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سلايدات

خرسانة مسلحة 1

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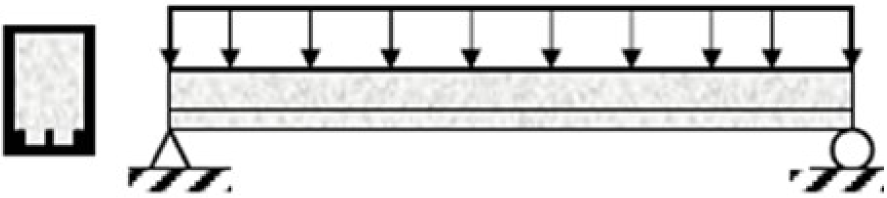
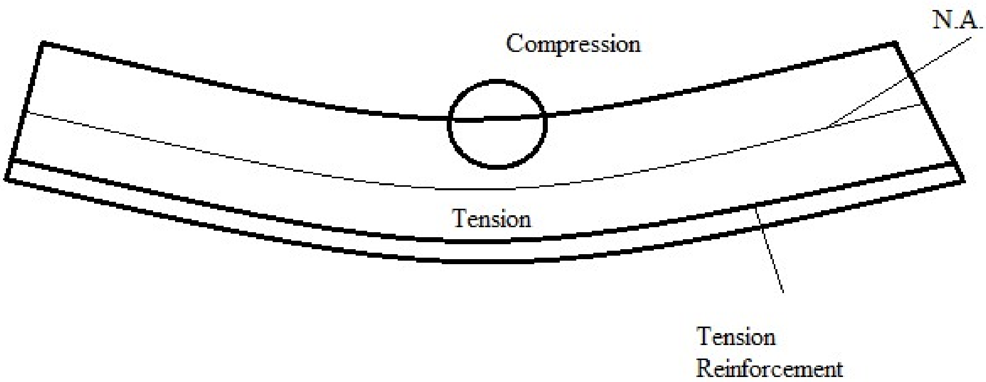
CHAPTER ONE

INTRODUCTION

Concrete = Cement + Water + Aggregate (Coarse and Fine = 70-75%) with or without additives (admixtures, fibers).

Concrete is strong in compression (high compressive strength), and weak in tension (low tensile strength).





Advantages of RC:

1. Relatively low cost material (*material, labor, time*)
2. Fire resistance (1 – 3hrs) fire rating without special fire proofing. (*steel structures or timber buildings must be fire proofed to attain similar fire rating*)
3. Suitability of material for architectural and structural functions. (*Concrete could be placed in a plastic condition and given the desired shape and texture by means of forms and finishing techniques. Size and shape are governed by the designer not by the availability of standard manufactured members*)
4. Rigidity. (*vibrations are rarely a problem*)
5. Low maintenance. (*especially if the proper design, mix design and material are used for the proper atmosphere*)
6. Availability of materials.

Disadvantages of RC:

1. Low tensile strength of concrete. (*compressive strength is 8 to 10 time the tensile strength*)
2. Forms and Shoring. (*extra labor and material not required by other forms of construction*)
3. Relatively low strength per unit weight or volume. (*concrete structures require larger volume and member sections when compared to steel structures*);

   Concrete compressive strength ≅ (5-10%) steel strength.

   Concrete unit weight ≅ 30% Steel unit weight
4. Time – Dependent volume changes

   Creep: deflection under sustained loads, deflection will tend to increase, possibly doubling, due to creep.

   Drying shrinkage: caused by hydration during the strength reactions.

Importance of Steel:

1. Steel has good bond with the concrete.
2. Concrete and steel have nearly equal coefficients of thermal expansion.
3. Good dense concrete protects steel from corrosion / rusting.

Sources of Uncertainty:

1. Actual load magnitude and distribution may differ from those assumed in the design.
2. Assumptions and simplifications in the analysis may result in different internal forces.
3. Actual behavior maybe different.
4. Actual member dimensions may differ from those specified in design.
5. Reinforcement may not be in its proper position.
6. Actual material strength may be different from that specified in design.

Safety Philosophy:

$$\Phi S_n \geq \gamma Q_d$$

ΦS_n = resistance, reduced nominal strength

γQ_d = load effects, ultimate load, factored load

S_n = nominal strength

Q_d = design load

Φ = strength reduction factor (*always less than one*)

γ = over load factor (*most of the time more than one*)

Load factors and load combinations from ACI – Code:

U = 1.2DL + 1.6LL (*DL, LL = principal variable loads*)

U = 0.9DL + 1.6WL (*DL = companion action variable loads*) dead load stabilize overturning and sliding

U ≅ Ultimate load (*sum of factored loads*)

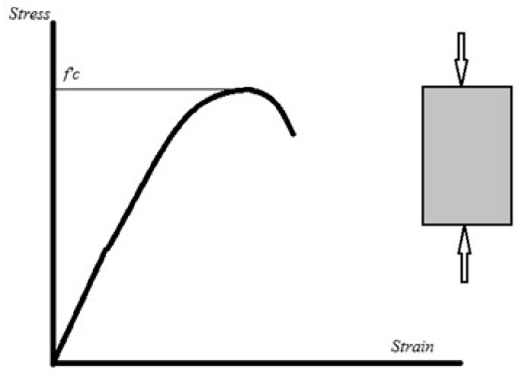
CHAPTER TWO

MATERIALS

Concrete is a composite material composed of cement, aggregate and water.

Mechanism of Failure in Concrete Loaded in Compression:

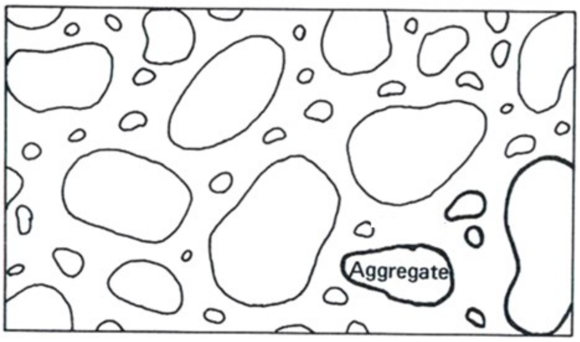
Stages of microcracking and failure in concrete subjected to uniaxial compression loading:



1. Shrinkage of the paste during hydration (***No – load bond cracks***)

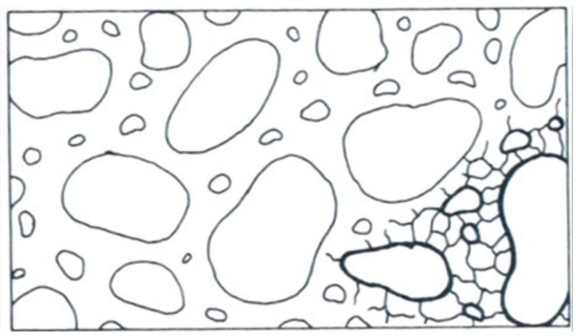
   Stress – strain curve remains linear up to 30% of the compressive strength

2. At stresses up to 30 to 40% of the compressive strength (***Bond cracks***)



The beginning of stage 3 is referred to as the discontinuity limit.

3. At stresses up to 50 to 60% of the compressive strength (***Mortar/Matrix cracks***)

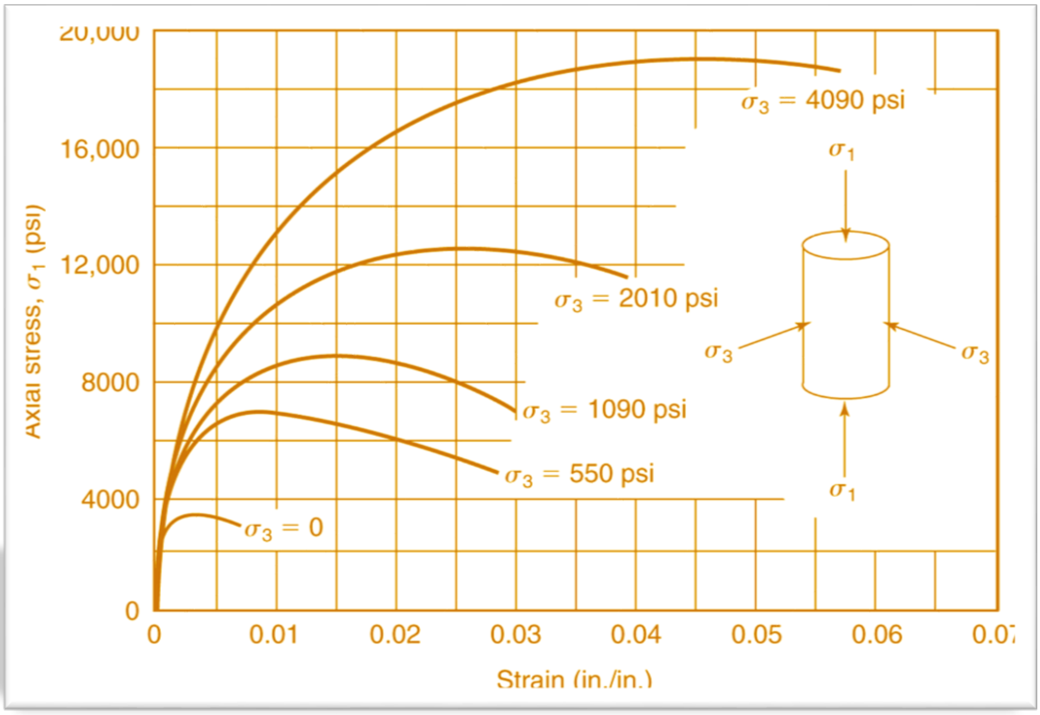


At this stage cracking increases only when increasing the load

4. At 75% of the ultimate load, the number of mortar cracks increases leaving fewer undamaged portions of the specimen to resist/carry the load. Stress at this stage is referred to as the ***Critical Stress*** because it is the onset of the ***Unstable crack propagation***. The ***Stable crack propagation*** is at stresses below the critical stress.

Triaxial Loading:

Composed of both axial and lateral (confining stress).



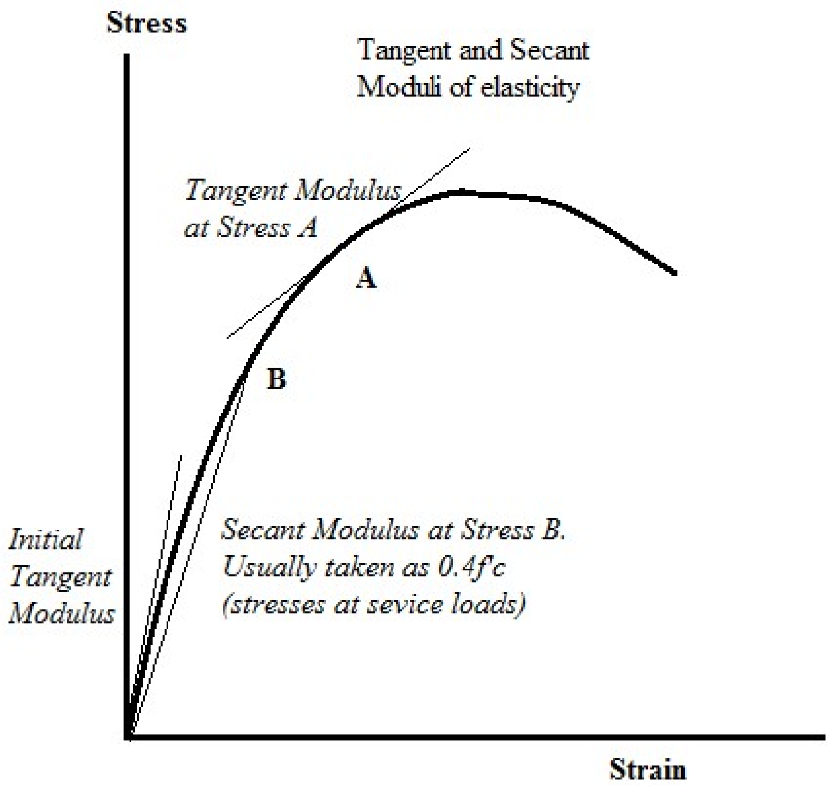
The higher the confining pressure the higher the compressive strength

Stress – Strain Curves for Concrete:

Three ways of defining the modulus of elasticity are illustrated in the figure below. Usually, the slope of the initial portion of the stress – strain curve defines the modulus of elasticity.

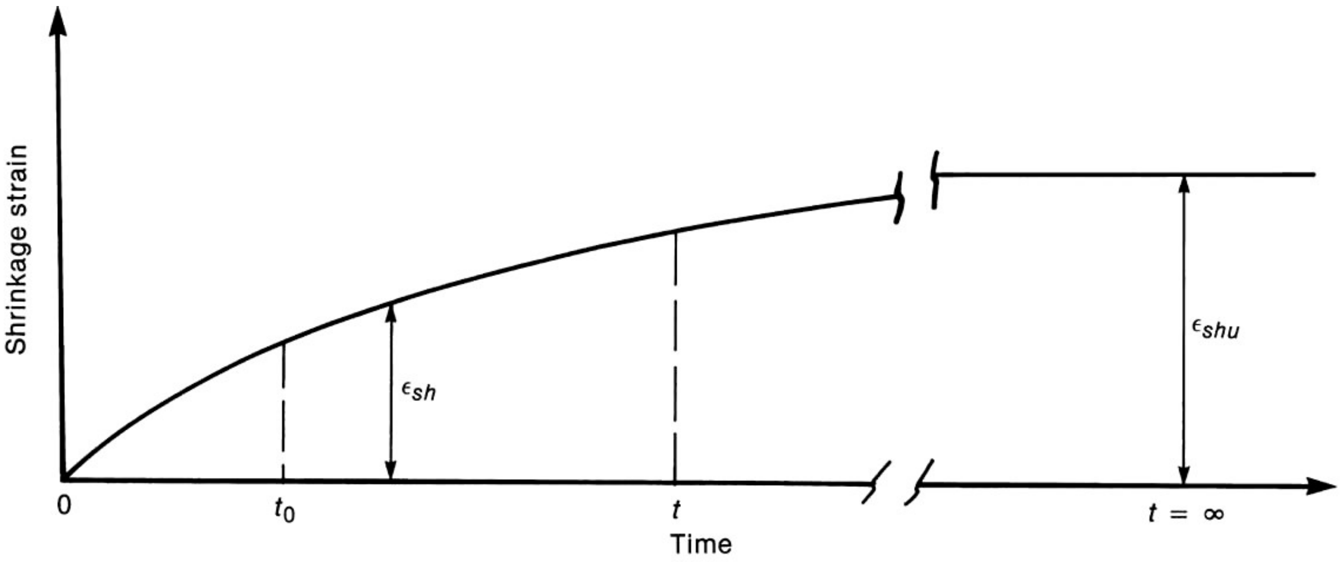
$E_c = 4700\sqrt{f_c^{'}}$, for normal weight concrete

Tangent and secant moduli of elasticity



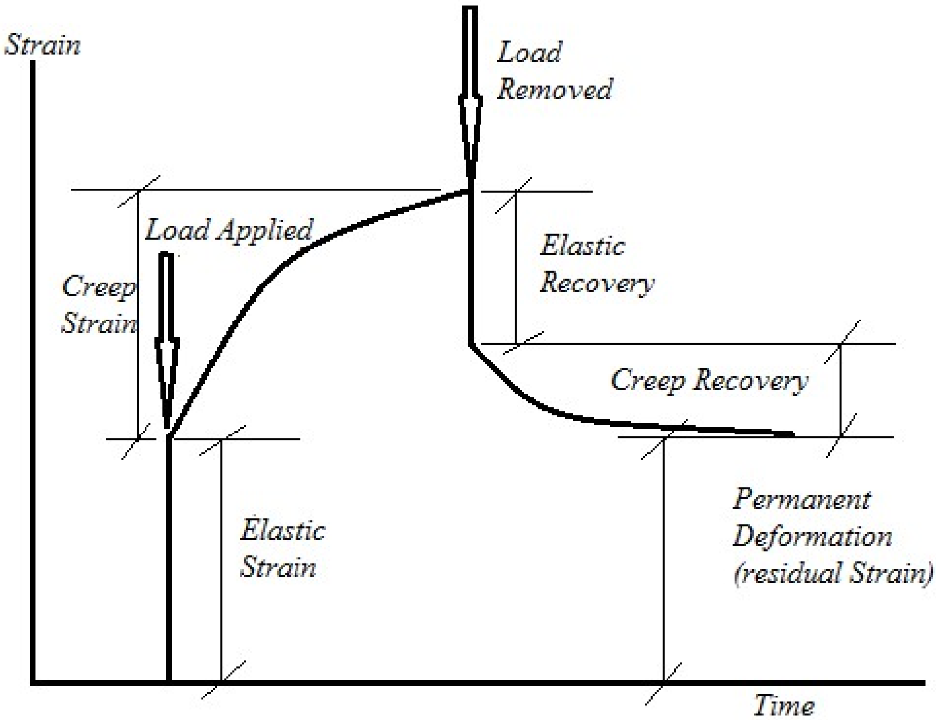
Time – Dependent Volume Changes:

Drying Shrinkage or simply Shrinkage: is the decrease in the volume of concrete during hardening and drying under constant temperature. The amount of shrinkage increases with time. The figure below shows shrinkage of unloaded specimen.

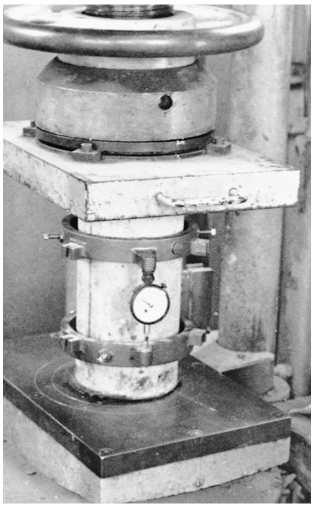


(a) Shrinkage of an unloaded specimen.

Creep: Deformations under sustained load. Figure below shows the instantaneous elastic and creep strains due to loading and unloading.



Standard Compression & Tension Tests:



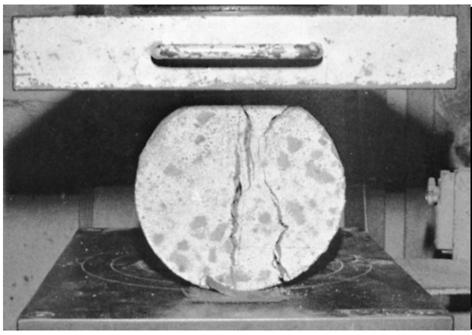
Standard Compression Test: In which cubical and cylindrical plain concrete samples are loaded in uniaxial compression

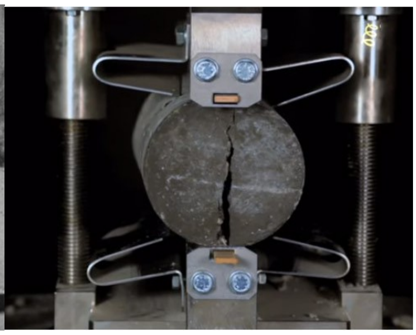
Modulus of Rupture (Flexural Test):

Plain concrete beam samples are loaded in flexure at the third points until it fails due to cracking on the tension face. The flexural tensile strength or modulus of rupture, f_r, is calculated. $f_r = 0.62\sqrt{f_c^{'}}$

Split Cylinder Test:

Plain concrete cylinder samples are placed on their side and loaded in compression along a diameter. The splitting tensile strength, fct, is calculated. $f_{ct} = 0.53\sqrt{f_c^{'}}$

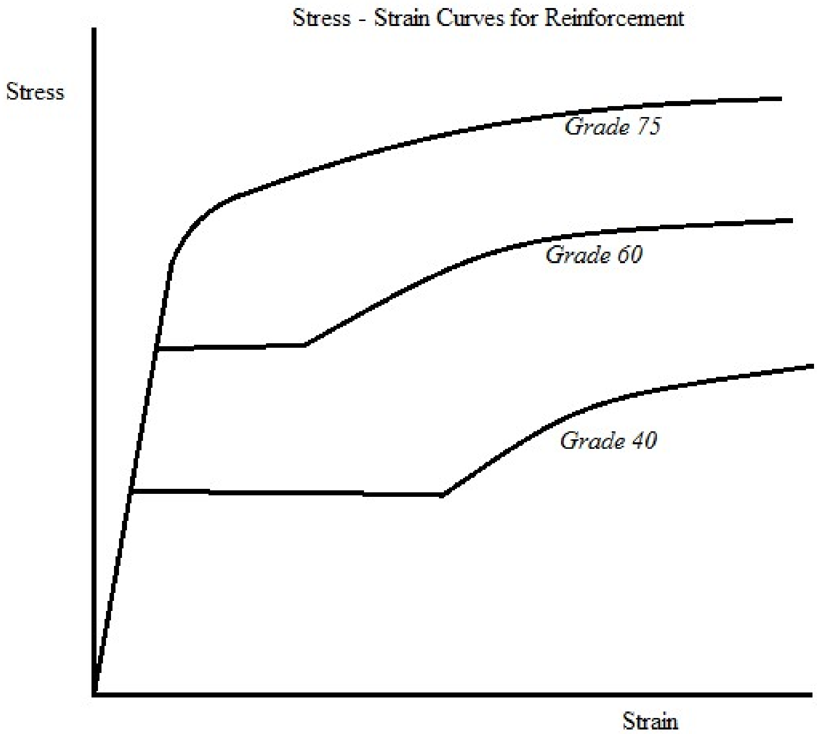




Reinforcement:

Because concrete is weak in tension, it is reinforced with steel bars or wires that resist the tensile stresses.

Special coatings are applied to steel, such as galvanizing or epoxy coating (green bar above) to protect against the exposure to salts in sea water.



The grade represents the yield strength in (ksi). As shown, the higher the grade of reinforcement the higher the strength and the lower the ductility.

THE DESIGN PROCESS

OBJECTIVES OF DESIGN:

The structure should satisfy four major criteria:

♦ Appropriateness: designed to serve its intended use
♦ Economy: overall cost should not exceed client's budget
♦ Structural Adequacy: the structure must:
  - Be strong enough to support all anticipated loadings safely
  - Not deflect, tilt, vibrate or crack in a way that affects its usefulness
♦ Maintainability: require minimum and simple maintenance procedures

THE DESIGN PROCESS:

♦ Phase I: defining the client's needs and priorities
♦ Phase II: development of project concept:
  - Number of possible layouts
  - Preliminary cost estimates
♦ Phase III: design of individual systems

LIMIT STATES AND THE DESIGN OF REINFORCED CONCRETE:

When a structure or an element becomes unfit for its intended use, it is said to have reached a limit state. Groups of limit states for reinforced concrete include:

♦ Ultimate Limit State:

Involves structural collapse of part or all of the structure. Major ultimate limit states may include:

- Loss of equilibrium of part or all the structure (tipping or sliding)
- Rupture of critical parts of the structure (partial or complete collapse)
- Progressive collapse (collapse of a member due to overload transferring the load to adjacent members and so on)
- Formation of plastic mechanism (yielding of reinforcement, plastic hinges)
- Instability due to deformations (buckling)
- Fatigue: fracture due to repeated stress cycles

♦ Serviceability Limit State:

Involves disruption of the functional use of the structure but not collapse. Major serviceability limit states include:

- Excessive deflections
- Excessive crack widths (water tanks, water getting to steel leads to rusting)
- Undesirable vibrations (vertical vibrations of floors or bridges)

♦ Special Limit States:

Involves damage or failure due to abnormal conditions; extreme earthquakes, fire, explosions, vehicular collisions

LIMIT STATES DESIGN PROCESS

Limit – states design is a process that involves:

- ♦ Identification of all potential modes of failure

    Water tank ⇒ excessive cracking ⇒ serviceability limit state

- ♦ Determination of acceptable level of safety per the ACI – Code

STRUCTURAL SAFETY

Safety factors are required for:

♦ Variability in resistance: actual strength of the members of the structure differ from those calculated

- Variability of strengths of concrete and reinforcement
- Differences in the as – built dimensions and those in the design
- Simplifying assumptions in the derived equations

♦ Variability in loading: specially live loads and environmental loads

♦ Consequences of failure:

- Loss of life
- Cost of clearing debris
- Cost to society in lost time

DESIGN PROCEDURES SPECIFIED IN THE ACI – CODE

♦ Strength Design: (2011 ACI – Code)

Based on required strength computed from combinations of factored loads and the reduced design strengths

♦ Working – Stress – Design

Based on working loads (service loads) or un-factored loads. In 2002 the appendix that allows the use of this method was deleted.

♦ Plastic Design, Limit Design, Capacity Design

Considers redistribution of moments as successive cross sections yield

LOADING AND ACTIONS

Loads classifications:

♦ Permanent loads: soil pressure, self – weight

♦ Accidental loads: vehicular collisions, explosions

♦ Variable loads:

- Sustained loads (long duration): Furniture
- Short duration: people

CHAPTER FOUR

FLEXURE: BASIC CONCEPTS, RECTANGULAR BEAMS

Loads transfer among the structure from top to bottom:

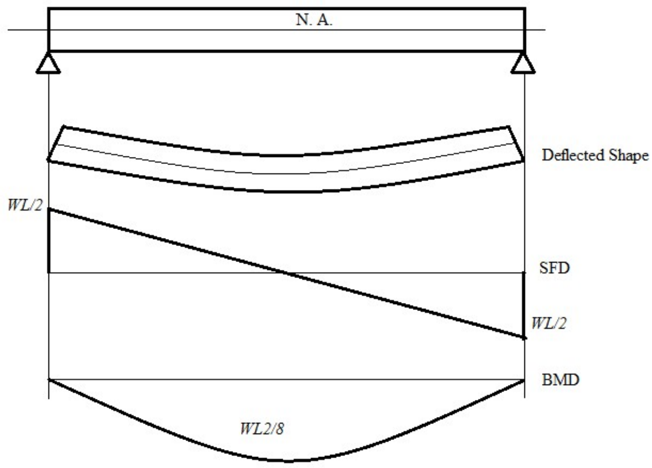
Slabs → Beams → Columns → Footings → Earth.

Slabs & Beams → Flexure + Shear

Columns → Axial + Flexure

Basic Safety Equation for Flexure: $\boldsymbol{\Phi M_n \geq M_u}$

M_n = nominal moment capacity, Φ = strength reduction factor (0.9 in design), ΦM_n = design moment, design flexural strength, factored moment capacity/resistance, M_u = required ultimate moment, moment due to factored loads.

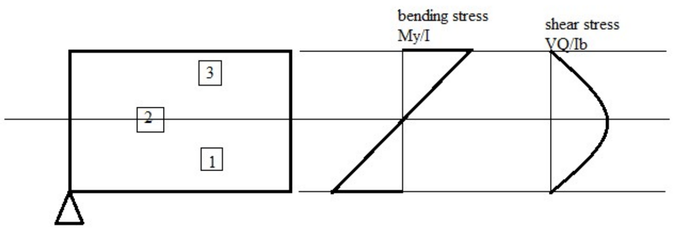


<u>Flexure in Beams:</u>

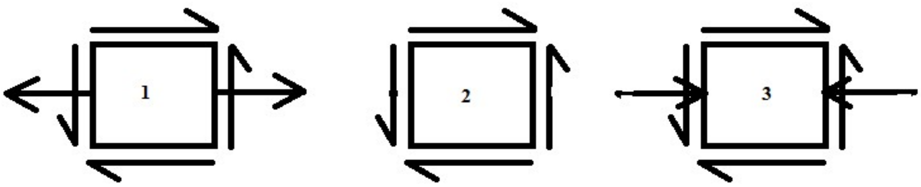
Stresses on element 1 (below the neutral axis) include tensile normal stresses (due to bending) and shear stresses.

Stresses on element 2 (on the neutral axis) include shear stresses only because the bending stress is zero at the neutral axis.

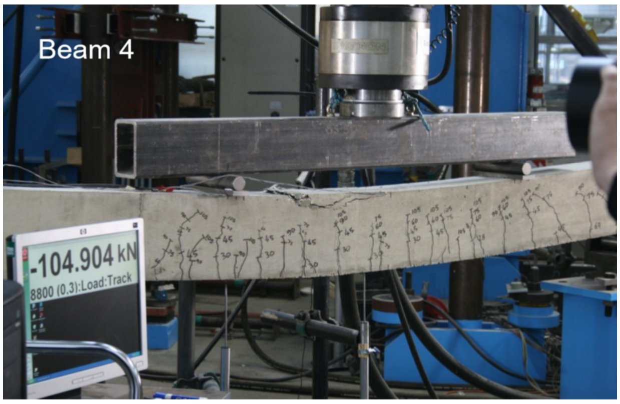
Stresses on element 3 (above the neutral axis) include compressive normal stresses (due to bending) and shear stresses.



The equations shown below to calculate the shear and normal stresses can only be used if the beam is made of linear elastic, homogenous and isotropic material. Reinforced concrete is neither homogenous nor elastic, therefore these formulas cannot be used to calculate the stresses.



Flexural Behavior (Laboratory Testing):

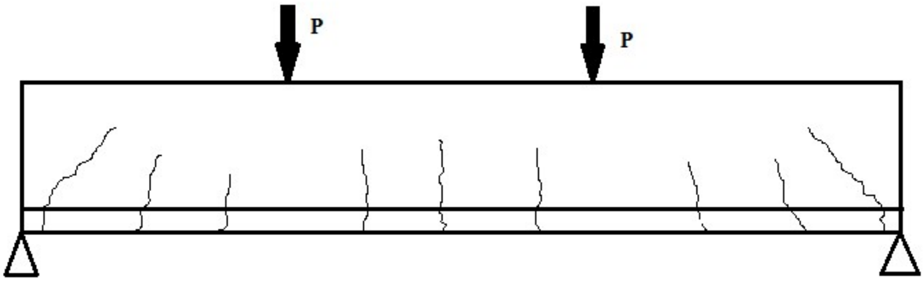


Stage A: Before Cracking

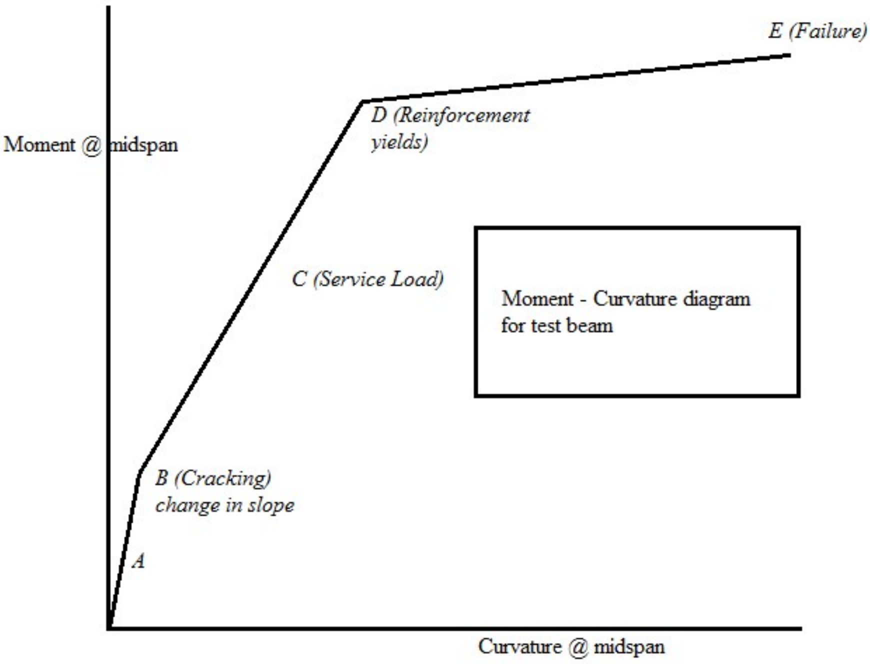
Stage B: Cracking

Stage C: After cracking before yielding of reinforcement (service load): in this stage the tensile force is transferred from the concrete to the steel.

Stage D: Yielding of reinforcement: in this stage the reinforcement reaches the yielding point. Curvature increases rapidly with very little increase in the moment.

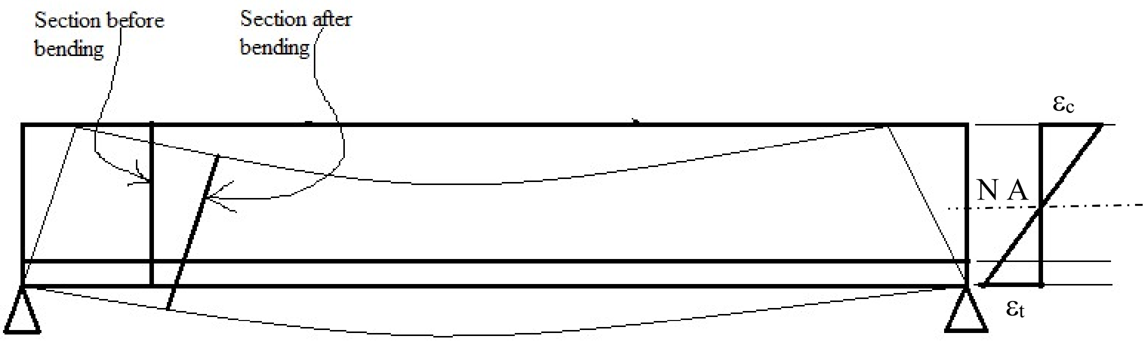


Stage E: Failure: beam failed as a result of crushing of the concrete on the top of the beam.



Basic Assumptions in Flexure Theory:

1. Sections perpendicular to the axis of bending that are plane before bending remain plane after bending.



2. The strain in the reinforcement is equal to the strain in the concrete at the same level. (*perfect bond*)

3. The stresses in the concrete and reinforcement can be computed from the strains by using the stress – strain curves for concrete and steel.

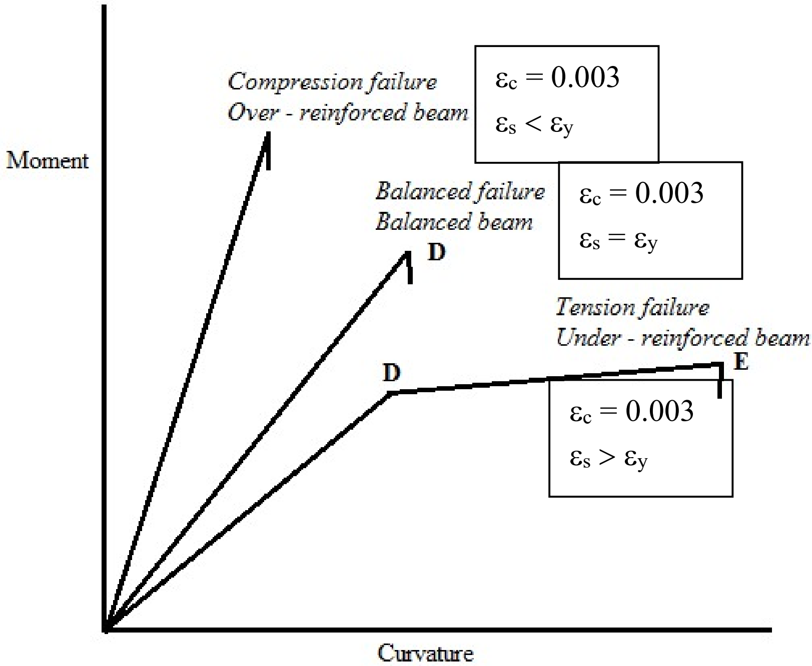
4. The tensile strength of concrete is neglected in flexural – strength calculations.

5. Concrete is assumed to fail when the maximum compressive strain reaches a limiting value (ε_{cu} = ***0.003*** per ACI – Code).

Flexural failure may occur in three different ways:

1. Compression failure: when the compressive strain in the concrete reaches the ultimate ($\varepsilon_{cu}=0.003$) the tensile strain in the steel is less than the yield strain ($\varepsilon_s < \varepsilon_y$). For grade 60 steel f_y = 420MPa, $\varepsilon_y = \sigma_y/E$ = **420/200,000 = 0.0021**.

2. Balanced failure: when the compressive strain in the concrete reaches the ultimate ($\varepsilon_{cu}=0.003$) the tensile strain in

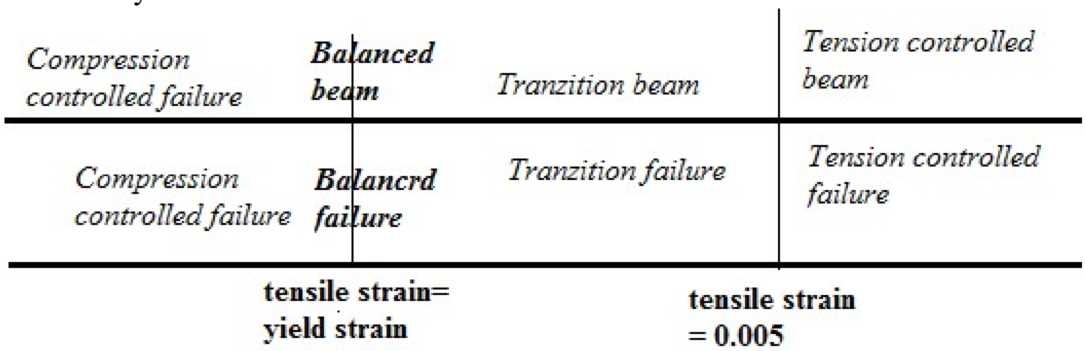


the steel is equal to the yield strain ($\varepsilon_s = \varepsilon_y$).

3. Tension failure: when the compressive strain in the concrete reaches the ultimate ($\varepsilon_{cu}=0.003$) the tensile strain in the steel exceeds the yield strain ($\varepsilon_s > \varepsilon_y$).

Strain Limits Method for Analysis and Design:

Per ACI – Code four types of beams depending on the anticipated mode of failure. Failure occurs when the compressive strain in the concrete reaches the ultimate ($\varepsilon_{cu}=0.003$). The type of failure is defined by the tensile strain at failure.



Flexure Theory:

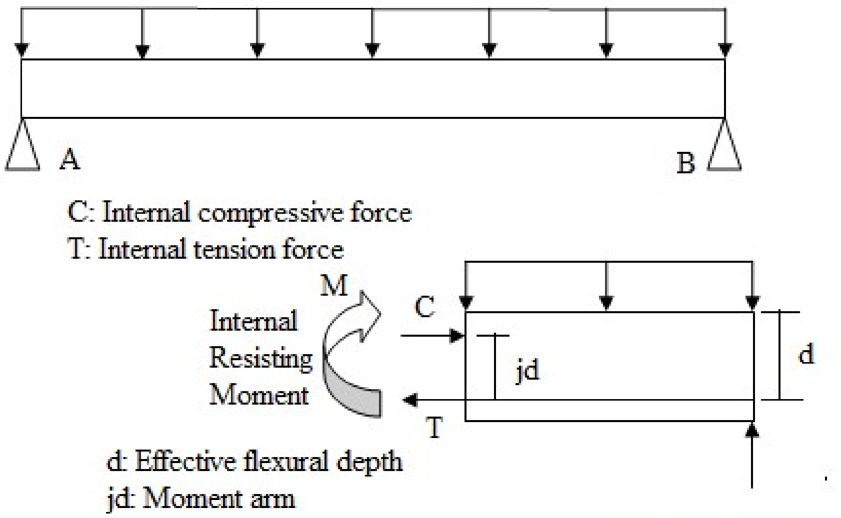
$\sum f_x = 0.0$

C = T (couple forces)

Produce couple moment (Internal resisting moment, M)

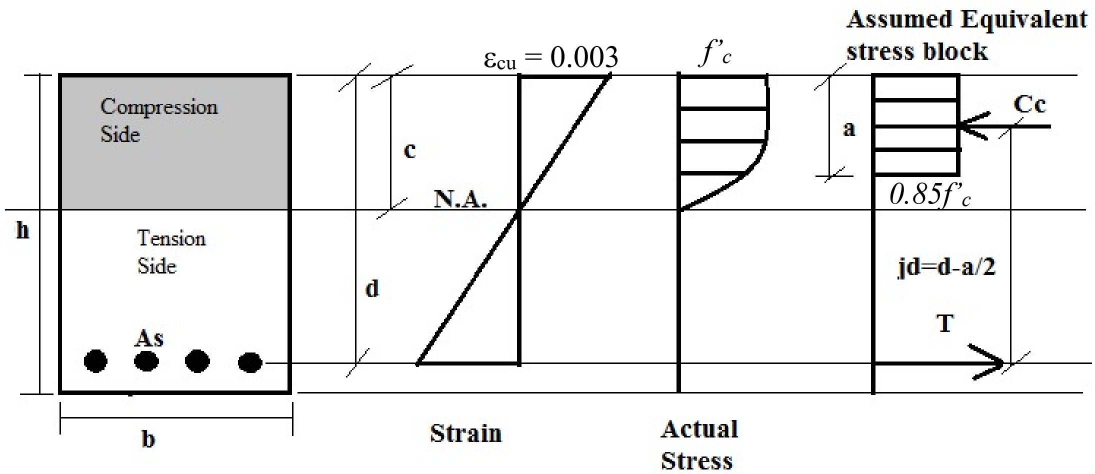
$M = C\,jd = T\,jd$

$M = M_n$



c = depth of compression area, or depth of NA

a= depth of the assumed equivalent stress block



a = β_1 c

The moment arm, $jd = (d-a/2)$, for rectangular beams only, and to be more specific, for rectangular compression areas only. ε_s

$T = A_s f_s$ (Always assume steel yielded $\varepsilon_s \geq \varepsilon_y \rightarrow f_s = f_y$)

From strain compatibility, $\varepsilon_s = 0.003\left(\dfrac{d-c}{c}\right)$

$C_c = 0.85 f'_c\, a\, b$

$T = C_c$

$As\, fy = 0.85 f'_c\, a\, b \rightarrow a = A_s f_y / (0.85 f'_c\, b)$

Calculate (***c***) and check if $\varepsilon_s \geq \varepsilon_y$

Nominal moment capacity, $M_n = A_s f_y\, jd = A_s f_y\, (d - a/2)$

Design moment capacity $\Phi M_n = \Phi (A_s f_y\, (d - a/2))$

If $\varepsilon_s < \varepsilon_y \rightarrow f_s \neq f_y \rightarrow$ Recalculate (***a***) from equilibrium equation,

Substitute $f_s = E\varepsilon_s = E0.003\left(\dfrac{d-c}{c}\right)$ instead of f_y and solve for ***c*** or ***a***. (Quadratic Equation)

$As\, E\, 0.003\, ((d\text{-}c)/c) = 0.85 f'_c\, a\, b \qquad (a = \beta_1 c)$

$$\left(0.85 f'_c b \beta_1\right)c^2 + \left(A_s E 0.003\right)c - A_s E 0.003 d = 0.0)$$

$$Ax^2 + Bx + C = 0.0$$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

<u>Minimum amount of tension reinforcement per ACI – Code:</u>

When a beam section cracks in tension, the crack usually propagates to a point near the centroid of the section and there is a sudden transfer of the tension force from the concrete to the reinforcing steel in the tension zone. Unless a minimum amount of reinforcement is present in the tension zone, the beam would fail suddenly.

$$As_{\min} = \begin{cases} \dfrac{0.25\sqrt{f'_c}}{f_y} b_w d \\ \dfrac{1.4}{f_y} b_w d \end{cases} \Rightarrow f'_c, f_y (MPa)$$

Values of β_1:

$$f_c^{'} \leq 28MPa \Rightarrow \beta_1 = 0.85$$

$$28MPa \leq f_c^{'} \leq 56MPa \Rightarrow \beta_1 = 0.85 - 0.05\frac{f_c^{'} - 28}{7}$$

$$f_c^{'} \geq 56MPa \Rightarrow \beta_1 = 0.65$$

Values of Strength Reduction Factor (Φ):

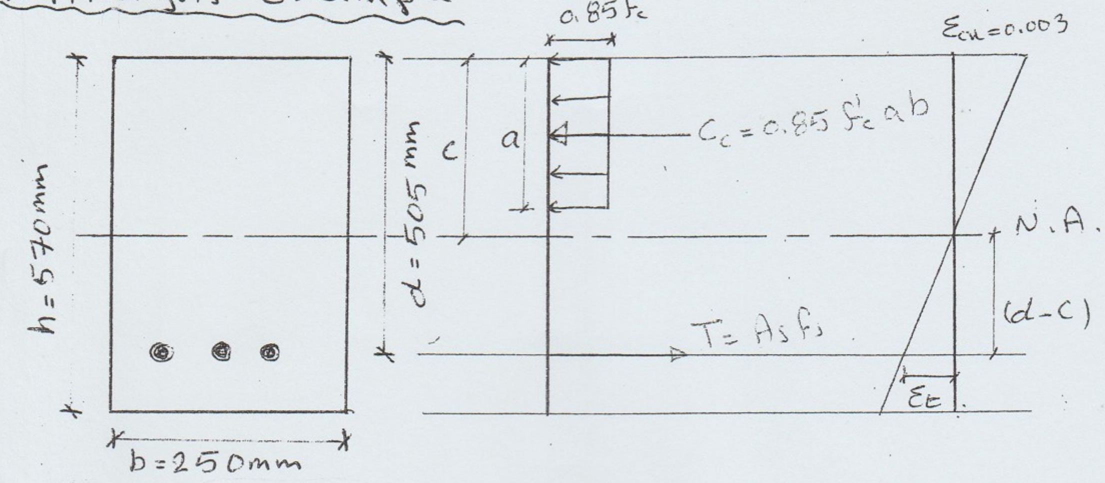
In beam analysis problems, the strength reduction factor is determined by the type of failure on the beam. The beam type of failure is determined by the tensile strain of the extreme tension reinforcement layer.

$\varepsilon_s \geq 0.005$ (Tension Controlled Failure) → $\Phi = 0.9$

$\varepsilon_y < \varepsilon_s < 0.005$ (Transition Controlled Failure) → $\Phi = 0.65 + (\varepsilon_s - 0.002)250/3$

$\varepsilon_s \leq \varepsilon_y$ (Balanced and Compression Controlled Failure) → $\Phi = 0.65$

* Analysis Example



A_s = ~~3 NO.25 M~~ = $1530\,mm^2$ (Table A4M)

$f'_c = 20\,Mpa.$

$f_y = 420\,Mpa.$

Determine the design moment capacity. (ϕM_n)

① Compute a:

assume steel yielded ⇒ $f_s = f_y$

Equilibrium ~~of~~ forces ⇒ $C_c = T$

$$0.85\, f'_c\, a\, b = A_s f_y$$

$$a = \frac{A_s f_y}{0.85\, f'_c\, b} = \frac{1530\,mm^2 * 420\,Mpa}{0.85 * 20\,Mpa * 250\,mm} = 151.2\,mm$$

Calculate c, $a = \beta_1 c$

$\beta_1 = 0.85$ ($f'_c = 20\,Mpa < 28\,Mpa$)

$$c = \frac{151.2}{0.85} = 177.9\,mm$$

② Check if steel yielded:

from similar triangles in the linear strain distribution

$$\frac{0.003}{\varepsilon_t} = \frac{c}{d-c} \Rightarrow \varepsilon_t = 0.003\left(\frac{d-c}{c}\right)$$

$$\varepsilon_t = 0.003\left(\frac{505-177.9}{177.9}\right) = 0.0055$$

$$\varepsilon_y = \frac{f_y}{E} = \frac{420\ Mpa}{200{,}000\ Mpa} = 0.0021$$

⇒ Steel has yielded ($\varepsilon_t > \varepsilon_y$) assumption o.k.

Tension controlled beam ($\phi = 0.9$) ($\varepsilon_t > 0.005$)

③ Compute M_n, ϕM_n

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$= 1530\ mm^2 * 420\ \frac{N}{mm^2}\left(505\ mm - \frac{151.2\ mm}{2}\right)$$

$$= 275.9 \times 10^6\ N.mm$$

$$= 275.9\ kn.m$$

$$\phi M_n = 0.9 * 275.9 = 248.3\ kn.m$$

④ Check $A_{s\,min}$:-

(When a beam section cracks in tension, the crack usually propagates to a point near the centroid of the section and there is a sudden transfer of the tension force from the concrete to the reinforcing steel in the tension zone Unless a minimum amount of reinforcement is present in the tension zone, the beam would fail suddenly)

ACI-Code $A_{s\,min} = \frac{0.25\sqrt{f'_c}}{f_y} b_w d$

$A_{s\,min} = \frac{1.4\, b_w d}{f_y}$

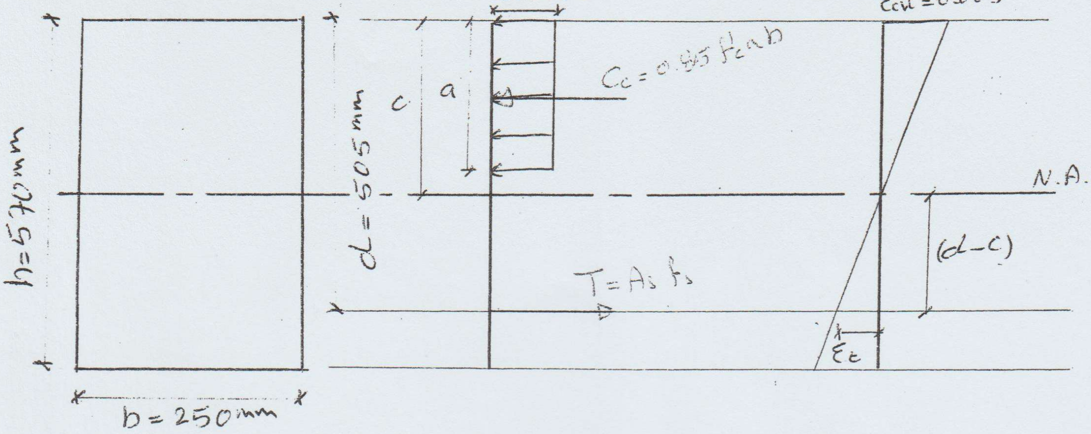
f'_c & f_y in Mpa.

$$A_{s\,min} = \frac{0.25\sqrt{20} * 250 * 505}{420} = 336\ mm^2$$

$$A_{s\,min} = \frac{1.4 * 250 * 505}{420} = 420\ mm^2$$

$A_s = 1530\ mm^2 > A_{s\,min}$ o.K.

Analysis Example



A_s = 6 No. 25M = 3060 mm² (Table A4M)

f'_c = 20 Mpa

f_y = 420 Mpa

Calculate the design moment capacity.

① Compute a:

assume steel yielded ⇒ $f_s = f_y$

Equilibrium of force ⇒ $a = \dfrac{A_s f_y}{0.85 f'_c b}$

$$a = \frac{3060 \times 420}{0.85 \times 20 \times 250} = 302.4 \text{ mm}$$

$$c = a / \beta_1 = \frac{302.4}{0.85} = 355.8 \text{ mm} \quad (f'_c < 28 \text{ Mpa} \Rightarrow \beta_1 = 0.85)$$

② Check if steel yielded:

$$\varepsilon_t = 0.003 \left(\frac{d-c}{c}\right) = 0.003 \left(\frac{505 - 355.8}{355.8}\right) = 0.00126$$

$\varepsilon_t < \varepsilon_y \Rightarrow f_s \neq f_y$ (Steel is not yielded)

$f_s = E\,\varepsilon_t = E(0.003)\left(\dfrac{d-c}{c}\right)$ (linear portion of σ-ε curve)

Equilibrium of forces $C_c = T$

$$0.85 f'_c\, a\, b = A_s \times E \times 0.003 \times \frac{d-c}{c} \qquad (a = \beta_1 c)$$

$$0.85 f'_c\, \beta_1\, c\, b = A_s \times E \times 0.003\, \frac{d-c}{c}$$

$$0.85 f'_c b \beta_1 c^2 + A_s E * 0.003 c - A_s E * 0.003 d = 0.0$$

$$3612.5 c^2 + (1836000) c - 927180000 = 0.0$$

$$c = \frac{-1836000 \pm \sqrt{(1836000)^2 - 4 * 3612.5 * (-927180000)}}{2 * 3612.5}$$

$$C = -254.12 \pm 566.78$$

$$\Rightarrow C = 312.66 \text{ mm}$$

$$a = 0.85 * 312.66 = 265.76 \text{mm} \quad (f'_c < 28 \text{MPa} \Rightarrow \beta_1 = 0.$$

$$\varepsilon_t = 0.003 \left(\frac{505 - 312.66}{312.66} \right) = 0.00184$$

$$f_s = E \varepsilon_t = 200{,}000 (0.00184) = 368 \text{ Mpa.}$$

③ Compute M_n & ϕM_n

$$M_n = A_s f_s (d - \frac{a}{2}) = 3060 * 368 * (505 - \frac{265.76}{2})$$

$$= 419 \text{ kn.m}$$

$\varepsilon_t < \varepsilon_y \Rightarrow$ Compression - Controlled failure

$\Rightarrow \phi = 0.65$

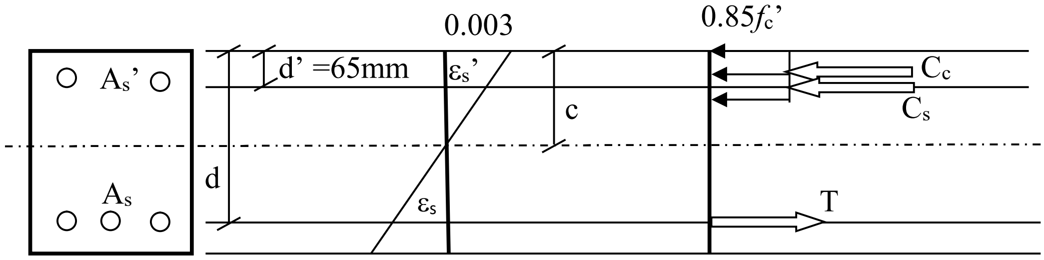
$$\phi M_n = 0.65 * 419 = 272.4 \text{ kn.m.}$$

④ Check $A_{s\,min}$

$A_{s\,min} = 336 \text{ mm}^2$
$A_{s\,min} = 420 \text{ mm}^2$
} $A_s = 3060 \text{ mm}^2 > A_{s\,min}$

o.k.

Beams with Compression Reinforcement (Doubly Reinforced Beams):



$T = C_c + C_s$ (assume $\varepsilon_s > \varepsilon_y$ & $\varepsilon_s' > \varepsilon_y$)

From strain compatibility, $\varepsilon_s = 0.003\left(\frac{d-c}{c}\right)$, $\varepsilon_s' = 0.003\left(\frac{c-d'}{c}\right)$

$A_s f_y = 0.85 f_c' ab + A_s' f_y$, Solve for ***a***

Calculate ***c*** and check first if ($\varepsilon_s' > \varepsilon_y$)

If first check is OK., check if ($\varepsilon_s > \varepsilon_y$)

If both checks are OK., calculate nominal and design moment capacities from equations below.

If $\varepsilon_s' < \varepsilon_y \rightarrow$ go back to equilibrium equation and use $f_s' = E\,\varepsilon_s' = E0.003\left(\frac{c-d'}{c}\right)$ instead of $\boldsymbol{f_y}$.

Solve equation for ***a***, and then check if ($\varepsilon_s > \varepsilon_y$), if OK., calculate moments from equations below

If $\varepsilon_s < \varepsilon_y \rightarrow$ go back to equilibrium equation and use $f_s = E\,\varepsilon_s = E0.003\left(\frac{d-c}{c}\right)$ instead of $\boldsymbol{f_y}$.

Solve equation for a, and calculate moments from equations below

Nominal moment capacity, $M_n = C_c(d - a/2) + C_s(d - d')$

Design moment capacity, $\phi M_n = \phi(C_c(d - a/2) + C_s(d - d'))$

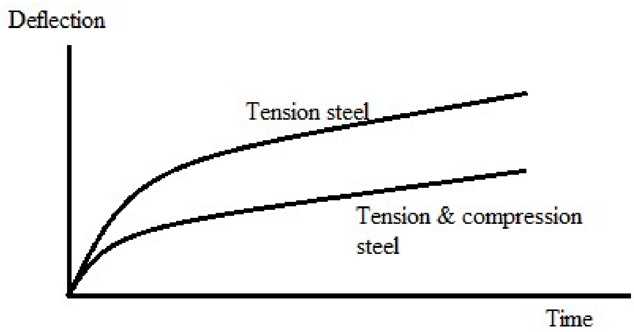
Note:

The strength reduction factor (***Φ***) is determined by the tensile strain in the tension reinforcement only not the strain in the compression reinforcement ($\boldsymbol{\varepsilon_s}$ not $\boldsymbol{\varepsilon_s'}$).

__Reasons for providing compression reinforcement:__

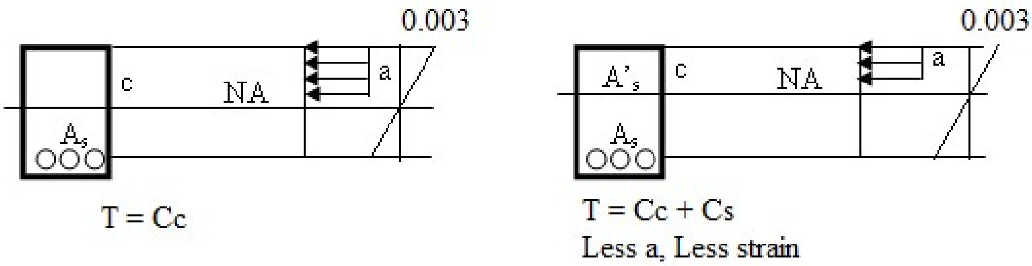
1. Reduce sustained load deflections: reduces the long term deflections in beams subjected to sustained loads.

   Creep of the concrete in the compression zone transfers the load to the compression steel → reducing the stress in the concrete → less creep in the concrete → reducing the sustained load deflections.



2. Increase ductility (ε_s):

   Addition of A_s' → Reduction in the concrete compression area → Reduction in the depth of the compression zone → more strain in the tension reinforcement.



3. Change mode of failure from compression/balanced/transition to tension:

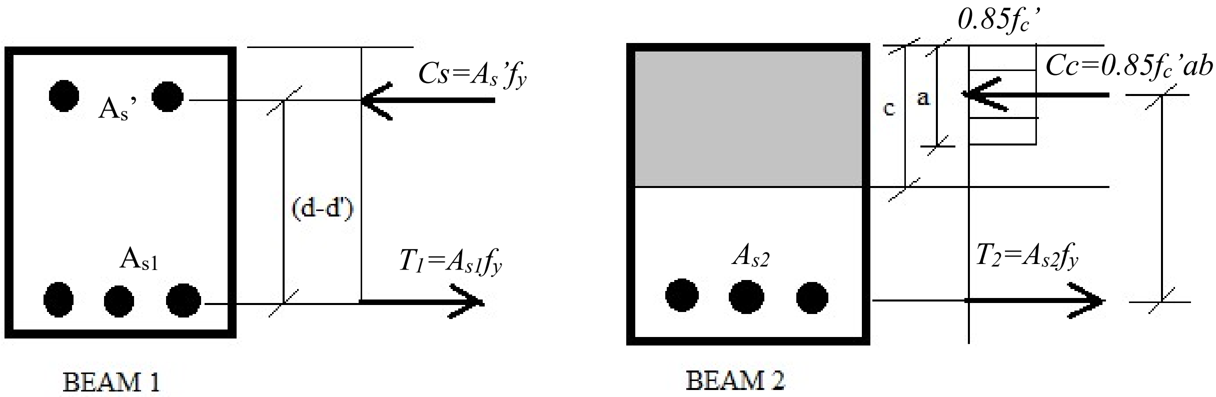
   This is achieved by increasing the ductility of the tension steel. Adding enough A_s' strengthens the compression zone sufficiently to allow the tension steel to reach a strain of 0.005 before the concrete crushes in the compression.

4. Fabrication ease:

   Providing small bars at the corners of the stirrups to hold the stirrups in place and to help anchor the stirrups

Analysis of beams with tension and compression reinforcement:

Case I: Tension steel yielded & compression steel yielded



Equilibrium in beam 1: $T_1 = C_s \rightarrow A_{s1}f_y = A_s'f_y \rightarrow A_{s1} = A_s'$

$$M_{n1} = A_{s1} f_y \left(d - d'\right)$$

$A_{s1} + A_{s2} = A_s \rightarrow A_{s2} = A_s - A_{s1}$

Equilibrium in beams 2: $T_2 = C_c \rightarrow \; a = \dfrac{A_{s2} f_y}{0.85 f_c' b}$

$$M_{n2} = A_{s2} f_y \left(d - \frac{a}{2}\right)$$

Total nominal moment capacity, $M_n = M_{n1} + M_{n2}$

Case II: Tension steel yielded & compression steel not yielded:

$T = C_c + C_s$

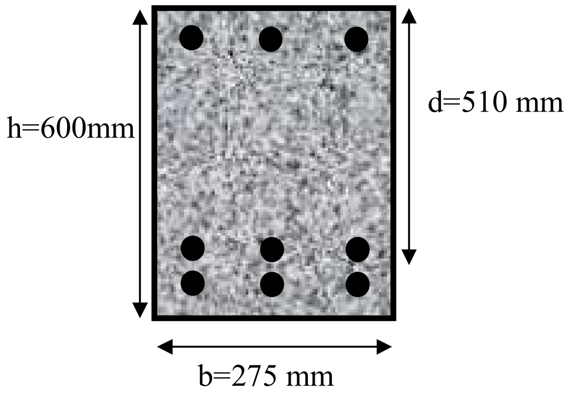
$$f_s' \neq f_y \rightarrow f_s' = E\varepsilon_s', \; \varepsilon_s' = 0.003\left(\frac{c - d'}{c}\right)$$

$$A_s f_y = 0.85 f_c' ab + A_s' E 0.003\left(\frac{c - d'}{c}\right), \; c = a/\beta_1$$

$$\left(0.85 f_c' b\right)a^2 + \left(0.003EA_s' - A_s f_y\right)a - \left(0.003EA_s'\beta_1 d'\right) = 0.0$$

$M_n = C_c\,(d - a/2) + C_s\,(d - d')$

Analysis Example: Determine the design moment capacity (ΦMn)



As' = 852 mm^2

As = 3060 mm^2

fc' = 20 MPa

fy = 420 MPa

d' = 65 mm

1. **COMPUTE (a):**

Assume $\varepsilon_s \geq \varepsilon_y$ and $\varepsilon_{s'} \geq \varepsilon_y$

Beam 1 : $As'fy = As_1 fy \Rightarrow$ As' = 852 mm^2

Beam 2: $As_2 = As - As_1$ = 3060 – 852 = 2208 mm^2

$a = As_2 fy / 0.85 fc' b$ = 2208 * 420 / (0.85 * 20 * 275) = 198 mm

c = a / β_1 = 198 / 0.85 = 233 mm (fc' ≤ 28 MPa → β_1 = 0.85)

OR:

$Asfy = As'fy + 0.85fc'ab$

3060*420 = 852*420 + 0.85*20*a*275

a = 198 mm

2. **CHECK IF COMPRESSION STEEL HAS YIELDED**

$\varepsilon_s' = 0.003 ((c - d') / c)$ = 0.003 ((233 – 65) / 233) = 0.0022 > ε_y assumption is OK

3. **CHECK IF TENSION STEEL HAS YIELDED**

$\varepsilon_s = 0.003 ((d - c) / c)$ = 0.003 ((510 – 233) / 233) = 0.0036 > ε_y assumption is OK

4. **COMPUTE MOMENT CAPACITY**

$Mn_1 = As_1 fy (d - d')$ = 852*420 (510 – 65) = 159.24 Kn.m

$Mn_2 = As_2 fy (d - a/2)$ = 2208*420 (510 – 198/2) = 381.14 Kn.m

Mn = $Mn_1 + Mn_2$ = 540.38 Kn.m

OR: at d_t = 600 – 65 = 535 mm

$\varepsilon_s = 0.003 ((d_t - c) / c)$

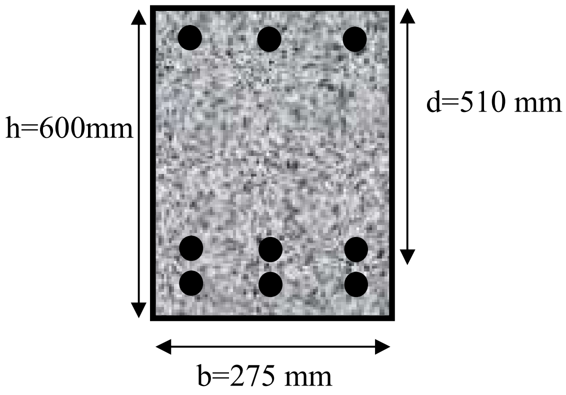
= 0.003 ((535 – 233) / 233) = 0.0039

$\varepsilon_y < \varepsilon_s$ = 0.0036 < 0.005 (Transition Controlled failure)

Φ = 0.65 + [(0.0036 – 0.002)*(250/3)] = 0.783

ΦMn = 0.783 * 540.38 = 423.1 Kn.m

Analysis Example: Determine the design moment capacity *(ΦM_n)*



$A_s' = 1704\ mm^2$

$A_s = 3060\ mm^2$

$f_c' = 20$ MPa

$f_y = 420$ MPa

$d' = 65$ mm

> OR:
>
> $A_s f_y = A_s' f_y + 0.85 f_c' a b$
>
> $3060*420 = 1704*420 + 0.85*20*a*275$
>
> $a = 121.8$ mm

5. **COMPUTE (a):**

Assume $\varepsilon_s \geq \varepsilon_y$ and $\varepsilon_{s'} \geq \varepsilon_y$

Beam 1 : $A_s' f_y = A_{s1} f_y \Rightarrow A_s' = 1704\ mm^2$

Beam 2: $A_{s2} = A_s - A_{s1} = 3060 - 1704 = 1356\ mm^2$

$a = A_{s2} f_y / 0.85 f_c' b = 1356 * 420 / (0.85 * 20 * 275) = 121.8$ mm

$c = a / \beta_1 = 121.8 / 0.85 = 143.32$ mm ($f_c' \leq 28$ MPa $\rightarrow \beta_1 = 0.85$)

6. **CHECK IF COMPRESSION STEEL HAS YIELDED**

$\varepsilon_s' = 0.003\ ((c - d') / c) = 0.003\ ((143.32 - 65) / 143.32) = 0.0016 < \varepsilon_y$ assumption is NOT OK

$A_s f_y = A_s' f_s + 0.85 f_c' a b$, ($f_s = E\varepsilon_s$)

$3060*420 = 1704*200{,}000*0.003*((c - 65) / c) + 0.85*20*0.85*c*275$

$3973.75\ c2 - 262800\ c - 66456000 = 0.0$

$C = 33.07 \pm 133.48 \Rightarrow c = 166.55$ mm $\Rightarrow a = 141.57$ mm

$\varepsilon_s' = 0.003\ ((c - d') / c) = 0.003\ ((166.55 - 65) / 166.55) = 0.0018 < \varepsilon_y$

$f_s' = 0.0018 * 20{,}000 = 360$ MPa

7. **CHECK IF TENSION STEEL HAS YIELDED**

$\varepsilon_s = 0.003\ ((d - c) / c) = 0.003\ ((510 - 166.55) / 166.55) = 0.0062 > \varepsilon_y$ assumption is OK

8. COMPUTE MOMENT CAPACITY

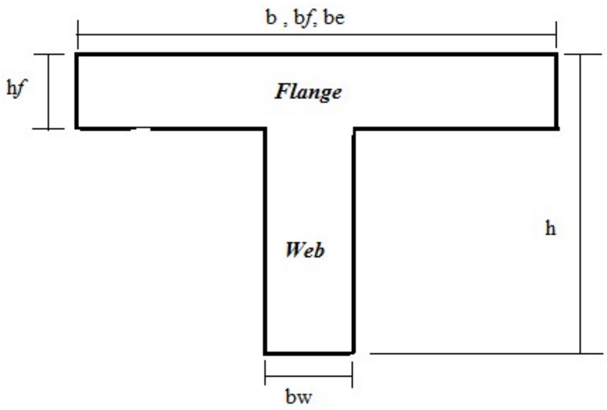
Mn = Cc (d – a/2) + As' (d – d')

0.85*20*141.57*(510 – 141.57/2) + 1704*360*(510 – 65) = 563.7 Kn.m

ε_s = 0.0062 > 0.005 (Tension Controlled failure, Φ = 0.9)

ΦMn = 0.9 * 563.7 = 507.3 Kn.m

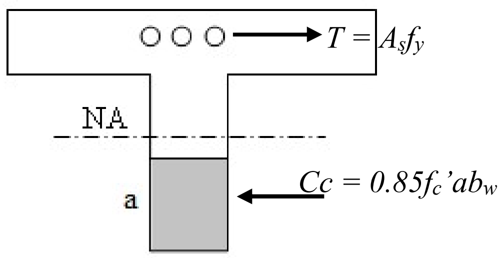
Analysis of T – Beams:



Generally the compression zone of a T – Beam is rectangular. Therefore, it is analyzed and designed as a rectangular beam. (Cases 1 and 2 below)

Unusual cases where the compression zone of a T – beam is T – shaped. Therefore, it is analyzed and designed as a T – beam. (Case 3 below)

Case 1: Beam under negative moment (Flange in tension)



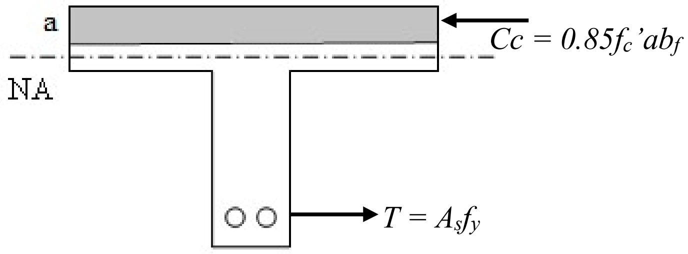
$$T = Cc$$

$$A_s f_y = 0.85 f_c^{'} a b_w$$

$$a = \frac{A_s f_y}{0.85 f_c^{'} b_w}$$

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$

Case 2: Beam under positive moment (Flange in compression; a ≤ h_f)



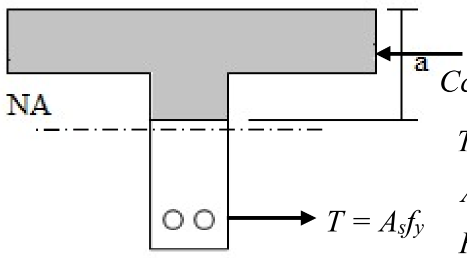
$$T = Cc$$

$$A_s f_y = 0.85 f_c^{'} a b_f$$

$$a = \frac{A_s f_y}{0.85 f_c^{'} b_f}$$

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$

Case 3: Beam under positive moment (Flange in compression; a > h_f)



$Cc = 0.85f_c'((b-b_w)h_f + ab_w))$

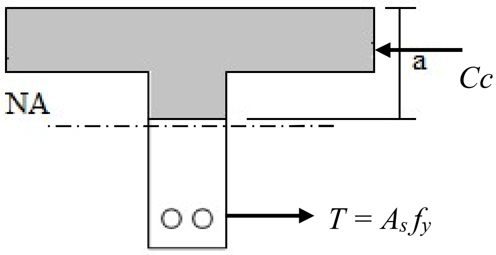
$$T = Cc$$

$$A_s f_y = 0.85 f_c^{'} \left[(b - b_w) h_f + a b_w \right]$$

$$Find \rightarrow a$$

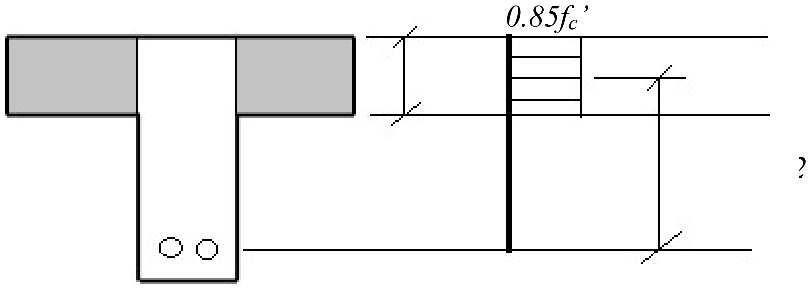
$$M_n = 0.85 f_c^{'} (b - b_w) h_f \left(d - \frac{h_f}{2} \right) + 0.85 f_c^{'} a b_w \left(d - \frac{a}{2} \right)$$

Analysis of nominal moment capacity for flanged sections in Positive Bending:



To avoid the need for locating the centroid of the compression force (***Cc***), the beam is divided into two beams as below:

<u>Beam Flange (***Beam F***):</u>



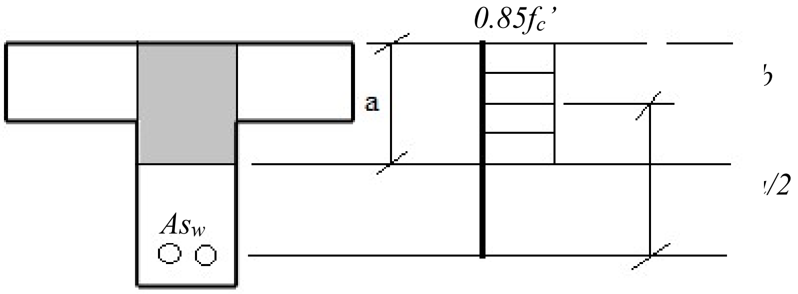
$$Cc_f = 0.85 f_c^{'} (b - b_w) h_f$$

$$T_f = Cc_f$$

$$As_f = \frac{0.85 f_c^{'} (b - b_w) h_f}{f_y}$$

$$Mn_f = Cc_f (d - \frac{h_f}{2})$$

<u>Beam Web (***Beam W***):</u>



$$As_w = As - As_f$$

$$T_w = Cc_w$$

$$a = \frac{As_w f_y}{0.85 f_c^{'} b_w}$$

$$Mn_w = Cc_w (d - \frac{a}{2})$$

$$Mn = Mn_f + Mn_w$$

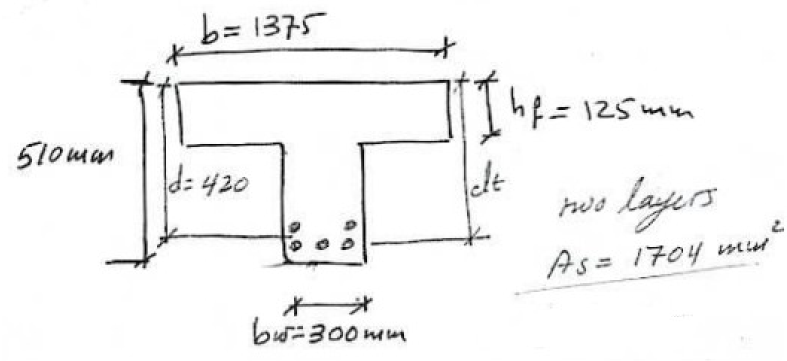
$$Mn = Cc_f (d - \frac{h_f}{2}) + Cc_w (d - \frac{a}{2})$$

$$\phi Mn = \phi \left(Cc_f (d - \frac{h_f}{2}) + Cc_w (d - \frac{a}{2}) \right)$$

Modification to As_{min}:

For statically determinate beams where the flange portion is in tension, ACI – Code recommends that (***b_w***), in As_{min} equations, be replaced by smaller of (***$2b_w$, b_f***).

<u>Analysis Example 1 (+ve moment capacity):</u> Determine the design moment capacity (ΦMn), *fc'=20MPa, fy=300MPa*



1. **Compute a**

Assume rectangular beam action ($a \le h_f$),
Assume $\varepsilon_s \ge \varepsilon_y$

T = Cc

$A_s f_y = 0.85\, f_c'\, a\, b_f$

1704 * 300 = 0.85 * 20 * a * 1375

a = 21.9 mm < h_f (assumption OK)

2. **Check of steel yielded**

$c = a / \beta_1 = 21.9 / 0.85 = 25.8$ mm

$\varepsilon_s = 0.003\, ((d - c) / c)$ = 0.003 ((420 – 25.8) / 25.8) = 0.046 > ε_y assumption is OK

3. **Compute *ΦMn***

$\varepsilon_s > 0.005$ (Tension Controlled Failure, Φ = 0.9)

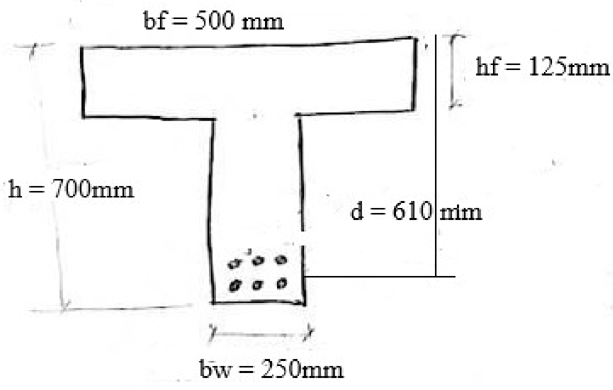
ΦMn = Φ As fy (d – a/2) = 0.9 * 1704 * 300 * (420 – 21.9/2) = 188.2 kN.m

4. **Check As_{min}**

As_{min} = 470 mm^2 and **588 mm^2**

As = 1704 mm^2 > As_{min} = 588 mm^2 (OK)

<u>Analysis Example 2 (+ve moment capacity):</u> Determine the design moment capacity (ΦM_n), *fc'=20MPa, fy=420MPa*



1. **Compute a**

Assume rectangular beam action ($a \le h_f$),
Assume $\varepsilon_s \ge \varepsilon_y$

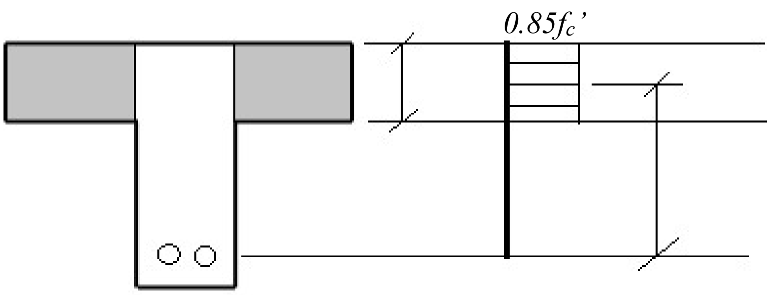
T = Cc

$A_s f_y = 0.85 f_c' a b_f$

3060 * 420 = 0.85 * 20 * a * 500

a = 151.2 mm > h_f (assumption NOT OK)

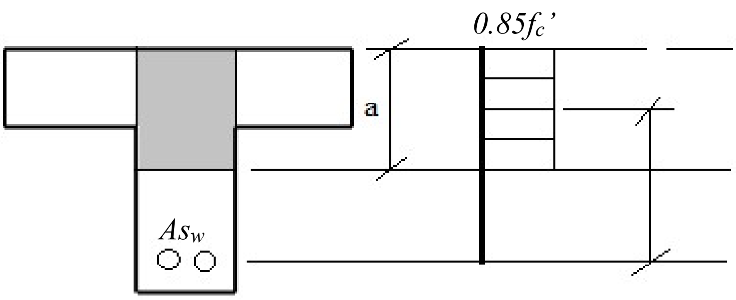
T – beam action. Divide beam into beam F and beam W



BEAM F

$T_f = Cc_f$

As_f * 420 = 0.85 * 20 * 125 * (500 – 250) = 1264.9 mm^2



BEAM W

As_w = 3060 – 1264.9 = 1795.1 mm^2

Tw = Ccw

1795.1 * 420 = 0.85 * 20 * a * 250

$a = 177.4$ mm

2. **Check of steel yielded**

$c = a / \beta_1 = 177.4 / 0.85 = 208.7$ mm

$\varepsilon_s = 0.003\ ((d - c) / c)$ = 0.003 ((610 – 208.7) / 208.7) = 0.0058 > ε_y assumption is OK

3. **Compute *ΦMn***

$\varepsilon_s > 0.005$ (Tension Controlled Failure, Φ = 0.9)

Mn_f = 1264.9 * 420 * (610 – 125/2) = 290.9 kN.m

Mn_w = 1795.1 * 420 * (610 – 177.4/2) = 393 kN.m

Mn = 683.9 Kn.M

Φ Mn = 0.9 * 683.9 = 615.5 kN.m

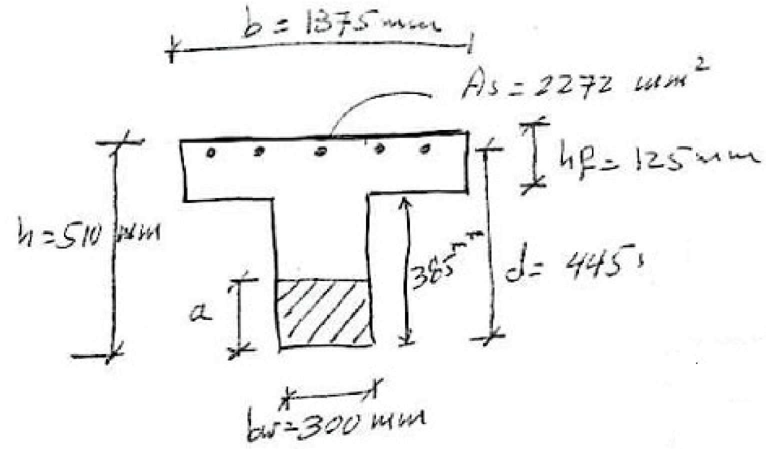
4. **Check As_{min}**

As_{min} = 406 mm^2 and **509 mm^2**

As = 3060 mm^2 > As_{min} = 509 mm^2 (OK)

<u>Analysis Example 3 (-ve moment capacity):</u> Determine the design moment capacity (ΦMn), *fc'=20MPa, fy=300MPa.*

Indeterminate Beam



1. **Compute a**

Assume $\varepsilon_s \geq \varepsilon_y$

T = Cc

As fy = 0.85 fc' a b_w

2272 * 300 = 0.85 * 20 * a * 300

a = 133.6 mm

2. **Check of steel yielded**

c = a / β_1 = 133.6 / 0.85 = 157.1 mm

$\varepsilon_s = 0.003\ ((d - c) / c)$ = 0.003 ((445 – 157.1) / 157.1) = 0.055 > ε_y assumption is OK

3. **Compute *ΦMn***

ε_s > 0.005 (Tension Controlled Failure, Φ = 0.9)

ΦMn = Φ As fy (d – a/2) = 0.9 * 2272 * 300 * (445 – 133.6/2) = 232 kN.m

4. **Check As_{min}**

As_{min} = 470 mm^2 and **588 mm^2**

As = 1704 mm^2 > As_{min} = 588 mm^2 (OK)

<u>PROBLEM</u>

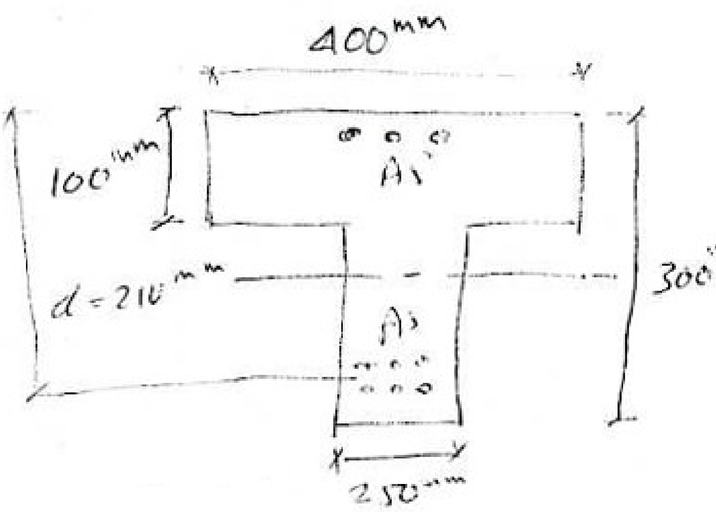
As' = 852 mm^2

As = 3060 mm^2

fc' = 20 MPa

fy = 420 MPa

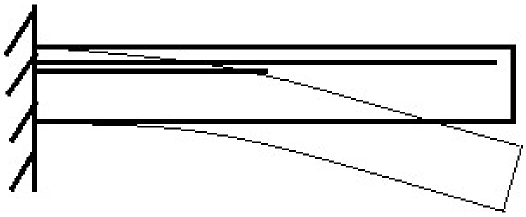
<u>ANSWER</u>: ΦMn = 100 kN.m

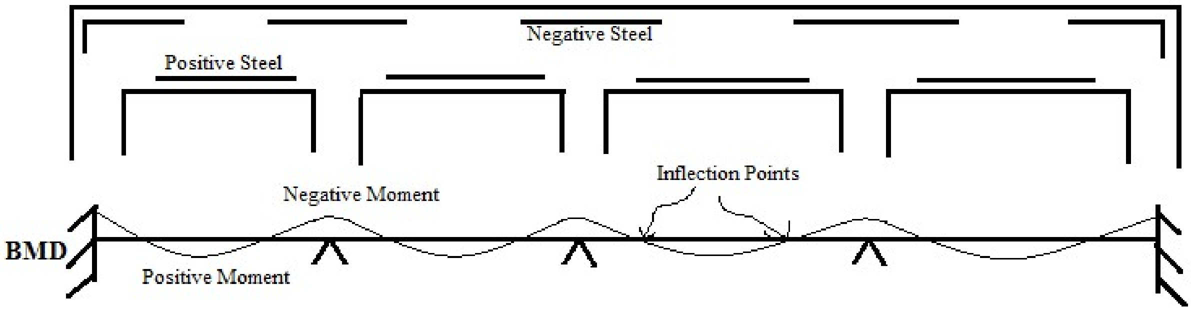


CHAPTER 5

A. DESIGN OF RECTANGULAR BEAMS

It is important that designers be able to visualize the deflected shape of a structure. The reinforcing bars for flexure are placed on the tensile face of the member.





Relationship between Beam Depth and Deflection:

The ACI – Code defines minimum beam thicknesses that generally are sufficient to limit beam deflection to acceptable values.

The selected beam depth, ***h***, from design will need to be checked against the minimum beam depth provided by the table.

TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE CALCULATED

	Minimum thickness, ***h***			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

Concrete Cover and Bar Spacing Per ACI – Code:

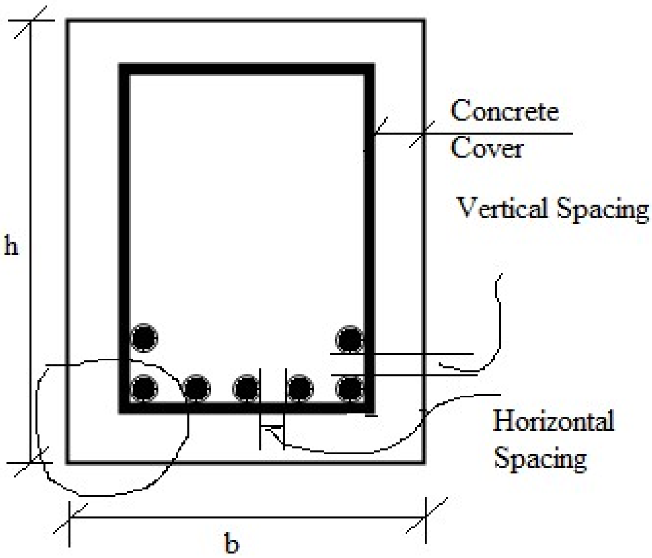
Minimum concrete cover for beams under normal exposure = **40mm**

Reasons for cover:

1. To bond the reinforcement to the concrete so that the two elements act together.
2. To protect the reinforcement against corrosion/rusting.
3. To protect the reinforcement from strength loss due to overheating in the case of fire.
4. On the top of slabs in garages and factories additional cover is provided so that abrasion and wear due to traffic will not reduce the cover below the required.

Minimum Horizontal spacing (S_h) = larger of:

1. Bar Diameter, db
2. 1.33 Maximum Size of Coarse Aggregate
3. 25mm
4. Diameter of Vibrator (desirable, not specified)

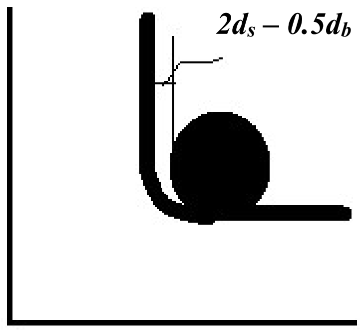


Minimum Vertical spacing (S_v) = larger of:

1. 1.33 Maximum Size of Coarse Aggregate
2. 25mm

Minimum beam width to fit the reinforcement in one layer can be calculated as follows:

b_{min} = 2 x **cover** + 2 x $\mathbf{d_s}$ + *number of bars* x $\mathbf{d_b}$ + *number of spacing's* x $\mathbf{S_h}$ + 2 x ($2d_s - 0.5d_b$)



If beam width from design is larger than b_{min} → one layer of reinforcement

If beam width from design is smaller than b_{min} → two layers of reinforcement (*or increase bar diameter to reduce number of bars and spacing's*)

Estimating the Effective Depth of a Beam:

Effective depth: the distance from the extreme compression fiber to the centroid of longitudinal reinforcement.

For beams with one layer of tension reinforcement → **d = h – 65** (mm) (*based on No.30M steel and No.10M stirrups, 40 + 10 + 15 = 65mm*)

For beams with two layers of tension reinforcement → **d = h = 90** (mm)

Generally speaking, the width of reinforced concrete beams (**b**) should not be less than **250mm** and ***preferably*** not less than **300mm**.

Minimum Amount of Tension Reinforcement:

To prevent a sudden failure with little or no warning when the beam cracks in flexure, the ACI – Code requires a minimum amount of tension reinforcement to be placed in the section.

$$As_{\min} = \begin{cases} \dfrac{0.25\sqrt{f_c^{'}}}{f_y} b_w d \\ \dfrac{1.4}{f_y} b_w d \end{cases} \Rightarrow f_c^{'}, f_y (MPa)$$

General Strength Design Requirements for Beams:

For gravity loading on a typical continuous floor system, the required combinations of factored loads should be determined from the first two equations in the ACI – Code.

U = 1.4 DL

U = 1.2 DL + 1.6 LL

The general strength requirement in the design of beam cross sections $\phi M_n \geq M_u$

Beam sections are normally designed to be tension – controlled. In design problems Φ is initially assumed as **0.9** to be confirmed at the end of the design process.

A. Design of Tension Reinforcement when Section Dimensions are known

$$\phi M_n = \phi A_s f_y (d - \frac{a}{2}) \geq M_u$$

$$\phi A_s f_y jd = M_u \Rightarrow A_s = \frac{M_u}{\phi f_y jd}$$

, (*the value of j generally ranges between* ***0.87*** *to* ***0.91****, for beams with narrow compression zones, and usually taken as* ***0.9***)

* Design Example / Beam Dimensions known

$f'_c = 25$ Mpa , $f_y = 420$ Mpa

DL = 14 kN/m (excluding self wt.) h = 600 mm

LL = 25 kN/m

$\gamma_{conc.} = 24$ kN/m^3

S.S. Beam
L = 8 m

b = 600 mm

$A_s = M_u / \phi f_y j d$

- Calc. self. wt. = 0.6 × 0.6 × 24 = 8.64 kN/m
- Calc. ultimate load $w_u = 1.2DL + 1.6LL$

  $w_u = 1.2(8.64 + 14) + 1.6(25)$

  $= 67.2$ kN/m
- Calc. ultimate Moment ; $M_u = w_u L^2 / 8$

  $M_u = 67.2 \times 8^2 / 8 = 537.3$ kN.m
- Calc. the effective depth , d

  assume one-layer of tension reinforcement

  $d = h - 65 = 600 - 65 = 535$ mm
- Calculate the required area of tension reinforcement

$A_s = 537.3 \times 10^6 / 0.9 \times 420 \times 0.9 \times 535 = 2952.1 \text{ mm}^2$

$a = 2952.1 \times 420 / 0.85 \times 25 \times 600 = 97.2$ mm

$A_s = M_u / \phi f_y (d - a/2) = \dfrac{537.5 \times 10^6}{0.9 \times 420 \times (535 - \frac{97.2}{2})} = 2922.5 \text{ mm}^2$

$a = 2922.5 \times 420 / 0.85 \times 25 \times 600 = 96.3$ mm

$A_s = 537.5 \times 10^6 / 0.9 \times 420 (535 - \frac{96.3}{2}) = 2919.6 \text{ mm}^2$

Convergence ⇒ O.K.

Select steel from table (A_s provided)

Try 6 No. 25M ; $A_s = 2945 \, mm^2$

check the one-layer assumption

$b_{min} = 2*40 + 2*10 + 6*25 + 5*25$
$+ 2*(2*10 - 0.5*25) = 390^{mm}$

$b_{min} < 600^{mm}$ o.k.

check Min. Area of tension reinforcement

$$A_{s\,min} = \begin{cases} \dfrac{0.25*\sqrt{25}}{420} * 600*535 = 955.4 \, mm^2 \\ \dfrac{1.4}{420} * 600*535 = 1070 \, mm^2 \end{cases}$$

$A_{s\,provided} = 2945 > A_{s\,min} = 1070^{mm^2}$

- check tension-Controlled & Moment Capacity.

$T = C_c \Rightarrow 2945*420 = 0.85*25*a*600$

$a = 97.^{mm}$; $c = a/\beta_1 = 114.13^{mm}$

$\varepsilon_s = 0.003 \left(\dfrac{535 - 114.13}{114.13} \right) = 0.011 > 0.005$

$\Rightarrow$ Tension - Controlled $\phi = 0.9$

$M_n = 2945*420(535 - 97/2) = 601.75$ kN.m

$\phi M_n = 0.9 * 601.75 = 541.6$ kN.m

$> M_u = 537.3$ kN.m

o.k.

B. Design of Tension Reinforcement when Section Dimensions are not known

$$Cc = T \Rightarrow a = \frac{A_s f_y}{0.85 f_c' b} \text{.................1}$$

Steel ratio or reinforcement ratio $\rho = \frac{A_s}{bd} \Rightarrow A_s = \rho bd$

Substituting in equation 1 above, $a = \rho \frac{f_y}{f_c'}(\frac{d}{0.85})$ 2

$w = \rho \frac{f_y}{f_c'}$, defined as mechanical reinforcement ratio. Substituting into equation 2 above

$$a = \frac{wd}{0.85} \text{.......................3}$$

For rectangular compression zone, $\phi M_n = \phi 0.85 f_c' ab(d - \frac{a}{2})$ 4

Substituting equation 3 in 4,

$$\phi M_n = \phi\left[bd^2 f_c' w(1 - 0.59w)\right]$$

$$k_n = f_c' w(1 - 0.59w)$$

$$\phi M_n = M_u = \phi bd^2 k_n$$

$$bd^2 = \frac{M_u}{\phi k_n}$$

k_n = flexural – resistance factor.

After determining the section dimensions using the previous equation, the tension reinforcement can be determined using the equation from the previous section, $A_s = \frac{M_u}{\phi f_y jd}$

Estimate self weight of rectangular beams:

Method 1: The weight of a rectangular beam will roughly be 10 to 15% of the unfactored loads it must carry.

Method 2: Beam overall depth, ***h*** **=** (***1/18 to 1/12***) of the span length of the beam

Beam width, ***b = 0.5h*** (previously) **=** ***0.8h*** (lately) = usually same as column width.

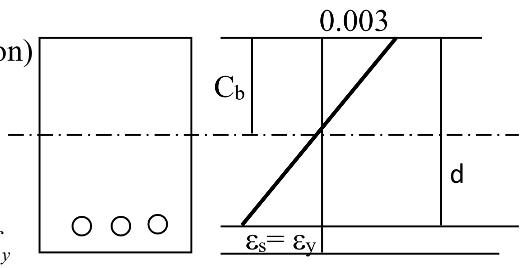
Self weight = γbh

<u>Selection of a Trial Steel Ratio:</u>

The tension reinforcement ratio can be initially selected based on the following considerations:

1. Economic Consideration ***(ρ = 0.01***)
2. Ductility Consideration; for a good level of ductility ($0.35\rho_b \le \rho \le 0.4\rho_b$)

   ρ_b = balanced steel ratio (achieves balanced condition)



   Strain compatibility: $C_b = \dfrac{0.003}{0.003+\varepsilon_y} d$

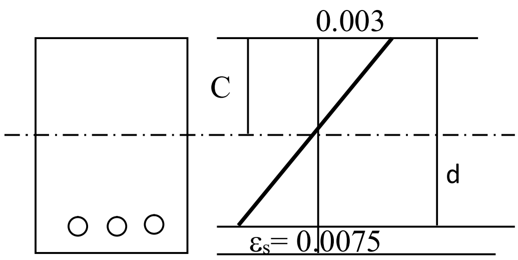
   $A_{sb} = \rho_b bd$

   Equilibrium: $\rho_b bdf_y = 0.85 f_c^{'} \beta_1 C_b b \Rightarrow C_b = \dfrac{\rho_b df_y}{0.85\beta_1 f_c^{'}}$

$$\rho_b = \frac{0.85\beta_1 f_c^{'}}{f_y}\left(\frac{0.003}{0.003+\varepsilon_y}\right) = \frac{0.85\beta_1 f_c^{'}}{f_y}\left(\frac{600}{600+f_y}\right)$$

3. By placing consideration: it may be hard to place the reinforcement if (ρ exceeds 0.015)

<u>**OR:**</u> select a reasonable starting value for the reinforcement ratio that will result in a tension – controlled section (suggested value of tension reinforcement strain, ε_s = 0.0075, this will result in results similar to past practices, shown above).



Strain Compatibility, $C = 0.286d$

$$\rho bdf_y = 0.85 f_c^{'} \beta_1 Cb$$

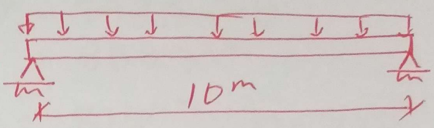
Equilibrium, $\rho = \dfrac{0.24\beta_1 f_c^{'}}{f_y} \cong \dfrac{\beta_1 f_c^{'}}{4 f_y}$

Design Example // Beam Dimension not known

$f'_c = 20 MPa$

$f_y = 420 MPa$; $LL = 25 kN/m$

$DL = 15 kN/m$ (excluding self wt.)



- Calculate ultimate Moment $M_u = w_u L^2/8$

Self. wt. → ① $(10-15)\% \cdot (15+25) = 4-6 kN/m$

② $h = (\frac{1}{18} - \frac{1}{12})(10,000) = 555.5 - 833.3 mm$

Try $h = 800 mm$; $b = 0.5h = 400 mm$

self. wt. $= 0.4 * 0.8 * 24 = 7.68 kN/m$

assume self wt. $= 8 kN/m$

$W_u = 1.2(8+15) + 1.6(25) = 67.6 kN/m$

$M_u = 67.6 * 10^2/8 = 845 kN.m$

- Calc. Beam dimensions $bd^2 = M_u/\phi k_n$

$k_n = f'_c w(1-0.59w)$; $w = \rho f_y/f'_c$

$\rho = 0.01$; $w = 0.01 * \frac{420}{20} = 0.21$

$k_n = 20 * 0.21(1-0.59*0.21) = 3.68$

$\phi k_n = 0.9 * 3.68 = 3.31$

$bd^2 = 845*10^6/3.31 = 255.16*10^6 mm^3$

$b = 400 mm \Rightarrow d = 798.7 mm$

$b = 450 mm \Rightarrow d = 753 mm$

$b = 500 mm \Rightarrow d = 714 mm$

choose $b = 400^{mm}$; $d = 798.7^{mm}$

assume Two layer ⇒ $h = 798.7 + 90$

$= 888.7^{mm}$

Use $h = 900^{mm}$; $d = 810^{mm}$; $b = 400^{mm}$

- Calculate New Mu & revise.

new self wt. $= 0.4 * 0.9 * 24 = 8.64$ kN/m

$W_{u\,new} = 1.2(8.64 + 15) + 1.6(25)$

$= 68.4$ kN/m

$M_{u\,(new)} = 68.4 * 10^2/8 = 855.$ kN.m

If Mu is increased by 10% or more repeat the design.

$\frac{855 - 845}{845} * 100\% = 1.2\% < 10\%$ ok.

proceed with new $M_u = 855$ kN.m

- Calculate $A_s = M_u / \phi f_y j d$

$A_s = 855 * 10^6 / 0.9 * 420 * 0.9 * 810 = 3102.8^{mm^2}$

$a = 3102.8 * 420 / 0.85 * 20 * 400 = 191.64^{mm}$

~~$A_s = 3102.8 * 420$~~

$A_s = 855 * 10^6 / 0.9 * 420 (810 - 191.64/2) = 3167.14^{mm^2}$

$a = 195.62$; $A_s = 3175.98^{mm^2}$

$a = 196.2$; $A_s = 3177.2^{mm^2}$

7 NO.25M
$A_s = 3436^{mm^2}$

Using table A – 4M, 7 No.25M is selected with A_s (provided) = 3436 mm^2

AREA OF BARS (mm^2)

Size of bar (mm)	Number of bars							
	1	2	3	4	5	6	7	8
8	50	101	151	201	251	302	352	402
10	79	157	236	314	393	471	550	628
12	113	226	339	452	566	679	792	905
14	154	308	462	616	770	924	1078	1232
16	201	402	603	804	1005	1206	1407	1609
18	255	509	763	1018	1272	1527	1781	2036
20	314	628	943	1257	1571	1885	2199	2513
22	380	760	1140	1521	1901	2281	2661	3041
25	491	982	1473	1964	2454	2945	3436	3927
32	804	1609	2413	3217	4021	4826	5630	6434
50*	1964	3927	5891	7854	9818	11781	13745	15708

Now we check the two layers assumptions by calculating the minimum width required to fit seven bars in one later based on minimum cover and spacing requirements, as follows:

b_{min} = 2 x 40 + 2 x 10 + 7 x 25 + 6 x 25 + 2 (2 x 10 – 0.5 x 25) = 440 mm > b = 400 mm, therefore, the two layers assumption is okay.

Now we check the minimum area of tension reinforcement:

As_{min} = (0.25 x √20 / 420) x 400 x 810 = 862.5 mm2

As_{min} = (1.4 / 420) x 400 x 810 = 1080 mm^2

As = 3436 mm^2 > As_{min} = 1080 mm^2 (OK)

As was mentioned previously, if the provided area of steel is smaller than the minimum area of steel, then the minimum area of steel should be used as the required, and from table A – 4M a new provided area of steel should be selected based on the minimum.

Now we need to check the tension controlled assumption ($\varepsilon s \geq 0.005$) and the moment capacity ($\varphi Mn \geq Mu$)

T = Cc

a = 3436 x 420 / (0.85 x 20 x 400) = 212.2mm, c = 212.2 / 0.85 = 249.7 mm

εs = 0.003 (810 – 249.7) / 249.7 = 0.0067 > 0.005 (tension controlled, φ = 0.9)

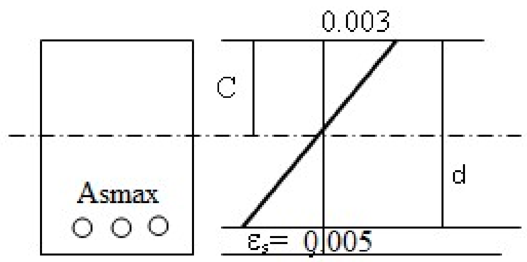
Please note that if the beam was not tension controlled then compression reinforcement should be used. An example will be provided later on.

Mn = 3436 x 420 x (810 – /212.2 / 2) = 1015.8 kN.m

φMn = 0.9 x 1015.8 = 914.2 kN.m > Mu = 857.1 kN.m (OK)

Maximum Area of Tension Reinforcement (A_{smax}):

A_{smax} can be defined as the area of tension reinforcement placed in the section that achieves a tensile strain of ***0.005*** at the ultimate condition.



From strain compatibility and equilibrium:

$$A_{s\max} = \frac{0.319\beta_1 f_c'}{f_y} bd$$

As was explained previously, by increasing the area of steel the strain will decrease, and by decreasing the area of steel the strain will increase. Therefore, if A_{smax} achieves a tensile strain of 0.005 any increase to the area of steel will reduce the strain below 0.005 and the beam will not be in a tension controlled condition, which is rejected by the ACI – Code. That's why A_{smax} is called the maximum area of tension reinforcement.

This information can be used in any of the previous design examples to determine if the beam is in tension controlled or not from the beginning of the design. After calculating As required the As_{max} formula can be used as a check. If As required and provided are less than As_{max} then the beam is tension controlled. If As required is larger than As_{max} the beam will not be in tension controlled and As' should be used, as explained in details in the following example.

Example:

Design the reinforcement for a rectangular beam to resist an ultimate moment of 740 kN.m. Beam width 400mm and overall depth of 600mm. Use f'c = 25MPa, fy = 420MPa.

Assume two layers of tension reinforcement (**small b, and large moment**)

d = 600 – 90 = 510mm

$As = Mu / (\varphi\ fy\ j\ d) = 740000000 / (0.9 \times 420 \times 0.9 \times 510) = 4265.1\ mm^2$

Check As_{max} before doing any iteration

$As_{max} = 0.319 \times 0.85 \times 25 \times 400 \times 510 / 420 = 3292.5\ mm^2$

Since $As > As_{max}$, the beam is not going to be in a tension controlled condition. Therefore, it should be designed as a doubly reinforced beam. Use As'

The suggested procedure for doubly reinforced beams is by dividing the beam into two beams (as was explained previously) and using $As_2 = As_{max}$

$As_2 = 3292.5\ mm^2$.

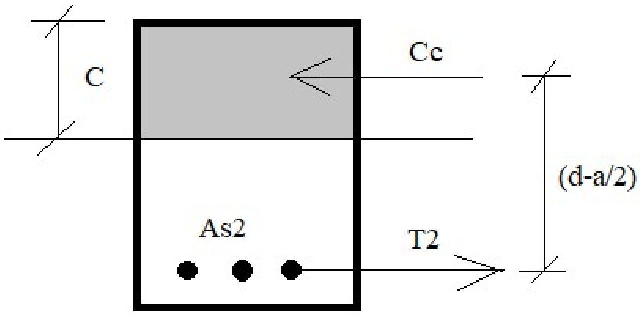
From equilibrium on beam 2

$T_2 = Cc$

$As_2\ fy = 0.85\ f'c\ a\ b$

$3292.5 \times 420 = 0.85 \times 25 \times a \times 400$

a = 162.69 mm, c = 191.4 mm



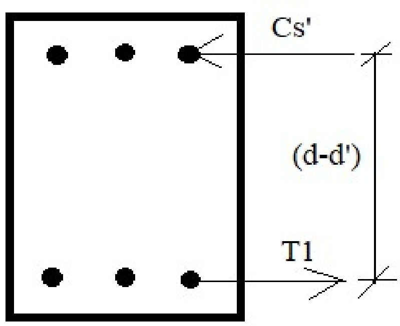
Beam 2

check strain $\varepsilon s = 0.003 \times (510 – 191.4) / 191.4 = 0.005$ (**tension controlled limit. Remember that the As_{max} formula was derived based on $\varepsilon_s = 0.005$. This could be used as a check to your solution**)

$\varphi Mn_2 = \varphi As_2\ fy\ (d – a/2) = 0.9 \times 3292.5 \times 420 \times (510 – 162.69 / 2)$ = 533.5 kN.m

$\varphi Mn = Mu = 740$

$\varphi Mn_1 + \varphi Mn_2 = 740$



Beam 1

φMn_1 + 533.5 = 740

φMn_1 = 206.5 kN.m

The beam is required to resist an ultimate moment of 740 kN.m. Using the maximum area of tension reinforcement that can be placed in a beam, beam2 can provide 533.5 kN.m. Beam 1 is required to provide the remaining moment of 206.5 kN.m and it should be designed for that moment.

Going back to beam 2, As_2 provided in the beam gets the beam to the tension controlled limit, εs = 0.005, but it is not enough to provide enough moment. The problem is that any increase to As over As_2 will reduce the strain and the beam becomes not in tension controlled anymore. Therefore, As_1 has to be added to the beam. Adding As_1 to As_2 will reduce the strain below the tension controlled limit, but with the presence of As', the effect of adding As_1 to the tension side will be eliminated by the presence of As' on the compression side.

<u>Beam 2</u> **$T_2 = Cc$ ($\varepsilon_s = 0.005$)**

Whole beam $T_1 + T_2 = Cc + Cs'$ (the effect of T_1 as a tension force is being eliminated by Cs' as a compressive force, therefore, ε_s remains 0.005)

φMn_1 = 206.5 kN.m

$\varphi Mn_1 = \varphi$ As1 fy (d – d')

206.5 x 106 = 0.9 x As1 x 420 x (510 – 65)

As_1 = 1227.6 mm^2

As (required) = $As_1 + As_2$ = 1227.6 + 3292.5 = 4520.1 mm^2

Select steel from table A – 4M

Use 10 No.25M, As (provided) = 4910 mm^2

AREA OF BARS (mm^2)

Size of bar (mm)	Number of bars							
	1	2	3	4	5	6	7	8
8	50	101	151	201	251	302	352	402
10	79	157	236	314	393	471	550	628
12	113	226	339	452	566	679	792	905
14	154	308	462	616	770	924	1078	1232
16	201	402	603	804	1005	1206	1407	1609
18	255	509	763	1018	1272	1527	1781	2036
20	314	628	943	1257	1571	1885	2199	2513
22	380	760	1140	1521	1901	2281	2661	3041
25	491	982	1473	1964	2454	2945	3436	3927
32	804	1609	2413	3217	4021	4826	5630	6434
50*	1964	3927	5891	7854	9818	11781	13745	15708

Check the two layer assumption

b_{min} = 2 x 40 + 2 x 10 + 10 x 25 + 9 x 25 + 2 x (2 x 10 – 0.5 x 25) = 590 mm > b = 400mm (2-layers assumption is ok)

6No.25M on the first layer from the bottom and 4No.25M on the second layer

b_{min} (for 6No.25M) = 390mm < b = 400mm (OK)

Based on the provided area of tension reinforcement determine a new value for As1

As_1 = As (provided) – As_2 = 4910 – 3292.5 = 1617.5 mm^2

εs' = 0.003 x (191.4 – 65) / 191.4 = 0.00198 < εy

Fs' = E εs' = 200000 x 0.00198 = 396.24 MPa

From equilibrium in beam1 determine As'

T_1 = Cs'

As_1 fy = As' fs'

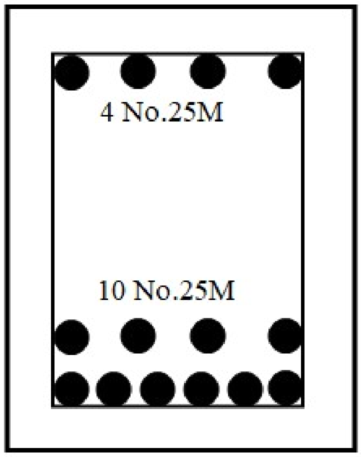
1617.5 x 420 = As' x 396.24

As' = 1714.5 mm^2

Select steel from table A – 4M

Use 4 No.25M, As’ = 1964 mm^2.

A complete analysis should be performed to confirm that the beam is in tension controlled and that $\varphi Mn \geq Mu$



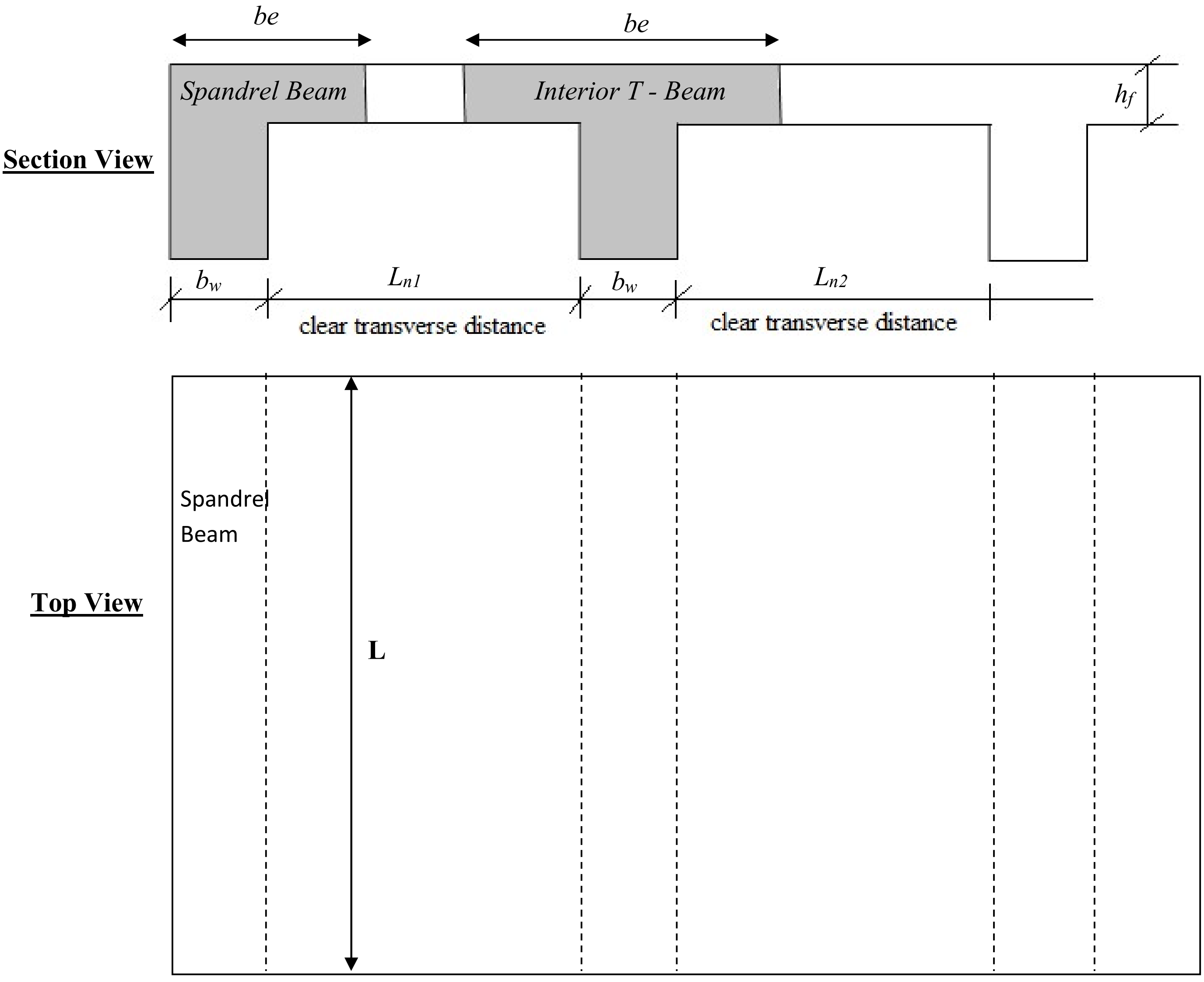
B. DESIGN of T – BEAMS:

Concrete is placed in the beams and slabs in a monolithic pour (**poured at the same time**) as a result the slab serves as the top of the beam. Portion of the slab will work with the beam as a flange and as a result a T – beam will be formed.

Two types of T – beams: (**based on the beam location in the slab**)

1. Interior T – beam has a flange on both sides.
2. Spandrel beam (exterior or inverted L – Shaped beam) has a flange on one side.

The following drawing determines the width of the slab that will work with the beam to form the T – beam. This width is named the effective flange width (***be***). is determined by the ACI – Code as follows:



The ACI – Code definitions for the effective compression flange width for interior and exterior T – beams in continuous floor systems as illustrated in the figure above are calculated as below:

Inverted L – Shaped: $b_e = smaller \begin{Bmatrix} b_w + l_{n1}/2 \\ b_w + 6h_f \\ b_w + L/12 \end{Bmatrix}$

Interior T – beam: $b_e = smaller \begin{Bmatrix} b_w + \dfrac{l_{n1} + l_{n2}}{2} \\ b_w + 2(8h_f) \\ L/4 \end{Bmatrix}$, *L= length of beam span as shown on top view*

In design, if the flange portion of the T – beam is in compression, the value of **j** is to be taken as **0.95** (wide compression area), otherwise, the T – beam is designed exactly as a rectangular beam with **j = 0.9**.

Therefore, it is very important before you start the design process to determine whether the flange portion is in tension or compression.

Per ACI – Code, as was mentioned previously in the analysis of T – beams, for statically determinate members with a flange in tension, A_{smin} shall not be less than value provided in A_{smin} equations for rectangular beams, except that $\mathbf{b_w}$ is replaced by either $\mathbf{2b_w}$ or the width of the flange **be**, whichever is smaller.

In the design of T – beams you are only required to know how to determine the effective flange width and the reinforcement.

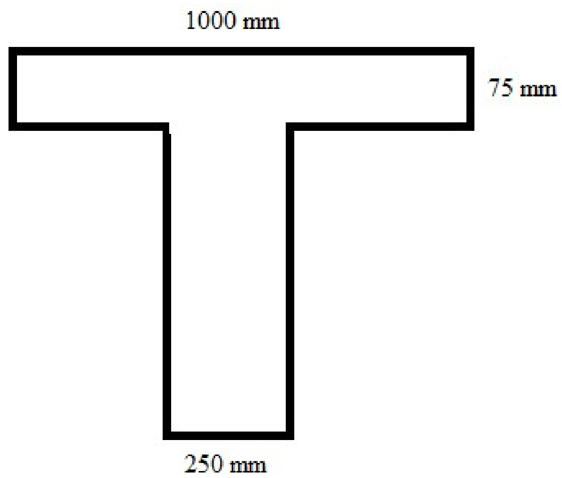
<u>EXAMPLE:</u>

Design the reinforcement of the T – beam shown to resist an ultimate positive moment of 740 kN.m. Use No.25M steel, f'c = 25 MPa, fy = 420 MPa, d = 500 mm.

As = Mu / φ fy j d

As = 740 x 10^6 / (0.9 x 420 x 0.95 x 500) = 4121.4 mm^2

Iterations cannot be performed because if **a** exceeds h_f the moment arm will not be (d – a/2) Therefore *As* required need to be checked based on the computed value of **a** later.



Use 9 No.25M, As = 4418 mm^2

Check As_{min} = 0.25 x √25 x 250 x 500 / 420 = 372 mm^2

= 1.4 x 250 x 500 / 420 = 416.7 mm^2

As = 4418 mm^2 > As_{min} = 416.7 mm^2 (OK)

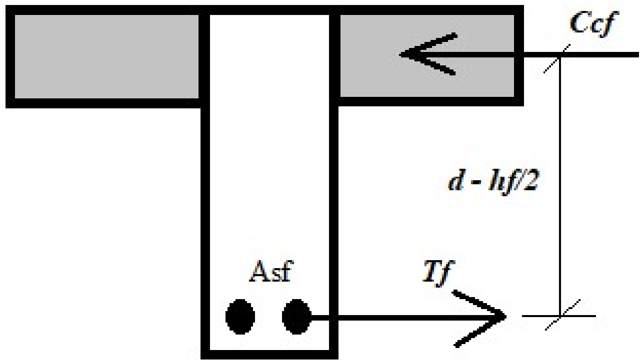
Check the tension controlled condition and φMn

Assume $a \leq h_f$

a = 4418 x 420 / (0.85 x 25 x 1000) = 87.3 mm > h_f

Then analyze as a T – beam

<u>Beam flange (F):</u>



$T_f = Cc_f$

As_f x 420 = 0.85 x 25 x 75 x (1000 – 250)

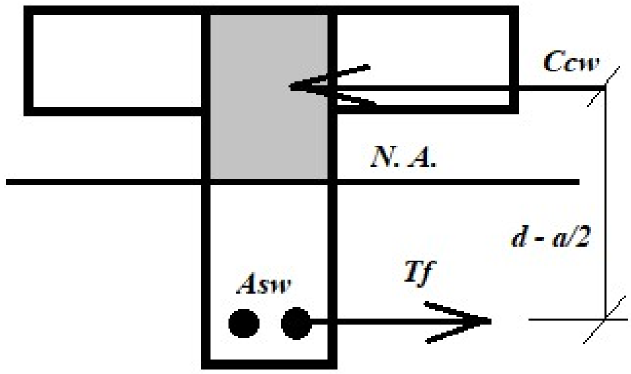
As_f = 2846 mm^2

As = As_f + Asw

4418 = 2846 + As_w

As_w = 1572 mm^2

<u>Beam web (W):</u>



$T_w = Cc_w$

1572 x 420 = 0.85 x 25 x a x 250

a = 124.3 mm, c = 146.2 mm

ε_s = 0.003 (500 – 146.2) / 146.2 = 0.0072 > 0.005 (OK)

Tension controlled

φMn_f = 0.9 x 2846 x 420 x (500 – 75/2) = 497.6 kN.m

φMn_w = 0.9 x 1572 x 420 x (500 – 124.3/2) = 260.2 kN.m

$\varphi Mn = \varphi Mn_f + \varphi Mn_w$

$\varphi Mn = 757.8$ kN.m > $Mu = 740$ kN.m (OK)

Check As required based on the computed value of a.

This step is required since there were no iterations in calculating the required area of steel on the first step. Since beam w is the only beam that has (a) the required φMn_w is calculated as follows:

$\varphi Mn = Mu$

$\varphi Mnf + \varphi Mn_w = Mu$

$497.6 + \varphi Mn_w = 740$

$\varphi Mn_w = 242.4$ kN.m (required) < $\varphi Mn_w = 260.5$ kN.m (provided)

Beam w is required to provide a moment of 242.4 kN.m

$0.9 \times As_w \times 420 \times (500 – 124.3/2) = 242.4 \times 10^6$

$As_w = 1464.6 mm^2$ (**required**) < $As_w = 1572\ mm^2$ (**provided**)

CHAPTER 6

ONE WAY SOLID SLABS

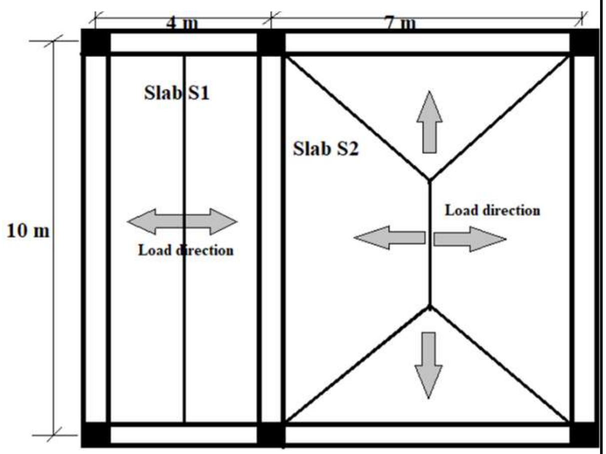
- Slabs are two dimensional elements, called area elements, that can transfer their load to the surrounding beams in one way or two ways. One way solid slabs are designed to transfer applied loads in only one direction. Since flexural stiffness is inversely proportional to the span length, slab panels would be much stiffer in their shorter span direction than in their longer span direction. Therefore, if the slab is surrounded by four beams, only two beams will be carrying the slab loads in one way slabs. Two way slabs, on the other hand, are designed to carry the load in two directions and the four beams surrounding the slab will be carrying the slab loads. In this chapter only one way solid slabs will be considered. Two way slabs will be studies in details in RC 2 course.

- Solid slabs contain no hollow bricks and consist of concrete and steel reinforcement only.
- If the ratio of the longer span length to the shorter span length of the slab is greater than or equal to **2**, it is considered as a one way slab. If the ratio is less than **2** the slab is considered as a two way slab. A common practice in one way solid slabs is to provide flexural reinforcement to resist the entire load in the short direction and only provide minimum steel for temperature and shrinkage effects in the long direction.

- Figure 1: Explanatory Example

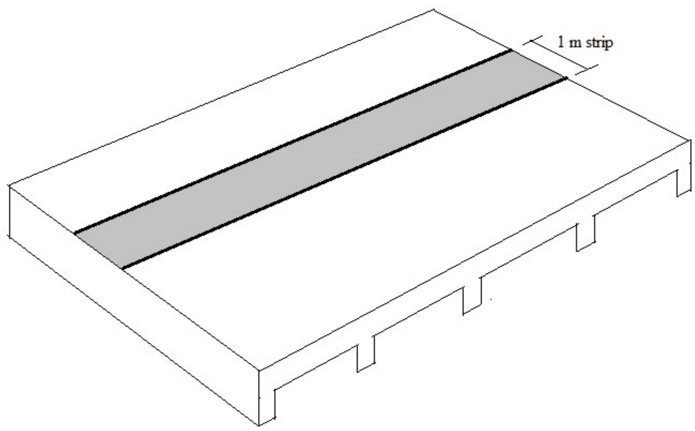
The drawing shows a floor system that contains two solid slabs. Slab S1 (4x10m) and slab S2 (7x10m). Each slab is surrounded by four beams.

Slab S1 is considered one way slab (10/4 = 2.5 > 2.0)
Slab S2 is considered two way slab (10/7 = 1.43 < 2.0)
The one way slab S1 will be transferring the load along the short direction (4m) to the two long beams on the sides of the slab as the arrows clearly show.
Half of the slab load will be transferred to each beam

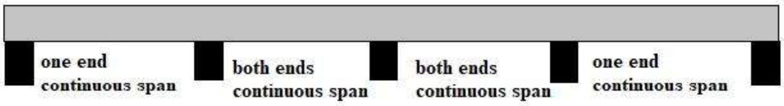


- The two way slab S2 will be transferring the load along the two directions to the four beams surrounding the slab as the arrows clearly show. Trapezoidal and triangular load distribution as shown.
- Assume the ultimate load on the slab = $10kN/m^2$.
- Load transferred from slab S1 to the two long beams = 10 * 4 /2 = 20kN/m
- Load transferred from slab S1 to the two short beams = 0.0

The one way solid slabs are simply designed by assuming a one meter wide strip spanning along the short direction as shown in the figure below. The strip is designed as a rectangular beam.



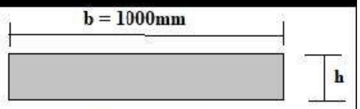
The drawing below show a longitudinal and cross sectional sections of the 1 m strip. The width of the strip is 1000mm and the depth (h) is to be determined based on table 9.5a (h_{min}). To determine the minimum depth, h_{min} for each span of the longitudinal section is to be determined using the table and the larger value of all of them should be the one selected.



Longitudinal section in the 1-m strip

TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE CALCULATED

	Minimum thickness, *h*			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	ℓ/20	ℓ/24	ℓ/28	ℓ/10
Beams or ribbed one-way slabs	ℓ/16	ℓ/18.5	ℓ/21	ℓ/8



cross section in the 1-m strip

- **<u>Slab Thickness:</u>**
- A one meter wide strip is assumed along the short direction and then designed as a wide rectangular beam of 1 m in width. ***The thickness of the slab is selected based on the minimum thickness provided by the ACI – Code (Table 9.5 a)*** to avoid deflection calculations or so that deflection is within the ACI – Code limits. Except for heavily loaded slabs (supporting several feet of earth) the slab thickness is chosen so that deflections will not be a problem.
- **<u>Minimum Concrete Cover:</u>**
- Minimum concrete cover = 20 mm (normal exposure; not exposed to weather or in contact with the ground)
- Minimum concrete cover = 40 mm (exposed to weather or ground)

- **Minimum Amount of Tension Reinforcement:**

For grade 60 steel As_{min} = 0.0018 b h

For grade 40 or 50 steel As_{min} = 0.002 b h

- **Per ACI – Code Shrinkage and Temperature Reinforcement** (As_{min}) is required perpendicular to the span of the slab (long direction). Slabs are thinner than beams supporting it. Therefore, concrete in slabs shrinks more rapidly than concrete in beams.

ACI Moment and Shear Coefficients for Analysis and Design of Non-Prestressed One Way Slabs and Continuous Beams:

Large parts of the ACI – Code were developed and written before the accessibility to structural analysis software's. Limitations of the method include:

- There are two spans or more
- Spans are approximately equal, difference in length less than 20% of the shorter span
- Loads are uniformly distributed
- Unfactored live load does not exceed three time the unfactored dead load
- Members are prismatic (have same cross section along the whole length of the beam or slab)

$$Mu = Cm(WuLn^2)$$

$$Vu = Cv(\frac{WuLn}{2})$$

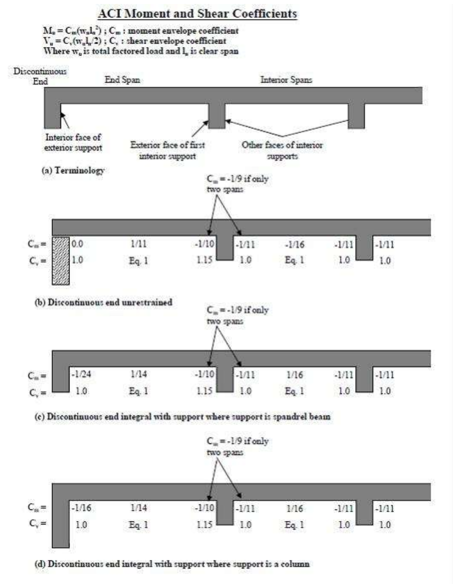
Cm and Cv values are obtained from the chart on the next pages, based on the supporting system and the location moment or shear is being considered.

ACI Moment and shear coefficients method (per ACI – Code)

CODE

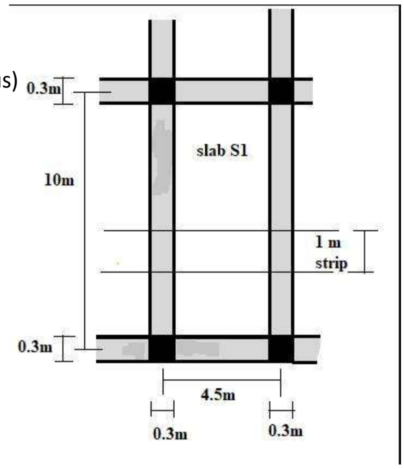
Discontinuous end integral with support	$w_u \ell_n^2/14$
Interior spans	$w_u \ell_n^2/16$
Negative moments at exterior face of first interior support	
Two spans	$w_u \ell_n^2/9$
More than two spans	$w_u \ell_n^2/10$
Negative moment at other faces of interior supports	$w_u \ell_n^2/11$
Negative moment at face of all supports for	
Slabs with spans not exceeding 10 ft; and beams where ratio of sum of column stiffnesses to beam stiffness exceeds eight at each end of the span	$w_u \ell_n^2/12$
Negative moment at interior face of exterior support for members built integrally with supports	
Where support is spandrel beam	$w_u \ell_n^2/24$
Where support is a column	$w_u \ell_n^2/16$
Shear in end members at face of first interior support	$1.15\, w_u \ell_n/2$
Shear at face of all other supports	$w_u \ell_n/2$

ACI Moment and shear coefficients method (per ACI – Code)



EXAMPLE

Slab S1 shown below is part of a floor system that resists a service dead load of 3 kN/m^2 (excluding self weight) and a service live load of $4 kN/m^2$. Use f'c = 28 MPa, fy = 414 MPa. Use No. 16M steel.

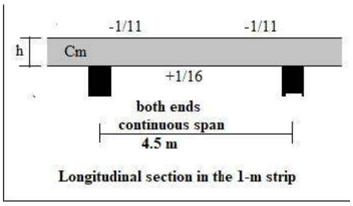


Estimate thickness of slab:

From table 9.5a, h_{min} = L / 28 (both ends continuous)

h_{min} = 4500 / 28 = 160 mm

Use h = 180mm



- Compute the factored loads and moments:

Self weight = 0.18 * $24 kN/m^3$ = $4.32 kN/m^2$

Wu = 1.2 (3 + 4.32) + 1.6 (4) = $15.18 kN/m^2$

Ln = 4.5 – (0.3/2) – (0.3/2) = 4.2 m

Mu = Cm Wu Ln^2

Mu (+ve) = (1/16) x 15.18 x 4.2^2 = 16.74 kN.m

Mu (-ve) = (1/11) x 15.18 x 4.2^2 = 24.34 kN.m

- <u>Design the reinforcement</u> for the positive and negative moments. Remember that the positive steel is at the bottom and the negative steel is at the top.

As = Mu / φ fy j d (**j = 0.95**, wide compression area)

d = 180 – 20 – 16/2 = 152mm

In slabs d $\neq$ h – 65, it should be calculated based on cover and steel diameter (no stirrups in slabs)

- <u>Design for Mu (+ve) = 24.34 kN.m</u>

As (required) = 24.34 x 10^6 / (0.9 * 414 * 0.95 * 152) = 452.4mm^2/m (**this area of steel is per one meter width of the slab, 1 – m strip**)

As_{min} = 0.0018 x 1000 x 180 = 324mm^2/m (**generally, most of one way solid slabs are deigned based on As_{min}. Small loads, small moments and therefore, small steel is required**)

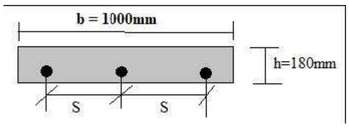
As > As_{min} (do iterations)

a = 452.4 * 414 / (0.85 x 28 x 1000) = 7.87mm

As = 24.34 x 10^6 / (0.9 x 414 x (152 – 7.87/2)) = 441.2 mm^2/m

a = 7.67 mm, As = 440.9 mm^2/m, a = 7.67mm (convergence)

In determining the reinforcement we do not select number of bars, instead spacing between the steel bars is selected based on the following
There is one bar (Ab) in each spacing (S) and in 1000mm there is As, therefore,
S = Ab x 1000 / As,   Ab = cross sectional area of one bar (Ab = 199mm^2 for No. 16M)
S = 199 x 1000 / 440.9 = 451 mm (**required spacing**)
The spacing needs to be rounded up to a number to simplify construction. **Say 450mm**.
Rounding up should be to a smaller number not to a larger number. Note that less spacing means more *As* in one meter, and more spacing means less *As* in one meter. Therefore, to achieve *As* provided more than *As* required, the spacing should be rounded up to a smaller number.
Per ACI – Code, S_{max} = smaller of (3h or 450mm) (**ACI limits maximum spacing**)
3h = 3 x 180 = 540mm. Therefore, S_{max} = 450 mm
USE S = 450mm (**provided spacing**), this spacing should be used for the whole slab.
The provided *As* in one meter strip can be calculated using the provided spacing as follows:
As (provided) = 1000 x 199 / 450 = 442.22 mm^2/m > As (required) = 440.9 mm^2/m.



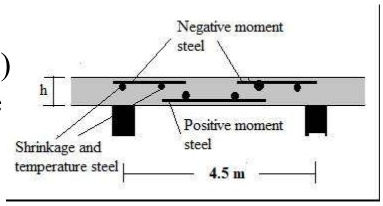
- Design reinforcement for the (+ve) Mu = 16.74 kN.m

As = 16.74 x 10^6 / (0.9 x 414 x 0.95 x 152) = 311 mm^2/m
As_{min} = 324 mm^2/m
As < As_{min}. Therefore, Use As_{min}
As = As_{min} = 324mm^2/m
S = 1000 x 199 / 324 = 614.2mm > S_{max} = 450mm
Use S = S_{max} = 450mm

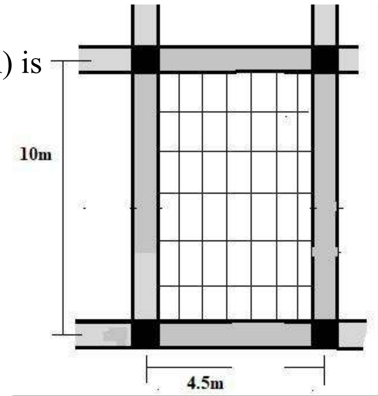
- Per ACI – Code, shrinkage and temperature reinforcement (As_{min}) is required perpendicular to the span of the slab. (**along the longer direction**)

As_{min} = 324 mm^2/m
Using No.12M steel for the shrinkage and temperature reinforcement (Ab = 113mm^2)
S = 1000 x 113 / 324 = 348.8mm (round to 330mm)
Per ACI – Code S_{max} (for shrinkage and temperature steel) = smaller of (5h or 450mm)
5h = 5 x 180 = 900mm. Therefore, S_{max} = 450mm
S = 330mm < Smax = 450mm. **Use S = 330mm**.

Steel along the short direction (4.5m) is based on the positive and negative moments on the slab. **1No.16M at 450mm** (top and bottom)



Steel along the long direction (10m) is for shrinkage and temperature reinforcement an always is As_{min}. **1No.12M at 330mm**.



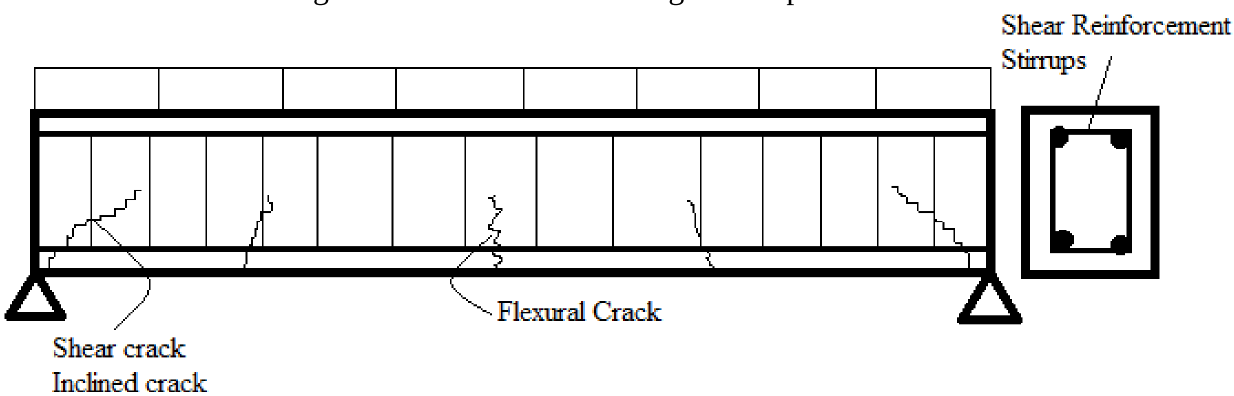
CHAPTER 7

SHEAR IN BEAMS

So far we have learned how to use the BMD to design rectangular and none rectangular beam sections and the amount of reinforcement; tension only or tension and compression.

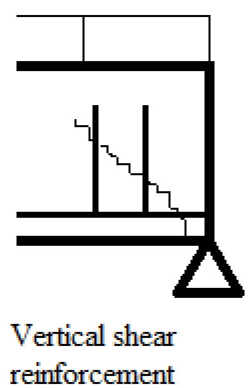
In this chapter we will learn how to use the SFD to design the beam section in order to resist the shear forces generated from loads.

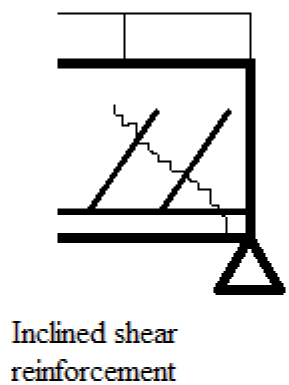
Shear reinforcement, web reinforcement or stirrups are used to resist the shear forces on the beam and to hold the longitudinal flexural reinforcing bars in place.



The beam above shows a simply supported beam with uniform distributed loading. Flexural and inclined cracks are generated near the midpoint and the ends of the beam, respectively. Flexural cracks are caused by flexure (flexural stresses exceed the flexural capacity of the beam). They occur near the midspan of the beam, point of maximum moment and zero shear. Inclined / Shear cracks are caused by shear (shear stresses exceed the shear capacity of the beam). They occur near the ends of the beam, points of maximum shear and zero moments. The two figures below show the types of shear reinforcement:

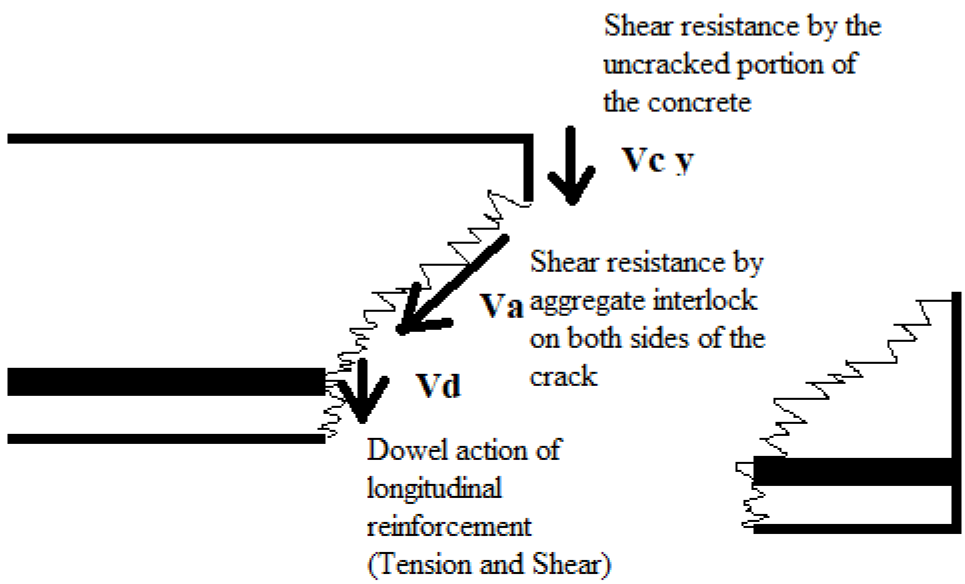
Inclined shear reinforcement is the one will be explained in this chapter.





Internal Forces in a Beam with and without Stirrups:

The figure below shows a beam with shear / inclined crack. The figure shows the resistance forces that generate the total resistance provided by the section if the section has no stirrups. The beam section resistance is generated by Vc_y, Va_y (Y – component of Va), and V_d. These three forces provide the concrete shear strength V_c.



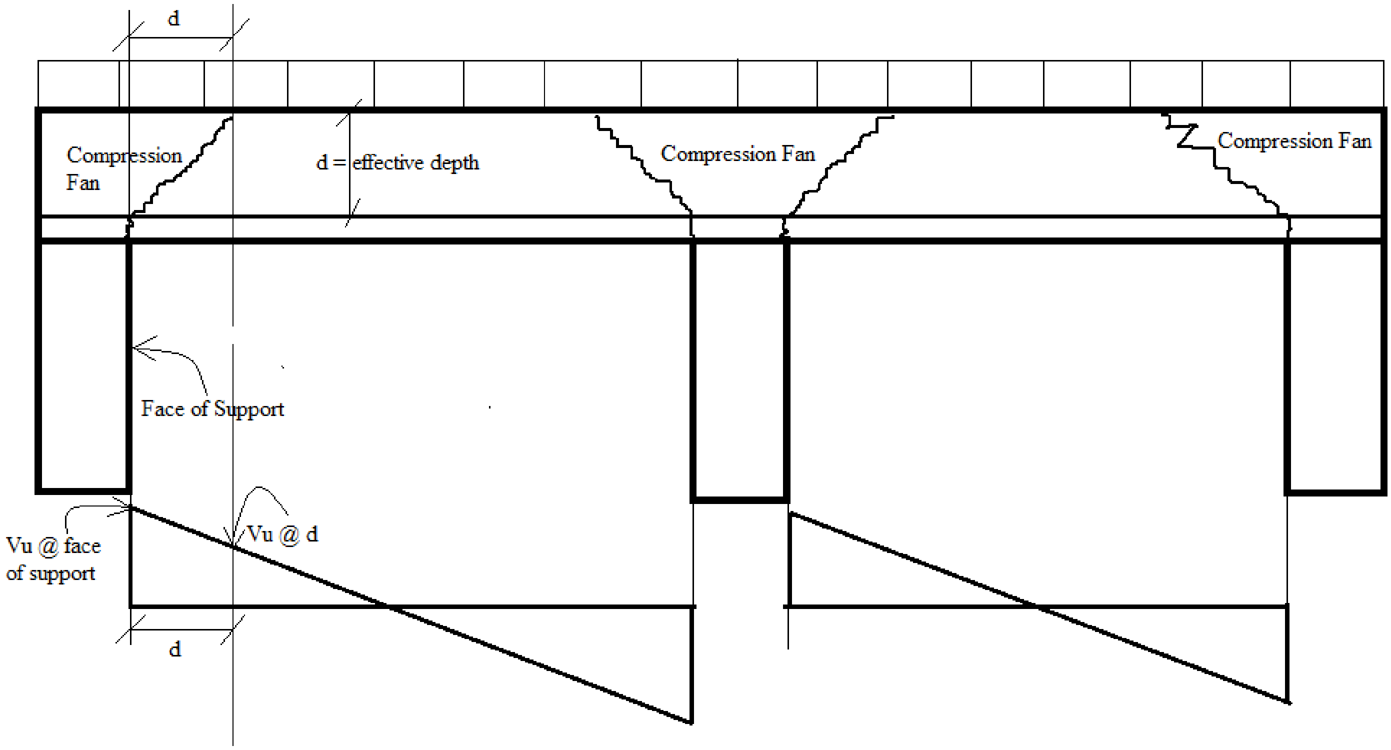
$$\mathbf{V_c = Vc_y + Va_y + V_d}$$

By the presence of shear reinforcement / stirrups, the nominal shear strength, V_n becomes:

$\mathbf{V_n = V_c + V_s}$, where Vc is the shear carried by the concrete and Vs is the shear carried by the stirrups.

Basic safety equation in shear is $\boldsymbol{\varphi V_n \geq V_u}$, where V_u is the ultimate / maximum shear generated by the load from SFD.

Location of Maximum Shear / Critical Section for the Design of Beams



Loads applied to the beam within a distance d from the face of support will be transmitted directly to the support by the compression fan above the 45° crack and will not affect the stresses in the stirrups crossing the cracks.

For non – prestressed members, sections located less than a distance d from face of support may be designed for the same shear, V_u @d, as that computed at a distance d from face of support.

This means that the ultimate shear V_u can be either at the face of support or at a distance **d** from face of support. V_u is at a distance d from face of support only if:

1. Support reaction introduces compression into the end region of a member.
2. Loads are applied on the top of the beam.
3. No concentrated forces within a distance d from face of support.

Analysis and Design of Reinforced Concrete Beams for Shear

$\varphi \mathbf{V_n} \geq \mathbf{V_u}$, $\varphi = 0.75$

$\mathbf{V_n} = \mathbf{V_c} + \mathbf{V_s}$, $\quad V_c = \frac{\sqrt{f_c'}}{6} b_w d$

Shear reinforcement is provided by spacing not number of stirrups. In a way similar to solid slab reinforcement by providing spacing between bars not number of bars.

Case I: $V_u \le \varphi V_c / 2$

No need for shear reinforcement. **Stirrups will need to provided only to hold the longitudinal reinforcement in place, not to resist shear forces.**

Case II: $\varphi V_c / 2 < V_u < \varphi V_c$

Minimum shear reinforcement is required. Spacing of stirrups should be determined based on the maximum spacing determined from following options:

$Av = 2Av_1$

Av_1 is the cross sectional area of 1 No.10M (**the steel used for the stirrups**)

$$S_{\max} = smaller\ of \begin{cases} \dfrac{16A_v f_y}{\sqrt{f_c^{'}}b_w} \\ \dfrac{A_v f_y}{0.33b_w} \\ \dfrac{d}{2} \\ 600\text{mm} \end{cases}$$

Case III: $V_u > \varphi V_c$, **and** $V_s \le \frac{2}{3}\sqrt{f_c^{'}}b_w d = 4V_c$

$V_u = \varphi V_n = \varphi (V_c + V_s)$

V_s **(required)** $= (V_u / \varphi) - V_c$

Spacing between stirrups should be determined based on

$$S_{\max} = smaller\ of \begin{cases} \dfrac{A_v f_y d}{V_s} \\ \dfrac{16A_v f_y}{\sqrt{f_c^{'}}b_w} \\ \dfrac{A_v f_y}{0.33b_w} \\ \dfrac{d}{2} \text{ or } 600\text{mm},\ for\, V_s \le 2V_c \\ \dfrac{d}{4} \text{ or } 300\text{mm},\ for\, V_s > 2V_c \end{cases}$$

If V_s exceeds $4V_c$ the section needs to be enlarged, the concrete section (determined previously beased on the moment) is not carrying enough shear. This means that $4V_c$ is considered as Vs_{max}. Therefore,

$Vu_{max} = \varphi(V_c + Vs_{max}) = \varphi(V_c + 4V_c) = \varphi\ 5\ V_c \ge V_u$ (**either @ face of support or @d**)

This means that for the section to be enough V_u (from SFD) should not exceed (φ 5 V_c), otherwise the section should be enlarged (increase b or d).

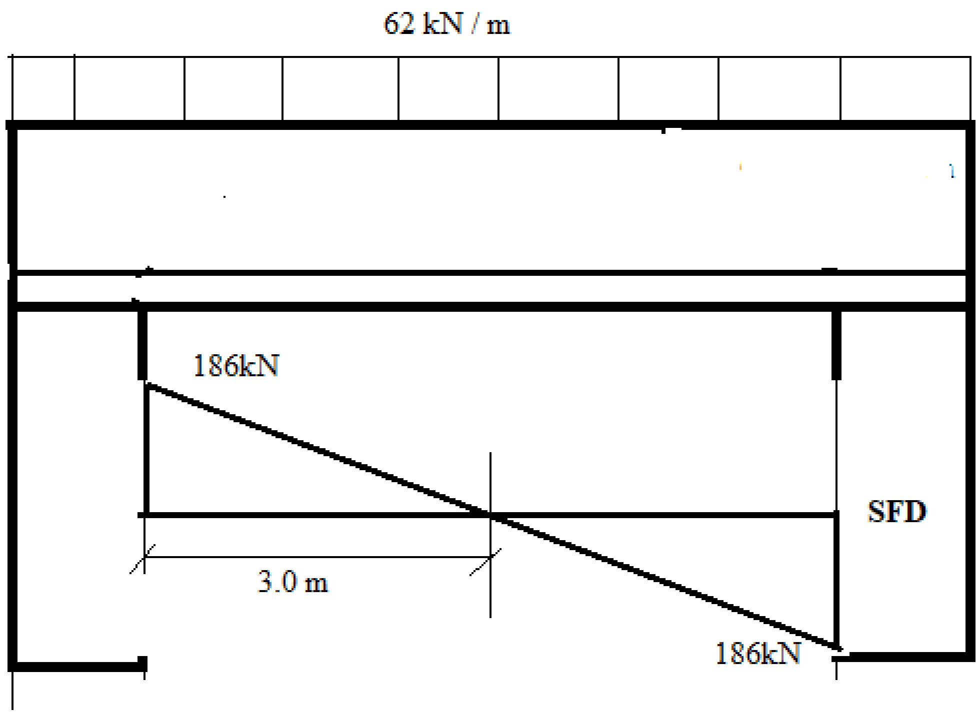
Example: A 6m long simply supported beam is loaded by ultimate load of 62 kN/m. The beam has a rectangular section of 400mm in width with an effective depth of 500mm. Design the beam for shear using No.10M for stirrups. Use f'c = 28MPa and fy = 300MPa.

First of all determine the reactions for the beam, which was found to be 186kN.

Draw the SFD as shown.

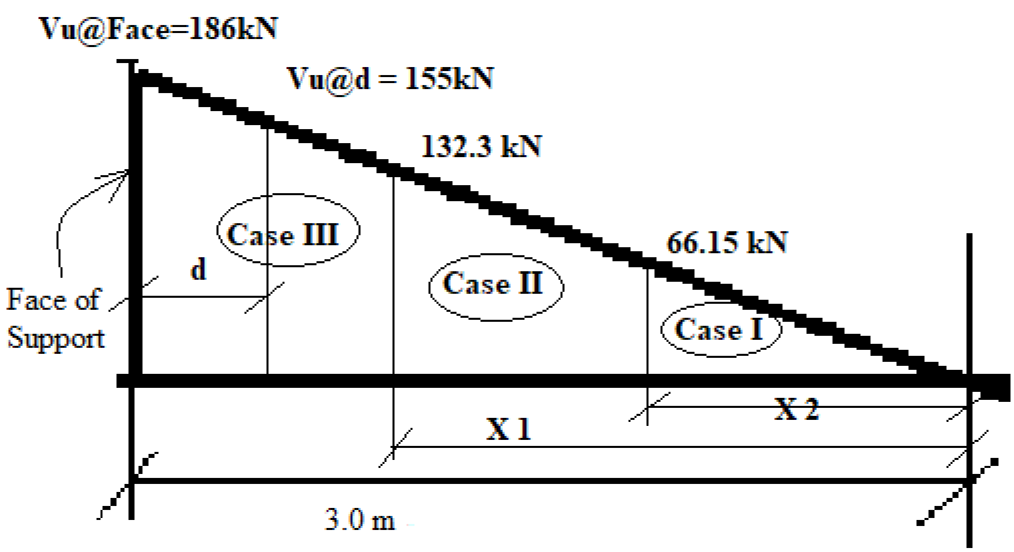
The three conditions explained previously applies, so, Vu to be used in the design should be calculated at a distance **d** from face of support.

Vu @ face of support = 186 kN



Vu @ d = 186 – 62 x 0.5 = 155 kN

Since the SFD is symmetrical, half of the beam will be designed and then reflected on the other half.



Calculate Vc, $Vc = \frac{\sqrt{28}}{6} x400x500 / 1000 = 176.4kN$

Calculate φVc = 0.75 x 176.4 = 132.3 kN and φVc/2 = 66.15 kN

Show φVc and φVc/2 on the SFD, as shown above

Check if the section is large enough

Vu_{max} = 0.75 x 5 x 176.4 = 661.4 kN

Vu @ d = 155 kN < Vu_{max} (**Section is okay**)

Determine distances *X1* and *X2*

186 / 3 = 132.3 / X1 = 66.15 / X2 (X1 = 2.13m, X2 = 1.07m)

Design for Case I, Vu ≤ φVc / 2

No need for shear reinforcement, but stirrups should be provided to hold the longitudinal reinforcement in place. So use

S = 300 mm

Design for Case II, φVc / 2 < Vu < φVc

Based on the calculated spacing on the side, S_{max} = 250mm.

Therefore, Use S = 250mm

$$S_{max} = smaller\ of \begin{cases} \dfrac{16A_v f_y}{\sqrt{f_c^{'}}b_w} = \dfrac{16x157x300}{\sqrt{28}x400} = 356mm \\ \dfrac{A_v f_y}{0.33b_w} = \dfrac{157x300}{0.33x400} = 356.8mm \\ \dfrac{d}{2} = \dfrac{500}{2} = 250mm \\ 600\text{mm} \end{cases}$$

Design for Case III, Vu > φVc, and $V_s \leq \frac{2}{3}\sqrt{f_c^{'}}b_w d = 4V_c$

$$Vs(required) = \frac{155}{0.75} - 176.4 = 30.3kN$$

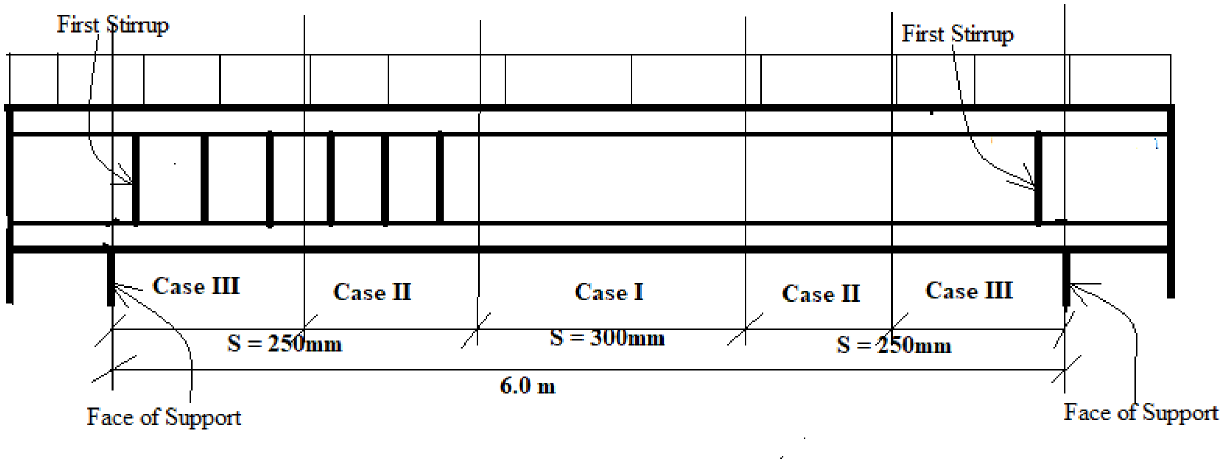
Vs < 2Vc, then ignore 300mm and d/4 from spacing choices.

Based on the calculated spacing, S_{max} = 250mm.

Therefore, Use S = 250mm

$$S_{max} = smaller\ of \begin{cases} \dfrac{A_v f_y d}{V_s} = \dfrac{157x300x500}{30.3x1000} = 777.2mm \\ \dfrac{16A_v f_y}{\sqrt{f_c^{'}}b_w} = 356mm \\ \dfrac{A_v f_y}{0.33b_w} = 356.8mm \\ \dfrac{d}{2} = 250\text{mm} \\ 600\text{mm} \end{cases}$$

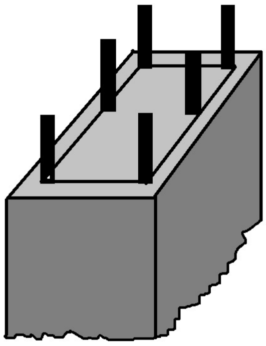
Note that the first stirrup is to be placed at a distance either 100mm or S / 2 from face of support.



CHAPTER 8

COLUMNS: Combined Axial Load and Bending

SHORT COLUMNS

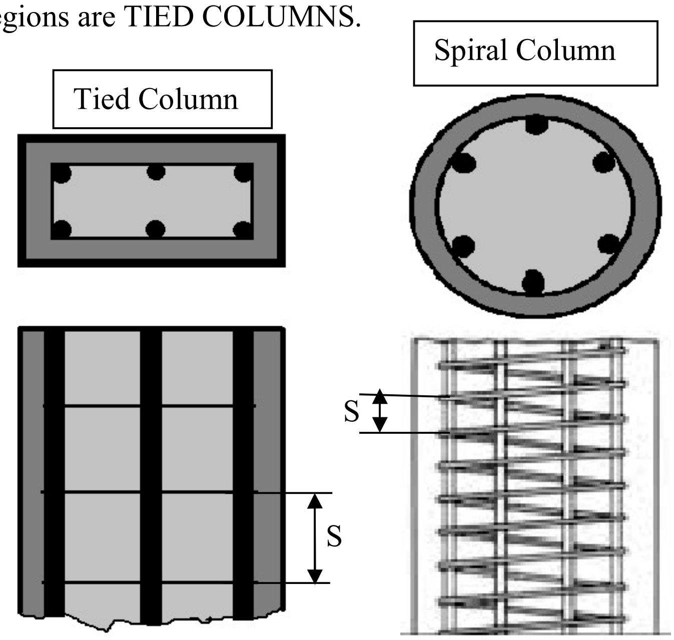


A column is a vertical structural member supporting axial compression loads with or without moments. Moments could be induced by misalignment of the load on the column, or unbalanced moments at the end of beams supported by the column.

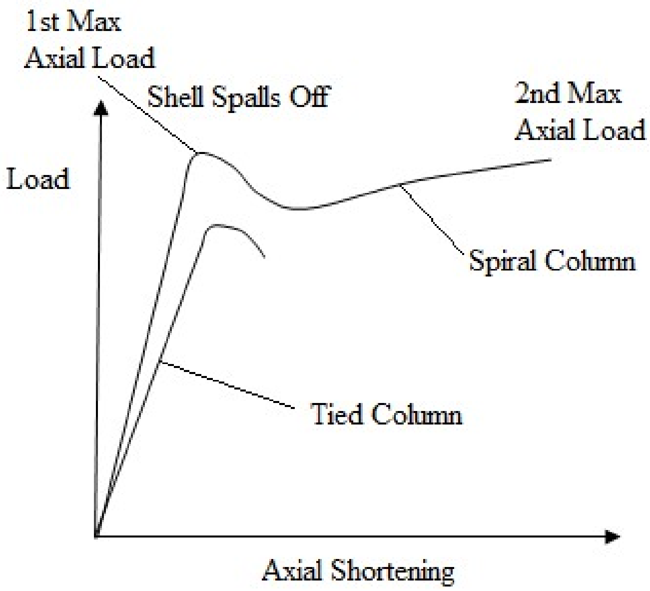
If the moments induced by slenderness effects weaken a column appreciably, it is referred to as slender column or long column. The great majority of concrete columns are sufficiently stocky that slenderness can be ignored. Such columns are referred to as short columns.

Over 95% of columns in buildings in non – seismic regions are TIED COLUMNS.

When high strength and / or high ductility (seismic regions) are required SPIRAL COLUMNS are used.



<u>Behavior of Tied and Spiral Columns:</u>



Initial parts if tied and spiral are similar. As the maximum load is reached, vertical cracks and crushing develop in the concrete shell outside the ties and spirals and the concrete spalls off.

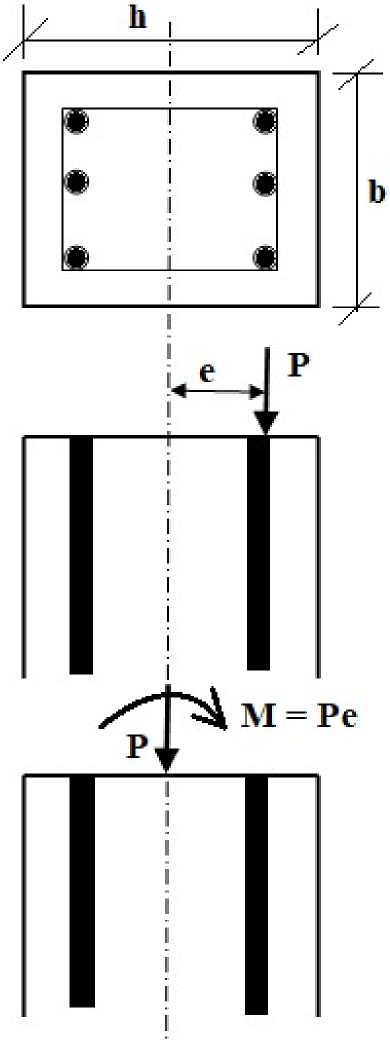
When this occurs in a tied column, the capacity of the core is less than the load on the column, the concrete in the core is crushed and reinforcement buckles outward between ties. This occurs suddenly without warning in a brittle manner.

When this occurs in a spiral column, the column does not fail immediately because the strength of the core has been enhanced by spirals. The column can undergo large deformations; eventually reaching a second maximum load when the spiral breaks and the column collapses.

In tied columns, **TIES** provide relatively little lateral restraint to the core, have little effect on the strength of the core, and they do however act to reduce the unsupported length of the longitudinal bars, thus reducing the danger of buckling of those bars. In spiral columns, **SPIRALS** act to restrain the lateral expansion of the column core, therefore, delaying the failure of the core, and making the column more ductile.

INTERACTION DIAGRAM (I.D.):

Almost all compression members in concrete structures are subjected to moments in addition to axial loads.



Compressive Strength $f_{cu} = \frac{P}{A} + \frac{My}{I}$

$$\frac{P}{f_{cu} A} + \frac{My}{f_{cu} I} = 1.0 \text{..........(1)}$$

If M = 0.0, $P / (f_{cu}\, A) = 1.0$, $P = f_{cu}\, A = P_{max}$

P_{max} = the maximum axial load the column can support (in the absence of moment)

$f_{cu} = P_{max} / A$**...........(2)**

If P = 0.0, $M\, y / (f_{cu}\, I) = 1.0$, $M = f_{cu}\, I / y = M_{max}$

M_{max} = the maximum moment the column can support (in the absence of axial load)

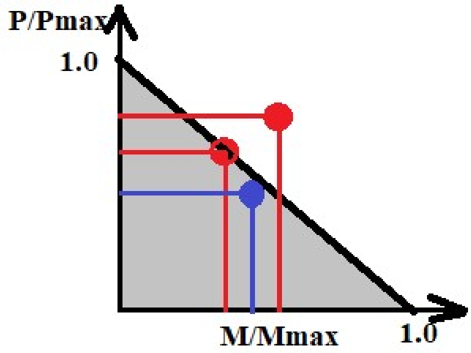
$f_{cu} = M_{max}\, y / I$**..........(3)**

Substitute eq. (2) in first part of eq. (1), and eq. (3) in second part of eq. (1)

$$\frac{P}{P_{max}} + \frac{M}{M_{max}} = 1.0 \text{ (Interaction Equation)}$$

Drawing the interaction equation will produce the interaction diagram (I. D.)

Points inside the diagram represent a combination of axial load and bending moment that will not cause failure to the column. It will be within the capacity of the column.

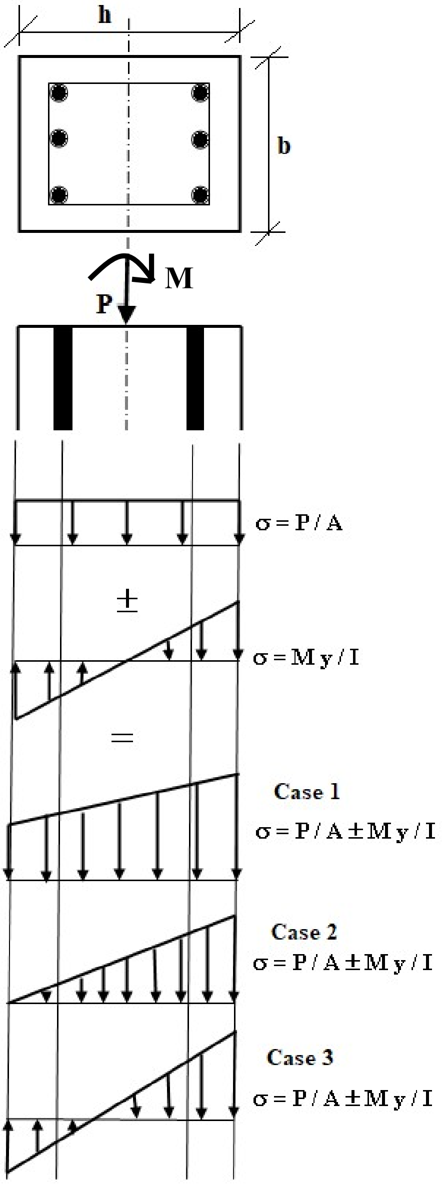


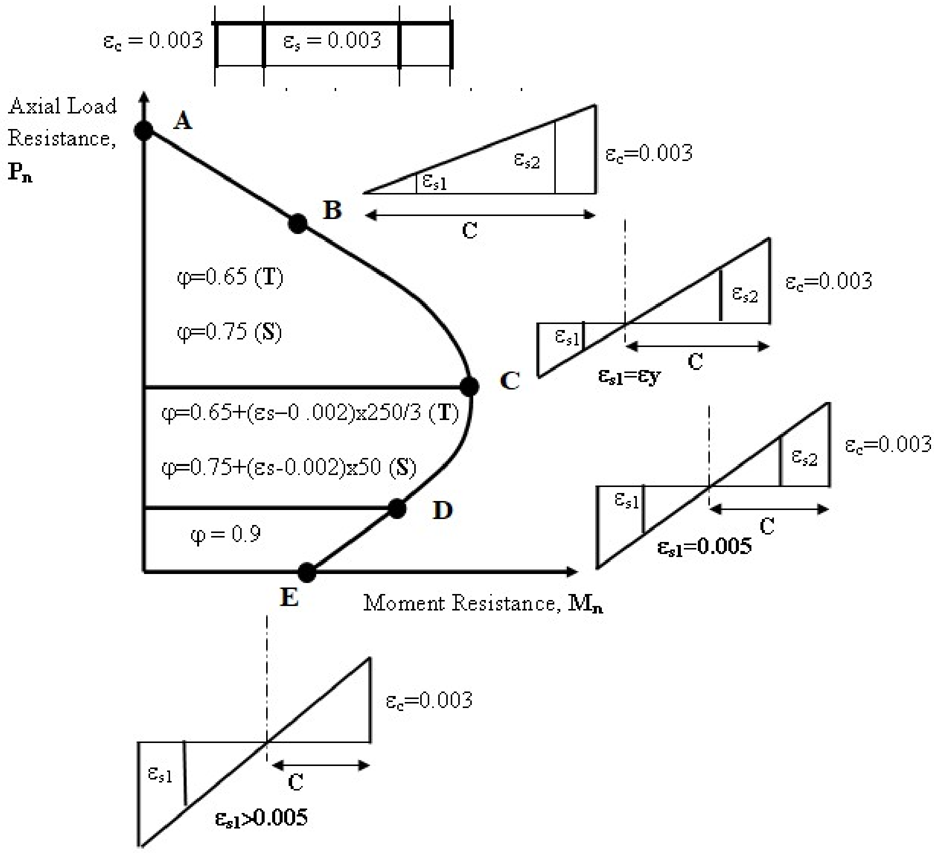
Points falling on or outside the line will equal or exceed the capacity of the resistance on the column section and hence will cause failure.

This shape of I.D. is valid for elastic columns.

Interaction Diagram for Concrete Columns:

Since concrete is not an elastic material, the calculation of the interaction diagram is more complex. Five points are determined on the interaction diagram:





Point A: Pure Axial Load, (***M_n = 0.0***)

Point B: Zero Tension / Onset of Tension

Point C: Balanced Condition ($\varepsilon_s = \varepsilon_y$)

Point D: Tension Controlled Condition (***ε_s = 0.005***)

Point E: Pure Moment, (***P_n = 0.0***)

Coordinates of each point (***M_n, P_n***) need to be determined in order to construct the I.D. Each point can be determined based on certain strain distribution.

This I. D. is called Nominal I. D. because it's build based on **M_n** and **P_n**. After introducing φ into **P_n** and **M_n**, the Design I. D. is built.

At point **A** (pure axial load), the axial load is at its maximum value and the moment is zero. So the stress is only axial stress **P / A**.

When we move from point A towards point B, the axial load is reduced and the moment has a small value. So two stresses will affect on the column; compressive axial stress **P / A** and bending stress **M y / I** tension of the left side and compression on the right side of the section. Since the value of the axial stress is larger than the bending stress, the resultant stress will still be compression on the whole section, as shown in **case 1** in the figure.

As we move closer to point B, more reduction to the axial load and the moment is increased. This means less value for **P / A** and more value for **M y / I** until **P / A** is equal to **M y / I**, as shown in **case 2** in the figure. This is point **B** on the I. D.

As we move from point B towards point C on the I. D. more reduction to the axial load and the moment is increased even more. This means that **M y / I** becomes larger than **P / A**, therefore, the resultant on the tension side becomes tension, as shown in **case 3** in the figure.

Point D represents the tension controlled limit, where the tensile strain equals 0.005

Point E represents the pure moment condition, where $P_n = 0.0$. The tensile strain at this point exceeds the tension controlled limit ($\varepsilon s > 0.005$). In determining M_n the column is analyzed as a beam with ignoring the steel in the compression side of the column.

POINT A:

Pure Axial Load

$M_n = 0.0$

$P_n = P_{n\,max} = 0.85\,f_c'\,(Ag - A_s) + A_s\,f_y$

POINT B:

Zero tension / onset of tension

C = h (depth of the neutral axis)

Point B represents the point after which tension starts in the section. Up to point B the whole section is still under compression.

POINT C:

Balanced Condition

$\varepsilon_{s1} = \varepsilon y$

C is determined based on strain compatibility.

POINT D:

Tension Controlled Limit Condition

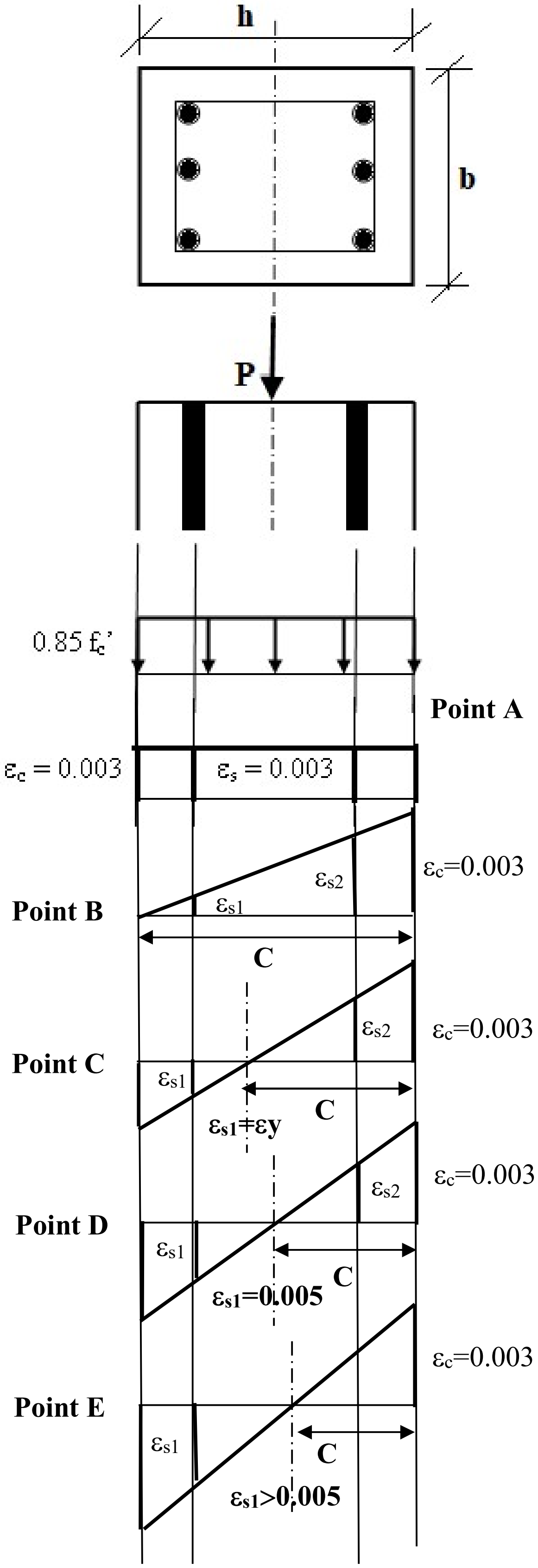
$\varepsilon_{s1} = 0.005$

C is determined based on strain compatibility.

POINT E:

Pure Moment Condition

$\varepsilon_{s1} > 0.005$

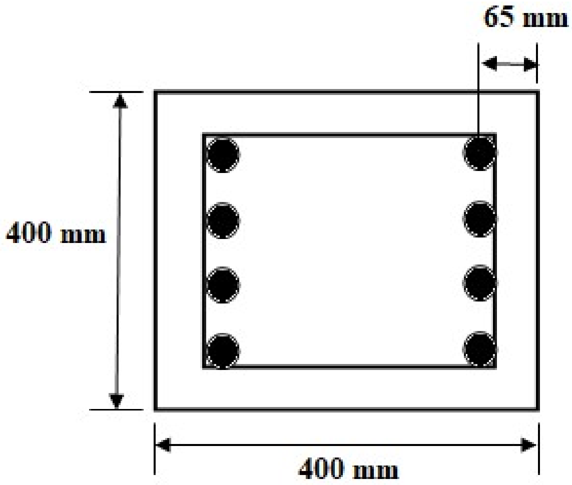


Maximum Axial Load:

The maximum axial load is achieved at point A on the I. D.

$$\varphi P_n \max = \varphi\left[0.85 f'_c \left(A_g - A_{st}\right) + A_s f_y\right]$$

To account for the effect of accidental moments the ACI – Code specify that the maximum load on a column must not exceed **0.85 φP_n max** for spiral columns and **0.80 φP_n max** for tied columns.



EXAMPLE: Calculation of I. D. of a Tied Column

$f'_c = 35$ MPa, $f_y = 420$ MPa, $A_s = 8$ No. 29M $= 5160\ mm^2$

Find the coordinates of the five points on the I. D. As explained previously, each point corresponds to a specific strain distribution.

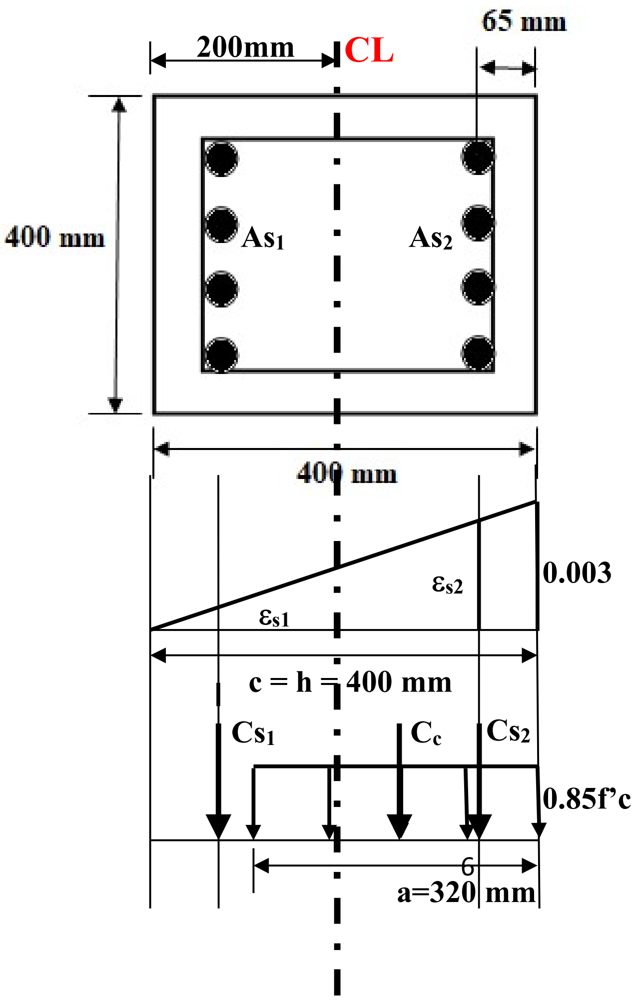
POINT A: (Pure Axial Load)

$M_n = 0.0$

$P_n = 0.85\ f_c'\ (A_g - A_s) + A_s\ f_y$

$P_n = 0.85 \times 35 \times (400 \times 400 - 5160) + 5160 \times 420$

$P_n = 6773.7$ kN.



POINT B: (Zero Tension)

$c = h = 400$mm

$a = \beta_1 c.\ \beta 1 = 0.85 - 0.05 \times (35 - 28) / 7 = 0.80$

$a = 0.8 \times 400 = 320$ mm

From strain compatibility determine strains

$0.003/400 = \varepsilon s_1/65 = \varepsilon s_2/(400-65)$

$\varepsilon s_1 = 0.0004875$ ($fs_1 \neq fy$) $fs_1 = 97.5$ MPa

$\varepsilon s_2 = 0.00251 > \varepsilon y$, $fs_2 = fy$

$As_1 = As_2 = As / 2 = 5160 / 2 = 2580\ mm^2$

$P_n = Cc + Cs_1 + Cs_2$

$Cc = 0.85 \times 35 \times 320 \times 400 = 3808$ kN

$Cs_1 = As_1\ fs_1 = 2580 \times 97.5 = 251.55$ kN

$Cs_2 = As_2\ (fs_2 - 0.85f_c') = 2580\ (420 - 0.85 \times 35) = 1006.8$ kN

As_2 is included in the compression area (axb). As_2 should have been excluded from Cc force calculation.

$Cc = 0.85\ f_c'\ (ab - As_2)$, what should've been subtracted is $0.85\ fc'\ As_2$

But if we do that the location of Cc force will not be at a/2. Therefore, to avoid that what should've been excluded from Cs has been excluded from Cs_2.

<u>P_n</u> = 3808 + 251.55 + 1006.8 = **<u>5066.35 kN</u>**

M_n = Sum of moments of all the above forces about the centroid of the section (NOT N. A.)

$M_n = Cc\ (h / 2 - a / 2) + Cs2\ (h / 2 - d') - Cs1\ (h / 2 - d')$

<u>M_n</u> = 3808 (200 – 320 / 2) + 1006.8 (200 – 65) – (200 – 65) = **<u>254.28 kN.m</u>**

<u>POINT C:</u> (Balanced Condition) **$\varepsilon s_1 = \varepsilon y = 0.0021$**

$0.003 / C = 0.0021 / (400 - 65 - C)$

C = 197mm, a = 0.8 x 197 = 157.6mm

$0.003 / C = \varepsilon s_2 / (C - 65)$, $\varepsilon s_2 = 0.00201 < \varepsilon y$

$fs_2 = 0.00201 \times 200000 = 402$ MPa

$Cc = 0.85\ fc'\ a\ b = 0.85 \times 35 \times 157.6 \times 400 = 1875.44$ kN

$Cs_2 = As_2\ (fs_2 - 0.85\ fc') = 2580 \times (402 - 0.85 \times 35) =$ 960.4 kN

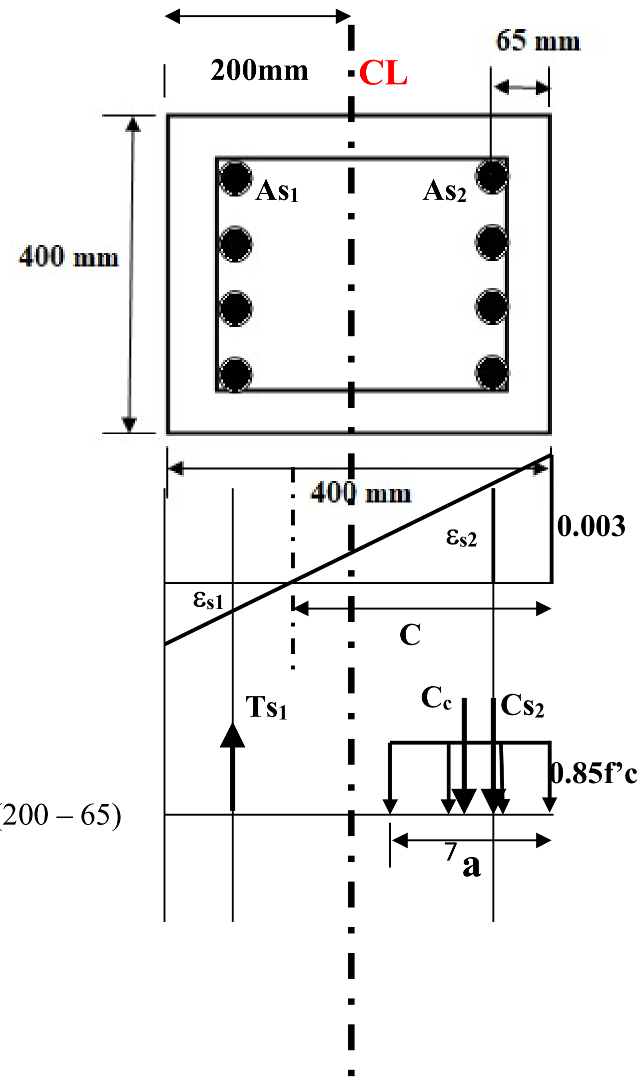
$Ts_1 = As_1\ fy = 2580 \times 420 = 1083.6$ kN

$P_n = Cc + Cs_2 - Ts_1$

<u>P_n</u> = 1875.44 + 960.4 – 1083.6 = **<u>1752.24 kN</u>**

$M_n = Cc\ (200 - a / 2) + Cs_2\ (200 - d') + Ts_1\ (200 - d')$

M_n = 1875.44 (200 – 157.6 / 2) +960.4 (200 – 65) + 1083.6 (200 – 65)



M_n = 503.24 kN.m

POINT D: (Tension Controlled Limit) **$\varepsilon s_1 = 0.005$**

Very similar to point C

0.003 / C = 0.005 / (400 – 65 – C), C = 125.6mm, a = 100.48mm

0.003 / C = εs_2 / (C – 65), εs_2 = 0.001447 < εy, fs_2 = 289.5 MPa

Cc = 0.85 x 35 x 100.48 x 400 = 1195.7 kN

Cs_2 = 2580 x (289.5 – 0.85 x 35) = 670.16 kN

Ts_1 = 2580 x 420 = 1083.6 kN

P_n = 1195.7 + 670.16 – 1083.6 = **782.3 kN**

M_n = 1195.7 x (200 – 100.48 / 2) + 670.16 x (200 – 65) + 1083.6 x (200 – 65) = **415.9 kN.m**

POINT E: (Pure Moment), **$\varepsilon s > 0.005$**

P_n = 0.0

Ignore compression reinforcement (As_2) and find M_n considering the column as a single reinforced rectangular beam.

T = Cc

As_1 fy = 0.85 fc' a b

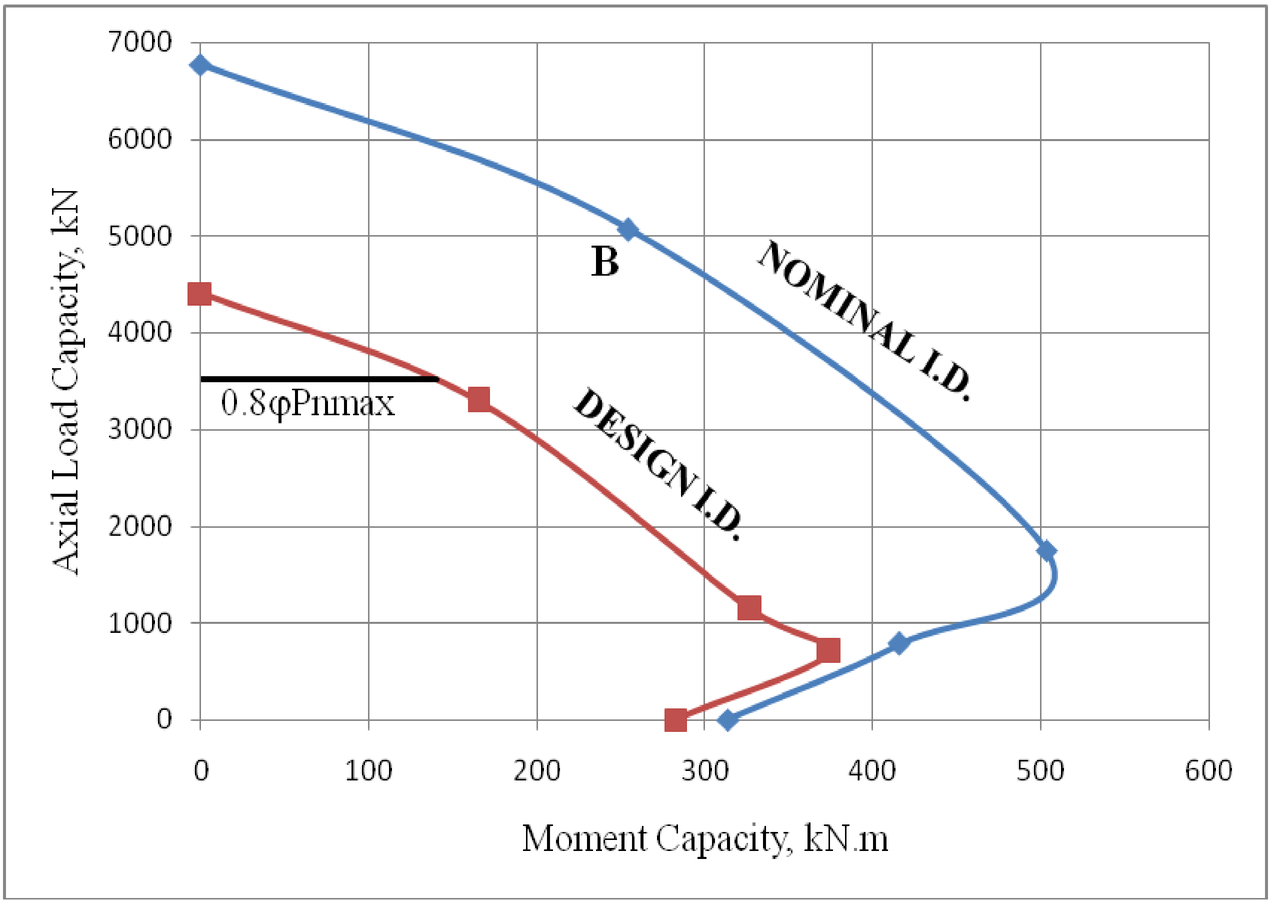
2580 x 420 = 0.85 x 35 x a x 400, a = 91.05mm, C = 113.8mm

εs = 0.003 (334 – 113.8) / 113.8 = 0.0058

M_n = As fy (d – a / 2) = 2580 x 420 x (335 – 91.05 / 2) = **313.67 kN.m**

Point	φ	Pn	Mn	φ Pn	φ Mn
A	0.65	6773.7	0	4402.9	0.0
B	0.65	5066.35	254.28	3293.1	165.3
C	0.65	1752.24	503.24	1139.0	327.1
D	0.9	782.3	415.9	704.1	374.3
E	0.9	0	313.67	0.0	282.3

0.8 φPn max = 0.8 x 4402.9 = 3522.3 kN



7000
6000
5000
4000
3000
2000
1000
0
0
100
200
300
400
500
600
Axial Load Capacity, kN
Moment Capacity, kN.m
B
NOMINAL I.D.
DESIGN I.D.
0.8φPnmax

Design of Short Columns

Two types of columns: short and slender columns.

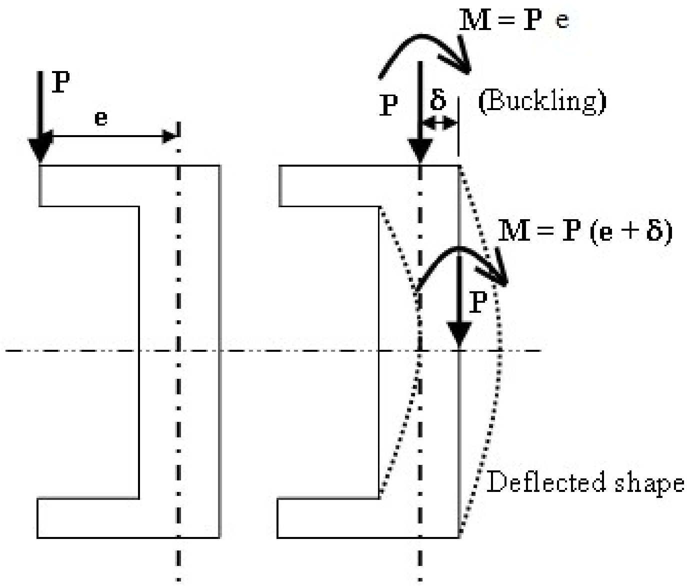
Moment @ ends = P e

Moment @ midspan = P (e + δ)

The deflection / buckling increases the moment for which the column must be designed for.

Because of the increase in the maximum moment (at the midspan of the column) due to buckling, the axial load capacity is reduced. The reduction in the axial load capacity results from what is referred to slenderness effect.

A slender column is defined as a column that has a significant reduction (> 5%) in its axial load capacity due to moments resulting from lateral deflections of the column.



Choice of materials properties and reinforcement ratios:

In small buildings, fc' in columns ≅ fc' in floors ≅ 28 – 31 MPa.

In tall buildings, fc' in columns > fc' in floors, to reduce the column size.

Per ACI – Code in general ($0.01 \leq \rho_{st} \leq 0.08$). although the code allows $\rho_{max} = 0.08$ it is generally very difficult to place this amount of steel in a column, particularly if lapped splices are used.

EXAMPLE: 400mm x 400mm column, As = 0.08 x 400 x 400 = 12800mm^2. If No.25M steel is to be used, 25No.25M bars are required. Therefore, it will be difficult to place 25 bars.

Tables A – 10 and A – 11 in the text book provide ρmax for various column sizes for square and circular columns (3 – 5% or 6%, respectively).

The most economical steel ratio for tied columns (1 – 2%), 1.5% is used. For spiral columns (2.5 – 5%), since they resist higher loads.

Per ACI – Code minimum number of bars in a tied column is 4 and in a spiral column is 6.

Almost universally, an even number of bars is used in a rectangular column to maintain symmetry about the axis of bending. All bars are the same size.

Estimating the column size:

For very small values of moment, the column size is governed by the maximum axial load

For tied columns $Ag(trial) = \dfrac{Pu}{0.4(f_c^{'} + \rho f_y)}$

For spiral columns $Ag(trial) = \dfrac{Pu}{0.5\left(f_c^{'} + \rho f_y\right)}$

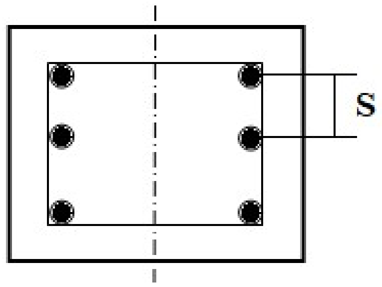
Although the ACI – Code does not specify a minimum column size, the minimum dimension of a cast in place tied column should not be less than 8 in (200 mm) and preferably not less than 10 in (250 mm). The diameter of a spiral column should not be less than 12 in (300 mm).

Bar spacing and cover requirements:

Per ACI – Code the clear cover ≥ 1.5 in (40 mm)

Minimum clear distance between longitudinal bars is based on:

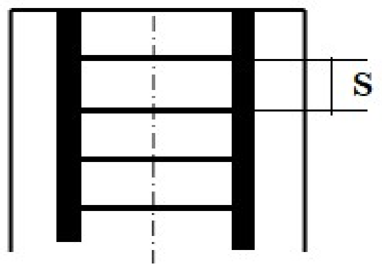
$$S\min = largerof\begin{cases} 1.5d_b \\ 1.5in(40mm) \\ 1\tfrac{1}{3}\,Max.SizeofC.A. \end{cases}$$, d_b = bar diameter



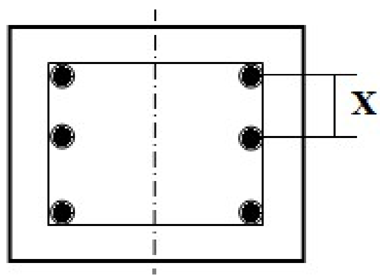
Clear distance limitation also apply to lap – spliced bars.

Spacing for ties:

$$S\max = SmallerOf\begin{cases} 16d_b \\ 48d_t \\ ColumLeastDimension(b) \end{cases}$$, d_t = tie diameter

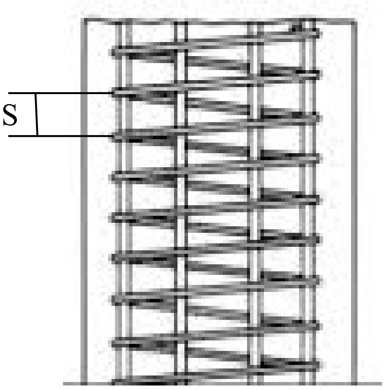


A bar is adequately supported against lateral movements if it is located at a corner of a tie and if the distance X is 6 in (150 mm) or less.



Spacing for spiral:

The maximum spacing that will result in a second maximum load that equals or exceeds the initial maximum load:



$$S \le \frac{\pi d_{sp}^2 f_y}{0.45 D_c f_c' \left(\frac{A_g}{A_c} - 1 \right)}$$

d_{sp} = diameter of spiral

D_c = diameter of the core (out to out of spiral)

A_c = area of the core, Ag = gross cross sectional area.

S_{max} = 75mm (to confine the core effectively)

S_{min} = larger of (25mm or 1.33 max. C. A. size) to avoid problems in concrete placing.

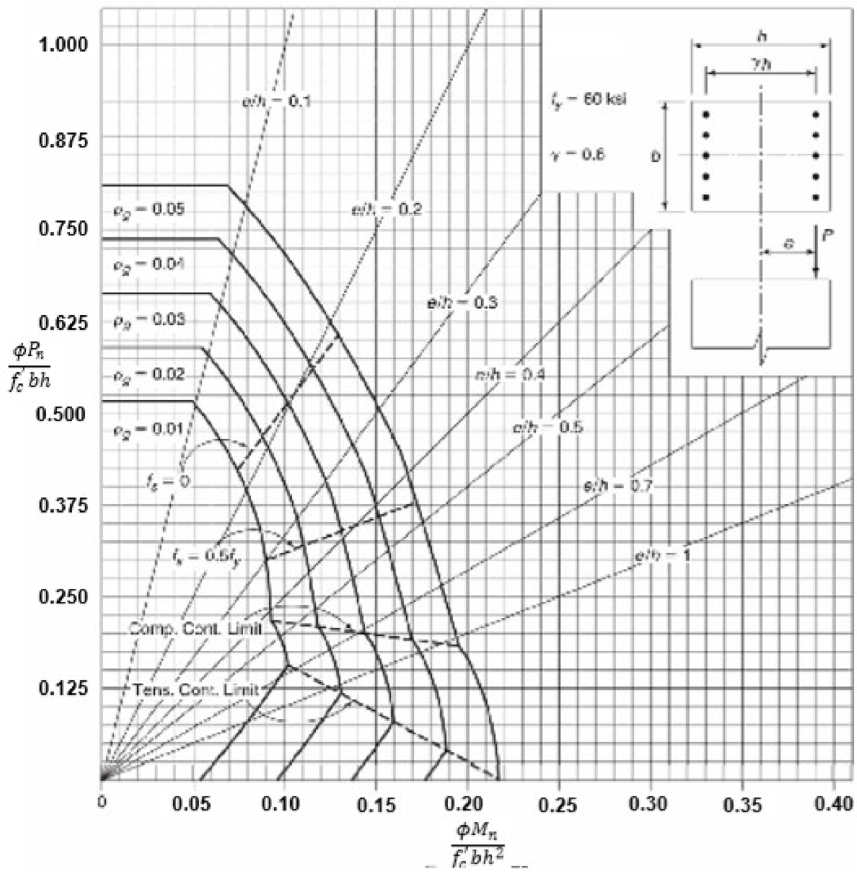
Choice of column type:

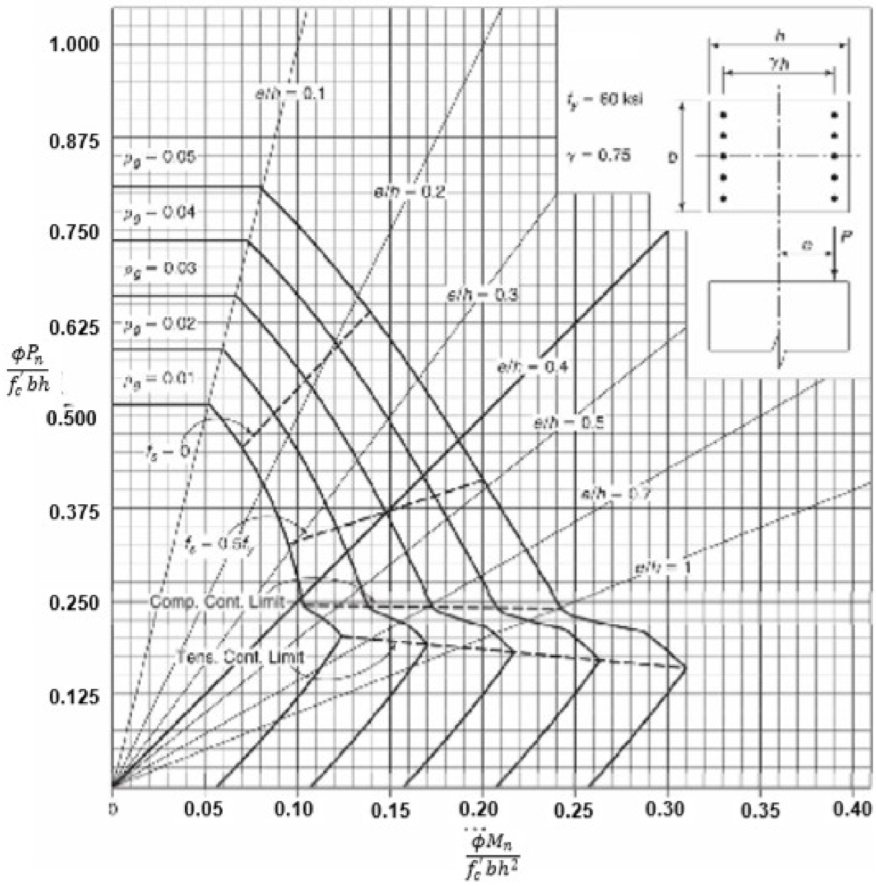
e / h = eccentricity ratio, **e = Mu / Pu**

<u>If e / h < about 0.1</u> → spiral column is more efficient. In terms of load capacity, φ = 0.75 for spiral and 0.65 for tied columns. In terms of maximum axial load capacity 0.8φPn for tied and 0.85φPn for spiral columns

<u>If e / h > 0.2</u> → a tied column with bars in the faces farthest from the axis of bending is more efficient. Even more efficiency can be obtained by using a rectangular column to increase the depth perpendicular to the axis of bending.

<u>If e / h < 0.2</u> and moment exists about both axes (biaxial bending) → a tied column with bars in four faces are used.


f_y = 60 ksi
γ = 0.6
h
γh
e
P
ρ_g = 0.05
ρ_g = 0.04
ρ_g = 0.03
ρ_g = 0.02
ρ_g = 0.01
e/h = 0.1
e/h = 0.2
e/h = 0.3
e/h = 0.4
e/h = 0.5
e/h = 0.7
e/h = 1
f_s = 0
f_s = 0.5f_y
Comp. Cont. Limit
Tens. Cont. Limit
1.000
0.875
0.750
0.625
0.500
0.375
0.250
0.125
$\frac{\phi P_n}{f_c' bh}$
0
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
$\frac{\phi M_n}{f_c' bh^2}$


f_y = 60 ksi
γ = 0.75
h
γh
e
P
ρ_g = 0.05
ρ_g = 0.04
ρ_g = 0.03
ρ_g = 0.02
ρ_g = 0.01
e/h = 0.1
e/h = 0.2
e/h = 0.3
e/h = 0.4
e/h = 0.5
e/h = 0.7
e/h = 1
f_s = 0
f_s = 0.5f_y
Comp. Cont. Limit
Tens. Cont. Limit
1.000
0.875
0.750
0.625
0.500
0.375
0.250
0.125
$\frac{\phi P_n}{f_c' bh}$
0
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
$\frac{\phi M_n}{f_c' bh^2}$

EXAMPLE: design of a square tied column. P_u = 1550 kN, M_u = 150 kN.m, f_c' = 20 MPa, fy = 420 MPa. Use No.25M steel.

Use ρ = 0.015 (most economical)

$$Ag(trial) = \frac{Pu}{0.4(f_c^{'} + \rho f_y)}$$ = 1550 x 10^3 / 0.4 (20 + 0.015 x 420) = 147338 mm^2

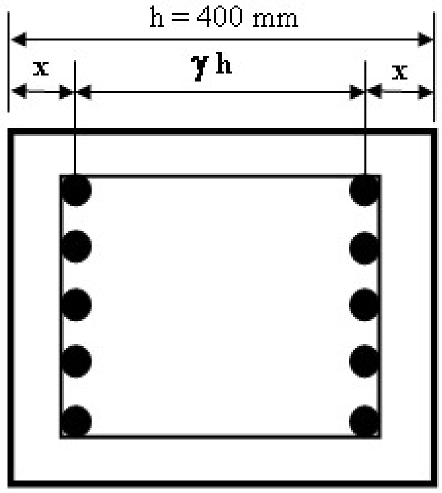
Ag = b x h (b = h for square column)

b = h = √147338 = 384 mm, Use 400 mm, b = h = 400 mm

Estimate the value of γ

x = cover + tie + d_b/2 = 40 + 10 + 25/2 = 62.5 mm

h = 400 = 2 x + γ h = 2 x 62.5 + γ x 400 → γ = 0.69



Two I. D.'s will need to be used to determine ρ, for γ = 0.6 and γ = 0.75. Then by interpolation ρ is calculated for γ = 0.69.

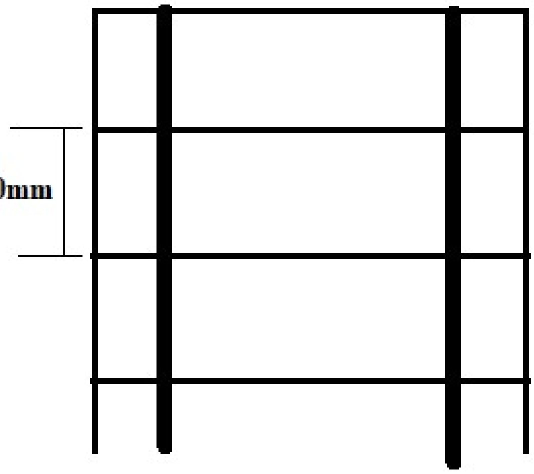
$$\frac{\varphi P_n}{f_c^{'} bh} = \frac{1550x10^3}{20x400x400} = 0.48$$

$$\frac{\varphi M_n}{f_c^{'} bh^2} = \frac{150x10^6}{20x400x400^2} = 0.12$$

From I.D. for γ = 0.6, ρ = 0.033

From I.D. for γ = 0.75, ρ = 0.028

By interpolation, for γ = 0.69, ρ = 0.03 (if ρ computed exceeds 0.03 – 0.04, larger section should be chosen)



ρ > ρ_{min} = 0.01 (OK)

As = ρ b h = 0.03 x 400 x 400 = 4800 mm^2 (Use 10 No.25M, As = 4910 mm^2, **5 on each side**)

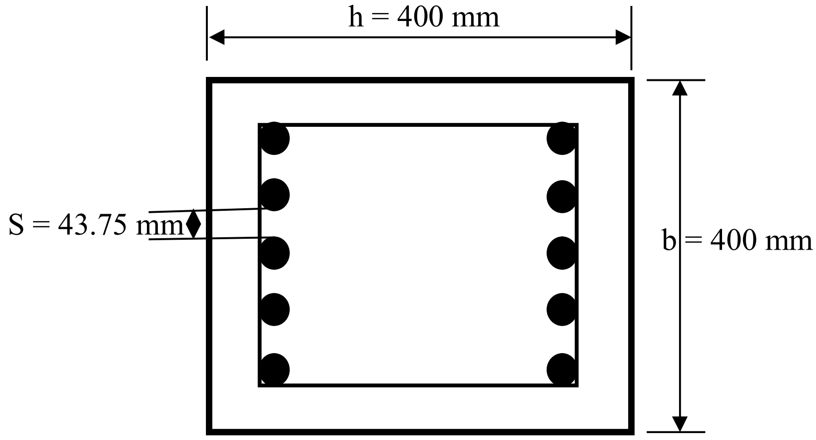
Spacing for ties = smaller of $\begin{cases} 16x25 = 400mm \\ 48x10 = 480mm, \\ b = 400mm \end{cases}$

S = 400mm

Spacing between longitudinal bars

400 = 2 x 40 + 2 x 10 + 5 x 25 + 4 x S $\rightarrow$ S = 43.75 mm < 150 mm (OK)

S = 43.75 mm > S_{min} = larger of $\begin{cases} 1.5x25 = 37.5mm \\ 40mm \end{cases}$, (OK)

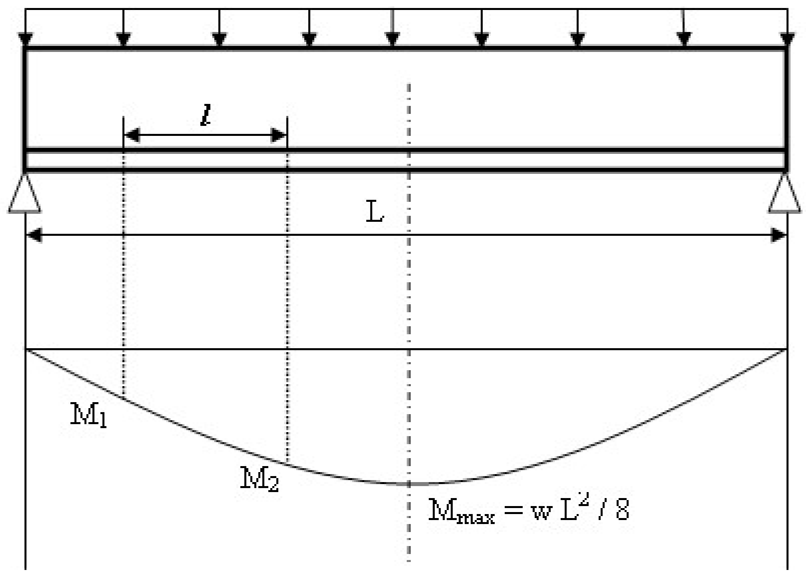


CHAPTER 9

Development, Anchorage and Splicing of Reinforcement

In a reinforced concrete beam the flexural compressive forces are being resisted by concrete, while the flexural tensile forces are being resisted by reinforcement.

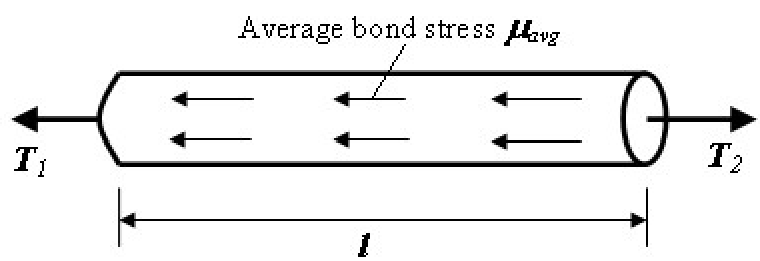
Take for example the simply supported beam shown with a uniform distributed loading. The beam will be under the effect of positive bending moment and reinforced with certain amount of tension reinforcement at the bottom of the beam as shown.



A segment of one bar with certain length (***l***) is being taken out of the area of steel and its free body diagram is drawn as shown below.

For one bar As = A_b (area of one bar)

The bar end segments are being acted upon by tensile forces T_1 and T_2, generated from the bending moment on the beam.



As explained before the tensile force is calculated as the area multiplied by the stress acting on the area. So,

$T_1 = fs_1 A_b$, (fs_1 = stress in steel at end 1, based on the value of M_1)

$T_2 = fs_2 A_b$, (fs_2 = stress in steel at end 2, based on the value of M_2)

Equilibrium should be satisfied on the bar segment. But $T_1 \neq T_2$ since there are two different values of the moment and the bending stress at each end of the bar segment M_1, fs_1, and M_2, fs_2. The equilibrium will be achieved by the bonding of the bar with the concrete. μ_{avg} is the average bond stress between the steel and concrete which is acting on the circumference area of the bar (contact area between the steel bar and the concrete).

Contact area = $\pi\, \mathbf{d_b}$. Therefore, the bonding force = $\mu_{avg}\,(l)\,\pi\, d_b$.

Therefore, $\mathbf{T_1} + \mu_{avg}\,(l)\,\pi\, d_b = \mathbf{T_2}$ (*equilibrium equation*)

This equation shows that there is a certain length of the steel bar that should be provided in the beam to provide enough bonding force to achieve equilibrium in the bar. This length is called **development length**, l_d.

Therefore, the development length, l_d, can be defined as the shortest length of bar in which the bar stress can increase from **zero** to **fy**. The bar stress is **zero** at the ends of the bar, and **fy** at the location of the maximum moment. Therefore, the distance from the point of the maximum moment to the end of the bar in the beam should not be less than the development length for equilibrium to be achieved.

If the distance from a point where the bar stress equals fy (maximum moment) to the end of the bar (bar stress is zero) is less than the development length, the bar will pull out of the concrete. Bond stress is not enough to produce equilibrium, ($T_2 > T_1$ + bonding force)

The development length for bars in tension is calculated using one of the formulas in the following table:

Development Length of Straight Bars and Standard Hooks

For deformed bars, ACI318-05 Section 12.2.2 defines the development length l_d given in the tab below. Note that l_d shall not be less than 300 mm.

Case	$\leq \phi 20$	$> \phi 20$
Case 1: Clear spacing of bars being developed not less than db, clear cover not less than db, and stirrups throughout l_d not less than code minimum **or** **Case 2:** Clear spacing of bars being developed not less than 2db and clear cover not less than db	$l_d = \frac{12 f_y \psi_t \psi_e \lambda}{25\sqrt{f_c'}} d_b$	$l_d = \frac{12 f_y \psi_t \psi_e \lambda}{20\sqrt{f_c'}} d_b$
Other cases	$l_d = \frac{18 f_y \psi_t \psi_e \lambda}{25\sqrt{f_c'}} d_b$	$l_d = \frac{18 f_y \psi_t \psi_e \lambda}{20\sqrt{f_c'}} d_b$

Development length calculation procedures and requirements for bars in compression are provided in the text book…

The terms in the foregoing equations are as follows:

ψ_t = **reinforcement location factor**

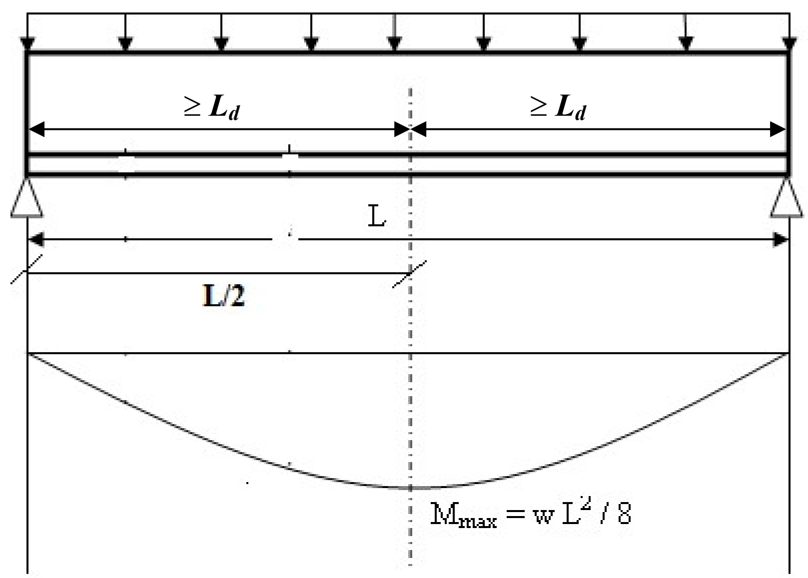
Horizontal reinforcement so placed that more than 300 mm of fresh concrete is cast in the member *below* the development length 1.3
Other reinforcement....................1.0

ψ_e = **coating factor**

Epoxy-coated bars with cover less than $3d_b$, or clear spacing less than $6d_b$ 1.5
All other epoxy-coated bars 1.2
Uncoated reinforcement.................... 1.0

λ = **lightweight aggregate concrete factor**

When all-lightweight aggregate concrete is used 1.3
When sand-lightweight aggregate concrete is used 1.2
Normal weight concrete is used.................... 1.0

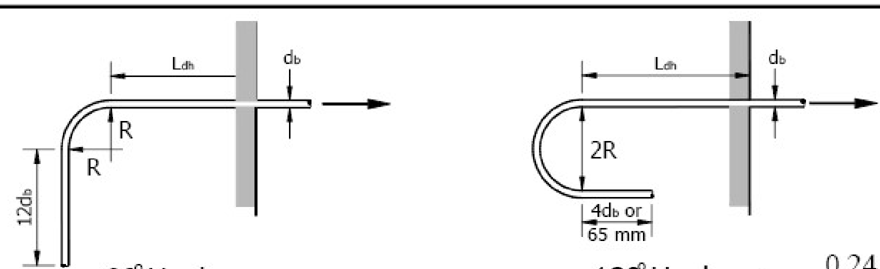


The calculated development length from the above formulas shall not be less than 300 mm.

After calculating the development length from one of the above formulas, each bar of the tension reinforcement must be checked to make sure that the provided length of the bar is more than or equal the calculated development length. As shown on the drawing the distance between the maximum moment to the end of the bar should be more than or equal the calculated development length. **If that distance is less than l_d, the bar should be extended to provide for l_d.**

When there is insufficient length available to develop a straight bar, standard hooks are used.

Standard Hooks – ACI sections 7.1 and 7.2.1



$$l_{dh} = \frac{0.24 f_y \psi_e \lambda}{\sqrt{f'_c}} d_b \geq \text{larger of} \begin{vmatrix} 8d_b \\ 150mm \end{vmatrix}$$

$\phi 10$ to $\phi 25$: $R = 3d_b$
$\phi 28$ and $\phi 32$: $R = 4d_b$
$\phi 50$: $R = 5d_b$

Two types of standard hooks are used: 90° hook and 180° hook. Design procedures and requirements for standard hooks are provided on the previous figure.

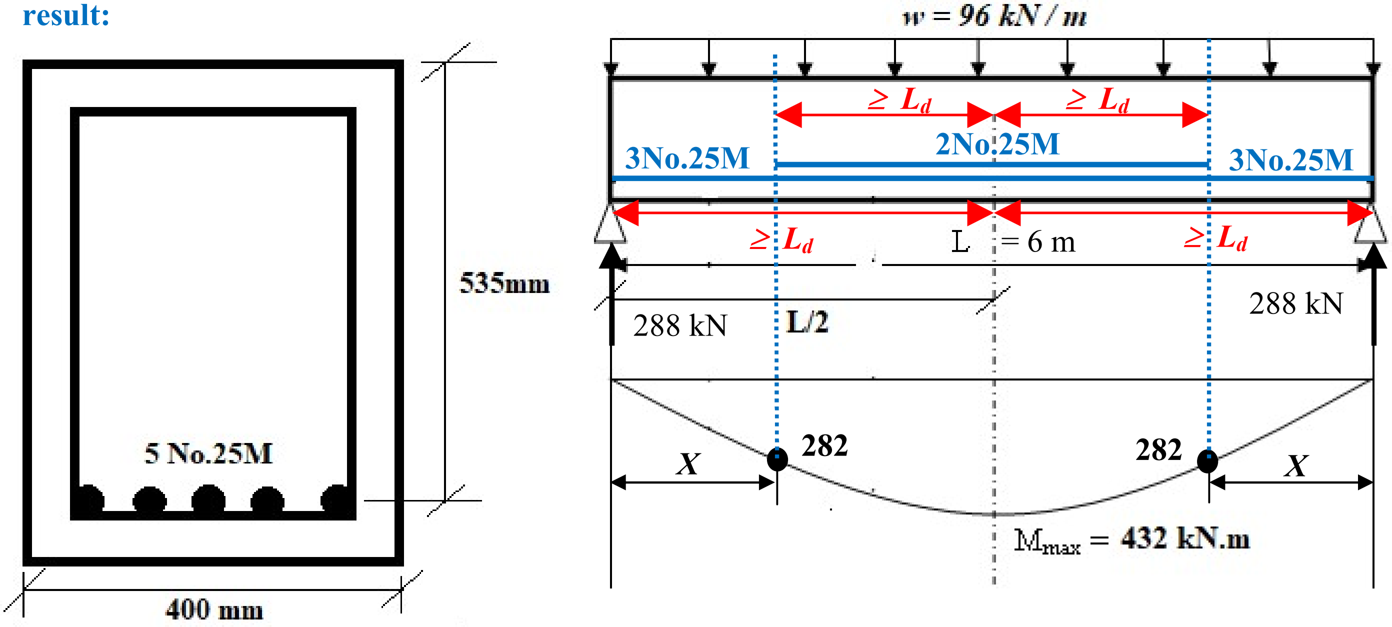
Bar cut offs and development of bars in tension

For economic reasons some tension bars in flexural members can be terminated or cut off where they are no longer needed.

EXAMPLE:

The simply supported beam shown below is loaded by an ultimate uniform distributed loading of 96 kN/m.

If this beam is to be designed for the ultimate moment (432 kN.m), the following beam will result:



The design moment capacity for the above section is calculated

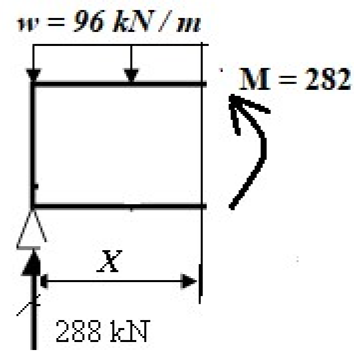
$\varphi M_n = 440$ kN.m $> M_u = 432$ kN.m

The maximum moment is located at the mid span of the beam. Based on the value of the maximum moments it was determined to use 5 bars of 25M steel. Moving away from the mid span the moment value is reduced and therefore less reinforcement is required.

If two bars are to be cut off, three bars will remain in the section. The design moment capacity of the section with three bars only is calculated (**$\varphi M_n = 282$ kN.m**) and drawn on the BMD above. This point is called theoretical cut off point, it's a cut off point that determines where the two bars are no longer needed.

Note that based on the BMD moments from supports to the theoretical cut off points are less than 282 kN.m. So, only 3 bars will be used after cutting two bars. For the remainder of the beam 5 bars will be used, as shown above.

To determine the location of the theoretical cut off point make a section at the location of the point and determine the value of ***X***.



Σ M = 0.0

282 – 288 X + 96 X (X / 2) = 0.0

X = 1.23 m (from each support)

Check L_d for the three bars and for the two bars.

Major factors affecting the location or bar cut off:

Bars can be cut off where they are no longer needed to resist tensile forces or where the remaining bars are adequate to do so.

There must be sufficient extension of each bar on each side of every cut off section to develop the force in the bar at that section.

Tension bars cut off in a region of moderately high shear force may cause a major stress concentration, which can lead to a major inclined cracks at the bar cut off location.

Some bar cut off rules:

Each bar must continue a distance larger of (***d or 12*** d_b) beyond the theoretical cut off point except at supports or at the ends of cantilevers.

At least **1/3** of the positive moment steel at simple beams and ¼ of the positive moment steel at continuous beams but not less than **2** bars must extend at least **150mm** into supports.

MORE RULES ARE AVAILABLE IN THE TEXT BOOK