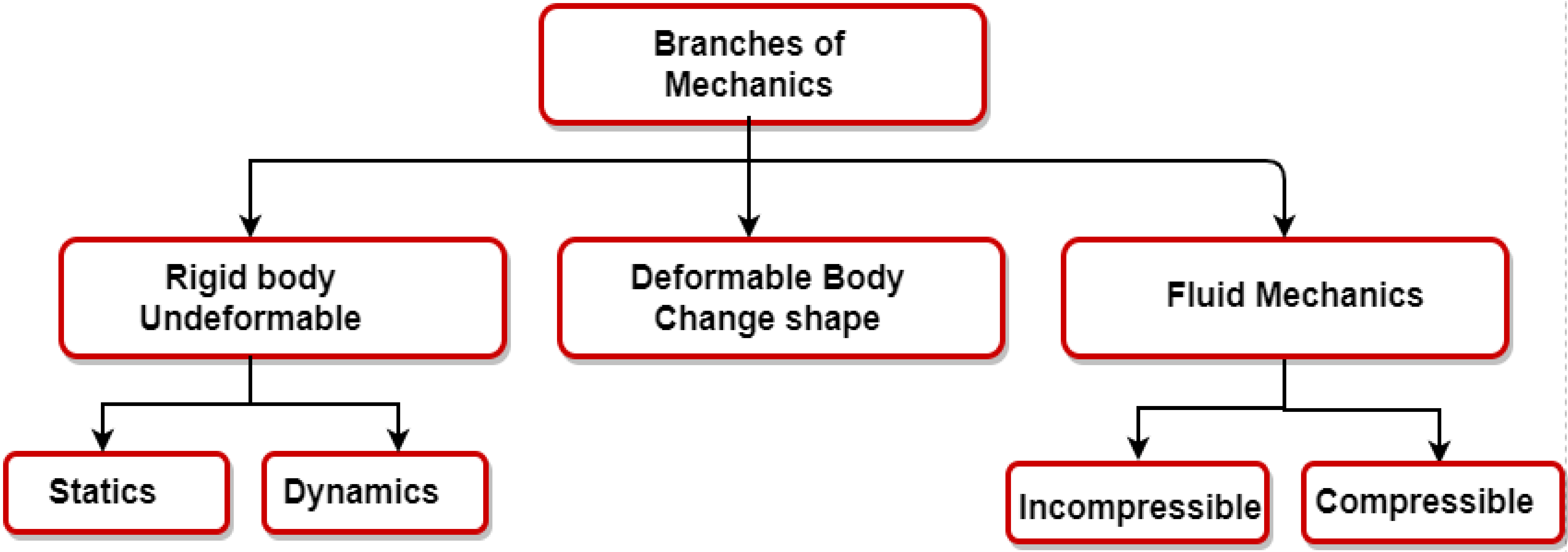
Statics

Chapter 1: Introduction

I. Marie

Textbook: Hibbeler, R.C., Engineering Mechanics- STATICS, 14th SI- Edition, Prentice Hall

1.1 Engineering Mechanics



Statics: bodies at rest (equilibrium)

Dynamics: accelerated motion of bodies

1.2 Fundamental concepts

Basic Quantities.

Length. ***Length*** is used to locate the position of a point in space and thereby describe the size of a physical system.

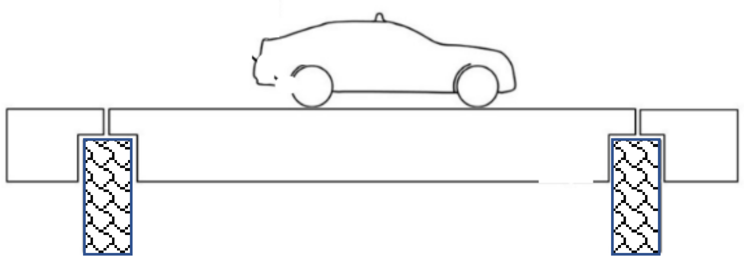
Time. ***Time*** is conceived as a succession of events. Although the principles of statics are time independent, this quantity plays an important role in the study of dynamics.

Mass. ***Mass*** is a measure of a quantity of matter

Force. In general, ***force*** is considered as a “push” or “pull” exerted by one body on another.

Idealizations.

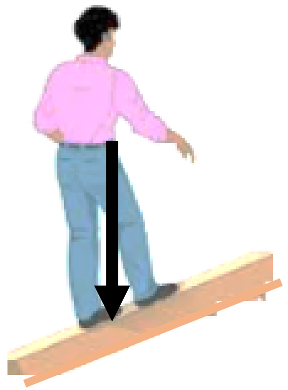
Models or idealizations are used in mechanics in order to simplify application of the theory. Here we will consider three important idealizations



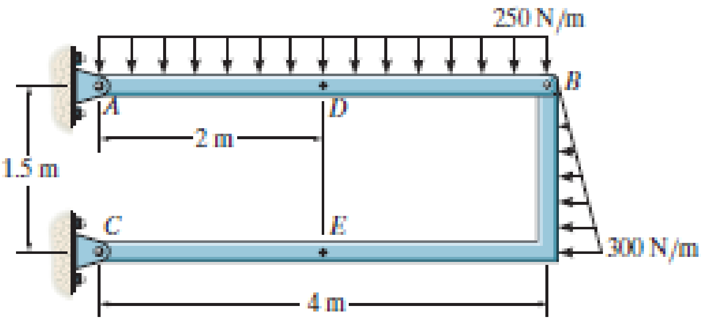
Particle. A ***particle*** has a mass, but a size that can be neglected

Rigid Body. A ***rigid body*** can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one another, both before and after applying a load

Concentrated Force. A ***concentrated force*** represents the effect of a loading which is assumed to act at a point on a body. We can represent a load by a concentrated force, provided the area over which the load is applied is very small compared to the overall size

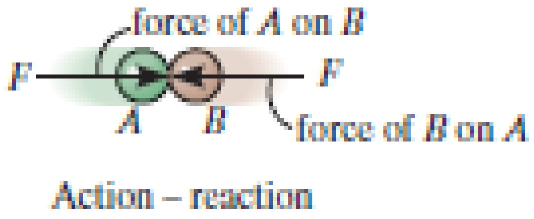


Distributed load.



Newton's Three Laws of Motion

Third Law. The mutual forces of action and reaction between two particles are equal, opposite, and collinear



1.3 Units of Measurement

The SI system of units is used extensively in this book since it is intended to become the worldwide standard for measurement

SI Units. The International System of units

Mass: kilogram, kg -(1Kg=1000g)
Length: meter, m -(1m=1000mm)
Force: Newton, N-(kN=1000N)
Weight: (N) =m (kg) * g (m/s^2)
Gravitational Acceleration, g=9.81 (m/s^2)

1.4 The International System of Ur

Prefixes. When a numerical quantity is either very large or very small, the units used to define its size may be modified by using a prefix.

Table 1.3 Prefixes

	Exponential Form	Prefix	SI Symbol
Multiple			
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	K
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

1.5 Numerical Calculations

It is important, however, that the answers to any problem be reported with justifiable accuracy using appropriate significant figures.

1.6 General Procedure for Analysis

- Read the problem carefully and try to correlate the actual physical situation with the theory studied.
- Tabulate the problem data and *draw to a large scale* any necessary diagrams.
- Apply the relevant principles, generally in mathematical form. When writing any equations, be sure they are dimensionally homogeneous.
- Solve the necessary equations, and report the answer with no more than three significant figures.
- Study the answer with technical judgment and common sense to determine whether or not it seems reasonable.

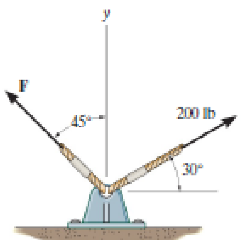
Name: ....
ID: ....

Hw. No. ....
Statics

Section :

Time u took to
solve the problem

Req. ....



solution

sketch

....

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Final answer

A4
210x297mm

Statics

Chapter 2 -Force Vectors

Iqbal Marie

CHAPTER OBJECTIVES

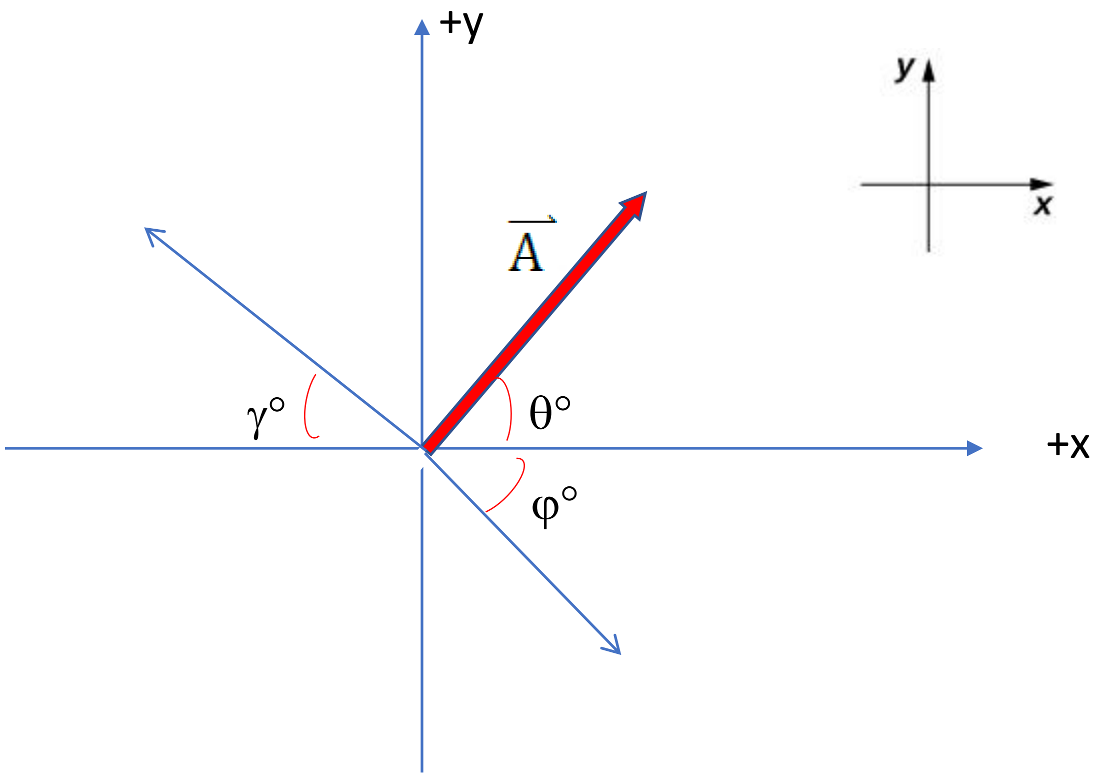
- To show how to add forces and resolve them into components using the Parallelogram Law.
- To express force and position in Cartesian vector form and explain how to determine the vector's magnitude and direction.
- To introduce the dot product in order to use it to find the angle between two vectors or the projection of one vector onto another.

2.1 Scalars and Vectors

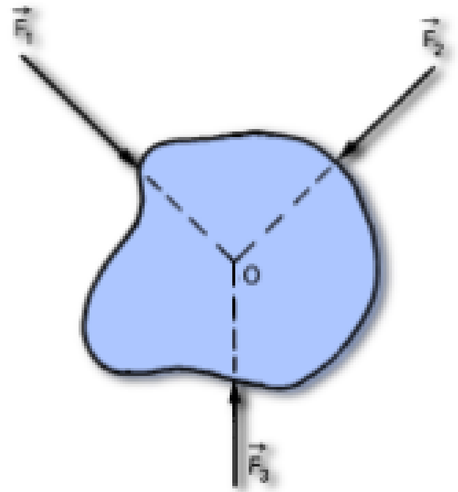
scalar is any **positive** or **negative physical quantity** **(magnitude)**

A **vector** is any physical quantity that is described by both a **magnitude** and **direction**

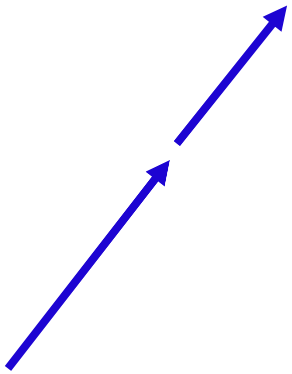
A vector is shown **graphically** by an **arrow**.



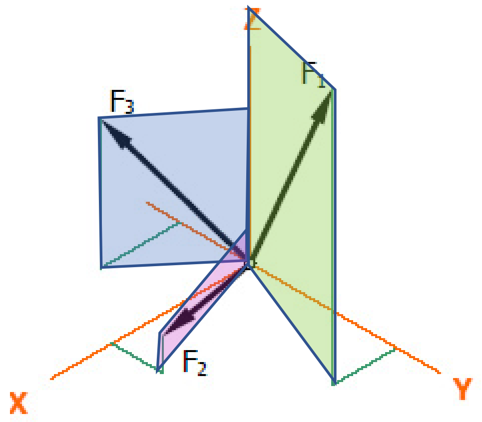
System of forces



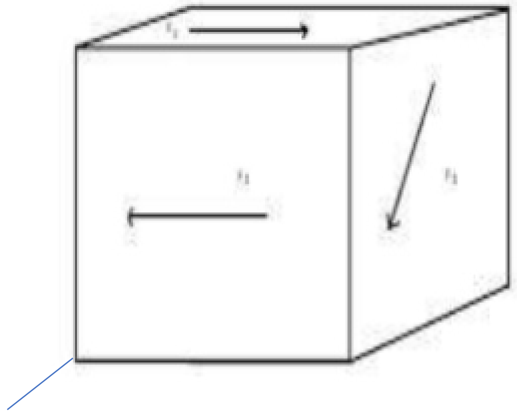
Concurrent coplanar forces



Colinear forces



Concurrent Non coplanar forces



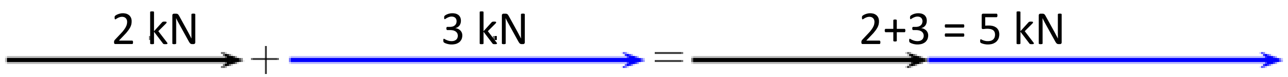
Non coplanar forces

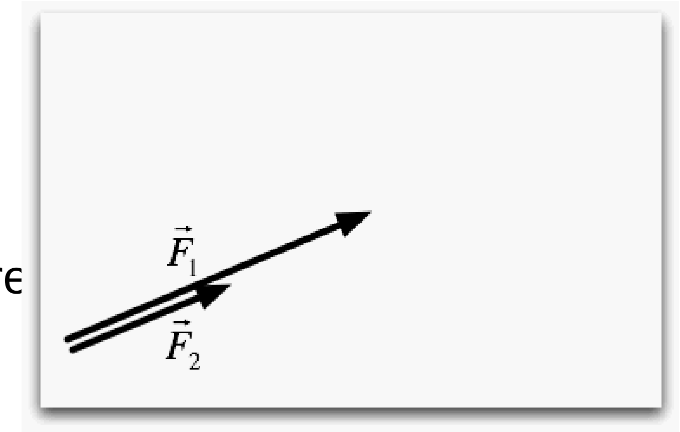
2.3 Vector Addition of Forces

Finding a Resultant Force.

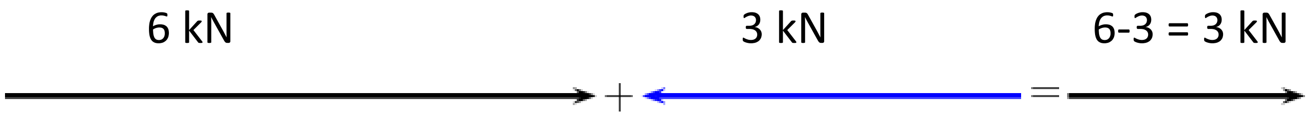
The resultant vector is the single vector whose effect is the same as the individual vectors acting together.

If the vectors are collinear, the resultant is formed by an algebraic or scalar addition





Subtracting colinear vectors is the same as adding a vector in the opposite dire



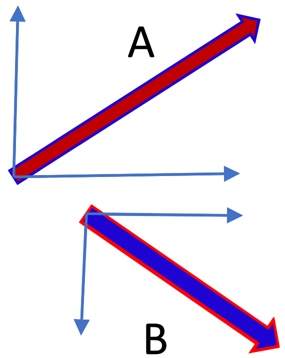
techniques of vector addition.

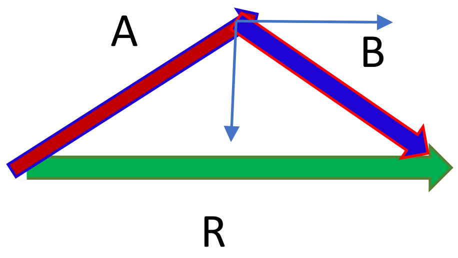
- graphical
  - **Head-to-Tail Method**
- algebraic techniques

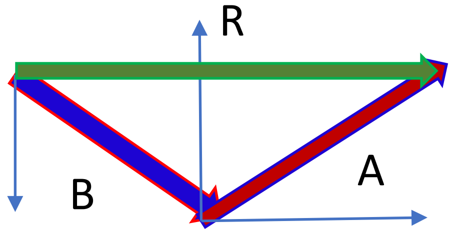
Addition of Coplanar force system

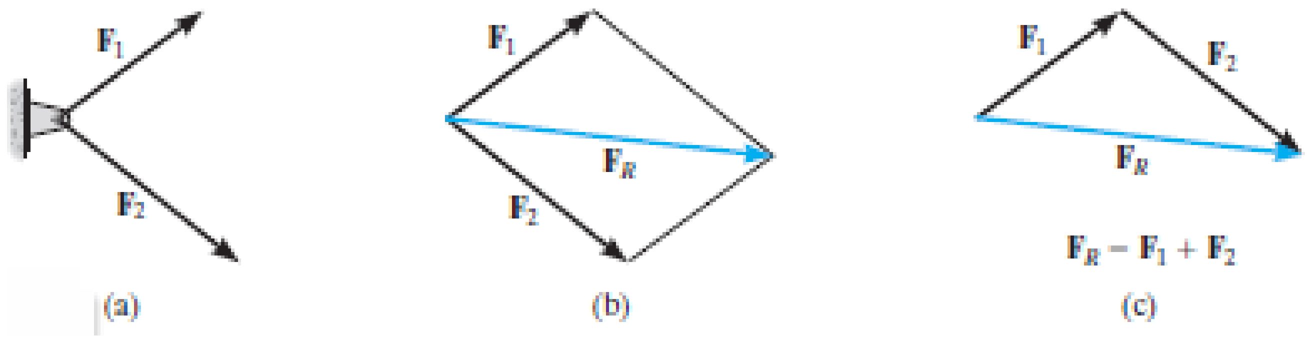
Vector addition : Finding the resultant force

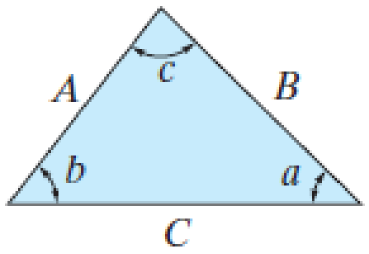
Find the resultant









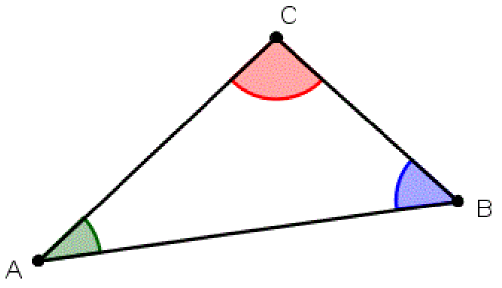


Cosine law:

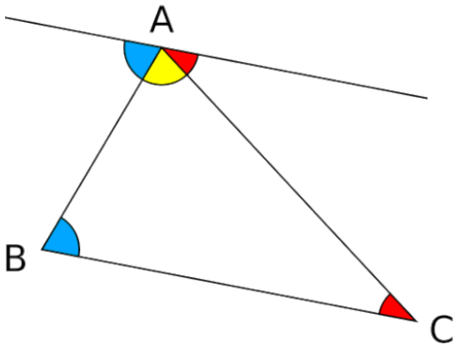
$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine law:

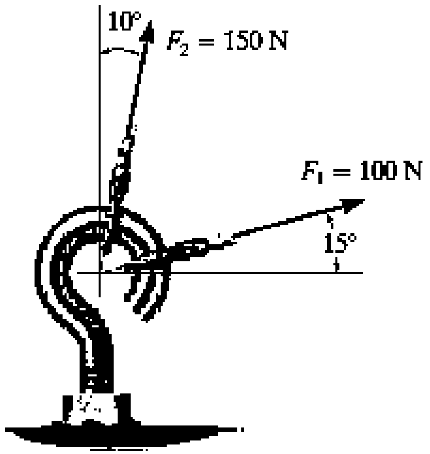
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

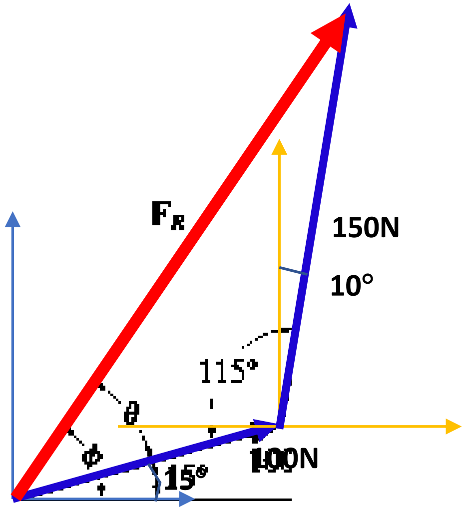


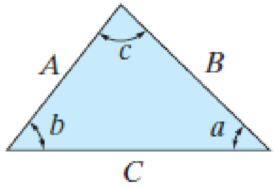
$\angle A + \angle B + \angle C = 180$



Ex.2.1 The screw eye in is subjected to two forces, **F**$_1$ and **F**$_2$. Determine the magnitude and direction of the resultant force.







Cosine law:
$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$
Sine law:
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Using the law of cosines

$$F_R = \sqrt{(100\text{ N})^2 + (150\text{ N})^2 - 2(100\text{ N})(150\text{ N})\cos 115^\circ}$$
$$= \sqrt{10\,000 + 22\,500 - 30\,000(-0.4226)} = 212.6\text{ N}$$
$$= 213\text{ N}$$

Applying the law of sines to determine θ,

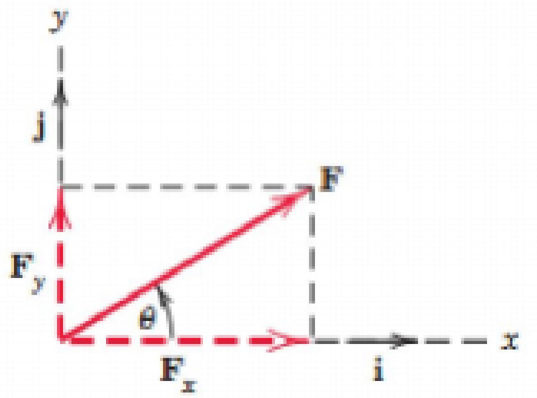
$$\frac{150\text{ N}}{\sin\theta} = \frac{212.6\text{ N}}{\sin 115^\circ} \qquad \sin\theta = \frac{150\text{ N}}{212.6\text{ N}}(\sin 115^\circ)$$
$$\theta = 39.8^\circ$$

Thus, the direction ϕ (phi) of $\mathbf{F}_R$, measured from the horizontal, is

$$\phi = 39.8^\circ + 15.0^\circ = 54.8^\circ$$

Finding the Components of a Force.

Rectangular Components:



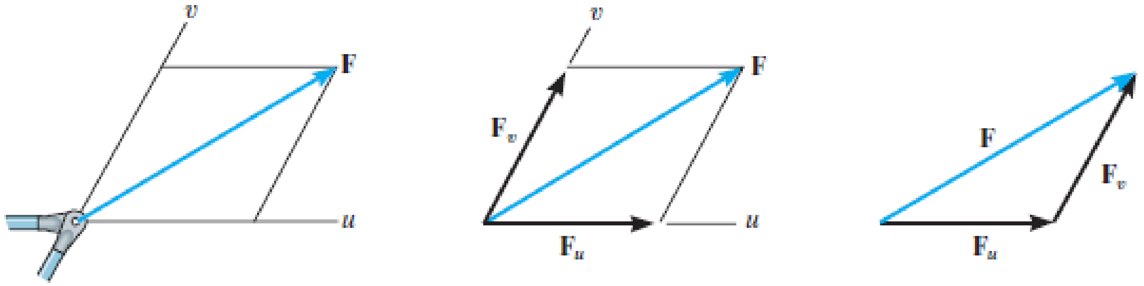
$$F_x = F\cos\theta$$

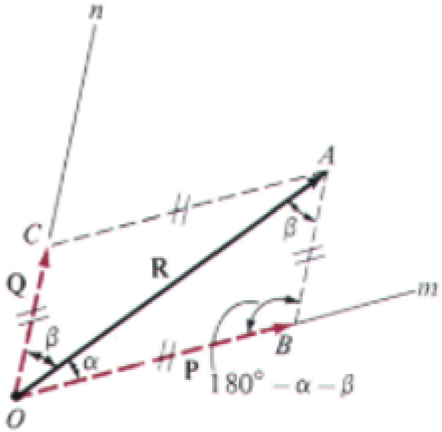
$$F_y = F\sin\theta$$

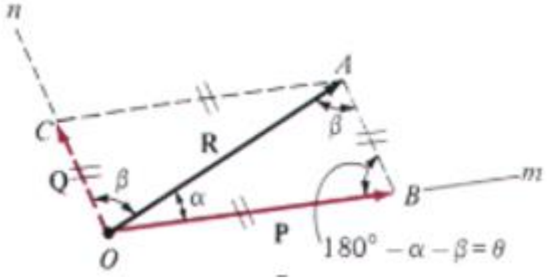
$$F = \sqrt{F_x^{\,2} + F_y^{\,2}}$$

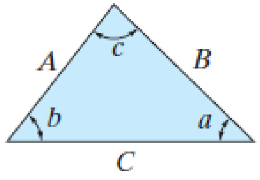
$$\theta = \tan^{-1}\frac{F_y}{F_x}$$

Non-rectangular Components:









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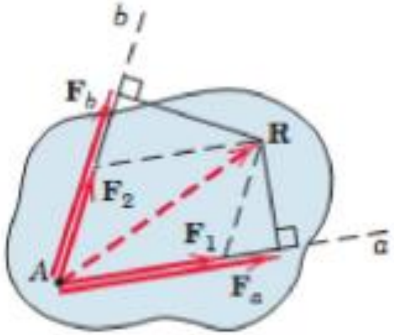
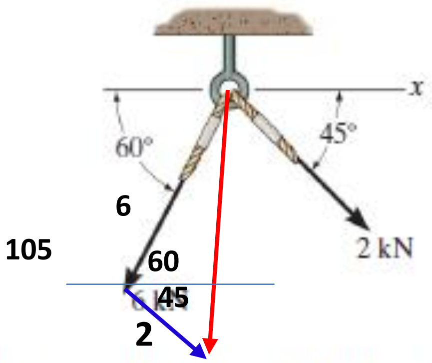
The components and projections of R are equal only when the axes a and b are perpendicular.

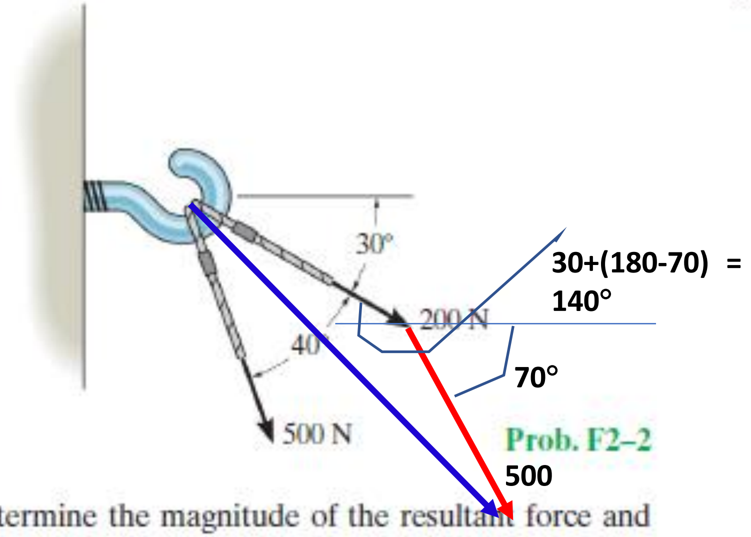


F2–1. Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x axis.



Prob. F2–1

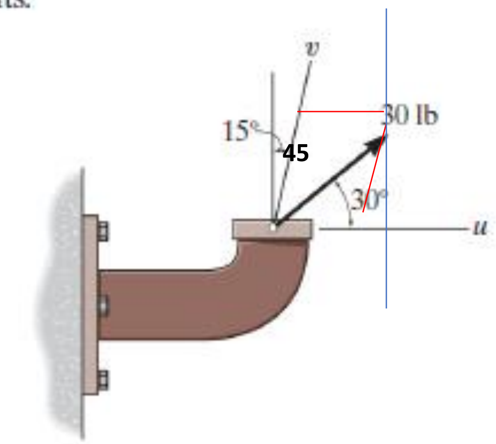
F2–2. Two forces act on the hook. Determine the magnitude of the resultant force.



Prob. F2–2

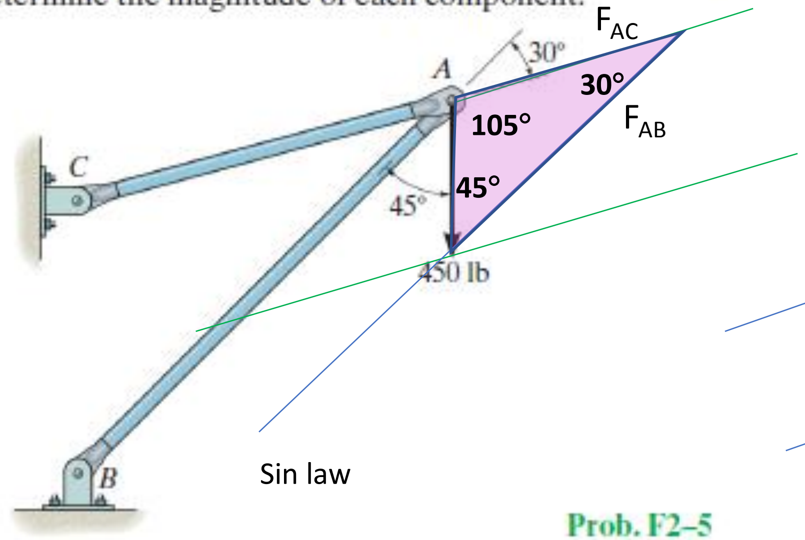
F2–3. Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.

F2–4. Resolve the 30-lb force into components along the u and v axes, and determine the magnitude of each of these components.



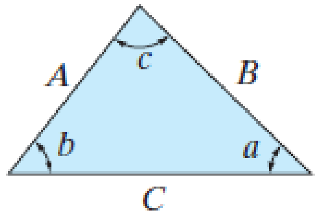
Prob. F2–4

F2–5. The force $F = 450$ lb acts on the frame. Resolve this force into components acting along members AB and AC, and determine the magnitude of each component.



Prob. F2–5

F2–6. If force **F** is to have a component along the u axis of $F_u = 6$ kN, determine the magnitude of **F** and the magnitude of its component $\mathbf{F}_v$ along the v axis.



Cosine law:

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Sine law:

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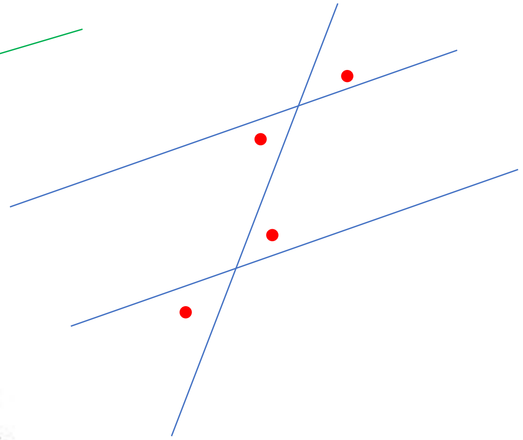
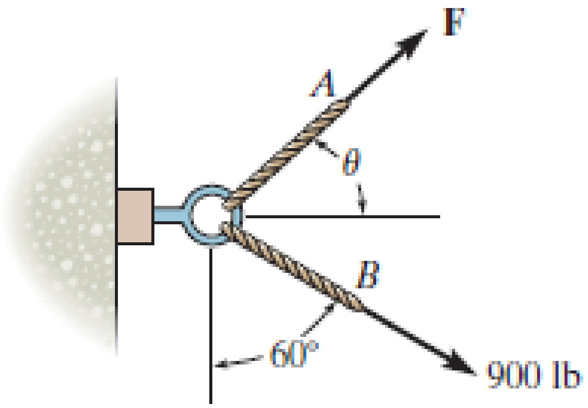
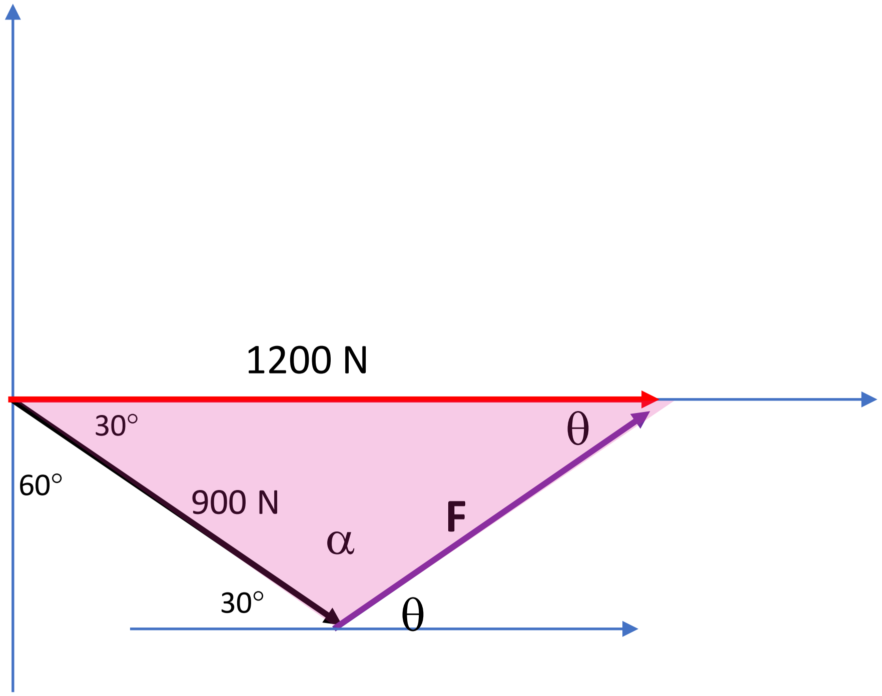
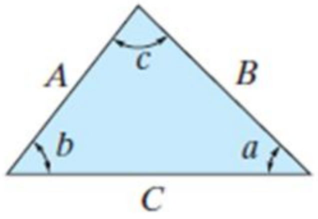


2–9. If the resultant force acting on the support is to be 1200 lb, directed horizontally to the right, determine the force **F** in rope A and the corresponding angle θ.







Cosine law:

$C = \sqrt{A^2 + B^2 - 2AB\cos c}$

Sine law:

$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$

$F = \sqrt{1200^2 + 900^2 - 2(1200)(900)\cos 30}$ = <u>615.94 N</u>

$$\frac{615.94}{\sin 30} = \frac{900}{\sin \theta}$$

<u>θ= 46.94°</u>

EXAMPLE 2.4

It is required that the resultant force acting on the eyebolt in Fig. 2–14*a* be directed along the positive *x* axis and that $\mathbf{F}_2$ have a *minimum* magnitude. Determine this magnitude, the angle θ, and the corresponding resultant force.

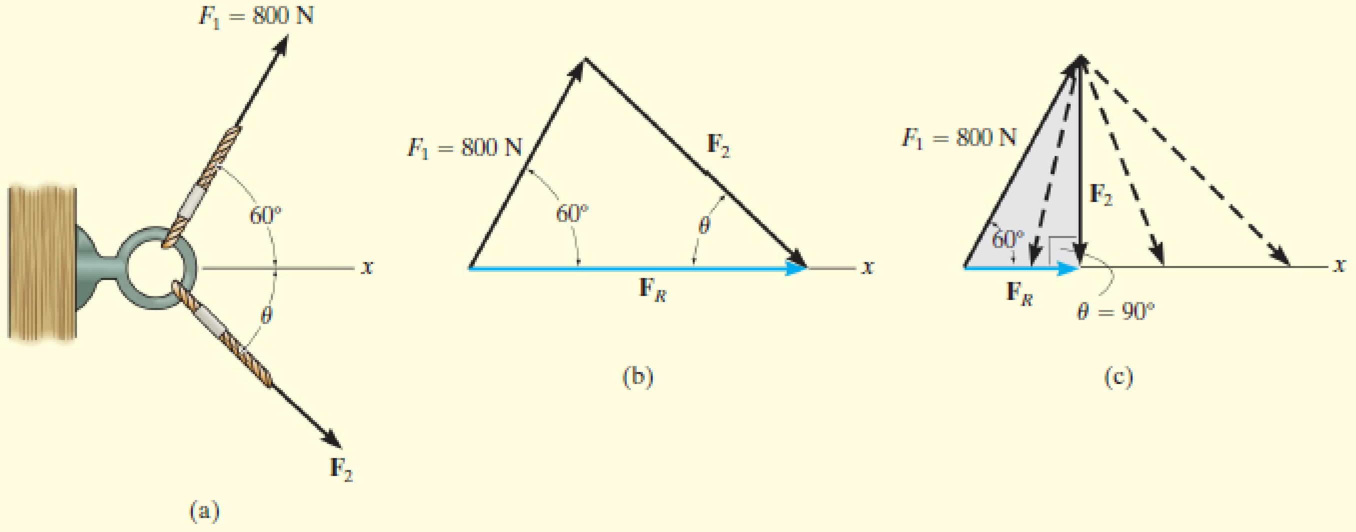


Fig. 2–14

SOLUTION

The triangle rule for $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ is shown in Fig. 2–14*b*. Since the magnitudes (lengths) of $\mathbf{F}_R$ and $\mathbf{F}_2$ are not specified, then $\mathbf{F}_2$ can actually be any vector that has its head touching the line of action of $\mathbf{F}_R$, Fig. 2–14*c*. However, as shown, the magnitude of $\mathbf{F}_2$ is a *minimum* or the shortest length when its line of action is *perpendicular* to the line of action of $\mathbf{F}_R$, that is, when

$$\theta = 90^\circ \qquad \textit{Ans.}$$

Since the vector addition now forms the shaded right triangle, the two unknown magnitudes can be obtained by trigonometry.

$$F_R = (800\text{ N})\cos 60^\circ = 400\text{ N} \qquad \textit{Ans.}$$

$$F_2 = (800\text{ N})\sin 60^\circ = 693\text{ N} \qquad \textit{Ans.}$$

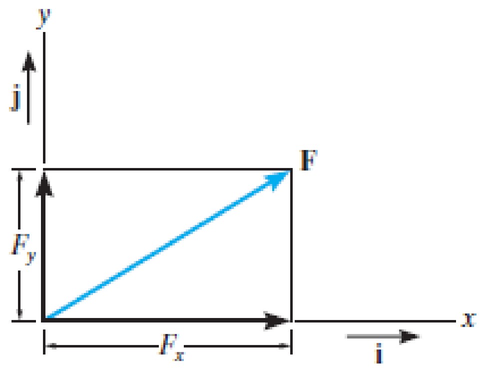
2.4 Addition of a System of Coplanar Forces

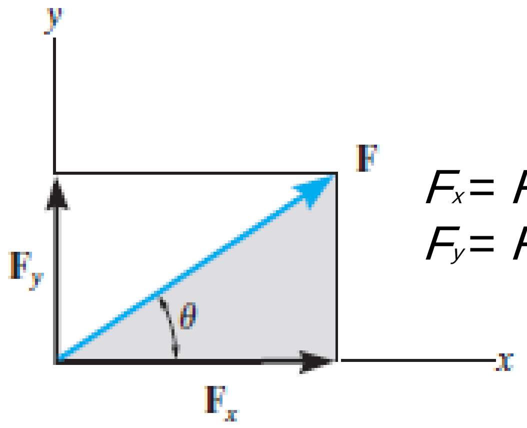
techniques of vector addition.

- graphical
  - **Head-to-Tail Method**
- algebraic techniques

Cartesian Vector Notation

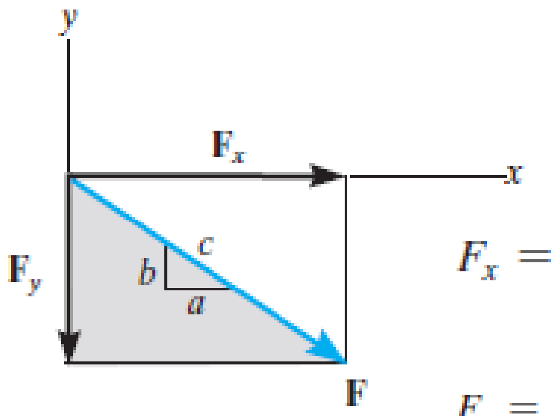
$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$





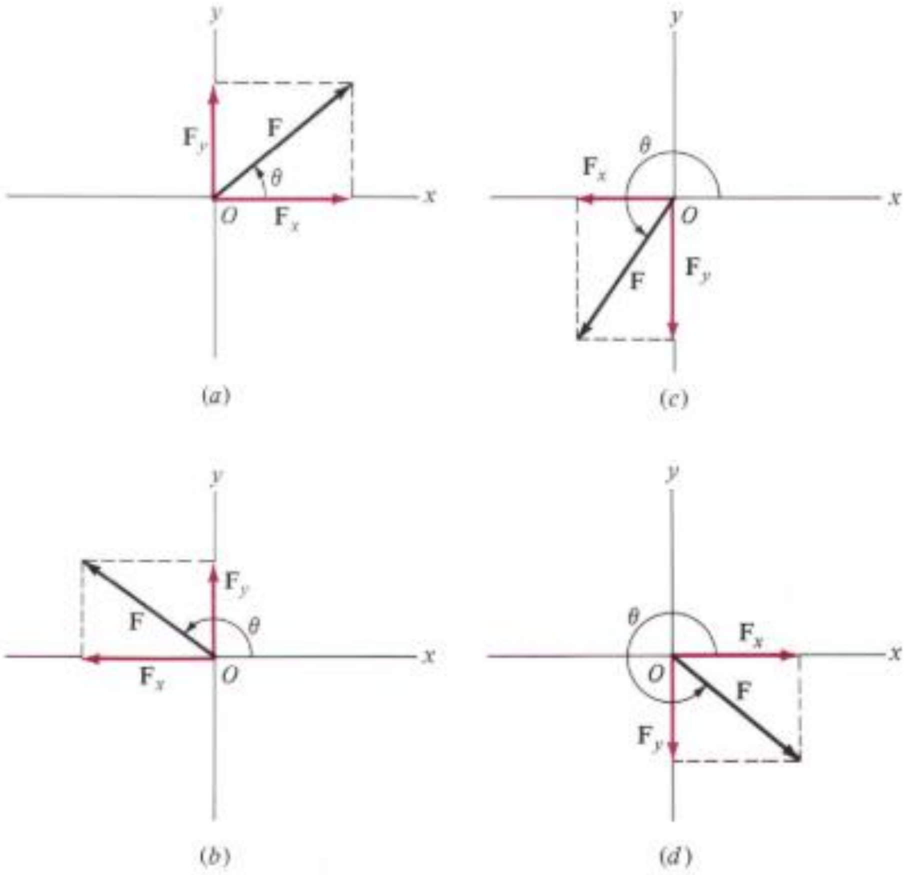
$F_x = F\cos\theta$

$F_y = F\sin\theta$



$$F_x = F\left(\frac{a}{c}\right)$$

$$F_y = -F\left(\frac{b}{c}\right)$$



Coplanar Force Resultants.

$\mathbf{F}_1 = F_{1x}\mathbf{i} + F_{1y}\mathbf{j}$
$\mathbf{F}_2 = -F_{2x}\mathbf{i} + F_{2y}\mathbf{j}$
$\mathbf{F}_3 = F_{3x}\mathbf{i} - F_{3y}\mathbf{j}$

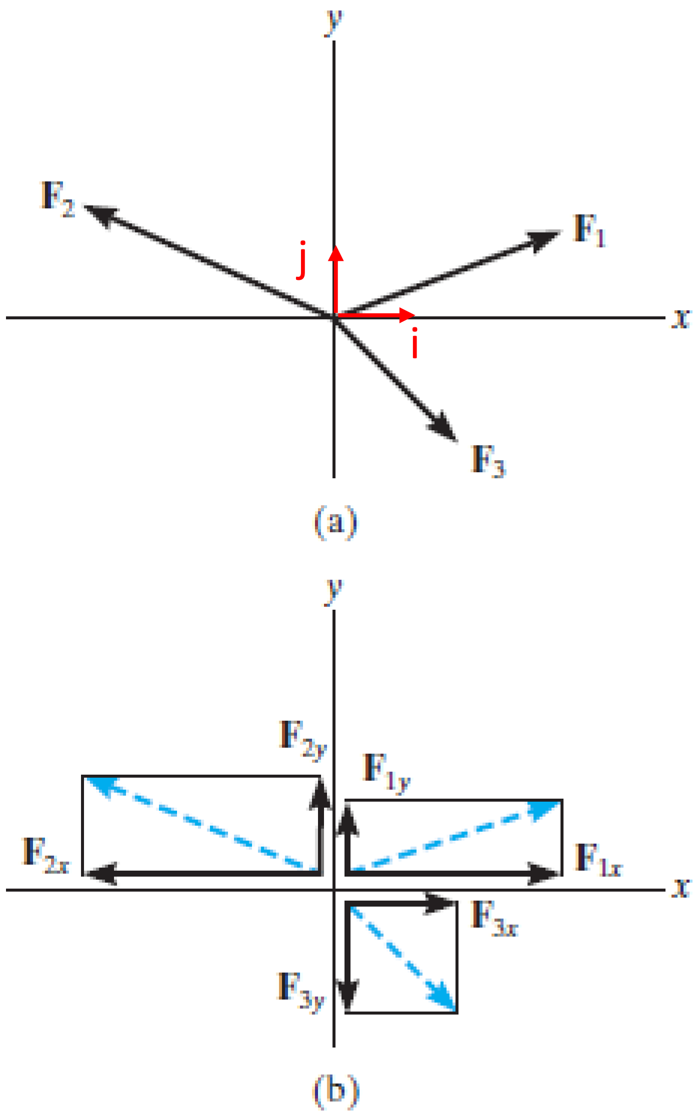
The vector resultant is therefore

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$
$= F_{1x}\mathbf{i} + F_{1y}\mathbf{j} - F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{3x}\mathbf{i} - F_{3y}\mathbf{j}$
$= (F_{1x} - F_{2x} + F_{3x})\ \mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\ \mathbf{j}$
$= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}$

$$(F_R)_x = \Sigma F_x$$
$$(F_R)_y = \Sigma F_y$$

Magnitude of F_R is $$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

Direction of the resultant force $$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$



Ex. Determine the *x* and *y* components of **F**$_1$ and **F**$_2$ acting on the boom shown and find the resultant as cartesian vector

Each force as a Cartesian vector.

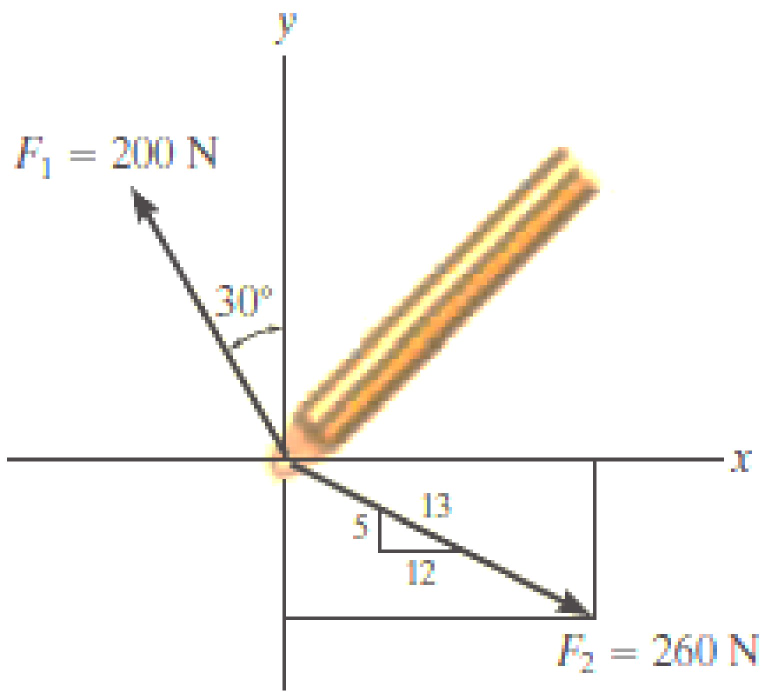
$\boldsymbol{F}_1$ = -200 sin 30i + 200 cos 30j   **=** { -100**i** + 173**j**} N

$\boldsymbol{F}_2 = 260\text{ N}\left(\frac{12}{13}\right)$ i - $260\text{ N}\left(\frac{5}{13}\right)$ j   = { 240**i** - 100**j**}N

$$(F_R)_x = \Sigma F_x$$
$$(F_R)_y = \Sigma F_y$$

resultant a s cartesian vector

F$_R$ **=** { 140**i** + 73**j**} N



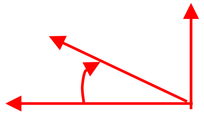
The end of the boom O is subjected to three concurrent and coplanar forces. Determine the magnitude and direction of the resultant force.

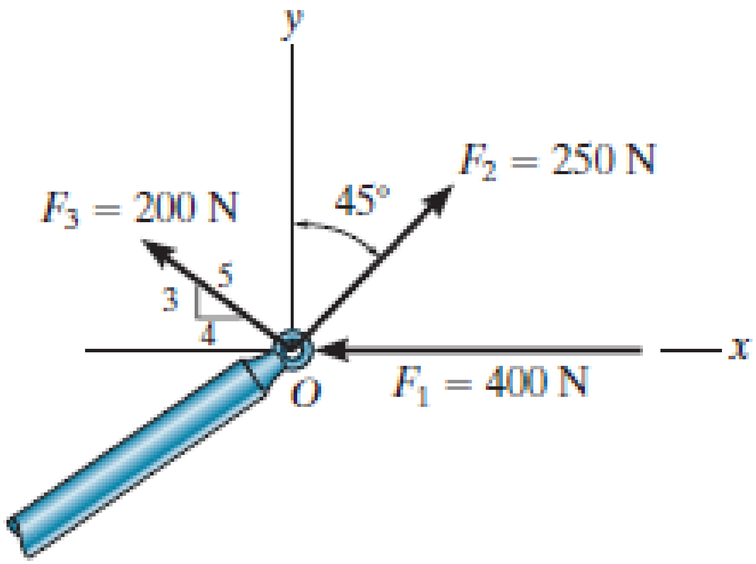
$\xrightarrow{+}$ $(F_R)_x = -400 + 250 \sin 45 - 200(4/5) = -383.2$ N

$+\uparrow$ $(F_R)_y = 250 \cos 45 + 200\,(3/5) = 296.8$ N

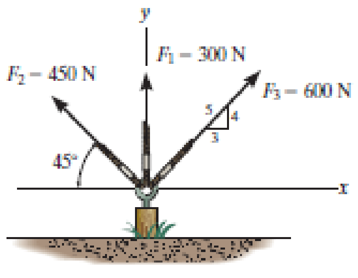
$$F_R = \sqrt{(-383.2\text{ N})^2 + (296.8\text{ N})^2} = 484.7\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{296.8}{383.2}\right) = 37.8^\circ$$



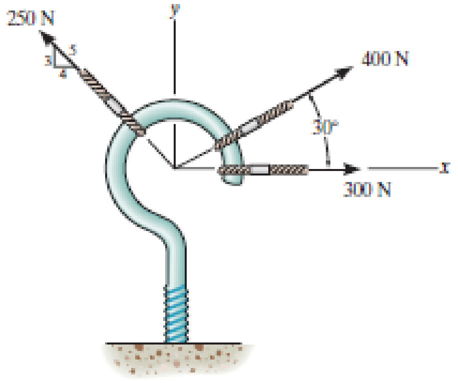


F2–7. Resolve each force acting on the post into its x and y components.



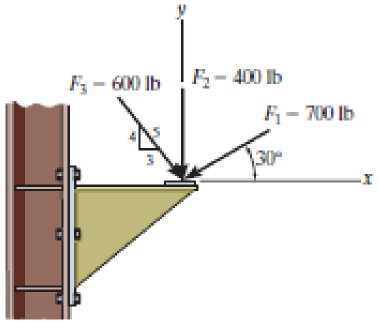
Prob. F2–7

F2–8. Determine the magnitude and direction of the resultant force.



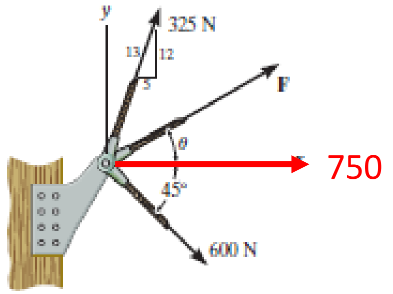
Prob. F2–8

F2–9. Determine the magnitude of the resultant force acting on the corbel and its direction θ measured counterclockwise from the x axis.



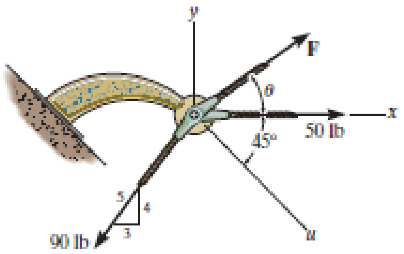
Prob. F2–9

F2–10. If the resultant force acting on the bracket is to be 750 N directed along the positive x axis, determine the magnitude of **F** and its direction θ.



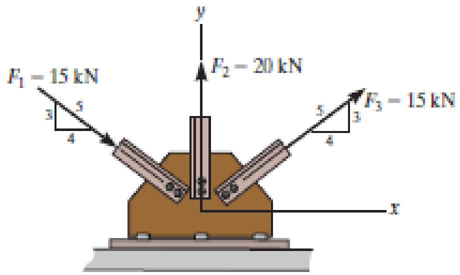
Prob. F2–10

F2–11. If the magnitude of the resultant force acting on the bracket is to be 80 lb directed along the u axis, determine the magnitude of **F** and its direction θ.



Prob. F2–11

F2–12. Determine the magnitude of the resultant force and its direction θ measured counterclockwise from the positive x axis.



Prob. F2–12

$Fr_x = 325/13 \times 5 + F\cos\theta + 600 \cos 45 = 750$ …..(1)

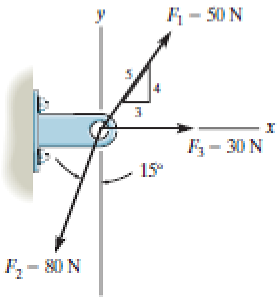
$Fr_y = 325/13 \times 12 + F\sin\theta - 600 \sin 45 = 0$ ……(2)

Divide eq. 2 by 1 ……..

θ = …..

Sub in 1 find F

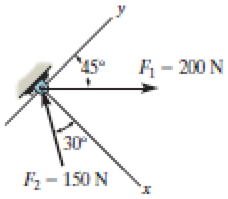
2–38. Express each of the three forces acting on the support in Cartesian vector form and determine the magnitude of the resultant force and its direction, measured clockwise from positive x axis.



Prob. 2–38

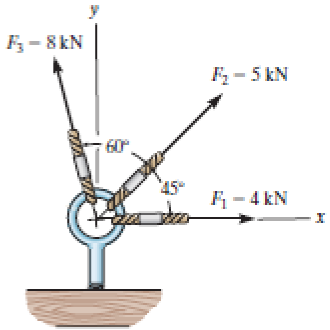
2–39. Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$.

***2–40.** Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



Probs. 2–39/40

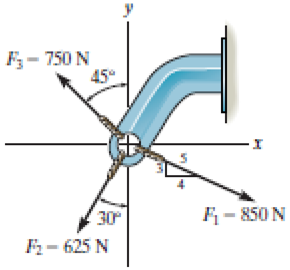
2–41. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



Prob. 2–41

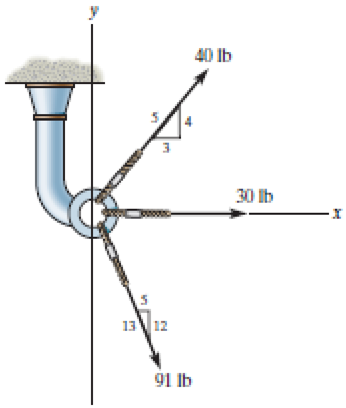
2–42. Express $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ as Cartesian vectors.

2–43. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



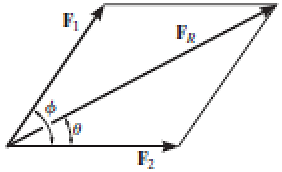
Probs. 2–42/43

***2–44.** Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.



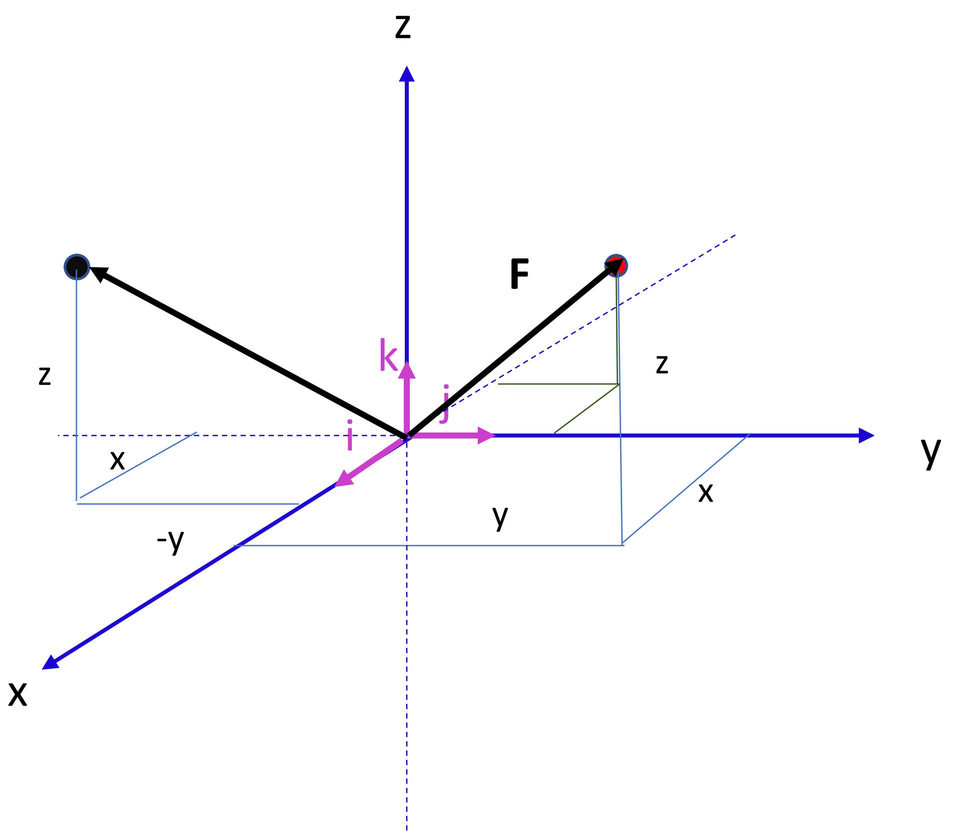
Prob. 2–44

2–45. Determine the magnitude and direction θ of the resultant force $\mathbf{F}_R$. Express the result in terms of the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ and the angle ϕ.



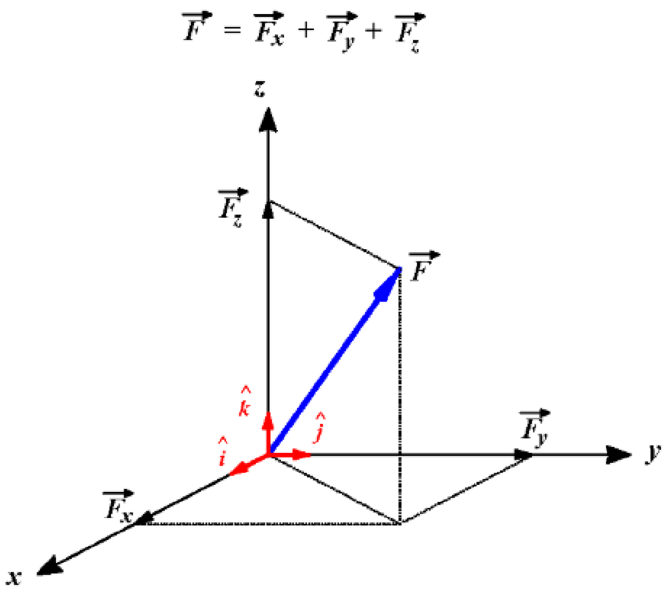
Prob. 2–45

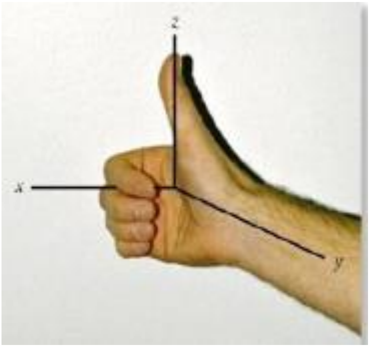
Force Vectors in 3D



z
F
z
k
j
z
i
x
y
x
-y
y
x

2.5 Cartesian Vectors





The right-hand rule

Rectangular Components of a Vector.

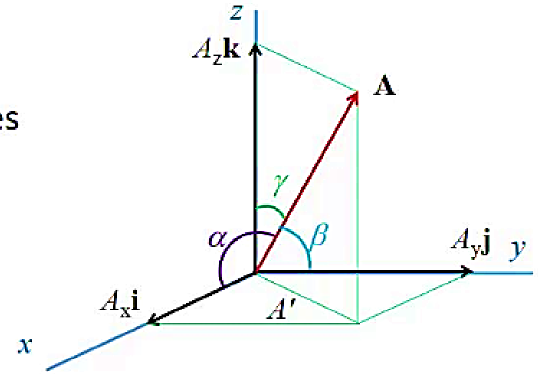
Cartesian Vector Representation

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

Magnitude of a Cartesian Vector

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Coordinate direction angles α, β and γ.



Coordinate Direction Angles.

We will define the *direction* of **A** by the ***coordinate direction angles*** a (alpha), b (beta), and g (gamma), measured between the *tail* of **A** and the *positive x, y, z* axes

$$\cos\alpha = \frac{A_x}{A} \quad \cos\beta = \frac{A_y}{A} \quad \cos\gamma = \frac{A_z}{A}$$

direction cosines of **A.**

the ***coordinate direction angles*** α , β , and γ , measured between the *tail* of **A** and the *positive x, y, z* axes provided they are located at the tail of **A**,

Unit Vector

A = A_x**i** + A_y**j** + A_z**k**, then

u$_A$ will have a magnitude of one and be dimensionless provided **A** is divided by its magnitude,

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$

components of **u**$_A$ *represent the direction cosines of* **A**,

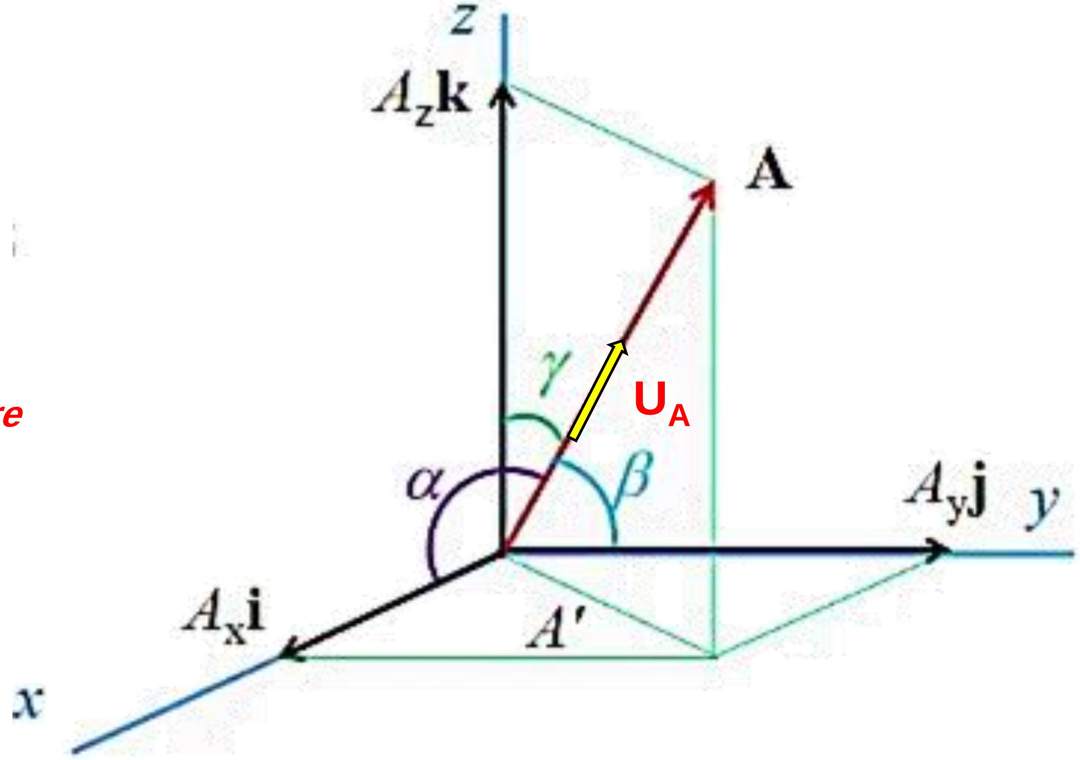
u$_A$= cos α**i** + cos β **j** + cos γ **k**

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

If two angles are given , use to find the third

$$\cos\alpha = \frac{A_x}{A} \quad \cos\beta = \frac{A_y}{A} \quad \cos\gamma = \frac{A_z}{A}$$

$$\begin{aligned}\mathbf{A} &= A\mathbf{u}_A \\ &= A\cos\alpha\mathbf{i} + A\cos\beta\mathbf{j} + A\cos\gamma\mathbf{k} \\ &= A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}\end{aligned}$$



Express the force F as a Cartesian vector

Only 2 of the direction angles are known. The 3rd angle α must satisfy

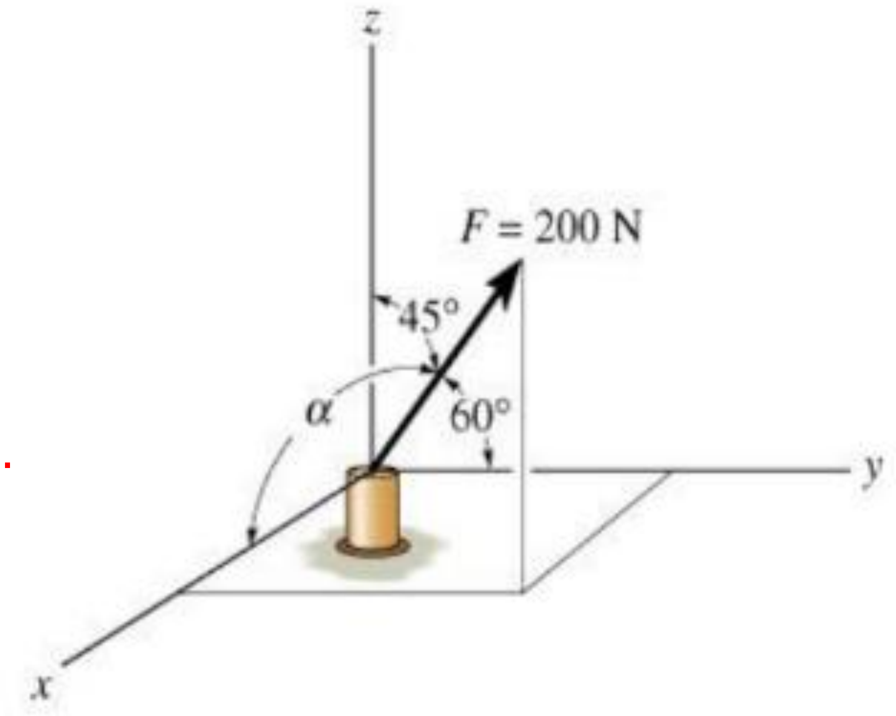
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$\cos^2\alpha + \cos^2 60 + \cos^2 45 = 1$

$\cos^2\alpha = 1 - 0.75 = 0.25$

$\cos\alpha = \pm\sqrt{0.25} = \pm 0.5$

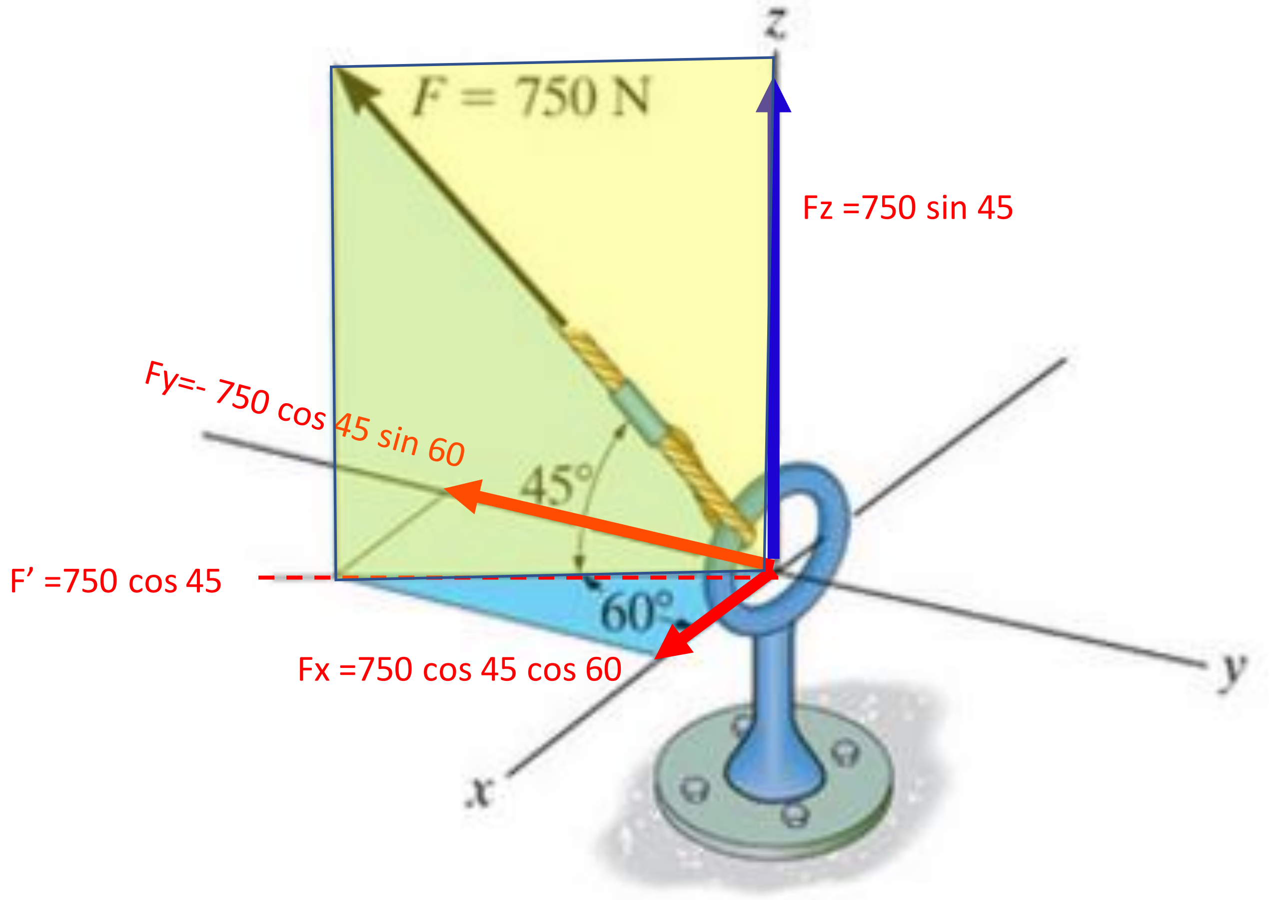
α = 60 ° or 120° since Fx is in the +x direction.



F = F cos(α) i + F cos(β) j + F cos(γ) k

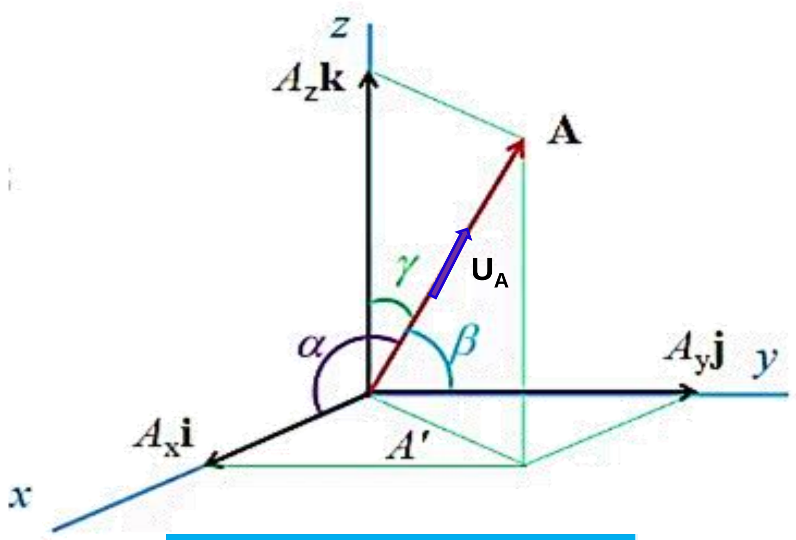
F = (100i + 100j + 141.4k) N

Express F as cartesian Vector



F = {750 cos 45 cos 60i - 750 cos 45 sin 60 j+ 750 sin 45 k} N

From last lecture



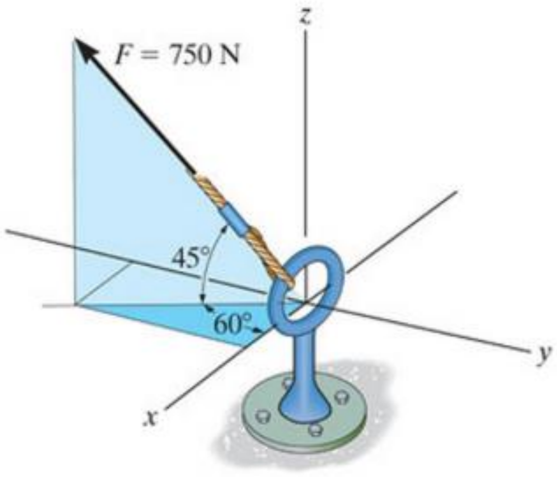
$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

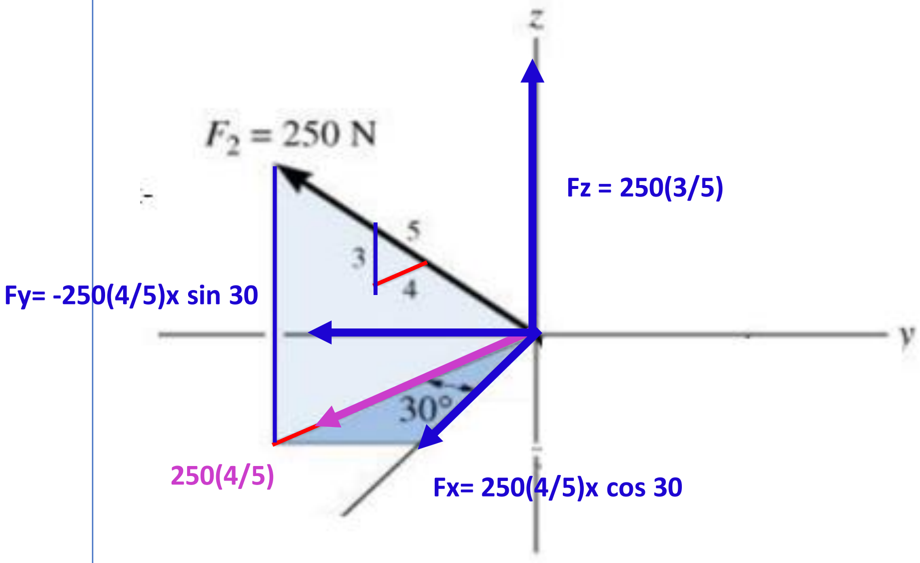
$$\cos\alpha = \frac{A_x}{A} \quad \cos\beta = \frac{A_y}{A} \quad \cos\gamma = \frac{A_z}{A}$$

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$

$$\begin{aligned}\mathbf{A} &= A\mathbf{u}_A \\ &= A\cos\alpha\,\mathbf{i} + A\cos\beta\,\mathbf{j} + A\cos\gamma\,\mathbf{k} \\ &= A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}\end{aligned}$$



F = {750 cos 45 cos 60i - 750 cos 45 sin 60 j+ 750 sin 45 k} N



Resolve $\mathbf{F_2}$ vertically and horizontally

F_{2z} = 250 (3/5) = 150 N, $F_{2'}$ = 250 (4/5) = 200 N

$F_{2'}$ can be further resolved as F_{2x} = 200cos30° & F_{2y} = -200sin30°

→ $\mathbf{F_2}$ = (173.2**i** - 100**j** + 150**k**) N

2.6 Addition of Cartesian Vectors

$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$

$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k},$

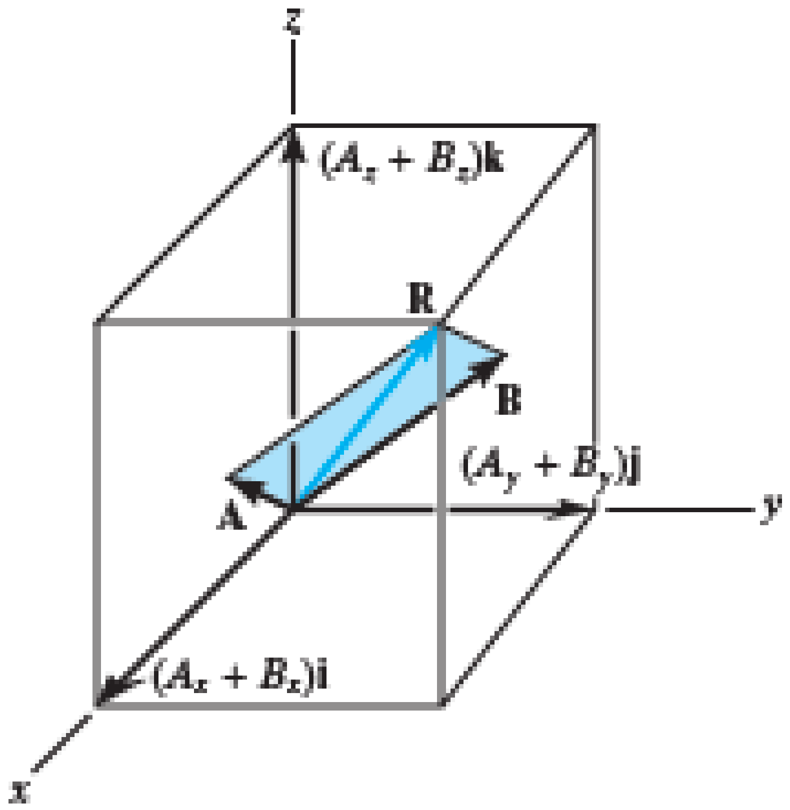
$\mathbf{F_R} = \mathbf{A} + \mathbf{B} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} + (A_z + B_z)\mathbf{k}$

$$\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$

Magnitude of the resultant $|F| = \sqrt{F_x^2 + F_y^2 + F_z^2}$

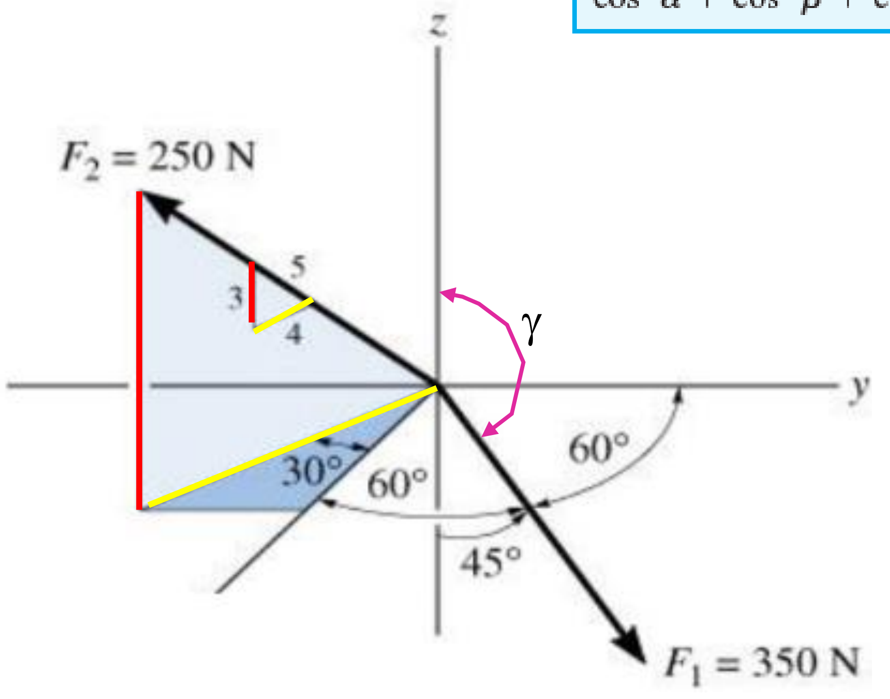
Direction of the resultant

$$\alpha = \cos^{-1}(F_{Rx} / F_R)$$
$$\beta = \cos^{-1}(F_{Ry} / F_R)$$
$$\gamma = \cos^{-1}(F_{Rz} / F_R)$$



$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Determine the magnitude and direction angles of the resultant force



Force $\mathbf{F_1}$: $\alpha = 60°$, $\beta = 60°$, $\gamma = ?$

$\mathbf{F_1} = F\cos 60^\circ\ \mathbf{i} + F\cos 60^\circ\ \mathbf{j} + F\cos 135^\circ\ \mathbf{k}$

$= (175\mathbf{i} + 175\mathbf{j} - 247.5\mathbf{k})$ N

Resolve $\mathbf{F_2}$ vertically and horizontally

$F_{2z} = 250\ (3/5) = 150$ N, $F_{2'} = 250\ (4/5) = 200$ N

$F_{2'}$ can be further resolved as $F_{2x} = 200\cos 30^\circ$ & $F_{2y} = -200\sin 30^\circ$

→ $\mathbf{F_2} = (173.2\mathbf{i} - 100\mathbf{j} + 150\mathbf{k})$ N

$$\mathbf{F_R} = \{348.21\mathbf{i} + 75.0\mathbf{j} - 97.487\mathbf{k}\}\ \text{N}$$

$$F_R = \sqrt{(348.21)^2 + (75.0)^2 - (97.487)^2} = 369.29\ \text{N}$$

direction

$$\alpha = \cos^{-1}\left(\frac{348.21}{369.29}\right) = 19.5°$$

$$\beta = \cos^{-1}\left(\frac{75.0}{369.29}\right) = 78.3°$$

$$\gamma = \cos^{-1}\left(\frac{-97.487}{369.29}\right) = 105°$$

Find the resultant force in cartesian vector

$F_{2XY} = 800 * \cos(45^\circ) = 565.69$ lb

$F_{2x} = 565.69 * \cos(45^\circ) = 489.90$ lb

$F_{2y} = 565.69 * \sin(30^\circ) = 282.84$ lb

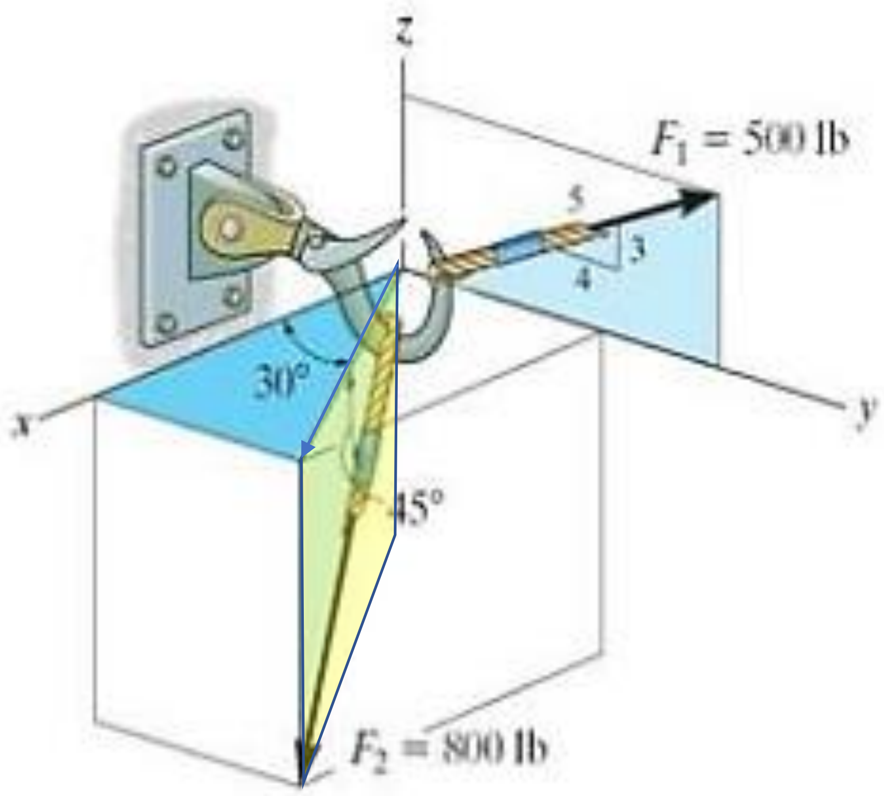
$F_{2z} = -800 * \sin(45^\circ) = -565.69$ lb

$\boldsymbol{F_1} = \{0\, \boldsymbol{i} + 400\, \boldsymbol{j} + 300\, \boldsymbol{k}\}$ lb

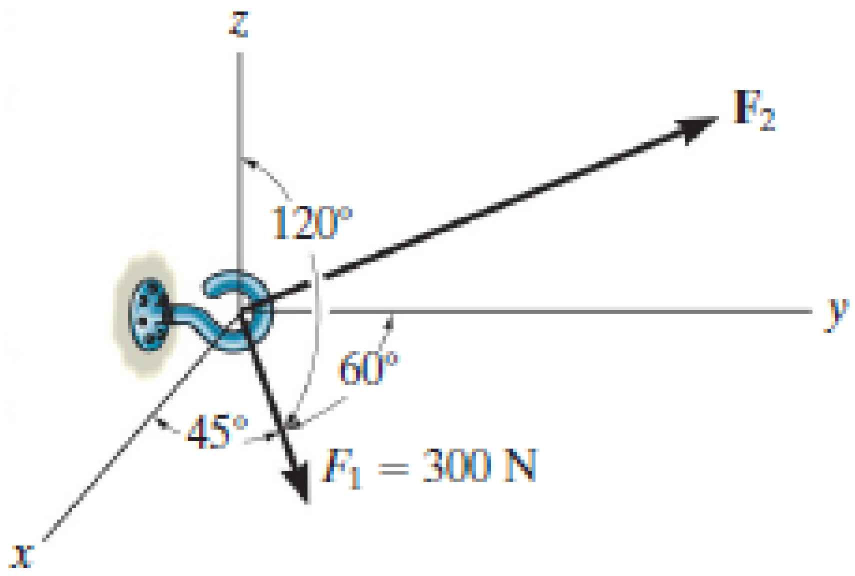
$\boldsymbol{F_2} = \{489.9\, \boldsymbol{i} + 282.8\, \boldsymbol{j} - 565.7\, \boldsymbol{k}\}$ lb

Now, $\boldsymbol{R} = \boldsymbol{F_1} + \boldsymbol{F_2}$ **or**

$\boldsymbol{R} = \{489.9\, \boldsymbol{i} + 682.8\, \boldsymbol{j} - 265.7\, \boldsymbol{k}\}$ lb



Two forces act on the hook shown. Specify the magnitude of **F**$_2$ and its coordinate direction angles so that the resultant force **F**$_R$ acts along the positive *y* axis and has a magnitude of 800 N.



$$\mathbf{F}_1 = F_1 \cos \alpha_1 \mathbf{i} + F_1 \cos \beta_1 \mathbf{j} + F_1 \cos \gamma_1 \mathbf{k}$$
$$= 300 \cos 45° \,\mathbf{i} + 300 \cos 60° \,\mathbf{j} + 300 \cos 120° \,\mathbf{k}$$
$$= \{212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k}\} \text{ N}$$
$$\mathbf{F}_2 = F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}$$

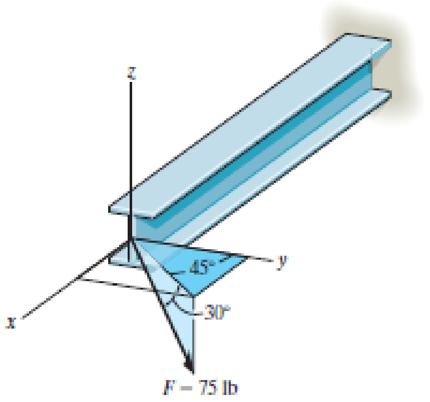
$$\mathbf{F_R} = \{0\mathbf{i} + 800\mathbf{j} + 0\mathbf{k}\} \text{ N}$$

$$F_{Rx} = \sum Fx \qquad 0 = 212.1 + F_{2x} \qquad F_{2x} = -212.1 \text{ N}$$
$$F_{Ry} = \sum Fy \qquad 800 = 150 + F_{2y} \qquad F_{2y} = 650 \text{ N}$$
$$F_{Rz} = \sum Fz \qquad 0 = -150 + F_{2z} \qquad F_{2z} = 150 \text{ N}$$

$$F_2 = \sqrt{(-212.1 \text{ N})^2 + (650 \text{ N})^2 + (150 \text{ N})^2}$$
$$= 700 \text{ N}$$

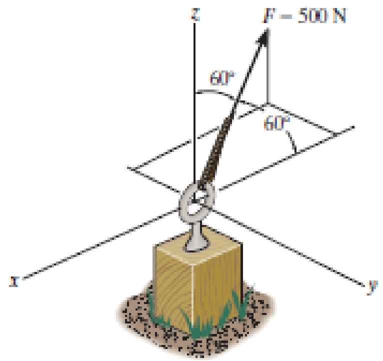
$$\cos \alpha_2 = \frac{-212.1}{700}; \qquad \alpha_2 = 108°$$
$$\cos \beta_2 = \frac{650}{700}; \qquad \beta_2 = 21.8°$$
$$\cos \gamma_2 = \frac{150}{700}; \qquad \gamma_2 = 77.6°$$

F2–13. Determine the coordinate direction angles of the force.



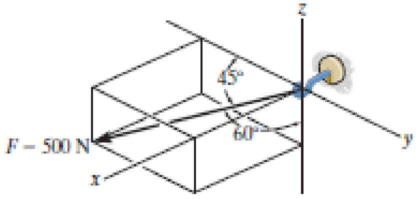
Prob. F2–13

F2–14. Express the force as a Cartesian vector.



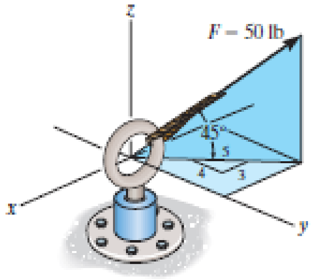
Prob. F2–14

F2–15. Express the force as a Cartesian vector.



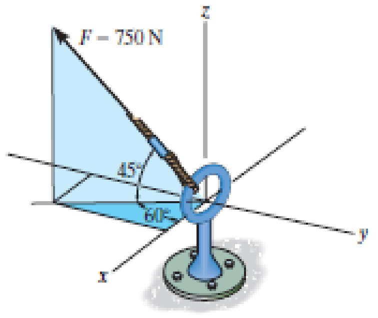
Prob. F2–15

F2–16. Express the force as a Cartesian vector.



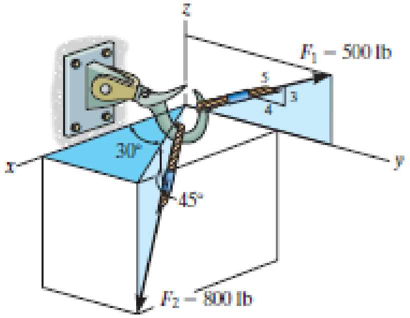
Prob. F2–16

F2–17. Express the force as a Cartesian vector.



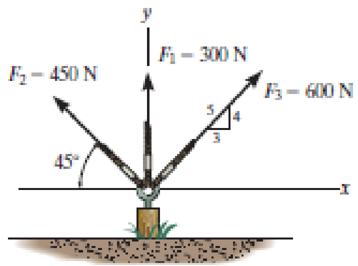
Prob. F2–17

F2–18. Determine the resultant force acting on the hook.



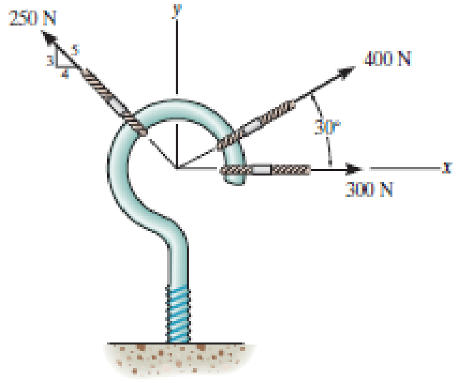
Prob. F2–18

F2–7. Resolve each force acting on the post into its x and y components.



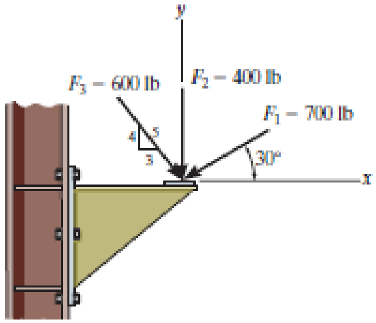
Prob. F2–7

F2–8. Determine the magnitude and direction of the resultant force.



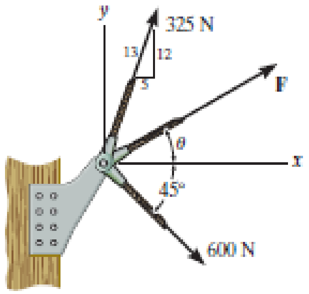
Prob. F2–8

F2–9. Determine the magnitude of the resultant force acting on the corbel and its direction θ measured counterclockwise from the x axis.



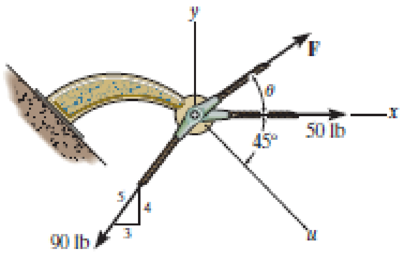
Prob. F2–9

F2–10. If the resultant force acting on the bracket is to be 750 N directed along the positive x axis, determine the magnitude of **F** and its direction θ.



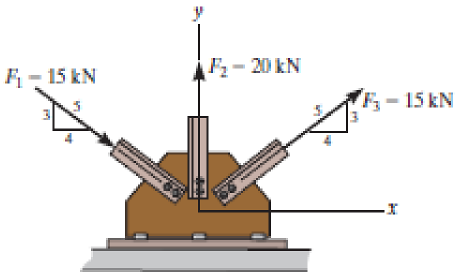
Prob. F2–10

F2–11. If the magnitude of the resultant force acting on the bracket is to be 80 lb directed along the u axis, determine the magnitude of **F** and its direction θ.



Prob. F2–11

F2–12. Determine the magnitude of the resultant force and its direction θ measured counterclockwise from the positive x axis.



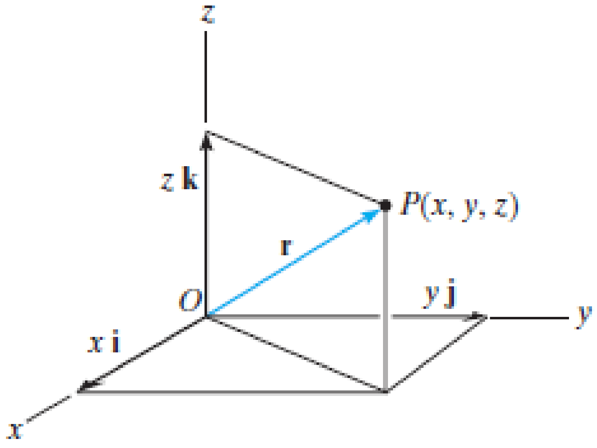
Prob. F2–12

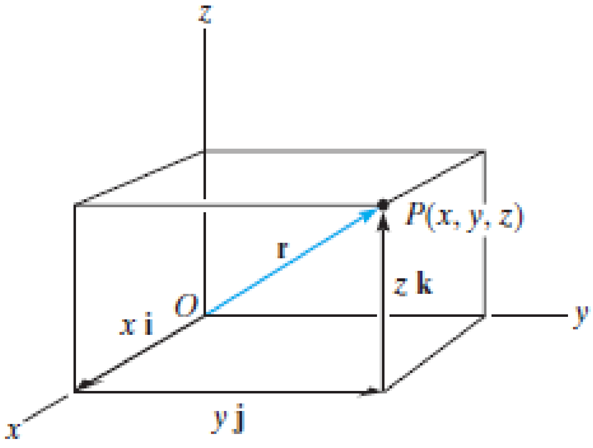
Chapter 2- Force Vectors

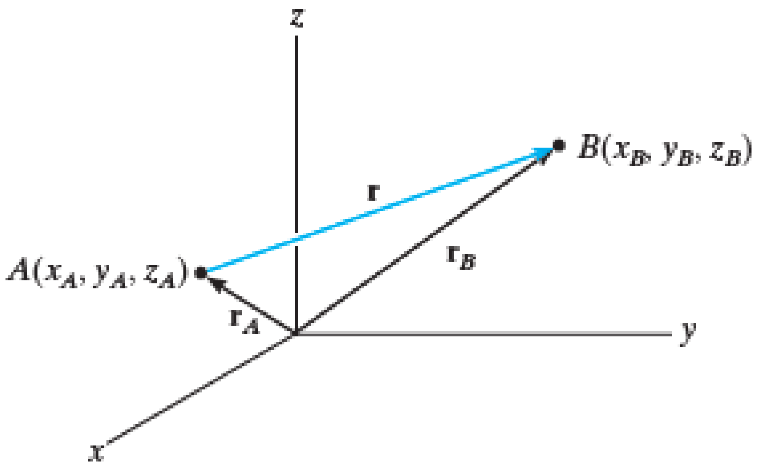
I. Marie

2.7 Position Vectors

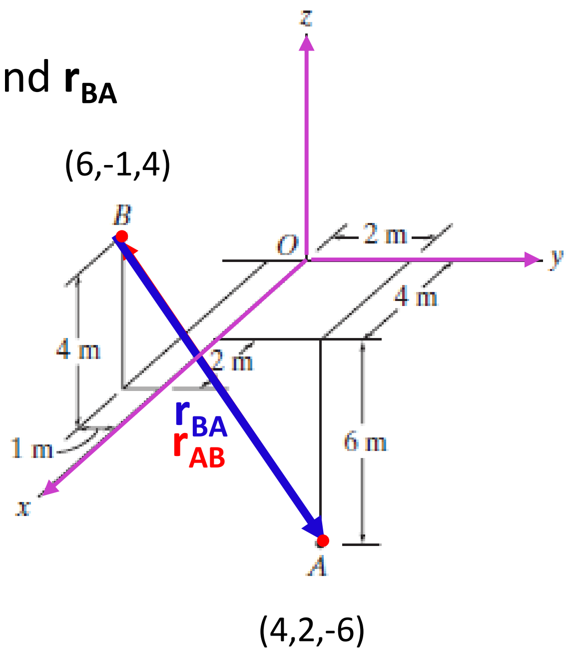
A ***position vector*** **r** is defined as a fixed vector which locates a point in space relative to another point.







Find $\mathbf{r}_{AB}$ and $\mathbf{r}_{BA}$

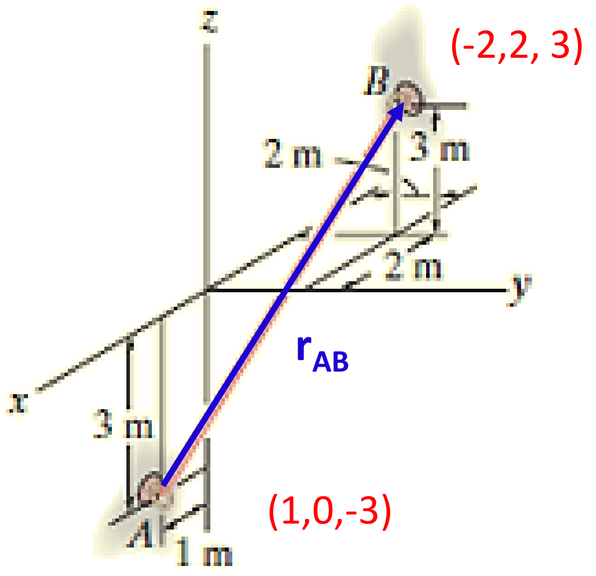


$\mathbf{r}_{AB}$ = (6-4)i + (-1-2)j + (4- -6)k = 2i -3j +10k

$\mathbf{r}_{BA}$ = (4-6)i + (2- -1)j + (-6 -4)k = -2i +3j -10k

$$\mathbf{r} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$$

Determine the length of AB and its direction



$\mathbf{r}_{AB}$ = {-3i +2j +6k} m

Length of AB

$$r_{AB} = \sqrt{3^2 + 2^2 + 6^2} = 7m$$

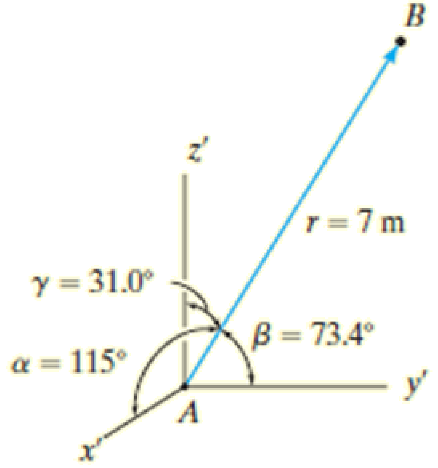
Unit vector = $\frac{\bar{r}}{|r|} = -\frac{3}{7}i + \frac{2}{7}j + \frac{6}{7}k$

Direction

α =cos $^{-1}$ (-3/7) = 115 °

β = cos $^{-1}$ (2/7) = 73.4 °

γ = = cos $^{-1}$ (6/7) = 31.0 °



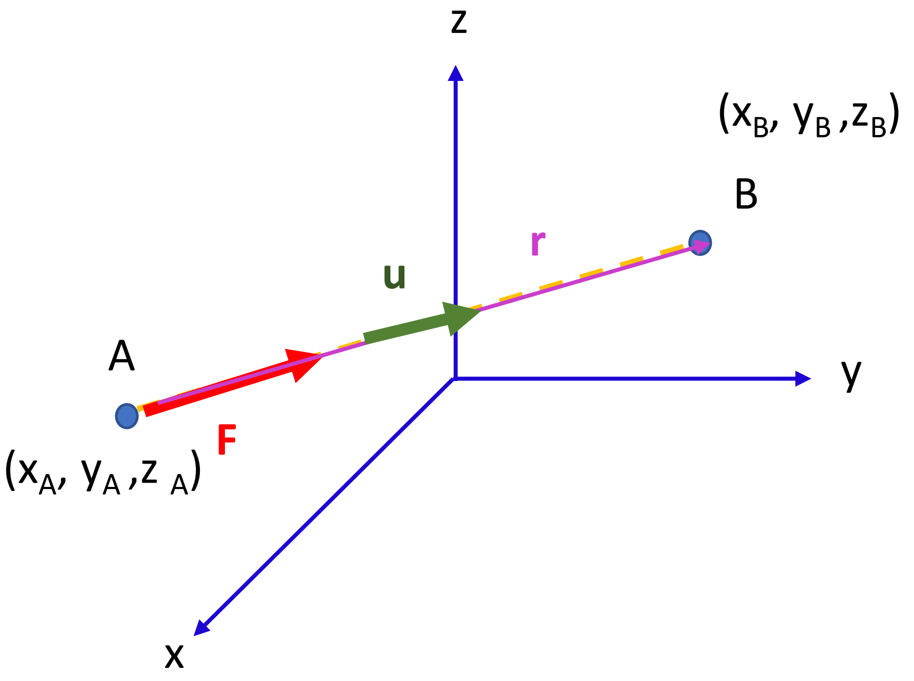
2.8 Force Vector Directed Along a Line

In many cases, the direction of a force is defined by the coordinates of two points along its line of action.

1. Form a position vector r along the line of action of force
2. Find the corresponding unit vector **u** = **r**/r that defines the direction of the line
3. Multiply the unit vector by the magnitude of the force

$$\mathbf{F} = F\cdot\mathbf{u} = F\,(\mathbf{r}/r)$$

$$\mathbf{F} = F\,\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right) = F\left(\frac{(x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}\right)$$



Express F as a Cartesian vector.

$\mathbf{F} = F \cdot \mathbf{u} = F\,(\mathbf{r}/r)$

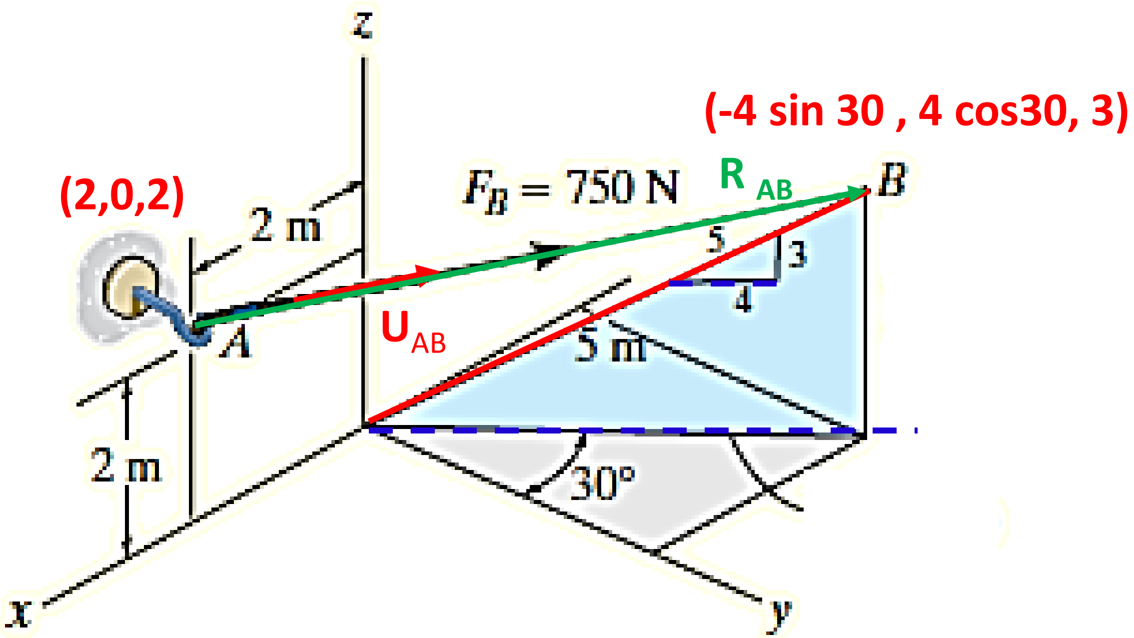
$$\mathbf{u}_B = \left(\frac{\mathbf{r}_B}{r_B}\right) = \frac{\{-4\mathbf{i} + 3.464\mathbf{j} + 1\mathbf{k}\}\ \text{m}}{\sqrt{(-4\ \text{m})^2 + (3.464\ \text{m})^2 + (1\ \text{m})^2}}$$

$$= -0.7428\mathbf{i} + 0.6433\mathbf{j} + 0.1857\mathbf{k}$$

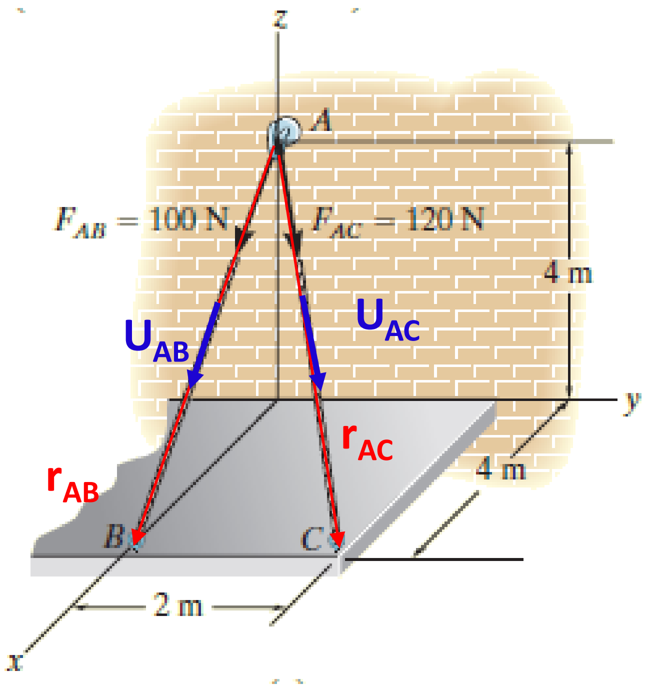
Force $\mathbf{F}_B$ expressed as a Cartesian vector becomes

$$\mathbf{F}_B = F_B\,\mathbf{u}_B = (750\ \text{N})(-0.74281\mathbf{i} + 0.6433\mathbf{j} + 0.1857\mathbf{k})$$

$$= \{-557\mathbf{i} + 482\mathbf{j} + 139\mathbf{k}\}\ \text{N}$$



Determine the resultant force acting at A as cartesian vector.



$A(0,0,4)$
$B(4,0,0)$
$C(4,2,0)$

$$\mathbf{r}_{AB} = \{4\mathbf{i} - 4\mathbf{k}\}\ \text{m}$$

$$r_{AB} = \sqrt{(4\ \text{m})^2 + (-4\ \text{m})^2} = 5.66\ \text{m}$$

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = (100\ \text{N})\left(\frac{4}{5.66}\mathbf{i} - \frac{4}{5.66}\mathbf{k}\right)$$

$$\mathbf{F}_{AB} = \{70.7\mathbf{i} - 70.7\mathbf{k}\}\ \text{N}$$

$$\mathbf{r}_{AC} = \{4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\}\ \text{m}$$

$$r_{AC} = \sqrt{(4\ \text{m})^2 + (2\ \text{m})^2 + (-4\ \text{m})^2} = 6\ \text{m}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = (120\ \text{N})\left(\frac{4}{6}\mathbf{i} + \frac{2}{6}\mathbf{j} - \frac{4}{6}\mathbf{k}\right)$$

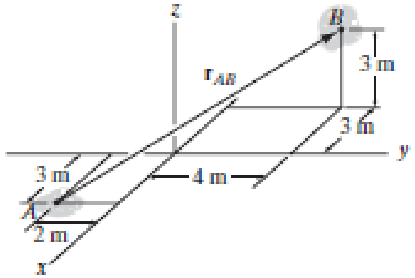
$$= \{80\mathbf{i} + 40\mathbf{j} - 80\mathbf{k}\}\ \text{N}$$

The resultant force is therefore

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC} = \{70.7\mathbf{i} - 70.7\mathbf{k}\}\ \text{N} + \{80\mathbf{i} + 40\mathbf{j} - 80\mathbf{k}\}\ \text{N}$$

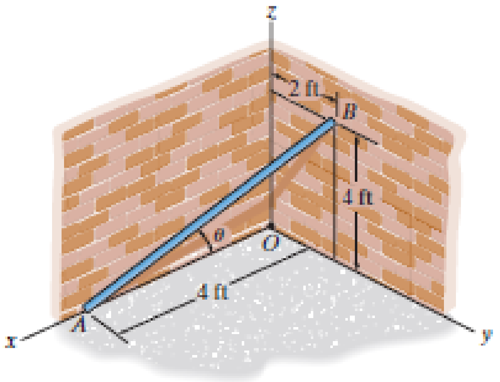
$$= \{151\mathbf{i} + 40\mathbf{j} - 151\mathbf{k}\}\ \text{N}$$

F2–19. Express the position vector $\mathbf{r}_{AB}$ in Cartesian vector form, then determine its magnitude and coordinate direction angles.



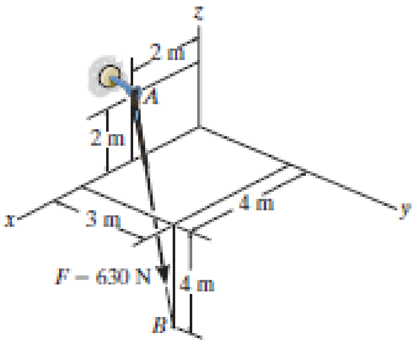
Prob. F2–19

F2–20. Determine the length of the rod and the position vector directed from A to B. What is the angle θ?

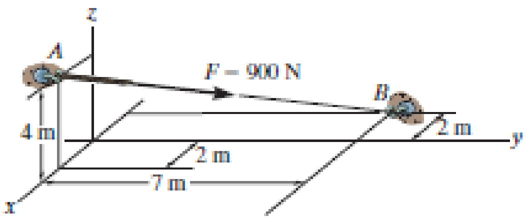


Prob. F2–20

F2–21. Express the force as a Cartesian vector.

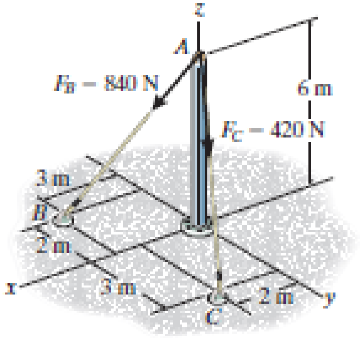


F2–22. Express the force as a Cartesian vector.



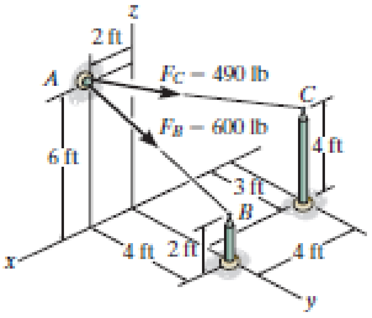
Prob. F2–22

F2–23. Determine the magnitude of the resultant force at A.



Prob. F2–23

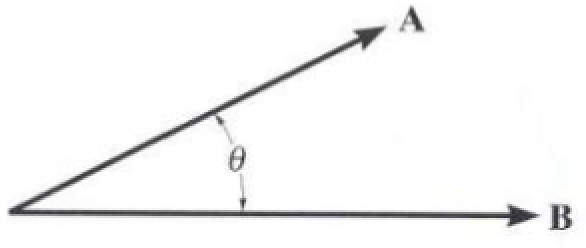
F2–24. Determine the resultant force at A.



2.9 Dot Product

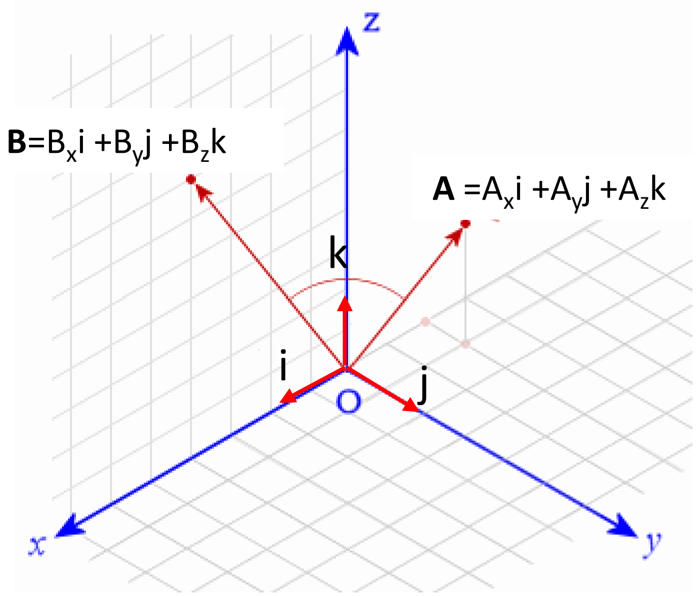
$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$$

The dot product is often referred to as
the ***scalar*** *product* of vectors



Laws of Operation.

1. Commutative law: $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$
2. Multiplication by a scalar: $a(\mathbf{A} \cdot \mathbf{B}) = (a\mathbf{A}) \cdot \mathbf{B} = \mathbf{A} \cdot (a\mathbf{B})$
3. Distributive law: $\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D})$



Cartesian Vector Formulation.

$$i \bullet j = j \bullet i = i \bullet k = k \bullet i = j \bullet k = k \bullet j = 0$$

$$i \bullet i = j \bullet j = k \bullet k = 1$$

$$\underline{A} \cdot \underline{B} = (A_x \underline{i} + A_y \underline{j} + A_z \underline{k}) \cdot (B_x \underline{i} + B_y \underline{j} + B_z \underline{k})$$
$$= A_x B_x + A_y B_y + A_z B_z$$

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

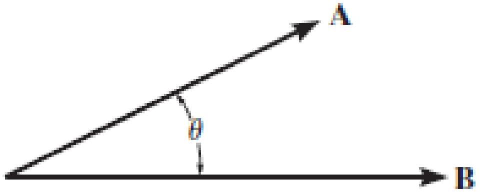
A = 2i -3j+1k

B = 4i +2j+2k

$$\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} . \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix} = 2.4 + (-3.2) + 1.2 = 4$$

Applications.

a. The angle formed between two vectors or intersecting l



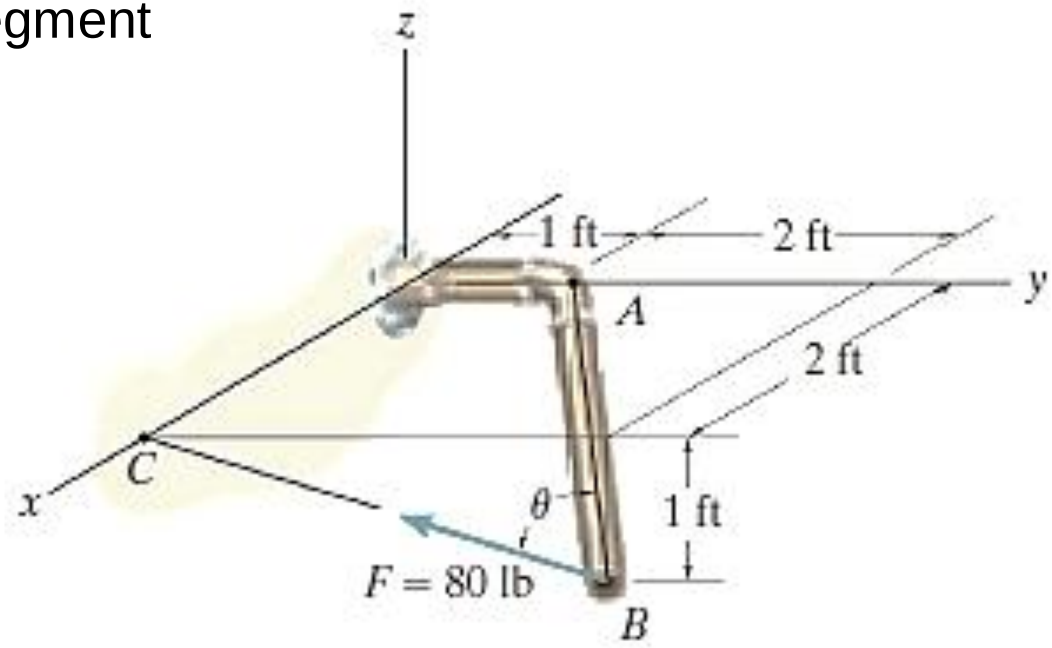
$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{A} \cdot \mathbf{B}}{AB}\right) \quad 0^\circ \leq \theta \leq 180^\circ$$

Ex. Determine the angle θ between **F** and the pipe segment *BA*

$$\mathbf{r}_{BA} = \{-2\mathbf{i} - 2\mathbf{j} + 1\mathbf{k}\} \text{ ft}, \; r_{BA} = 3 \text{ ft}$$

$$\mathbf{r}_{BC} = \{-3\mathbf{j} + 1\mathbf{k}\} \text{ ft}, \; r_{BC} = \sqrt{10}\, ft$$



$$\cos \theta = \frac{\mathbf{r}_{BA} \cdot \mathbf{r}_{BC}}{r_{BA} r_{BC}} = \frac{(-2)(0) + (-2)(-3) + (1)(1)}{3\sqrt{10}} = 0.7379$$

$$\theta = 42.5^\circ$$

Determine the angle between the edges of the sheet-metal bracket

$r_1 = \{400i + 0j + 250k\}$ mm

$r_2 = \{50i + 300j + 0k\}$ mm

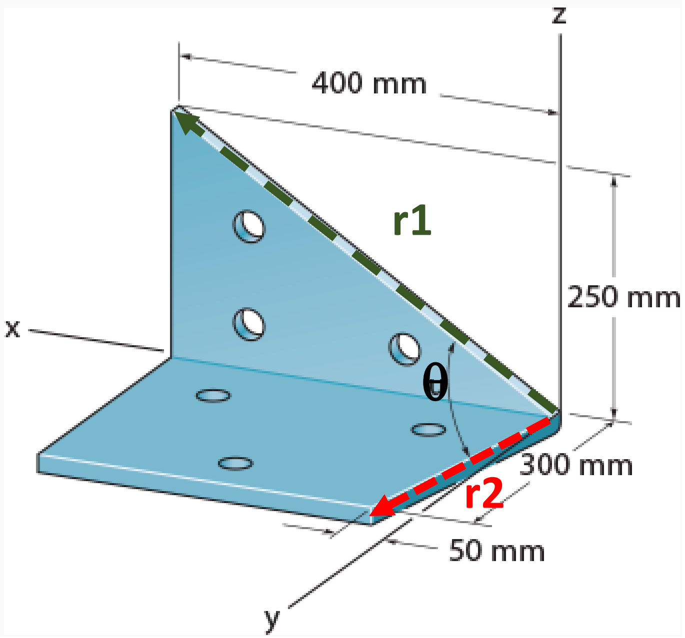
$$r_1 = \sqrt{400^2 + 250^2} = 471.7 \text{ mm}$$

$$r_2 = \sqrt{50^2 + 300^2} = 304.1\ mm$$

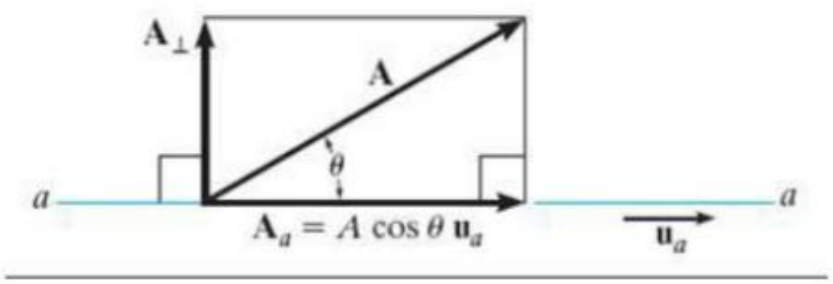
$$\theta = \cos^{-1}\left(\frac{\mathbf{A} \cdot \mathbf{B}}{AB}\right) \quad 0^\circ \leq \theta \leq 180^\circ$$

$$r_1 \cdot r_2 = [(400)(50) + (0)(300) + (250)(0)] = 20000$$

$$\theta = \cos^{-1}\left(\frac{20000}{(471.7)(304.1)}\right) = 82^0$$



b. The components of a vector parallel and perpendicular to a line.



You can determine the components of a vector parallel and perpendicular to a line using the dot product.

Steps:

1. Find the unit vector, ***uaa´*** along line aa´

2. Find the scalar projection of **A** along line aa´ by

   A|| = ***A* • *uaa*** = AxUx + AyUy + Az Uz   Magnitude of the component of A parallel to a-a

3. If needed, the projection can be written as a vector, **A||** , by using the unit vector ***uaa´*** and the magnitude found in step 2.

   A|| = A|| ***uaa´***   the component of A parallel to a-a s cartesian vector

4. The scalar and vector forms of the perpendicular component can easily be obtained by

   $|A_\perp| = \sqrt{A^2 - A_{||}^2}$ the magnitude of the perpendicular component of A to a-a

   A = **A** - **A**$_{||}$ the perpendicular component of A to a-a as cartesian vector

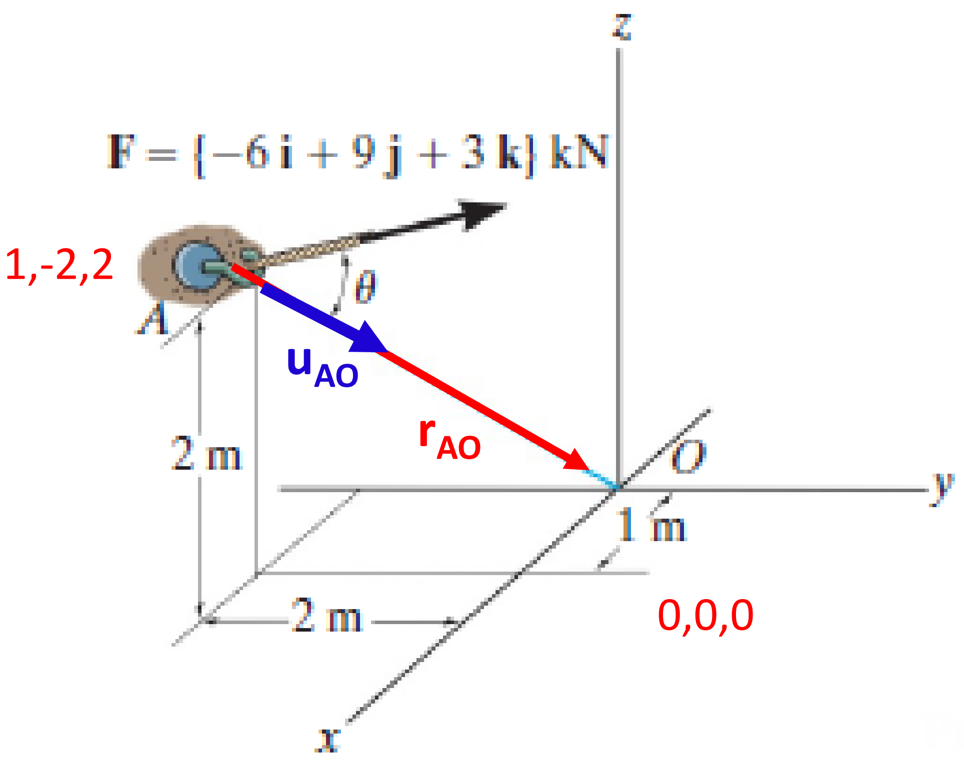
Ex. Find the magnitude of the projection of F along line AO

$\mathbf{r}_{A0}$ = -1i +2j -2k

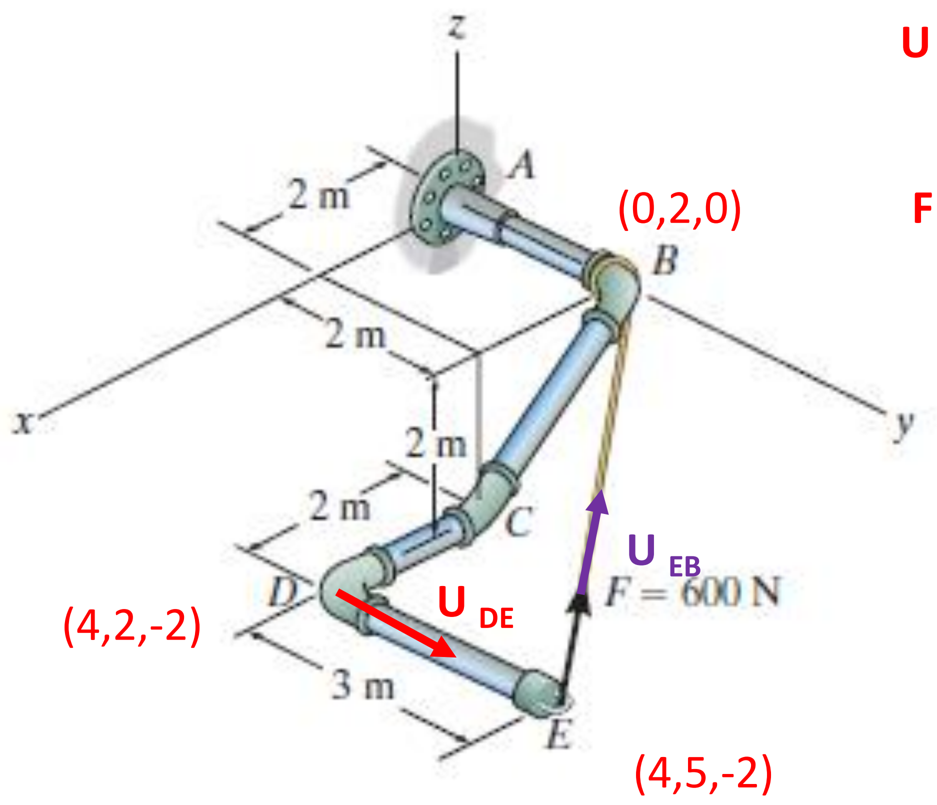
$$r_{AO} = \sqrt{1^2 + 2^2 + 2^2} = 3$$

$$U_{AO} = \frac{-1\mathrm{i} + 2\mathrm{j} - 2k}{3} = \frac{-1}{3}i + \frac{2}{3}j - \frac{2}{3}k$$

$$F_{||} = \{-6i + 9j + 3k\} . \{\frac{-1}{3}i + \frac{2}{3}j - \frac{2}{3}k\} = 6\ kN$$



2–113. Determine the magnitudes of the components of $F = 600$ N acting along and perpendicular to segment DE of the pipe assembly.



$\mathbf{U}_{DE} = \{0i + 3j + 0k\} / 3 = 0i + 1j + 0k$

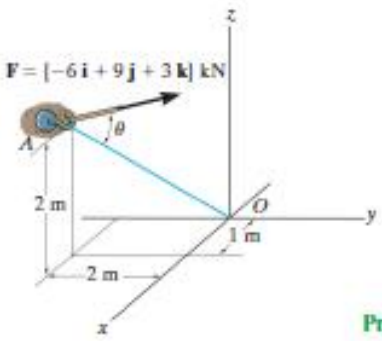
$\mathbf{U}_{EB} = \{-4i - 3j + 2k\} / \sqrt{4^2 + 3^2 + 2^2} = -0.72i - 0.54j + 0.36k$

$\mathbf{F}_{EB} = 600\{-0.72i - 0.54j + 0.36k\} = \{-432i - 324j + 216k\}$ N

$F_{\parallel DE} = \{-432i - 324j + 216k\}.\{0i + 1j + 0k\} =$ **-324 N**

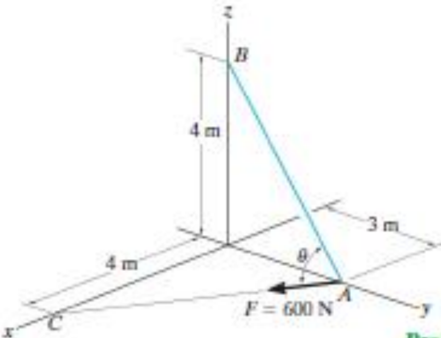
$F_{\perp DE} = \sqrt{600^2 - 324^2} = 505N$

F2–25. Determine the angle θ between the force and the line *AO*.



Prob. F2–25

F2–26. Determine the angle θ between the force and the line *AB*.



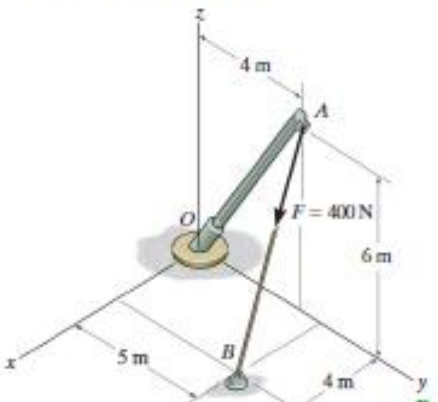
Prob. F2–26

F2–27. Determine the angle θ between the force and the line *OA*.

F2–28. Determine the projected component of the force along the line *OA*.

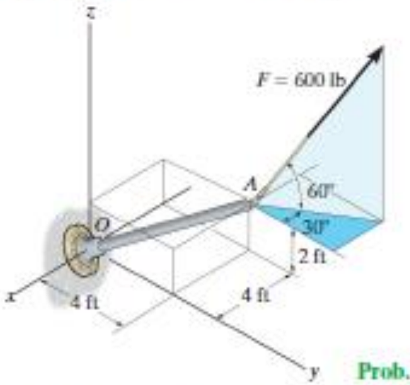
Probs. F2–27/28

F2–29. Find the magnitude of the projected component of the force along the pipe *AO*.



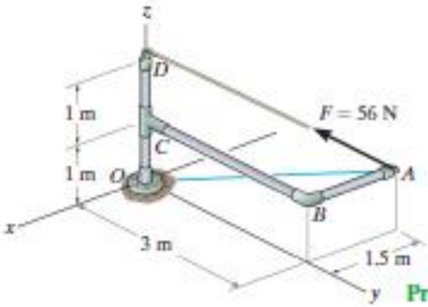
Prob. F2–29

F2–30. Determine the components of the force acting parallel and perpendicular to the axis of the pole.



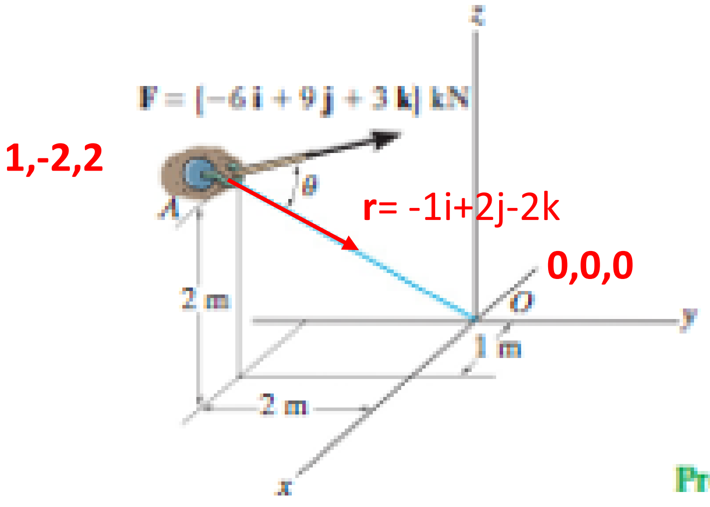
Prob. F2–30

F2–31. Determine the magnitudes of the components of the force $F = 56$ N acting along and perpendicular to line *AO*.



Prob. F2–31

F2–25. Determine the angle θ between the force and the line AO.

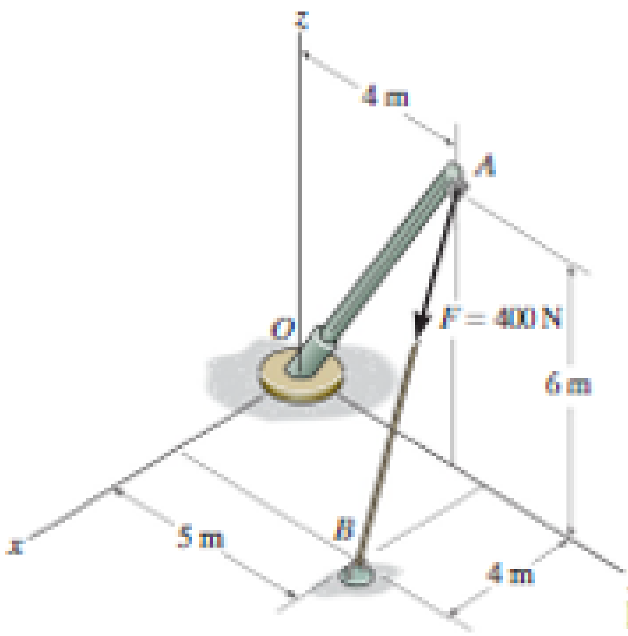


Prob. F2–25

$$\cos\theta = \frac{(-6\,i + 9j + 3k).(-1i + 2j - 2k)}{\sqrt{6^2 + 9^2 + 3^2}\,.\sqrt{1^2 + 2^2 \cdot 2^2}} = 0.534$$

θ= 57.69°

F2–29. Find the magnitude of the projected component of the force along the pipe AO.



Prob. F2–29

$$\mathbf{F} = (400\text{ N})\frac{\{4\,\mathbf{i} + 1\,\mathbf{j} - 6\,\mathbf{k}\}\text{m}}{\sqrt{(4\text{ m})^2 + (1\text{ m})^2 + (-6\text{ m})^2}}$$

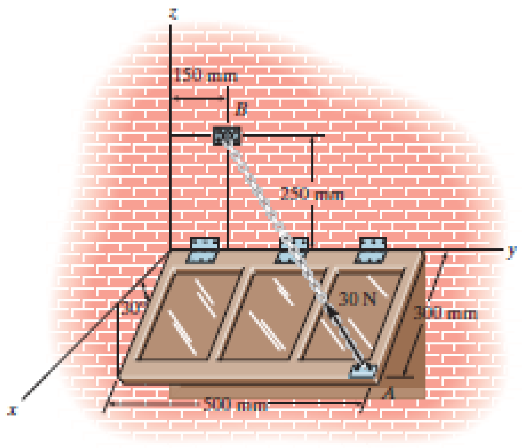
$$= \{219.78\mathbf{i} + 54.94\mathbf{j} - 329.67\mathbf{k}\}\text{ N}$$

$$\mathbf{u}_{AO} = \frac{\{-4\,\mathbf{j} - 6\,\mathbf{k}\}\text{ m}}{\sqrt{(-4\text{ m})^2 + (-6\text{ m})^2}}$$

$$= -0.5547\mathbf{j} - 0.8321\mathbf{k}$$

$$(F_{AO})_{\text{proj}} = \mathbf{F}\cdot\mathbf{u}_{AO} = 244\text{ N}$$

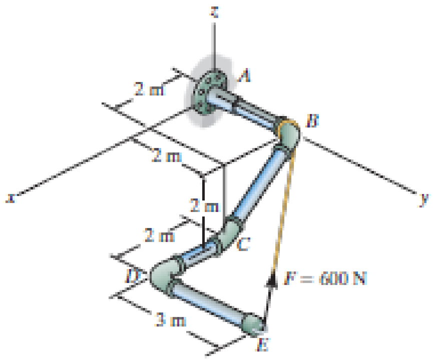
2–111. The window is held open by cable AB. Determine the length of the cable and express the 30-N force acting at A along the cable as a Cartesian vector.



Prob. 2–111

***2–112.** Given the three vectors **A**, **B**, and **D**, show that $\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D})$.

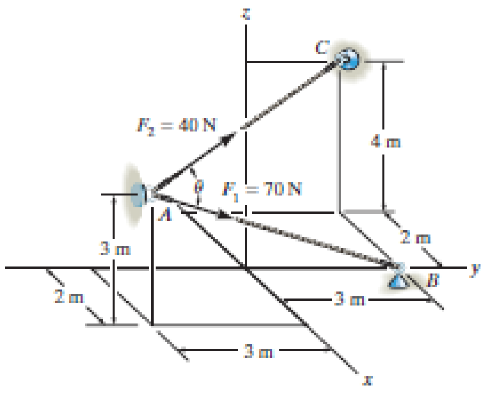
2–113. Determine the magnitudes of the components of $F = 600$ N acting along and perpendicular to segment DE of the pipe assembly.



Probs. 2–112/113

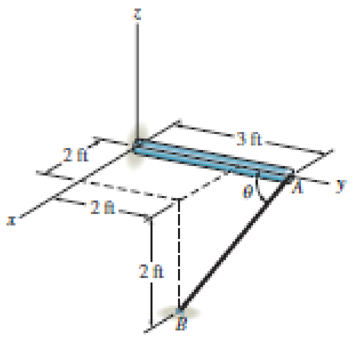
2–114. Determine the angle θ between the two cables.

2–115. Determine the magnitude of the projection of the force $\mathbf{F}_1$ along cable AC.



Probs. 2–114/115

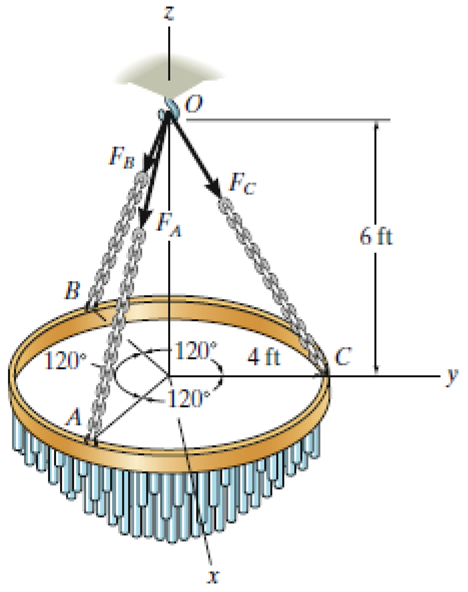
***2–116.** Determine the angle θ between the y axis of the pole and the wire AB.



Prob. 2–116

2–109.

The chandelier is supported by three chains which are concurrent at point O. If the resultant force at O has a magnitude of 130 lb and is directed along the negative z axis, determine the force in each chain.



SOLUTION

$$\mathbf{F}_C = F\frac{(4\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{4^2 + (-6)^2}} = 0.5547\,F\mathbf{j} - 0.8321\,F\mathbf{k}$$

$$\mathbf{F}_A = \mathbf{F}_B = \mathbf{F}_C$$

$$F_{Rz} = \Sigma F_z; \qquad 130 = 3(0.8321F)$$

$$F = 52.1\text{ lb}$$

Statics
Chapter 3
Equilibrium of a particle

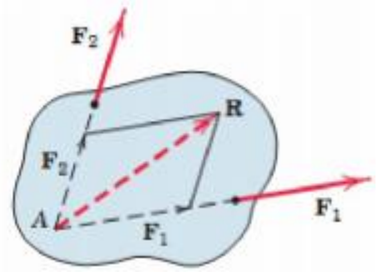
Iqbal Marie

3.1 Condition for the Equilibrium of a Particle

A particle: An object with mass but of negligible dimensions.

A particle is in **equilibrium** if the resultant of all forces acting on the particle is equal to zero.

Equation of equilibrium in (2D) $\Sigma F = 0$ ➡ $\sum F_x = 0 \; \sum F_y = 0$

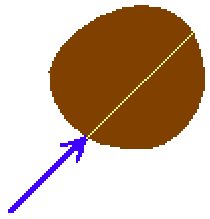


Concurrent Forces:

Two or more forces are said to be concurrent at a point if their lines of action intersect at that point.

Principle of Transmissibility:

a force may be applied at any point on its given line of action without altering the resultant effects



3.2 The Free-Body Diagram (FBD)

To apply equilibrium equations we must account for all **known and unknown forces** acting on the **particle** using FBD .

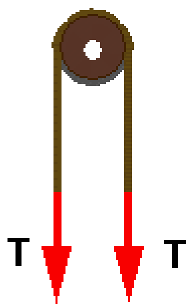
FBD is a sketch that shows the particle “free” from its surroundings with all the forces acting on it.

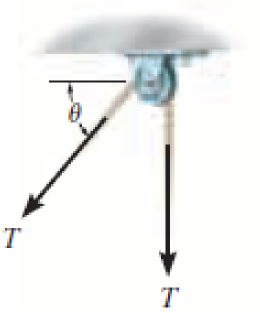
Three types of supports often encountered in particle equilibrium problems.

1. Cables and Pulleys.

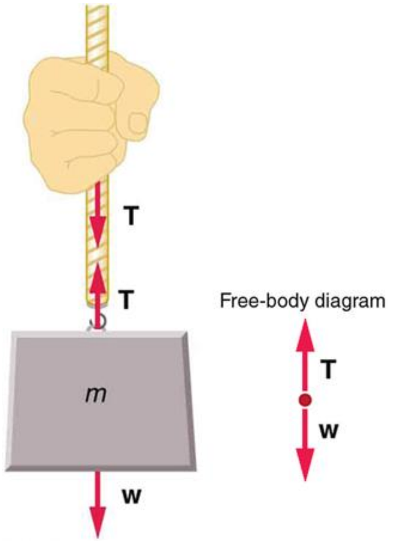
String or cable: A mechanical device that can only transmit a **tensile force (T)** along itself. The tension force is always directed in the direction of the cable.

Pulleys are assumed to be frictionless.

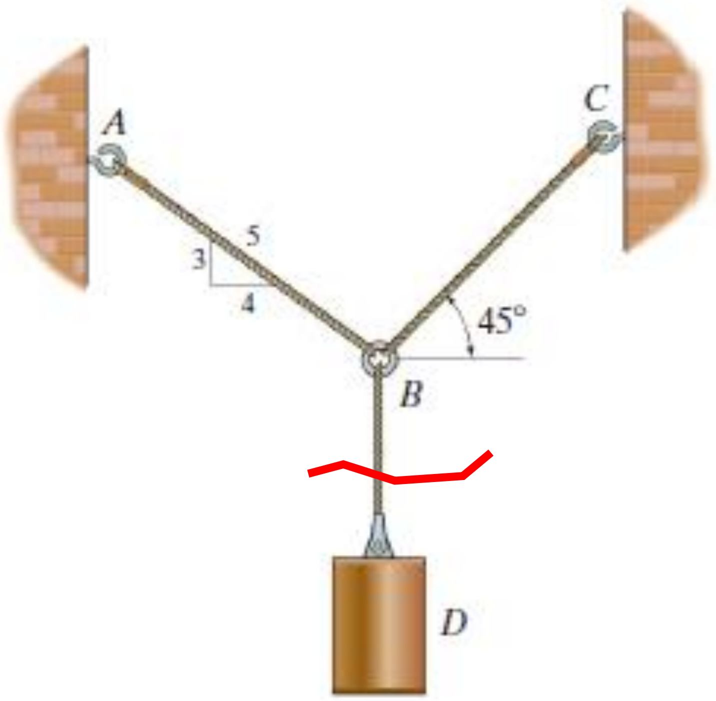


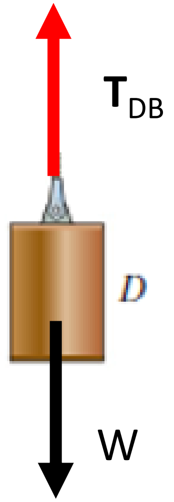


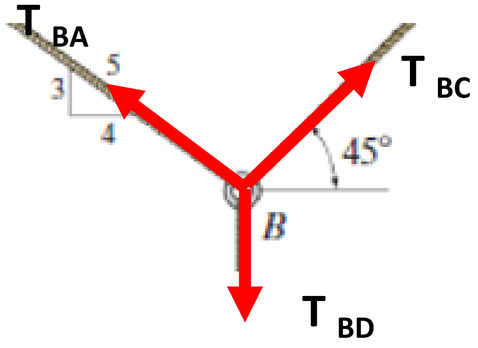
Cable is in tension

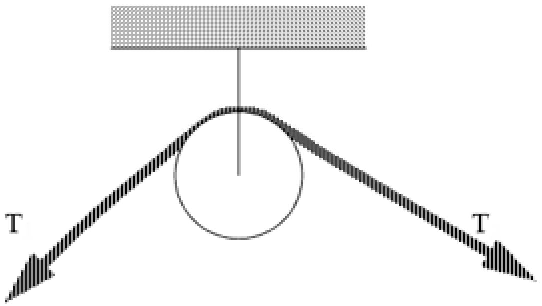


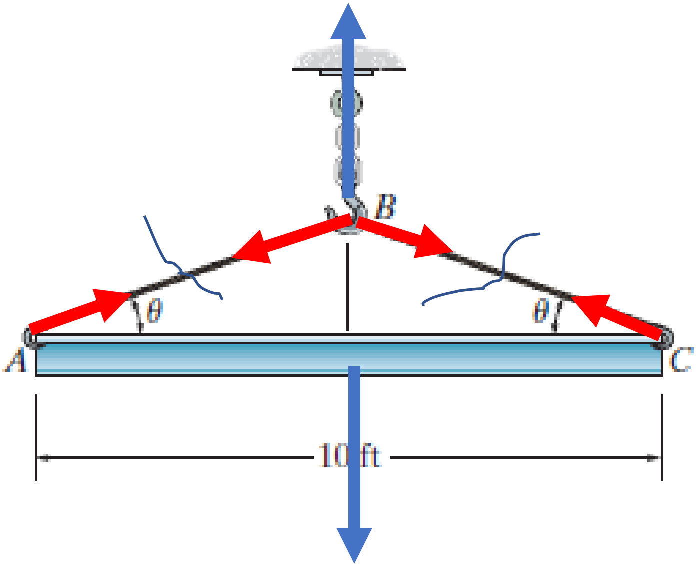
Draw FBD of Cylinder D and point B

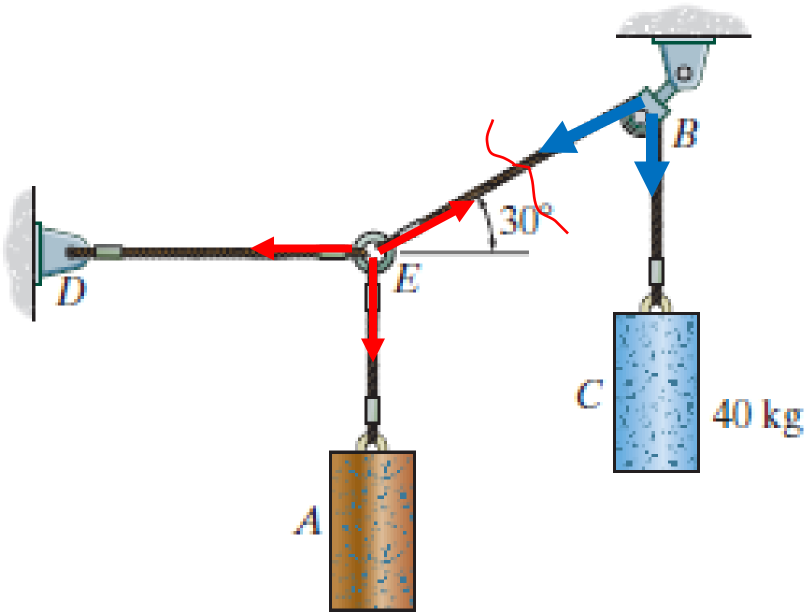


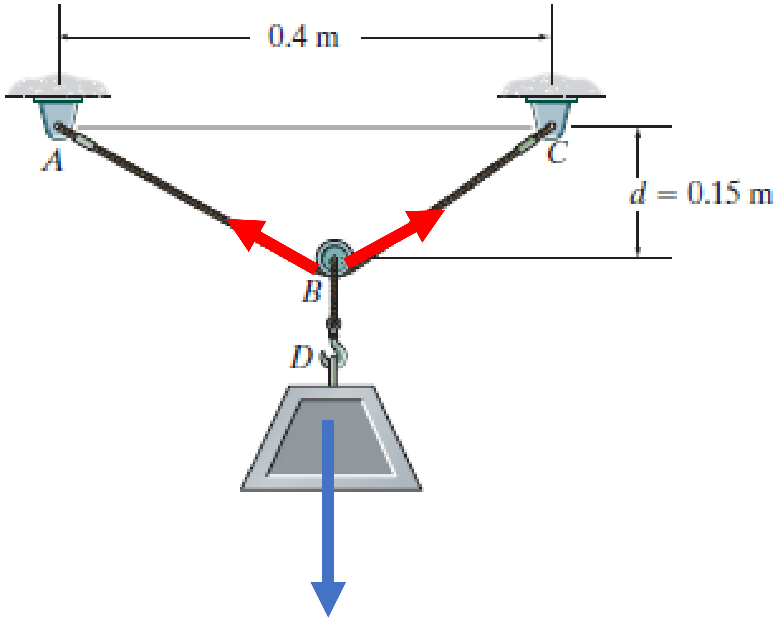





T
T


B
θ
θ
A
C
10 ft


B
30°
D
E
C
40 kg
A


0.4 m
A
C
d = 0.15 m
B
D

2. Springs. If a *linearly elastic spring*

A mechanical device which exerts a force (**tension or compression**) along its line of action and proportional to its extension **(F = k Δ).**

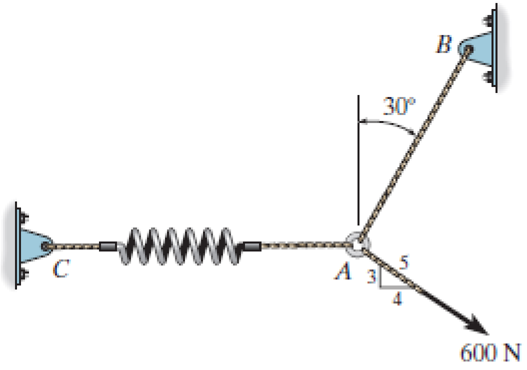
K : constant of proportionality which is a measure of **stiffness** or strength

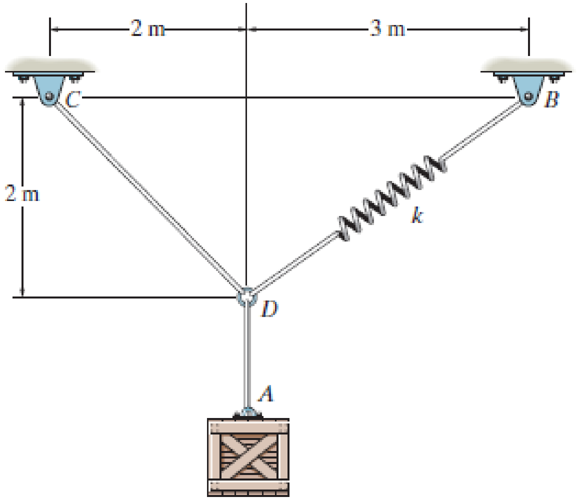
Δ : (stretch or deformation

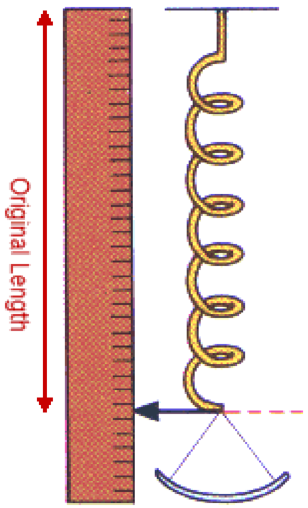
l′ : Stretched length , final length

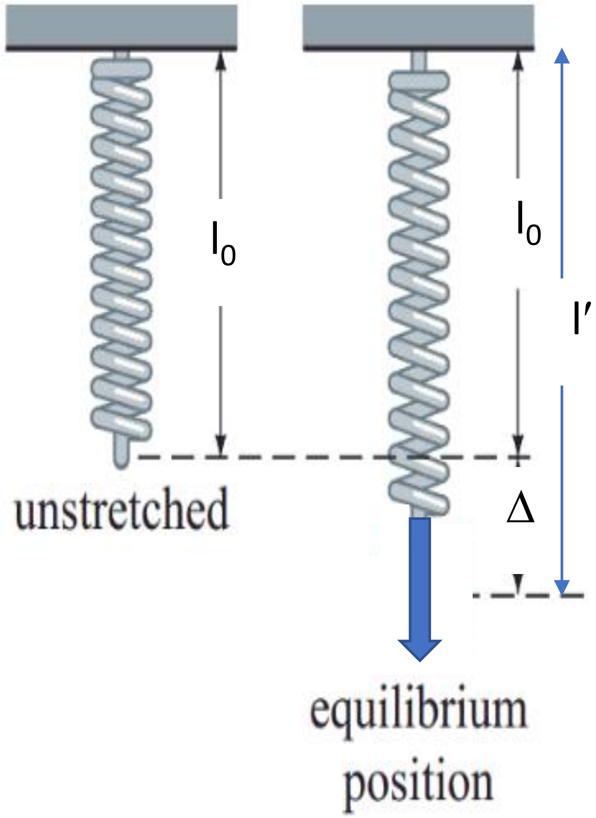
l_o : unstretched length, initial length

$$\Delta = l' - l_o$$



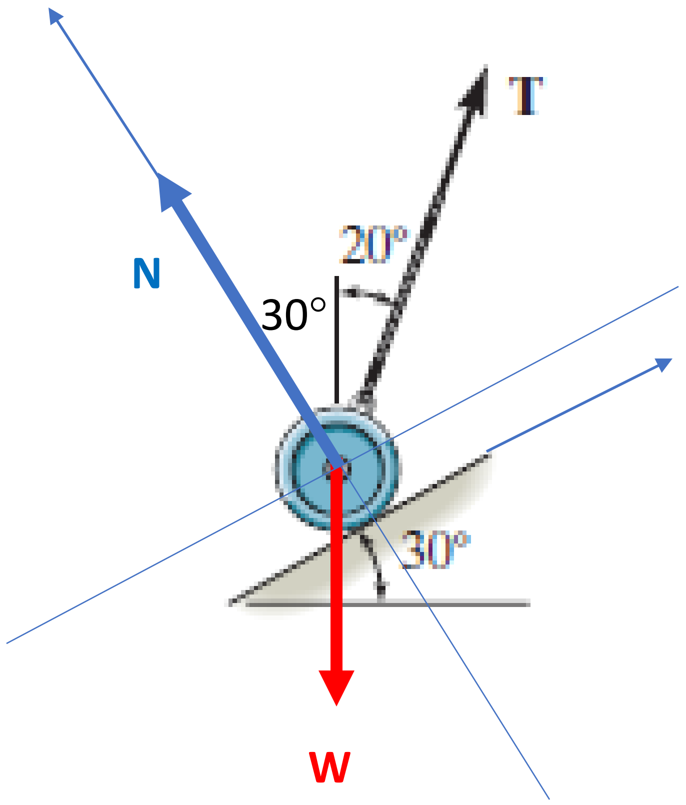


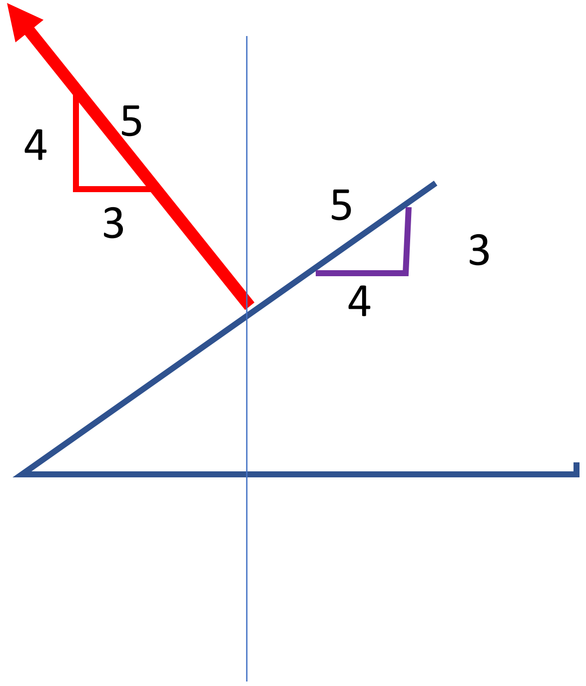




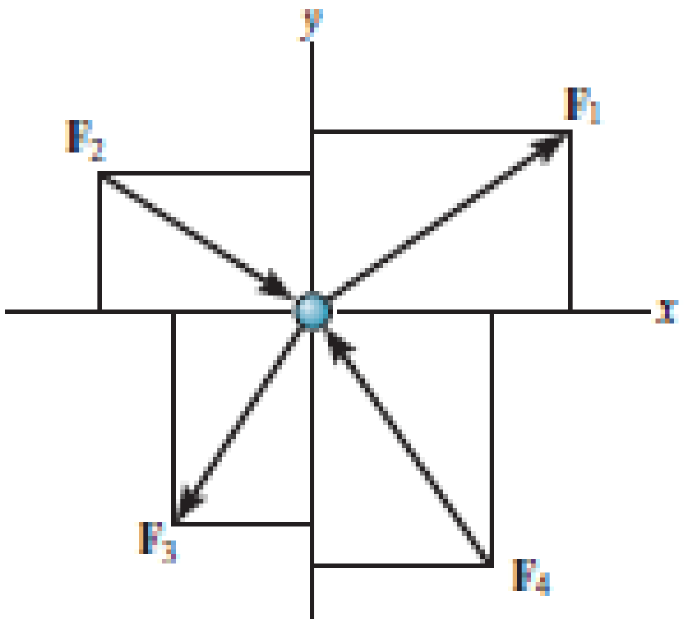
3. Smooth surface

If an object rests on a *smooth surface*, then the surface will exert a **force on the object that is normal to the surface** at the point of contact.





3.3 Coplanar Force Systems (2D)



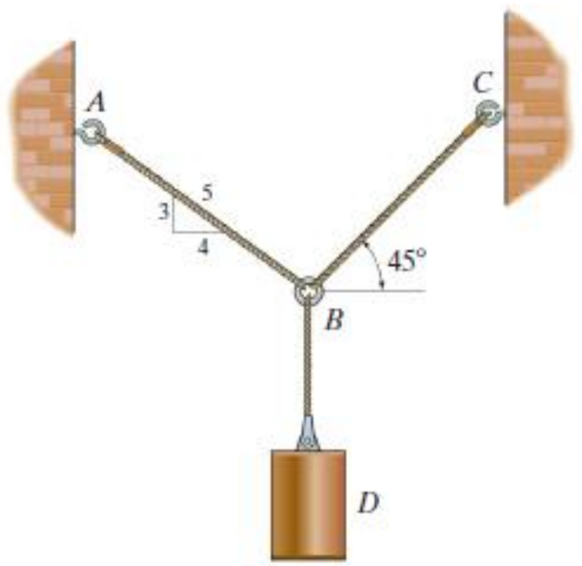
The first step in solving any equilibrium problem is:

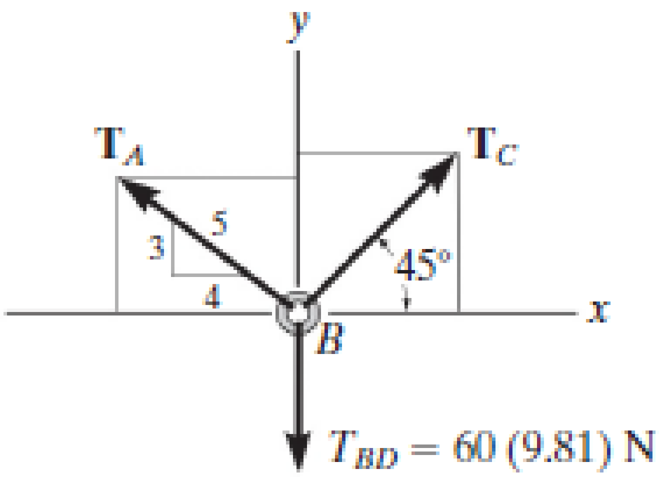
1. to draw the particle's free-body diagram
2. Then apply equilibrium equations

$\overset{+}{\rightarrow}\ \Sigma F_x = 0$ **and** $+\uparrow\ \Sigma F_y = 0.$

Components are positive if they are directed along a positive axis, and negative if they are directed along a negative axis.

Determine the tension in cables *BA* and *BC* necessary to support the 60-kg cylinder





$\xrightarrow{+} \Sigma F_x = 0; \quad T_C \cos 45 - 4/5\, T_A = 0$……………………….(1)

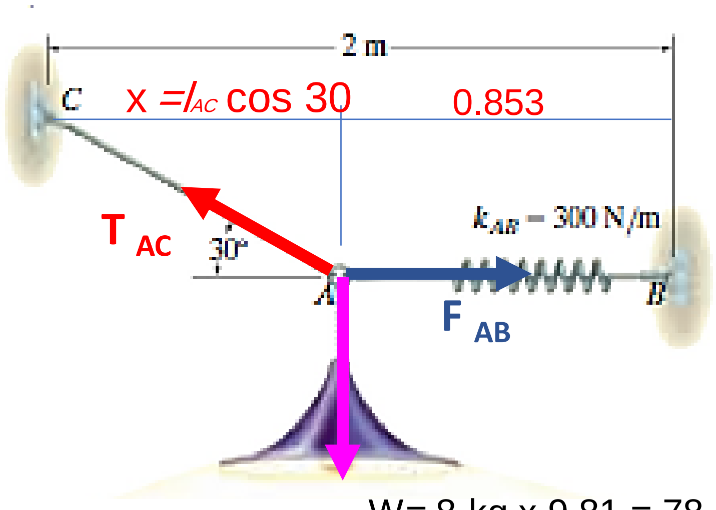
$+\uparrow \Sigma \; F_y = 0; \quad T_C \sin 45 + 3/5\, T_A - 60(9.81) = 0$ .... (2)

From (1) $T_A = 0.8839\, T_C$. Substituting this into Eq. (2)

T_C = 475.66 N substituting this result into Eq. (1) or Eq. (2), we get

T_A = 420 N

Determine the **required length of cord *AC*** so that the 8-kg lamp can be suspended in the position shown. The *undeformed* length of spring *AB* is $l_{AB} = 0.4$ m, and the spring has a stiffness of $k_{AB} = 300$ N/m.



W= 8-kg x 9.81 = 78.5 N

$+\rightarrow \Sigma F_x = 0$

$F_{AB} - T_{AC} \cos 30 = 0$ ....(1)

$+\uparrow \; \Sigma F_y = 0.$

$T_{AC} \sin 30 - 78.5 = 0$ .... (2)

Solving

$F_{AB} = 135.9$ N

$F_{AB} = k_{AB} \Delta_{AB}$; .... $135.9 = 300\, \Delta_{AB}$

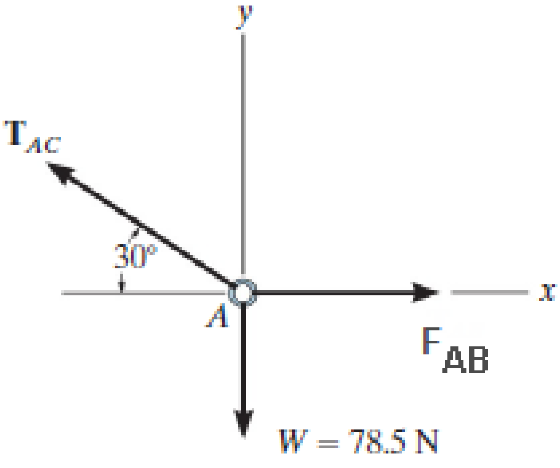
$\Delta_{AB} = 0.453$ m

$l'_{AB} = l_{0AB} + \Delta_{AB}$

$L'_{AB} = 0.4 + 0.453 = 0.853$ m

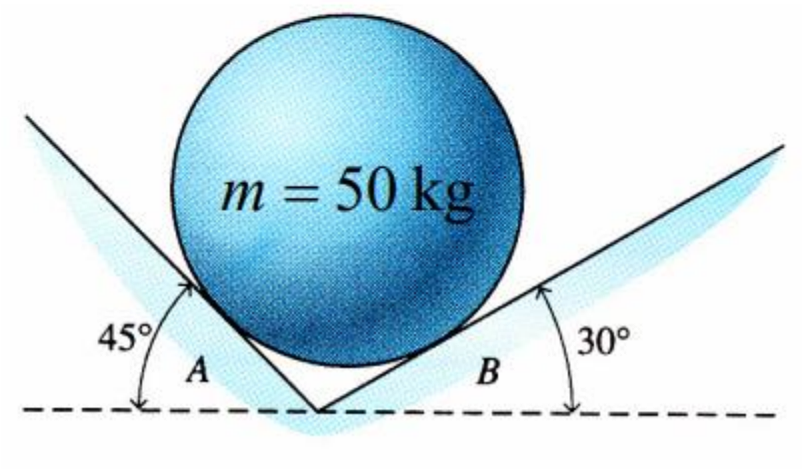
$2 - 0.853 = l_{AC} \cos 30$

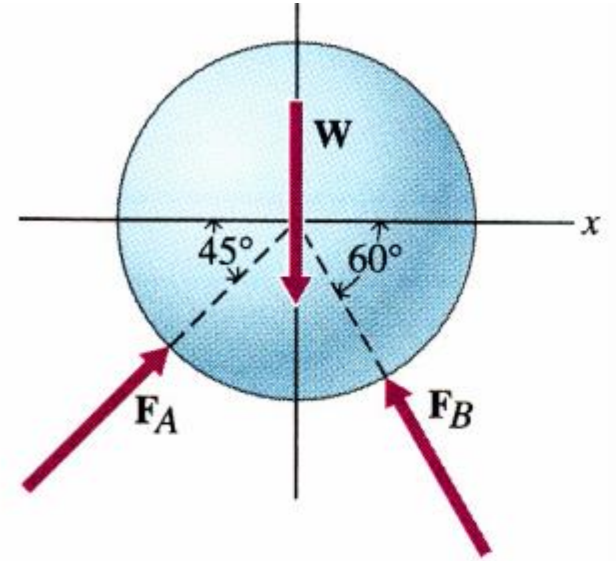
$\underline{l_{AC} = 1.32 \text{ m}}$



FBD

Determine the forces exerted by the smooth surfaces at A and B

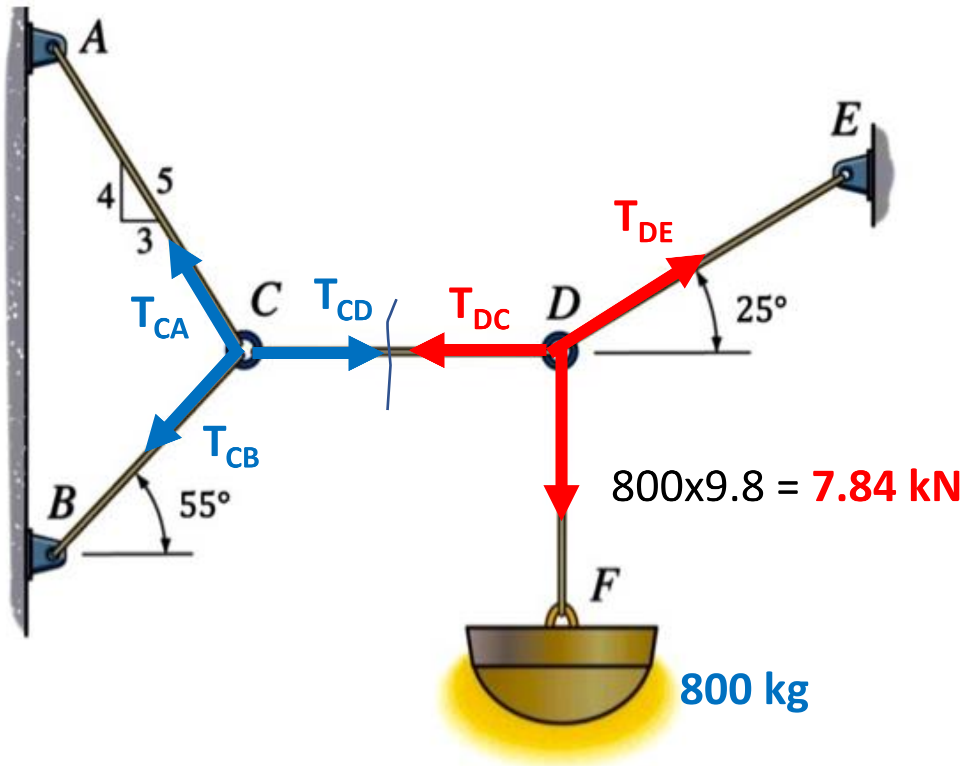




$$\sum F_x(+\rightarrow) = F_A \cos 45^\circ - F_B \cos 60^\circ = 0$$

$$\sum F_y(+\uparrow) = F_A \sin 45^\circ + F_B \sin 60^\circ - mg = 0$$

Find the tension in each cable



Point D.

$\xrightarrow{+} \Sigma F_x = 0$

$T_{DE} \cos 25 - T_{DC} = 0$ ....(1)

$+\uparrow \Sigma F_y = 0.$

$T_{DE} \sin 25 - 7.84 = 0$ ....T_{DE} = 18.55 kN

Sub. In 1.

T_{DC} = 16.81 kN = T_{CD}

Point C.

$\xrightarrow{+} \Sigma F_x = 0$

16.81 - T_{CA} (3/5) - $T_{CB} \cos 55 = 0$........................(1)

$+\uparrow \Sigma F_y = 0.$

T_{CA} (4/5) - $T_{CB} \sin 55 = 0$ .... T_{CA} = 1.02 T_{CB} kN

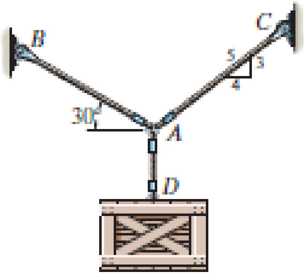
Sub. In 1.

T_{CB} = 14.18 kN

T_{ED} = 14.46 kN

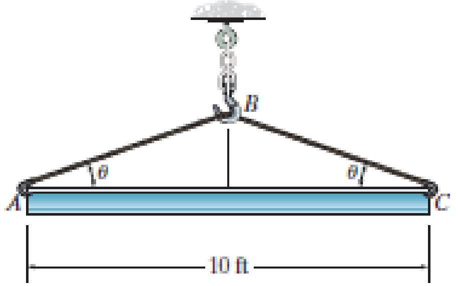
All problem solutions must include an FBD.

F3–1. The crate has a weight of 550 lb. Determine the force in each supporting cable.



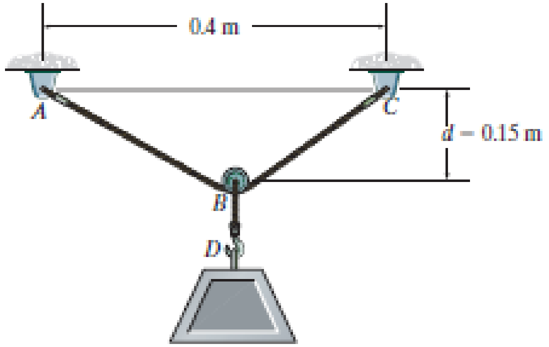
Prob. F3–1

F3–2. The beam has a weight of 700 lb. Determine the shortest cable ABC that can be used to lift it if the maximum force the cable can sustain is 1500 lb.

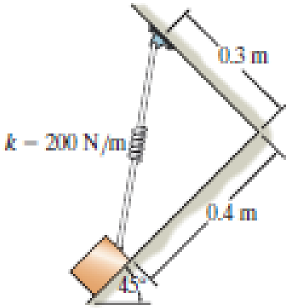


Prob. F3–2

F3–3. If the 5-kg block is suspended from the pulley B and the sag of the cord is $d = 0.15$ m, determine the force in cord ABC. Neglect the size of the pulley.

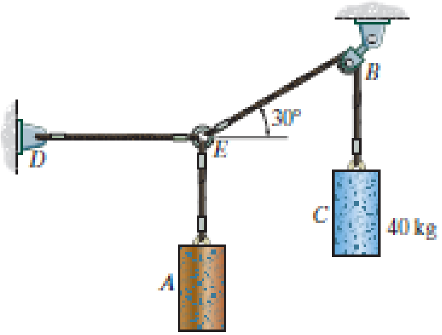


F3–4. The block has a mass of 5 kg and rests on the smooth plane. Determine the unstretched length of the spring.



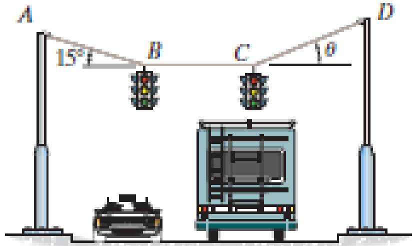
Prob. F3–4

F3–5. If the mass of cylinder C is 40 kg, determine the mass of cylinder A in order to hold the assembly in the position shown.

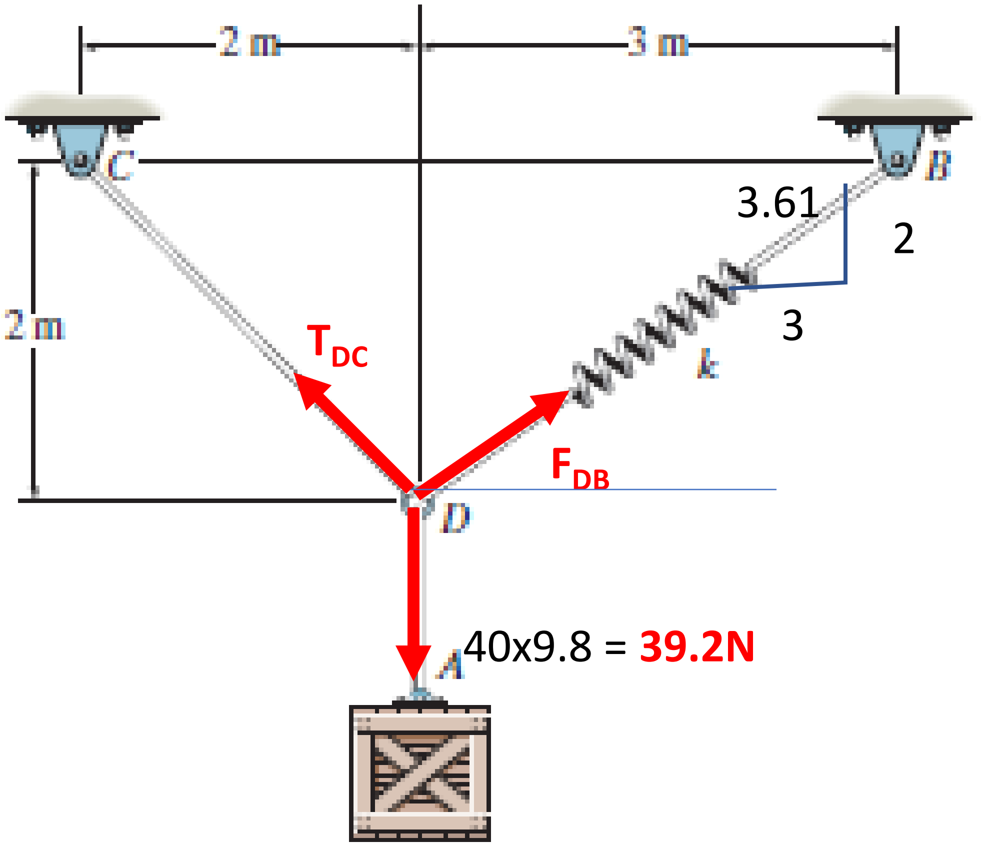


Prob. F3–5

F3–6. Determine the tension in cables AB, BC, and CD, necessary to support the 10-kg and 15-kg traffic lights at B and C, respectively. Also, find the angle θ.



Determine the **unstretched length (L_o)** of DB to hold 40kg crate. **K =180N/m**



Point D.

$+\rightarrow \Sigma F_x = 0$

$F_{DB}(3/3.61) - T_{DC}(\cos 45) = 0$ ....(1)

$+\uparrow \Sigma F_y = 0.$

$F_{DB}(2/3.61) + T_{DC}(\sin 45) - 39.2 = 0$ .... (2)

F_{DB} =....................

T_{DC} = ...

$F_{DB} = K\Delta = 180$ .... Δ = ...

$Lo = L' - \Delta = 3.61 - \Delta$

The unstretched length of spring *AB* is 3 m. If the block is held in the equilibrium position shown, determine the mass of the block at *D*.

$F_{AB} = K\Delta = 30(5-3) = 60N$

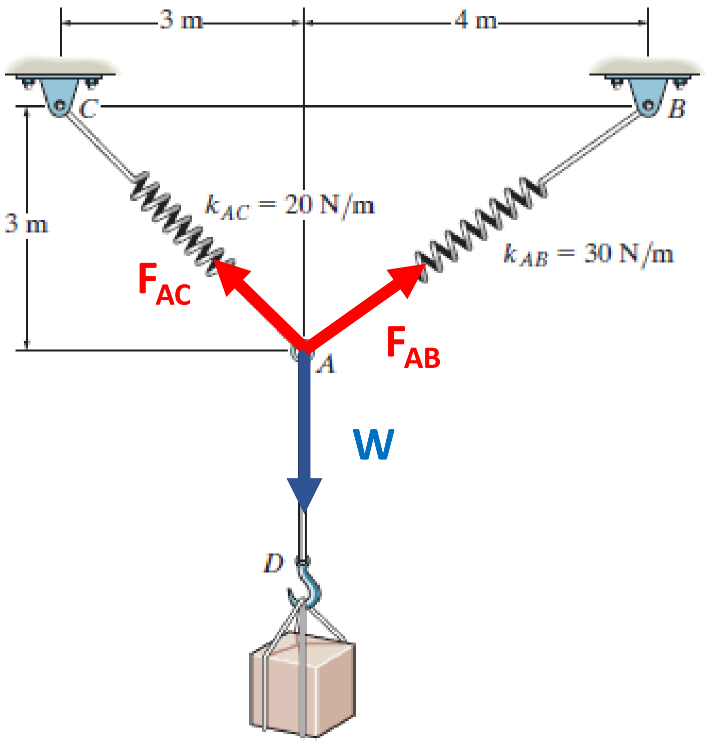
$+\rightarrow \Sigma F_x = -F_{AC} \cos 45 + 60(4/5) = 0$

$F_{AC} = 67.88$ N

$+\uparrow \Sigma F_y = -W + 67.88 \sin 45 + 60(3/5) = 0$

W = 84 N

Mass = <u>84/9.81 = 8.56 kg</u>



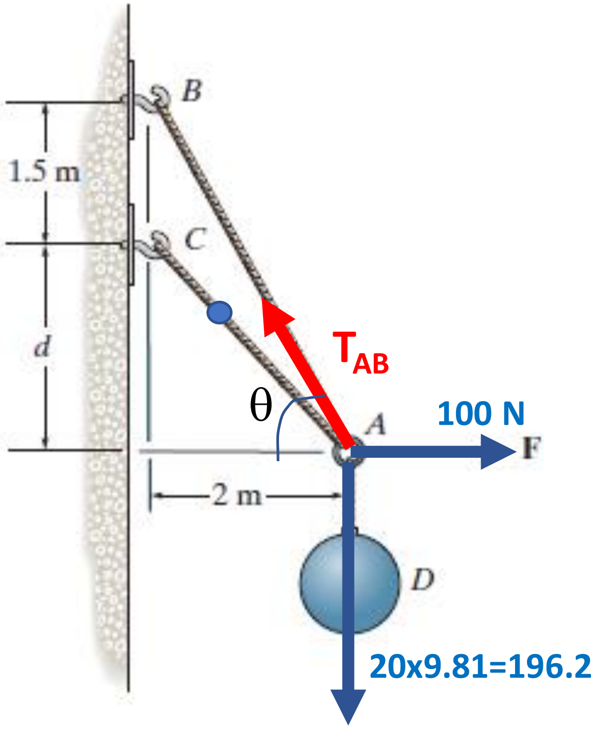
The ball D has a mass of 20 kg. If a force of is applied horizontally to the ring at A, determine the
dimension d so that the force in cable AC is zero. F = 100 N

$+\rightarrow \Sigma F_x = -T_{AB} \cos\theta + 100 = 0$ ……………………….1

$+\uparrow \Sigma F_y = -196.2 + T_{AB} \sin\theta = 0$ ……………………….2

Solving 1 and 2

$\theta = 62.99 \qquad T_{AB} = 220.2$



From geometry
Tan θ= (1.5+d)/2
d = 2.42

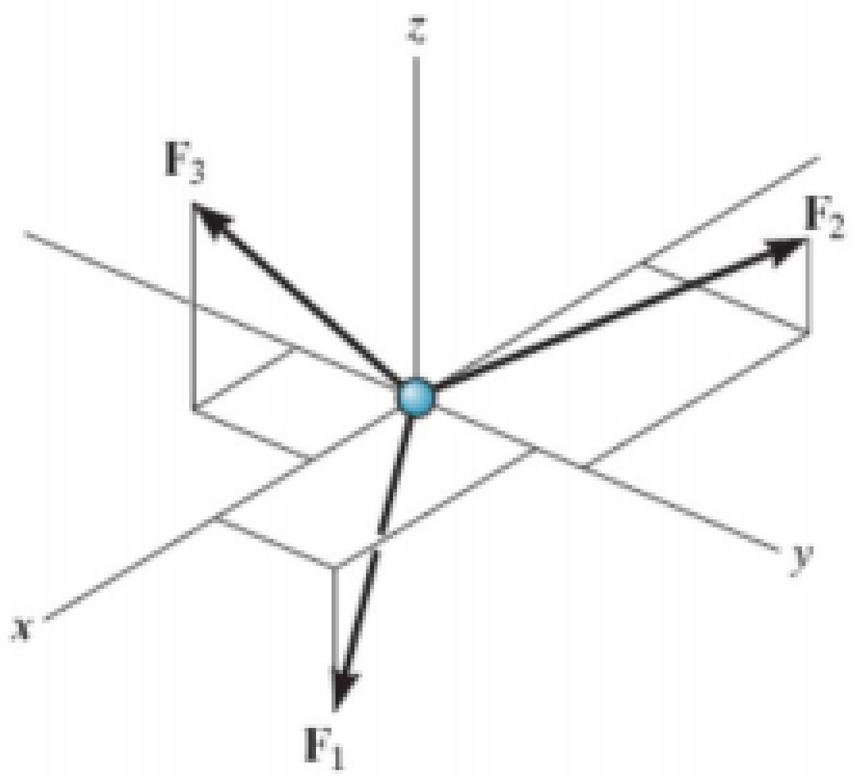
3.4 Three-Dimensional Force Systems

For particle equilibrium $\sum \mathbf{F} = 0$

Resolving into i, j, k components

$$\sum F_x i + \sum F_y j + \sum F_z k = 0$$

$$\sum F_x i = 0$$
$$\sum F_y j = 0$$
$$\sum F_z k = 0$$



If the solution for a force yields a negative result, this indicates that its sense is the reverse of that shown on the free-body diagram.

A 90N load is suspended from the hook shown. If the load is supported by two cables and a spring having a stiffness *k* = 500 N/m, determine the force in the cables and the stretch of the spring for equilibrium.

$\Sigma F_x = 0;$ $T_D \sin 30 - 4/5\, T_c = 0$ (1)

$\Sigma F_y = 0;$ $-T_D \cos 30 + F_B = 0$ (2)

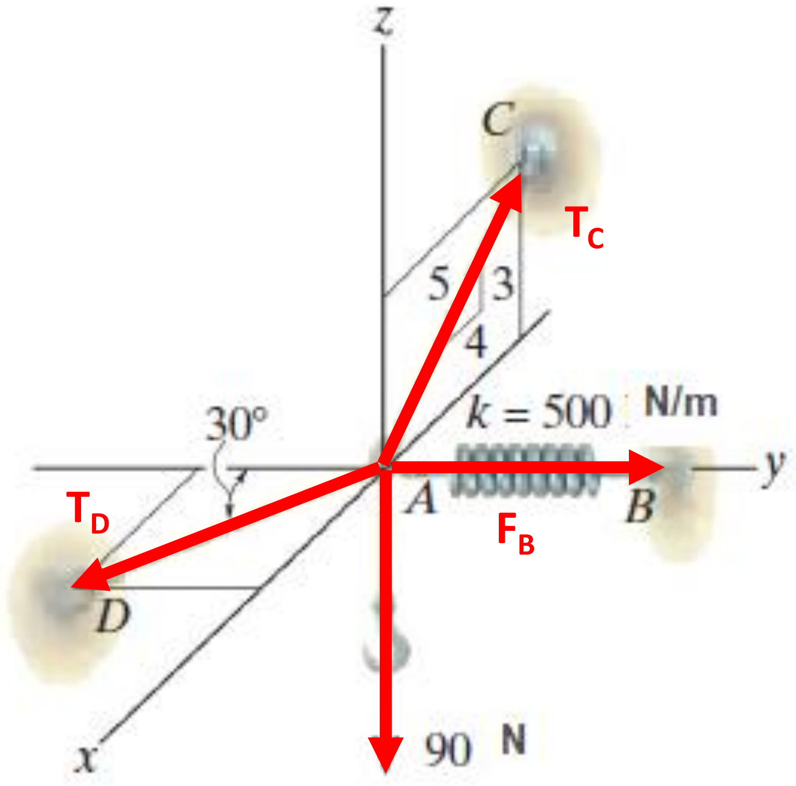
$\Sigma F_z = 0;$ $3/5\, T_c - 90 = 0$ (3)

Solving the equation

$T_C = 150$ N

$T_D = 240$ N

$F_B = 207.8$ N

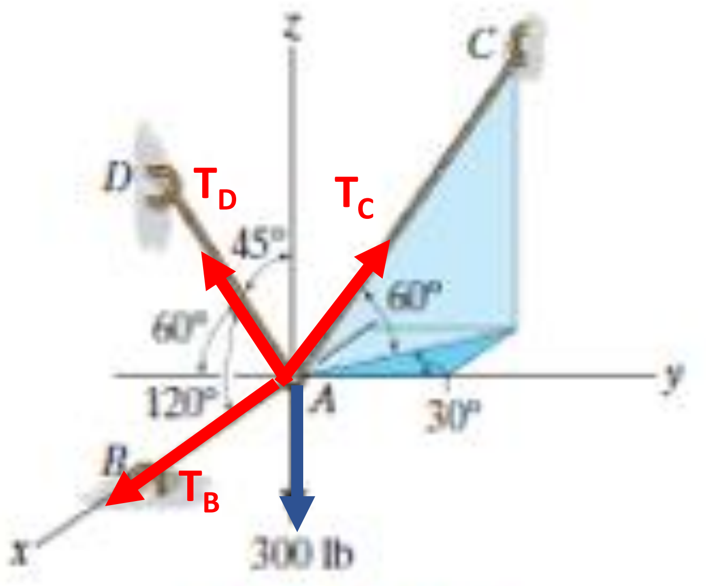


$F_B = k\, \Delta_{AB}$

$207.8 = (500)(\Delta_{AB})$

$\Delta_{AB} = 0.416$ m

F3–10. Determine the tension developed in cables *AB*, *AC*, and *AD*.

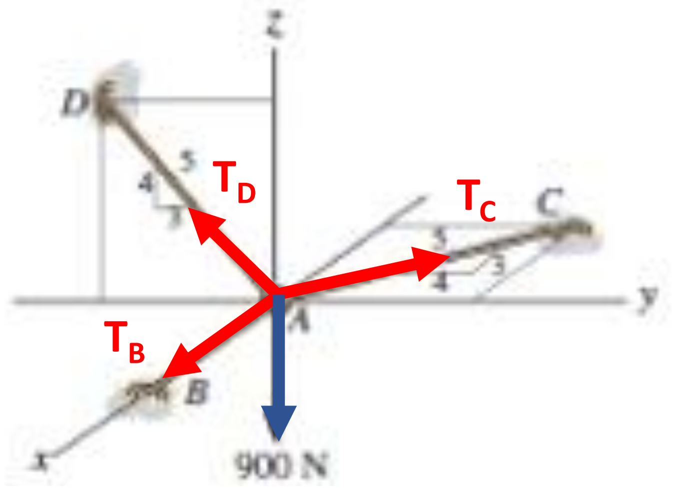


$\Sigma F_x = 0;$ $T_B + T_D \cos 120 - T_c \cos 60 \sin 30 = 0$ (1)

$\Sigma F_y = 0;$ $-T_D \cos 60 + T_c \cos 60 \cos 30 = 0$ (2)

$\Sigma F_z = 0;$ $T_D \cos 45 + T_c \sin 60 - 300 = 0$ (3)

F3–8. Determine the tension developed in cables *AB*, *AC*, and *AD*.

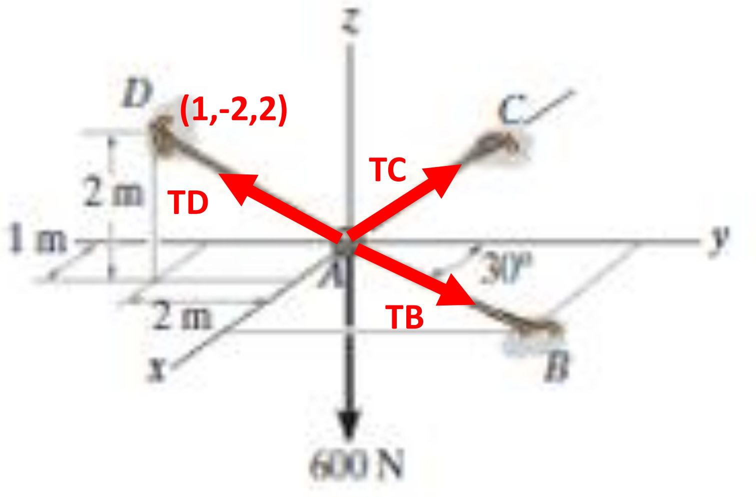


$\Sigma F_x = 0;$ $T_B + - T_c (3/5) = 0$ (1)

$\Sigma F_y = 0;$ $-T_D (3/5) + T_c (4/5) = 0$ (2)

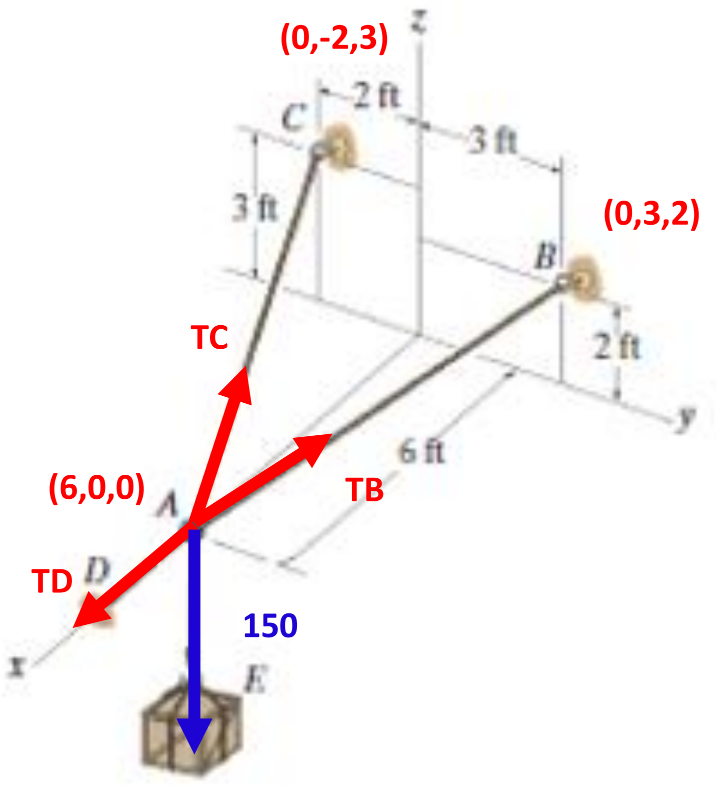
$\Sigma F_z = 0;$ $T_D (4/5) - 900 = 0$ (3)

F3–9. Determine the tension developed in cables AB, AC, and AD.



Prob. F3–9

F3–11. The 150-lb crate is supported by cables AB, AC and AD. Determine the tension in these wires.



$\Sigma F_x = 0;$ $\quad -T_C + 1/3T_D - + T_B \sin 30 = 0$ (1)

$\Sigma F_y = 0;$ $\quad -2/3T_D + T_B \sin 30 = 0$ (2)

$\Sigma F_z = 0;$ $\quad 2/3T_D = 0$ (3)

Statics

Chapter 4

Force system resultant

Iqbal Marie

4.1 Moment of a Force— Scalar Formulation

The **Moment is a vector**

The **magnitude** of the moment is directly proportional to the
magnitude of **F** and the perpendicular distance or ***moment arm***

Magnitude. The magnitude of $\mathbf{M}_O$ in 2-D is

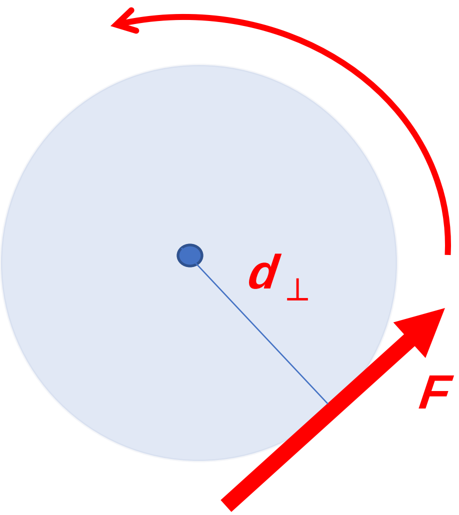
$$M_O = F d_{\perp}$$

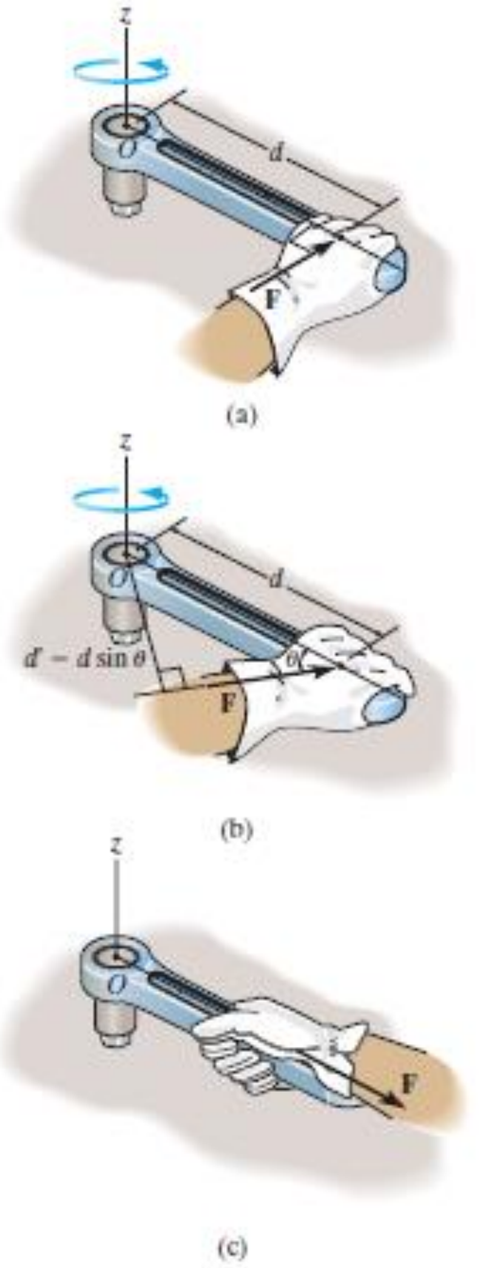
Units of moment magnitude is N.m

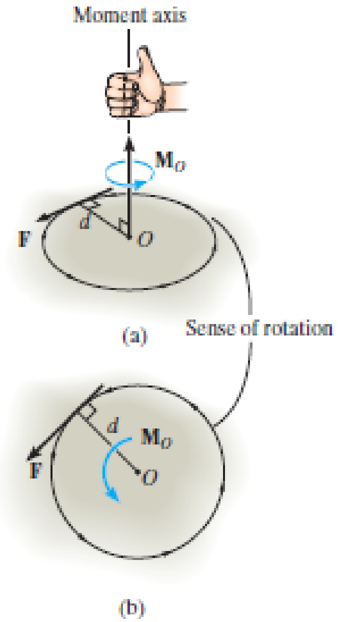
$d_{\perp}$ is the ***moment arm*** *(perpendicular distance* from the axis at point *O* to the line of action of the force

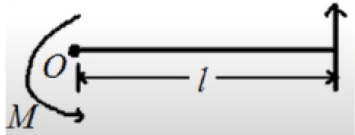
Direction. The by its ***moment axis***, which is perpendicular to the plane that contains the force **F** and its moment arm *d*. **The right-hand rule**

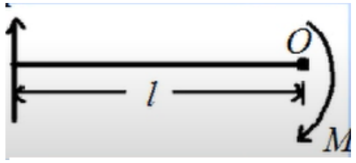
In **2-D**, the direction of M_O is either clockwise (CW) or counter-clockwise (CCW) depending on the tendency for rotation



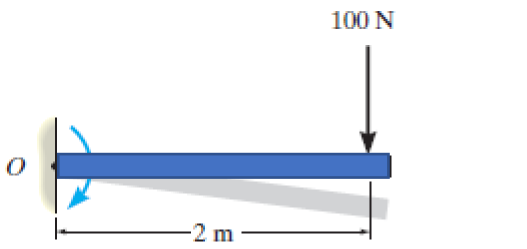




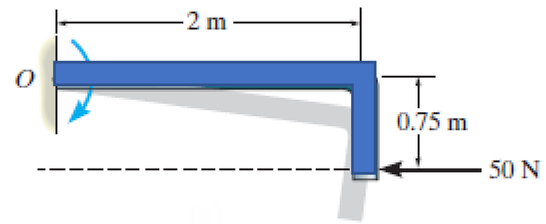




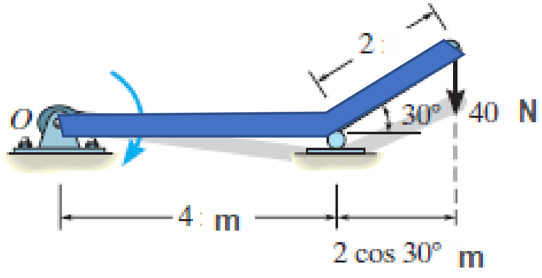
Ex. Find the moment about point **O**.



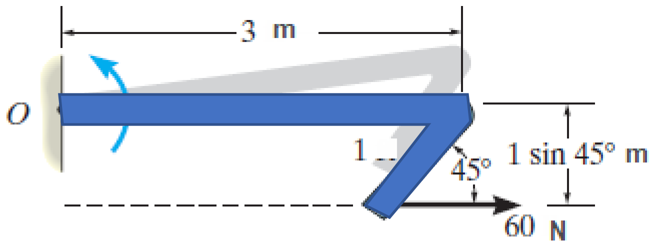
$M_O = (100\quad)(2\quad) = 200\ \text{N}\cdot\text{m}$ ↻



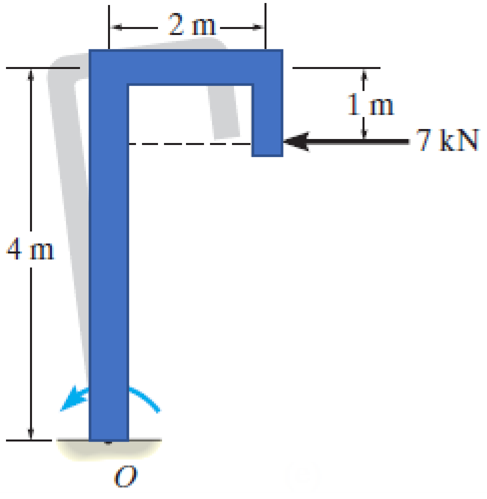
$M_O = (50\quad)(0.75\quad) = 37.5\ \text{N}\cdot\text{m}$ ↻



$M_O = (40\quad)(4\quad + 2\cos 30°\quad) = 229$ N.m ↻



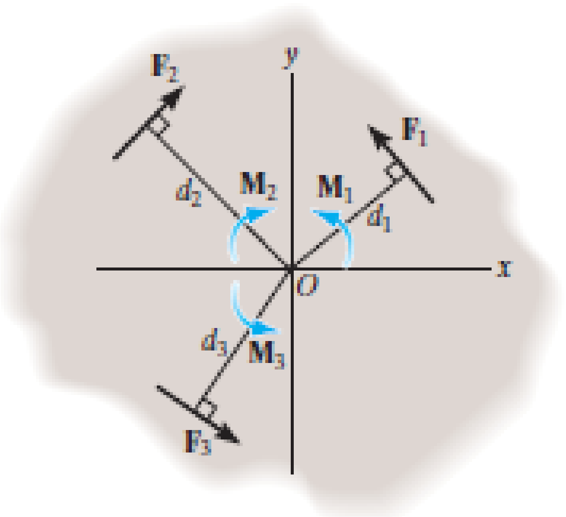
$M_O = (60\quad)(1\sin 45°\quad) = 42.4$ N.m ↺



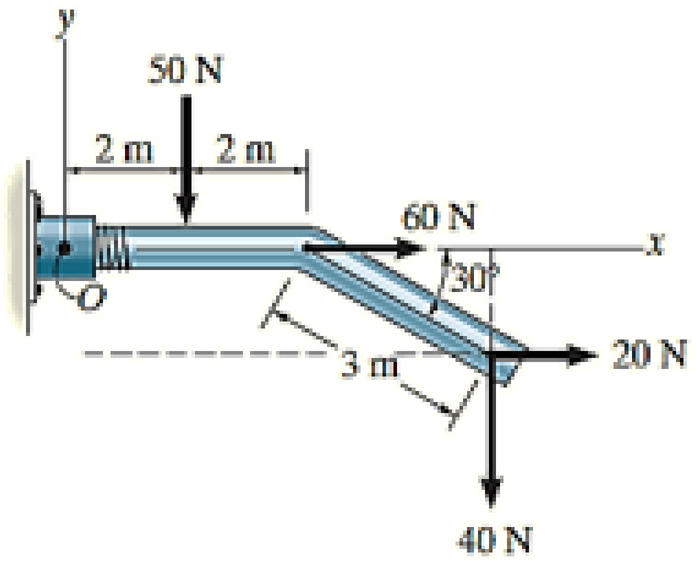
$M_O = (7\quad)(4\quad - 1\quad) = 21.0\ \text{kN}\cdot\text{m}$ ↺

Resultant Moment.

$$\circlearrowleft +(M_R)_o = \Sigma Fd; \qquad (M_R)_o = F_1d_1 - F_2d_2 + F_3d_3$$



Ex. Determine the resultant moment of the four forces a bout point O.



$$\circlearrowleft + (M_R)_o = \Sigma Fd;$$

$$(M_R)_o = -50\text{ N}(2\text{ m}) + 60\text{ N}(0) + 20\text{ N}(3\sin 30^\circ\text{ m})$$

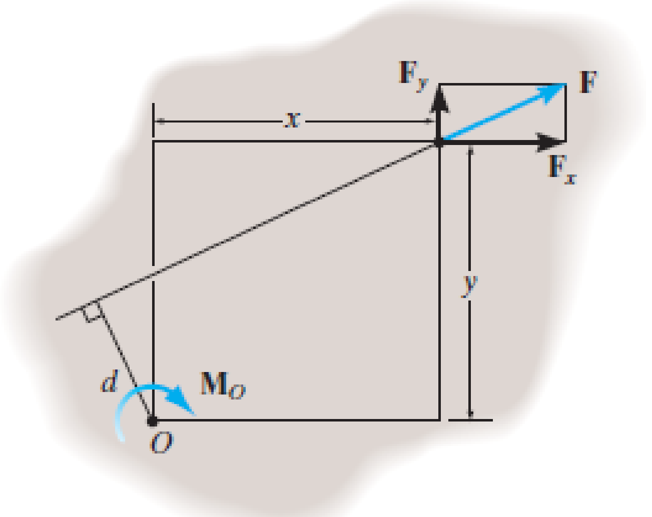
$$-40\text{ N}(4\text{ m} + 3\cos 30^\circ\text{ m})$$

$$(M_R)_o = -334\text{ N}\cdot\text{m} = 334\text{ N}\cdot\text{m}\circlearrowright$$

4.4 Principle of Moments

Varignon's theorem

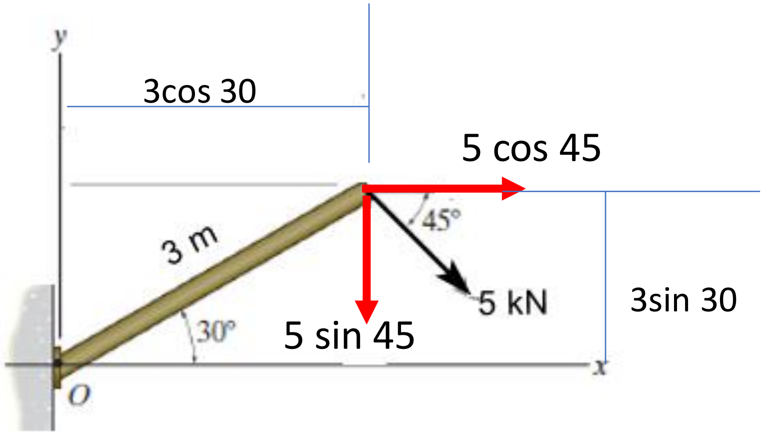
The moment of a force about a point is equal to the sum of
the moments of the components of the force about the point.



$M_O = F_x y - F_y x$

$M_O = Fd.$

Ex. Determine the moment of the force about point *O*.



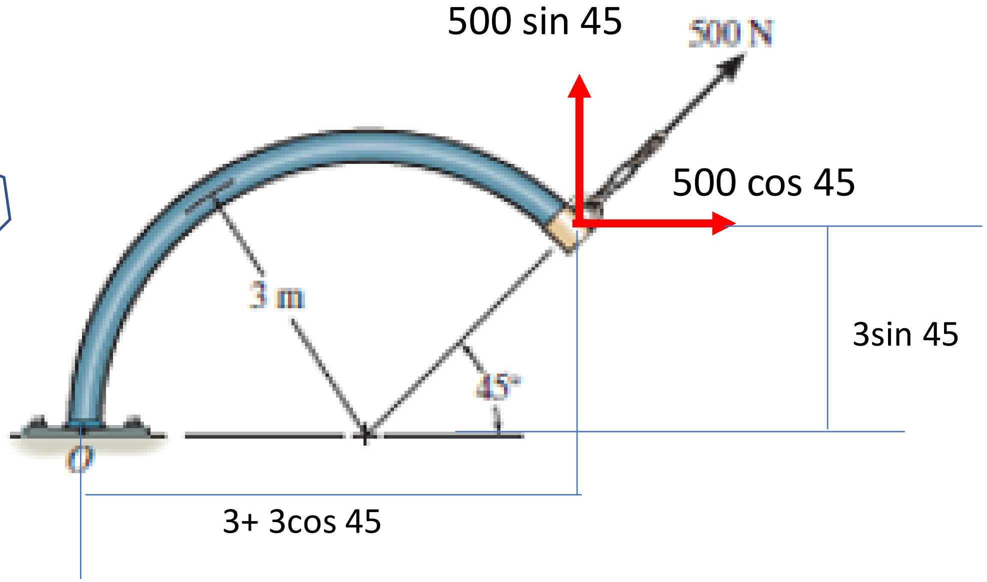
$$\circlearrowleft + M_O =$$

$$= -(5 \cos 45^\circ\ \text{kN})(3 \sin 30^\circ\ \text{m}) - (5 \sin 45^\circ\ \text{kN})(3 \cos 30^\circ\ \text{m})$$

$$= -14.5\ \text{kN}\cdot\text{m} = 14.5\ \text{kN}\cdot\text{m} \circlearrowright$$

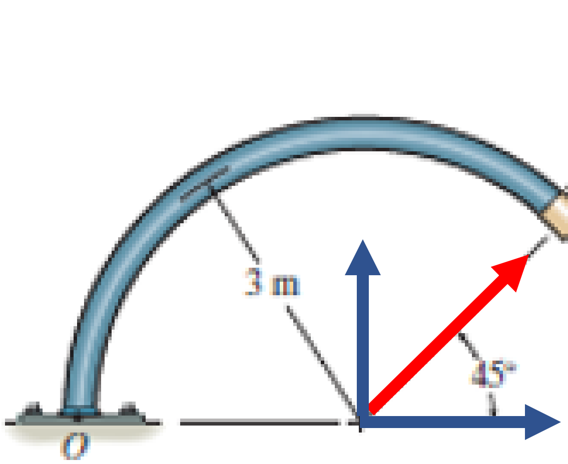
Determine the moment of the fore about point O.

Mo=- 500cos45 (3cos45) + 500sin45(3+3cos45) =1060.66 N.m +

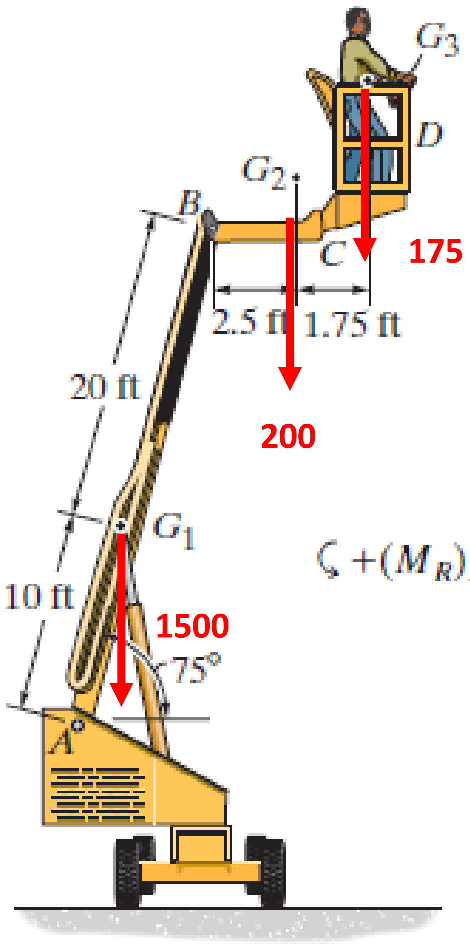


Or: apply the principle of transmissibility

Mo= + 500sin45(3) =1060.66 N.m +



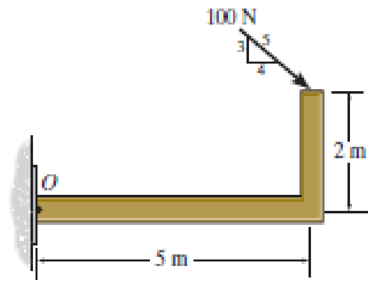
4–26. If the 1500-lb boom AB, the 200-lb cage BCD, and the 175-lb man have centers of gravity located at points G_1, G_2, and G_3, respectively, determine the resultant moment produced by all the weights about point A.



$$\zeta +(M_R)_A = \Sigma Fd; \qquad (M_R)_A = -1500(10\cos 75°) - 200(30\cos 75° + 2.5) - 175(30\cos 75° + 4.25)$$

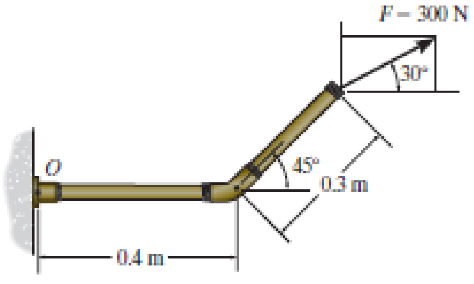
$$= -8037.75\ \text{lb}\cdot\text{ft} = 8.04\ \text{kip}\cdot\text{ft}\ \textit{(Clockwise)}$$

F4–1. Determine the moment of the force about point *O*.



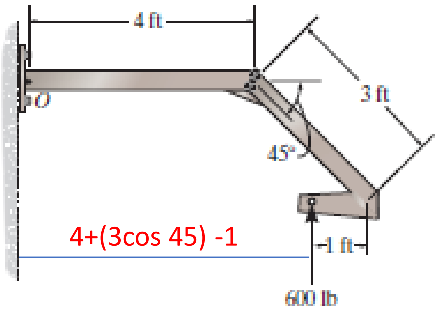
Prob. F4–1

F4–2. Determine the moment of the force about point *O*.



Prob. F4–2

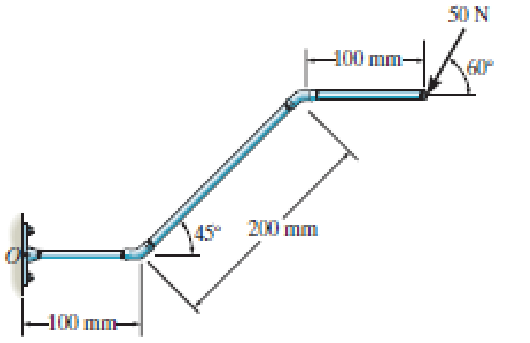
F4–3. Determine the moment of the force about point *O*.



Mo =600 x (4 +(3cos 45) -1)

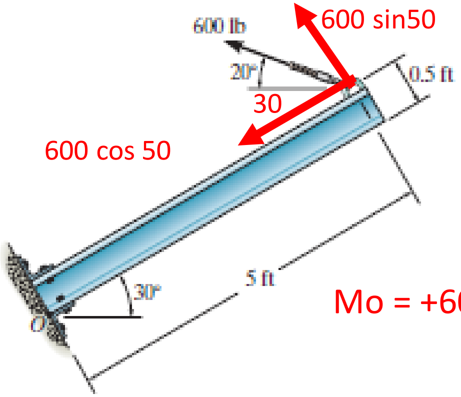
Prob. F4–3

F4–4. Determine the moment of the force about point *O*. Neglect the thickness of the member.



Prob. F4–4

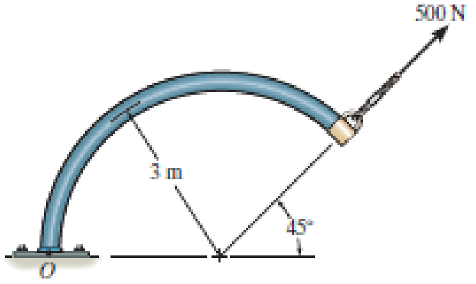
F4–5. Determine the moment of the force about point *O*.



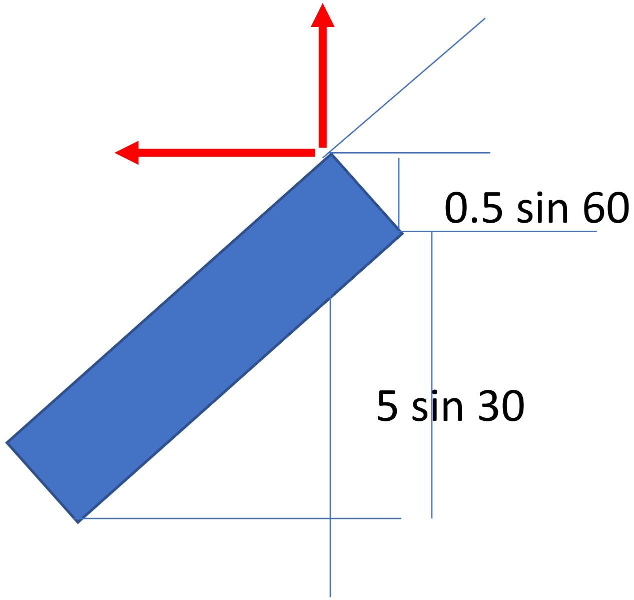
Mo = +600cos 50 x 0.5 + 600 sin50 x 5 =

Prob. F4–5

F4–6. Determine the moment of the force about point *O*.



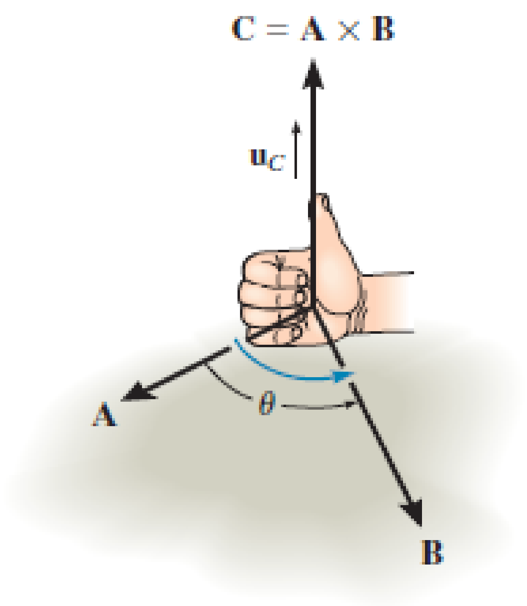
Prob. F4–6



4.2 Cross Product

The ***cross product*** of two vectors **A** and **B** yields the vector **C**,

C = A X B



Magnitude. $C = AB \sin \theta.$ $(0 \leq \theta \leq 180)$.

Direction. Vector **C** has a direction that is perpendicular to the plane containing **A** and **B** the (right-hand rule)

A X **B** ≠ **B** X **A**.

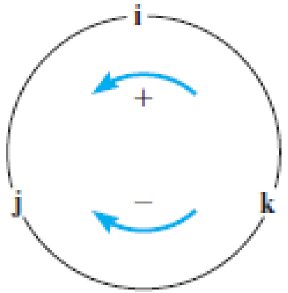
A X (**B** + **D**) = (**A** X **B**) + (**A** X **D**)

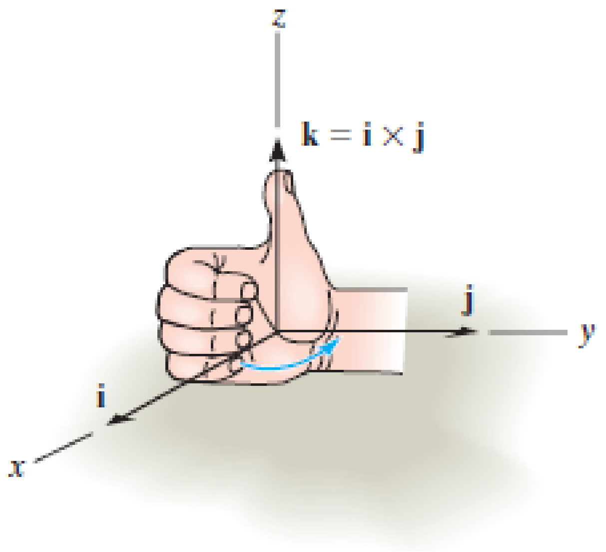
Cartesian Vector Formulation.

$\mathbf{i} \times \mathbf{j} = \mathbf{k}$ $\mathbf{i} \times \mathbf{k} = -\mathbf{j}$ $\mathbf{i} \times \mathbf{i} = 0$

$\mathbf{j} \times \mathbf{k} = \mathbf{i}$ $\mathbf{j} \times \mathbf{i} = -\mathbf{k}$ $\mathbf{j} \times \mathbf{j} = 0$

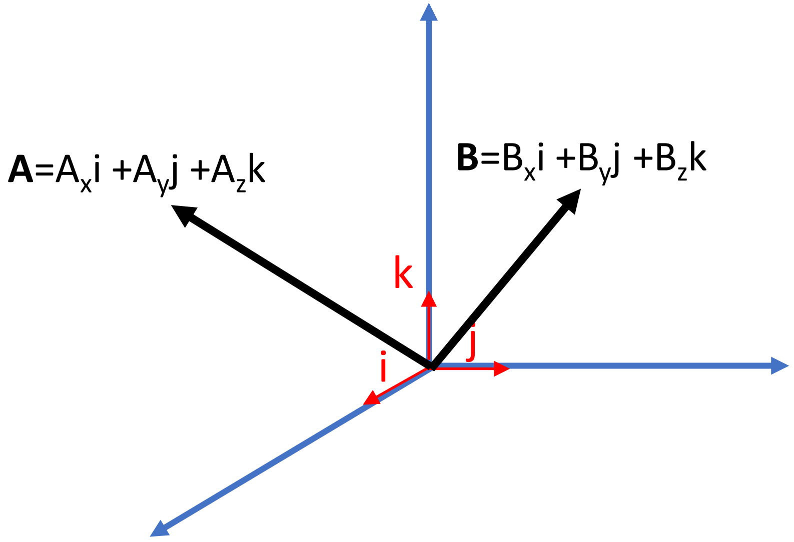
$\mathbf{k} \times \mathbf{i} = \mathbf{j}$ $\mathbf{k} \times \mathbf{j} = -\mathbf{i}$ $\mathbf{k} \times \mathbf{k} = 0$

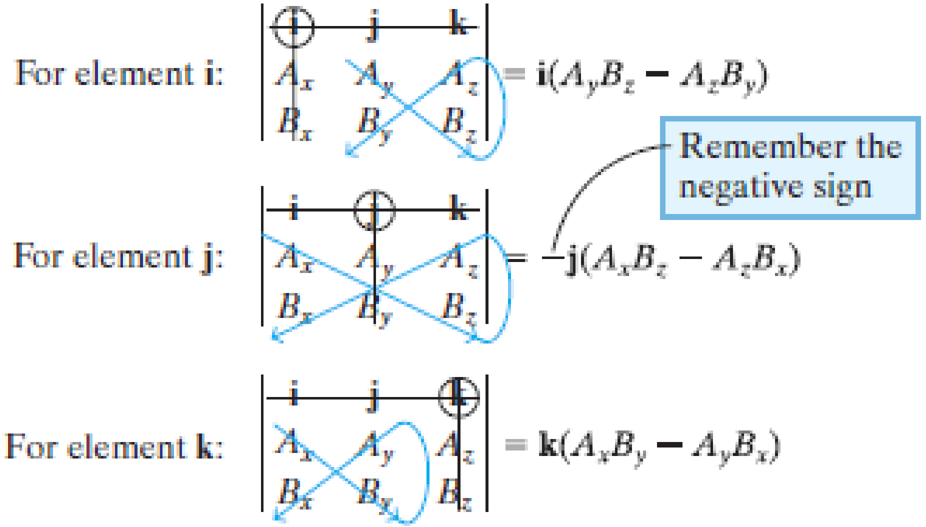




$$\mathbf{A} \times \mathbf{B} = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k})$$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \overset{+}{\mathbf{i}} & \overset{-}{\mathbf{j}} & \overset{+}{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$





A = 2i -3j+1k
B = -1i +3j+4k
Find **A x B**

C=A x B= $\begin{vmatrix} \overset{+}{i} & \overset{-}{-j} & \overset{+}{k} \\ 2 & -3 & 1 \\ -1 & 3 & 4 \end{vmatrix} = -15i - 9j + 3k$

$$|\boldsymbol{C}| = \sqrt{15^2 + 9^2 + 3^2} = 17.75$$

Direction of C

$$\cos\alpha = \frac{-15}{17.75}$$

$$\cos\beta = \frac{-9}{17.75}$$

$$\cos\gamma = \frac{3}{17.75}$$

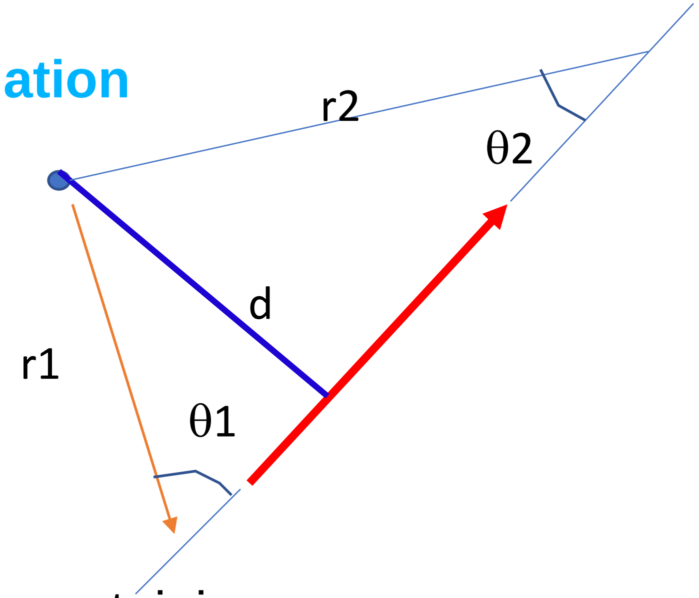
4.3 Moment of a Force—Vector Formulation

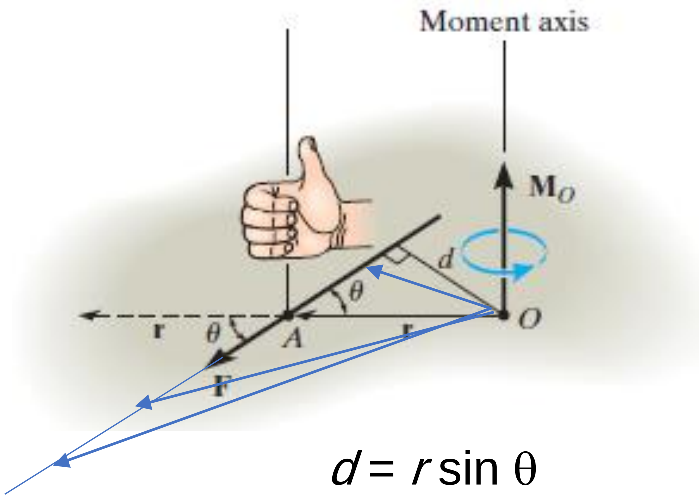
$$M_O = Fd = rF\sin\theta = F(r\sin\theta)$$

$$\mathbf{M_O} = \mathbf{r_1} \times \mathbf{F} = \mathbf{r_2} \times \mathbf{F} = \mathbf{r_3} \times \mathbf{F}$$

$$\mathbf{M_O} = \mathbf{r} \times \mathbf{F} \quad = \mathbf{M_x\, i + M_y\, j + M_z\, k}$$

$$M_O = \sqrt{M_x^2 + M_y^{2} + M_z^{2}}$$





$d = r\sin\theta$

Direction of **Mo** is perpendicular to the plane containing **r** and **F**

Cartesian Vector Formulation

$\mathbf{M_O = r \times F \neq F \times r}$

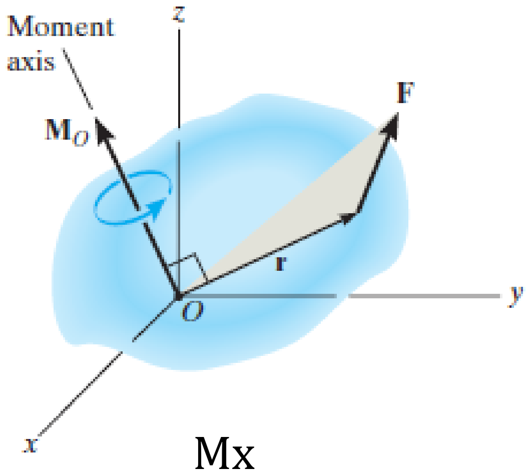
+ - +

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

components of the position vector drawn from point *O* to *any point* on the line of action of the force

the x, y, z components of the force vector

$$M_O = \sqrt{M_x^2 + M_y^{2} + M_z^{2}}$$



Direction of M

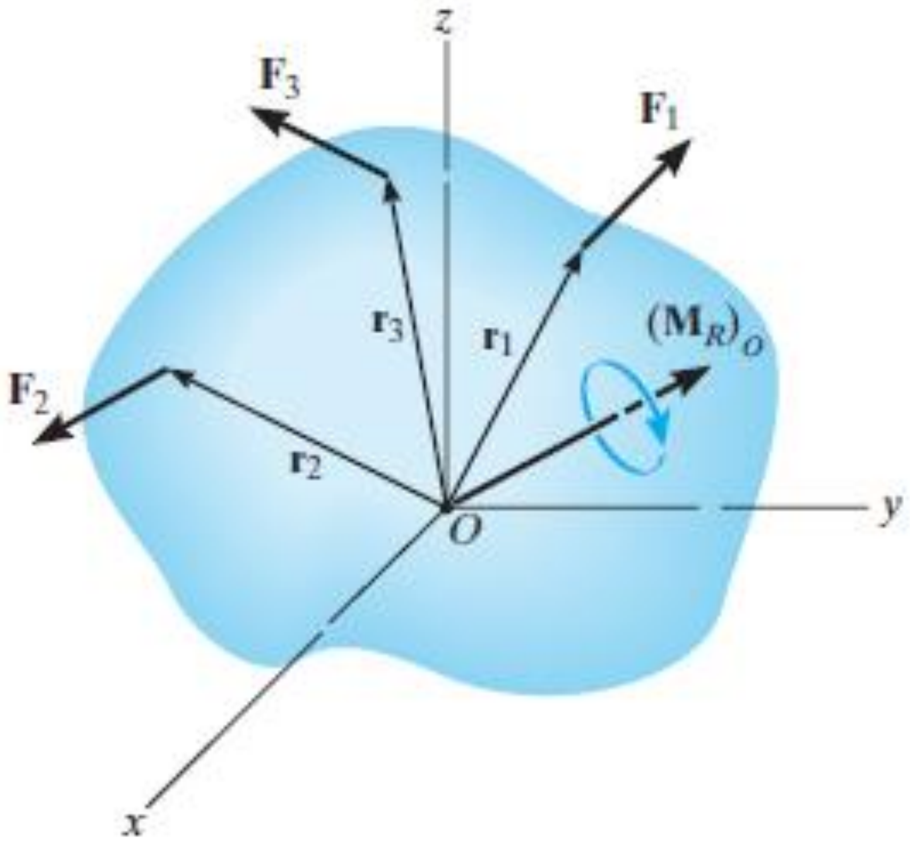
$$\cos\alpha = \frac{\mathrm{Mx}}{\mathrm{M}}$$

$$\cos\beta = \frac{\mathrm{My}}{M}$$

$$\cos\gamma = \frac{\mathrm{Mz}}{\mathrm{M}}$$

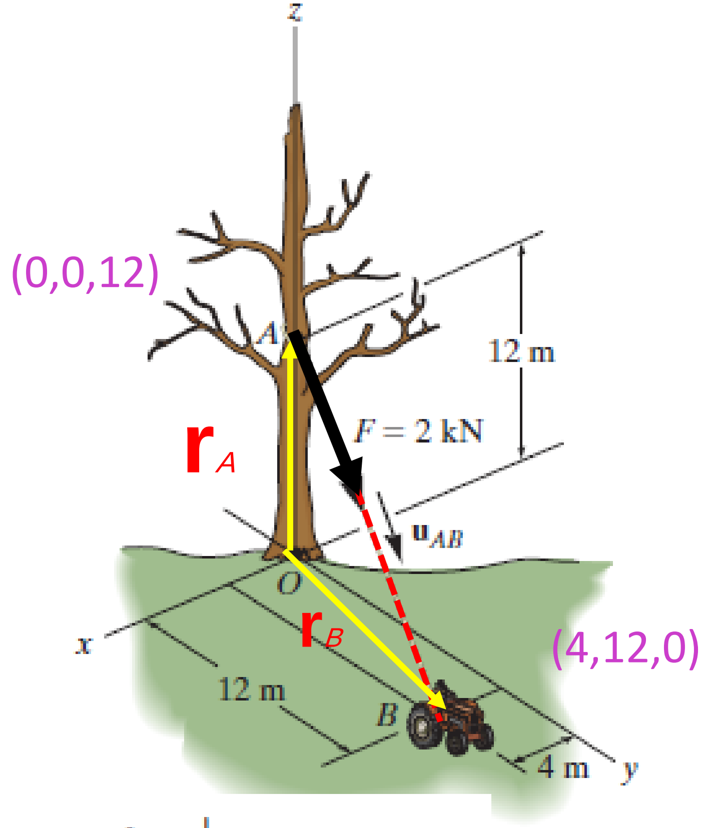
Resultant Moment of a System of Forces.

$$(M_r)_o = \Sigma\,(\mathbf{r} \times \mathbf{F})$$



Ex. Determine the moment produced by the force about point *O*. Express the result as a Cartesian vector.

$\mathbf{M_O} = \mathbf{r} \times \mathbf{F}$



F expressed as a Cartesian vector is

$$\mathbf{F} = F\mathbf{u}_{AB} = 2\left[\frac{\{4\mathbf{i} + 12\mathbf{j} - 12\mathbf{k}\}}{\sqrt{(4\quad)^2 + (12\quad)^2 + (-12\quad)^2}}\right]$$

$$= \{0.4588\mathbf{i} + 1.376\mathbf{j} - 1.376\mathbf{k}\}\ \text{kN}$$

$\mathbf{r}_A = \{12\mathbf{k}\}$ m and $\mathbf{r}_B = \{4\mathbf{i} + 12\mathbf{j}\}$ m

$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix}$$

$$= [0(-1.376) - 12(1.376)]\mathbf{i} - [0(-1.376) - 12(0.4588)]\mathbf{j} + [0(1.376) - 0(0.4588)]\mathbf{k}$$

$$= \{-16.5\mathbf{i} + 5.51\mathbf{j}\}\ \text{kN}\cdot\text{m}$$

Or

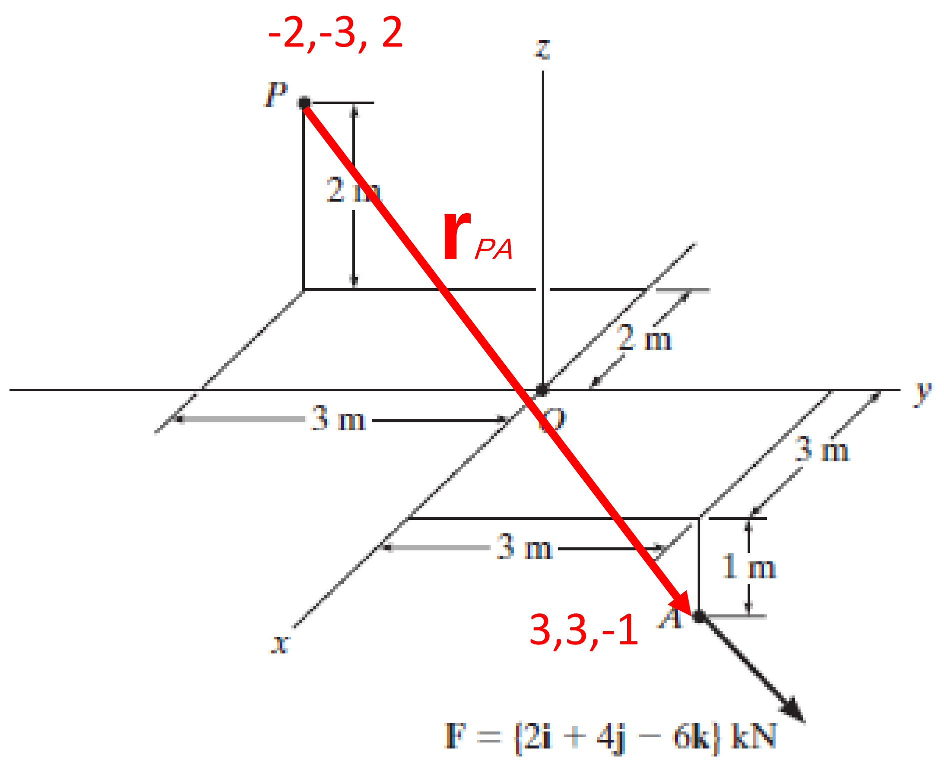
$$\mathbf{M}_O = \mathbf{r}_B \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 12 & 0 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix}$$

$$= [12(-1.376) - 0(1.376)]\mathbf{i} - [4(-1.376) - 0(0.4588)]\mathbf{j} + [4(1.376) - 12(0.4588)]\mathbf{k}$$

$$= \{-16.5\mathbf{i} + 5.51\mathbf{j}\}\ \text{kN}\cdot\text{m}$$

$\mathbf{r}_{PA} = 5i + 6j - 3k$

4–31. Determine the moment of the force **F** about point *P*. Express the result as a Cartesian vector.



$$\mathbf{M} = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \overset{+}{i} & \overset{-}{-j} & \overset{+}{k} \\ 5 & 6 & -3 \\ 2 & 4 & -6 \end{vmatrix} = -24i + 24j + 8k$$

The magnitude of the moment

$$\boldsymbol{M} = \sqrt{24^2 + 24^2 + 8^2} = 34.87\boldsymbol{Nm}$$

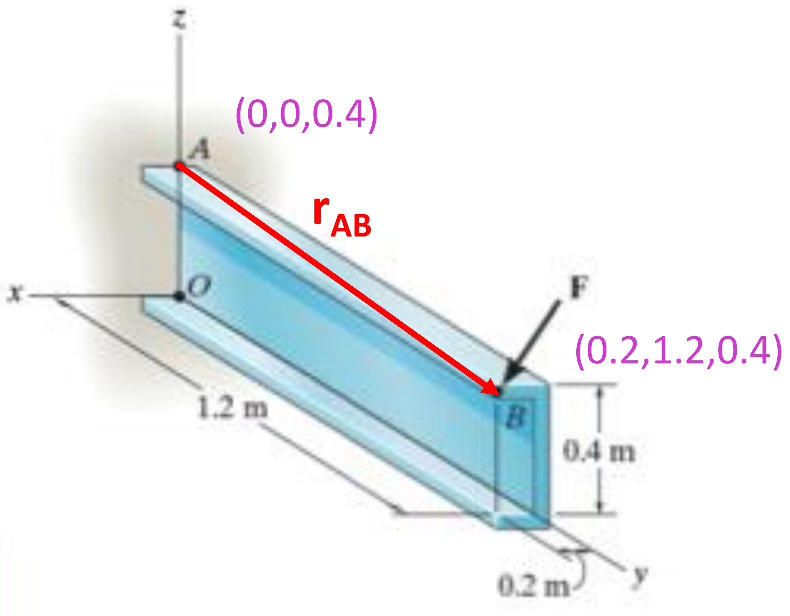
Direction of the moment

$$\cos\alpha = \frac{-24}{34.87} = \dots\dots\dots \quad \alpha = 133.49^o$$

$$\cos\beta = \frac{24}{34.87} = \dots\dots\dots\dots. \quad \beta = 46.51^o$$

$$\cos\gamma = \frac{8}{34.87} = \cdots\dots\dots\dots\dots.\gamma = 76.74^o$$

The force **F** = {600**i** + 300**j** - 600**k**} N acts at the end of the the beam. Determine the moment of the force about point *A*.



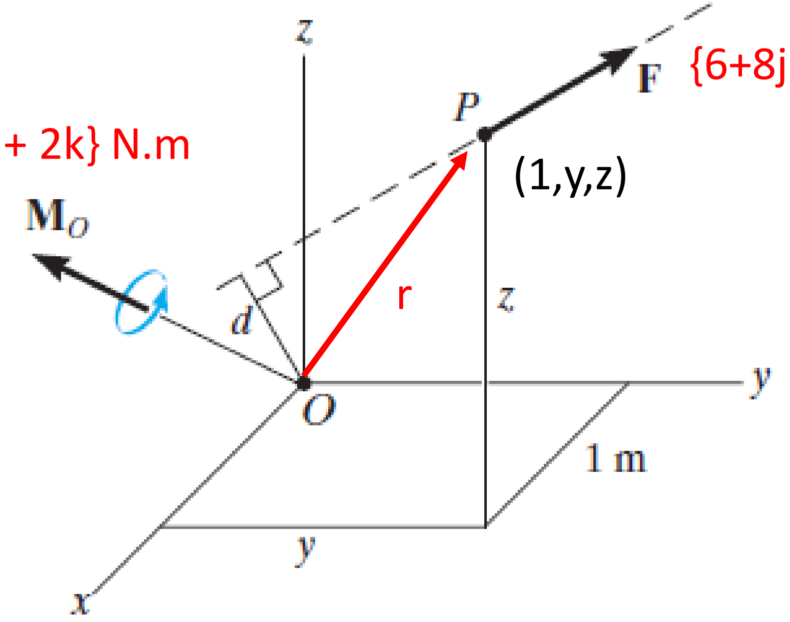
$$\mathbf{r} = \{0.2\mathbf{i} + 1.2\mathbf{j}\}\text{ m}$$

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2 & 1.2 & 0 \\ 600 & 300 & -600 \end{vmatrix}$$

$$\mathbf{M}_O = \{-720\mathbf{i} + 120\mathbf{j} - 660\mathbf{k}\}\text{ N}\cdot\text{m}$$

4–46. The force $\mathbf{F} = \{6\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}\}$ N creates a moment about point O of $\mathbf{M}_O = \{-14\mathbf{i} + 8\mathbf{j} + 2\mathbf{k}\}$ N·m. If the force passes through a point having an x coordinate of 1 m, determine the y and z coordinates of the point. Also, realizing that $M_O = Fd$, determine the perpendicular distance d from point O to the line of action of **F**. *Note:* The figure shows **F** and $\mathbf{M}_O$ in an arbitrary position.

= {-14i +8j + 2k} N.m



{6+8j + 10k} N

r= 1 i + y j + z k

$$\textbf{Mo= rxF}\begin{vmatrix} i & -j & k \\ 1 & y & z \\ 6 & 8 & 10 \end{vmatrix} = (10y - 8z)i - (10 - 6z)j + (8 - 6y)k$$

= {-14i +8j + 2k} N.m

Z= 3m
Y=1m

d= M/F

$$d = \frac{\sqrt{(14^2 + 8^2 + 2^2)}}{\sqrt{6^2 + 8^2 + 10^2}} = \frac{16.25}{14.14} = 1.15\ m$$

4.5 Moment of a Force about a Specified Axis

$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}.$

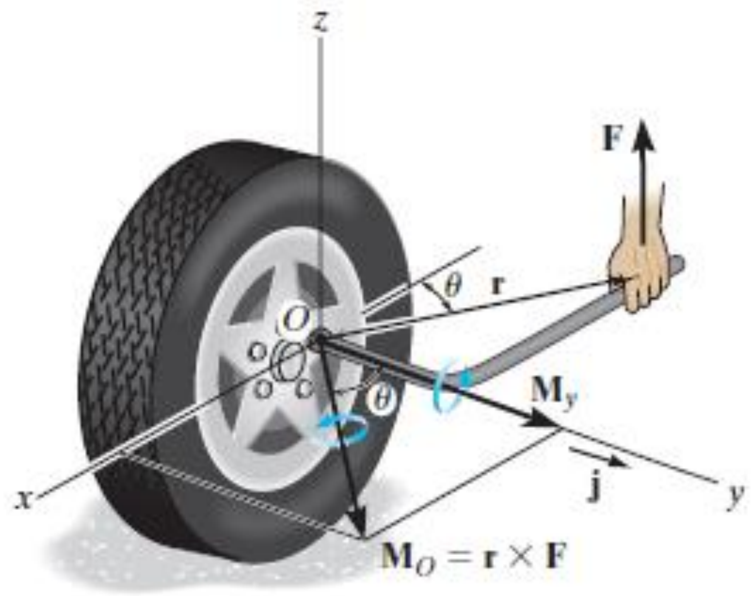
The component **M***y* along the *y* axis is the *projection* of $\mathbf{M_O}$ onto the *y* axis.

the projection of this moment onto the *a* axis is $M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F})$

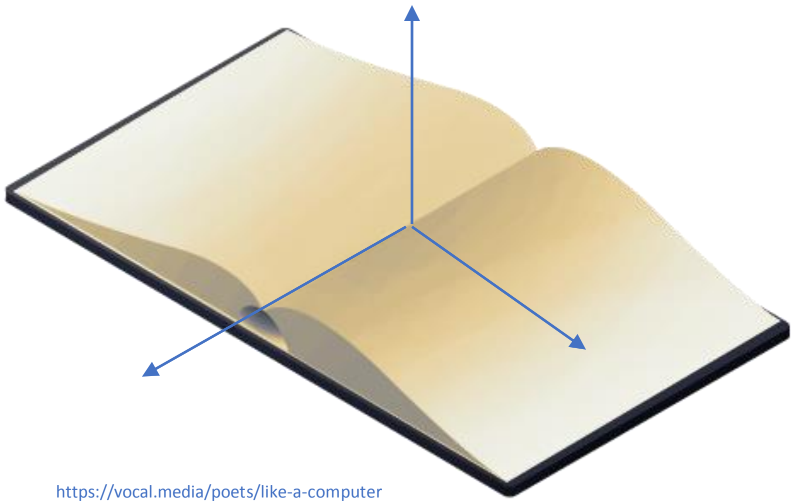
$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F})$ *scalar triple product.*

$$M_a = [u_{a_x}\mathbf{i} + u_{a_y}\mathbf{j} + u_{a_z}\mathbf{k}] \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

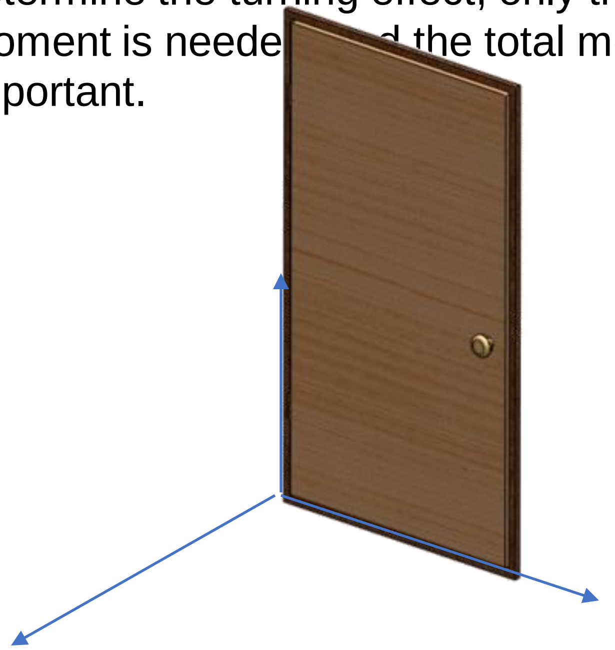
$$= u_{a_x}(r_yF_z - r_zF_y) - u_{a_y}(r_xF_z - r_zF_x) + u_{a_z}(r_xF_y - r_yF_x)$$



the nut can only rotate about the *y* axis. Therefore, to determine the turning effect, only the *y* component of the moment is neede … the total moment produced is not important.



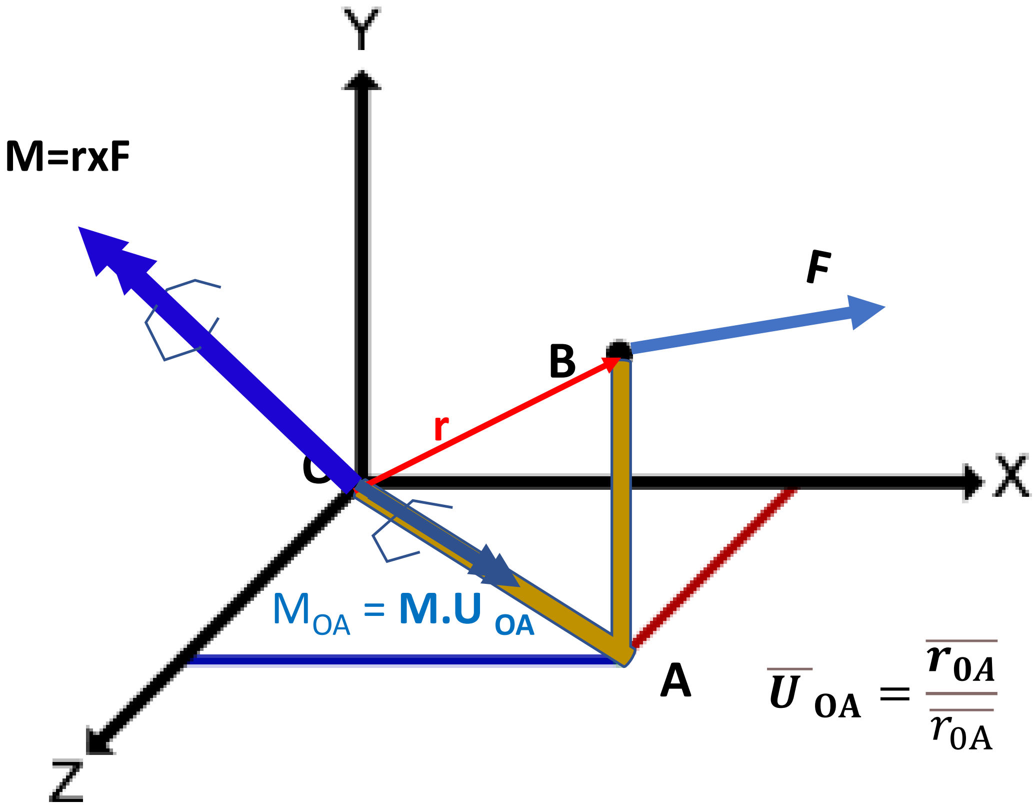
https://vocal.media/poets/like-a-computer



$$\begin{matrix} \color{red}{+} & \color{red}{-} & \color{red}{+} \end{matrix}$$

M=rxF .U = $\begin{vmatrix} 0 & 0.6 & 0.8 \\ 2 & -3 & 1 \\ -1 & 3 & 4 \end{vmatrix} =$

$$0 - 0.6x(8+1) + 0.8x(6-3) = -3$$



$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

components of the unit vector defining the direction of the *a-a* axis

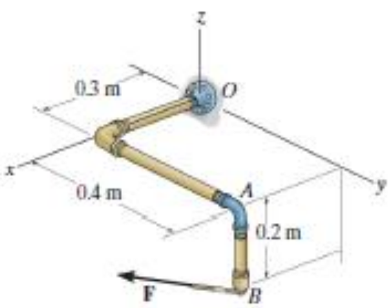
the position vector extended from *any point O* on the *a-a* axis to *any point A* on the line of action of the force

components of the force vector.

If it is positive, then **M***a* will have the same sense as **u***a*, whereas if it is negative, then **M***a* will act opposite to **u***a*

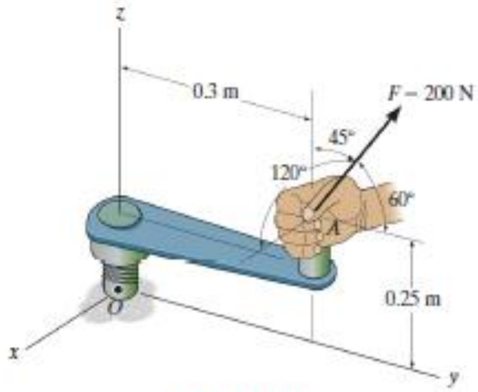
F4–13. Determine the magnitude of the moment of the force $\mathbf{F} = \{300\mathbf{i} - 200\mathbf{j} + 150\mathbf{k}\}$ N about the x axis.

F4–14. Determine the magnitude of the moment of the force $\mathbf{F} = \{300\mathbf{i} - 200\mathbf{j} + 150\mathbf{k}\}$ N about the OA axis.



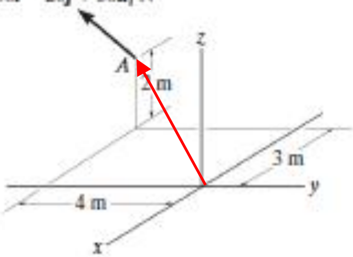
Probs. F4–13/14

F4–15. Determine the magnitude of the moment of the 200-N force about the x axis. Solve the problem using both a scalar and a vector analysis.



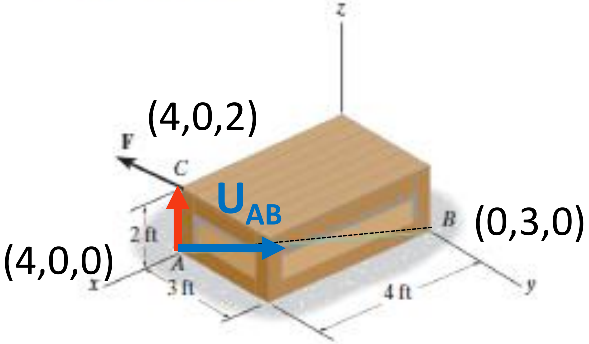
Prob. F4–15

F4–16. Determine the magnitude of the moment of the force about the y axis.



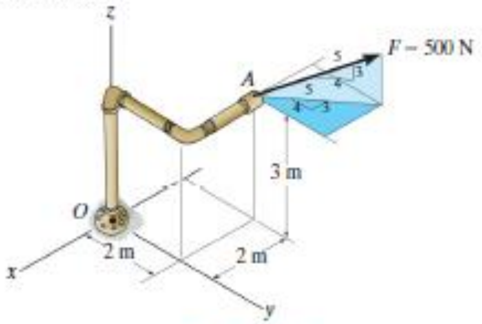
Prob. F4–16

F4–17. Determine the moment of the force $\mathbf{F} = \{50\mathbf{i} - 40\mathbf{j} + 20\mathbf{k}\}$ lb about the AB axis. Express the result as a Cartesian vector.



Prob. F4–17

F4–18. Determine the moment of force **F** about the x, the y, and the z axes. Solve the problem using both a scalar and a vector analysis.



Prob. F4–18

$$\textbf{My=(rxF).U}\begin{vmatrix} 0 & 1 & 0 \\ -3 & -4 & 2 \\ 30 & -20 & 50 \end{vmatrix} = 0 - (-150 - 60) + 0$$

My =210 N.m

U_{AB}= (-4i +3j+0k) /√ $(4^2 + 3^2)$ = -0.8i + 0.6j +0k

$$\mathbf{M_{AB}= (rxF).U_{AB}}\begin{vmatrix} -0.8 & 0.6 & 0 \\ 0 & 0 & 2 \\ 50 & -40 & 20 \end{vmatrix} =$$

-0.8x(80) -0.6x(-100) +0 = -4 lb.ft

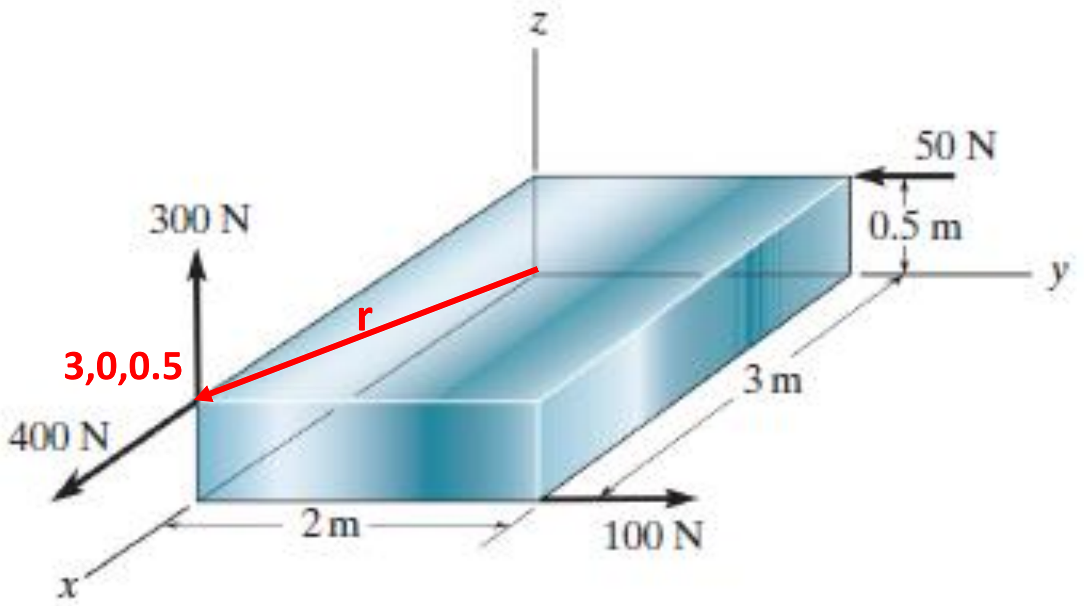
Find the resultant moment about the y- axis

$M_y = -300x3+400x0.5 = -700$ N.m

Or

$$M_y = \Sigma(r \times F).U \begin{vmatrix} 0 & 1 & 0 \\ 3 & 0 & 0.5 \\ 400 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 0 \\ 3 & 0 & 0.5 \\ 0 & 0 & 300 \end{vmatrix} =$$

$+400x0.5 -3x300 = -700$ N.m

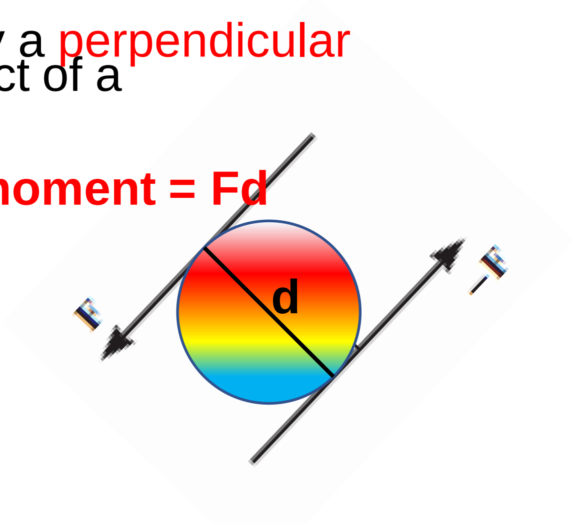


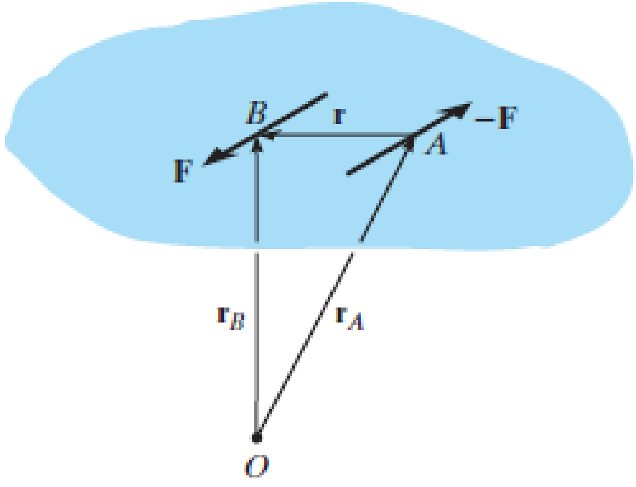
4.6 Moment of a Couple

A **couple** is defined as two parallel forces that have the same magnitude,
but opposite directions, and are separated by a perpendicular distance ,

Since the resultant force is zero, the only effect of a couple is to
produce an actual rotation due to a **couple moment = Fd**

couple moment is a ***free vector***,





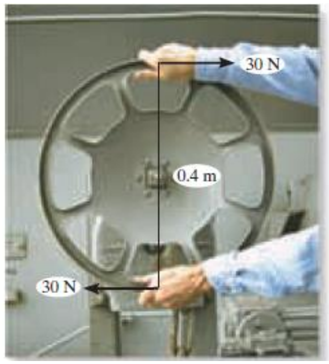
In 3D

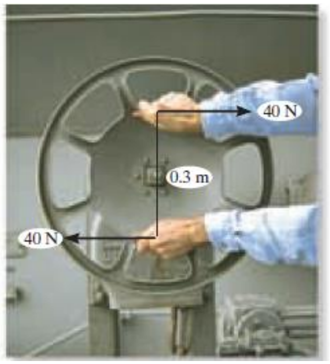
$$\mathbf{M} = \mathbf{r}_B \times \mathbf{F} + \mathbf{r}_A \times -\mathbf{F} = (\mathbf{r}_B - \mathbf{r}_A) \times \mathbf{F}$$

However $\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}$ or $\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A$, so that

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

where **r** is directed from *any point* on the line of action of one of the forces to any point on the line of action of the other force **F**.





Equivalent Couples. If two couples produce a moment with the *same magnitude and direction*

$$M = (30\text{ N})(0.4\text{ m}) = 12\text{ N.m}$$

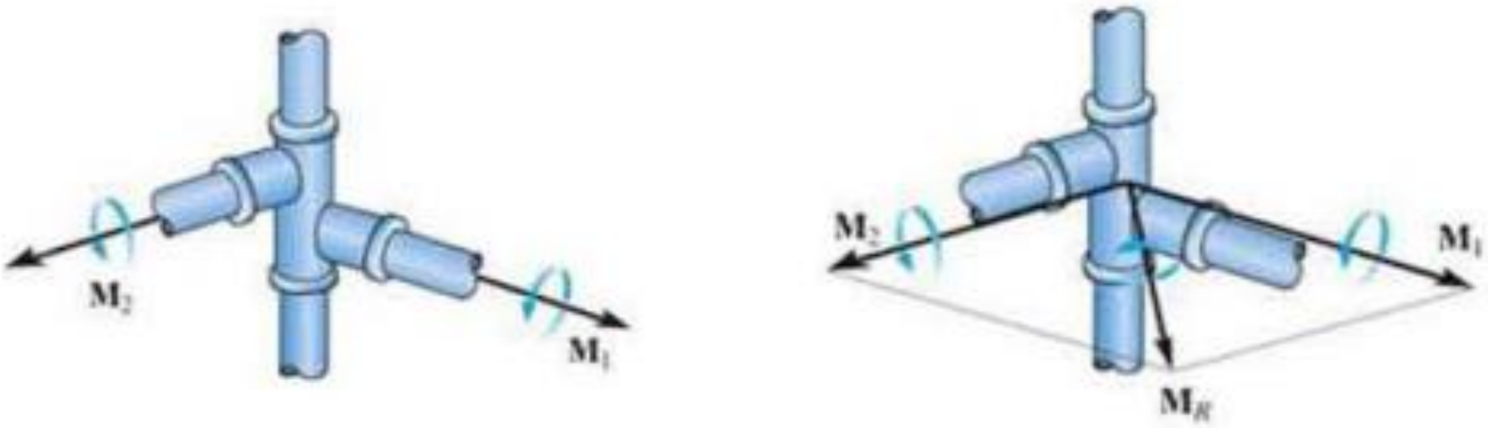
$$M = (40\text{ N})(0.3\text{ m}) = 12\text{ N.m}$$

Resultant Couple Moment.

- Resultant Couple Moment: It is simply the vector sum of all the couple moments of the system.

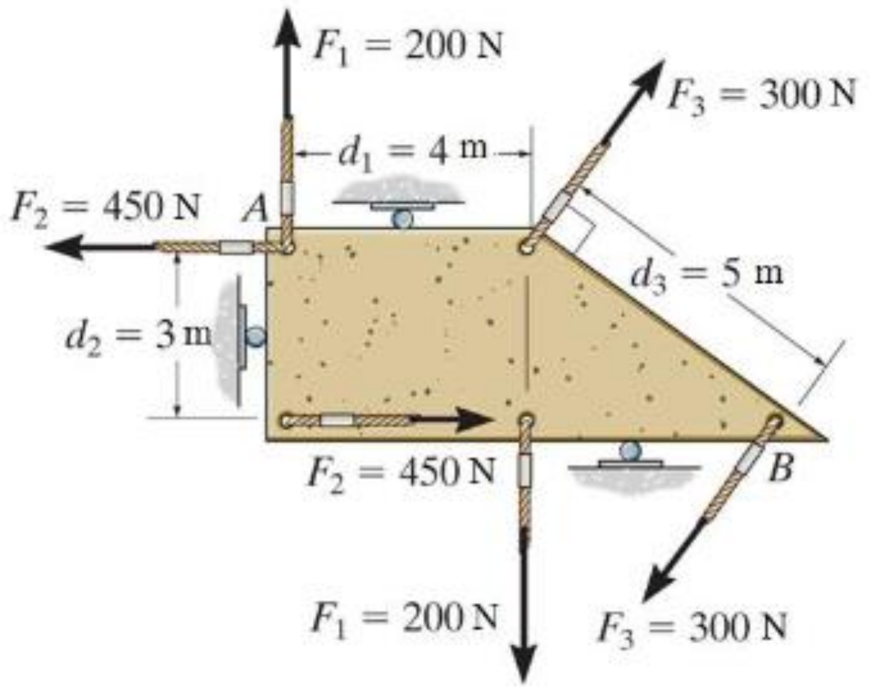
$$\mathbf{M}_R = \sum (\mathbf{r} \times \mathbf{F})$$

- For example, consider the couple moments $\mathbf{M}_1$ and $\mathbf{M}_2$ acting on the pipe. We can join their tails at any arbitrary point and find the resultant couple moment.



$$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$$

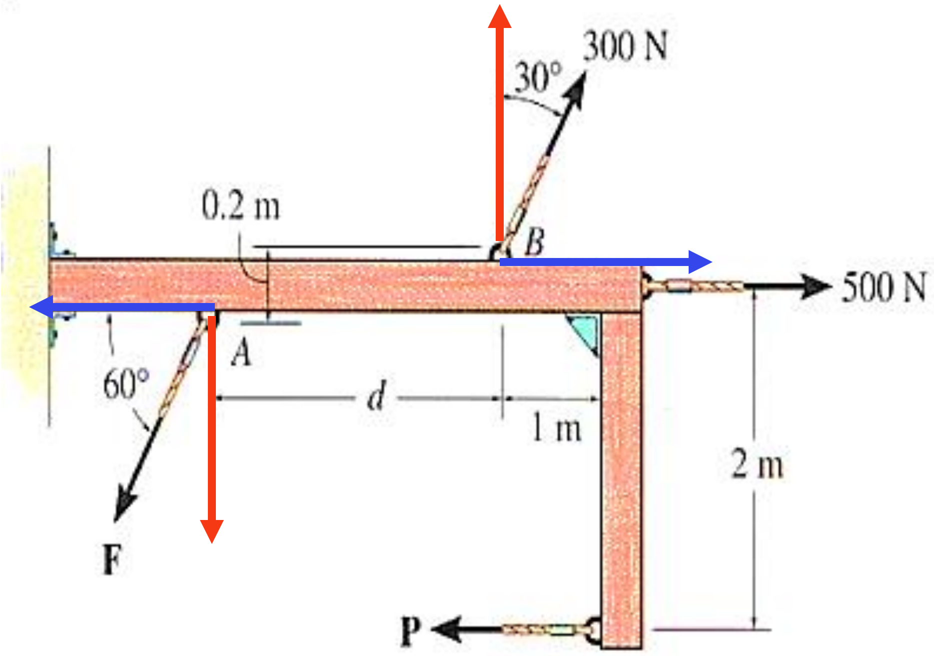
Determine the resultant couple moment of the three couples acting on the plate



$$\circlearrowleft + M_R = \sum M =$$

$$= -(200\text{ N})(0.4\text{ m}) + (450\text{ N})(0.3\text{ m}) - (300\text{ N})(0.5\text{ m})$$
$$= -95\text{ N.m} = 95\text{ N.m} \circlearrowright$$

Two couples act on the beam. If the resultant couple is zero.
Find: The magnitudes of the forces P and F and the distance d

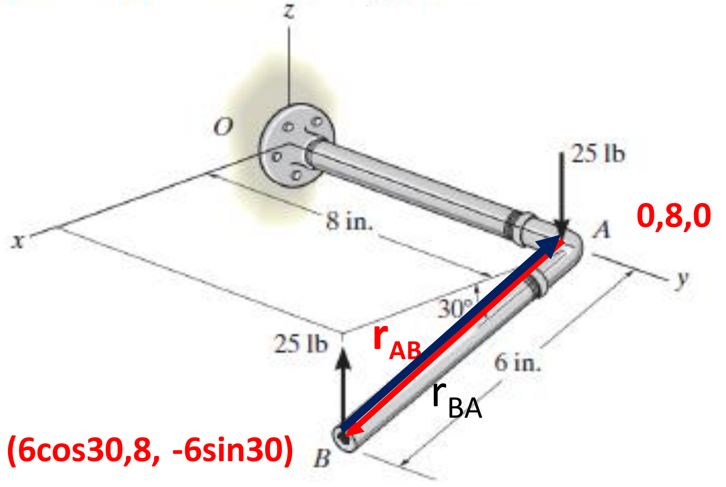


P = 500 N and F = 300 N

$$+\circlearrowright \Sigma M = -(500)(2) + (300 \cos 30^\circ)(d) - (300 \sin 30^\circ)(0.2) = 0$$

d = 3.96 m

Determine the couple moment acting on the pipe shown in Fig. 4–32*a*. Segment *AB* is directed 30° below the *x*–*y* plane.



(a)

$$M= (\mathbf{r} \times \mathbf{F}) \begin{vmatrix} i & -j & k \\ 5.2 & 0 & -3 \\ 0 & 0 & 25 \end{vmatrix} =$$

-0i- (5.2x25)j+0k = - 130j N.m

OR

$$M= (\mathbf{r} \times \mathbf{F}) \begin{vmatrix} i & -j & k \\ -5.2 & 0 & +3 \\ 0 & 0 & -25 \end{vmatrix} =$$

-0i- (-5.2x-25)j+0k = - 130j N.m

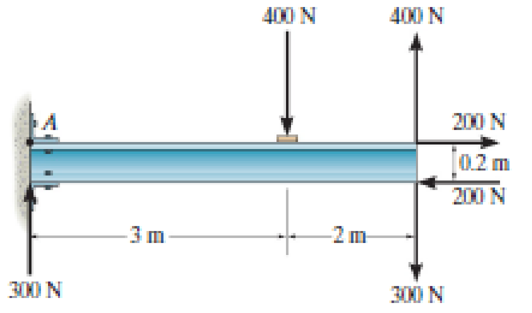
OR

SOLUTION I (VECTOR ANALYSIS)

The moment of the two couple forces can be found about *any point*. If point *O* is considered, Fig. 4–32*b*, we have

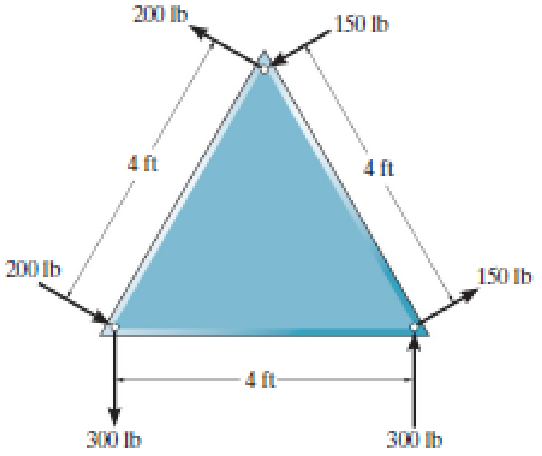
$$\mathbf{M} = \mathbf{r}_A \times (-25\mathbf{k}) + \mathbf{r}_B \times (25\mathbf{k})$$
$$= (8\mathbf{j}) \times (-25\mathbf{k}) + (6\cos 30^\circ\mathbf{i} + 8\mathbf{j} - 6\sin 30^\circ\mathbf{k}) \times (25\mathbf{k})$$
$$= -200\mathbf{i} - 129.9\mathbf{j} + 200\mathbf{i}$$
$$= \{-130\mathbf{j}\}\ \text{lb}\cdot\text{in.}$$

F4–19. Determine the resultant couple moment acting on the beam.



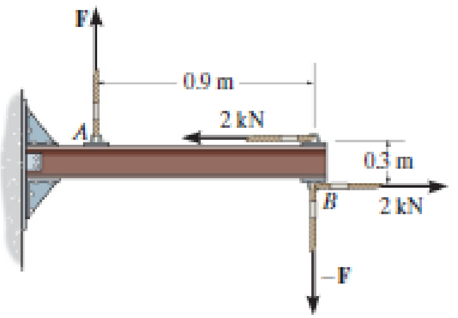
Prob. F4–19

F4–20. Determine the resultant couple moment acting on the triangular plate.

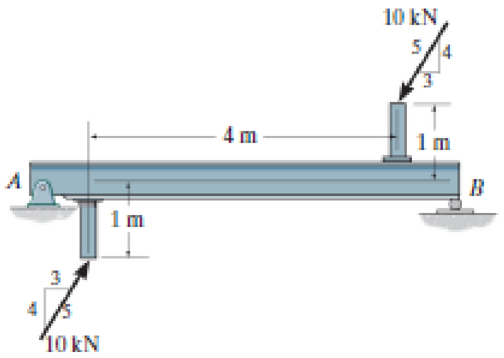


Prob. F4–20

F4–21. Determine the magnitude of **F** so that the resultant couple moment acting on the beam is 1.5 kN · m clockwise.

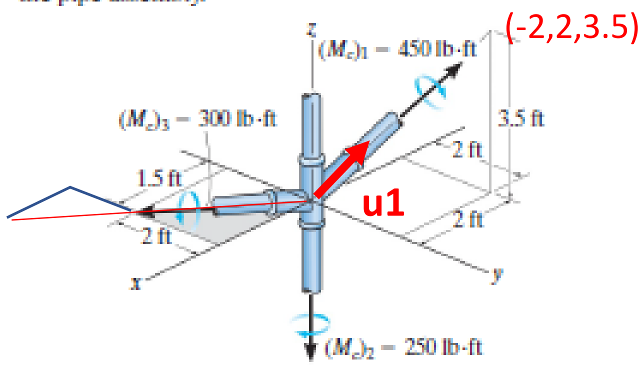


F4–22. Determine the couple moment acting on the beam.



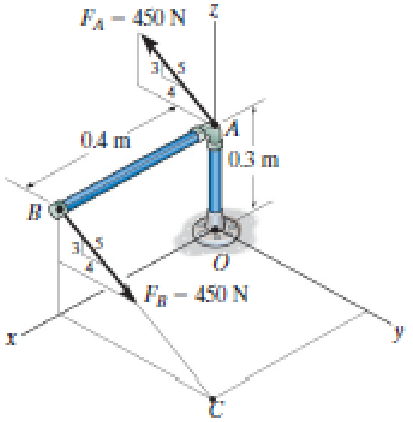
Prob. F4–22

F4–23. Determine the resultant couple moment acting on the pipe assembly.



Prob. F4–23

F4–24. Determine the couple moment acting on the pipe assembly and express the result as a Cartesian vector.



$$\mathbf{u}_1 = \frac{\mathbf{r}_1}{r_1} = \frac{\{-2\mathbf{i} + 2\mathbf{j} + 3.5\mathbf{k}\}}{\sqrt{(-2\quad)^2 + (2\quad)^2 + (3.5\quad)^2}}$$

$$= -\frac{2}{4.5}\mathbf{i} + \frac{2}{4.5}\mathbf{j} + \frac{3.5}{4.5}\mathbf{k}$$

$$(\mathbf{M}_c)_1 = (M_c)_1\mathbf{u}_1$$

$$= (450\quad)\left(-\frac{2}{4.5}\mathbf{i} + \frac{2}{4.5}\mathbf{j} + \frac{3.5}{4.5}\mathbf{k}\right)$$

$$= \{-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k}\}\ \text{lb}\cdot\text{ft}$$

M_2 =0i +0j -250 k

M_3 = 300(1.5/2.5)I -300(2/2.5) j + 0k
= 180i -240 j + 0k

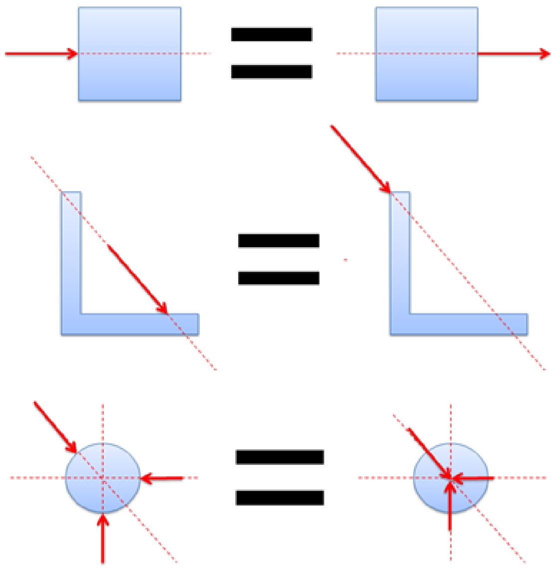
$$(\mathbf{M}_c)_R = \Sigma \mathbf{M}_c;$$

$$(\mathbf{M}_c)_R = \{-20\mathbf{i} - 40\mathbf{j} + 100\mathbf{k}\}\ \text{lb}\cdot\text{ft}$$

4.7 Simplification of a Force and Couple System

principle of transmissibility

sliding vector

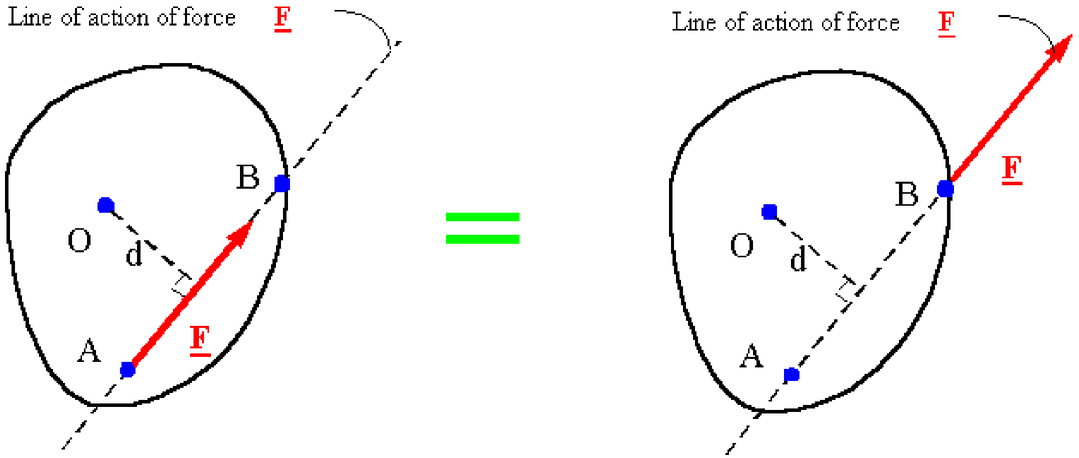


Sometimes it is convenient to reduce a system of forces and couple moments acting on a body to a simpler form by replacing it with an ***equivalent system***,

A system is equivalent if the ***external effects*** it produces on a body are the same as those caused by the original force and couple moment system.

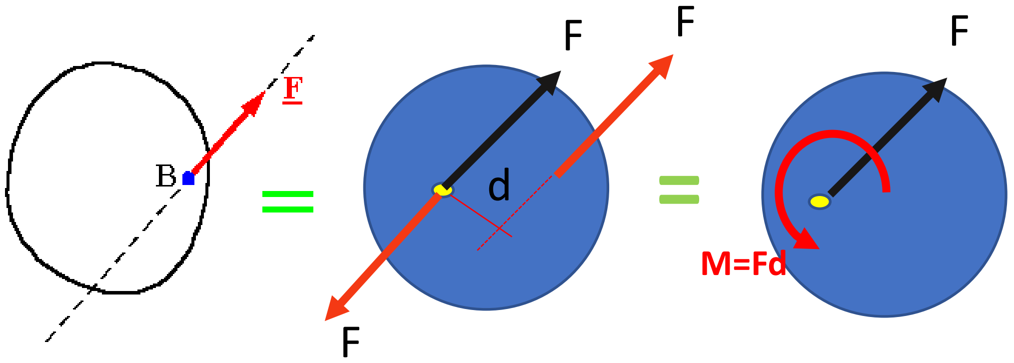
$$\begin{cases} \sum \underline{\mathbf{F}} \text{ for system 1} = \sum \underline{\mathbf{F}} \text{ for system 2} \\ \sum \underline{\mathbf{M}}_O \text{ for system 1} = \sum \underline{\mathbf{M}}_O \text{ for system 2} \end{cases} \Leftrightarrow \text{The two force systems are equivalent}$$

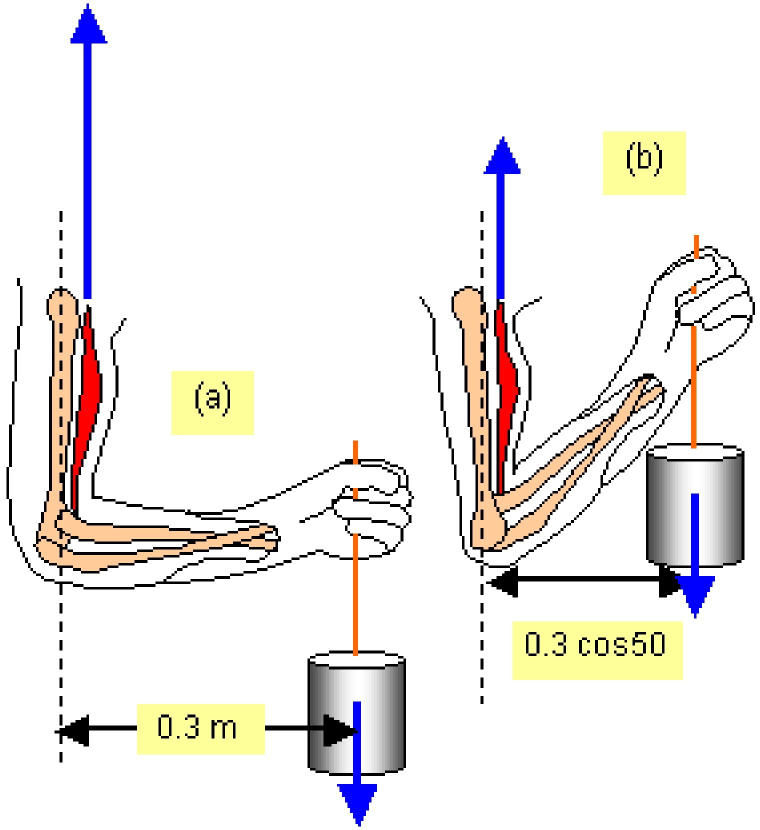
Moving a force along its line of action



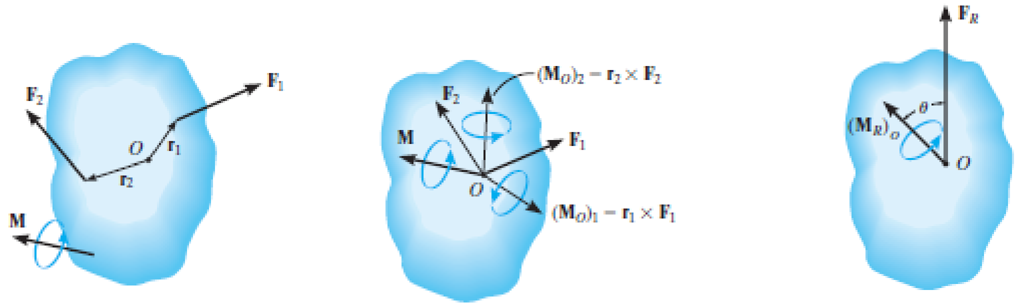
System of Forces and Couple Moments.

Moving a force off its line of action





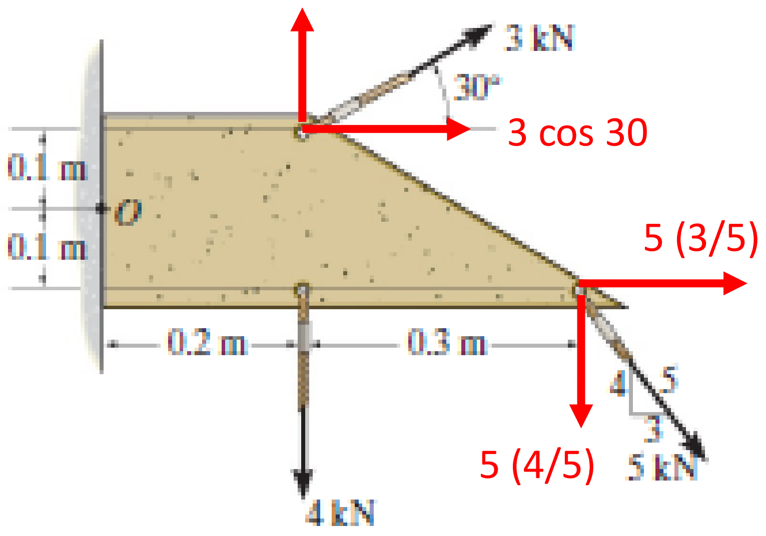
The resultant of a force and couple system



$$(F_R)_x = \Sigma F_x$$
$$(F_R)_y = \Sigma F_y$$
$$(M_R)_O = \Sigma M_O + \Sigma M$$

$$\mathbf{F}_R = \Sigma \mathbf{F}$$
$$(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O + \Sigma \mathbf{M}$$

Replace the force and couple system by an **equivalent resultant force and couple moment acting at point O**



F_R =

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = (3\text{ kN})\cos 30^\circ + \left(\tfrac{3}{5}\right)(5\text{ kN}) = 5.598\text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = (3\text{ kN})\sin 30^\circ - \left(\tfrac{4}{5}\right)(5\text{ kN}) - 4\text{ kN} = -6.50\text{ kN} = 6.50\text{ kN}\downarrow$$

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{(5.598\text{ kN})^2 + (6.50\text{ kN})^2} = 8.58\text{ kN}$$

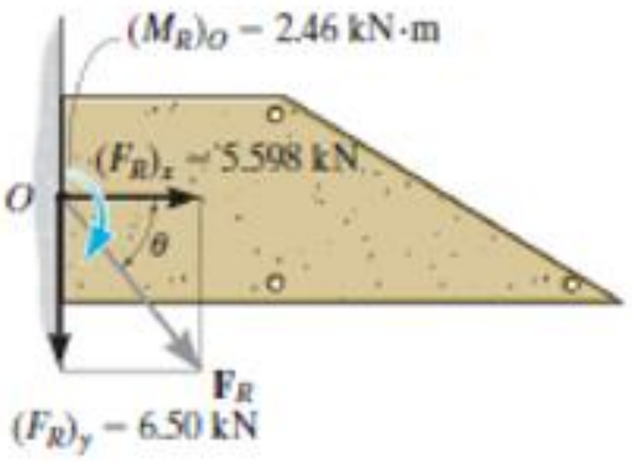
Its direction θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{6.50\text{ kN}}{5.598\text{ kN}}\right) = 49.3^\circ$$

$$\circlearrowleft + (M_R)_O = \Sigma M_O;$$

$$(M_R)_O = (3\text{ kN})\sin 30^\circ(0.2\text{ m}) - (3\text{ kN})\cos 30^\circ(0.1\text{ m}) + \left(\tfrac{3}{5}\right)(5\text{ kN})(0.1\text{ m}) - \left(\tfrac{4}{5}\right)(5\text{ kN})(0.5\text{ m}) - (4\text{ kN})(0.2\text{ m})$$

$$= -2.46\text{ kN}\cdot\text{m} = 2.46\text{ kN}\cdot\text{m} \circlearrowright$$



NOTE: Realize that the resultant force and couple moment in will produce the same external effects or reactions at the supports as those produced by the force system.

Replace the loading by an equivalent resultant force and moment at A

Fr

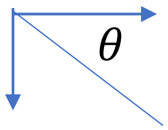
ΣFx= 900 sin30 = 450 N →

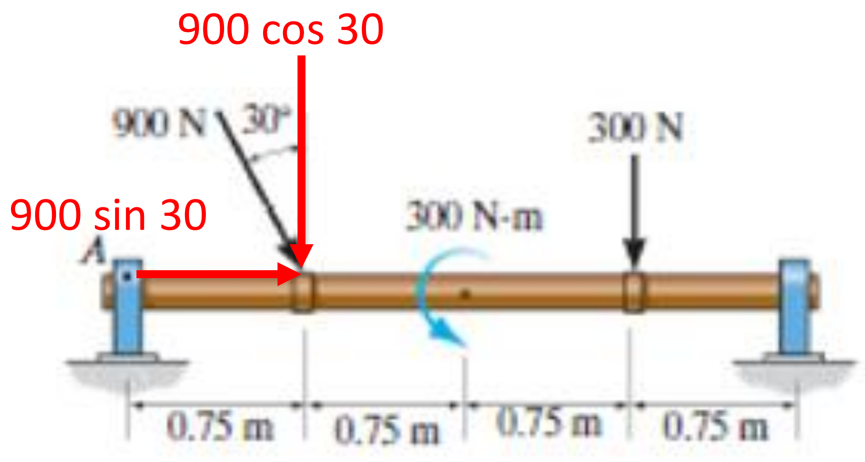
ΣFy = -900cos 30 -300 = -1079.42 ↓

Fr= $\sqrt{450^2 + 1079.42^2} = 1163N$

$$\tan\theta = 1079.42/450$$

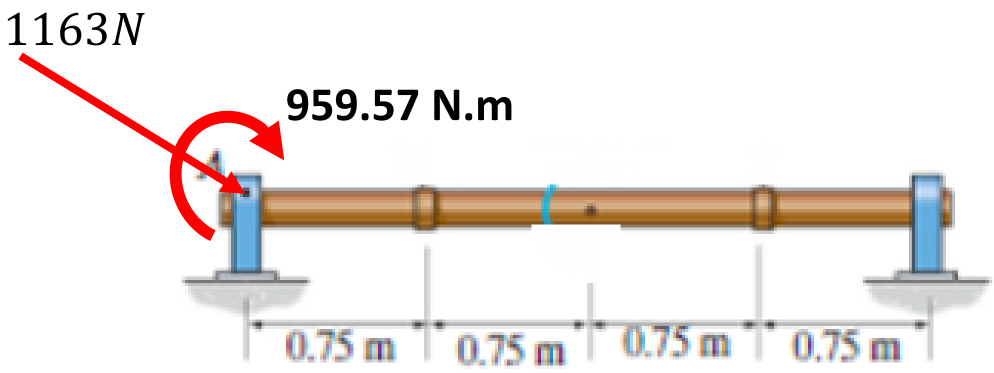
$$\theta = 67.37°$$



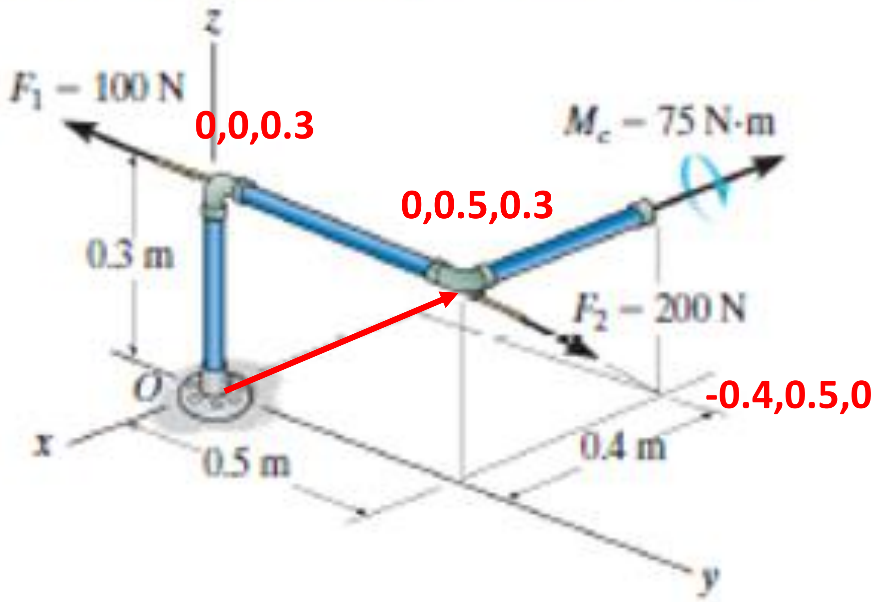


$M_A r$

ΣM_A = -900cos30 x 0.75 +300 -300x2.25 = -959.57 N.m



F4-30. Replace the loading system by an equivalent resultant force and couple moment acting at point *O*.



F1= 0i -100j +0k

$$F2 = \frac{200(-0.4i + 0j - 0.3k)}{\sqrt{(0.4^2 + 0.3^2)}} = -160i + 0j - 120k$$

FR = ΣF = -160i -100j –120k

$$FR = \sqrt{160^2 + 100^2 + 120^2} = 223.6 \text{ N}$$

$\mathbf{M_{oF1}}$= 100x0.3 i + 0j +0k = 30i +0j +0k

Mc = -75i +0j +0k

$$\mathbf{M_{F2}}\text{= (rxF)} \begin{vmatrix} i & -j & k \\ 0 & 0.5 & 0.3 \\ -160 & 0 & -120 \end{vmatrix} =$$

-0.5x120i -160x0.3j +160x0.5k =

MR = Σ**M**= -105i -48j –80k

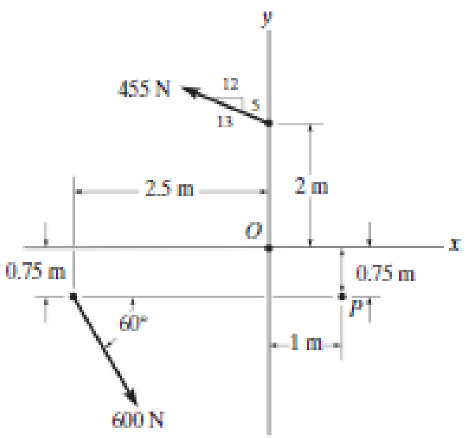
$$MR = \sqrt{105^2 + 48^2 + 80^2} = 140.46 \text{ N.m}$$

$$\mathbf{M_{F1}}\text{= (rxF)} \begin{vmatrix} i & -j & k \\ 0 & 0 & 0.3 \\ 0 & -100 & 0 \end{vmatrix} =$$

100x0.3i + 0j +0k = 30i +0j +0k

4–97. Replace the force system by an equivalent resultant force and couple moment at point O.

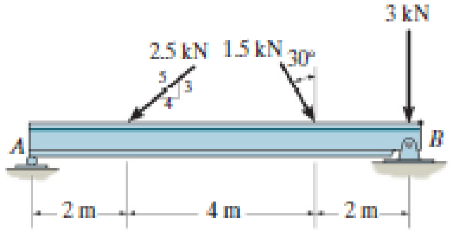
4–98. Replace the force system by an equivalent resultant force and couple moment at point P.



Probs. 4–97/98

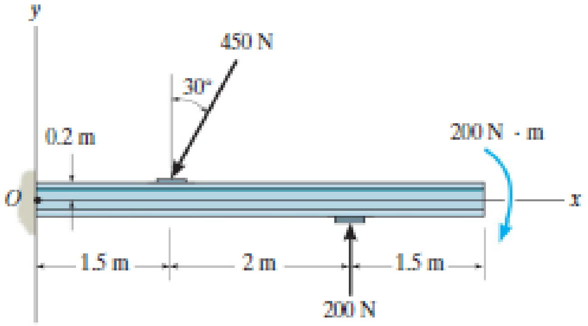
4–99. Replace the force system acting on the beam by an equivalent force and couple moment at point A.

***4–100.** Replace the force system acting on the beam by an equivalent force and couple moment at point B.



Probs. 4–99/100

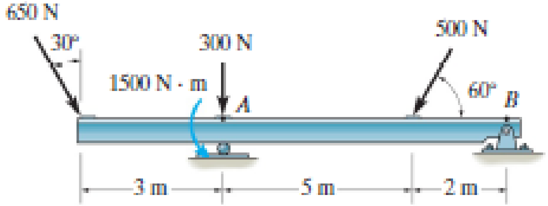
4–101. Replace the loading system acting on the beam by an equivalent resultant force and couple moment at point O.



Prob. 4–101

4–102. Replace the loading system acting on the post by an equivalent resultant force and couple moment at point A.

4–103. Replace the loading system acting on the post by an equivalent resultant force and couple moment at point B.



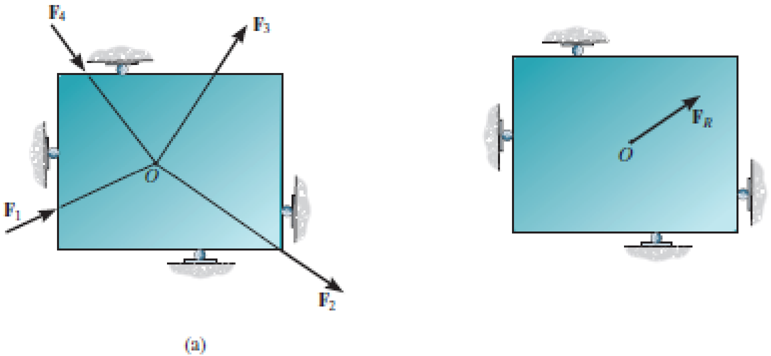
Probs. 4–102/103

4.8 Further Simplification of a Force and Couple System

The force system can be further reduced to an ***equivalent single resultant*** force provided the lines of action of $\mathbf{F}_R$ and $(\mathbf{M}_R)_O$ are *perpendicular* to each other.

Concurrent Force System.

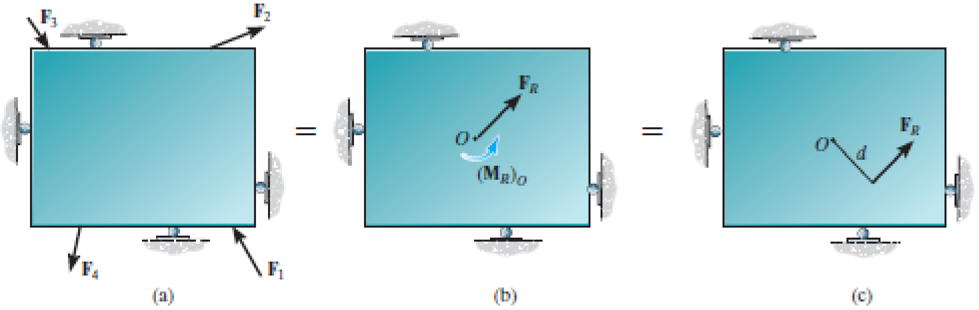
$$\mathbf{F}_R = \Sigma \mathbf{F}$$



Coplanar Force System.

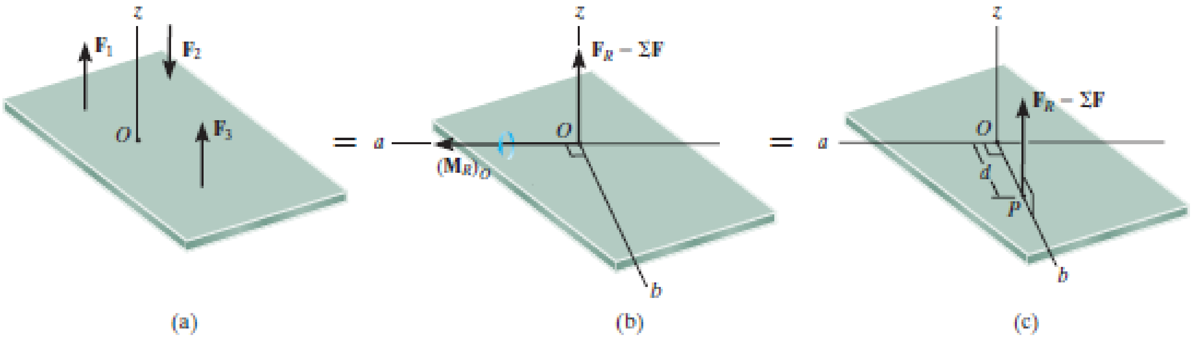
$$\mathbf{F}_R = \Sigma \mathbf{F}$$

$$(M_R)_O = \Sigma\, F d = \Sigma\, M_O$$



The resultant moment can be replaced by moving the resultant force $\mathbf{F}_R$ a perpendicular or moment arm distance d away from point O such that $\mathbf{F}_R$ produces the *same moment* $(\mathbf{M}_R)_O$ about point O,

$$M_{R\,O} = F_R\, d \qquad d = (M_R)_O / F_R.$$



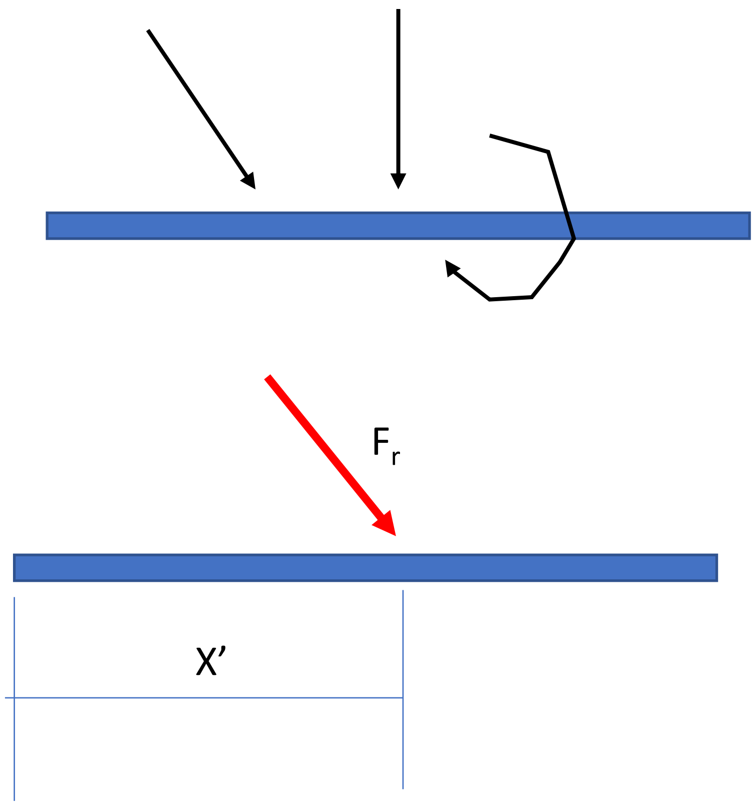
Parallel Force System.

$$\mathbf{F}_R = \Sigma\, \mathbf{F}$$

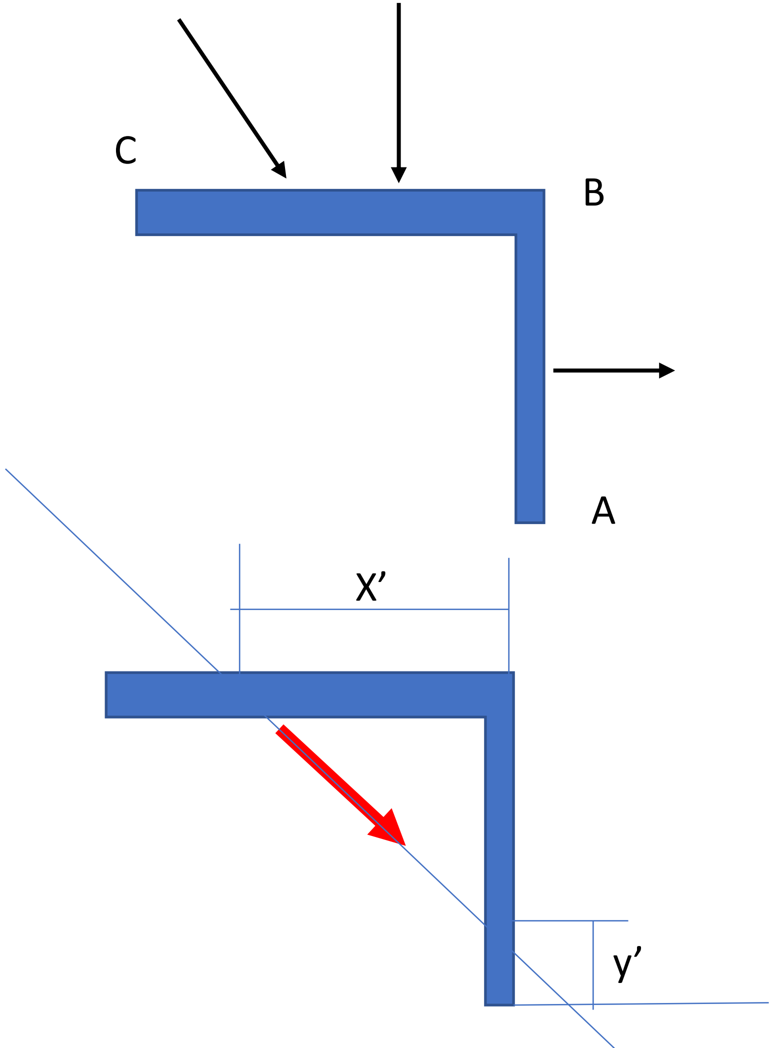
$$M_R)_O = F_R\, d \qquad d = (M_R)_O / F_R.$$

3 cases to be discussed

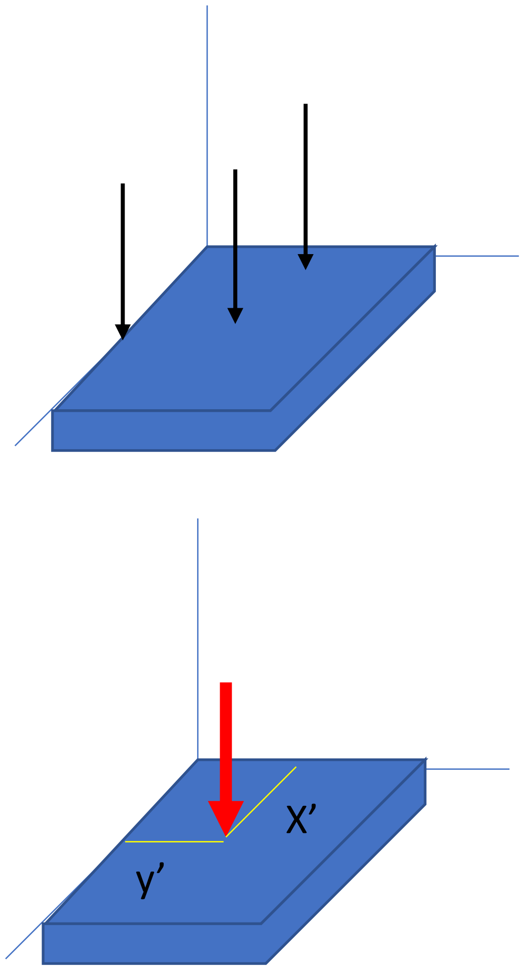
1. Replace the loading by single resultant force and where to be located from point A



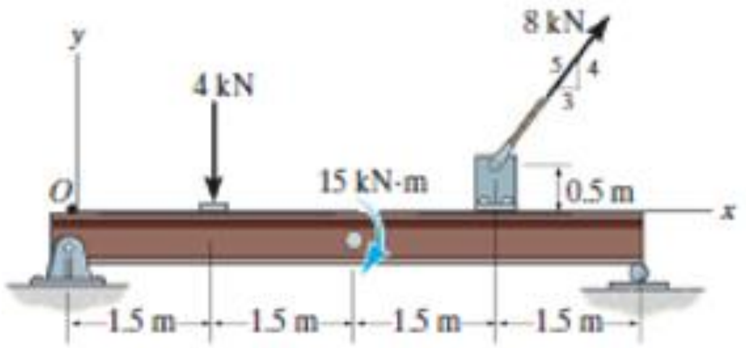
2. Replace the loading by single resultant force and where its line of action intersect members AB and BC from A

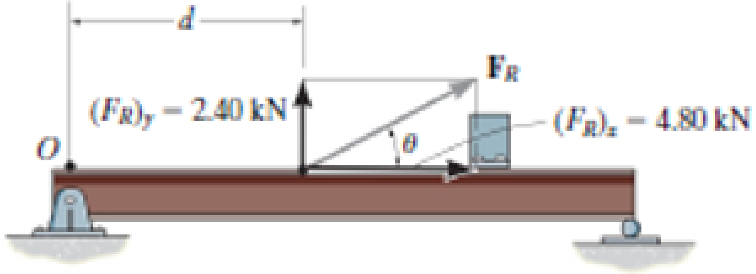


3. Replace the loading by single resultant force and and locate its point of application



Replace the force and couple moment system acting on the beam by an equivalent resultant force, and find where its line of action intersects the beam, measured from point O.





F_R

Force Summation. Summing the force components,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 8\text{ kN}\left(\tfrac{3}{5}\right) = 4.80\text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -4\text{ kN} + 8\text{ kN}\left(\tfrac{4}{5}\right) = 2.40\text{ kN}\uparrow$$

From Fig. 4–44*b*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(4.80\text{ kN})^2 + (2.40\text{ kN})^2} = 5.37\text{ kN}$$

The angle θ is

$$\theta = \tan^{-1}\left(\frac{2.40\text{ kN}}{4.80\text{ kN}}\right) = 26.6^\circ$$

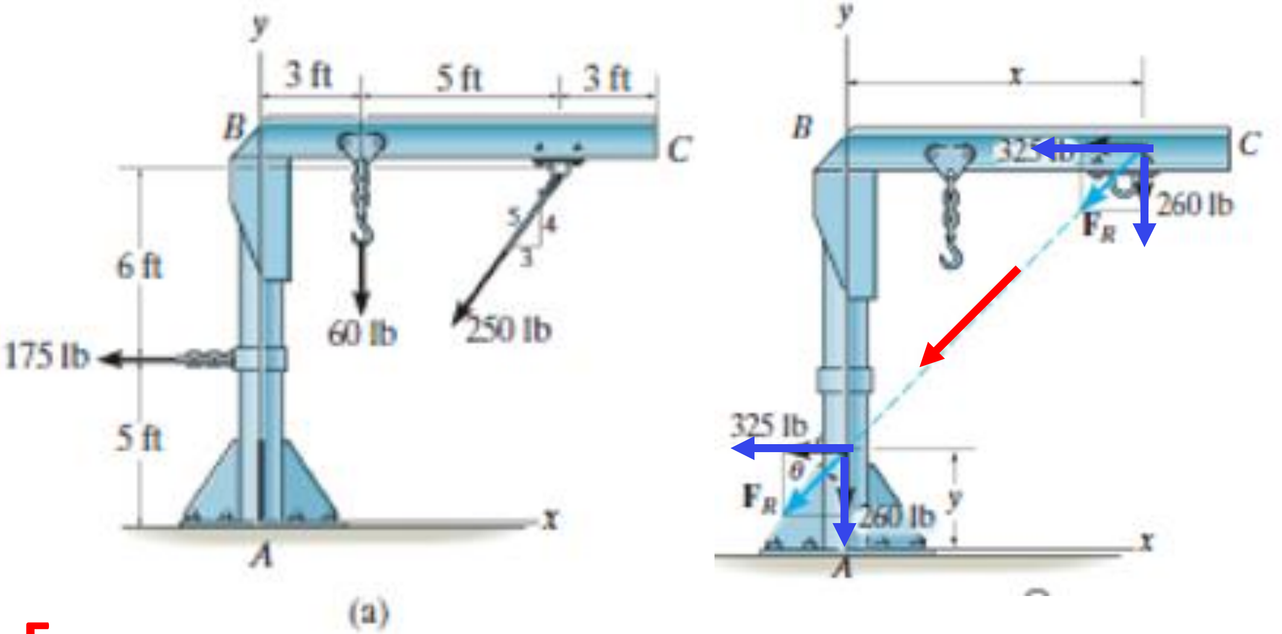
M_R

$$\circlearrowleft + (M_R)_O = \Sigma M_O; \qquad 2.40\text{ kN}(d) = -(4\text{ kN})(1.5\text{ m}) - 15\text{ kN}\cdot\text{m}$$

$$-\left[8\text{ kN}\left(\tfrac{3}{5}\right)\right](0.5\text{ m}) + \left[8\text{ kN}\left(\tfrac{4}{5}\right)\right](4.5\text{ m})$$

$$d = 2.25\text{ m}$$ *Ans.*

The jib crane shown is subjected to three coplanar forces. Replace this loading by an equivalent resultant force and specify where the resultant's line of action intersects the column *AB* and boom *BC*.



Move the resultant force to intersect member AB

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad 325\text{ lb}\,(y) + 260\text{ lb}\,(0)$$
$$= 175\text{ lb}\,(5\text{ ft}) - 60\text{ lb}\,(3\text{ ft}) + 250\text{ lb}\left(\tfrac{3}{5}\right)(11\text{ ft}) - 250\text{ lb}\left(\tfrac{4}{5}\right)(8\text{ ft})$$
$$y = 2.29\text{ ft}$$

$\mathbf{F}_R$

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \quad (F_R)_x = -250\text{ lb}\left(\tfrac{3}{5}\right) - 175\text{ lb} = -325\text{ lb} = 325\text{ lb} \leftarrow$$
$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = -250\text{ lb}\left(\tfrac{4}{5}\right) - 60\text{ lb} = -260\text{ lb} = 260\text{ lb}\downarrow$$

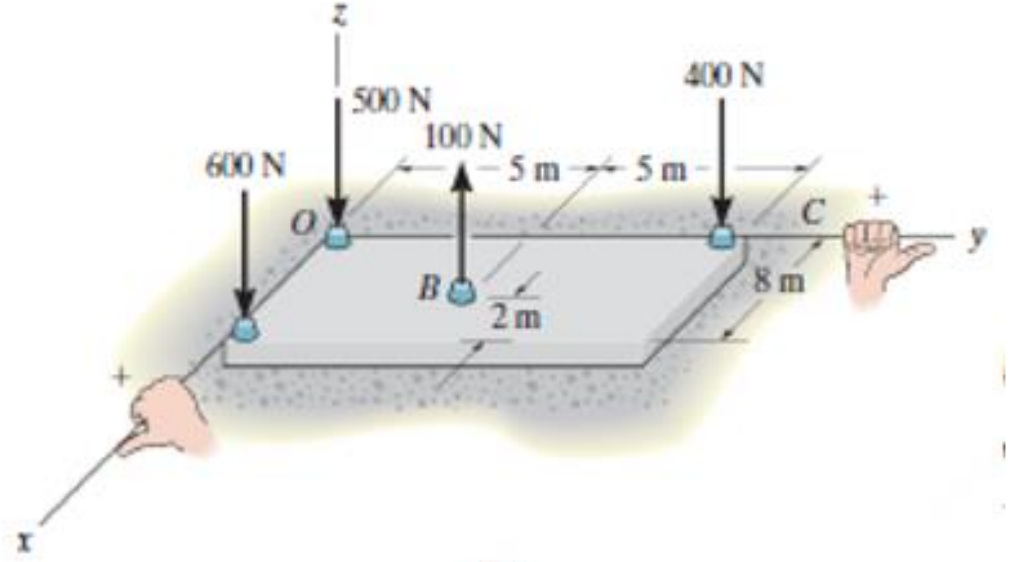
As shown by the vector addition in Fig. 4–45*b*,

$$F_R = \sqrt{(325\text{ lb})^2 + (260\text{ lb})^2} = 416\text{ lb}$$
$$\theta = \tan^{-1}\left(\frac{260\text{ lb}}{325\text{ lb}}\right) = 38.7^\circ$$

By the principle of transmissibility, $\mathbf{F}_R$ can be placed at a distance *x* where it intersects *BC*, Fig. 4–45*b*. In this case we have

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad 325\text{ lb}\,(11\text{ ft}) - 260\text{ lb}\,(x)$$
$$= 175\text{ lb}\,(5\text{ ft}) - 60\text{ lb}\,(3\text{ ft}) + 250\text{ lb}\left(\tfrac{3}{5}\right)(11\text{ ft}) - 250\text{ lb}\left(\tfrac{4}{5}\right)(8\text{ ft})$$
$$x = 10.9\text{ ft} \qquad \textit{Ans.}$$

The slab is subjected to four parallel forces. Determine the magnitude and direction of a resultant force equivalent to the given force system, and locate its point of application on the slab.



$+\uparrow F_R = \Sigma F;$ $\quad F_R = -600\text{ N} + 100\text{ N} - 400\text{ N} - 500\text{ N}$

$= -1400\text{ N} = 1400\text{ N}\downarrow$

$(M_R)_x = \Sigma M_x;$

$-(1400\text{ N})y = 600\text{ N}(0) + 100\text{ N}(5\text{ m}) - 400\text{ N}(10\text{ m}) + 500\text{ N}(0)$

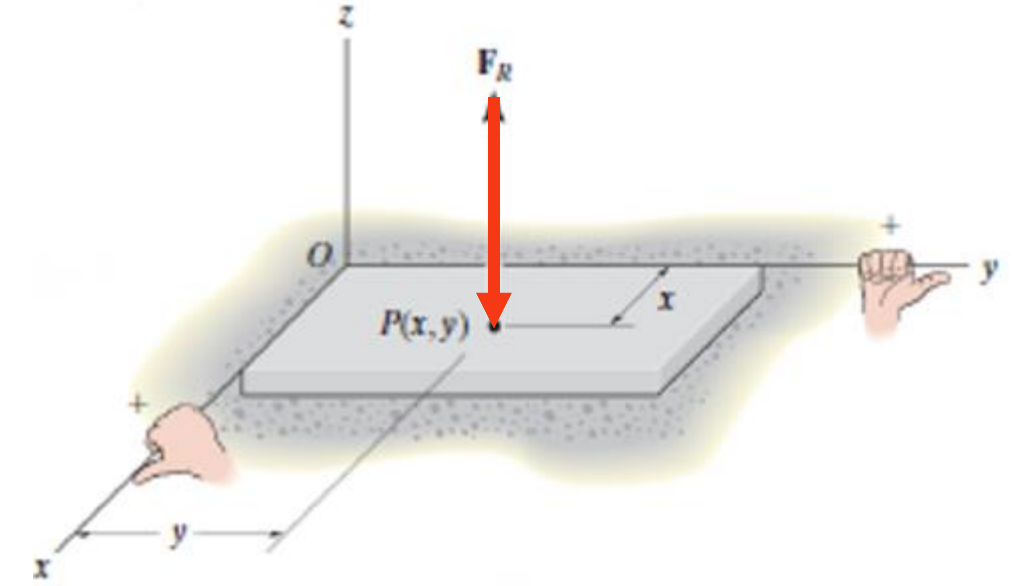
$-1400y = -3500 \qquad y = 2.50\text{ m}$ *Ans.*

$(M_R)_y = \Sigma M_y;$

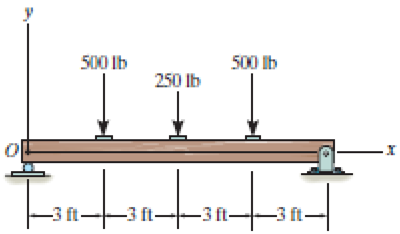
$(1400\text{ N})x = 600\text{ N}(8\text{ m}) - 100\text{ N}(6\text{ m}) + 400\text{ N}(0) + 500\text{ N}(0)$

$1400x = 4200$

$x = 3\text{ m}$ *Ans.*

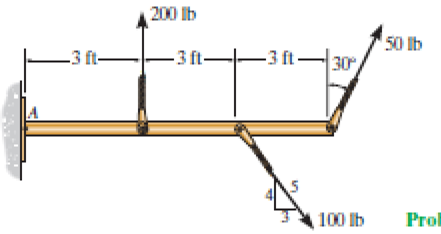


F4–31. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the beam measured from O.



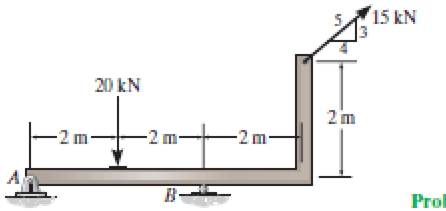
Prob. F4–31

F4–32. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the member measured from A.



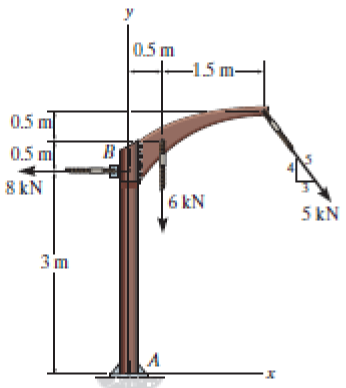
Prob. F4–32

F4–33. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the horizontal segment of the member measured from A.



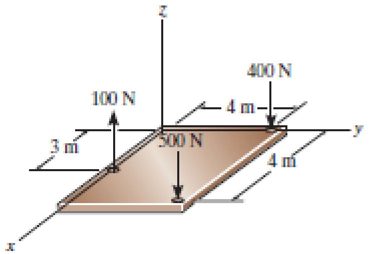
Prob. F4–33

F4–34. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the member AB measured from A.



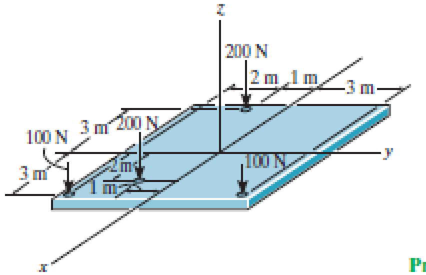
Prob. F4–34

F4–35. Replace the loading shown by an equivalent single resultant force and specify the x and y coordinates of its line of action.



Prob. F4–35

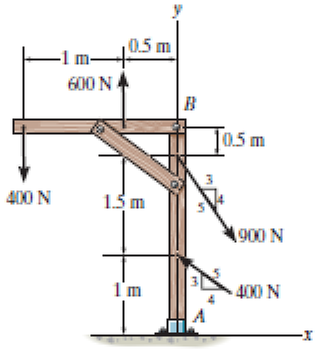
F4–36. Replace the loading shown by an equivalent single resultant force and specify the x and y coordinates of its line of action.



Prob. F4–36

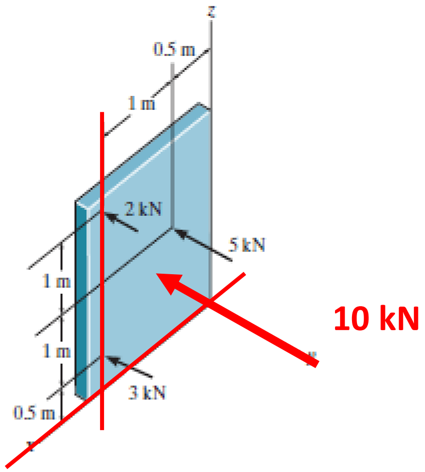
*4–120. Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a vertical line along member *AB*, measured from *A*.

4–121. Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a horizontal line along member *CB*, measured from end *C*.



Probs. 4–120/121

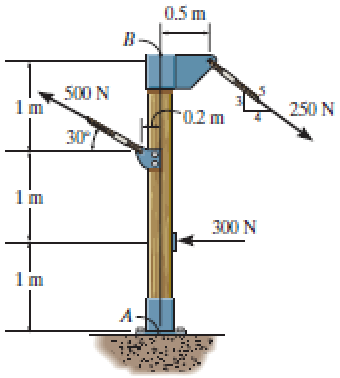
*4–124. Replace the parallel force system acting on the plate by a resultant force and specify its location on the *x*-*z* plane.



Prob. 4–124

4–122. Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post *AB* measured from point *A*.

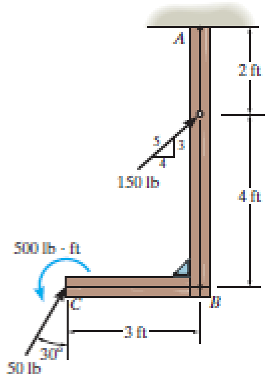
4–123. Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post *AB* measured from point *B*.



Probs. 4–122/123

4–125. Replace the force and couple system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member *AB*, measured from *A*.

4–126. Replace the force and couple system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member *BC*, measured from *B*.



Probs. 4–125/126

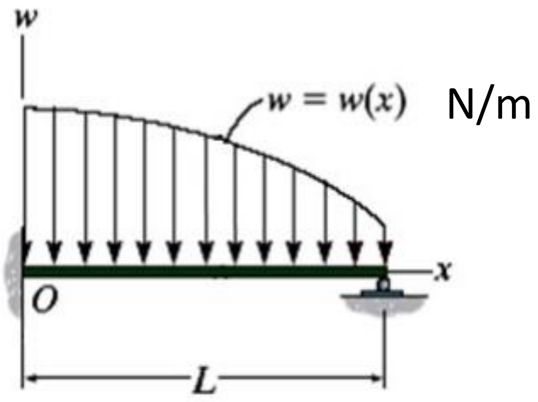
$\Sigma M_{z'}$ .... $5 \times 1 = 10x'$ ....$x' = 0.5$ m

$X = 1.5 - 0.5 = \underline{1\text{ m}}$

ΣM_x ...... $5 \times 1.5 + 2 \times 2.5 + 3 \times 0.5 = 10z$

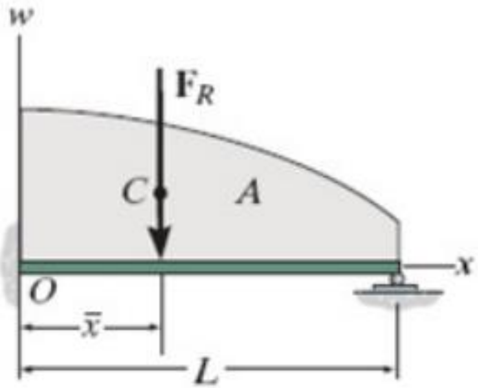
$Z = 14/10 = \underline{1.4\text{ m}}$

4.9 Reduction of a Simple Distributed Loading

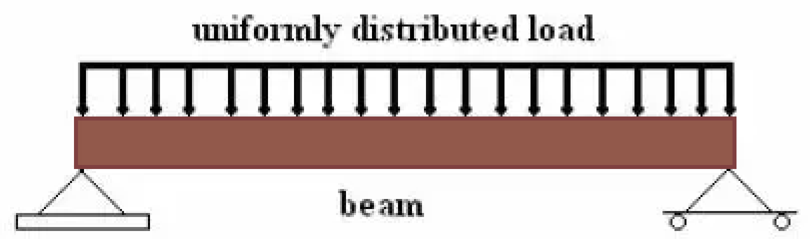


A body may be subjected to a loading that is distributed over its surface,

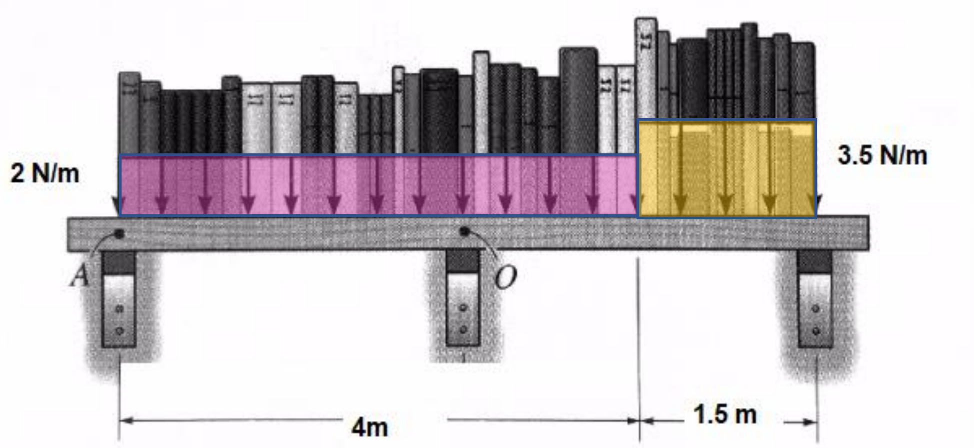
- wind pressure on a sign board.
- water pressure on the tank surface
- weight of sand on the floor of a storage container



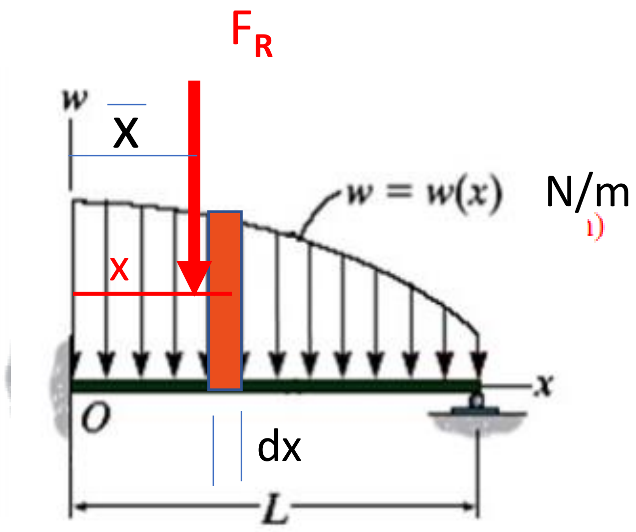
The most common type of distributed loading is uniform loading along a single axis (kN/m)



example of this is the **weight of the beam itself**



The coplanar distributed load can be further reduced to a single equivalent resultant force F_R.



$$dA = w(x)\,dx = dF$$

$$A = F_R = \int dA = \int w(x)\,dx$$

The magnitude of the resultant force is equal to the total area A under the loading diagram.

$$F_R = \int_L w(x)\,dx = \int_A dA = A$$

- The line of action of F_R passes through the centroid (geometric center) of this area.

$$\bar{x} = \frac{\int_L x w(x)\,dx}{\int_L w(x)\,dx} = \frac{\int_A x\,dA}{\int_A dA}$$

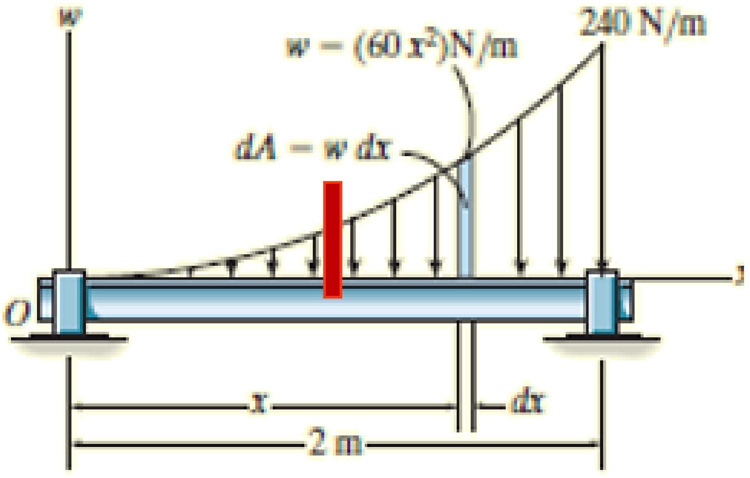
$$\circlearrowleft + \quad (M_R)_O = \Sigma M_O:$$

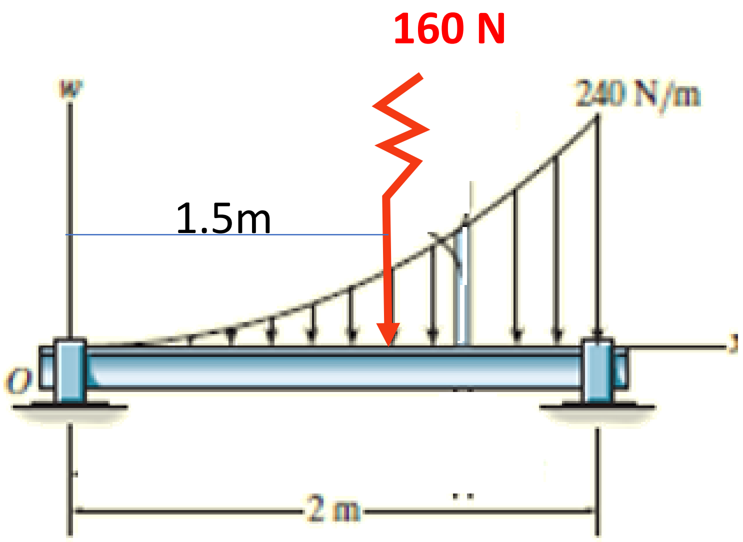
$$-\bar{x}\,F_R = -\int_L x\,dF$$

$$\bar{x}\int_L w(x)\,dx = \int_L x\,w(x)\,dx$$

$$\bar{x} = \frac{\int_L x\,w(x)\,dx}{\int_L w(x)\,dx} = \frac{\int_L x\,dA}{\int_L dA}$$

Determine the magnitude and location of the equivalent resultant force on the shaft





$dF = dA = w(x)\,dx = 60x^2\,dx$

$$F_R = \int_A dA = \int_0^{2\text{ m}} 60x^2\,dx = 60\left(\frac{x^3}{3}\right)\Big|_0^{2\text{ m}} = 60\left(\frac{2^3}{3} - \frac{0^3}{3}\right)$$

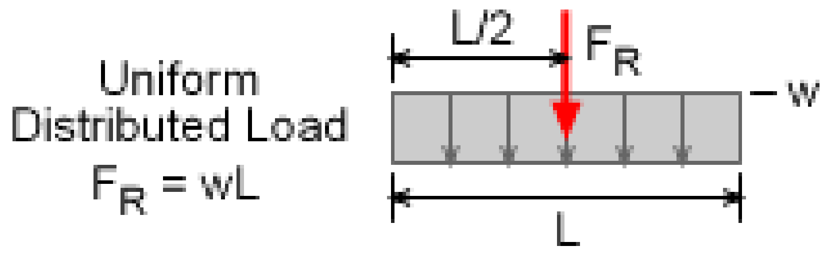
$$= 160\text{ N}$$

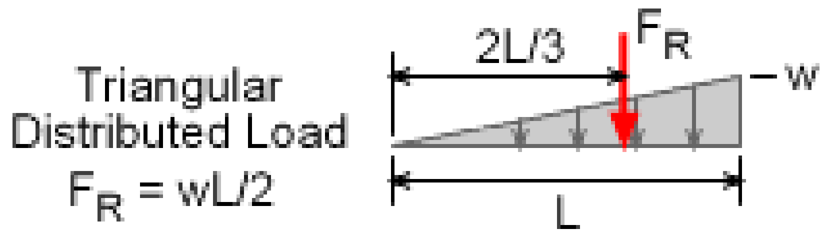
$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} = \frac{\int_0^{2\text{ m}} x(60x^2)\,dx}{160\text{ N}} = \frac{60\left(\frac{x^4}{4}\right)\Big|_0^{2\text{ m}}}{160\text{ N}} = \frac{60\left(\frac{2^4}{4} - \frac{0^4}{4}\right)}{160\text{ N}}$$

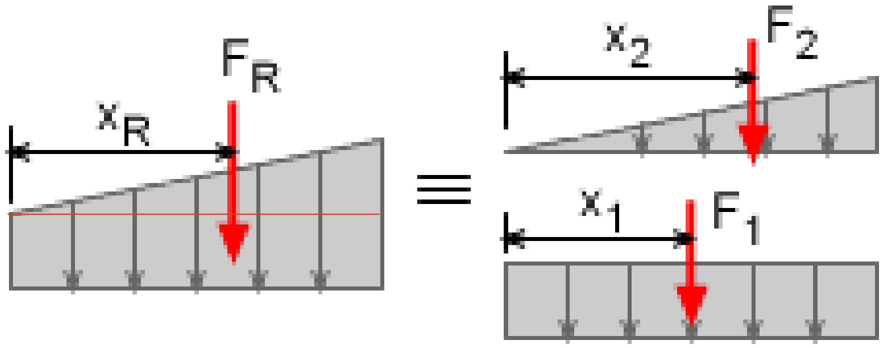
$$= 1.5\text{ m}$$

Composite Loading

a composite load can be generated with a combination of simpler loads such as a uniform and a triangular line load.



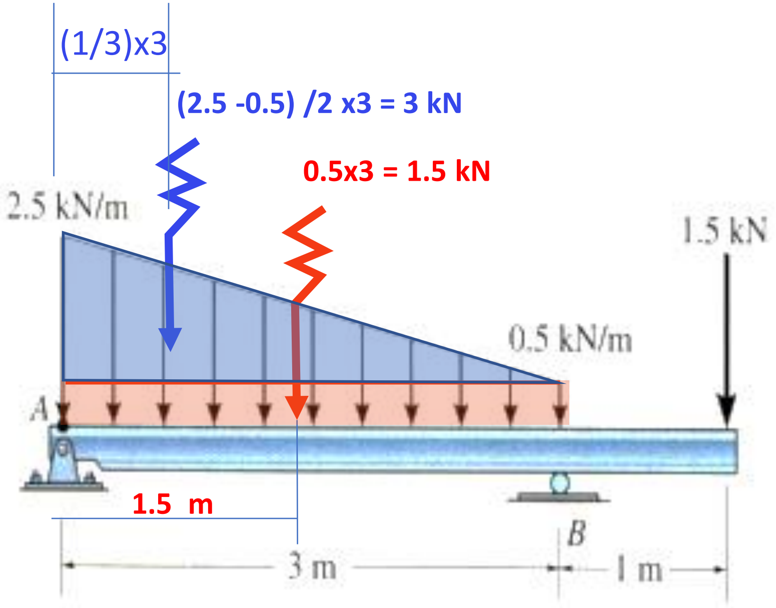




$F_R = F_1 + F_2$

$x_R\, F_R = x_1\, F_1 + x_2\, F$

Find The equivalent force and its location from point A.

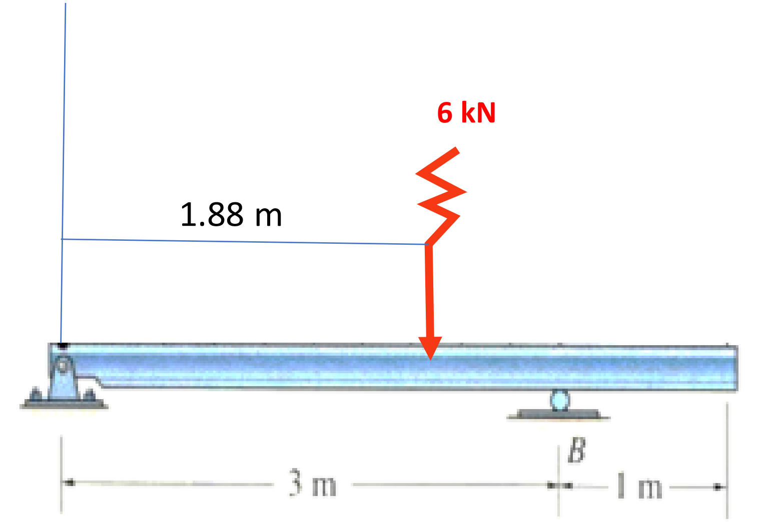


F_R = 1.5 + 3 + 1.5 = 6 kN

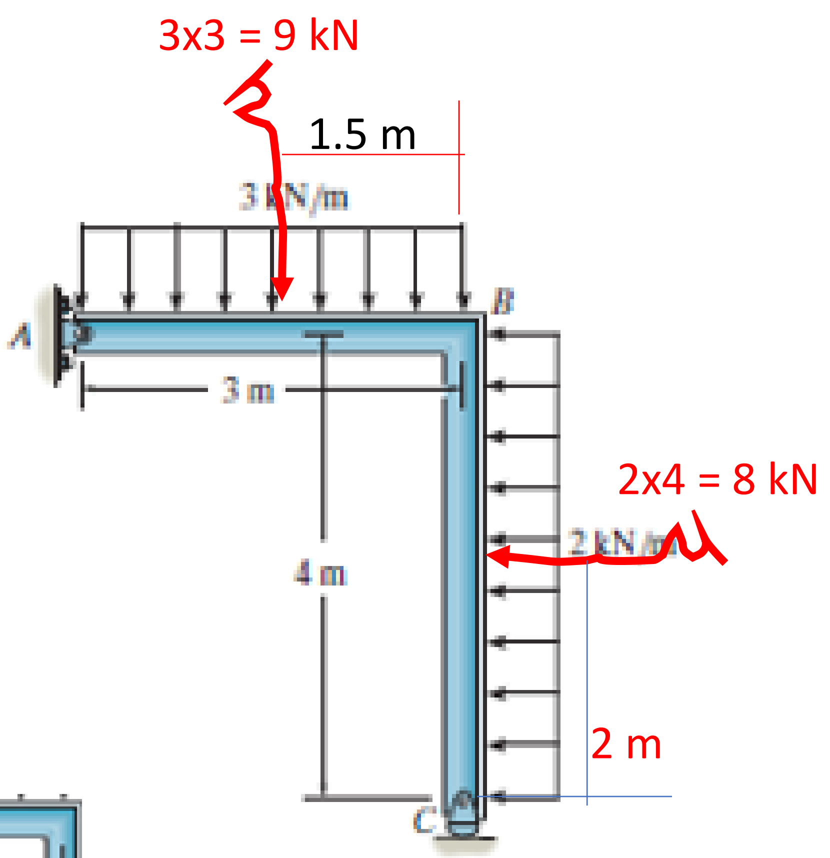
MR_A = (1.5) (1.5) + 3 (1) + (1.5) 4 = 11.25 kN • m

X F_R = 11.25 kN • m

X = (11.25) / (6) = 1.88 m from A.



4–155. Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects a vertical line along member BC, measured from C.



Fx = -8kN

Fy = -9

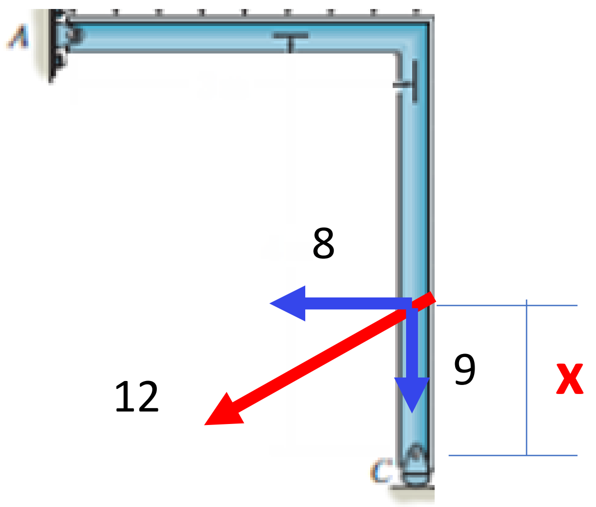
FR = $\sqrt{8^2+9^2} = 12\ kN$

$$\tan\theta = \frac{9}{8} \qquad \theta = 48°$$

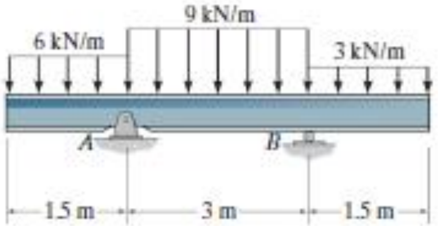
$$\sum M_c = 8\times 2 + 9\times 1.5 = 29.5\ kN.m$$

MFR =8 x = 29.55

X= 3.7 m

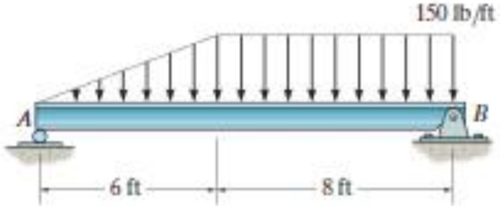


F4–37. Determine the resultant force and specify where it acts on the beam measured from A.



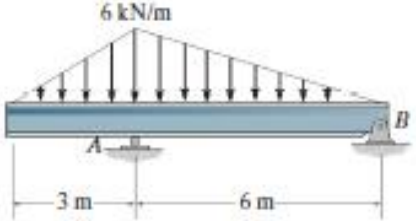
Prob. F4–37

F4–38. Determine the resultant force and specify where it acts on the beam measured from A.

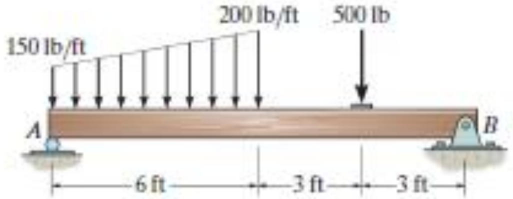


Prob. F4–38

F4–39. Determine the resultant force and specify where it acts on the beam measured from A.

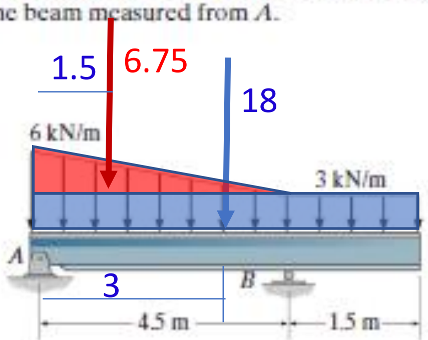


F4–40. Determine the resultant force and specify where it acts on the beam measured from A.



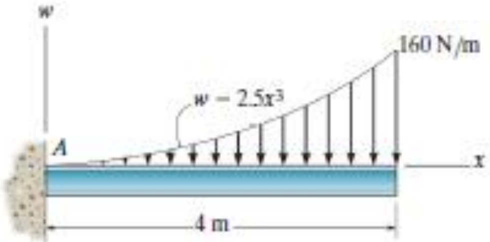
Prob. F4–40

F4–41. Determine the resultant force and specify where it acts on the beam measured from A.



Prob. F4–41

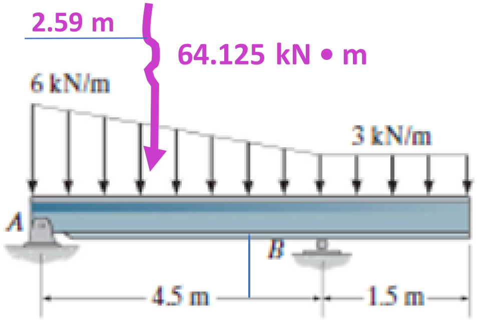
F4–42. Determine the resultant force and specify where it acts on the beam measured from A.



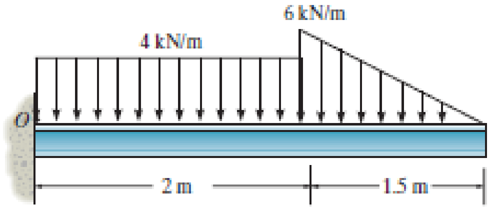
F_R = 6.75 + 18 = 24.75 kN

MR_A = (1.5) (6.75) + (18)(3) = 64.125 kN • m

X' = 64.125/24.75 = 2.59 m

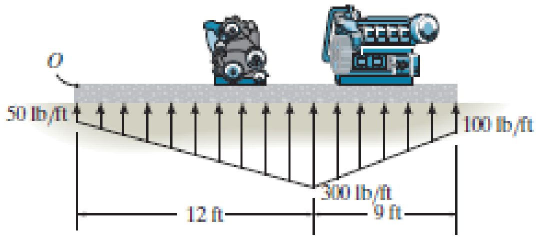


4–143. Replace this loading by an equivalent resultant force and specify its location, measured from point O.



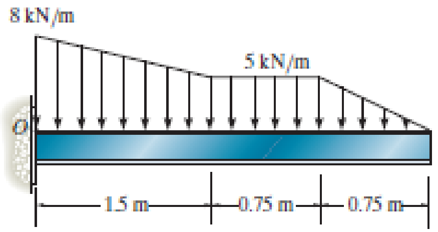
Prob. 4–143

***4–144.** The distribution of soil loading on the bottom of a building slab is shown. Replace this loading by an equivalent resultant force and specify its location, measured from point O.



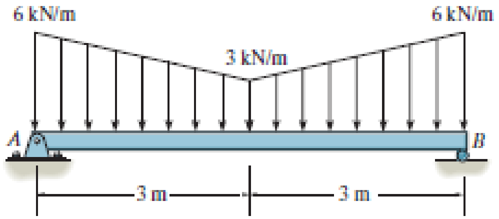
Prob. 4–144

4–145. Replace the loading by an equivalent resultant force and couple moment acting at point O.



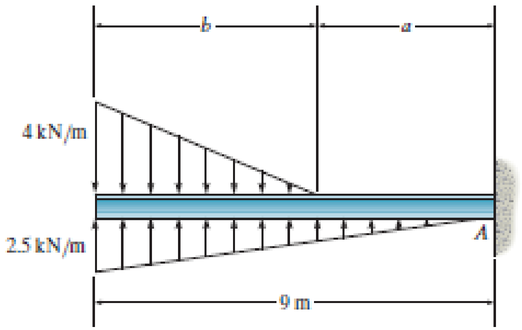
Prob. 4–145

4–146. Replace the distributed loading by an equivalent resultant force and couple moment acting at point A.



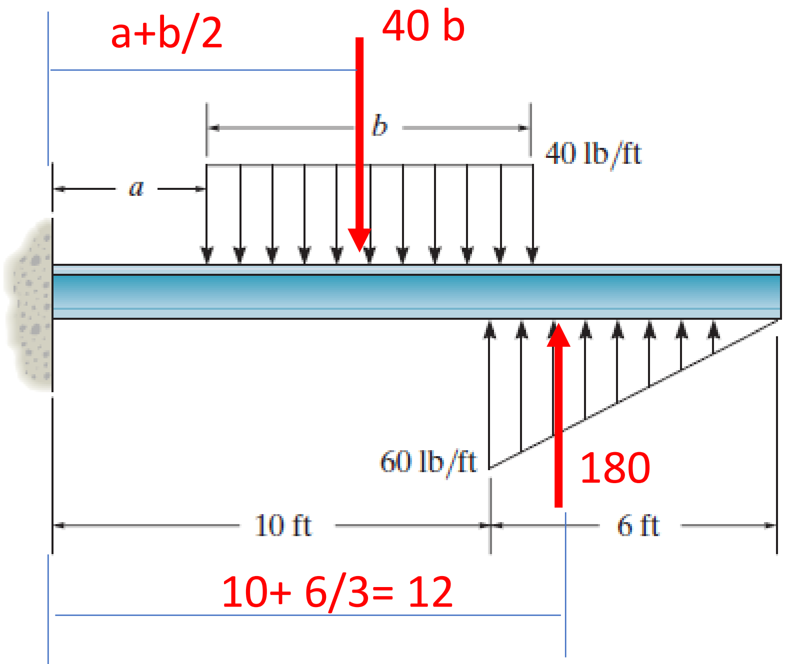
Prob. 4–146

4–147. Determine the length b of the triangular load and its position a on the beam such that the equivalent resultant force is zero and the resultant couple moment is 8 kN · m clockwise.



Prob. 4–147

The beam is subjected to the distributed loading. Determine the length *b* of the uniform load and its position *a* on the beam such that the resultant force and couple moment acting on the beam are zero.



Require $F_R = 0$.

$+\uparrow F_R = \Sigma F_y; \quad 0 = 180 - 40b$

$$b = 4.50 \text{ ft}$$

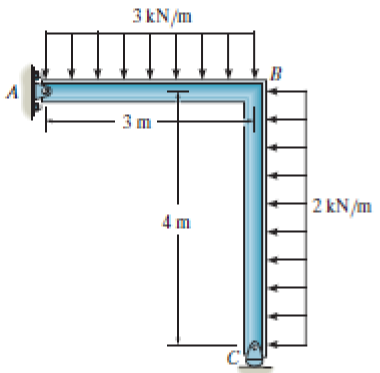
Require $M_{R_A} = 0$. Using the result $b = 4.50$ ft, we have

$\circlearrowleft + M_{R_A} = \Sigma M_A; \quad 0 = 180(12) - 40(4.50)\left(a + \frac{4.50}{2}\right)$

$$a = 9.75 \text{ ft}$$

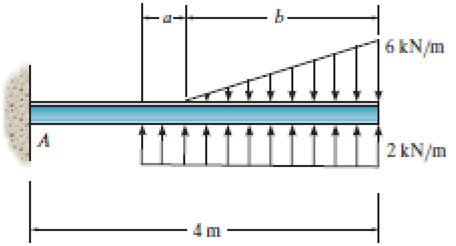
4–154. Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects a horizontal line along member *AB*, measured from *A*.

4–155. Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects a vertical line along member *BC*, measured from *C*.



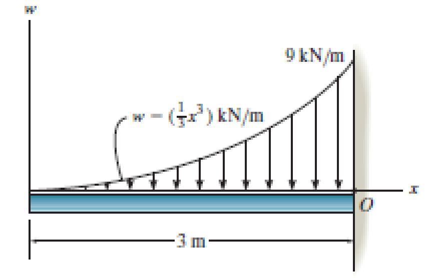
Probs. 4–154/155

***4–156.** Determine the length *b* of the triangular load and its position *a* on the beam such that the equivalent resultant force is zero and the resultant couple moment is 8 kN · m clockwise.



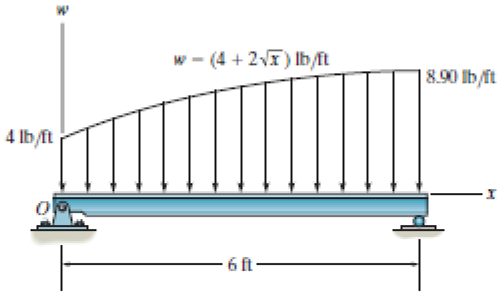
Prob. 4–156

4–157. Determine the equivalent resultant force and couple moment at point *O*.



Prob. 4–157

4–158. Determine the magnitude of the equivalent resultant force and its location, measured from point *O*.



Prob. 4–158

Chapter 5

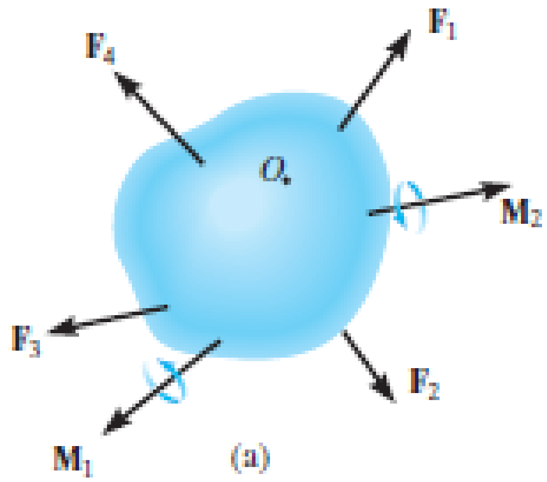
Equilibrium of a Rigid Body

I. Marie

5.1 Conditions for Rigid-Body Equilibrium

EQUILIBRIUM IN TWO DIMENSIONS

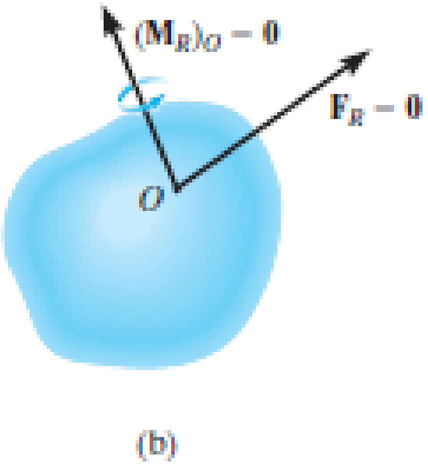
If this resultant force and couple moment are both equal to zero, then the body is said to be in ***equilibrium***.



$$\mathbf{F}_R = \Sigma \mathbf{F} = 0$$

$$(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O = 0$$

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$



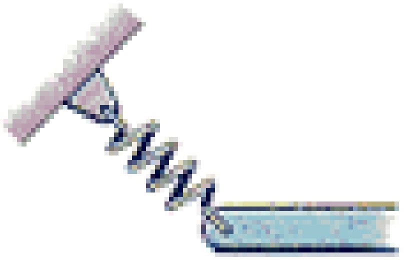
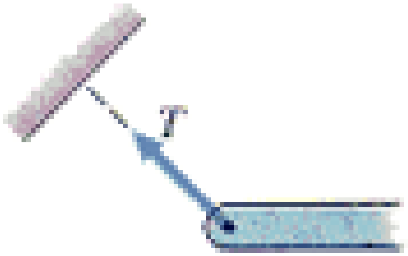
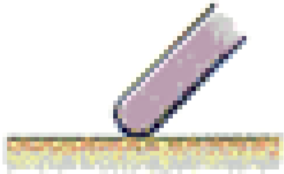
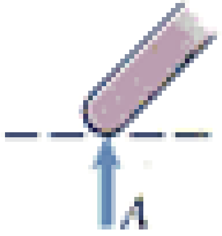
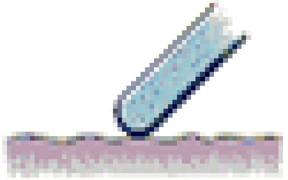
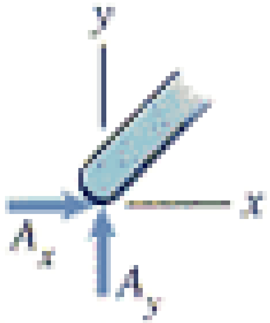
5.2 Free-Body Diagrams

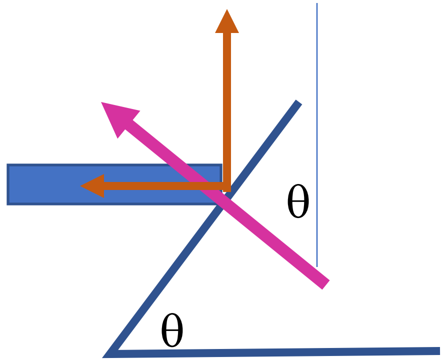
You should know **types of reactions** that can occur at the **supports**. (**Table 5.1)**

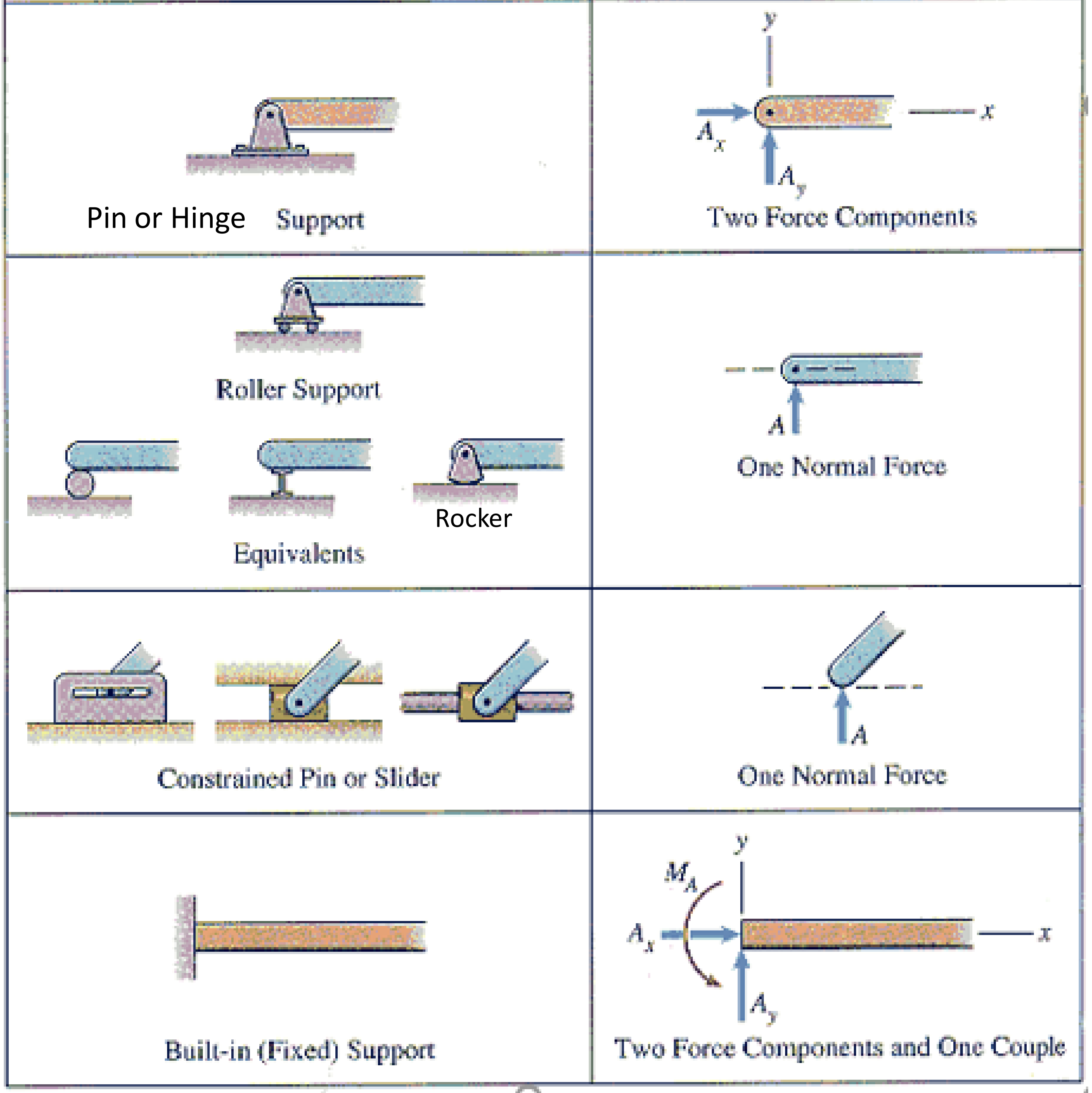
A reaction force developed in a body in the direction where support prevents translation . Similarly,
A support prevents the rotation of a body in a given direction by exerting a couple moment on the body in the opposite direction

No equilibrium problem should be solved without *first drawing the free-body diagram*

.

Supports	Reactions
 Rope or Cable  Spring	 One Colinear Force
 Contact with a Smooth Surface	 One Normal Force
 Contact with a Rough Surface	 Two Force Components




Pin or Hinge
Support
y
x
A_x
A_y
Two Force Components
Roller Support
Rocker
Equivalents
A
One Normal Force
Constrained Pin or Slider
A
One Normal Force
Built-in (Fixed) Support
y
M_A
A_x
x
A_y
Two Force Components and One Couple


Pin or Hinge

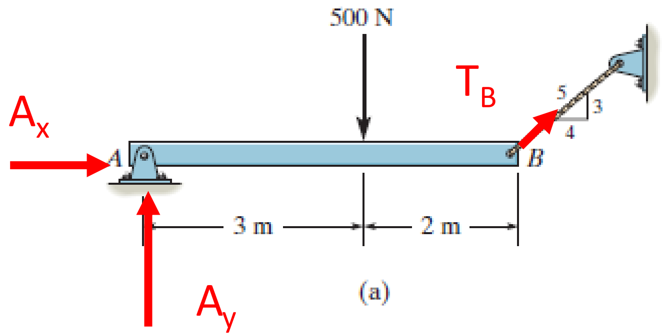

Rocker

5.3 Equations of Equilibrium

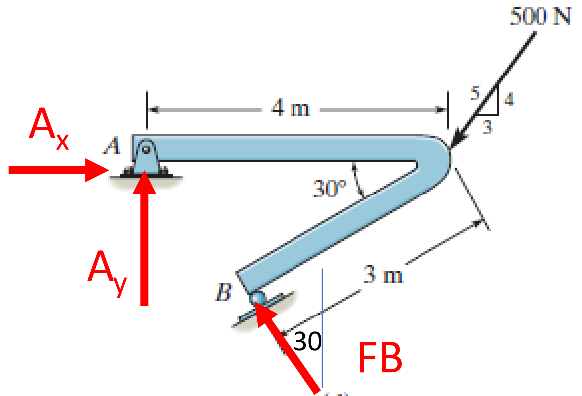
$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$

$$\Sigma F_x = 0$$
$$\Sigma M_A = 0$$
$$\Sigma M_B = 0$$

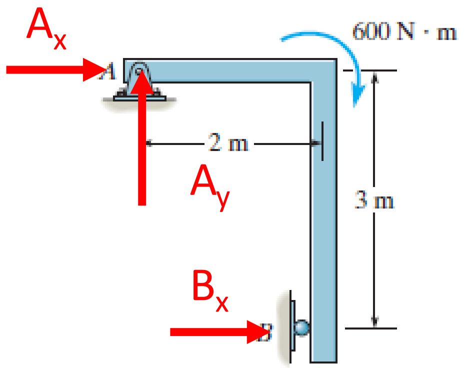
Draw FBD



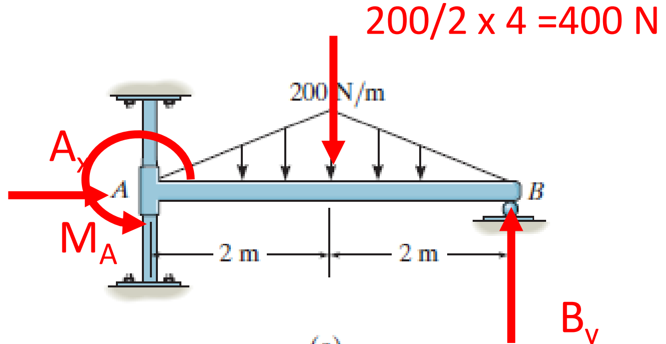
(a)



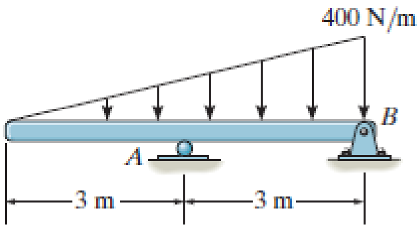
(d)



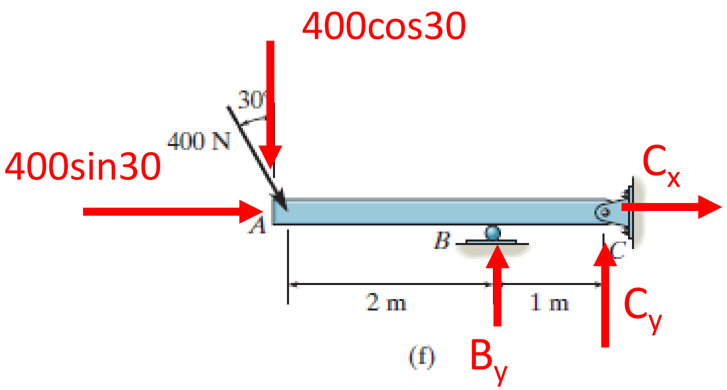
(b)



(e)



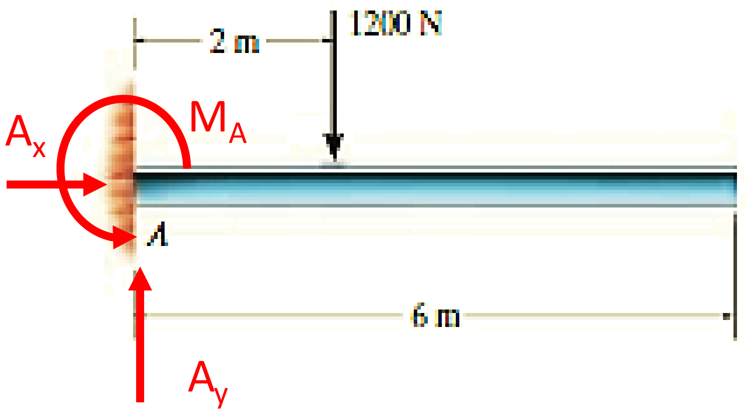
(c)



(f)

Draw FBD

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$



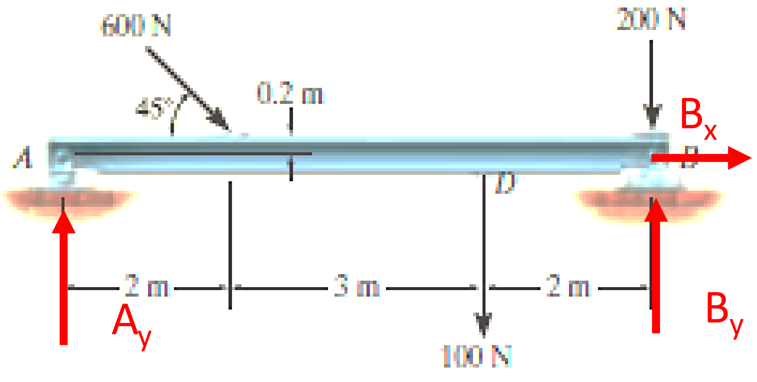
$\Sigma M_A = 0$ $\quad$ $- 1200x2 + M_A = 0$

$M_A = 2400$ N.m

$\Sigma Fx = 0$ $\quad$ Ax 0

$\Sigma Fy = 0$ $\quad$ Ay -1200 = 0

Ay = 1200 N



$\xrightarrow{+} \Sigma F_x = 0;$ $\quad$ $600 \cos 45° \text{ N} - B_x = 0$

$$B_x = 424 \text{ N}$$

A direct solution for $\mathbf{A}_y$ can be obtained by applying the moment equation $\Sigma M_B = 0$ about point B.

$\zeta + \Sigma M_B = 0;$ $\quad$ $100 \text{ N}(2 \text{ m}) + (600 \sin 45° \text{ N})(5 \text{ m})$

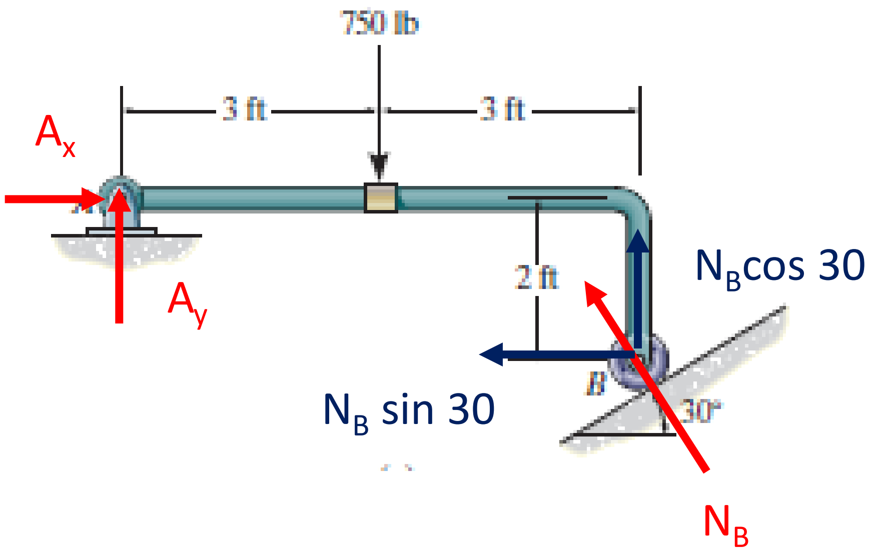
$$- (600 \cos 45° \text{ N})(0.2 \text{ m}) - A_y(7 \text{ m}) = 0$$

$$A_y = 319 \text{ N}$$

Summing forces in the y direction, using this result, gives

$+\uparrow \Sigma F_y = 0;$ $\quad$ $319 \text{ N} - 600 \sin 45° \text{ N} - 100 \text{ N} - 200 \text{ N} + B_y = 0$

$$B_y = 405 \text{ N}$$



$\zeta + \Sigma M_A = 0;$

$$[N_B \cos 30°](6 \text{ ft}) - [N_B \sin 30°](2 \text{ ft}) - 750 \text{ lb}(3 \text{ ft}) = 0$$

$$N_B = 536.2 \text{ lb} = 536 \text{ lb}$$

Using this result,

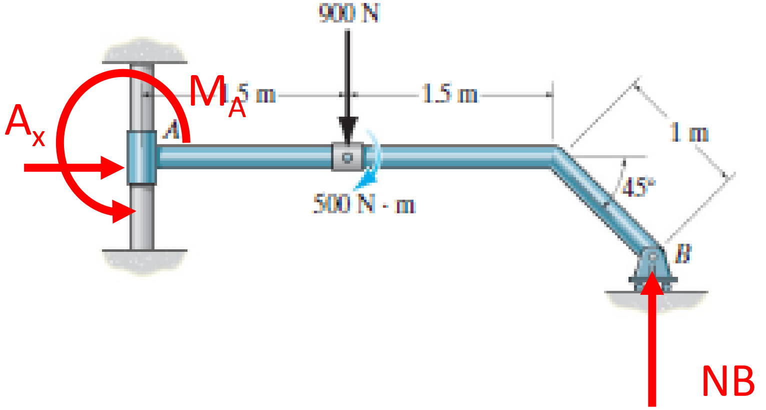
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad$ $A_x - (536.2 \text{ lb}) \sin 30° = 0$

$$A_x = 268 \text{ lb}$$

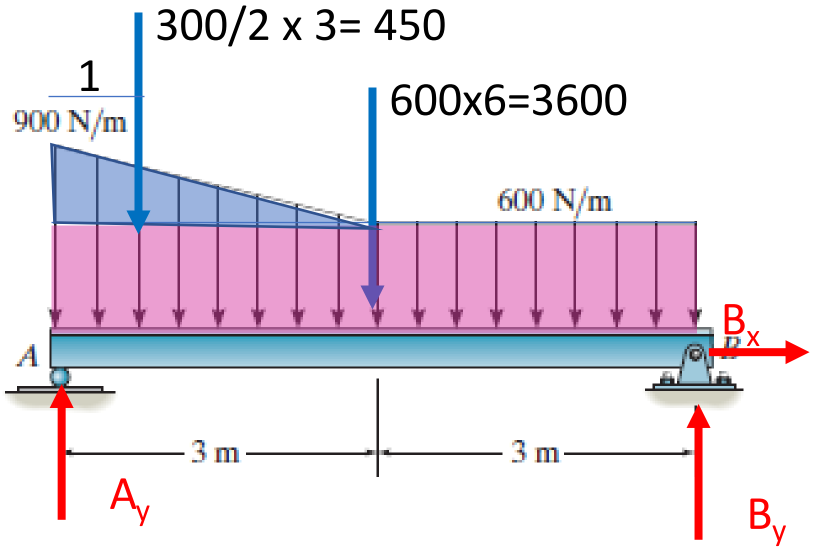
$+\uparrow \Sigma F_y = 0;$ $\quad$ $A_y + (536.2 \text{ lb}) \cos 30° - 750 \text{ lb} = 0$

$$A_y = 286 \text{ lb}$$

Determine the support reactions



$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$



$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad N_B - 900\text{ N} = 0$

$N_B = 900\text{ N}$

The moment M_A can be determined by summing moments either about point A or point B.

$\circlearrowleft + \Sigma M_A = 0;$

$M_A - 900\text{ N}(1.5\text{ m}) - 500\text{ N}\cdot\text{m} + 900\text{ N}[3\text{ m} + (1\text{ m})\cos 45°] = 0$

$M_A = -1486\text{ N}\cdot\text{m} = 1.49\text{ kN}\cdot\text{m} \circlearrowright$

or

$\circlearrowleft + \Sigma M_B = 0;$ $\quad M_A + 900\text{ N}[1.5\text{ m} + (1\text{ m})\cos 45°] - 500\text{ N}\cdot\text{m} = 0$

$M_A = -1486\text{ N}\cdot\text{m} = 1.49\text{ kN}\cdot\text{m} \circlearrowright$

$\Sigma M_A = 0$ $\quad$ - 450x1 -3600x3+6By= 0

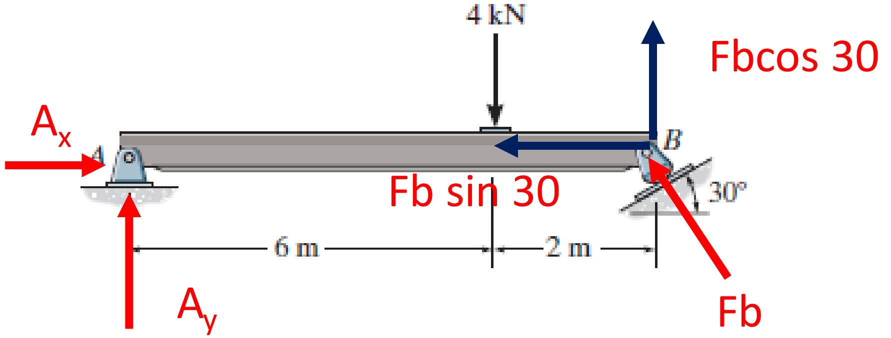
By = 1875 N

$\Sigma Fx = 0$ $\quad$ Bx =0

$\Sigma Fy = 0$ $\quad$ Ay -450 -3600 + 1875 = 0

Ay = 2175 N

*5–12. Determine the horizontal and vertical components of reaction at the pin A and the reaction of the rocker B on the beam.



ΣM_A = -4x6 + FB cos 30 x 8 = 0

FB = 3.46 N

ΣFx = Ax – FB sin 30 = 0

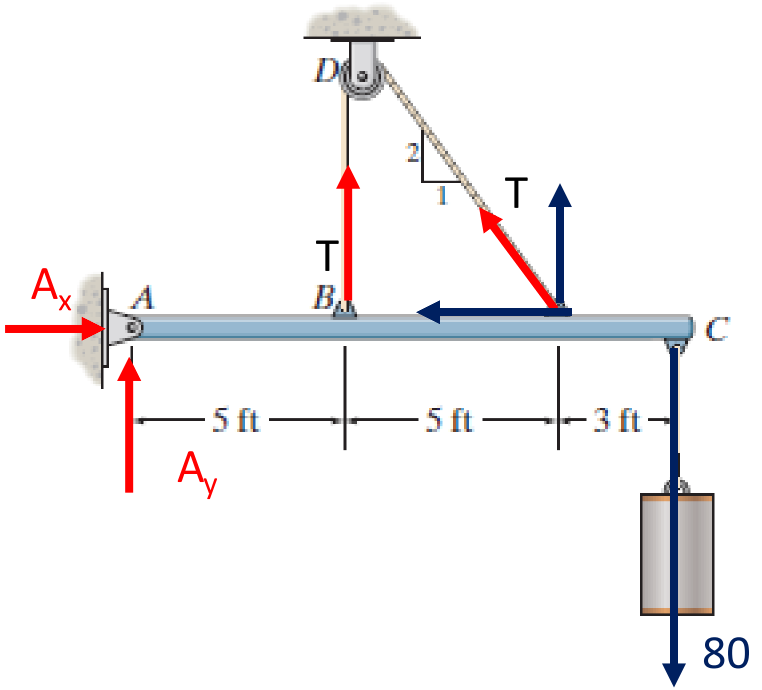
Ax = 1.73 N

ΣFy = Ay- 4 + FBcos30 = 0

Ay = 1 N

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$

*5–16. Determine the tension in the cable and the horizontal and vertical components of reaction of the pin A. The pulley at D is frictionless and the cylinder weighs 80 lb.



ΣM_A = 5T + (2T/2.24)(10) -13x80 = 0

T = 74.66 N

ΣFx = Ax – (T/2.24)= 0

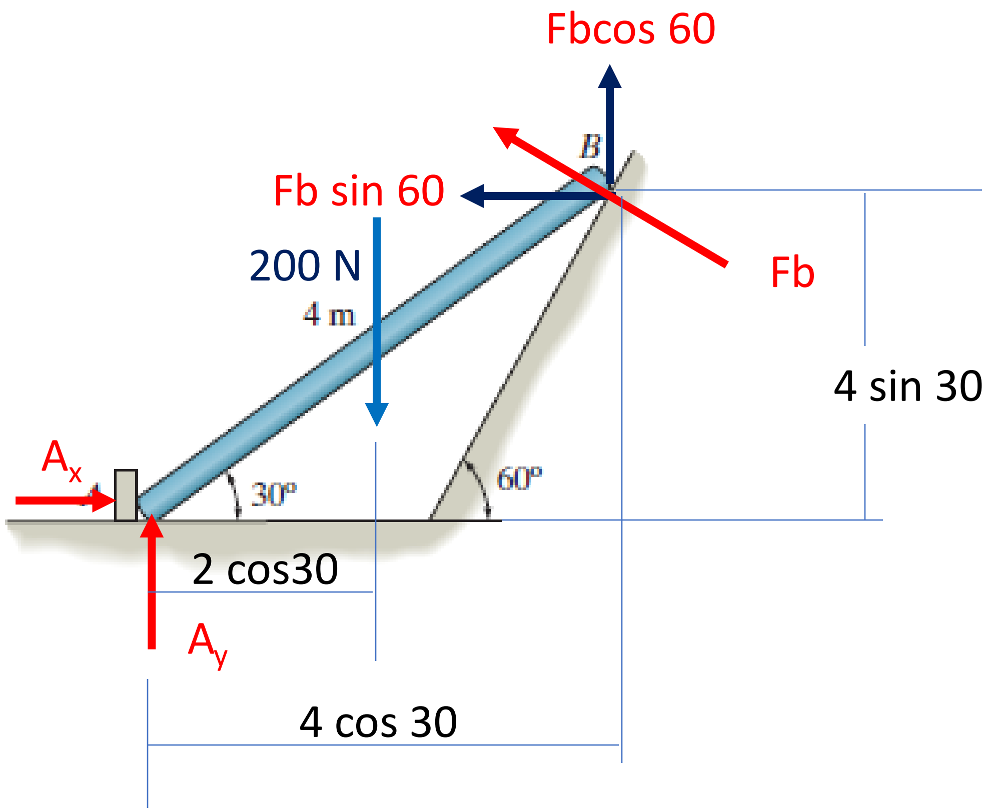
Ax = 33.33 N

ΣFy = Ay + T + (2T/2.24 -80 = 0

Ay = 61.32 N

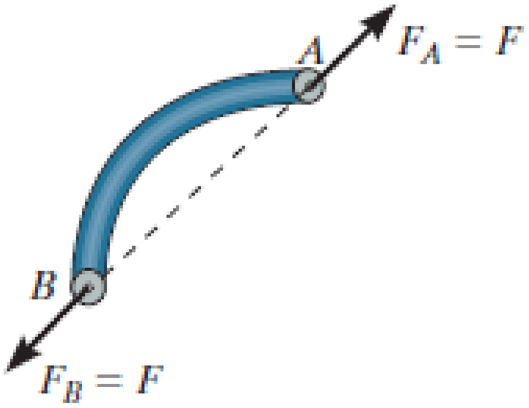
5–27. Determine the reactions acting on the smooth uniform bar, which has a mass of 20 kg.

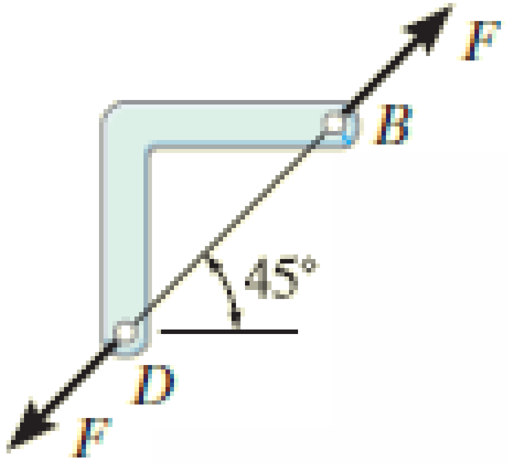
$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$

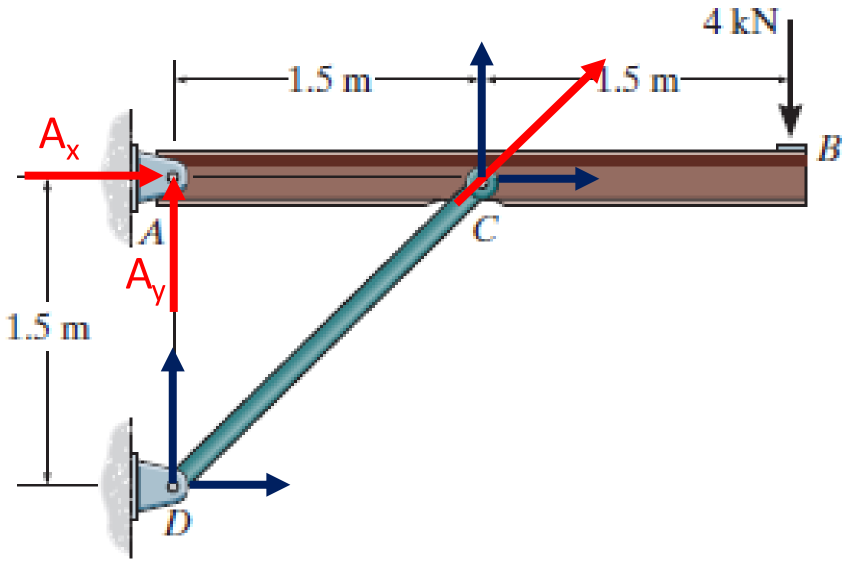


5.4 Two- Force Members

has forces applied at only two points on the member and *must have the same magnitude, act in opposite directions, and have the same line of action, directed along the line joining the two points where these forces act.*

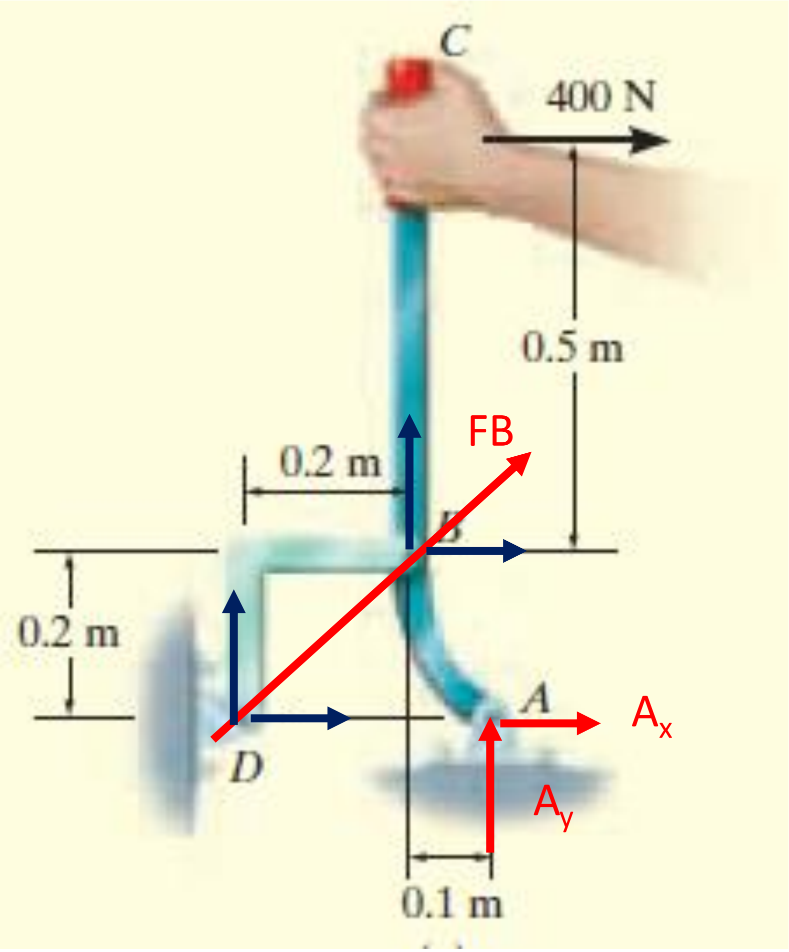






The lever *ABC* is pin supported at *A* and connected to a short link *BD*. If the weight of the members is negligible, determine the force of the pin on the lever at *A*.

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$



Σ MA = -400x0.7- FB x 0.707 x0.3 = 0

FB =- 1320 N (opposite to the assumed direction

Σ Fy = 0 = Ay +(- 1320 x0.707) = 0

Ay = 933.24 N

Σ Fx = 0 = Ax + 400 (- 1320 x0.707) = 0

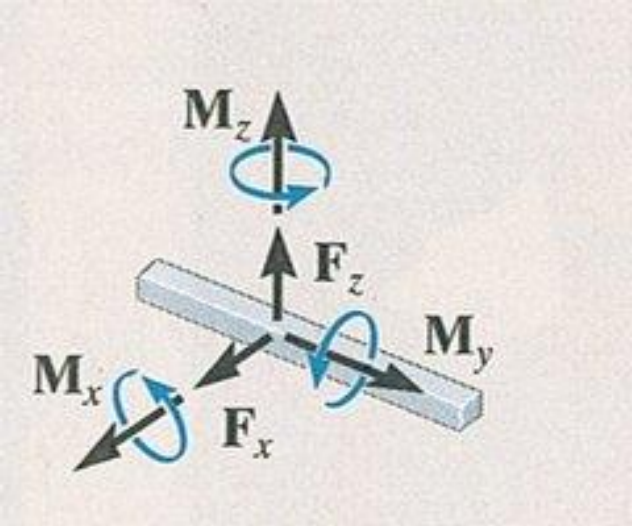
Ax = 533.24 N

$$F\,A = \sqrt{933.24^2 + 533.24^2} = 1075\ N$$

EQUILIBRIUM IN THREE DIMENSIONS

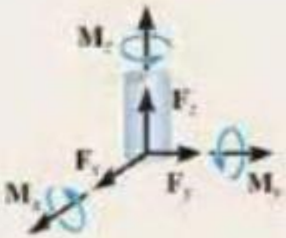
5.5 Free-Body Diagrams

The first step in solving three-dimensional equilibrium problems, as in the case of two dimensions, is to draw a free-body diagram. Before we can do this, however, it is first necessary to discuss the types of reactions that can occur at the supports.

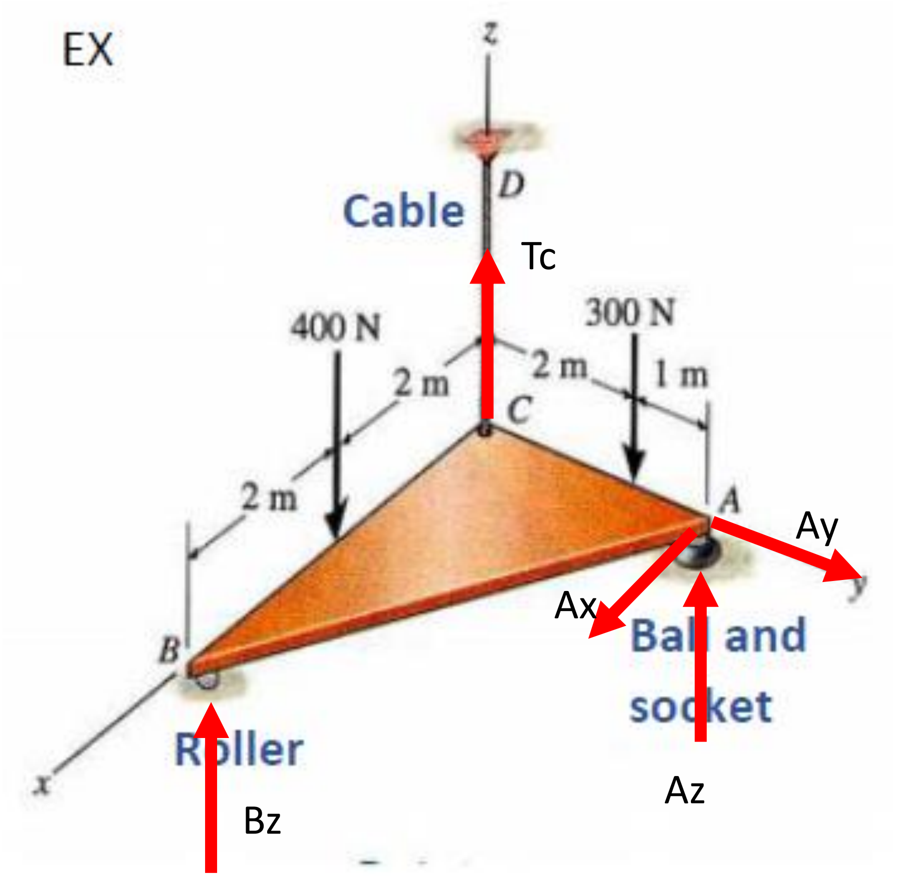


Support Reactions.

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems		
Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  ball and socket		Three unknowns. The reactions are three rectangular force components.

(8)  single smooth pin		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments are generally not applied if the body is supported elsewhere. See the examples.
(9)  single hinge		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments are generally not applied if the body is supported elsewhere. See the examples.
(10)  fixed support		Six unknowns. The reactions are three force and three couple-moment components.

Draw FBD


EX
z
D
Cable
Tc
400 N
300 N
2 m
2 m
2 m
1 m
C
A
Ay
y
Ax
B
Ball and socket
Roller
x
Az
Bz

5.6 Equations of Equilibrium

When a body is in equilibrium, the net force and the net moment equal zero:

$\Sigma F_X = 0 \quad \Sigma F_Y = 0 \quad \Sigma F_Z = 0$

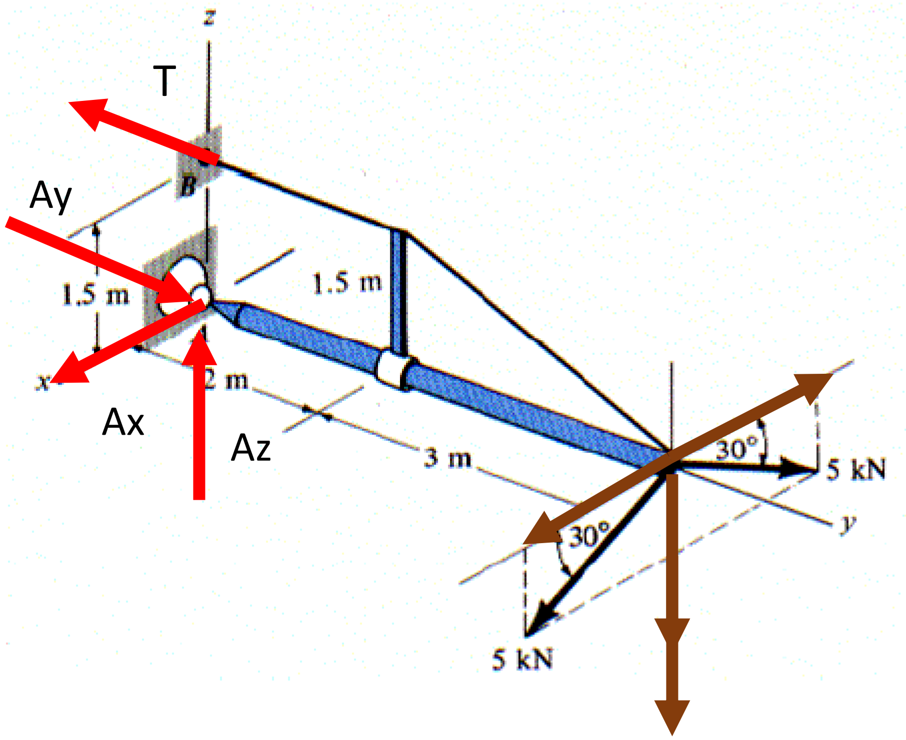
$M_X = 0 \quad \Sigma M_Y = 0 \quad \Sigma M_Z = 0$

You should write all forces and reactions as cartesian vectors (determine the x, y and z components)

The moment equations can be determined about any point.

Choose the point where the maximum number of unknown forces are present Or a line or axis of maximum unknowns

Ex. Determine the tension in the cable and support reactions exerted by the ball ansd socket at A.



$\Sigma M_x = 1.5T - 2x\ 5\sin30x5 = 0$

T = 16.67 kN

$\Sigma F_x = Ax + 5\cos30 - 5\cos 30 = 0$

Ax = 0

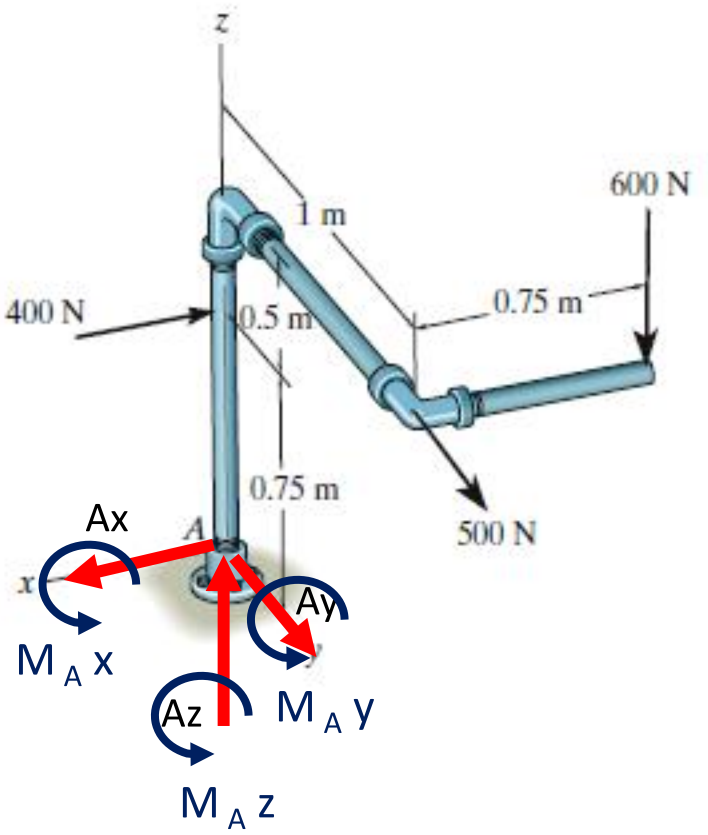
$\Sigma F_y = Ay - T = 0$ ……. $Ay - 16.67 = 0 \quad = 0$

Ay = 16.67

$\Sigma F_z = Az - 2x5\sin30 = 0$

Az = 5 kN

Determine the components of reaction at the fixed support *A*.



$\Sigma F_x = 0;\ A_x - 400 = 0$

$A_x = 400$ N

$\Sigma F_y = 0;\ 500 + A_y = 0$

$A_y = -500$ N

$\Sigma F_z = 0;\ A_z - 600 = 0$

$A_z = 600$ N.

$\Sigma M_x = 0;\ (M_A)_x - 500(1.25) - 600(1) = 0$

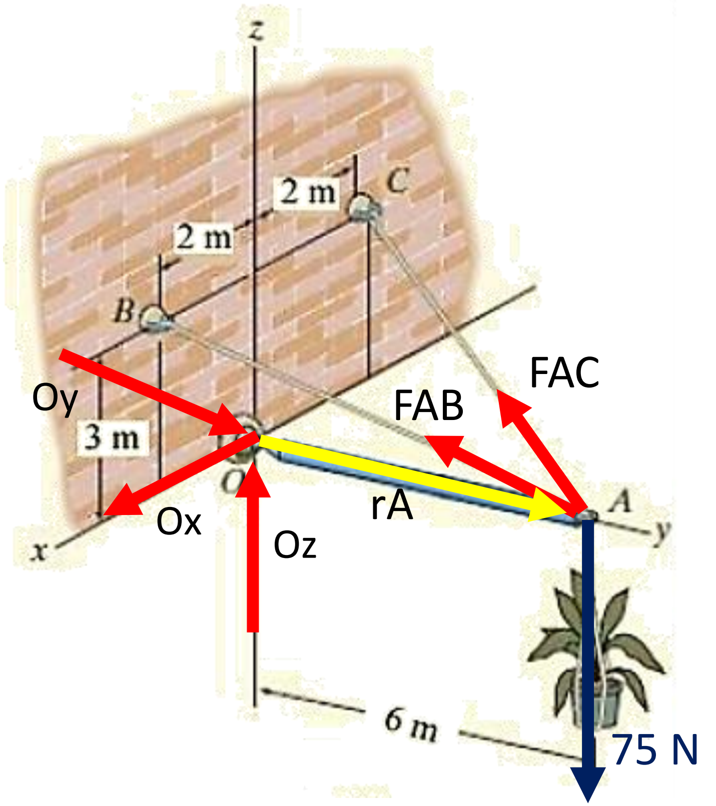
$(M_A)_x = 1225$ N .m

$\Sigma M_y = 0;\ (M_A)_y - 400(0.75) - 600(0.75) = 0$

$(M_A)_y = 750$ N .m

$\Sigma M_z = 0;\quad (M_A)_z = 0$

Determine the tension in wires AB and AC to support 75 N flower pot



$$\mathbf{F}_{AB} = F_{AB}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = F_{AB}\left(\frac{\{2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}\}\text{m}}{\sqrt{(2\text{ m})^2 + (-6\text{ m})^2 + (3\text{ m})^2}}\right)$$
$$= \tfrac{2}{7}F_{AB}\mathbf{i} - \tfrac{6}{7}F_{AB}\mathbf{j} + \tfrac{3}{7}F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = F_{AC}\left(\frac{\{-2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}\}\text{m}}{\sqrt{(-2\text{ m})^2 + (-6\text{ m})^2 + (3\text{ m})^2}}\right)$$
$$= -\tfrac{2}{7}F_{AC}\mathbf{i} - \tfrac{6}{7}F_{AC}\mathbf{j} + \tfrac{3}{7}F_{AC}\mathbf{k}$$

$\Sigma \mathbf{M}_O = \mathbf{0};$ $\quad \mathbf{r}_A \times (\mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{W}) = \mathbf{0}$

$$(6\mathbf{j}) \times \left[\left(\tfrac{2}{7}F_{AB}\mathbf{i} - \tfrac{6}{7}F_{AB}\mathbf{j} + \tfrac{3}{7}F_{AB}\mathbf{k}\right) + \left(-\tfrac{2}{7}F_{AC}\mathbf{i} - \tfrac{6}{7}F_{AC}\mathbf{j} + \tfrac{3}{7}F_{AC}\mathbf{k}\right) + (-75\mathbf{k})\right] = \mathbf{0}$$

$$\left(\tfrac{18}{7}F_{AB} + \tfrac{18}{7}F_{AC} - 450\right)\mathbf{i} + \left(-\tfrac{12}{7}F_{AB} + \tfrac{12}{7}F_{AC}\right)\mathbf{k} = \mathbf{0}$$

$\Sigma M_x = 0;$ $\quad \tfrac{18}{7}F_{AB} + \tfrac{18}{7}F_{AC} - 450 = 0$

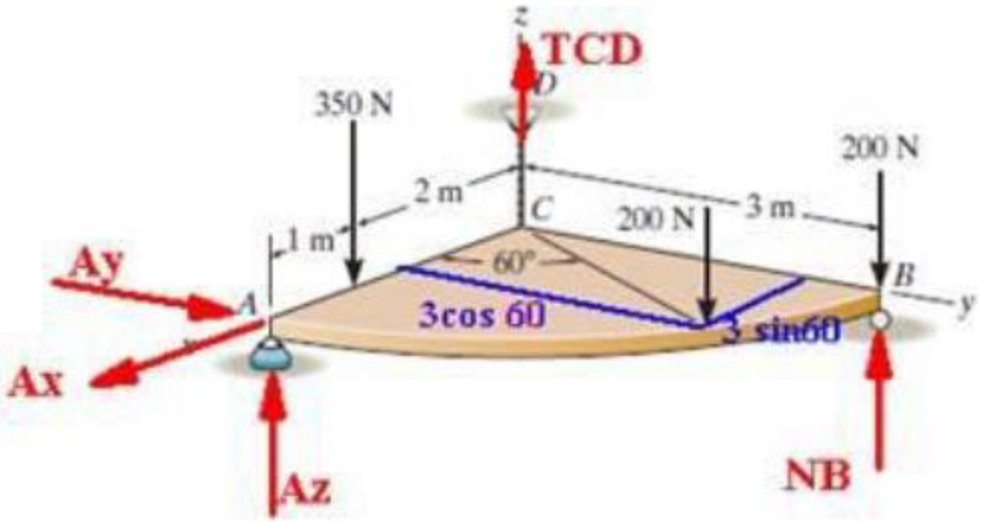
$\Sigma M_y = 0;$ $\quad 0 = 0$

$\Sigma M_z = 0;$ $\quad -\tfrac{12}{7}F_{AB} + \tfrac{12}{7}F_{AC} = 0$

Solving Eqs. (1) and (2) simultaneously,

$$F_{AB} = F_{AC} = 87.5\text{ N}$$

Determine the force components acting on the ball-and-socket at A, the reaction at the roller B and the tension on the cord CD needed for equilibrium of the quarter circular plate.



$\Sigma M_x = 0;\quad N_B(3) - 200(3) - 200(3\sin 60°) = 0$

$N_B = 373.21\text{ N} = 373\text{ N}$

$\Sigma M_y = 0;\quad 350(2) + 200(3\cos 60°) - A_z(3) = 0$

$A_z = 333.33\text{ N} = 333\text{ N}$

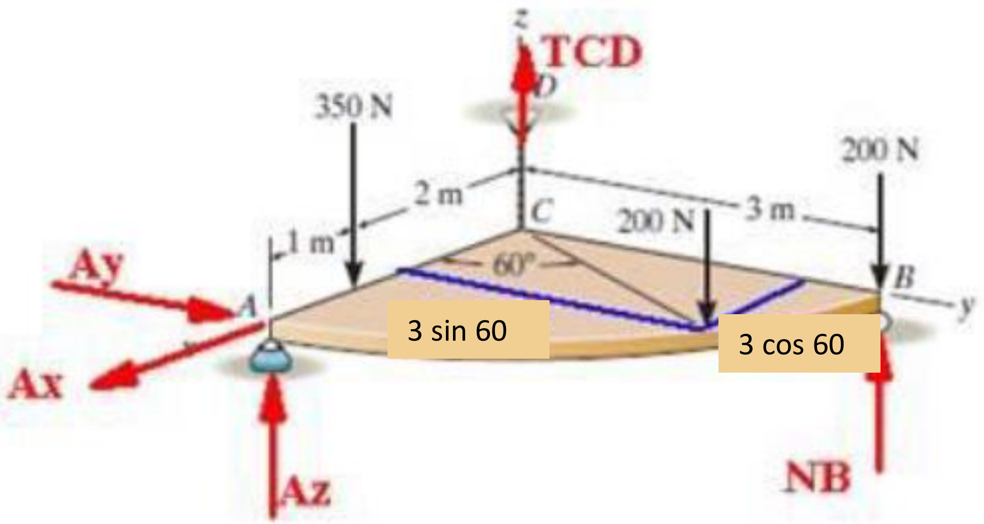
$\Sigma F_z = 0;\quad T_{CD} + 373.21 + 333.33 - 350 - 200 - 200 = 0$

$T_{CD} = 43.5\text{ N}$

$\Sigma F_x = 0;\quad A_x = 0$

$\Sigma F_y = 0;\quad A_y = 0$

Determine the force components acting on the ball-and-socket at A, the reaction at the roller B and the tension on the cord CD needed for equilibrium of the quarter circular plate.



$\Sigma M_x = 0;\quad N_B(3) - 200(3) - 200(3\sin 60°) = 0$

$N_B = 373.21\text{ N} = 373\text{ N}$

$\Sigma M_y = 0;\quad 350(2) + 200(3\cos 60°) - A_z(3) = 0$

$A_z = 333.33\text{ N} = 333\text{ N}$

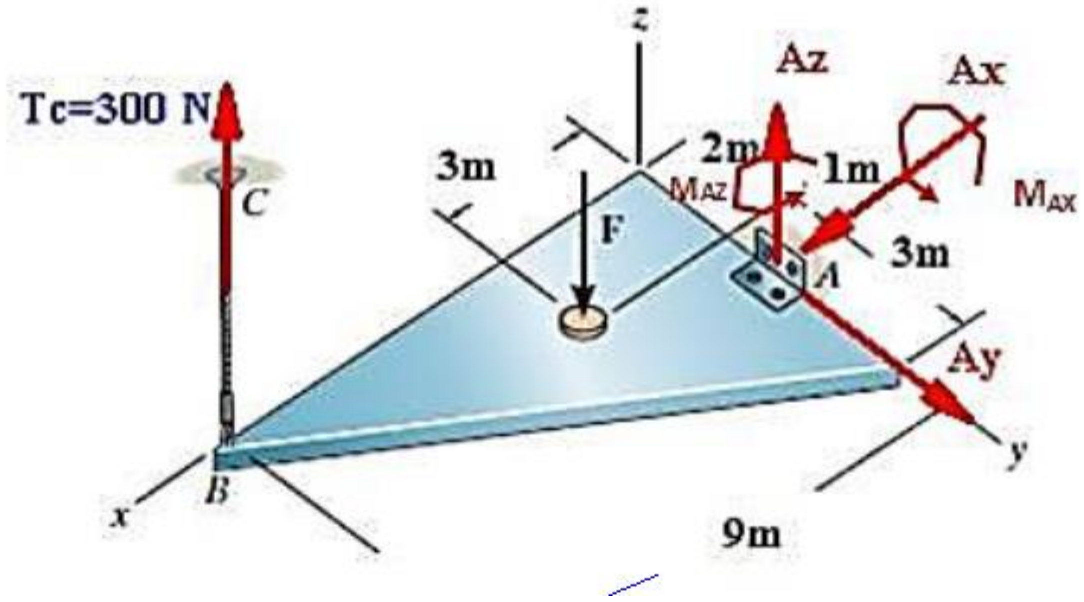
$\Sigma F_z = 0;\quad T_{CD} + 373.21 + 333.33 - 350 - 200 - 200 = 0$

$T_{CD} = 43.5\text{ N}$

$\Sigma F_x = 0;\quad A_x = 0$

$\Sigma F_y = 0;\quad A_y = 0$

If the cable can be subjected to a maximum tension of 300 N determine the maximum force F which may be applied to the plate. Compute the x, y, z components of reaction at the hinge A for this loading.



$$\Sigma M_y = 0; \quad 3(F) - 300(9) = 0$$

$$F = 900 \text{ N}$$

$$\Sigma F_x = 0; \quad A_x = 0$$

$$\Sigma F_y = 0; \quad A_y = 0$$

$$\Sigma F_z = 0; \quad -900 + 300 + A_z = 0; \quad A_z = 600 \text{ N}$$

$$\Sigma M_{Ax} = 0; \quad M_{Ax} + 900(1) - 3(300) = 0; \quad M_{Ax} = 0$$

$$\Sigma M_{Az} = 0; \quad M_{Az} = 0$$

Ex. The homogeneous plate has a mass of 100 kg and is subjected to a force and couple moment along its edges. If it is supported in the horizontal plane by a roller at *A*, a ball-and-socket joint at *B*, and a cord at *C*, determine the components of reaction at these supports.

Equations of Equilibrium. Since the three-dimensional geometry is rather simple, a *scalar analysis* provides a *direct solution* to this problem. A force summation along each axis yields

$\Sigma F_x = 0; \quad B_x = 0$ *Ans.*

$\Sigma F_y = 0; \quad B_y = 0$ *Ans.*

$\Sigma F_z = 0; \quad A_z + B_z + T_C - 300\text{ N} - 981\text{ N} = 0$ (1)

Recall that the moment of a force about an axis is equal to the product of the force magnitude and the perpendicular distance (moment arm) from the line of action of the force to the axis. Also, forces that are parallel to an axis or pass through it create no moment about the axis. Hence, summing moments about the positive *x* and *y* axes, we have

$\Sigma M_x = 0; \quad T_C(2\text{ m}) - 981\text{ N}(1\text{ m}) + B_z(2\text{ m}) = 0$ (2)

$\Sigma M_y = 0; \quad 300\text{ N}(1.5\text{ m}) + 981\text{ N}(1.5\text{ m}) - B_z(3\text{ m}) - A_z(3\text{ m}) - 200\text{ N}\cdot\text{m} = 0$ (3)

The components of the force at *B* can be eliminated if moments are summed about the *x′* and *y′* axes. We obtain

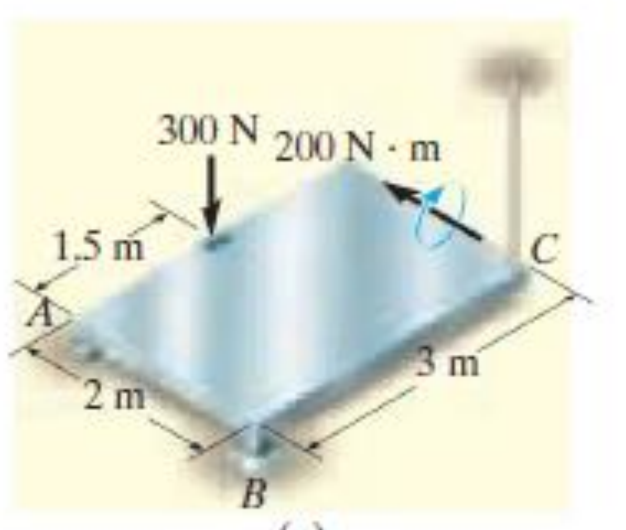
$\Sigma M_{x'} = 0; \quad 981\text{ N}(1\text{ m}) + 300\text{ N}(2\text{ m}) - A_z(2\text{ m}) = 0$ (4)

$\Sigma M_{y'} = 0; \quad -300\text{ N}(1.5\text{ m}) - 981\text{ N}(1.5\text{ m}) - 200\text{ N}\cdot\text{m} + T_C(3\text{ m}) = 0$ (5)

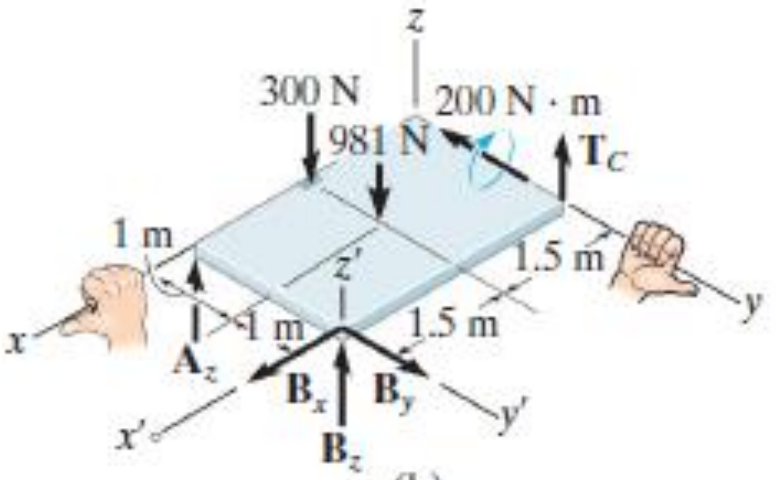
Solving Eqs. 1 through 3 or the more convenient Eqs. 1, 4, and 5 yields

$A_z = 790\text{ N} \quad B_z = -217\text{ N} \quad T_C = 707\text{ N}$ *Ans.*

The negative sign indicates that $\mathbf{B}_z$ acts downward.

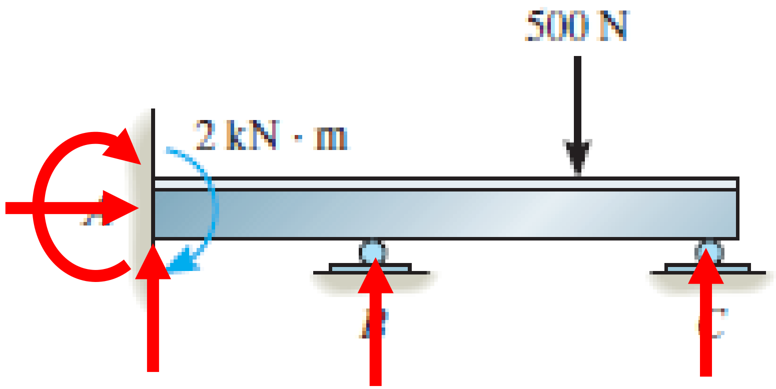


(a)



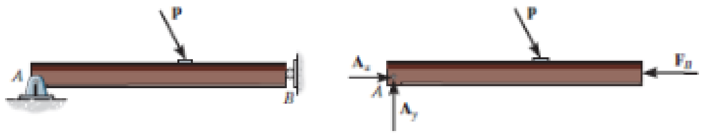
(b)

5.7 Constraints and Statical Determinacy



Statically indeterminate : there will be more unknowns than the equations of equilibrium

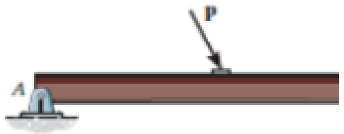
Improper Constraints.



lines of action of the reactive forces are *concurrent*

the beam is improperly constrained, $\Sigma M_A \neq 0$

improper constraining leads to instability occurs when the *reactive forces* are all *parallel*



body may have *fewer* reactive forces than equations of equilibrium that must be satisfied. The body then becomes only *partially constrained*. (unstable)

In engineering practice, these situations should be avoided at all times since they will cause an unstable condition.

chapter 6

Structure Analysis

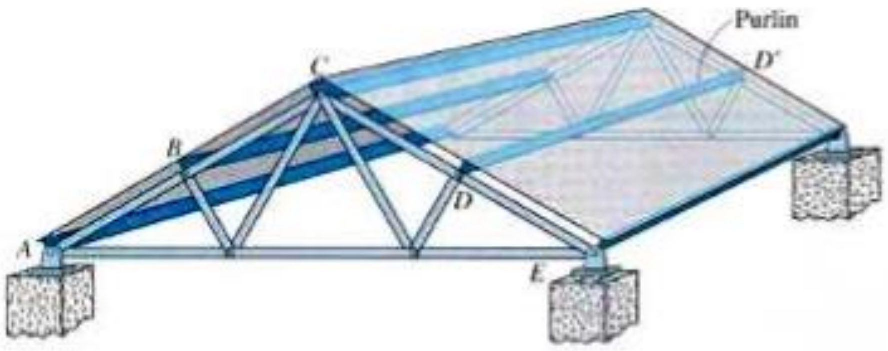
(Truss- frames and machines)

Lecture 1

27/02/2022

Iqbal Marie

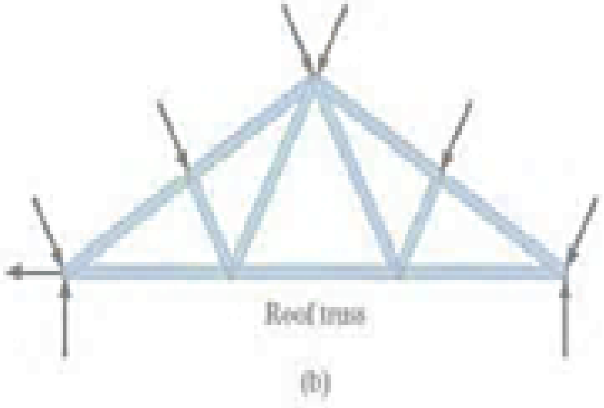
6.1 Simple Trusses

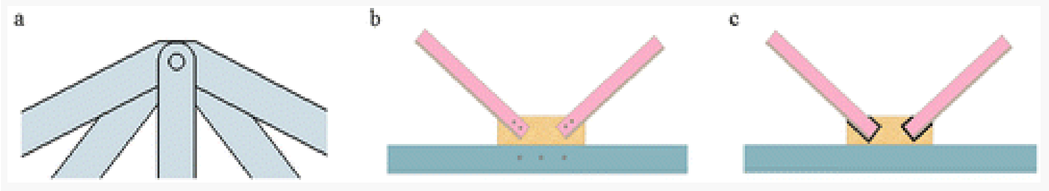


A simple truss is a planar truss which composed of **triangular elements.** Often used to support **roofs** and **bridges**.

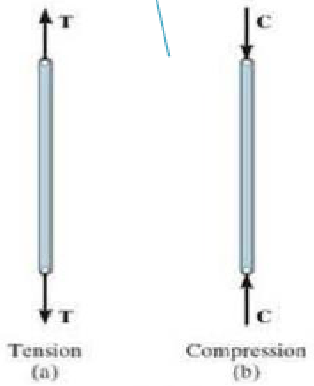
truss is a **structure assumed to :**

- **composed of: slender members joined** together **at frictionless joints (smooth pins)**
- the members act as **two-force members**. They are in either **tension or compression**.
- Compressive members are made thicker to prevent buckling
- All loads are applied at the joints.
- The weight of the truss members is often neglected as the weight is usually small



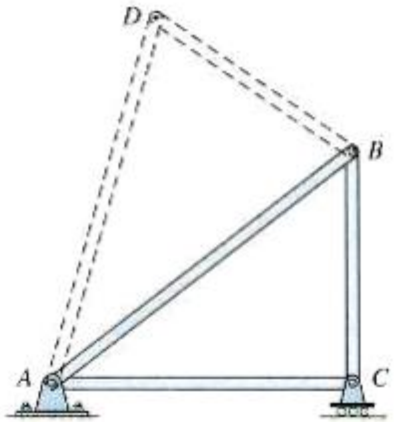


Assumed smooth Pin joint

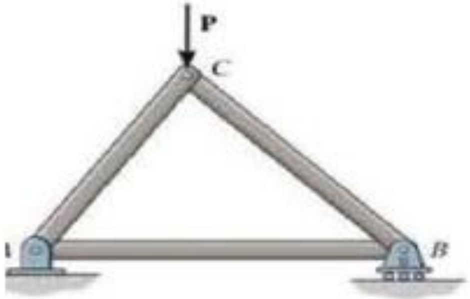


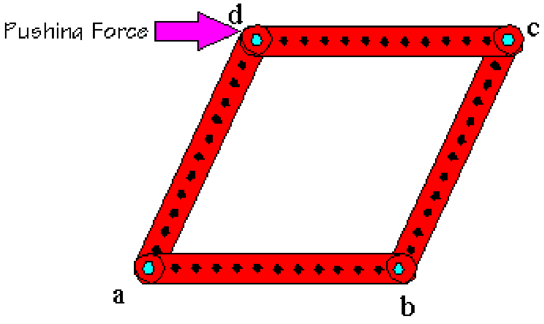
A simple truss is a planar truss which begins with a triangular element and can be expanded by adding two members and a joint.

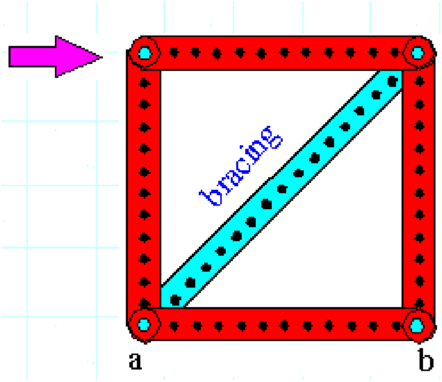
For truss design ; (members and joints)

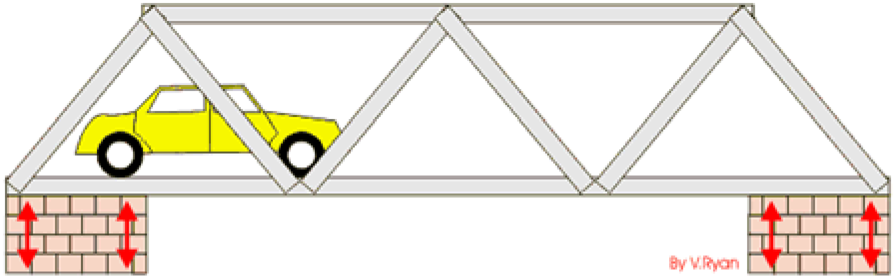


It is necessary to determine the forces in each truss member.
This is called the Truss analysis.









https://sites.google.com/site/structuresinigoandsofia/triangi

6.2 Method of joints (to analyse or design a truss, it is necessary to determine the force in each of its members)

This method is based on the fact that **if the entire truss is in equilibrium**, then each of **its joints is also in equilibrium**

To analyse any truss:

1. **Draw the FBD of the entire truss to determine the supports reactions applying the equilibrium equations (show them on the truss)**
   $\Sigma F_x = 0$ and $\Sigma F_y = 0$ and $\Sigma M = 0$

2. Draw FBD for each joint assuming tension in each member (start with the one of two members)

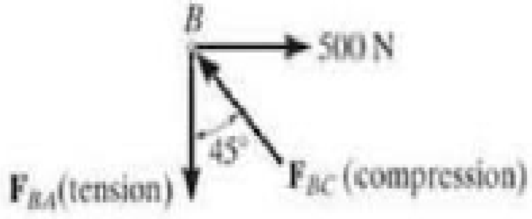
3. Apply equilibrium equation for each joint to find the force in each member

$\Sigma F_x = 0$ and $F_y = 0$... if the result is positive then the member is in tension and if negative the member is in compression.

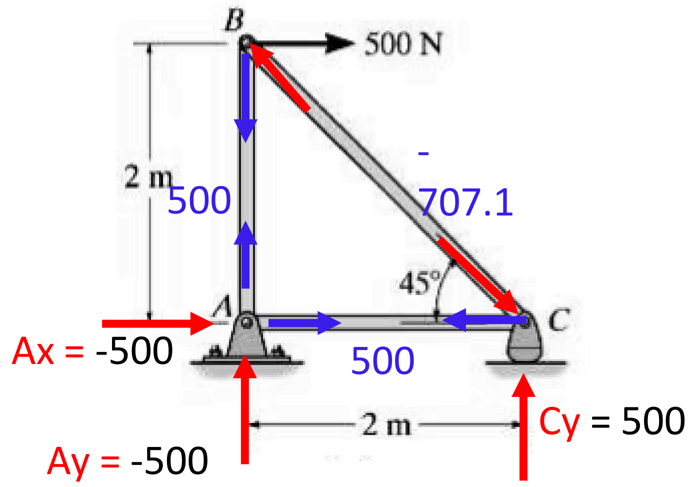
4. Show the results on the truss (show the arrow tip pointing to the joint for compression and away from the joint for tension)

5. Go to the next joint until you finish all the joints and obtain the forces in all members

6. Use table to arrange your solution



Find the forces in all members and specify if tension or compression



$M_A = 0 \quad -500 \times 2 + 2Cy = 0$

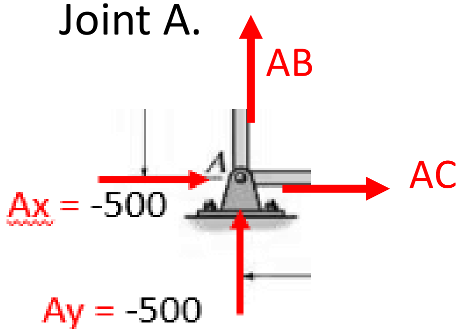
$Cy = 500$ N

$Fx = 0 \quad Ax = -500$ N

$Fy = 0 \quad Ay + 500 = 0$

$Ay = -500$N

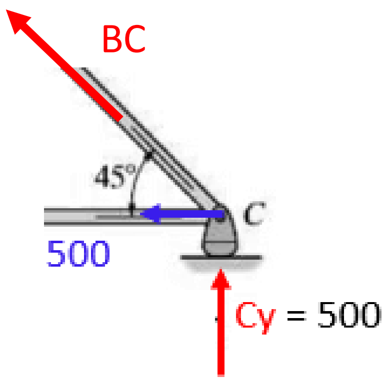
Joint A.



$\Sigma Fx = 0 \quad -500 + AC = 0 \quad$ AC = +500 N (T)

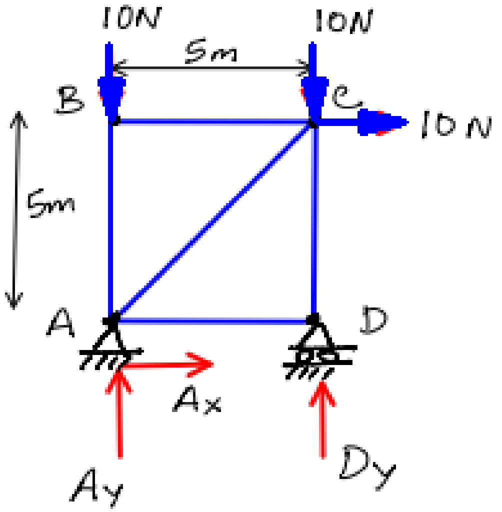
$\Sigma Fy = 0 \quad AB - 500 = 0 \quad$ AB = + 500 N (T)

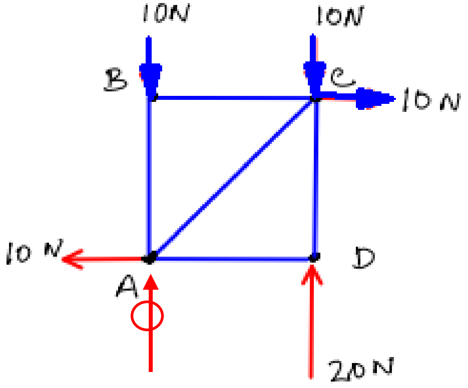
Joint C.



$\Sigma Fx = 0 \quad -500 - BC \cos 45 = 0 \quad$ BC = -707.1N (C)

Ex. Analyse the truss shown (Find the internal forces in all the members)
Show the results on the truss



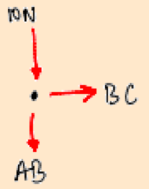
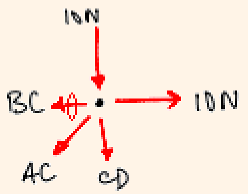
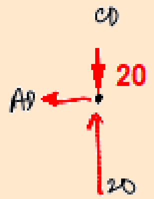


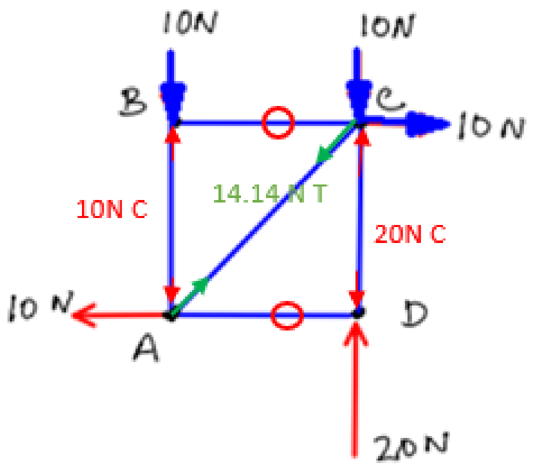
1. Support reactions

$$\sum F_x = 0 \Rightarrow A_x + 10 = 0 \Rightarrow \boxed{A_x = -10N}$$

$$\sum F_y = 0 \Rightarrow A_y + D_y - 10 - 10 = 0 \Rightarrow \boxed{A_y = 0}$$

$$\sum M_A = 0 \Rightarrow -(10\times5) - (10\times5) + D_y\times5 = 0 \Rightarrow \boxed{D_y = 20N}$$

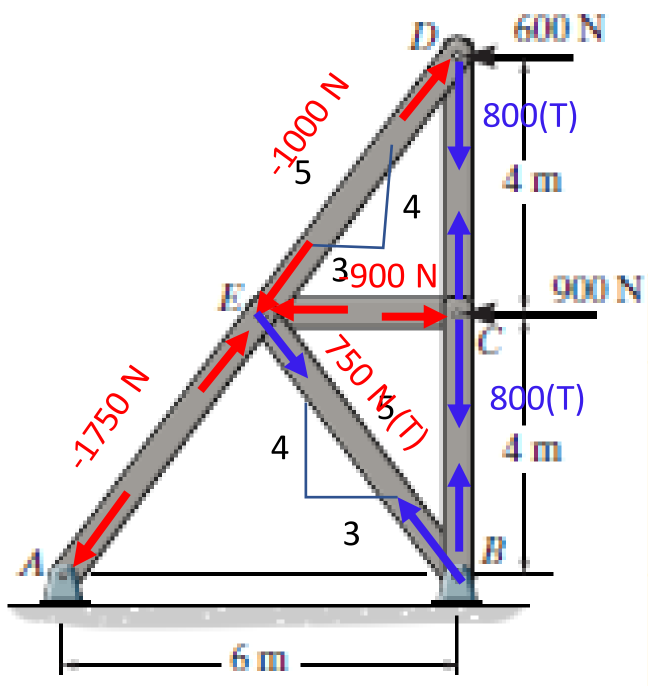
Joint	FBD	Equ. Equ
B		$\sum F_x = 0 \Rightarrow \boxed{BC = 0}$ $\sum F_y = 0 \Rightarrow -AB - 10 = 0 \Rightarrow \boxed{AB = -10}$ Compresion
C		$\sum F_x = 0 \Rightarrow -AC(\cos 45^\circ) + 10 = 0$ $\Rightarrow AC = 10\sqrt{2} = \boxed{14.14N}$ T $\sum F_y = 0 \Rightarrow -10 - AC\sin 45^\circ - CD = 0$ $\Rightarrow CD = -10N - 10 = \boxed{-20\ N}$ C
D		$\sum F_x = 0 \Rightarrow \boxed{AD = 0}$

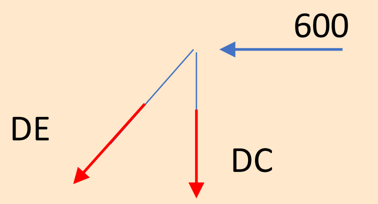
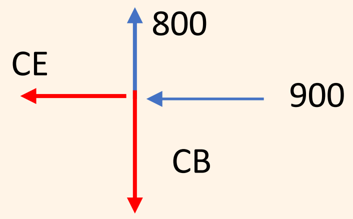
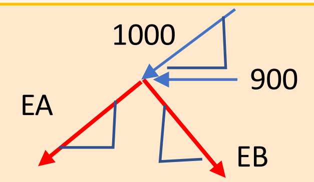


The results shown on the truss

Determine the forces in all the members and specify if it is in tension or compression

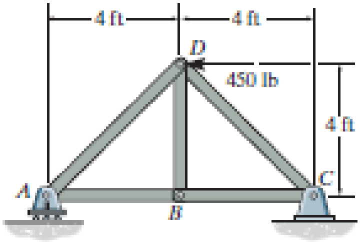
Show your results on the truss



Joint	FBD	Equ. Equ
D		Fx=0 -600-3DE/5 =0 DE = -1000 N (C) Fy=0 -DC- (-1000x4/5=0 DC = 800(T)
C		Fx=0 -CE - 900=0 DE = -900 N (C) FY=0 800- CB =0 CB=800 N (T)
E		Fx=0 $-\frac{3EA}{5}$ -900 - $\frac{3000}{5}$ + $\frac{3EB}{5}$ = 0 Fy=0 $-\frac{4EA}{5}$ - $\frac{4000}{5}$ - $\frac{4EB}{5}$ = 0 EB = 750 N (T) EA = 1750 N (C)

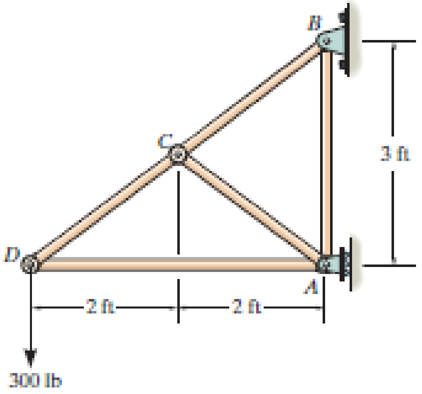
All problem solutions must include FBDs.

F6–1. Determine the force in each member of the truss. State if the members are in tension or compression.



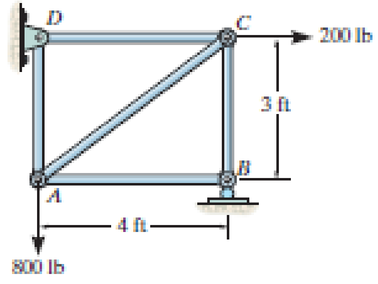
Prob. F6–1

F6–2. Determine the force in each member of the truss. State if the members are in tension or compression.



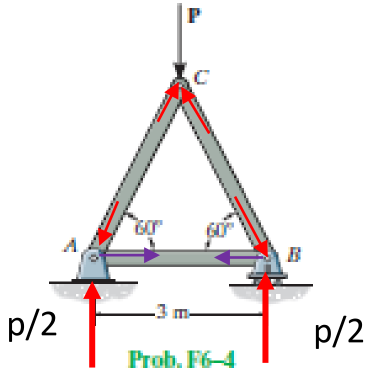
Prob. F6–2

F6–3. Determine the force in each member of the truss. State if the members are in tension or compression.



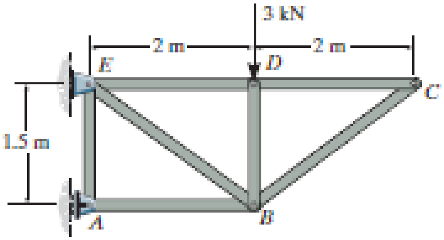
Prob. F6–3

F6–4. Determine the greatest load *P* that can be applied to the truss so that none of the members are subjected to a force exceeding either 2 kN in tension or 1.5 kN in compression.



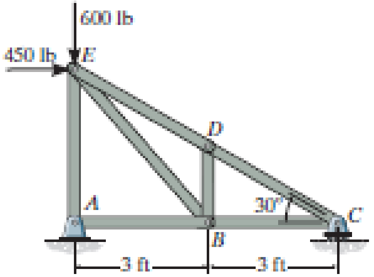
Prob. F6–4

F6–5. Identify the zero-force members in the truss.



Prob. F6–5

F6–6. Determine the force in each member of the truss. State if the members are in tension or compression.



Prob. F6–6

Ac sin 60 +p/2 = 0

AC = -P/(2sin 60) = -0.58 P (c)

AC cos60 +AB = 0

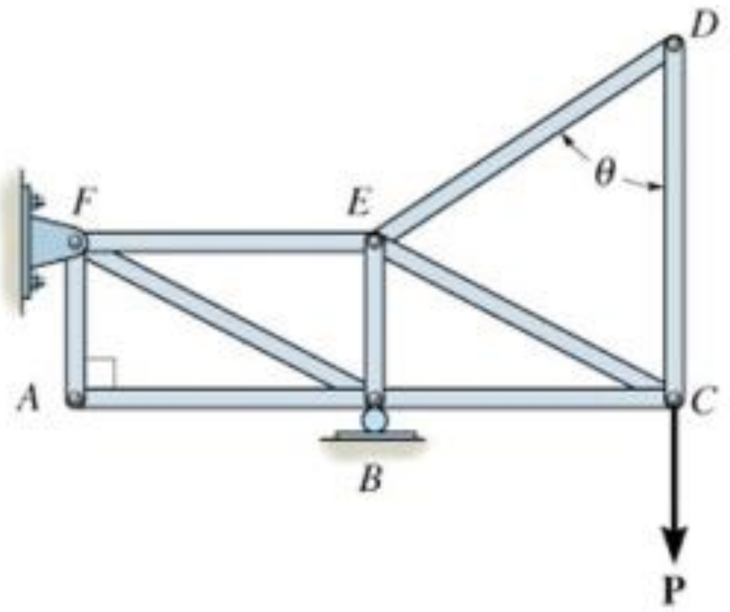
AB = -P cos 60/(2sin60) = 0.29 P (T)

0.58P = 1.5	P = 2.6 kN
0.29 P = 2	**P = 0.69 kN**

6.3 Zero force members

To identify zero force members by inspection

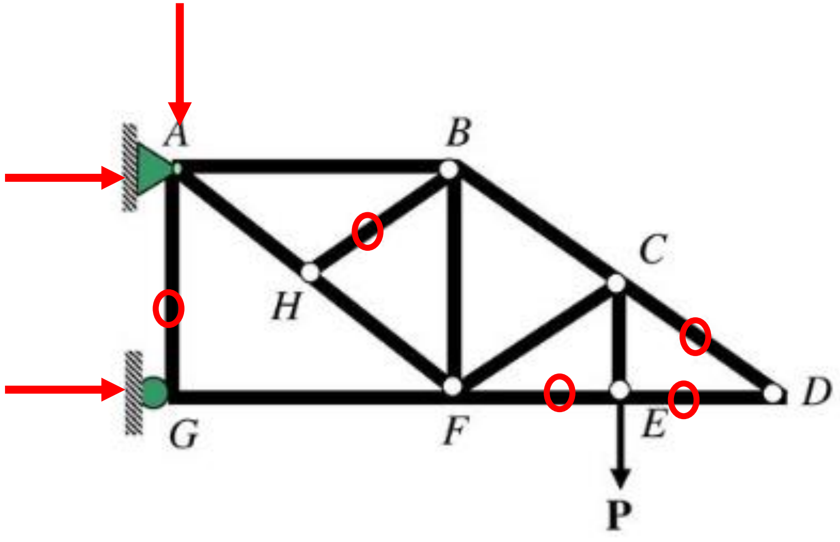
Please note that zero-force members are used to increase stability and rigidity of the truss, and to provide support for various different loading conditions.

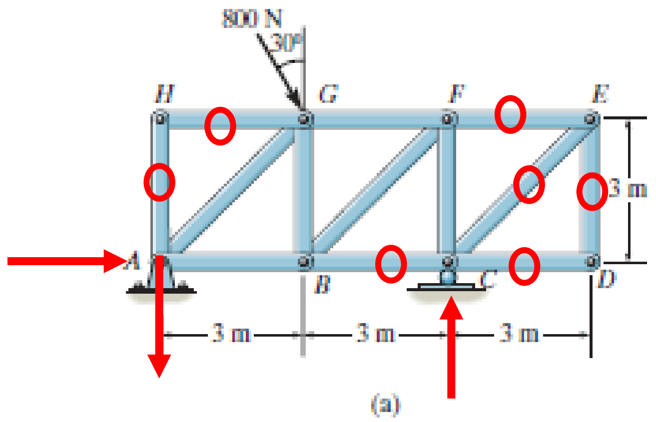


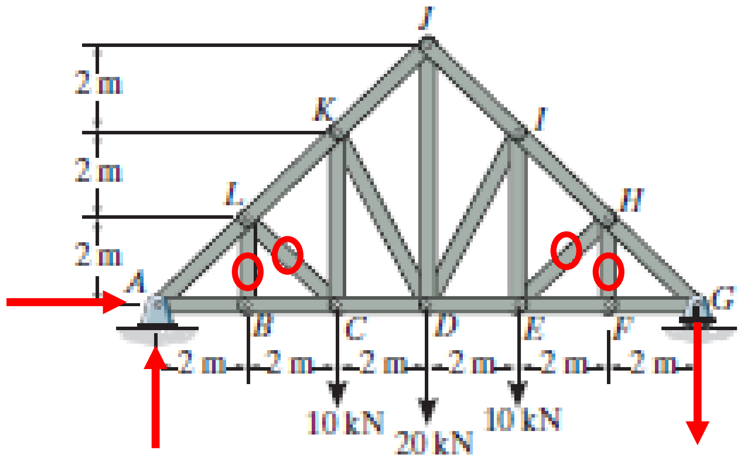
If a joint has only two non-collinear members and there is no external load or support reaction at that joint, then those two members are zero-

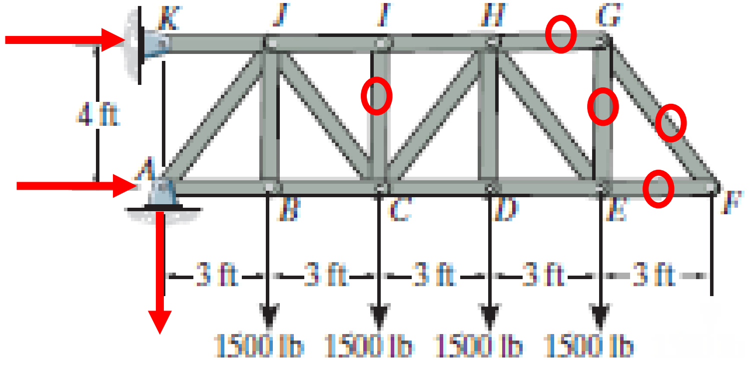
If three members form a truss joint for which two of the members are collinear and there is no external load or reaction at that joint, then the third non-collinear member is a zero force member.

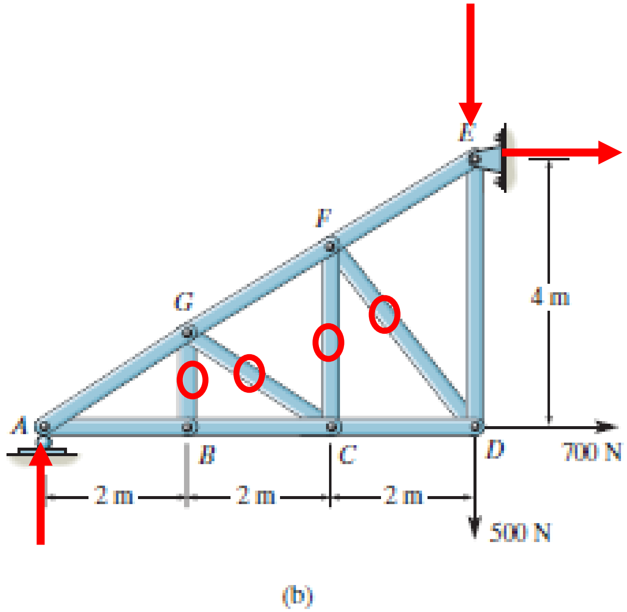
find Zero force members





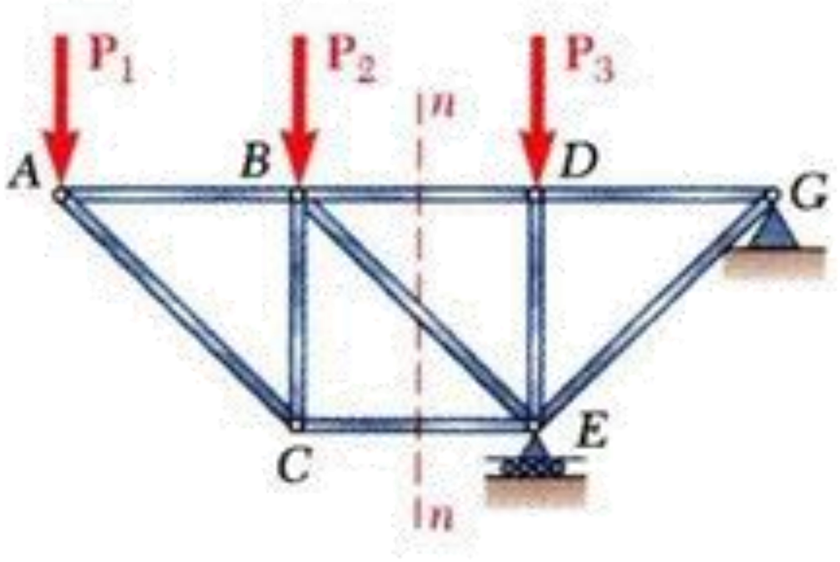






6.4 The Method of Sections

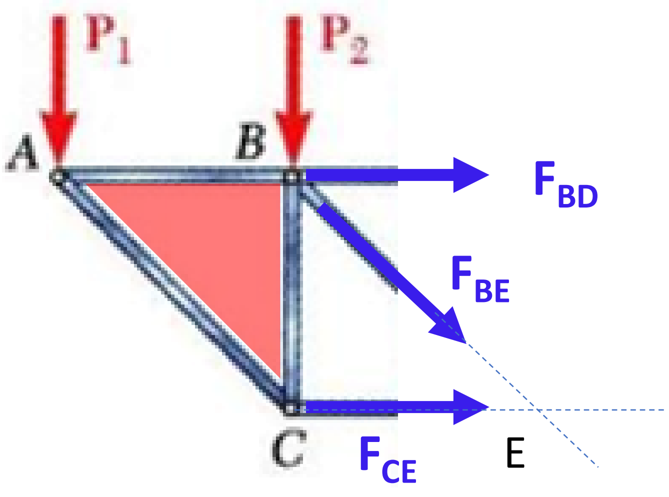
~~6.5 Space Truss (محذوف)~~



The **method of joints** is used to find the internal forces in all the truss members.
The **method of sections** is used to find the internal forces only in a few specific members of a truss,

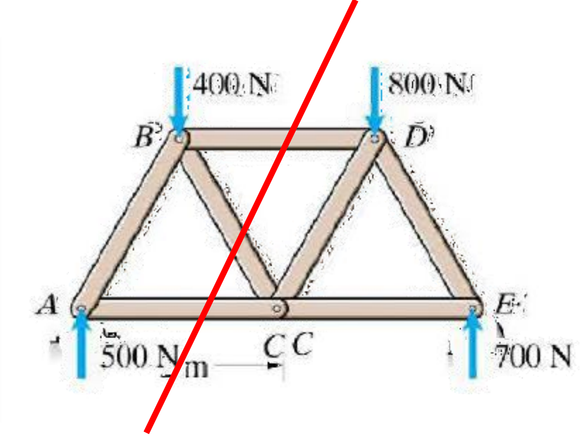
Procedure;

- Find support reactions if needed
- Make cut (section) passing through the required members
- Don't cut more than three members (most of cases) (the section must cut the truss into 2 separate parts)
- Draw FBD of one of the 2 different parts of the truss (take the simplest) showing all external loads and reactions on it , assume tension in the 3 members that are cut.
- Apply Equilibrium to find the forces in the required members $\sum F_x = \sum F_y = \sum M_z = 0$



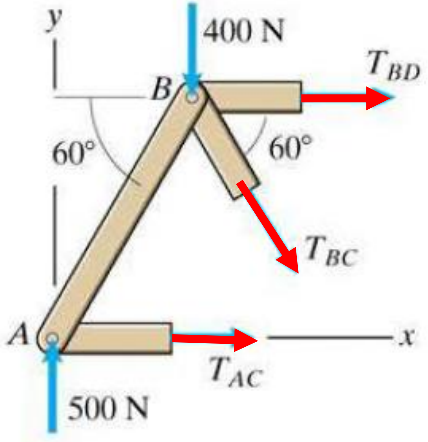
Ex. Find the internal forces in members BD, BC and AC and indicate if they are in tension or compression

1. Find support reactions



2. make section through the required members

3. Draw FBD of either parts of the truss.. assume tension in the cut members



4. Apply equilibrium equations on the part of the truss

$$\sum F_x = \sum F_y = \sum M_z = 0$$

$$\sum M_B = (2\sin 60^\circ)T_{AC} - (2\cos 60^\circ)(500) = 0$$

$$\Rightarrow T_{AC} = 289\ N$$

$$\sum F_y = 500\ N - 400\ N - T_{BC}\sin 60^\circ = 0$$

$$T_{BC} = 115\ N$$

$$\sum F_x = T_{AC} + T_{BD} + T_{BC}\cos 60^\circ = 0$$

$$T_{BD} = -346\ N$$

(+) = tension …….. (-) = compression)

Determine the force in members *BE, EF*, and *CB*, and state
if the members are in tension or compression.

$$\circlearrowleft + \Sigma M_E = 0; \qquad -5\,(4) + F_{CB}\,(4) = 0$$

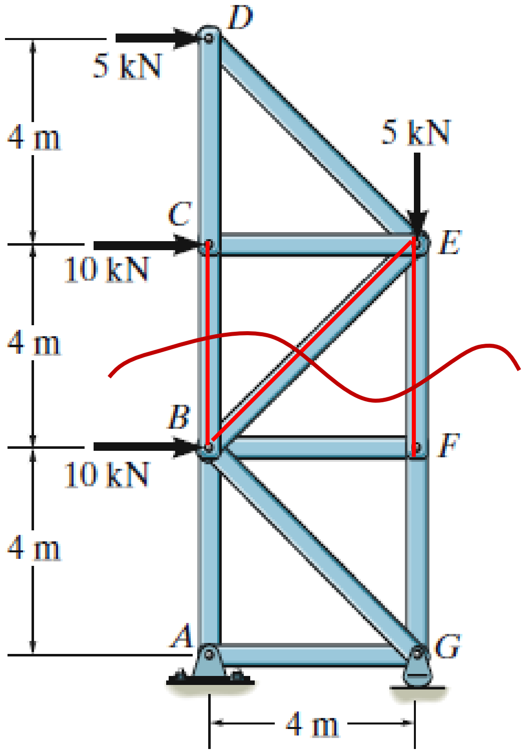
$$F_{CB} = 5\text{ kN (T)}$$

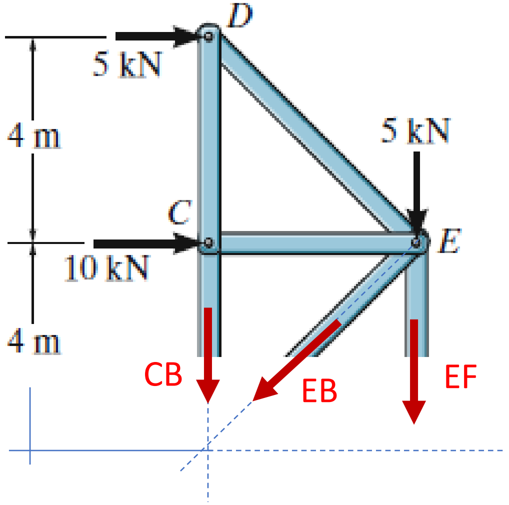
$$\circlearrowleft + \Sigma M_B = 0; \qquad -5\,(8) - 10\,(4) - 5\,(4) + F_{EF}\,(4) = 0$$

$$F_{EF} = 25\text{ kN (C)}$$

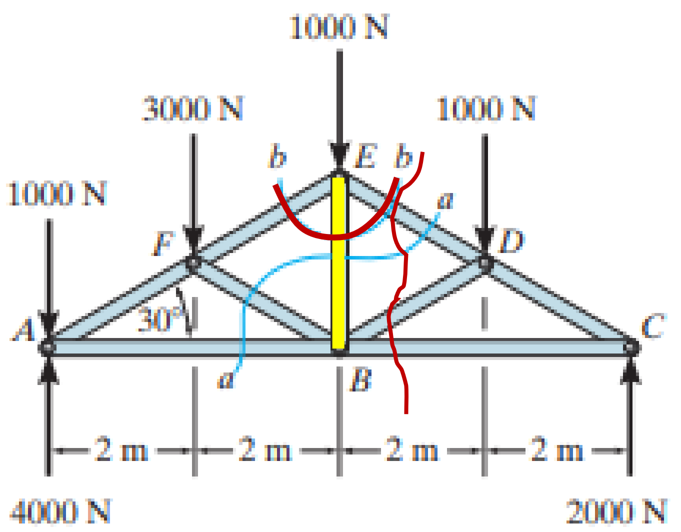
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 5 + 10 - F_{BE}\cos 45^\circ = 0$$

$$F_{BE} = 21.2\text{ kN (T)}$$



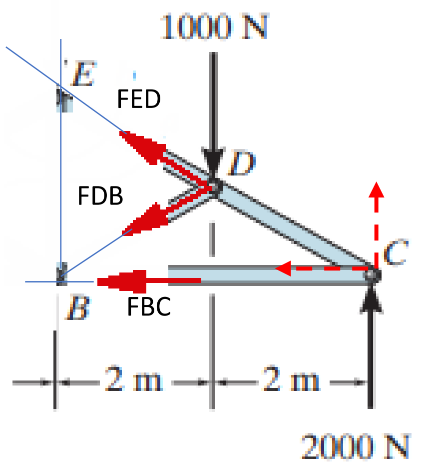


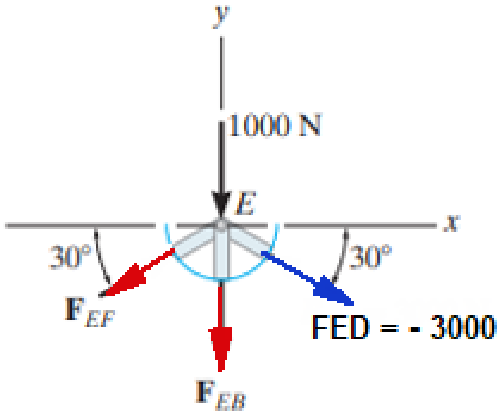
Determine the force in member **EB** of the roof truss shown. Indicate whether the member is in tension or compression.



(a)

Book solution





(c)

From the section

$\Sigma M_B = 0$ ……4 F_{ED} sin 30 -1000x2+2000x4 = 0 F_{ED} = - 3000 N (C)

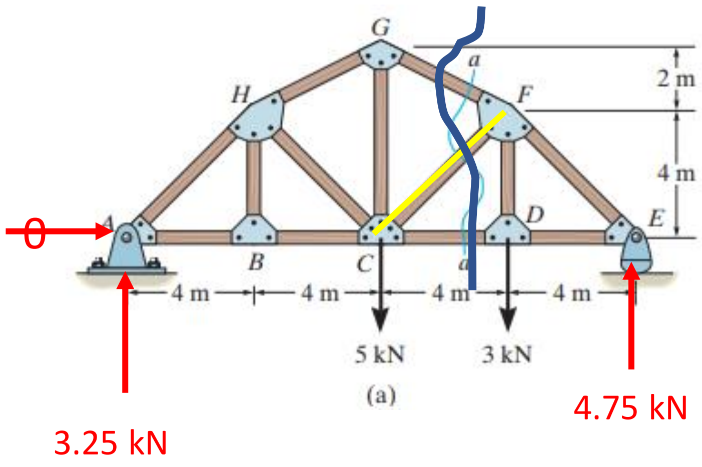
From section B (joint)

Considering now the free-body diagram of section bb, Fig. c, we have

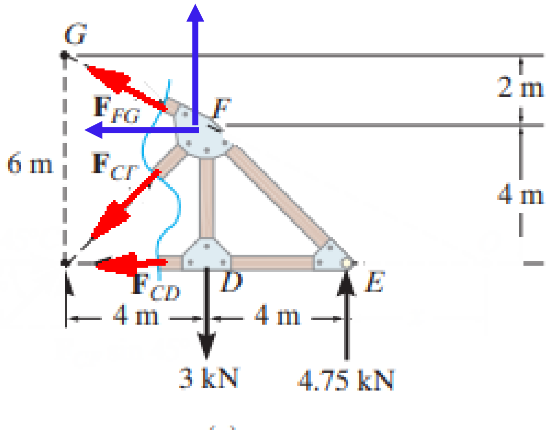
Σ Fx = 0; - F_{EF} cos 30 +(- 3000 cos 30) = 0 F_{EF} = - 3000 N (C)

Σ Σ Fy = 0; - 2(-3000 sin 30) - 1000 N - FEB = 0 **FEB = 2000 N (T)**

Determine the force in **member CF** of the truss. Indicate whether the member is in tension or compression. Assume each member is pin connected.



(a)

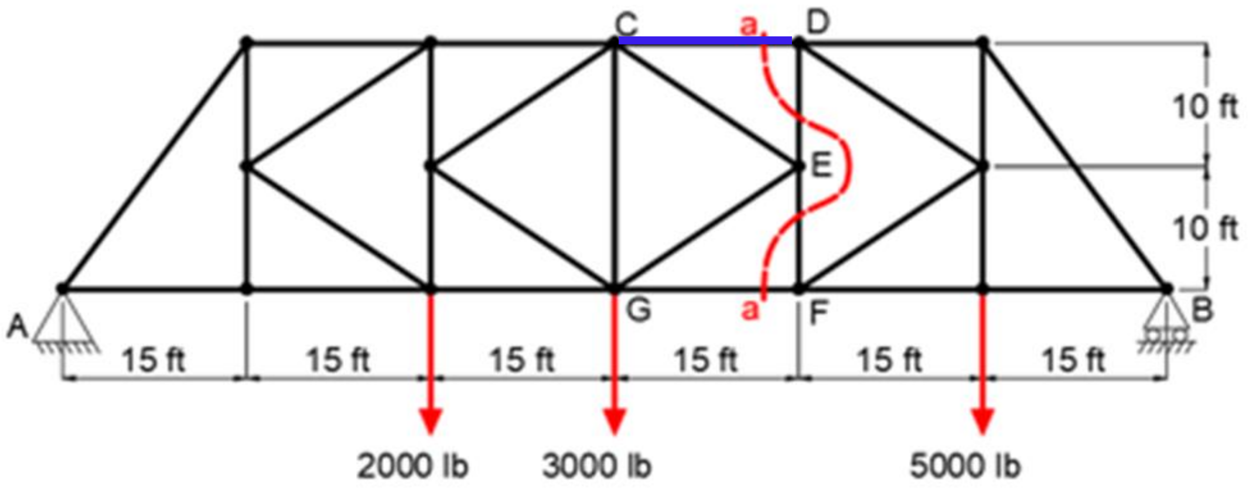


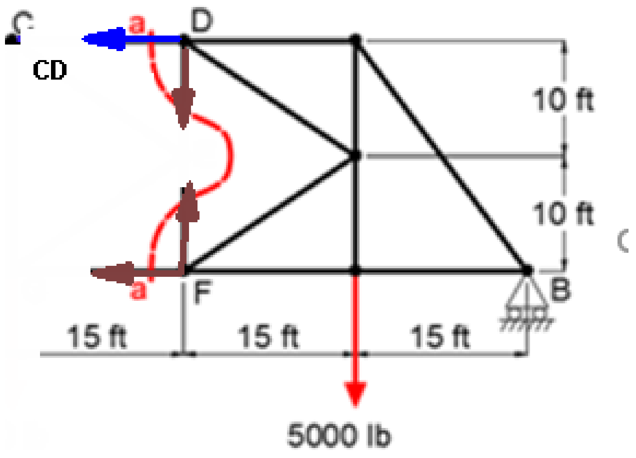
(c)

$\Sigma M_c = 0;$ $F_{FG}\cos 26.5(6) + -(3)(4) + (4.75)(8) = 0$ $F_{FG} = -4.84$ kN (C)

$\Sigma F_y = 0;$ $-4.84 \sin 26.5 + -(3) + 4.75 - F_{CF} \sin 45 = 0$ $\mathbf{F_{CF} = -0.58}$ **kN (C)**

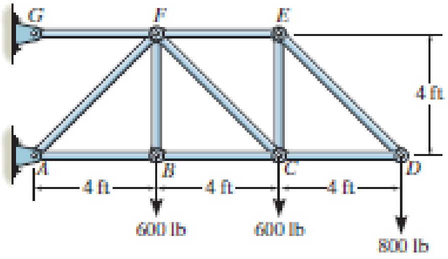
Determine the force in member **CD** of the K-trusstruss shown. Indicate whether the member is in tension or compression.





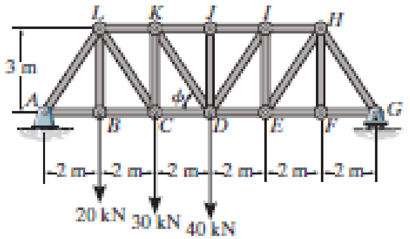
All problem solutions must include FBDs.

F6–7. Determine the force in members *BC*, *CF*, and *FE*. State if the members are in tension or compression.



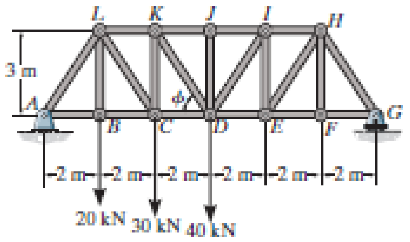
Prob. F6–7

F6–8. Determine the force in members *LK*, *KC*, and *CD* of the Pratt truss. State if the members are in tension or compression.



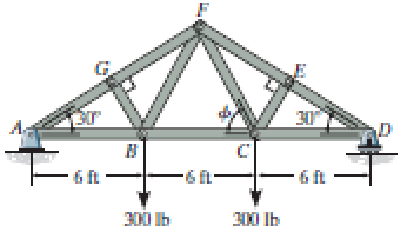
Prob. F6–8

F6–9. Determine the force in members *KJ*, *KD*, and *CD* of the Pratt truss. State if the members are in tension or compression.



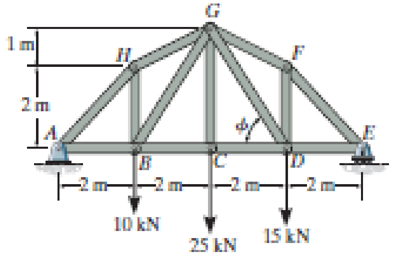
Prob. F6–9

F6–10. Determine the force in members *EF*, *CF*, and *BC* of the truss. State if the members are in tension or compression.



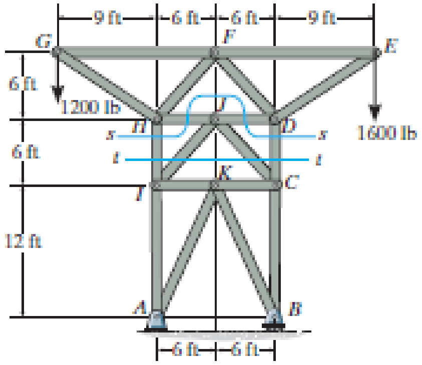
Prob. F6–10

F6–11. Determine the force in members *GF*, *GD*, and *CD* of the truss. State if the members are in tension or compression.



Prob. F6–11

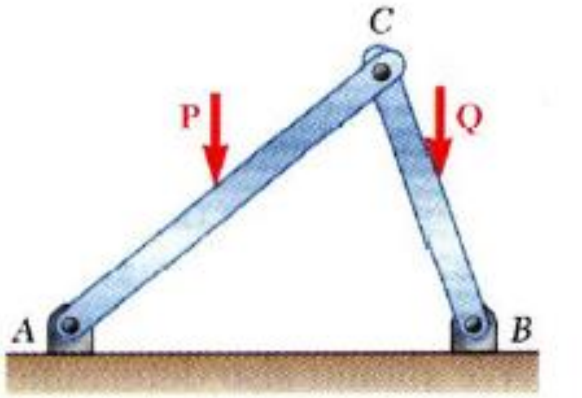
F6–12. Determine the force in members *DC*, *HI*, and *JI* of the truss. State if the members are in tension or compression. *Suggestion:* Use the sections shown.



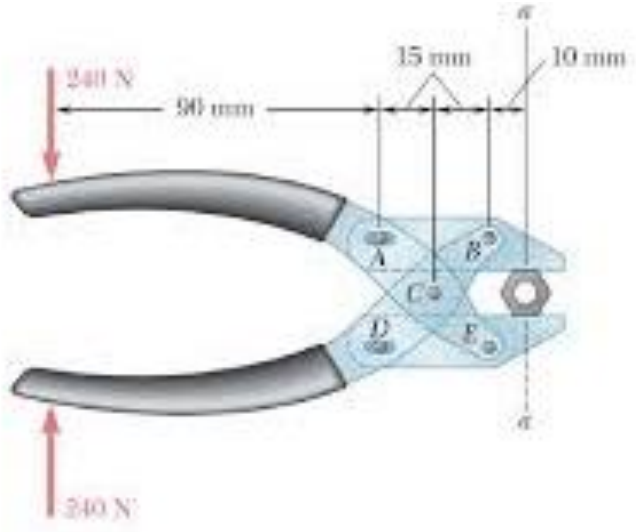
Prob. F6–12

6.6 Frames and Machines

Frames and Machines are two common type of structures which are often composed of pin-connected multiforce members.



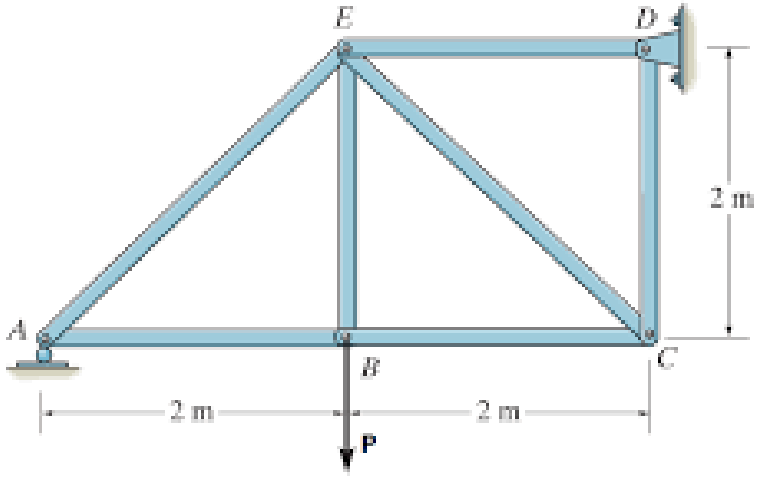
Frames are generally stationary and support external loads.
Machines contain moving parts and are designed to alter the effect of forces.



Requirements

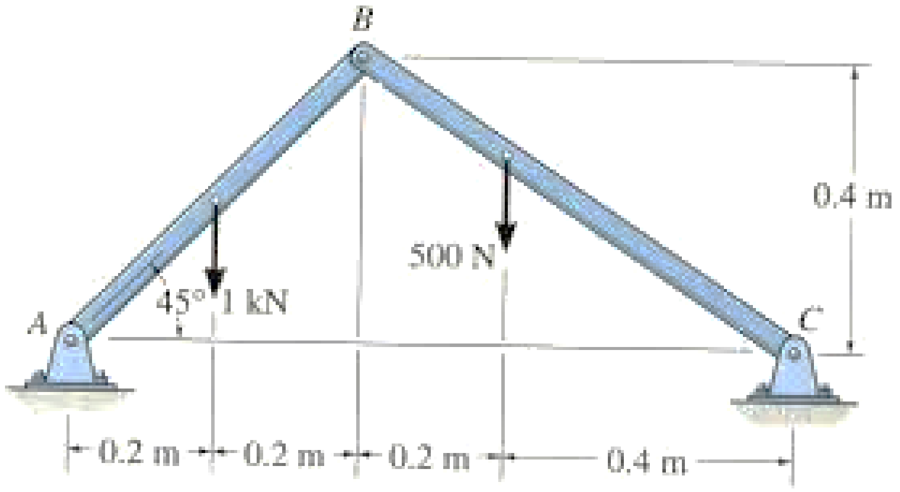
- **Draw the free body diagram of a frame or machine and its members.**
- **Determine the forces acting at the joints and supports of a frame or machine.**

Difference between a truss and a frame



Truss

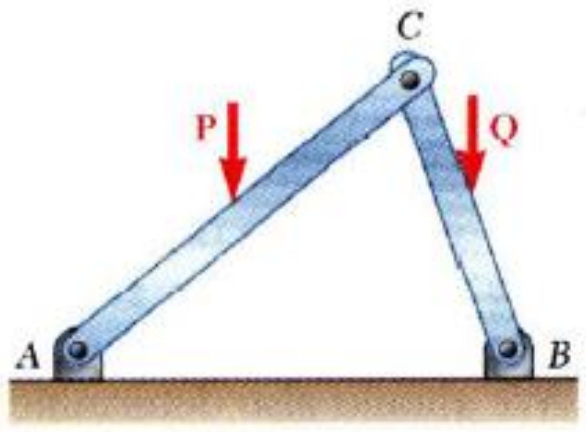
All members are two force members
Forces are applied at the joints

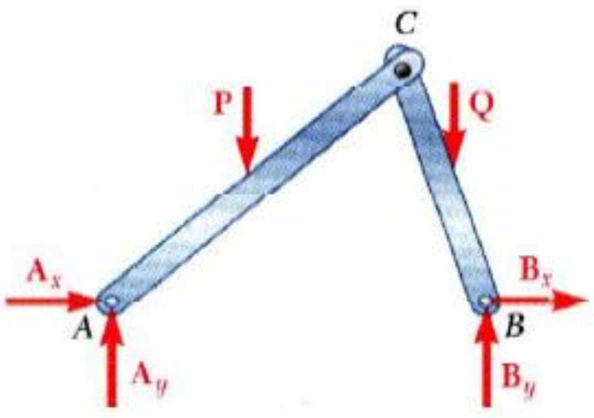


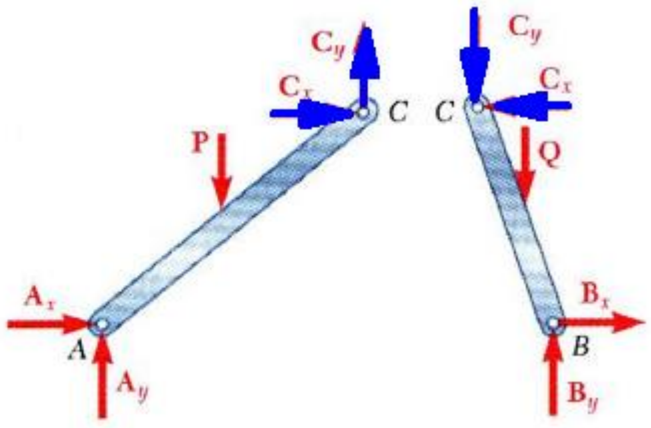
Frame

Some of members are multiforce members..
Forces can be applied at members

Draw FBD of each component of the frame







FBD of the entire frame

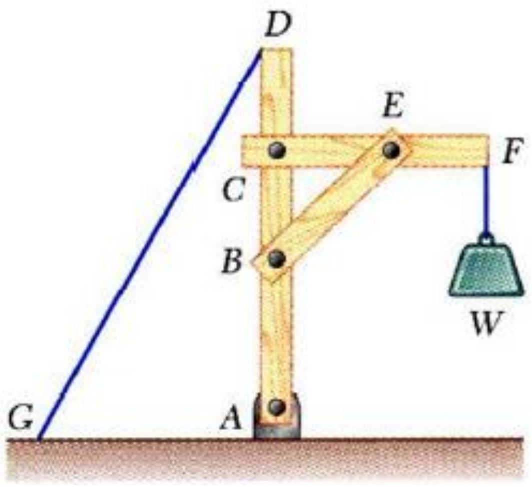
FBD of each part

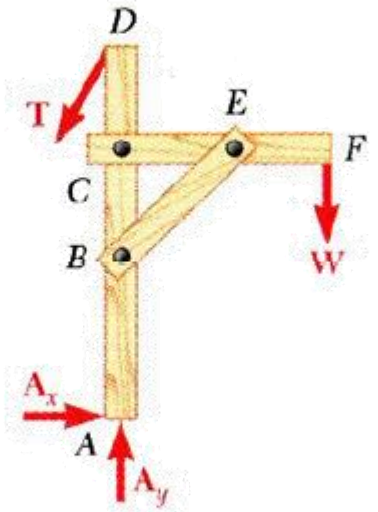
External support reactions

Internal forces at the pins

Frame analysis Steps:

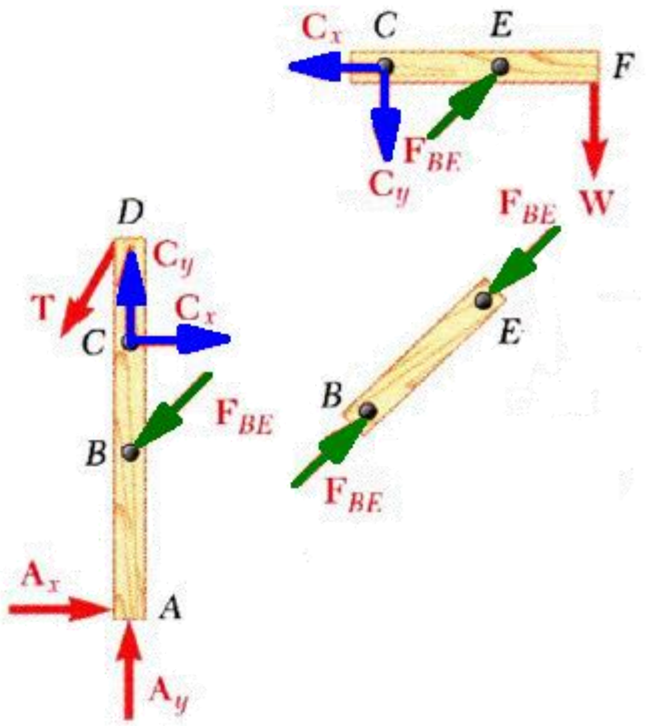
1. Draw FBD of the whole frame and determine the as much external reactions as possible





External support reactions

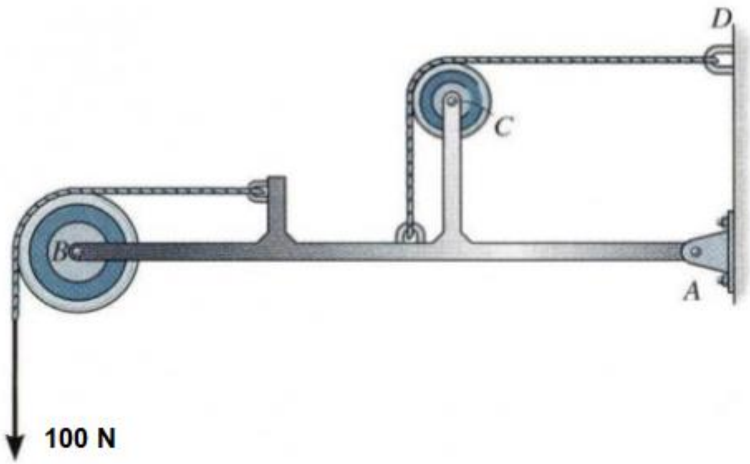
3. Apply the equilibrium equations to each part separately and find all the unknowns

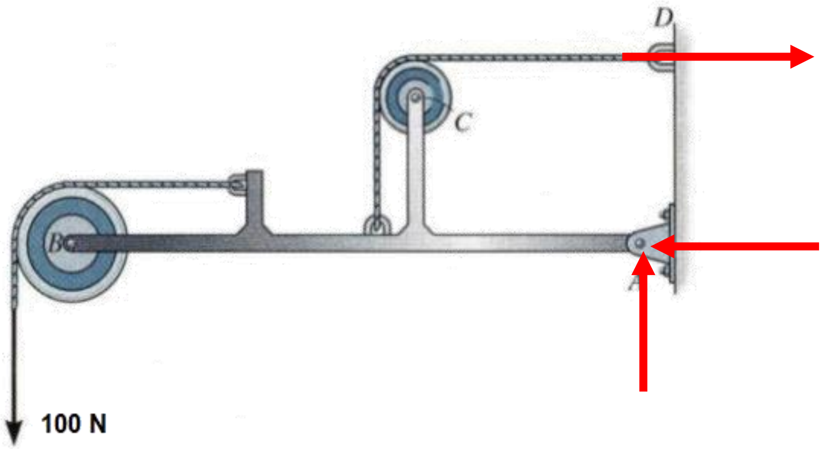


Internal forces at the pins

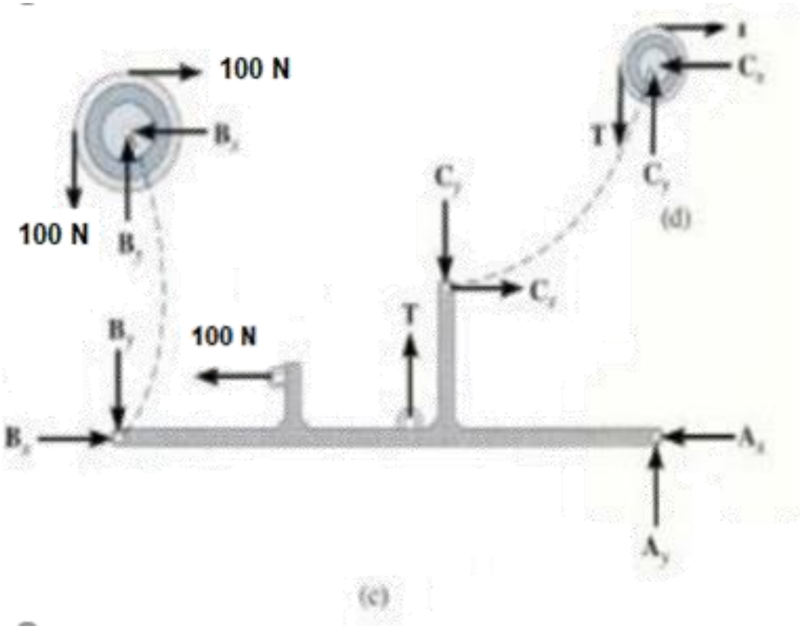
2. Dismantle the frame and draw FBD of each member applying internal forces at each pin (joint)

Draw FBD of the entire frame and each part

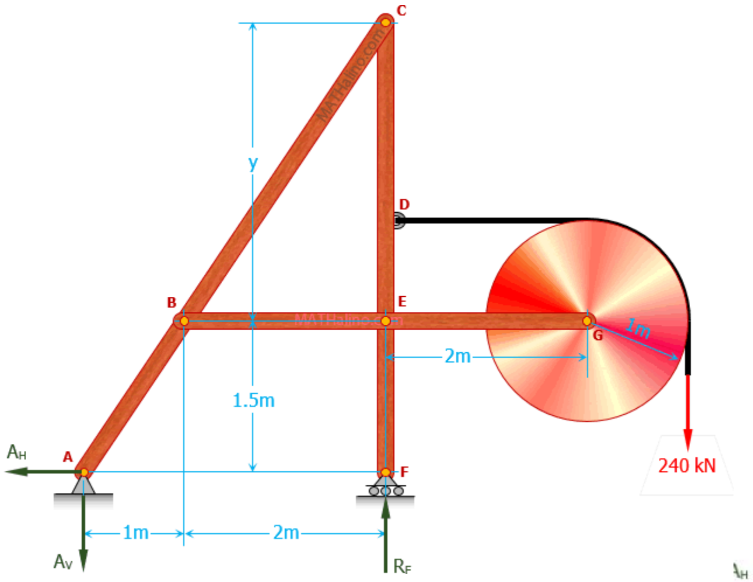


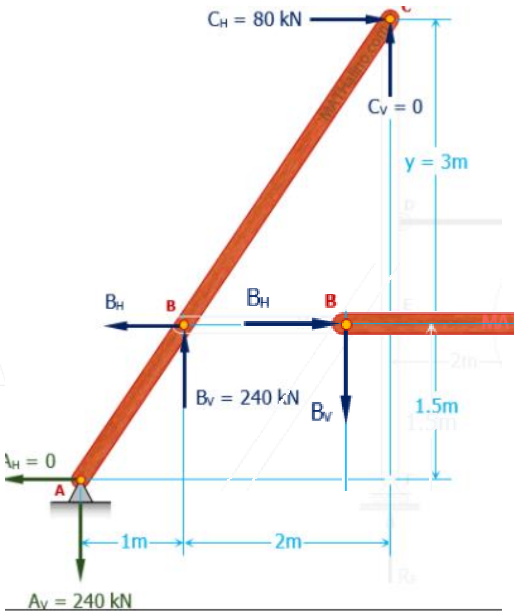


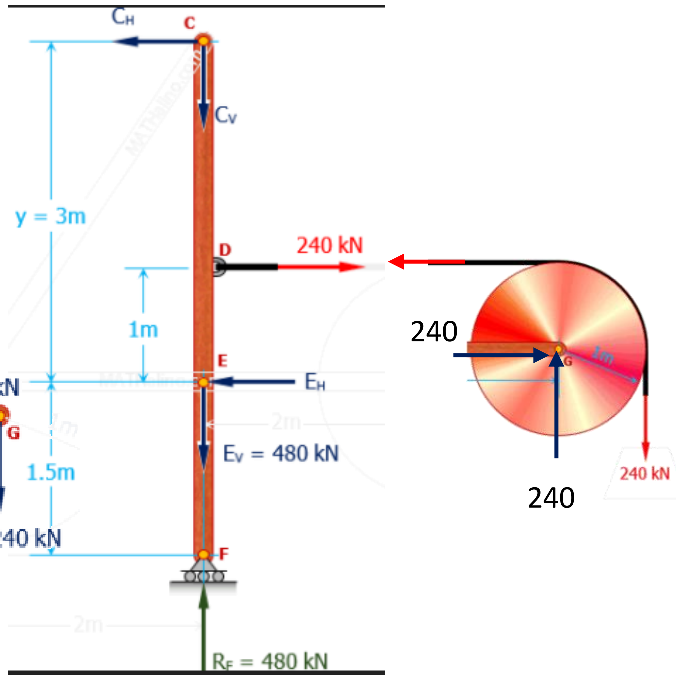
FBD of the entire frame



FBD of the each part







$\Sigma M_A = 0$

$3R_F = 6(240)$

$R_F = 480$ kN

$\Sigma M_F = 0 \Sigma MF = o$

$3A_V = 3(240)$

$A_V = 240$ kN

From the FBD of member BG

$\Sigma M_B = 0$

$2E_V = 4(240)$

$E_V = 480$ kN

$$\Sigma M_A = 0$$

$$1.5B_H + 1(240) = 4.5(80)$$

$$B_H = 80 \text{ kN}$$

Summary

$B_H = 80$ kN and $B_V = 240$ kN

$C_H = 80$ kN and $C_V = 0$

$E_H = 160$ kN and $E_V = 480$ kN

Thus,

$R_B = \sqrt{80^2 + 240^2} = 252.98$ kN ← *answer*

$R_C = \sqrt{80^2 + 0^2} = 80$ kN ← *answer*

$R_E = \sqrt{160^2 + 480^2} = 505.96$ kN ← *answer*

$$\Sigma M_E = 0$$

$$2B_V = 2(240)$$

$$B_V = 240 \text{ kN}$$

From the FBD of member CF

$$\Sigma F_V = 0$$

$$C_V + 480 = 480$$

$$C_V = 0$$

$$\Sigma M_E = 0$$

$$3C_H = 1(240)$$

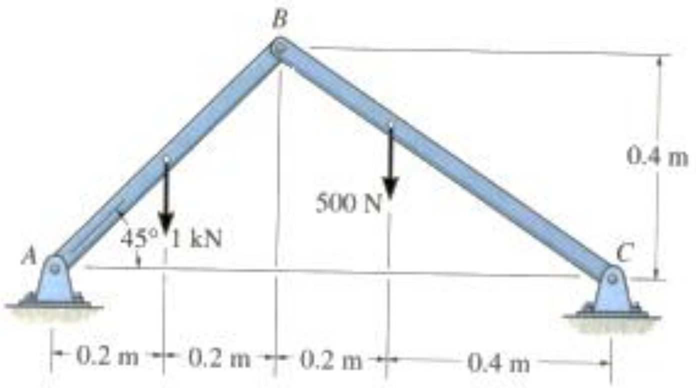
$$C_H = 80 \text{ kN}$$

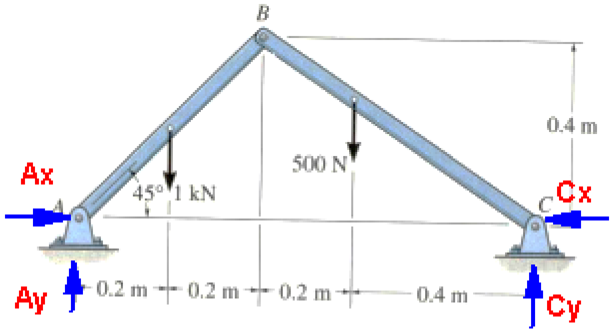
$$\Sigma M_C = 0$$

$$3E_H = (3-1)(240)$$

$$E_H = 160 \text{ kN}$$

Determine the external support reactions at hinges at A and C
Determine the internal forces exerted by Pin at B.

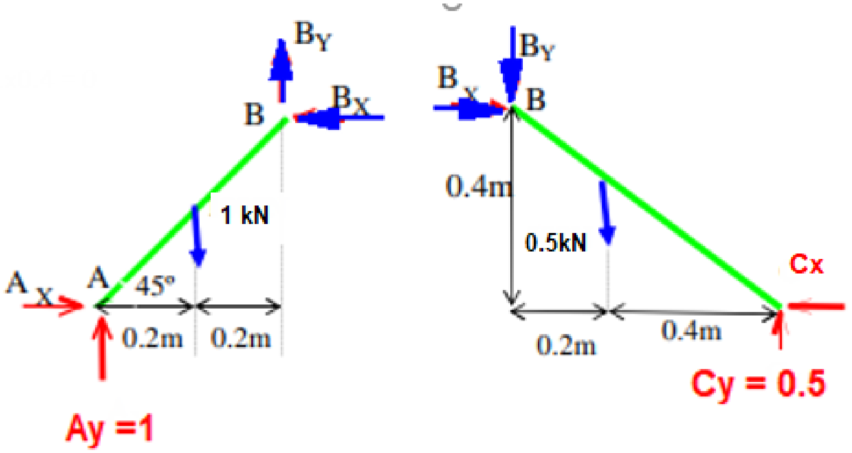




FBD of the entire frame

From the entire frame
There are 4 unknows and 3 equilibrium equations so try to find as much unknows as you can if possible

$\Sigma M_C = 0$ .... 0.5x.4 +1x.8 -1xAy =0
<u>Ay = 1 kN</u>
$\Sigma F_y = 0$ .... -1-0.5 +1 +Cy = 0
<u>Cy = 0.5 kN</u>
Show the results on the frame then draw FBD for each part

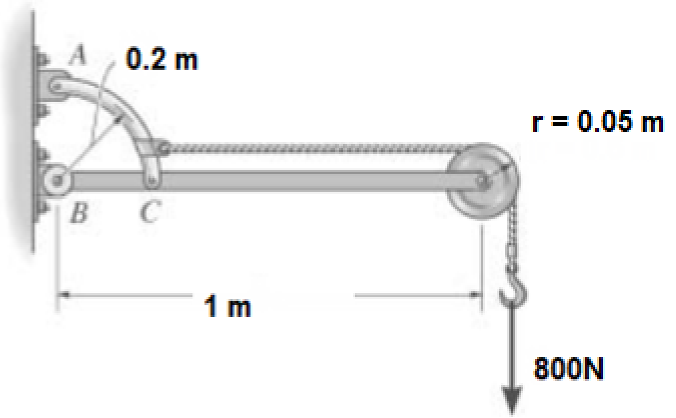


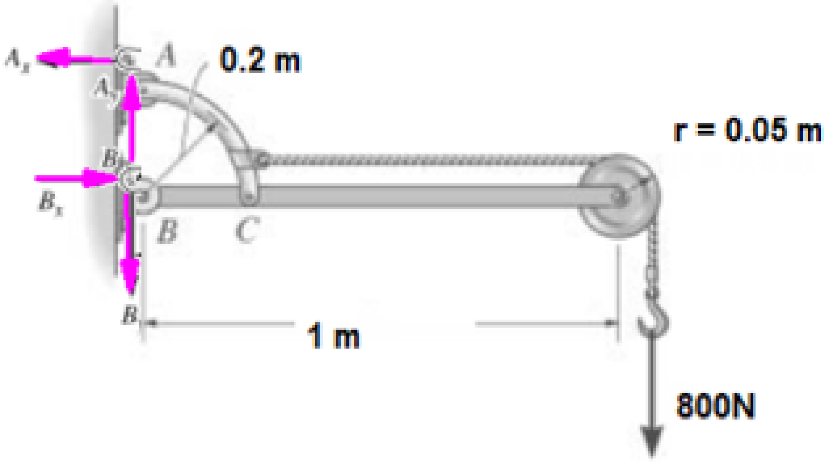
FBDs of each Part

Apply equilibrium equations to each part and solve for unknowns

Member AB
$\Sigma M_B = 0$ .... 1x0.2+0.4AX-1x0.4 = 0
<u>Ax = 0.5 kN</u>
$\Sigma F_x = 0$ .... O.5-Bx = 0
<u>Bx= 0.5 kN</u>
$\Sigma F_y = 0$ .... 1-1+By=0
<u>By = 0</u>
Member BC
$\Sigma F_x = 0$ .... –Cx+0.5= 0
<u>Cx= 0.5 kN</u>

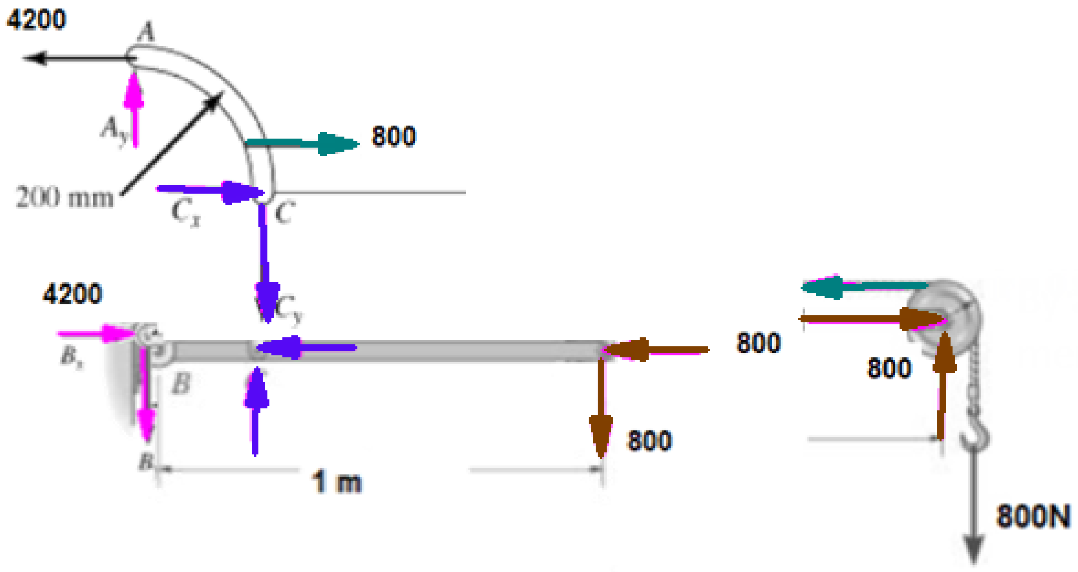
Determine the horizontal and vertical components of force that the pins at A, B, and C exert on their connecting members.





$A_x = 4.2\,\text{kN}$

$B_x = 4.2\,\text{kN}$



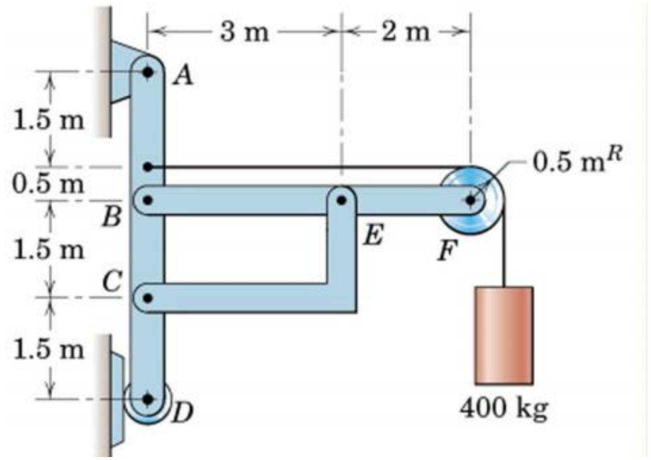
By applying equilibrium equations to member **AC** or **BC** D

$B_y = 3.2\,\text{kN}$

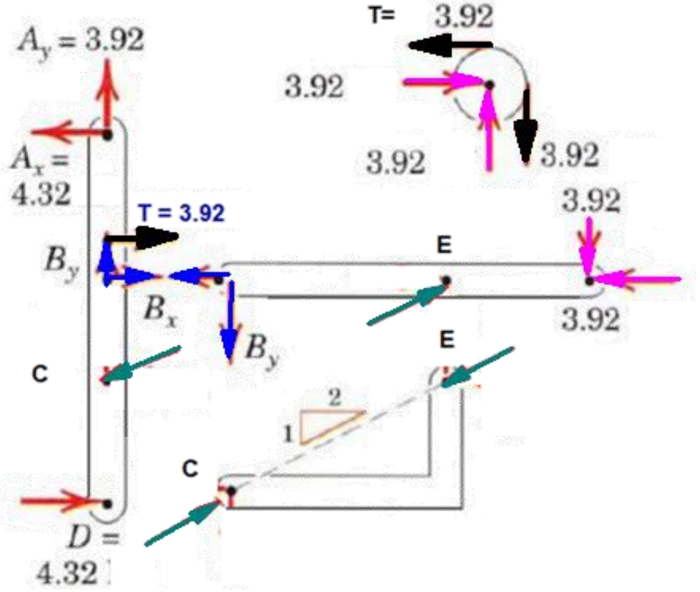
$C_x = 3.4\,\text{kN}$

$C_y = 4\,\text{kN}$

Determine external support reactions and all internal forces at all pins

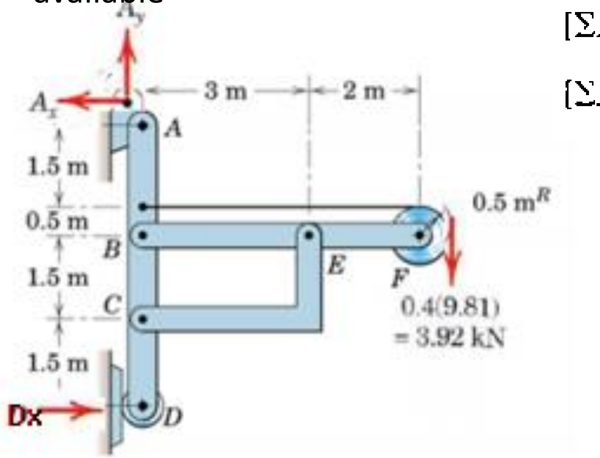


2. Draw FBD of each part knowing that CE is a two force member



1. Draw FBD of the entire frame

3 Equilibrium equations are available



$[\Sigma M_A = 0]$ $\quad 5.5(0.4)(9.81) - 5D = 0$

$[\Sigma F_x = 0]$ $\quad A_x - 4.32 = 0$

$[\Sigma F_y = 0]$ $\quad A_y - 3.92 = 0$

$D = 4.32$ kN

$A_x = 4.32$ kN

$A_y = 3.92$ kN

start with Member **BEF** (has 3 unknowns)

$[\Sigma M_B = 0]$ $\quad 3.92(5) - \frac{1}{2}E_x(3) = 0 \qquad E_x = 13.08$ kN

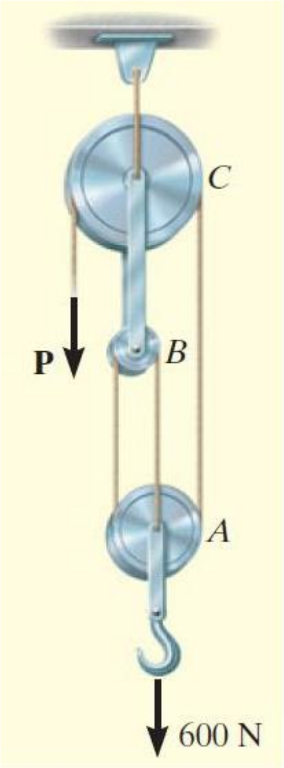
$[\Sigma F_y = 0]$ $\quad B_y + 3.92 - 13.08/2 = 0 \qquad B_y = 2.62$ kN

$[\Sigma F_x = 0]$ $\quad B_x + 3.92 - 13.08 = 0 \qquad B_x = 9.15$ kN

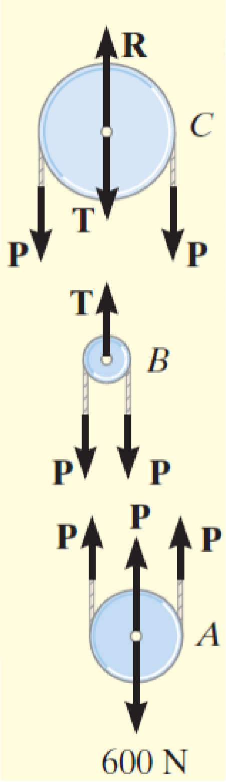
Member CE

∑Fx = 0 ……….. Cx = Ex = 13.08

Find **the tension in the cables** and the **force P** required to support the 600 N force using the frictionless pulley system (neglect self weight)



Draw FBD of each pulley (cutting all the cables) and apply equilibrium equations



Pulley A

$+\uparrow\Sigma F_y = 0;$ $3P - 600\text{ N} = 0$ $P = 200\text{ N}$

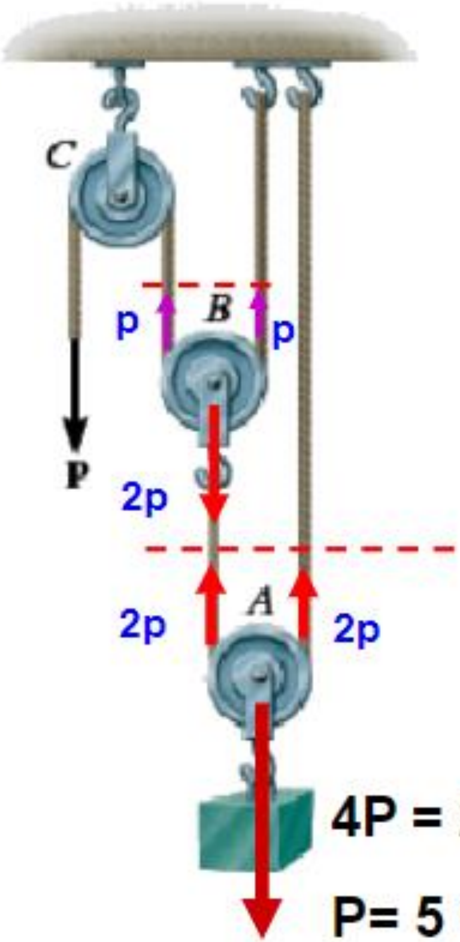
Pulley B

$+\uparrow\Sigma F_y = 0;$ $T - 2P = 0$ $T = 400\text{ N}$

Pulley C

$+\uparrow\Sigma F_y = 0;$ $R - 2P - T = 0$ $R = 800\text{ N}$

Determine the force **P** needed to hold the **20kN** block in equilibrium.

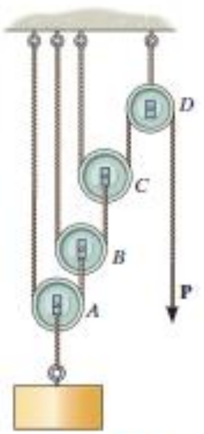


4P = 20

P= 5 kN

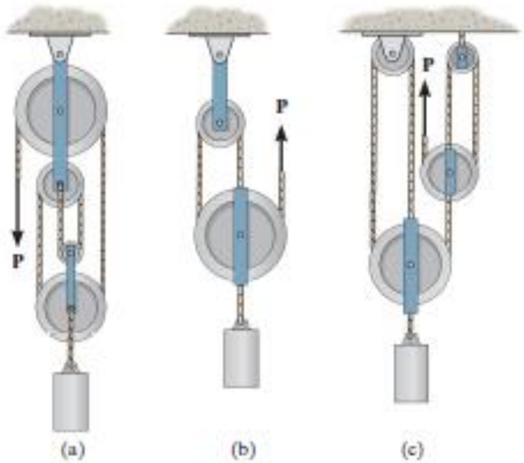
All problem solutions must include FBDs.

6–61. Determine the force **P** required to hold the 100-lb weight in equilibrium.



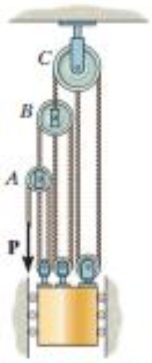
Prob. 6–61

6–62. In each case, determine the force **P** required to maintain equilibrium. The block weighs 100 lb.



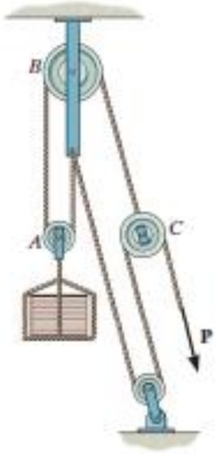
Prob. 6–62

6–63. Determine the force **P** required to hold the 50-kg mass in equilibrium.



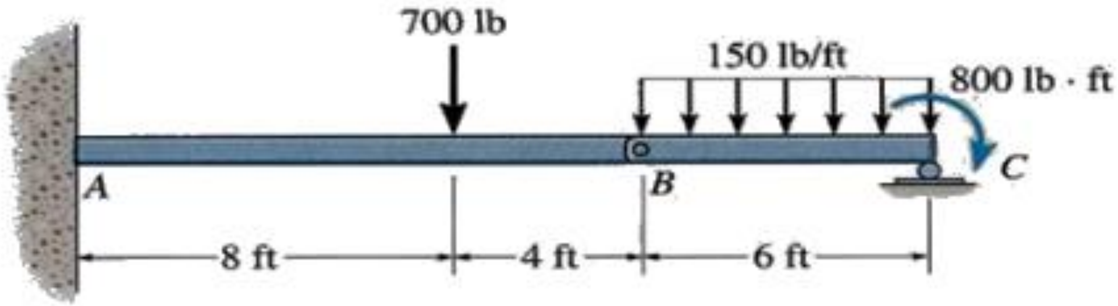
Prob. 6–63

***6–64.** Determine the force **P** required to hold the 150-kg crate in equilibrium.



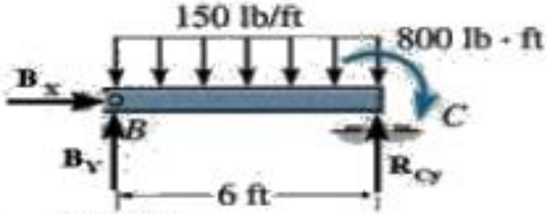
Prob. 6–64

Find the reactions

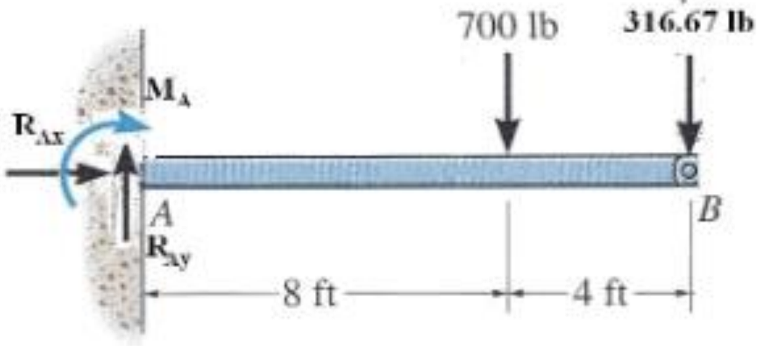


$$\sum F_x = 0 = R_{Ax}$$

$$\sum F_y = 0 = R_{Ay} - 700\text{ lb}$$
$$-316.67\text{ lb}$$



$$\sum M_B = 0 = -M_A - 700\text{ lb}(8\text{ ft})$$
$$-316.67\text{ lb}(12\text{ ft})$$



$$R_{Ay} = 1016.67\text{ lb}$$
$$R_{Ax} = 0\text{ lb}$$
$$M_A = -9400\text{ lb-ft}$$

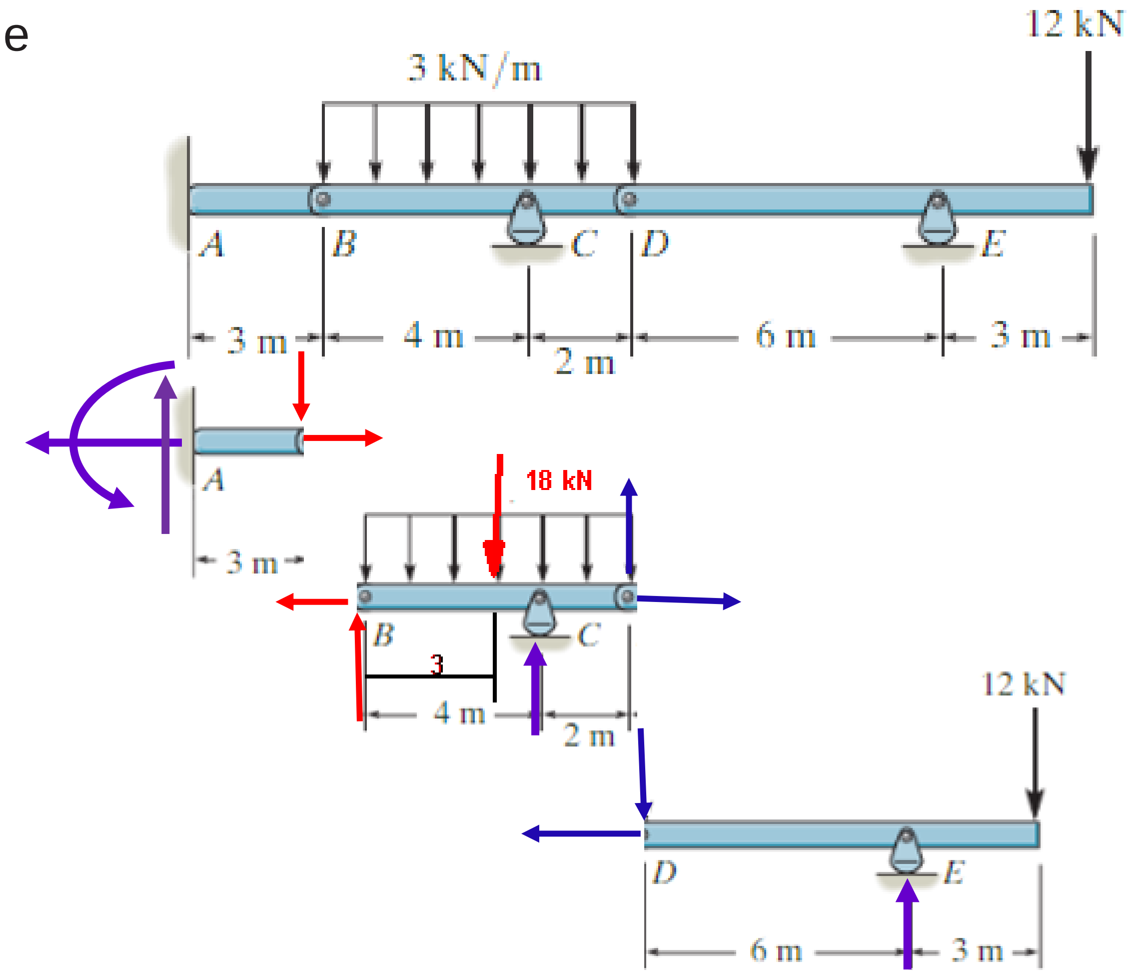
$$\sum F_x = 0 = B_X$$

$$\sum F_y = 0 = R_{Cy} + B_Y$$
$$-150\text{ lb/ft}(6\text{ ft})$$

$$\sum M_B = 0 = R_{Cy}(6\text{ ft})$$
$$-150\text{ lb/ft}(6\text{ ft})(3\text{ ft}) - 800\text{ lb-ft}$$

$$R_{Cy} = 583.33\text{ lb}$$
$$B_X = 0\text{ lb}$$
$$B_Y = 316.67\text{ lb}$$

Determine the reactions at the supports A, C, and E of the compound beam



Equations of Equilibrium. Equilibrium of member *DE* will be considered first by referring to Fig. *c*.

$\circlearrowleft + \Sigma M_D = 0;\quad N_E(6) - 12(9) = 0 \quad N_E = 18.0\text{ kN}$

$\circlearrowleft + \Sigma M_E = 0;\quad D_y(6) - 12(3) = 0 \quad D_y = 6.00\text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad D_x = 0$

Next, member *BD*, Fig. *b*.

$\circlearrowleft + \Sigma M_C = 0;\quad 6.00(2) + 3(6)(1) - B_y(4) = 0 \quad B_y = 7.50\text{ kN}$

$\circlearrowleft + \Sigma M_B = 0;\quad N_C(4) + 6.00(6) - 3(6)(3) = 0 \quad N_C = 4.50\text{ kN}$

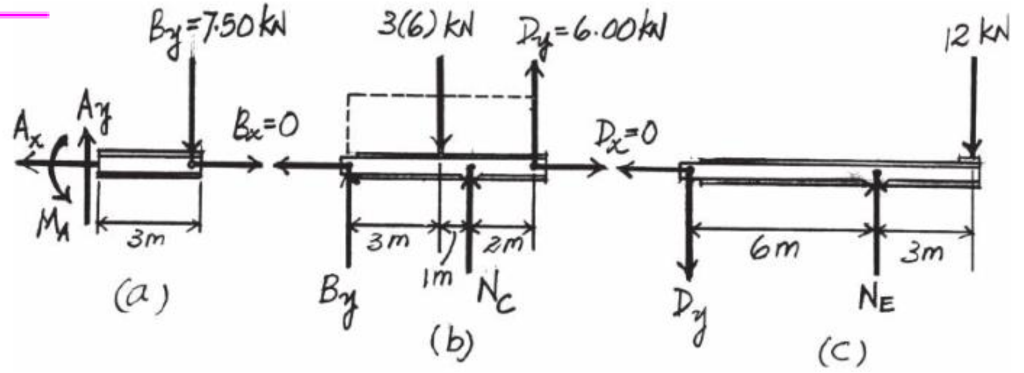
$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad B_x = 0$

Finally, member *AB*, Fig. *a*

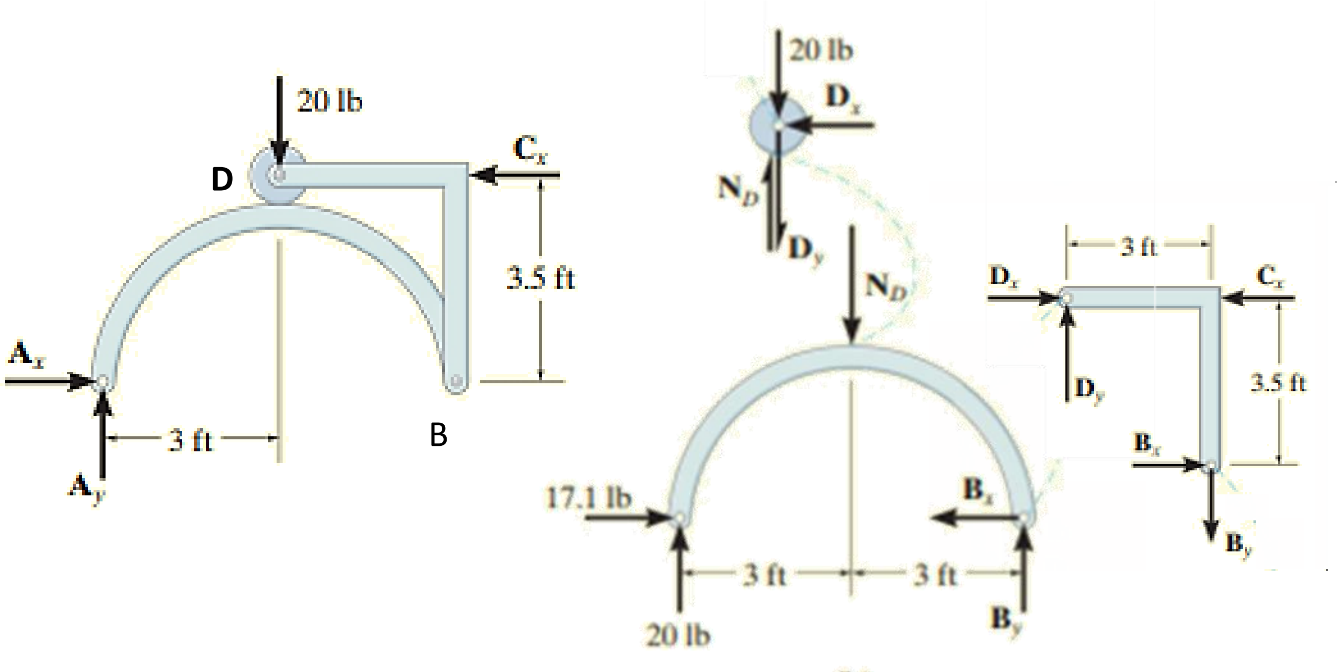
$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x = 0$

$+\uparrow \Sigma F_y = 0;\quad A_y - 7 - 50 = 0 \quad A_y = 7.50\text{ kN}$

$\circlearrowleft + \Sigma M_A = 0;\quad M_A - 7.50(3) = 0 \quad M_A = 22.5\text{ kN}\cdot\text{m}$



The smooth disk shown in is pinned at D and has a weight of 20 lb. Neglecting the weights of the other members, ***determine the horizontal and vertical components of reaction at pins B and D.***



(b)

Free-Body Diagrams. The free-body diagrams of the entire frame and each of its members

Entire Frame

$\circlearrowleft +\Sigma M_A = 0; \quad -20 \text{ lb } (3 \text{ ft}) + C_x(3.5 \text{ ft}) = 0 \quad C_x = 17.1 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 17.1 \text{ lb} = 0 \quad A_x = 17.1 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad A_y - 20 \text{ lb} = 0 \quad A_y = 20 \text{ lb}$

Member AB

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 17.1 \text{ lb} - B_x = 0 \quad B_x = 17.1 \text{ lb}$

$\circlearrowleft +\Sigma M_B = 0; \quad -20 \text{ lb } (6 \text{ ft}) + N_D(3 \text{ ft}) = 0 \quad N_D = 40 \text{ lb}$

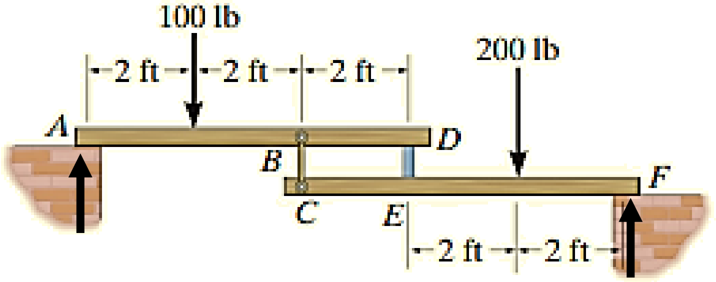
$+\uparrow \Sigma F_y = 0; \quad 20 \text{ lb} - 40 \text{ lb} + B_y = 0 \quad B_y = 20 \text{ lb}$

Disk

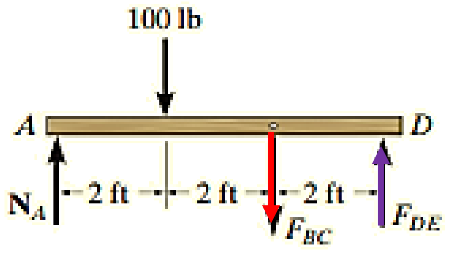
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad D_x = 0$

$+\uparrow \Sigma F_y = 0; \quad 40 \text{ lb} - 20 \text{ lb} - D_y = 0 \quad D_y = 20 \text{ lb}$

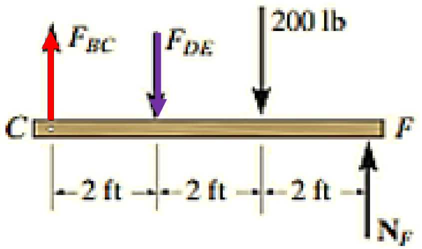
The two planks in a are connected together by cable BC and a smooth spacer DE. Determine the reactions at the smooth **supports A and F**, and also find the **force developed in the cable and spacer.**



(a)



120 lb



For the entire beam

$\Sigma M_F = 0$ …… $2x200 + 100x8 - N_A\ x10 = 0$ $N_A = 120$ lb

$\Sigma F_y = 0$; $-100-200+120+N_F = 0$ $N_F = 180$ lb

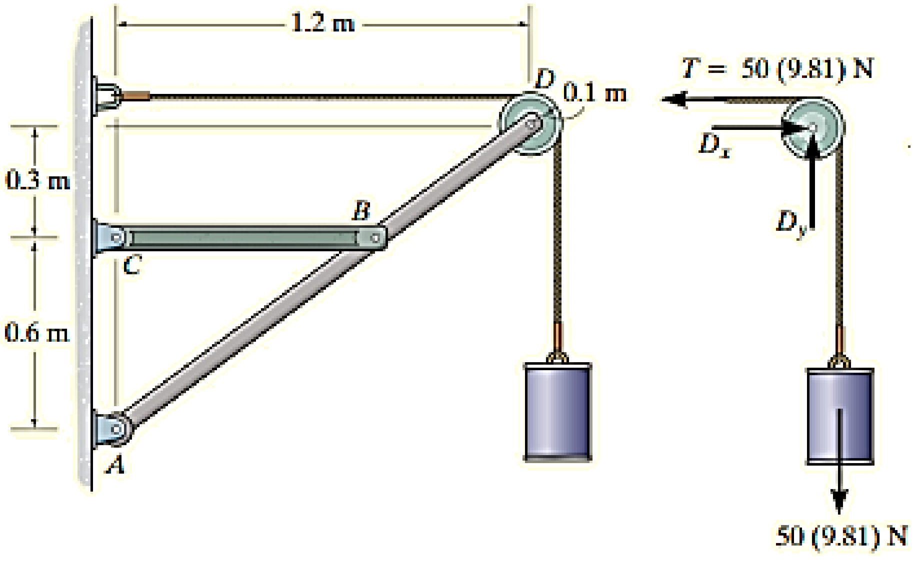
Member AD

Considering now the free-body diagram of section AD we have

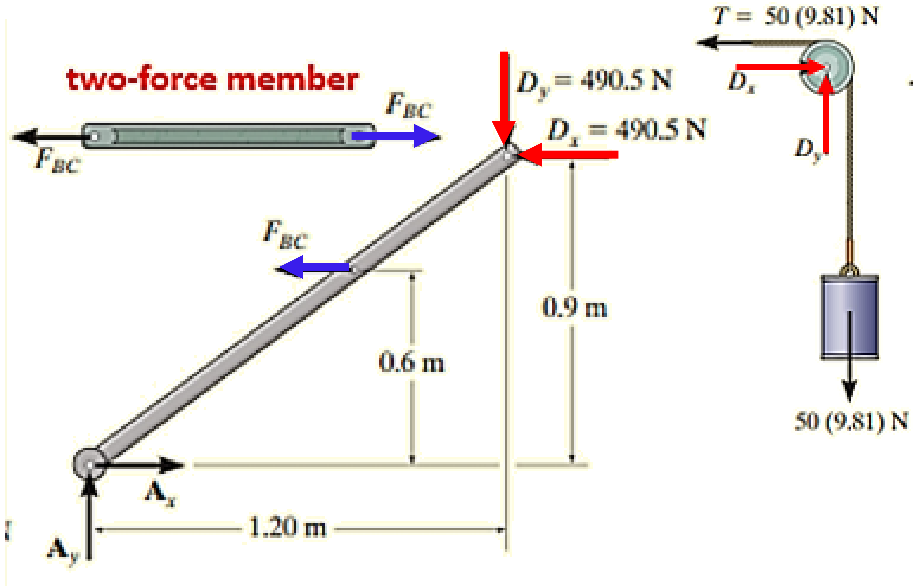
$\Sigma M_D = 0$ …… $F_{BC}\ x2 + 100x4 - 120x6 = 0$ $F_{BC} = 160$ lb

$\Sigma F_y = 0$; $-100-160+120+F_{DE} = 0$ FDE= 140 lb

The frame in Figa supports the 50-kg cylinder. Determine the **horizontal and vertical components of reaction at A and the force at C.**



(a)



(b)

Equations of Equilibrium. We will begin by analyzing the equilibrium of the pulley. The moment equation of equilibrium is automatically satisfied with $T = 50(9.81)$ N, and so

$$\xrightarrow{+} \Sigma F_x = 0; \qquad D_x - 50(9.81)\text{ N} = 0 \quad D_x = 490.5\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad D_y - 50(9.81)\text{ N} = 0 \quad D_y = 490.5\text{ N} \qquad \textit{Ans.}$$

Using these results, F_{BC} can be determined by summing moments about point A on member ABD.

$$\circlearrowleft + \Sigma M_A = 0; F_{BC}(0.6\text{ m}) + 490.5\text{ N}(0.9\text{ m}) - 490.5\text{ N}(1.20\text{ m}) = 0$$

$$F_{BC} = 245.25\text{ N} \qquad \textit{Ans.}$$

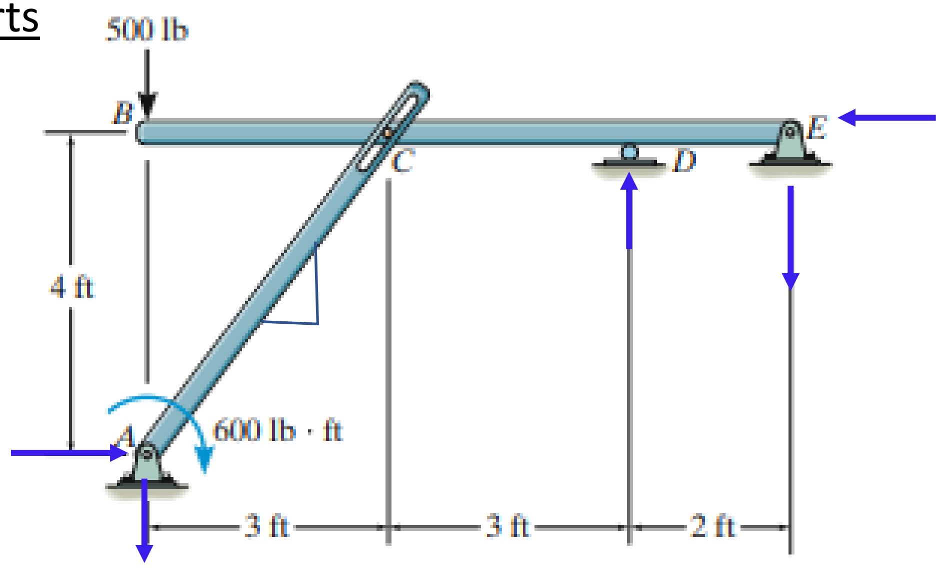
Now A_x and A_y can be determined by summing forces.

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 245.25\text{ N} - 490.5\text{ N} = 0 \quad A_x = 736\text{ N} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 490.5\text{ N} = 0 \quad A_y = 490.5\text{ N} \qquad \textit{Ans.}$$

Equatic

The two-member structure is connected at C by a pin, which is fixed to BDE and passes through the smooth slot in member AC. Determine the horizontal and vertical components of reaction at the supports



Member *AC*:

$\circlearrowleft + \Sigma M_A = 0;\quad N_C(5) - 600 = 0$

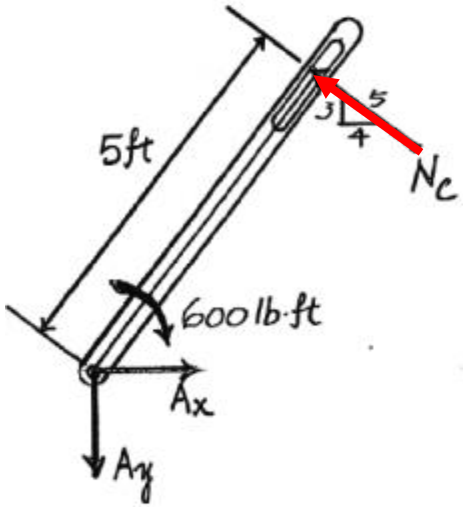
$N_C = 120 \text{ lb}$

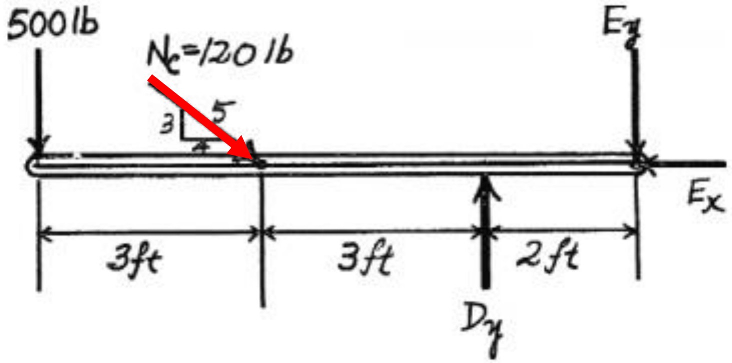
$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x - 120\left(\frac{4}{5}\right) = 0$

$A_x = 96 \text{ lb}$

$+\uparrow \Sigma F_y = 0;\quad -A_y + 120\left(\frac{3}{5}\right) = 0$

$A_y = 72 \text{ lb}$





Member *BDE*:

$\circlearrowleft + \Sigma M_E = 0;\quad 500(8) + 120\left(\frac{3}{5}\right)(5) - D_y(2) = 0$

$D_y = 2180 \text{ lb} = 2.18 \text{ kip}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad -E_x + 120\left(\frac{4}{5}\right) = 0$

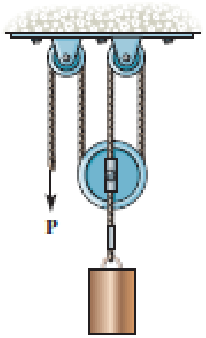
$E_x = 96 \text{ lb}$

$+\uparrow \Sigma F_y = 0;\quad -500 - 120\left(\frac{3}{5}\right) + 2180 - E_y = 0$

$E_y = 1608 \text{ lb} = 1.61 \text{ kip}$

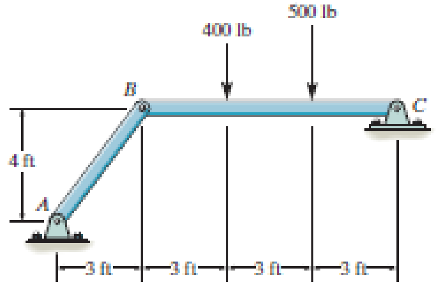
All problem solutions must include FBDs.

F6–13. Determine the force P needed to hold the 60-lb weight in equilibrium.



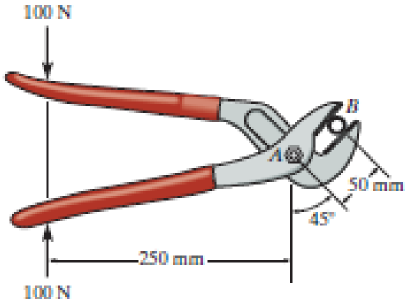
Prob. F6–13

F6–14. Determine the horizontal and vertical components of reaction at pin C.



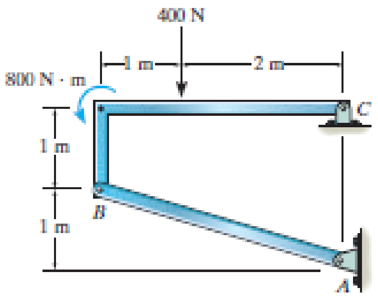
Prob. F6–14

F6–15. If a 100-N force is applied to the handles of the pliers, determine the clamping force exerted on the smooth pipe B and the magnitude of the resultant force that one of the members exerts on pin A.



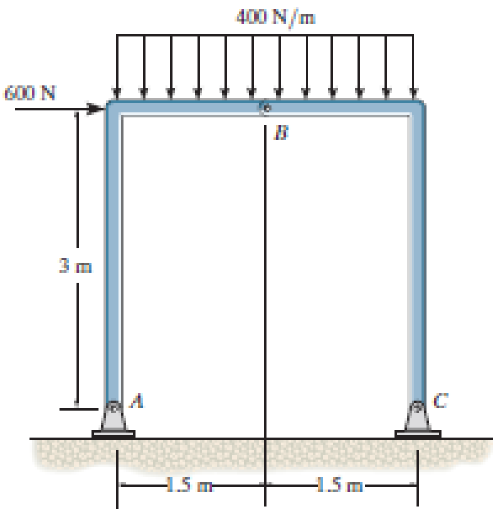
Prob. F6–15

F6–16. Determine the horizontal and vertical components of reaction at pin C.



Prob. F6–16

F6–21. Determine the components of reaction at A and C



Prob. F6–21

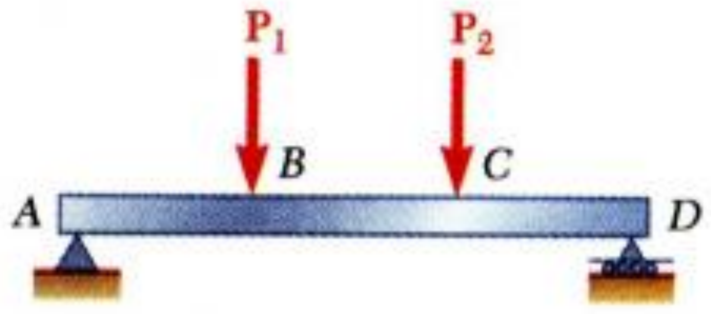
Chapter 7

Internal Forces

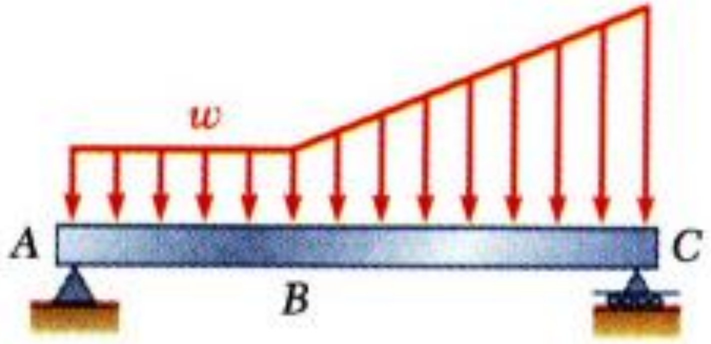
Iqbal Marie

Beams

A beam is defined as a structural member designed primarily to support forces acting perpendicular to the axis of the member.



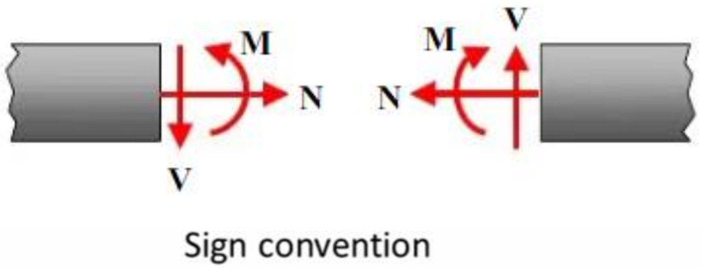
(a) Concentrated loads

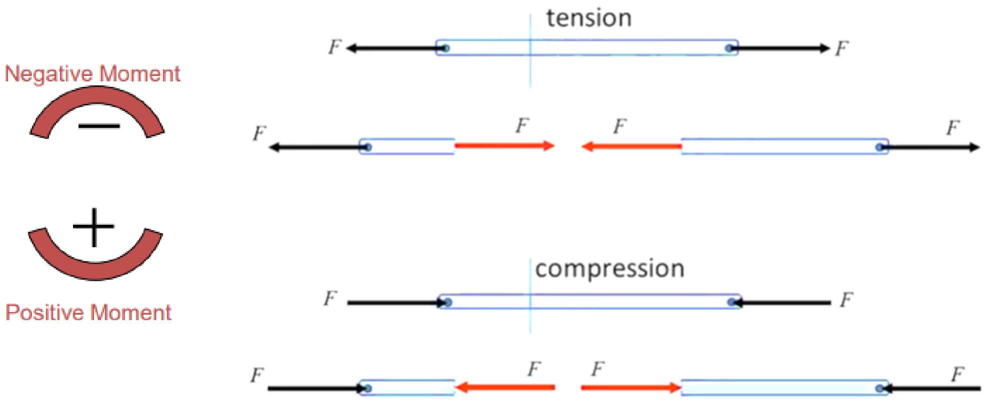


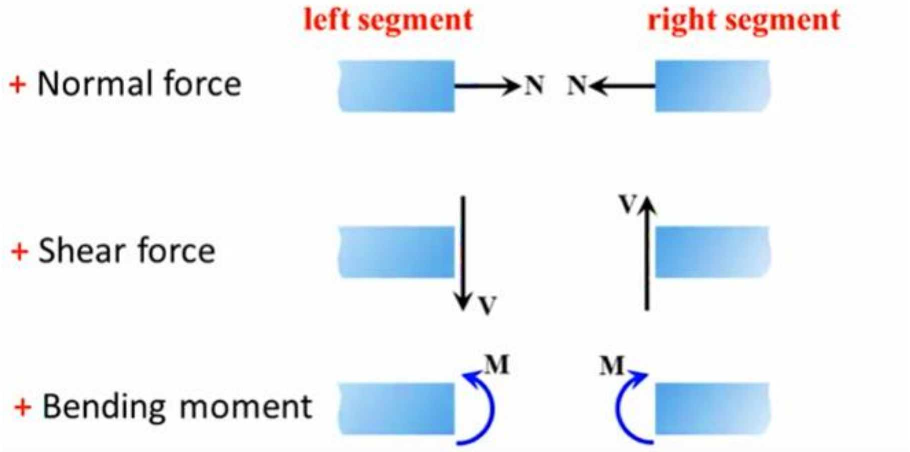
(b) Distributed load

Internal Force in Structures

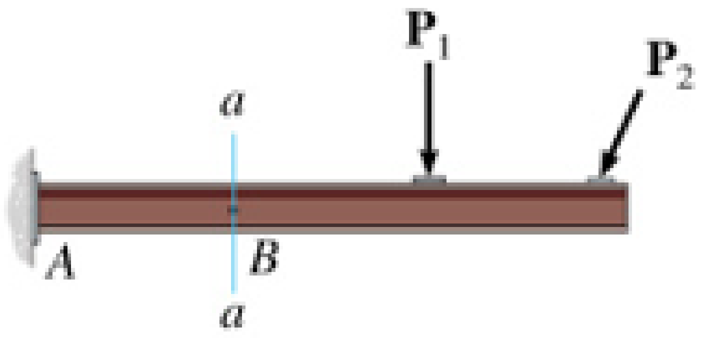
- Shear Forces (V)
- Bending Moment (M)
- Normal Forces (N Tension or compression)

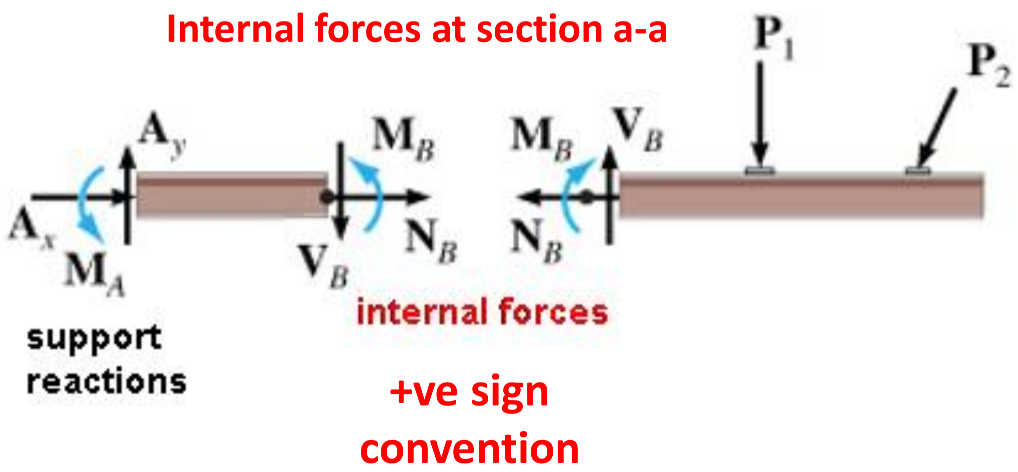


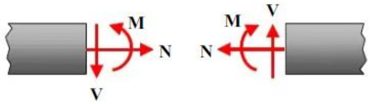




For each cross section, there is a shear force **V** and a bending moment **M** and a normal force **N**.







Solution Procedure

The general scheme for finding the internal set of forces at certain point is:

a) Draw the free-body diagram

b) Determine the support reactions by applying equations of equilibrium

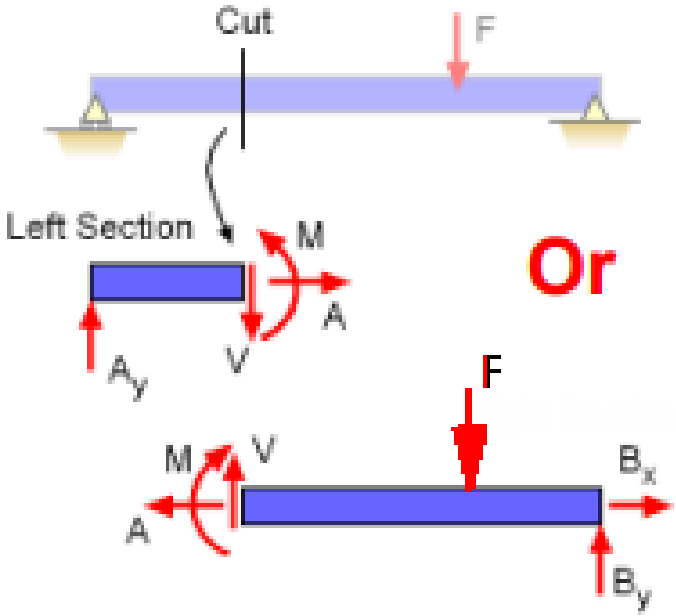
$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_z = 0$$

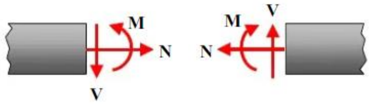
d) Make section passing through the point

e) Draw free body diagram for either part of the beam (the one to the right or left of the section) showing the positive internal forces on it (N, V, M)

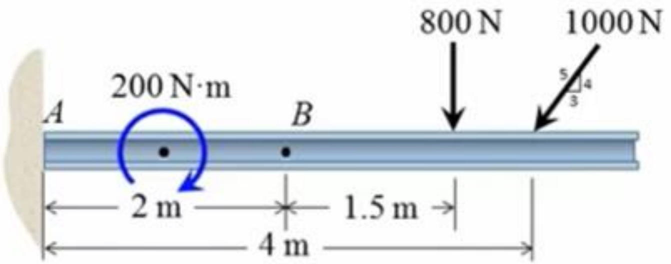
f) Apply equilibrium equations for that part and find the internal forces

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_z = 0$$

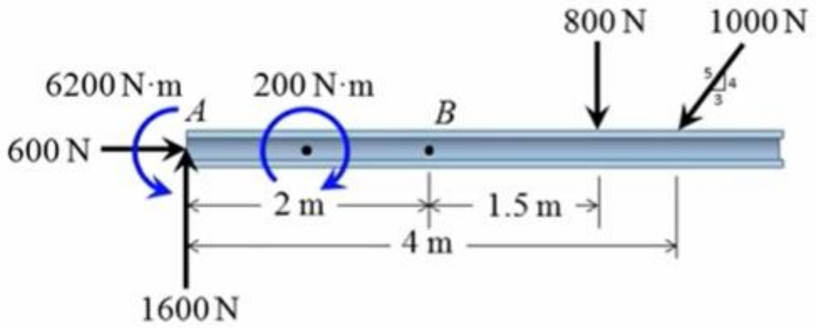




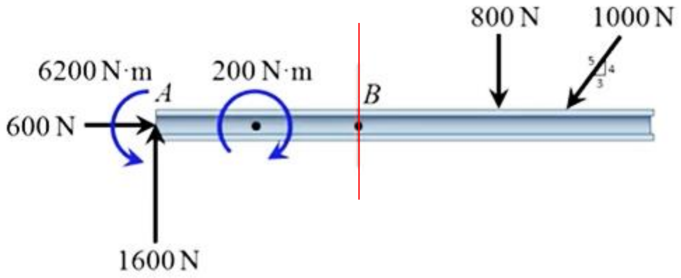
Determine the internal forces at point B of the cantilever beam.



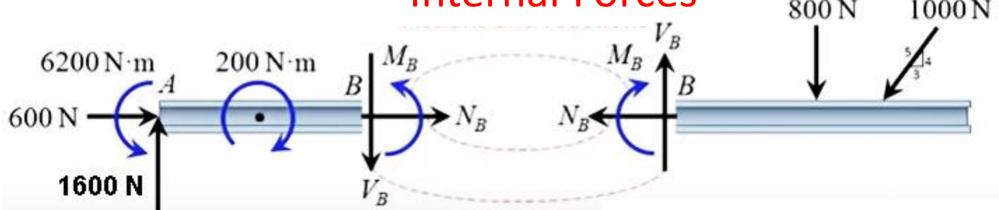
Step 1: If necessary, determine external support reaction(s).



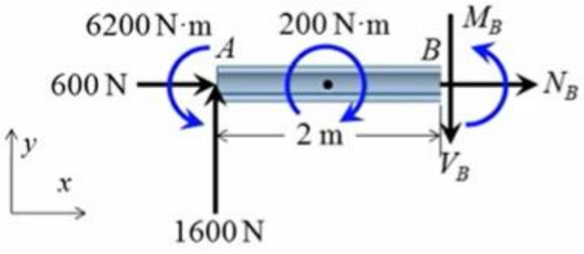
Step 2: "Cut" the member at the specified point and note the unknowns.



Internal Forces



Solving the left segment:



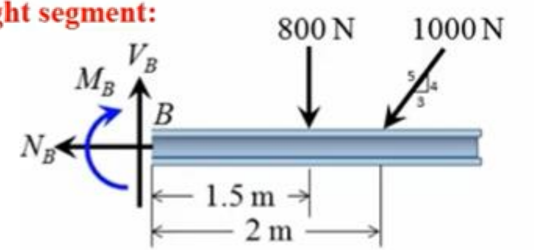
$$\sum F_x = 600\text{N} + N_B = 0$$

$$\sum F_y = 1600\text{N} - V_B = 0$$

$$\sum M_B = 6200\text{N}\cdot\text{m} - 200\text{N}\cdot\text{m} - 1600\text{N}\cdot 2\text{m} + M_B = 0$$

$$N_B = -600 \text{ N}$$
$$V_B = 1600 \text{ N}$$
$$M_B = -2800 \text{ N}\cdot\text{m}$$

right segment:



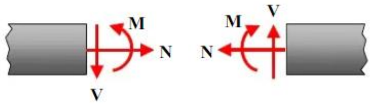
$$\sum F_x = -N_B - \frac{3}{5}\cdot 1000\text{N} = 0$$

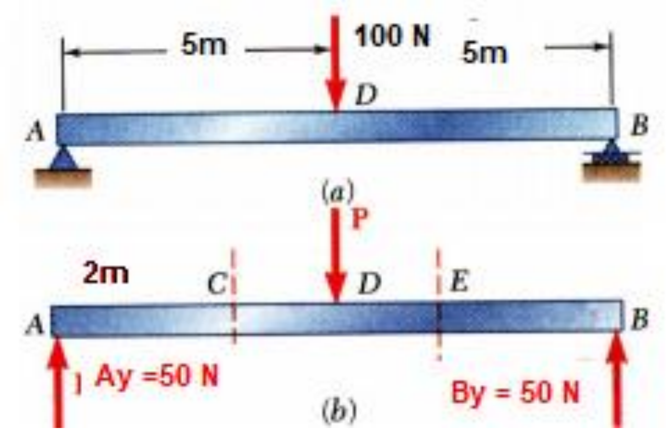
$$\sum F_y = V_B - 800\text{N} - \frac{4}{5}\cdot 1000\text{N} = 0$$

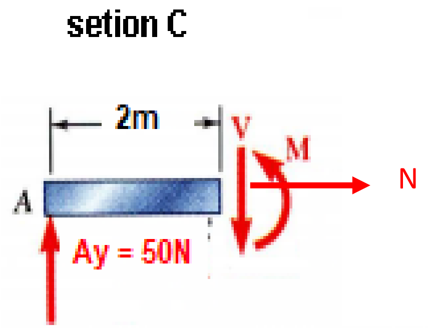
$$\sum M_B = -M_B - 800\text{N}\cdot 1.5\text{m} - \frac{4}{5}\cdot 1000\text{N}\cdot 2\text{m} = 0$$

$$N_B = -600 \text{ N}$$
$$V_B = 1600 \text{ N}$$
$$M_B = -2800 \text{ N}\cdot\text{m}$$

find internal forces **point c and E**



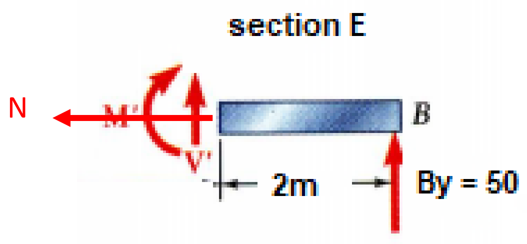




Σ Fy = 0 ……. 50 – Vc = 0 ……. Vc = + 50N

Σ Fx= 0 ……. . Nc=0

Σ Mc=0 -50x2 +Mc = 0 …… Mc = +100 N.m

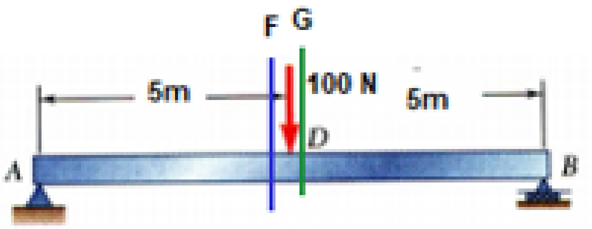


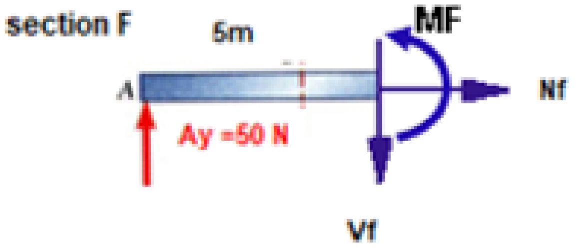
Σ Fy = 0 ……. 50 + VE = 0 VE = -50N

Σ Fx= 0 ……. NE=0

Σ ME=0 +50x2 -ME = 0 ME = +100 N.m

To find internal forces **at the concentrated load**. (**Pint D**.)
Make two sections; one just to the left of the load and another just to the right of the load

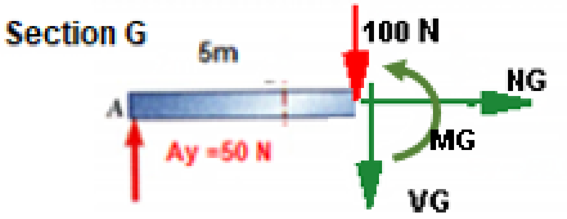




Σ Fy = 0 ……. 50 – VF = 0 ……. VF = + 50N

Σ Fx= 0 ……. . NF=0

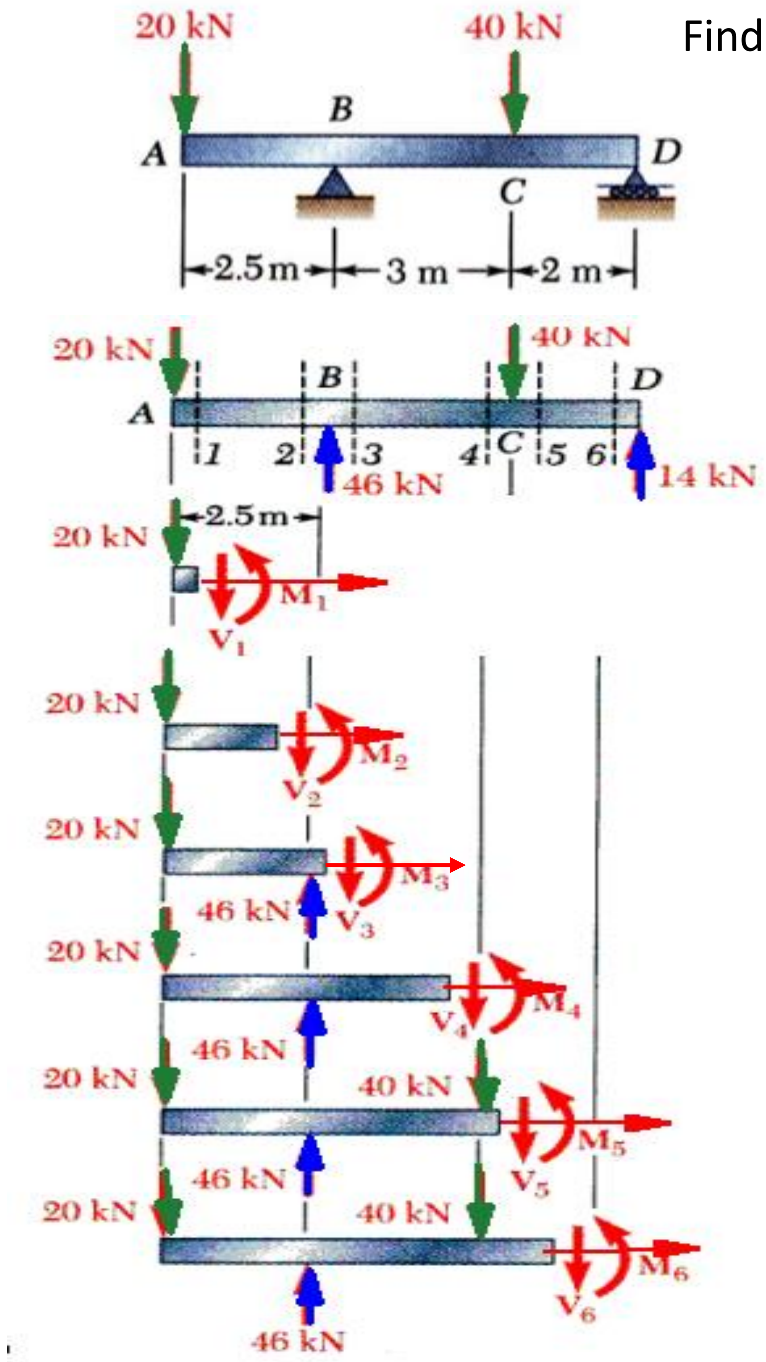
Σ MF=0 -50x5 +MF = 0 …… MF = +250 N.m



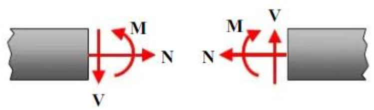
Σ Fy = 0 ……. 50 – 100 - VG = 0 VG = - 50N

Σ Fx= 0 ……. NG=0

Σ MG=0 -50x5 +MG = 0 MG = +250 N.m



Find internal forces at the shown sections



Taking entire beam as a free-body, calculate reactions at B and D.

Section 1

$$\sum F_y = 0: \quad -20\,\text{kN} - V_1 = 0 \qquad \boxed{V_1 = -20\,\text{kN}}$$

$$\sum M_1 = 0: \quad (20\,\text{kN})(0\,\text{m}) + M_1 = 0 \qquad \boxed{M_1 = 0}$$

Apply equilibrium equations for all section

V_2 = -20 kN M_2 = -50 kN.m

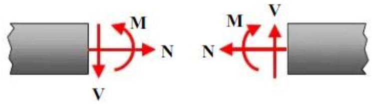
$V_3 = 26\,\text{kN}$ $M_3 = -50\,\text{kN}\cdot\text{m}$

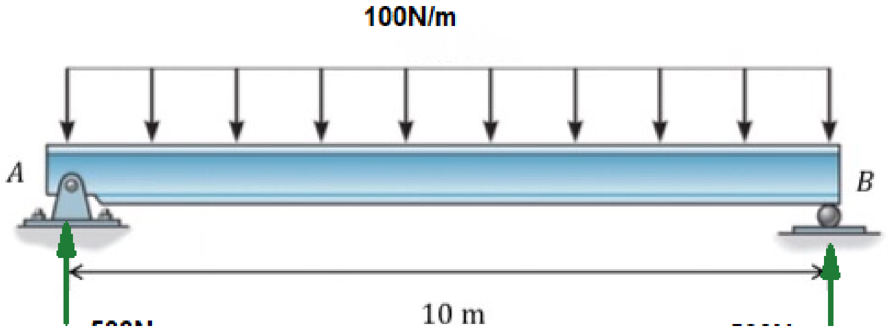
$V_4 = 26\,\text{kN}$ $M_4 = +28\,\text{kN}\cdot\text{m}$

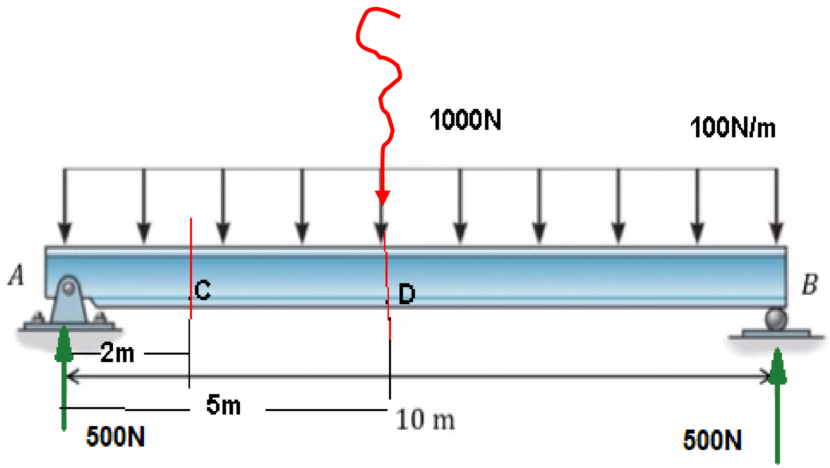
$V_5 = -14\,\text{kN}$ $M_5 = +28\,\text{kN}\cdot\text{m}$

$V_6 = -14\,\text{kN}$ $M_6 = 0$

Find internal forces at point C and D which are 2m and 5m to the right of A respectively

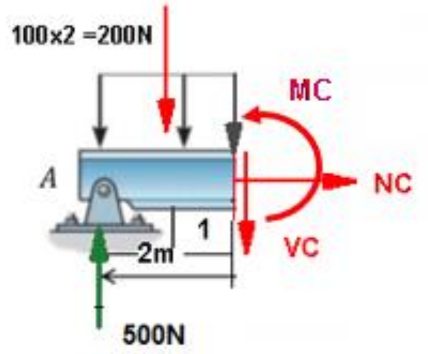






Support reactions

Section C

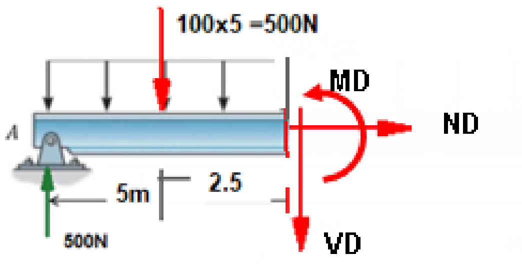


Σ Fy = 0 ……. 500-200- Vc = 0 Vc = 300N

Σ Fx= 0 ……. Nc=0

Σ Mc=0 -500x2 + 200x1 +Mc = 0 Mc = +800 N.m

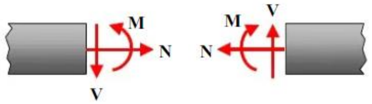
Section D



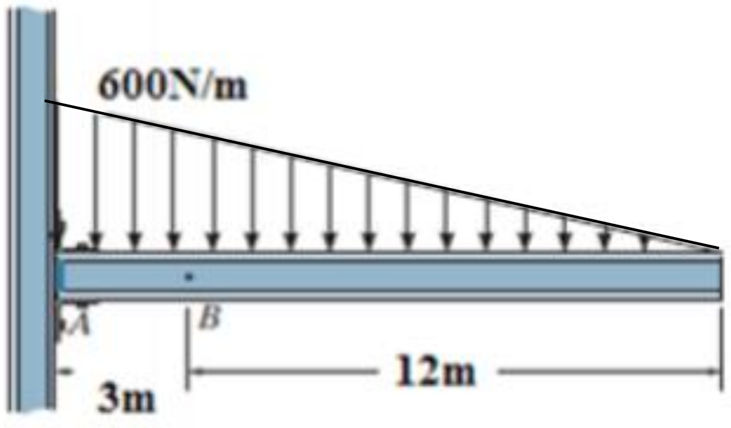
Σ Fy = 0 … 500-500- VD = 0 VD = 0

Σ Fx= 0 ……. ND=0

Σ MD=0 -500x5 + 500x 2.5 +MD = 0 MD = +1250 N.m

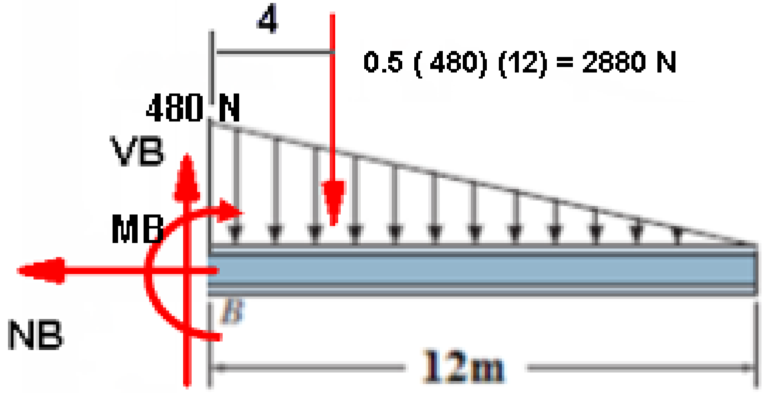


Determine the internal normal force, shear force, and moment in the cantilever beam at point B.



No need to find the support reaction if taking the right part of the section

600/15 = y/ 12
Y= 480 N/m



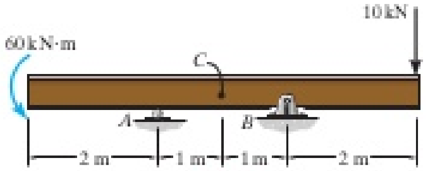
Σ Fy = 0 ……. -2800+ VB = 0 — VB = + 2800 N

Σ Fx= 0 ……. — NB=0

Σ MB=0 -2880x4 - MB = 0 — MB = -11520 N.m

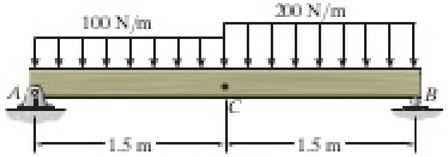
FUNDAMENTAL PROBLEMS

F1–1. Determine the internal normal force, shear force, and bending moment at point *C* in the beam.



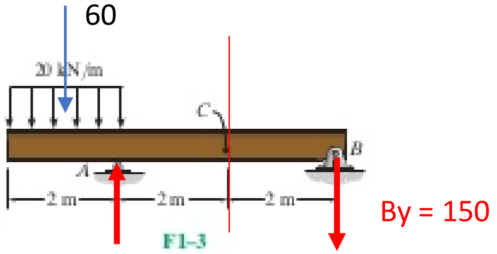
F1–1

F1–2. Determine the internal normal force, shear force, and bending moment at point *C* in the beam.



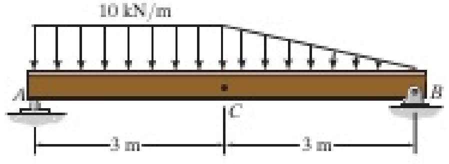
F1–2

F1–3. Determine the internal normal force, shear force, and bending moment at point *C* in the beam.



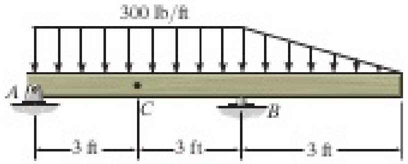
F1–3

F1–4. Determine the internal normal force, shear force, and bending moment at point *C* in the beam.

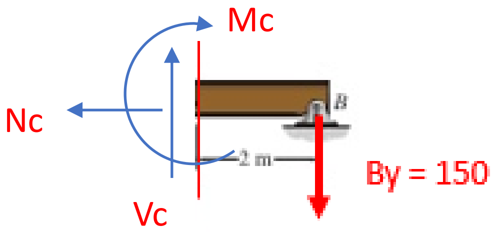


F1–4

F1–5. Determine the internal normal force, shear force, and bending moment at point *C* in the beam.

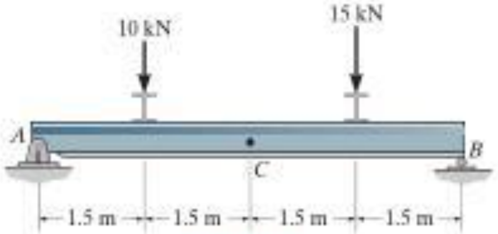


F1–5



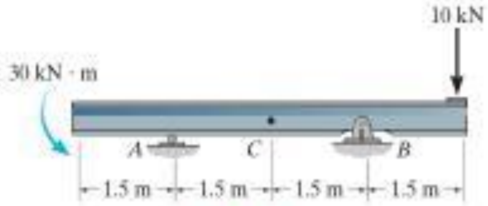
All problem solutions must include FBDs.

F7–1. Determine the normal force, shear force, and moment at point *C*.



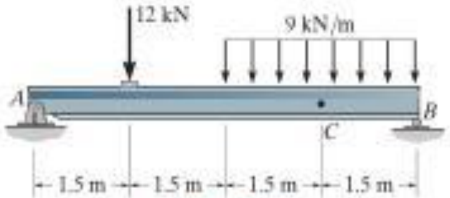
Prob. F7–1

F7–2. Determine the normal force, shear force, and moment at point *C*.



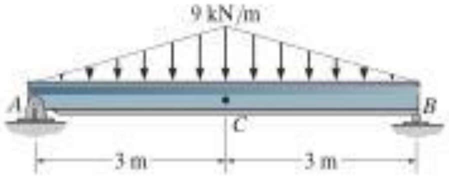
Prob. F7–2

F7–4. Determine the normal force, shear force, and moment at point *C*.



Prob. F7–4

F7–5. Determine the normal force, shear force, and moment at point *C*.



Prob. F7–5

Σ Fy = 0 ……. -150+ Vc = 0

Σ Fx= 0 …….

Σ MB=0 -150x2 - MB = 0

<u>Vc = + 150 N</u>

<u>Nc=0</u>

<u>Mc = -300 N.m</u>

7.2 and 7.3 Shear and Bending moment Diagrams

a) Draw the free-body diagram

b) Determine the support reactions by applying equations of equilibrium

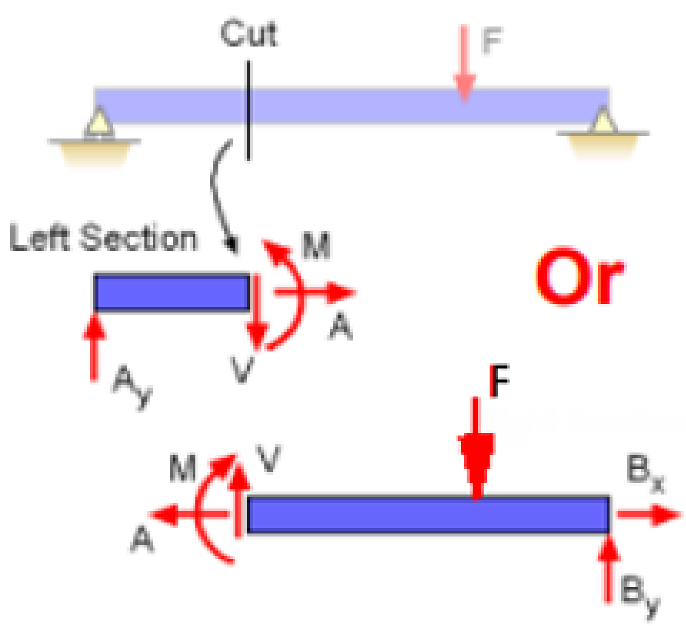
$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_z = 0$$

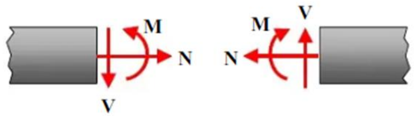
d) Make section at points (where needed) along the axis of the beam

e) Draw free body diagram for either part of the beam (the one to the right or left of the section) showing the positive internal forces on it (N, V, M)

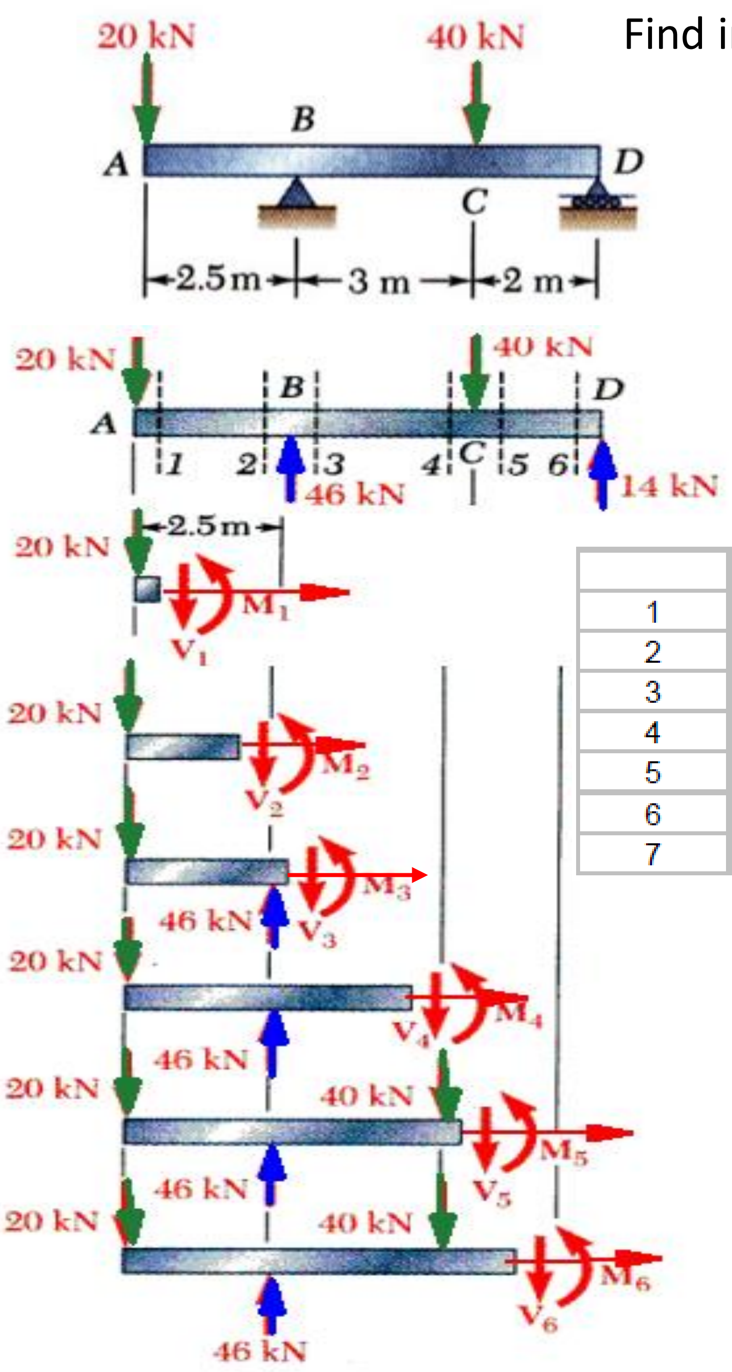
f) Apply equilibrium equations for that part and find the internal forces

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_z = 0$$

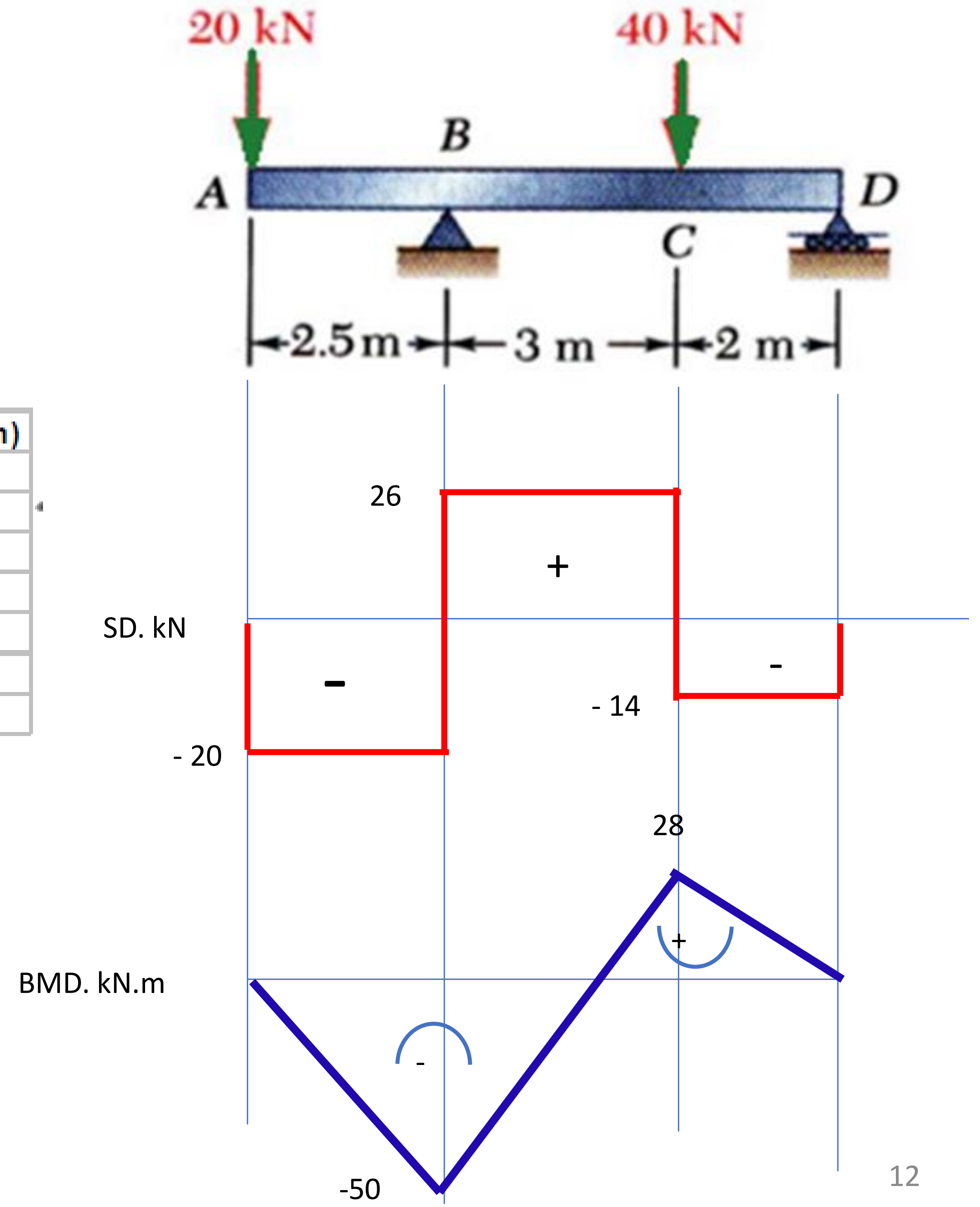


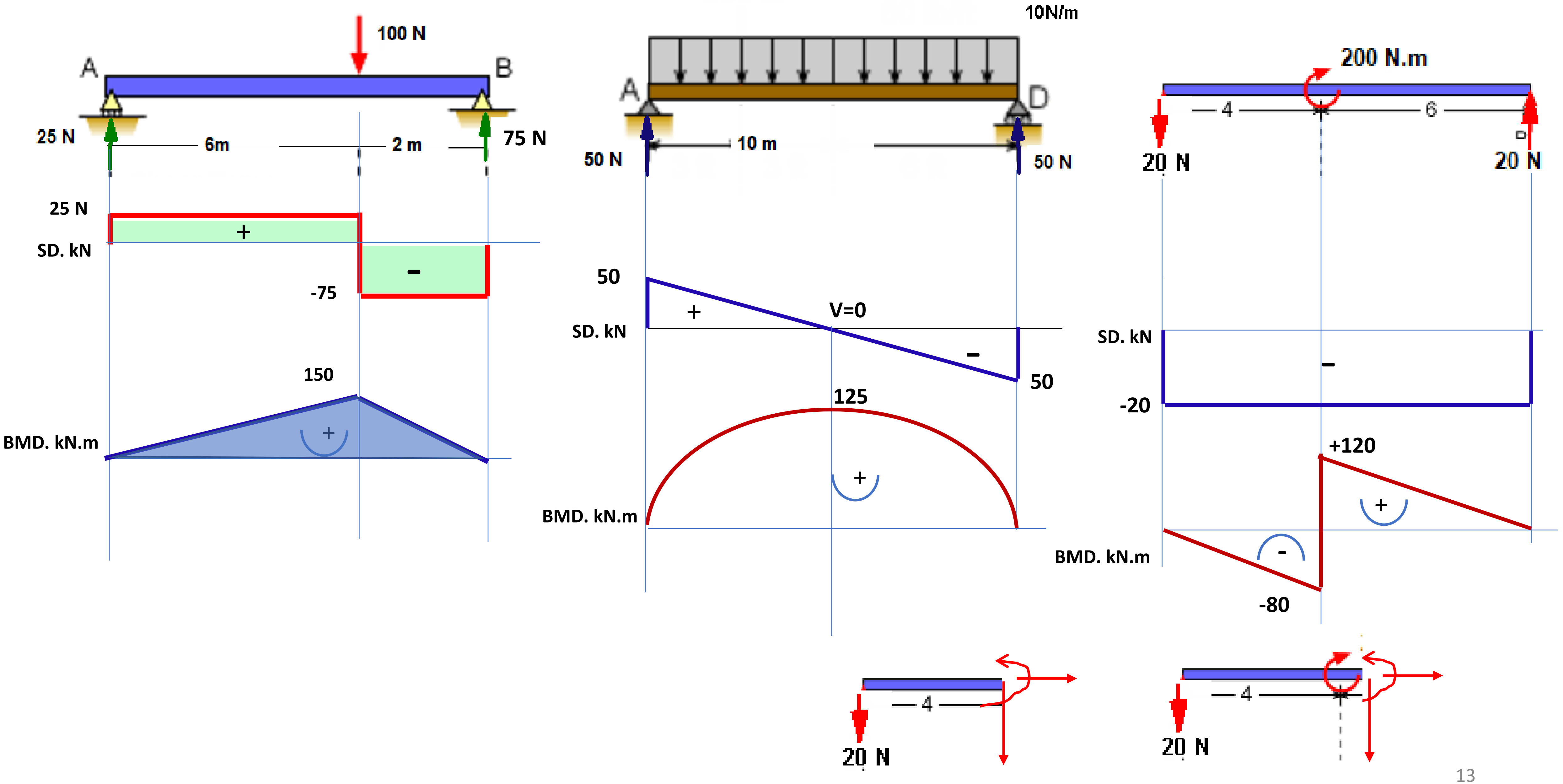


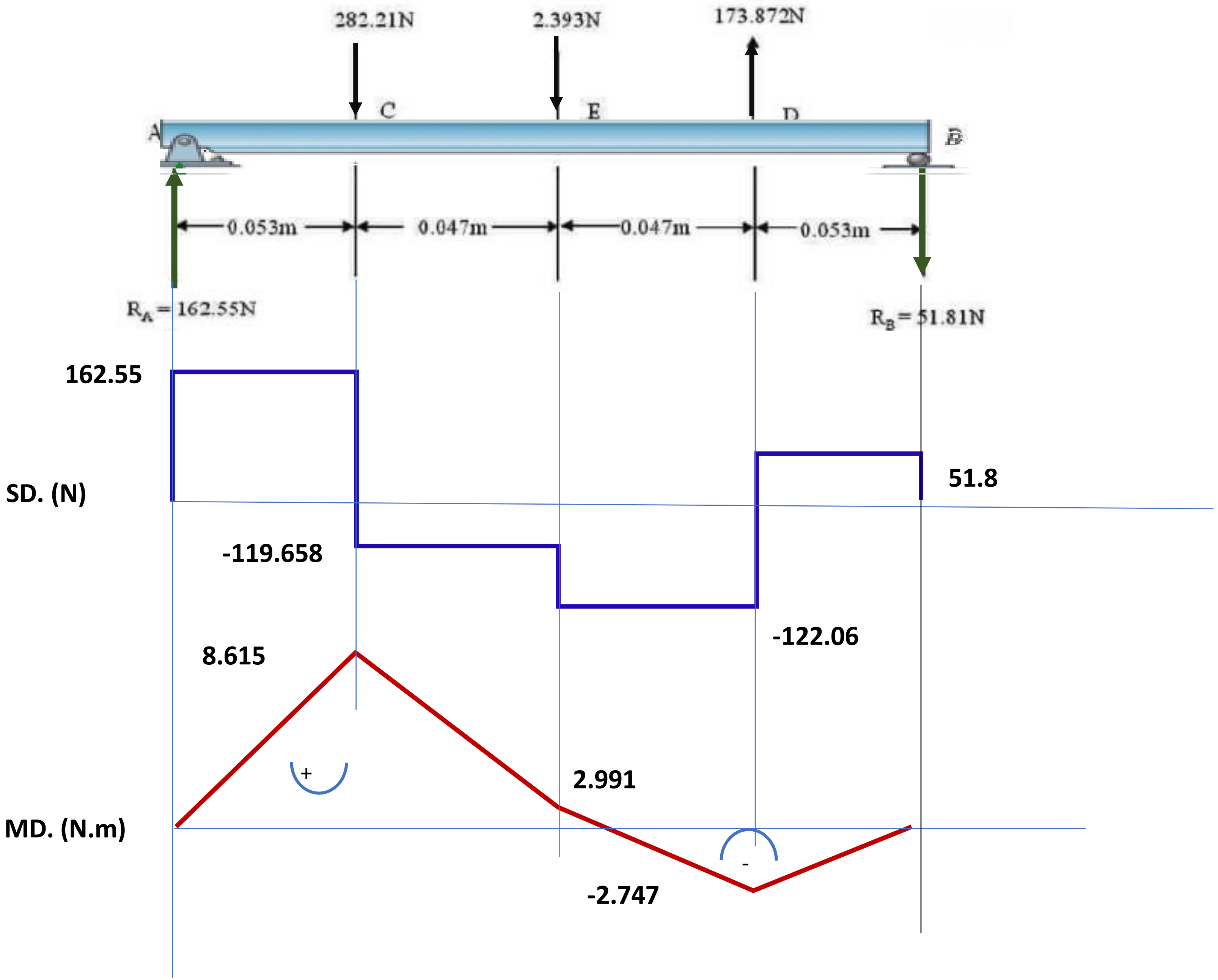
Find internal forces at the shown sections

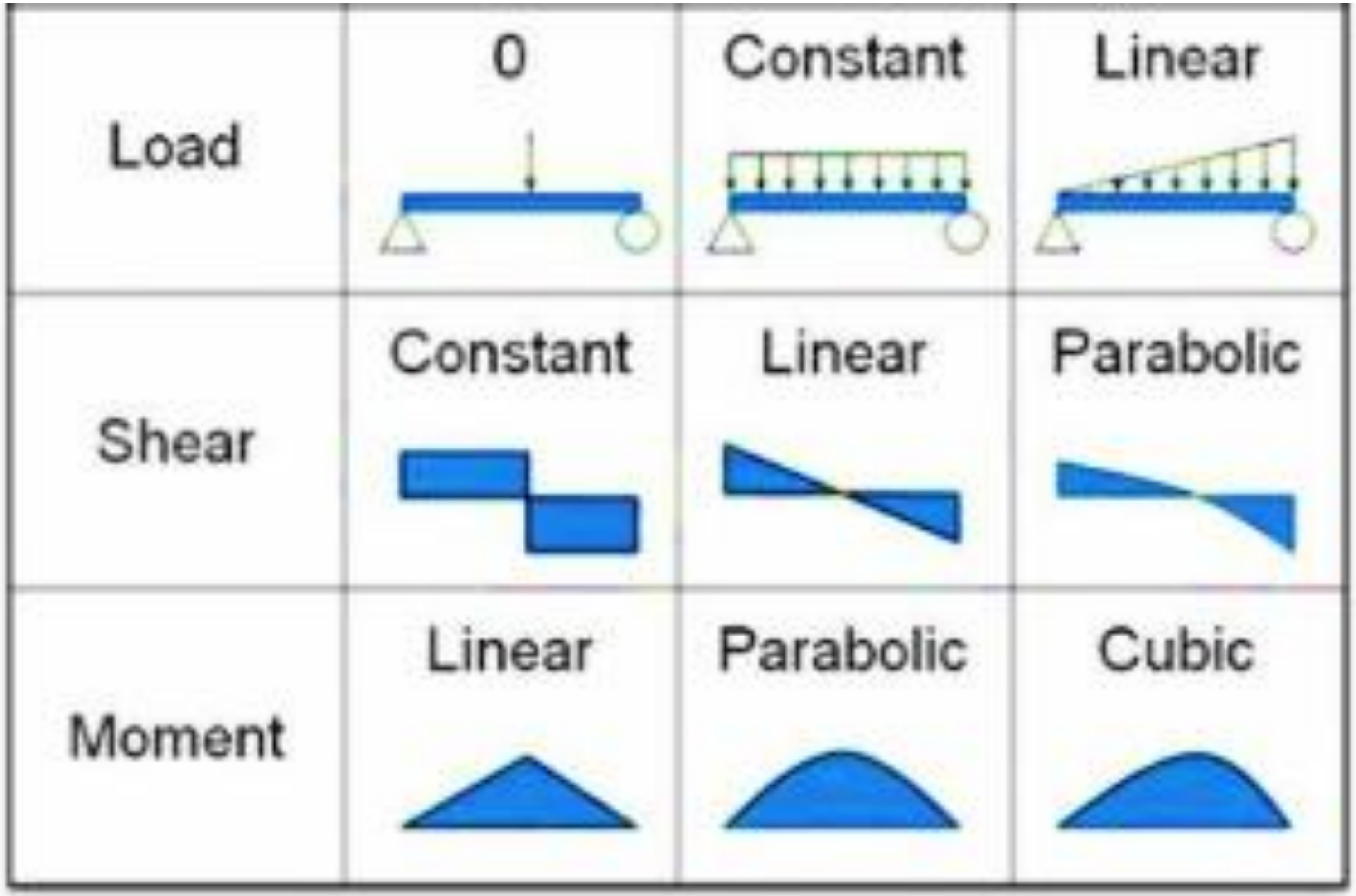


	Location (m)	Shear (kN)	Moment (kN-m)
1	0	-20	0
2	2.5	-20	-50
3	2.5	26	-50
4	5.5	26	28
5	5.5	-14	28
6	7.5	-14	0
7	7.5	0	0

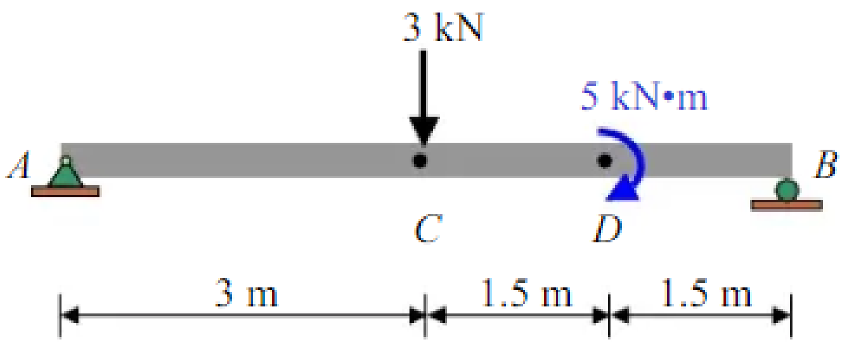


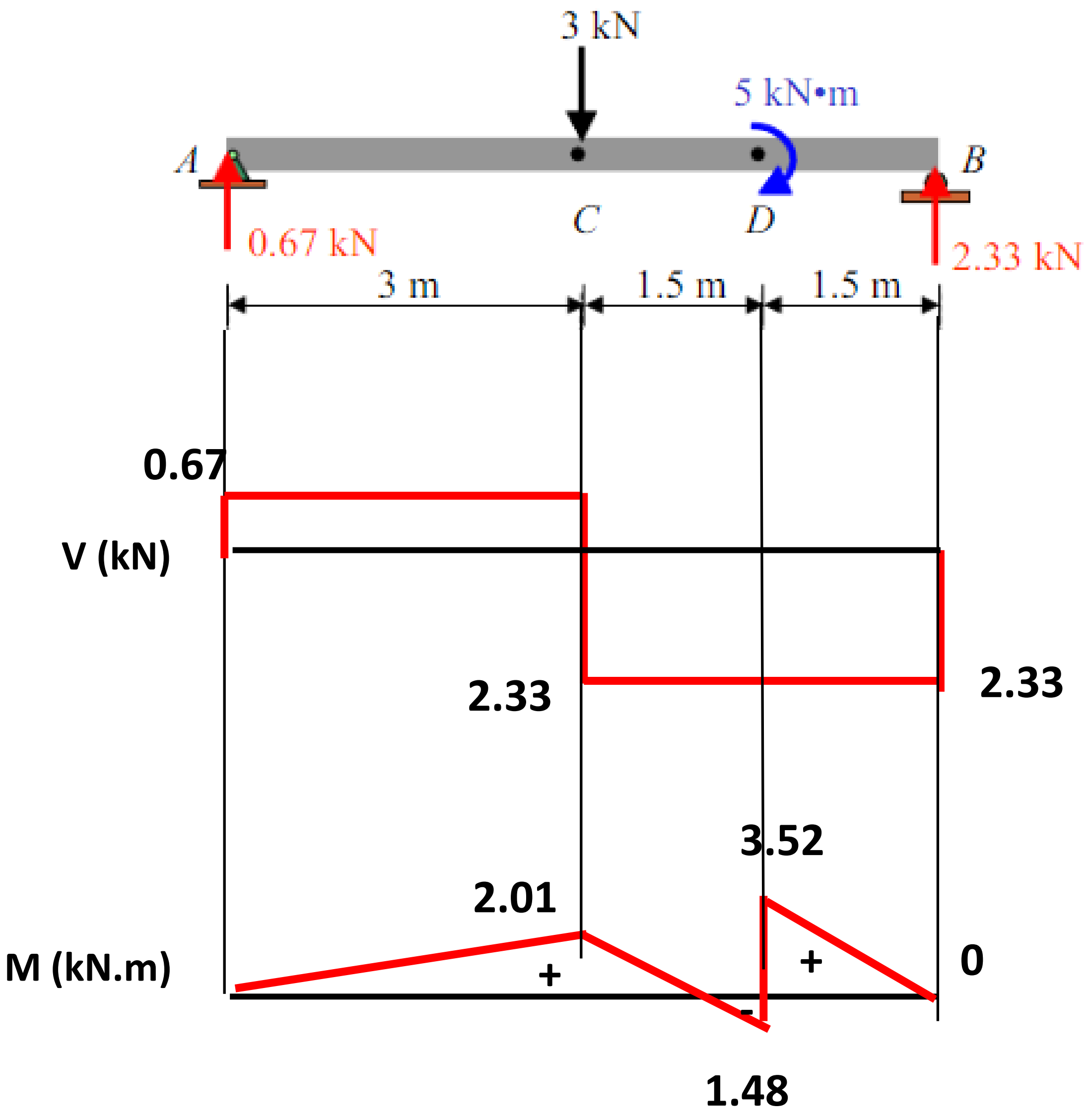

100 N
A
B
25 N
6m
2 m
75 N
25 N
SD. kN
+
-
-75
150
BMD. kN.m
+
10N/m
A
D
50 N
10 m
50 N
50
SD. kN
+
V=0
-
50
125
BMD. kN.m
+
200 N.m
4
6
20 N
20 N
SD. kN
-
-20
+120
+
BMD. kN.m
-
-80
4
20 N
4
20 N

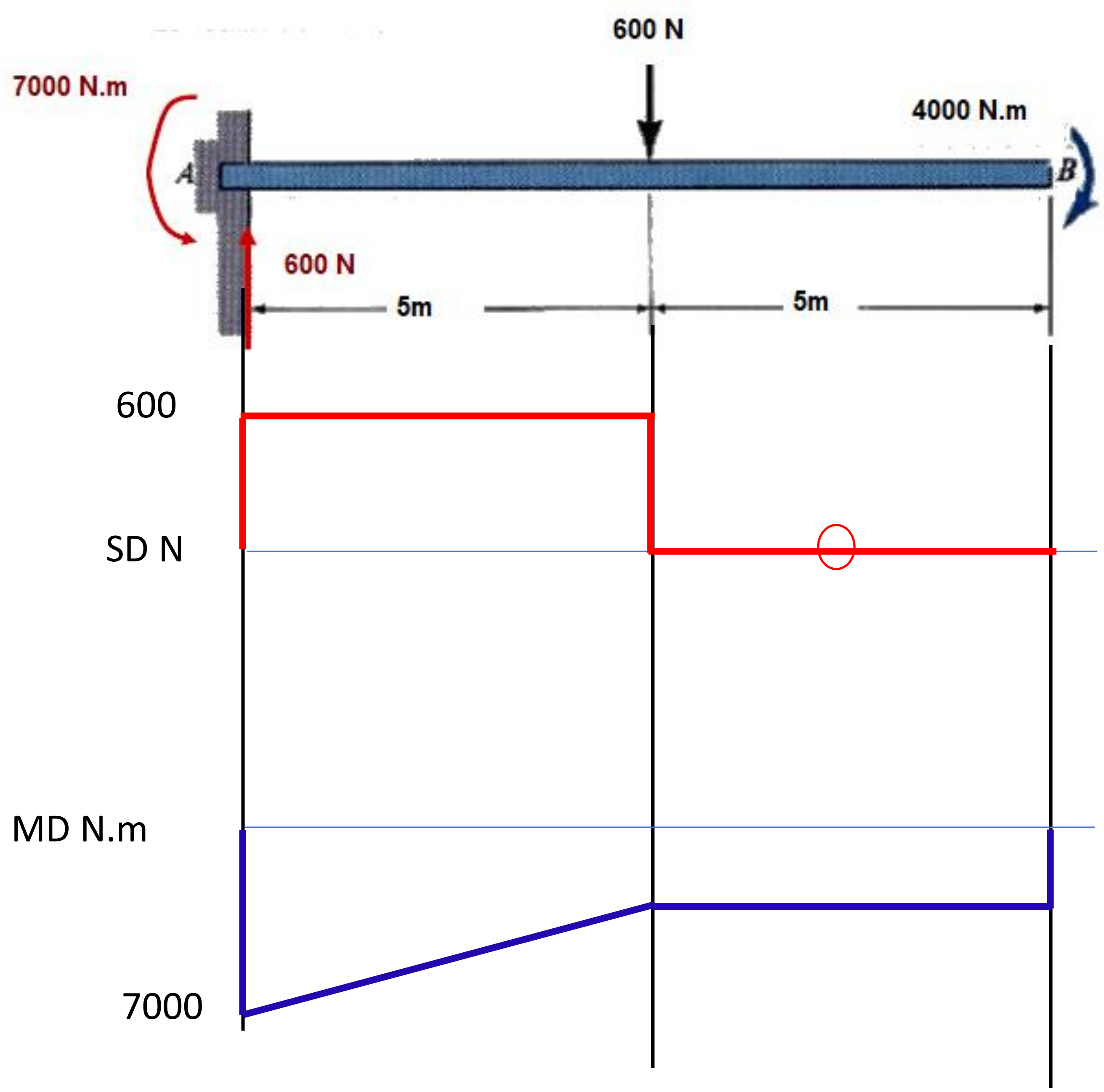

282.21N
2.393N
173.872N
C
E
D
A
B
0.053m
0.047m
0.047m
0.053m
RA = 162.55N
RB = 51.81N
162.55
SD. (N)
51.8
-119.658
-122.06
8.615
+
2.991
MD. (N.m)
-
-2.747


Load
0
Constant
Linear
Shear
Constant
Linear
Parabolic
Moment
Linear
Parabolic
Cubic

Draw the shear and moment diagrams for the beam shown i






600 N
7000 N.m
4000 N.m
A
B
600 N
5m
5m
600
SD N
MD N.m
7000

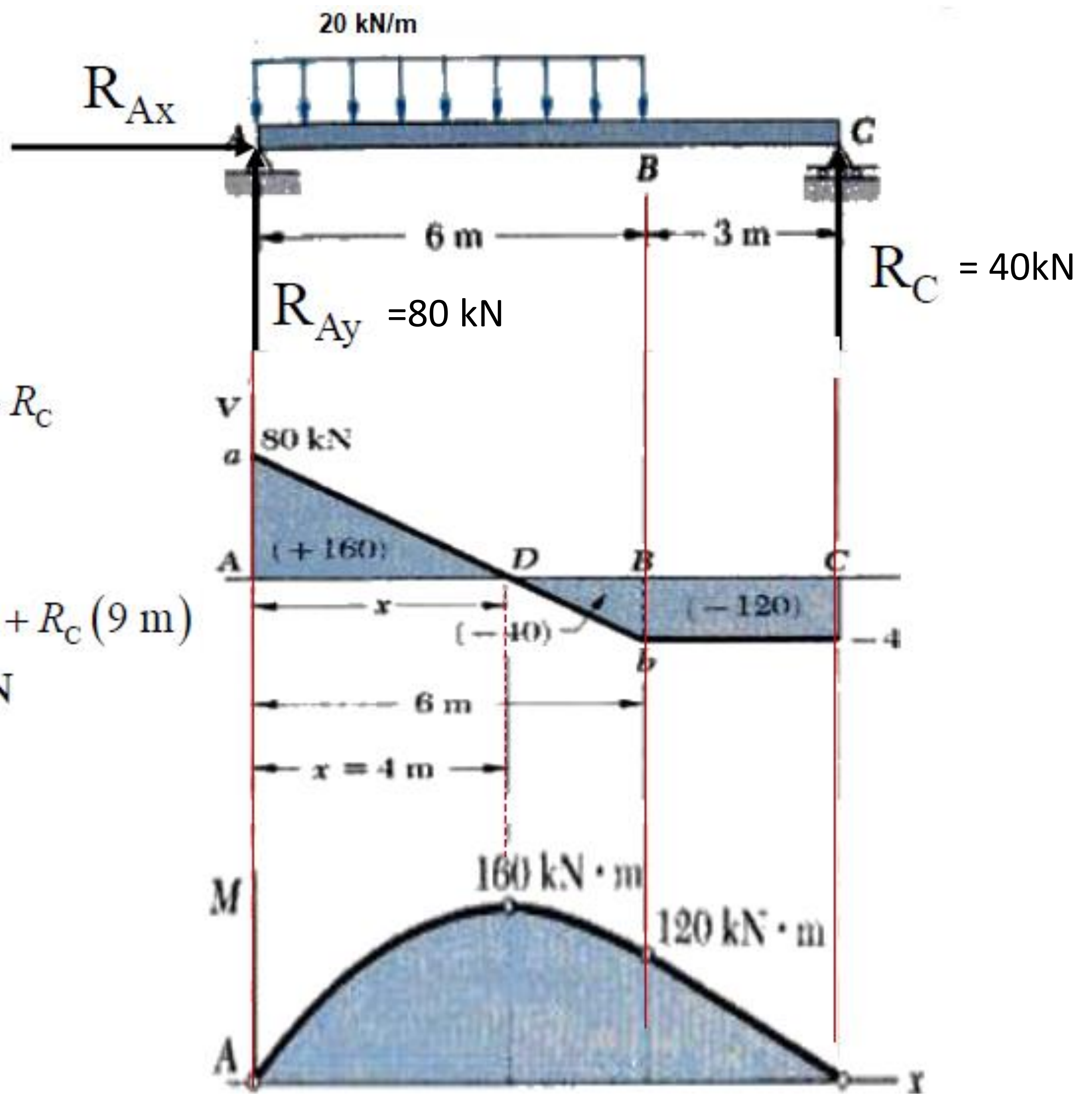
$$\sum F_x = 0 = R_{Ax}$$

$$\sum F_y = 0 = R_{Ay} - 20\ \text{kN/m}(6\ \text{m}) + R_C$$

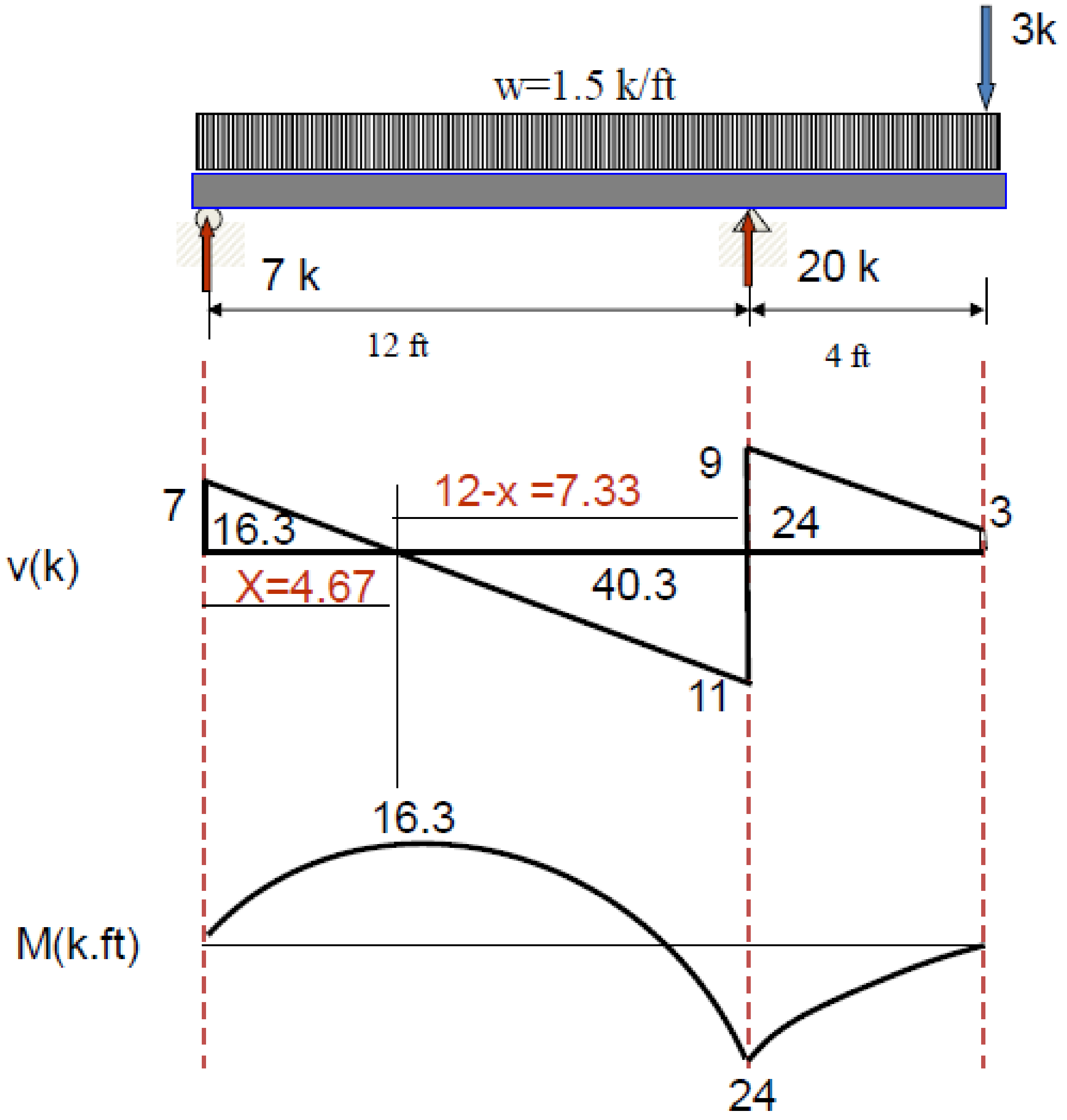
$$\Rightarrow R_{Ay} + R_C = 120\ \text{kN}$$

$$\sum M_A = 0 = -20\ \text{kN/m}(6\ \text{m})(3\ \text{m}) + R_C(9\ \text{m})$$

$$\Rightarrow R_C = 40\ \text{kN}\ \&\ R_{Ay} = 80\ \text{kN}$$



For the beam shown here draw the shear and moment diagram:



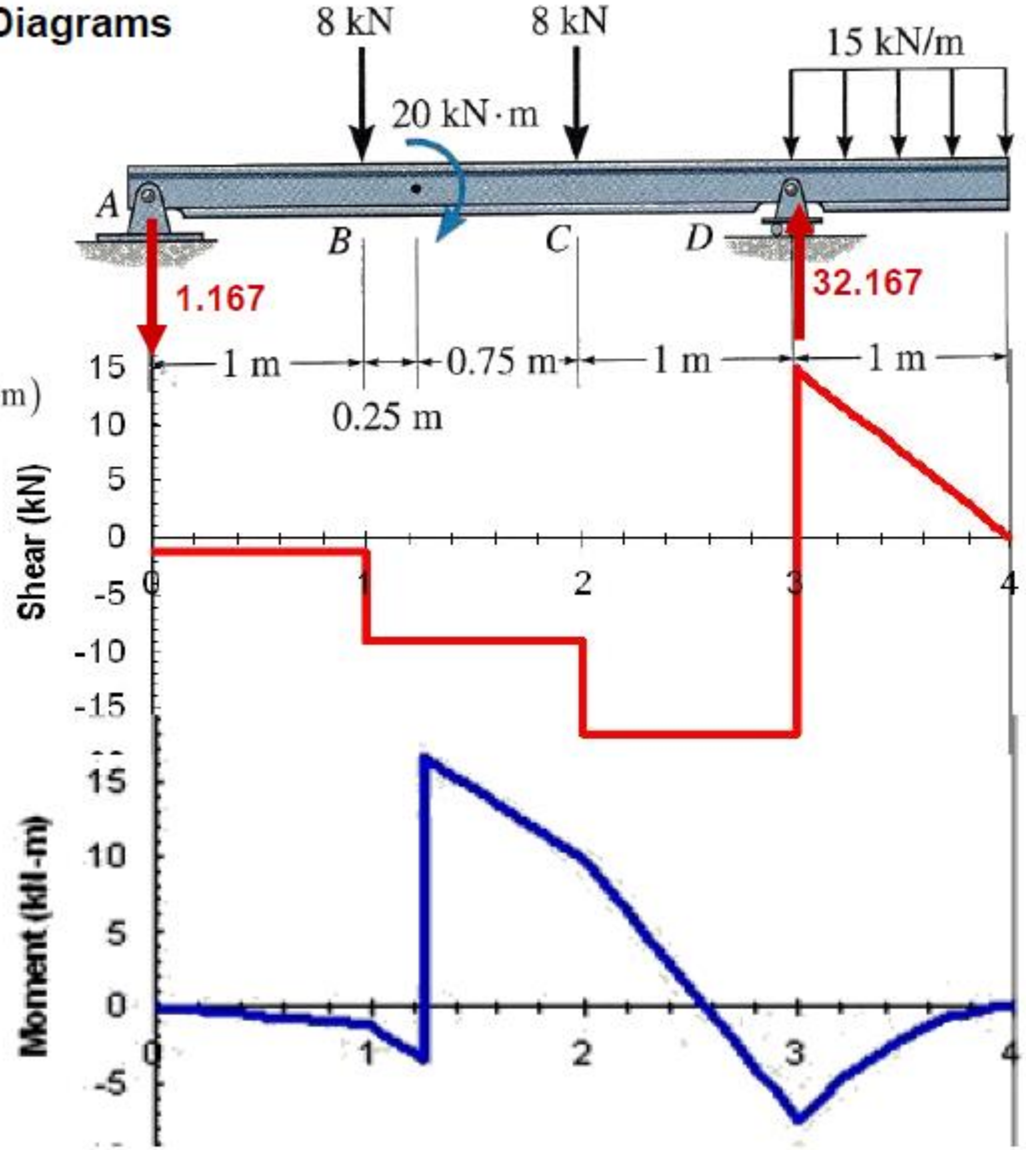
Draw shear and moment Diagrams

$$\sum F_x = 0 = R_{Ax}$$

$$\sum F_y = 0 = R_{Ay} - 8\text{ kN}$$
$$-8\text{ kN} - 15\text{ kN/m}(1\text{ m}) + R_{Ey}$$

$$\sum M_A = 0 = -8\text{ kN}(1\text{ m}) - 20\text{ kN-m}$$
$$-8\text{ kN}(2\text{ m})$$
$$-15\text{ kN/m}(1\text{ m})(3.5\text{ m}) + R_{Dy}(3\text{ m})$$

$$R_{Dy} = 32.167\text{ kN}$$
$$R_{Ax} = 0\text{ kN}$$
$$R_{Ay} = -1.167\text{ kN}$$



$$\sum F_x = 0 = B_X$$

$$\sum F_y = 0 = R_{Cy} + B_Y - 150\text{ lb/ft}(6\text{ ft})$$

$$\sum M_B = 0 = R_{Cy}(6\text{ ft}) - 150\text{ lb/ft}(6\text{ ft})(3\text{ ft}) - 800\text{ lb-ft}$$

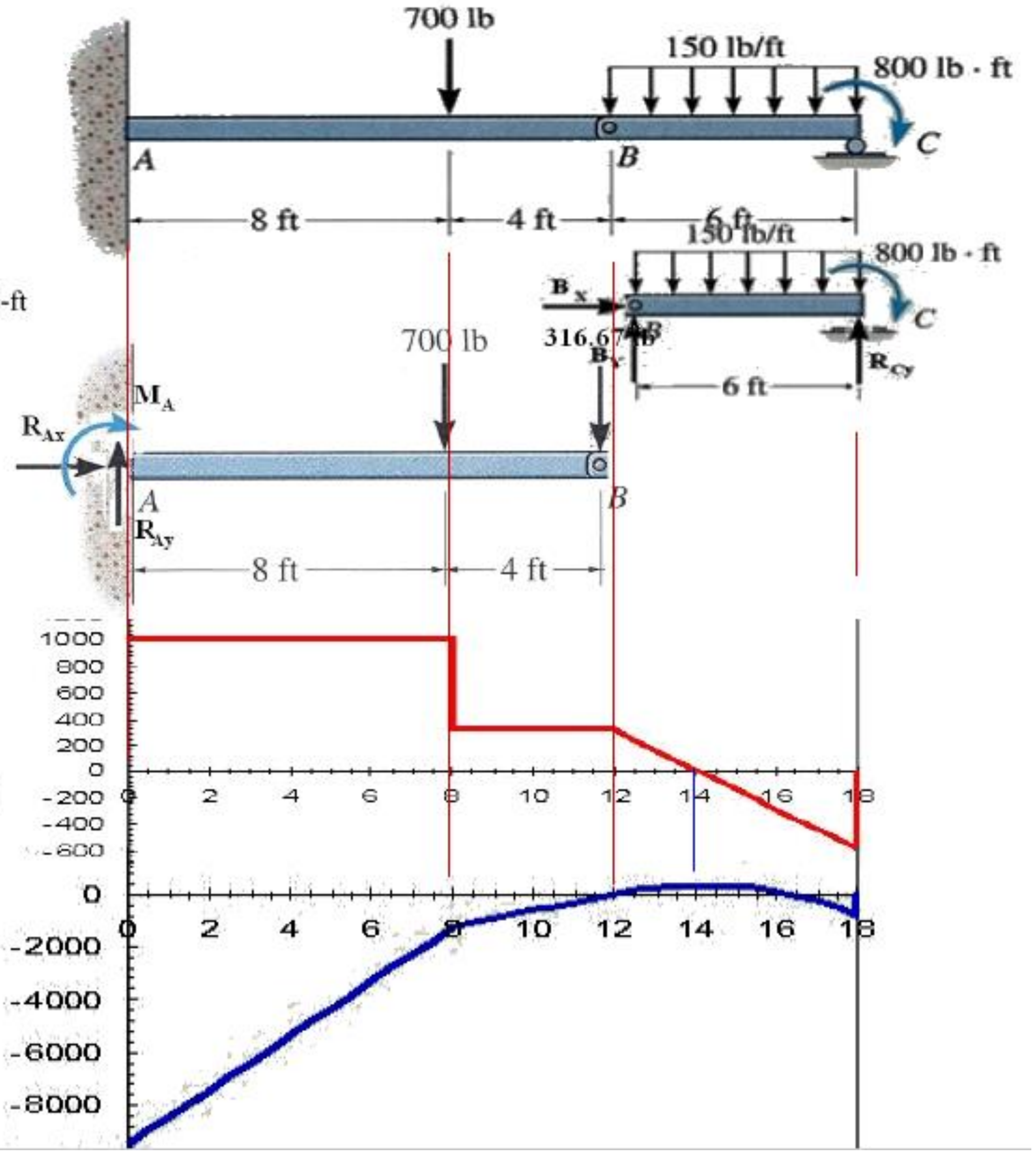
$R_{Cy} = 583.33$ lb
$B_X = 0$ lb
$B_Y = 316.67$ lb

$$\sum F_x = 0 = R_{Ax}$$

$$\sum F_y = 0 = R_{Ay} - 700\text{ lb} - 316.67\text{ lb}$$

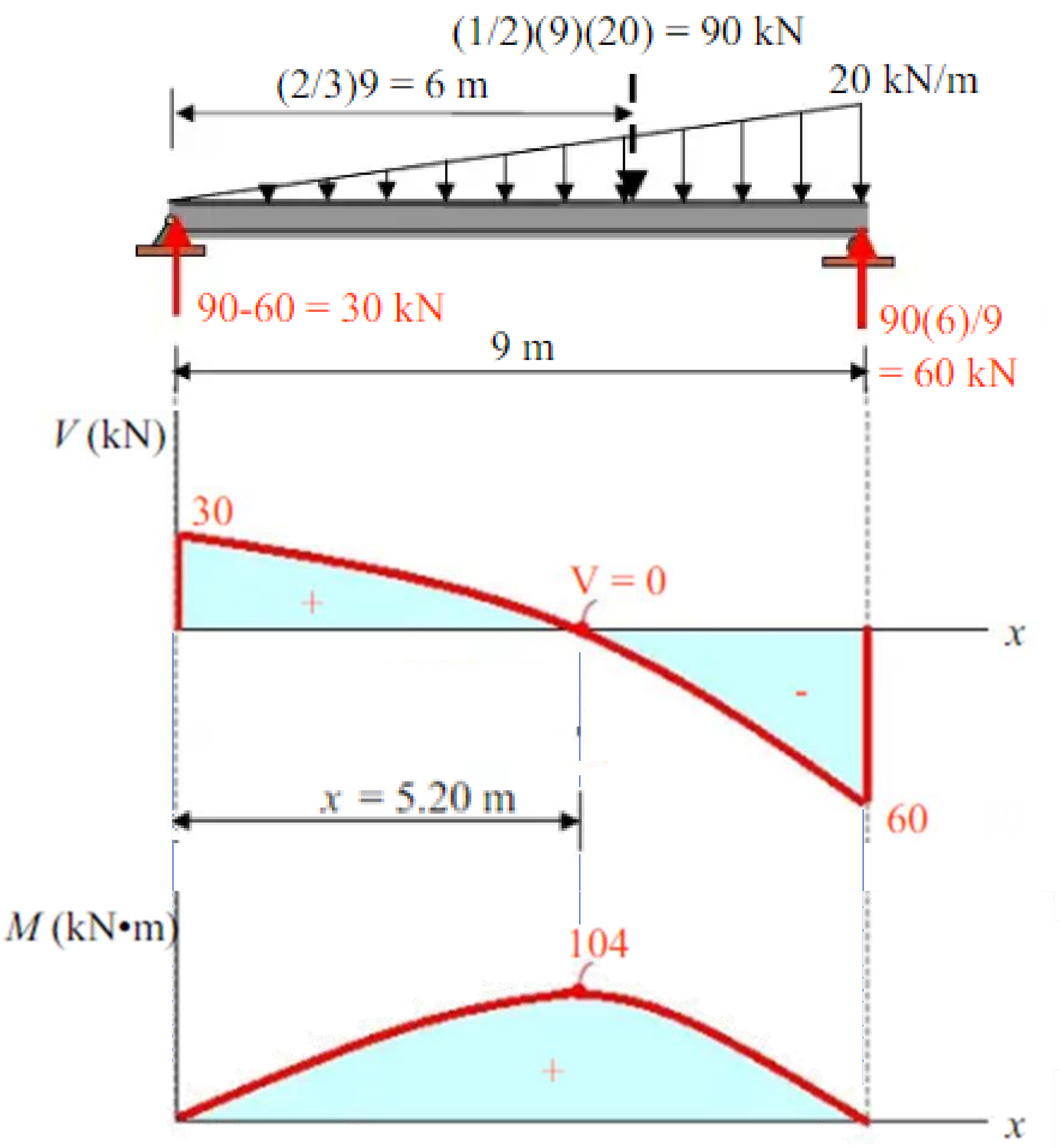
$$\sum M_B = 0 = -M_A - 700\text{ lb}(8\text{ ft}) - 316.67\text{ lb}(12\text{ ft})$$

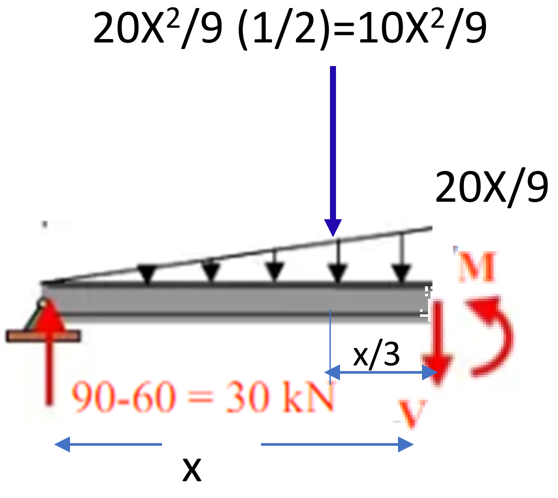
$R_{Ay} = 1016.67$ lb
$R_{Ax} = 0$ lb
$M_A = -9400$ lb-ft



7.3 Relation Between Load, Shear and Moment

Draw the shear and moment diagrams for the beam shown





ΣFy=0 30 -10X²/9 -V= 0

V= 30 –(10X²/9)

ΣM=0 - 30X +10X²/9(X/3) + M= 0

M= 30X –(10/9) (X³/3)

V=0 ……. 30 –(10X²/9) =0

X=5.2 m

$\circlearrowleft \Sigma M_x = 0:$

$$[(\frac{1}{2})(5.2)(20\frac{5.2}{9})](\frac{5.2}{3}) - 30(5.2) = 0$$

$M = 104$ kN•m

$$\Delta V = \int w(x)\, dx$$

Change in shear = Area under loading curve

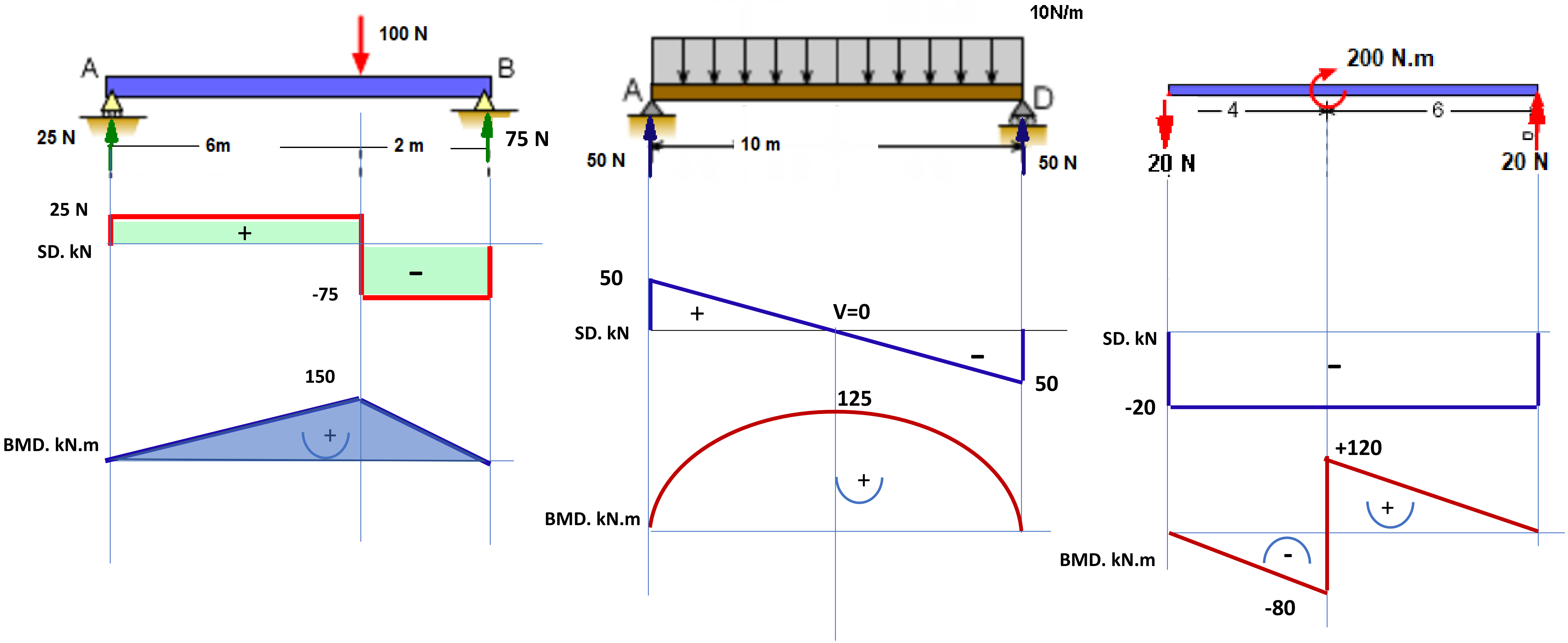
$$\frac{dM}{dx} = V$$

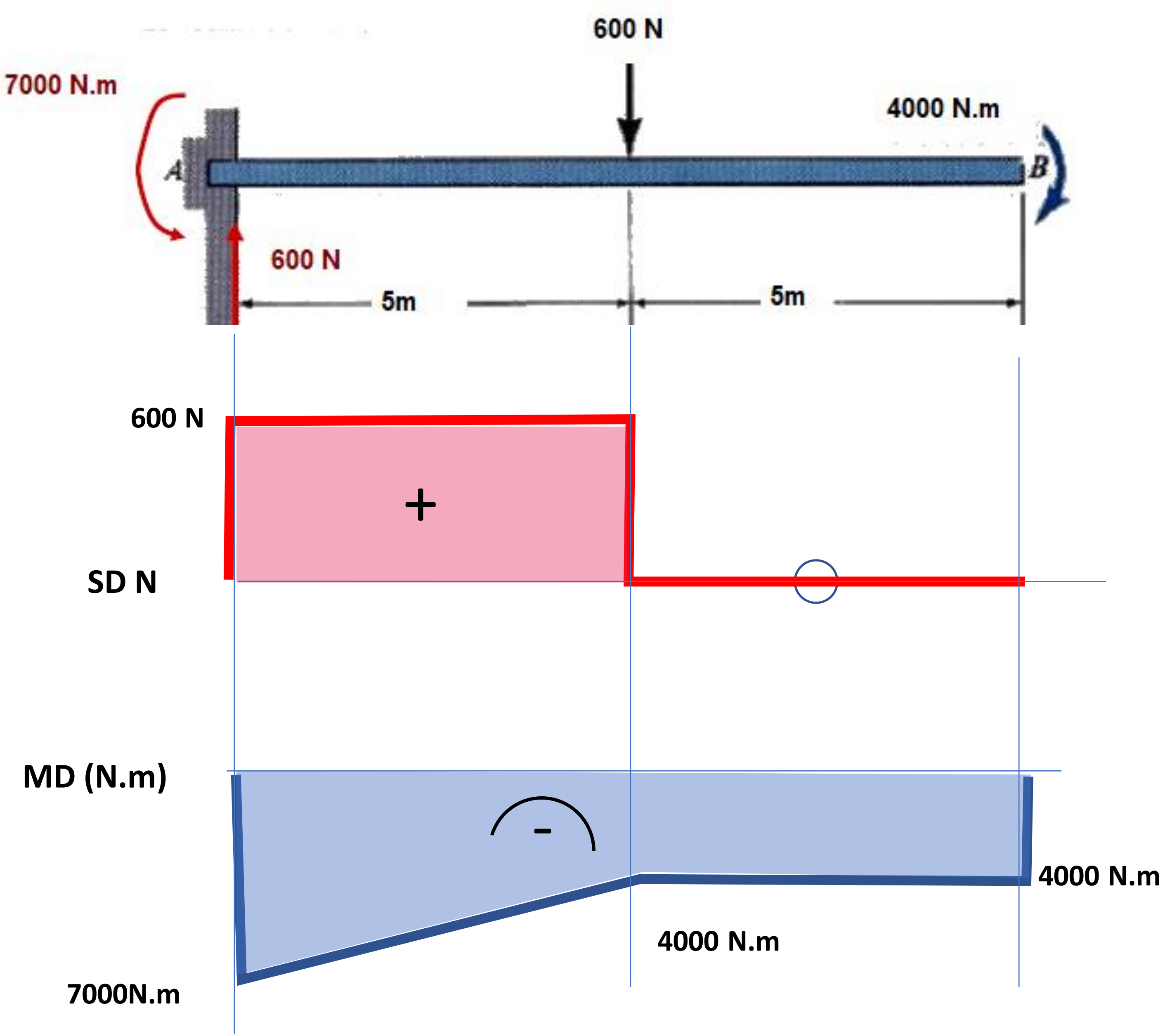
Slope of moment diagram = Shear

$$\Delta M = \int V\, dx$$

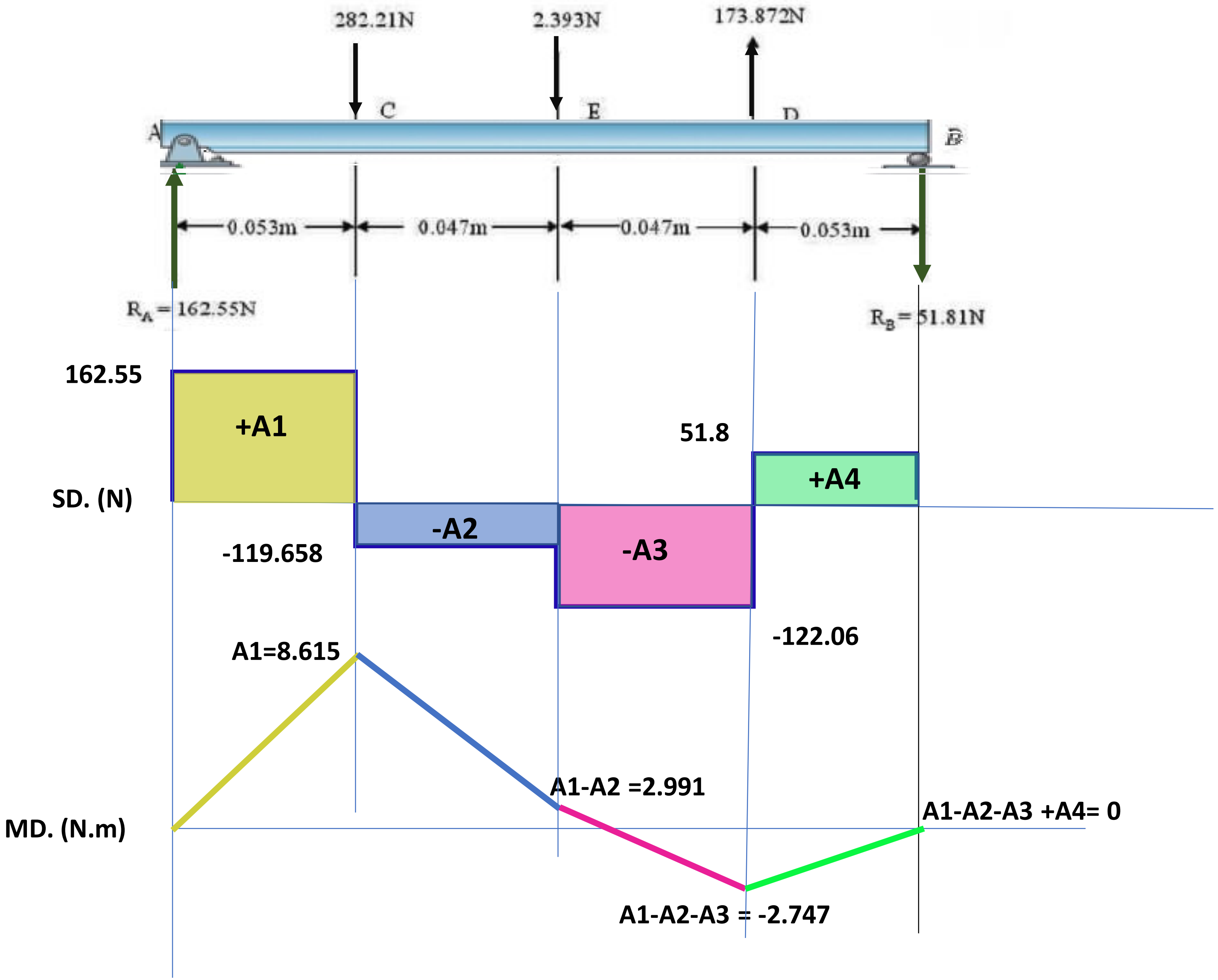
Change in moment = Area under shear diagram

Draw SD and MD

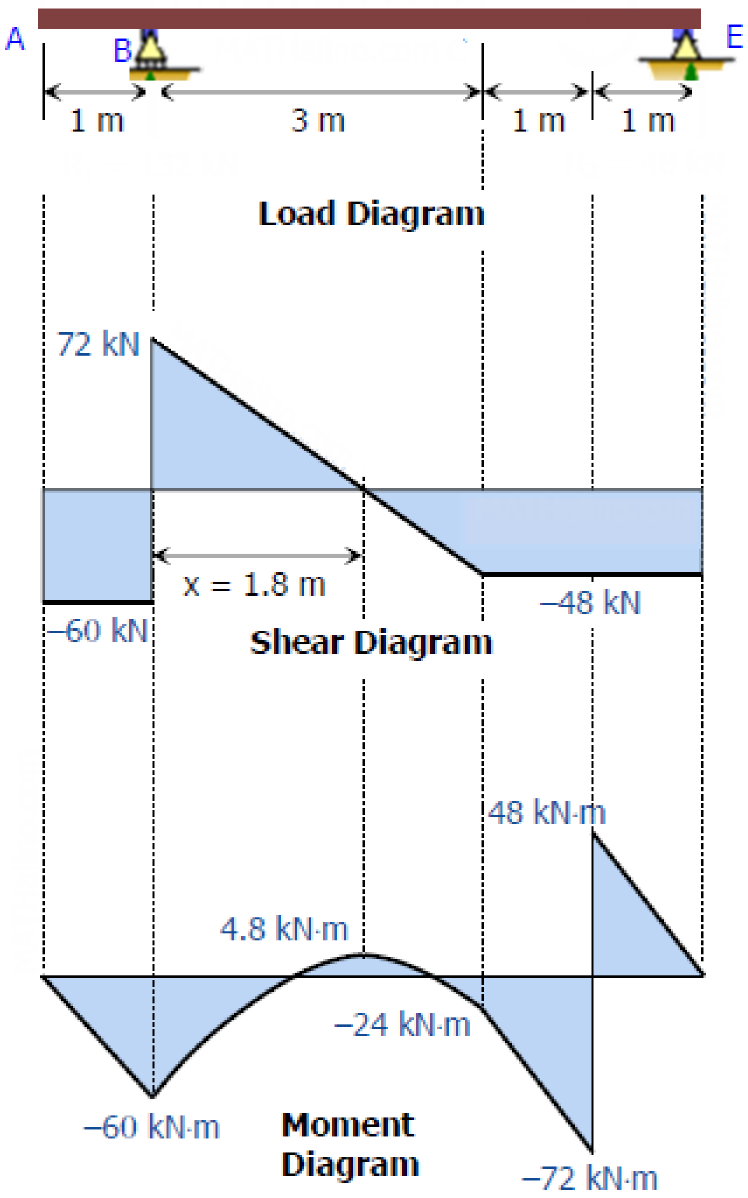


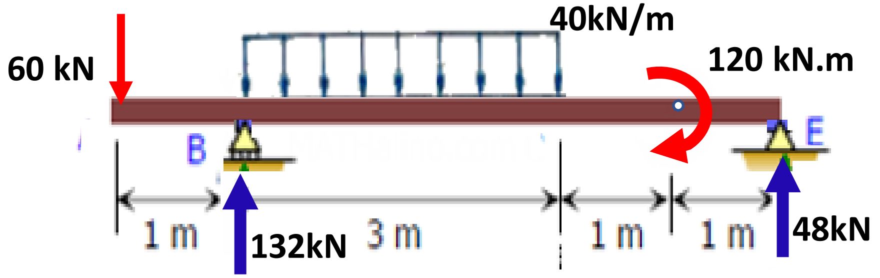

600 N
7000 N.m
4000 N.m
A
B
600 N
5m
5m
600 N
+
SD N
MD (N.m)
-
4000 N.m
4000 N.m
7000N.m

Draw SD and MD

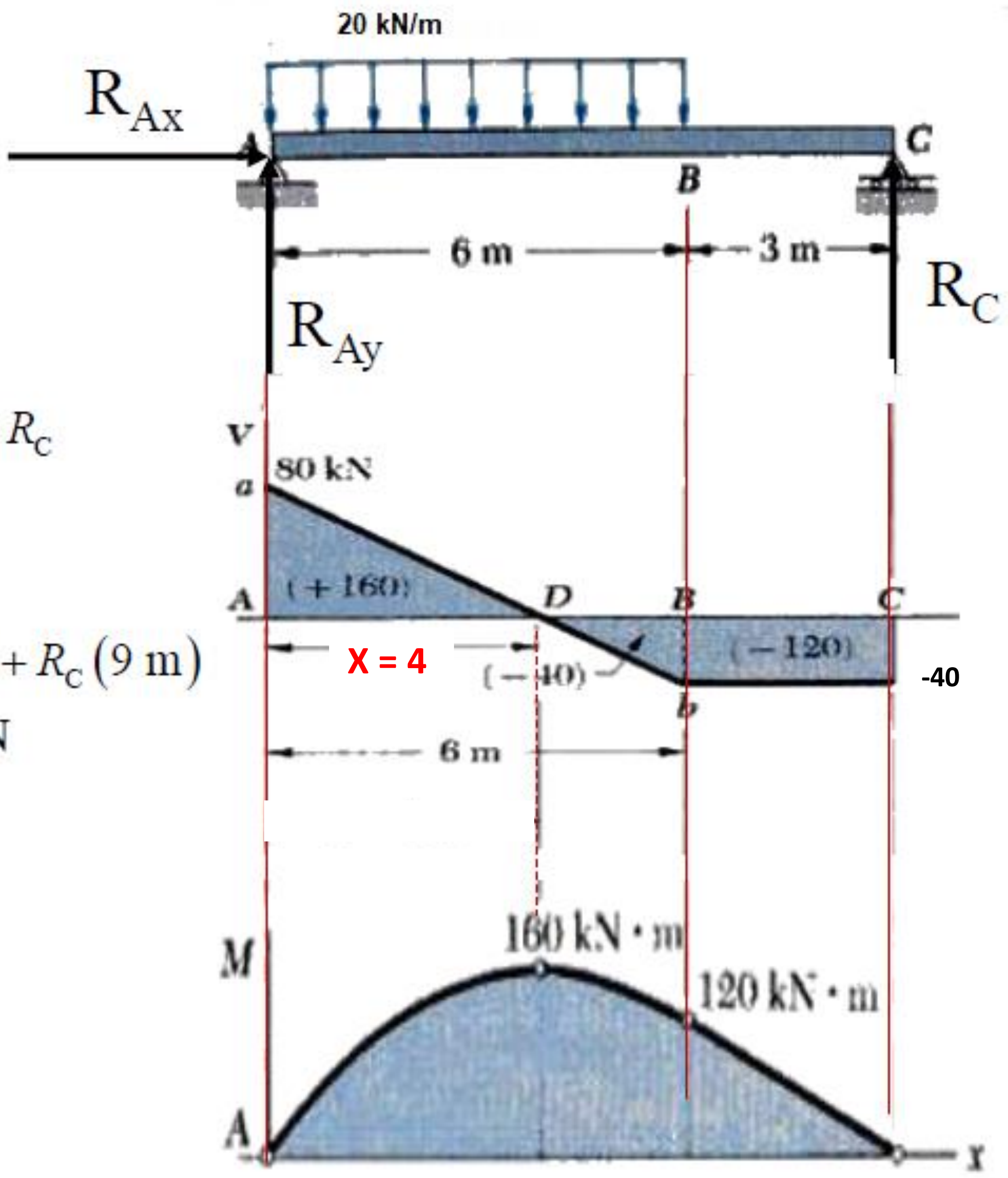


Show the loading system on the beam and show support reactions . Shear and moment diagrams are given





Draw SD and MD



$$\sum F_x = 0 = R_{Ax}$$

$$\sum F_y = 0 = R_{Ay} - 20\text{ kN/m}(6\text{ m}) + R_C$$

$$\Rightarrow R_{Ay} + R_C = 120\text{ kN}$$

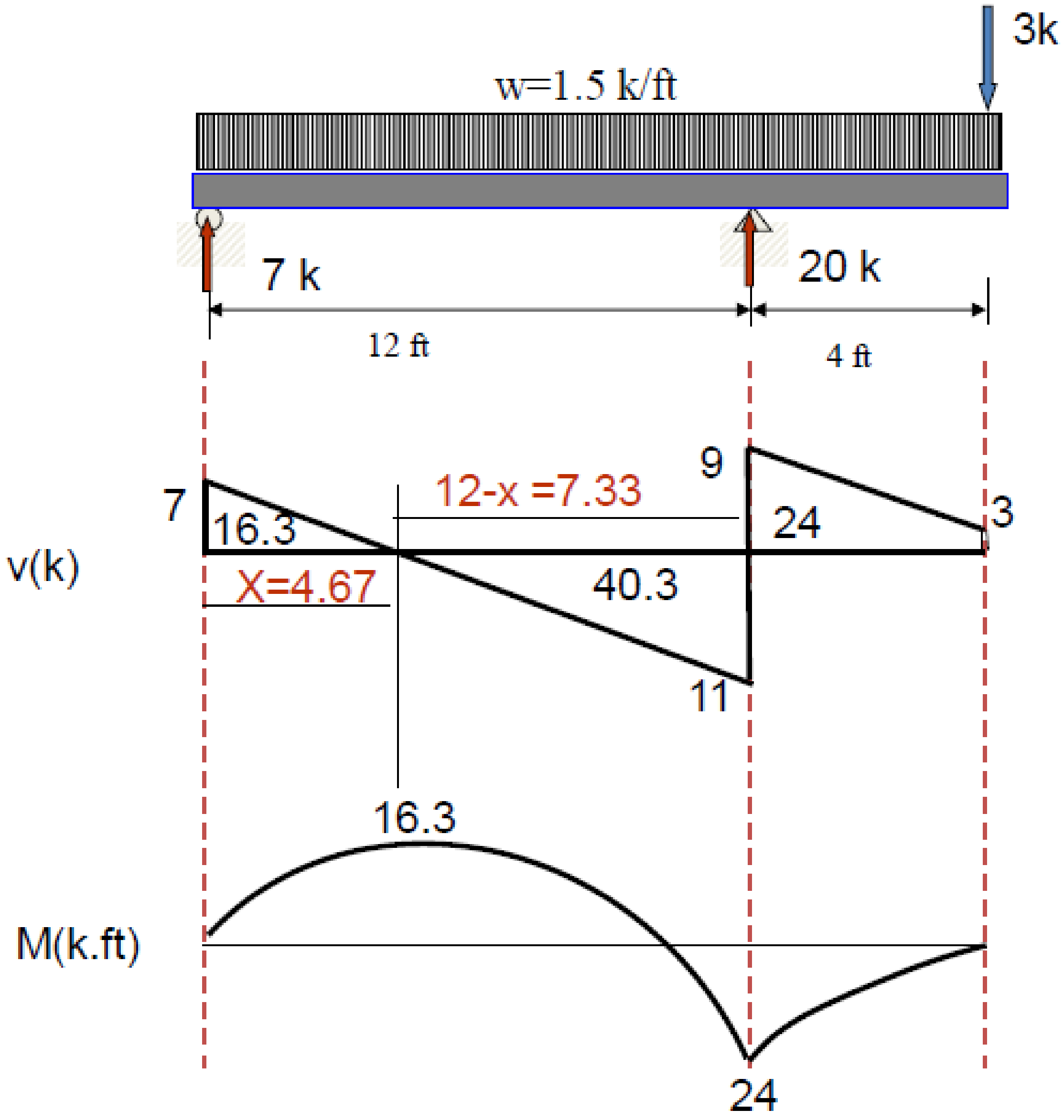
$$\sum M_A = 0 = -20\text{ kN/m}(6\text{ m})(3\text{ m}) + R_C(9\text{ m})$$

$$\Rightarrow R_C = 40\text{ kN} \;\&\; R_{Ay} = 80\text{ kN}$$

80 /X = 20
X = 4

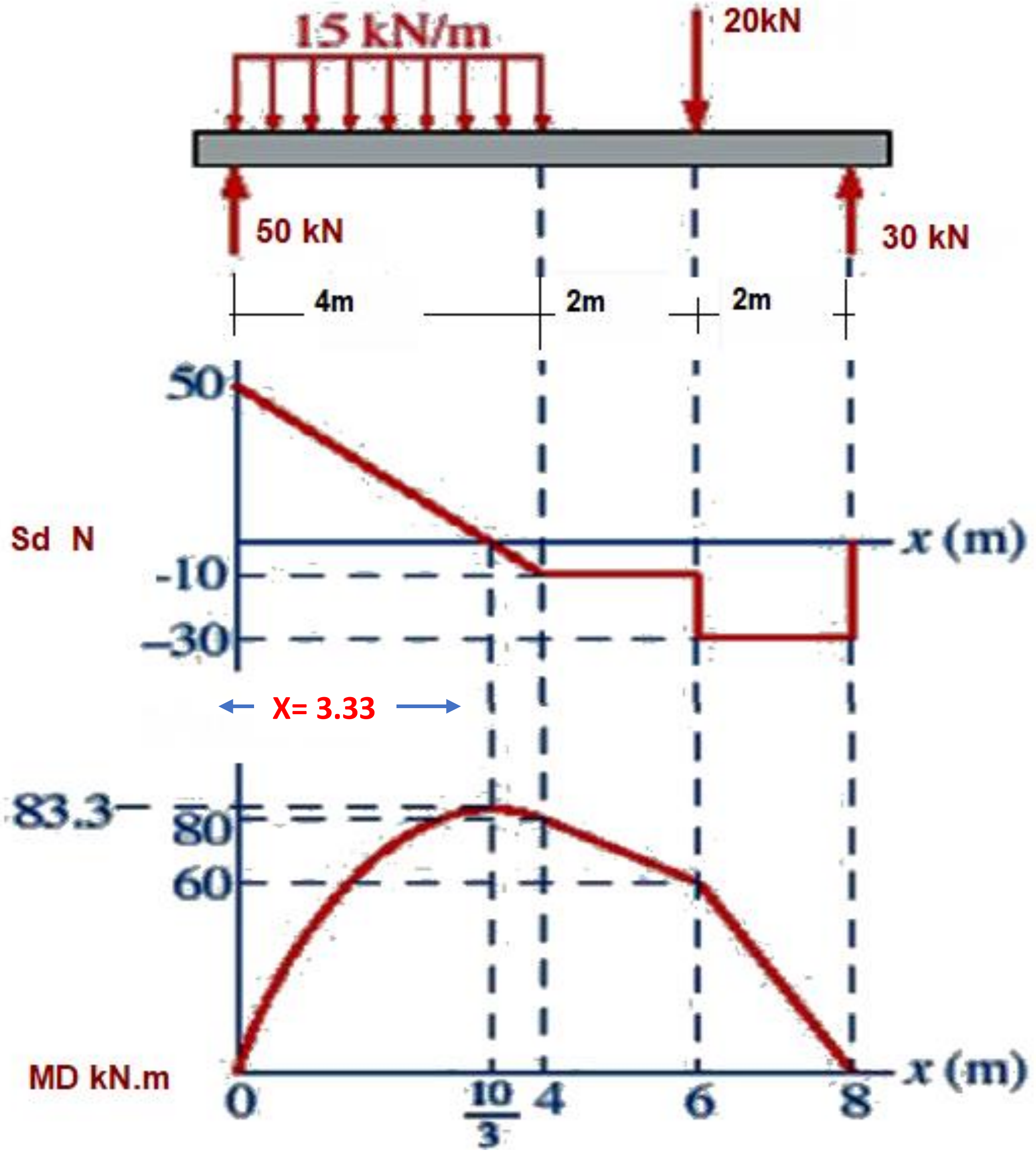
Draw SD and MD

For the beam shown here draw the shear and moment diagram:



7/X=1.5
X= 4.67

Draw SD and MD



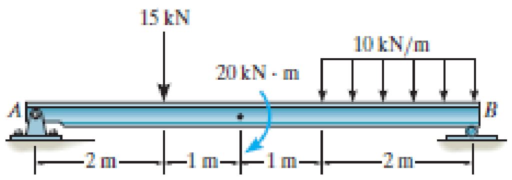
50 /X=15

X= 3.33

Important Points

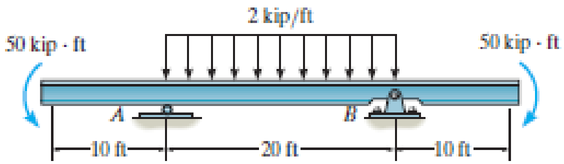
- The slope of the shear diagram at a point is equal to the intensity of the distributed loading, where positive distributed loading is upward, i.e., $dV/dx = w(x)$.
- The change in the shear ΔV between two points is equal to *the area* under the distributed-loading curve between the points.
- If a concentrated force acts upward on the beam, the shear will jump upward by the same amount.
- The slope of the moment diagram at a point is equal to the shear, i.e., $dM/dx = V$.
- The change in the moment ΔM between two points is equal to the *area* under the shear diagram between the two points.
- If a *clockwise* couple moment acts on the beam, the shear will not be affected; however, the moment diagram will jump *upward* by the amount of the moment.
- Points of *zero shear* represent points of *maximum or minimum moment* since $dM/dx = 0$.
- Because two integrations of $w = w(x)$ are involved to first determine the change in shear, $\Delta V = \int w(x)\,dx$, then to determine the change in moment, $\Delta M = \int V\,dx$, then if the loading curve $w = w(x)$ is a polynomial of degree n, $V = V(x)$ will be a curve of degree $n + 1$, and $M = M(x)$ will be a curve of degree $n + 2$.

*7–76. Draw the shear and moment diagrams for the beam.



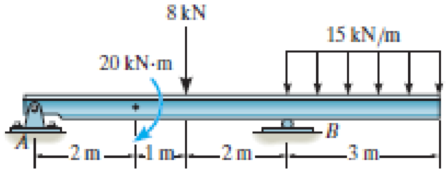
Prob. 7–76

7–77. Draw the shear and moment diagrams for the beam.



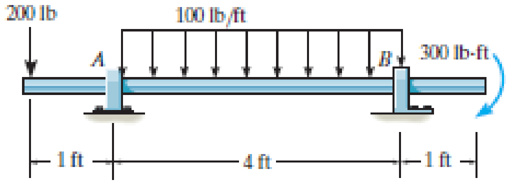
Prob. 7–77

7–78. Draw the shear and moment diagrams for the beam.



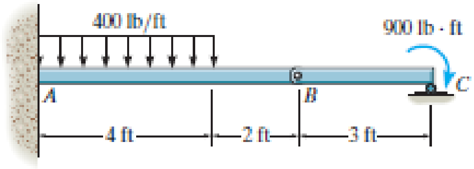
Prob. 7–78

7–79. Draw the shear and moment diagrams for the shaft. The support at A is a journal bearing and at B it is a thrust bearing.



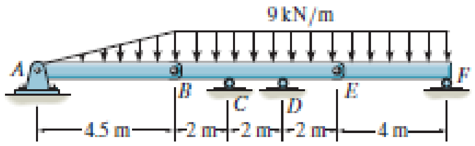
Prob. 7–79

*7–80. Draw the shear and moment diagrams for the beam.



Prob. 7–80

7–81. The beam consists of three segments pin connected at B and E. Draw the shear and moment diagrams for the beam.



Prob. 7–81

Chapter 8
Friction

8.1 and 8.2

Iqbal Marie

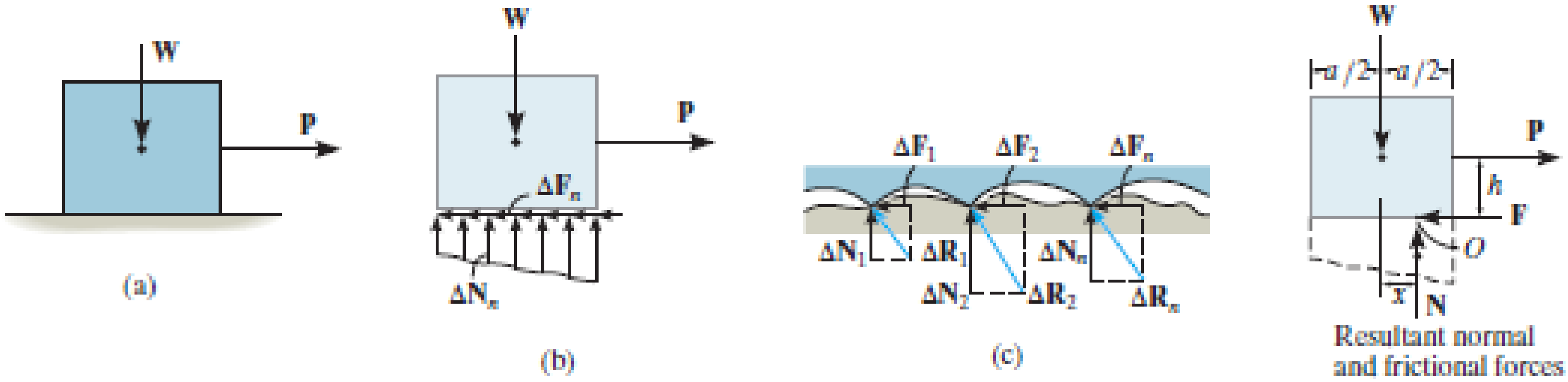
8.1 Characteristics of Dry Friction

Friction is a force that resists the movement of two contacting surfaces that slide relative to one another.

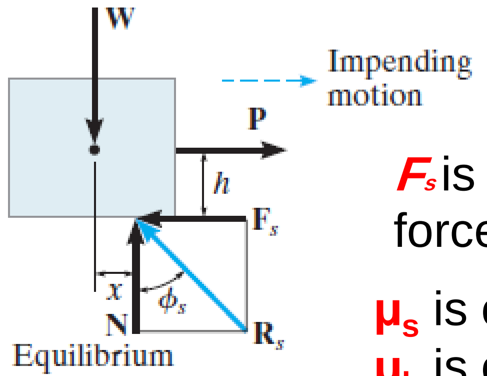
Acts *tangent* to the surface at the points of contact and is directed so as to oppose the possible or existing motion between the surfaces.

Dry friction, called ***Coulomb friction*** since its characteristics were studied extensively by the French physicist Charles-Augustin de Coulomb in 1781.

Dry friction occurs between the contacting surfaces of bodies when there is no lubricating fluid.



$$F_s = \mu_s N$$



F_s is *directly proportional* to the resultant normal force N. Expressed mathematically, $F_s = \mu_s N$

μ_s is called the ***coefficient of static friction***

μ_k is called the ***coefficient of Kinatic friction***

angle of static friction =

$$\phi_s = \tan^{-1}\left(\frac{F_s}{N}\right) = \tan^{-1}\left(\frac{\mu_s N}{N}\right) = \tan^{-1}\mu_s$$

- Maximum static-friction force:

  $F_m = \mu_s N$

- Kinetic-friction force:

  $F_k = \mu_k N$

  $\mu_k \cong 0.75\mu_s$

- Maximum static-friction force and kinetic-friction force are:
  - proportional to normal force
  - dependent on type and condition of contact surfaces
  - independent of contact area

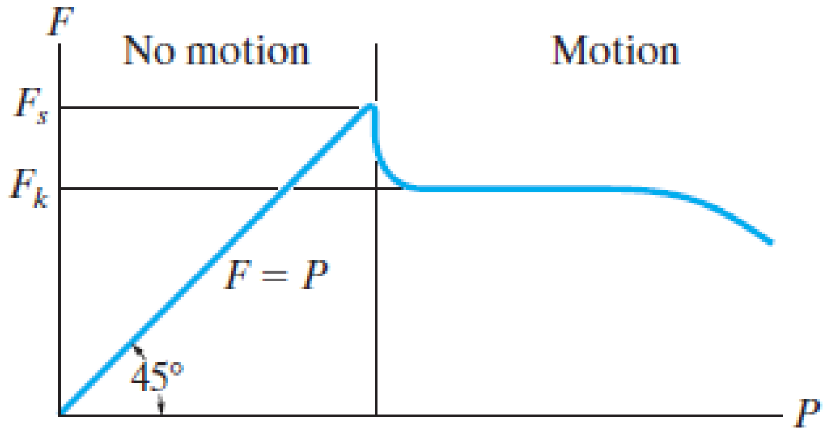


Table 8–1 Typical Values for μ_s

Contact Materials	Coefficient of Static Friction (μ_s)
Metal on ice	0.03–0.05
Wood on wood	0.30–0.70
Leather on wood	0.20–0.50
Leather on metal	0.30–0.60
Copper on copper	0.74–1.21

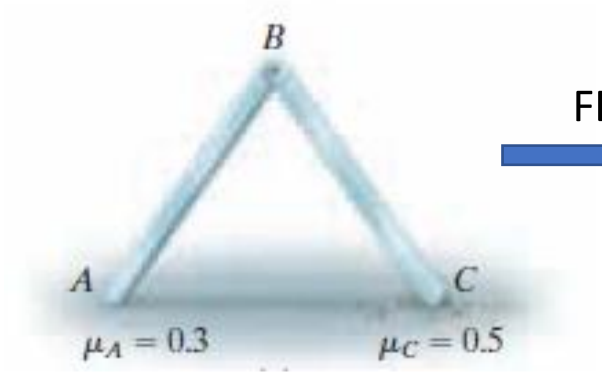
the coefficient of friction should be determined directly by an experiment that involves the two materials to be used.

Motion. If the magnitude of **P** acting on the block is increased so that it becomes slightly greater than F_s, the frictional force at the contacting surface will drop to a smaller value F_k, **called the *kinetic frictional force*.**

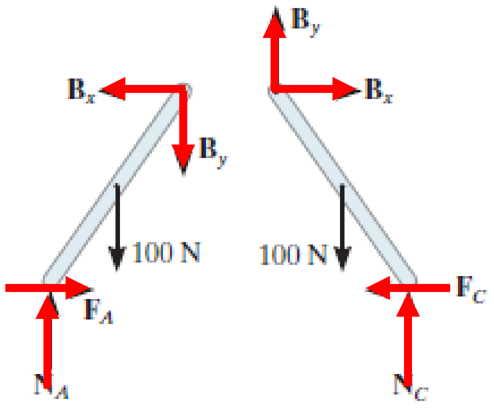
8.2 Problems Involving Dry Friction

Types of Friction Problems. In general, there are three types of static problems involving dry friction

1. No Apparent Impending Motion **number of unknowns to be *equal* to the number of available equilibrium equations**



FBD

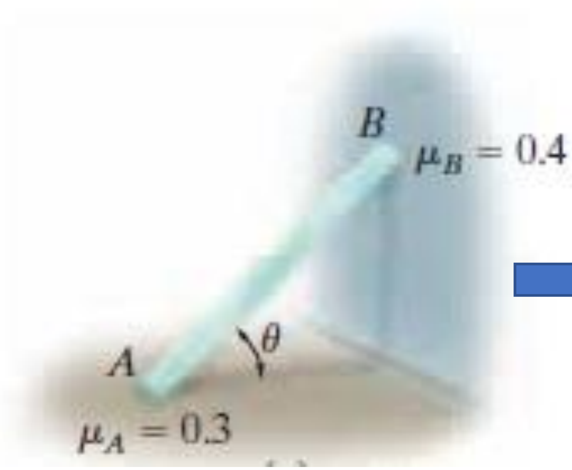


Solve the problem and find F_A and F_C using equilibrium equation
Check their values for friction force at A and C ,
If $F_A \geq 0.3 N_A$ or $F_C \geq 0.5 N_C$ then slipping will occur and the body will not remain in equilibrium

6 unknown
3 equilibrium equation per each member = 6 equations
2 friction equation : $F_A = 0.3 N_A$ or $F_C = 0.5 N_C$

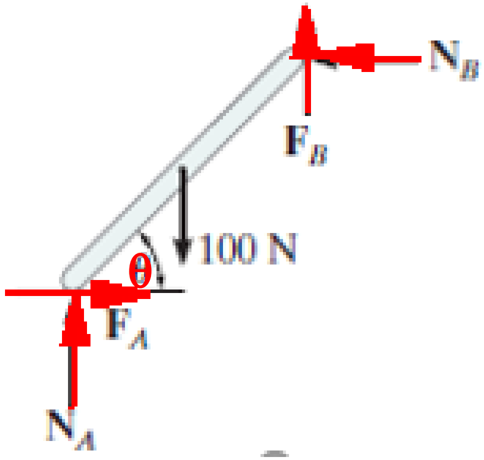
2. Impending Motion at All Points of Contact

The total number of unknowns will *equal* the total number of available equilibrium equations + the total number of available frictional equations, $F = \mu_s N$.



finding the smallest angle θ

FBD



F *must always* be shown acting with its *correct sense* on the free-body diagram, *whenever* the frictional equation is used for the solution of a problem

***5* unknowns**

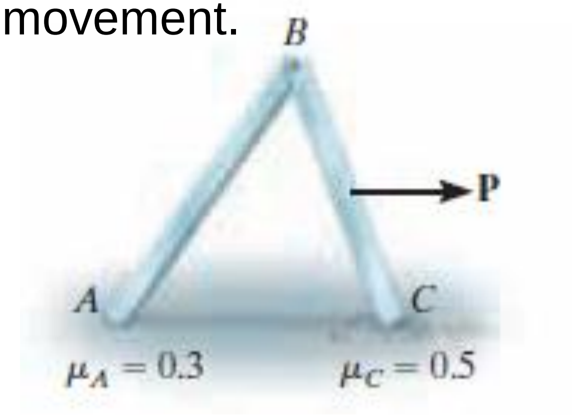
3 equilibrium equations

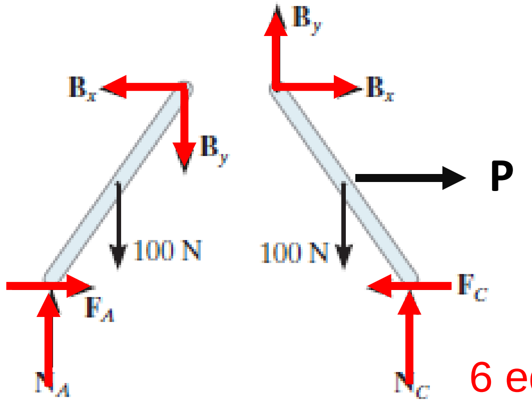
2 static frictional equations

$F_A = 0.3 N_A$ and $F_B = 0.4 N_B$.

3. Impending Motion at Some Points of Contact

determine the horizontal force P needed to cause movement.





7 unknowns

6 equilibrium equations +
2 possible static frictional equations.
$F_A = 0.3N_A$ and $F_C = 0.5N_C$

number of unknowns will be *less* than the number of available equilibrium equations plus the number of available frictional equations or conditional equations for tipping.

several possibilities for motion or impending motion will exist and the problem will involve a determination of the kind of motion which actually occurs

it will either cause slipping at *A* and no slipping at *C*,

$F_A = 0.3N_A$ and $F_C \leq 0.5N_C$;

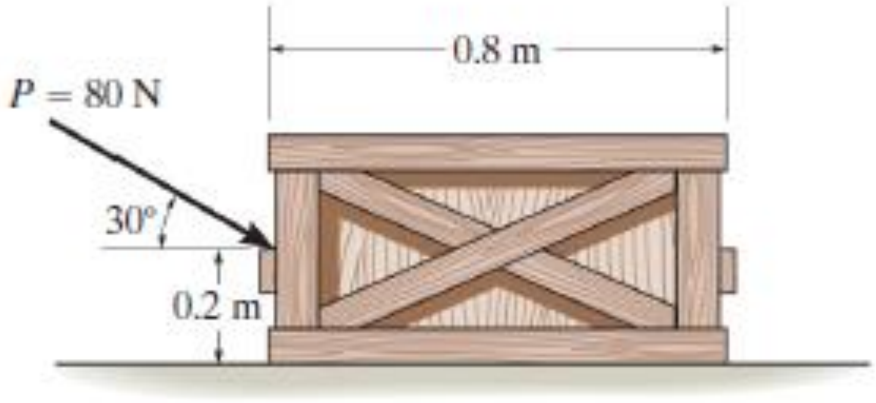
or

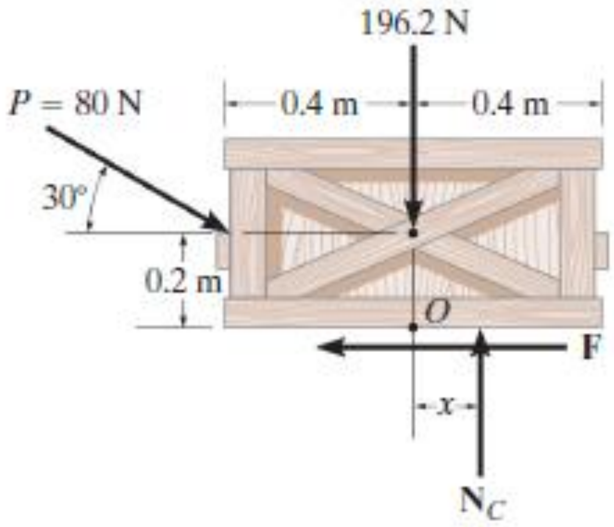
slipping occurs at *C* and no slipping at *A*, in which

$F_C = 0.5N_C$ and $F_A \leq 0.3N_A$.

calculating *P* for each case and then choosing the case for which *P* is *smaller*.

The uniform crate has a mass of 20 kg. If a force $P = 80$ N is applied to the crate, determine if it remains in equilibrium. The coefficient of static friction is $u_S = 0.3$.





$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 80\cos 30^\circ\ \text{N} - F = 0$$

$$+\uparrow\Sigma F_y = 0; \quad -80\sin 30^\circ\ \text{N} + N_C - 196.2\ \text{N} = 0$$

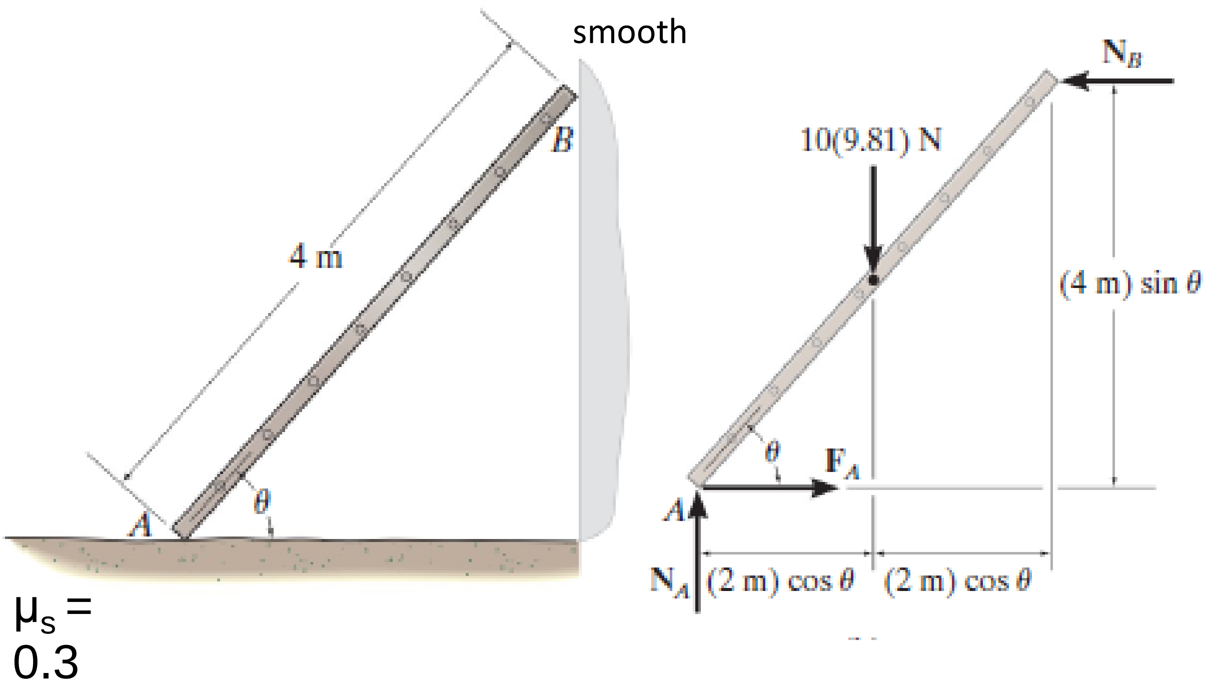
$$\circlearrowleft +\Sigma M_O = 0; \quad 80\sin 30^\circ\ \text{N}(0.4\ \text{m}) - 80\cos 30^\circ\ \text{N}(0.2\ \text{m}) + N_C(x) = 0$$

$F = 69.3$ N $< F_s = 0.3\text{x}\ 236.3 = 70.89$ → No slipping

$N_C = 236.2$ N

$x = -0.00908\ \text{m} = -9.08\ \text{mm}$ → $X < 0.4$ .....No tipping

The uniform 10-kg ladder in Fig. 8–9*a* rests against the smooth wall at *B*, and the end *A* rests on the rough horizontal plane for which the coefficient of static friction is $\mu_s = 0.3$. Determine the angle of inclination θ of the ladder and the normal reaction at *B* if the ladder is on the verge of slipping.



Equations of Equilibrium and Friction. Since the ladder is on the verge of slipping, then $F_A = \mu_s N_A = 0.3N_A$. By inspection, N_A can be obtained directly.

$$+\uparrow\Sigma F_y = 0; \qquad N_A - 10(9.81)\text{ N} = 0 \qquad N_A = 98.1\text{ N}$$

Using this result, $F_A = 0.3(98.1\text{ N}) = 29.43\text{ N}$. Now N_B can be found.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 29.43\text{ N} - N_B = 0$$

$$N_B = 29.43\text{ N} = 29.4\text{ N} \qquad \textit{Ans.}$$

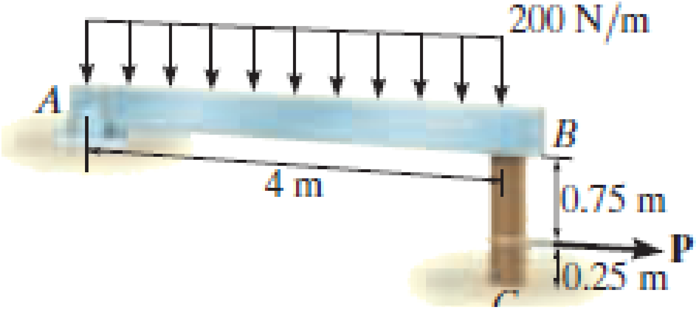
Finally, the angle θ can be determined by summing moments about point *A*.

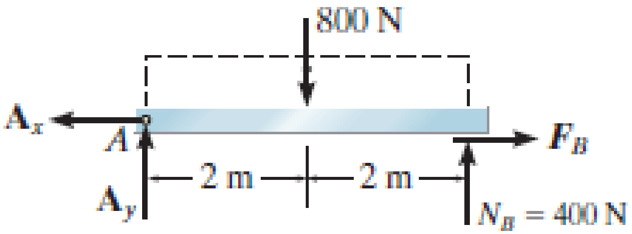
$$\circlearrowleft +\Sigma M_A = 0; \qquad (29.43\text{ N})(4\text{ m})\sin\theta - [10(9.81)\text{ N}](2\text{ m})\cos\theta = 0$$

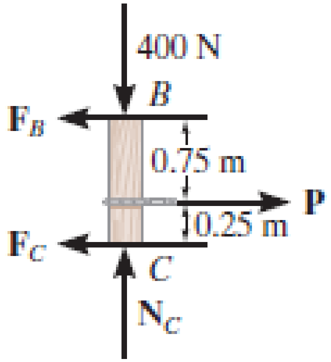
$$\frac{\sin\theta}{\cos\theta} = \tan\theta = 1.6667$$

$$\theta = 59.04^\circ = 59.0^\circ \qquad \textit{Ans.}$$

Beam AB is subjected to a uniform load of 200 N/m and is supported at B by post BC, Fig. 8–10a. If the coefficients of static friction at B and C are $\mu_B = 0.2$ and $\mu_C = 0.5$, determine the force **P** needed to pull the post out from under the beam. Neglect the weight of the members and the thickness of the beam.







7 unknowns
6 equilibrium equations
2 friction equation
$F_B = 0.2\ N_B$
$F_C = 0.5\ N_C$

Member AB

$\Sigma M_A = 0$, $N_B = 400N$

Member BC

Assume post slip at B

$F_B = 0.2\ N_B = 0.2 \times 400 = 80N$

$\Sigma M_C = 0$, $-0.25\ P + 1 \times 80 = 0$ … **$P = 320N$**

$\Sigma F_x = 0$, $-Fc - 80 + 320 = 0$ ……. **$F_C = 240N$**

$\Sigma F_y = 0$, $Nc - 400 = 0$ ……. **$N_C = 400N$**

Check friction at C

Since $F_C = 240\text{ N} > \mu_C N_C = 0.5(400\text{ N}) = 200\text{ N}$, slipping at C occurs. Thus the other case of movement must be investigated.

(Post Slips at C and Rotates about B.) Here $F_B \le \mu_B N_B$ and

$F_C = \mu_C N_C$; $F_C = 0.5 N_C$

$P = 267\text{ N}$ *Ans.*

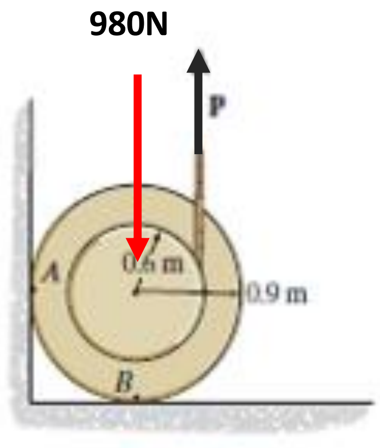
$N_C = 400\text{ N}$

$F_C = 200\text{ N}$

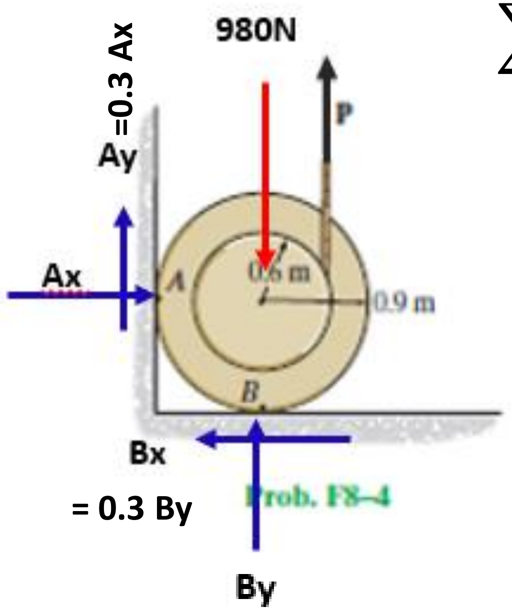
$F_B = 66.7\text{ N}$

Obviously, this case occurs first since it requires a *smaller* value for P.

F8–4. If the coefficient of static friction at contact points A and B is $\mu_s = 0.3$, determine the maximum force P that can be applied without causing the 100-kg spool to move.



Prob. F8–4



$$\sum M_A = (-980)(0.9) - 0.9(0.3\,By) + (By)(0.9) + (1 \cdot 5)P = 0$$

$$\sum F_x = Ax - 0.3\,By = 0$$

$$Ax = 0.3By$$

$$\sum F_y = 0.3Ax + By - 980 + P = 0$$

$$0.3Ax + By - 980 + P = 0$$

$$P = -By - 0.3(0.3By) + 980$$

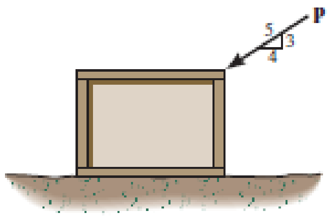
$$(-980)(0.9) - 0.9(0.3By) + (B_y)(0.9) + (1 \cdot 5)(-1.09By + 980) = 0$$

$$By = 585$$

$$\boldsymbol{P = 342.35N}$$

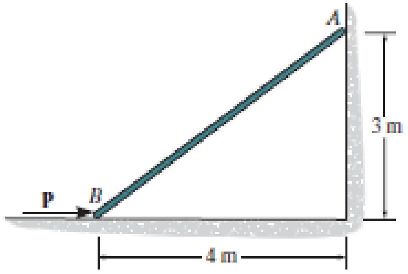
All problem solutions must include FBDs.

F8–1. Determine the friction developed between the 50-kg crate and the ground if a) $P = 200$ N, and b) $P = 400$ N. The coefficients of static and kinetic friction between the crate and the ground are $\mu_s = 0.3$ and $\mu_k = 0.2$.



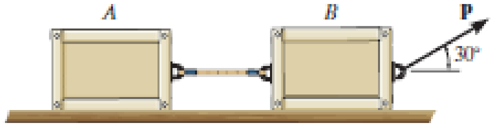
Prob. F8–1

F8–2. Determine the minimum force P to prevent the 30-kg rod AB from sliding. The contact surface at B is smooth, whereas the coefficient of static friction between the rod and the wall at A is $\mu_s = 0.2$.



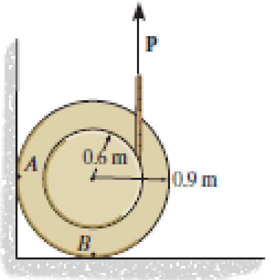
Prob. F8–2

F8–3. Determine the maximum force P that can be applied without causing the two 50-kg crates to move. The coefficient of static friction between each crate and the ground is $\mu_s = 0.25$.



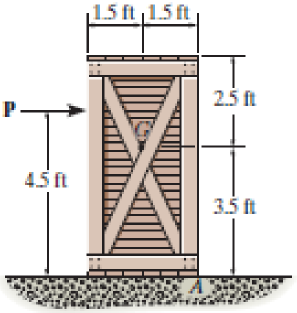
Prob. F8–3

F8–4. If the coefficient of static friction at contact points A and B is $\mu_s = 0.3$, determine the maximum force P that can be applied without causing the 100-kg spool to move.



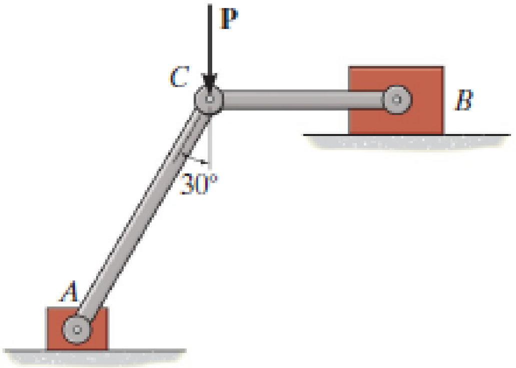
Prob. F8–4

F8–5. Determine the maximum force P that can be applied without causing movement of the 250-lb crate that has a center of gravity at G. The coefficient of static friction at the floor is $\mu_s = 0.4$.



Prob. F8–5

Blocks A and B have a mass of 3 kg and 9 kg, respectively, and are connected to the weightless links shown in Fig. 8–11a. Determine the largest vertical force **P** that can be applied at the pin C without causing any movement. The coefficient of static friction between the blocks and the contacting surfaces is $\mu_s = 0.3$.



(a)

Equations of Equilibrium and Friction. The force in links AC and BC can be related to P by considering the equilibrium of pin C.

$$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\cos 30° - P = 0; \qquad F_{AC} = 1.155P$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 1.155P \sin 30° - F_{BC} = 0; \qquad F_{BC} = 0.5774P$$

Using the result for F_{AC}, for block A,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_A - 1.155P \sin 30° = 0; \qquad F_A = 0.5774P \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad N_A - 1.155P\cos 30° - 3(9.81\text{ N}) = 0;$$

$$N_A = P + 29.43\text{ N} \qquad (2)$$

Using the result for F_{BC}, for block B,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad (0.5774P) - F_B = 0; \qquad F_B = 0.5774P \qquad (3)$$

$$+\uparrow \Sigma F_y = 0; \qquad N_B - 9(9.81)\text{ N} = 0; \qquad N_B = 88.29\text{ N}$$

Movement of the system may be caused by the initial slipping of *either* block A or block B. If we assume that block A slips first, then

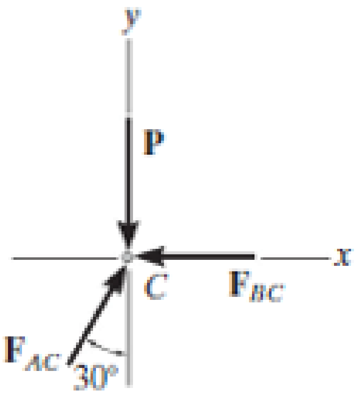
$$F_A = \mu_s N_A = 0.3N_A \qquad (4)$$

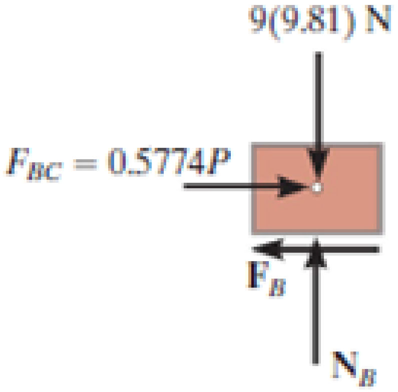
Substituting Eqs. 1 and 2 into Eq. 4,

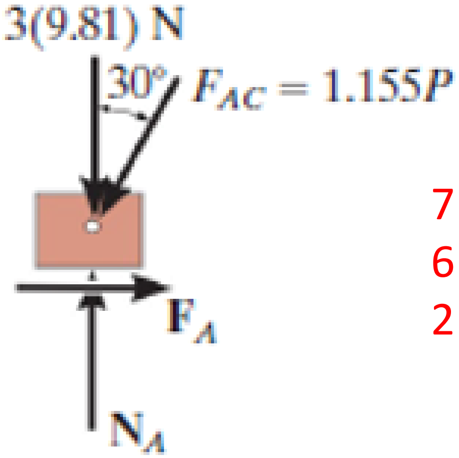
$$0.5774P = 0.3(P + 29.43)$$

$$P = 31.8\text{ N} \qquad \textit{Ans.}$$

Substituting this result into Eq. 3, we obtain $F_B = 18.4$ N. Since the maximum static frictional force at B is $(F_B)_{max} = \mu_s N_B = 0.3(88.29\text{ N}) = 26.5\text{ N} > F_B$, block B will not slip. Thus, the above assumption is correct. Notice that if the inequality were not satisfied, we would have to assume slipping of block B and then solve for P.







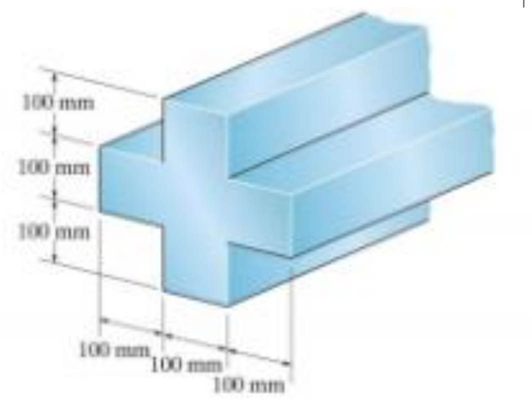
7 unknowns
6 equation
2 friction

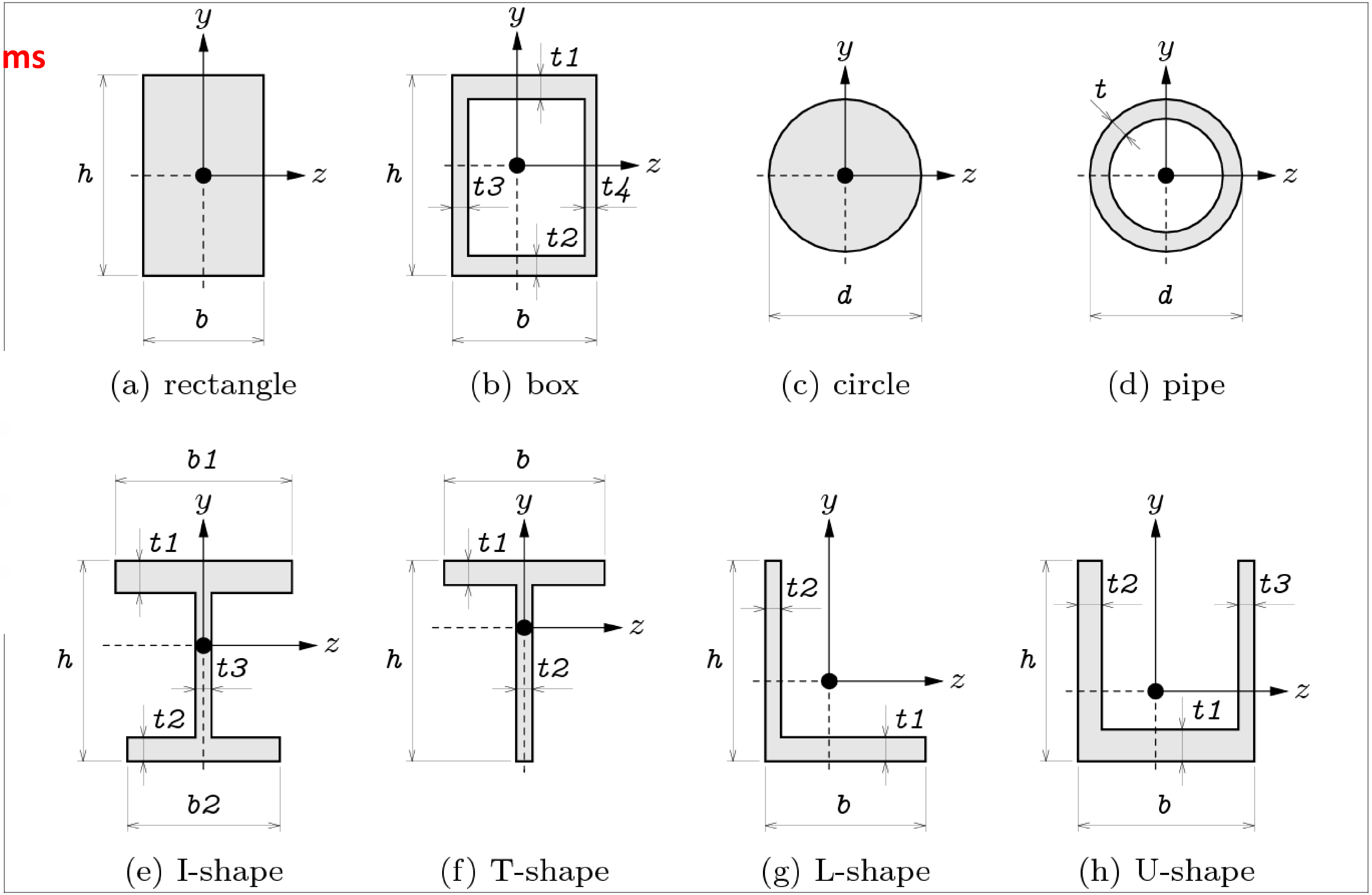
Chapter 9

Center of Gravity & Centroid

Iqbal Marie

Cross section of beams

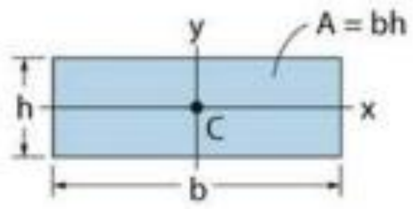




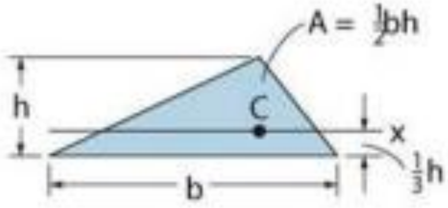
9.1 Centroid of an Area

The centroid, is a point defining the geometric center of an object.

This point coincides with the center of mass or the center of gravity only if the material composing the body is uniform or homogeneous.

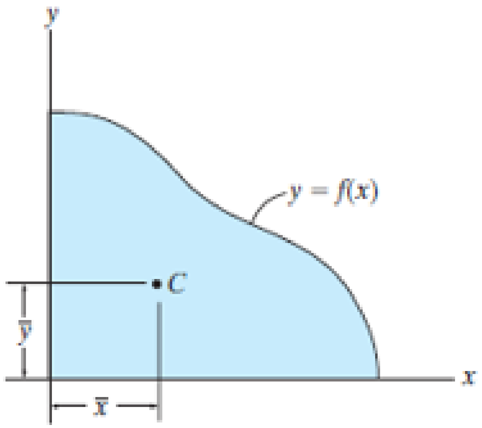


Rectangular area

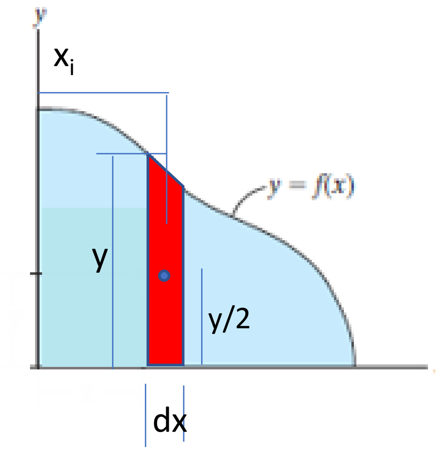


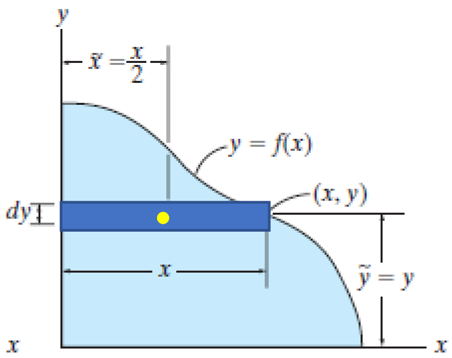
Triangular area

Required X’, and Y’



Select deferential element





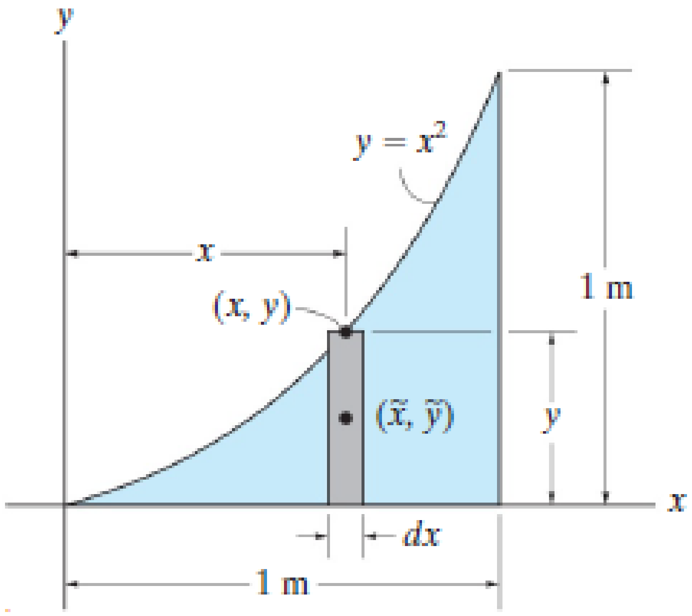
dA = y dx = f(x) dx

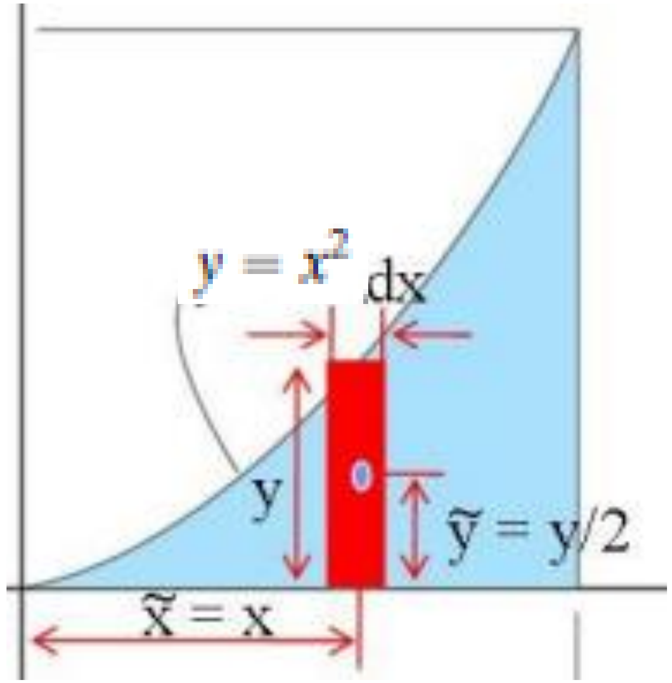
x_i =x

y_i =y/2 = f(x)/2

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} \qquad \bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA}$$

The centroid location (x’ , y’)





dA = y dx = x^2 dx
x’ = x
y’ = y / 2 = x^2/ 2

$$\int_A dA = \int_0^{1\,m} y\,dx = \int_0^{1\,m} x^2\,dx = 0.333$$

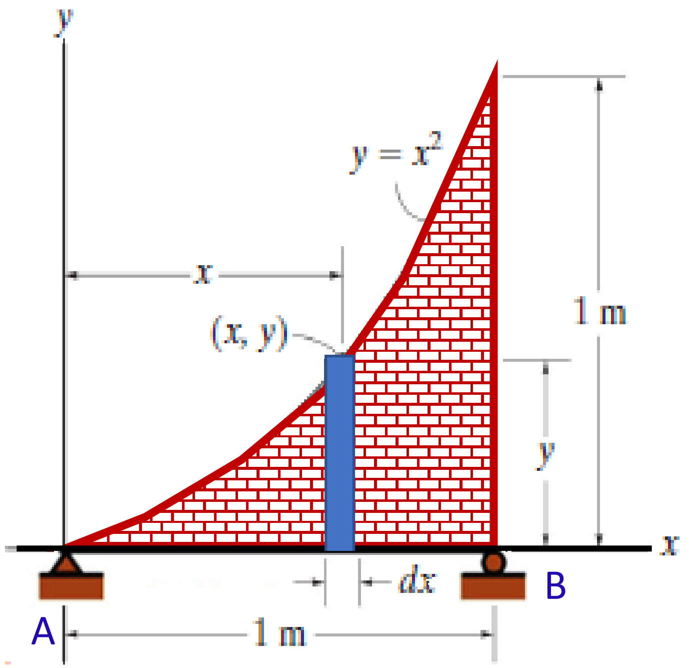
$$\int_A \tilde{x}\,dA = \int_0^{1\,m} xy\,dx = \int_0^{1\,m} x^3\,dx = 0.25$$

$$\int_A \tilde{y}\,dA = \int_0^{1\,m} (y/2)y\,dx = \int_0^{1\,m} (x^2/2)x^2\,dx = 0.10$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{0.25}{0.333} = 0.75\ m$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{0.1}{0.333} = 0.3\ m$$

Find support reactions if the unit weight of the material is $\gamma = 20\ kN/m^2$

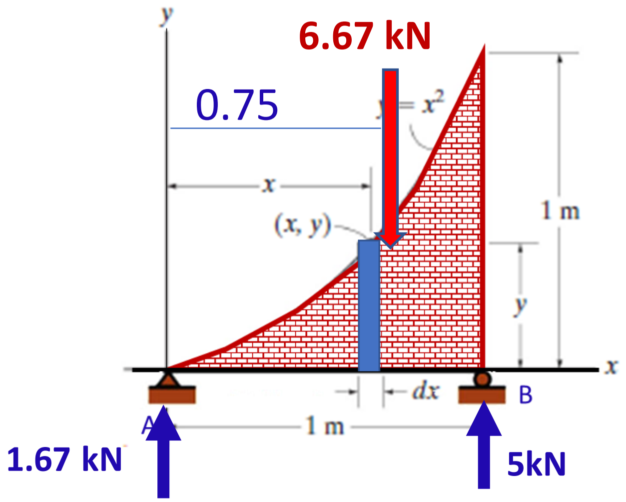


$$A = \int_A dA = \int_0^{1\,m} y\,dx = \int_0^{1\,m} x^2\,dx = 0.333$$

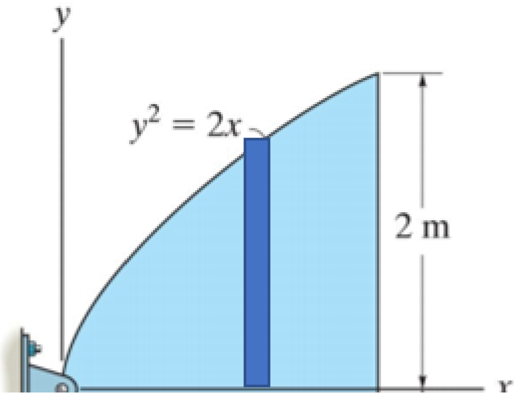
Weight W= 0.333x 20 = 6.67 kN

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{1\,m} xy\,dx}{\int_0^{1\,m} y\,dx} = \frac{\int_0^{1\,m} x^3\,dx}{\int_0^{1\,m} x^2\,dx} = \frac{0.250}{0.333} = 0.75\ m$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{1\,m} (y/2)y\,dx}{\int_0^{1\,m} y\,dx} = \frac{\int_0^{1\,m} (x^2/2)x^2\,dx}{\int_0^{1\,m} x^2\,dx} = \frac{0.100}{0.333} = 0.3\ m$$

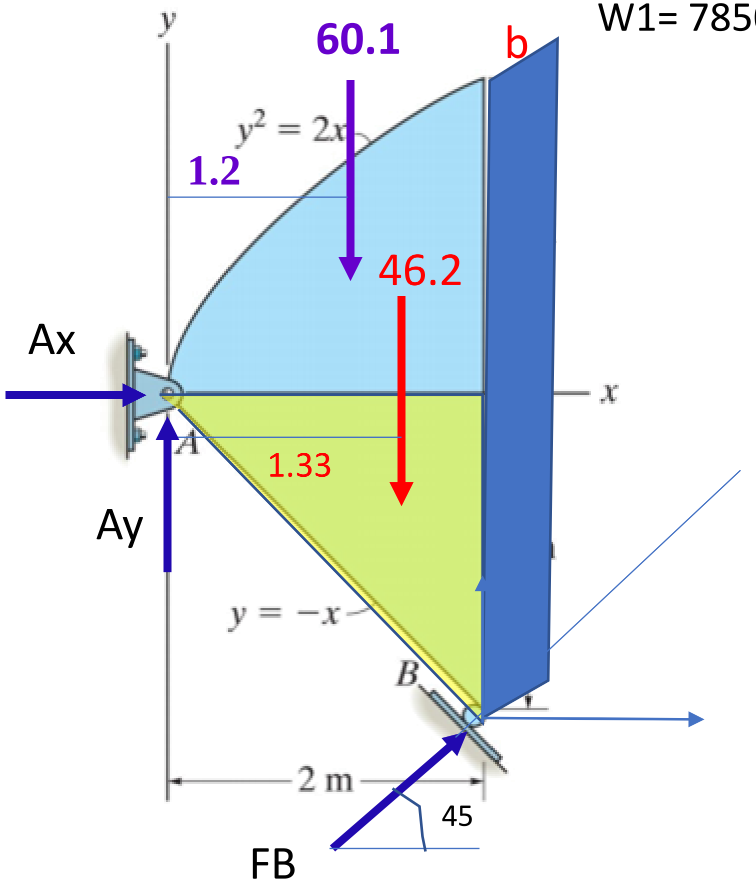


The steel plate is b= 0.3 m thick and has a density of 7850 kg/m³.
find the reactions at the pin and roller support.



W= ρ g volume = ρ g A b

W1= 7850x9.81x0.3x0.5x2x2) = 46205.1N = 46.2 kN



$$A2 = (2)^{0.5} \int (x)^{0.5}\, dx = (2^{(1.5)}/1.5) \times 2^{0.5} = 2.67 m^2$$

W2= 7850x9.81x0.3x2.67) = 60066.63N = 60.1 kN

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} = \frac{(2)^{0.5} \int (x^3)^{0.5}\, dx = (2^{(2.5)}/2.5) \times 2^{0.5}}{2.67} = 3.2/2.67 = 1.2 \text{ m}$$

$\circlearrowleft + \Sigma M_A = 0;$ FB = 47.9

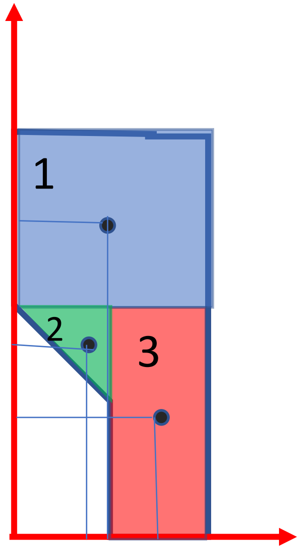
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ Ax = 33.9

$+\uparrow \Sigma F_y = 0;$ Ay =73.9

9.2 ***composite body :*** consists of a series of connected "simpler" shaped bodies, which may be rectangular, triangular, semicircular

Procedure of Analysis

The location of the center of gravity of a body or the centroid of a composite geometrical object represented by area



- **Locate reference axes**
- **Divide the area into a number of simple shapes.**
- **Find the area of each shape then sum all the areas** ΣA.
- **If a composite body has a *hole*, or a geometric region having no material, then consider the composite body without the hole and consider the hole as a *negative* area**
- **Determine the coordinates x , y , z of the center of gravity or centroid of each part.**
- **Determine X $\bar{Y}$ by applying the center of gravity equations,**

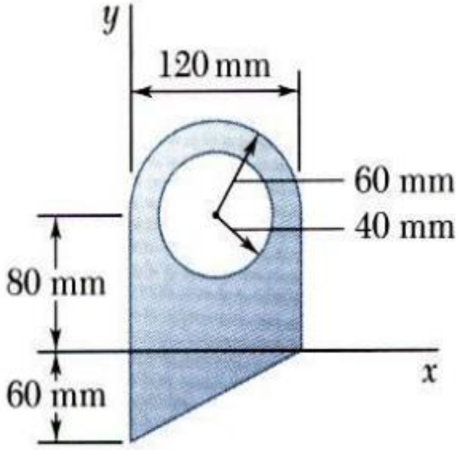
$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A}$$

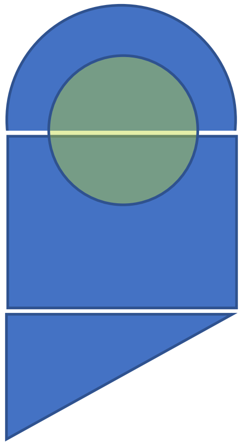
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

If an object is *symmetrical* about an axis, the centroid of the object lies on

If desired, the calculations can be arranged in tabular form

Component	A, mm²	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm³	$\bar{y}A$, mm³
	$\Sigma A =$			$\Sigma\bar{x}A =$	$\Sigma\bar{y}A =$

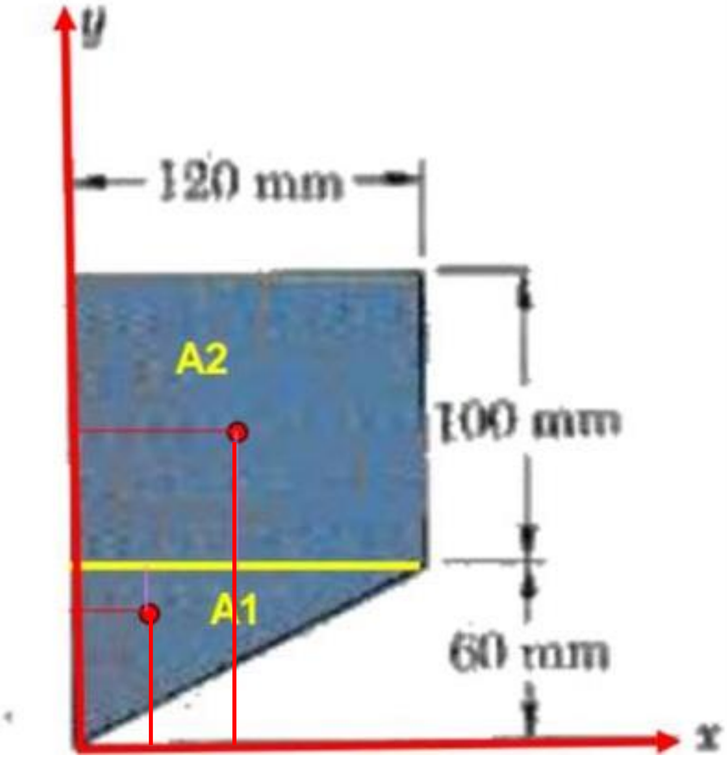




Find the centroid of the given body from the shown x- and y- axes

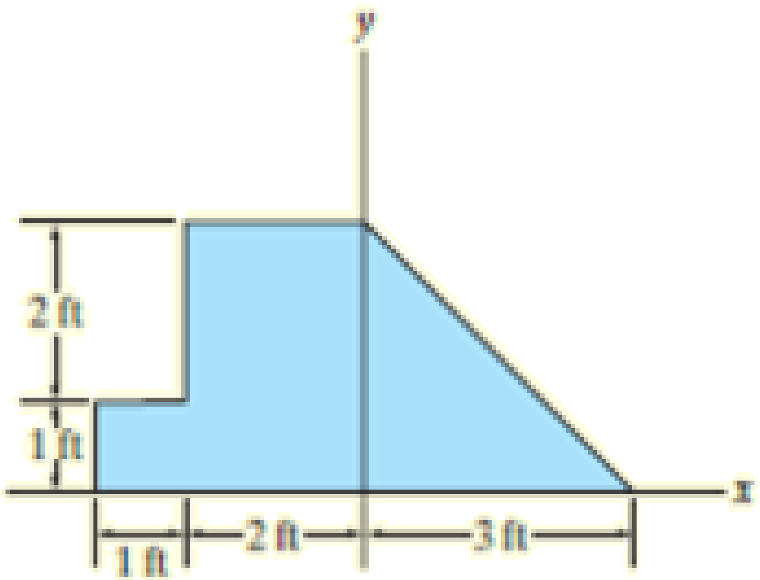
$$\bar{x} = \frac{1}{A_T}\sum \bar{x}_i A_i$$

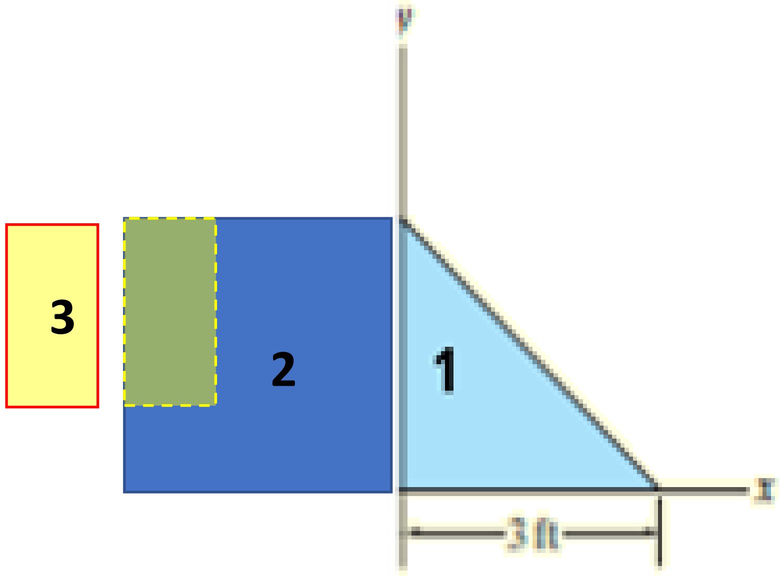
$$\bar{y} = \frac{1}{A_T}\sum \bar{y}_i A_i$$



Body	Area(mm^2)	x (mm)	y(mm)	x*Area (mm^3)	y*Area (mm^3)
A1	3600	40	40	144000	144000
A2	12000	60	110	720000	1320000
Sum	15600			864000	1464000
centroid (x)	55.38 mm				
centroid (y)	93.85 mm				

Locate the centroid of the plate area from the given axes





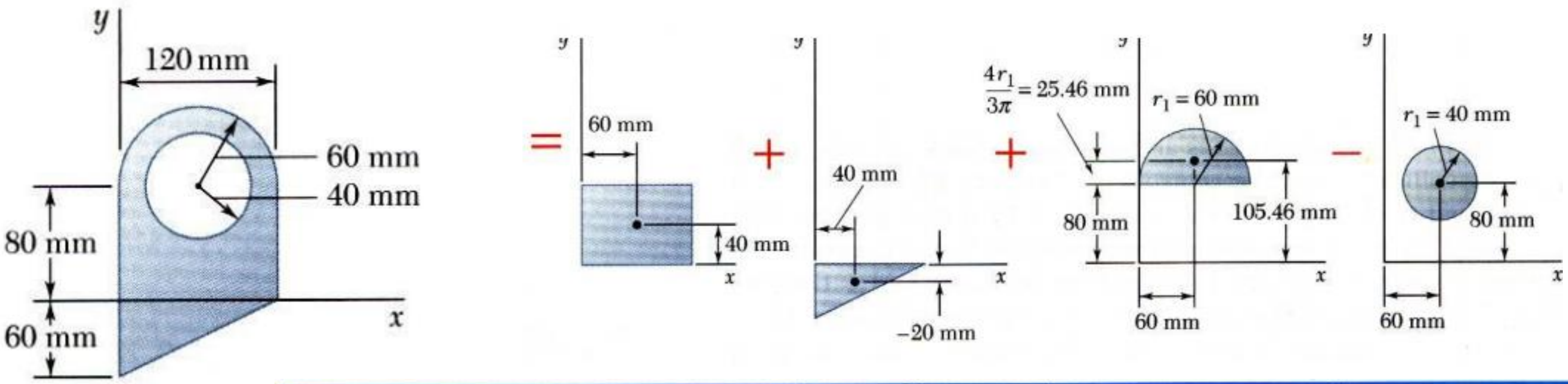
Segment	A (ft²)	$\tilde{x}$ (ft)	$\tilde{y}$ (ft)	$\tilde{x}A$ (ft³)	$\tilde{y}A$ (ft³)
1	$\frac{1}{2}(3)(3) = 4.5$	1	1	4.5	4.5
2	$(3)(3) = 9$	−1.5	1.5	−13.5	13.5
3	$-(2)(1) = -2$	−2.5	2	5	−4
	$\Sigma A = 11.5$			$\Sigma\tilde{x}A = -4$	$\Sigma\tilde{y}A = 14$

Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{-4}{11.5} = -0.348 \text{ ft} \qquad \textit{Ans.}$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{14}{11.5} = 1.22 \text{ ft} \qquad \textit{Ans.}$$

Determine the location of the centroid from the given axes.



Component	A, mm²	x̄, mm	ȳ, mm	x̄A, mm³	ȳA, mm³
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	−20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$

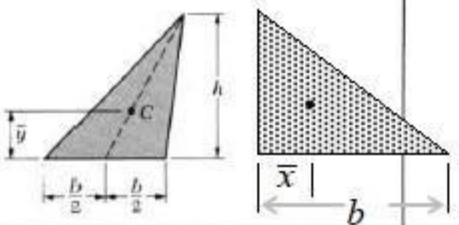
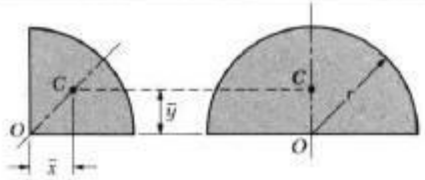
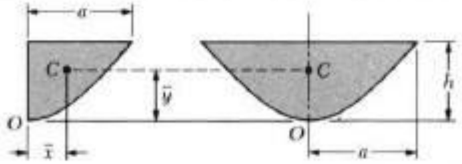
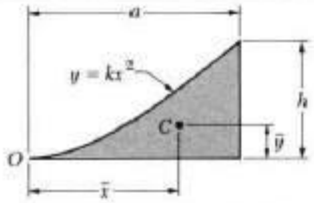
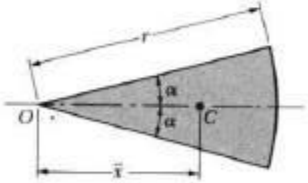
$$\bar{X} = \frac{\sum \bar{x}A}{\sum A} = \frac{+757.7 \times 10^3\ \text{mm}^3}{13.828 \times 10^3\ \text{mm}^2}$$

$$\bar{X} = 54.8\ \text{mm}$$

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{+506.2 \times 10^3\ \text{mm}^3}{13.828 \times 10^3\ \text{mm}^2}$$

$$\bar{Y} = 36.6\ \text{mm}$$

Centroids of Common Shapes of Areas and Lines

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic span-drel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2

9.6 Fluid Pressure

According to Pascal's law, a fluid at rest creates a pressure *p* at a point that is the ***same* in *all* directions**

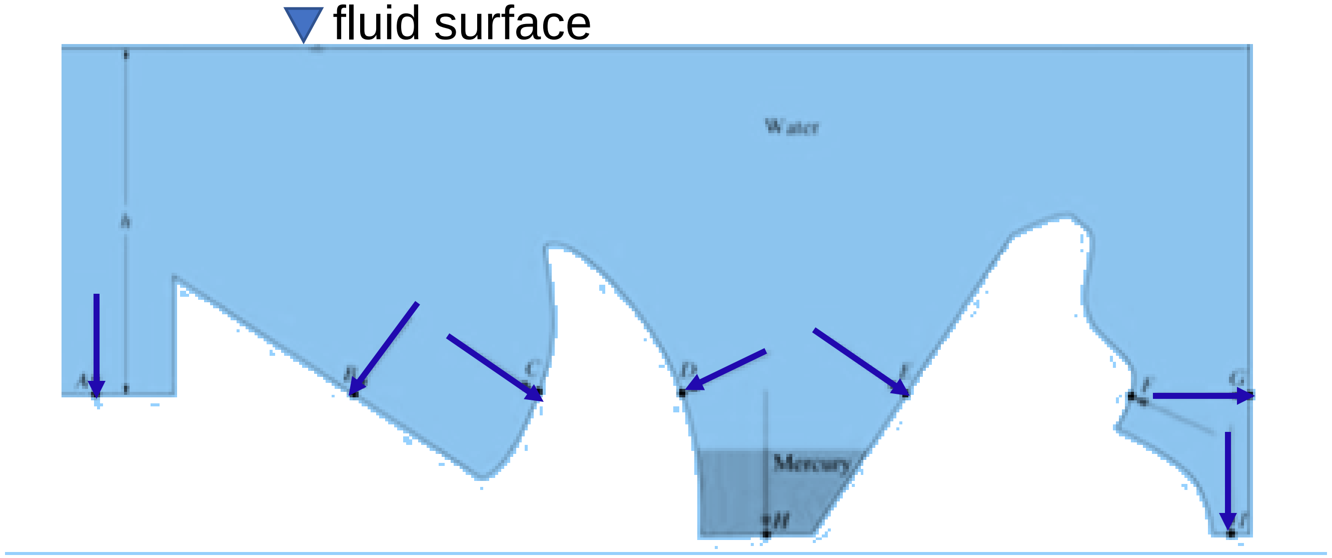
The magnitude of ***p***, depends on the specific weight γ or mass density ρ of the fluid and the depth **z** of the point from the fluid surface

$\rho_w = 1000\ kg/m^3$

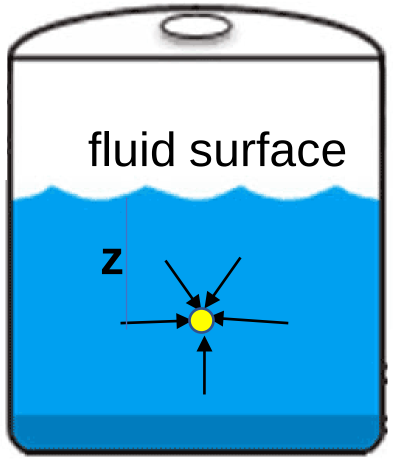
for water or $\gamma_w = \rho_w . g = 9810\ N/m^3 = 9.81 kN/m^3$

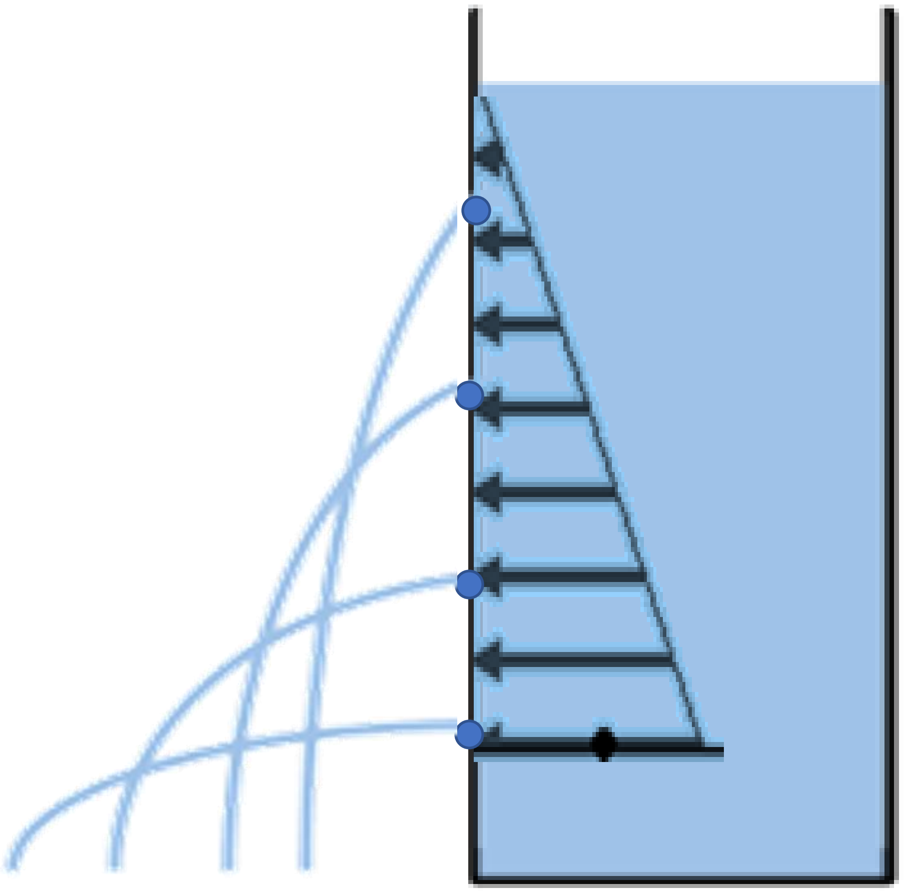
$$p = \gamma z = \rho g z$$

Any Free surface open to the atmosphere has atmospheric pressure p = 0

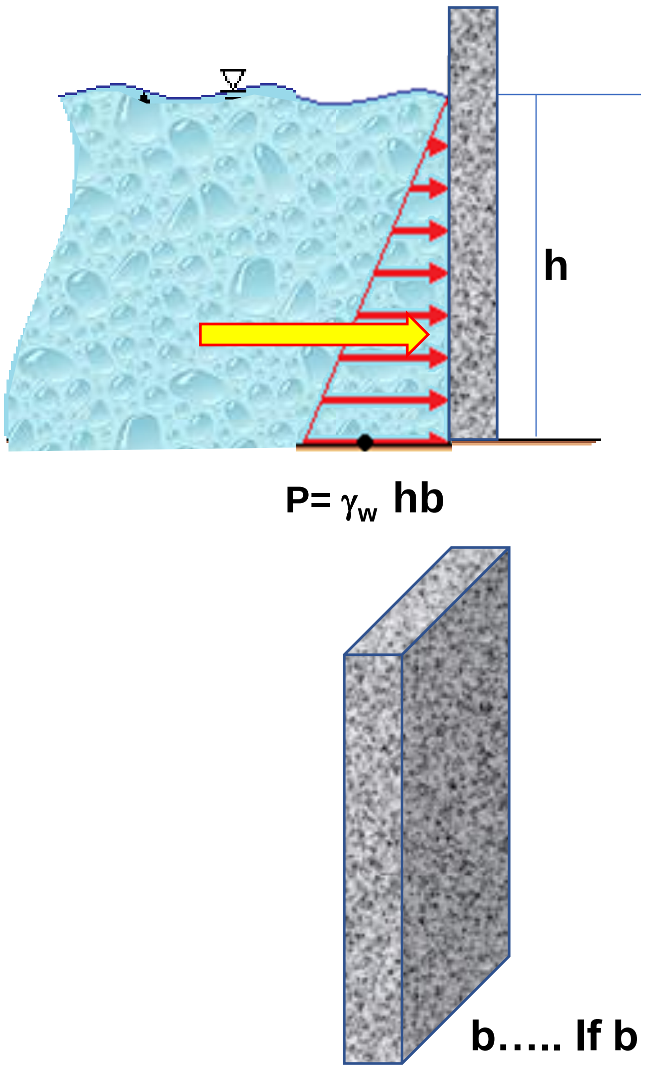


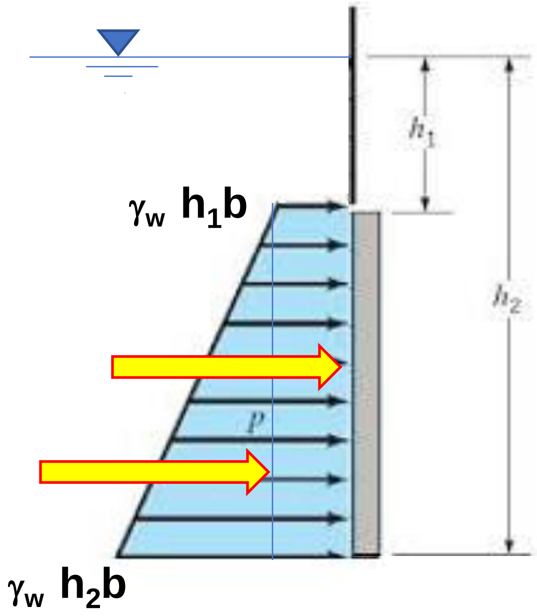
https://www.google.com/url?sa=i&url=https%3A%2F%2Fwww.watervision.co.nz%2F&psig=AOvVaw20GQgmSH5obuun80XaGWFg&ust=1586409226065000&source=images&cd=vfe&ved=0CAIQjRxqFwoTCIC9gpeJ2OgCFQAAAAAdAAAAABAD

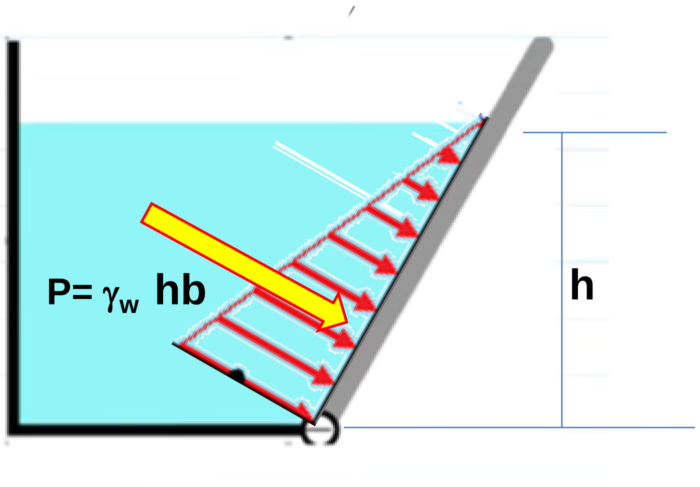




Draw the hydrostatic forces on the surfaces shown

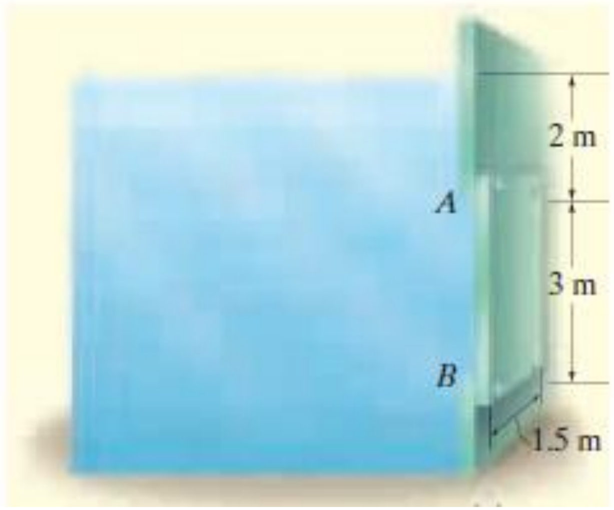






b….. If b not given … assume b = 1 unit length

Determine the magnitude and location of the **resultant hydrostatic fc** acting on the submerged rectangular **plate *AB*** shown
The plate has a **width of 1.5 m**; ρ_w= 1000 kg/m^3



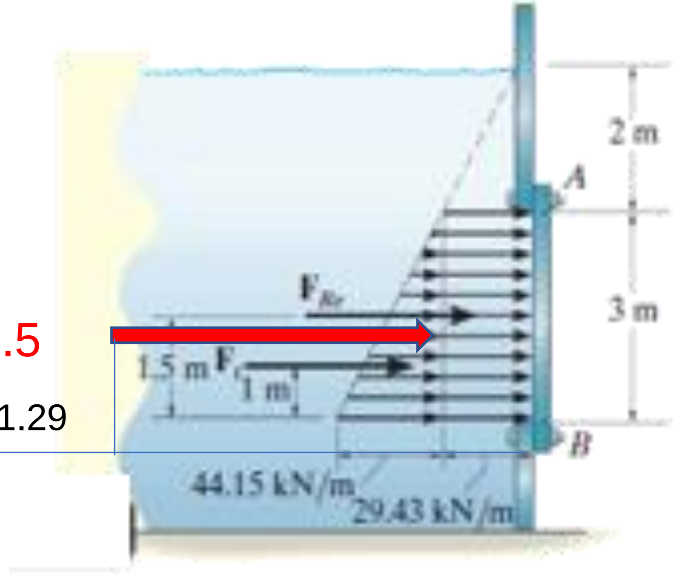
γ_w= 1000x9.8=9800 N/m^3 =9.81 kN/m^3

w_A= 9.81 x1.5 x2 = 29.43 kN/m

w_B= 9.81 x1.5 x5 = 73.58 kN/m

F1 = 29.43x3 = 88.29 kN
F2 = (73.58- 29.43) x3/2 = 66.23 kN
FR = 88.29 +66.23 = 154.5 kN

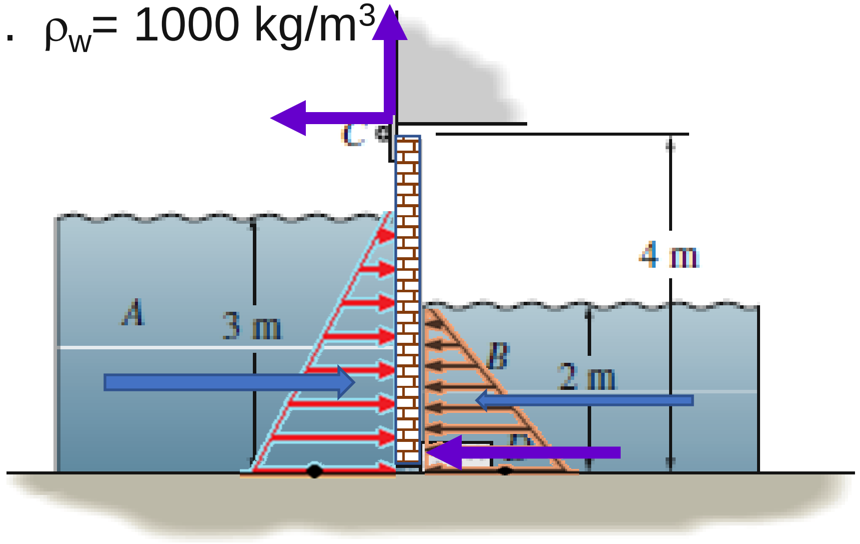
h= = (88.29 x1.5 +66.23x 1)/ 154.5 = 1.29 m



FR=154.5 kN

h=1.29 m

When the tide water *A* subsides, the tide gate automatically swings open to drain the marsh *B*. For the condition of high tide shown, determine the horizontal reactions developed at the hinge *C* and stop block *D*. The length of the gate is 6 m and its height is 4 m. ρ_w= 1000 kg/m³



Fluid Pressure: The fluid pressure at points D and E can be determined using Eq. 9–13, $p = \rho g z$.

$$p_D = 1.0(10^3)(9.81)(2) = 19\,620 \text{ N/m}^2 = 19.62 \text{ kN/m}^2$$

$$p_E = 1.0(10^3)(9.81)(3) = 29\,430 \text{ N/m}^2 = 29.43 \text{ kN/m}^2$$

Thus,

$$w_D = 19.62(6) = 117.72 \text{ kN/m}$$

$$w_E = 29.43(6) = 176.58 \text{ kN/m}$$

$$F_{R_1} = \frac{1}{2}(176.58)(3) = 264.87 \text{ kN}$$

$$F_{R_2} = \frac{1}{2}(117.72)(2) = 117.72 \text{ kN}$$

$\zeta + \Sigma M_C = 0;\quad 264.87(3) - 117.72(3.333) - D_x(4) = 0$

$$D_x = 100.55 \text{ kN} = 101 \text{ kN}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad 264.87 - 117.72 - 100.55 - C_x = 0$

$$C_x = 46.6 \text{ kN}$$

The **2-m-wide** rectangular gate is pinned at its center *A* and is prevented from rotating by the block at *B*. Determine the reactions at these supports due to hydrostatic pressure. $\rho_w = 1000 \text{ kg/m}^3$.

$$w_1 = 1000(9.81)(3)(2) = 58\,860 \text{ N/m}$$

$$w_2 = 1000(9.81)(3)(2) = 58\,860 \text{ N/m}$$

$$F_1 = \frac{1}{2}(3)(58\,860) = 88\,290$$

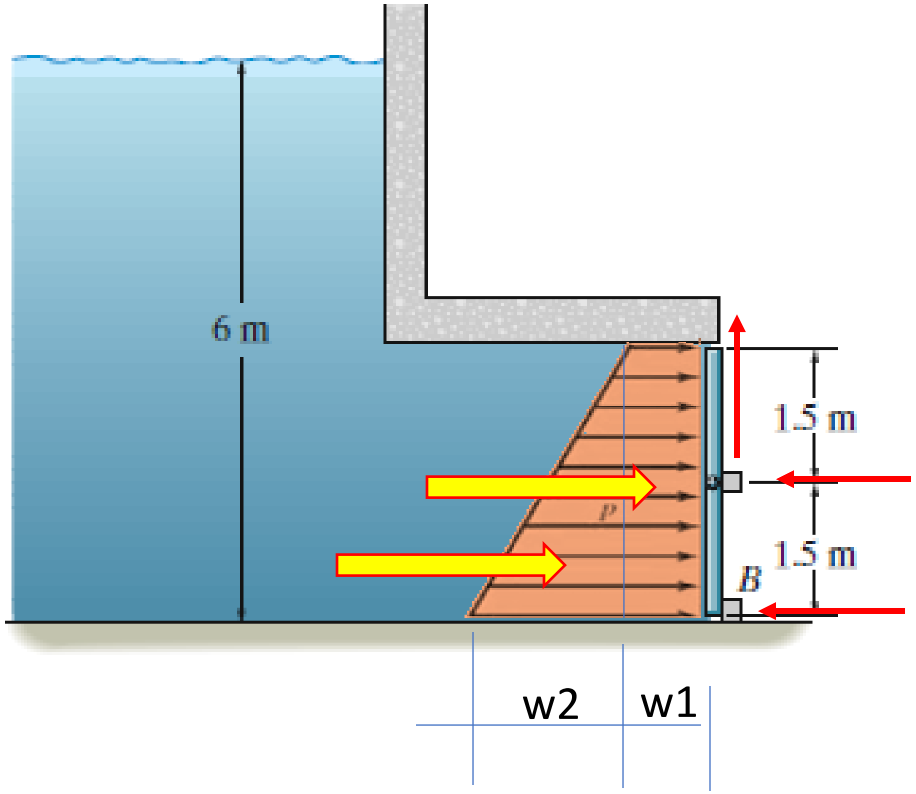
$$F_2 = (58\,860)(3) = 176\,580$$

$\zeta + \Sigma M_A = 0;$ $\quad 88\,290(0.5) - F_B(1.5) = 0$

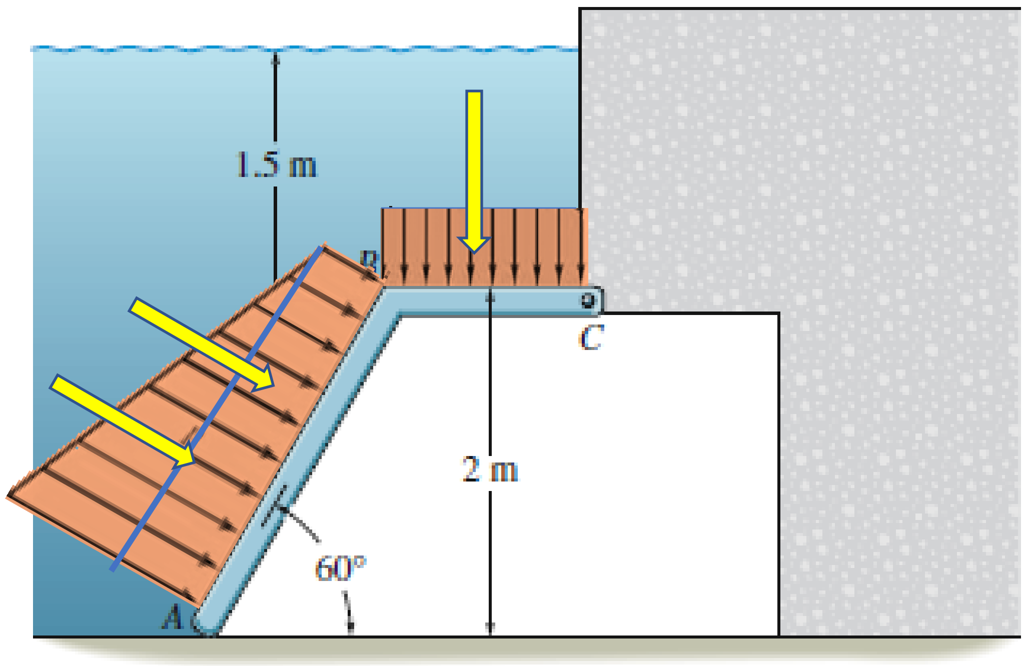
$$F_B = 29\,430 \text{ N} = 29.4 \text{ kN}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 88\,290 + 176\,580 - 29\,430 - F_A = 0$

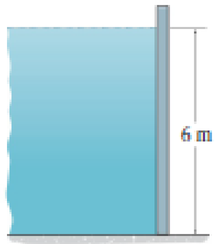
$$F_A = 235\,440 \text{ N} = 235 \text{ kN}$$



Determine the magnitude of the resultant force acting on the gate *ABC* due to hydrostatic pressure. The gate has a width of **1.5 m**. $\rho_w = 1000 kg/m^3$

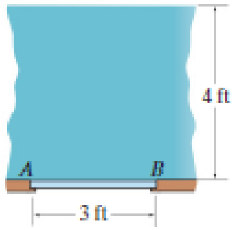


F9–17. Determine the magnitude of the hydrostatic force acting per meter length of the wall. Water has a density of $\rho = 1\ \mathrm{Mg/m^3}$.



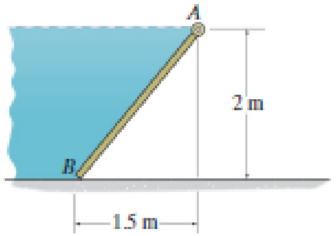
Prob. F9–17

F9–18. Determine the magnitude of the hydrostatic force acting on gate AB, which has a width of 4 ft. The specific weight of water is $\gamma = 62.4\ \mathrm{lb/ft^3}$.



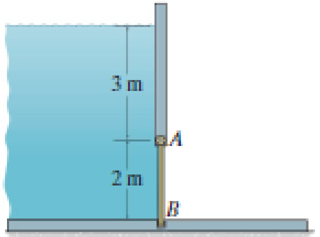
Prob. F9–18

F9–19. Determine the magnitude of the hydrostatic force acting on gate AB, which has a width of 1.5 m. Water has a density of $\rho = 1\ \mathrm{Mg/m^3}$.



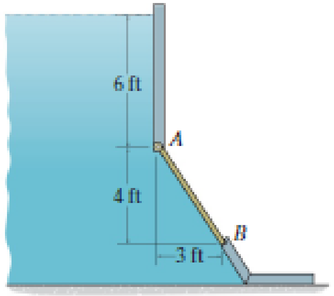
Prob. F9–19

F9–20. Determine the magnitude of the hydrostatic force acting on gate AB, which has a width of 2 m. Water has a density of $\rho = 1\ \mathrm{Mg/m^3}$.



Prob. F9–20

F9–21. Determine the magnitude of the hydrostatic force acting on gate AB, which has a width of 2 ft. The specific weight of water is $\gamma = 62.4\ \mathrm{lb/ft^3}$.



Prob. F9–21

Chapter 10

Moments of Inertia

MOI

Iqbal Marie

10.1 Definition of Moments of Inertia for Areas (MOI)

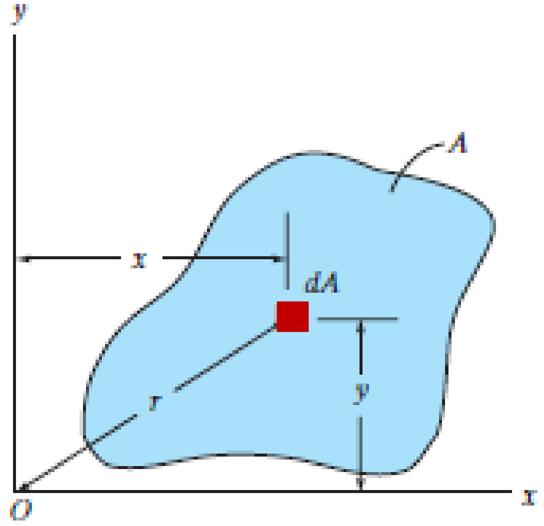
The integrals ($\int y^2 dA$, $\int x^2 dA$) is sometimes referred to as the "**second moment" of the area about an axis** , but more often it is called the ***moment of inertia of the area***.

Although for an area this integral has no physical meaning, it often arises in formulas used in fluid mechanics, mechanics of materials, structural mechanics, and mechanical design,

The **moments of inertia** of a differential area ***dA*** about the ***x*** **and** ***y*** axes are $dI_x = y^2\, dA$

$dI_y = x^2\, dA$,

$$I_x = \int_A y^2\, dA$$
$$I_y = \int_A x^2\, dA$$



The ***polar moment of inertia***. It is defined as $dJ_o = r^2 dA$,

$r^2 = x^2 + y^2$

$$J_O = \int_A r^2\, dA = I_x + I_y$$

The units for moment of inertia involve length raised to the fourth power, (m^4, mm^4)

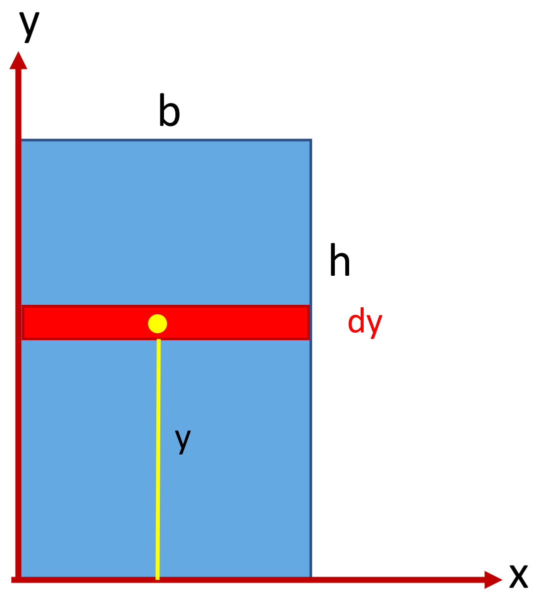
a. Find moment of inertia about the x axis (passing through the base of the rectangle) (I_x)

Take a differential element of area that is *parallel* to the *x* axis

$$I_x = \int_A y^2\, dA$$

$$I_x = \int y^2 dA = \int_0^h y^2 \,(b\, dy) = by^3/3 \Big]_0^h = bh^3/3$$

$$I_x = \frac{1}{3} bh^3$$



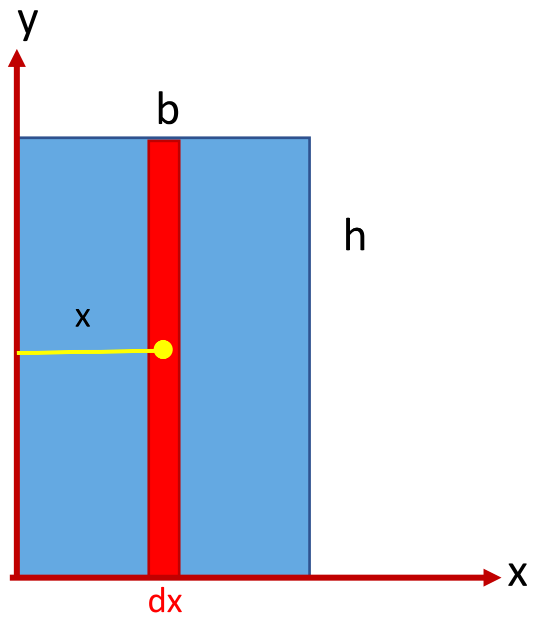
b. Find moment of inertia about the y axis (I_y)

Take a differential element of area that is *parallel* to the y axis

$$I_y = \int_A x^2\, dA$$

$$I_y = \int x^2 dA = \int_0^b x^2 \,(h\, dx) = hx^3/3 \Big]_0^b = hb^3/3$$

$$I_y = \frac{1}{3} hb^3$$



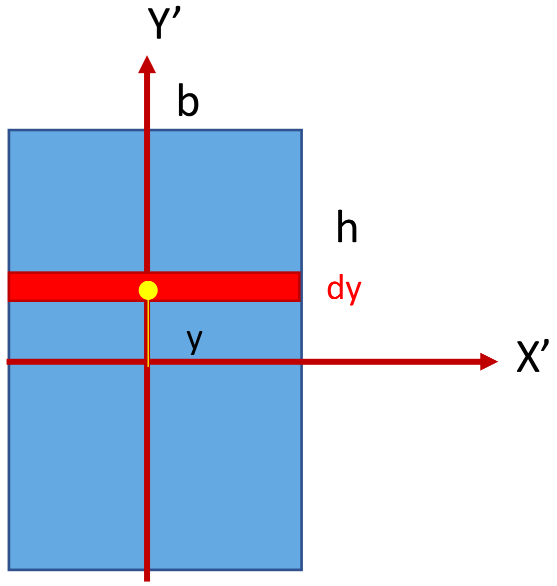
a. Find moment of inertia about the x' axis (passing through the centroid of the rectangle)

A differential element of area that is *parallel* to the *x* axis

$$I_x = \int_A y^2\,dA$$

$$I_x = \int y^2 dA = \int_{-h/2}^{h/2} y^2 \; (b\,dy) = by^3/3 \Big|_{-h/2}^{h/2} = bh^3/12$$

$$\bar{I}_{x'} = \frac{1}{12} b h^3$$

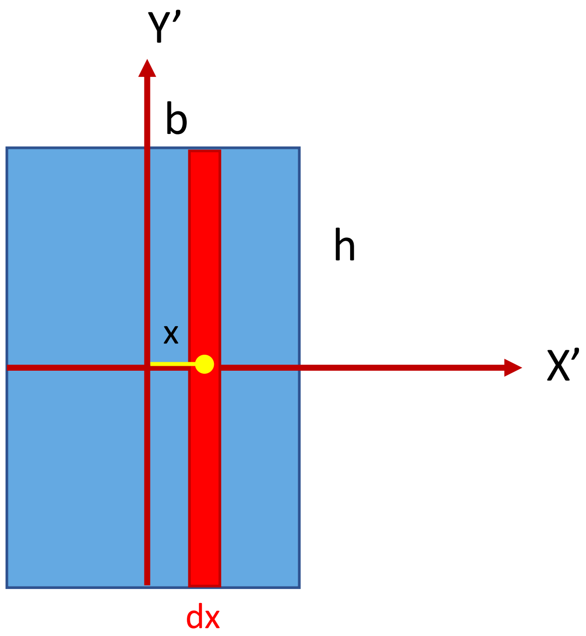


b. Find moment of inertia about the y axis axis (passing through the centroid of the rectangle)

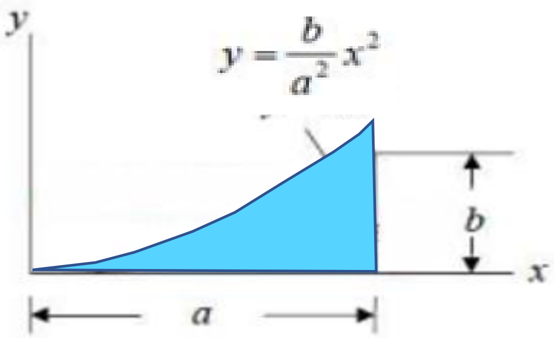
A differential element of area that is *parallel* to the y axis

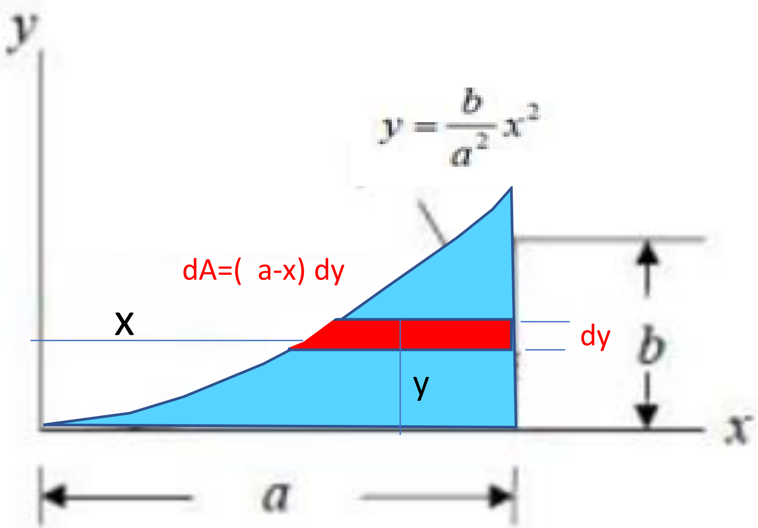
$$I_y = \int x^2 dA = \int_{-b/2}^{b/2} x^2 \; (h\,dx) = hx^3/3 \Big|_{-b/2}^{b/2} = hb^3/12$$

$$\bar{I}_{y'} = \frac{1}{12} h b^3$$



Ex. Find moment of inertia about the x and Y axes

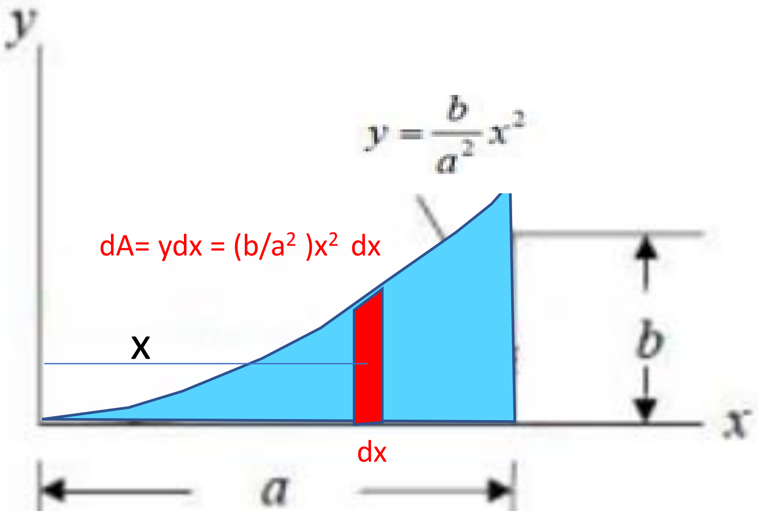




$$dI_x = y^2 dA$$

$$= y^2\left(a - \frac{a}{b^{\frac{1}{2}}}y^{\frac{1}{2}}\right)dy$$

$$I_x = \int_0^b \left(ay^2 - \frac{a}{b^{\frac{1}{2}}}y^{\frac{5}{2}}\right)dy = \left|\frac{ay^3}{3} - \frac{2a}{7b^{\frac{1}{2}}}y^{\frac{7}{2}}\right|_0^b = \frac{ab^3}{21}$$



$$dI_y = x^2 dA = x^2(y)dx = x^2\frac{b}{a^2}x^2 dx$$

$$dI_y = \left(\frac{b}{a^2}x^4\right)dx$$

$$I_y = \int_0^a \left(\frac{b}{a^2}x^4\right)dx = \left|\frac{b}{5a^2}x^5\right|_0^a = \frac{a^3b}{5}$$

10.2 Parallel-Axis Theorem for an Area

The ***parallel-axis theorem*** can be used to find the moment of inertia of an area about *any axis* that is parallel to an axis passing through the centroid and about which the moment of inertia is known

$$I_x = \int_A y^2 \, dA$$

$$I_x = \int_A (y' + d_y)^2 \, dA = \int_A y'^2 \, dA + 2d_y \int_A y' \, dA + d_y^2 \int_A dA$$

0, Y' = 0

$$I_x = \bar{I}_{x'} + A d_y^2$$

$$I_y = \bar{I}_{y'} + A d_x^2$$

MOI about the centroidal axis

A: area
dy: the distance between the two parallel axes

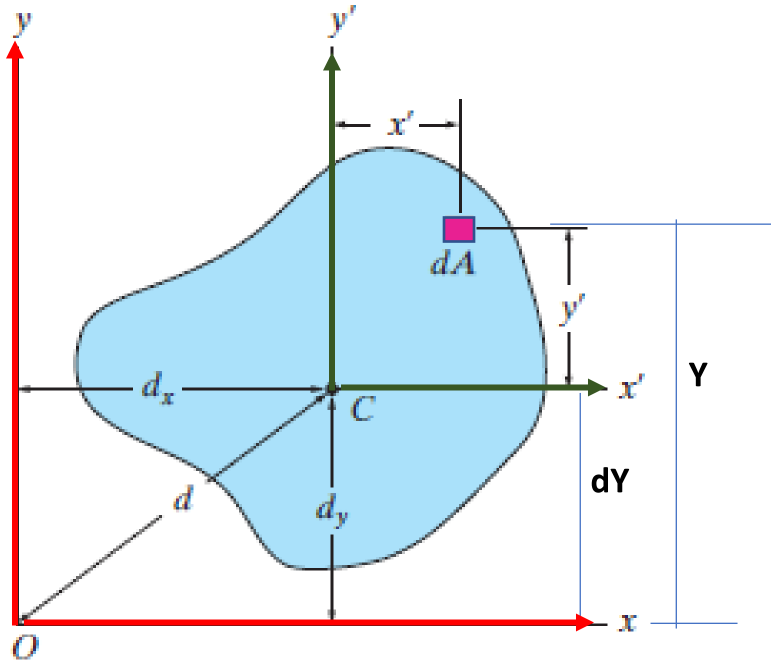
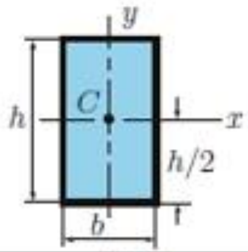
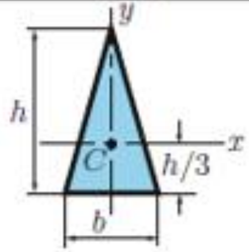


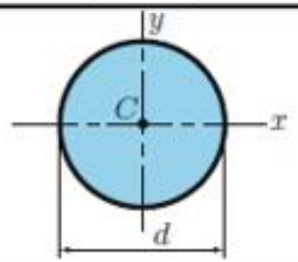
Table: Area inertia properties for some common cross sections



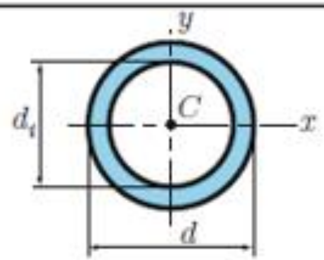
$$A = bh$$
$$I_{xx} = \frac{bh^3}{12}$$
$$I_{yy} = \frac{b^3h}{12}$$



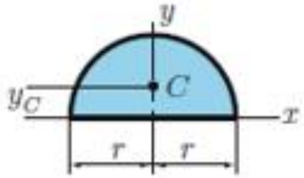
$$A = \frac{bh}{2}$$
$$I_{xx} = \frac{bh^3}{36}$$
$$I_{yy} = \frac{b^3h}{36}$$



$$A = \frac{\pi d^2}{4}$$
$$I_{xx} = I_{yy} = \frac{\pi d^4}{64}$$



$$A = \frac{\pi}{4}(d^2 - d_i^2)$$
$$I_{xx} = I_{yy} = \frac{\pi}{64}(d^4 - d_i^4)$$



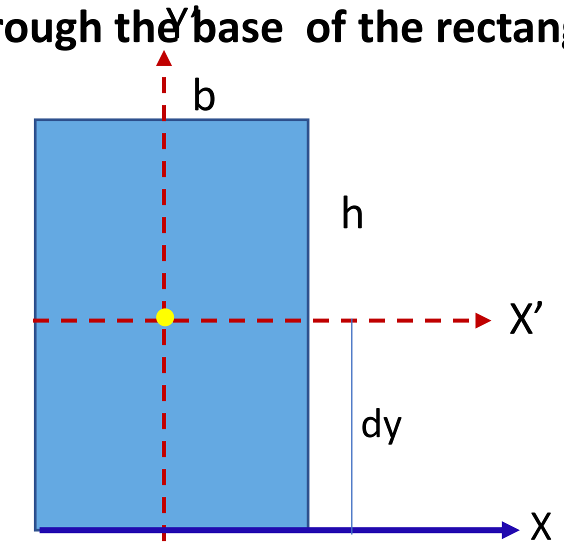
$$A = \frac{\pi r^2}{2}$$
$$I_{xx} = I_{yy} = \frac{\pi r^4}{8}$$
$$y_C = \frac{4r}{3\pi}$$

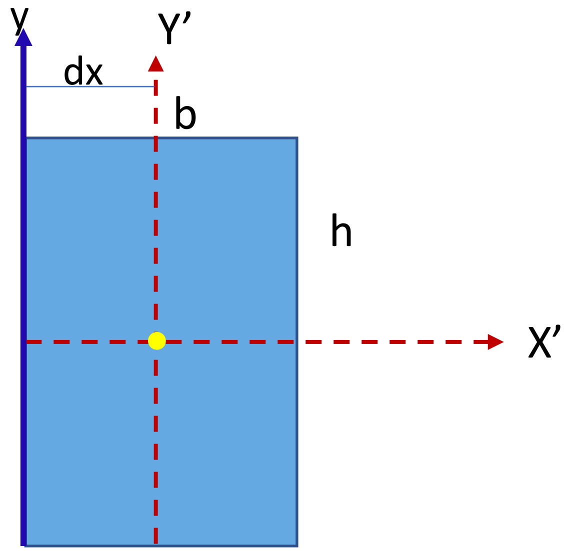
Find moment of inertia about the x axis (passing through the base of the rectangle)

$$I_{x} = I_{x'} + A d_y^2$$

$$= \frac{1}{12}bh^3 + bh\left(\frac{h}{2}\right)^2 = \frac{1}{3}bh^3$$

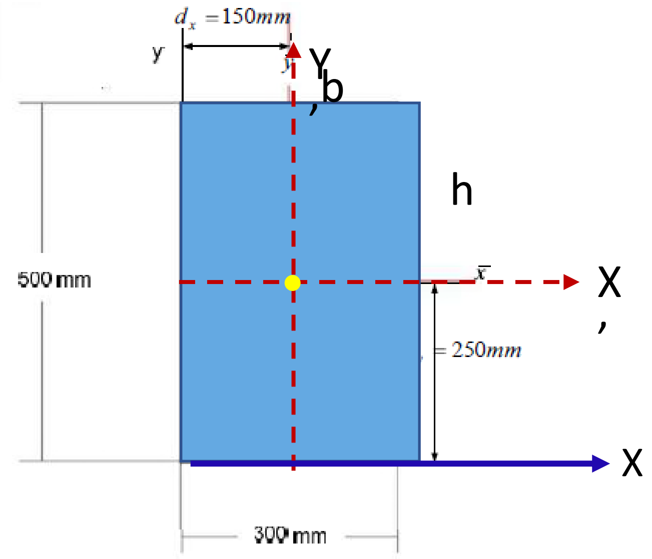
Find moment of inertia about the y axis





$$I_{y'} = \frac{1}{3}hb^3$$

Use the ***parallel-axis theorem*** to find Ix and Iy

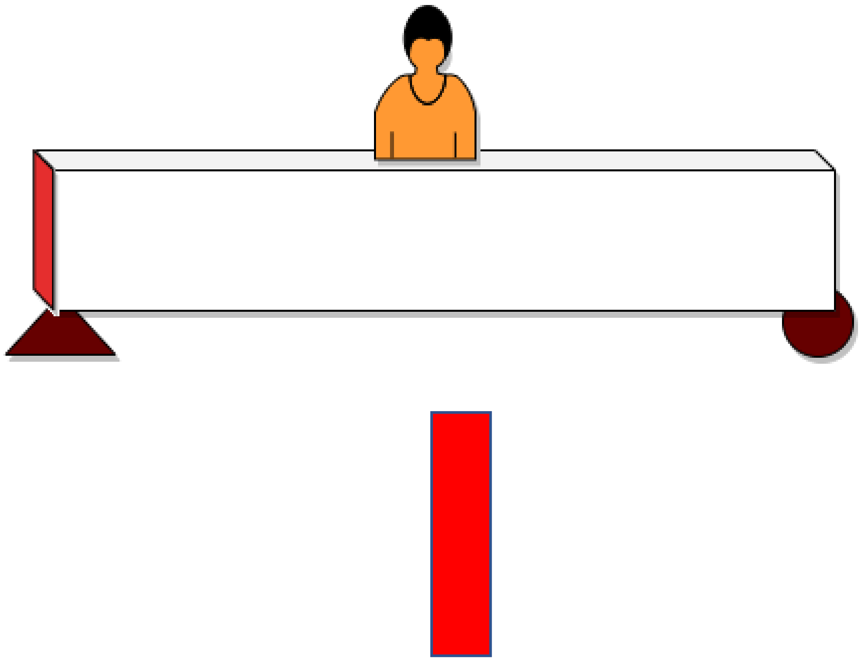


$$I_{\bar{x}} = \frac{1}{12}bh^3 = \frac{1}{12}(300)(500)^3 = 3.125\times10^9 mm^4$$

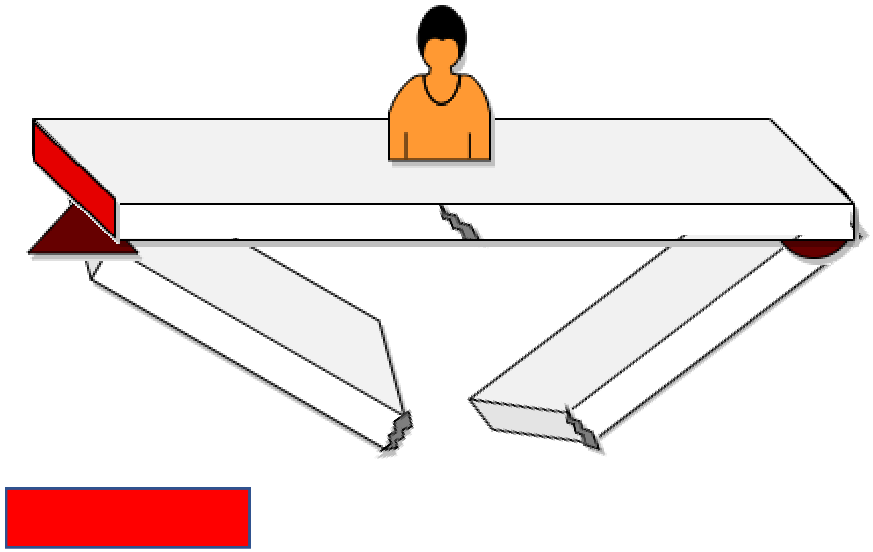
$$I_{\bar{y}} = \frac{1}{3}hb^3 = \frac{1}{12}(500)(300)^3 = 1.125\times10^9 mm^4$$

$$I_x = I_{\bar{x}} + Ad_y^2 = 3.125\times10^9 + (300\times500)\times250^2 = 1.25\times10^{10} mm^4$$

$$I_y = I_{\bar{y}} + Ad_x^2 = 1.125\times10^9 + +(300\times500)\times150^2 = 4.5\times10^9 mm^4$$



$$\bar{I}_{x'} = \frac{1}{12}bh^3$$



$$\bar{I}_{x'} = \frac{1}{12}bh^3$$

10.3 Radius of Gyration of an Area (k)

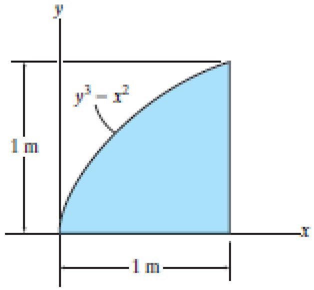
The ***radius of gyration*** of an area about an axis has units of length ***(m, cm or mm)*** and is a quantity that is often used for the design of columns in structural mechanics. Provided the areas and moments of inertia are *known.*

$$k_x = \sqrt{\frac{I_x}{A}}$$

$$k_y = \sqrt{\frac{I_y}{A}}$$

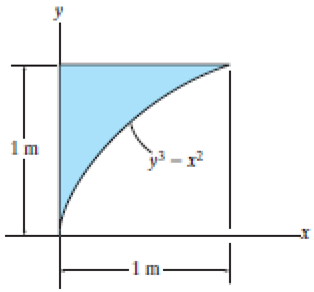
$$k_O = \sqrt{\frac{J_O}{A}}$$

F10–1. Determine the moment of inertia of the shaded area about the x axis.



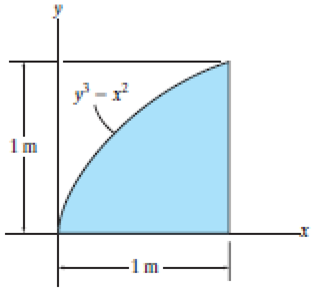
Prob. F10–1

F10–2. Determine the moment of inertia of the shaded area about the x axis.



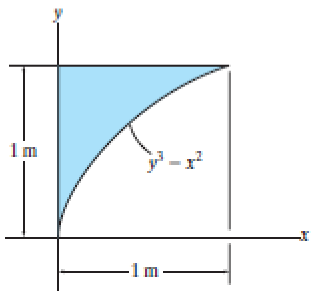
Prob. F10–2

F10–3. Determine the moment of inertia of the shaded area about the y axis.



Prob. F10–3

F10–4. Determine the moment of inertia of the shaded area about the y axis.



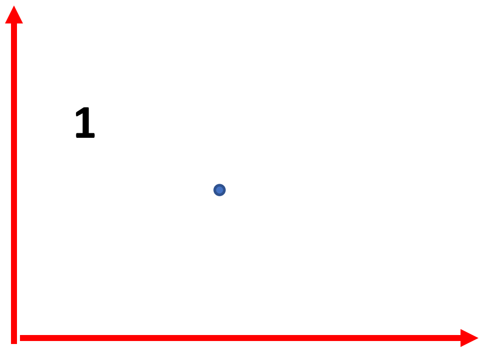
Prob. F10–4

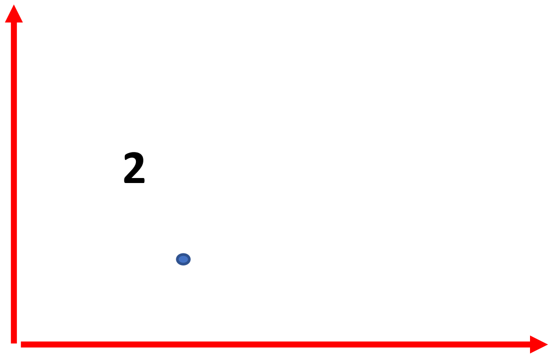
10.4 Moments of Inertia for Composite Areas

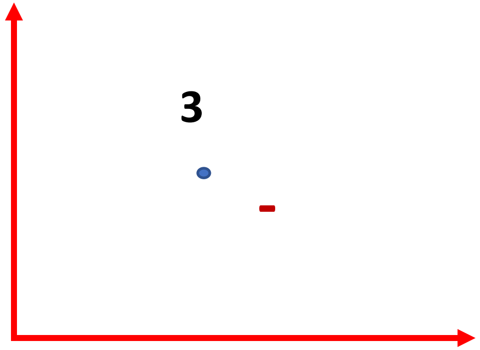
A composite area:
consists of a series of connected "**simpler" shapes,** such as rectangles, triangles, and circles The **moment of inertia of each of these parts about their centroidal axes is known** the Parallel-Axis Theorem Is used to determine the MOI of the composite area

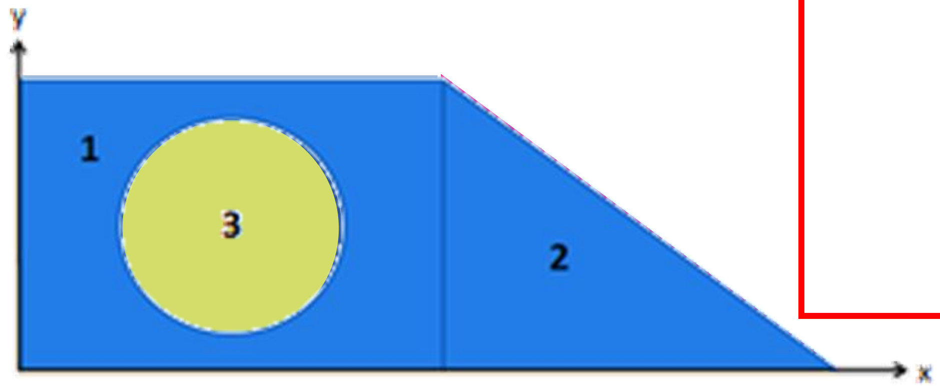
$$I_x = \sum I_{\bar{x}} + \sum A D_y^2$$

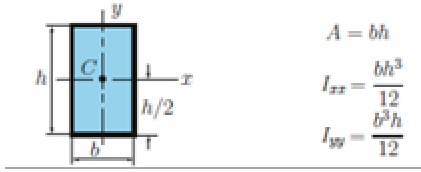
$$I_y = \sum I_{\bar{y}} + \sum A D_x^2$$

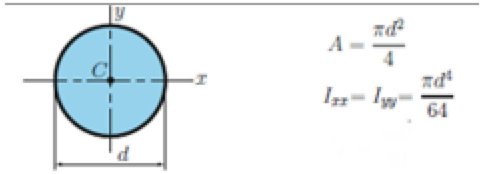


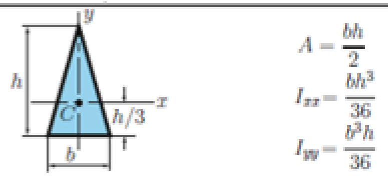






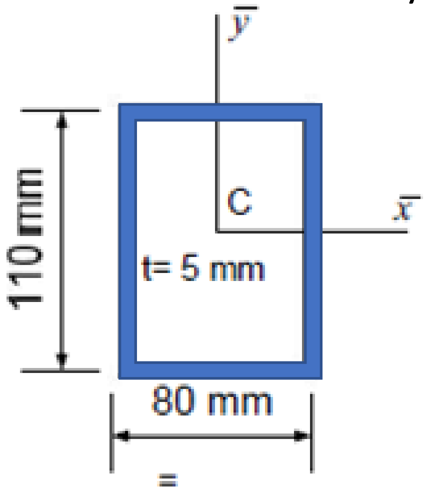


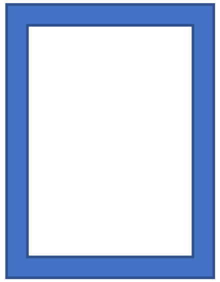


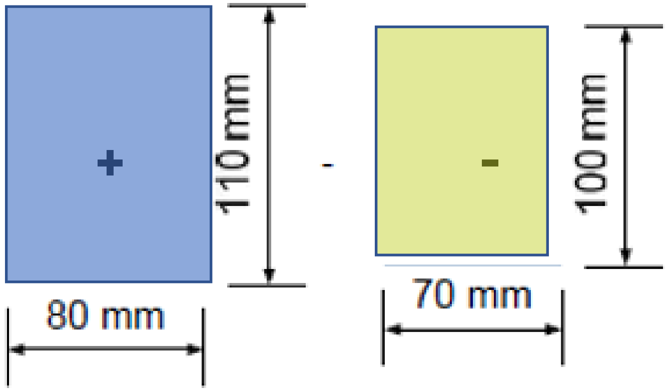


Shape	Area	$I_{x'}$ bout Centroid	$I_{y'}$ about Centroid	distance correction x (d_x)	distance correction y (d_y)	Corrected Ixx $I_{x'} + A\,dy^2$	Corrected Iyy $I_{y'} + A\,dx^2$
1							
2							
3							
Total						Σ	Σ

Find moment of inertia about the x and y axes passing through its centroid







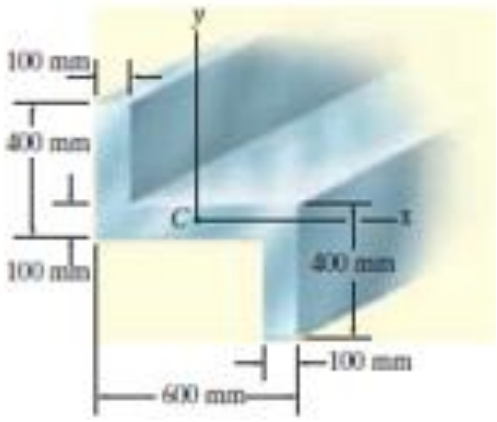
$$I_x = \sum I_{\bar{x}}$$

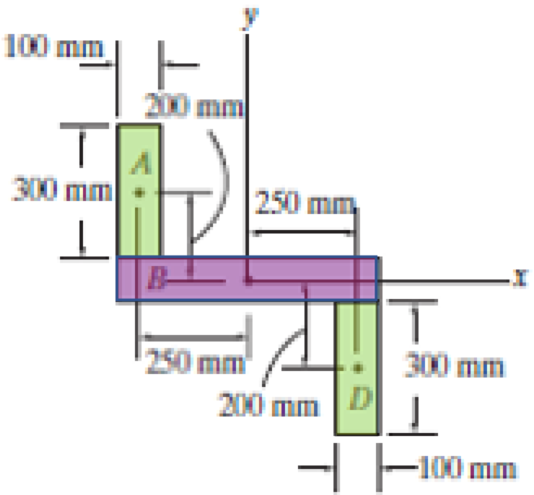
$$I_y = \sum I_{\bar{y}}$$

$$I_{\bar{x}} = \frac{1}{12}(80)(110)^3 - \frac{1}{12}(70)(100)^3 = 3.04\times10^6 mm^4$$

$$I_{\bar{y}} = \frac{1}{12}(110)(80)^3 - \frac{1}{12}(100)(70)^3 = 1.835\times10^6 mm^4$$

Determine the moments of inertia for the cross-sectional area of the member shown in Fig. 10–9*a* about the *x* and *y* centroidal axes.





SOLUTION

Composite Parts. The cross section can be subdivided into the three rectangular areas *A*, *B*, and *D* shown in Fig. 10–9*b*. For the calculation, the centroid of each of these rectangles is located in the figure.

Parallel-Axis Theorem. From the table on the inside back cover, or Example 10.1, the moment of inertia of a rectangle about its centroidal axis is $\bar{I} = \frac{1}{12}bh^3$. Hence, using the parallel-axis theorem for rectangles *A* and *D*, the calculations are as follows:

Rectangles A and D

$$I_x = \bar{I}_{x'} + Ad_y^2 = \frac{1}{12}(100)(300)^3 + (100)(300)(200)^2$$
$$= 1.425(10^9)\text{ mm}^4$$
$$I_y = \bar{I}_{y'} + Ad_x^2 = \frac{1}{12}(300)(100)^3 + (100)(300)(250)^2$$
$$= 1.90(10^9)\text{ mm}^4$$

Rectangle B

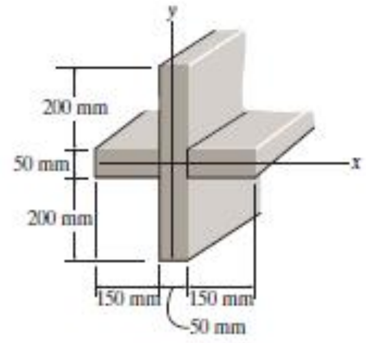
$$I_x = \frac{1}{12}(600)(100)^3 = 0.05(10^9)\text{ mm}^4$$
$$I_y = \frac{1}{12}(100)(600)^3 = 1.80(10^9)\text{ mm}^4$$

Summation. The moments of inertia for the entire cross section are thus

$$I_x = 2[1.425(10^9)] + 0.05(10^9)$$
$$= 2.90(10^9)\text{ mm}^4 \qquad \textit{Ans.}$$
$$I_y = 2[1.90(10^9)] + 1.80(10^9)$$
$$= 5.60(10^9)\text{ mm}^4 \qquad \textit{Ans.}$$

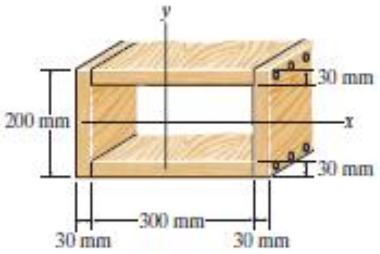
part	area	Ix’	Iy’	dx	dy	Ix’+Ady²	Iy’ +Adx²
A	100x300	1/12(100x 300³)	1/12(100³x 300)	-250	200	1.424 x10⁹	1.9x10⁹
B	100x600	1/12(600x 100³)	1/12(100x 600³)	0	0	0.05x10⁹	1.8x10⁹
D	100x300	1/12(100x 300³)	1/12(100x 300³)	250	-200	1.424 x10⁹	1.9x10⁹
						2.9x10⁹	5.6x10⁹

F10–5. Determine the moment of inertia of the beam's cross-sectional area about the centroidal x and y axes.



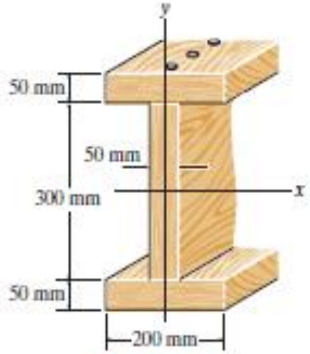
Prob. F10–5

F10–6. Determine the moment of inertia of the beam's cross-sectional area about the centroidal x and y axes.



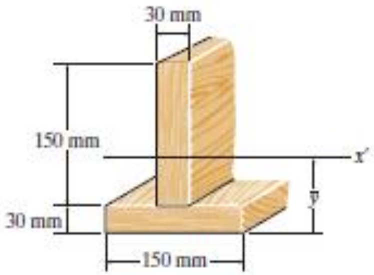
Prob. F10–6

F10–7. Determine the moment of inertia of the cross-sectional area of the channel with respect to the y axis.

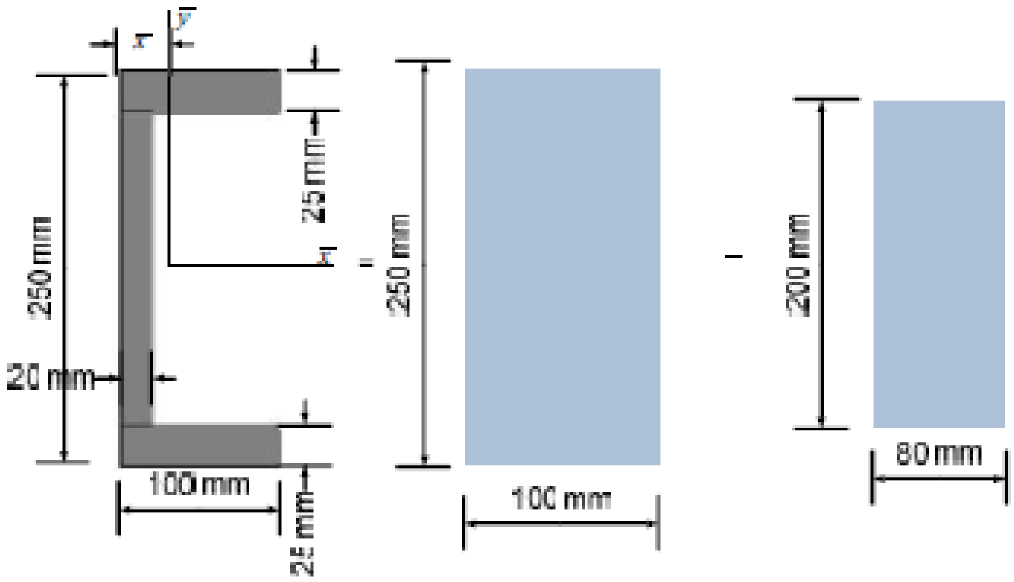


Prob. F10–7

F10–8. Determine the moment of inertia of the cross-sectional area of the T-beam with respect to the x' axis passing through the centroid of the cross section.



Prob. F10–8



	A, mm^2	$\bar{x}$, mm	$\bar{x}A$, mm^3
1	$100\times 250 = 25000$	50	1250×10^3
2	$-80\times 200 = -16000$	60	-960×10^3
	$\sum A = 9000$		$\sum \bar{x}A = 290\times 10^3$

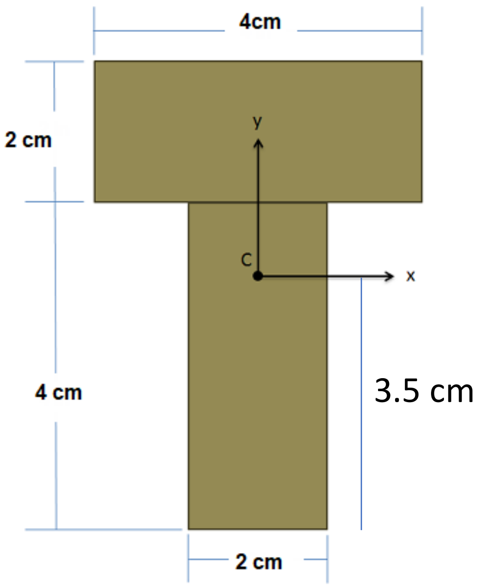
$$\bar{x} = \frac{\sum \bar{x}A}{\sum A} = \frac{290\times 10^3}{9000} = 32.222\ mm$$

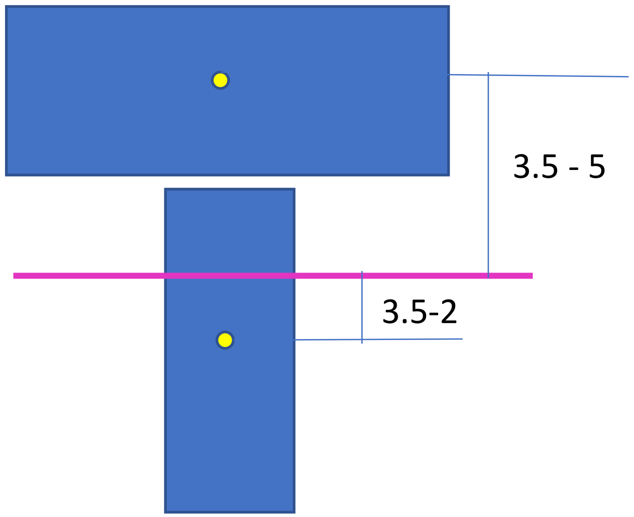
$$I_{\bar{x}} = \left(\tfrac{1}{12}100\times 250^3\right) - \left(\tfrac{1}{12}80\times 200^3\right) = 76.875\times 10^6 mm^4$$

$$I_{\bar{y}} = \left(\tfrac{1}{12}250\times 100^3\right) + (250\times 100)(50-32.222)^2$$

$$-\left(\tfrac{1}{12}200\times 80^3\right) + (200\times 80)(60-32.222)^2 = 7.8556\times 10^6 mm^4$$

Find Ix about centroidal axis





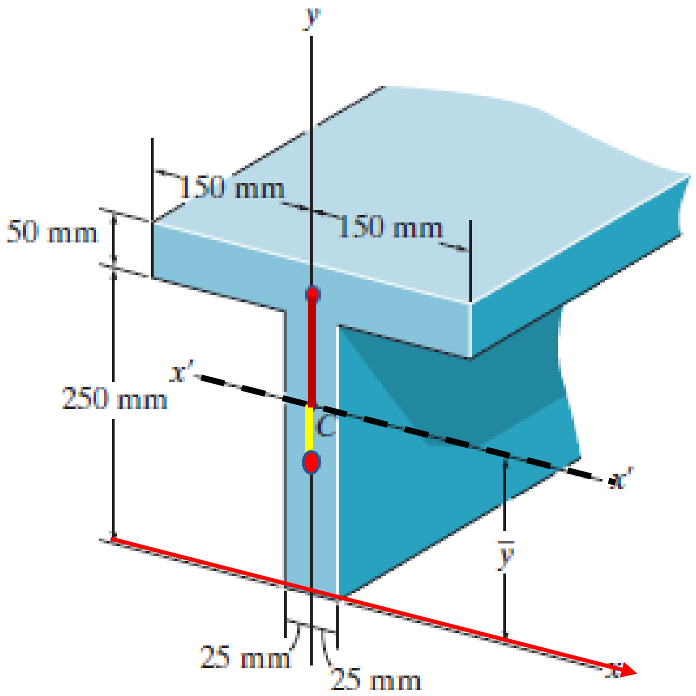
$$\bar{y} = \frac{8 \times 2 + 5 \times 8}{16} = 3 \cdot 5$$

$$\bar{Y} - \bar{y}_i$$

part	area	Ix’	Iy’	dx	dy	$Ix' + Ady^2$
A						
B						

$$I_x = 49.334\ cm^4$$

Determine $\bar{y}$, which locates the centroidal axis x' for the cross-sectional area of the T-beam, and then find the moment of inertia about x' axis

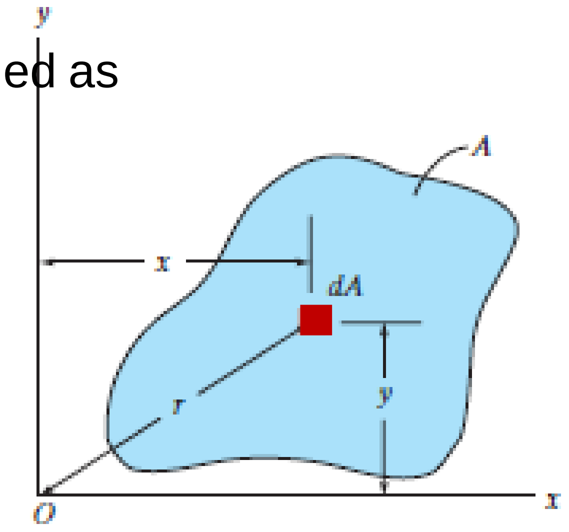


Part No	Area	Y'	Ay'	Ix'	Dy = $\bar{y} - y'$	Ix'+ADy^2
	$\sum$ A =		$\sum$ Ay' =			$\sum$

$$\bar{y} = \frac{\sum A\, y'}{\sum A}$$

10.5 Product of Inertia for an Area

The ***product of inertia*** of the area with respect to the x and y axes is defined as



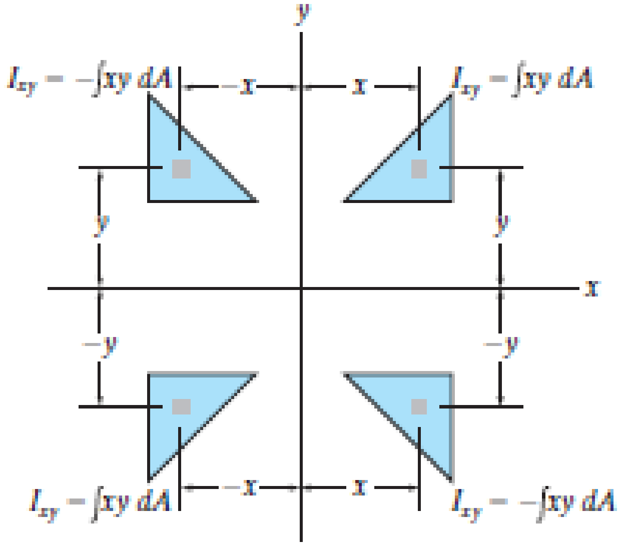
$$I_{xy} = \int xy\, dA$$

The units for I_{XY} (m^4, mm^4)

$$I_x = \int y^2\, dA$$

$$I_y = \int x^2\, dA$$

I_{xy} May have –ve or +ve sign depends on the quadrant where the area is located.



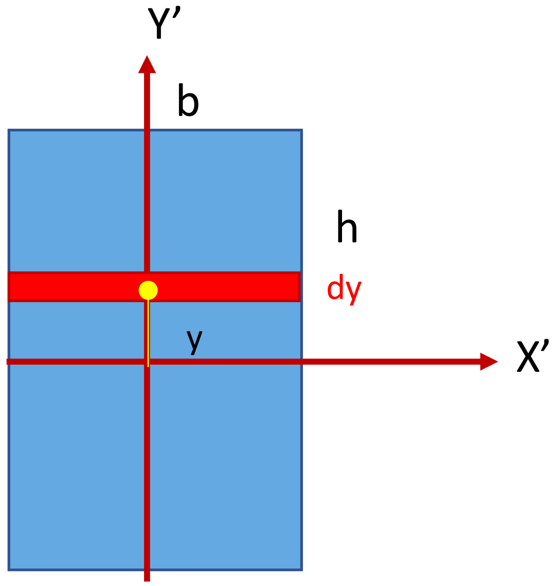
It is important that the *algebraic signs* for d_x and d_y be maintained when applying this equation.

Parallel axis theorem for products of inertia:

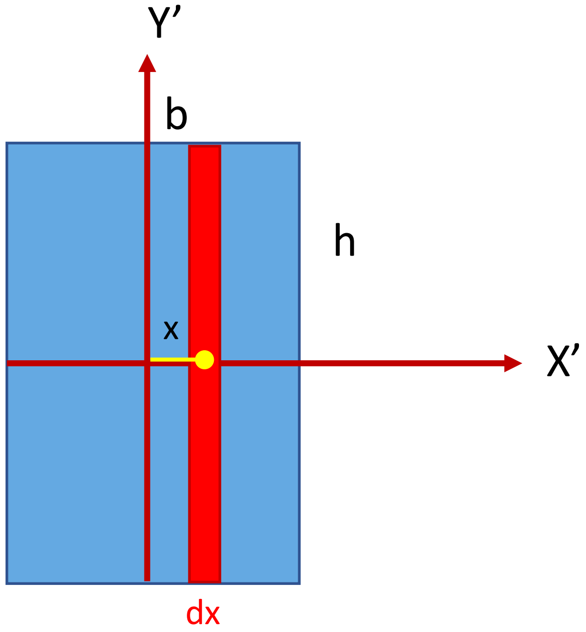
0

$$I_{xy} = I_{x'y'} + A d_x d_y$$

$$I_{xy} = \int xy \, dA$$



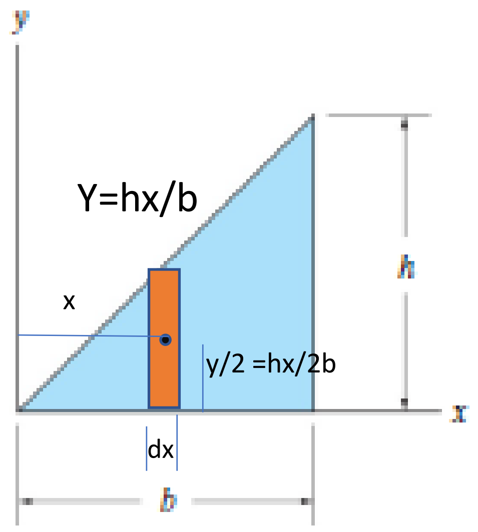
$$I_{xy} = \int xy \, dA$$



When <u>the x axis, the y axis, or both are an axis of symmetry</u>, the product of inertia is **zero**.

Determine the product of inertia I_{xy} for the triangle shown in

or



A differential element that has a **thickness *dx***, has an area ***dA* = *y dx***.
The product of inertia of this element with respect to the *x and y* axes is determined using the parallel-axis theorem.

$$dI_{xy} = d\bar{I}_{x'y'} + dA\,\tilde{x}\,\tilde{y}$$

where $\tilde{x}$ and $\tilde{y}$ locate the *centroid* of the element or the origin of the x', y' axes. Since $d\bar{I}_{x'y'} = 0$, due to symmetry, and $\tilde{x} = x$, $\tilde{y} = y/2$, then

$$dI_{xy} = 0 + (y\,dx)x\left(\frac{y}{2}\right) = \left(\frac{h}{b}x\,dx\right)x\left(\frac{h}{2b}x\right)$$
$$= \frac{h^2}{2b^2}x^3\,dx$$

Integrating with respect to x from $x = 0$ to $x = b$ yields

$$I_{xy} = \frac{h^2}{2b^2}\int_0^b x^3\,dx = \frac{b^2h^2}{8} \qquad \textit{Ans.}$$

$dA = y\,dx.$ $= hx/b\ dx$

$$I_{xy} = \int xy\,dA$$

$$I_{xy} = \int_0^b x\left(\frac{h}{2b}x\right)\frac{h}{b}x\,dx = \frac{h^2}{2b^2}\frac{x^4}{4} = \frac{h^2\,b^2}{8}$$

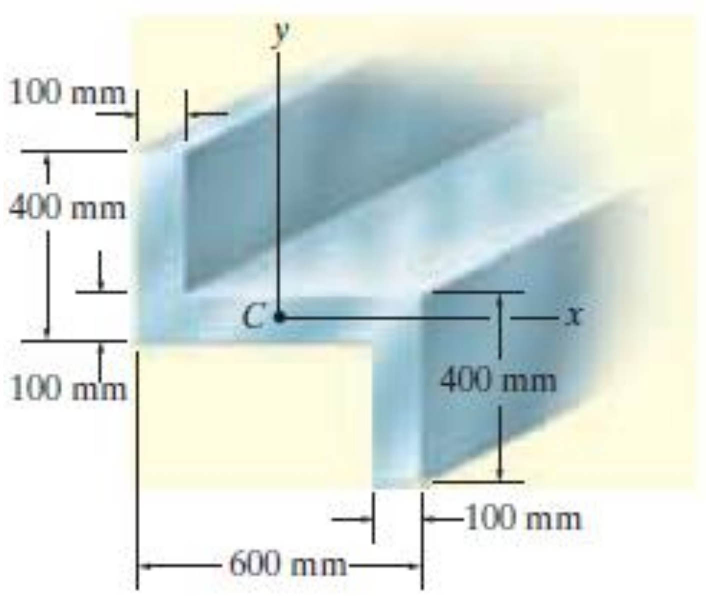
Determine the product of inertia for the cross-sectional area of the member shown, about the *x* and *y* centroidal axes.

$$I_x = \sum I_{x'} + \sum AD_y^2$$

$$I_y = \sum I_{y'} + \sum AD_x^2$$

$$I_{xy} = \sum I_{x'y'} + \sum A d_x d_y$$

(0)



A&D

$$I_{x1} = \frac{100 \times 300^3}{12} + 10\,0\text{x}300 \times 200^2 = 1.424\,x10^9$$

$$I_{y1} = \frac{300 \times 100^3}{12} + 10\,0\text{x}300 \times 250^2 = 1.9\,x10^9$$

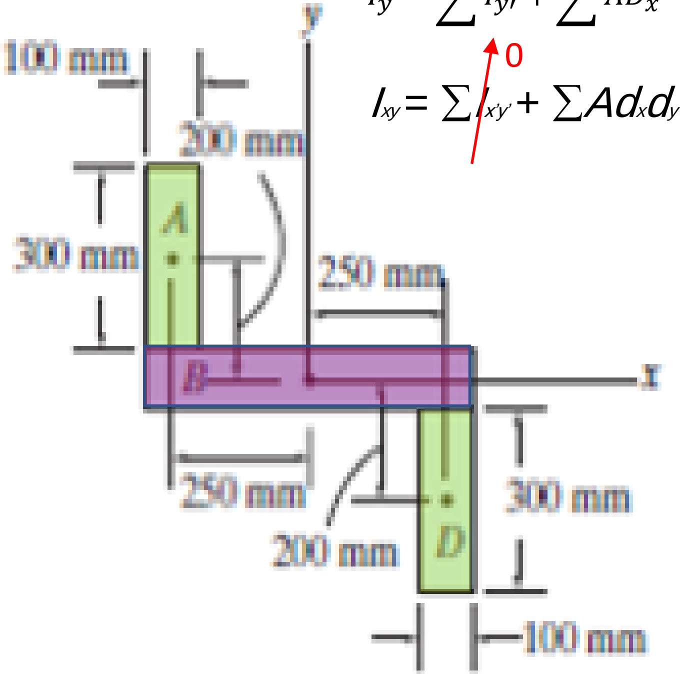
$$I_{xy1} = 0 + 10\,0\text{x}300 \times -250\text{x}200 = -1.5x10^9$$

B

$$I_{x1} = \frac{600 \times 10000^3}{12} = 0.05\,x10^9$$

$$I_{y1} = \frac{100 \times 600^3}{12} = 1.8\,x10^9$$

$$I_{xy1} = 0 + 0 = 0$$



part	A	Ix’	Iy’	dx	dy	Ix= Ix’+Ady^2	Iy =Iy’ +Adx^2	Ixy=0+ Adxdy
A	100x300	1/12(100x 300^3)	1/12(100^{3}x 300)	-250	200	1.424 x10^9	1.9x10^9	-1.5x10^9
B	100x600	1/12(600x 100^3)	1/12(100x 600^3)	0	0	0.05x10^9	1.8x10^9	0
D	100x300	1/12(100x 300^3)	1/12(100x 300^3)	250	-200	1.424 x10^9	1.9x10^9	-1.5x10^9
						2.9x10^9	**5.6x10^9**	**-3x10^9**

10.6 Moments of Inertia for an Area about Inclined Axes

In structural and mechanical design, it is sometimes necessary to calculate the moment of inertia with respect to a set of inclined u, v, axes **when the values of θ , Ix , Iy , Ixy are known.**

$$I_x = \int y^2 dA \quad I_y = \int x^2 dA$$

$$I_{xy} = \int xy\, dA$$

$$u = x \cos\theta + y \sin\theta$$

$$v = y \cos\theta - x \sin\theta$$

$$I_u = I_x \cos^2\theta + I_y \sin^2\theta - 2I_{xy} \sin\theta \cos\theta$$

$$I_v = I_x \sin^2\theta + I_y \cos^2\theta + 2I_{xy} \sin\theta \cos\theta$$

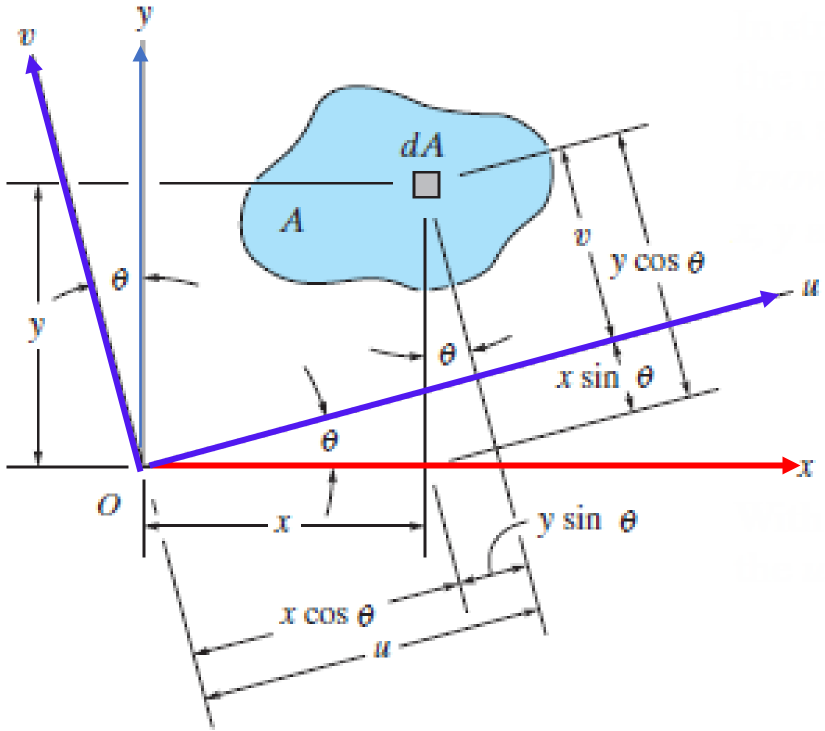
$$I_{uv} = I_x \sin\theta \cos\theta - I_y \sin\theta \cos\theta + I_{xy}(\cos^2\theta - \sin^2\theta)$$

Using the trigonometric identities $\sin 2\theta = 2 \sin\theta \cos\theta$ and $\cos 2\theta = \cos^2\theta - \sin^2\theta$ we can simplify the above expressions, in which case

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy} \sin 2\theta$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy} \sin 2\theta$$

$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy} \cos 2\theta$$



Principal Moments of Inertia.

when the orientation of these axes about which the moments of inertia for the area are **maximum and minimum,** this particular set of axes is called the ***principal axes* of the area**, and the corresponding moments of inertia with respect to these axes are called the ***principal moments of inertia*.**

$$\frac{dI_u}{d\theta} = -2\left(\frac{I_x - I_y}{2}\right)\sin 2\theta - 2I_{xy}\cos 2\theta = 0$$

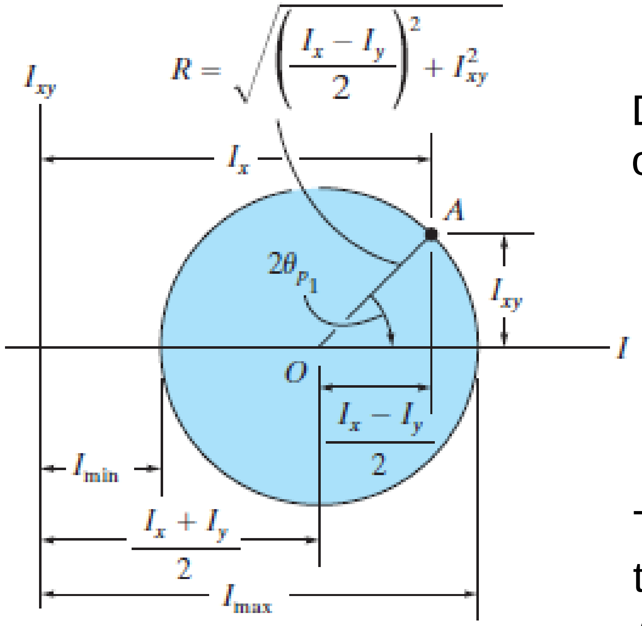
$\theta = \theta_p,$

$$\tan 2\theta_p = \frac{-I_{xy}}{(I_x - I_y)/2}$$

$$I_{\substack{\max \\ \min}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

10.7 Mohr's Circle for Moments of Inertia
Graphical method

Mohr's circle may be used to graphically or analytically determine the moments and product of inertia for any rectangular axes including the principal axes and principal moments and products



Determine I_x, I_y, and I_{xy}.

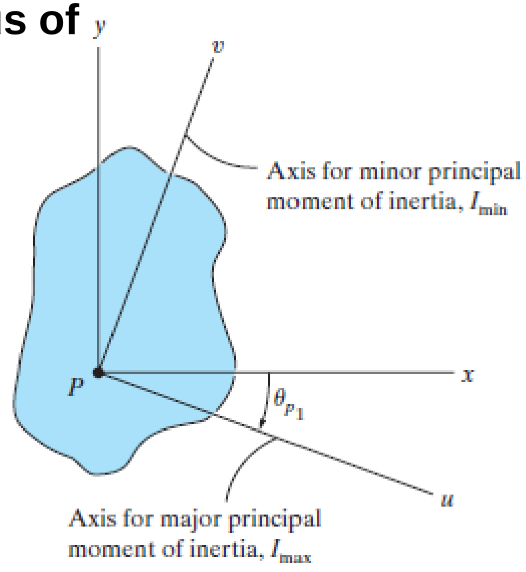
Determine the **center of the circle, *O***, which is located at a **distance (I_x+ I_y)/2** from the origin,

plot the reference point ***A* having coordinates (I_x, I_{xy}).** Remember, **I_x is always positive, whereas I_{xy} can be either positive or negative**

Connect the reference point *A* with the center of the circle and determine the distance *OA* by trigonometry. This distance represents **the radius of the circle**

To find the orientation of the major principal axis, use trigonometry to find the **angle 2θ$_p$**, ***measured from the radius OA to the positive I axis***,

The axis for minimum moment of **inertia I $_{max}$ is perpendicular to the axis for I $_{min}$.**

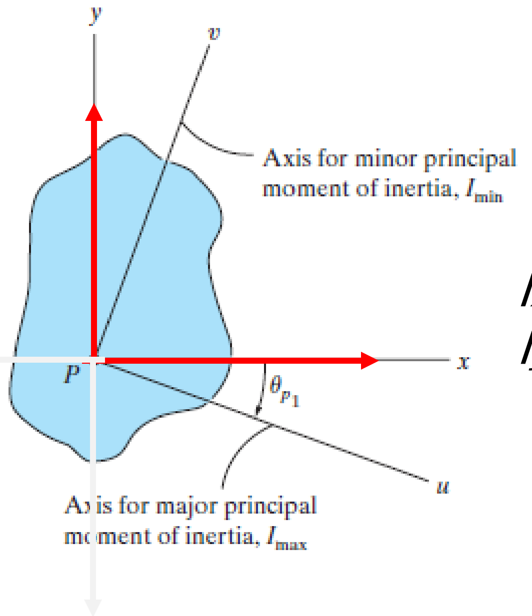


Mohr’s circle

Find I_u, Iv and I_{uv} at angle θ= 30° CW

2θ= 60°

Orientation of the principal axes

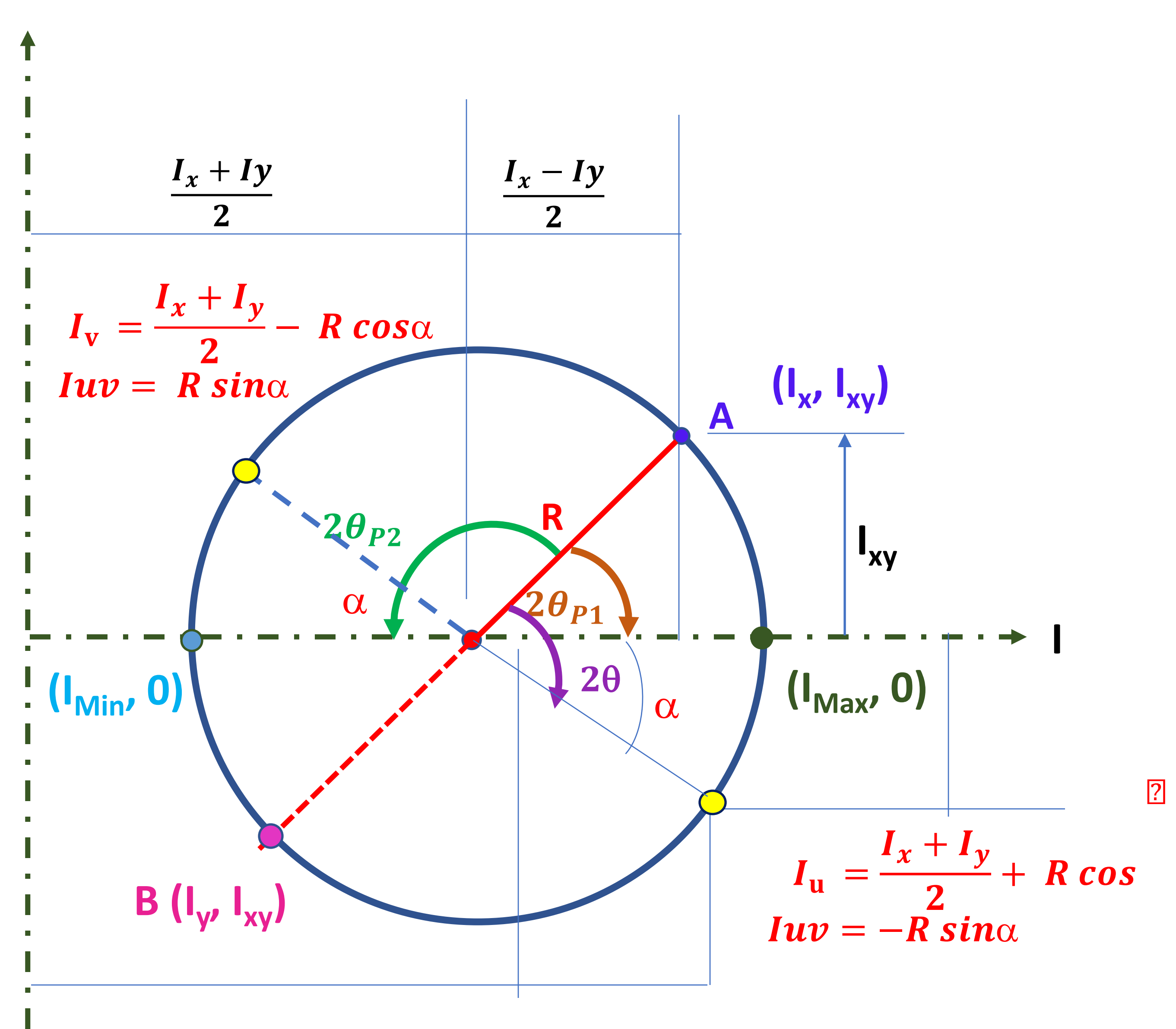


I_x mm ^{4},
I_y mm ^{4}
I_{xy}mm ^{4}.

$$R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

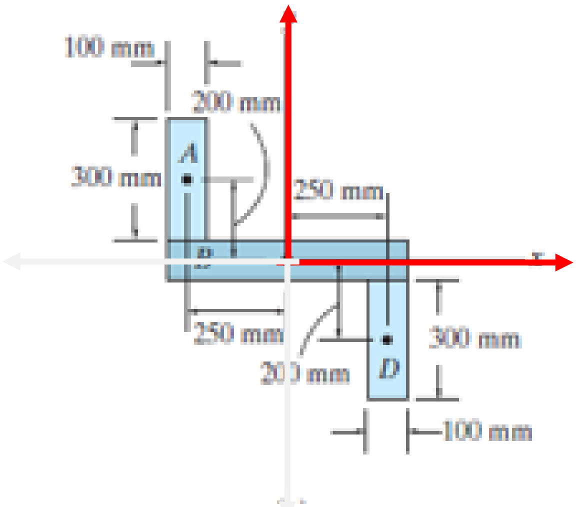
$$I_{\substack{max \\ min}} = \frac{I_x + I_y}{2} \pm R$$

$$Tan\ 2\theta_p = \frac{Ixy}{\frac{I_x - Iy}{2}}$$



Using Mohr's circle, determine the principal moments of inertia and the orientation of the major principal axes for the cross-sectional area of the member shown with respect to an axis passing through the centroid.

$I_x = 2.90(10^9)$ mm^4,
$I_y = 5.60(10^9)$ mm^4
$I_{xy} = -3.00(10^9)$ mm^4.



$(I_x + I_y)/2 = (2.90 + 5.60)/2 = 4.25 \times 10^9$

$(I_x - I_y)/2 = (2.90 - 5.60)/2 = -1.35 \times 10^9$

$$R = \sqrt{(1.35)^2 + 3^2} = 3.29$$

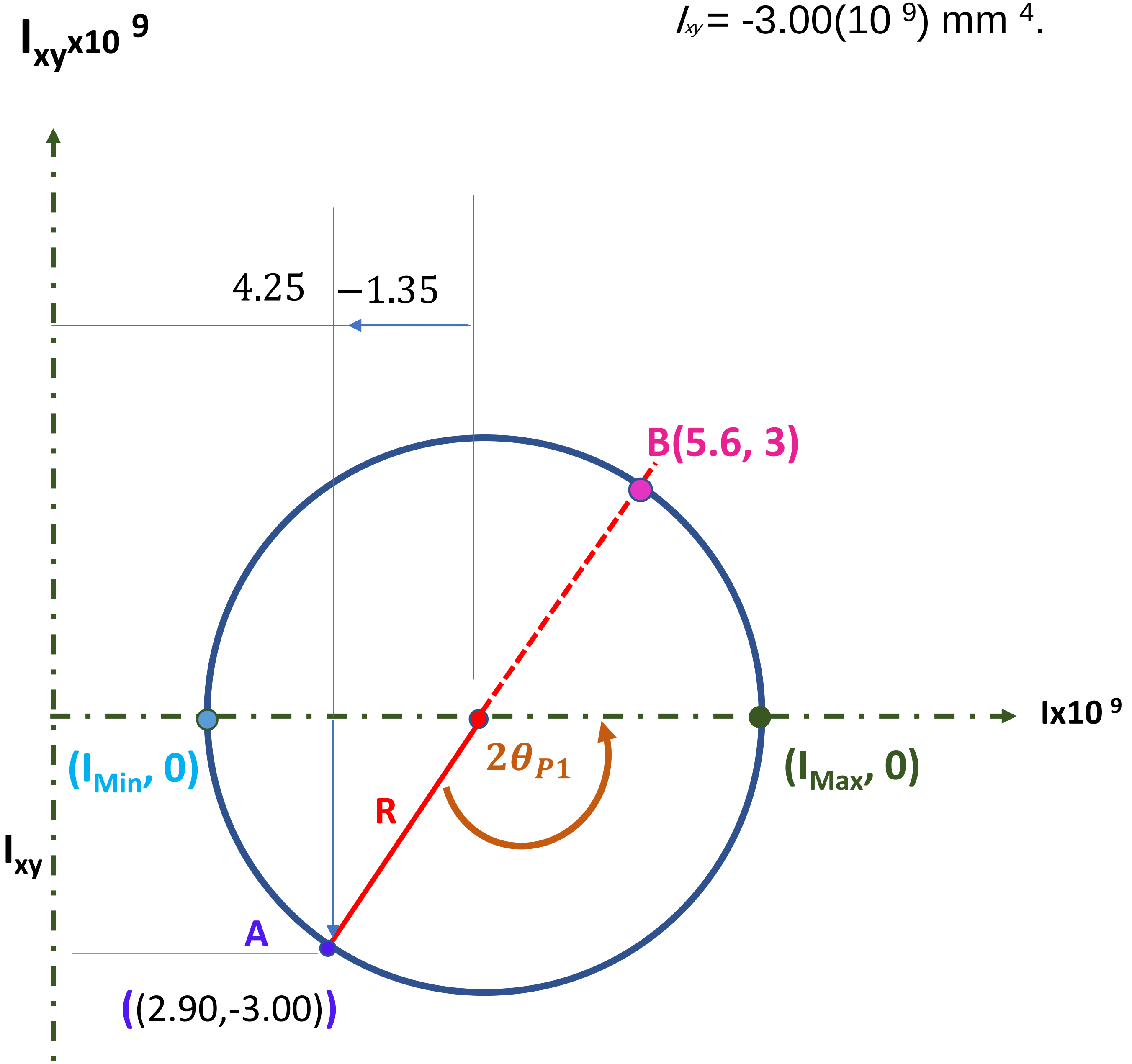
$$I_{\max/\min} = 4.25 \pm 3.29 =$$

7.54×10 mm^4

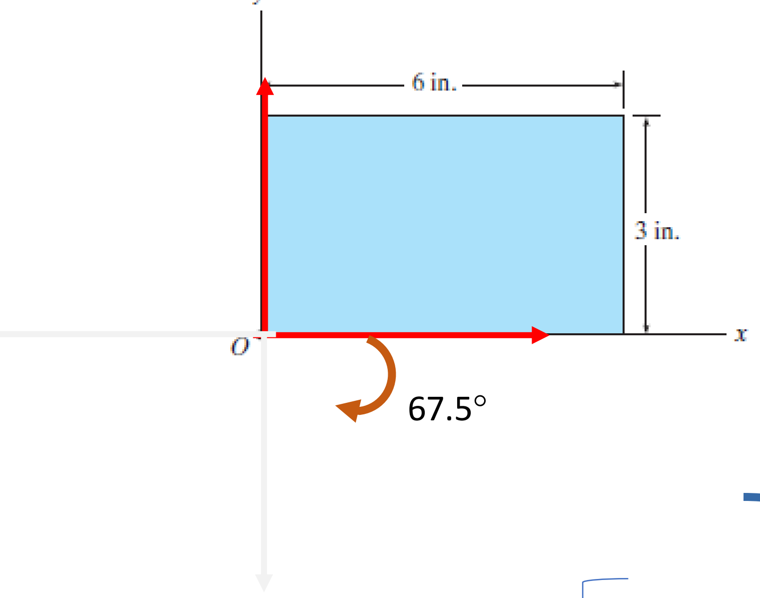
0.96×10^9 mm^4

$$\tan 2\theta_{p2} = \frac{3}{1.35} = 2.22 \quad \ldots\ \theta_{p2} = 32.88^\circ$$

$\theta_{p1} = 90 - 32.88 = 57.12^\circ$ CCW



Determine the orientation of the principal axes having an origin at point *O*, and the principal moments of inertia for the rectangular area about these axes. using Mohr's circle.

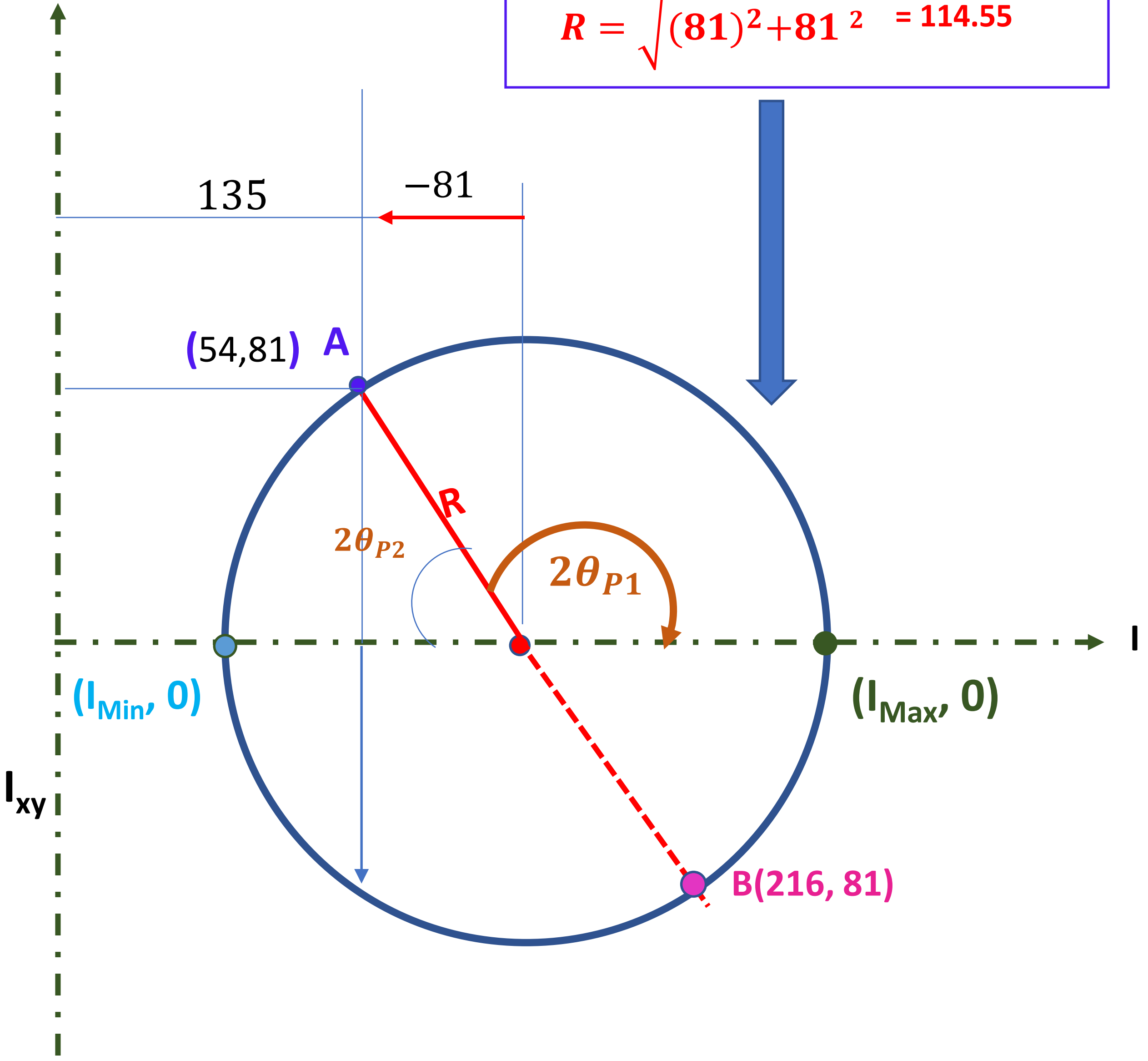


$I_x = 54$
$I_y = 216$
$I_{xy} = 81$

$(I_x + I_y)/2 = (54+216)/2 = 135$

$(I_x - I_y)/2 = (54-216)/2 = -81$

$R = \sqrt{(81)^2 + 81^2} = 114.55$



$$I_{\substack{max \\ min}} = 135 \pm 114.555 = \begin{cases} 249.55 \\ 20.45 \end{cases}$$

$$Tan\ 2\theta_{p2} = \frac{81}{81}$$

$\theta_{p2} = 22.5°$

$\theta_{p1} = 90 - 22.5 = 67.5°$ CW