



اللجنة الأكاديمية للهندسة المدنية

دفتر

# فيزياء ا

سالي بني ياسين

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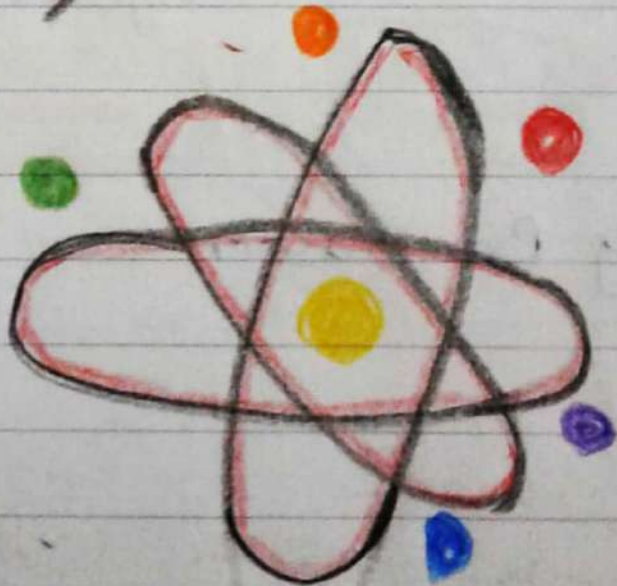
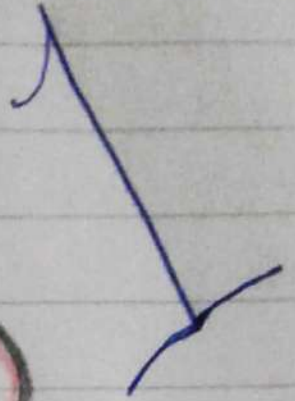
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First

PHYSICS



#Sally Bani Yaseen

# ch 1: Dimensional Analysis :-

\* International units system :-

- |            |   |         |
|------------|---|---------|
| ① mass     | M | kg      |
| ② distance | L | m       |
| ③ Time     | T | second. |

Ex. ① average speed =  $\frac{\text{Distance}}{\text{Time}}$

$$S = \frac{D}{T} \rightarrow [S] = \frac{[D]}{[T]}$$

$$= \frac{m}{s} = m/s$$

$$= m s^{-1} \quad (L T^{-1})$$

② acceleration =  $\frac{\text{speed}}{\text{Time}}$

$$a = \frac{S}{t} \rightarrow [a] = \frac{[S]}{[t]}$$

$$[a] = \frac{\frac{m}{s}}{s} = \frac{m}{s^2}$$

$$= m/s^2 = m s^{-2} \quad (L T^{-2})$$

③ Force = mass \* acceleration

$$F = ma$$

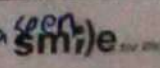
$$[F] = [m][a]$$

$$= \text{kg } m s^{-2}$$

$$= \text{Newt on (N)}$$

$$N = \text{kg } m s^{-2}$$

$$= M L T^{-2}$$

\* Sally Bani 

④ Work = Force  $\times$  dis

$$W = Fd$$

$$[W] = [F][d] \rightarrow [W] = N \cdot m = \text{Joule } \underline{J}$$

$$\begin{aligned} J &= Nm \\ &= kg \, m \, s^{-2} \cdot m \\ J &= kg \, m^2 \cdot s^{-2} \end{aligned}$$

Ex. Let  $F = \frac{G m_1 m_2}{r^2}$

$F$  = Force

$m$  = mass

$r$  = distance

Find  $[G]$

$$G = \frac{F r^2}{m_1 m_2}$$

$$[G] = \frac{[F][r^2]}{[m_1][m_2]}$$

$$\begin{aligned} &= \frac{N \cdot m^2}{kg^2} = N \cdot m^2 \cdot kg^{-2} \\ &= kg \, m \, s^{-2} \cdot m^2 \cdot kg^{-2} \\ &= m^3 \, kg^{-1} \, s^{-2} \end{aligned}$$

Ex. Let  $A = e^{\alpha t}$

Find  $[\alpha]$

Since  $e^{\alpha t}$  is unit less  $[\alpha t]$  unit less

$$S [\alpha] = S^{-1}$$

-1

#Sally Bani Yaseen

\*For any equation to be correct

$$[L.H.S] \equiv [R.H.S]$$

لا تكون الوحدات المتجانسة  
تكون وحدات القياس متكافئة

Ex. Check the equation if correct or not :-

$$v = at^2$$

وحداتها  
وحداتها  
وحداتها

سرعة  $v \equiv$  speed

$a \equiv$  acceleration

$t \equiv$  time

$$[v] \equiv [a][t^2]$$

$$\frac{m}{s} \equiv \frac{m}{s^2} * s^2$$

$$\frac{m}{s} \neq m$$

not correct

Ex.  $X = v_i t + \frac{1}{2} at^2$

time acceleration

تجسري

$$m \equiv \frac{m \cdot s}{s} + \frac{m \cdot s^2}{s^2}$$

\* يجب ان تكون وحدات القياس متكافئة

$$m \equiv m \quad \checkmark \text{ correct}$$

Ex. Consider the equation :-

$$S = \frac{1}{2} a^k t^h$$

Find the values of k & h that fit the equation

where  $S \equiv$  distance

$a \equiv$  acceleration

$t \equiv$  time

$$[s] = [a^k] \cdot [t^h]$$

$$m = \left(\frac{m}{s^2}\right)^k \cdot s^h$$

$$m = \frac{m^k}{s^{2k}} \cdot s^h$$

$$m = m^k \cdot s^{-2k} \cdot s^h$$

$$m^1 = m^k \cdot s^{h-2k}$$

$$k = 1$$

تجسري

اس

10

$$\rightarrow h - 2k = 0 \Rightarrow h = 2$$

\* Sally Bani yaseen smile...

$$\text{Let } a = v^k r^h$$

$a \equiv$  acceleration

$v \equiv$  speed

$r \equiv$  radius

Find  $k, h$

$$[a] \equiv [v^k] \cdot [r^h]$$

$$\frac{m}{s^2} \equiv (m s^{-1})^k \cdot (m)^h$$

$$m s^{-2} \equiv m^k \cdot s^{-1k} \cdot m^h$$

$$m s^{-2} \equiv m^{k+h} \cdot s^{-1k}$$

$$\frac{-2}{-1} = \frac{-1k}{-1}$$

$$k = 2$$

$$h + k = 1$$

$$h + \frac{2}{-2} = \frac{1}{-2}$$

$$h = -1$$

$$a = \frac{v^2}{r}$$

تسارع  
مرکزی

H.w Let  $T = 2\pi L^h g^k$

$T \equiv$  time

$L \equiv$  length

$g \equiv$  acceleration

$$[T] \equiv [L^h] [g^k]$$

$$s \equiv m^h \cdot \left(\frac{m}{s^2}\right)^k$$

$$[s] \equiv m^h \cdot m^k \cdot s^{-2k}$$

$$\frac{1}{-2} = \frac{-2k}{-2}$$

$$k = -\frac{1}{2}$$

$$h + k = 0$$

$$h - \frac{1}{2} = 0$$

$$h = \frac{1}{2}$$

#

# Sally Bari yaseen

Ex. Let  $v = c_1 t + c_2 t^2 + c_3 t^4$

Find the dimensions of  $c_1, c_2, c_3$ .

$$[v] = [c_1 t] = [c_2 t^2] = [c_3 t^4]$$

$$\frac{1}{s} \times \frac{m}{s} = [c_1] s \times \frac{1}{s} \rightarrow [c_1] = \frac{m}{s^2} = m s^{-2}$$

acceleration

$$\text{Also } \frac{1}{s^2} \times \frac{m}{s} = [c_2] s^2 \times \frac{1}{s^2} \rightarrow [c_2] = m s^{-3}$$

$$\text{Also } \frac{1}{s^4} \times \frac{m}{s} = [c_3] s^4 \times \frac{1}{s^4} \rightarrow [c_3] = m s^{-5}$$

H.w

(9) Which of the following equations are dimensionally correct?

(a)  $v_f = v_i + ax$

(b)  $y = (2m) \cos(kx)$  where  $k = 2m^{-1}$

(11) Kinetic energy  $K$  has dimensions  $kg \cdot m^2/s^2$

It can be written in terms of the momentum  $p$  and mass  $m$  as

$$K = \frac{p^2}{2m}$$

# Sally Banivaseen

- 13] The position of a particle moving under uniform acceleration is some function of time and the acceleration. Suppose we write this position as  $x = ka^m t^n$ , where  $k$  is a dimensionless constant. Show by dimensional analysis that this expression is satisfied if  $m=1$  and  $n=2$ . Can this analysis give the value of  $k$ ?

- 14] @ Assume the equation  $x = At^3 + Bt$  describes the motion of a particular object, with  $x$  having the dimension of length and  $t$  dimension of time. Determine the dimensions of the constants  $A$  &  $B$ .

$$x = At^3 + Bt$$

$$[x] \equiv [A][t]^3 + [B][t]$$

$$m \equiv [A][s]^3 + [B]s$$

[A] dimension of

$$m \equiv \frac{m \cdot s^3}{s^3} + \frac{m \cdot s}{s}$$

[B] dimension of speed

- (b) Determine the dimensions of the derivative

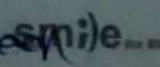
$$dx/dt = 3At^2 + B$$

$$[dx]/[dt] \equiv [A][t]^2 + [B]$$

$$m \equiv \frac{m \cdot s^2}{s^2} + \frac{m}{s}$$

$$[dx][dt] \equiv m$$

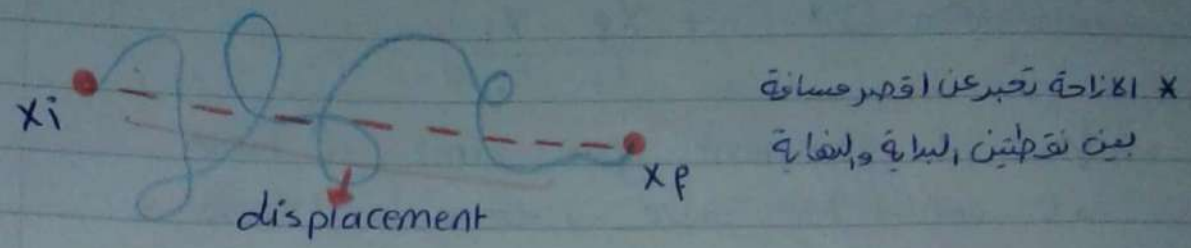
dimension of speed

# Sally Bani Vase 



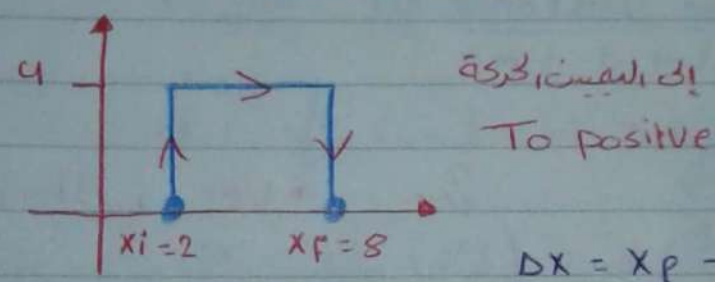
No. 7  
**Ch: 2 Motion in One dimension**

\* Displacement  $\Delta x$  الإزاحة



$\Delta x \equiv$  change in position and it is the shortest distance between two points

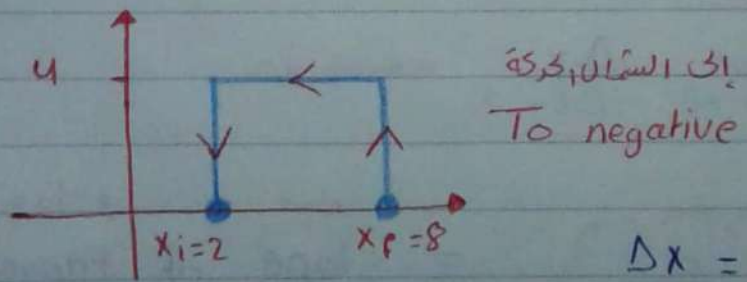
$\Delta x = x_f - x_i$



$\Delta x = x_f - x_i$   
 $= 8 - 2 = +6 \text{ m}$

لأنه الحركة إلى اليمين  $\rightarrow$

المسافة الكلية Distance =  $4 + 6 + 4 = 14 \text{ m}$



$\Delta x = 2 - 8 = -6 \text{ m}$

لأنه الحركة إلى اليسار، على عكس اتجاه  $\leftarrow$

Distance =  $4 + 6 + 4 = 14 \text{ m}$

\* Displacement has direction & magnitude  
 it could be +ve, -ve or Zero

\* Distance has only magnitude and it is always +ve

## \* Average Velocity

السرعة المتوسطة

$$v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

$v \equiv$  displacement per unit of time

- $v$  could be +ve, -ve or zero and it has magnitude & direction

## \* Average speed

 $S$ 

السرعة القياسية

$$S \equiv \frac{\text{total distance}}{\text{total time}}$$

$$S = \frac{D}{t}$$

&amp; +ve, direction

## \* Instantaneous Velocity

السرعة اللحظية

لها  
 $\Delta t \rightarrow 0$

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$$

تعريف مشتقة

$$v_{ins} = \frac{dx}{dt}$$

ميل  
 جـا-ه  
 $\equiv$  slope of tangent

\* Sally Bari Vaseen

## \* average acceleration

التسارع

- change in velocity per unit of time

تغير السرعة بالنسبة للزمن

$$a = \frac{\Delta v}{\Delta t} \rightarrow \frac{v_f - v_i}{\Delta t} \quad +ve$$

- $v_f > v_i \rightarrow a > 0$  acceleration زيادة
- $v_f = v_i \rightarrow a = 0$  constant velocity
- $v_f < v_i \rightarrow a < 0$  deceleration نقصان

## \* Instantaneous acceleration

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$$

$$a = \frac{dv}{dt} \quad ; \quad v = \frac{dx}{dt}$$

$$a = \frac{d}{dt} \left( \frac{dx}{dt} \right) \rightarrow a = \frac{d^2 x}{dt^2} \quad \text{مشتقة ثانية}$$

\* Sally Bani vaseen

\* equation of motion :-

معادلات الحركة

شروط معادلات

الحركة 1] Constant acceleration

\* التسارع الثابت في فترة زمنية

فترة زمنية

2] We have constant mass

\* تعويض الكتلة

$$a = \frac{v_f - v_i}{t_f - t_i} \rightarrow \text{في البداية يكون Zero}$$

$$a = \frac{v_f - v_i}{t_f} \Rightarrow v_f - v_i = at$$

1]

$$v_f = v_i + at$$

Constant speed  $\rightarrow$  Constant acceleration

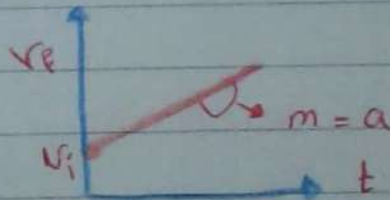
معادلة الخط المستقيم

$$y = mx + b$$

slope

y-inter step

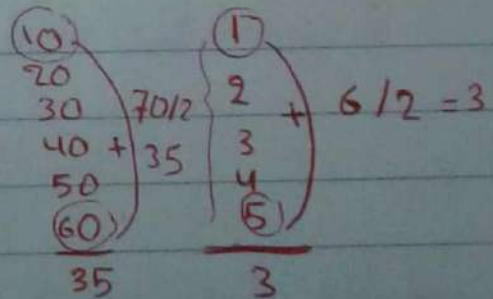
$$v_f = at + v_i$$



\* Since we have constant acceleration

$$\bar{v} = \frac{v_f + v_i}{2}$$

but  $\bar{v} = \frac{\Delta x}{\Delta t}$



\* Sally Bari Vaseen

$$\frac{\Delta x}{\Delta t} = \frac{1}{2} (v_i + v_f)$$

$$\boxed{2} \quad \Delta x = \frac{1}{2} (v_i + v_f) t$$

But  $v_f = v_i + at$  تعويض

$$\Delta x = \frac{1}{2} (v_i + v_i + at) t$$

$$\boxed{3} \quad \Delta x = v_i t + \frac{1}{2} at^2$$

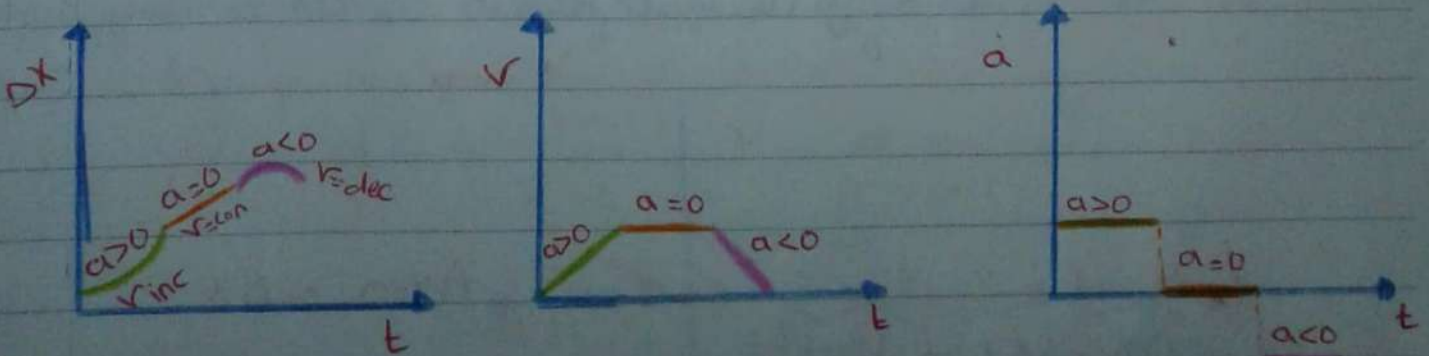
\* Now using e.1

$$t = \frac{v_f - v_i}{a} \quad \text{So} \quad \Delta x = v_i \left( \frac{v_f - v_i}{a} \right) + \frac{1}{2} a \left( \frac{v_f - v_i}{a} \right)^2$$

$$\boxed{4} \quad v_f^2 = v_i^2 + 2a \Delta x$$

\* Now  $\Delta x = \underbrace{v_i t}_{\text{constant}} + \frac{1}{2} \underbrace{at^2}_{\text{constant}}$

$$Ax^2 + Bx + C = 0 \quad \text{بواسطة التربيعية}$$



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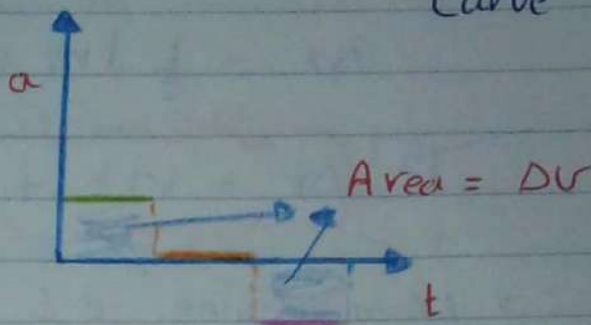
$$a = \frac{dv}{dt}$$

$$\int_{v_i}^{v_f} dv = \int_0^t a dt$$

$$\Delta v = \int_0^t a dt$$

area

$\Delta v \equiv$  Area under  $a \text{ vs } t$  curve



$\hookrightarrow \Delta v = a \Delta t$

$$v_f - v_i = a(t_f - t_i)$$

$$v_f = v_i + at$$

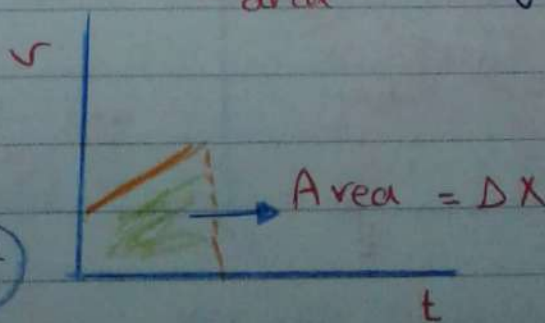
Bul  $v_i = \frac{dx}{dt}$

$$\frac{dx}{dt} = v_i + at$$

$$\int_{x_i}^{x_f} dx = \int_0^t (v_i + at) dt$$

$$\Delta x = \int_0^t (v_i + at) dt$$

$\Delta x \equiv$  Area under  $v \text{ vs } t$  curve



$\hookrightarrow \Delta x = v_i t + \frac{1}{2} at^2$

\* Sally Bani Yaseen smile

\* Free Falling :-

سقوط حر (تسارع الجاذبية)

- motion due to gravity

حركة في مجال الجاذبية الأرضية

$$a \equiv g = 9.8 \text{ ms}^{-2} \approx 10 \text{ ms}^{-2} \text{ down}$$

$$v_f = v_i + gt$$

$$\Delta y = \frac{1}{2} (v_i + v_f) t$$

$$\Delta y = v_i t + \frac{1}{2} gt^2$$

$$v_f^2 = v_i^2 + 2g \Delta y$$

$$g = 10 \text{ down} \\ = -10$$

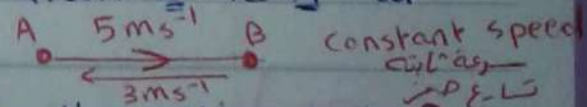
12-2-2018

\* Velocity  $\rightarrow$  Magnitude  $\rightarrow$  speed  
 $\searrow$  direction

$$v = -5 \text{ ms}^{-1}$$

التي هي أولًا، أو التنازل speed

Q3) A person walks first at a constant speed of 5 m/s along a straight line from point A to point B and then back along the line from B to A at a constant speed of 3 m/s



a) what is her average speed over the entire trip?

Average speed =  $\frac{\text{total } d}{\text{total } t}$

$$S = \frac{d}{t} = \frac{X + X}{t + t}$$

نصف المسافة  
الوقت فاصل

$$= \frac{2X}{t_1 + t_2} \Rightarrow X = 5t_1 \rightarrow t_1 = \frac{X}{5}$$

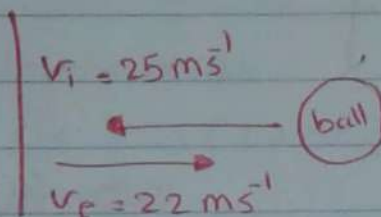
$$B \rightarrow A \rightarrow X = 3t_2 \rightarrow t_2 = \frac{X}{3}$$

$$S = \frac{2X}{\frac{X}{5} + \frac{X}{3}} = ( \quad ) \text{ ms}^{-1}$$

\* Sally Bani Vaseen smile...

⑥ What is her average velocity over the entire trip?  
Average velocity = 0       $\Delta x / \Delta t = 0$

Q14) A 500-g Super Ball traveling at  $25 \text{ m s}^{-1}$  bounces off a brick wall and rebounds at  $22 \text{ m s}^{-1}$ . A high-speed camera records this event. If the ball is in contact with the wall for 3.5 ms, what is the magnitude of the average acceleration of the ball during this time interval?



ارتدت

$$a = \frac{v_f - v_i}{\Delta t}$$

$$= \frac{22 - (-25)}{3.5 \times 10^{-3}}$$

$$= \frac{22 + 25}{3.5 \times 10^{-3}}$$

$$a = 13.4 \times 10^3 \text{ m s}^{-2}$$

Q21) A particle moves along the x axis according to the equation  $x = 2 + 3t - t^2$  where x is in meters and t is in seconds. At  $t = 3 \text{ s}$ , find :-

a) the position of the particle?

← تحديد السرعة  
القطعية  
تحديد الزمن  
المعطى

$$x = 2 + 3t - t^2 \quad \rightarrow \quad t = 3 \text{ s}$$

$$x = 2 + 3 \times 3 - 3^2$$

$$x = 3 \text{ m}$$

#Sally Bani yaseen  
smile



b) its velocity ?

مستوية التغير  
للمعادلة الخطية

$$\text{Velocity} = \frac{dx}{dt} = 3 - 2t$$

$$= 3 - 2 \times 3$$

$$= -3 \text{ m s}^{-1}$$

$$\text{speed} = \underline{3 \text{ m}}$$

c) its acceleration ?

مستوية التغير  
للانحنائية  
Velocity

$$a = \frac{dv}{dt} = -2 \text{ m s}^{-2}$$

Q29) An object moving with uniform acceleration has a velocity of  $12 \text{ cm s}^{-1}$  in the positive X direction when its X coordinate is  $3 \text{ cm}$ . If its X coordinate 2 s later is  $-5 \text{ cm}$  what is its acceleration ?

$$v = 12 \text{ cm s}^{-1}, \quad X_i = 3 \text{ cm}, \quad t = 2 \text{ s}, \quad X_f = -5 \text{ cm}$$

$$\Delta X = -8 \text{ cm}$$

$$\Delta X = v_i t + \frac{1}{2} a t^2$$

$$-8 = 12 \times 2 + \frac{1}{2} a \cdot 4$$

$$-8 = 24 + 2a \rightarrow \frac{2a}{2} = \frac{-32}{2}$$

$$a = -16 \text{ cm s}^{-2}$$

\* Find  $v_f$

$$v_f = v_i + at$$

$$v_f = 12 + -16 \times 2$$

$$v_f = -20 \text{ cm s}^{-1}$$

التسارع في اتجاه الخاسر جان هيرك زادت عندي السرعة

# Sally Bani Yaseen

smile

Q38) A particle moves along the x axis. Its position is given by the equation  $X = 2 + 3t - 4t^2$  with x in meters and t in seconds. Determine

- its position when it ~~returns to the~~ changes direction
- velocity " " returns to the position it had at  $t=0$

$$X = 2 + 3t - 4t^2$$

Position when change direction  $v=0$

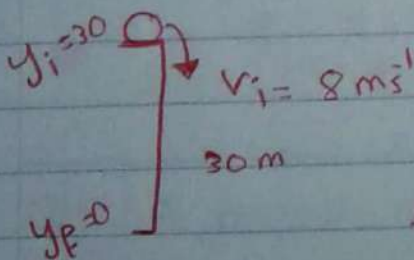
$$v = \frac{dx}{dt} \rightarrow v = 3 - 8t$$

$$0 = 3 - 8t$$

$$3 = 8t \rightarrow t = \frac{3}{8} \text{ Sec.}$$

$$X = 2 + 3 \times \frac{3}{8} - 4 \left( \frac{9}{64} \right) = ( ) \text{ m}$$

Q51) A ball is thrown directly downward with an initial speed of  $8 \text{ ms}^{-1}$  from a height of 30 m. After what time interval does it strike the ground?



$$\Delta y = y_f - y_i = -30 \text{ m}$$

$$v_i = -8 \text{ ms}^{-1}$$

$$g = -10$$

$$\Delta y = v_i t - \frac{1}{2} g t^2$$

$$-30 = -8t - \frac{1}{2} 10 t^2$$

$$5t^2 + 8t - 30 = 0$$

\* Sally Bani Yaseen  
smile

عالم الكون  $-b \pm \sqrt{b^2 - 4ac}$   $\rightarrow$   $ac$

$\swarrow$   $\searrow$   
 جداول  $2a$   $\swarrow$   $\searrow$   
 $t$   $t_2$

$$t = \frac{-8 \pm \sqrt{64 + 4 \times 5 \times 30}}{2 \times 5}$$

$$t = \frac{-8 \pm 25.7}{10}$$

نقل 8 الى اليمين  $\leftarrow$   
 حاسبه سابق

$$t = \frac{-8 + 25.7}{10} \rightarrow t = 1.77 \text{ sec.}$$

\* Find its speed when striking the ground

$$v_f = v_i + gt$$

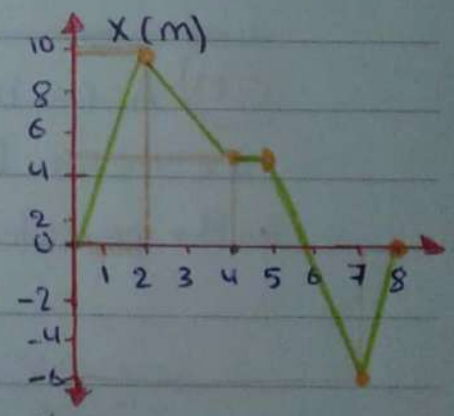
$$= -8 - 10 \times 1.77$$

$$= -25.7 \text{ ms}^{-1}$$

speed =  $25.7 \text{ ms}^{-1}$

**How**

Q1) The position versus time for a certain particle moving along the x axis is shown. Find the average velocity in the time intervals



(a) 0 to 2s

$$v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

$$= \frac{10 - 0}{2 - 0} \Rightarrow v = \frac{10}{2} = 5 \text{ ms}^{-1}$$

(b) 0 to 4s

$$v = \frac{\Delta x}{\Delta t} = \frac{5.5 - 0}{4 - 0} = 1.37 \text{ ms}^{-1}$$

(c) 2s to 4s

$$v = \frac{5.5 - 10}{4 - 2} = \frac{-4.5}{2} = -2.25 \text{ ms}^{-1}$$

(d) 4s to 7s

$$v = \frac{-6 - 5.5}{7 - 4} = \frac{-11.5}{3} = -3.8 \text{ m/s}$$

(e) 0 to 8s  $v = 0$

\* Sally Bari vsisten

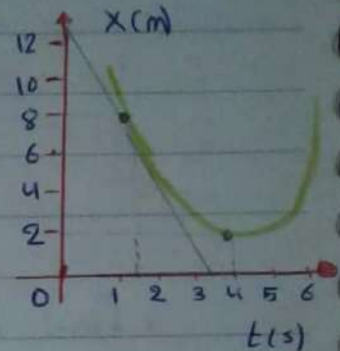
(Q7) A position-time graph for a particle moving along the x axis.

(a) Find the average velocity in the time interval

$$t = 1.50 \text{ to } t = 4 \text{ s}$$

$$v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

$$= \frac{2 - 8}{4 - 1.5} = \frac{-6}{2.5} = -2.4 \text{ m s}^{-1}$$



(b) Determine the instantaneous velocity at  $t = 2 \text{ s}$  by measuring the slope of the tangent line shown in the graph

$$v = \frac{\Delta x}{\Delta t} = \frac{2 - 4}{1} = -2 \text{ m s}^{-1}$$

(c) At what value of  $t$  is the velocity zero  
 $t = 3 \text{ s to } t = 5 \text{ s}$

(Q19) A particle starts from rest and acceleration as shown. Determine

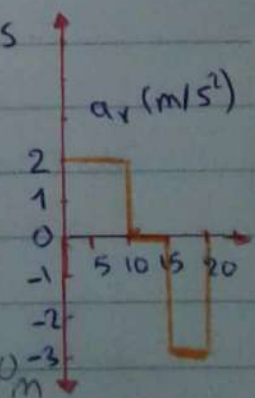
(a) the particle's speed at  $t = 10.0 \text{ s}$  and at  $t = 20.0 \text{ s}$

$$a = \frac{\Delta v}{\Delta t}$$

$$v = a \Delta t$$

$$v = (-3 - 0) \times (20 - 10)$$

$$v = -3 \times 10 = -30 \text{ m s}^{-1} \text{ speed} = 30 \text{ m s}^{-1}$$



(b) the distance traveled in the first 20 s

$$s = \frac{D}{t}$$

$$30 = \frac{D}{20}$$

$$D = 600 \text{ m}$$

smile

#Sally Bani Yaseen

Q20) An object moves along the x axis according to the equation  $x = 3t^2 - 2t + 3$  where x is in meters and t is in seconds. Determine.

(a) The average velocity between  $t = 2\text{ s}$  to  $t = 3\text{ s}$

$$v = \frac{\Delta x}{\Delta t}$$

$$x_f = 3 \times 9 - 2 \times 3 + 3 = 24\text{ m}$$

$$x_i = 3 \times 4 - 2 \times 2 + 3 = 11\text{ m}$$

$$v = \frac{24 - 11}{3 - 2} = 13\text{ m}\cdot\text{s}^{-1}$$

(b) the instantaneous speed at  $t = 2\text{ s}$  and  $t = 3\text{ s}$

$$v = \frac{dx}{dt} \Rightarrow v = 6t - 2$$

$$\text{at } t = 2\text{ s} \rightarrow v = 6 \times 2 - 2 = 10\text{ m}\cdot\text{s}^{-1} \rightarrow \text{speed} = 10\text{ m}\cdot\text{s}^{-1}$$

$$\text{at } t = 3\text{ s} \rightarrow v = 6 \times 3 - 2 = 16\text{ m}\cdot\text{s}^{-1} \rightarrow \text{speed} = 16\text{ m}\cdot\text{s}^{-1}$$

(c) The average acceleration between  $t = 2\text{ s}$  and  $t = 3\text{ s}$

$$a = \frac{\Delta v}{\Delta t} \Rightarrow$$

d) the instantaneous acceleration at  $t = 2\text{ s}$  and  $t = 3\text{ s}$

$$a = \frac{dv}{dt} \Rightarrow a = 6\text{ m}\cdot\text{s}^{-2}$$

e) At what time is the object at rest?

# Sally Bani Vaseen  
smile

(Q28) A truck covers 40 m in 8.50 s while smoothly slowing down to a final speed of  $2.8 \text{ m s}^{-1}$

(a) Find its original speed  $X = 40 \text{ m}$ ,  $t = 8.5 \text{ s}$ ,  $v_f = 2.8 \text{ m s}^{-1}$

$$\Delta x = \frac{1}{2}(v_i + v_f)t \Rightarrow 40 = \frac{1}{2}(v_i + 2.8)8.5 \quad 4.25$$

$$9.41 = v_i + 2.8$$

$$v_i = 6.61 \text{ m s}^{-1}$$

(b) Find its acceleration

$$v_f = v_i + at$$

$$2.8 = 6.61 + a * 8.5$$

$$6.61 - 6.61 -$$

$$\frac{a * 8.5 = -3.81}{8.5 \quad 8.5}$$

$$a = 0.44 \text{ m s}^{-2}$$

(Q35) The driver of a car slams on the brakes when he sees a tree blocking the road. The car slows uniformly with an acceleration of  $-5.6 \text{ m s}^{-2}$  for 4.2 s making straight skid marks 62.4 m long, all the way to the tree. with what speed does the car then strike the tree?

$$X = 62.4 \text{ m}, \quad a = -5.6 \text{ m s}^{-2}, \quad t = 4.2 \text{ s}$$

$$\Delta x = v_i t + \frac{1}{2} a t^2 \Rightarrow 62.4 = v_i * 4.2 + \frac{1}{2} * -5.6 * (4.2)^2$$

$$62.4 = v_i * 4.2 + 49.3$$

$$\frac{13.1}{4.2} = \frac{v_i * 4.2}{4.2}$$

$$v_i = 3.1 \text{ m s}^{-1}$$

$$v_f = v_i + at$$

$$= 3.1 + -5.6 * 4.2$$

$$v_f = -10.5 \text{ m s}^{-1}$$

\* Sally Bani Vaseen

smile for me

(Q48) A baseball is hit so that it travels straight upward after being struck by the bat. A fan observes that it takes 3s for the ball to reach its maximum height. Find

(a) the ball's initial velocity

(b) the height it reaches

---

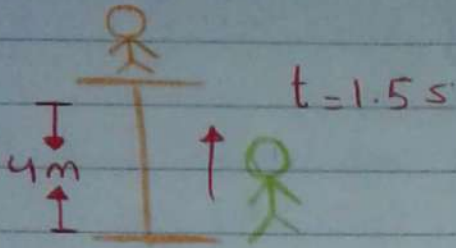
(Q49) It possible to shoot an arrow at a speed as high as  $100 \text{ m}\cdot\text{s}^{-1}$

(a) IF Friction can be ignored, how high would an arrow launched at this speed rise if shot straight up?

(b) How long would the arrow be in the air

\* Sally Bani Vaseen  
smile...

Q 53 A student throws a set of keys vertically upward to her sorority sister, who is in a window 4 m above. The second student catches the keys 1.5 s later.



a) with what initial velocity were the keys thrown?

$$\Delta y = +4 \text{ m}$$

$$t = 1.5 \text{ s}$$

$$g = -10 \text{ m s}^{-2}$$

$$v_i = ??$$

$$\Delta y = v_i t + \frac{1}{2} g t^2$$

$$4 = v_i * 1.5 + \frac{1}{2} (-10) * (1.5)^2$$

$$v_i \approx 10 \text{ m s}^{-1}$$

b) what was the velocity of the keys just before they were caught?  $v_f = ??$

$$v_f = v_i + g t$$

$$v_f = 10 - 10 * 1.5$$

$$v_f = -5 \text{ m s}^{-1}$$

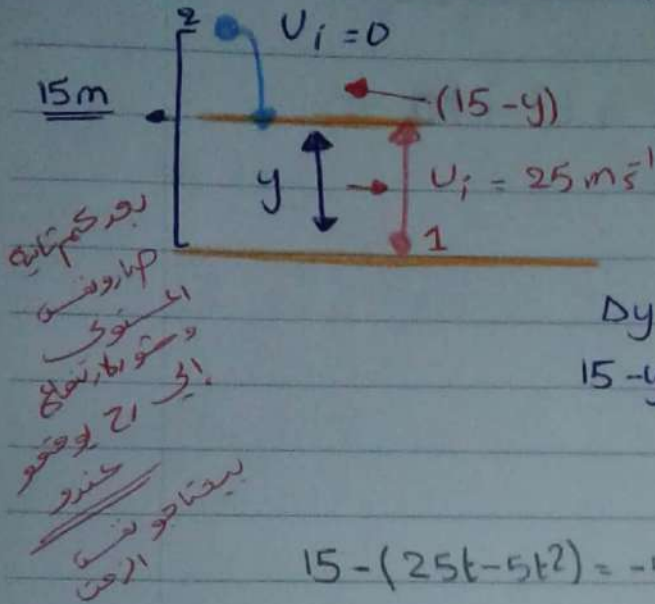
سرعة الارتفاع

# Sally Bani Yaseen

smile...



Q52 A ball is thrown upward from the ground an initial speed of  $25 \text{ m s}^{-1}$  at the same instant, another ball is dropped from building 15 m high. After how long will the balls be at the same height above the ground?



ball 2  
 $\Delta y = -(15 - y)$       $g = -10 \text{ m s}^{-2}$   
 $u_i = 0$

$$\Delta y = u_i t + \frac{1}{2} g t^2$$

$$15 - y = 0t - 5t^2$$

ball 1  
 $\Delta y = y$   
 $u_i = 25 \text{ m s}^{-1}$   
 $\Delta y = u_i t + \frac{1}{2} g t^2$   
 $y = 25t - 5t^2$

$$15 - (25t - 5t^2) = -5t^2$$

$$15 - 25t = 0$$

$$25t = 15$$

$$t = 0.6 \text{ s} \Rightarrow$$

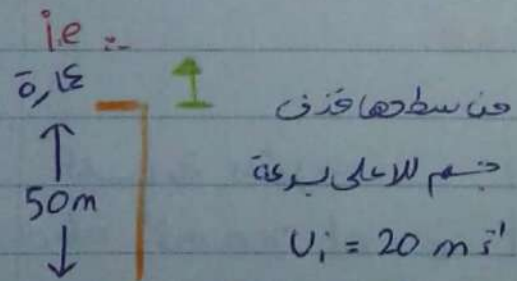
$$y = 25t - 5t^2$$

$$= 25 \times 0.6 - 5(0.6)^2$$

$$= 15 - 5 \times 0.35$$

$$= 15 - 1.8$$

$$= 13.2 \text{ m}$$



$\Delta y = -50$       $u_i = +20$       $t = ??$   
 $g = -10 \text{ m s}^{-2}$

$$\Delta y = u_i t + \frac{1}{2} g t^2$$

$$-50 = 20t + 5t^2$$

$$5t^2 - 20t - 50 = 0$$

$$t^2 - 4t - 10 = 0$$

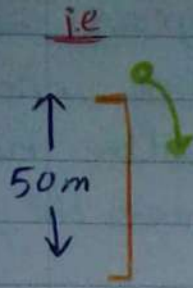
رجع كم الوقت الذي استغرقه في الت

على المميز  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\frac{-4 \pm \sqrt{16 - 4 \times -10}}{2} \Rightarrow$$

$t = 1.75 \text{ s}$  \* Sally Bari Vavee

smile



قذف للأعلى

$$u_i = 20 \text{ m}$$

$$t = ??$$

الأضلاع  $u_i = -20$

$$50 = -20t - 5t^2$$

$$\frac{-20t}{-5} - \frac{5t^2}{-5} + \frac{50}{-5} = 0$$

$$4t + t^2 + 10 = 0$$

$$t^2 + 4t + 10 = 0$$

مع المميز

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-4 \pm \sqrt{16 - 4 \times 1 \times 10}}{2}$$

t

i.e

$$v_i = 100 \text{ m/s}$$

t = ??

\* الارتفاع صفر مرة زعمنا الزوال

الزحف للارتفاع

\* اح ورجح للارتفاع الزاوية = 0

للعودة

على الأرض

$$\Delta y = v_i t + \frac{1}{2} g t^2$$

$$0 = 100t + 5t^2$$

$$0 = t(100 + 5t)$$

$$t = 20 \text{ sec.}$$

الارتفاع المقطوعة total أقصى ارتفاع

\* at what time the object is at 200 m above ground

$$200 = 100t - 5t^2$$

$$t^2 - 20t + 40 = 0$$

$$5t^2 - 100t + 200 = 0$$

$$t = \frac{20 \pm \sqrt{400 - 4 \times 1 \times 40}}{2}$$

$$t = \frac{20 \pm \sqrt{240}}{2}$$

$$\rightarrow t = 2.25 \text{ sec}$$

# Sally Bari Yasser

$$\text{or } t = 17.7 \text{ sec}$$

smile

14-2-2018

No.

25

i.e

التقاء

t

y

$u_i = 10 \text{ m s}^{-1}$

t-2

y

$u_i = 20 \text{ m s}^{-1}$

After 2 sec

\* الازاحة فتاوه

ليس زحنا مختلف

$y = 20(t-2) - 5(t-2)^2$

$y = 10t - 5t^2$

#sally Bani yaseen

# Ch 3:- Vectors & Scalars

قياسات  
بها مقدار واتجاه

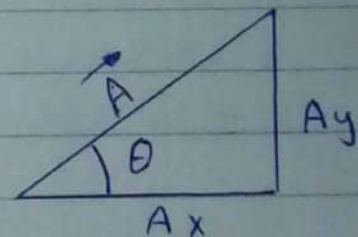
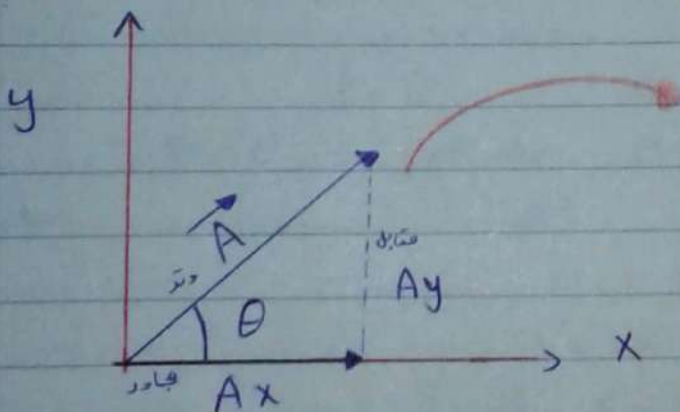
قياس  
بها مقدار، واتجاه جمعاً جبرياً

\* **Vectors** :- quantities that characterized by magnitude & direction

\* **Scalars** :- quantities that characterized by magnitude only

19-2-2018

\* **Component method** \*



$\frac{\text{الجوار}}{\text{وتر}} \cos \theta = \frac{Ax}{A} \Rightarrow Ax = A \cos \theta$

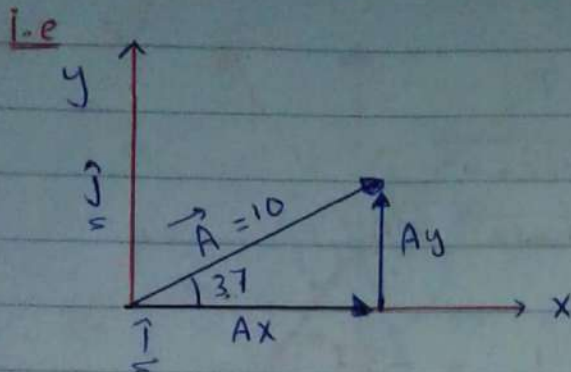
$\frac{\text{المقابل}}{\text{وتر}} \sin \theta = \frac{Ay}{A} \Rightarrow Ay = A \sin \theta$

$\frac{\text{المقابل}}{\text{الجوار}} \tan \theta = \frac{Ay}{Ax} \Rightarrow \theta = \tan^{-1} \frac{Ay}{Ax}$

$\frac{\text{قياسا فوسل}}{|\vec{A}|} = \sqrt{Ax^2 + Ay^2}$

# Sally Bari yaseen

smile



$$A_x = |\vec{A}| \cos \theta = 10 \cos 37 = 6$$

$$A_y = |\vec{A}| \sin \theta = 10 \sin 37 = 8$$

$$\vec{A} = 6\hat{i} + 8\hat{j}$$

unit vector
unit vector  
x-axis
y-axis

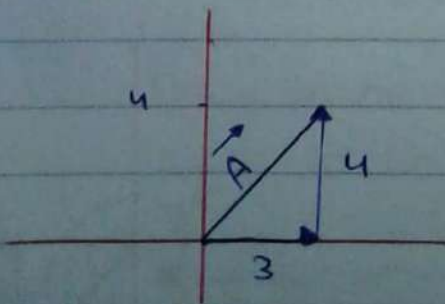
i.e Consider the vector  $\vec{A} = 3\hat{i} + 4\hat{j}$   
 Find the magnitude & direction of  $\vec{A}$

$$* \text{ Magnitude } \equiv |\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$|\vec{A}| = \sqrt{3^2 + 4^2} = 5$$

$$* \text{ direction } \equiv \theta = \tan^{-1} \frac{A_y}{A_x}$$

$$\theta = \tan^{-1} \frac{4}{3} = 37^\circ$$



# Sally Bani vaseen

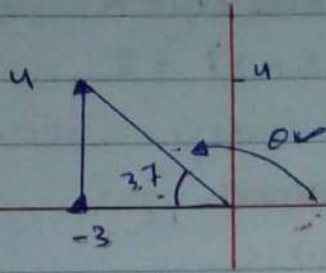
i.e  $\vec{A} = -3\hat{i} + 4\hat{j}$

$$\text{Magnitude} = |\vec{A}| = \sqrt{(-3)^2 + 4^2}$$

$$= \sqrt{9 + 16} = 5$$

$$\text{direction} = \theta = \tan^{-1} \frac{4}{-3} = -37^\circ$$

نقيس الزاوية  
عناقص السيلان  
الموجب على عقارب  
الساعة

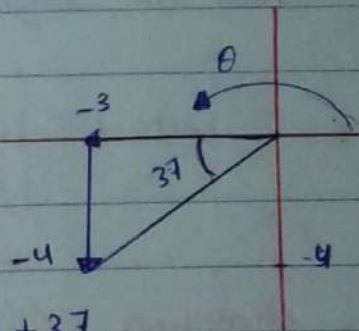


$$\theta = 180 - 37 = 143^\circ$$

i.e  $\vec{A} = -3\hat{i} + 4\hat{j}$

$$|\vec{A}| = 5$$

$$\theta = \tan^{-1} \frac{-4}{-3} = 37^\circ$$

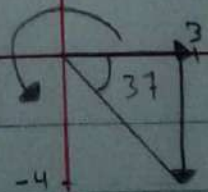


$$\theta = 180 + 37$$

H.w  $\vec{A} = 3\hat{i} - 4\hat{j}$

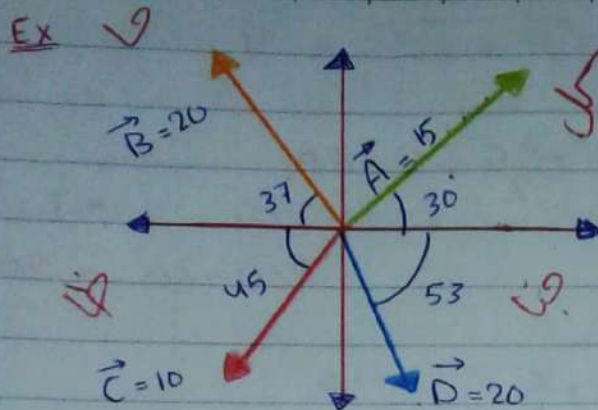
$$|\vec{A}| = 5$$

$$\theta = \tan^{-1} \frac{-4}{3} = -37^\circ$$



$$\theta = 360 - 37$$

#Sally Bani Vaseen



الجهة الحارة  
ذلك كل حثبه للمركبات

Find the resultant vector

**A** →  $A_x = |\vec{A}| \cos \theta = 15 \cos 30 = 13$   
 $A_y = |\vec{A}| \sin \theta = 15 \sin 30 = 7.5$   
 $(\vec{A} = 13\hat{i} + 7.5\hat{j})$

**B** →  $B_x = -20 \cos 37 = -16$   
 $B_y = 20 \sin 37 = 12$   
 $(\vec{B} = -16\hat{i} + 12\hat{j})$

**C** →  $C_x = -10 \cos 45 = -7$   
 $C_y = -10 \sin 45 = -7$   
 $(\vec{C} = -7\hat{i} + 7\hat{j})$

**D** →  $D_x = 20 \cos 53 = 12$   
 $D_y = -20 \sin 53 = -16$   
 $(\vec{D} = 12\hat{i} - 16\hat{j})$

Now

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

$$\vec{R} = (13 + -16 + -7 + 12)\hat{i} + (7.5 + 12 + -7 + -16)\hat{j}$$

$$\vec{R} = 2\hat{i} - 3.5\hat{j}$$

# Sally Bani Vaseen  
 smile

magnitude  $|\vec{R}| = \sqrt{2^2 + 3.5^2} = \underline{4}$

direction  $\theta = \tan^{-1} \frac{-3.5}{2} = -60$

ع.ل.ع

$$360 - 60 = 300^\circ$$

Ex Let  $\vec{A} = 2\hat{i} - 3\hat{j}$

$$\vec{B} = -6\hat{i} + 8\hat{j}$$

Find  $|\vec{A}|$ ,  $|\vec{B}|$ ,  $|\vec{A} + \vec{B}|$

Find  $\vec{R} = 3\vec{A} - 2\vec{B}$

$$|\vec{A}| = \sqrt{4+9} = \sqrt{13} = 3.6$$

$$|\vec{B}| = \sqrt{36+64} = 10$$

$$\vec{A} + \vec{B} = -4\hat{i} + 5\hat{j}$$

$$(2 - 6)\hat{i} + (-3 + 8)\hat{j} \\ = -4\hat{i} + 5\hat{j}$$

$$|\vec{A} + \vec{B}| = \sqrt{16+25} = 6.4$$

$$|\vec{A} + \vec{B}| \neq |\vec{A}| + |\vec{B}|$$

$$\vec{R} = 3\vec{A} - 2\vec{B}$$

$$3\vec{A} = 6\hat{i} - 9\hat{j}$$

$$2\vec{B} = -12\hat{i} + 16\hat{j}$$

$$3\vec{A} - 2\vec{B} = (6 - (-12))\hat{i} + (-9 - 16)\hat{j}$$

$$\vec{R} = 18\hat{i} - 25\hat{j}$$

# Sally Bani. vareen

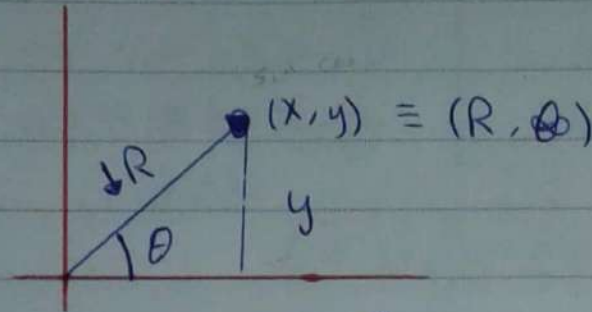
smile...



in general  $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

في الاتجاهات  $\hat{i}, \hat{j}, \hat{k}$

\* Polar coordinates -  $\theta$  - المناء القطبي



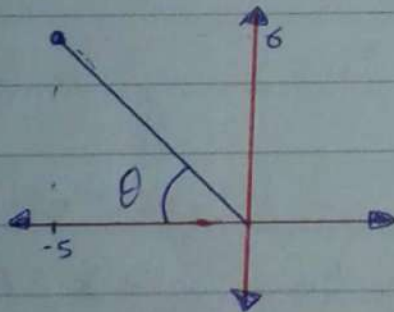
$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\theta = \tan^{-1} \frac{y}{x}$$

$$|\vec{r}| = \sqrt{x^2 + y^2}$$

$(-5, 6)$  Convert to  $(r, \theta)$



$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{25 + 36} \approx 7$$

$$\theta = \tan^{-1} \frac{6}{-5} = -50$$

$$\theta = 180 - 50$$

$$\theta = 130$$

$$(-5, 6) \equiv (7, 130^\circ)$$

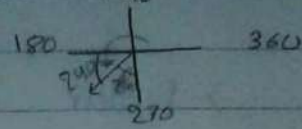
\* Sally Bani yaseen

smile

H-w

32

Q1 The polar coordinates of a point are  $r=5.50$  m and  $\theta=240^\circ$ . what are the Cartesian coordinates of this point?



$$\begin{aligned} \theta - 180 \\ 240 - 180 \end{aligned}$$

$$x = r \cos \theta$$

$$x = 5.50 \cos 60$$

$$x = 5.50 \times 0.5$$

$$x = 2.75$$

$$y = r \sin \theta$$

$$y = 5.50 \sin 60$$

$$y = 5.50 \times 0.86$$

$$y = 4.73$$

$$(5.50, 240^\circ) \equiv (2.75, 4.73)$$

Q2 The rectangular coordinates of a point are given by  $(2, y)$  and its polar coordinates  $(r, 30^\circ)$ .

Determine :-

a, b) the value of y and r

$$x = r \cos \theta$$

$$2 = r \cos 30^\circ$$

$$2 = r \times 0.86$$

$$\frac{2}{0.86} = \frac{r \times 0.86}{0.86}$$

$$\boxed{r = 2.3}$$

$$y = r \sin \theta$$

$$y = 2.3 \sin 30$$

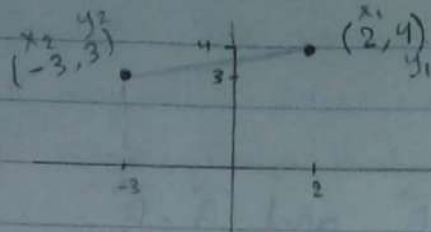
$$y = 2.3 \times 0.5$$

$$\boxed{y = 1.15}$$

\* Sally Bani Vaseen  
smile

Q3 Two points in the xy plane have Cartesian coordinates  $(2, 4) m$  and  $(-3, 3) m$ . Determine

a) the distance between these points



$$\begin{aligned} \text{Distance} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-3 - 2)^2 + (3 - 4)^2} \\ &= \sqrt{25 + 1} = \sqrt{26} = 5.09 \text{ m} \end{aligned}$$

b) their polar coordinates

(2, 4)  $r = \sqrt{x^2 + y^2}$   
 $r = \sqrt{4 + 16} = \sqrt{20} = 4.4$

$\theta = \tan^{-1} \frac{y}{x} = 63.4^\circ$

$(4.4, 63.4^\circ)$

(-3, 3)  $r = \sqrt{9 + 9} = \sqrt{18}$   
 $r = 4.2$

$\theta = \tan^{-1} \frac{-3}{3} = -45^\circ$

$\theta = 180 - 45$

$(4.2, 135^\circ)$

Q10 A force  $\vec{F}_1$  of magnitude 6 units acts on an object at the origin in a direction  $\theta = 30^\circ$  above the positive x axis. A second force  $\vec{F}_2$  of magnitude 5 units acts on the object in the direction of the positive y axis.

Find graphically the magnitude and direction of the resultant force  $\vec{F}_1 + \vec{F}_2$

$\vec{F}_1 = 6\hat{i} + 3.75\hat{j}$

$x = r \cos \theta$   
 $6 = r \cos 30 \rightarrow 6 = r \times 0.866$   
 $r = 7.5$



$\vec{F}_2 = 0\hat{i} + 5\hat{j}$

$\vec{R} = \vec{F}_1 + \vec{F}_2$

$y = r \sin 30$

$y = 7.5 \times 0.5$

$\vec{R} = 6\hat{i} + 8.75\hat{j}$

$|\vec{R}| = \sqrt{36 + 76.5}$   
 $= 10.6$

$y = 3.75$

$\theta = \tan^{-1} \frac{8.75}{6} = 55.5^\circ$  smile...

Q23 Consider the two vectors  $\vec{A} = 3\hat{i} - 2\hat{j}$  and  $\vec{B} = -\hat{i} - 4\hat{j}$  calculate

a)  $\vec{A} + \vec{B}$

b)  $\vec{A} - \vec{B}$

c)  $|\vec{A} + \vec{B}|$

d)  $|\vec{A} - \vec{B}|$

e) the directions of  $\vec{A} + \vec{B}$  and  $\vec{A} - \vec{B}$

a)  $\Rightarrow \vec{A} + \vec{B} = 2\hat{i} - 6\hat{j}$

$$\theta = \tan^{-1} \frac{-6}{2} = -71.6^\circ \quad \theta = 180 - 71.6^\circ$$

$$\theta = 108.4^\circ$$

b)  $\Rightarrow \vec{A} - \vec{B} = 4\hat{i} + 2\hat{j}$

$$\theta = \tan^{-1} \frac{2}{4} = 26.5^\circ$$

c)  $\vec{A} + \vec{B} = 2\hat{i} - 6\hat{j}$

$$|\vec{A} + \vec{B}| = \sqrt{4 + 36} = \sqrt{40} = 6.32$$

d)  $\vec{A} - \vec{B} = 4\hat{i} + 2\hat{j}$

$$|\vec{A} - \vec{B}| = \sqrt{16 + 4} = \sqrt{20} = 4.47$$

e)  $\vec{A} + \vec{B} \quad \theta = 108.4^\circ$

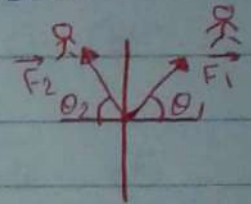
$\vec{A} - \vec{B} \quad \theta = 26.5^\circ$

\*Sally Bani yaseen

smile...

Q29 The helicopter view in Fig shows two people pulling on a stubborn mule. The person on right pulls with a force  $\vec{F}_1$  of magnitude 120 N and direction of  $\theta_1 = 60^\circ$ . The person on the left pulls with a force  $\vec{F}_2$  of magnitude 80 N and direction of  $\theta_2 = 75^\circ$ . Find

- the single force that is equivalent to the two forces shown
- the force that third person would have to exert on the mule to make resultant force equal to zero



Q31 Consider the three displacement vectors  $\vec{A} = (3\hat{i} - 3\hat{j})\text{ m}$ ,  $\vec{B} = (1\hat{i} - 4\hat{j})\text{ m}$  and  $\vec{C} = (-2\hat{i} + 5\hat{j})\text{ m}$ . Use the component method to determine.

- the magnitude and direction of  $\vec{D} = \vec{A} + \vec{B} + \vec{C}$
- " " " " "  $\vec{E} = -\vec{A} - \vec{B} + \vec{C}$

a)  $\vec{D} = \vec{A} + \vec{B} + \vec{C} \Rightarrow \vec{D} = 2\hat{i} - 2\hat{j}$

magnitude  $|\vec{D}| = \sqrt{4+4} = \sqrt{8} = 2.8$

Direction  $\theta = \tan^{-1} \frac{-2}{2} = -45^\circ$  or  $180 - 45^\circ$

b)  $\vec{E} = -\vec{A} - \vec{B} + \vec{C} \Rightarrow \vec{E} = 6\hat{i} + 12\hat{j}$   $\theta = 135^\circ$

Magnitude  $|\vec{E}| = \sqrt{36+144} = \sqrt{180} = 13.4$

Direction  $\theta = \tan^{-1} \frac{12}{6} = 63.4^\circ$

$\theta = 180 - 63.4^\circ$  or  $\theta = 116.6^\circ$

Q36 Given the displacement vectors  $\vec{A} = (3\hat{i} - 4\hat{j} + 4\hat{k})\text{m}$  and  $\vec{B} = (2\hat{i} + 3\hat{j} - 7\hat{k})\text{m}$  Find the magnitudes of following vectors and express each in terms of its rectangular components

a)  $\vec{C} = \vec{A} + \vec{B}$

b)  $\vec{D} = 2\vec{A} - \vec{B}$

a)  $\vec{C} = \vec{A} + \vec{B} \Rightarrow \vec{C} = 5\hat{i} - 1\hat{j} - 3\hat{k}$

$$|\vec{C}| = \sqrt{5^2 + (-1)^2 + (-3)^2} = \sqrt{32} = 5.9$$

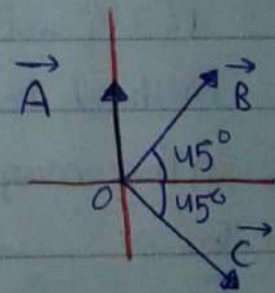
b)  $\vec{D} = 2\vec{A} - \vec{B} \Rightarrow \vec{D} = 2\hat{i} - 14\hat{j} + 22\hat{k}$

$$|\vec{D}| = \sqrt{2^2 + (-14)^2 + 22^2} = 684$$

Q38 Three displacement vectors of a croquet ball where  $|\vec{A}| = 20$  units,  $|\vec{B}| = 40$  units and  $|\vec{C}| = 30$  units

Find a) the resultant in unit vector notation

b) the magnitude and direction of the resultant displacement



#Sally Bari yaseen

smile for life

Q20 A girl delivering newspapers covers her route by traveling 3 blocks west, 4 blocks north and then 6 blocks east.   
 غرب شمال شرق

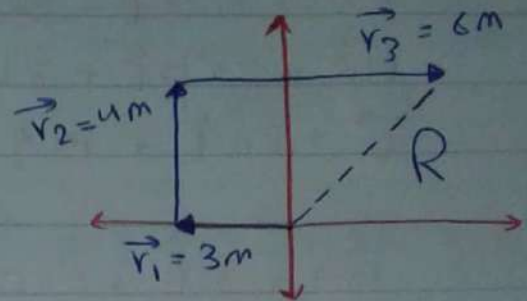
a) what is her resultant displacement

$$\vec{R} = \vec{r}_1 + \vec{r}_2 + \vec{r}_3$$

$$\vec{r}_1 = -3\hat{i} + 0\hat{j}$$

$$\vec{r}_2 = 0\hat{i} + 4\hat{j}$$

$$\vec{r}_3 = 6\hat{i} + 0\hat{j}$$



$$\vec{R} = 3\hat{i} + 4\hat{j}$$

magnitude  $|\vec{R}| = \sqrt{9+16} = 5$

direction  $\theta = \tan^{-1} \frac{4}{3} = 25.3$

b) What is the total distance she travels?

$$\text{Distance} = 4 + 3 + 6 = \underline{13 \text{ m}}$$

Q32 vector  $\vec{A}$  has x and y components of  $-8.70 \text{ cm}$  and  $15.0 \text{ cm}$  respectively. Vector  $\vec{B}$  has x and y components of  $13.2 \text{ cm}$  and  $-6.6 \text{ cm}$  respectively.

If  $\vec{A} - \vec{B} + 3\vec{C} = 0$  what are the components of  $\vec{C}$ ?

$$\vec{A} = -8.7\hat{i} + 15\hat{j}, \quad \vec{B} = 13.2\hat{i} - 6.6\hat{j}$$

$$\vec{A} - \vec{B} + 3\vec{C} = 0$$

$$3\vec{C} = \vec{B} - \vec{A} \Rightarrow \vec{C} = \frac{1}{3}(\vec{B} - \vec{A}) = \frac{1}{3}[(21.9)\hat{i} + (21.6)\hat{j}]$$

$$\vec{C} = 7.3\hat{i} + 7.2\hat{j}$$

H.w  $\vec{A} - \vec{B} + 3\vec{C} = 4\hat{i} + 6\hat{j}$

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smile...

Q37  $\vec{A} = 6\hat{i} - 8\hat{j}$   
 $\vec{B} = -8\hat{i} + 3\hat{j}$   
 $\vec{C} = 26\hat{i} + 19\hat{j}$

if  $a\vec{A} + b\vec{B} + \vec{C} = 0$   
 where  $a, b$  constants  
 Find  $a, b$

$a\vec{A} = 6a\hat{i} - 8a\hat{j}$   
 $b\vec{B} = -8b\hat{i} + 3b\hat{j}$   
 $\vec{C} = 26\hat{i} + 19\hat{j}$

$a\vec{A} + b\vec{B} + \vec{C} = 0$   
 $(6a - 8b + 26)\hat{i} + (-8a + 3b + 19)\hat{j} = 0$

$6a - 8b + 26 = 0$   
 $-8a + 3b + 19 = 0$   
 $3b = \frac{1}{3}(8a - 19)$

$b = \frac{1}{3}(8a - 19)$   
 $6a - 8b + 26 = 0$

$6a - 8 \times \frac{1}{3}(8a - 19) + 26 = 0$

$6a - 2.6(8a - 19) + 26 = 0$

$6a - (20.8a - 49.4) + 26 = 0$

$6a - 20.8a + 49.4 + 26 = 0$

$-14.8a + 75.4 = 0$

$\frac{-14.8a}{-14.8} = \frac{-75.4}{-14.8}$

$a = 5.09$

$6 \times 5.09 - 8b + 26 = 0$

$30.54 - 8b + 26 = 0$

$\frac{-8b}{-8} = \frac{-56.54}{-8}$

$b = 7.06$

smile...



H.W  $a\vec{A} + b\vec{B} + \vec{C} = 6\hat{i} + 10\hat{j}$

$$a(6\hat{i} - 8\hat{j}) + b(-8\hat{i} + 3\hat{j}) + (26\hat{i} + 19\hat{j}) = (6\hat{i} + 10\hat{j})$$

$$a(6\hat{i} - 8\hat{j}) + b(-8\hat{i} + 3\hat{j}) + (20\hat{i} - 9\hat{j}) = 0$$

$$(6a - 8b - 20)\hat{i} + (-8a + 3b - 9)\hat{j} = 0$$

$$6a - 8b - 20 = 0$$

$$-8a + 3b - 9 = 0$$

$$\frac{3b}{3} = \frac{8a + 9}{3}$$

$$b = \frac{1}{3}(8a + 9)$$

$$6a - 8b - 20 = 0$$

$$6a - 8 \times \frac{1}{3}(8a + 9) - 20 = 0 \Rightarrow 6a - 2.6(8a + 9) - 20 = 0$$

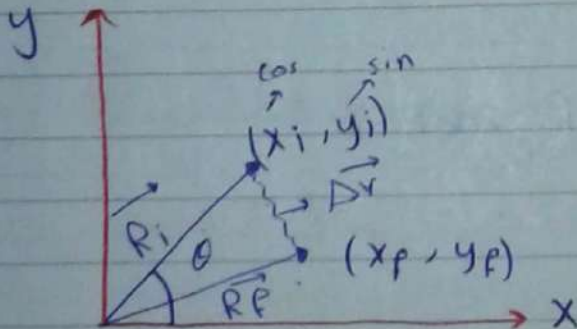
$$6a - 20.8a + 23.4 - 20 = 0$$

$$-14.8a + 3.4 = 0$$

$$-14.8a = -3.4$$

$$a = .22$$

### Ch:-4 Motion in 2-D



Position Vector

$$\vec{r} = x\hat{i} + y\hat{j}$$

Displacement  $\Delta\vec{r}$

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$$= (x_f\hat{i} + y_f\hat{j}) - (x_i\hat{i} + y_i\hat{j})$$

$$= (x_f - x_i)\hat{i} + (y_f - y_i)\hat{j}$$

$\Delta x$

$\Delta y$

Displacement vector

$$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j}$$

# Sally Bani vaseen

- Velocity Vector  $\vec{v}$

$$\vec{v} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x \hat{i} + \Delta y \hat{j}}{\Delta t}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j} \quad \text{or} \quad \vec{v} = \frac{d\vec{r}}{dt}$$

- acceleration vector

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a} = a_x \hat{i} + a_y \hat{j}$$

\* So equations of motion

$$\boxed{1} \quad \vec{v}_f = \vec{v}_i + \vec{a}t$$

$$\boxed{2} \quad \Delta \vec{r} = \frac{1}{2} (\vec{v}_i + \vec{v}_f) t$$

$$\boxed{3} \quad \Delta \vec{r} = \vec{v}_i t + \frac{1}{2} \vec{a} t^2$$

#Sally Baniyaseen

Q6 A particle initially located at the origin has an acceleration  $\vec{a} = 3\hat{j} \text{ m/s}^2$  and an initial velocity of  $\vec{v}_i = 5\hat{i} \text{ m/s}$ . Find  $\vec{v}_f$ , speed

$$\vec{a} = 0\hat{i} + 3\hat{j}, \quad \vec{v}_i = 5\hat{i} + 0\hat{j}, \quad \vec{r}_i = 0\hat{i} + 0\hat{j}$$

at  $t=2$   $\Delta \vec{r} = \vec{v}_i t + \frac{1}{2} \vec{a} t^2$

$$\vec{r}_f - \vec{r}_i = (5\hat{i} + 0\hat{j}) * 2 + \frac{1}{2} (0\hat{i} + 3\hat{j}) * 4$$

$$\vec{r}_f = 10\hat{i} + 6\hat{j}$$

$$\vec{v}_f = \vec{v}_i + \vec{a} t$$

$$\vec{v}_f = (5\hat{i} + 0\hat{j}) + (0\hat{i} + 3\hat{j}) * 2$$

$$\vec{v}_f = 5\hat{i} + 6\hat{j}$$

$$\text{Speed } |\vec{v}_f| = \sqrt{25 + 36}$$

Q9 A fish swimming in a horizontal plane has velocity  $\vec{v}_i = 4\hat{i} + 1\hat{j} \text{ m s}^{-1}$  at a point in the ocean where the position relative to a certain rock is  $\vec{r}_i = 10\hat{i} - 4\hat{j} \text{ m}$ . After the fish swims with constant acceleration for 20s its velocity is ~~at~~  
 $\vec{v}_f = (20\hat{i} - 5\hat{j}) \text{ m s}^{-1}$

a) What are the components of the acceleration of the fish & the  $\vec{v}_f$

$$\vec{v}_i = 4\hat{i} + 1\hat{j}, \vec{v}_f = 20\hat{i} - 5\hat{j}$$

$$\vec{r}_i = 10\hat{i} - 4\hat{j}, t = 20 \text{ s}$$

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

$$(20\hat{i} - 5\hat{j}) = (4\hat{i} + 1\hat{j}) + \vec{a} \times 20$$

$$\frac{16\hat{i} - 6\hat{j}}{20} = \frac{20\vec{a}}{20}$$

$$\vec{a} = \frac{0.8\hat{i} - 0.3\hat{j}}{\text{x y}}$$

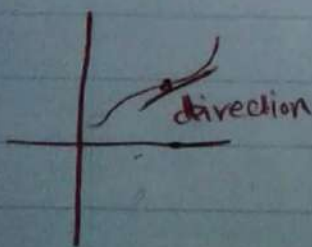
$$\vec{a} = \frac{1}{2}(\vec{v}_i + \vec{v}_f) t$$

$$\vec{v}_f - (10\hat{i} - 4\hat{j}) = \frac{1}{2}(4\hat{i} - 1\hat{j} + 20\hat{i} - 5\hat{j})$$

$$\vec{v}_f - (10\hat{i} - 4\hat{j}) = 24\hat{i} - 4\hat{j}$$

$$\vec{v}_f = 25\hat{i} - 4\hat{j}$$

b) What is the direction of motion at direction of the velocity  $\vec{v}_f$  t=25 sec



$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

$$\vec{v}_f = (4\hat{i} + 1\hat{j}) + (0.8\hat{i} - 0.3\hat{j}) \times 25$$

$$\vec{v}_f = (4\hat{i} + 1\hat{j}) + (20\hat{i} - 7.5\hat{j})$$

$$\vec{v}_f = (24\hat{i} - 6.5\hat{j})$$

$$|\vec{v}_f| = \sqrt{576 + 43.56} = 24.8$$

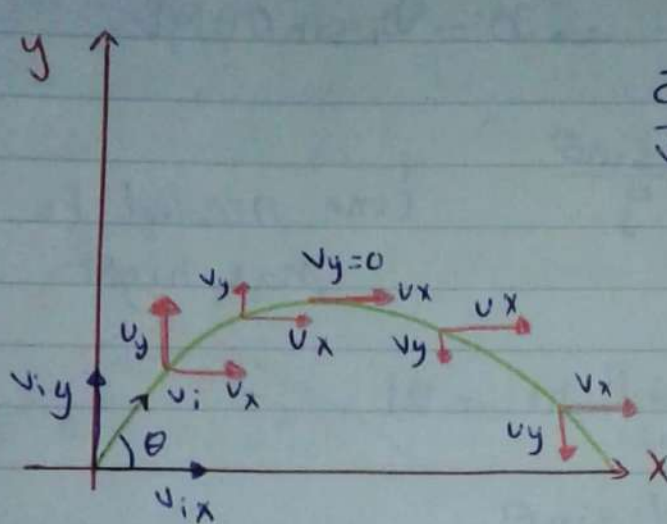
$$\theta = \tan^{-1} \frac{-6.5}{24} = -15^\circ$$

$$360 - 15 = 345$$

smile...

## \* Projectial motion :-

حركة المقذوفات



$$\vec{a} = 0\hat{i} - 10\hat{j}$$

$$\vec{v}_i = \underbrace{v_i \cos \theta}_{v_{ix}} \hat{i} + \underbrace{v_i \sin \theta}_{v_{iy}} \hat{j}$$

X-axis

$$v_{xf} = v_{yi} + a_x t$$

$$v_{xf} = v_{yi} \Rightarrow v_{xf} = v_i \cos \theta$$

No change on  $v_x$  the  $v_x$   $a_x = 0$ 

$$\Delta x = v_{ix} t + \frac{1}{2} a_x t^2$$

$$\Delta x = v_i \cos \theta t$$

y-axis

$$v_{yf} = v_{yi} + a_y t^2$$

$$v_{yf} = v_i \sin \theta - g t^2$$

$$\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$$

$$\Delta y = v_i \sin \theta t - \frac{1}{2} g t^2$$

\* Now of Max height  $y_{\max}$  (h)

$$v_y f = 0 \rightarrow 0 = v_i \sin \theta - gt$$

$$t = \frac{v_i \sin \theta}{g}$$

time needed to reach  
Max height

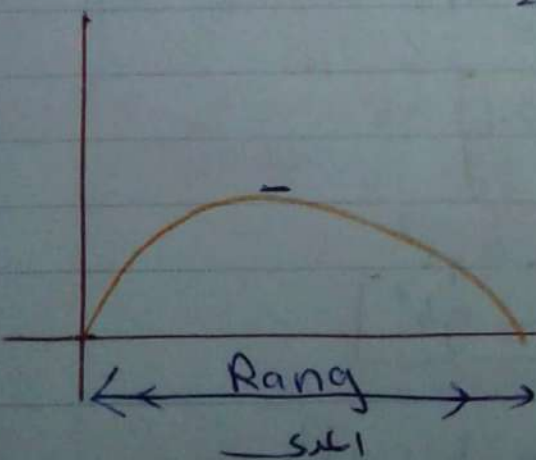
↳ time of flight =  $2t$

$$t = 2 \frac{v_i \sin \theta}{g}$$

↳ Max height (h)

$$y_{\max} = v_i \sin \theta \frac{v_i \sin \theta}{g} - \frac{1}{2} g \frac{v_i^2 \sin^2 \theta}{g^2}$$

$$h = \frac{v_i^2 \sin^2 \theta}{2g}$$



The Range R برو زفنا الطيران كامل

$$t = 2 \frac{v_i \sin \theta}{g}, \Delta x = v_i \cos \theta t$$

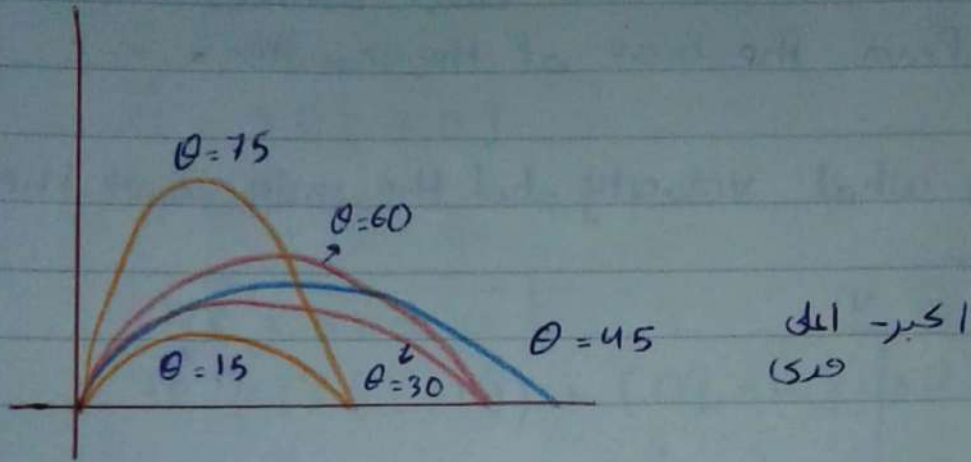
$$R = \frac{v_i \cos \theta \cdot 2 v_i \sin \theta}{g}$$

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

smile...

$$R \text{ max} \rightarrow \sin 2\theta = 1$$

$$2\theta = 90 \rightarrow \theta = 45^\circ \rightarrow \text{له اعلى قدرى}$$



$$\text{Now } \Delta x = v_i \cos \theta t$$

$$t = \frac{\Delta x}{v_i \cos \theta}$$

$$\Delta y = v_i \sin \theta t - \frac{1}{2} g t^2$$

$$\Delta y = v_i \sin \theta \frac{\Delta x}{v_i \cos \theta} - \frac{1}{2} g \frac{\Delta x^2}{v_i^2 \cos^2 \theta}$$

$$y = x \tan \theta - \frac{g}{2v_i^2 \cos^2 \theta} x^2$$

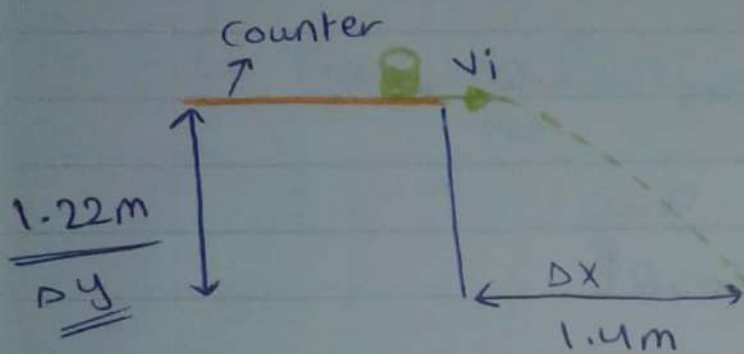
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#Sally Bani Yaseen

smile...

Q13 In a local bar, a customer slides an empty beer mug down the counter for a refill. The height of the counter is 1.22m. The mug slides off the counter and strikes the floor 1.4 m from the base of the counter.

a) With what velocity did the mug leave the counter?



$$a_x = 0 \quad a_y = -g$$

$$v_{ix} = v_i \quad v_{iy} = 0$$

$$\underline{v_i = ?}$$

$$\Delta x = v_{ix} t + \frac{1}{2} a_x t^2$$

$$1.4 = v_i t$$

$$\text{but } \Delta y = v_{iy} t + \frac{1}{2} a_y t^2$$

$$-1.22 = -\frac{1}{2} \times 10 \times t^2$$

$$-1.22 = -5 t^2$$

$$t = 0.5 \text{ sec}$$

$$\underline{So} \quad 1.4 = v_i \times 0.5$$

$$v_i = 2.8 \text{ m/s}^1$$

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smile...



b) what was the direction of the mug's velocity just before it hit the floor?

$$\vec{v}_f = ??$$

$$\vec{a} = 0\hat{i} - 10\hat{j}$$

$$\vec{v}_i = 2.8\hat{i} + 0\hat{j}$$

$$t = 0.5 \text{ sec}$$

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

$$\vec{v}_f = (2.8\hat{i} + 0\hat{j}) + (0\hat{i} - 10\hat{j}) \times 0.5$$

$$\vec{v}_f = \underbrace{2.8\hat{i}}_x - \underbrace{5\hat{j}}_y$$

$$|\vec{v}_f| = \sqrt{7.84 + 25} = \sqrt{32.84} = 5.7$$

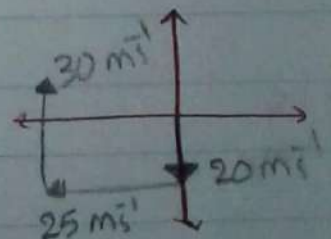
$$\theta = \tan^{-1} \frac{-5}{2.8} = -60.7$$

H.w Q1 A motorist drives south at  $20 \text{ m s}^{-1}$  for 3 min then turns west and travels at  $25 \text{ m s}^{-1}$  for 2 min and finally travels northwest at  $30 \text{ m s}^{-1}$  for 1 min. For this 6 min trip find.

a) the total vector displacement

Displacement vector

$$\vec{\Delta r} = \Delta x \hat{i} + \Delta y \hat{j}$$



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smile

b) The average speed

c) The average velocity

Q3 Suppose the position vector for a particle is given as a function of time by  $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$  with  $x(t) = at + b$  and  $y(t) = ct^2 + d$  where  $a = 1 \text{ m s}^{-1}$ ,  $b = 1 \text{ m}$ ,  $c = 0.125 \text{ m s}^{-2}$  and  $d = 1 \text{ m}$

a) Calculate the average velocity during the time interval from  $t = 2 \text{ s}$  to  $t = 4 \text{ s}$

$$\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{v} = v_x \hat{i} + v_y \hat{j}$$

$$\vec{v} = (at + b) \hat{i} + (ct^2 + d) \hat{j}$$

$$\vec{v} = (1 \times 2 + 1) \hat{i} + (0.125 \times 16 + 1) \hat{j}$$

b) Determine the velocity and the speed at  $t = \underline{\underline{2.5}}$

Q7 The vector position of a particle varies in time according to the expression  $\vec{r} = 3\hat{i} - 6t^2\hat{j}$  where  $\vec{r}$  is in meters and  $t$  in second.

a) Find an expression for the velocity of the particle as a function of time

b) Determine the acceleration of the particle as a function of time

c) Calculate the particle's position and velocity at  $t = 1$  sec.

\* Sally Bari yaseen  
smile...

Q10 A snowmobile is originally at the point with position vector 29 m at  $95^\circ$  counterclockwise from the x-axis, moving with velocity  $4.5 \text{ m s}^{-1}$  at  $40^\circ$ . It moves with constant acceleration  $1.9 \text{ m s}^{-2}$  at  $200^\circ$ . After 5 s have elapsed find

a) its velocity

b) its position vector

Q12 An astronaut on a strange planet finds that she can jump a maximum horizontal distance of 15 m if her initial speed is  $3 \text{ m s}^{-1}$ . What is the free fall acceleration on the planet?

$$V_i = 3 \text{ m s}^{-1}, R = 15 \text{ m}$$

#Sally Bani Yuseen

smile...

Q15 A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. What is the angle of projection?

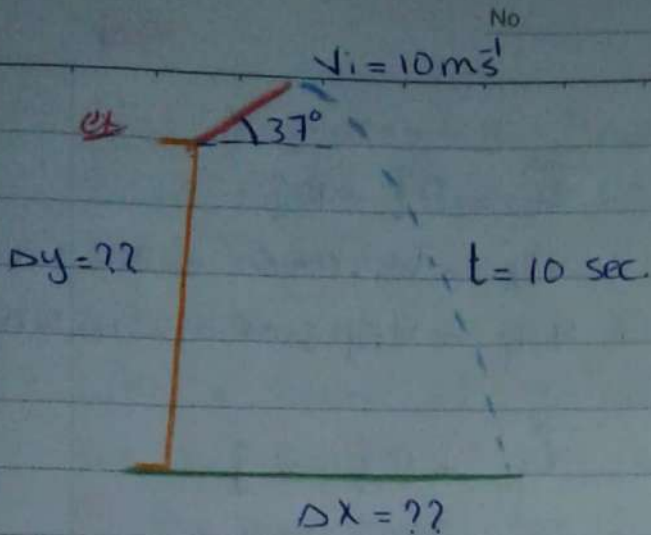
Q33 The athlete rotates a 4.0 kg discus along a circular path of radius 1.06 m. The maximum speed of the discus is 20 m.s<sup>-1</sup>. Determine the magnitude of the maximum radial acceleration of the discus.

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smile...

Q38 An athlete swings a ball, connected to the end of a chain in a horizontal circle. The athlete is able to rotate the ball at the rate of  $8 \text{ rev. s}^{-1}$  when the length is  $0.9 \text{ m}$ . He is able to rotate the ball only  $6 \text{ rev. s}^{-1}$ .

- a) Which rate of rotation gives the greater speed for ball?
- b) What is the centripetal acceleration of the ball at  $8 \text{ rev. s}^{-1}$ ?
- c) What is the centripetal acceleration at  $6 \text{ rev. s}^{-1}$ ?

\* Sally Bani Yaseen  
smile



$$\vec{a} = 0\hat{i} - 10\hat{j}$$

$$\bar{v}_{ix} = v_i \cos \theta = 10 \cos 37 = 8$$

$$v_{iy} = v_i \sin \theta = 10 \sin 37 = 6$$

$$\vec{v}_i = 8\hat{i} + 6\hat{j}$$

$$\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$$

$$\Delta y = 6 \times 10 + \frac{1}{2} (-10) \times 100$$

$$\Delta y = 60 - 500 \Rightarrow \Delta y = -440$$

$$\Delta x = v_{ix} t + \frac{1}{2} a_x t^2$$

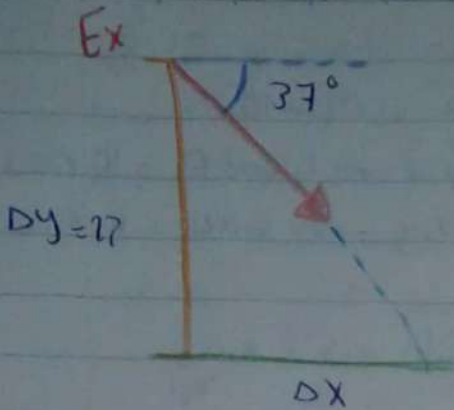
$$\Delta x = 8 \times 10 = 80 \text{ m}$$

$$\vec{\Delta r} = 80\hat{i} - 440\hat{j}$$

$$\text{or } \vec{\Delta r} = \vec{v}_i t + \frac{1}{2} \vec{a} t^2$$

$$\vec{\Delta r} = (8\hat{i} + 6\hat{j}) \times 10 + \frac{1}{2} (0\hat{i} - 10\hat{j}) \times 100$$

$$\vec{\Delta r} = \frac{\Delta x}{80\hat{i}} - \frac{\Delta y}{440\hat{j}}$$



$$\vec{a} = 0\hat{i} - 10\hat{j}$$

$$v_{ix} = v_{ix} \cos \theta = +10 \cos 37 = 8$$

$$v_{iy} = v_{iy} \sin \theta = -10 \sin = -6$$

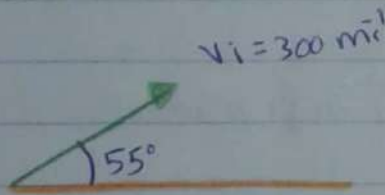
$$\vec{v}_i = 8\hat{i} - 6\hat{j}$$

$$\Delta y = -6 \times 10 - 5 \times 100 = -560 \text{ m}$$

$$\Delta x = 8 \times 10 = 80 \text{ m}$$

$$\vec{\Delta r} = 80\hat{i} - 560\hat{j}$$

Q16 To start an avalanche on mountain slope an artillery shell is fired with an initial velocity of  $300 \text{ m}\cdot\text{s}^{-1}$  at  $55^\circ$  above the horizontal. It explodes on the mountainside  $42 \text{ s}$  after firing. What are the  $x$  &  $y$  coordinates of the shell where it explodes relative to its firing point.



القوة بفر  $t = 42 \text{ sec}$

$$\vec{a} = 0\hat{i} - 10\hat{j}$$

$$\vec{v}_i = 300 \cos 55 \hat{i} + 300 \sin 55 \hat{j}$$

$$\vec{v}_i = 172\hat{i} + 246\hat{j}$$

$$\vec{\Delta r} = \vec{v}_i t + \frac{1}{2} a t^2$$

$$\vec{\Delta r} = (172\hat{i} + 246\hat{j}) \times 42 + \frac{1}{2} (0\hat{i} - 10\hat{j}) \times (42)^2$$

$$\vec{\Delta r} =$$

نبي احب زونا  
اقصا ارتفاع مشان  
اعرف وقت الصر

$$\vec{v}_p = \vec{v}_i + at \Rightarrow \vec{v}_p = (172\hat{i} + 246\hat{j}) + (0\hat{i} - 10\hat{j}) \times 42$$

$$\vec{v}_p = (172\hat{i} - 174\hat{j})$$

$\Delta x \quad \Delta y$

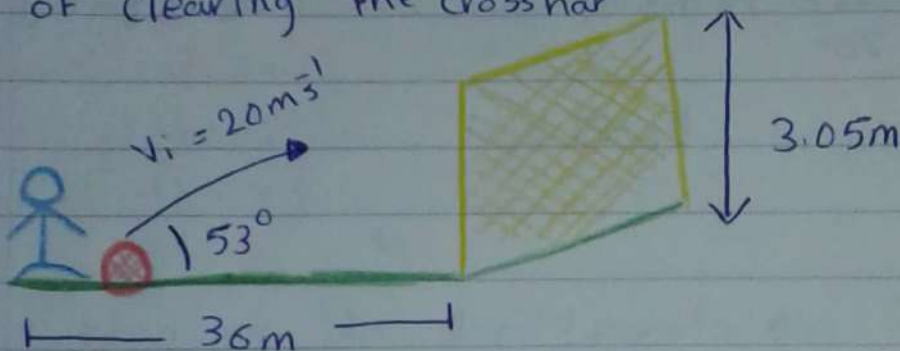
سالب القوة وهو نازلة

smile...



Q23 A placekicker must kick a football from a point 36 m (about 40 yards) from the goal. Half the crowd hopes the ball will clear the crossbar, which is 3.05 m high. When kicked, the ball leaves the ground with a speed of  $20 \text{ m/s}$  at an angle of  $53.0^\circ$  to the horizontal.

a) By how much does the ball clear or fall short of clearing the crossbar



حسب  $\Delta y$  اذا كانت  $3 \text{ m}$  ببقية الهدف

$$\Delta y = v_{iy}t + \frac{1}{2} a_y t^2$$

$$\Delta y = 20 \sin 53 \times t - 5 t^2$$

$$\Delta y = 16 \times 3 - 5 \times 9$$

$$\Delta y = 48 - 45$$

$$\Delta y = 3 \text{ m} \times$$

الزمن هو

سوى  $\Delta x$

$$\Delta x = v_{ix}t + \frac{1}{2} a_x t^2$$

$$\Delta x = v_{ix}t$$

$$36 = 20 \cos 53 \times t$$

$$\frac{36}{12} = \frac{12}{12} t$$

$$t = 3 \text{ sec}$$

~~Sally~~ Bari yaseen

smile

H.W

No

56

Q20 A ball is tossed from an upper-story window of a building. The ball is given an initial velocity of  $8 \text{ m/s}$  at an angle of  $20^\circ$  below the horizontal. It strikes the ground  $3 \text{ s}$  later.

a) How far horizontally from the base of the building does the ball strike the ground



$$v_i = 8 \text{ m/s}$$

$$20^\circ$$

$$\vec{a} = 0\hat{i} - 10\hat{j}$$

$$t = 3 \text{ s}$$

$\Delta x = ??$

$$\Delta x = v_{ix} \times t + \frac{1}{2} a_x t^2$$

$$\Delta x = 8 \cos 20^\circ \times 3$$

$$\Delta x = 22.5 \text{ m}$$

b) Find the height from which the ball was thrown

$\Delta y = ??$

$$\Delta y = v_{iy} \times t + \frac{1}{2} a_y t^2$$

$$= 8 \sin 20^\circ \times 3 - 5 \times 9$$

$$= 8.2 - 45$$

$$\Delta y = -36.8 \text{ m}$$

c) How long does it take the ball to reach a point  $10 \text{ m}$  below the level of launching?

$$\Delta y = v_{iy} \times t + \frac{1}{2} a_y t^2$$

$$-10 = 8 \sin 20^\circ \times t - 5t^2$$

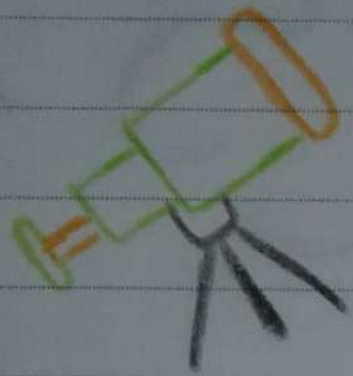
$$-10 = 2.7t - 5t^2$$

$$t^2 - 0.54t - 2$$

$$\frac{5t^2}{5} - \frac{2.7t}{5} - \frac{10}{5} = 0$$

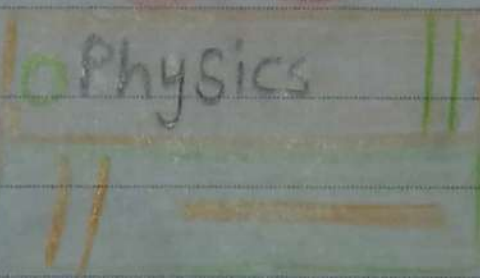
$$\text{smile}_{\text{for me}}$$

$$2.7 \pm 11$$



$$E = mc^2$$

E



# Sally Baniyaseen

\* Circular motion :-

الحركة الدائرية

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \rightarrow \begin{matrix} \text{متغير بتغير} \\ \text{مقدراً واتجاهاً} \end{matrix} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Call  $\vec{v}$  is a vector

↳ changing  $\vec{v}$  → change the magnitude (speed)  
 ↳ changing the direction

مثلاً  $v = 2 \text{ m s}^{-1}$

$v_i = 2 \hat{i}$

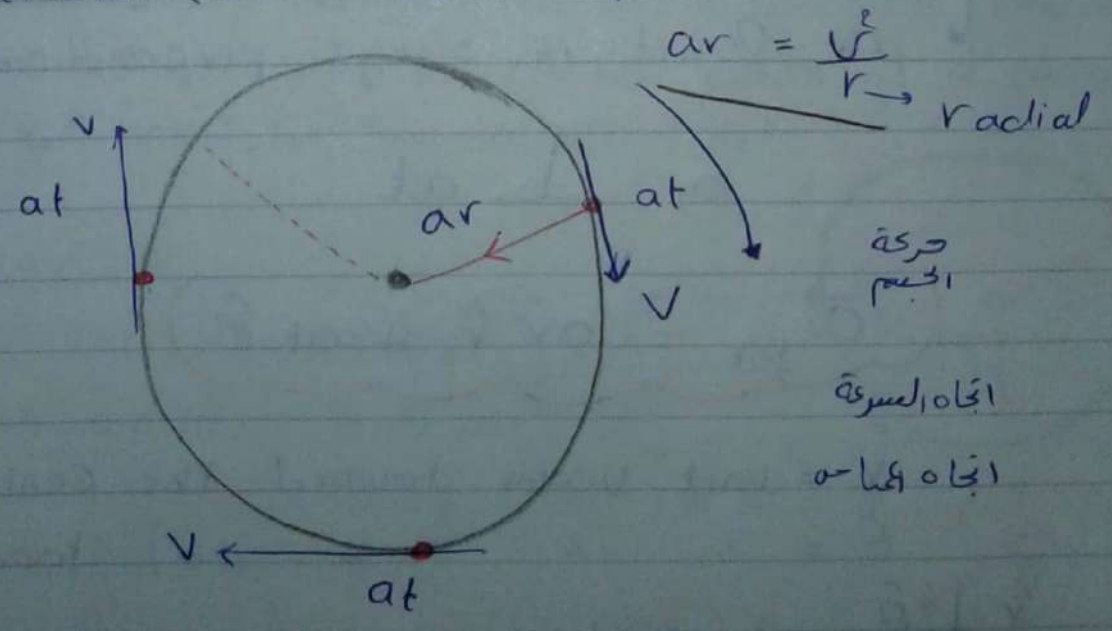
تغير في الاتجاه

$v_f = 2 \hat{j}$

↳ changing the speed greats the linear acceleration (tangential acceleration)

$$a_t = \frac{\Delta |\vec{v}|}{\Delta t}$$

↳ while changing the direction greats the Centratiple acceleration (radial acceleration)



$$a_t = \frac{\Delta v}{\Delta t} = \frac{d\vec{v}}{dt}$$

is always toward the tangent at  $a_r = \frac{v^2}{r}$

is always toward the center

### • Uniform circular motion

$a_t = 0$  → No change in speed only change in direction

### • Non uniform circular motion

change in speed & change in direction

→  $a_r$  &  $a_t$

↳  $a_r$  &  $a_t$  is always perpendicular قطب‌مماسی

$$a_r \perp a_t$$

$$\vec{a}_{tot} = a_r \hat{r} + a_t \hat{\theta}$$

$\hat{r}$  ≡ unit vector toward the center

$\hat{\theta}$  ≡ " " " " tangent

$$\hat{r} \perp \hat{\theta}$$

$$|\vec{a}_{tot}| = \sqrt{a_r^2 + a_t^2}$$

Q36 A tire 0.5 m in radius rotates at a constant rate of 200 rev/min. Find the speed & acceleration of a small stone lodged in the tread of the tire

$$r = 0.5 \text{ m}, \quad \omega = 200 \text{ rev/min} \quad \text{تعبير عن السرعة}$$

\* فنطرح محيط العجل للحول، السرعة

$$v = \frac{\omega \cdot 2\pi r}{t}$$

$$= \frac{200 \cdot 2\pi \cdot \frac{1}{2}}{1 \cdot 60}$$

$$v = 10.5 \text{ m s}^{-1}$$

$$a_r = \frac{v^2}{r} = \frac{(10.5)^2}{0.5} = 214 \text{ m s}^{-2}$$

Q40 The total acceleration of a particle moving clockwise in a circle of radius 2.5 m at a certain instant of time, for that instant find

$$a_r = a \cos \theta$$

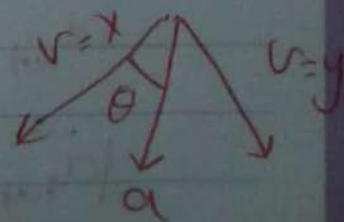
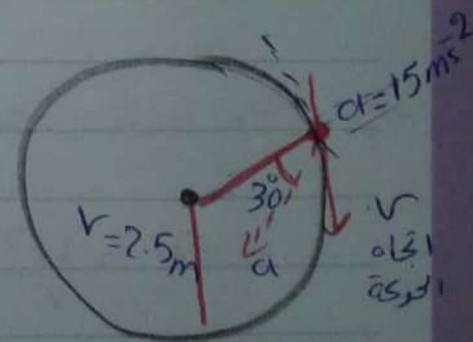
$$= 15 \cos 30 = 13 \text{ m s}^{-2}$$

$$a_t = 15 \sin 30 = 7.5 \text{ m s}^{-2}$$

$$\vec{a}_{\text{tot}} = 13 \hat{r} + 7.5 \hat{\theta}$$

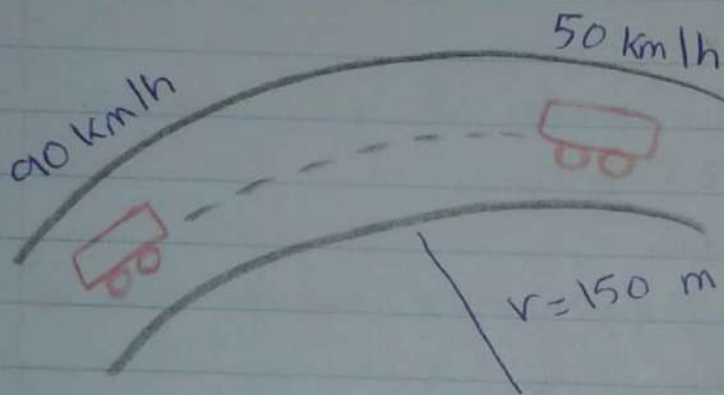
$$a_r = \frac{v^2}{r} \Rightarrow v = \sqrt{13 \times 2.5}$$

$$13 = \frac{v^2}{2.5} \Rightarrow \sqrt{v^2} = \sqrt{13 \times 2.5}$$



smile...

Q41 A train slows down as it rounds a sharp horizontal turn, going from 90 km/h to 50 km/h in the 15 s it takes to round the bend. The radius of the curve is 150 m. Compute the acceleration at the moment the train speed reaches 50 km/h. Assume the train continues to slow down at this time at the same rate.



t نسا في خطي  
r ر مركزي

لعمامة نون 30  
total  
t = 15 sec.

$$a_t = \frac{|\vec{v}_f| - |\vec{v}_i|}{t}$$

$$= \frac{14 - 25}{15}$$

$$= -0.73 \text{ m/s}^2$$

$$v_f = \frac{50 \text{ km/h}}{h}$$

$$= \frac{50 * 1000 \text{ m}}{3600 \text{ sec}}$$

$$v_f = 14$$

$$v_i = \frac{90 * 1000}{3600}$$

$$v_i = 25$$

$$a_r = \frac{v^2}{r} = \frac{(14)^2}{150} = 1.3 \text{ m/s}^2$$

$$\vec{a}_{tot} = 1.3 \hat{r} - 0.73 \hat{\theta}$$

$$|\vec{a}_{tot}| = \sqrt{(1.3)^2 + (0.73)^2}$$

## Ch:5 Newton's laws :-

### 1st law :-

Object at rest or moving with constant velocity stay as it's unless an-external force acts on

#### Uniforme

↳ Constant velocity  
     ↓  
     constant

in magnitude  $\equiv$  speed  
 and direction

↳ if  $\vec{a} = 0 \iff \sum \vec{F}_{ext} = 0$

if  $\sum \vec{F}_{ext} = 0$  then the object either at rest  
 for moving with constant velocity.

Now

$$\sum \vec{F}_{ext} = 0 \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases}$$

اذا ساكن او سرعته ثابتة

### 2nd law :-

if  $\sum \vec{F}_{ext} \neq 0$  ??

then there is an acceleration





$$\sum \vec{F}_{\text{ext}} \propto \vec{a}$$

$$\sum \vec{F}_{\text{ext}} = \text{const} \vec{a}$$

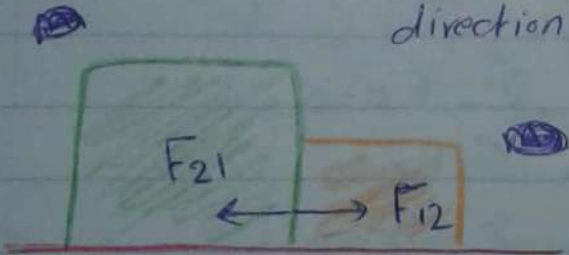
$$\sum \vec{F}_{\text{ext}} = \text{mass} * \vec{a}$$

$$\sum \vec{F}_{\text{ext}} = m\vec{a} \quad \text{ausgewähltes}$$

$$\sum F_x = ma_x \quad / \quad \sum F_y = ma_y \quad / \quad \sum F_z = ma_z$$

3rd law :-

For every action there is a reaction equal in magnitude & opposite in direction

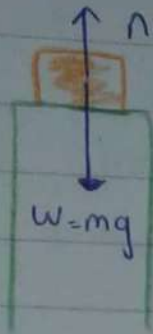


$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\vec{F}_{12} + \vec{F}_{21} = 0$$

$$\sum \vec{F} = 0 \Rightarrow a = 0$$

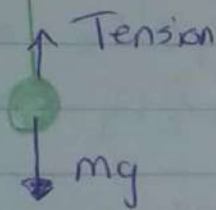
\* Free body diagram :-



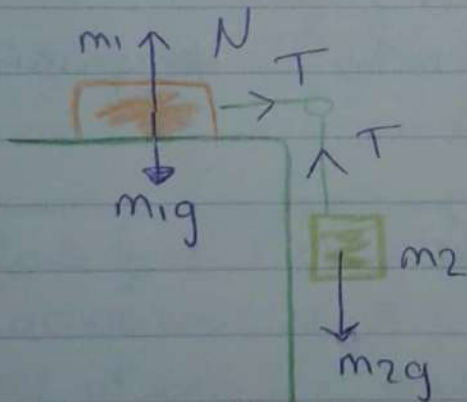
عاموری قوت / سطح

$$+ N - mg = 0$$

$$N = mg$$



$$T - mg = 0$$



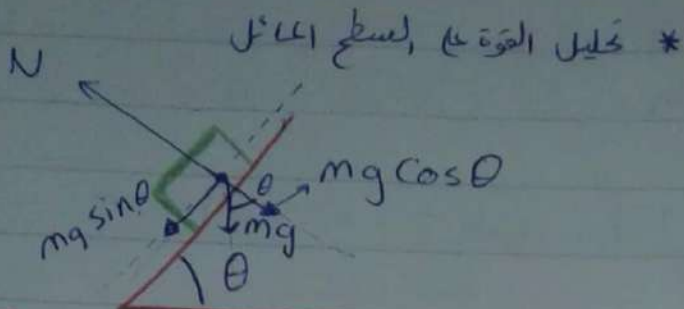
For  $m_1$

$$\sum F_x = m_1 a_x \rightarrow T = m_1 a_x$$

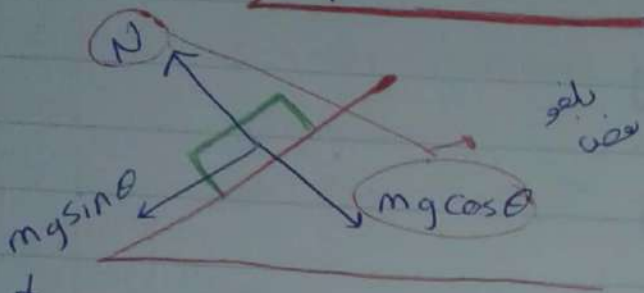
$$\sum F_y = m_1 a_y \rightarrow N - m_1 g = 0$$

For  $m_2$

$$\sum F_y = m_2 a_y \rightarrow T - m_2 g = -m_2 a$$



الزاوية

صورة y

بعض

بعض  
الجزء

$$\vec{\Sigma F} = m\vec{a} \begin{cases} \Sigma F_x = ma_x \\ \Sigma F_y = ma_y \end{cases}$$

$$\Sigma F_x = ma_x$$

$$mg \sin \theta = ma_x \Rightarrow g \sin \theta = a_x$$

$$\Sigma F_y = ma_y$$

$$N - mg \cos \theta = 0 \quad \text{بعض الصورة}$$

$$N = mg \cos \theta$$

Q12 Besides the gravitational force, a 2.8 kg object is subjected to one other constant force. The object starts from rest and in 1.2 s experiences a displacement of  $(4.2 \hat{i} - 3.3 \hat{j})$  m, where the direction of  $\hat{j}$  is the upward vertical direction. Determine the other force

$$m = 2.8 \text{ kg}, \quad \vec{v}_i = 0, \quad \vec{D}r = 4.2\hat{i} - 3.3\hat{j}$$

$$t = 1.2 \text{ sec.}$$

$$\sum \vec{F} = m\vec{a}$$

$$\vec{F}_1 + \vec{F}_2 = m\vec{a}$$

$$\vec{F}_1 = w = mg = -28\hat{j}$$

$$D_r = v_i t + \frac{1}{2} a t^2$$

$$4.2\hat{i} - 3.3\hat{j} = \frac{1}{2} \vec{a} \times 1.44$$

$$\vec{a} = 5.8\hat{i} - 4.6\hat{j}$$

$$-28\hat{j} + \vec{F}_2 = 2.8 \times (5.8\hat{i} - 4.6\hat{j})$$

$$\begin{array}{r} -28\hat{j} + \vec{F}_2 = (15.4\hat{i} - 12.88\hat{j}) \\ +28\hat{j} \qquad \qquad \qquad +28\hat{j} \\ \hline \vec{F}_2 = 15.4\hat{i} + 15.12\hat{j} \end{array}$$

Q18 A force  $\vec{F}$  applied to an object of mass  $m_1$  produces an acceleration of  $3 \text{ m/s}^2$ . The same force applied to a second object of mass  $m_2$  produces an acceleration of  $1 \text{ m/s}^2$

$$F \rightarrow m_1 \rightarrow a_1 = 3 \text{ m/s}^2 \quad / \quad F \rightarrow m_2 \rightarrow a_2 = 1 \text{ m/s}^2$$

1] what is the value of the ratio  $\frac{m_1}{m_2}$ ?

$$\begin{array}{l} F = m_1 a_1 \Rightarrow F = 3m_1 \\ F = m_2 a_2 \Rightarrow F = 1m_2 \end{array} \quad \left. \vphantom{\begin{array}{l} F = m_1 a_1 \\ F = m_2 a_2 \end{array}} \right\} \rightarrow \frac{3m_1}{m_2} = \frac{1m_2}{m_2}$$

$$1 = \frac{3m_1}{m_2} \Rightarrow \frac{m_1}{m_2} = \boxed{\frac{1}{3}}$$

2] If  $m_1$  and  $m_2$  are combined into an object find the acceleration under the action of the force  $\vec{F}$   $M \rightarrow m_1 + m_2 / F \rightarrow M \rightarrow a_M$

$$F = M a$$

$$F = \left(\frac{F}{3} + F\right) a$$

$$F = (m_1 + m_2) a$$

$$\rightarrow F = \frac{4F}{3} a$$

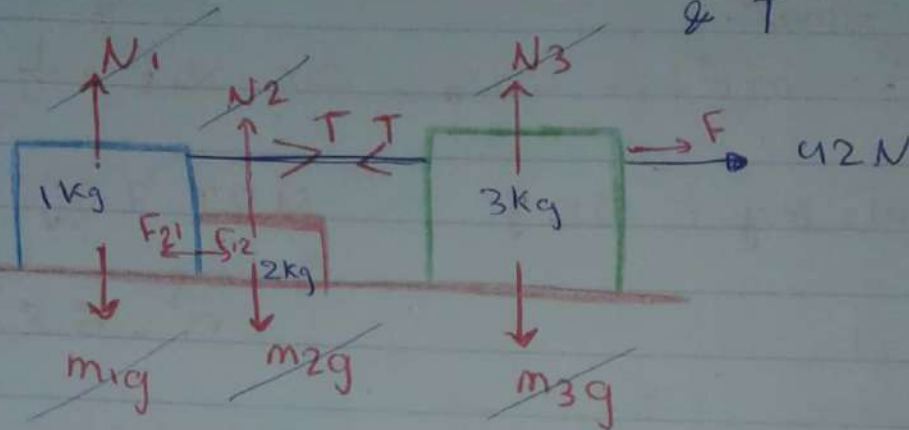
$$\text{but } F = 3m_1, \quad m_1 = \frac{F}{3}$$

$$F = m_2, \quad m_2 = F$$

$$a = \frac{4}{3} \text{ m/s}^2$$

smile...

Q 29 Assume the three blocks portrayed move on a frictionless surface and a 42 N Force acts as shown on the 3 kg block. Determine the acceleration & T



Take  $m_3$

$$\sum \vec{F} = m_3 a$$

$$F - T = m_3 a$$

$$42 - T = 3a$$

Take  $m_1$

$$\sum \vec{F} = m_1 a$$

$$T - F_{21} = m_1 a$$

$$T - F_{21} = a$$

Take  $m_2$

$$\sum \vec{F} = m_2 a$$

$$F_{12} = 2a$$

OR

باعتبار كل جسم واحد  
والتساوي بينهم

$$\sum F = Ma$$

$$42 = (m_1 + m_2 + m_3) a$$

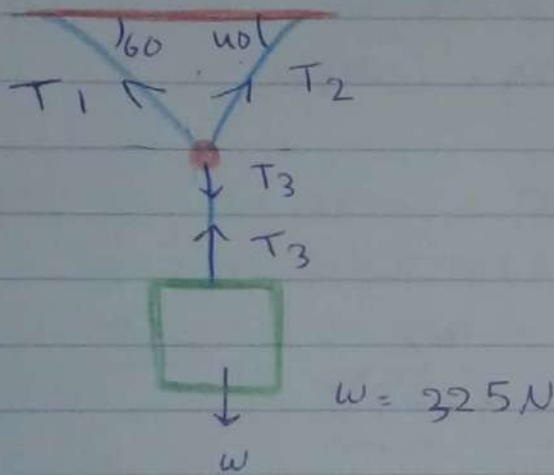
$$a = 7 \text{ m/s}^2$$

$$42 - T = 3a$$

$$\frac{42}{42} - T = \frac{3 \times 7}{42}$$

$$T = 63$$

Q33 A bag of cement weighing 325 N hangs in equilibrium from three wires as suggested in Fig. Two of the wires make angles  $\theta_1 = 60^\circ$ ,  $\theta_2 = 40^\circ$  with the horizontal. Assuming the system is ~~in~~ in equilibrium find the tensions  $T_1$ ,  $T_2$ ,  $T_3$  in the wires.



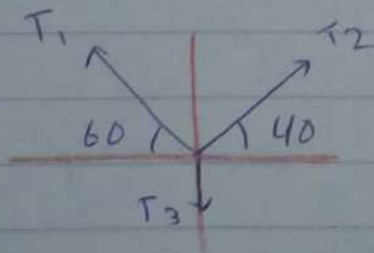
Equilibrium قوتون  
 $\Sigma F = 0$

$$\Sigma \vec{F}_x = 0 \quad \Sigma \vec{F}_y = 0$$

$$T_3 - w = 0$$

$$T_3 = w$$

$$T_3 = 325 \text{ N}$$



x (axis)

$$\Sigma F_x = 0$$

$$T_2 \cos 40 - T_1 \sin 60 = 0$$

y (axis)

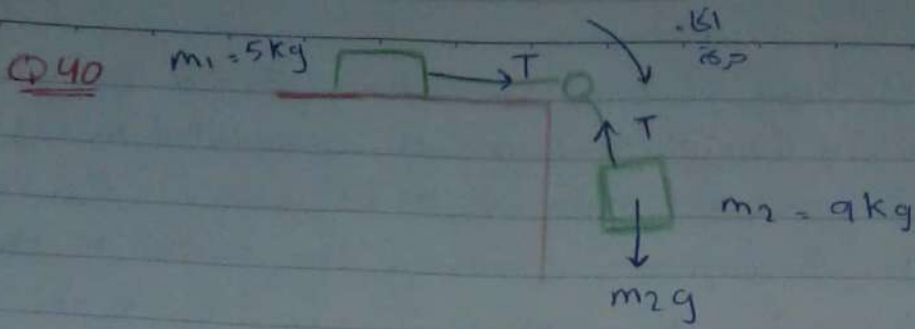
$$T_2 \sin 40 + T_1 \sin 60 - T_3 = 0$$

$$T_2 * 0.76 - T_1 * 0.8 = 0$$

$$0.76 T_2 = T_1 * 0.8$$

$$T_2 = T_1 * 1.05$$

$$(T_1 * 1.05) * 0.6 + T_1 * 0.5 - 325 = 0$$



$$m_1 \Rightarrow T = 5a$$

المركبات

$$m_2 \Rightarrow T - m_2 g = -m_2 a$$

$$5a - 90 = -9a$$

$$14a = 90$$

$$a = 6.4 \text{ m/s}^2$$

$$T = 6.4 \times 5 = 32 \text{ N}$$



$$m_1 \Rightarrow T - m_1 g = -m_1 a$$

$$T - 20 = -2a$$

$$m_2 \Rightarrow m_2 g \sin \theta - T = m_2 a$$

$$6 \times 10 \sin 55 - T = 6a$$

$$49 - T = 6a$$

$$-20 + T = -2a$$

$$2a = -8a$$

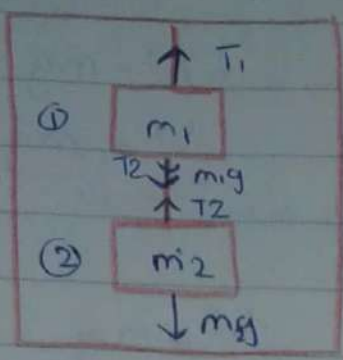
$$a = -3.6 \text{ m/s}^2$$

smile

$$T - 20 = -2 \times 3.6 \rightarrow T = 27.2 \text{ N}$$

Q43

$m = 3.5 \text{ kg}$



$a = 1.6 \text{ m/s}^2$

اَسْرِي

$\Sigma F = ma$

1]  $T_1 - (T_2 + mg) = ma$

$T_1 - T_2 - 35 = 3.5 \times 1.6$

$T_1 - 40.6 - 35 = 5.6$

$T_1 - 75.6 = 5.6$

$T_1 = 81.2$

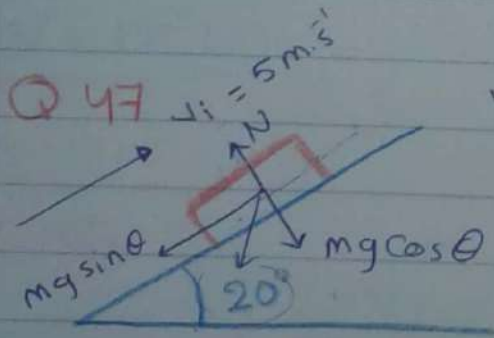
2]  $T_2 - mg = ma$

$T_2 - 35 = 3.5 \times 1.6$

$T_2 - 35 = 5.6$   
 $\quad + 35 \quad + 35$

$T_2 = 40.6$

Q47



$\Sigma F = ma$

$-mg \sin \theta = ma$

$a = -3.4 \text{ m/s}^2$

$\Delta x$  we

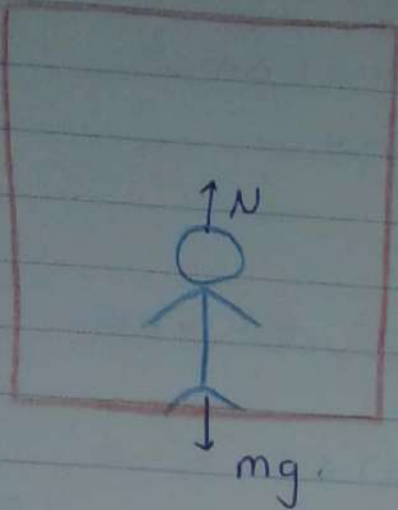
$v_f^2 = v_i^2 + 2a \Delta x$

$0 = 25 - 2 \times 3.4 \Delta x$

$\Delta x = 3.7 \text{ m}$

-10x





$$N - mg = ma$$

$$N = mg + ma \quad \text{عجلة}$$

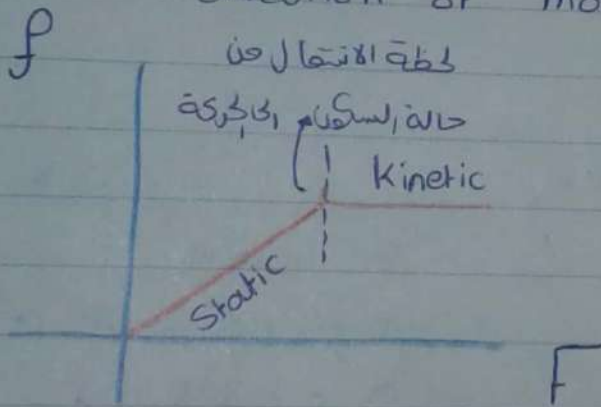
$$N - mg = -ma$$

$$N = mg - ma \quad \text{توقف}$$

\* Friction force  $f$  :- قوة الاحتكاك

دائماً تكون عكس اتجاه الحركة

it is a resistive force always opposite to the direction of motion



1. Static friction  $f_s$

$$f_s \propto N \quad \text{وحد تجريبياً}$$

$$f_s = \mu_s N \quad \mu_s \text{ coefficient of static friction}$$

2. Kinetic friction  $f_k$

$$f_k \propto N \rightarrow f_k = \mu_k N \quad \text{coefficient of kinetic friction}$$

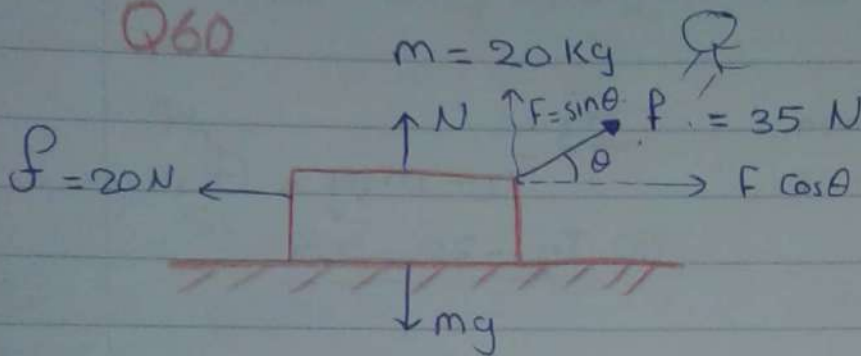
In general

$$f = \mu N$$

↙ معامل الاحتكاك

↗ M: ديمترى طبيعة الاسفل وقياس تجريبياً قيمه

Q60



$$\theta = ??$$

$$N = ??$$

constant speed  $a = 0$

y-axis

$$N + F \sin \theta - mg = 0$$

$$N = mg - F \sin \theta$$

$$N = 171 \text{ N}$$

$\Sigma F = ma$  x-axis

$$F \cos \theta - f = 0$$

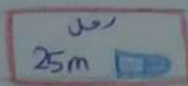
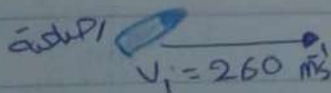
$$35 \cos \theta - 20 = 0$$

$$\cos \theta = 0.57 \quad 35 \cos \theta = 20$$

$$\theta = 55^\circ$$

Q53

$$m = 12 \text{ g}$$



$$v_f = 0$$

لدى قوة الاحتكاك

$$\Sigma F = m(a) \rightarrow v_f^2 = v_i^2 + 2a \Delta x$$

$$0 = 67600 + 50 a$$

$$\frac{-50 a}{-50} = \frac{67600}{-50}$$

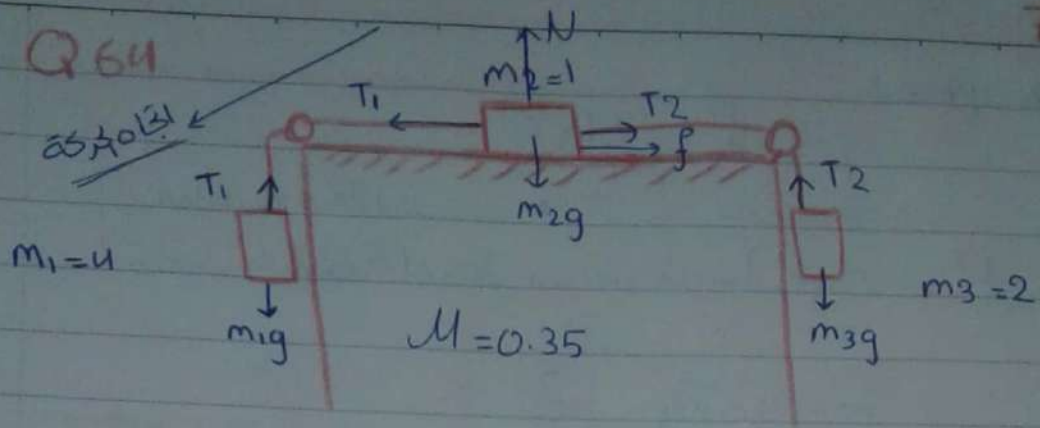
$$f = ma$$

$$f = 12 \times 1.352$$

$$f = 16.224 \text{ N}$$

$$a = -1352 \text{ m/s}^2$$

Q64



$\underline{m_1}$       down

$$T_1 - m_1g = -m_1a$$

①  $T_1 - 40 = -4a$

$\underline{m_3}$       up

$$T_2 - m_3g = +m_3a$$

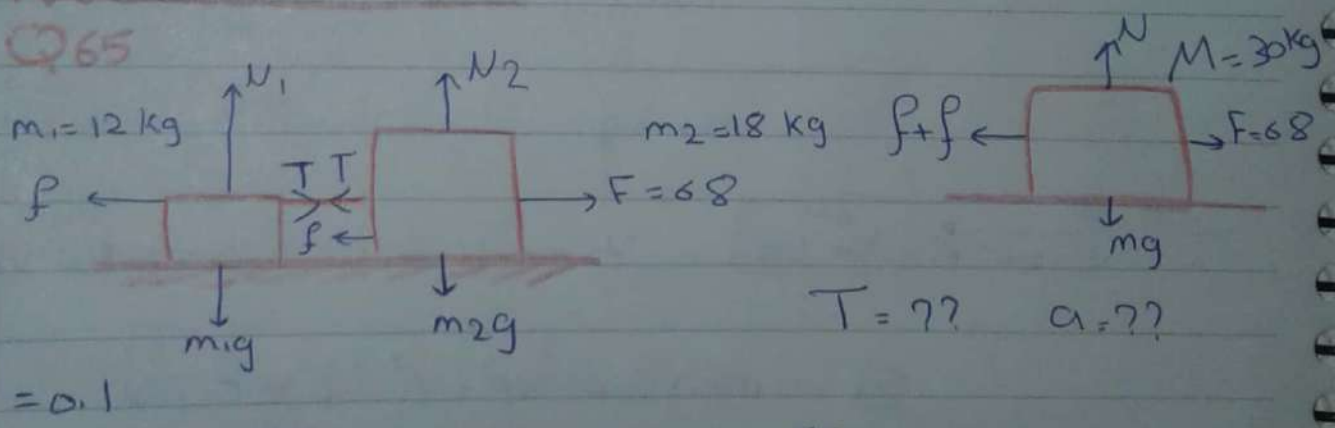
②  $T_2 - 20 = 2a$

$\underline{m_2}$

$$T_2 - T_1 + \mu m_2g = -m_2a$$

③  $T_2 - T_1 + 3.5 = -a$

Q65



↓  $\mu$   $\rightarrow$   $\mu$

$$\Sigma F = ma$$

$$F - f = Ma$$

$$F = \mu N = Ma$$

$$F - \mu Mg = Ma$$

$$68 - 0.1 \times 300 = 30a$$

$$a = 1.2 \text{ m/s}^2$$

$m_1$        $\Sigma F = m_1a$

$$T - f = m_1a$$

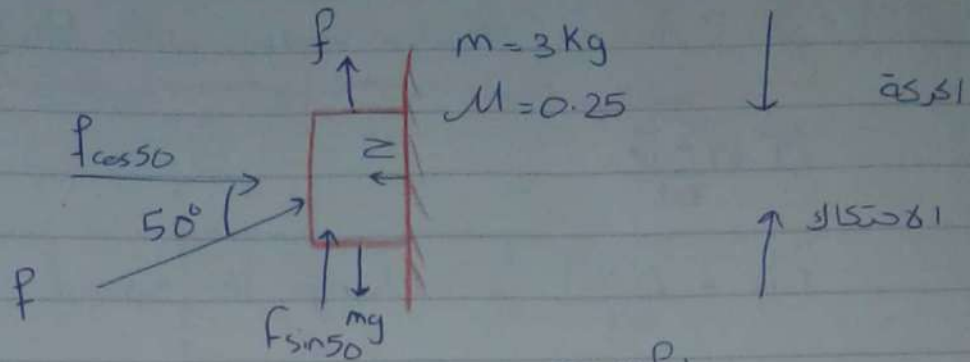
$$T - 0.1 \times 12 \times 10 = 12 \times 1.2$$

$$T - 0.1 \times 120 = 14.4$$

$$T = 26.4 \text{ N}$$

$T = ??$        $a = ??$

Q66



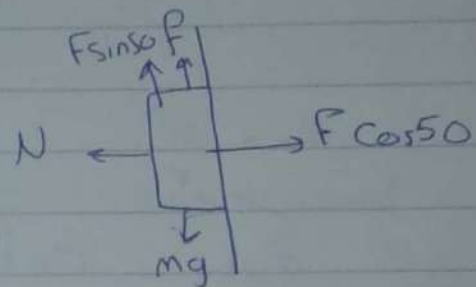
$\Sigma F = 0$

x axis

$F \cos 50 = N$

y axis

$F \sin 50 + f = mg$

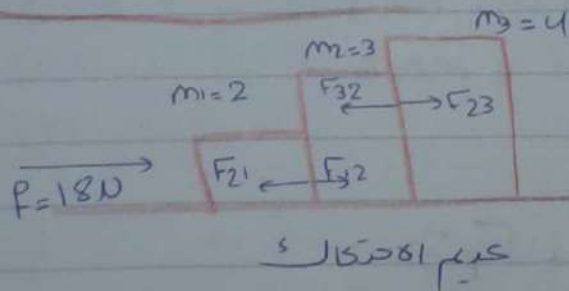


$F \sin 50 + \mu N = mg$

$F \sin 50 + \mu F \cos 50 = mg$

$F \sin 50 + 0.25 * F \cos 50 = 3 * 10$

Q83



$F = ma$

$18 = (2 + 3 + 4) a$

$a = 2 \text{ m/s}^2$

$m_1 \rightarrow F - F_{21} = m_1 a$

$18 - F_{21} = 4$

$F_{21} = 14 \text{ N}$

$F_{12} = 14 \text{ N}$

$m_2$

$F_{12} - F_{32} = m_2 a$

$14 - F_{32} = 3 * 2$

$F_{32} = 8 \text{ N}$

$F_{23} = 8 \text{ N}$

## Ch: 6: Circular motion with Newton laws

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

$$a_t = \frac{\Delta |\vec{v}|}{\Delta t} \quad \text{change in speed}$$

$$a_r = \frac{v^2}{r} \quad \text{change in direction}$$

$$\vec{a}_{\text{total}} = a_r \hat{r} + a_t \hat{\theta}$$

$$a_t \perp a_r \rightarrow |\vec{a}_{\text{total}}| = \sqrt{a_r^2 + a_t^2}$$

Since  $\sum \vec{F} = m\vec{a}$

$$\begin{aligned} \vec{F}_{\text{total}} &= m(a_r \hat{r} + a_t \hat{\theta}) \\ &= m a_r \hat{r} + m a_t \hat{\theta} \end{aligned}$$

قوة الطرد المركزي

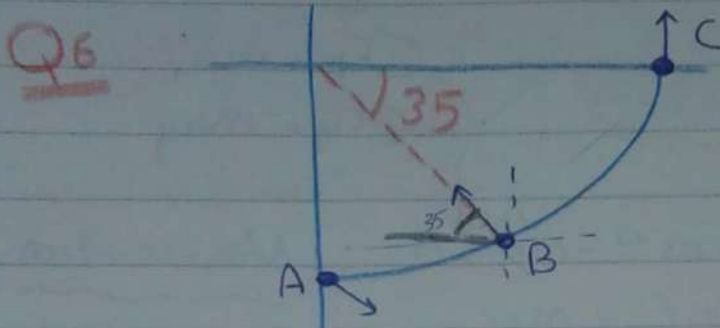
$$\vec{F}_{\text{total}} = F_r \hat{r} + F_t \hat{\theta}$$

where  $F_r = m a_r = m \frac{v^2}{r}$  radial force

$$F_t = m a_t = m \frac{d|\vec{v}|}{dt} \quad \text{tangential force}$$

$$F_r \perp F_t$$

- Uniform circular motion at  $t=0 \rightarrow F_t = 0$
- Non uniform at  $t \neq 0 \rightarrow a_r \neq 0$



ABC = 235m مسافة Constant speed  
 $t = 36s$  زمن

$$v = \frac{s}{t}$$

$$a_r = \frac{v^2}{r}$$

$$v = \frac{ABC}{t} = \frac{235}{36} = \underline{\underline{6.5 \text{ m s}^{-1}}}$$

and  $\frac{ABC}{r} = \frac{1}{4} 2\pi r$

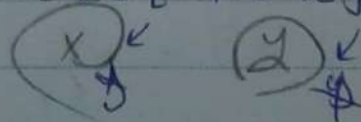
$$235 = \frac{1}{2} \pi r$$

$$r = \underline{\underline{150}}$$

$$a_r = \frac{(6.5)^2}{1.5}$$

$$a_r = 0.28 \text{ m s}^{-2}$$

$$a_r = -0.23 \hat{i} + 0.16 \hat{j} \quad \text{في } (x, y)$$



Average acceleration

$$a = \frac{v_f - v_i}{\Delta t}$$

$$\vec{a} = \frac{6.5 \hat{j} - 6.5 \hat{i}}{36}$$

$$v_f = 6.5 \hat{j}$$

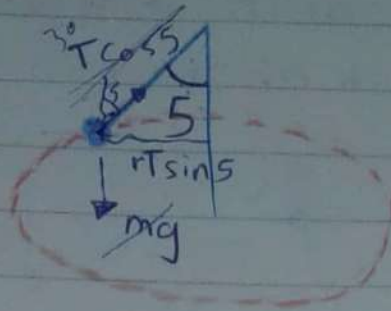
$$v_i = 6.5 \hat{i}$$

$$\vec{a} = -0.18 \hat{i} + 0.18 \hat{j}$$

Q8

$$L = 30 \text{ m}$$

$$m = 80 \text{ kg}$$



$$\sum \vec{F} = m\vec{a}$$

$$\sum F_x = m a_x$$

$$\sum F_y = m a_y$$

y-axis  $\Rightarrow T \cos \theta - mg = 0$  No motion

$$T \cos \theta = mg$$

$$T \cos 5 = 80 \times 10$$

$$\underline{T = 803 \text{ N}}$$

x-axis  $\Rightarrow T \sin 5 = ??$  *القوة الجاذبة*

$$T \sin 5 = m a_r$$

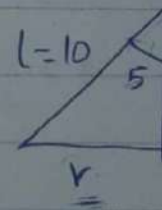
$$\frac{803 \sin 5}{8} = \frac{8 a_r}{8}$$

$$a_r = 7.7 \text{ m/s}^2$$

Velocity  $\Rightarrow a_r = \frac{v^2}{r}$

$$7.7 = \frac{v^2}{0.87}$$

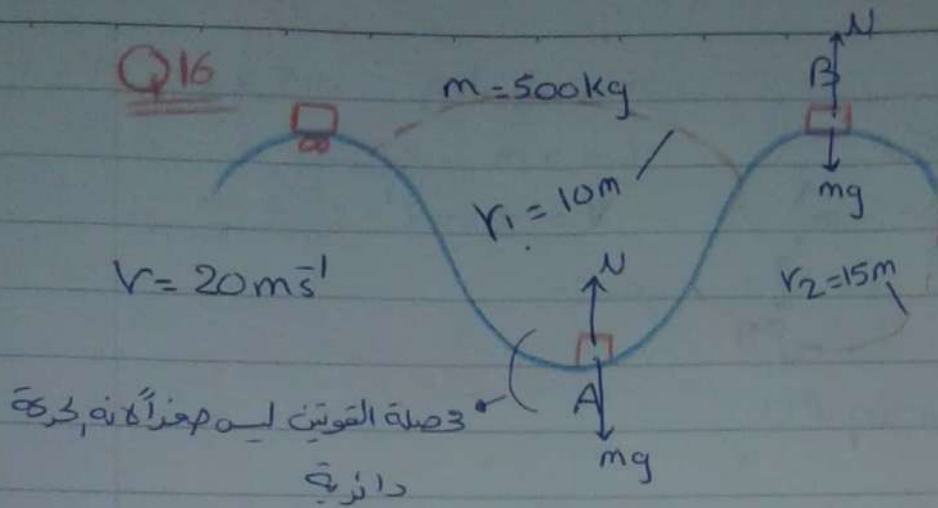
$$v^2 = \text{---} \text{ m/s}^{-1}$$



$$\sin 5 = \frac{r}{10}$$

$$r = 0.87 \text{ m}$$

Q16



(A)

$$N = ??$$

$$\Sigma F = + mar$$

$$N - mg = m \frac{v^2}{r}$$

$$N = mg + m \frac{v^2}{r} \Rightarrow N = 5000 + \frac{500 \times 400}{10}$$

$$N = 25000 \text{ N}$$

$$N = 25 \text{ kN}$$

$$(B) \Sigma F = mar$$

$$N - mg = - \frac{mv^2}{r} \quad \underline{N=0}$$

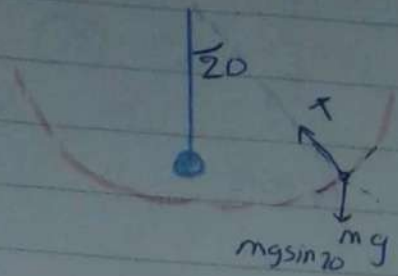
$$+mg = - \frac{mv^2}{r}$$

$$v = \sqrt{rg} \Rightarrow v = \sqrt{150} = 12.24 \text{ m/s}$$



Q18

78



$m = \frac{1}{2} \text{ kg} = 0.5 \text{ kg}$

$L = 2 \text{ m}$

$v = 8 \text{ m s}^{-1}$

$T - mg \cos \theta = \frac{mv^2}{r}$

$T = \frac{mv^2}{r} + mg \cos 20$

$T = 20.1 \text{ N}$

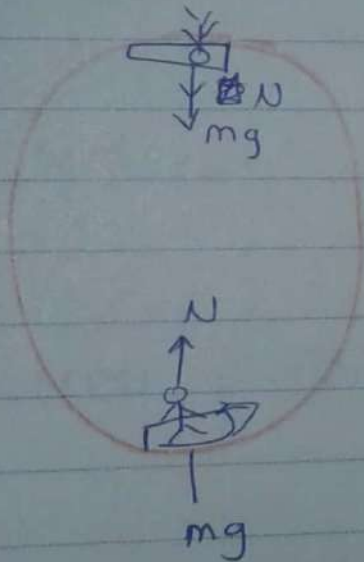
$-mg \sin 20 = ma_t$

$a_t = -3.4 \text{ m s}^{-2}$

$\vec{a}_{\text{tot}} = \frac{v^2}{r} \hat{r} - 3.4 \hat{\theta}$

$\vec{a}_{\text{tot}} = 32 \hat{r} - 3.4 \hat{\theta}$

loop to loop

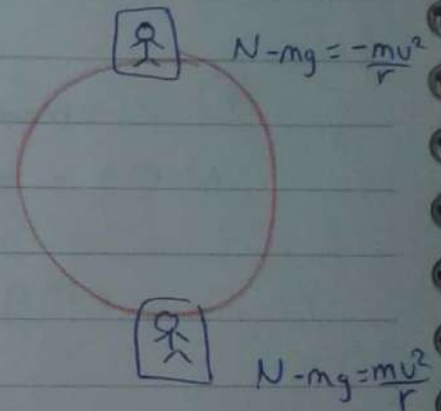


$-N + mg = + \frac{mv^2}{r}$

$N = \frac{mv^2}{r} - mg$

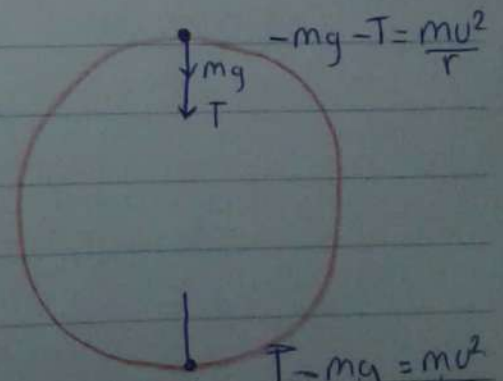
$N - mg = + \frac{mv^2}{r}$

$N = mg + \frac{mv^2}{r}$



$N - mg = -\frac{mv^2}{r}$

$N - mg = \frac{mv^2}{r}$

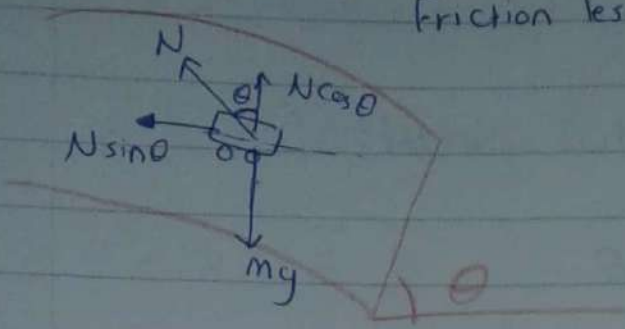


$-mg - T = \frac{mv^2}{r}$

$T - mg = \frac{mv^2}{r}$

smile for you

Friction less



$$N \cos \theta - mg = 0$$

$$N = \frac{mg}{\cos \theta}$$

$$N \sin \theta = \frac{mU^2}{r}$$

$$mg \tan \theta = \frac{mU^2}{r}$$

$$U = \sqrt{rg \tan \theta}$$

$$r = 10$$

$$\theta = 20 \rightarrow U = 85 \text{ m/s}^1$$

28-3-2018

## Ch: 7 Work & Energy

Vectors producte

1- Cross producte

$$\text{Vector} \times \text{Vector} = \text{Vector}$$

2- dot (scalar) producte

$$\text{Vector} \cdot \text{Vector} = \text{Scalar}$$

Dot product

let  $\vec{A}$  &  $\vec{B}$  to be two vectors

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

$\theta$  is angle between  $\vec{A}$  &  $\vec{B}$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\begin{aligned} \vec{A} \cdot \vec{B} &= (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) \\ &= A_x B_x \underbrace{\hat{i} \cdot \hat{i}}_1 + A_x B_y \underbrace{\hat{i} \cdot \hat{j}}_0 + A_x B_z \underbrace{\hat{i} \cdot \hat{k}}_0 \end{aligned}$$

$$A_y B_y \underbrace{\hat{j} \cdot \hat{j}}_1 + A_y B_x \underbrace{\hat{j} \cdot \hat{i}}_0 + A_y B_z \underbrace{\hat{j} \cdot \hat{k}}_0$$

$$A_z B_x \underbrace{\hat{k} \cdot \hat{i}}_0 + A_z B_y \underbrace{\hat{k} \cdot \hat{j}}_0 + A_z B_z \underbrace{\hat{k} \cdot \hat{k}}_1$$

Now  $\hat{i} \cdot \hat{i} = |\hat{i}| |\hat{i}| \cos \theta$

$$= 1 * 1 * \cos 0$$

$$= \boxed{1}$$

$$\hat{i} \cdot \hat{j} = |\hat{i}| |\hat{j}| \cos \theta$$

$$= 1 * 1 * \cos 90$$

$$= \boxed{0}$$

$$\hat{i} \cdot \hat{k} = |\hat{i}| |\hat{k}| \cos \theta$$

$$= 1 * 1 * \cos 90$$

$$= \boxed{0}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

Ex

$$\text{let } \vec{A} = 3\hat{i} + 4\hat{j}$$

$$\vec{B} = -5\hat{i} - 7\hat{j}$$

Find  $\vec{A} \cdot \vec{B}$  and the angle between  $\vec{A}$  &  $\vec{B}$

$$\begin{aligned} \vec{A} \cdot \vec{B} &= A_x B_x + A_y B_y \\ &= (3 * -5) + (4 * -7) \\ &= -15 - 28 = \boxed{-43} \end{aligned}$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \Rightarrow \cos \theta = \frac{-43}{\sqrt{9+16} \sqrt{25+49}} = \frac{-43}{43}$$

$$\boxed{\theta = 180^\circ}$$

Ex let  $\vec{A} = 5\hat{i} - 8\hat{j} + 7\hat{k}$

Find the angle between  $\vec{A}$  &

(a) the x-axis

$$\hat{i} \cdot \vec{A} = |\hat{i}| |\vec{A}| \cos \theta_x$$

$$\begin{aligned} \hat{i} \cdot \vec{A} &= \hat{i} \cdot (5\hat{i} - 8\hat{j} + 7\hat{k}) \\ &= 5 + 0 + 0 = \boxed{5} \end{aligned}$$

$$\hat{i} \cdot \vec{A} = |\hat{i}| |\vec{A}| \cos \theta_x$$

$$5 = 1 * \sqrt{25+64+49} \cos \theta_x$$

$$\theta_x = 64.8^\circ$$

$\vec{A}$

No.

28-3-2018

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(b) the y-axis

$$\begin{aligned}\hat{j} \cdot \vec{A} &= \hat{j} \cdot (5\hat{i} - 8\hat{j} + 7\hat{k}) \\ &= 0 - 8 + 0 \\ &= -8\end{aligned}$$

$$\hat{j} \cdot \vec{A} = |\hat{j}| |\vec{A}| \cos \theta_y$$

$$-8 = 1 \cdot 11.75 \cos \theta_y$$

$$\theta_y = 133^\circ$$

(c) the z-axis

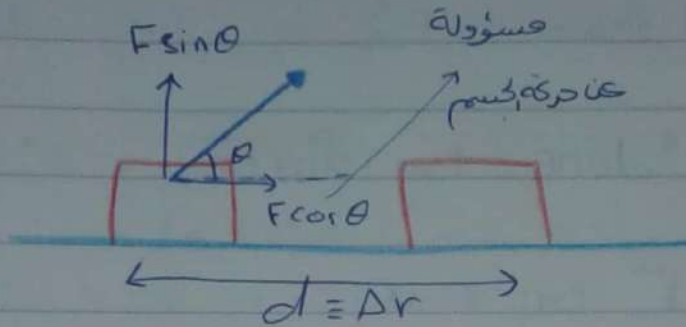
$$\begin{aligned}\hat{k} \cdot \vec{A} &= \hat{k} \cdot (5\hat{i} - 8\hat{j} + 7\hat{k}) \\ &= \boxed{7}\end{aligned}$$

$$7 = 1 \cdot 11.75 \cos \theta_k$$

$$\theta_k =$$

## \* Work done by const. force

Const. force = const. in magn. & direction



Define

work  $\equiv$  Force that caused the motion \* displacement

$$W = F \cos \theta \Delta r$$

$$W = F \Delta r \cos \theta$$

$$W = \vec{F} \cdot \vec{\Delta r}$$

الوحدة

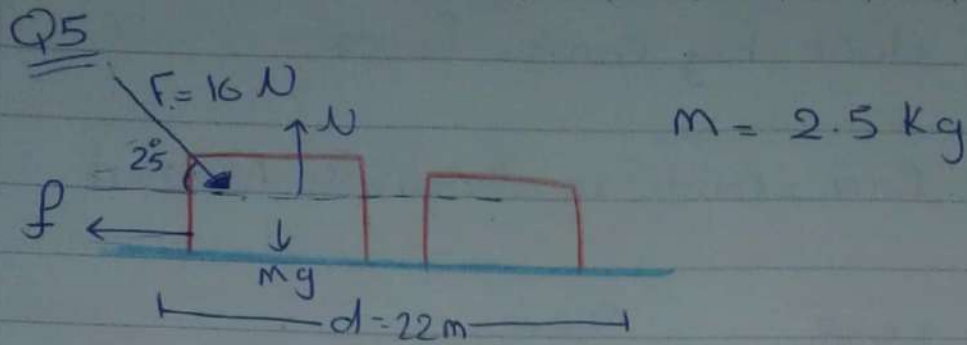
$$[W] = N \cdot m = J$$

Note :-

$$W = F \Delta r \cos \theta$$

if  $W = 0$   $\left\{ \begin{array}{l} \Delta r = 0 \\ \cos \theta = 0, \theta = 90 \end{array} \right.$  إما لا يوجد حركة أو ما يقرب

work is scalar could be +ve, -ve, zero



a work done by force

$$W_F = F \Delta r \cos \theta$$

$$= 16 (2.2) \cos 25 = 24 \text{ J}$$

b work done by Normal force

$$W_N = N \Delta r \cos \theta$$

$$= N \Delta r \cos 90 = 0$$

c work done by gravity

$$W_{mg} = 0$$

d If the friction force = 10 N Find work done by friction

$$W_f = f \Delta r \cos \theta$$

$$= 10 (2.2) \cos 180 = -22 \text{ J}$$

Q11

$$\vec{F} = 6\hat{i} - 2\hat{j}$$
$$\vec{Dr} = 3\hat{i} + \hat{j}$$

Find w &  $\theta$ 

$$w = \vec{F} \cdot \vec{Dr}$$
$$= (6 \times 3) + (-2 \times 1)$$
$$= 18 + -2 = 16 \text{ J}$$

$$w = F \cdot Dr \cos \theta$$

$$16 = \sqrt{40} \cdot \sqrt{10} \cos \theta$$

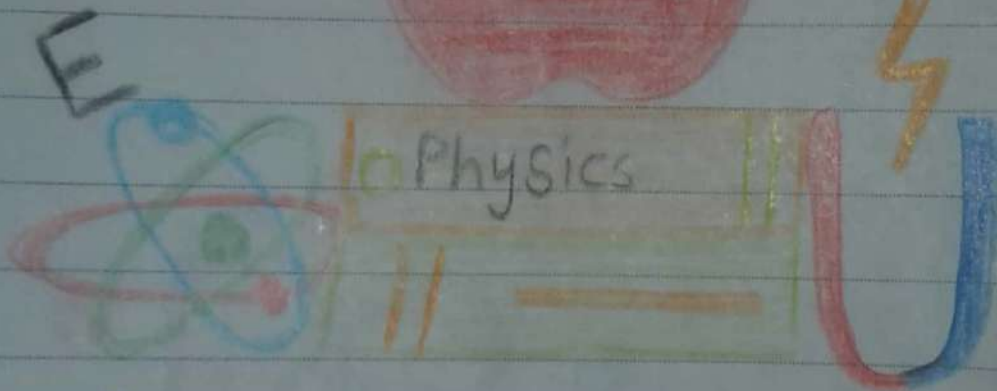
$$\theta = \underline{\underline{37^\circ}}$$

Second direction





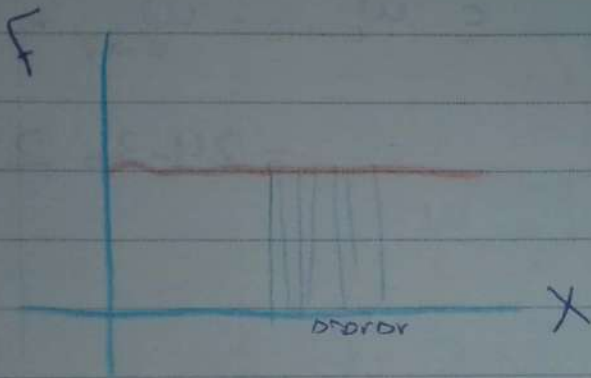
$$E = mc^2$$



\* Sally Bariyaseen

① Final

• Work done by Vary Force



$$\Delta w = F \cdot \Delta x$$

$$W \approx \sum \Delta w = \sum F \cdot \Delta x$$

$$W = \sum_{\Delta x \rightarrow 0} F \Delta x$$

Integral

$$W = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x}$$

In general

$$W = \int_{p_i}^{p_f} \vec{F} \cdot d\vec{r}$$

smile...

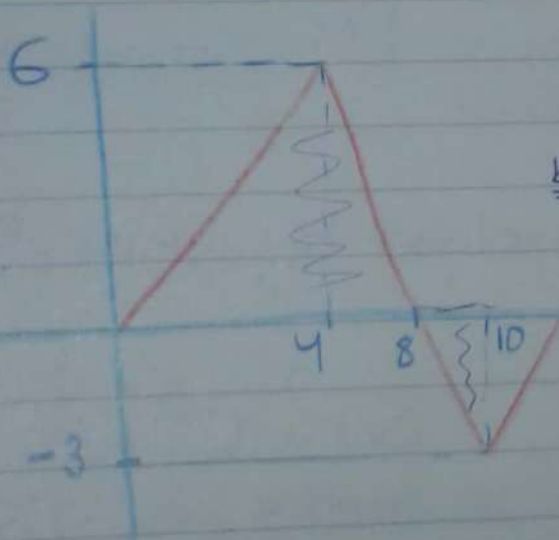
work = Area under  $r$

$F$  &  $r$  Curve

Note if  $F = \text{Constant}$   
 Work =  $\vec{F} \cdot \int_{r_i}^{r_f} dr$

$$W = \vec{F} \cdot \Delta r$$

Q14



$$a \quad W_{0 \rightarrow 8} = \frac{1}{2} \times 8 \times 6 = 24 \text{ J}$$

$$b \quad W_{8 \rightarrow 10} = \frac{1}{2} \times 2 \times -3 = -3 \text{ J}$$

$$c \quad W_{0 \rightarrow 10} = W_{0 \rightarrow 8} + W_{8 \rightarrow 10} \\ = 24 - 3 = 21 \text{ J}$$

H.W 15

Q26 Vary in force

$$F = 8x - 16 \quad x_i = 0, \quad x_f = 3$$

$$W = \int_{x_i}^{x_f} F dx = \int_0^3 (8x - 16) dx$$

$$4x^2 - 16x \Big|_0^3$$

$$W = -12 \text{ J}$$

Q2a  $\vec{F} = 4x\hat{i} + 3y\hat{j}$   
 $x_i = 0 \quad x_f = 5$

only in x-axis

Find W

$$\begin{aligned} \vec{F} &= F_x \hat{i} + F_y \hat{j} \\ \vec{r} &= x\hat{i} + y\hat{j} \\ d\vec{r} &= dx\hat{i} + dy\hat{j} \end{aligned} \quad \left\{ \begin{aligned} W &= \int \vec{F} \cdot d\vec{r} \\ &= \int (F_x \hat{i} + F_y \hat{j}) (dx\hat{i} + dy\hat{j}) \\ W &= \int F_x dx + F_y dy \end{aligned} \right.$$

$$W = \int \vec{F} \cdot d\vec{r} = \int 4x\hat{i} + 3y\hat{j} (dx\hat{i} + dy\hat{j})$$

$$W = \int_0^5 4x dx + \int_0^0 3y dy$$

$$W = 2x^2 \Big|_0^5 = 50 \text{ J}$$

$$\begin{aligned} \vec{r}_i &= 0\hat{i} + 0\hat{j} \\ \vec{r}_f &= 5\hat{i} + 8\hat{j} \end{aligned}$$

$$W = \int_0^5 4x dx + \int_0^8 3y dy$$

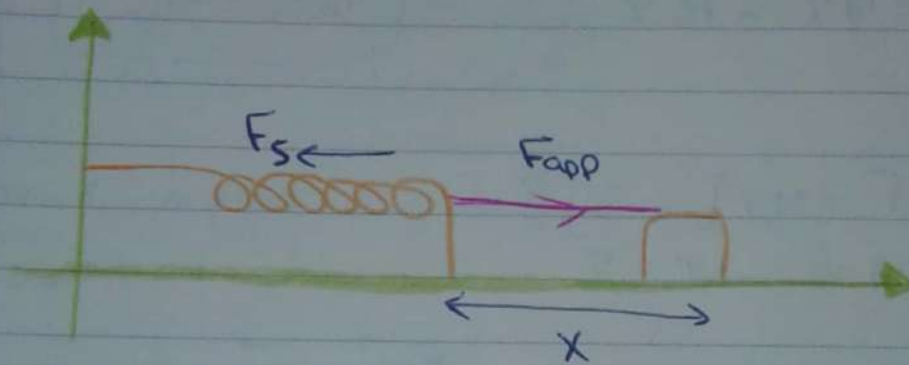
$$W = 2x^2 \Big|_0^5 + \frac{3}{2} y^2 \Big|_0^8$$

o smile...

$$W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$$

as a good example

Spring Force



$$F_{app} \propto x$$

$$F_{app} = kx$$

$k \equiv$  Spring Const

$$[k] = N/m$$

The spring force ( $F_s$ ) is always opposite to the displacement

$$\boxed{F_s = -kx} \text{ Hook law.}$$

$$W_{app} = \int_{x_i}^{x_f} F \, dx$$

$$= \int_{x_i}^{x_f} kx \, dx \quad \rightsquigarrow \quad \left. \frac{1}{2} kx^2 \right]_{x_i}^{x_f}$$

$$W_{app} = \frac{1}{2} kx_f^2 - \frac{1}{2} kx_i^2$$

Define: the elastic potential energy  $U_s$   
طاقة اللفج المرنيح

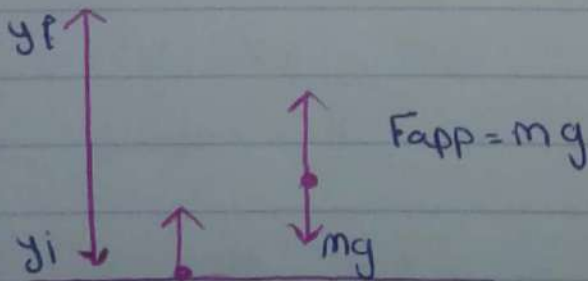
$$F = kx$$

$$U_s = \frac{1}{2} kx^2$$

$$\Rightarrow W_{app} = U_f - U_i \quad \Rightarrow \quad W_{app} = \Delta U_s$$

$$W_s = -\Delta U_s$$

\* gravitational potential energy  $U_g$ :



$$W_{app} = F_{app} \Delta y$$

$$= mg(\uparrow) \cdot (y_f - y_i) \downarrow$$

$$W_{app} = mg y_f - mg y_i$$

Define:  $U = mgy$

$$W_{app} = U_f - U_i$$

$$W_{app} = \Delta U$$

$$W_g = -\Delta U$$

\* if the only change is in the height then the work done by the applied force is equal to the change in gravitational potential energy

$$W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$$

16-4-2018

$$U_s = \frac{1}{2} k x^2$$

$$W_{app} = \Delta U$$

$$W_{app} = \Delta U_s$$

$$W_g = -\Delta U$$

$$W_s = -\Delta U_s$$

$$U = mgy$$

### \* Work - Kinetic energy theorem

$$W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$$

$$F = ma = m \frac{dv}{dt}$$

Now  $a = \frac{dv}{dt} = \frac{dv}{dr} \frac{dr}{dt}$  chain rules

$$\hookrightarrow W = \int_{r_i}^{r_f} m \frac{dv}{dr} \left( \frac{dr}{dt} \right) dr \rightarrow dr$$

$$W = m \int_{v_i}^{v_f} v dv \rightarrow W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

Define the kinetic energy K

$$K = \frac{1}{2} m v^2$$

$$\hookrightarrow W = K_f - K_i$$

$$W_{app} = \Delta K$$

if the only change is in the velocity in the speed then the work done by the applied force is equal to the change in the kinetic energy.



\* Total energy. (mechanical energy)

$$E = K + U$$

$$E = \frac{1}{2}mv^2 + mgy$$

if No external force No total energy change

↳ No energy loss or gain

$$\Delta E = 0$$

$$E_f = E_i$$

$$K_f + U_f = K_i + U_i$$

$$K_f - K_i = U_i - U_f$$

$$\Delta K = -\Delta U$$

$$\Delta U = -\Delta K$$

Conservative system  
or force

نظام و قوت محافظه کارانه

i.e Spring gravity.

$m = 10 \text{ kg}$  Ex

10m

أسفل

	$v$	$K$	$E$	
	1000	0	1000	} $\Delta E = 0$
	800	200	1000	
	300	700	1000	} $\Delta E = 0$
	0	1000	1000	

$\Delta E = \Delta$

- the work done by the applied force

$$W_{app} = \Delta E$$

↳ there is a change in height and speed

In general work done by applied -  
work done by friction =  $\Delta E$

$$W_{app} - \int f_k d = \Delta E$$

$$\text{if } f_k = 0 \rightarrow \Delta E = W_{app}$$

$$\text{if } f_k = 0, \overset{w}{F}_{app} = 0 \rightarrow \Delta E = 0 \text{ conservative}$$

$$\text{if } \overset{w}{F}_{app} = 0 \Rightarrow \Delta E = -\int f_k d$$

### \* Conservative Force

$$(1) \Delta E = 0$$

$$\Delta K = -\Delta U \quad \text{or} \quad \Delta U = -\Delta K$$

$$(2) W = \int_{r_i}^r \vec{F} \cdot d\vec{r}$$

work done by conservative force is

independent on the track is depends only

on displacement

$$(3) W = \oint \vec{F} \cdot d\vec{r} = 0 \quad \text{على مغلق}$$

The work done by conservative force

on a closed track is Zero

$$W = \int_r^r \vec{F} \cdot d\vec{r} = 0$$

(4) For any conservative force we can define a potential energy function  $U$  such that

$$W_{\text{cons}} = -\Delta U$$

$$W_s = -\Delta U_s$$

$$\Delta y = -\Delta U_y$$

$$\text{Now } -\Delta U = W_{\text{cons}}$$

في

$$U_i = 0$$

$$-U = \int \vec{F}_{\text{cons}} \cdot d\vec{r}$$

$$-du = \vec{F}_{\text{cons}} \cdot d\vec{r}$$

$$\vec{F} = -\frac{du}{dr}$$

$$F_x = -\frac{du}{dx}$$

$$F_y = -\frac{du}{dy}$$

$$F_z = -\frac{du}{dz}$$

EX

$$\text{let } U(x, y, z) = 3x^2yz^3 + 5x^2y$$

to be a potential energy function for some conservative force find  $\vec{F}$  at (1, 1, 1)

$$F_x = -\frac{dU}{dx} = -(6xyz^3 + 10xy)$$

$\downarrow$   
 at (1, 1, 1)

$$= -16$$

$$F_y = \frac{-du}{dy} = -(3x^2z^3 + 5x^2) = -8$$

$$F_z = \frac{-du}{dz} = -(3x^2y3z^2 + 5x^2) = -9$$

$$\vec{F} = -16\hat{i} - 8\hat{j} - 9\hat{k}$$

\* Power

القوة

Work (energy) done per unit of time

$$P = \frac{W}{t}$$

$$[P] = J/s = \text{Watt.}$$

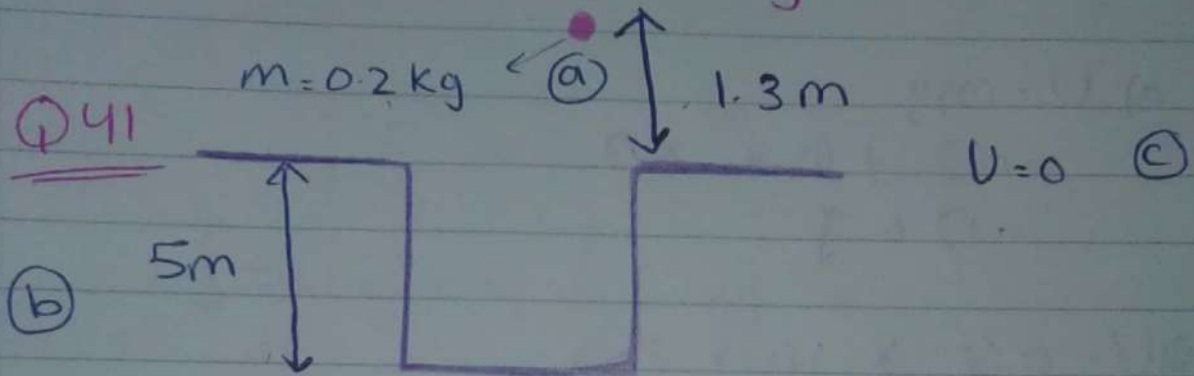
$$P = \frac{W}{t} = \frac{\vec{F} \cdot \vec{r}}{t} = \vec{F} \cdot \vec{v}$$

$$P = \vec{F} \cdot \vec{v}$$

$$P_{\text{average}} = \frac{\Delta W}{\Delta t} ; P_{\text{ins}} = \frac{dW}{dt}$$

H.w ch7 [ 5, 6, 9, 10, 11, 12, 15  
17, 31, 33, 42, 50, 31 ]

ch8 [ 6, 12, 22 ]



a)  $U = mgy$   
 $= 0.2 * 10 * (1.3)$   
 $= 2.6 \text{ J}$

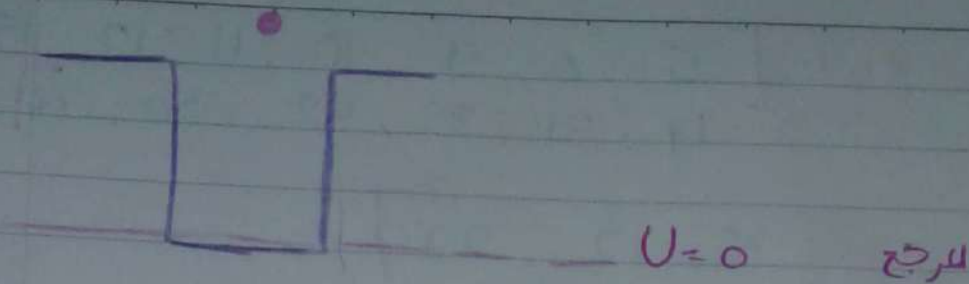
b)  $U = 0.2 * 10 * (-5)$   
 $= -10 \text{ J}$

c)  $\Delta U = U_p - U_i$   
 $= -10 - 2.6$   
 $= -12.6 \text{ J}$

What the work done by the gravitation  
 work

$$W_{\text{gra}} = -\Delta U = +12.6 \text{ J}$$

$W_{\text{gr}}$

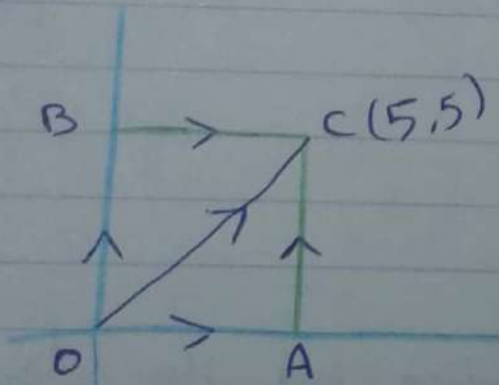


$$\begin{aligned} \text{a) } U &= mgy \\ &= 0.2 * 10 * 6.3 \\ &= 12.6 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{b) } U &= 0.2 * 10 * \text{Zero} \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{c) } \Delta U &= U_f - U_i = 0 - 12.6 = -12.6 \text{ J} \\ W_{\text{gra}} &= -\Delta U = +12.6 \text{ J} \end{aligned}$$

Q 45



$$\vec{F} = 2y\hat{i} + x^2\hat{j}$$

work done in  
each case

$$\text{d) } W_{O \rightarrow A \rightarrow C} = W_{OA} + W_{AC}$$

$$W_{OA} = \int \vec{F} \cdot d\vec{r}$$

$$= \int (2y\hat{i} + x^2\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$= \int_{y=0}^0 2y dx + \int x^2 dy = 0$$

$$W_{AC} = \int \vec{F} \cdot d\vec{r}$$

$$= \int_{x=5}^0 2y dx + \int x^2 dy$$

$$x=5 \rightarrow dx=0$$

$$= \int_0^5 25 dy = 25(5-0) = 125 \text{ J}$$

$$\textcircled{b} W_{ABC} = W_{AB} + W_{AC}$$

$$W_{AB} = \int_{x=0}^0 2y dx + \int x^2 dy = 0$$

$$W_{AC} = \int_{y=5}^0 2y dx + \int x^2 dy$$

$$= \int_0^5 10 dx = 10(5-0) = 50 \text{ J}$$



$$\textcircled{c} \text{ Woc} = \int 2y dx + \int x^2 dy$$

but  $y = mx + b$

$$m = \text{slope} = \frac{5-0}{5-0} = 1$$

$$y = x \quad dy = dx$$

$$\text{Woc} = \int_0^5 2y dx + \int_0^5 y^2 dy$$

$$= 10(5-0) + 25(5-0) = \underline{175}$$

Q119

$$U = 3x^3y - 7x$$

Find  $\vec{F}$

$$F_x = \frac{-du}{dx} = 9x^2y - 7$$

$$= 7 - 9x^2y$$

$$F_y = \frac{-du}{dy} = 3x^3 - 0$$

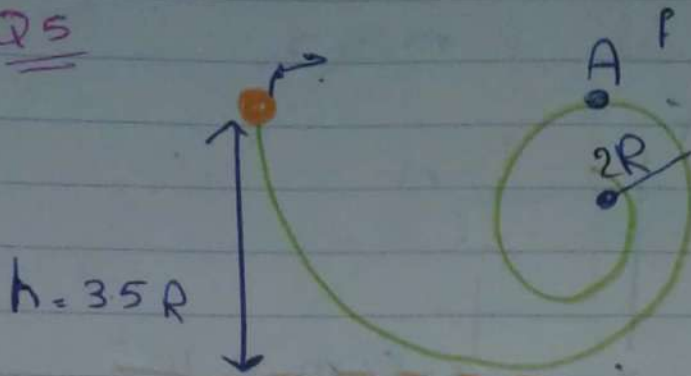
$$= -3x^3$$

$$(7 - 9x^2y) \hat{i} - 3x^3 \hat{j}$$

Find  $\vec{F}$  at  $(2, 3)$

Ch 8

No

Q5

$$E_i = E_f$$

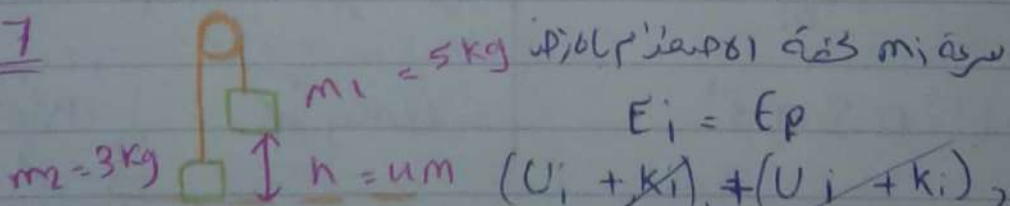
$$U_i + \cancel{k_i} = U_f + k_f$$

$$U_i = U_f + k_f$$

$$mgy_i = mgy_f + \frac{1}{2}mv^2$$

$$10 \times 35R = 10 \times 2R + \frac{1}{2}v^2$$

$$v = \sqrt{30R} \text{ m s}^{-1}$$

Q7

$$E_i = E_f$$

$$(U_i + k_i)_1 + (U_i + k_i)_2 = (U_f + k_f)_1 + (U_f + k_f)_2$$

$$(U_i)_1 = (k_f)_1 + (U_f + k_f)_2$$

$$m_1gh = \frac{1}{2}m_1v_f^2 + m_2gh + \frac{1}{2}m_2v_f^2$$

$$50 \times 4 = \frac{1}{2}5v_f^2 + 3 \times 10 \times 4 + \frac{1}{2}3v_f^2$$

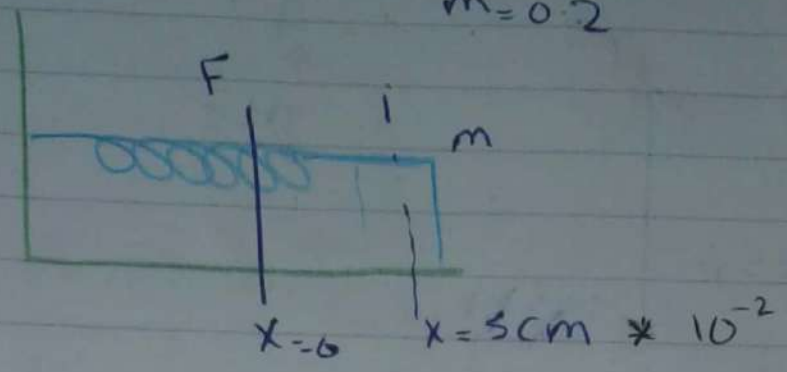
$$v_f = \underline{\hspace{2cm}}$$

smile...

Q15

$k = 5000 \text{ N/m}$

$m = 0.2$



$U = mgy$   
 $U_s = \frac{1}{2} k x^2$

a) smooth

$E_i = E_f$

$k_i + U_{i1} = k_f + U_{f1}$

$U_{si} = kF_1$

$\frac{1}{2} k x^2 = \frac{1}{2} m v^2$

$v = \sqrt{\frac{k}{m}}$

$v = 0.8 \text{ m/s}$

b) Friction

$\mu_k = 0.35$

Work done

$w = F_d$

$w_{app} - F_k d = \Delta E$

$-F_k d = \Delta E$

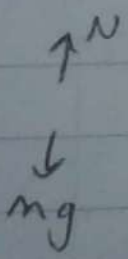
$-W d = E_f - E_i$

$-W mg d = kF - U_{si}$

$-W mg d = \frac{1}{2} m v^2 - \frac{1}{2} k x^2$

Work done

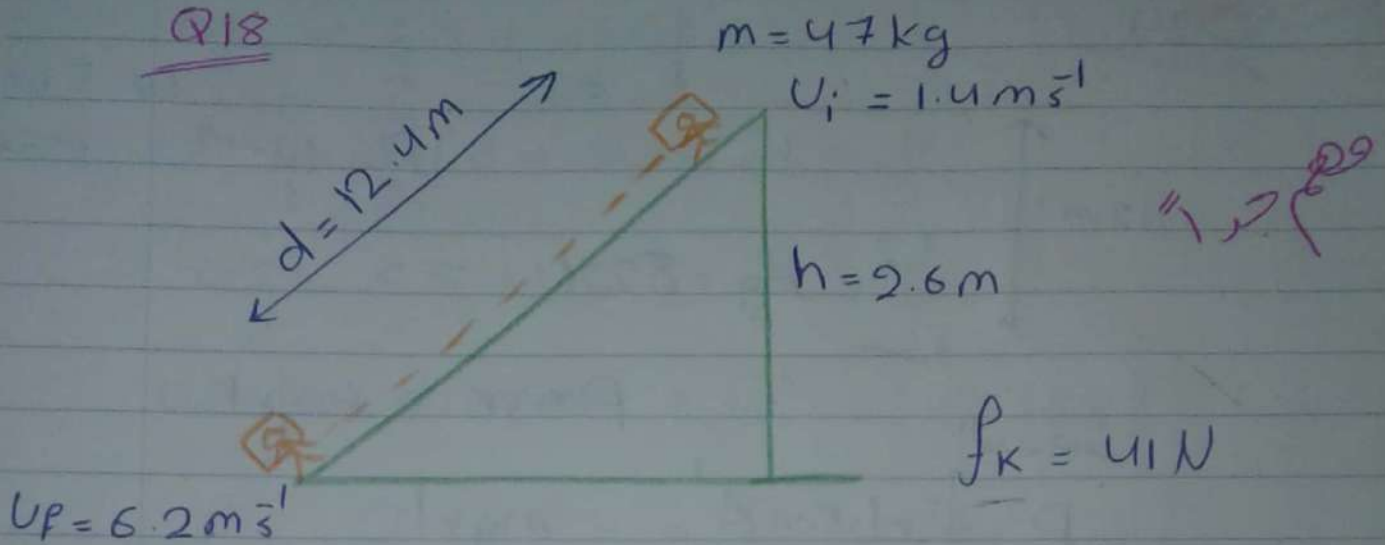
$v = \dots$



$w_{app} - F_k d = \Delta E$

$F = W$

$\Delta E = E_f - E_i$   
 $kF - U_i$

Q18

\* work done by the motor

$$W_{\text{app}} - (f_k d) = \Delta E$$

$$W_{\text{app}} - 41 \times 12.4 = E_f - E_i$$

$$= K_f - (K_i + U_i)$$

$$W_{\text{app}} - 508.4 = \frac{1}{2} m v_f^2 - \left( \frac{1}{2} m u_i^2 + mgh \right)$$

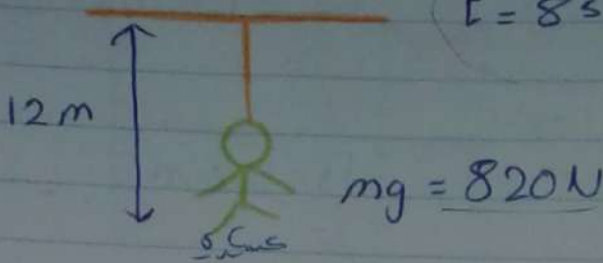
$$W_{\text{app}} - 508.4 = \frac{1}{2} 47 (6.2)^2 - \left( \frac{1}{2} 47 (1.4)^2 + 47 \times 10 \times 2.6 \right)$$

$$W_{\text{app}} - 508.4 = 145.7 - (46.06 + 1222)$$

$$W_{\text{app}} - 508.4 = 145.7 - 1268.06$$

$$W_{\text{app}} = -613.96 \text{ J}$$

Q29



$W = F \cdot D$

$P = \frac{W}{t} \rightarrow 820 \times 12$

الوزن فقط

Power =  $\frac{\text{Work}}{t}$

$P = \frac{F \cdot d \cdot \cos \theta}{t} = \frac{mgd}{t}$

$= \frac{820 \times 12}{8} = 1230$

OR  $W = \Delta U = U_f - U_i \rightarrow 0$

$= mgy = 820 \times 12$

$P = \frac{W}{t} = \frac{820 \times 12}{8} = 1230 \text{ Watt}$

Q30

$m = 0.875 \text{ kg}$     $U_i = 0$     $U_f = 0.62 \text{ m/s}^2$

$t = 21 \text{ ms}$    power?

$P = \frac{W}{t}$  ;  $W = (F \cdot d)$

$m \cdot a$     $m \cdot a$

قوة التسارع    $ma$

but  $F = ma$

$U_f = U_i + at$

$0.62 = 0 + a \cdot 21 \times 10^{-3}$

$a = 29.5 \text{ m/s}^2$

$F = 0.875 \times 29.5$

$= \underline{\underline{25.8 \text{ N}}}$

21

23-4-2018

No.

$$\Delta x = \frac{1}{2} (v_i + v_f) t$$

$$= \frac{1}{2} (0 + 0.62) 21 \times 10^{-3}$$

$$\Delta x = 6.5 \times 10^{-3} \text{ m}$$

$$W = 25.8 \times 6.5 \times 10^{-3} = 0.168 \text{ J}$$

$$P = \frac{0.168}{21 \times 10^{-3}} = 8 \text{ watt}$$

OR

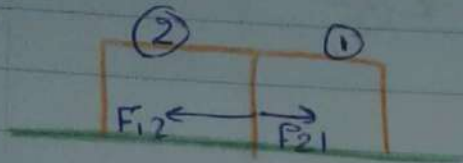
$$W = \Delta K = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$= 0.168 \text{ J}$$

$$P = \frac{0.168 \text{ J}}{21 \times 10^{-3}} = 8 \text{ watt}$$

Ch 8 done

## (ch 9: Momentum at collisions)



$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\vec{F}_{12} + \vec{F}_{21} = 0$$

$$m_2 \vec{a}_2 + m_1 \vec{a}_1 = 0$$

$$m_2 \frac{d\vec{v}_2}{dt} + m_1 \frac{d\vec{v}_1}{dt} = 0$$

$$\frac{d}{dt} (m_2 \vec{v}_2 + m_1 \vec{v}_1) = 0$$

Define  $\rightarrow$  the linear momentum  $\vec{p}$

$$\vec{p} = m\vec{v}$$

$$\hookrightarrow \frac{d}{dt} (\vec{p}_2 + \vec{p}_1) = 0$$

$$\vec{p}_1 + \vec{p}_2 = \text{const}$$

$$\vec{p}_{\text{total}} = \text{const}$$

$$\Delta \vec{p}_{\text{total}} = 0$$

$$\hookrightarrow \vec{p}_i = \vec{p}_f$$

23

For an isolated system the total momentum is

conserved

$$\hookrightarrow d\vec{P} = 0$$

$$\sum \vec{P}_i = \sum \vec{P}_f$$

$$\text{Now } \vec{P} = m\vec{v}$$

$$P^2 = m^2 v^2 \rightarrow \frac{P^2}{2m} = \frac{m^2 v^2}{2m}$$

$$\frac{P^2}{2m} = \frac{1}{2} m v^2$$

$$\hookrightarrow K = \frac{P^2}{2m}$$

$$\text{Now } \vec{p} = m\vec{v}$$

$$\frac{d\vec{p}}{dt} = \frac{d}{dt} m\vec{v}$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

So Newton's 2nd

$$\sum \vec{F}_{\text{ext}} = \frac{d\vec{p}}{dt}$$

$$\sum \vec{F}_{\text{ext}} = \frac{d}{dt} (m\vec{v})$$



We can write

$$\vec{F} = \frac{D\vec{p}}{dt}$$

So  $\vec{F} \neq \frac{d\vec{p}}{dt}$

$$\int_{P_i}^{P_f} d\vec{p} = \int_0^t \vec{F} dt$$

$$\Delta\vec{p} = \int_0^t \vec{F} dt$$

Impulse  $\vec{I}$  الدفع

Define the Impulse  $\vec{I}$

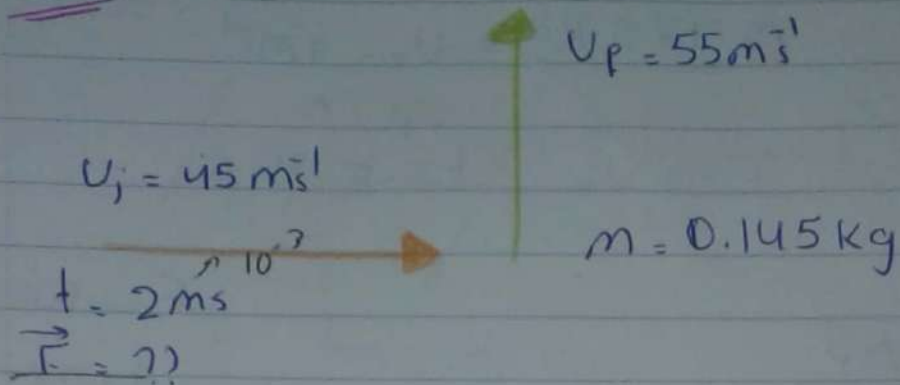
$$\vec{I} = \Delta\vec{p} = \int_0^t \vec{F} dt$$

$|\vec{I}| \equiv$  Area under the curve

H.w

2, 4, 8, 13, 19, 20

23, 25, 33, 34

Q5

$$\vec{F} = \frac{D\vec{P}}{Dt}$$

$$\begin{aligned} D\vec{P} &= \vec{P}_f - \vec{P}_i \\ &= m\vec{u}_f - m\vec{u}_i \end{aligned} \quad \text{vectors}$$

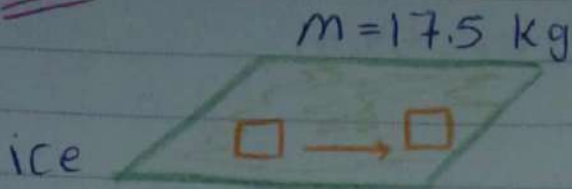
$$= 0.145 * 55 \hat{j} - 0.145 * 45 \hat{i}$$

$$D\vec{P} = 8 \hat{j} - 6.5 \hat{i}$$

$$\vec{F} = \frac{8 \hat{j} - 6.5 \hat{i}}{2 * 10^{-3}}$$

$$\vec{F} = 4000 \hat{j} - 3250 \hat{i}$$

$$\vec{F} = -3250 \hat{i} + 4000 \hat{j}$$

Q3

$$v_i = 3 \text{ m s}^{-1}$$

$$v_f = 0$$

$$t = 8.75 \text{ s}$$

$$\vec{F} = \frac{\Delta \vec{P}}{\Delta t}$$

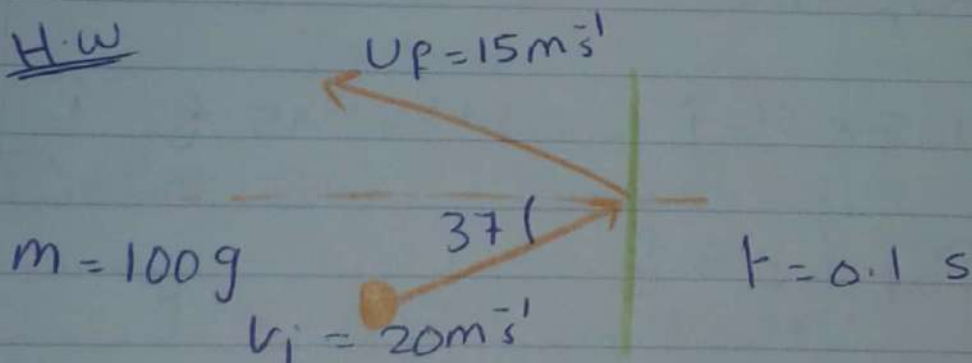
$$= \frac{\vec{P}_f - \vec{P}_i}{\Delta t}$$

$$= \frac{0 - 3 \times 17.5}{8.75}$$

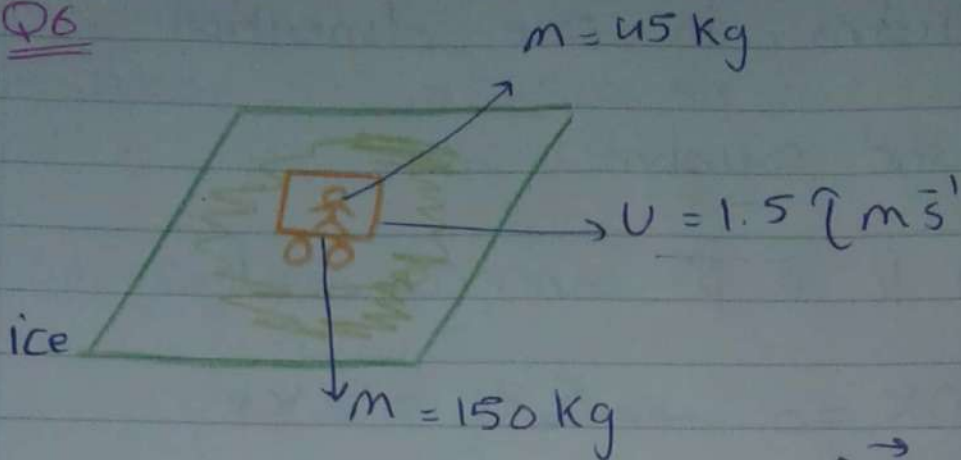
$$\vec{F} = -6 \hat{j} \text{ N}$$

$$F = P$$

$$P = m v$$

H.W

$$\vec{F} = ??$$

Q6

$$\Delta P = 0$$

$$\begin{aligned} \vec{P}_i &= \vec{P}_f \\ 0 &= \vec{P}_G + \vec{P}_p \end{aligned}$$

$$= m_G U_G + m_p v_p$$

$$= 45 \times 1.5 \hat{i} + 150 v_p$$

$$v_p = -0.45 \hat{i}$$

Q11

$$\begin{aligned} \Delta P &= 0 \\ \vec{P}_i &= \vec{P}_f \end{aligned}$$

$$0 = m\vec{v} + 3m \cdot 2 \hat{i}$$

$$\vec{v} = -6 \hat{i} \text{ m/s}$$

## → \* Collisions in one dimension

### ① elastic collisions

Both  $K$  &  $\vec{P}$  are conserved

$$\Delta K = 0 \rightarrow \sum K_i = \sum K_f$$

$$\Delta P = 0 \rightarrow \sum \vec{P}_i = \sum \vec{P}_f$$

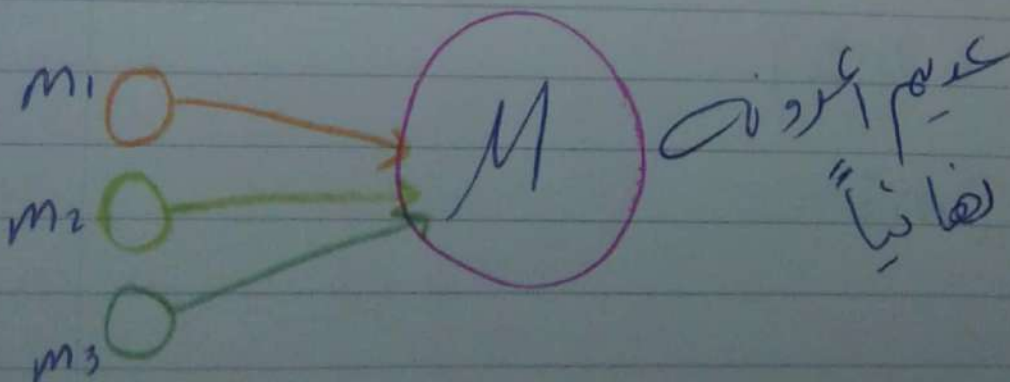
### ② in elastic collisions

$K$  is not conserved  $\Delta K \neq 0$   
but  $\vec{P}$  is conserved

$$\Delta \vec{P} = 0 \rightarrow \sum \vec{P}_i = \sum \vec{P}_f$$

So  $\vec{P}$  is always conserved

Completely inelastic



Q 22

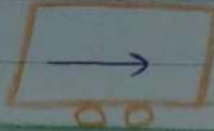
$$m_1 = 1200 \text{ Kg}$$

$$U_{1i} = 25 \text{ m/s}$$



$$m_2 = 9000 \text{ Kg}$$

$$U_{2i} = 20 \text{ m/s}$$



$$U_{1f} = 18 \text{ m/s}$$

كم سرعة  
السيارة بعد التصادم

$$\Sigma P_i = \Sigma P_f$$

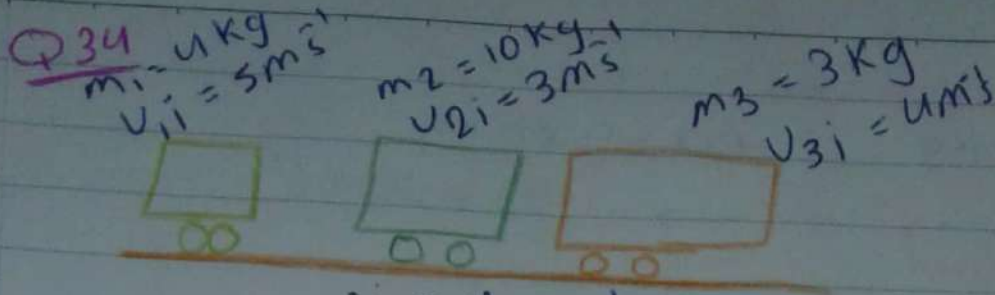
$$\Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}$$

$$m_1 \vec{U}_{1i} + m_2 \vec{U}_{2i} = m_1 \vec{U}_{1f} + m_2 \vec{U}_{2f} \rightarrow \text{بدي اياها}$$

$$1200 * 25 \hat{i} + 9000 * 20 \hat{i} = 1200 * 18 \hat{i} + 9000 * \vec{U}_{2f}$$

$$\vec{U}_{2f} = 21 \hat{i} \text{ m/s}$$



لصاروح جسم واحد سرعة الجسم، لزي نتج

$$\sum \vec{P}_i = \sum \vec{P}_f$$

$$\vec{P}_{1i} + \vec{P}_{2i} + \vec{P}_{3i} = \vec{P}$$

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} + m_3 \vec{v}_{3i} = M \vec{v}_f$$

$$4 \times 5 \hat{i} + 10 \times 3 \hat{i} + 3 \times (-4) \hat{i} = 17 \vec{v}_f$$

$$38 \hat{i} = 17 \vec{v}_f$$

$$\vec{v}_f = 2.2 \hat{i} \text{ m/s}$$

للبيّن

let  $m_3 = 50 \text{ kg}$

$$20 \hat{i} + 30 \hat{i} - 200 \hat{i} = 63 \vec{v}_f$$

$$-150 \hat{i} = 63 \vec{v}_f$$

$$\vec{v}_f = -2.4 \hat{i} \text{ m/s}$$

للبيّن