



اللجنة الأكاديمية للهندسة المدنية

دفتر

## فيزياء 2

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# Physics 2

①

30/1/2017

Ch 23

Tuesday

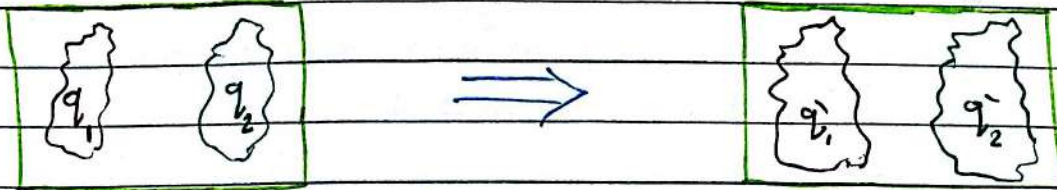
## The Coulomb's law

1) The charge properties (أصناف الشحنات):

A) The charge is conserved (الحفظ)

Charge =  $Q / q$

System.

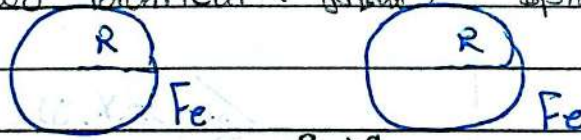


$$Q_i = q_1 + q_2$$

$$Q_i = Q_f$$

$$Q_f = q_1' + q_2'$$

E.X: Two identical (متساوية) spheres



$$q_1' = q_2' = \frac{q_1 + q_2}{2}$$

B) The charge is quantized (التكمين)

$$q = N \cdot e$$

$$N = \pm 1, \pm 2, \pm 3, \dots$$

$$e = -1.6 \times 10^{-19} \text{ C}$$

E.X: Find the number of electrons on the charged

object  $Q = 3.2 \times 10^{-15} \text{ C}$

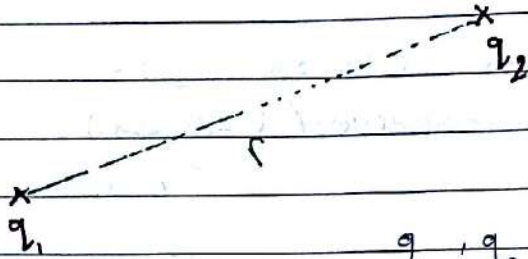
sol.

$$Q = N \cdot e$$

$$N = \frac{Q}{e} = \frac{3.2 \times 10^{-15}}{1.6 \times 10^{-19}} = 2 \times 10^4 \text{ electrons}$$

2)

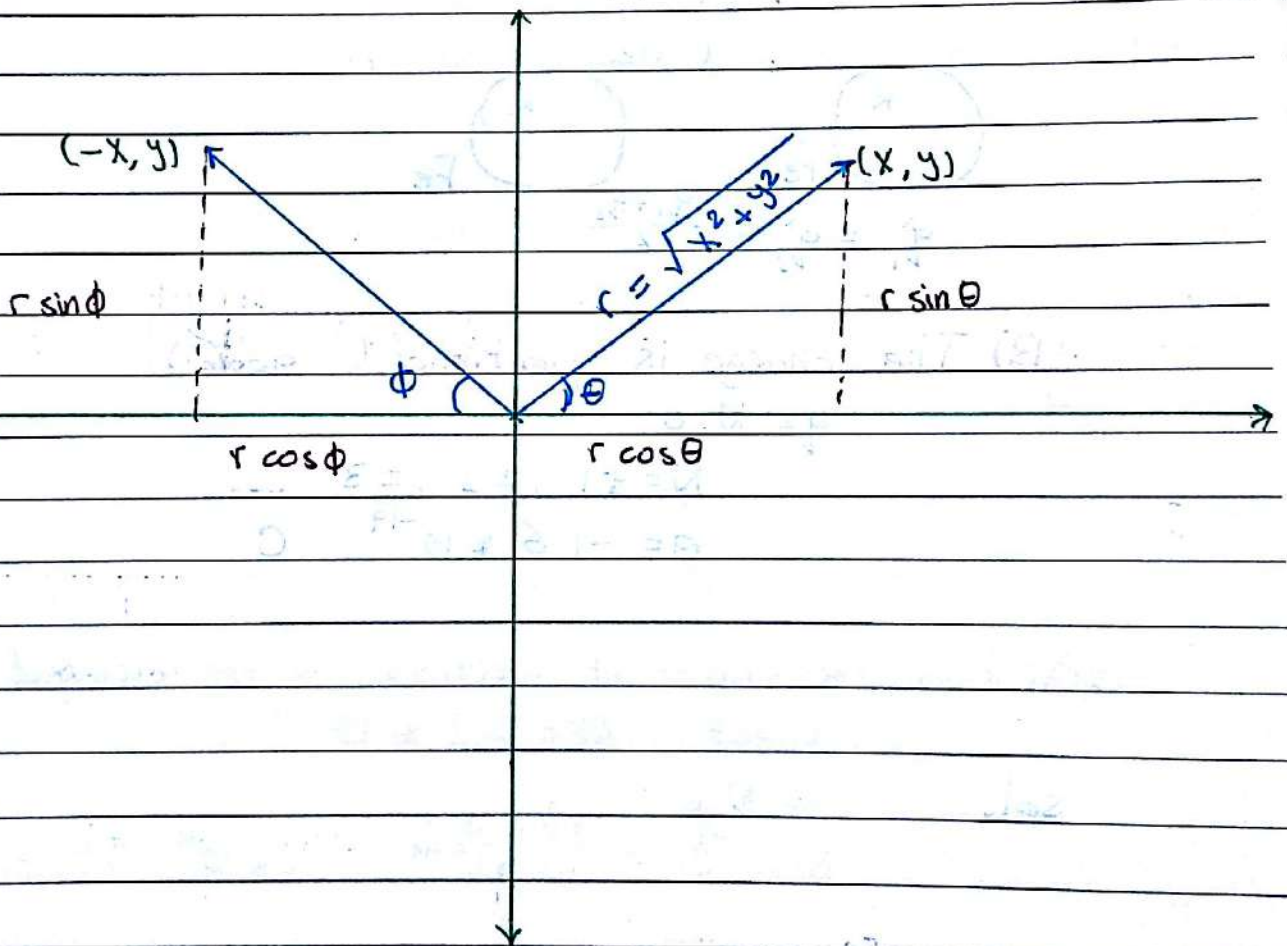
## 2) Coulomb's law



$q_1, q_2$  are point charges  
 $r$  is distance or separation (المسافة)

Attractive force = قوة جذب

Repulsive force = قوة دفع



1/2/2018

Thursday

$$F \sim \frac{q_1 q_2}{r^2}$$

↳ إنتقالية الجسيمات

$q_1, q_2 \sim$  proportional  $\rightarrow$  متناسبة  
 $r^2 \sim$  inversly proportional  $\rightarrow$  عكس التناسل

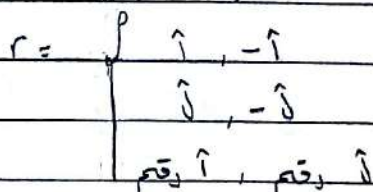
$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2$$

$$F = \frac{K_e q_1 q_2}{r^2}$$

$$K_e = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9$$

$$\vec{F} = K_e \frac{q_1 q_2}{r^2} \hat{r}$$



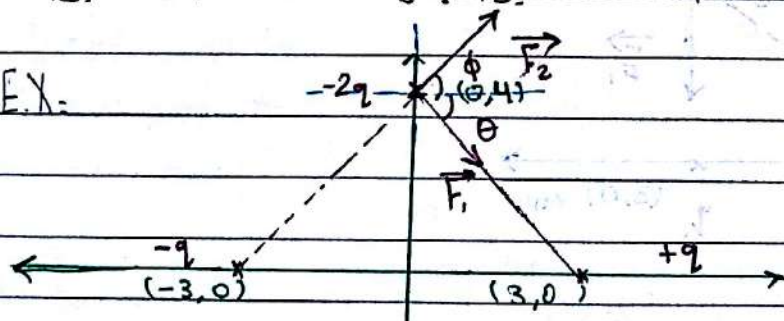
EX:  $q_1$   $\circ$   $\xrightarrow{r}$   $\circ$   $q_2$

Which of the following statement is correct

- A)  $\vec{F}_{12} = \vec{F}_{21}$
- B)  $F_{12} = -F_{21}$
- C)  $\vec{F}_{12} = -3\vec{F}_{21}$
- D)  $\vec{F}_{12} = -\vec{F}_{21}$

vector إنتقالية  $\rightarrow$  vector إنتقالية  $\rightarrow$  vector إنتقالية  $\rightarrow$  vector إنتقالية

EX:



$q = -5 \mu\text{C}$   
Find the force  
on the charge  
 $-2q$

$$\vec{F}_1 = \frac{9 \times 10^9 \times 5 \times 10^{-6} \times 10 \times 10^{-6}}{25} (\cos\theta \hat{i} - \sin\theta \hat{j})$$

$$= 18 \times 10^{-3} \left( \frac{3}{5} \hat{i} - \frac{4}{5} \hat{j} \right)$$

$$\vec{F}_2 = \frac{9 \times 10^9 \times 5 \times 10^{-6} \times 10 \times 10^{-6}}{25} (\cos\theta \hat{i} + \sin\theta \hat{j})$$

$$= 18 \times 10^{-3} \left( \frac{3}{5} \hat{i} + \frac{4}{5} \hat{j} \right)$$

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2$$

$$= 2 \times 18 \times 10^{-3} \times 3 \hat{i} \text{ N}$$

4/2/2018

Sunday

### 3) The electric field

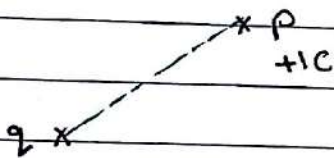
الحقل الكهربائي

$\times q_0 = \text{test charge } (+1C)$

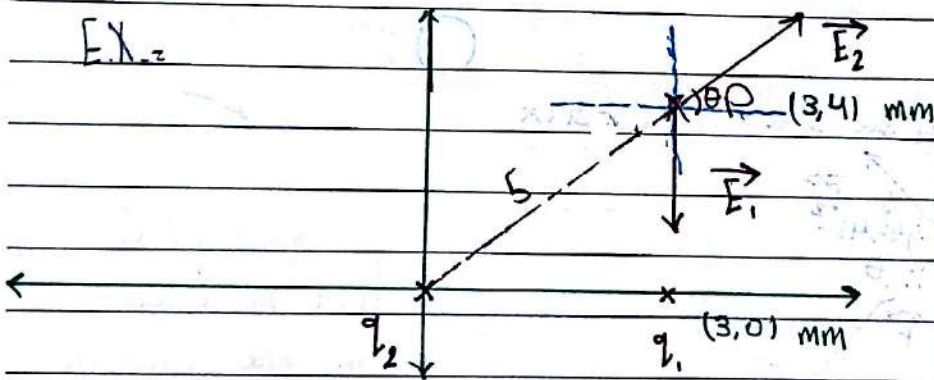
$q \times$

(E) The electric field is the electric force exerted (تؤثر) on a positive unit charge.

$$\vec{E} = \frac{\vec{F}}{q_0} = k_e \frac{q}{r^2} \hat{r}$$



Remember  $\vec{F} = \vec{E} \cdot q_0$



- $q_1 = -5 \text{ nC}, q_2 = 10 \text{ nC}$
- Find the net electric field at the point P
  - Find the force on the point charge  $Q = -4 \text{ nC}$  located at P

sol/ ①  $\vec{E}_1 = \frac{9 \times 10^9 \times 5 \times 10^{-9}}{(4 \times 10^{-3})^2} = \frac{45}{16} \hat{j} \text{ M N/C}$

$\vec{E}_2 = \frac{9 \times 10^9 \times 10 \times 10^{-9}}{(5 \times 10^{-3})^2} (\cos \theta \hat{i} + \sin \theta \hat{j})$

$= \frac{90}{125} (3\hat{i} + 4\hat{j}) \text{ M N/C}$

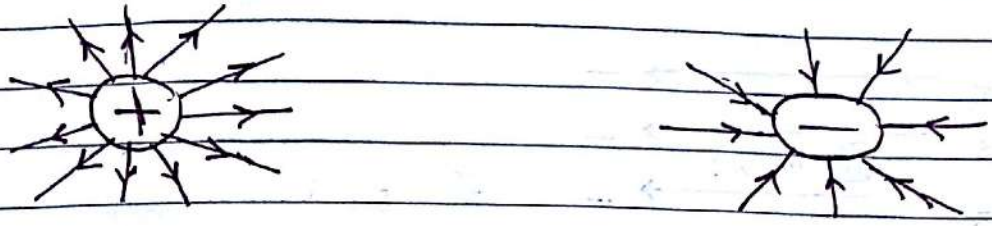
$\vec{E}_p = \frac{270}{125} \hat{i} + \left( \frac{360}{125} - \frac{45}{16} \right) \hat{j} = \frac{270}{125} \hat{i} + \frac{675}{10000} \hat{j} \text{ M N/C}$

②  $\vec{F}_e = q_0 \vec{E}_p$

$= -4 \times 10^{-9} \left( \frac{270}{125} \hat{i} + \frac{675}{10000} \hat{j} \right) \text{ M N}$

4) The electric field lines properties

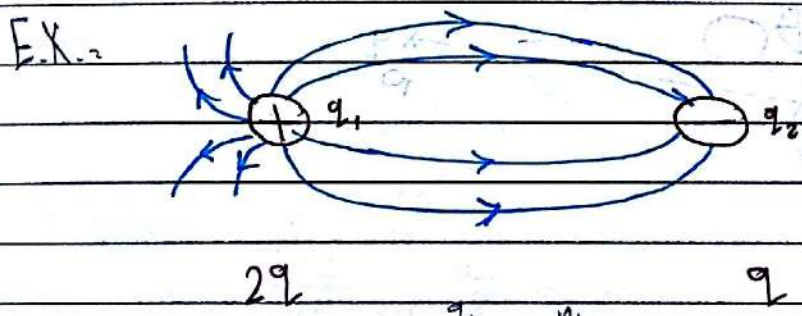
A) The electric field lines emerged (تخرج) From positive charges and end to negative charges



B) The electric field lines intensity (شدة) proportional to the charge magnitude.

# of electric field lines  $\sim q$

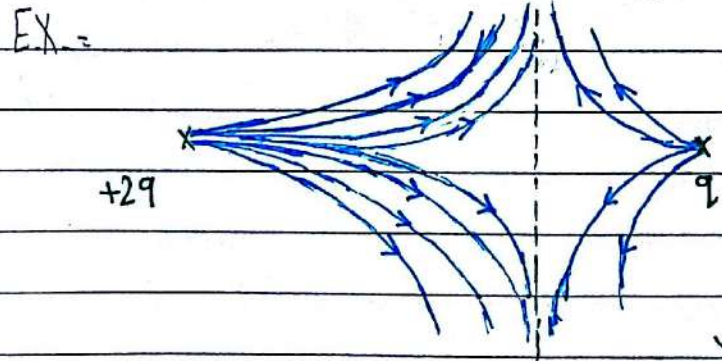
C) The electric field lines don't intersect (لا تقاطع)



$$\frac{q_1}{q_2} = \frac{n_1}{n_2}$$

$$\frac{q_1}{q_2} = \frac{8}{4}$$

$$q_1 = 2q_2$$



مناطق	مناطق	المنطقة
خارجها	بينها	منطقة
و اقرب	و اقرب	الاصغر
للاهمى في	للاهمى في	
المقدار	المقدار	

vanshing (تلاشي) electric field  
 vanshing electric force  
 $E=0$  OR  $F=0$

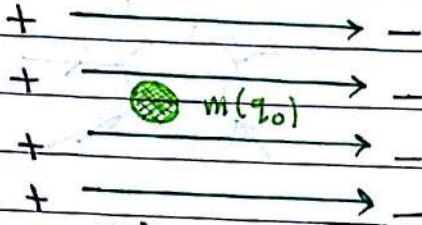
6

6/2/2018

Tuesday

### 5) Motion of a charge particle in a uniform E-field

حركة جسيم مشحون في مجال كهربائي متجانس



المجال الكهربائي بين اللوحين المتوازيين

$$\vec{E} = E_0 \hat{i}$$

$$\vec{F} = q_0 \vec{E} = q_0 E_0 \hat{i}$$

$$\vec{F} = m \vec{a} = q_0 E_0$$

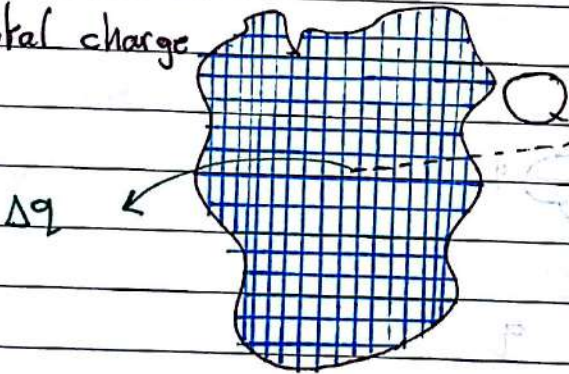
$$a = \frac{q_0 E_0}{m}$$

يولد قوة التسارع في اتجاه المجال الكهربائي

### 6) The E-field of a continuous charge distribution (density)

المجال الكهربائي لتوزيع متجانس (كثافة)

Total charge



$$\Delta \vec{E} = k_e \frac{\Delta q_i}{r_i^2} \hat{r}_i$$

$$E_p = \sum_{i=1}^N k_e \frac{\Delta q_i}{r_i^2}$$

$$\xrightarrow[N \rightarrow \infty]{\Delta q \rightarrow 0} \int k_e \frac{dq}{r^2}$$

$$E_p = \int \frac{k_e dq}{r^2}$$

charge

# Charge density

## 1) 3D

volume charge density

(ع)  $\rho = \frac{\text{charge}}{\text{volume}} = \frac{Q}{V}$

1) sphere (كرة)

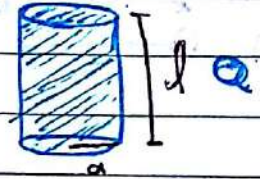
R = Radius (نصف القطر)

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3}$$



2) cylinder (أسطوانة)

$$\rho = \frac{Q}{\pi a^2 l}$$



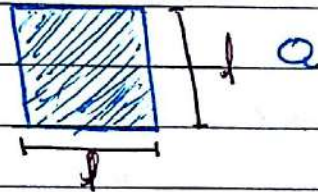
## 2) 2D

surface charge density

(ب)  $\sigma = \frac{Q}{\text{Area}}$

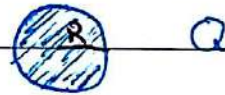
1) square

$$\sigma = \frac{Q}{l^2}$$



2) disk (قرص)

$$\sigma = \frac{Q}{\pi R^2}$$



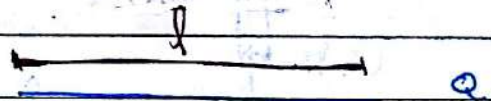
## 3) 1D

linear charge density

(ج)  $\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{l}$

1) wire (سلك)

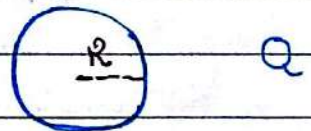
$$\lambda = \frac{Q}{l}$$



2) Ring (حلقة)

$$l = 2\pi R$$

$$\lambda = \frac{Q}{l} = \frac{Q}{2\pi R}$$





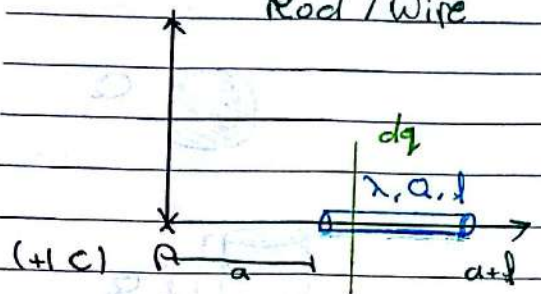
8

8/2/2018

Thursday

# Linear charge density ( $\lambda$ )

## Rod / Wire



$l =$  length of wire  
 $Q =$  charge

$$E_P = \int_{\text{charge}} k_e \frac{dq}{r^2}$$

$$= \int_{\text{charge}} k_e \frac{dq}{x^2}$$

$$\lambda = \frac{q}{x}$$

$$q = \lambda x$$

$$dq = \lambda dx$$

$$= k_e \int_a^{a+l} \frac{\lambda dx}{x^2}$$

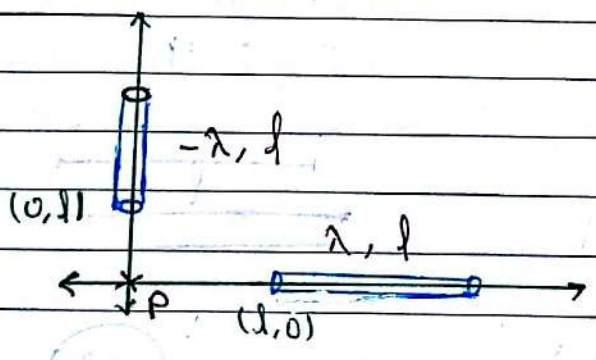
$$= k_e \lambda \left[ -\frac{1}{x} \right]_a^{a+l}$$

$$= k_e \lambda \left[ \frac{1}{a} - \frac{1}{a+l} \right]$$

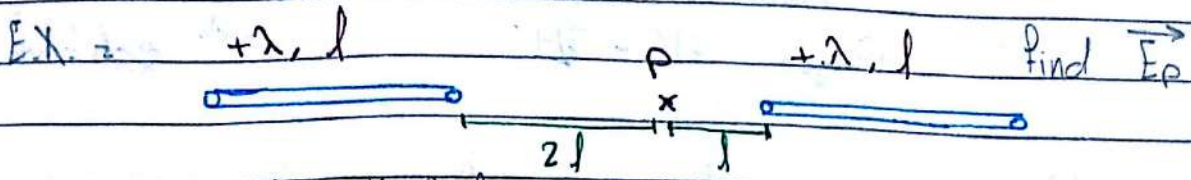
$$= \frac{k_e \lambda l}{a(a+l)} = \frac{k_e Q}{a(a+l)}$$

Ans

$E_{x_2}$



- A)  $\frac{k_e \lambda}{2l} (\hat{i} + \hat{j})$     **B)  $\frac{k_e \lambda}{2l} (-\hat{i} + \hat{j})$**     C)  $\frac{k_e \lambda}{2l} (-\hat{i} - \hat{j})$     d)  $\frac{k_e \lambda}{2l} (\hat{i} - \hat{j})$



$$\vec{E}_1 = \frac{K\epsilon\lambda l}{2l(2l+l)}$$

$$= \frac{K\epsilon\lambda}{6l} \hat{i}$$

$$\vec{E}_2 = \frac{K\epsilon\lambda l}{l(l+l)}$$

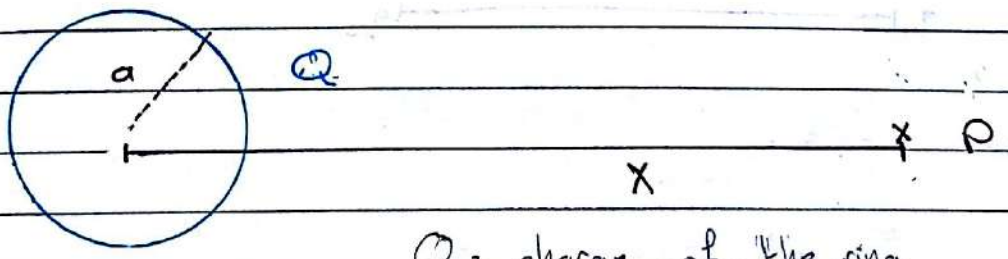
$$= \frac{K\epsilon\lambda}{2l} (-\hat{i})$$

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2$$

$$= \left( \frac{K\epsilon\lambda}{6l} - \frac{K\epsilon\lambda}{2l} \right) \hat{i}$$

$$= \frac{2K\epsilon\lambda}{6l} (-\hat{i}) = \frac{1}{3} \frac{K\epsilon\lambda}{l} (-\hat{i}) \text{ N/C}$$

### Charged Ring



Q = charge of the ring

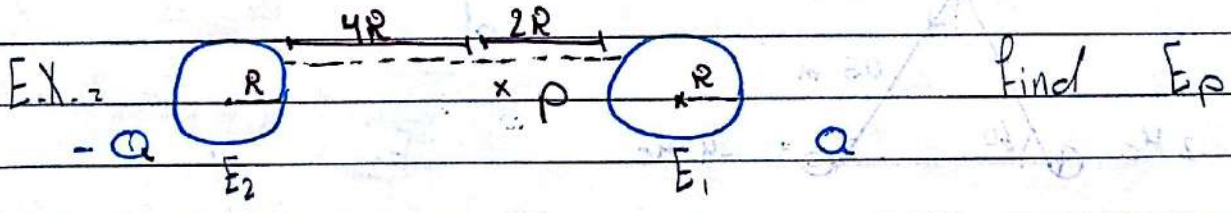
a = Radius

x = The distance between the ring's center to the point P

$$E_P = \frac{K\epsilon Q x}{(x^2 + a^2)^{3/2}}$$

Direction = +ve z outwards  $\hat{i}$

-ve z inwards  $-\hat{i}$



sol

$$\vec{E}_1 = \frac{K\epsilon Q 2R}{(4R^2 + R^2)^{3/2}} \hat{i}$$

$$\vec{E}_2 = \frac{K\epsilon Q 4R}{(16R^2 + R^2)^{3/2}} (-\hat{i})$$

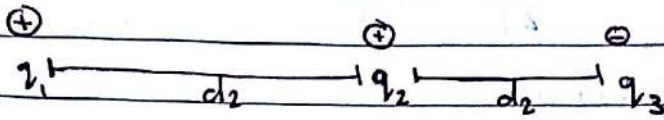
$$\vec{E}_P = \vec{E}_1 + \vec{E}_2$$

# Problems of Ch 23

Pages 716 - 724

in 9<sup>th</sup> edition

12)



$$q_1 = 6 \mu\text{C}, q_2 = 15 \mu\text{C}$$

$$q_3 = -2 \mu\text{C}, d_1 = 3 \text{ cm}$$

$$d_2 = 2 \text{ cm}$$

A) Find the net force on  $q_1$

sol.

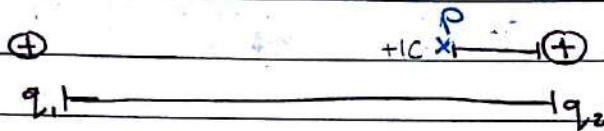
$$\vec{F}_{21} = \frac{kq_1q_2}{(2 \times 10^{-2})^2} \quad (-\hat{i}) \quad \text{N}$$

$$\vec{F}_{31} = \frac{kq_1q_3}{(5 \times 10^{-2})^2} \quad (\hat{i}) \quad \text{N}$$

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 \quad (\hat{i})$$

$$= -202.5 + 43.2 = -159.3 \quad (-\hat{i}) \quad \text{N}$$

13)



$$q_1 = 3 \mu\text{C}$$

$$q_2 = 9 \mu\text{C}$$

$$d = 1.5 \text{ m}$$

sol.

$$F_1 = F_2$$

$$\frac{kq_1q_1}{(d-x)^2} = \frac{kq_2q_2}{x^2}$$

$$\frac{3 \times 3}{(d-x)^2} = \frac{9 \times 9}{x^2}$$

$$\frac{3}{(d-x)^2} = \frac{9}{x^2}$$

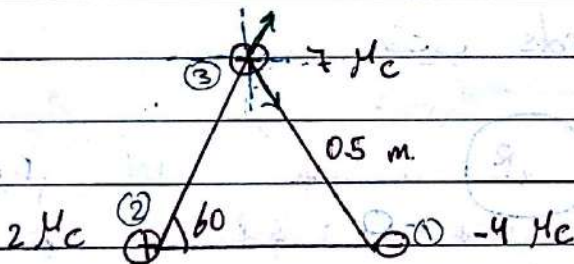
$$3x^2 = (d-x)^2$$

$$\sqrt{3}x = d-x$$

$$(1+\sqrt{3})x = \frac{3}{2}$$

$$x = \frac{3}{2(1+\sqrt{3})}$$

15)



sol.

$$\vec{F}_{13} = \frac{kq_1q_3}{r^2}$$

$$= 1(\cos 60^\circ \hat{i} - \sin 60^\circ \hat{j}) \text{ N/C}$$

$$\vec{F}_{23} = \frac{kq_2q_3}{r^2}$$

$$= 0.5(\cos 60^\circ \hat{i} + \sin 60^\circ \hat{j}) \text{ N/C}$$

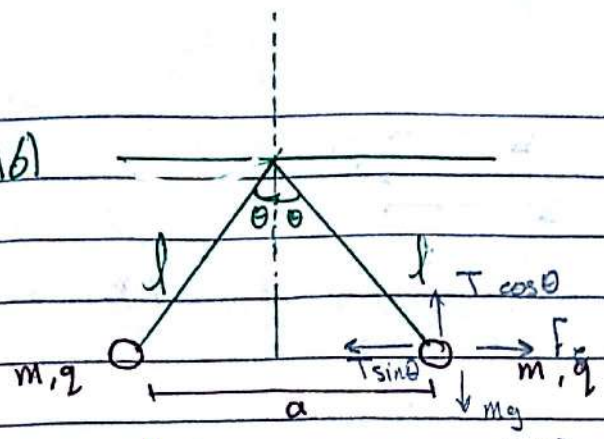
$$\vec{F}_{\text{Total}} = \vec{F}_{13} + \vec{F}_{23}$$

$$= (0.5 + 1)\cos 60^\circ \hat{i} - (1 - 0.5)\sin 60^\circ \hat{j}$$

$$= 0.75 \hat{i} - 0.433 \hat{j}$$

$$|\vec{F}| = 0.866, 330^\circ$$

16)



المسألة  
 المسألة  
 $\theta = 5^\circ$   
 $l = 15 \text{ cm}$   
 $m = 0.2 \text{ g}$   
 Find  $q$

sol.  $m = 0.2 \times 10^{-3} \text{ Kg}$

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$T \sin \theta = F_e$

$T \cos \theta = mg$

إذن

$F_e = mg \tan \theta$

$a = 2 l \sin \theta$

$\frac{k_e q^2}{4 l^2 \sin^2 \theta} = mg \tan \theta$

$q = \pm \sqrt{\frac{4 l^2 mg \sin^2 \theta \tan \theta}{k_e}} = \pm \sqrt{3.59 \times 10^{-17}}$

21)

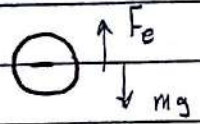
A)  $|F| =$

$\frac{k_e q_1 q_2}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9} \times 18 \times 10^{-9}}{(0.3)^2} = 216 \times 10^{-7} \text{ N}$

B)

$q = \frac{q_1 + q_2}{2} = \frac{12 - 18}{2} = -3 \text{ nC}$

24)



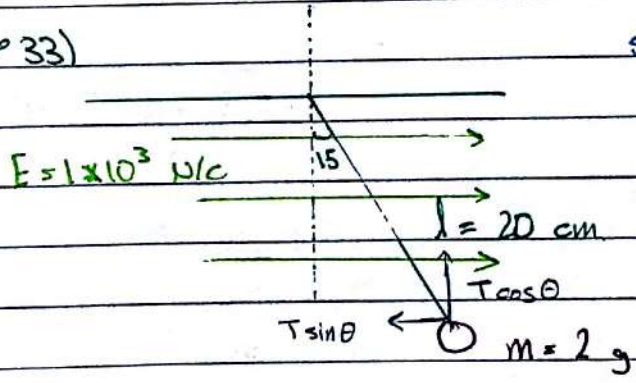
sol.  $\Sigma F_y = 0$

$mg = F_e$

$3.8 \times 10^{-3} \times 10 = F_e \times 18 \times 10^{-6}$

$F_e = 3.8 \times 10^4 \text{ N/C}$

33)



sol.  $\Sigma F_y = 0$

$\Sigma F_x = 0$

$mg = T \cos \theta$

$T \sin \theta = F_e$

$2 \times 10^{-3} \times 10 = T \cos 15$

$T \sin 15 = 1 \times 10^3 \times q$

إذن

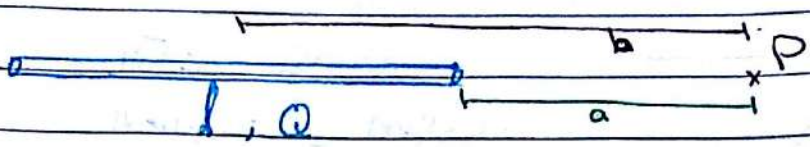
$\tan 15 = \frac{1}{2} \times 10^6 q$

$q = 2 \tan 15 \times 10^{-6}$

$= 0.535 \times 10^{-6} \text{ C}$

12

37)



$Q = -22 \text{ } \mu\text{C}$   
 $l = 14 \text{ cm}$   
 $b = 36 \text{ cm}$

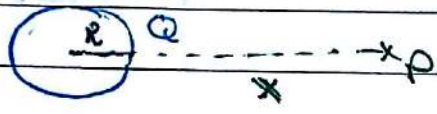
sol.  $a = 36 - 7 = 29 \text{ cm}$

$$E_p = \frac{k_e Q}{a(a+l)}$$

$$= \frac{9 \times 10^9 \times 22 \times 10^{-6}}{29 \times 10^{-2} (29 \times 10^{-2} + 14 \times 10^{-2})} = 1.59 \times 10^6 \text{ } \mu\text{C}$$

or toward the rod.

39)



$Q = 75 \text{ } \mu\text{C}$   
 $R = 10 \text{ cm}$

$$c) E_p = \frac{k_e Q x}{(x^2 + R^2)^{3/2}}$$

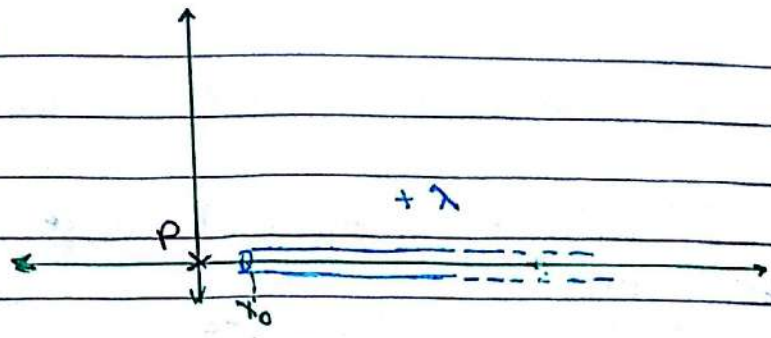
$$= \frac{9 \times 10^9 \times 75 \times 10^{-6} \times 30 \times 10^{-2}}{((30 \times 10^{-2})^2 + (10 \times 10^{-2})^2)^{3/2}} = 6.4 \times 10^6 \text{ } \mu\text{C}$$

or outward

42)

←→ also into

43)



Infinite wire

Sol.

$$E_p = \int_{\text{charge}} K_e \frac{dq}{r^2}$$

$$= \int_{\text{charge}} K_e \frac{dq}{x^2}$$

$$= \int_{\text{charge}} K_e \frac{dq}{x^2}$$

$$= K_e \lambda \int_{x_0}^{\infty} \frac{dx}{x^2}$$

$$= K_e \lambda \lim_{b \rightarrow \infty} \int_{x_0}^b \frac{1}{x^2} dx$$

$$= K_e \lambda \lim_{b \rightarrow \infty} \left[ -\frac{1}{x} \right]_{x_0}^b$$

$$= K_e \lambda \lim_{b \rightarrow \infty} \left( -\frac{1}{b} + \frac{1}{x_0} \right)$$

$$= K_e \lambda \left( 0 + \frac{1}{x_0} \right) = \frac{K_e \lambda}{x_0}, \text{ outward.}$$

$\lambda = \frac{Q}{L}$
$dq = \lambda dx$

45)

Charge Arc (wire)



$l = R \alpha$      $\alpha = \text{Radian}$

$\lambda = \frac{q}{l} = \frac{q}{R \alpha}$

$\pi = 3.14$

**Sol**  $E_0^{\text{ARC}} = \frac{2 K_e \lambda \sin(\alpha/2)}{R}$

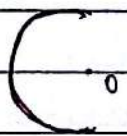
Sol

$l = 14 \text{ cm}$      $\alpha = \pi$

$R = \frac{l}{\alpha} = \frac{0.14}{3.14} = 0.045$

$\lambda = \frac{-7.5}{0.14} = -53.57 \text{ } \mu\text{C/m}$

$E_0 = \frac{2 \times 9 \times 10^9 \times 53.57 \times 10^{-6} \times \sin(\pi/2)}{0.045} = 2.14 \times 10^6 \text{ N/C}$



inward (left)

49) A)  $\frac{q_1}{q_2} = \frac{n_1}{n_2}$

$= \frac{18}{6} = 3$     ( $q_1 = 3q_2$ )

B)  $q_1 = +ve$     ,     $q_2 = -ve$

14

51) A)  $F = ma$

$$E_0 \times q = m_p \times a$$
$$a = \frac{E_0 \times q}{m_p} = \frac{1640 \times 1.6 \times 10^{-14}}{1.67 \times 10^{-27}} = 613 \times 10^8 \text{ m/s}^2$$

B)  $v_f = v_i + at$

$$t = \frac{v_f - v_i}{a} = \frac{1.2 \times 10^6 - 0}{613 \times 10^8} = 1.9 \times 10^{-5} \text{ s}$$

C)  $\Delta x = v_i t + \frac{1}{2} at^2$

$$= 0 + \frac{1}{2} \times 613 \times 10^8 \times (1.9 \times 10^{-5})^2 = 11.06 \text{ m}$$

D)  $K = \frac{1}{2} m v^2$

$$= \frac{1}{2} \times 1.67 \times 10^{-27} \times (1.2 \times 10^6)^2 = 2.4 \times 10^{-15} \text{ J}$$

52) A)  $F = ma$

$$a = \frac{E_0 \times q}{m_p} = 5.74 \times 10^{13} \text{ (-) m/s}^2$$

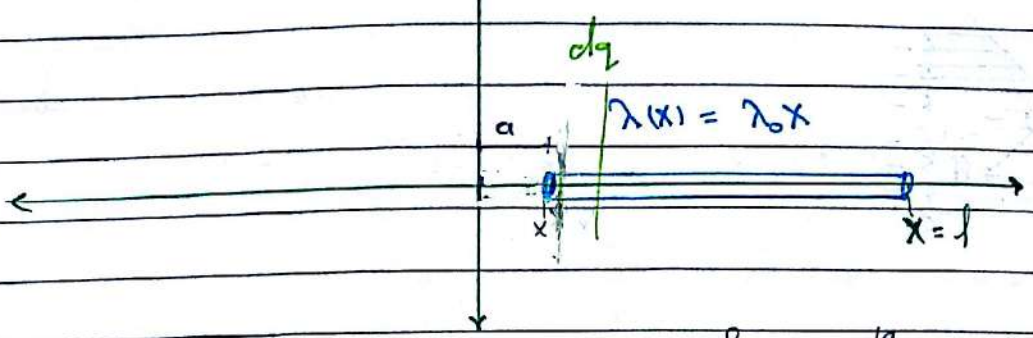
B)  $v_f^2 = v_i^2 + 2a\Delta x$

$$v_i = \sqrt{-2a\Delta x} = 28.3 \times 10^6 \text{ m/s}$$

C)  $v_f = v_i + at$

$$t = \frac{v_i}{a} = 4.93 \times 10^{-7} \text{ s}$$

# E.X. Non-uniform charge density.



$$E_p = \int_{\text{charge}} K_e \frac{dq}{x^2}$$

$$\lambda(x) = \frac{q}{x}$$

$$dq = \lambda(x) dx = \lambda_0 x dx$$

$$= \int_0^l K_e \lambda_0 \frac{x}{(x+a)^2} dx$$

$$y = x+a$$

$$x = y-a$$

$$dx = dy$$

$$= K_e \lambda_0 \int \frac{y-a}{y^2} dy$$

$$= K_e \lambda_0 \left[ \int \frac{1}{y} dy - \int \frac{1}{y^2} dy \right]$$

$$= K_e \lambda_0 \left[ \ln|y| + \frac{1}{y} \right]$$

$$= K_e \lambda_0 \left[ \ln|x+a| + \frac{1}{x+a} \right]_0^l$$

$$= K_e \lambda_0 \left( \ln\left|\frac{l+a}{a}\right| + \frac{a}{l+a} - 1 \right)$$



16

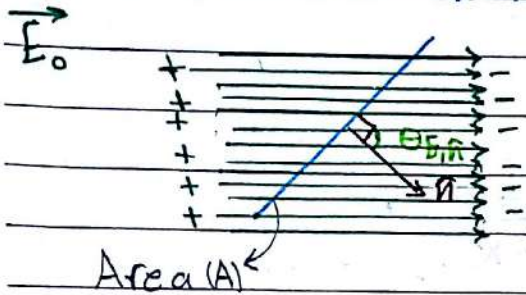
15/2/2018

Chapter 24

Thursday

### Gauss's Law

1. The electric flux =  $\Phi_E$  الكهربائي التدفق



We define the electric flux

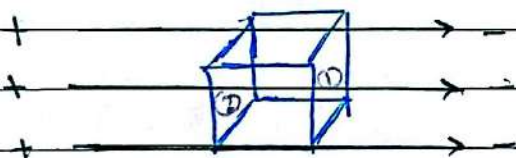
$$\Phi_E = E_0 A \cos \theta_{E, \hat{n}}$$

$$= \vec{E} \cdot \vec{A}, \quad \vec{A} = A \hat{n}$$

For arbitrary (شبهية) shape object

$$\Phi_E = \int_{\text{Area}} (E \cos \theta_{E, \hat{n}}) dA$$

E.X = Cube (مكعب) in electric field find the total flux from the cube



sol.

$$\Phi_1 = E_0 \cdot A$$

$$= E_0 \cdot l$$

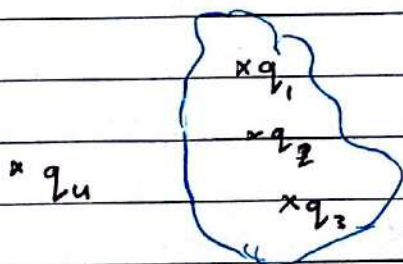
$$\Phi_2 = -E_0 \cdot A$$

$$= -E_0 \cdot l$$

$$\Phi_T = \text{Zero}$$

\* Conclusion = The total flux for uncharge object immersed in electric field is Zero

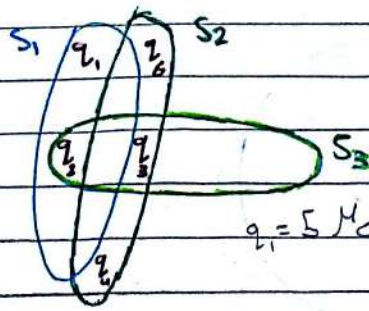
### Gauss's Law of charged object.



Gaussian surface.

$$\Phi_E = \frac{\sum q_{in}}{\epsilon_0} = \frac{q_1 + q_2 + q_3}{\epsilon_0}$$

EX-2



Find the Flux throughout the surfaces

$q_1 = 5 \text{ Mc}$ ,  $q_2 = 3 \text{ Mc}$ ,  $q_3 = -10 \text{ Mc}$ ,  $q_4 = 5 \text{ Mc}$   
 $q_5 = 12 \text{ Mc}$ ,  $q_6 = -8 \text{ Mc}$

Sol

$$\Phi_1 = \frac{\sum q_{in}}{\epsilon_0} = \frac{q_1 + q_2}{\epsilon_0} = \frac{(5+3) \times 10^{-6}}{8.85 \times 10^{-12}} = 0.9 \times 10^6$$

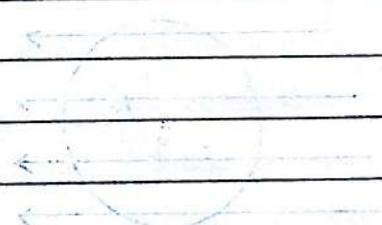
$$\Phi_2 = \frac{q_3 + q_4 + q_5}{\epsilon_0} = \frac{(3+5-8) \times 10^{-6}}{8.85 \times 10^{-12}} = \text{zero}$$

$$\Phi_3 = \frac{q_2 + q_3}{\epsilon_0} = \frac{(3-10) \times 10^{-6}}{8.85 \times 10^{-12}} = 0.79 \times 10^6$$

Gauss's Law

$$\int E \cos \theta \, dA = \frac{q_{in}}{\epsilon_0}$$

Note: The Gauss's law applicable only to obtain an electric field for symmetrical charge distribution.

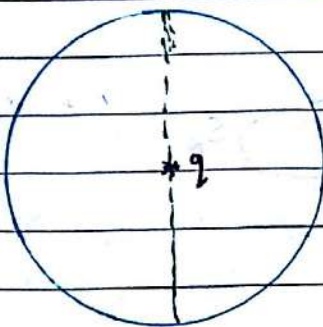


18/2/2018

Sunday

Ex 12

Gaussian surface.



Find the Flux Through the 1) Sphere

2) Hemi-sphere (نصف كروي)

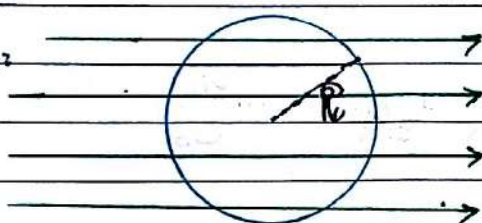
3) Quarter.

Sol. 1)  $\Phi_{\text{sphere}} = \frac{q}{\epsilon_0}$

2)  $\Phi_{\text{hemi}} = \frac{q}{2\epsilon_0}$

3)  $\Phi_{\text{quarter}} = \frac{q}{4\epsilon_0}$

Ex 22



$\vec{E} = E_0 \hat{i}$

Find the Flux through the 1) Sphere

2) Hemi-sphere

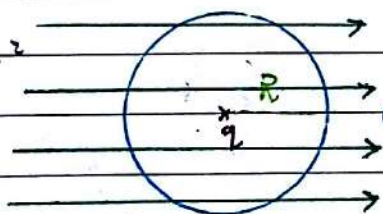
3) quarter

Sol. 1)  $\Phi_T = \text{Zero}$

2)  $\Phi_{\text{hemi}} = \vec{E} \cdot A_{\text{hemi}} = E_0 \times \frac{4\pi R^2}{2}$

3)  $\Phi_{\text{quarter}} = \vec{E} \cdot A_{\text{quarter}} = E_0 \times \frac{4\pi R^2}{4}$

Ex 32



$\vec{E} = E_0 \hat{i}$

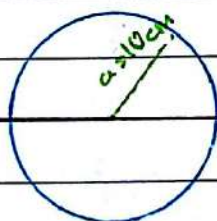
Gaussian surface

Find the Flux through the 1) Sphere 2) Hemi-sphere

Sol.  $\Phi_T = \frac{q}{\epsilon_0}$

$\Phi_{\text{hemi}} = E_0 \frac{4\pi R^2}{2} + \frac{q}{2\epsilon_0}$

Ex 42



Infinite wire

Find  $\Phi_T$

$\lambda = -27 \text{ nC/m}$

Sol

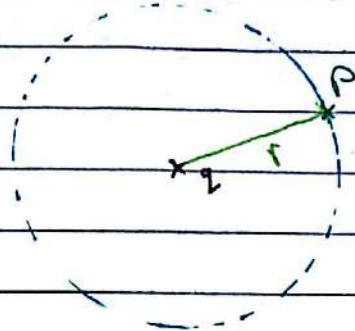
$\lambda = \frac{q_{\text{in}}}{l}$

$q_{\text{in}} = \lambda \times l = -27 \times 10^{-9} \times 20 \times 10^{-1} = -5.4 \text{ nC}$

$\Phi_T = \frac{\lambda l}{\epsilon_0} = \frac{-5.4 \times 10^{-9}}{8.85 \times 10^{-12}} = 0.61 \times 10^3$

### Application on Gauss's law

Ex. 1: Find the electric field a point charge  
sol



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$\vec{E} \cdot \int d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

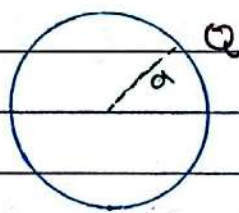
$$E \cdot (4\pi r^2) = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi r^2 \epsilon_0} = k_e \frac{q}{r^2} \#$$

Ex. 2: Sphere

- A) Conducting sphere (  $\vec{a}, \vec{b}, \vec{c}$  ) (Copper, Metallic, Al, ...)
- 1) Conducting solid sphere (  $\vec{a}, \vec{b}, \vec{c}$  )
  - 2) Conducting spherical shell (  $\vec{a}, \vec{b}, \vec{c}$  )

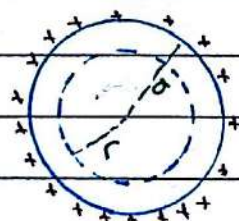
total charge: Q  
Radius: a



Find the electric field inside the spherical shell

a) (  $E_{in} = ?$  ,  $r < a$  )

sol



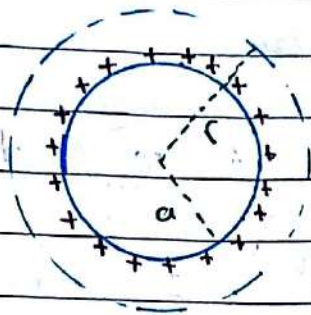
$$Q = \frac{Q}{4\pi a^2}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{0}{\epsilon_0}$$

$$E = \text{zero}$$

sol.



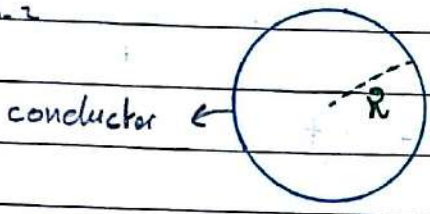
b)  $E_{out}$  s??,  $r > a$

$$\int \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$E (4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q \epsilon_0}{r^2}$$

EX. 2

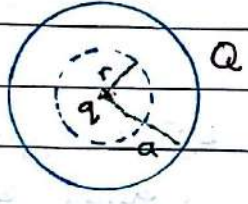


$Q = -7 \mu C$   
 $r = 50 \text{ cm}$

find  $E$  at  $r = 10 \text{ cm}$

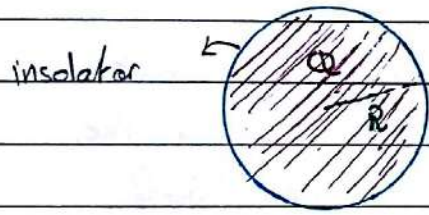
sol.  $E = 0$

EX. 3



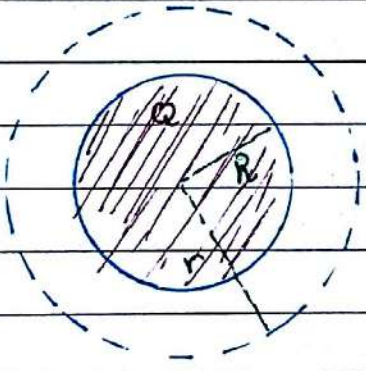
sol.  $E_{in} = \frac{k_e q}{r^2}$   
 $E_{out} = \frac{k_e (Q + q)}{r^2}$ ,  $R > a$

### B) Non-conducting sphere (isolated (single))



$$\rho = \frac{4}{3} \pi R^3$$

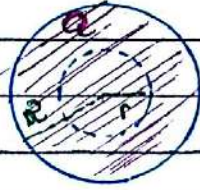
a)  $E_{out}$  s??,  $r > R$



sol.  $\int \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$   
 $E (4\pi r^2) = \frac{Q}{\epsilon_0}$   
 $E = \frac{k_e Q}{r^2}$ ,  $r > R$   
 $Q = \rho \frac{4}{3} \pi R^3$

b)  $E_{in} = ?$ ,  $r < R$

$\rho, Q$



sol  $\rho = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{q_{in}}{\frac{4}{3}\pi r^3}$

$$q_{in} = \rho \left( \frac{4}{3}\pi r^3 \right)$$

$$= \frac{Q}{\frac{4}{3}\pi R^3} \left( \frac{4}{3}\pi r^3 \right)$$

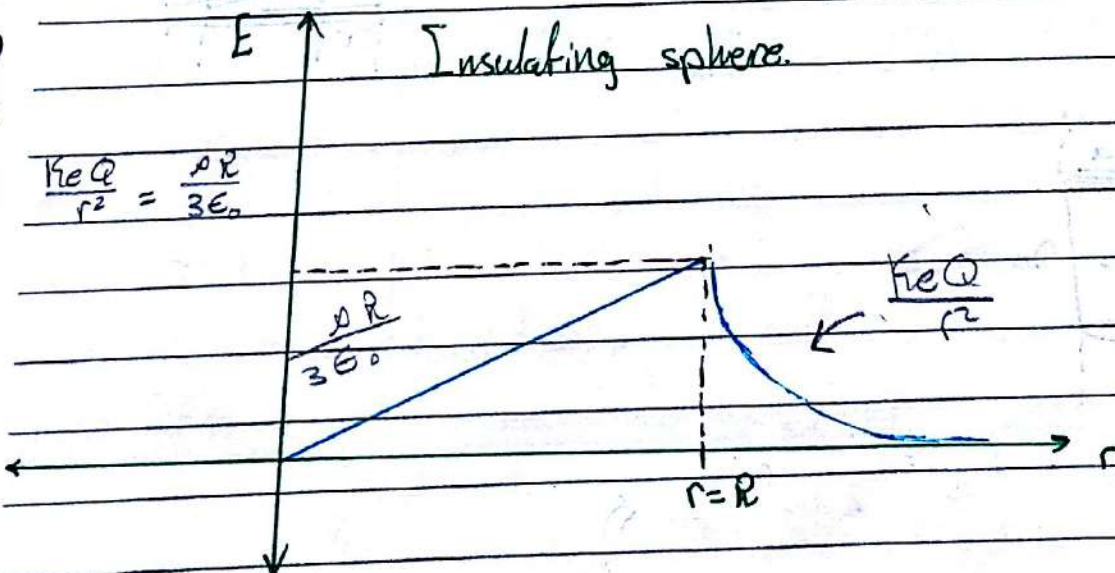
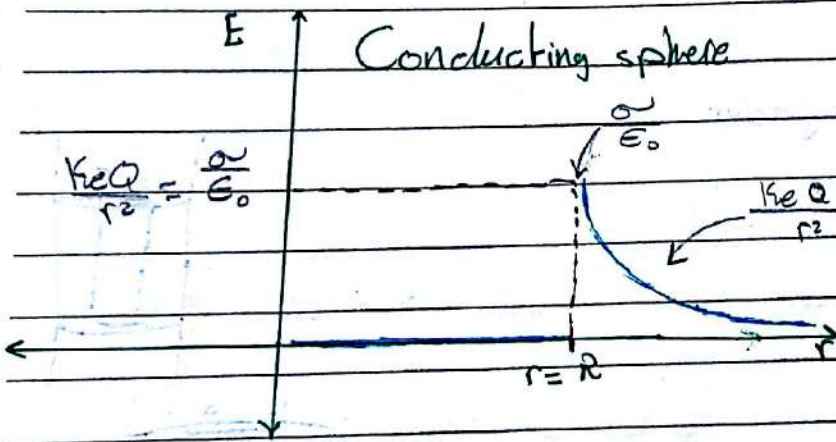
$$= \frac{Q}{R^3} r^3$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$E (4\pi r^2) = \rho \left( \frac{4}{3}\pi r^3 \right) \times \frac{1}{\epsilon_0}$$

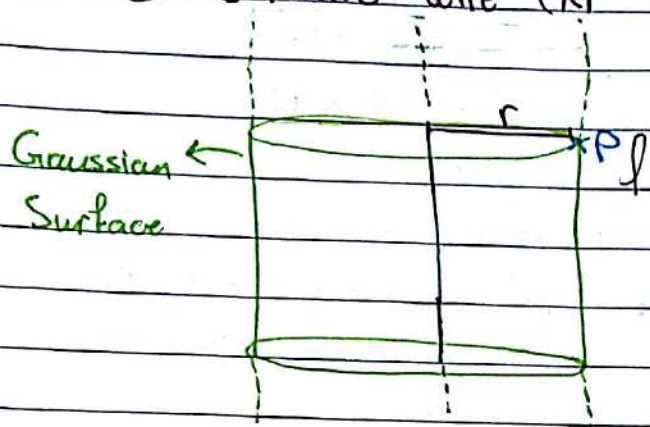
$$E_{in} = \frac{\rho r}{3\epsilon_0}, \quad r < R$$

$$= \frac{k_e Q r}{R^3}$$



# Cylindrical symmetry

① Infinite wire ( $\lambda$ )

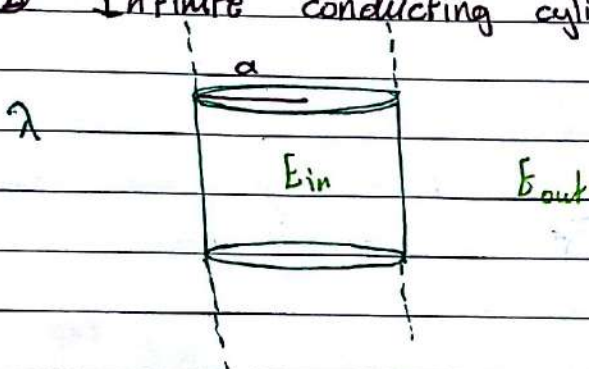


$$\int E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

$$E(2\pi r l) = \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi r \epsilon_0} = \frac{2k\epsilon\lambda}{r}$$

② Infinite conducting cylinder



①  $E_{in} = 0, r < a$

$$\int E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

$E_{in} = 0, r < a$

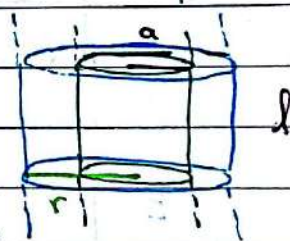


②  $E_{out} = ??, r > a$

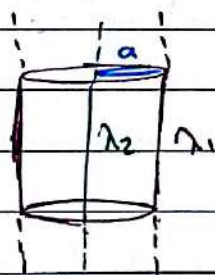
$$\int E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

$$E(2\pi r l) = \frac{\lambda l}{\epsilon_0}$$

$$E_{out} = \frac{2k\epsilon\lambda}{r}, r > a$$



$E_{in} = 0$



$a = 50 \text{ cm}$

$\lambda_1 = 10 \text{ MC/m}, \lambda_2 = 5 \text{ MC/m}$

Find the electric field at 1) 25 cm

2) 75 cm

sol. 1)  $E|_{25} = \frac{2k\epsilon\lambda_2}{r} = 3.6 \times 10^5$

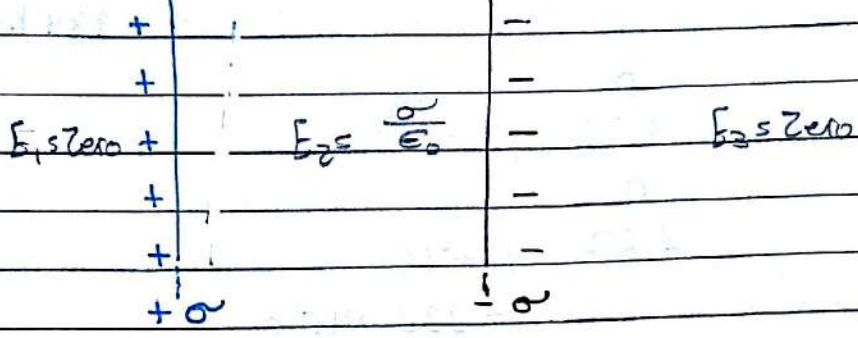
2)  $E|_{75} = \frac{2k\epsilon(\lambda_1 + \lambda_2)}{r} = 3.6 \times 10^5$

Calculation

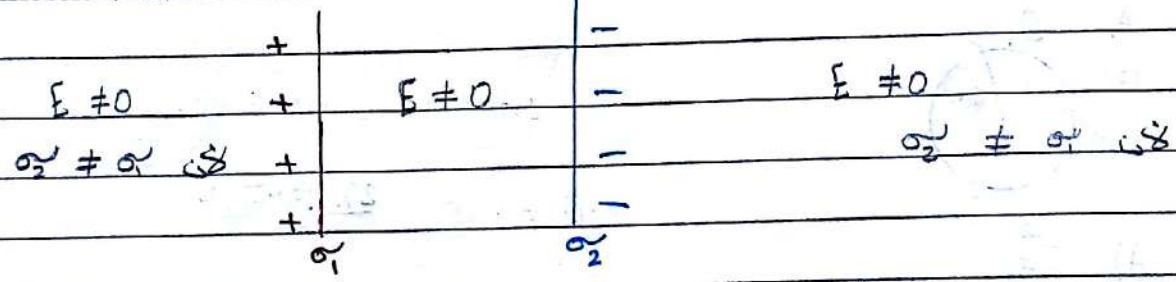
③ Infinite/Large sheet, plane, disk Dipole moment  $\vec{p}$  USA

1. non-conductor (Insulator)  $\rightarrow E = \frac{\sigma}{2\epsilon_0}$
2. conducting  $\rightarrow E = \frac{\sigma}{\epsilon_0}$

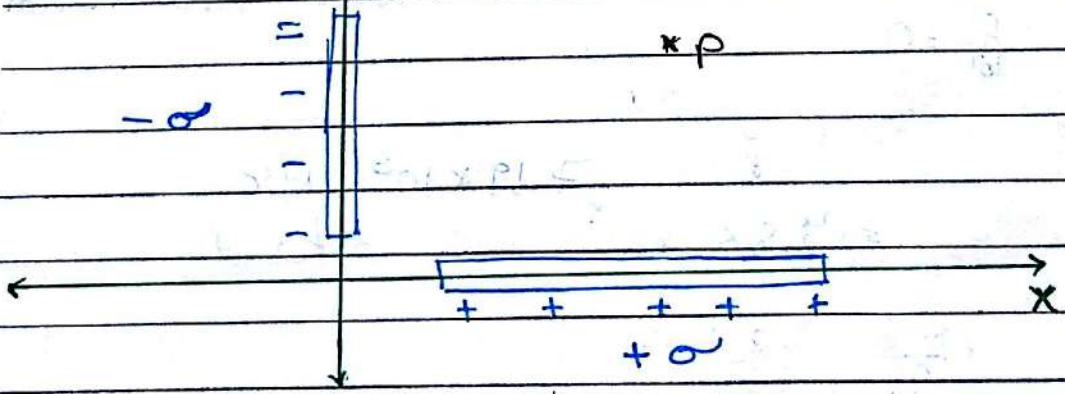
EX-2 non-conducting infinite sheet



EX-2



EX-2 non-conducting infinite sheet  
Find  $\vec{E}_p$



sol.  $\vec{E}_p = \frac{\sigma}{2\epsilon_0} (-\hat{i} + \hat{j})$



24

22/2/2018

# Problems of Ch 24

Thursday

3)  $\Phi_{max} = E A \cos \theta$

$5.2 \times 10^5 = E \times (0.2)^2 \times \pi$

$E = \frac{5.2}{\pi (0.2)^2} \times 10^5 \text{ N/C}$

4) A)  $\Phi = E A \cos 180 = -2.34 \text{ K Nm}^2/\text{C}$

B)  $\Phi = E A \cos 60 = 7.8 \times 10^4 \times (0.3 \times \frac{0.1}{\cos 60}) \cos 60 = 2.34 \text{ K Nm}^2/\text{C}$

C)  $\Phi = E A \cos 90 = 0$

5) A)  $\Phi = E A \cos 0 = 858 \text{ Nm}^2/\text{C}$

B)  $\Phi = E A \cos 90 = 0$

C)  $\Phi = E A \cos 40 = 657 \text{ Nm}^2/\text{C}$

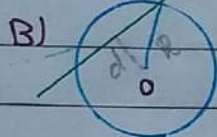
8)  $\Phi = \frac{q_{in}}{\epsilon_0} = \frac{(1-3) \times 10^{-9}}{8.85 \times 10^{-12}} = -226 \text{ Nm}^2/\text{C}$

12) B)  $\Phi_T = \frac{q_{in}}{\epsilon_0} = 1.92 \times 10^7 \text{ Nm}^2/\text{C}$

A)  $\Phi_{surface} = \frac{\Phi_T}{3} = 3.2 \times 10^6 \text{ Nm}^2/\text{C}$

17)

A) 0



length of the wire =  $2 \sqrt{R^2 - d^2}$

$\lambda = \frac{Q}{L} \Rightarrow Q = \lambda \cdot 2 \sqrt{R^2 - d^2}$

$\Phi = \frac{q_{in}}{\epsilon_0} = \frac{2\lambda \sqrt{R^2 - d^2}}{\epsilon_0}$

21)

A)  $\frac{Q}{2\epsilon_0}$

B)  $\Phi_{curved} + \Phi_{flat} = 0$

$\Phi_{flat} = \frac{-Q}{2\epsilon_0}$

24) A)  $E = \frac{2k\lambda}{r} = \frac{2 \times 9 \times 10^9 \times 90 \times 10^{-6}}{0.1} = 162 \times 10^5 \text{ N/C inward}$

29)

A)

$E = 0$

B)

$E = \frac{kqQ}{r^2} = \frac{9 \times 10^9 \times 32 \times 10^{-6}}{(0.2)^2} = 7.19 \times 10^6 \text{ N/C}$

30)

A)  $E = \frac{\sigma}{2\epsilon_0} = 4.86 \times 10^9 \text{ N/C outward}$

B) No

34)

A)  $E = \frac{2kq\lambda}{r} = \frac{2kqQ}{r^2}$

$E = \frac{2 \times 9 \times 10^9 \times Q}{2.4 \times 0.19} = 3.6 \times 10^3$

$Q = +9.12 \times 10^{-9} \text{ C} = +9.12 \text{ nC}$

B) 0

35) A)  $E_{in} = \frac{\rho}{3\epsilon_0} r$ ,  $\rho = \frac{3Q}{4\pi r^3} = 9.7 \times 10^{-5} \text{ C/m}^3$

$E_0 = 0$

B)  $E_0 = \frac{9.7 \times 10^{-5}}{3 \times 8.85 \times 10^{-12}} \times 0.1 = 3.65 \times 10^5 \text{ N/C}$

D)  $E_{out} = \frac{k_e Q}{r^2}$   
 $E_0 = \frac{9 \times 10^9 \times 26 \times 10^{-6}}{(0.1)^2} = 1.46 \times 10^6 \text{ N/C}$

سب 39)

44) A)  $E = \frac{\sigma}{\epsilon_0} \Rightarrow \sigma = E \epsilon_0$

$\sigma = 7.08 \times 10^{-7} \text{ C/m}^2$  one +ve and one -ve

B)  $\sigma = \frac{Q}{2A}$  (A pain 2A لیکو) د جسے 2A لیکو

$Q = 2\sigma A = 354 \times 10^{-9} \text{ C}$

سب 47)

A) 0

B)  $E = \frac{k_e Q}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(0.03)^2} = 8 \times 10^7 \text{ N/C outward}$

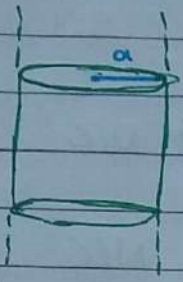
C) 0

D)  $E = \frac{9 \times 10^9 \times (8-4) \times 10^{-6}}{(0.03)^2} = 73.3 \times 10^5 \text{ N/C outward}$

سب 4) غیر مطلوب



### Insulating Cylinder



$Q, \rho$

$$E_{out} = \frac{\rho A^2}{2\epsilon_0 r}, \quad r > a$$

$$E_{in} = \frac{\rho r}{2\epsilon_0}, \quad r < a$$

EX:  $Q = 170 \text{ } \mu\text{C}, a = 5 \text{ cm}, l = 2.4$

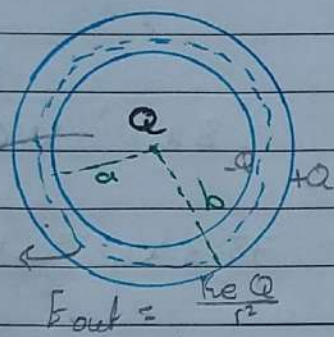
① charge density

$$\rho = \frac{Q}{V} = \frac{170}{\pi a^2 l} = 90 \text{ } \mu\text{C/m}^2$$

②

$$E(r=1 \text{ cm}) = \frac{\rho r}{2\epsilon_0} = \frac{90 \times 0.01 \times 10^{-6}}{2 \times 8.85 \times 10^{-12}} = 10.1 \times 10^6 \text{ N/C}$$

EX:



spherical conducting shell

$a = 20 \text{ cm}, b = 50 \text{ cm}, Q = 70 \text{ } \mu\text{C}$

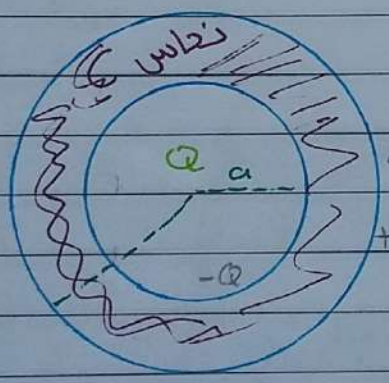
$E_{in} = \frac{k_e Q}{r^2}$

$E = 0$

$E_{out} = \frac{k_e Q}{r^2}$

Induced charge: إذا  $Q$  موجبة ← الموجب - الموجب + السالبة للسالبة  
 إذا  $Q$  سالبة ← الموجب + الموجب - السالبة للسالبة

EX:



$Q_{net} = 5 \text{ } \mu\text{C}$   
 $Q_s = 70 \text{ } \mu\text{C}$

Find  $Q_x$

$\times Q_x$

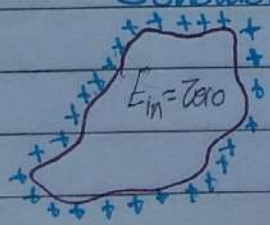
sol

$$Q_{net} = Q + Q_x$$

$$5 = 70 + Q_x$$

$$Q_x = -65 \text{ } \mu\text{C}$$

### Conductors in electro-static equilibrium.



$$E_{out} (\text{outside conductor}) = \frac{\sigma}{\epsilon_0}$$

25/2/2018

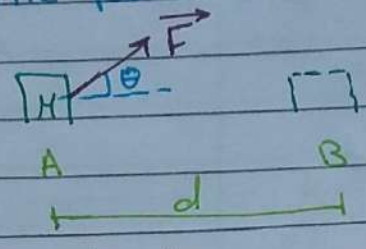
Sunday

# Chapter 25 The electric potential

الطاقة الكهربية

## 1. The electric potential energy

Remember:



The work done

$$W_{A \rightarrow B} = \vec{F} \cdot \vec{x}$$

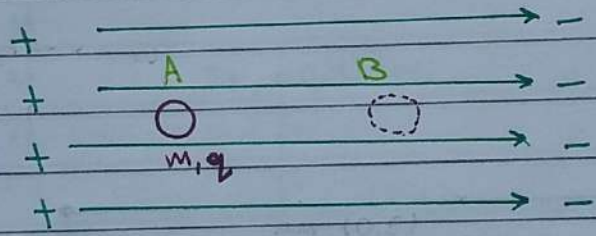
$$= F(\vec{B} - \vec{A}) \cdot \cos \theta = Fd \cos \theta$$

$$W_{A \rightarrow B} = q \int_A^B \vec{E} \cdot d\vec{s}$$

$d\vec{s} = \begin{cases} dx \hat{i} \\ dy \hat{j} \\ dx \hat{i} + dy \hat{j} \end{cases}$

$$\Delta U + \Delta K = 0$$

$$W_{A \rightarrow B} = -\Delta U_{A \rightarrow B}$$



The electric work done on the particle is

$$W_{A \rightarrow B} = q \int_A^B \vec{E}_0 \cdot d\vec{s}$$

$$= q \int_A^B \epsilon_0 \vec{E} \cdot d\vec{s} = \epsilon_0 q \int_A^B E \cos \theta ds$$

E.X:  $\vec{E} = E_0 \hat{i}$

$$W = \epsilon_0 E_0 q \int_A^B \cos \theta ds = \epsilon_0 E_0 d$$

We define the electric potential energy difference

$$\Delta U_{A \rightarrow B} = -W_{A \rightarrow B} = -\epsilon_0 q \int_A^B \vec{E} \cdot d\vec{s}$$

$$\Delta U_{A \rightarrow B} = -\epsilon_0 q \int_A^B \vec{E} \cdot d\vec{s}$$

We define the electric potential difference

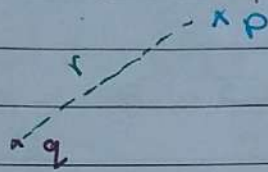
$$\Delta V_{A \rightarrow B} = \frac{\Delta U_{A \rightarrow B}}{q_0}$$

$$\Delta V_{A \rightarrow B} = - \int_A^B \vec{E} \cdot d\vec{s}$$

EX:  $\vec{E} = E_0 \hat{i}$

$\Delta V_{A \rightarrow B} = -E_0 d$

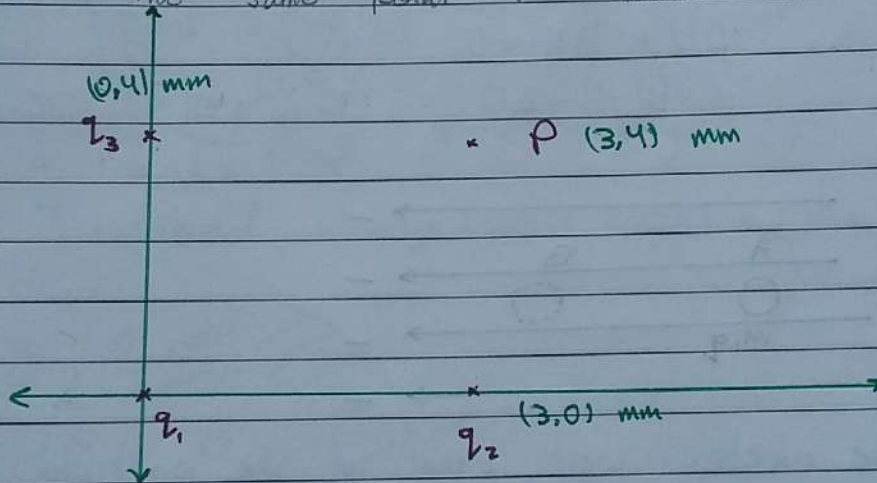
EX: The electric potential of a point charge



$$E_P = \frac{k_e q}{r^2}$$

$$V_P = \frac{k_e q}{r}$$

EX: Find the electric potential of 3 charges at the same point P shown in the figure



- $q_1 = -25 \mu C$
- $q_2 = 16 \mu C$
- $q_3 = -9 \mu C$

**sol.**  $V_P = V_1 + V_2 + V_3$

$$= \frac{q \times 10^9 \times 10^{26}}{10^{-3}} \left( \frac{-25}{5} + \frac{16}{4} + \frac{-9}{3} \right) = -36 \text{ MV}$$

② Find the work required to bring a proton (+e) from infinity to the point P

**sol.**  $W_{A \rightarrow B} = -U_{A \rightarrow B} = -q_0 \Delta V_{A \rightarrow B}$

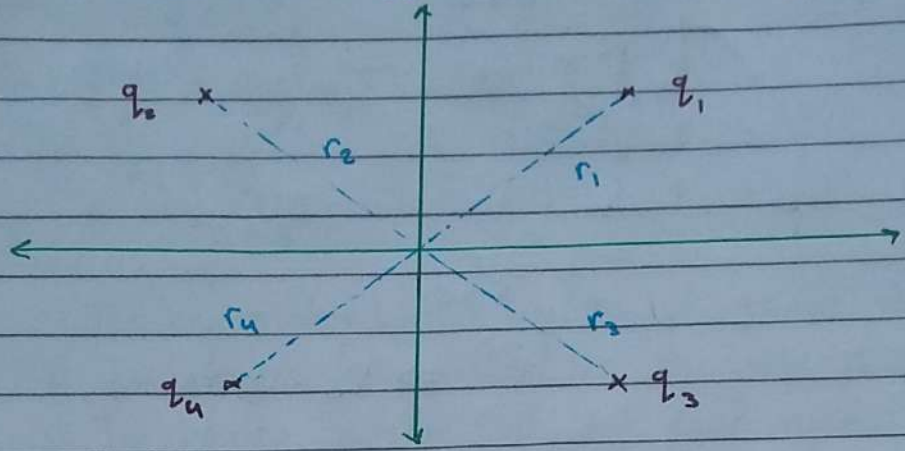
$$W_{\infty \rightarrow P} = -e (V_P - V_\infty)$$

$$= -1.6 \times 10^{-19} (-36 \times 10^6 - 0) = +57.6 \times 10^{-13} \text{ J}$$

26/2/2018

Monday

The electric energy stored in a system of charges.



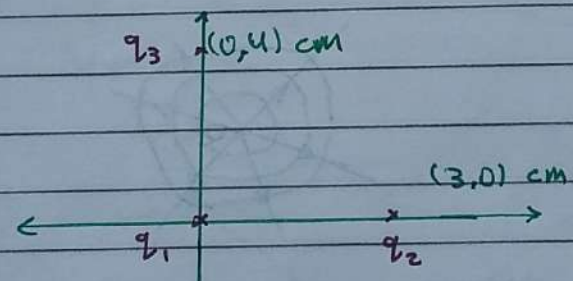
- السؤال
- 1) What is the energy stored in the charges shown in the figure
  - 2) What is the work require to assemble (تجميع) charges in the figure

The answer is ( )

$$U = \frac{k_e q_1 q_2}{r_{12}} + \frac{k_e q_1 q_3}{r_{13}} + \frac{k_e q_1 q_4}{r_{14}} + \frac{k_e q_2 q_3}{r_{23}} + \frac{k_e q_2 q_4}{r_{24}} + \frac{k_e q_3 q_4}{r_{34}}$$

(index) بالترتيب من الأول إلى الأخير

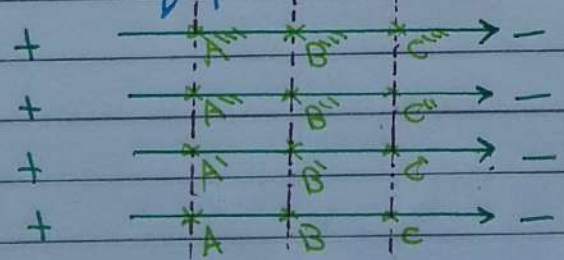
Ex-2



- $q_1 = -10 \text{ nC}$
- $q_2 = 20 \text{ nC}$
- $q_3 = -15 \text{ nC}$

sol.  $U = \frac{q \times 10^{-9} \times 10^{-18}}{10^{-2}} \left( \frac{-10 \times 20}{3} + \frac{-10 \times -15}{4} + \frac{20 \times -15}{5} \right) \text{ J}$

Equipotential surface (سطح الجهد المتساوي)



كل سطح متساوي الجهد له فرق الجهد

$$\Delta V = V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{s}$$

$\uparrow V$   $\downarrow K$  في اتجاه المجال الكهربائي  $\downarrow V$   $\uparrow K$

المجال الكهربائي في اتجاه زيادة الجهد، والعكس بالعكس

Find the work required to move the charge  $q_0$  from A to C

sol  $W_{A \rightarrow C} = W_{A \rightarrow B} + W_{B \rightarrow C}$

$$= q_0 \int_A^C \vec{E} \cdot d\vec{s} + 0$$

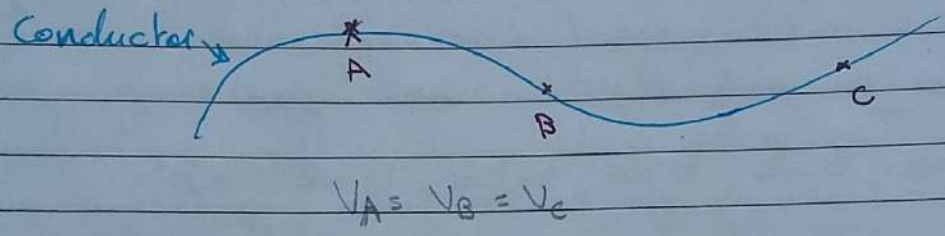
$$= q_0 \int_A^C E_0 \hat{i} \cdot dx \hat{i}$$

$$= q_0 E_0 (A-C) = -\Delta U_{A \rightarrow C}$$

$$= -q_0 \Delta V_{A \rightarrow C}$$

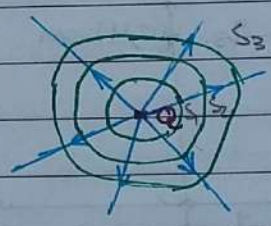
$$q_0 E_0 d = -q_0 \Delta V_{A \rightarrow C} \Rightarrow V_C - V_A = -E_0 d$$

$$W_{A \rightarrow B} = q_0 \Delta V$$



Point charge

	$S_1$	$S_2$	$S_3$
+ Q 1st	1000	800	600
- Q 1st	-1000	-800	-600

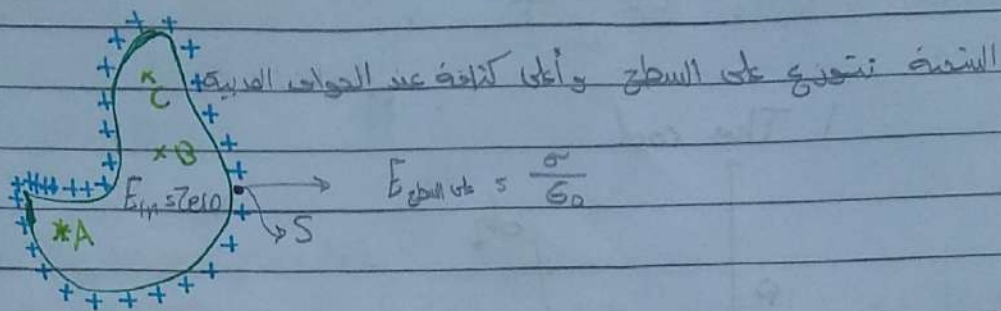


Move a proton from  $S_1$  to  $S_2$

$$W_{S_1 \rightarrow S_2} = -\Delta U = -q_0 \Delta V_{S_1 \rightarrow S_2}$$

$$= -q_0 (V_{S_2} - V_{S_1})$$

# Conductor in electrostatic equilibrium (موصِّل في اتزان كهواستاتيكي)



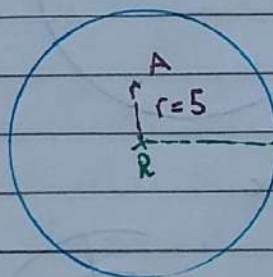
## Properties -

- 1) The charge density is maximum at minimum curvature (شد) (الشد)
- 2) The electric field inside the conductor is Zero
- 3) The electric potential of any point inside the conductor is the same as the surface ( $V_A = V_B = V_C = V_S$ )

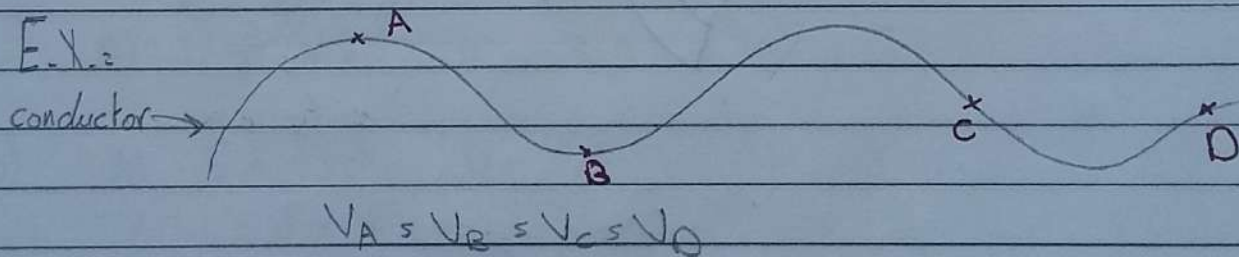
## Sphere conductor

$Q = 50 \mu C$

$R = 30 \text{ cm}$



$V_A \Big|_{r=5} = V_S = \frac{kqQ}{r}$   
 $E \Big|_S = \text{Zero}$



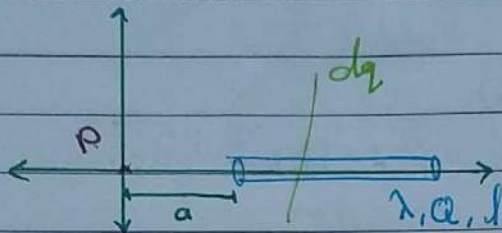


32

⑤ The electric potential of a continuous charge distribution

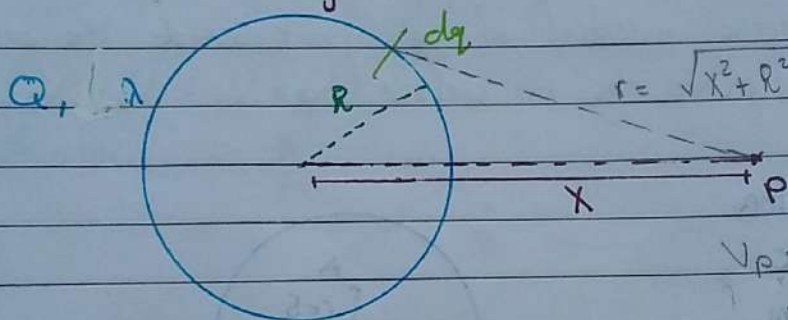
$$V_p = K_e \int_{\text{charge}} \frac{dq}{r}$$

1. The rod



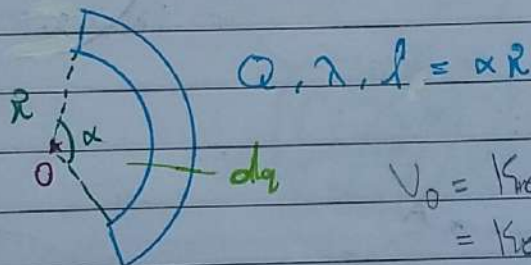
$$\begin{aligned} V_p &= K_e \int \frac{dq}{x+a} \\ &= K_e \int \frac{\lambda dx}{x+a} \\ &= K_e \lambda (\ln|x+a|) \Big|_0^l \\ &= K_e \lambda \ln \left| \frac{a+l}{a} \right| \end{aligned}$$

2. The ring



$$\begin{aligned} V_p &= K_e \int \frac{dq}{\sqrt{x^2 + R^2}} \\ &= K_e \frac{Q}{\sqrt{x^2 + R^2}} \end{aligned}$$

3. Arc



$$\begin{aligned} V_p &= K_e \int \frac{dq}{R} \\ &= K_e \frac{Q}{R} \\ &= K_e \lambda \alpha \end{aligned}$$

27/2/2018

## Problems of Ch. 25.

33

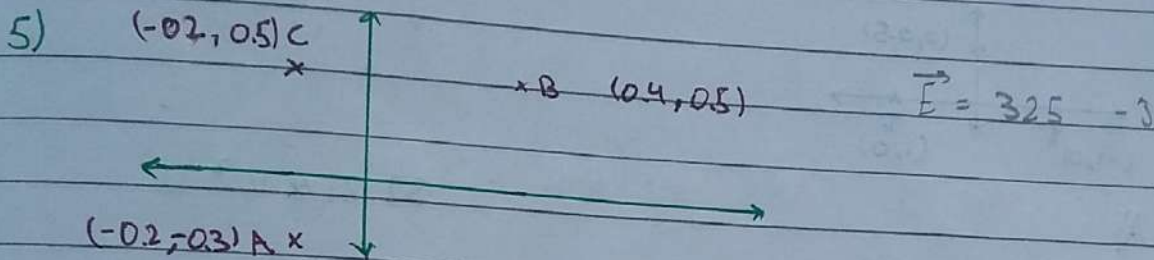
Tuesday

3)  $\Delta K + \Delta U = 0$

$\Delta K = -\Delta U$

$\frac{1}{2} m v_f^2 + \frac{1}{2} m v_i^2 = -\Delta U$

$$v_f = \sqrt{\frac{-2\Delta U}{m}} = \sqrt{\frac{-2 \times 1.6 \times 10^{-19} \times (-120)}{1.67 \times 10^{-27}}} = 15.16 \times 10^4 \text{ m/s}$$



sol.  $V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$

$\Delta V = -\int_{-0.3}^{0.5} E_0 \hat{j} \cdot d\vec{s} = -\int_{-0.3}^{0.5} E_0 \hat{j} \cdot d\vec{s}$

$= -\int_{-0.3}^{0.5} E_0 \hat{j} \cdot d\vec{s} = -\int_{-0.3}^{0.5} E_0 \hat{j} \cdot d\vec{s}$

$= -\int_{-0.3}^{0.5} E_0 \hat{j} \cdot d\vec{s} = -\int_{-0.3}^{0.5} E_0 \hat{j} \cdot d\vec{s}$

$= 325 \times 0.8 = 260 \text{ V}$

16) A)  $V = \frac{kq_1}{d/2} + \frac{kq_2}{d/2}$

$= \frac{k(q_1 + q_2)}{d/2} = \frac{9 \times 10^9 \times 2 \times 10^{-9}}{0.175} = 103 \text{ V}$

B)  $V = \frac{kq_1 q_2}{r_{12}} = -3.85 \times 10^{-7} \text{ J}$

20) A)  $E = \frac{kq}{r^2}$ ,  $V = \frac{kq}{r}$

$V = E \times r$

$r = \frac{V}{E} = \frac{3000}{500} = 6 \text{ m}$

B)  $V = \frac{kq}{r}$

$q = \frac{Vr}{k} = 2 \times 10^{-6} \text{ C}$

23)  $\leftarrow \begin{array}{c} P_1 \quad x \quad +q \quad a \quad -2q \\ \times \quad \times \quad \times \quad \times \end{array} \rightarrow$

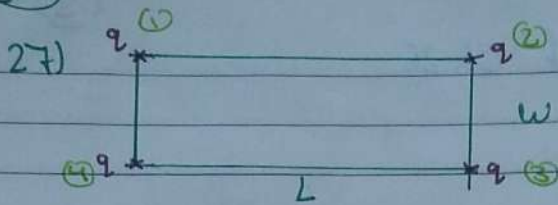
A)  $E_P = 0 \Rightarrow E_q = -E_{-2q} \Rightarrow \frac{kq}{x^2} = \frac{-k(-2q)}{(x+2)^2}$

$\frac{1}{x^2} = \frac{2}{(x+2)^2} \Rightarrow 2x^2 = (x+2)^2 \Rightarrow x = \frac{-2}{\sqrt{2}-1} = -4.83 \text{ m}$

B)  $V_B = 0 \Rightarrow V_q = -V_{-2q} \Rightarrow \frac{kq}{a} = \frac{-k(-2q)}{2-a}$

$2a = 2-a \Rightarrow 3a = 2 \Rightarrow a = 0.67$

34



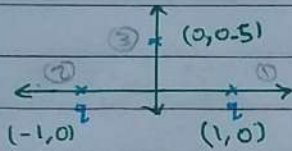
$q = +10 \text{ Mc}$   
 $L = 60 \text{ cm}$   
 $W = 15 \text{ cm}$

$$\Delta U = U_f - U_i$$

$$= \frac{K_e q^2}{10^{-2}} \left( \frac{1}{L} + \frac{1}{\sqrt{L^2+W^2}} + \frac{1}{W} + \frac{1}{W} + \frac{1}{\sqrt{L^2+W^2}} + \frac{1}{L} \right) - \frac{K_e q^2}{10^{-2}} \left( \frac{1}{L} + \frac{1}{\sqrt{L^2+W^2}} + \frac{1}{W} + \frac{1}{W} \right)$$

$$= \frac{K_e q^2}{10^{-2}} \left( \frac{1}{\sqrt{L^2+W^2}} + \frac{1}{L} \right) = 274 \times 10^3 \text{ J}$$

25



$q = 2 \text{ Mc}$

A)  $V = \frac{K_e q}{d} = \frac{9 \times 10^9 \times 2 \times 2 \times 10^{-6}}{1.12} = 32.2 \times 10^3 \text{ V}$

B)  $\Delta U = U_f - U_i$   
 $= K_e q_1 \left( \frac{q_2}{2} + \frac{q_3}{1.12} + \frac{q_3}{1.12} \right) - \frac{K_e q_1^2}{2} = 96.4 \times 10^{-3} \text{ J}$

34

$\Delta K + \Delta U = 0$

$K_f + U_f = K_i + U_i$

$4 \left( \frac{1}{2} m v^2 \right) + K_e q^2 \left( \frac{1}{2L} + \frac{1}{2\sqrt{2}L} + \frac{1}{2L} + \frac{1}{2L} + \frac{1}{2\sqrt{2}L} + \frac{1}{2L} \right) = 0 + K_e q^2 \left( \frac{1}{L} + \frac{1}{\sqrt{2}L} + \frac{1}{L} + \frac{1}{\sqrt{2}L} + \frac{1}{L} \right)$

$2 m v^2 = \left( \frac{1}{\sqrt{2}} + 2 \right) \frac{K_e q^2}{L}$

$v = \sqrt{\frac{\left( \frac{1}{\sqrt{2}} + 2 \right) K_e q^2}{2 m}} = \sqrt{\frac{\left( \frac{1}{\sqrt{2}} + 1 \right) K_e q^2}{m}}$

39) A)  $\Delta U = - \int \vec{E} \cdot d\vec{s}$   
 $E = - \frac{\partial V}{\partial x}$

$E_x = - \frac{\partial V}{\partial x} = -5 + 6xy$

$E_y = - \frac{\partial V}{\partial y} = 3x^2 - 2z^2$

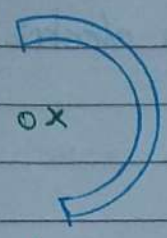
$E_z = - \frac{\partial V}{\partial z} = -4yz$

B) At  $(1, 0, -2)$

$E_x = -5 \text{ J}, E_y = -5 \text{ J}, E_z = 0$

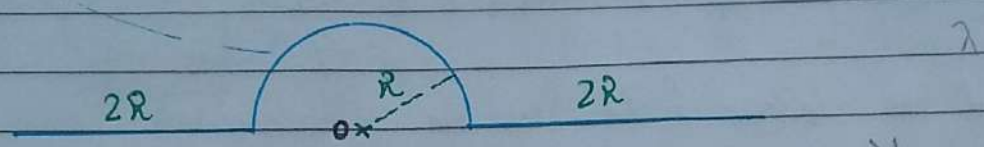
$|E| = \sqrt{-5^2 + 0^2 + -5^2} = 7.07$

44)



sol.  $V_0 = K_e \lambda \alpha = K_e \frac{Q}{L} \alpha$   
 $= \frac{9 \times 10^9 \times (-7.5) \times 10^{-6} \times 3.14}{0.14} = -29.7 \times 10^3$

47)



$V_0 = V_1 + V_2 + V_3$   
 $= K_e \lambda \ln \left| \frac{a+l}{a} \right| + K_e \lambda \ln \left| \frac{a+l}{a} \right| + K_e \lambda \alpha$   
 $= K_e \lambda \left( \ln \left| \left( \frac{a+l}{a} \right)^2 \right| + 3.14 \right) = K_e \lambda \left( \ln \left| \frac{R+2R}{R} \right|^2 + 3.14 \right)$

49)

$V_s = \frac{K_e Q}{R}$   
 $7500 = \frac{9 \times 10^9 \times Q}{0.3}$   
 $Q = 2.5 \times 10^{-7} \text{ C}$

$Q = N \cdot q_e$   
 $N = \frac{Q}{q_e} = \frac{2.5 \times 10^{-7}}{1.6 \times 10^{-19}} = 1.56 \times 10^{12} \text{ e}^-$

50)

A)  $r=10$

$E_{10} = \text{Zero}$   
 $V_{10} = V_s = \frac{K_e Q}{R} = 16.7 \times 10^{11}$

B)  $r=20$

$E_{20} = \frac{K_e Q}{r^2} = 54 \times 10^{11}$   
 $V_{20} = \frac{K_e Q}{r} = 10.8 \times 10^{11}$

C)  $r=14$

$E_{14} = \frac{K_e Q}{r^2} = 11.02 \times 10^{11}$   
 $V_{14} = \frac{K_e Q}{r} = 16.7 \times 10^{11}$

6/3/2018

Tuesday

## Obtaining the electric field from electric potential

Rem:  $\Delta V = -\int \vec{E} \cdot d\vec{x}$   
 $E_x = -\frac{\partial V}{\partial x}$

The electric field  $\vec{E}$   
 $\vec{E}(x, y, z) = -\frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j} - \frac{\partial V}{\partial z} \hat{k}$

E.X. = Find the electric field from the potential

$$V(x, y, z) = 5xyz$$

$$\frac{\partial V}{\partial x} = 5yz, \quad \frac{\partial V}{\partial y} = 5xz, \quad \frac{\partial V}{\partial z} = 5xy$$

$$\vec{E} = -5yz \hat{i} - 5xz \hat{j} - 5xy \hat{k}$$

$$\vec{E}_{(1, -1, 0)} = 0 - 0 - 5 \times 1 \times -1 = 5 \hat{k}$$

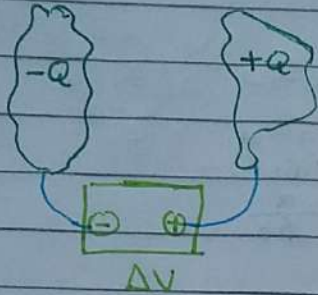
$$|\vec{E}| = 5 \text{ V/m}$$

E.X. =  $V(x) = \frac{1}{x}$

$$E_x \Big|_{x=5} = -\frac{\partial V}{\partial x} \Big|_{x=5} = \frac{+1}{x^2} \Big|_{x=5} = \frac{1}{25}$$

# Chapter 26 The capacitors

## 1. Capacitance definition.



We define the capacitance "C" as

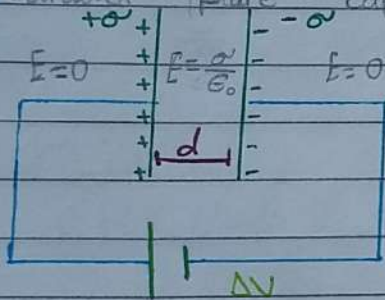
$$C = \frac{Q}{\Delta V}$$

\* Note = C doesn't depend on the charge

$$[C] = \frac{C}{V} = \text{Farad}$$

## Examples on capacitors

### 1. Parallel plate capacitor (المكثف ذو اللوحين المتوازيين)



d: separation between the plates

$$C = \frac{Q}{\Delta V}$$

but  $\Delta V = E \cdot (\text{distance})$

$$= \frac{\sigma}{\epsilon_0} \cdot d = \frac{Qd}{\epsilon_0 A}$$

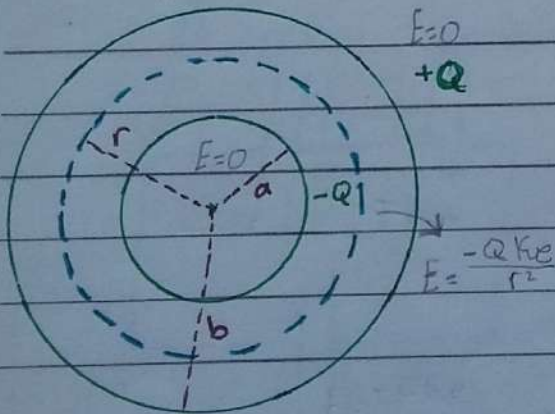
$$C = \frac{Q}{\frac{Qd}{\epsilon_0 A}} = \frac{\epsilon_0 A}{d}$$

$$C_{\text{parallel}} = \frac{\epsilon_0 A}{d}$$

$$A = X \text{ cm}^2 = X \times 10^{-4} \text{ m}^2$$

$$X \text{ mm}^2 = X \times 10^{-6} \text{ m}^2$$

### 2. Spherical capacitor



$$\Delta V = -\int_a^b E \cdot ds$$

$$= + \int_a^b \frac{Qk\epsilon}{r^2} dr$$

$$= Qk\epsilon a \int_a^b \frac{1}{r^2} dr$$

$$= Qk\epsilon \left( -\frac{1}{r} \right) \Big|_a^b = k\epsilon Q \left( \frac{1}{a} - \frac{1}{b} \right)$$

$$C = \frac{Q}{\Delta V} = \frac{4\pi\epsilon_0 ab}{b-a}$$

$$= \frac{4\pi\epsilon_0 ab}{b-a}$$

Special case

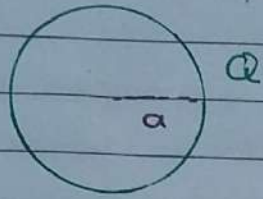
Isolated sphere

1. isolated spherical capacitor

يعطيات السؤال هو

2. isolated drop of water

نصف قطر كره واحد



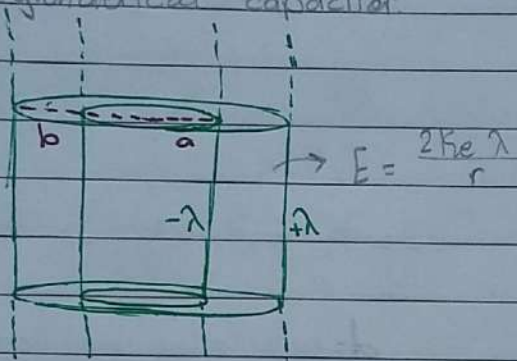
$$C_{\text{isolated sphere}} = \lim_{b \rightarrow \infty} \left( \frac{C_{\text{sph}}}{b-a} \right)$$

$$= \lim_{b \rightarrow \infty} \frac{4\pi\epsilon_0 a b}{b-a}$$

$$= 4\pi\epsilon_0 a$$

$$= \frac{a}{k\epsilon_0}$$

3. Cylindrical capacitor



$$\Delta V = -\int E \cdot dr$$

$$= -2k\epsilon\lambda \int_a^b \frac{dr}{r}$$

$$= -2k\epsilon\lambda \ln\left|\frac{b}{a}\right|$$

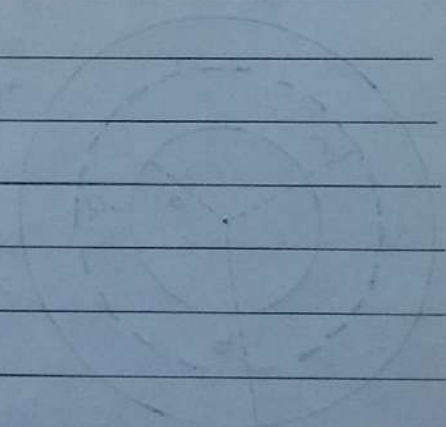
$$C = \frac{Q}{\Delta V}$$

$$= \frac{\lambda l}{-2k\epsilon\lambda \ln\left|\frac{b}{a}\right|}$$

$$= -2k\epsilon\lambda \ln\left|\frac{b}{a}\right| \quad \text{Kie}$$

Capacitance per unit length (السؤال يطلب -1 لكل وحدة طول)

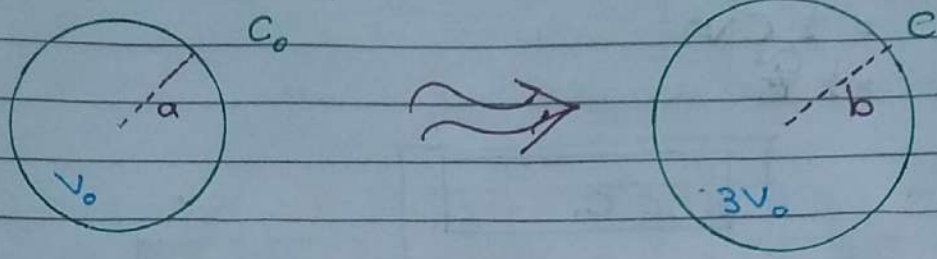
$$\frac{C}{l} = \frac{2k\epsilon\lambda \ln\left|\frac{b}{a}\right|}{l}$$



8/3/2018

Thursday

E.X. = isolated capacitor find  $\frac{C}{C_0}$



$C_0 = 4\pi\epsilon_0 a$  ,  $V(C_0) = \frac{4}{3}\pi a^3$

$C = 4\pi\epsilon_0 b$  ,  $V(C) = \frac{4}{3}\pi b^3$

$V(C_0) = 3V(C)$

$3(\frac{4}{3}\pi a^3) = \frac{4}{3}\pi b^3$

$3a^3 = b^3$

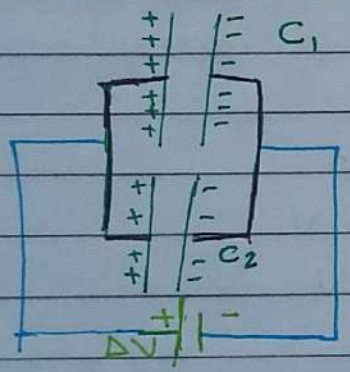
$b = \sqrt[3]{3} a$

$C = 4\pi\epsilon_0 \sqrt[3]{3} a$

$\frac{C}{C_0} = \frac{4\pi\epsilon_0 \sqrt[3]{3} a}{4\pi\epsilon_0 a} = \sqrt[3]{3}$

2. Combination of capacitors (توحيد الموصلات)

1) Parallel combination (توحيد على التوالي)



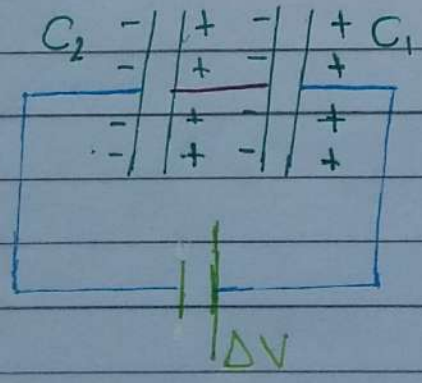
الوجوه  
الكل في  
الموصلتين  
على نفس  
القطر

A) The charge on each capacitor =  $Q_1 \neq Q_2$

B) The potential difference across each capacitor =  $V_1 = V_2$

C) Equivalent capacitance (الموصلات)  
 $C_{eq} = C_1 + C_2$

2) Series combination (توحيد على التوازي)



A)  $Q_1 = Q_2 = Q_{eq}$

B)  $\Delta V = \Delta V_1 + \Delta V_2$

C)  $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$



40

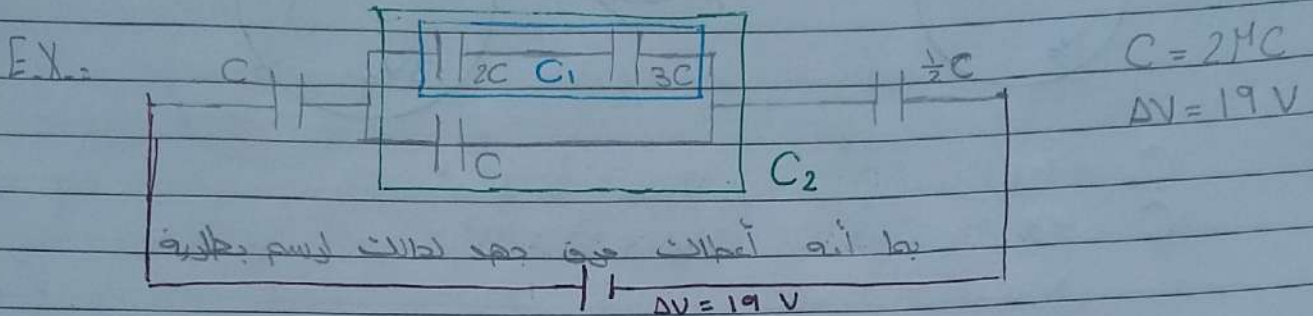
Energy stored in a capacitor (U)

$$U = \frac{1}{2} q V$$

$$= \frac{1}{2} C V^2$$

$$= \frac{1}{2} \frac{Q^2}{C}$$

$$Q = CV$$



1) The equivalent capacitance

$$\frac{1}{C_1} = \frac{1}{2C} + \frac{1}{3C} = \frac{1}{4} + \frac{1}{6} = \frac{10}{24} = \frac{5}{12}$$

$$C_1 = \frac{12}{5} \mu C$$

$$C_2 = C_1 + C = \frac{12}{5} + 2 = \frac{22}{5} \mu C$$

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C_2} + \frac{1}{\frac{1}{2}C} = \frac{1}{2} + \frac{5}{22} + \frac{1}{1} = \frac{38}{22} = \frac{19}{11}$$

$$C_{eq} = \frac{11}{19} \mu C$$

2) Find the potential difference across each capacitor.

$$Q_{eq} = C_{eq} \Delta V$$

$$= \frac{11}{19} \times 19 = 11 \mu C$$

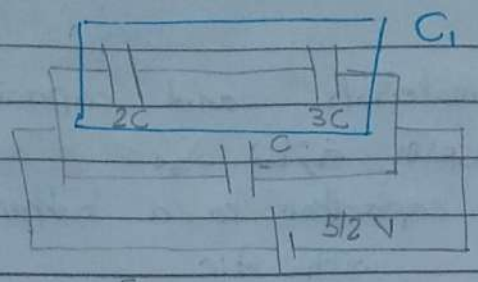
$$Q_{\frac{1}{2}C} = Q_{C_2} = Q_C = Q_{eq} = 11 \mu C$$

$$\Delta V_{\frac{1}{2}C} = \frac{Q_{\frac{1}{2}C}}{C_{\frac{1}{2}C}} = \frac{11}{1} = 11 V$$

$$\Delta V_{C_2} = \frac{Q_{C_2}}{C_{C_2}} = \frac{11}{22} \times 5 = \frac{5}{2} V$$

$$\Delta V_C = \frac{Q_C}{C_C} = \frac{11}{2} V$$

$$U_{\frac{1}{2}C} = \frac{1}{2} Q_{\frac{1}{2}C} \Delta V_{\frac{1}{2}C} = \frac{1}{2} \times 11 \times 11 = 60.5 \mu J$$



$$\Delta V_C = \Delta V_{C_1} = \frac{5}{2} \text{ V}$$

$$Q_C = C_C \Delta V_C = 2 \times \frac{5}{2} = 5 \text{ } \mu\text{C}$$

$$Q_{C_1} = C_{C_1} \Delta V_{C_1} = \frac{12}{5} \times \frac{5}{2} = 6 \text{ } \mu\text{C}$$

$$Q_{2C} = Q_{3C} = Q_{C_1} = 6 \text{ } \mu\text{C}$$

$$\Delta V_{2C} = \frac{Q_{2C}}{C_{2C}} = \frac{6}{4} = 1.5 \text{ V}$$

$$\Delta V_{3C} = \frac{Q_{3C}}{C_{3C}} = \frac{6}{6} = 1 \text{ V}$$

Look to EX 264 in the book.

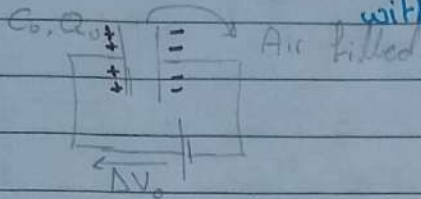
Rewiring - فتح وإغلاق المفاتيح  
 قبل = بعد  
 كفاءة الدارة

\* إذا كان هناك موازنة مشحونة و أخرى لا ويتم توصيلهم  
 يكون التوصيل على التوازي

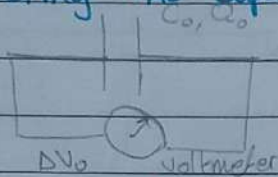
### 3. Dielectric materials and capacitance

دور كابتور عازل

1. Connecting the capacitor to a battery while is filled with Air

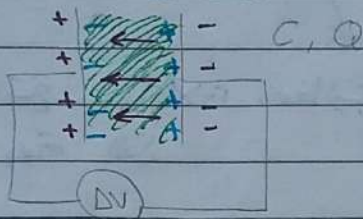


2. Disconnecting the capacitor from the battery



3. Inserting the dielectric material inside the capacitor.

إدخال العازل



1) The potential difference across the capacitor =

$$\Delta V < \Delta V_0 \quad \Delta V = \frac{\Delta V_0}{K}, \quad K = \text{dielectric constant, } (K > 1)$$

2) The charge on the capacitor =  $Q = Q_0$

3) The capacitance  $C = C_0 K$

4) The energy stored in the capacitor =

$$U_0 = \frac{1}{2} Q \Delta V_0 \quad , \quad U_f = \frac{1}{2} Q \Delta V = \frac{1}{2} Q \frac{\Delta V_0}{K} = \frac{U_0}{K}$$

The work required to insert the dielectric material inside the capacitor

$$W = \Delta U = U_f - U_0 = \left(\frac{1}{K} - 1\right) U_0$$

كلية ك

Problems

2) A)  $C = \frac{Q}{\Delta V} = \frac{10 \times 10^{-6}}{10} = 1 \text{ MF}$

B)  $1 \times 10^{-6} = \frac{100 \times 10^{-6}}{\Delta V} = 100 \text{ V}$

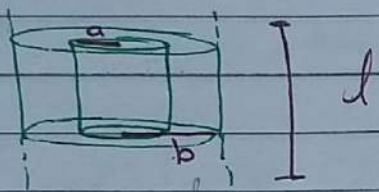
3) A)  $Q = CV = 4 \times 12 = 48 \text{ MC}$

B)  $Q = CV = 4 \times 1.5 = 6 \text{ MC}$

4) A)  $C = \frac{ab}{K \epsilon_0 (b-a)} = \frac{7 \times 10^{-2} \times 14 \times 10^{-2}}{9 \times 10^9 (14-7) \times 10^{-2}} = 15 \text{ PF}$

B)  $\Delta V = \frac{Q}{C} = \frac{4 \times 10^{-6}}{15 \times 10^{-12}} = 26 \times 10^4 \text{ V}$

5)



$l = 50 \text{ m}, Q = 8.1 \text{ MC}$

$2a = 2.58 \text{ mm}$

$2b = 7.27 \text{ mm}$

A)  $C = \frac{2 \pi \epsilon_0 \ln \frac{b}{a}}{l} = \frac{2 \times 9 \times 10^9 \times \ln \frac{7.27}{2.58}}{50} = 269 \text{ nF}$

B)  $\Delta V = \frac{Q}{C} = \frac{8.1 \times 10^{-6}}{269 \times 10^{-12}} = 3.02 \text{ kV}$

7)

سؤال عربي

$Q = CV \text{ --- (1)}, C = \frac{\epsilon_0 A}{d} \text{ --- (2)}$

$Q = \frac{\epsilon_0 A \Delta V}{d}$

charge density ( $\sigma$ ) =  $\frac{Q}{A}$

$\sigma = \frac{\epsilon_0 A \Delta V}{Ad}$

$d = \frac{\epsilon_0 \Delta V}{\sigma} = \frac{8.85 \times 10^{-12} \times 150}{30 \times 10^{-9} \times 10^4} = 44.3 \text{ Mm}$

$\sigma = 30 \times 10^{-9} \text{ nC/cm}^2$   
 $= 30 \times 10^{-9} \times 10^4 \text{ C/m}^2$

9) A)  $\Delta V = E \cdot d \Rightarrow E = \frac{\Delta V}{d} = \frac{20}{1.8 \times 10^{-3}} = 11 \times 10^3 \text{ N/C}$

B)  $E = \frac{\sigma}{\epsilon_0} \Rightarrow \sigma = E \epsilon_0 = 11 \times 10^3 \times 8.85 \times 10^{-12} = 98.3 \times 10^{-9} \text{ nC/m}^2$

C)  $C = \frac{Q}{V} = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 7.6 \times 10^{-4}}{1.8 \times 10^{-3}} = 37.4 \times 10^{-13} \text{ F}$

D)  $Q = CV = 74.7 \text{ pC}$

OR  $Q = \sigma A = 74.7 \text{ pC}$

8 (44)

11) A)  $E = \frac{k_e Q}{r^2} \Rightarrow Q = \frac{E r^2}{k_e} = \frac{0.21^2 \times 4.9 \times 10^4}{9 \times 10^9} = 2.4 \times 10^{-7} \text{ C}$   
 $\sigma = \frac{Q}{4\pi R^2} = \frac{2.4 \times 10^{-7}}{4\pi \times 0.16^2} = 7.46 \times 10^{-7} \text{ C/m}^2$

B)  $C = 4\pi \epsilon_0 R = 17.8 \text{ pF}$

21)  $C_p = 100 C_s$

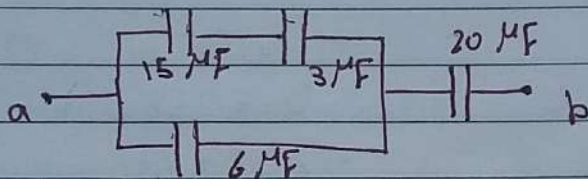
$C_p = C + C + C + \dots + C = nC$

$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \dots + \frac{1}{C} = \frac{n}{C} \Rightarrow C_s = \frac{C}{n}$

$nC = 100 \frac{C}{n}$

$n^2 = 100 \Rightarrow n = 10$

23) 50



A)  $\frac{1}{C_1} = \frac{1}{15} + \frac{1}{3} = \frac{6}{15} \Rightarrow C_1 = \frac{15}{6} = 2.5 \text{ } \mu\text{F}$

$C_2 = \frac{15}{6} + 6 = \frac{51}{6} = 8.5 \text{ } \mu\text{F}$

$\frac{1}{C_{eq}} = \frac{6}{51} + \frac{1}{20} = \frac{171}{1020} \Rightarrow C_{eq} = \frac{1020}{171} = 5.96 \text{ } \mu\text{F}$

B)  $Q = CV = 89.5 \text{ } \mu\text{C}$

$Q_{C_{eq}} = Q_{20} = Q_{C_2} = 89.5 \text{ } \mu\text{C}$

$V_{C_2} = \frac{Q_{C_2}}{C_2} = 10.5 \text{ V}$

$V_{C_2} = V_6 = V_{C_1}$

$Q_6 = C_6 V_6 = 63.16 \text{ } \mu\text{C}$

$Q_{C_1} = C_1 V_{C_1} = 26.32 \text{ } \mu\text{C}$

OR  $Q_{C_1} = Q_{C_{eq}} - Q_6 = 26.32$

$Q_{C_1} = Q_{15} = Q_3 = 26.32 \text{ } \mu\text{C}$

27)  $C_p = C_1 + C_2 = 9 \text{ pF}$

$C_s = 2 \text{ pF} \Rightarrow \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2}$

$\frac{C_1 + C_2}{C_1 C_2} = \frac{1}{2} \Rightarrow \frac{9}{C_1 C_2} = \frac{1}{2}$

$C_1 C_2 = 18 \Rightarrow C_1 (9 - C_1) = 18$

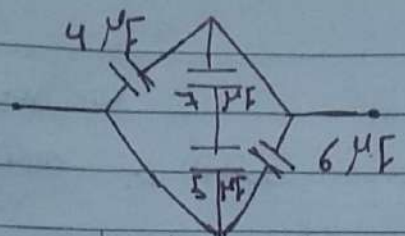
$C_1^2 - 9C_1 + 18 = 0$

$(C_1 - 3)(C_1 - 6) = 0$

$C_1 = 3 \quad C_1 = 6$

$C_2 = 6 \quad C_2 = 3$

251  
حلها



$$\frac{1}{C_1} = \frac{1}{7} + \frac{1}{5} = \frac{12}{35} \Rightarrow C_1 = \frac{35}{12}$$

$$C_{eq} = \frac{35}{12} + 4 + 5 = \frac{143}{12} = 11.9 \mu F$$

32)  
حلها

A)  $U = \frac{1}{2} CV^2 = \frac{1}{2} \times 3 \times 12^2 = 216 \text{ J}$

B)  $U = \frac{1}{2} CV^2 = \frac{1}{2} \times 3 \times 10^2 = 150$

47)  
حلها

A)  $C = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 25 \times 10^{-4}}{1.5 \times 10^{-2}} = 1475 \times 10^{-14} \text{ F}$

$Q = CV = 147.5 \times 10^{-14} \times 250 = 369 \text{ pC}$

$Q_{\text{before}} = Q_{\text{after}} = 369 \text{ pC}$

B)  $C_p = C_0 \times K = 147.5 \times 10^{-14} \times 80 = 11800 \text{ pF}$

$= 1.2 \times 10^{-10} \text{ F}$

$V_p = \frac{V_0}{K} = \frac{250}{80} = 3.125 \text{ V}$

C)  $\Delta U = (\frac{1}{K} - 1) U_0$

$= (\frac{1}{80} - 1) \times \frac{1}{2} \times 369 \times 10^{-12} \times 250 = -45.5 \text{ nJ}$

63)  
حلها  
سؤال  
ثاني

initially:  $Q = CV$

$= 10 \times 15 = 150 \text{ MC}$

then:  $V = \frac{q_1}{C_1} + \frac{q_2}{C_2}$

$50 = \frac{q}{5} + \frac{150+q}{10}$

$50 = \frac{2q}{10} + \frac{150+q}{10}$

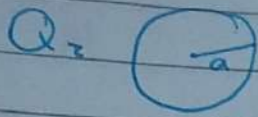
$3q = 350$

$q = 117 \text{ MC}$

in  $5 \mu F$ :  $\Delta V = \frac{q}{C} = \frac{117}{5} = 23.3 \text{ V}$

in  $10 \mu F$ :  $\Delta V = \frac{q}{C} = \frac{117+150}{10} = 26.7 \text{ V}$

46



$C = 1 \text{ pF}$

هذا سؤال في  
الجزء الثاني من الامتحان edition 11.

1) If its radius is

$C = \frac{Q}{V}$

$a = 9 \times 10^{-3} \text{ m}$

2) If  $r = 2 \text{ cm}$ , what is the capacitance?

$C = \frac{2 \times 10^{-2}}{9 \times 10^{-4}}$

$= 2.2 \text{ pF}$

3)  $\Delta V = 100$ , find charge.

$Q = CV$

$= 10 \text{ nC}$

Q2



$Q = 48 \text{ MC}$

هذا سؤال في  
الجزء الثاني من الامتحان edition 11.

$\Delta V = 12 \text{ V}$

1) Find C

$C = \frac{Q}{V}$

$= \frac{48}{12} = 4 \text{ MF}$

b) Teflon ( $K = 2.1$ )

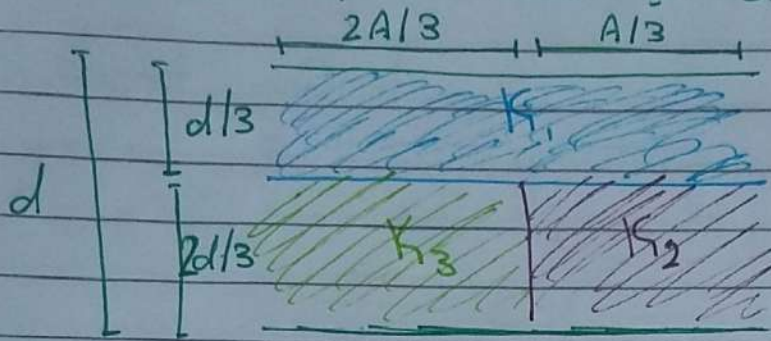
1)  $Q = Q_0 = 48 \text{ MF}$

2)  $C = K C_0 = 8.4 \text{ MF}$

3)  $\Delta V = \frac{\Delta V_0}{K} = 5.7 \text{ V}$

4)  $U_p = \frac{U_0}{K} = \frac{1}{2} \frac{Q \Delta V_0}{K} = 137.1 \text{ J}$

موجود في نسخة الكتاب القديمة E.X



خذ رسمه لمواسعة تحتوي 3 مواد dielectric  
 المواد التي حوتها بعض يتكون على التوالي  
 و التي جنب بعض على التوازي

$$C_1 = K_1 \frac{\epsilon_0 A}{d/3} = \frac{3K_1 \epsilon_0 A}{d}$$

$$C_2 = K_2 \frac{\epsilon_0 A/3}{2d/3} = \frac{K_2 \epsilon_0 A}{2d}$$

$$C_3 = K_3 \frac{\epsilon_0 2A/3}{2d/3} = \frac{K_3 \epsilon_0 A}{d}$$

$$C_{23} = C_2 + C_3 = \frac{\epsilon_0 A}{d} \left( \frac{K_2}{2} + K_3 \right)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_{23}} + \frac{1}{C_1}$$

إذا كان عندك مواسعات المشحونة و دولتها بعضها بعضها  
 (أ) على التوالي  $Q_p = V (C_2 - C_1)$   
 (ب) على التوازي  $Q_p = V (C_2 + C_1)$   
 دائماً  $Q_i \leq Q_p$



15/3/2018

Chapter 27

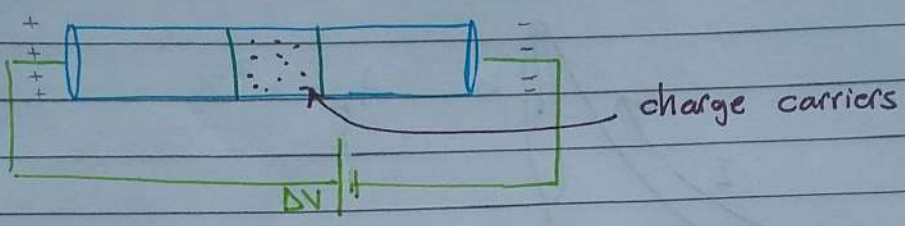
Thursday

Current and Resistance

التيار والمقاومة

1) Current definition

تعريف التيار



1) The average current

التيار المتوسط

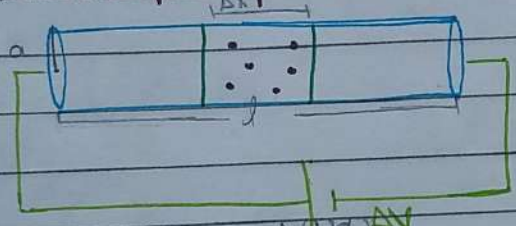
$$I_{avg} = \frac{\Delta Q}{\Delta T} = \frac{Q_f - Q_i}{T_f - T_i} = \frac{C}{s}, [I] = \text{Amper}$$

2) Instantaneous current

التيار اللحظي

$$I = \lim_{\Delta T \rightarrow 0} \frac{\Delta Q}{\Delta T} = \frac{dQ(t)}{dt}$$

2) The microscopic picture of the current



Rem:  $I = \frac{\Delta Q}{\Delta T} = \frac{\Delta(Nq_0)}{\Delta T}$  ,  $N$ : number of charge carriers  
 $q_0$ : charge of charge carriers

We define the number density =  $n$  as  
 $n = \frac{\# \text{ of charge carriers}}{\text{volume}} = \frac{N}{A \Delta x} = \frac{\text{electrons}}{m^3}$

$$I = \frac{\Delta(n A \Delta x q_0)}{\Delta T} = n A q_0 v_d$$

$v_d$  = drift velocity      السرعة الانسيابية

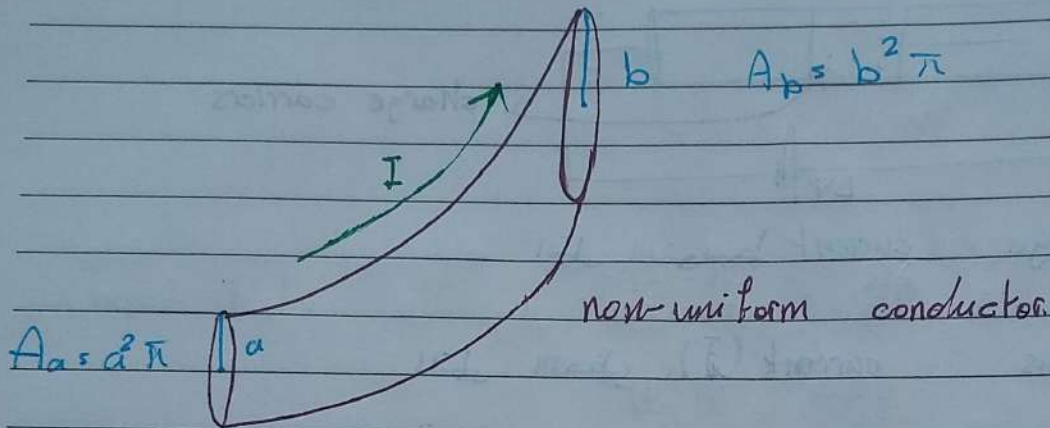
copper , الكثافة هي 27.1 الج/سم<sup>3</sup>

$$I = n A q_0 v_d$$

3) The current density ( $\vec{J}$ )

The current ( $I$ ) = scalar

current density ( $\vec{J}$ ) = vector



$$J = \frac{I}{A}$$

$$J_a = \frac{I}{a^2 \pi}, \quad J_b = \frac{I}{b^2 \pi}$$

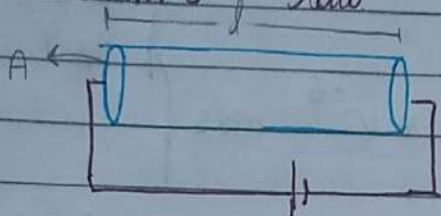
$$J = \frac{I}{A} = n q_0 v_d$$

18/3/2018

Sunday

$$\vec{J} = nq_0 \vec{v}_d$$

4) Ohm's Law



$$E = \frac{\Delta V}{l}$$

$$\vec{J} \sim \vec{E}$$

Ohm's Law

$$J = \sigma E$$

$\sigma$  = conductivity = التوصيلية

$\rho = \frac{1}{\sigma}$  = resistivity = المقاومية

$$J = \frac{E}{\rho} = \frac{\Delta V}{\rho l}$$

$$\frac{I}{A} = \frac{\Delta V}{\rho l}$$

$$\Delta V = \left( \frac{\rho l}{A} \right) I$$

$$\Delta V = R I$$

$R$  is called the resistance (المقاومة)  $(\Omega) = \frac{\rho l}{A}$

What is the power delivered in the resistance

OR What is the heat dissipated (الحرارة المتبددة) in the resistance

Answer,  $P = I^2 R$   
 $= I \Delta V$

[Watt]

5) Resistance temperature dependent

المقاومة المعتمدة على درجة الحرارة

$$R(T) = R_0 (1 + \alpha (T - T_0)) \quad \text{where}$$

$R(T)$  = resistance of temperature  $T$

$R_0$  = resistance of temperature  $T_0$  ( $T_0 = 20^\circ C$ )

$\alpha$  = thermal resistance coefficient (معامل المقاومة الحرارية)  $[^\circ C^{-1}]$

$$\rho(T) = \rho_0 (1 + \alpha (T - T_0))$$

52

# Problems

3)

Al و Cu = المعدل في السلك

$$n_{Cu} = 8.49 \times 10^{28} \text{ e/m}^3$$

a)  $v_d = ?$

$$I = n A q_0 v_d$$

$$v_d = \frac{I}{n A q_0} = 9.2 \times 10^{-5} \text{ m/s}$$

d)  $\Delta V = ?$

$$\Delta V = E l$$

b) Current density

$$J = \frac{I}{A} = \frac{5}{4 \times 10^{-6}} = 1.25 \times 10^6 \text{ A/m}^2$$

c) electric field in the wire, resistivity =  $1.7 \times 10^{-8}$

$$J = \sigma E = \frac{E}{\rho}$$

$$E = \rho J = 2.125 \times 10^{-2} \text{ V/m}$$

7)

$$I(t) = I_0 e^{-t/\beta}, \quad \beta = 70$$

a)  $I(t) = \frac{dq}{dt}$

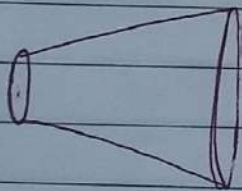
$$\int dq = \int I(t) dt$$

$$Q = \int_0^{\beta} I_0 e^{-t/\beta} dt = I_0 (-\beta) e^{-t/\beta} \Big|_0^{\beta} = I_0 \beta (1 - e^{-1})$$

b)  $Q = I_0 \beta (1 - e^{-10})$

c)  $Q = \lim_{x \rightarrow \infty} (I_0 \beta (1 - e^{-x})) = I_0 \beta$

8)



a)  $J_1 = \frac{I}{A_1} = \frac{5}{\pi \times 16 \times 10^{-6}} = 9.8 \times 10^4 \text{ A/m}^2$

b) the same

c) smaller

d)  $J_2 = \frac{J_1}{4}$

$$= \frac{5}{4\pi \times 16 \times 10^{-6}} = 24.5 \times 10^4 \text{ A/m}^2$$

9)  $q(t) = 4t^3 + 5t + 6$

a)  $I = \dot{q}(t) = 12t^2 + 5$

$$I(t) = 17 \text{ A}$$

b)  $\vec{J} = \frac{I}{A} = \frac{17}{2 \times 10^{-3}} = 8.5 \times 10^3 \text{ A/m}^2$

15)  $\Delta V = IR$   
 $\Delta V = I \frac{\rho l}{A} \Rightarrow \rho = \frac{\Delta V A}{I l} = 1.589 \times 10^{-8}$   
 the wire is made from silver

16)  $I = \frac{A \Delta V}{\rho l} = \frac{0.6 \times 10^{-6} \times 0.9}{5.6 \times 10^{-8} \times 15} = 64 A$

19) سؤال فيزياء عن السلك الموصل و...  
 السلك

23)  $J = \sigma E$   
 $\sigma = \frac{J}{E} = 6 \times 10^{-15} \frac{1}{m}$

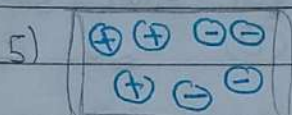
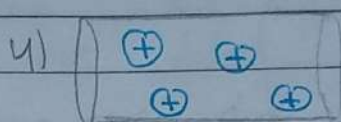
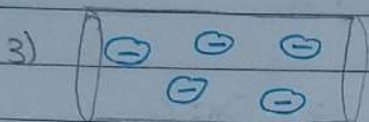
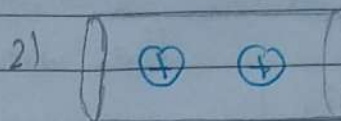
26)  $R(T) = R_0 (1 + \alpha (T - T_0))$   
 $T = \frac{\frac{R(T)}{R} - 1}{\alpha} + T_0 = 1435 ^\circ C$

31) a)  $I = \frac{A \Delta V}{\rho l} = 30 A$   
 b)  $R \text{ at } 20^\circ C = 1.7 \times 10^{-8} \Omega \cdot m$   
 $R(30) = 1.7 \times 10^{-8} (1 + \alpha (10))$   
 $I = \frac{\Delta V}{R}$

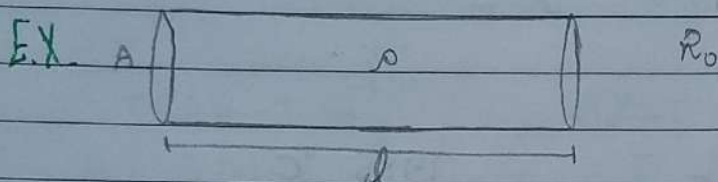
39) a)  $P = I \Delta V$   
 $I = \frac{P}{\Delta V} = \frac{1000}{120} = 8.3 A$   
 b)  $P = I^2 R$   
 $R = \frac{P}{I^2} = \frac{1000}{(8.3)^2} = 14.5 \Omega$

Q2:  $P = 600 W$ ,  $\Delta V = 120 V$ , find  $I$   
 sol:  $I = \frac{P}{\Delta V} = 5 A$

Ex. Rank (رتب) the current from the largest to least.



$$I_5 > I_3 > I_1 = I_4 > I_2$$



تم تقص هذا السلك الى n اسلاك بنفس الطول و معزم بطول  
 Find  $\frac{R_n}{R_0}$  بميزم

$$R_0 = \frac{\rho l}{A}$$

$$R_n = \frac{\rho \cdot n l}{n A}$$



$$\frac{R_0}{R_n} = \frac{1}{n^2}$$

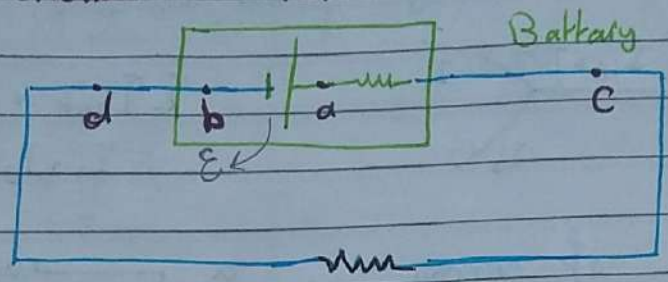
بتقدر تظ السؤال على انه  
 توصيل توازي

22/3/2018

# Chapter 28

Thursday

## [1] electromotive force ( $\mathcal{E}_{mf}$ ) القوة الدافعة الكهربية



$R =$  load resistance

مقاومة الحمل

$$\mathcal{E} = \mathcal{E}_{mf}$$

$$\Delta V_{cd} = \mathcal{E} - I r = I R$$

$\Delta V_{cd} =$  terminal voltage

$r =$  internal resistance of the battery

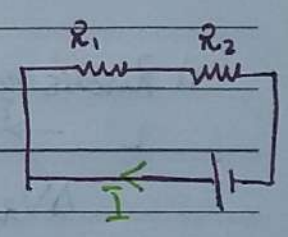
$$I = \frac{\mathcal{E}}{r + R}$$

## [2] Combination of resistance

### 1. the series combination

a) the current across each resistance  $I = I_1 = I_2$

b) potential difference across each resistance  $\mathcal{E} = \Delta V_1 + \Delta V_2$



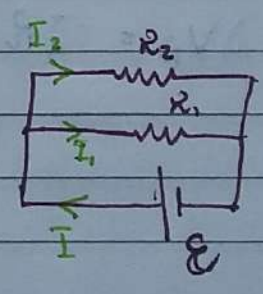
c) equivalent resistance  $R_{eq} = R_1 + R_2$

### 2. the parallel combination

a)  $I = I_1 + I_2$

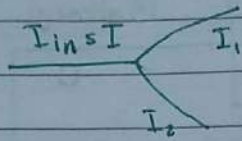
b)  $\Delta V_1 = \Delta V_2 = \mathcal{E}$

c)  $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$



[3] Kirchoff's Rules

1) Conservation of charge (حفظ الشحنة)

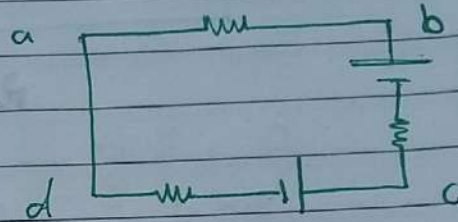


$$\sum I_{out} = \sum I_{in}$$

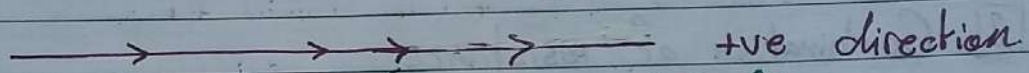
$$I_1 + I_2 = I$$

2) Conservation of energy

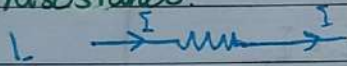
$$\sum_{\text{closed loop}} \Delta V_{abcda} = 0$$



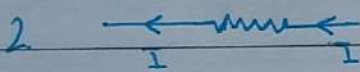
Kirchoff convention في حل الدوائر الكهربائية



1) Resistance

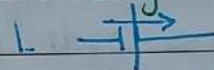


$$\Delta V_R = -IR$$

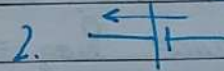


$$\Delta V_R = IR$$

2) Battery (EMF)



$$\Delta V = +\mathcal{E}$$



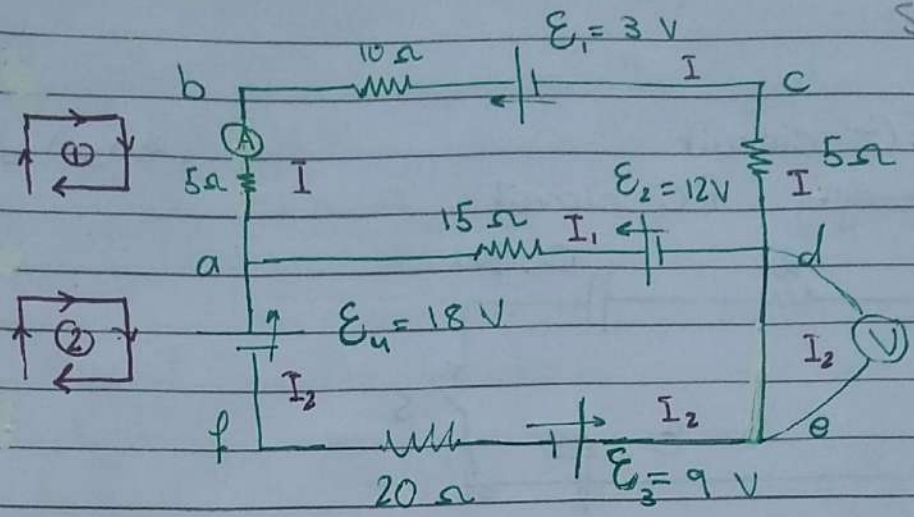
$$\Delta V = -\mathcal{E}$$



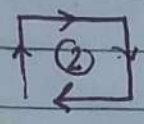
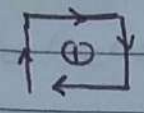
25/3/2018

Sunday

EX



أنت اخذنا  
انتظام الحركة  
الطوبى



$$I = I_1 + I_2 \quad \text{--- (1)}$$

بالتى من اليمين

$$\sum \Delta V_{abceda} = 0$$

$$-5I + (-10I) + (-3) + (-5I) + 12 + (-15I_1) = 0$$

$$-20I - 15I_1 + 9 = 0 \quad \text{--- (2)}$$

$$\sum \Delta V_{fadeff} = 0$$

$$18 + 15I_1 + (-12) + (-9) + (-20I_2) = 0$$

$$15I_1 - 20I_2 - 3 = 0 \quad \text{--- (3)}$$

① into ②

$$-35I_1 - 20I_2 + 9 = 0 \quad \text{--- (4)}$$

③ into ④

$$-35 \left( \frac{1}{5} + \frac{4}{3} I_2 \right) - 20I_2 + 9 = 0 \Rightarrow I_2 = \frac{3}{100} \text{ A}$$

$$\text{(4)} \Rightarrow I_1 = \frac{9}{35} - \frac{20}{35} \times \frac{3}{100} = \frac{24}{100} = \frac{6}{25} \text{ A}$$

$$\text{(1)} \Rightarrow I = I_1 + I_2 = \frac{27}{100} \text{ A}$$

Find  $V_d - V_e$

sol.  $V_e + \text{-----} = V_d$

$$V_e + (-9) + (-20 \times \frac{3}{100}) + 18 + (15 \times \frac{24}{100}) - 12 = V_d$$

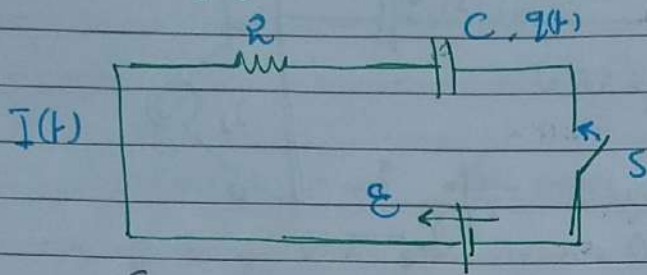
$$V_e + 3 - 3 = V_d \Rightarrow V_e - V_d = \text{Zero}$$

H.W Find  $V_d - V_b$

## RC - circuit

زمن خزان الطارء و العوامة

## a) charging RC-circuit



$$\begin{aligned}\mathcal{E} &= \Delta V_R + \Delta V_C \\ &= R I(t) + \frac{q(t)}{C}\end{aligned}$$

$$\text{but } I(t) = \frac{dq(t)}{dt}$$

$$\begin{aligned}\mathcal{E} &= R \frac{dq}{dt} + \frac{q(t)}{C} \\ \frac{dq}{dt} + \frac{q(t)}{RC} &= \frac{\mathcal{E}}{R}\end{aligned}$$

we define a time constant or characteristic time  $\tau$ ,  $\tau = RC$  [sec]

$$\frac{dq}{dt} + \frac{q(t)}{\tau} = \frac{\mathcal{E}}{R}$$

$$\Rightarrow q(t) = Q_{\max} (1 - e^{-t/\tau})$$

$$Q_{\max} = \mathcal{E}C, \text{ at time} = \infty$$

$$q(t) = \mathcal{E}C (1 - e^{-t/RC})$$

The current in the circuit:

$$\begin{aligned}I(t) &= \frac{dq}{dt} \\ &= \frac{\mathcal{E}}{R} e^{-t/\tau} = I_{\max} e^{-t/\tau}\end{aligned}$$

$$I_{\max} = \frac{\mathcal{E}}{R}, \text{ when } t=0.$$

1) the potential difference across the capacitor

$$V_C(t) = \frac{q(t)}{C} = \mathcal{E} (1 - e^{-t/\tau})$$

2) the potential difference across the resistance

$$V_R(t) = R I(t) = \mathcal{E} e^{-t/\tau}$$

3) the energy stored in a capacitor

$$\begin{aligned}
 U(t) &= \frac{1}{2} q_c(t) V_c(t) \\
 &= \frac{1}{2} C \mathcal{E}^2 (1 - e^{-t/\tau})^2 \\
 &= \frac{1}{2} U_{\max} (1 - e^{-t/\tau})^2
 \end{aligned}$$

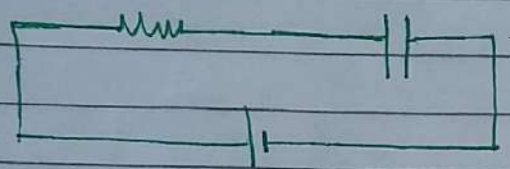
$U_{\max}$  when  $t = \infty$

4) the power delivered in the resistance

$$\begin{aligned}
 P(t) &= R I^2(t) \\
 &= \frac{\mathcal{E}^2}{R} e^{-2t/\tau} = P_{\max} e^{-2t/\tau}
 \end{aligned}$$

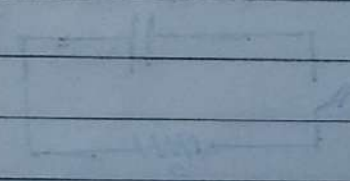
$P_{\max}$  when  $t = 0$

E.X the figure shows an RC-circuit the  $R = 2 \text{ M}\Omega$  and  $C = 4 \text{ nF}$  and  $\mathcal{E} = 12 \text{ V}$ , initially the capacitor is uncharged



- 1) what is the time constant?
- 2) what is the maximum charge in capacitor?
- 3) what is the maximum current?
- 4) what is the charge on capacitor when  $T = 2\tau = 16 \text{ ms}$ .

- sol.
- 1)  $\tau = RC = 2 \times 10^6 \times 4 \times 10^{-9} = 8 \text{ ms}$
  - 2)  $Q_{\max} = \mathcal{E}C = 12 \times 4 = 48 \text{ nC}$
  - 3)  $I_{\max} = \frac{\mathcal{E}}{R} = \frac{12}{2 \times 10^6} = 6 \text{ }\mu\text{A}$
  - 4)  $q(t) = Q_{\max} (1 - e^{-t/\tau})$   
 $= 48 (1 - e^{-2}) \text{ nC}$



29/3/2018

Thursday

charging RC-circuit.

⇒ 1. initially uncharged capacitor.

2.  $R, C, \mathcal{E}$  are constant.

$$q = Q_{\max} (1 - e^{-t/\tau})$$

5) Find the time at which the energy stored in the capacitor is 25% of its maximum value

$$\text{sol. } U(t) = U_{\max} (1 - e^{-t/\tau})^2$$

$$\frac{U(t)}{U_{\max}} = (1 - e^{-t/\tau})^2 = 0.25$$

$$1 - e^{-t/\tau} = \pm 0.5$$

$$e^{-t/\tau} = 1 \mp 0.5$$

$$e^{-t/\tau} = 0.5, 1.5$$

$$\frac{-t}{\tau} = \ln 0.5, \ln 1.5$$

$$t = -\tau \ln 0.5 = 0.69 \tau$$

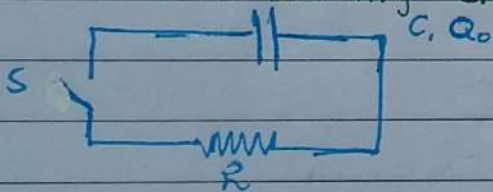
6) Find the time at which the charge on the capacitor is 0.3 of its maximum value

$$\text{sol. } \frac{q(t)}{Q_{\max}} = 1 - e^{-t/\tau} = 0.3$$

$$e^{-t/\tau} = 0.7$$

$$t = -\tau \ln 0.7$$

b) Discharging the RC-circuit

⇒ 1) initially charged capacitor. 2)  $C, R$  are constant.

$$\Delta V_C + \Delta V_R = 0$$

$$\frac{q(t)}{C} + R I(t) = 0$$

$$\frac{dq}{dt} + \frac{1}{RC} q(t) = 0$$

$$q(t) = Q_0 e^{-t/\tau}$$

$$I(t) = \frac{-Q_0}{RC} e^{-t/\tau} = I_{\max} e^{-t/\tau}$$

$$I_{\max} = \frac{-Q_0}{RC}, \text{ when } t = 0$$

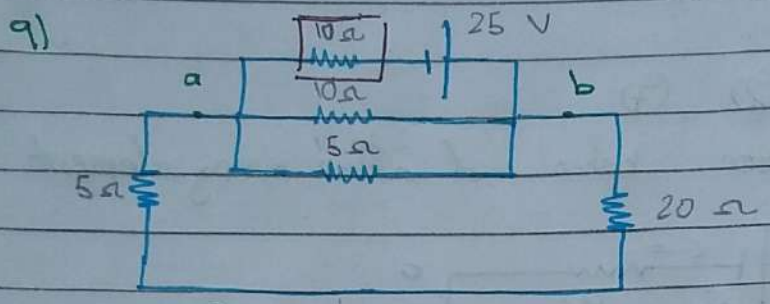
$$U(t) = \frac{q(t)^2}{2C} = \frac{Q_0^2}{2C} e^{-2t/\tau}$$

$$= U_{\max} e^{-2t/\tau}$$

Problems

1) a)  $P_R = \frac{\Delta V^2}{R} \Rightarrow R = \frac{\Delta V^2}{P} = \frac{11.6^2}{20} = 6.728 \Omega$

b)  $\mathcal{E} = I_r + I_r$   
 $P = I^2 R \Rightarrow I = \sqrt{\frac{P}{R}} = 1.72 \text{ A}$   
 $r = \frac{\mathcal{E} - IR}{I} = \frac{15 - 11.6}{1.72} = 1.982 \Omega$



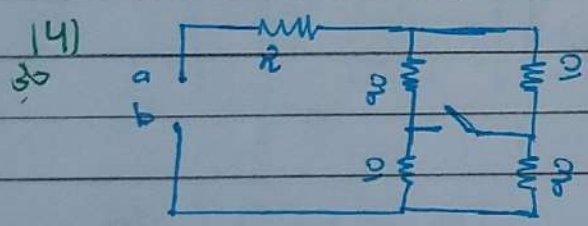
a)  $R_1 = 20 + 5 = 25 \Omega$   
 $\frac{1}{R_2} = \frac{1}{10} + \frac{1}{5} + \frac{1}{25} = \frac{17}{50} \Rightarrow R_2 = \frac{50}{17} \Omega$   
 $R_{eq} = \frac{50}{17} + 10 = \frac{220}{17} \Omega$   
 $I = \frac{\Delta V}{R} = 25 \times \frac{17}{220} = \frac{85}{44} = 1.9318 \text{ A}$

$\Delta V_{10} = I_{10} \times R_{10} = 19.318 \text{ V}$

$\Delta V_{R_2} = 25 - 19.318 = 5.682 \text{ V}$

$I_{25} = I_{20} = I_5 = \frac{\Delta V_{R_2}}{25} = \frac{5.682}{25} = 0.22 \text{ A} = 220 \text{ mA}$

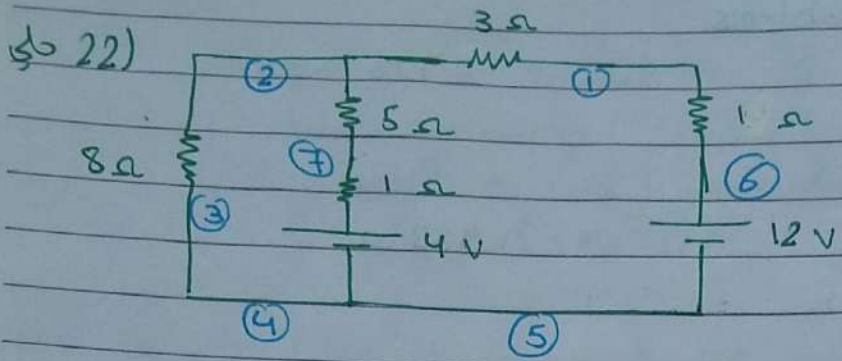
b)  $\Delta V_{ab} = \Delta V_{25} = \Delta V_{R_2} = 5.682 \text{ V}$



a) before closing the key  
 $R_1 = 10 + 90 = 100 \Omega, R_2 = 10 + 90 = 100 \Omega$   
 $\frac{1}{R_3} = \frac{1}{100} + \frac{1}{100} = \frac{2}{100} \Rightarrow R_3 = \frac{100}{2} = 50 \Omega$   
 $R_{eq} = 50 + R$

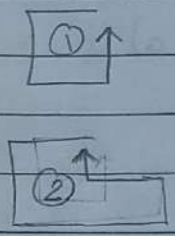
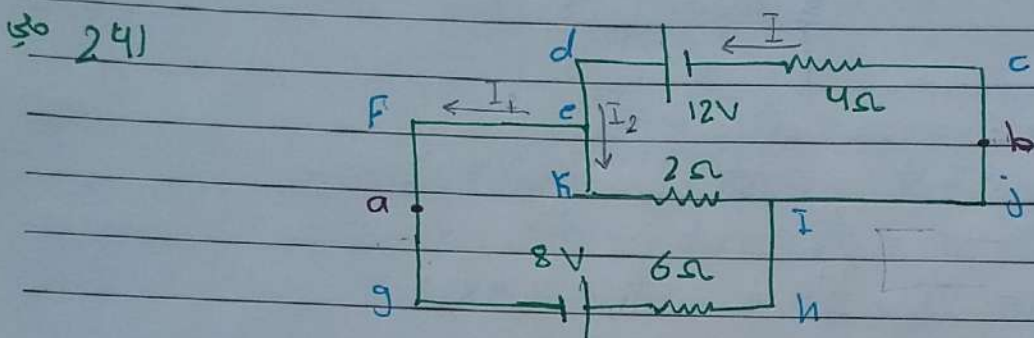
after closing key  
 $\frac{1}{R_1} = \frac{1}{10} + \frac{1}{90} = \frac{9}{90} \Rightarrow R_1 = R_2 = 10 \Omega$   
 $R_{eq} = R + R_1 + R_2 = 20 + R$   
 = decrease

b)  $20 + R = \frac{1}{2} (50 + R) \Rightarrow \frac{1}{2} R = 5 \Rightarrow R = 10 \Omega$



التيار في أرقام الأسلاك

Ammeter in (5), (2), (7)  
 Voltmeter in all number before and after every element



a)  $I = I_1 + I_2$  — (1)  
 $\sum \Delta V_{cdkjc} = 0$

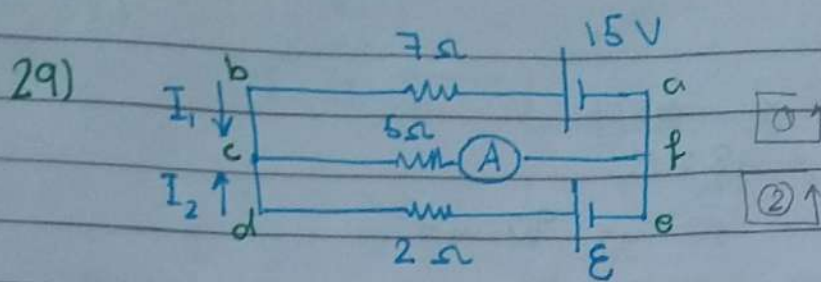
$-4I + 12 - 2I_2 = 0$  — (2)

$\sum \Delta V_{efghike} = 0$   
 $8 - 6I_1 + 2I_2 = 0$  — (3)

(1) into (2)  
 $-4I_1 - 6I_2 + 12 = 0$  — (4)

$-2 \times (3) + 3 \times (4)$   
 $-16 + 12I_1 - 4I_2 + 12I_1 - 18I_2 + 36 = 0$   
 $22I_2 = 20 \Rightarrow I_2 = 0.909$

b)  $V_b + \dots = V_a$   
 $V_b + 2I_2 = V_a$   
 $V_b - V_a = 2I_2 = -1.82$



$$I = I_1 + I_2 = 2 \Rightarrow I_2 = 2 - I_1 \quad \text{--- (1)}$$

$$\sum \Delta V_{abcfa} = 0$$

$$15 - 7I_1 - 5I_2 = 0$$

$$15 - 7I_1 - 10 = 0 \Rightarrow I_1 = \frac{5}{7} \text{ A}$$

$$\frac{5}{7} + I_2 = 2 \Rightarrow I_2 = \frac{9}{7} \text{ A}$$

$$\sum \Delta V_{fcdcf} = 0$$

$$5I_1 + 2I_2 - \mathcal{E} = 0$$

$$10 + \frac{18}{7} = \mathcal{E} \Rightarrow \mathcal{E} = \frac{88}{7} \text{ V}$$

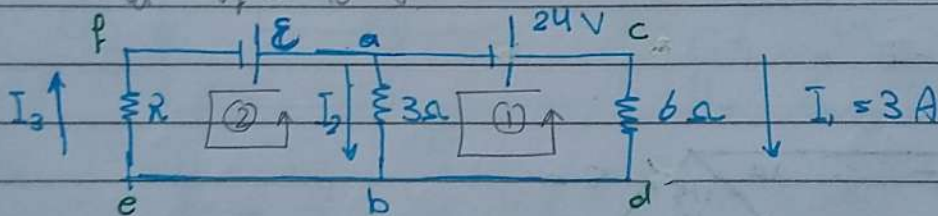
$$V_d - V_f = ??$$

فدع زياد من الكوندر

$$V_d + 2 \times \frac{9}{7} - \frac{88}{7} = V_f$$

$$V_d - V_f = 10 \text{ V}$$

So 32)



a)  $\sum \Delta V_{cabdc} = 0$

$$-24 - 3I_2 + 3 \times 6 = 0 \Rightarrow I_2 = -2 \text{ A}$$

b)  $I_3 = I_2 + I_1$

$$= -2 + 3 = 1 \text{ A}$$

c) No, we can't

37) a)  $T = RC = 2 \text{ ms}$

b)  $Q_{\text{max}} = EC = 180 \mu\text{C}$

c)  $I_{\text{max}} = \frac{E}{R} = \frac{9}{100} = 90 \text{ mA}$

d)  $q(t) = Q_{\text{max}} (1 - e^{-t/\tau})$   
 $t = \tau = 180(1 - e^{-1}) \mu\text{C}$

e) Find the charge on capacitor after long time.  
 $Q_{\text{max}} = 180 \mu\text{C}$

39) a)  $I(t) = \frac{Q_0}{RC} e^{-t/RC}$   
 $= \frac{5.1}{2.6} e^{-t/2.6}$

b)  $Q(t) = Q_0 e^{-t/\tau}$   
 $= 5.1 e^{-8126 t}$

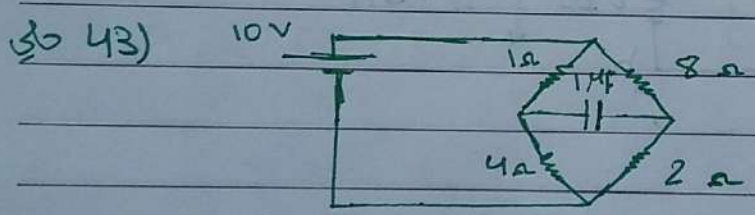
c)  $I_{\text{max}} = \frac{Q_0}{RC} = \frac{5.1}{2.6}$

d) Find the time at which the energy stored in the capacitor is 10% of its maximum value.

$U(t) = U_{\text{max}} e^{-2t/\tau}$   
 $\frac{1}{10} = e^{-t/\tau}$

$-2t/\tau = \ln 0.1$

$t = -\frac{\tau}{2} \ln 0.1$



a)  $R_1 = 1 + 4 = 5 \Omega \Rightarrow I_1 = \frac{\Delta V}{R_1} = \frac{10}{5} = 2 \text{ A} \Rightarrow V_1 = 10 - 1 \times 2 = 8 \text{ V}$

$R_2 = 8 + 2 = 10 \Omega \Rightarrow I_2 = \frac{\Delta V}{R_2} = \frac{10}{10} = 1 \text{ A} \Rightarrow V_2 = 10 - 8 \times 1 = 2 \text{ V}$

$\Delta V = V_1 - V_2 = 6 \text{ V}$

b)  $R_1 = 4 + 2 = 6 \Omega$

$R_2 = 1 + 8 = 9 \Omega$

$\frac{1}{R_{\text{eq}}} = \frac{1}{6} + \frac{1}{9} = \frac{15}{54} \Rightarrow R_{\text{eq}} = \frac{54}{15} = 3.6 \Omega$

$q(t) = Q_0 e^{-t/RC} \Rightarrow \frac{q(t)}{Q_0} = e^{-t/RC}$

$0.1 = e^{-t/3.6}$

$t = -3.6 \ln 0.1 = 8.29 \mu\text{s}$

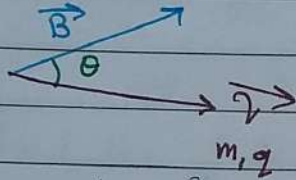


Note:

1. The electric affects both stationary or moving charge particle.
2. The magnetic field ( $\vec{B}$ ) [Tesla] affect only moving particles.

القوة الكهربائية تؤثر على الجسيمات الساكنة والحركة = المجال المغناطيسي يؤثر على الجسيمات المتحركة فقط

[1] The magnetic force on a moving charged particle.



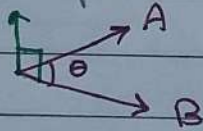
The magnetic force on the charge particle is

$$\vec{F}_B = q \vec{v} \times \vec{B}$$

$$|\vec{F}_B| = q v B \sin \theta$$

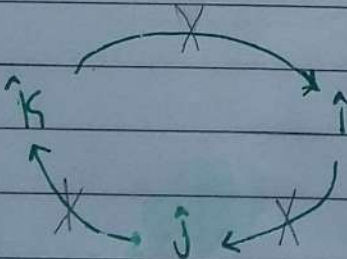
↳ cross product

cross product



$$1) \vec{C} = \vec{A} \times \vec{B}$$

$$2) \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$



$$\begin{cases} \hat{i} \times \hat{j} = \hat{k} \\ \hat{j} \times \hat{k} = \hat{i} \\ \hat{k} \times \hat{i} = \hat{j} \end{cases} \quad \begin{cases} \hat{j} \times \hat{i} = -\hat{k} \\ \hat{k} \times \hat{j} = -\hat{i} \\ \hat{i} \times \hat{k} = -\hat{j} \end{cases}$$

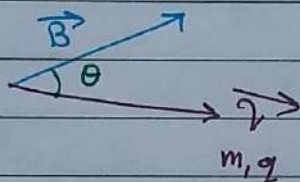
The Magnetic Field ( $\vec{B}$ )

Note:

1. The electric affects both stationary ( $v=0$ ) or moving charge particle.
2. The magnetic field ( $\vec{B}$ ) [Tesla] affect only moving particles.

القوة الكهربائية تؤثر على الجسيمات الساكنة والحركة = القوة المغناطيسية تؤثر على الجسيمات المتحركة فقط

[1] The magnetic force on a moving charged particle.



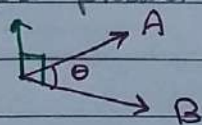
The magnetic force on the charge particle is

$$\vec{F}_B = q \vec{v} \times \vec{B}$$

└── cross product

$$|\vec{F}_B| = q v B \sin \theta$$

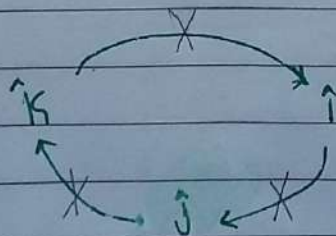
cross product



$$1) \vec{C} = \vec{A} \times \vec{B}$$

$$2) \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

تكون القوة المغناطيسية



$$\begin{aligned} \hat{i} \times \hat{j} &= \hat{k} & \hat{j} \times \hat{i} &= -\hat{k} \\ \hat{j} \times \hat{k} &= \hat{i} & \hat{k} \times \hat{j} &= -\hat{i} \\ \hat{k} \times \hat{i} &= \hat{j} & \hat{i} \times \hat{k} &= -\hat{j} \end{aligned}$$

EX = Find the magnetic force on a proton with  $\vec{v} = 3\hat{i} - \hat{k}$  in a magnetic field  $\vec{B} = 2\hat{k} + 5\hat{j}$  (T)

sol.  $\vec{v} = 3\hat{i} + 0\hat{j} - \hat{k}$   
 $\vec{B} = 0\hat{i} + 5\hat{j} + 2\hat{k}$

$\vec{F} = q \vec{v} \times \vec{B}$

= q	$\hat{i}$	$\hat{j}$	$\hat{k}$	} المصفوفة
	3	0	-1	
	0	5	2	

=  $q(10 \times 2 - (-1 \times 5))\hat{i} - (3 \times 2 - 0 \times 0)\hat{j} + (3 \times 5 - 0 \times (-1))\hat{k}$   
=  $q(5\hat{i} - 6\hat{j} + 15\hat{k})$  N

طريقة الميزان المصفوفة

الرتب طابقت المصفوفة الى  $\hat{i}, \hat{j}, \hat{k}$   
تحتسب كل ما  $\hat{i}, \hat{j}, \hat{k}$  كل وحدة (لحالات) وبالطريقة

- 1 2

- 3 4

الميزان  $4 \times 1$  و الميزان  $2 \times 3$

موضا كل وحدة مكانها  $(\hat{k}) + (\hat{j}) - (\hat{i})$   
حرفه فوق

5/4/2018

Thursday

### Right hand rule for positive charge.

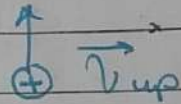
convention : 1)  $\odot$  : out of the page

2)  $\otimes$  : into the page.

اليد اليمنى للأطراف المتحركة  
في اتجاه المجال المغناطيسي  
تكون القوة المتحركة المتحركة

$E \cdot X_1 = +ve$

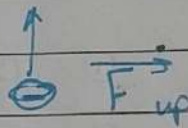
$\Rightarrow \vec{F} = \text{left}$



$\vec{B}_{in}$

$E \cdot X_2 = -ve$

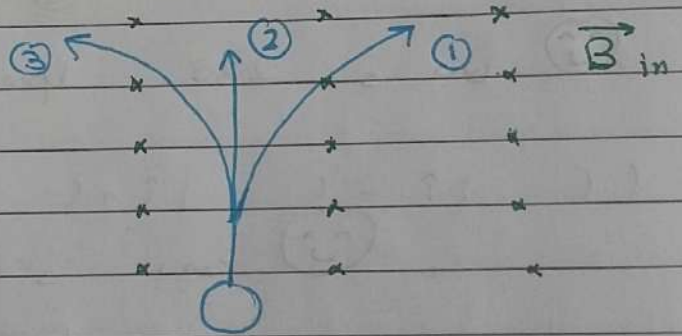
$\Rightarrow \vec{B} = \text{left}$



$\vec{V}_{out}$

أنت أو أي شيء متحرك

$E \cdot X_3 =$  ...

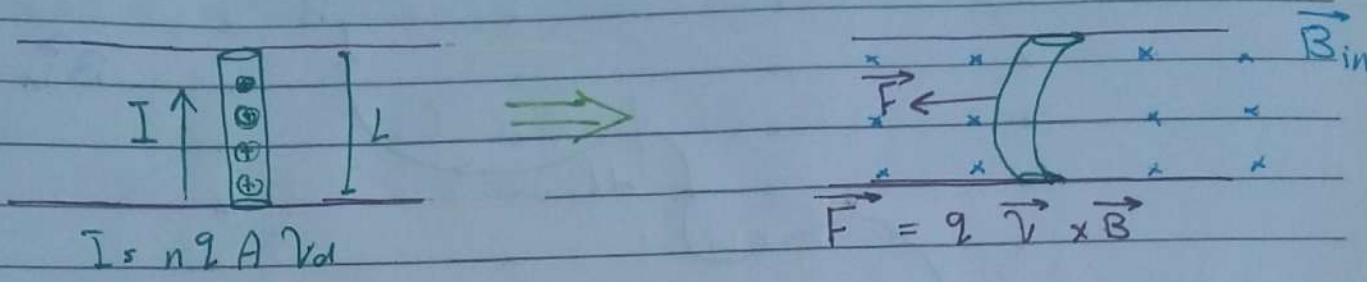


1 = electron (-ve)

2 = n,  $\alpha$  (نوترون و ألفا)

3 = proton (+ve)

[2] The magnetic force on a current-carrying conductor  
القوة المغناطيسية على السلك الحامل للتيار

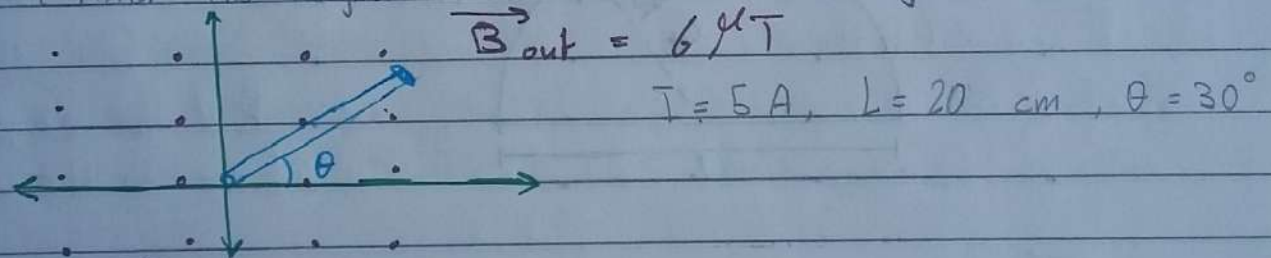


The magnetic force on a charge  
 $\vec{F}_q = q \vec{v} \times \vec{B}$

The magnetic force on a wire  
 $\vec{F}_I = q (\vec{v} \times \vec{B}) n A L$   
 $= (nq v_d A) \vec{L} \times \vec{B}$   
 $= I \vec{L} \times \vec{B}$

$|\vec{F}_I| = I L B \sin \theta_{L,B}$

EX. Find the magnetic force on the following wire

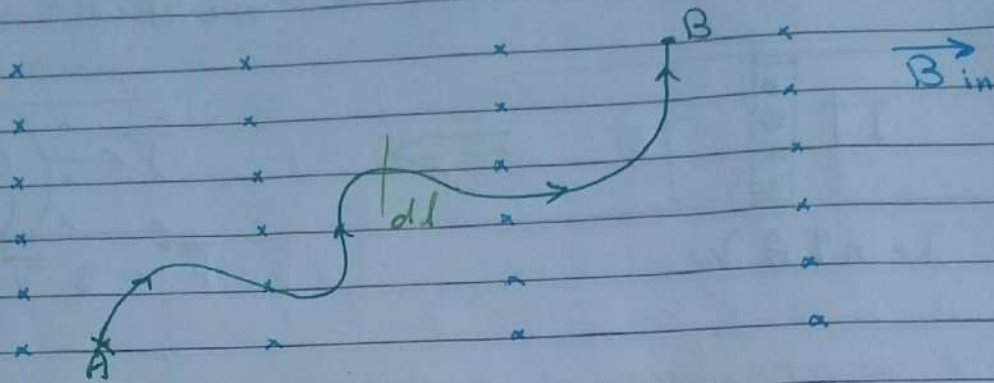


sol.  $\vec{L} = 20 \cos \theta \hat{i} + 20 \sin \theta \hat{j}$  cm

$\vec{F} = I \vec{L} \times \vec{B}$   
 $= 5 \left( \frac{20}{100} * \frac{\sqrt{3}}{2} \hat{i} + \frac{20}{100} * \frac{1}{2} \hat{j} \right) \times 6 \hat{k}$   
 $= 5 * \frac{20}{100} * \frac{1}{2} (\sqrt{3} \hat{i} + 1 \hat{j}) \times 6 \hat{k}$   
 $= 5 * \frac{20}{100} * \frac{1}{2} * 6 (-\sqrt{3} \hat{j} + 1 \hat{i}) \text{ MN}$   
 $\vec{F}_L = 3 (\hat{i} - \sqrt{3} \hat{j}) \text{ MN}$

70

A magnetic force on a non-uniform wire.



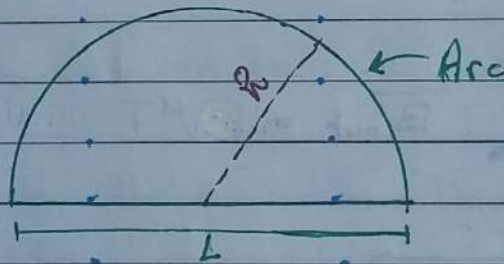
$$\vec{F}_L = I \int_A^B d\vec{l} \times \vec{B}$$

$$F_L = I \int_A^B B \sin\theta \, dl$$

closed loop

$$F_L \text{ closed loop} = \text{Zero}$$

EX =



$$I = 2 \text{ mA}$$

$$B = 50 \text{ MT}$$

$$L = 50 \text{ cm}$$

$$R = 2L$$

Find the magnetic force on the Arc

sol.  $\vec{F}_{\text{closed loop}} = 0$

$$\vec{F}_L + \vec{F}_{\text{Arc}} = 0$$

$$\vec{F}_{\text{Arc}} = -\vec{F}_L$$

$$|\vec{F}_L| = I L B \sin\theta$$

$$\vec{F}_L = I L B \sin\theta$$

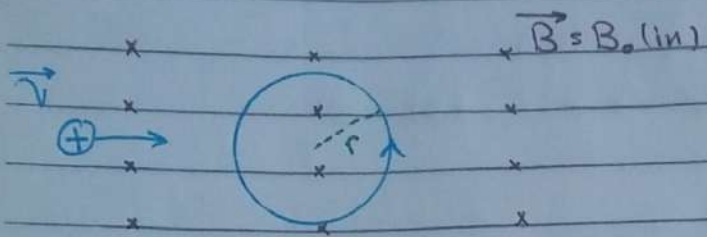
$$= 2 \times 10^{-3} \times 50 \times 10^{-2} \times 50 \times 10^{-6} \times \sin 90 = 5 \times 10^{-8} \text{ N (down)}$$

$$\vec{F}_{\text{Arc}} = 5 \times 10^{-8} \text{ N (up)}$$

12/4/2018

### [4] Motion of a charge particles in magnetic field circular path ( $\vec{B} \perp \vec{v}$ )

إذا الزاوية 90  $\leftarrow$  حركة دائرية ، إذا آلي أو أقل من 90  $\leftarrow$  حركة (helical) (دائرية)



1)  $F_{\text{mag}} = F_{\text{central}}$

$$q v B = m a_c$$

$$q v B = m \frac{v^2}{r}$$

$$r = \frac{m v}{q B} = \text{radius of the circular path (revolution) (دوران)}$$

2) periodic time (الوقت الدوري)

$$v = \frac{2\pi r}{T}$$

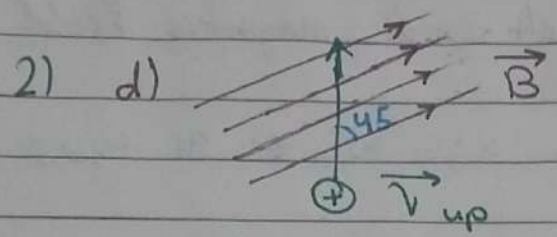
$$T = \frac{2\pi r}{v} = 2\pi r \frac{m}{q B} \quad \text{صفا}$$

3) Angular speed (velocity) (السرعة الزاوية) ( $\omega$ )

Angular Frequency (التردد الزاوي)

$$\omega = \frac{2\pi}{T} = \frac{q B}{m} \quad \text{صفا}$$

# Problems



انحراف - deflection

sol.  $B = B_0 \cos 45 \hat{i} + B_0 \sin 45 \hat{j}$

السرعة تكون في اتجاه  $\hat{j}$  لأن يكون لها تأثير  
 $\Rightarrow \vec{F} = qv \hat{j} \times B$

3) b)  $\vec{v} = v \hat{z}$  out,  $\vec{F}_c = F \hat{z}$  up  
 $\Rightarrow \vec{B} = B \hat{x}$  right

6)  $F = qvB \sin \theta$   
 $\theta = \sin^{-1} \frac{F}{qvB} = \sin^{-1} \left( \frac{8.2 \times 10^{-13}}{1.6 \times 10^{-19} \times 4 \times 10^6 \times 1.7} \right) = 49$

Find the central acceleration. (من عند الدائرة)

$F = ma_c$   
 $a_c = \frac{F}{m} = \frac{8.2 \times 10^{-13}}{1.67 \times 10^{-27}} = 4.9 \times 10^{10}$

7) a)  $F_{min} = 0$  ; when  $\theta = 0, 180$   
 b)  $F_{max} = qvB$  ; when  $\theta = \pi, \frac{3\pi}{2}$

$\Delta U + \Delta K = 0$   
 $v = \sqrt{\frac{2q \Delta V}{m_e}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 2400}{9.11 \times 10^{-31}}} = 2.9 \times 10^4$

$F_{max} = qvB$   
 $= 1.6 \times 10^{-19} \times 2.9 \times 10^4 \times 1.7 = 7.9 \times 10^{-15} \text{ N}$

8)  $\vec{F} = q \vec{v} \times \vec{B}$

$= q$	$\hat{i}$	$\hat{j}$	$\hat{k}$
	2	-4	1
	1	2	-1

$\vec{F} = e(2\hat{i} + 3\hat{j} + 8\hat{k})$   
 $|\vec{F}| = e\sqrt{4+9+64}$

أو البروتين نفس الرقم  
 بينه للـ  $\hat{i}, \hat{j}, \hat{k}$  اتجاه الانتظامات  
 (  $\hat{k}, \hat{j}, \hat{i}$  )



9) a)  $F = qvB \sin\theta$   
 $= 1.6 \times 10^{-19} \times 5.02 \times 10^6 \times 0.18 \times \sin 60 = 1.252 \times 10^{-13} \text{ N}$

b)  $F = ma$   
 $a = \frac{F}{m} = \frac{1.252 \times 10^{-13}}{1.67 \times 10^{-27}} = 7.4 \times 10^{13} \text{ m/s}^2$

11)  $F = qvB = mpa$   
 $B = \frac{mpa}{qv} = \frac{1.67 \times 10^{-27} \times 2 \times 10^{13}}{1.6 \times 10^{-19} \times 1 \times 10^7} = 2.1 \times 10^{-26} \text{ T}$

13) a)  $qvB = m \frac{v^2}{r}$  perpendicular to velocity  
 $r = \frac{mv}{qB} = \frac{9.11 \times 10^{-31} \times 1.5 \times 10^7}{1.6 \times 10^{-19} \times 2 \times 10^{-3}} = 4.3 \times 10^8 \text{ m}$

b)  $T = \frac{2\pi r m}{qv} = \frac{2\pi \times 4.3 \times 10^8 \times 9.11 \times 10^{-31}}{1.6 \times 10^{-19} \times 1.5 \times 10^7} = 102.6 \times 10^{-11}$

32) a)  $F = I L B \sin\theta$   
 $= 3 \times 14 \times 10^{-2} \times 0.28 \times \sin 90 = 0.1176$

33) a)  $\vec{F} = I \vec{L} \times \vec{B}$   
 $\frac{F}{L} = I B$   
 $0.12 = 15 B \Rightarrow B = 8 \times 10^{-3} \text{ T}$

b) up ( $\hat{k}$ )

34) a)  $F = I L B \sin\theta$   
 $= 5 \times 2.8 \times 0.39 \times \sin 60 = 4.728 \text{ N}$

35)  $\vec{F} = I \vec{L} \times \vec{B}$   
 $= 2.4 \times 0.75 \times 1.6 = 2.88 \text{ N}$

Q2. هذا سؤال رقم 1 بالبيضة التي مع الدكتور

+ve,  $\vec{B} = B_0 \hat{i}$ , find the direction when

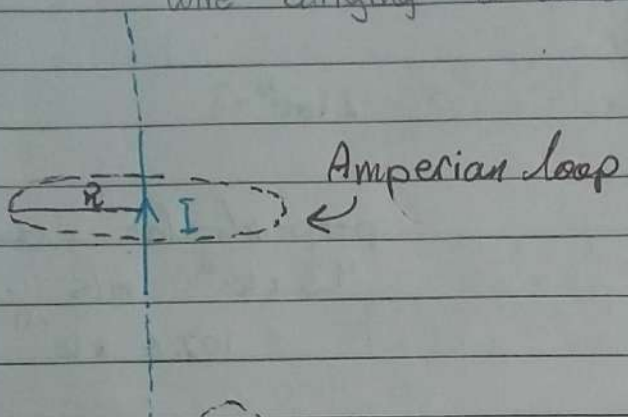
a)  $\vec{v} = v_0 \hat{j}$   
 $\hat{j} \times \hat{i} = -\hat{k}$

b)  $\vec{v} = v_0 (-\hat{j})$   
 $-\hat{j} \times \hat{i} = \hat{k}$

c)  $\vec{v} = v_0 \hat{i}$   
 no effect

[I] Amper's law .

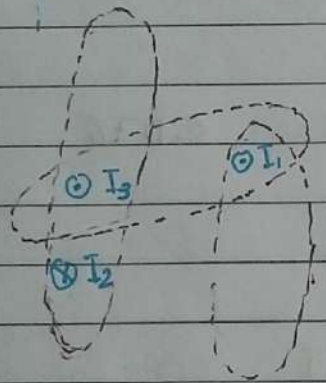
To calculate the magnetic field of an infinite wire carrying a current I



$$\oint B \cdot ds = \mu_0 \sum I_{in}$$

B = magnetic field  
s = circumference (2πr) (الطول)  
 $\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$   
is free space permeability  
البيضاء الكمال سوا

EX.



$$I_1 = 5 \text{ A}, I_2 = 10 \text{ A}, I_3 = 20 \text{ A}$$

Rank  $\oint B \cdot ds$  for the above loop from least to greatest

sol.  $B_2 > B_3 > B_1$

انتبه الى اتجاه التيار للكل في اللفج  
مما كان الجمع في اللفج

EX: a) find the magnetic field of an infinite wire carrying current I at distance r from it.

$$\oint B \cdot ds = \mu_0 \sum I_{in}$$

$$B \cdot 2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

b) find the magnetic force exerted by the conductor (I = 10A) at distance r = 100 mm on an electron moving perpendicular at the magnetic field with speed 4.11 m/s

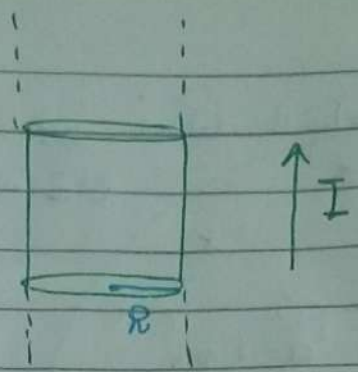
$$F = q \vec{v} \times \vec{B}$$
$$= q \vec{v} \left( \frac{\mu_0 I}{2\pi r} \right)$$

$$= 1.6 \times 10^{-19} \times 4 \times 10^6 \times \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 100 \times 10^{-3}} = 1.28 \text{ nN down}$$

19/4/2018

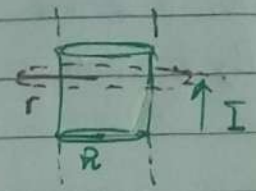
Thursday

E.X.



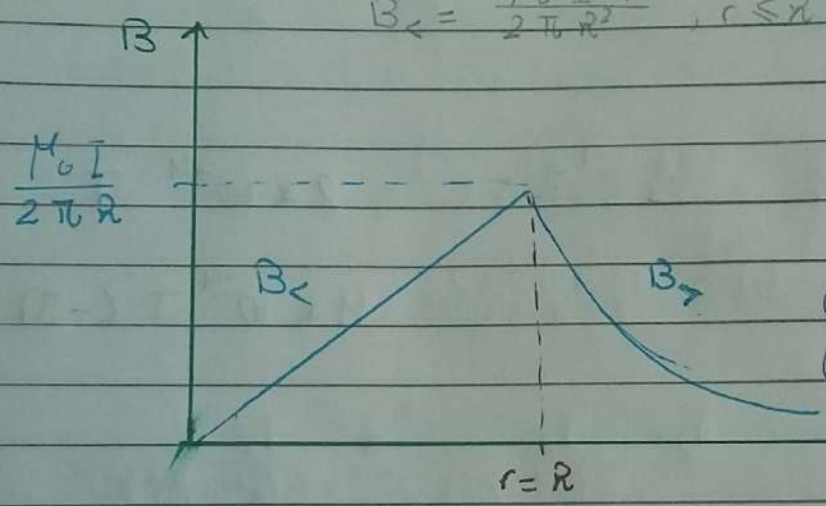
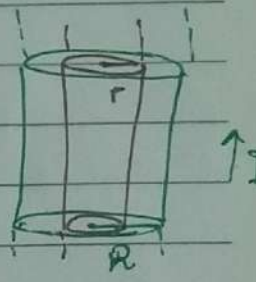
Find the magnetic field  
 a) outside the wire  $r > R$   
 b) inside the wire  $r \leq R$

sol a)  $\int B \cdot ds = \mu_0 I_{in}$   
 $B \times 2\pi r = \mu_0 I$   
 $B = \frac{\mu_0 I}{2\pi r}, r > R$

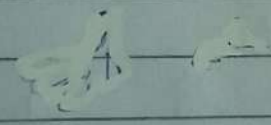


b)  $\int B \cdot ds = \mu_0 I_{in}$   
 $B \times 2\pi r = \mu_0 I$   
 $B(2\pi r) = \mu_0 I \frac{r^2}{R^2}$   
 $B = \frac{\mu_0 I}{2\pi R^2} r, r \leq R$

$J = \frac{I}{R^2 \pi}$
$J = \frac{I_{in}}{\pi r^2}$
$I_{in} = I \frac{r^2}{R^2}$



Slope =  $S = \frac{\mu_0 I}{2\pi R}$   
 $\frac{I}{R} = \frac{2\pi S}{\mu_0}$   
 انما تلك الحد الحسب انت



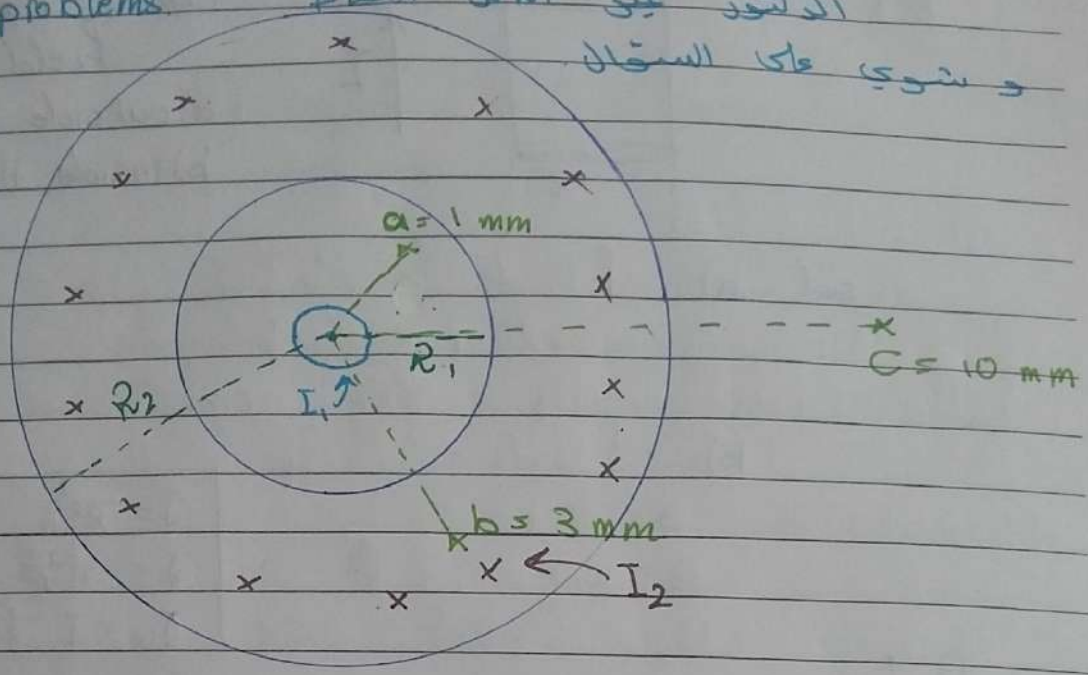
قاعدة اليد اليمنى للسلك  
 (1) الاتجاه انتاج التيار  
 (2) اتجاه المجال يكون مع اتجاه دوران  
 الأضلاع

Q31 in problems

الدكتور غير امكن النظام

و بشوي على السوال

- $I_1 = 1 A$
- $I_2 = 3 A$
- $R_1 = 2 mm$
- $R_2 = 5 mm$



a) B)  $a = 1 mm$

$$B_a = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 1}{2 \times \pi \times 1 \times 10^{-3}} = 2 \times 10^{-4} T$$

b) B)  $c = 10 mm$

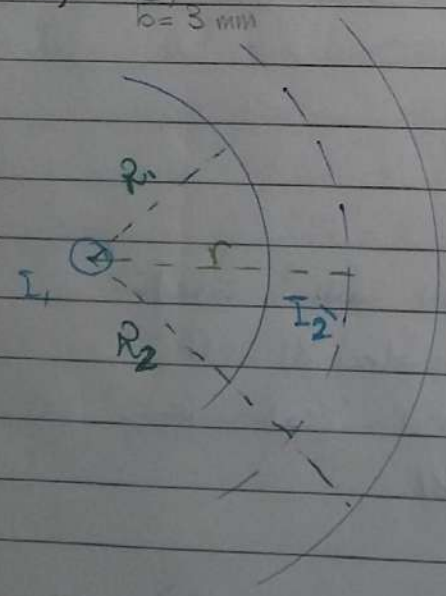
$$B_c = \frac{\mu_0 (I_2 - I_1)}{2\pi r} = \frac{4\pi \times 10^{-7} \times 2}{2\pi \times 10 \times 10^{-3}} = 4 \times 10^{-4} T (-j)$$

c) B)  $b = 3 mm$

$$I_2' = \frac{I_2}{\pi(R_2^2 - R_1^2)} = \frac{I_2'}{\pi(r^2 - R_1^2)}$$

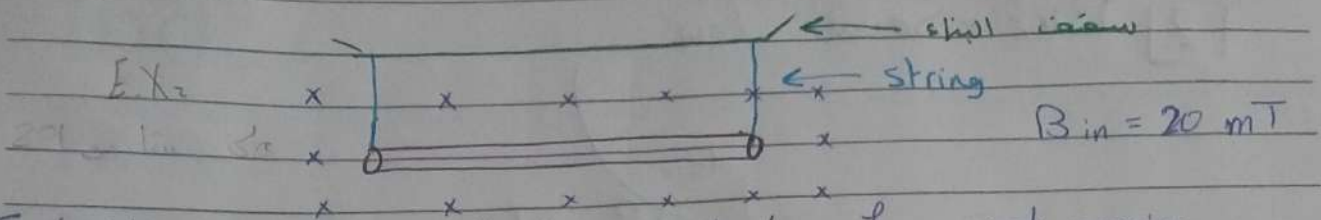
$$I_2' = \frac{I_2 (r^2 - R_1^2)}{(R_2^2 - R_1^2)} = \frac{3(9 - 4)}{(25 - 4)} = \frac{15}{21} < 1$$

$$B_b = \frac{\mu_0 (I_1 - I_2')}{2\pi r} = \frac{4\pi \times 10^{-7} (1 - \frac{15}{21})}{2\pi \times 3 \times 10^{-3}} = 0.38 \times 10^{-4} T$$



22/4/2018

Sunday



Find the direction and the magnitude of current passing the wire such that the tension in the string is zero if the mass per length of the wire =  $0.5 \text{ g/mm}$

sol.  $\Sigma \vec{F} = 0$

$$2T + I L B \sin \theta = mg$$

$$I L B = mg$$

$$I = \frac{mg}{L B} = \frac{\lambda g}{B} = \frac{0.5 \times 10^{-3}}{20 \times 10^{-3}} = 2.5 \times 10^{-3} \text{ (+)}$$

EX:  $q = 1 \text{ mC}$ ,  $\vec{v} = 2\hat{i} + \hat{k}$ ,  $\vec{F} = \hat{j}$ , find B  
assume  $B_x = 0$

sol.  $\vec{F} = q \vec{v} \times \vec{B}$

$\hat{j} = q$	$\hat{i}$	$\hat{j}$	$\hat{k}$
	2	0	1
	$B_x$	$B_y$	$B_z$

$$\hat{j} = q ( (-B_y) \hat{i} - (2B_z - B_x) \hat{j} + (2B_y) \hat{k} )$$

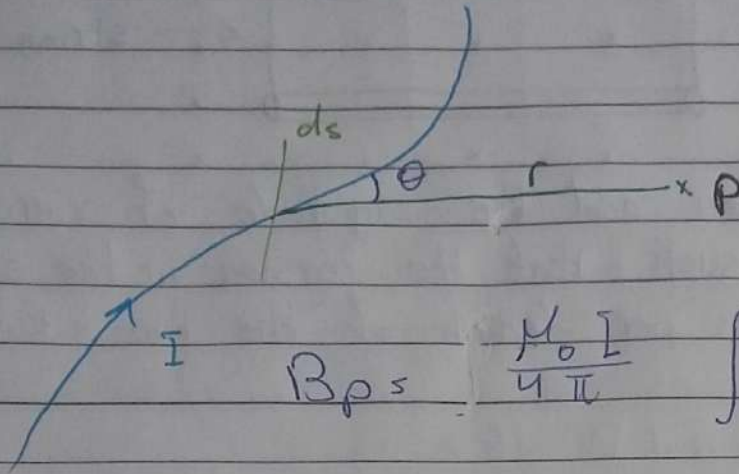
$B_y = 0 \Leftrightarrow 0 = \hat{i}, \hat{k} \Leftrightarrow \hat{j}$  ok!  $\vec{F}$  ok!

$$\hat{j} = -2q B_z \hat{j}$$

$$1 = -2q B_z$$

$$B = B_z = \frac{1}{-2q} \hat{k}$$

[2] Biot - Savart law.

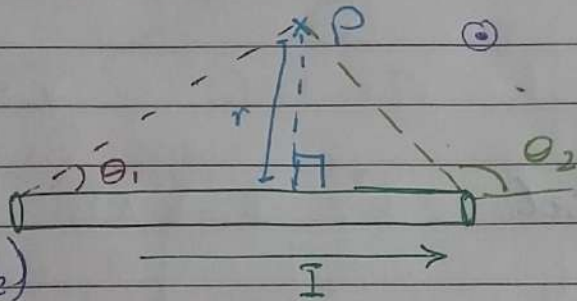


$$B_p = \frac{\mu_0 I}{4\pi} \int \frac{ds \times \hat{r}}{r^3}$$

1) The magnetic field of a finite wire

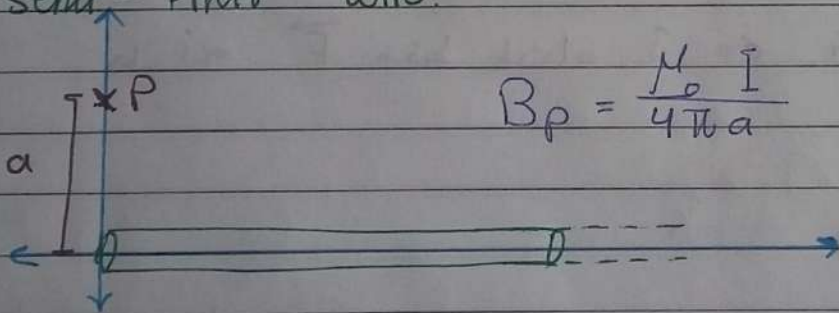
$\theta_1 =$  angle with wire

$\theta_2 =$  angle with wire



$$B_p = \frac{\mu_0 I}{4\pi r} (\cos \theta_1 - \cos \theta_2)$$

\* semi-finite wire.



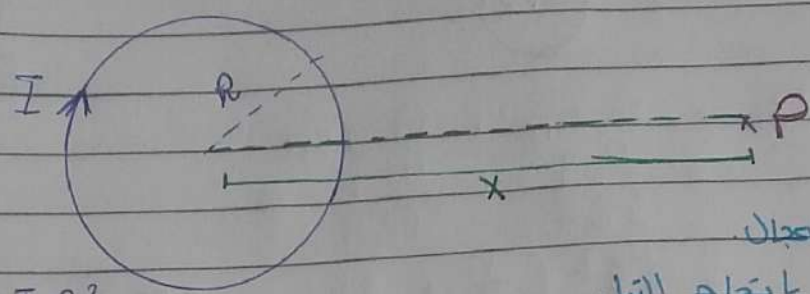
$$B_p = \frac{\mu_0 I}{4\pi a}$$

if the wire is infinite then  $\theta_1 = 0$  and  $\theta_2 = \pi$  then  $B_p = \frac{\mu_0 I}{2\pi r}$

$$B_p = \frac{\mu_0 I}{4\pi a \times 2}$$

24/4/2018

### 2) Circular loop carrying a current

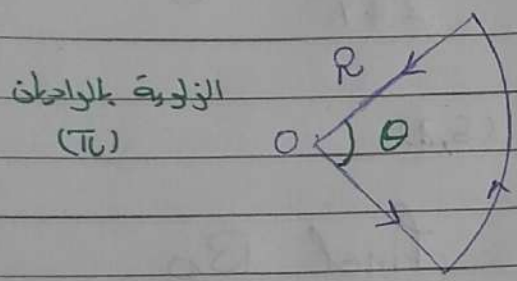


$$B_p = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

المعروفه اتجاه المجال  
 (ا) اتجاه اليد باتجاه التيار  
 (ب) يكون المجال باتجاه الابهام

لو كانت آثر من لفه من انبوب Bp بعدد اللفات

### 3) Circular Arc

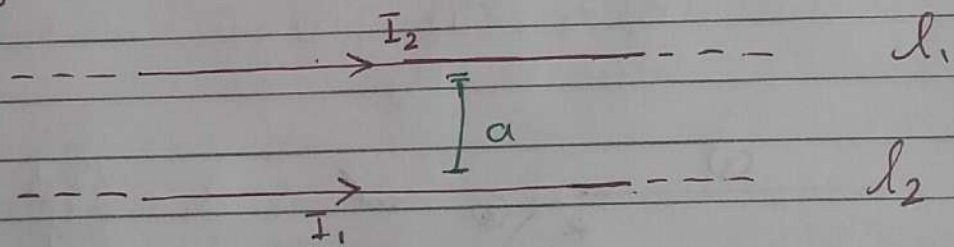


$$B_o = \frac{\mu_0 I \theta}{4\pi R}$$

الزاوية بالاركان  
 (π)

بتعريفه ان 3.14 π من 180  
 في ذلك

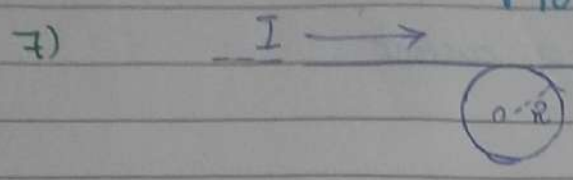
### The magnetic force between two conductors.



$$\begin{aligned} \vec{F}_{1 \rightarrow 2} &= I_2 l_2 \times \vec{B}_1 \\ &= I_2 l_2 \frac{\mu_0 I_1}{2\pi a} \end{aligned}$$

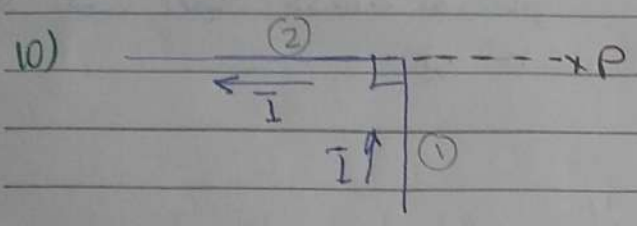
$= \frac{\mu_0 I_1 I_2}{2\pi a} l_2$   
 إذا التيارين نفس الاتجاه ← تجاذب  
 إذا عكس الاتجاه ← تنافر

# Problems

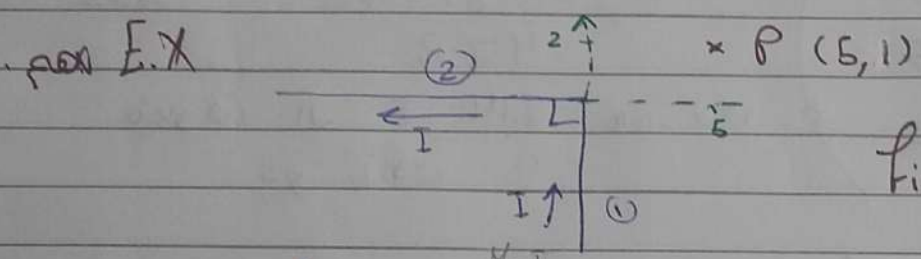


$I = 1A$   
 $R = 0.15 \text{ cm}$

sol.  $B_0 = B_{\text{wire}} + B_{\text{loop}}$   
 $= \frac{\mu_0 I}{2\pi R} + \frac{\mu_0 I}{4\pi R}$   
 $= \frac{\mu_0 I}{2R} \left( \frac{1}{\pi} + 1 \right) = 552 \text{ mT}$

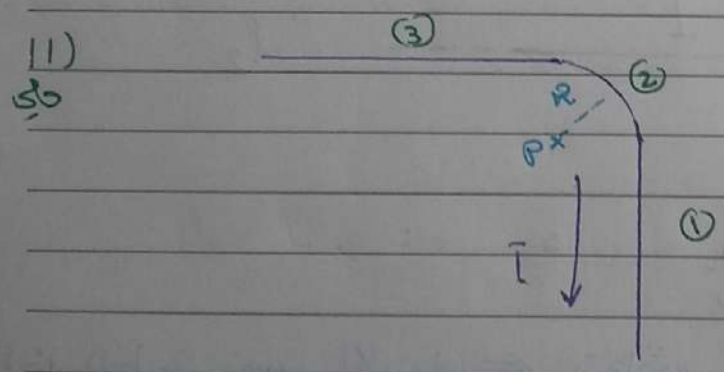


sol.  $B_p = \frac{\mu_0 I}{2\pi x}$  (⊗)



find  $B_p$

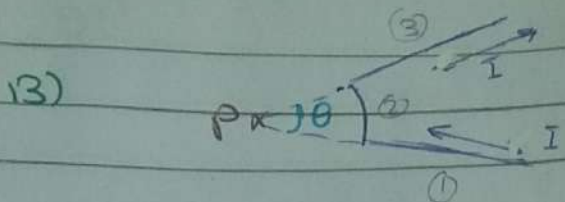
sol.  $B_1 = \frac{\mu_0 I}{2\pi (5)}$  (⊗)  
 $B_2 = \frac{\mu_0 I}{2\pi (1)}$  (⊗)  
 $B_p = B_1 + B_2$



sol.  $B_1 = \frac{\mu_0 I}{2\pi R}$  (⊗)  
 $B_2 = \frac{\mu_0 I}{2\pi R}$  (⊗)  
 $B_3 = \frac{\mu_0 I}{4\pi R} \theta = \frac{\mu_0 I}{8R}$  (⊗)

$B_p = B_1 + B_2 + B_3$   
 $= \frac{\mu_0 I}{2R} \left( \frac{1}{\pi} + \frac{1}{4} \right)$





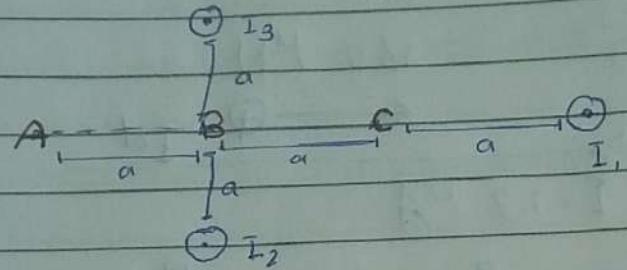
$\theta = 30^\circ = \frac{\pi}{6}$

$R = 0.3 \text{ m}, I = 3 \text{ A}$

الدائرة عند كل السؤالات

sol.  $B_p = \frac{\mu_0 I \theta}{4\pi R} = \frac{4\pi \times 10^{-7} \times 3 \times \frac{\pi}{6}}{4\pi \times 0.3} = \frac{\pi}{6} \mu T$

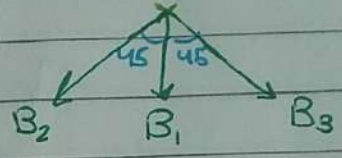
سؤال 15)



$I_1 = I_2 = I_3 = 2 \text{ A}$

$a = 1 \text{ cm}$

a)  $B_2 = B_3 = \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{2\pi a\sqrt{2}}$   
 $B_1 = \frac{\mu_0 I}{2\pi (3a)}$

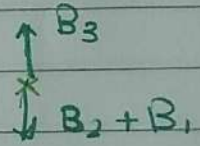


$B_3 \uparrow = -B_2 \uparrow \Rightarrow$  (مساوية في الزاوية)

$B_A = B_1 + B_2 \cos 45 + B_3 \cos 45$

$= \frac{\mu_0 I}{2\pi a} \left( \frac{1}{3} + \frac{1}{\sqrt{2}} \cos 45 + \frac{1}{\sqrt{2}} \cos 45 \right) = 53.3 \mu T \downarrow$

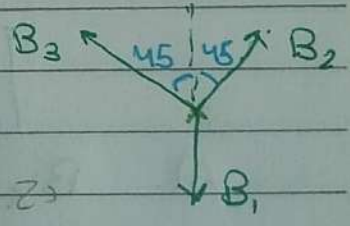
b)  $B_2 = B_3 = \frac{\mu_0 I}{2\pi a}$   
 $B_1 = \frac{\mu_0 I}{2\pi (2a)}$



(مساوية في الزاوية)  $B_2 = B_3$

$B_B = B_1 = 20 \mu T \downarrow$

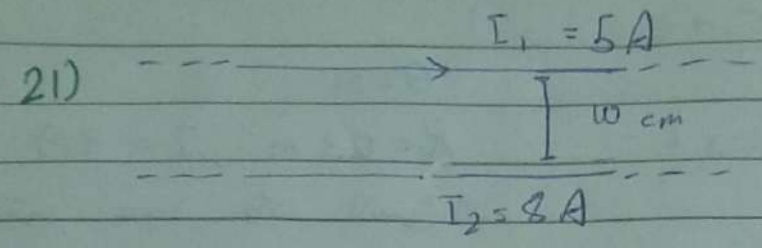
c)  $B_2 = B_3 = \frac{\mu_0 I}{2\pi (a\sqrt{2})}$   
 $B_1 = \frac{\mu_0 I}{2\pi a}$



$B_2 \uparrow = -B_3 \uparrow \Rightarrow$  (مساوية في الزاوية)

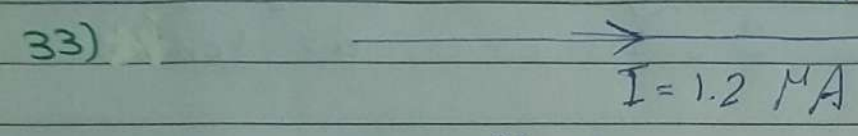
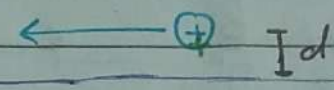
$B_C = B_1 - (B_2 + B_3)$

$= \frac{\mu_0 I}{2\pi a} \left( 1 - \left( \frac{1}{\sqrt{2}} \cos 45 + \frac{1}{\sqrt{2}} \cos 45 \right) \right) = \text{Zero}$



B)  $F = \frac{\mu_0 I_1 I_2 l}{2\pi a}$

$F = \frac{\mu_0 I_1 I_2}{2\pi a} = 160 \mu N$



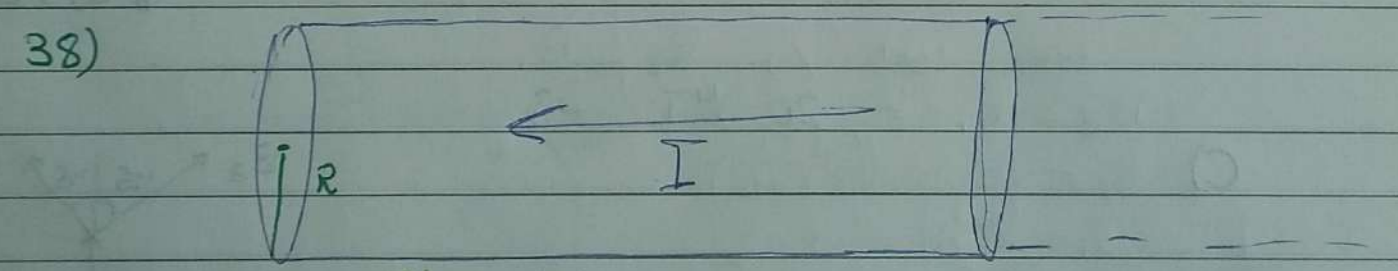
$m_p = 1.67 \times 10^{-27} \text{ Kg}$ ,  $q_e = 1.6 \times 10^{-19} \text{ C}$

$F_B = m_p g$

$\frac{\mu_0 I}{2\pi d} = m_p g$

$d = \frac{\mu_0 I}{2\pi m_p g} = 5.3 \text{ cm}$

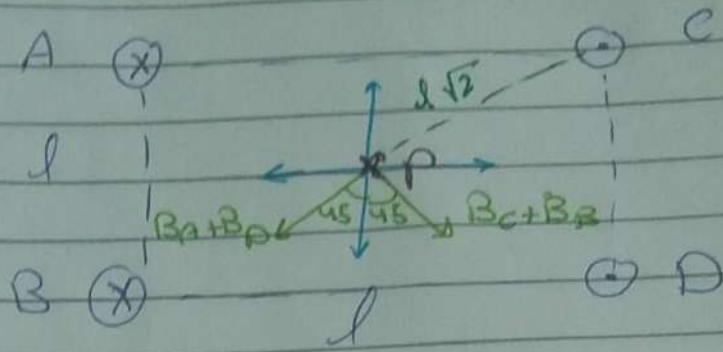
لما يكون proton يكون يتحرك عكس اتجاه التيار  
ولما يكون electron يكون يتحرك مع اتجاه التيار



A)  $B_{<} = \frac{\mu_0 I r}{2\pi R^2}$   
 $r = R/2$

$B_{>} = \frac{\mu_0 I}{2\pi r}$   
 $r = 2R$

39)

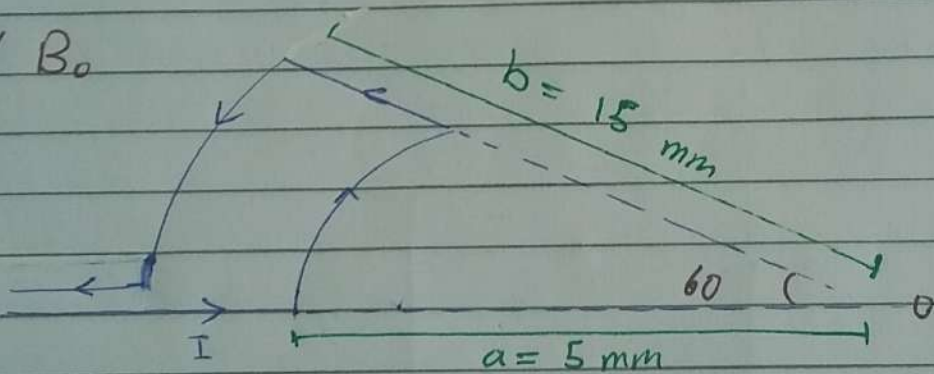


$$B_A = B_B = B_C = B_D = \frac{\mu_0 I}{2\pi a}$$

$$B_0 = \frac{4 \mu_0 I}{2\pi a} \cdot \cos 45 = 20 \mu T \hat{i}$$

EX : يوجد سلكان // بعضهما البعض

Find  $B_0$



$$I = 5A$$

$$\text{sol. } B_0 = B_a - B_b$$

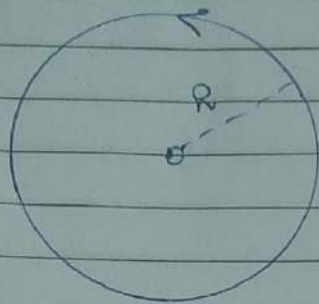
$$= \frac{\mu_0 I}{4\pi a} \cdot \frac{\pi}{3} - \frac{\mu_0 I}{4\pi b} \cdot \frac{\pi}{3}$$

$$= \frac{\mu_0 I}{4\pi} \left( \frac{1}{a} - \frac{1}{b} \right) = 13.96 \mu T$$

84

E.X

سؤال 7 : طبيعة الدائرة



$I = 5 \text{ A}$   
 $B_0 = 10 \text{ mT}$

Find R

sol  $B_0 = \frac{\mu_0 I}{2R}$

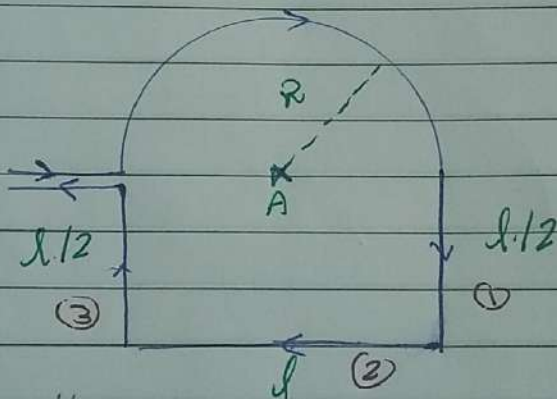
$$10 \times 10^{-6} = \frac{4\pi \times 10^{-7} \times 5}{2R}$$

$$R = \frac{\pi}{10} = 0.314 \text{ m}$$

سؤال 13 : طبيعة الدائرة

E.X

Find  $B_A$



$R = l/2$

sol  $B_{Arc} = \frac{\mu_0 I}{4\pi R} \theta = \frac{\mu_0 I}{4R}$

$B_1 = B_3 = \frac{\mu_0 I}{4\pi R} (\cos \frac{\pi}{4} - \cos 90)$

$B_2 = \frac{\mu_0 I}{4\pi R} (\cos \frac{\pi}{4} - \cos 135) = \frac{2\mu_0 I}{4\pi R} \cos \frac{\pi}{4}$