بسم الله الرحمن الرحيم

تقدم لكم لجنة سيفيلتي أسئلة سنوات سابقة محلولة لمادة تفاضل وتكامل 2 لمقرر الشهر الأول

كل الشكر للطالبين : عواد عزت و مؤيد زياد

ونذكر بأن هذا اجتهاد من زملائنا فمن وجد خطأً فليصلحه ودمتم بخير جميعاً

10. By using the trigonometric substitution
*

(2 Points)

$x = 3\sec(\theta)$, *where* $0 \leq \theta < \frac{\pi}{2}$,

the integral $\int \frac{27}{x^3\sqrt{x^2-9}}dx$ *can be transformed into one of the following integrals*

- $\int \csc^2(\theta).\,d\theta$
- $\int \sin^2(\theta).\,d\theta$
- $\int \cot^2(\theta).\,d\theta$
- $\int \cos^2(\theta).\,d\theta$ ✓
- $\int \cos(\theta).\,d\theta$

$$\int \frac{27}{x^3\sqrt{x^2-9}}\,dx \qquad x = 3\sec\theta$$

$$dx = 3\sec\theta\tan\theta\,d\theta$$

$$\int \frac{27 \cdot 3\sec\theta\tan\theta}{27 \cdot \sec^3\theta\sqrt{9\sec^2\theta - 9}}\,d\theta$$

$$\int \frac{3\tan\theta}{\sec^2\theta\sqrt{\tan^2\theta \cdot 9}} = \int \frac{3\tan\theta}{3\sec^2\theta\tan\theta}$$

$$= \int \cos^2\theta\,d\theta$$

17. What is the value of A in the partial fraction decomposition *

(3 Points)

$$\int \frac{4x}{(x-1)^2(x+1)}dx =$$

$$\int \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}.dx$$

- −4
- 1
- −3
- 2
- 4

$$\int \frac{4x}{(x-1)^2(x+1)}$$

$$\int \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$$

$$A(x-1)(x+1) + B(x+1) + C(x-1)^2 = 4x$$

when $x = 1 \rightarrow 2B = 4$ $\boxed{B = 2}$

when $x = -1 \rightarrow 4C = -4$ $\boxed{C = -1}$

when $x = 0 \rightarrow -A + 2 - 1 = 0$ $\boxed{A = 1}$

1. Suppose that *
(2 Points)

$f(0) = f(2),\ f'(2) = 2,\ and\ f'' \text{ is continuous.}$

$Find\ \int_0^2 x f''(x).dx$

- 2
- 1
- 3
- 5
- 4

$$\int_0^2 x \cdot f''(x)\, dx$$

$u = x \qquad du = dx$

$dv = f''(x)\,dx \qquad v = f'(x)$

$$x \cdot f'(x) - \int f'(x)\, dx$$

$$x f'(x) - f(x) \Big|_0^2$$

$$2 \times 2 - f(2) - (0 - f(0))$$

$$4 - f(2) + f(0) \qquad \text{and } f(2) = f(0)$$

$$= 4$$

4. Integrating *

(2 Points)

$\int x \sin^{-1}(x).\, dx$ *by parts*

in correct way we obtain

an expression of the form $A - \int B \,.\, dx$

then $B =$

- $-\frac{x^2}{2\sqrt{1-x^2}}$
- $\frac{x}{1+x^2}$
- $\frac{x^2}{2(1+x^2)}$
- $\frac{x^2}{2\sqrt{1-x^2}}$ ✓
- $\frac{x}{2\sqrt{1-x^2}}$

$$\int x \sin^{-1}(x)\, dx$$

$$u = \sin^{-1}(x) \qquad du = \frac{1}{\sqrt{1-x^2}}\, dx$$

$$-\int$$

$$dv = x\, dx \qquad v = \frac{x^2}{2}$$

$$\frac{x \sin^{-1}(x)}{2} - \int \frac{x^2}{2\sqrt{1-x^2}}\, dx$$

16. To integrate the function *
(2 Points)

$\frac{x^3}{(x-2)(x+2)}$ *by partial fractions*

one should try to express it in the form

- $1 + \frac{A}{x-2} + \frac{B}{x+2}$
- $\frac{A}{x-2} + \frac{B}{x+2}$
- $\frac{Ax+B}{x-2} + \frac{Cx+D}{x+2}$
- $x + \frac{A}{x-2} + \frac{B}{x+2}$
- $\frac{Ax}{x-2} + \frac{Bx}{x+2}$

$$\frac{x^3}{(x-2)(x+2)}$$

$$\begin{array}{r|l} & x \\ \hline x^2-4 & x^3 \\ & -(x^3-4x) \\ \hline & 4x \end{array}$$

$$x + \frac{4x}{(x-2)(x+2)}$$

$$x + \frac{A}{x-2} + \frac{B}{x+2}$$

9. Which substitution is needed to evaluate the integral *
(2 Points)

$\int sec^5(\theta)\, tan^7(\theta).\, d(\theta)$

- $u = sin(\theta)$
- $u = cos(\theta)$
- $u = cot(\theta)$
- $u = tan(\theta)$
- $u = sec(\theta)$

19. Question *
(2 Points)

$$\int \frac{x+2}{x-1} dx =$$

- $x + 3\ln|x-1| + C$ ✓
- $x + 5\ln|x-1| + C$
- $x + 4\ln|x-1| + C$
- $x + 2\ln|x-1| + C$
- $x + \ln|x-1| + C$

20. Find the Cartesian equation for the

$$\int \frac{x+2}{x-1}\,dx$$

$$\begin{array}{r|l} & 1 \\ \hline x-1 & x+2 \\ & -(x-1) \\ \hline & 3 \end{array}$$

$$\int \left(1 + \frac{3}{x-1}\right) dx$$

$$x + 3\ln|x-1| + c$$

13. what is the suitable substitution required to evaluate the integral *
(2 Points)

$$\int \frac{x}{\sqrt{2x-3+x^2}} dx$$

- $x = 2\,tan(\theta) + 1$
- $x = 2\,sec(\theta) + 1$
- $x = 2\,sec(\theta) - 1$
- $x = 2\,sin(\theta) + 1$
- $x = 2\,tan(\theta) - 1$

14. The two polar points *
(2 Points)

$$\int \frac{x}{\sqrt{2x-3+x^2}}$$

by Completing square

$$\int \frac{x}{\sqrt{(x+1)^2-4}}$$

$x+1 = 2\sec\theta$

$x = 2\sec\theta - 1$

17. *
(2 Points)

$\int cos(5x)cos(x).dx =$

- $\frac{1}{2}(\frac{sin(4x)}{4} + \frac{sin(6x)}{6}) + C$
- $\frac{1}{2}(\frac{cos(4x)}{4} - \frac{cos(6x)}{6}) + C$
- $-\frac{1}{2}(\frac{sin(4x)}{4} - \frac{sin(6x)}{6}) + C$
- $\frac{1}{2}(\frac{sin(4x)}{4} - \frac{sin(6x)}{6}) + C$
- $-\frac{1}{2}(\frac{sin(4x)}{4} + \frac{sin(6x)}{6}) + C$

18. Which one of the following polar

$$\int \cos(5x) \cdot \cos(x)\, dx$$

$$\int \left(\frac{1}{2} \left[\cos(4x) + \cos(6x) \right] \right) dx$$

$$\frac{1}{2} \left[\int \cos 4x\, dx + \int \cos 6x\, dx \right]$$

$$\frac{1}{2} \left(\frac{\sin 4x}{4} + \frac{\sin 6x}{6} \right) + C$$

[redacted]

16. By using the trigonometric substitution *
(3 Points)

$x = \frac{3}{2}\tan(\theta)$, *where* $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$,

the integral $\int \frac{16x^3}{3(4x^2+9)^{\frac{3}{2}}}dx$

can be transformed into

one of the following integrals

- $\int \frac{\tan^2(\theta)}{\sec^2(\theta)}d\theta$
- $\int \frac{\tan^3(\theta)}{\sec^3(\theta)}d\theta$
- $\int \frac{\tan^2(\theta)}{\sec^3(\theta)}d\theta$
- $\int \frac{\tan^3(\theta)}{\sec(\theta)}d\theta$
- $\int \frac{\tan^3(\theta)}{\sec^2(\theta)}d\theta$

Which one of the following integrals .3
* represent the area
(3 نقطة)

inside $r = 1 - \sin(\theta)$
and outside $r = 1$.

$$\frac{16}{3}\int \frac{x^3}{\sqrt{((2x)^2+3^2)^3}}\,dx \qquad x = \frac{3}{2}\tan\theta$$

$$dx = \frac{3}{2}\sec^2\theta\, d\theta$$

$$\frac{16}{3}\int \frac{27/8\ \tan^3\theta}{(\sqrt{9(\sec^2\theta)})^3} * \frac{3}{2}\sec^2\theta$$

$$\frac{16}{3}\int \frac{\tan^3\theta}{\sec^3\theta} * \frac{3}{16}\sec^2\theta\, d\theta$$

$$\int \frac{\tan^3\theta}{\sec\theta}\, d\theta$$

A suitable substitution u will give the .7 following *

(3 نقطة)

$$\int tan^3(x)\, sex^n(x)\, dx = \int u^6 - u^4 .\, du$$

$Then \;\; n =$

- 5
- 6
- 7
- 3
- 4

Find the value of the constant C for .5 which the following integral is convergent *

(3 نقطة)

$$\int_0^\infty \left(\frac{x}{x^2+1} - \frac{C}{3x+1}\right)dx$$

- 2
- 4
- 3
- 6
- 5

$$\int \tan^3 x \sec^n x \, dx = \int u^6 - u^4 \, du$$

بالتجريب

$n=5$

$$\int \tan^3 x \sec^5 x \, dx$$

$u = \sec x$

$dx = \frac{du}{\sec x \tan x}$

$$\int (\sec^2 x - 1) u^4 \, du \rightarrow \int u^6 - u^4$$

2. Which one of the following improper integrals is divergent *
(2 Points)

- $\int_1^\infty x^{\pi-7}\,dx$
- $\int_1^\infty \frac{1}{x^{\sqrt{2}}}\,dx$
- $\int_1^\infty \frac{1}{x^{e}}\,dx$
- $\int_1^\infty \frac{1}{x^{1.1}}\,dx$
- $\int_1^\infty x^{3-e}\,dx$

(a) $\int_1^{\infty} x^{\pi-7}\,dx$

$\frac{1}{x^p}$ $p>1$ con, $p\leq 1$ div

$\rightarrow \int_1^{\infty} \frac{1}{x^{7-\pi}}\,dx$ ✓ con

(b) con , (d) con , (c) con

e $\int_1^{\infty} x^{3-e}\,dx \rightarrow \int_1^{\infty} \frac{1}{x^{e-3}}\,dx$ ✓ divergant

14. If m, n are positive integers, where *

(1 Point)

$m = n$, then $\int_{-\pi}^{\pi} \sin(mx)\sin(nx)\,.dx = \pi$

◯ False

F

◯ True