f_c' a b_f (d_p - \frac{h_f}{2}) +$$
$$0.85 f_c' b_w (a - h_f)(d_p - h_f - (a - h_f)/2)$$

$\phi M_n = 1,160\,kN.m \gg M_u = 812\,kN.m$ *O.K.*

No need for mild steel

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CHECK ULTIMATE STRENGTH CAPACITY

Ultimate loading

γ_D		**1.2**
γ_L		**1.537**
$M_{u,D}$	kN.m	227
$M_{u,SD}$	kN.m	0
$M_{u,L}$	kN.m	584
$M_{u,T}$	kN.m	811
$V_{u,d/2}$	kN	172 120.9
$V_{i,d/2}$	kN	124
$V_{d,d/2}$	kN	40
$M_{u,d/2}$	kN.m	71
$V_{u,No\ need}$	kN	76 **5200**
v_h	Mpa	0.00

Failure stress in PS steel

assumed f_{ps}	MPa	**1,490**
Calc. f_{ps}	MPa	1,592
β_1		0.814
A_{comp}	mm²	59,680
Flanged Section		

Strain calculation

ε_e	mm/mm	0.00445
ε_{decomp}	mm/mm	0.00043
ε_3	mm/mm	0.00688
ε_T	mm/mm	0.01176
a	mm	198
c	mm	243
ω		0.210 **under-reinforced**

Tension reinforcement

d_p	mm	800
d	mm	810
A_s	mm²	0
f_y	MPa	414
16		4 804.2

Compression reinforcement

d'	mm	50.0
A_s'	mm²	0
f_y'	MPa	414

Moment strength

M_n	kN.m	1,288	
ϕM_n	kN.m	1,160	OK
M_{cr}	kN.m	154	OK
$A_{s,min}$	mm²	429	OK

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EXAMPLE 2 (MIN. & MAX. AREA OF STEEL)

Check for minimum area of steel :

$A = 450 \times 100 + 150 \times 350 = 97,500\,mm^2$

$A_{s,min} = 0.004(97500) = 390\,mm^2 < 1,192\,mm^2$ *O.K.*

Check for maximum area of steel:

$$\left[\omega_{pw} + \frac{d}{d_p}(\omega_w - \omega_w')\right] = \frac{0.85\,a}{d_p}$$

$$= \frac{0.85 \times 198}{800} = 0.210 < 0.36\beta_1 = 0.293 \quad O.K.$$

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EXAMPLE 2 (CHECK STRESS LIMITS)

Stresses at transfer:

$P_i = 12 \times 99.3 \times 1035 = 1,233\,kN$

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t}$$

$$f^t = -\frac{1,233,000}{195,000}\left(1 - \frac{350 \times 450}{96,218}\right) - \frac{190 \times 10^6}{41.69 \times 10^6}$$

$f^t = 4.027 - 4.557 = -0.530\,MPa(compression) < -12.5\,MPa$

$$f^b = -\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_D}{S^b}$$

$f^b = -16.678 + 4.456 = -12.13\,MPa < -12.5\,MPa$

$\therefore$ *At Transfer: no tension and compression stresses*

are less than the limit

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EXAMPLE 2 (CHECK STRESS LIMITS)

Stresses at Service:

$$P_e = 12 \times 99.3 \times 860 = 1,025\,kN$$

$$f^t = -\frac{P_e}{A_c}(1 - \frac{ec_t}{r^2}) + \frac{M_T}{S^t}$$

$$f^t = -\frac{1,025,000}{195,000}(1 - \frac{350 \times 450}{96,218}) - \frac{570 \times 10^6}{41.69 \times 10^6}$$

$$f^t = +3.347 - 13.660 = -10.313\,MPa < -11.0\,MPa \quad O.K.$$

$$f^b = -\frac{P_e}{A_c}(1 + \frac{ec_b}{r^2}) + \frac{M_T}{S^b}$$

$$f^b = -13.858 + 13.660 = -0.198\,MPa(compression) \quad O.K.$$

$\therefore$ *The section is Okay*

EXAMPLE 2 (FINAL DESIGNED SECTION AT MIDSPAN)
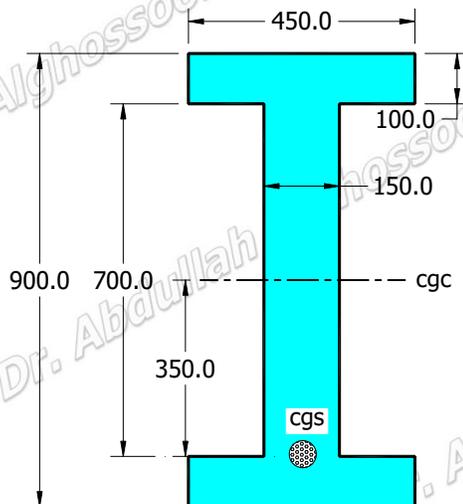
PRESTRESSED CONCRETE
-Shear Design-

- The Hashemite University
- Department of Civil Engineering
- Dr. Abdullah Alghossoon
- Abdullahm_ab@hu.edu.jo
- Fall 2022

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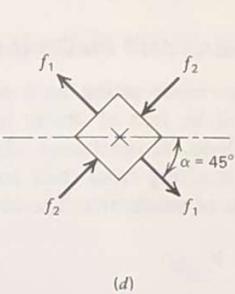## DESIGN SHEAR FORCE DIAGRAM
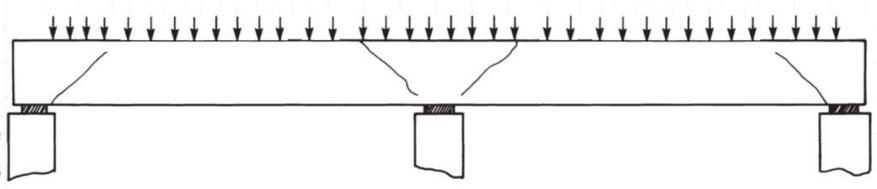
(a) Beam.
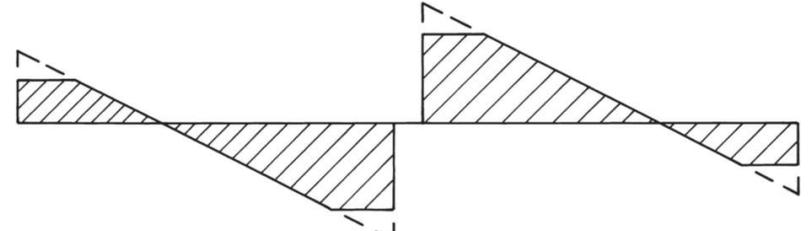
(b) Shear force diagram.

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DIAGONAL CRACKING IN R.C. BEAM

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DIAGONAL CRACKING IN R.C. BEAM

❖ Shear stresses in beams generate due to either bending, which is referred to as flexure-shear stress, or twisting, which is referred to as torsional shear stress.

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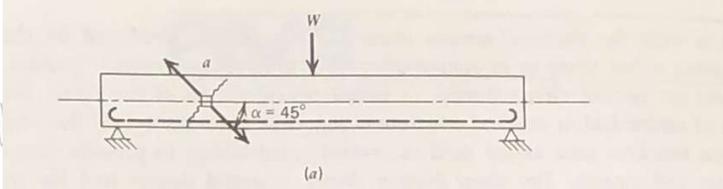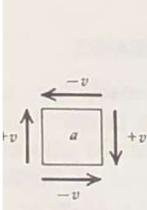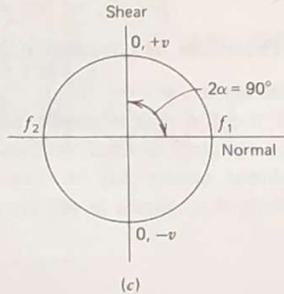
DIAGONAL CRACKING IN R.C. BEAM
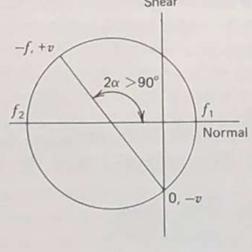

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DIAGONAL CRACKING IN PS BEAM

❖ In presence of prestressing force, the length and crack width of a diagonal crack is small. Thus, the aggregate interlock and compression zone of concrete are larger as compared to a non-prestressed beam under the same load.

❖ Hence, the shear strength of concrete (V_c) increases in presence of prestressing force. This is accounted for in the expression of V_c.
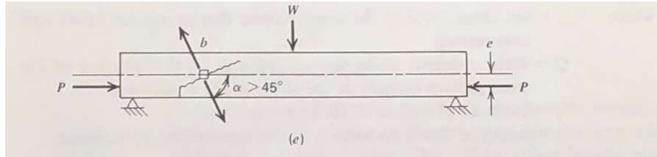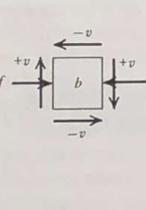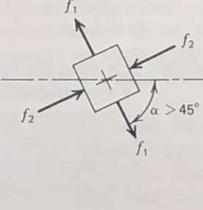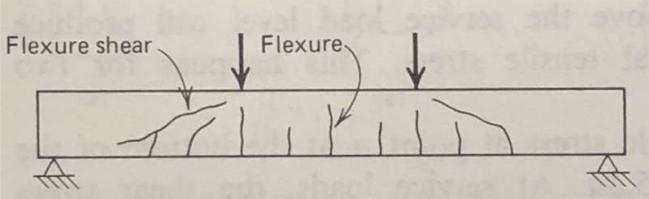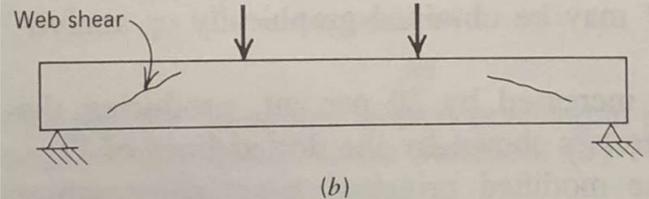

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TYPES OF DIAGONAL CRACKING IN PC BEAM

❖ Typically, for I-beams, cracking will initiate not at the N.A., but at the junction of the lower flange and the web (high shear stress, lower compression).

❖ Also, cracking will not initiate near the supports (high shear stress, but high pre-compression also).

❖ Therefore, diagonal cracking is likely at about the quarter span.

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MODES OF SHEAR FAILURE

❖ The occurrence of a mode of failure depends on the span-to-depth ratio, loading, cross-section of the beam, amount and anchorage of reinforcement.

1) Diagonal tension failure

2) Shear compression failure

3) Shear tension failure

4) Web-crushing failure

5) Arch rib failure

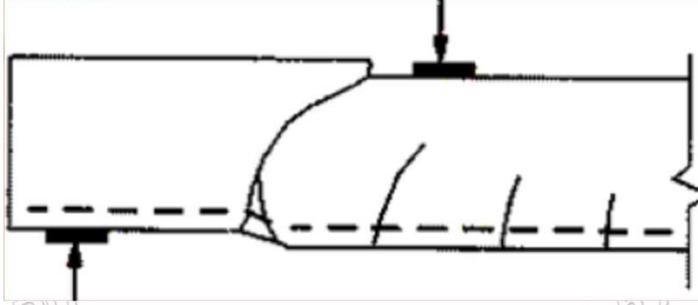Dr. Abdullah Alghossoon

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DIAGONAL TENSION FAILURE

❖ An inclined crack propagates rapidly due to inadequate shear reinforcement

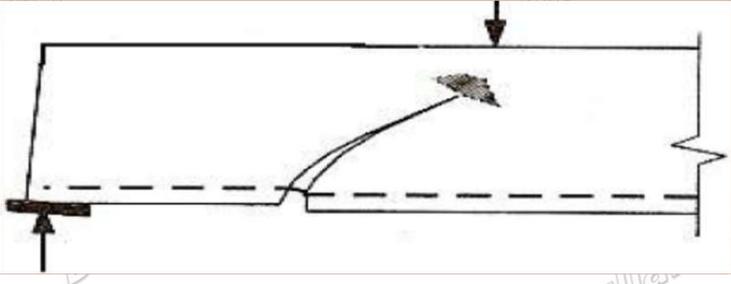Dr. Abdullah Alghossoon   Prestressed Concrete

SHEAR COMPRESSION FAILURE

❖ There is crushing of the concrete near the compression flange above the tip of the inclined crack.

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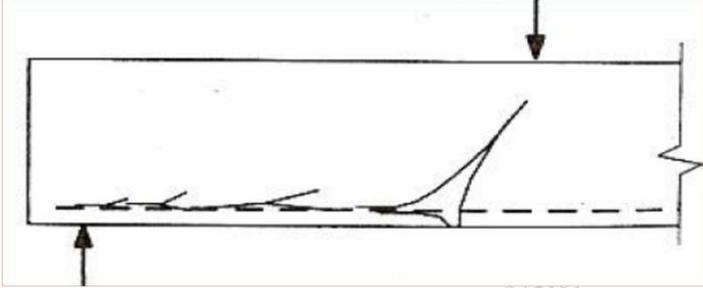# SHEAR TENSION FAILURE

❖ Due to inadequate anchorage of the longitudinal bars, the diagonal cracks propagate horizontally along the bars.

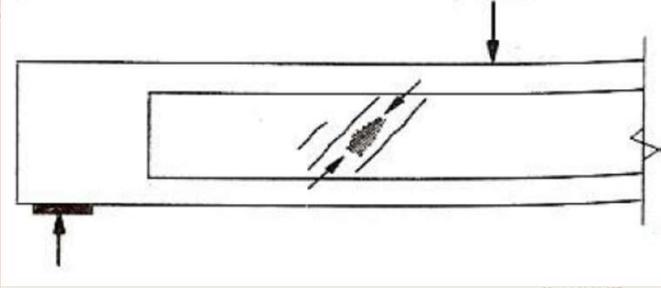Dr. Abdullah Alghossoon   Prestressed Concrete

WEB CRUSHING FAILURE

❖ The concrete in the web crushes due to inadequate web thickness.
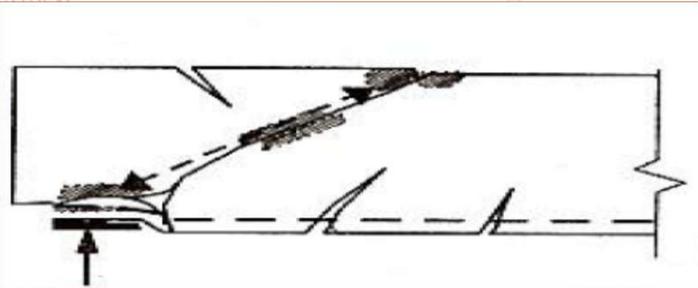

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ARCH RIB FAILURE

❖ For deep beams, the web may buckle and subsequently crush. There can be anchorage failure or failure of the bearing.

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SHEAR STRENGTH, V_c (318-19) SEC. 22.

❖ Sectional Strength 22.5.1.1 $V_n = V_c + V_s$

❖ Simplified Method for V_c : 22.5.6.2

22.5.6 V_c for prestressed members

22.5.6.1 This section shall apply to the calculation of V_c for post-tensioned and pretensioned members in regions where the effective force in the prestressed reinforcement is fully transferred to the concrete. For regions of pretensioned members where the effective force in the prestressed reinforcement is not fully transferred to the concrete, 22.5.7 shall govern the calculation of V_c.

> Same as f_{pe}

22.5.6.2 For prestressed flexural members with $A_{ps}f_{se} \geq 0.4(A_{ps}f_{pu} + A_s f_y)$, V_c shall be calculated in accordance with Table 22.5.6.2, but need not be less than $0.17\lambda\sqrt{f_c'}\,b_w d$. Alternatively, it shall be permitted to calculate V_c in accordance with 22.5.6.3.

> Minimun of V_{ci} and V_{cw}

> Use it when $f_{pe} > 0.4\, f_{pu}$

Table 22.5.6.2—Approximate method for calculating V_c

	V_c	
Least of (a), (b), and (c):	$\left(0.05\lambda\sqrt{f_c'} + 4.8\dfrac{V_u d_p}{M_u}\right)b_w d$ [1],[2]	(a)
	$(0.05\lambda\sqrt{f_c'} + 4.8)b_w d$	(b)
	$0.42\lambda\sqrt{f_c'}b_w d$	(c)

[1]M_u occurs simultaneously with V_c at the section considered.

[2]When calculating the $V_u d_p/M_u$ term in Eq. 22.5.6.2(a), d_p is the distance from the extreme compression fiber to the centroid of prestressed reinforcement. It shall not be permitted to take d_p as **0.80**h as in 22.5.2.1.

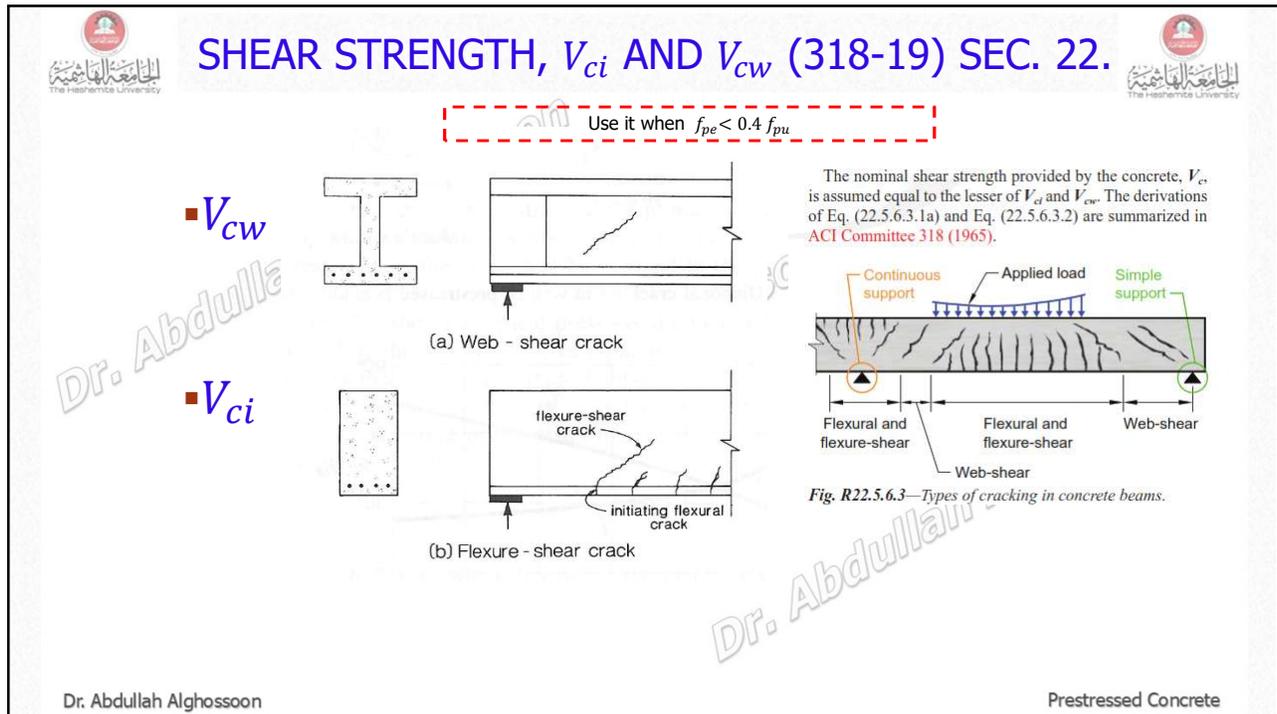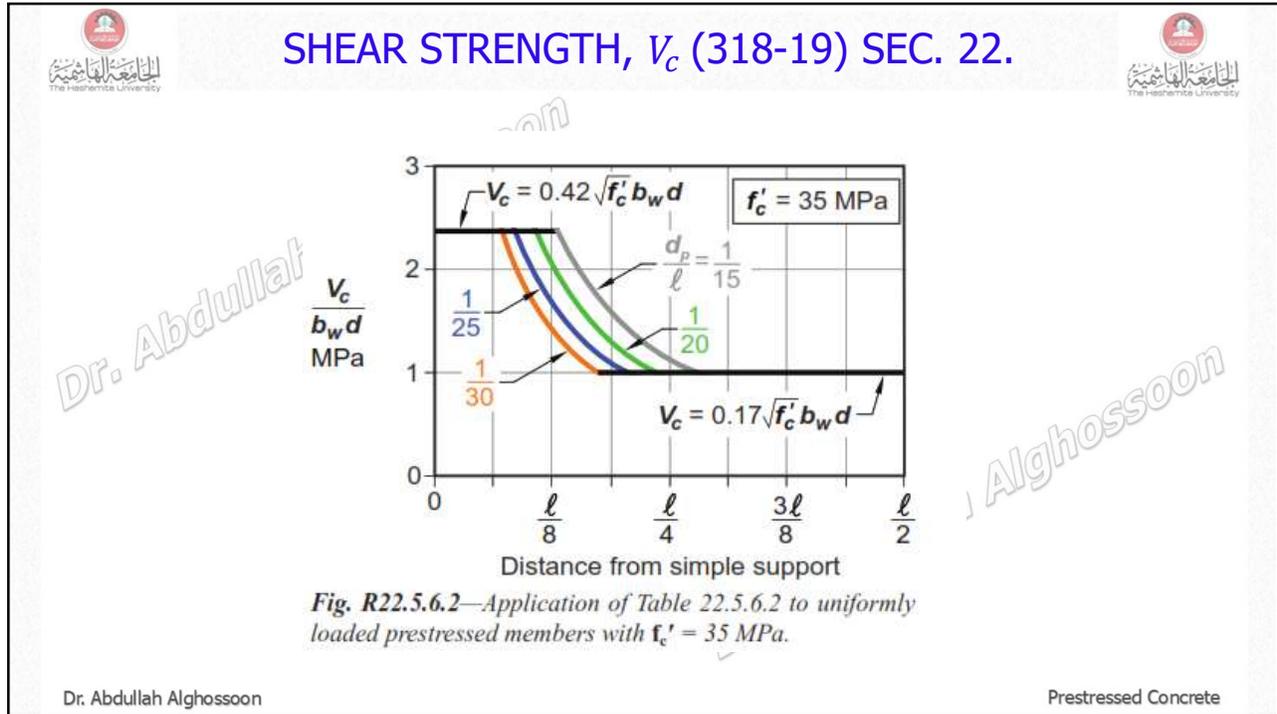$$\frac{V_u d_p}{M_u} = \frac{d_p(\ell - 2x)}{x(\ell - x)} \leq 1 \qquad (R22.5.6.2)$$

> d : can be taken as the average value of d_s and d_{ps} but should be > 0.8h

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SHEAR STRENGTH, V_c (318-19) SEC. 22.

Fig. R22.5.6.2—*Application of Table 22.5.6.2 to uniformly loaded prestressed members with* $f_c' = 35$ *MPa.*

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SHEAR STRENGTH, V_{ci} AND V_{cw} (318-19) SEC. 22.

Use it when $f_{pe} < 0.4 f_{pu}$

- V_{cw}

- V_{ci}

(a) Web - shear crack

(b) Flexure - shear crack

The nominal shear strength provided by the concrete, V_c, is assumed equal to the lesser of V_{ci} and V_{cw}. The derivations of Eq. (22.5.6.3.1a) and Eq. (22.5.6.3.2) are summarized in ACI Committee 318 (1965).

Fig. R22.5.6.3—*Types of cracking in concrete beams.*

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SHEAR STRENGTH, V_{ci} AND V_{cw} (318-19) SEC. 22.

❖ V_{ci} and V_{cw} Method: 22.5.6.3

V_d = shear force at section due to unfactored dead load.

V_i = factored shear force at section due to externally applied load causing M_{max}.

f_{ce} = concrete compressive stress due to P_e at extreme fibers of section.

f_d = Tensile stress due to unfactored dead load at extreme fiber resulting from self-weight only.

22.5.6.3.1 The flexure-shear strength V_{ci} shall be calculated by (a) but need not be taken less than (b) or (c):

(a) $V_{ci} = 0.05\lambda\sqrt{f_c'}b_w d_p + V_d + \dfrac{V_i M_{cre}}{M_{max}}$ (22.5.6.3.1a)

(b) For members with $A_{ps}f_{se} < 0.4(A_{ps}f_{pu} + A_s f_y)$,

$\qquad V_{ci} = 0.14\lambda\sqrt{f_c'}b_w d$ (22.5.6.3.1b)

(c) For members with $A_{ps}f_{se} \geq 0.4(A_{ps}f_{pu} + A_s f_y)$,

$\qquad V_{ci} = 0.17\lambda\sqrt{f_c'}b_w d$ (22.5.6.3.1c)

where d_p need not be taken less than **0.80h**, the values of M_{max} and V_i shall be calculated from the load combinations causing maximum factored moment to occur at section considered, and M_{cre} shall be calculated by:

$M_{cre} = \left(\dfrac{I}{y_t}\right)(0.5\lambda\sqrt{f_c'} + f_{pe} - f_d)$ (22.5.6.3.1d) te

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SHEAR STRENGTH, V_c (318-19) SEC. 22.

$$V_{ci} = 0.05\sqrt{f_c'}\,b_w\,d_p + V_d + \frac{V_i}{M_{max}}(M_{cr})$$

$$0.42\sqrt{f_c'}\,b_w\,d_p \geq V_{ci} \geq 0.14\sqrt{f_c'}\,b_w\,d_p$$

$$M_{cr} = S^b(0.5\sqrt{f_c'} + f_{ce} - f_d)$$

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SHEAR STRENGTH, V_c (318-19) SEC. 22.

❖ Vc for pretensioned beam in sections located in: 22.5.7

22.5.7 V_c for pretensioned members in regions of reduced prestress force

22.5.7.1 When calculating V_c, the transfer length of prestressed reinforcement, ℓ_{tr}, shall be assumed to be $50d_b$ for strand and $100d_b$ for wire.

22.5.7.2 If bonding of strands extends to the end of the member, the effective prestress force shall be assumed to vary linearly from zero at the end of the prestressed reinforcement to a maximum at a distance ℓ_{tr} from the end of the prestressed reinforcement.

22.5.7.3 At locations corresponding to a reduced effective prestress force in 22.5.7.2, V_c shall be calculated in accordance with (a) through (c):

(a) The reduced effective prestress force shall be used to determine the applicability of 22.5.6.2.
(b) The reduced effective prestress force shall be used to calculate V_{cw} in 22.5.6.3.
(c) The value of V_c calculated using 22.5.6.2 shall not exceed the value of V_{cw} calculated using the reduced effective prestress force.

22.5.7.4 If bonding of strands does not extend to the end of the member, the effective prestress force shall be assumed to vary linearly from zero at the point where bonding commences to a maximum at a distance ℓ_{tr} from that point.

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SHEAR STRENGTH, V_{cw} (318-19) SEC. 22.

❖ V_{cw} for pretensioned beam in sections located in: 22.5.7

22.5.6.3.2 The web-shear strength V_{cw} shall be calculated by:

$$V_{cw} = (0.29\lambda\sqrt{f_c'} + 0.3f_{pc})b_w d_p + V_p \qquad (22.5.6.3.2)$$

❖ V_p = the vertical component of the effective prestress at a particular section ≈ 0; since tendon slope is small.

❖ d_p = distance from extreme compression fiber to the centroid of prestressed steel or 0.8h whichever is greater (if mild steel exists, use the average depth value of prestressing and non-prestressing steel.

❖ f_{cgc} The resultant compressive stress at either the centroid of the section or at the junction of the web and flange.
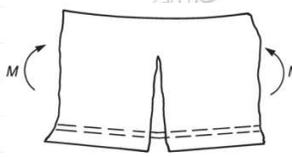
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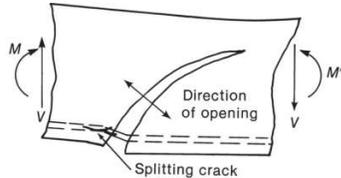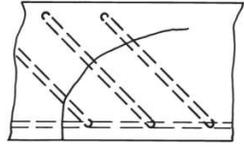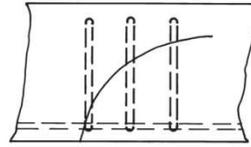
INCLINED CRACKS AND SHEAR REINFORCEMENT

(a) Flexural crack.

(b) Inclined crack.

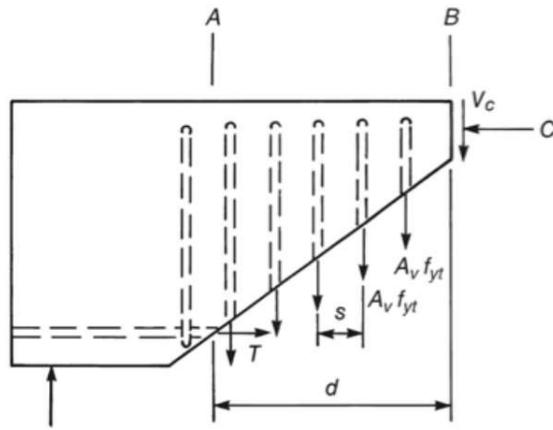(c) Inclined shear reinforcement.

(d) Vertical shear reinforcement.

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SHEAR RESISTED BY STIRRUPS

(a) Vertical stirrups.

See Sec. 22.5.8 in 318-19

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SHEAR STRENGTH V_s (318-19)

22.5.8.1 At each section where $V_u > \phi V_c$, transverse reinforcement shall be provided such that Eq. (22.5.8.1) is satisfied.

$$V_s \geq \frac{V_u}{\phi} - V_c \qquad (22.5.8.1)$$

22.5.8.2 For one-way members reinforced with transverse reinforcement, V_s shall be calculated in accordance with 22.5.8.5.

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VERTICAL STIRRUPS/TIES- V_s

22.5.8.5.3 V_s for shear reinforcement in 22.5.8.5.1 shall be calculated by:

$$V_s = \frac{A_v f_{yt} d}{s} \qquad (22.5.8.5.3)$$

where s is the spiral pitch or the longitudinal spacing of the shear reinforcement, and A_v is given in 22.5.8.5.5 or 22.5.8.5.6.

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MINIMUM AMOUNT OF SHEAR REINFORCEMENT

9.6.3.2 For prestressed beams, a minimum area of shear reinforcement, $A_{v,min}$, shall be provided in all regions where $V_u > 0.5\phi V_c$ except for the cases in Table 9.6.3.1. For these cases, at least $A_{v,min}$ shall be provided where $V_u > \phi V_c$.

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MINIMUM AMOUNT OF SHEAR REINFORCEMENT

Table 9.6.3.1—Cases where $A_{v,min}$ is not required if $V_u \leq \phi V_c$

Beam type	Conditions
Shallow depth	$h \leq 10$ in.
Integral with slab	$h \leq$ greater of $2.5t_f$ or $0.5b_w$ and $h \leq 24$ in.
Constructed with steel fiber-reinforced normalweight concrete conforming to 26.4.1.5.1(a), 26.4.2.2(i), and 26.12.7.1(a) and with $f_c' \leq 6000$ psi	$h \leq 24$ in. and $V_u \leq \phi 2\sqrt{f_c'}b_w d$
One-way joist system	In accordance with 9.8

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MINIMUM AMOUNT OF SHEAR REINFORCEMENT

$$S_{max} = smaller \ of \begin{cases} \begin{bmatrix} 600\,mm \\ \frac{3}{4}h \end{bmatrix} if \ V_s \le 0.33\sqrt{f_c'}b_w d_p \qquad V_s \le 4\sqrt{f_c'}b_w d_p \quad for \quad psi \\ \begin{bmatrix} 300\,mm \\ \frac{3}{8}h \end{bmatrix} if \ V_s > 0.33\sqrt{f_c'}b_w d_p \qquad V_s > 4\sqrt{f_c'}b_w d_p \quad for \quad psi \\ \frac{16\,A_v f_y}{b_w\sqrt{f_c'}} \\ \frac{A_v f_y}{0.35\,b_w} \\ \frac{80\,A_v f_y d_p}{A_{ps} f_{pu}}\sqrt{\frac{b_w}{d_p}} \end{bmatrix} if \ f_{pe} \ge 0.4 f_{pu} \end{cases}$$

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SUMMARY

Reinforced Concrete

$$V_u \le \phi V_n = \phi\left(V_c + V_s\right) \qquad V_s = \frac{A_v f_y d}{s}$$

Reinforced Concrete Strengthened with FRP
(Fiber-reinforced polymer)

$$V_u \le \phi V_n = \phi\left(V_c + V_s + \psi_f V_f\right) \qquad V_f = \frac{A_{fv} f_{fe} d_{fv}}{s_f}$$

Prestressed Concrete

$$V_u \le \phi V_n = \phi\left(V_c + V_s\right) \qquad V_s = \frac{A_v f_y d_p}{s}$$

1) Approximation Design Method

$$\left(f_{pe} > 0.4 f_{pu}\right) \qquad 0.17\sqrt{f_c'}b_w d_p \le V_c = \left(0.05\sqrt{f_c'} + 4.8\frac{V_u d_p}{M_u}\right)b_w d_p \le 0.42\sqrt{f_c'}b_w d_p$$

2) Detailed Design Method $\qquad V_c = \min\left(V_{cw}, V_{ci}\right)$

$$V_{cw} = \left(0.29\sqrt{f_c'} + 0.3 f_{pc}\right)b_w d_p + V_p \qquad V_{ci} = 0.05\sqrt{f_c'}b_w d_p + V_d + \frac{V_i M_{cr}}{M_{max}}$$

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SUMMARY

Critical section is at h/2 from face of support:

<u>Case I:</u> $V_u \leq fV_c/2$
No shear reinforcement is required if

<u>Case II:</u> $fV_c/2 \leq V_u \leq fV_c$
Minimum shear reinforcement is required

<u>Case III:</u> $V_u \geq fV_c$
Shear reinforcement is required

$$V_s = \frac{V_u}{\phi} - V_c$$

$$S_{req'd} = \frac{A_v f_y d_p}{V_s}$$

<u>Case IV:</u>
Enlarge the section

$$V_s > 0.66\sqrt{f_c'} b_w d_p \quad in \quad MPa \quad mm\,(results\ in\ N)$$

$$V_s > 8\sqrt{f_c'} b_w d_p \quad in \quad psi \quad and \quad in\,(results\ in\ Ib)$$

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MAXIMUM SPACING OF STIRRUPS
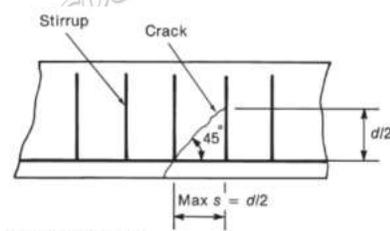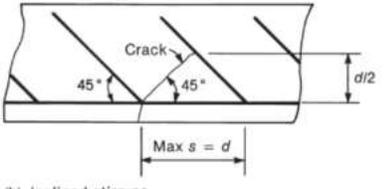
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MAXIMUM SPACING OF SHEAR REINFORCEMENT

9.7.6.2.2 Maximum spacing of legs of shear reinforcement along the length of the member and across the width of the member shall be in accordance with Table 9.7.6.2.2.

Table 9.7.6.2.2—Maximum spacing of legs of shear reinforcement

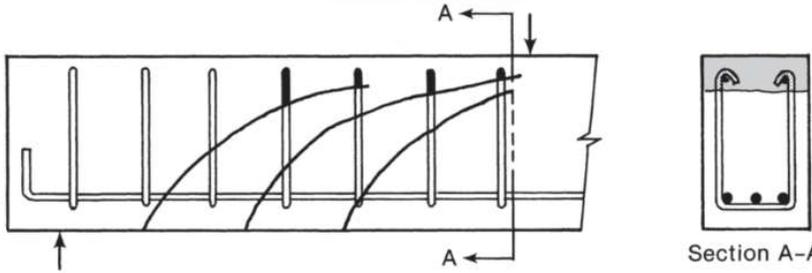| Required V_s		Maximum s, in.			
		Nonprestressed beam		Prestressed beam	
		Along length	Across width	Along length	Across width
$\leq 4\sqrt{f_c'}b_w d$	Lesser of:	$d/2$	d	$3h/4$	$3h/2$
		24 in.			
$> 4\sqrt{f_c'}b_w d$	Lesser of:	$d/4$	$d/2$	$3h/8$	$3h/4$
		12 in.			

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ANCHORAGE OF STIRRUPS

❖ The stirrups should be bent close to the compression and tension surfaces, satisfying the minimum cover.

Section A-A

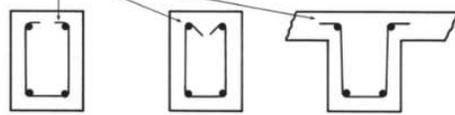Dr. Abdullah Alghossoon · Prestressed Concrete
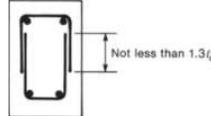
STIRRUP FREE ENDS ANCHORAGE

❖ Each bend of the stirrups should be around a longitudinal bar. The diameter of the longitudinal bar should not be less than the diameter of stirrups.

❖ The ends of the stirrups should be anchored by standard hooks.

Standard stirrup hook, ACI Section 25.3.2, Must enclose a bar, ACI Section 25.7.1.3

(b) Stirrup anchorage requirements for No. 5 and smaller bars as per ACI Sections 25.3.2 and 25.7.1.3. 135° or 90° hooks are preferred.

Not less than $1.3 l_d$

(c) Stirrup anchorage as per ACI Section 25.7.1.7.

(d) Two piece closed stirrup
—Beams with torsion or compression reinforcement. ACI Sections 9.7.6.4 and 25.

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STIRRUP FREE ENDS ANCHORAGE

Stirrups as close to compression and tension faces as cover and spacing requirements permit.

Between anchored ends, each bend shall enclose a longitudinal bar.
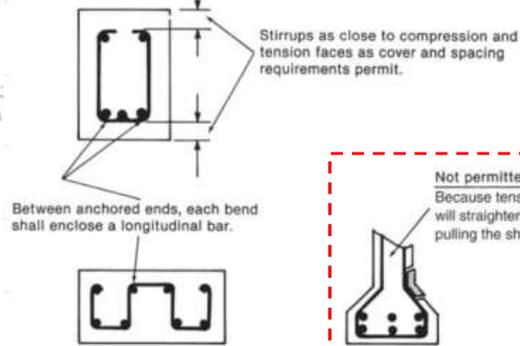
Not permitted
Because tension in the stirrup will straighten the bend, pulling the shaded piece off.

(a) General requirements.
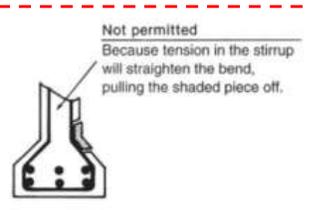
❖ There should not be any bend in a re-entrant corner. In a re-entrant corner, the stirrup under tension has the possibility to straighten, thus breaking the concrete cover.

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CRITICAL SECTION FOR SHEAR: H/2- AWAY (PRESTRESSED) FROM SUPPORT FACE, EXCEPT:
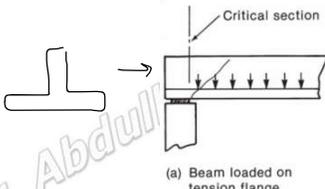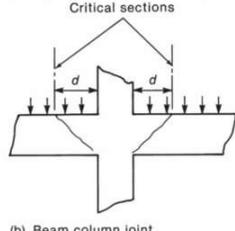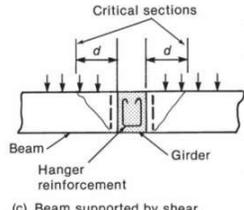
Sec. 9.4.3 ACI 318-19 17
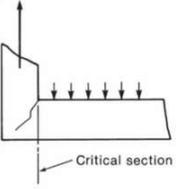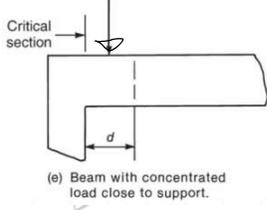
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HORIZONTAL SHEAR AT SERVICE

❖ For flanged section, although the web carries vertical shear, there is horizontal shear stress in the flange.

Max. horizontal Shear Stress, v_h,

$$v_h = \frac{VQ}{I_c b_v} \quad \text{(V: is the service shear force)}$$

Principal Tensile Stress f_t'

$$f_t' = \sqrt{\left(\frac{\bar{f}_c}{2}\right)^2 + v_h^2} - \frac{\bar{f}_c}{2}$$

$\bar{f}_c$:is the concrte stress due to effective prestressing force at location under consideration

AASHTO Limits v_h to 1.1MPa, if exceeded, special vertical ties or dowels are needed
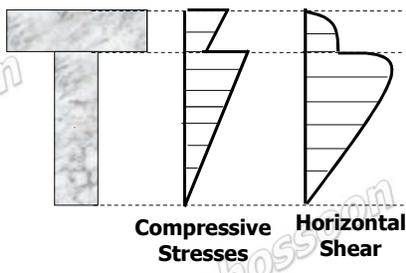
Compressive Stresses   **Horizontal Shear**
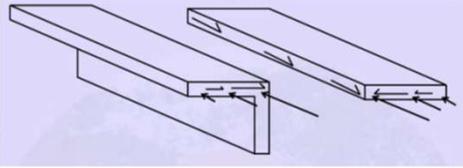
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HORIZONTAL SHEAR AT ULTIMATE

❖ Direct Design Method:

Case I: $V_u \leq V_{nh} = 0.55\,\phi b_v d_{pc}$

    no vertical ties are needed, only roughen the

    precast element surface.

Case II: $V_u \leq V_{nh} = 0.55\,\phi b_v d_{pc}$ for not roughened surface

    $V_u \leq V_{nh} = 3.50\,\phi b_v d_{pc}$ for roughened to 6mm

        amplitude

Use minimum dowels: $\dfrac{A_{vf}}{S} = \text{Larger of}$ $\begin{cases} \dfrac{0.35\,b_w}{f_y} \\[2mm] \dfrac{b_w\sqrt{f_c'}}{16\,f_y} \\[2mm] 600\,mm \end{cases}$

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HORIZONTAL SHEAR AT ULTIMATE

Case III: $V_u > V_{nh} = 3.50\,b_v d_{pc}$

Use shear friction theory, such that: $A_{vf} = \dfrac{V_{nh}}{\mu f_y}$ (This is the total steel area required

along the horizontal shear length (l_{vh}) as shown in the Figure).

Moment diagram $2l_{vh}$

Moment diagram l_{vh} $2l_{vh}$ l_{vh}
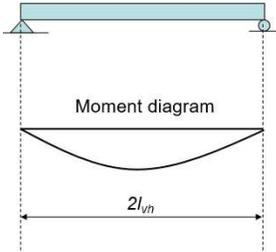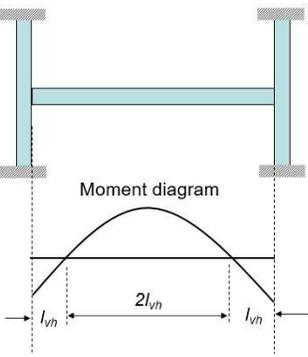
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HORIZONTAL SHEAR AT ULTIMATE

Surface Type	μ
Concrete placed monolithically	1.4λ
Concrete placed against hardened concrete with surface intentionally roughened to 6mm amplitude	1.0λ
Concrete placed against hardened concrete not intentionally roughened	0.6λ
Concrete anchored to as-rolled structural steel by headed studs or by reinforcing bars	0.7λ

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HORIZONTAL SHEAR AT ULTIMATE

where:

$\lambda=1.0$ for normal weight concrete

$\lambda=0.85$ for sand-lightweight concrete

$\lambda=0.75$ for all other lightweight concrete

For all cases: $V_{nh} \leq \begin{cases} 0.2 f_c' b_v l_{vh} \\ 5.50 b_v l_{vh} \end{cases}$

$b_v \equiv$ contact width of precast section web (wich horizontal shear is being calculated)

$d_{pc} \equiv$ depth from compression fiber of the composite section to the centroid cgs

$A_c \equiv$ Area of concrete resisting shear $= b_v d_{pc}$

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SHEAR DESIGN EXAMPLE

❖Design the stirrups of a 10.7m span simply supported prestressed beam with the shown section at midspan. Longitudinal ϕ reinforcement is used to hold the stirrups.

❖The properties of the section are as follows:
  ❖ A_c = 159,000 mm²
  ❖ I = 1.7808 x 10¹⁰ mm⁴
  ❖ A_{ps} = 960 mm²

❖Assume the concrete has f'_c=35MPa, and f_{ps} steel has f_{pu} = 1470MPa and f_{pe} = 860MPa.

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SHEAR DESIGN EXAMPLE

❖The service load including the beam self-weight is 30.2kN/m & the ultimate is 45.3kN/m.

❖The width of the bearings is 400 mm. The clear cover to longitudinal reinforcement is 30 mm.
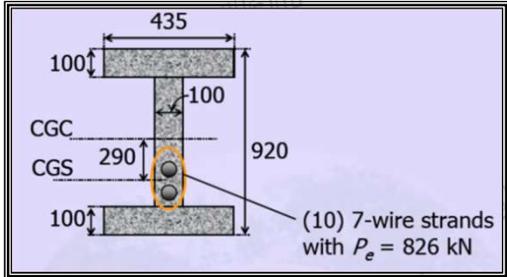
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SHEAR DESIGN EXAMPLE

(1) compute V_u at face of support:

$$V_u = \frac{w_u L}{2} = 243\,kN$$

$$V_n = \frac{V_u}{0.75} = 323\,kN$$

(2) compute V_u at critical section of h/2 from support:

$$V_u\ @\ h\ /\ 2 = 243 - 45.3(\,0.92\ /\ 2\,) = 221\,kN$$

$$(\,V_n\ @\ h\ /\ 2\,)\ /\ \phi = 296\,kN$$

(3) compute V_c at critical section:

$$M_u\ @\ h\ /\ 2 = 243(\,0.46\,) - 45.3(\,0.46\,)^2\ /\ 2$$

$$= 108\,kN.m$$

$$d_p = 750\,mm$$

$$\frac{V_u d_p}{M_u} = 1.56 \quad \therefore USE\ 1.0$$

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SHEAR DESIGN EXAMPLE

Since $f_{pe} > 0.4\,f_{pu}$ use ACI approximate equation

$$V_c = 0.17\sqrt{f_c'}\,b_w d_p < (\,0.05\sqrt{f_c'} + 4.8\frac{V_u d_p}{M_u}\,)b_w d_p < 0.42\sqrt{f_c'}\,b_w d_p$$

$$V_c = (\,0.05\sqrt{35} + 4.8 \times 1\,)(\,100\,)(\,750\,) = 382\,kN$$

$$0.17\sqrt{f_c'}\,b_w d_p = 75\,kN$$

$$0.42\sqrt{f_c'}\,b_w d_p = 186.3\,kN$$

Take $186.3\,kN$

(4) since $V_n > V_c$ Calculate area of shear reinforcement

   assume $\phi10$ closed stirrups, $A_v = 157\,mm^2$

$$V_s = \frac{V_u}{\phi} - V_c = 296 - 186.3 = 109.7\,kN$$

$$S_{req'd} = \frac{A_v f_y d_p}{V_s} = \frac{157\,mm^2 \times 250\,N\ /\ mm^2 \times 750\,mm}{109700\,N} = 268\,mm < min\ S_{min}$$

USE $\phi10$ closed stirrups at S = 250mm

A_v/S = 0.628 mm²/mm

$$S_{max} = smaller\ of \begin{cases} \begin{bmatrix} 600\,mm \\ \frac{3}{4}h \end{bmatrix} if\ V_s \le 0.33\sqrt{f_c'}\,b_w d_p \\ \begin{bmatrix} 300\,mm \\ \frac{3}{8}h \end{bmatrix} if\ V_s > 0.33\sqrt{f_c'}\,b_w d_p \\ \frac{16\,A_v f_y}{b_w\sqrt{f_c'}} = \frac{16 \times 157 \times 250}{100 \times \sqrt{35}} = 1061\,mm \\ \frac{A_v f_y}{0.35\,b_w} = \frac{157 \times 250}{0.35 \times 100} = 1121\,mm \\ \frac{80\,A_v f_y d_p}{A_{ps} f_{pu}}\sqrt{\frac{b_w}{d_p}} = 480\ if\ f_{pe} \ge 0.4\,f_{pu} \end{cases}$$

Since Vs=146

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SHEAR DESIGN EXAMPLE

(5) Dowel Design for Composite Action

service load Horizontal shear stress:

$$Q_f = (435)(100)(460 - 50) = 17{,}835 \times 10^3$$

$$V = 30.2 \times 10.7 / 2 = 162 kN$$

$$\tau_f = \frac{V Q_f}{Ib} = \frac{(162 \times 10^3)(17{,}835 \times 10^3)}{(1.7808 \times 10^{10})(100)} = 1.62 MPa$$

Ultimate load Horizontal Shear:

$$V_u = 242.4 kN$$

$$Provided\ V_{nh} = 3.5\, b_v d_{pc} = 3.5(100)(750) = 262.5 kN$$

$$Req'd\ V_{nh} = \frac{V_u}{\phi} = \frac{242.4}{0.75} = 323 kN > Provided\ V_{nh}$$

$$\therefore \frac{A_{vf}}{l_{vf}} = \frac{V_{nh}}{\mu f_y} = \frac{323 \times 10^3}{1.4 \times 414} = 558 mm^2\ /\!/(l_{vh} = 5.35\,m)$$

$$= 0.104\, mm^2 / mm$$

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SHEAR DESIGN EXAMPLE

Check min. dowels:

$$\frac{A_{vf}}{S} = Larger\ of \begin{cases} \dfrac{0.35\, b_w}{f_y} = 0.085\, mm^2\ /\ mm \\[3mm] \dfrac{b_w \sqrt{f_c'}}{16\, f_y} = 0.089\, mm^2\ /\ mm \quad \textbf{Controls} \end{cases}$$

Assume $\phi 10$ stirrups, $A_v = 157\, mm^2$

$$\therefore S = \frac{157}{0.104} = 1{,}510\, mm > 250\, mm > 0.75\, h$$

USE $\phi 10$ closed stirrup at S=250mm

Extend vertical shear stirrpus to work as dowels

Thus, USE $\phi 10$ closed stirrup at S=250mm

Dr. Abdullah Alghossoon   Prestressed Concrete

SHEER DESIGN EXAMPLE
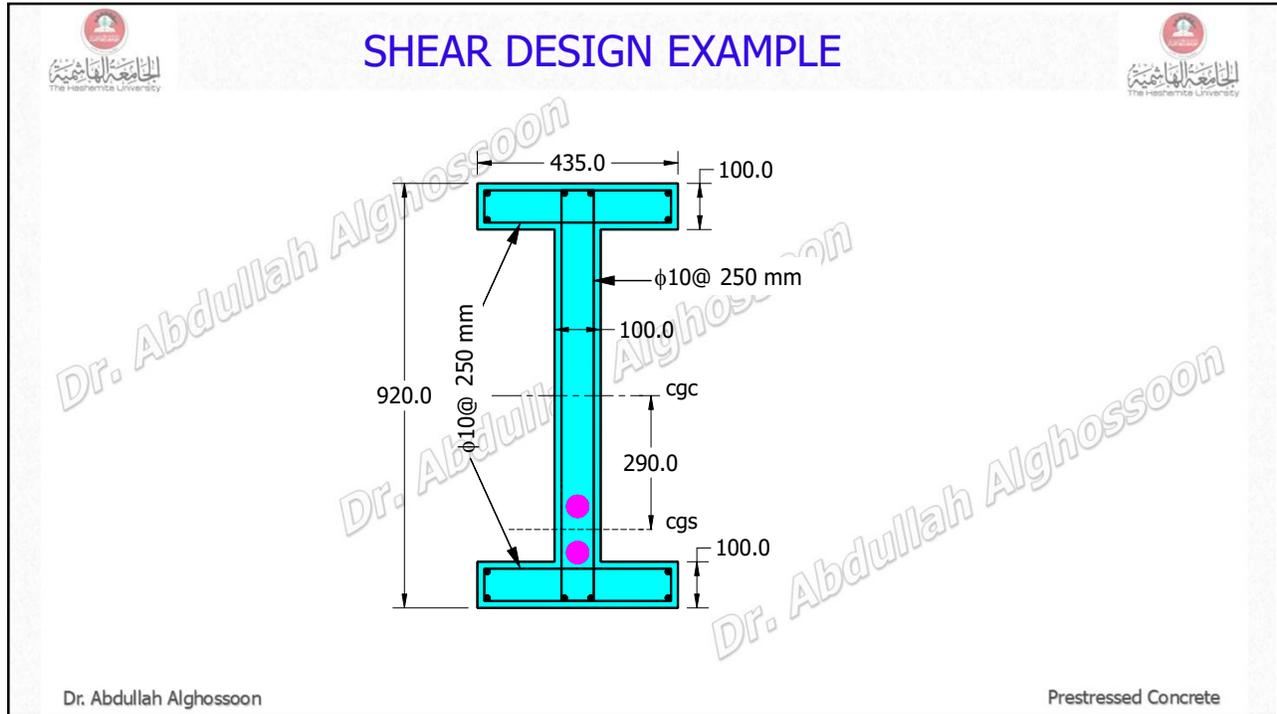
Inputs

Section properties

$$A_c := 449 in^2 \qquad I_c := 22469 in^4 \qquad c_b := 17.77 in \qquad c_t := 6.23 in$$

$$L := 64 ft \qquad A_{ps} := 0.153 in^2 \qquad N_A := 10 \qquad e := 14.77 in$$

(a)

(b)

Material properties

$$f_c := 6000 psi \qquad E_c := 115^{1.5} \left(33 \sqrt{\frac{f_c}{psi}} \right) psi = 3.152 \times 10^6 \cdot psi$$

$$f_{ci} := 0.8 \cdot f_c \qquad f_{ic} := -0.6 \cdot \frac{f_{ci}}{psi} \qquad f_c := -0.45 \cdot \frac{f_c}{psi} \qquad f_{it} := 6 \sqrt{\frac{f_{ci}}{psi}} \qquad f_t := 6 \cdot \sqrt{\frac{f_{ci}}{psi}}$$

$$E_{ps} := 28 \, 10^6 psi \quad f_{pu} := 270000 psi \quad f_{py} := 220000 psi \quad f_{pi} := 0.7 f_{pu} \quad f_e := 150000 psi$$

loading Info:

$$W_D := 359 plf \text{ Dead load} \qquad W_L := 420 plf \text{ Live load} \qquad W_{SD} := 0 plf \qquad \text{Super imposed}$$

$$S_t := \frac{I_c}{c_t} = 3.607 \times 10^3 \cdot in^3 \qquad \text{Top section modulus} \qquad r := \sqrt{\frac{I_c}{A_c}} = 7.1 \cdot in \text{ Radius of gyration}$$

$$S_b := \frac{I_c}{c_b} = 1.264 \times 10^3 \cdot in^3 \qquad \text{Bottom section modulus}$$

$$M_D := W_D \cdot \frac{L^2}{8} = 2.206 \times 10^6 \cdot in \cdot lbf \quad \text{Moment due to dead load}$$

$$M_L := W_L \cdot \frac{L^2}{8} = 2.58 \times 10^6 \cdot in \cdot lbf \quad \text{Moment due to live load}$$

$$M_{SD} := W_{SD} \cdot \frac{L^2}{8} = 0 \cdot in \cdot lbf \qquad \text{Moment due to super imposed load}$$

$$M_T := M_D + M_{SD} + M_L \qquad \text{Total external moment}$$

$$P_i := N_A \cdot A_{ps} \cdot f_{pi} = 1.286 \times 10^6 N \qquad P_e := N_A \cdot A_{ps} \cdot (f_e) = 1.021 \times 10^6 N \qquad \text{Prestressing load (intial and effective)}$$
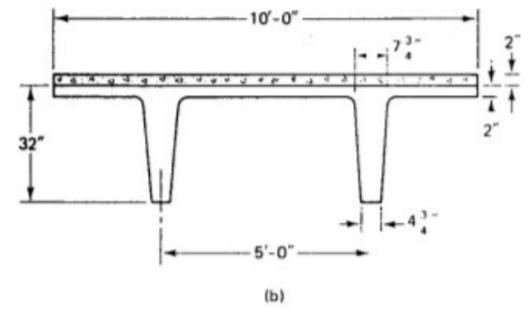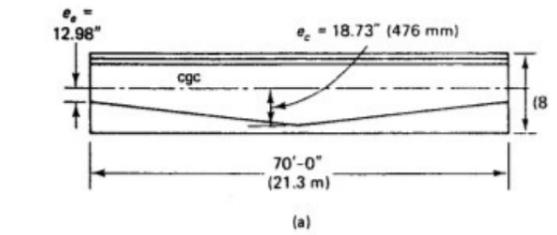
Analysis Methods

1- Basic Method (stress method)

$$f_{ti} := \frac{-P_i}{A_c} \left(1 - \frac{e \cdot c_t}{r^2} \right) - \frac{M_D}{S_t} = -71.4 \cdot psi$$

$$f_{bi} := \frac{-P_i}{A_c} \left(1 + \frac{e \cdot c_b}{r^2} \right) + \frac{M_D}{S_b} = -2.3 \times 10^3 \cdot psi$$

$$f_{tl} := \frac{-P_e}{A_c} \left(1 - \frac{e \cdot c_t}{r^2} \right) - \frac{M_D}{S_t} - \frac{M_{SD}}{S_t} - \frac{M_L}{S_t} = -898.3 \cdot psi$$

$$f_{bl} := \frac{-P_e}{A_c} \left(1 + \frac{e \cdot c_b}{r^2} \right) + \frac{M_D}{S_b} + \frac{M_{SD}}{S_b} + \frac{M_L}{S_b} = 593.3 \cdot psi$$

2- C-line Method (strength method)

$$a := \frac{M_T}{P_e} = 20.9 \cdot in \qquad e_e := a - e = 6.08 \cdot in$$

$$f_{tl} := \frac{-P_e}{A_c}\left(1 + \frac{e_e \cdot c_t}{r^2}\right) = -898.3 \cdot psi$$

$$f_{bl} := \frac{-P_e}{A_c}\left(1 - \frac{e_e \cdot c_b}{r^2}\right) = 593.3 \cdot psi$$

3- Load-Balance Method

$$P := P_e$$

$$W_b := 8 \cdot \frac{P \cdot e}{L^2} = 551.7 \cdot plf$$

$$W_T := W_D + W_{SD} + W_L = 779 \cdot plf$$

$$W_{ub} := W_T - W_b = 227.3 \cdot plf$$

$$M_{ub} := \frac{W_{ub} \cdot L^2}{8} = 1.396 \times 10^6 \cdot in \cdot lbf$$

$$f_{tl} := \frac{-P}{A_c} - \frac{M_{ub}}{S_t} = -898.3 \cdot psi$$

$$f_{bl} := \frac{-P}{A_c} + \frac{M_{ub}}{S_b} = 593.3 \cdot psi$$

How to plot stresses along the beam length

$$M_D(x) := W_D \cdot L \cdot \frac{x}{2} - \frac{W_D \cdot (x)^2}{2} \qquad \text{Moment as a function of x due to dead load}$$

$$M_L(x) := W_L \cdot L \cdot \frac{x}{2} - \frac{W_L \cdot (x)^2}{2} \qquad \text{Moment as a function of x due to live load}$$

$$M_{SD}(x) := W_{SD} \cdot L \cdot \frac{x}{2} - \frac{W_{SD} \cdot (x)^2}{2} \qquad \text{Moment as a function of x due to super imposed load}$$

$$e_x(x) := \frac{-4e}{L^2}x^2 + \frac{4e}{L}x \qquad \text{Eccentricity can be expressed as function (for example constant or polynomial)}$$

$$f_{bi}(x) := \frac{-P_i}{A_c}\left(1 + \frac{e_x(x) \cdot c_b}{r^2}\right) + \frac{M_D(x)}{S_b} \qquad \text{Initial stresses at extreme bottom fibers where only dead load (self weight acting)}$$

$$f_{ti}(x) := \frac{-P_i}{A_c}\left(1 - \frac{e_x(x) \cdot c_t}{r^2}\right) - \frac{M_D(x)}{S_t} \qquad \text{Initial stresses at extreme top fibers, where only dead load (self weight acting)}$$

$$f_{bl}(x) := \frac{-P_e}{A_c}\left(1 + \frac{e_x(x) \cdot c_b}{r^2}\right) + \frac{M_D(x)}{S_b} + \frac{M_{SD}(x)}{S_b} + \frac{M_L(x)}{S_b} \qquad \text{Final stresses at extreme bottom fiber (after losses and including all loads)}$$

$$f_{tl}(x) := \frac{-P_e}{A_c}\left(1 - \frac{e_x(x) \cdot c_t}{r^2}\right) - \frac{M_D(x)}{S_t} - \frac{M_{SD}(x)}{S_t} - \frac{M_L(x)}{S_t} \qquad \text{Final stresses at extreme top fiber (after losses and including all loads)}$$

Extreme fiber stresses along the length of SS beam (Immediately after transfer forces)
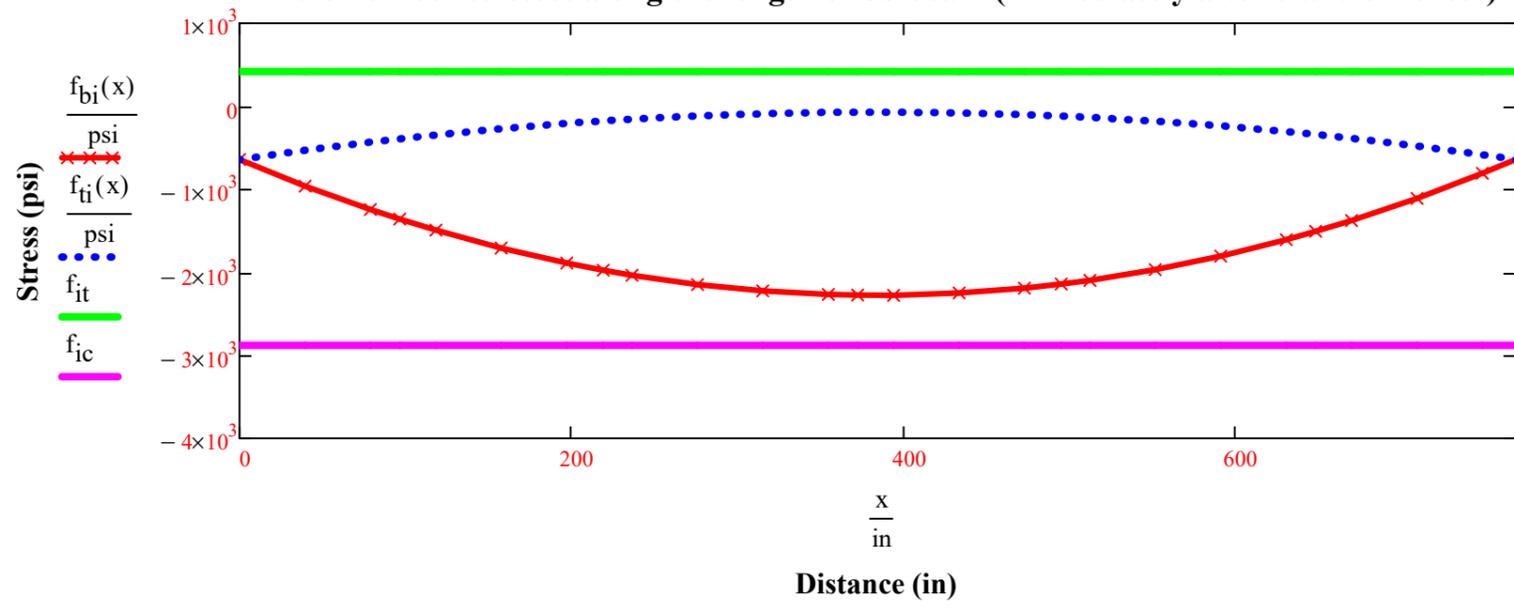
$\dfrac{f_{bi}(x)}{psi}$

$\dfrac{f_{ti}(x)}{psi}$

f_{it}

f_{ic}

Stress (psi)

$\dfrac{x}{in}$

Distance (in)

Extreme fiber stresses along the length of SS beam (After allowances of all losses)

$\dfrac{f_{bl}(x)}{psi}$

$\dfrac{f_{tl}(x)}{psi}$

f_t

f_c

Stress (psi)

$\dfrac{x}{in}$

Distance (in)
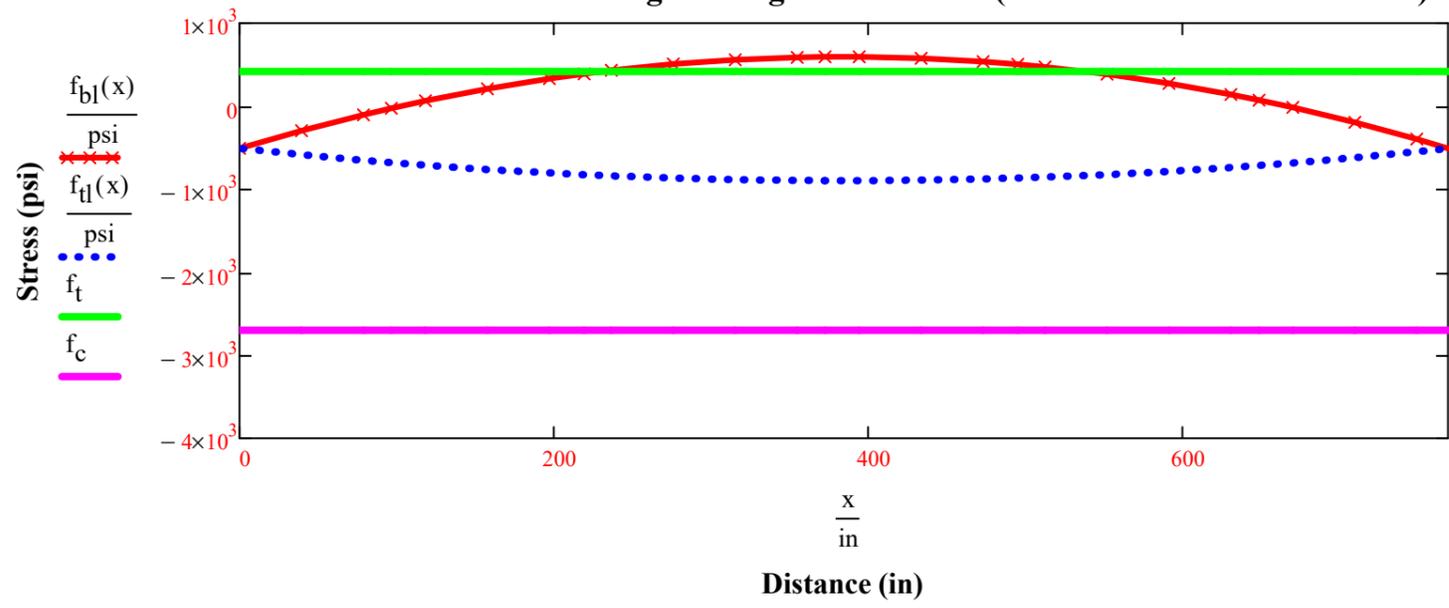
32"
1.2 cm)

RC Beam Design

Geometric and material properties

$w := 10 \dfrac{kN}{m}$ Distributed load $L := 8m$ Span

$b := 500mm$ width $n := 6$ No of rebars $fc := 25MPa$ Compressive strength

$h := 320mm$ Height $d_p := 16mm$ Dia of rebar $F_y := 414MPa$ Yield strength

$d := 0.9h$ Depth $E_s := 200000MPa$ Steel modulus $E_c := 0.043 \cdot 2500^{1.5} \cdot \sqrt{\dfrac{fc}{MPa}} \cdot MPa = 2.688 \times 10^4 \cdot MPa$ Concrete modulus

Note :
1- lb/ft^3 to kg/m^3 multiblied by 16
2- Divided by MPa because it is an imperical equation (units are meaningless)

Analysis

$A_s := n \cdot \dfrac{\pi\, d_p^{\,2}}{4} = 1.2 \times 10^3 \cdot mm^2$ Total area of steel

$A_{smin} := 1\% \cdot b \cdot h = 1.6 \times 10^3 \cdot mm^2$ Recommended min As to mitigate creep effect

$A_{smax} := 4\% \cdot b \cdot h = 6.4 \times 10^3 \cdot mm^2$ Recommended max As to avoid congestion

$M_u := 1.2 \cdot \dfrac{w \cdot L^2}{8} = 96 \cdot kN \cdot m$ Flexural Demand

Strength Design

$$T(F_y) := A_s \cdot F_y \qquad \text{from statics T=C}$$

$$a(fc, F_y) := \frac{T(F_y)}{b \cdot 0.85 \cdot fc} \qquad \text{Depth of whitney block}$$

$$M(fc, F_y) := T(F_y) \cdot \left(d - \frac{a(fc, F_y)}{2} \right) \qquad \text{Flexural Strength} \qquad M(fc, F_y) = 132.1 \cdot kN \cdot m \qquad \text{This is how we get value based on the selected fc and Fy above}$$

$$\beta := 0.85$$

$$f(\varepsilon_s) := \frac{\varepsilon_s}{d - \frac{a(fc, F_y)}{\beta}} - \frac{0.003}{\frac{a(fc, F_y)}{\beta}} \qquad \text{Calculating strain at steel to check tension controlled condition >0.005} \qquad x := 1 \quad \text{This is just an initial guess to solve the equations. You can pick different number}$$

$$\varepsilon_s := root(f(x), x) = 0.013 \qquad \text{is it >0.0055} \qquad \text{solving f(εs)}$$

Serviceability Design

$$I_g := \frac{b \cdot h^3}{12} = 1.365 \times 10^9 \cdot mm^4 \qquad \text{gross inertia}$$

$$I_{cr} := 0.3 I_g \qquad \text{ACI approximate method to calculate cracked moment of inertia for beams}$$

$$f(y) := \frac{(y)^2 \cdot b}{2} - \frac{E_s}{E_c} A_s \cdot (d - y) \qquad \text{centroid of the transformed cracked section} \qquad x := 1mm \quad \text{This is just an initial guess to solve the equations. You can pick different number}$$

$$Y := root(f(x), x) = 0.085 \, m \qquad \text{solving f(y)}$$

$$I_{cr} := \frac{b \cdot (Y)^3}{3} + \frac{E_s}{E_c} \cdot A_s \cdot (d - Y)^2 = 4.723 \times 10^8 \cdot mm^4$$ Calculated cracked moment of inertial $$\frac{I_{cr}}{I_g} = 0.346$$ Ratio of inertia

Note :
1- The calculated Icr is very close to the ACI assumption (Icr = 0.3 Ig). However, keep in mind we usually use the effective moment of inertial Not the cracked (Review RC1 and RC2 for more info)

$$\Delta_{all} := \frac{L}{180} = 44.4 \cdot mm$$ Allowable deflection

$$\frac{\Delta_{all} \cdot 384 \cdot E_c \cdot I_{cr}}{5 \cdot L^4} = 10.6 \cdot \frac{kN}{m}$$ This is how much load we can apply without violating servicability limits. This would limit spans of RC beams even when using high strength materials

This figure demonstrate the effect of f'c on the flexural strength
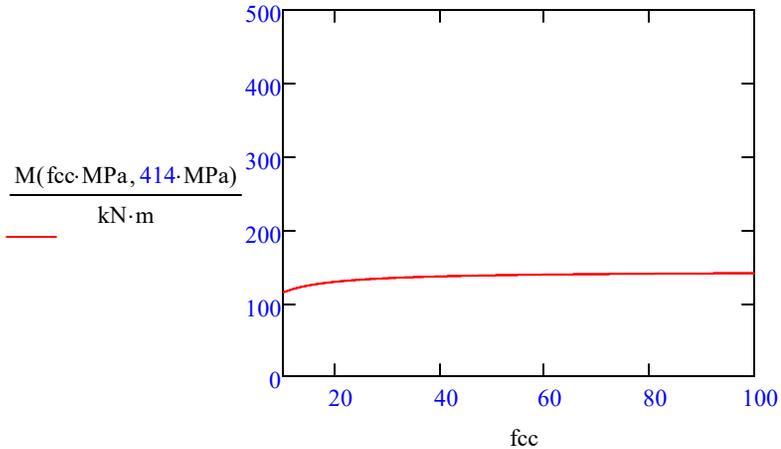
This figure demonstrate the effect of Fy on the flexural strength

$$\frac{M(fcc \cdot MPa, 414 \cdot MPa)}{kN \cdot m}$$

$$\frac{M(25MPa, F_{yy} \cdot MPa)}{kN \cdot m}$$
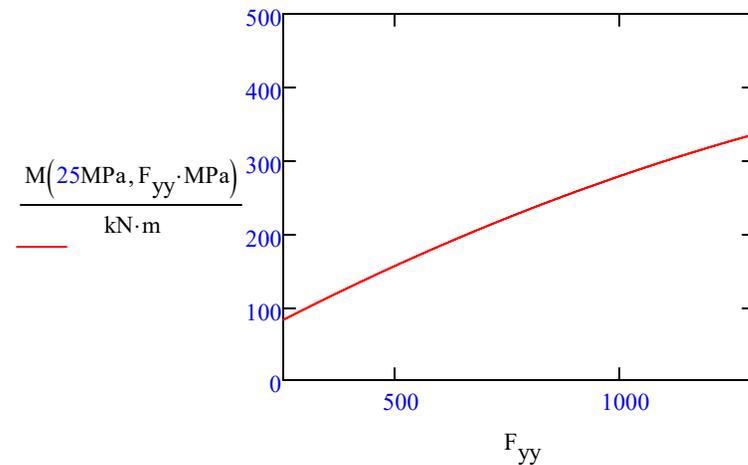
Note :
1- I used fcc and Fyy to avoid confusion with fc and Fy defined above (You can use any name excep the once you define before)

Same figures above but normalized (divided by flexural strength of beam using fc=25MPa and Fy=414MPa)

$$\frac{M(fcc \cdot MPa, 414 \cdot MPa)}{M(25MPa, 414MPa)}$$

$$\frac{M\left(25MPa, F_{yy} \cdot MPa\right)}{M(25MPa, 414MPa)}$$

fcc

F_{yy}
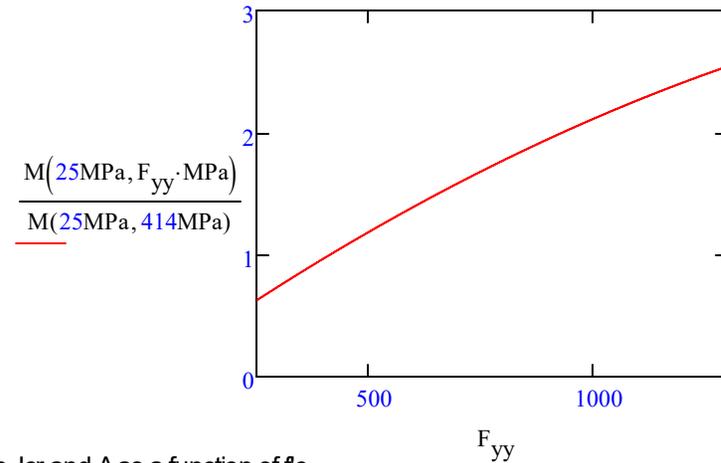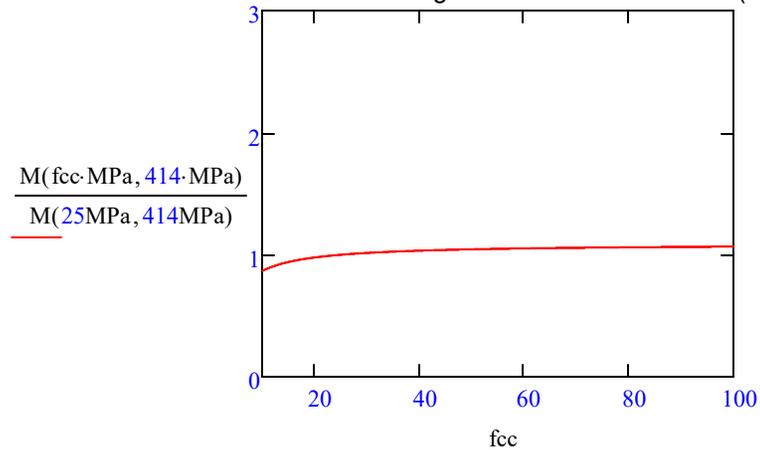
Now Let's see the influnce of fc on SS beam deflection. First we will define Ec, Icr and Δ as a function of f'c

$$E_c(fc) := 0.043 \cdot 2500^{1.5} \cdot \sqrt{\frac{fc}{MPa}} \cdot MPa$$

$$I_{cr}(fc) := \frac{b \cdot (Y)^3}{3} + \frac{E_s}{E_c(fc)} \cdot A_s \cdot (d - Y)^2$$

$$\Delta(fc) := \frac{5 \cdot w \cdot L^4}{384 \, E_c(fc) \cdot I_{cr}(fc)}$$

This figure demonstrate the effect of fc on SS beam deflection

$$\frac{\Delta(fcc \cdot MPa)}{mm}$$

fcc

**Note :
1- f'c has insignificant effect in reducing beam deflection

2- Increasing As reduces the deflection significantly but may violate tension control condition

* To check the ultimate capacity "flexural strength"

1- $M_n = A_{ps} \, f_{ps} \left[d_p - \dfrac{a}{2} \right] \cdot \, ?$

$7 * 54.8 \, mm^2$? $(150 - 25) \, mm$

total height   cover.

2- Let's find $f_{ps} \rightarrow$ use compatibility approach.

f_{pe}: effective f_{pi}: initial

$\varepsilon_1 = \dfrac{f_{pE}}{E_{ps}} = \dfrac{0.9 * 0.9 * 0.7 \, f_{pu}}{193100 \, MPa}$ f_{pj}: jacking $= 0.00546$

$405 \, KN = 1055 * 7 * 54.8$

$\varepsilon_2 = \dfrac{P}{A \, E_c} \left(1 + \dfrac{e^2}{r^2} \right) = 0.0022$

   $50.4 \, mm$

$30300 \, MPa$ $2547 \, mm^2$

$7 * 54.8 \, mm^2$ $1800 \, MPa$ assumed

$\varepsilon_3 \rightarrow$ assume $f_{ps} = 1800 \, MPa \rightarrow a = \dfrac{A_{ps} \, f_{ps}}{0.85 \, f'_c \, b} = 20 \, mm$

$c = \dfrac{a}{\beta} = \dfrac{20 \, mm}{0.81} = 24.4 \, mm$ $35 \, MPa$ $1170 \, mm$

$\varepsilon_3 = \dfrac{d_p - c}{c} * 0.003 = 0.01238$

$\varepsilon_T = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0.01806 \rightarrow f_{ps} = 1800 \, MPa$ ok ✓

③ $M_n = A_{ps} \, f_{ps} \left[d_p - \dfrac{a}{2} \right] = 79.5 \, KN \cdot m$

   $1800 \, MPa$ $\overset{20mm}{\underset{2m}{}}$ $23 * 0.119057$

④ $\underset{Test}{M} = \dfrac{PL}{4} + \dfrac{\omega_{so} \, l^2}{8} \longrightarrow \boxed{P = 156.3 \, KN}$

79.5   ?   2m

— Now remember that there are some differences between assumed values and specimen properties. for example

The cover was 30 mm not 25 mm, so $e = 75 - 30 = 45 \, mm$

and $d_p = 150 - 30 = 120 \, mm$. if you repeat same steps

$M_n = 76 \, KN \cdot m$

and
$P = 149 \, KN$ exactly same number

150 KN maximum load

HOLLOW-CORE
4'-0" x 6"
Normal Weight Concrete
4'-0"

Strand Pattern Designation
76-S
S = straight
Diameter of strand in 16ths
No. of Strand (7)

Safe loads shown include dead load of 10 psf for untopped members and 15 psf for topped members. Remainder is live load. Long-time cambers include superimposed dead load but do not include live load.

Capacity of sections of other configurations are similar. For precise values, see local hollow-core manufacturer.

$f'_c = 5,000 \, psi$
$f_{pu} = 270,000 \, psi$

Section Properties

	Untopped	Topped
A =	187 in.²	283 in.²
I =	763 in.⁴	1,640 in.⁴
y_b =	3.00 in.	4.14 in.
y_t =	3.00 in.	3.86 in.
S_b =	254 in.³	396 in.³
S_t =	254 in.³	425 in.³
DL =	49 psf	295 psf
wt =	195 psf	295 psf
V/S =	1.73 in.	74 psf

Key
444 — Safe superimposed service load, psf
0.1 — Estimated camber at erection, in.
0.2 — Estimated long-time camber, in.

4HC6

Table of safe superimposed service load (psf) and cambers (in.)   No Topping

Strand Designation Code	Span, ft																				
	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
66-S	444	382	333	282	238	203	175	151	131	114	100	88	77	68	59	52	46	40	33	28	
	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.1	0.0	-0.2	-0.4	-0.5	-0.7				
	0.2	0.2	0.2	0.3	0.3	0.3	0.2	0.2	0.2	0.1	0.1	0.0	-0.1	-0.3	-0.5	-0.7	-0.9	-1.2	-1.5	-1.9	
76-S	466	388	328	278	238	205	178	155	136	120	105	93	82	73	65	57	49	42	36	31	
	0.3	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.2	0.1	0.1	0.0	-0.1	-0.3	-0.4	-0.6		
	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.2	0.1	0.0	-0.1	-0.2	-0.4	-0.7	-0.9	-1.2	-1.6	-2.0		
96-S	466	421	386	338	292	263	229	201	177	157	139	124	110	99	88	78	68	60	53	46	
	0.3	0.3	0.3	0.4	0.4	0.4	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.4	0.3	0.3	0.1	0.0	-0.1		
	0.3	0.4	0.4	0.5	0.5	0.5	0.6	0.6	0.6	0.5	0.5	0.4	0.2	0.1	-0.1	-0.3	-0.6	-0.9	-1.3		
87-S	478	433	398	362	322	290	264	240	212	188	167	149	134	119	107	95	85	76	68	60	
	0.3	0.4	0.4	0.5	0.5	0.6	0.6	0.7	0.7	0.7	0.7	0.8	0.8	0.7	0.7	0.6	0.6	0.4	0.3		
	0.4	0.5	0.5	0.6	0.7	0.7	0.7	0.8	0.8	0.8	0.8	0.7	0.7	0.6	0.5	0.3	0.2	0.0	-0.3	-0.6	
97-S	490	445	407	374	346	311	276	242	220	203	186	166	148	133	119	107	96	86	78	70	
	0.4	0.4	0.5	0.5	0.6	0.7	0.7	0.8	0.8	0.9	0.9	0.9	1.0	0.9	0.9	0.8	0.7	0.5	0.3	0.0	
	0.5	0.6	0.6	0.7	0.7	0.8	0.9	1.0	1.0	1.0	1.0	0.9	0.9	0.8	0.7	0.5	0.3	0.1	-0.2		

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$2^\circ \rightarrow 120 \, psf \rightarrow$ live 110 psf $\times 1.6 = 176$
   super $= 10 \, psf \times 1.2 = 12$
   Self-weight $= 49 \, psf \times 1.2 = \dfrac{58.8}{246.8}$

$\phi M_n = 246.8 \, psf * 4 \, ft * (20 \, ft)^2 / 8 = 49360 \, lb.ft$

$\phi M_n = 49360 * 4.448 * 0.3048 = 67 \, KN.$

$M_n = \dfrac{67}{0.9} = 75 \, KN$

$M = \dfrac{PL}{4} \rightarrow P = 75 * 2 = \boxed{150 \, KN}$

$1' \rightarrow 465 \, psf \rightarrow$ live 435 psf $\times 1.6 = 696$
   super $= 10 \, psf \times 1.2 = 12$
   Self-weight $= 49 \, psf \times 1.2 = \dfrac{58.8}{766.8}$

$\phi M_n = 766.8 \, psf * 4 \, ft * (11 \, ft)^2 / 8 = 463914 \, lb.ft$

$\phi M_n = 463914 * 4.448 * 0.3048 = 63 \, KN.$

$M_n = \dfrac{63}{0.9} = 70 \, KN$

$M = \dfrac{PL}{4} \rightarrow P = 75 * 2 = \boxed{140 \, KN}$

whatever you try
it will be around
$\boxed{145 \, KN}$

PRESTRESSED CONCRETE
-Deflection of PS-

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- Dr. Abdullah Alghossoon
- Abdullahm_ab@hu.edu.jo
- Spring 2025
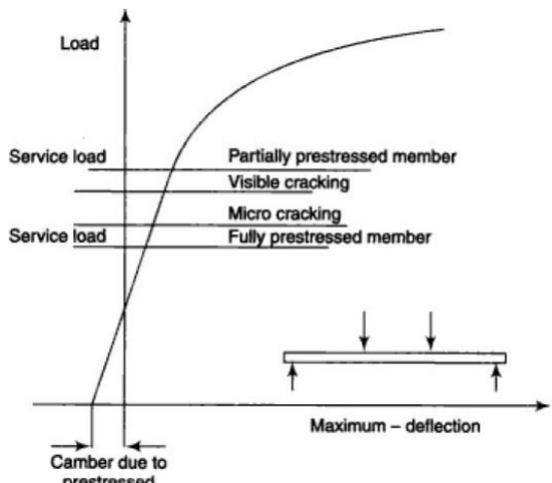
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DEFLECTION OF PS MEMBER

❖Philosophy of design – "Limit State Approach" adopted by the design code requires a proper knowledge of the behavior of structural concrete members at the multiple limit states, of which deflection forms an important criterion for the safety of the structures.

❖Structural members should be designed to have adequate stiffness to limit deflections, which may adversely affect the strength or serviceability of the structure at working loads.

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1

DEFLECTION OF PS MEMBER

❖Why Control On Deflections Is Required????

1. Structural analysis basically based on small deflection assumption.
2. Too much bending in main structural parts not only looks bad but can also make the floor unusable.
3. Large movements due to changing loads or vibrations may make people feel uncomfortable.
4. Too much deflection can damage finishes, walls, and connected structures.

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DEFLECTION OF PS MEMBER

❖FACTORS INFUENCING DEFLECTIONS

1. Imposed load and self-weight
2. Magnitude of the prestressing force
3. Cable profile
4. Second moment of area of cross-section
5. Modulus of elasticity of concrete
6. Shrinkage, creep and relaxation of steel stress
7. Span of the member
8. Fixity conditions

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DEFLECTION OF PS MEMBER

Computation of Deflection

Pre-cracking stage

Short term/ Instantaneous deflection

Use Gross moment of inertia

Post-cracking stage

Long-term deflection

Use Effective moment of inertia

❖ Long-term deflections under continuous loads are estimated using empirical methods. These methods either apply an effective (long-term) modulus of elasticity or adjust short-term deflections with appropriate factors.
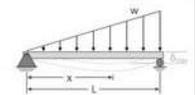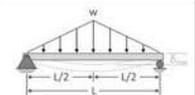
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DEFLECTION OF PS MEMBER

❖ Short term deflection of uncracked member

❖ short-term deflections are governed by the bending movement distribution along the span and flexural rigidity of the members.

❖ movement area theorem:

The vertical deviation between point A and point B on an elastic care equals to the moment of area under the *M/EI* diagram between these two points.

$$y_b = \int_A^B \frac{M}{EI} x \, dx$$

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DEFLECTION OF PS MEMBER

❖ Effect of Tendon Profile on Deflections

- In most prestressed beams, tendons are placed with eccentricity toward the bottom to counteract sagging due to gravity loads.
- As a result, the beam initially deflects upward (camber) when prestress is applied or transferred.
- Since the bending moment at any section depends on the prestressing force and tendon eccentricity, the tendon profile directly reflects the shape of the bending moment diagram (BMD).
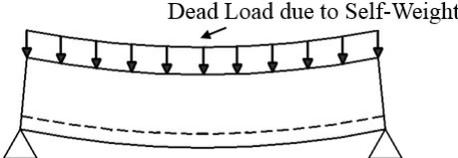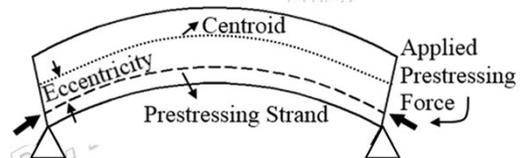
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DEFLECTION OF PS MEMBER

❖ Deflection due to typical loading conditions

Beam and load cases	Maximum Beam Deflection	Beam and load cases	Maximum Beam Deflection
	$\delta_{max} = \dfrac{PL^3}{48EI}$		$\delta_{max} = \dfrac{0.00652wL^4}{EI}$
	$\delta_{max} = \dfrac{Pb\left(3L^2 - 4b^2\right)}{48EI}$		$\delta_{max} = \dfrac{wL^4}{120EI}$
	$\delta_{max} = \dfrac{5wL^4}{384EI}$		$\delta_{max} = \dfrac{ML^2}{9\sqrt{3}EI}$
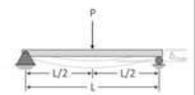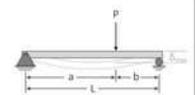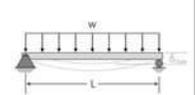
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DEFLECTION OF PS MEMBER

❖ Straight Tendons

$$y_c = -\frac{\left(Pe\dfrac{L}{2}\right)\dfrac{L}{4}}{EI} = \frac{PeL^2}{8EI}$$

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DEFLECTION OF PS MEMBER

❖ Trapezoidal Tendons

$$y_c = -\frac{Pe}{EI}\left[l_2\left(l_1 + \frac{l_2}{2}\right) + \left(\frac{l_1}{2}\right)\frac{2}{3}l_1\right]$$

$$= -\frac{Pe}{6EI}\left[2l_1^2 + 6l_1l_2 + 3l_2^2\right]$$

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DEFLECTION OF PS MEMBER

❖ Parabolic Tendons (concentric anchor)

$$y_c = -\left(\frac{5PeL^2}{48EI}\right)$$

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DEFLECTION OF PS MEMBER

❖ Parabolic Tendons (eccentric anchor)

$$y_c = -\left(\frac{5PL^2}{48EI}(e_1 + e_2)\right) + \left(\frac{Pe_2L^2}{8EI}\right)$$

$$= \frac{PL^2}{48EI}(-5e_1 + e_2)$$
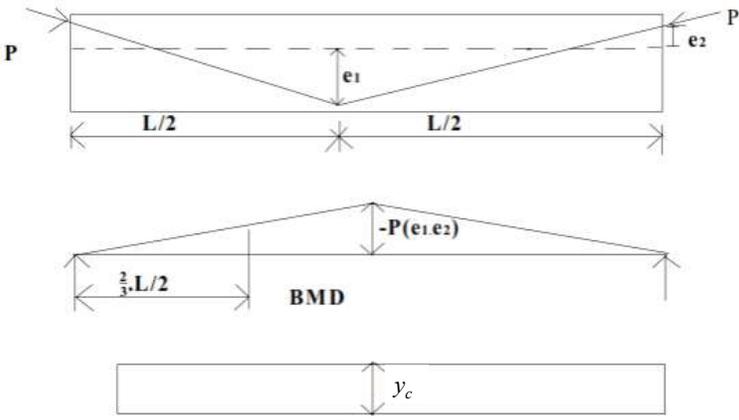
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DEFLECTION OF PS MEMBER

❖ Sloping Tendons (eccentric anchor)

$$y_c = -\left(\frac{PL^2}{12EI}(e_1 + e_2)\right) + \left(\frac{Pe_2L^2}{8EI}\right)$$

$$= \frac{PL^2}{24EI}(-2e_1 + e_2)$$

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DEFLECTION OF PS MEMBER

❖ Moment of Inertia

- Class U: $f_t \leq 0.62\sqrt{f_c'}$
  - Use gross section moment of inertia, I_g

- Class T: $0.62\sqrt{f_c'} \leq f_t \leq \sqrt{f_c'}$
  - Use effective moment of inertia, I_e

- Class C: $f_t > \sqrt{f_c'}$
  - Use effective moment of inertia, I_e

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DEFLECTION OF PS MEMBER

❖ Moment of Inertia

$$I_e = I_{cr} + \left(\frac{M_{cr}}{M_a}\right)^3 (I_g - I_{cr}) \le I_g$$

$$\frac{M_{cr}}{M_a} = 1 - \left(\frac{f_{tl} - f_r}{f_L}\right)$$

M_a : *Max. service unfactored live load moment*

f_{tl} : *Total service load concrete stress*

f_r : *Modulus of rupture*

f_L : *Service live load concrete stress*

The PCI Approach:

$$I_{cr} = \left(n_p A_{ps} d_p^2 + n_s A_s d^2\right)\left(1 - 1.6\sqrt{n_p \rho_p + n_s \rho_s}\right)$$

$$n_p = \frac{E_{ps}}{E_c}$$

$$n_s = \frac{E_s}{E_c}$$

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DEFLECTION OF PS MEMBER

❖ Estimation of long-term deflection

- Calculations should consider not only the increased deflections due to flexural stresses, but also the time-dependent deflections resulting from time-dependent shortening of the flexural member.

- Deformations in prestressed members change over time due to concrete creep, shrinkage, and the relaxation of stress in the steel. However, in practical cases, the change of stress is small

- The long term deflection can be computed as a function of the instantaneous deflection.

- For simplicity, creep is induced by the average prestress acting over the given time.

$$\Delta_{Final} = -\Delta_{pe} - \left(\frac{\Delta_{pi} + \Delta_{pe}}{2}\right)C_u$$

$$\Delta_{pe} = \frac{P_e}{P_i}\Delta_i$$

Long-term deflection due to prestressing

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DEFLECTION OF PS MEMBER

❖ Estimation of long-term deflection

$$\Delta_{Final} = -\Delta_{pe} + \left(\frac{\Delta_{pi} + \Delta_{pe}}{2}\right)C_u + (1 + C_u)(\Delta_D + \Delta_{SD}) + \Delta_L$$

Long-term deflection due to prestressing, self weight, superimposed dead load and live load

- Alternatively, use the long –term multipliers from PCI (Table 4.8).

PCI Deflection limitations

Table 4.8.1   Maximum permissible computed deflections

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to non-structural elements likely to be damaged by large deflections	Immediate deflection due to live load	$\ell/180^a$
Floors not supporting or attached to non-structural elements likely to be damaged by large deflections	Immediate deflection due to live load	$\ell/360$
Roof or floor construction supporting or attached to non-structural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of non-structural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)^c	$\ell/480^b$
Roof or floor construction supporting or attached to non-structural elements not likely to be damaged by large deflections		$\ell/240^d$

ACI 318-25 Deflection limitations

Table 24.2.2—Maximum permissible calculated deflections

Member	Condition		Deflection to be considered	Deflection limitation[1]
Flat roofs	Not supporting or attached to nonstructural elements likely to be damaged by large deflections		Immediate deflection due to maximum of L_r, S, and R	$\ell/180^{[2]}$
Floors			Immediate deflection due to L	$\ell/360$
Roof or floors	Supporting or attached to nonstructural elements	Likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements, which is the sum of the time-dependent deflection due to all sustained loads and the immediate deflection due to any additional live load[5]	$\ell/480^{[4]}$
		Not likely to be damaged by large deflections		$\ell/240^{[5]}$

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PRESTRESSED CONCRETE

-End Zone Design of PS-

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- Department of Civil Engineering
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- Spring 2025

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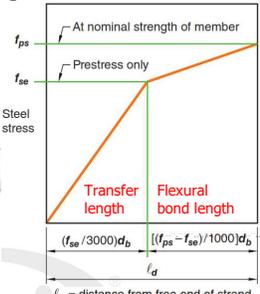Dr. Abdullah Alghossoon   Prestressed Concrete

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ANCHORAGE DESIGN OF PS

❖ End zone and development length (Pre-tensioned members)

- The bond between the prestressing tendon and the surrounding concrete over a certain length helps gradually transfer the prestressing force from the tendon into the concrete, moving it away from the end block and toward the middle of the span.

- Meaning that there must be a minimum to required (beyond the critical section) to transferee prestressing forces to the member, which we called the "development length".

- According to the (ACI318-25, section 25.4.8.1) The development length for pretensioned seven wire strands in tension shall be:

Value of stresses at nominal strength   Effective prestressing

$$Min.\ l_d = \left(\frac{f_{pe}}{20.7}\right)d_b + \left(\frac{f_{ps} - f_{pe}}{6.9}\right)d_b \quad ; MPa,\ mm$$

$$Min.\ l_d = \left(\frac{f_{pe}}{3000}\right)d_b + \left(\frac{f_{ps} - f_{pe}}{1000}\right)d_b \quad ; psi,\ in$$

$$\approx 150d_p - 200d_p$$

- **Twice** the development length is required for strand not bonded through to the end of a member unless members designed for zero tension in the concrete under service load conditions

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ANCHORAGE DESIGN OF PS

❖ End zone and development length (Pre-tensioned members)

- The Development length or deformed bars and deformed wires in compression shall be the greater of:

$$l_{dc} = \begin{cases} 8\,in \\ \dfrac{f_y \psi_r}{50\lambda\sqrt{f_c'}}\,d_p \\ 0.0003 f_y \psi_r d_p \end{cases}$$

Imperial Units !!

Table 25.4.9.3—Modification factors for deformed bars and wires in compression

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Confining reinforcement ψ_r	Reinforcement enclosed within (1), (2), (3), or (4): (1) a spiral (2) a circular continuously wound tie with $d_b \geq 1/4$ in. and pitch $\leq$ 4 in. (3) No. 4 bar or D20 wire ties in accordance with 25.7.2 spaced $\leq$ 4 in. on center (4) hoops in accordance with 25.7.4 spaced $\leq$ 4 in. on center	0.75
	Other	1.0

- The development length can be reduced if the provided reinforcement area is more than required. However, there are too many cases where this is not allowed (check ACI 25.4.10.2)

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ANCHORAGE DESIGN OF PS

❖ Spalling and bursting cracks near the end zone

- When the prestressing force is released, it is initially applied locally at the strand locations. But to reach equilibrium, this force must distribute (spread out) into the full concrete cross-section over a short length near the member end, known as the anchorage or end zone.

- Shear lag and localized bending mechanisms can cause very high principal tension stresses in the concrete in the development length region at the girder ends, particularly with the slender cross sections
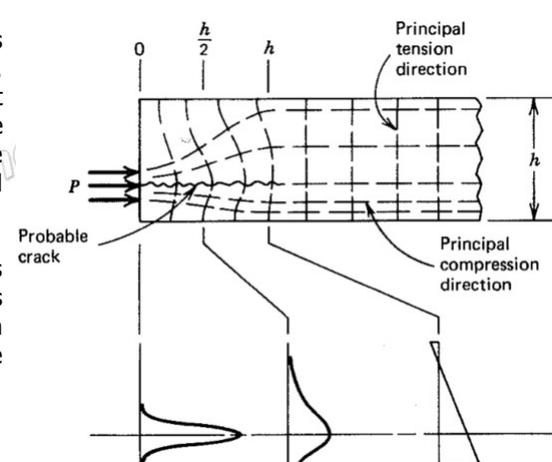
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ANCHORAGE DESIGN OF PS

❖ Spalling and bursting cracks near the end zone (Pre-tensioned)

▪ Shear lag and localized bending. Both of these mechanisms can cause very high principal tension stresses in the concrete in the development length region at the girder ends, particularly with the slender cross sections

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ANCHORAGE DESIGN OF PS

❖ Reinforcement against spalling and bursting cracks near the end zone (Pre-tensioned)
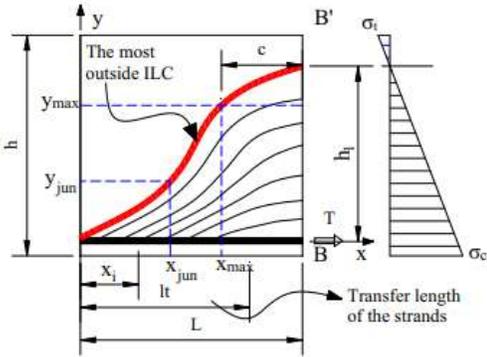
Prestressing force at transfer

Beam height

Total area of stirrups

$$A_t = 0.021 \frac{P_i h}{f_s l_t}$$

<138MPa (half of f_y to control cracking)

Length of transfer zone

PCI equation: a laboratory-based equations (empirical) but it works for both system of units SI and US

ANCHORAGE DESIGN OF PS

❖ End zone of Post-tensioned members

- Length of anchorage zone is at which the PS force transfer into a linear distribution across the section depth and according to St. Venant's principle is equal to 'h'.

- This zone consists of:
  - General zone: its length along span is 'h'
  - Local zone: it's the insert prism of concrete surrounding & immediately ahead of the anchorage device.

▪ Method of Analysis:

  1- Linear elastic (such as FEM)

  2-Equilibrium-based plasticity such as Strut-and-Tie model.

  3- Approximate method
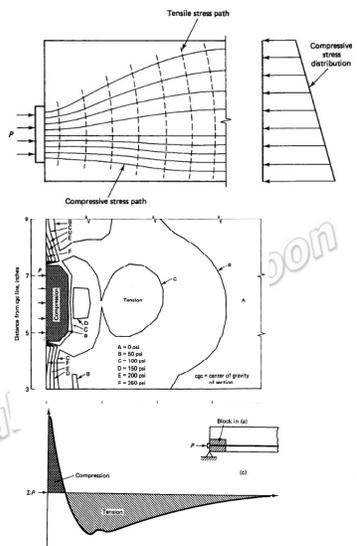
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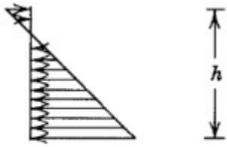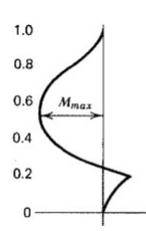
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ANCHORAGE DESIGN OF PS

❖ End zone of Post-tensioned members
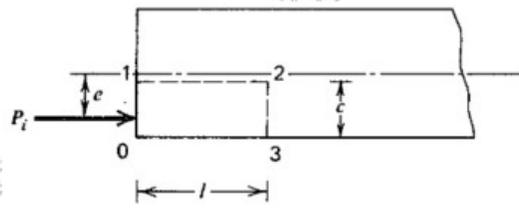
Approximately $l_m ≈ 2/3h$ to $7/8h$
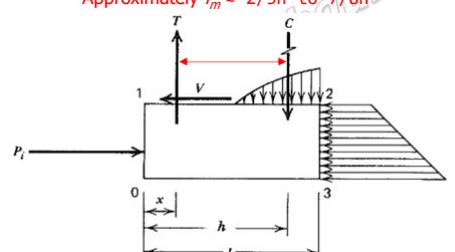
$$T = \frac{M_{max}}{h - x} = \frac{M_{max}}{l_m}$$

$$A = \frac{T}{f_s}$$

<138MPa (half of f_y to control cracking)

Moment on face 12

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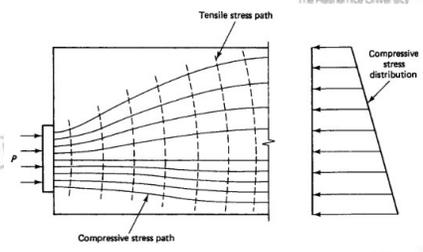
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ANCHORAGE DESIGN OF PS

❖ Local zone

- In addition to Vertical tensile stresses in the bursting and spelling zone. End zone distress may be caused by the high concentration of longitudinal compression under the bearing plates of the anchorages.

- The bearing stress on the concrete caused by post tensioning anchorages according to ACI code commentary should not exceed the following:

$$f_b = 0.8 f'_{ci} \sqrt{\frac{A_2}{A_1}} - 0.2 \le 1.25 f'_{ci} \quad \text{and} \quad f_b = 0.6 f'_c \sqrt{\frac{A_2}{A_1}} \le f'_c$$

- Supplementary reinforcement under anchor plates is needed to resist diagonal splitting cracks that radiate from the plate corners and may cause spalling. A continuous spiral is the most effective, acting like confinement steel to enhance local strength and resist splitting. The spiral should have a diameter roughly equal to the plate diagonal or as large as cover allows It should start near the anchor plate and extend at least twice the plate's side length.

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ANCHORAGE DESIGN OF PS

❖ End zone of Post-tensioned members (Example)

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