



اللجنة الأكاديمية للهندسة المدنية

دفتر

فيزياء 1

عمار الكبش

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Chapter (1)

Basic Units

1 mass كيلوغرام kilograms (kg)	2 length, distance الطول, المسافة meters (m)	3 Time زمن, الوقت seconds (s)
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Derived Units

1 Velocity السرعة m/s L/T	2 Force القوة kgm/s ²	3 acceleration التسارع m/s ² L/T ²	4 Momentum الزخم kg·m/s
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5 Work
الشغل
نيوتن U·m

Dimensions - الأبعاد

Mass = M
length = L
Time = T

* Note

Volume $\rightarrow m^3$
المساحة $\rightarrow m^2$

~~Dimensional analysis~~
Dimensional analysis 8-

جوابي، الجواب،

In any equation:-

The dimension of the right hand side = the dimension of the left hand side

(ex):- show that $x = \frac{u}{2} t + \frac{1}{2} a t^2$

↑ velocity ↑ acceleration $\text{ع} \text{L}$

↑ displacement

$$L = \frac{L}{T} * T + \frac{L}{T^2} * T^2$$

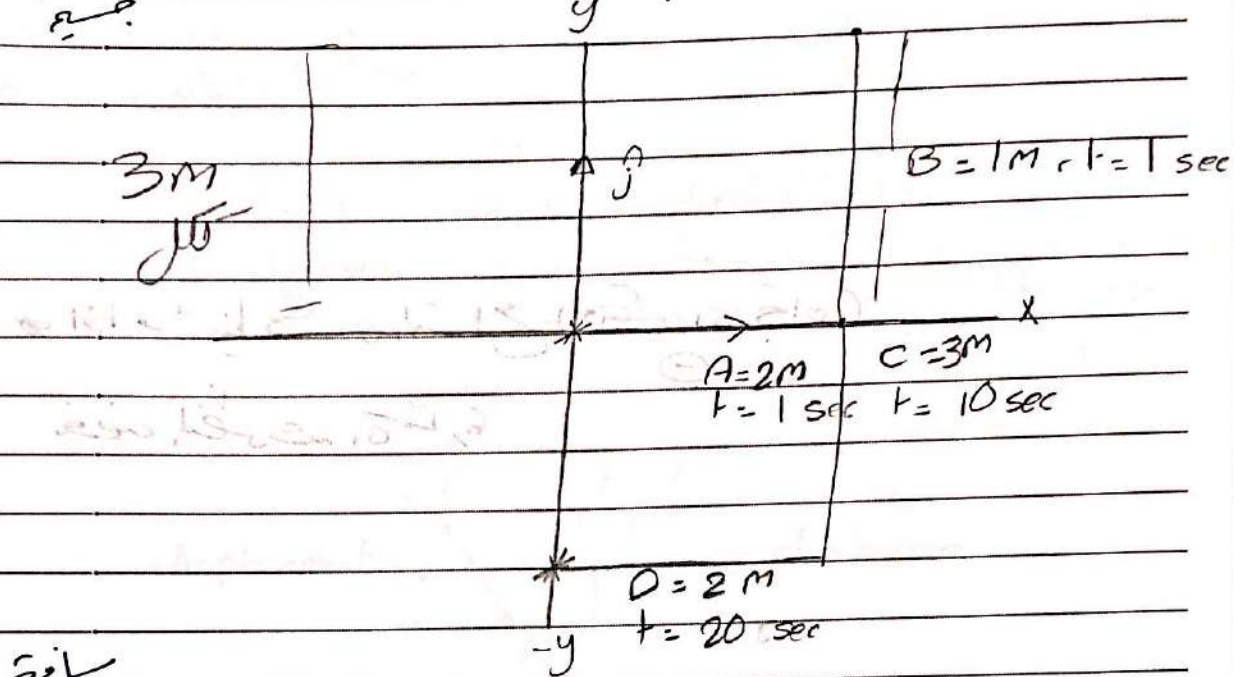
$$L = L + L$$

سواء كان الجواب بالسرعة أو التسارع $L = L$ correct dimensionally

Example 8 -

« الجسيم المتحرك »

a particle moves in the x-y plane as shown



المسافة

Distance = 8 m

Displacement = 2 meters on the negative y-axis = $-2\hat{j}$

average speed = $\frac{8}{32}$ m/s المتوسط

average velocity = $\frac{-2\hat{j}}{32}$ في اتجاه سالب
طاقة فقط

ساح = θ (زاوية) (في الحالة) في الاتجاهات
 θ وانما في

في الاتجاه = ساح = θ (زاوية) في حال (ب) و (س)
الزاوية فقط

Chapter (2)

Motion in one Dimension

الحركة في بعد واحد

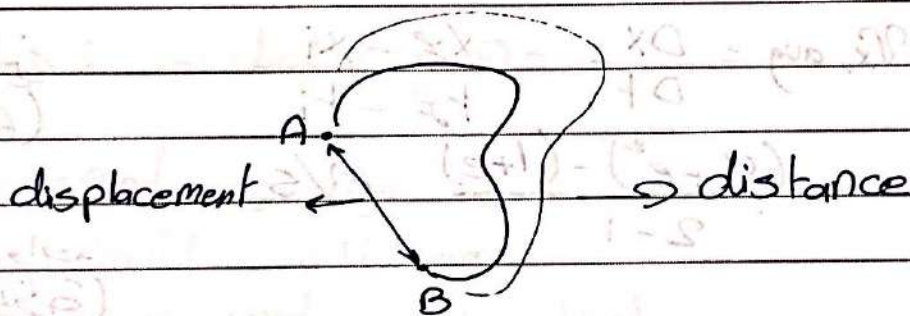
* مسافات الحركة

سرعة، اتجاه، موقع

1. distance :- مسافة (+) (كَمَّالِيَّة)

2. displacement :- إزاحة (+, -) (كَمَّالِيَّة وَجَهِيَّة) $x_f - x_i = \Delta x$

3. Position :- موقع (x)
 موقع نهائي - موقع ابتدائي



* average speed = $\frac{\text{distance}}{\text{time}}$

* الزمن الإجمالي
ليس واتجاهي

* average Velocity = $\frac{\text{displacement}}{\text{time}}$

distance = d

time = t

* average speed = $v_{avg} = \frac{d}{t}$

* average Velocity = $v_{x, avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$

موقع نهائي - موقع ابتدائي

$\Delta x = x_f - x_i$



* instantaneous Velocity

السرعة اللحظية

$$v_x = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

السرعة اللحظية

(ex) :- a particle moves with $x(t) = t^2 + e^t$

Find 1- the velocity between $t=1$ & $t=2$

$$v_{x, \text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$
$$= \frac{(4 + e^2) - (1 + e)}{2 - 1} \text{ m/s}$$

السرعة المتوسطة
(Average)

السرعة اللحظية
(instantaneous)

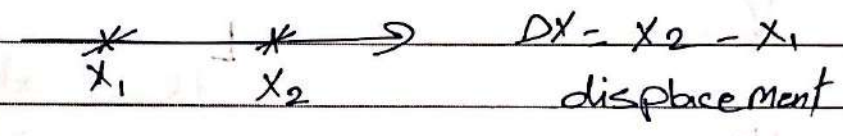
2- the velocity at $t=3$ sec

$$v_x = \frac{dx}{dt} = 2t + e^t$$

$$v_x = 2 \times 3 + e^3 = 6 + e^3 \text{ m/s}$$

Motion in one Dimension

\dot{x} Position \dot{s} avg speed \dot{v} avg velocity \dot{a} acceleration
 Δx displacement Δs distance



avg $s \rightarrow$ distance \oplus

avg speed \rightarrow v_{avg}
 avg velocity \rightarrow $v_{x, avg}$
 Instantaneous speed \rightarrow $v = |v_x|$

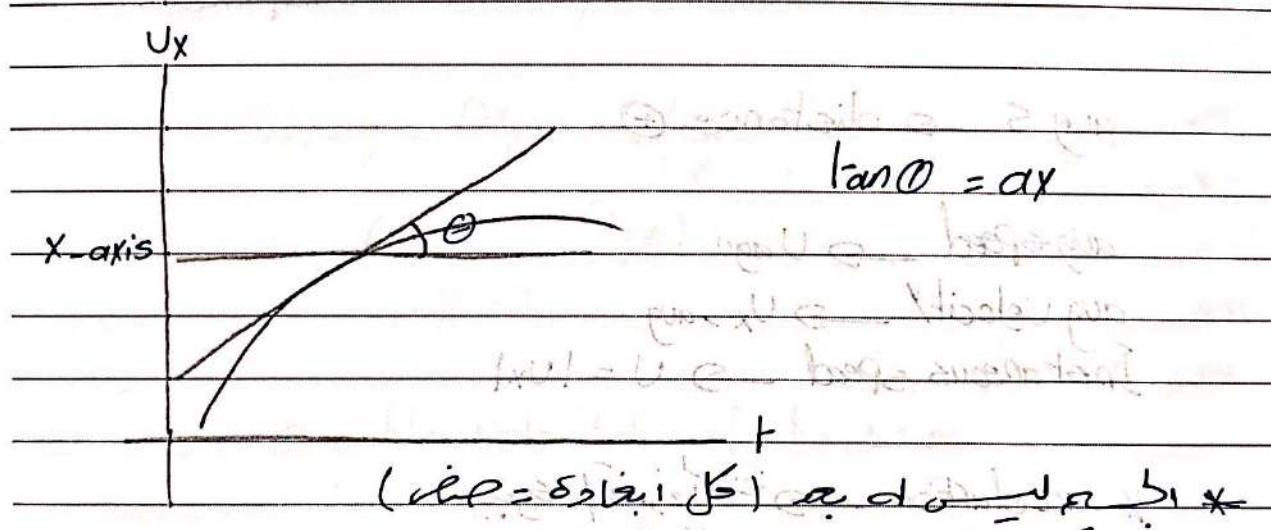
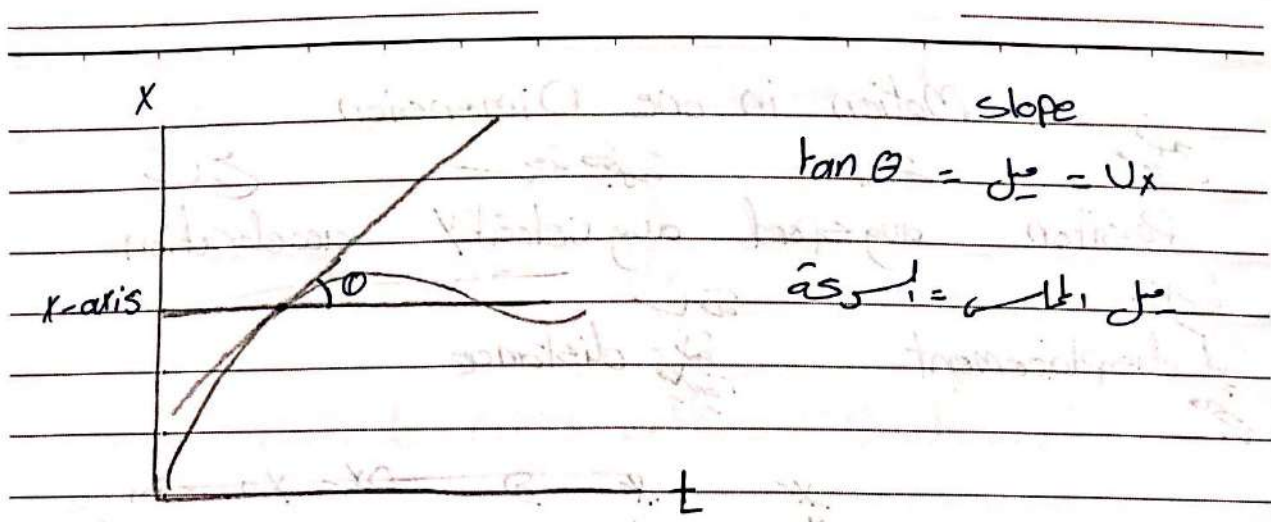
* acceleration \rightarrow \dot{v}
 * Velocity \rightarrow \dot{x}

average acceleration \rightarrow $a_{x, avg} = \frac{\Delta v_x}{\Delta t}$

instantaneous acceleration \rightarrow

$$a_x = \lim_{\Delta t \rightarrow 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt} \left(\frac{dx}{dt} \right) = \frac{d^2x}{dt^2}$$

\ominus ()



ex → the position of a particle is given by

$$x(t) = -\frac{t^2}{2} + 2t + 1$$

@ Find the displacement between $t=1$ and $t=2$

$$\Delta x = x_2 - x_1$$

$$x_2 = -4 + 4 + 1 = 1 \text{ m}$$

$$x_1 = -1 + 2 + 1 = 2 \text{ m}$$

$$\Delta x = 1 - 2 = -1 \text{ m}$$

b) Find the velocity between $t=1$ and $t=2$

$$v_{x, \text{avg}} = \frac{\Delta x}{\Delta t} = \frac{-1}{2-1} = -1 \text{ m/s}$$

c) Find the velocity at 3sec

$$v_x = \frac{dx}{dt} = 2t + 2$$

$$v_x(3) = -2 \cdot 3 + 2 = -4 \text{ m/s}$$

d) Find the average acceleration between $t=0$

$$a_{x, \text{avg}} = \frac{\Delta v_x}{\Delta t} = \frac{v_{x2} - v_{x1}}{\Delta t}$$

$$\left. \begin{array}{l} v_{x2} = -2 \cdot 1 + 2 = 0 \\ v_{x1} = -2 \cdot 0 + 2 = 2 \end{array} \right\} a_{x, \text{avg}} = \frac{0 - 2}{1} = -2 \text{ m/s}^2$$

e) Find the acceleration at $t=1$ sec and at $t=10$ s

$$a_x = \frac{dv_x}{dt} = -2$$

$$a_x(1) = -2 \text{ m/s}^2$$

$$a_x(10) = -2 \text{ m/s}^2$$

* Motion with constant speed or Velocity

$$U_x = U_{x, avg} = \frac{DX}{Dt}$$

$$DX = U_x Dt$$

$$x_2 - x_1 = U_x Dt$$

$$x_2 = x_1 + U_x Dt$$

السرعة ثابتة

السرعة = $\frac{dx}{dt}$
السرعة = $\frac{dx}{dt}$

* motion with constant acceleration

السرعة ثابتة

السرعة (متغيرة واتجاهها)

$$a_x = \frac{du_x}{dt}, \quad a_{x, avg} = \frac{DU_x}{Dt}$$

السرعة ثابتة

$$a_x = a_{x, avg} \Rightarrow \frac{DU_x}{Dt} = \frac{U_{xf} - U_{xi}}{Dt}$$

السرعة الزمنية
السرعة (متغيرة)
(متغير)

$$U_{xf} - U_{xi} = a_x Dt$$

$$U_{xf} = U_{xi} + a_x Dt \quad (1)$$

* لتكبير الزيادة في السرعة قبل a, c, w (السرعة ثابتة)

نصفه زوال سرعة و آخره متوقف وبعد ذلك (2) $\frac{1}{2}$
حال قبل التوقف

$$U_{x, avg} = \frac{U_{xf} + U_{xi}}{2} = \frac{DX}{Dt}$$

$$2DX = (U_{xi} + U_{xf}) Dt$$

displacement $\frac{DX}{Dt} = \frac{1}{2} (U_{xi} + U_{xf}) Dt$

معادلات (1)
بدل (U_{xf})

$$DX = \frac{1}{2} (U_{xi} + U_{xi} + a_x Dt) Dt$$

$$= (U_{xi} + \frac{1}{2} a_x dt) dt$$

$$DX = U_{xi} dt + \frac{1}{2} a_x dt^2 \quad (2)$$

دالة

$$(1) \quad dt = \frac{U_{xf} - U_{xi}}{a_x}$$

$$DX = U_{xi} \frac{(U_{xf} - U_{xi})}{a_x} + \frac{1}{2} a_x \left(\frac{(U_{xf} - U_{xi})}{a_x} \right)^2$$

$$= \frac{U_{xi} U_{xf}}{a_x} - \frac{U_{xi}^2}{a_x} + \frac{1}{2} a_x \frac{(U_{xf}^2 + U_{xi}^2 - 2U_{xi} U_{xf})}{a_x^2}$$

$$DX = \frac{U_{xi} U_{xf}}{a_x} - \frac{U_{xi}^2}{a_x} + \frac{1}{2} \frac{U_{xf}^2}{a_x} + \frac{1}{2} \frac{U_{xi}^2}{a_x} - \frac{U_{xi} U_{xf}}{a_x}$$

$$DX a_x = -U_{xi}^2 + \frac{1}{2} U_{xf}^2 + \frac{1}{2} U_{xi}^2$$

$$= -\frac{1}{2} U_{xi}^2 + \frac{1}{2} U_{xf}^2$$

$$2a_x DX + U_{xi}^2 = U_{xf}^2$$

ex → A drunk driver moves with 120 km/h in a straight street, suddenly a dog passed the street the driver hit breaks at 100m from the dog, with acceleration 2m/s². Does he hit dog?

$$a_x = -2 \text{ m/s}^2$$

$$U_{xi} = 120 \text{ km/h}$$

$$DX = ?$$

$$U_{xf} = 0$$

الكل

$U_{yf}^2 = U_{yi}^2 + 2ay \Delta y$
 إذا كانت سرعة = 0 / 0
 يعني ان السرعة
 انكسر

$120 \text{ km/h} \rightarrow \frac{120 * 1000}{60 * 60} = \frac{1200}{36} \text{ m/s}$

بالعرف
 $0 = \left(\frac{1200}{36}\right)^2 + 2 * -2 \Delta y$

$\Delta y = \frac{(1200/36)^2}{4} = 277 \text{ m}$

* إذا كانت سرعة الجسم في لحظة ما هي U_{yi} ونريد ان نعرف ما هي سرعة الجسم في لحظة t
 نأخذ الجاذبية g ونسقطها في المحور y

* Free falling objects

* (تأخر شيئاً من سرعة) يصبح عليها سرعة الجاذبية

$a_y = g = 9.8 \text{ m/s}^2 \rightarrow$ نأخذ الجاذبية

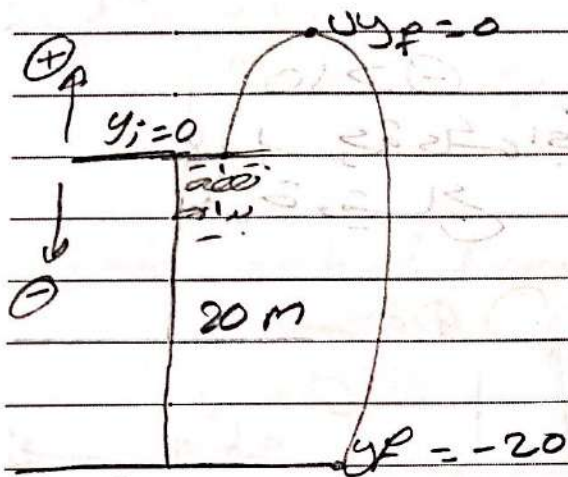
$U_{yf} = U_{yi} + g \Delta t \quad (1)$

$\Delta y = U_{yi} \Delta t + \frac{1}{2} g (\Delta t)^2 \quad (2)$

$U_{yf}^2 = U_{yi}^2 + 2g \Delta y \quad (3)$

Examples - A person throws upward from the top of a building of height 20m with initial velocity 10(m/s).

1. The maximum height from the ground
2. The time for the stone to hit the ground
3. The velocity at which the stone hit



$$(v_{yf} = 0 \text{ m/s}, t = 1 \text{ s})$$

$$y = 0$$

$$u_{iy} = 10 \text{ m/s}$$

$$a_y = 9.8 \text{ m/s}^2 \approx 10 \text{ m/s}^2$$

المعادلة

$$1) u_{yf}^2 = u_{yi}^2 + 2g \Delta y$$

$$a_y = 10 \text{ m/s}^2$$

$$0 = 100 + 2 \cdot (-10) (y_f - 0)$$

$$y_f - \frac{100}{20} = 5 \text{ m}$$

* The maximum height = 5 + 20 = 25 m

$$2) \Delta y = u_{iy} t + \frac{1}{2} g t^2$$

$$-20 = 10t - 5t^2 \Rightarrow 5t^2 - 10t - 20 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{10 \pm \sqrt{100 - 4 \cdot (-5) \cdot (-20)}}{2 \cdot 5}$$

$$- \frac{10 \pm \sqrt{500}}{10} = -1 \pm \sqrt{5} = 3,2$$

تجاه $-1,2$
 في اتجاه $-1,2$ في اتجاه $-1,2$

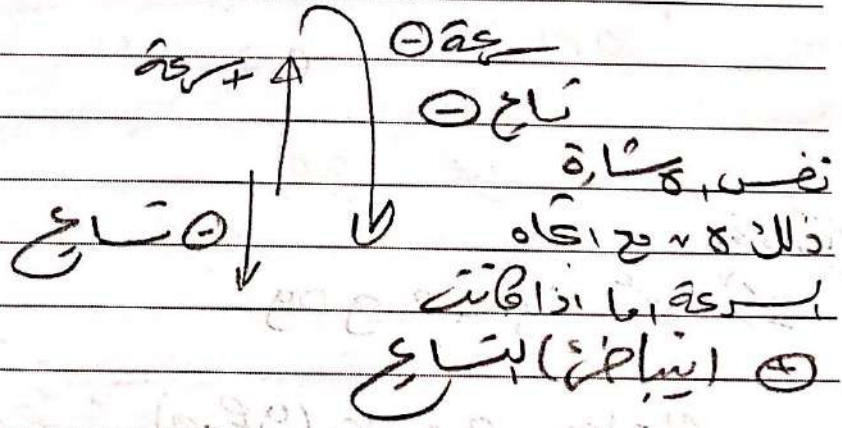
3) سرعة النهائية

$$v_{yf} = v_{yi} + gt$$

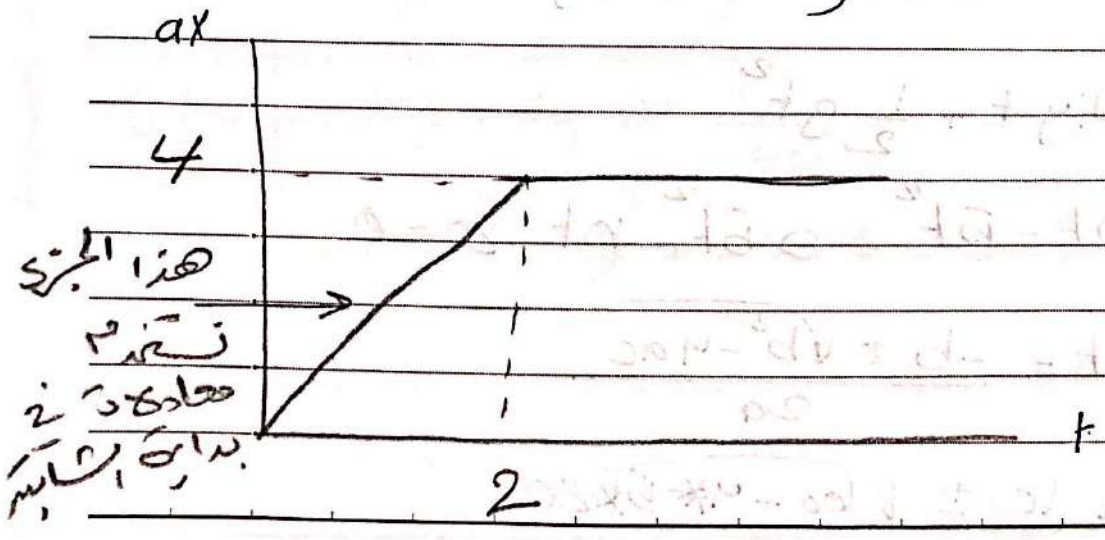
$$= 10 - 10 * 3,2$$

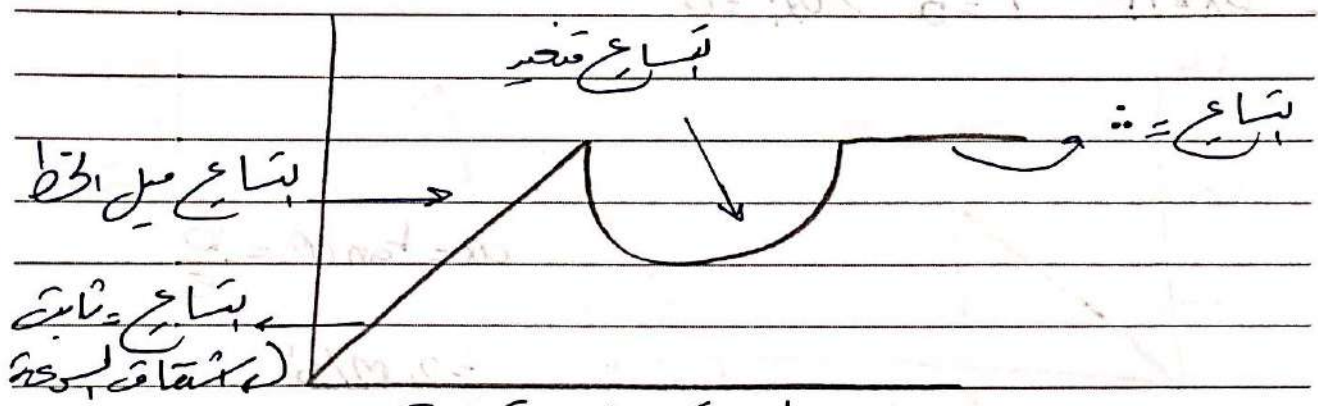
$$v_{yf} = -22 \text{ m/s}$$

$\theta > 10$
 سرعة في اتجاه
 الحركة في اتجاه



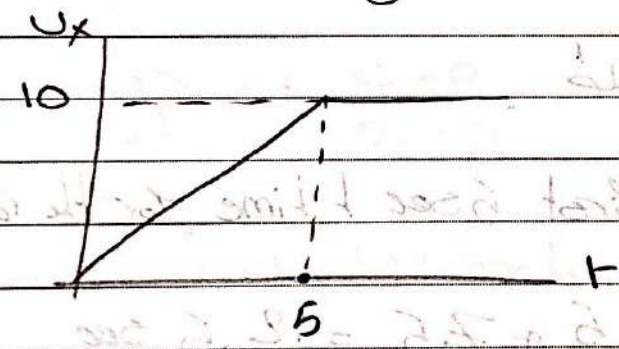
Examples - cart leaves starts the 100m race and acceleration as in the figure :-





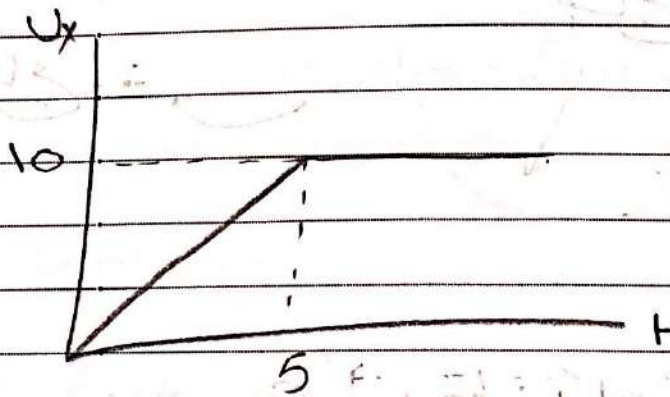
* ما عطف بَسَاج كالرغم مِلِ اِنْقَا
 كالرسم ونبطع فيها.

Examples- one hundred racer acceleration and his velocity is shown in the figure.



- 1- * Find the distance covered by the racer in the first 5 sec.
- 2- * How much time racer heads to cover 100 meters.

$\Delta x = 99 \quad t = 5 \quad u_{xi} = 0$



$a_x = \tan \theta = \frac{10}{5}$

$= 2 \text{ m/s}^2$

$\Delta x = u_{xi} t + \frac{1}{2} a_x t^2$

$= 0 + \frac{1}{2} \times 2 \times 25 = 25 \text{ m}$

2- در 5 ثانیه با شتاب
75

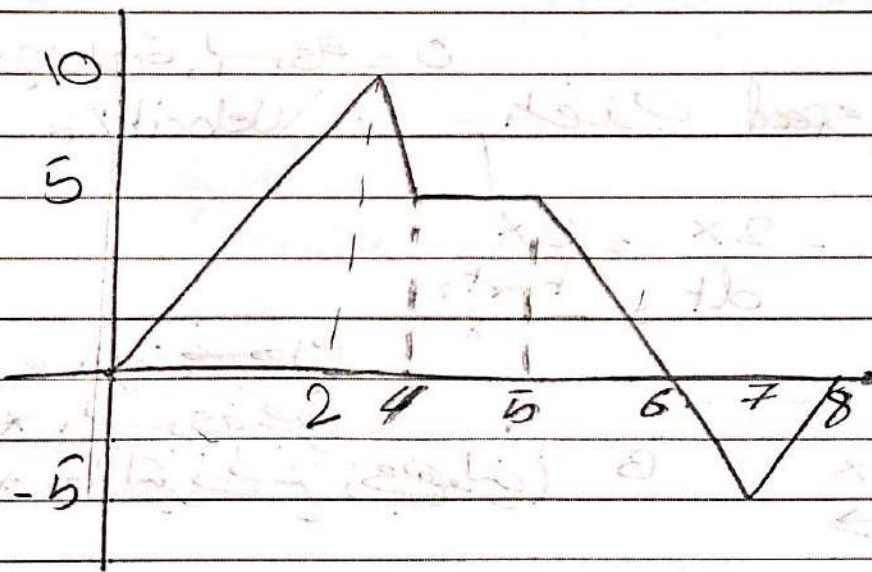
time = time for first 5 sec + time for the rest 75

$= 5 + \frac{75}{10} = 5 + 7.5 = 12.5 \text{ sec}$

as acceleration is constant, so we can use the equation $\Delta x = u_{xi} t + \frac{1}{2} a_x t^2$

$\Delta x = u_{xi} t$

as velocity is constant



(A) average velocity between $t=0$, $t=2$

$$u_x = \frac{\Delta x}{\Delta t} = \frac{5 - 0}{2 - 0} = 2.5 \text{ m/s}$$

(B) average velocity between $t=4$, $t=7$

$$u_x = \frac{\Delta x}{\Delta t} = \frac{-5 - 5}{7 - 4} = \frac{-10}{3} = -3.33 \text{ m/s}$$

displacement is Δx and Δt is time interval
 $\Delta x = \text{distance}$ $\Delta t = \text{time}$

(g) Find the velocity and acceleration at $t=6$ sec

$$u_x = \text{slope} = \frac{-5 - 5}{7 - 5} = \frac{-10}{2} = -5$$

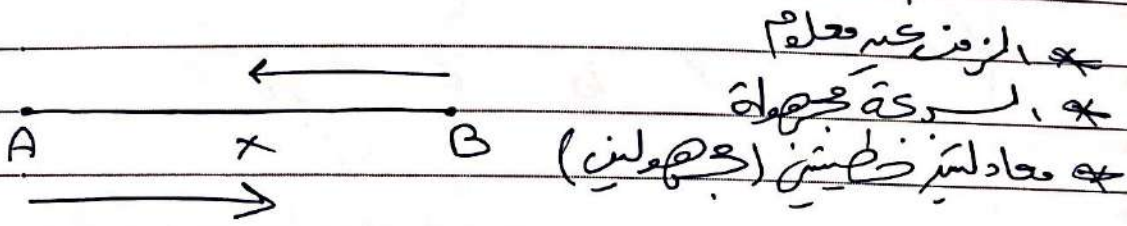
$a_x = 0$ ← acceleration (u_x)

(سؤال ٣)

Problem (3)

السرعة المتوسطة و السرعة
avg speed و Velocity

$$\textcircled{1} V_{avg} = \frac{d}{\Delta t} = \frac{2x}{t_1 + t_2} = \frac{2x}{t_1 + t_2}$$



$$x = 5t_1 \rightarrow t_1 = \frac{x}{5}$$

$$x = 3t_2 \rightarrow t_2 = \frac{x}{3}$$

$$V_{avg} = \frac{2x}{\frac{x}{5} + \frac{x}{3}} = \frac{2}{\frac{8}{15}} = \frac{30}{8} \text{ m/s}$$

Vectors and scalars

Physical Quantities

Scalars

Scalars

You need only

↳ Magnitude

* length, distance
temperature

Vectors

Vectors

You need both

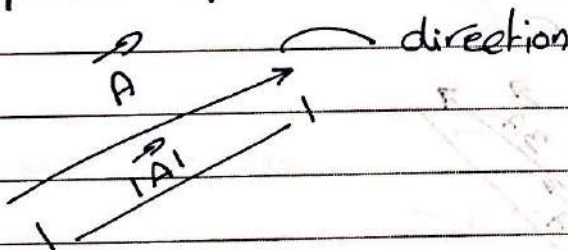
↳ Magnitude and direction

* Velocity, force
acceleration

* Any vector quantity = $A \vec{A}$, $|A|$ Magnitude of \vec{A}

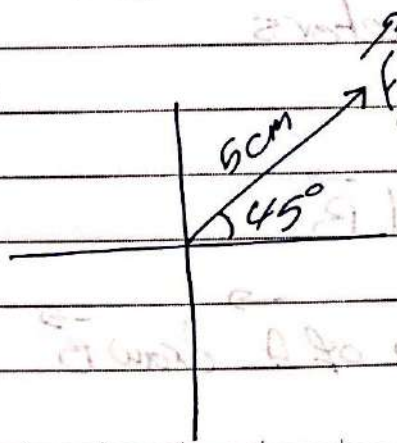
* s scalar = A

* Graphical representation



Example:- Force 10 N at 45° above the x-axis

* every 2N = 1cm



* Properties of vectors

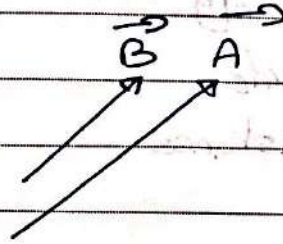
① $\vec{A} = \vec{B}$ if

and $|\vec{A}| = |\vec{B}|$

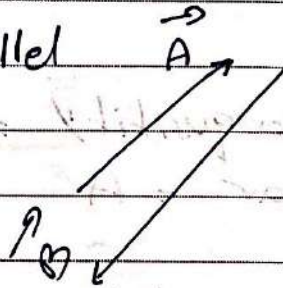
direction of \vec{A} is same direction of \vec{B}

② $\vec{A} \parallel \vec{B}$ if

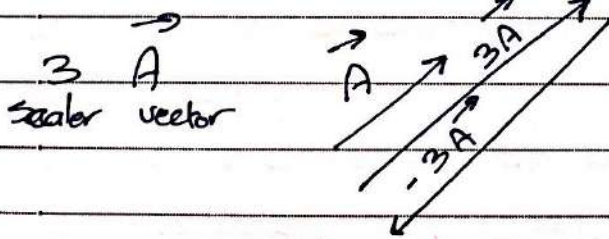
they have the same direction



③ \vec{A} and \vec{B} are anti-parallel



④ multiplication by a scalar



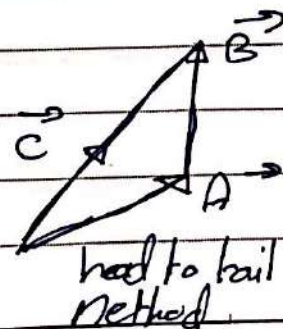
⑤ addition of vectors

$\vec{A} + \vec{B} = \vec{C}$

* to add \vec{A} and \vec{B}

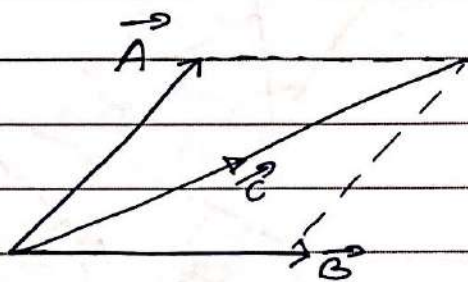
① draw \vec{A}

② From the top of \vec{A} draw \vec{B}

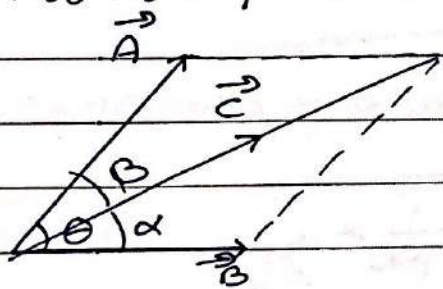


or using the parallelogram method

① draw \vec{A} and \vec{B} from the same point



* Addition of two vectors mathematically -



$$|\vec{C}| = \sqrt{A^2 + B^2 + 2|\vec{A}||\vec{B}|\cos\theta}$$

\leftarrow $\cos\theta = \cos(\pi - \theta)$

$$\frac{\sin\alpha}{A} = \frac{\sin(\pi - \theta)}{C}$$

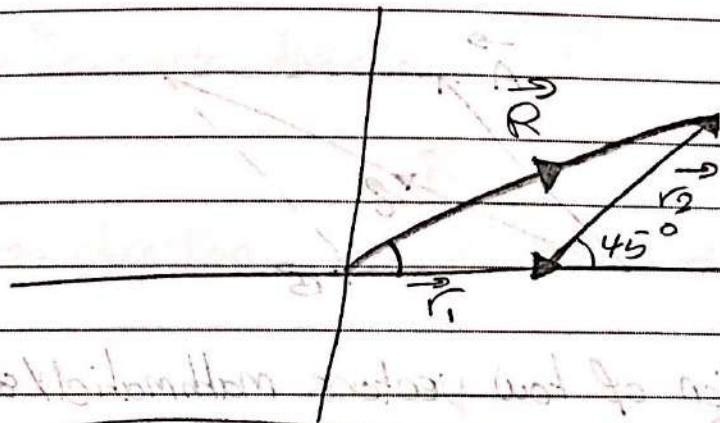
\leftarrow $\sin(\pi - \theta) = \sin\theta$
 (قوسین متساوی)

$$\frac{\sin\alpha}{A} = \frac{\sin\theta}{C}$$

$$\sin\alpha = \frac{A \sin\theta}{C}$$

$$\sin\beta = \frac{B \sin\theta}{C}$$

Example: a car moves east 10 km and then it moves east-North another 10 km. Find the resultant displacement



$$|\vec{R}| = \sqrt{r_1^2 + r_2^2 + 2r_1r_2 \cos \theta}$$

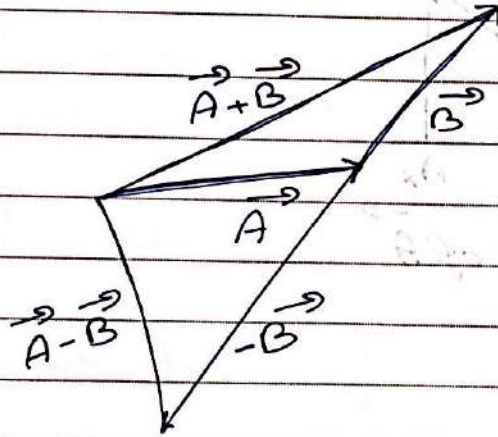
$$= \sqrt{100 + 100 + 2 \times 10 \times 10 \times \frac{1}{\sqrt{2}}}$$

$$\sin \alpha = \frac{r_2}{R} \sin 45^\circ$$

$$= \frac{10}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = x$$

$$\alpha = \sin^{-1} x$$

~~Subtraction~~ Subtracting vectors

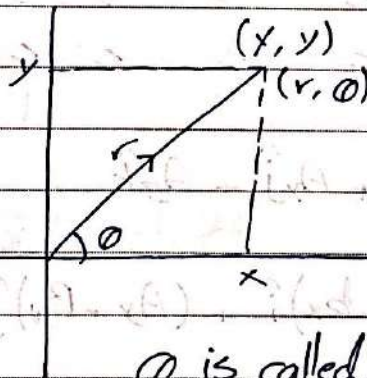


* Components of vectors

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

$$\tan \theta = \frac{y}{x}$$



θ is called the polar angle

$$x = r \cos \theta$$

$$y = r \sin \theta$$

* Example

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$$

$$r = \sqrt{9 + 9} = \sqrt{18}$$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\frac{-3}{3} = \tan^{-1}(-1) = -45^\circ \\ &= 360 - 45 \\ &= 315^\circ \end{aligned}$$

$$V_{2x} = V_2 \cos 120^\circ$$

$$= 2 \times \frac{1}{2} = 1$$

$$V_{2y} = V_2 \sin 120^\circ$$

$$= 2 \times 0.87 = 1.74$$

$$\vec{V}_2 = \hat{i} + 1.74\hat{j}$$

$$V_{3x} = V_3 \cos 240^\circ = 2 \times \frac{-1}{2} = -1$$

$$V_{3y} = V_3 \sin 240^\circ = 2 \times 0.87 = -1.74$$

$$\vec{V}_3 = -\hat{i} - 1.74\hat{j}$$

$$V_{4x} = V_4 \cos 300^\circ = 2 \times \frac{1}{2} = 1$$

$$V_{4y} = V_4 \sin 300^\circ = 2 \times -0.87 = -1.74$$

$$\vec{V}_4 = \hat{i} - 1.74\hat{j}$$

$$\vec{V}_1 + \vec{V}_2 + \vec{V}_3 + \vec{V}_4 = \vec{V}$$

$$= (1.74 - 1 - 1 + 1)\hat{i} + (1 + 1.74 - 1.74 - 1.74)\hat{j}$$

$$= 0.74\hat{i} - 0.74\hat{j}$$

$$V = \sqrt{(0.74)^2 + (-0.74)^2}$$

$$\theta = \tan^{-1} \frac{-0.74}{0.74} = \tan^{-1}(-1) = 365 - 45$$

315

Find

$$\vec{C} = 2\vec{V}_1 - 3\vec{V}_2 + 6\vec{V}_3 - \vec{V}_4$$

$$\begin{aligned} 2\vec{V}_1 &= 2 * 1.74\hat{i} + 2 * 1\hat{j} \\ &= 3.48\hat{i} + 2\hat{j} \end{aligned}$$

$$-3\vec{V}_2 = (-3 * -1)\hat{i} + -3 * 1.74\hat{j}$$

$$6\vec{V}_3 = (6 * -1)\hat{i} + (6 * -1.74)\hat{j}$$

$$-\vec{V}_4 = (-1 * 1)\hat{j} + (-1 * -1.74)\hat{j}$$

Chapter (3)

(1) Problem 8- $r = 5.5$ / $\theta = 240^\circ$

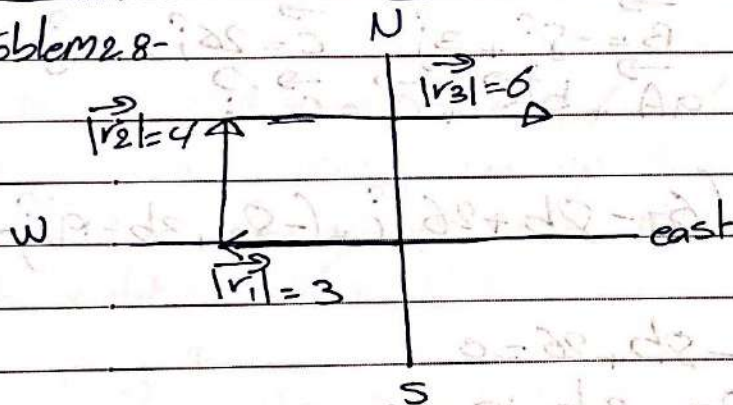
* Polar coordinates
 $r = \sqrt{x^2 + y^2}$
 $\theta = \tan^{-1} \frac{y}{x}$

* what are the Cartesian coordinates
 (x, y)

$$x = r \cos \theta = 5.5 * \cos 240$$

$$y = r \sin \theta = 5.5 * \sin 240$$

problem 2.8-



$$\left. \begin{array}{l} r_{1x} = -3 \\ r_{1y} = 0 \end{array} \right\} \vec{r}_1 = -3\hat{i}$$

$$\left. \begin{array}{l} r_{2x} = 0 \\ r_{2y} = 4 \end{array} \right\} \vec{r}_2 = 4\hat{j}$$

$$\left. \begin{array}{l} r_{3x} = 6 \\ r_{3y} = 0 \end{array} \right\} \vec{r}_3 = 6\hat{i}$$

$$\begin{aligned} \vec{r} &= \vec{r}_1 + \vec{r}_2 + \vec{r}_3 \\ &= 3\hat{i} + 4\hat{j} \end{aligned}$$

$$|\vec{r}| = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$\theta = \tan^{-1} \frac{4}{3}$$

problem 238- $\vec{A} = 3\hat{i} - 2\hat{j}$ $\vec{B} = -\hat{i} - 4\hat{j}$

a) $\vec{A} + \vec{B} = 2\hat{i} - 6\hat{j}$

$|\vec{A} + \vec{B}| = \sqrt{4 + 36} = \sqrt{40}$

$\theta = \tan^{-1} \frac{-6}{2} = \tan^{-1} -3 = 360 - 71.5 = 288.5^\circ$

b) $\vec{A} - \vec{B} = 4\hat{j} + 2\hat{j}$

$|\vec{A} - \vec{B}| = \sqrt{20}$

$\theta = \tan^{-1} \frac{-1}{2} = \tan^{-1} -0.5 = 26.5^\circ$

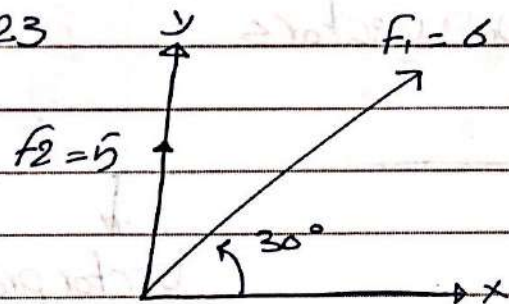
problem 378- $\vec{A} = 6\hat{i} - 8\hat{j}$ $\vec{B} = -8\hat{i} + 3\hat{j}$ $\vec{C} = 26\hat{i} - 19\hat{j}$

Find a and b such $a\vec{A} + b\vec{B} + \vec{C} = \vec{0}$?

$$\left. \begin{array}{l} \vec{aA} = 6a\hat{i} - 8a\hat{j} \\ \vec{bB} = -8b\hat{i} + 3b\hat{j} \\ \vec{C} = 26\hat{i} - 19\hat{j} \end{array} \right\} \begin{array}{l} (6a - 8b + 26)\hat{i} + (-8a + 3b - 19)\hat{j} = \vec{0} \\ 6a - 8b + 26 = 0 \\ -8a + 3b - 19 = 0 \end{array}$$

$a = -1.6$
 $b = 2$

Problem 23



$$F_{1x} = F_1 \cos 30 = 3\sqrt{3}$$

$$F_{1y} = 6 \sin 30 = 3$$

$$\vec{F}_1 = 3\sqrt{3}\hat{i} + 3\hat{j}$$

$$\vec{F}_2 = 5\hat{j}$$

$$\vec{F}_1 + \vec{F}_2 = 3\sqrt{3}\hat{i} + 8\hat{j}$$

$$|\vec{F}_1 + \vec{F}_2| = \sqrt{27 + 64}$$

$$\theta = \tan^{-1} = \frac{8}{3\sqrt{3}}$$

(Chapter 2) Example 8 - A guy try to hit a thief at the top of a building by stone. The guy throws the stone with initial velocity from the ground 20 m/s. Does the stone hit the thief of the height of building 20m.

$$v_{yf}^2 = v_{yi}^2 - 2g \Delta y$$

$$0 = 4 - 20 \Delta y$$

$$\Delta y = \frac{4}{20} = \frac{1}{5} \text{ m}$$

سؤال: هل يمسح الحجر لصاحبه السرقة في الطابق الثاني من المبنى؟

$$\Delta y = \frac{v_{yf}^2 - v_{yi}^2}{-2g} = \frac{0 - 20^2}{-2 \times 10} = 20 \text{ m}$$

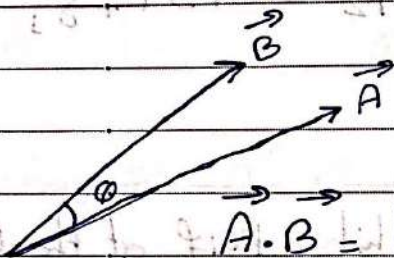
* Multiplication of two vectors

Scalar product
(dot product)

$$\vec{A} \cdot \vec{B} = \text{scaler}$$

Vector product
(cross product)

$$\vec{A} \times \vec{B} = \text{vector} = \vec{c}$$



$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta = \text{scaler (is scalar)}$$

$$= A_x B_x + A_y B_y + A_z B_z \text{ - scaler is}$$

* Example :- $\vec{A} = 2\vec{i} - \vec{j} + \vec{k}$ $\vec{B} = 4\vec{i} + \vec{k}$

What's the angle between them :-

$$\begin{aligned} \vec{A} \cdot \vec{B} &= 2 \times 4 + (-1) \times 0 + 1 \times 1 \\ &= 8 + 1 = 9 \end{aligned}$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

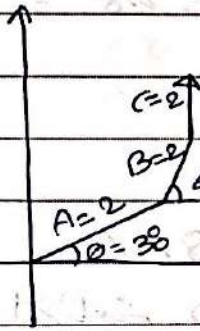
$$9 = \sqrt{2^2 + (-1)^2 + 1^2} * \sqrt{4^2 + 0^2 + 1^2} * \cos \theta$$

$$9 = \sqrt{6} * \sqrt{17} \cos \theta \rightarrow \cos \theta = \frac{9}{\sqrt{102}}$$

$$\theta = \cos^{-1} \frac{9}{\sqrt{102}}$$

* Rules: $\vec{A} = \hat{i}$ $\vec{A} \cdot \vec{B} = 0$ $\vec{A}, \vec{B}, \vec{C}$
 $\vec{B} = \hat{j}$ $\vec{A} \cdot \vec{C} = 0$ $\hat{i} \quad \hat{j} \quad \hat{k}$
 $\vec{C} = \hat{k}$ $\vec{B} \cdot \vec{C} = 0$ one perpendicular
 90° کے زاویے پر

* Example :-



$$A_x = A \cos 30 = 2 * \frac{\sqrt{3}}{2} = \sqrt{3}$$

$$A_y = A \sin 30 = 2 * \frac{1}{2} = 1$$

$$\vec{A} = \sqrt{3}\hat{i} + \hat{j}$$

$$\vec{C} = 2\hat{j}$$

$$B_x = B \cos 60 = 2 * \frac{1}{2} = 1$$

$$B_y = B \sin 60 = 2 * \frac{\sqrt{3}}{2} = \sqrt{3}$$

$$\vec{B} = \hat{i} + \sqrt{3}\hat{j}$$

$$\vec{A} + \vec{B} = (\sqrt{3} + 1)\hat{i} + (1 + \sqrt{3})\hat{j}$$

$$(\vec{A} + \vec{B}) \cdot \vec{C} = (1 + \sqrt{3}) * 2$$

* Chapter 2 problem 208-

$x(t) = 3t^2 - 2t + 3$ → Find the time at which the object changes direction

$$v_x = \frac{dx}{dt} = 6t - 2 = 0 \rightarrow t = 2/6 = 1/3 \text{ s}$$

* Problem 288-

$$Dx = 40 \text{ m}, Dt = 8.5, v_{xf} = 2.8 \text{ m/s}$$

* smoothly - constant - uniform → ناعماً ثابتاً

$$v_{xf} = v_{xi} + a_x Dt$$

$$2.8 = v_{xi} + a_x * 8.5 \quad \text{--- (1)}$$

$$Dx = v_{xi} Dt + 1/2 a_x Dt^2$$

$$40 = v_{xi} * 8.5 + 1/2 a_x (8.5)^2 \quad \text{--- (2)}$$

$$\textcircled{1} \quad 2.8 = v_{xi} + 8.5 a_x$$

$$\textcircled{2} \quad 40 = v_{xi} * 8.5 + 36.1 a_x$$

$$v_{xi} = 6.6$$

$$a_x = -0.4$$

chapter 4

* Motion in two and three dimension *

	one dimension	two and three dimension
1 position	X or y or Z	X and y and Z position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
2 average velocity	$v_{x, \text{avg}} = \frac{\Delta x}{\Delta t}$	$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t}$ $\rightarrow \frac{\Delta x\hat{i}}{\Delta t} + \frac{\Delta y\hat{j}}{\Delta t} + \frac{\Delta z\hat{k}}{\Delta t}$
3 instantaneous velocity	$v_x = \frac{dx}{dt}$	$\vec{v} = \frac{d\vec{r}}{dt}$ $\rightarrow \frac{dx\hat{i}}{dt} + \frac{dy\hat{j}}{dt} + \frac{dz\hat{k}}{dt}$
4 average acceleration	$a_{x, \text{avg}} = \frac{\Delta v_x}{\Delta t}$	$\vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t}$
5 instantaneous acceleration	$a_x = \frac{dv_x}{dt}$	$\vec{a} = \frac{d\vec{v}}{dt}$

* Motion with constant accel *

1 - $\vec{v}_f = \vec{v}_i + \vec{a}t$ $v_{xf} = v_{xi} + a_x t$
 $v_{yf} = v_{yi} + a_y t$
 $v_{zf} = v_{zi} + a_z t$

2 - $\vec{r} = \vec{v}_i t + \frac{1}{2} \vec{a} t^2$

3 - $|\vec{v}_f|^2 = |\vec{v}_i|^2 + 2\vec{a} \cdot \vec{r}$

Example 7: an airplane moves in the sky as
 $\vec{r}(t) = (t^2)\hat{i} + \sin(t)\hat{j} + \hat{k}$

Find the average accel between $t=0$, $t = \pi/2$

$$\vec{a}_{\text{avg}} = \frac{d\vec{v}}{dt} \rightarrow \vec{v} = \frac{d\vec{r}}{dt} = 2t\hat{i} + \cos(t)\hat{j}$$

$$v_f = \pi\hat{j}, \quad v_i = \hat{j} \rightarrow \vec{a}_{\text{avg}} = \frac{\pi\hat{i} - \hat{j}}{\pi/2} = \boxed{2\hat{i} - \frac{2}{\pi}\hat{j}}$$

Find the accel $t = \pi/2$

$$\vec{a} = \frac{d\vec{v}}{dt} = 2\hat{i} - \sin(t)\hat{j}$$

$$\vec{a} = 2\hat{i} - \hat{j} \quad / \quad |\vec{a}| = \sqrt{4+1} = \boxed{\sqrt{5}}$$

$$\theta = \tan^{-1}(-1/2) = 360 + (-26.5) = \boxed{333.5}$$

Example 8: A particle starts to move initial velocity $\vec{v}_i = 2\hat{i} - \hat{j} + \hat{k}$
 IF the motion was uniform accel $\vec{a} = 2\hat{i} - \hat{k}$

① Find disp. after 2sec

$$\vec{Dr} = \vec{v}_i t + \frac{1}{2}\vec{a} t^2 \quad |\vec{Dr}| = \sqrt{64+4}$$

$$\vec{Dr} = (2\hat{i} - \hat{j} + \hat{k})2 + \frac{1}{2}(2\hat{i} - \hat{k}) \times 4 = \sqrt{68}$$

$$= 4\hat{i} - 2\hat{j} + 2\hat{k} + (4\hat{i} - 2\hat{k})$$

$$\vec{Dr} = 8\hat{i} - 2\hat{j}$$

$$\theta = \tan^{-1} \frac{-2}{8} = \tan^{-1} \frac{-1}{4}$$

$$360 -$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cos \theta \quad \text{تعريف لـ \theta}$$

$$\vec{v}_f \cdot \vec{i} = |\vec{v}_f| \cdot |\vec{i}| \cos \alpha$$

2- find the magnitude and direction of the Velocity after 2 sec.

$$\vec{v}_p = \vec{v}_i + \vec{a}t$$

$$= (2\vec{i} - \vec{j} + \vec{k}) + (2\vec{i} - \vec{k}) * 2$$

$$= (2\vec{i} + 4\vec{i} - \vec{j} - \vec{k} + 2\vec{k})$$

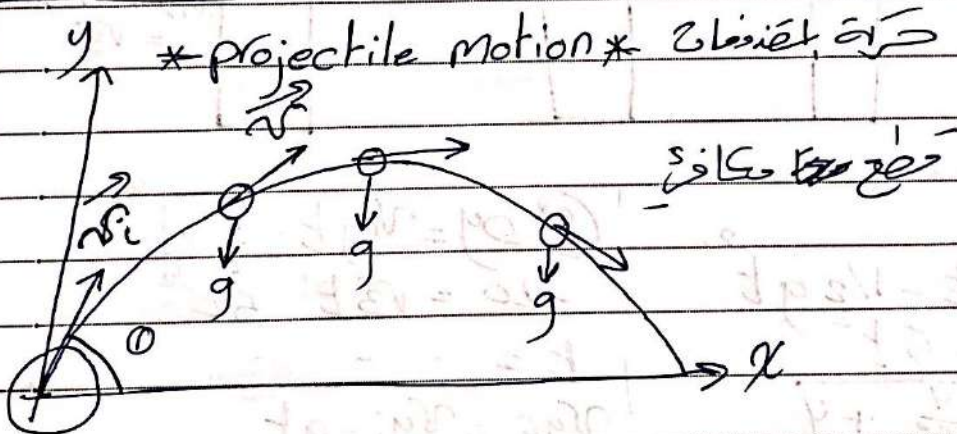
$$= 6\vec{i} - \vec{j} + \vec{k} \quad \Rightarrow |\vec{v}_f| = \sqrt{36 + 1 + 1} = \sqrt{38}$$

$$\vec{v}_f \cdot \vec{i} = (6\vec{i} - \vec{j} + \vec{k}) \cdot \vec{i}$$

$$= 6 \quad \Rightarrow 6 = |\vec{v}_f| |\vec{i}| \cos \alpha$$

$$6 = \sqrt{38} \cos \alpha \quad \Rightarrow \cos \alpha = \frac{6}{\sqrt{38}}$$

$$\alpha = \cos^{-1} \frac{6}{\sqrt{38}}$$



$$\vec{a} = -g\vec{j}$$

on the x-axis

$$x = v_x * t$$

(1)

حركة المقذوفات

x-axis

v_x

v_f

U high mom

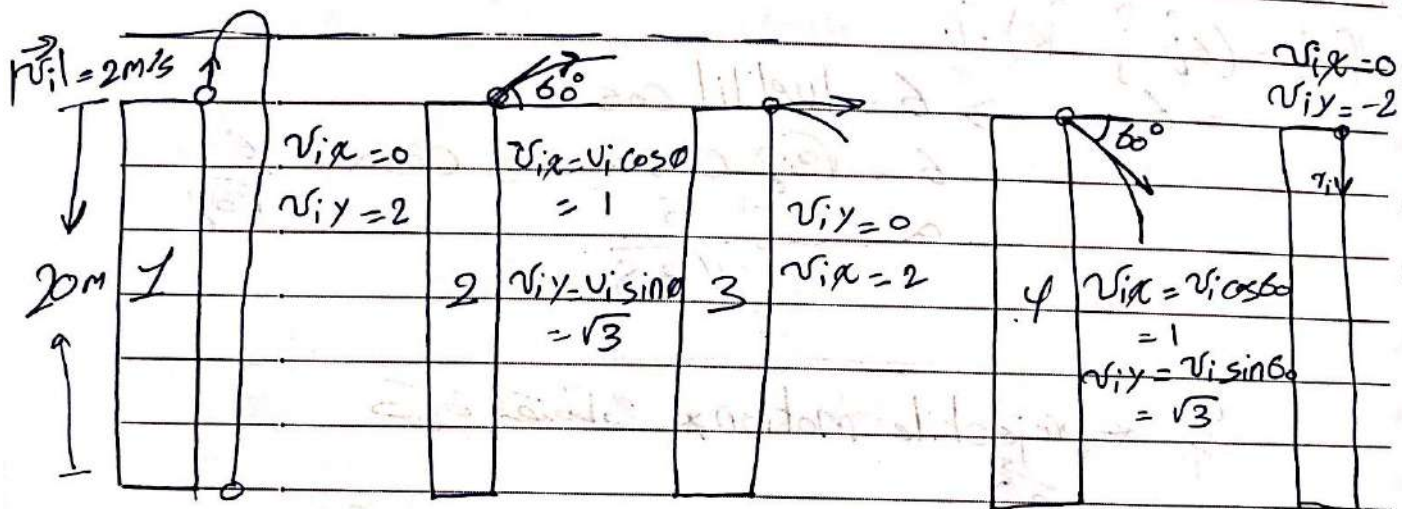
$$x = v_x * t = U_i \cos t$$

on the y-axis the motion is just freely falling object

$$1 - v_{yf} = v_{yi} - gt$$

$$2 - v_{yf}^2 = v_{yi}^2 - 2g \Delta y$$

$$3 - \Delta y = v_{yi} t - \frac{1}{2} gt^2$$



① Range = 0

$$\Delta y = v_{iy} t - \frac{1}{2} gt^2$$

$$-20 = 2t - 5t^2$$

$$t = \frac{2 \pm \sqrt{4 + 400}}{10}$$

$$v_{fy} = v_{iy} - gt$$

$$= 2 - 10 * \left(\frac{2 + \sqrt{4 + 400}}{10} \right)$$

$$R = 0$$

$$\text{speed} = |v_{iy}|$$

② $\Delta y = v_{iy} t - \frac{1}{2} gt^2$

$$-20 = \sqrt{3}t - 5t^2$$

$$t = \dots$$

$$v_{yf} = v_{iy} - gt$$

$$= \sqrt{3} - 10t$$

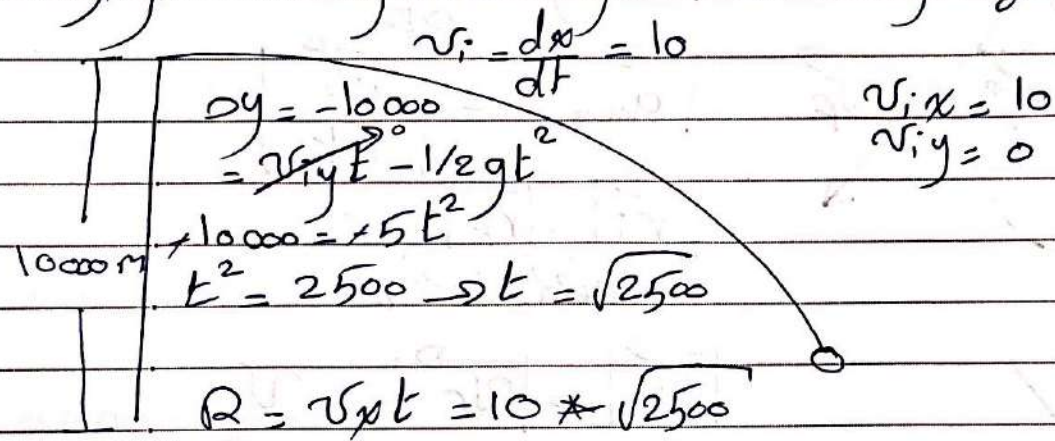
$$\text{speed} = |v|$$

$$= \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{1^2 + v_y^2}$$

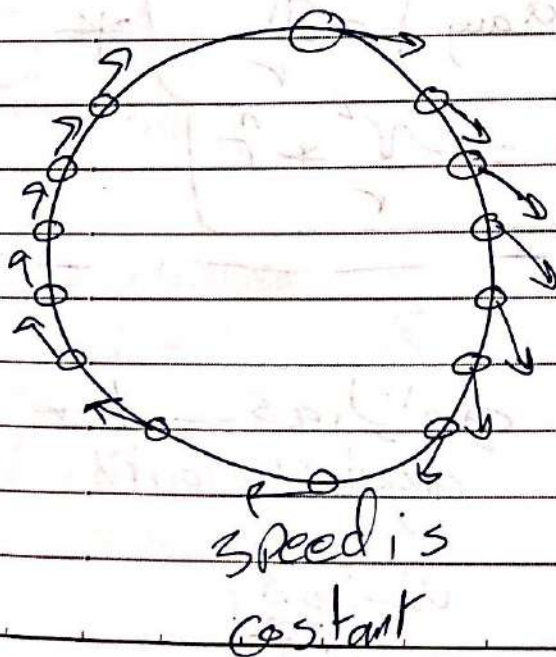
$$R = v_x t = 1 * t$$

Example 8 - an F-16 fighter moves horizontally with $x(t) = 10t$. The fighter relays a rocket at height 10 km trying to hit a ground target. find the range of the rocket.

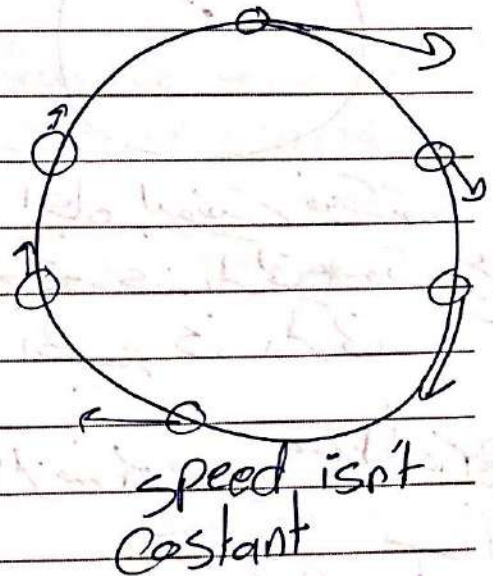


Circular motion

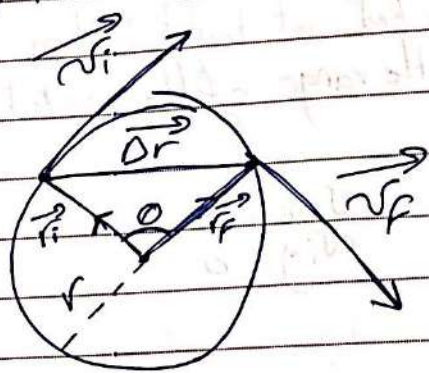
Uniform motion



non-uniform motion



Uniform Circular Motion - الحركة المنتظمة الدائرية

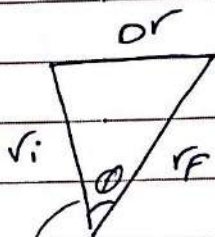


$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

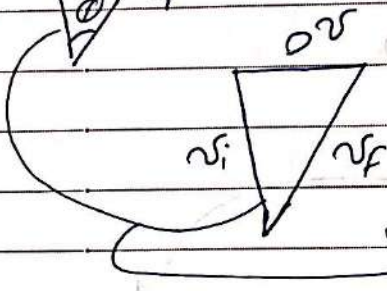
$$|\vec{a}_{avg}| = \left| \frac{\Delta \vec{v}}{\Delta t} \right|$$

$$|\vec{v}_i| = |\vec{v}_f| = v$$

السرعة ثابتة



$$|\vec{v}_i| = |\vec{v}_f| = v$$

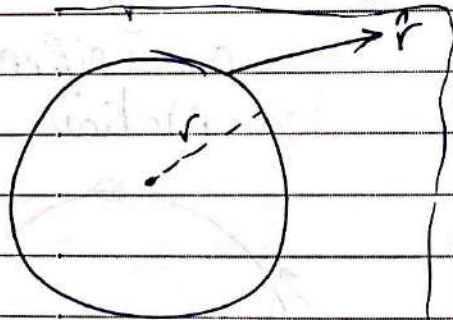


$$\Delta r = r \Delta \theta$$

$$\Delta v = v \Delta \theta$$

$$\frac{\Delta v}{\Delta r} = \frac{v}{r}$$

$$\Delta v = \frac{v}{r} \Delta r$$



$$|\vec{a}_{avg}| = \frac{v^2}{r}$$

$$a_c = -\frac{v^2}{r} \hat{r}$$

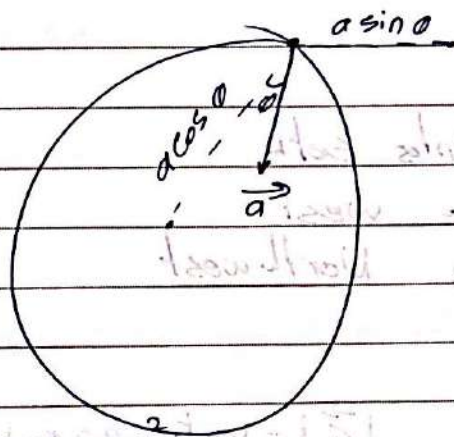
اتجاه التسارع في مركز
وتساوي القوة المركزية وتساوي
الجسم نحو المركز

Period time * الزمن الدوري

* السرعة الزاوية
angular velocity

$$T = \frac{2\pi r}{v}$$

$$\omega = \frac{2\pi}{T}$$

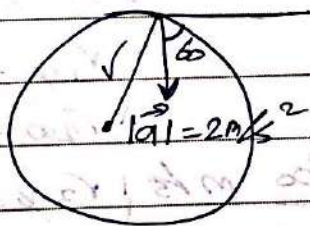


$$a \cos \theta = a_c = \frac{v^2}{r}$$

$$a \sin \theta = a_t \Rightarrow \frac{dv}{dt} = \frac{v}{r}$$

$$a_t = \frac{dv}{dt} \quad a = \sqrt{a_t^2 + a_c^2}$$

(Q) a particle moves in circular motion as the figure:-



find
 a_c , a_t
 $\frac{dv}{dt}$, $\frac{v}{r}$

Find the radius of the circular when the velocity = 10 m/s

$$a_c = a \sin 60$$

$$= 2 * \frac{\sqrt{3}}{2} = \sqrt{3}$$

$$a_c = \frac{v^2}{r} \Rightarrow 1 = \frac{100}{r} \Rightarrow r = 100 \text{ m}$$

$$a_t = a \cos 60$$

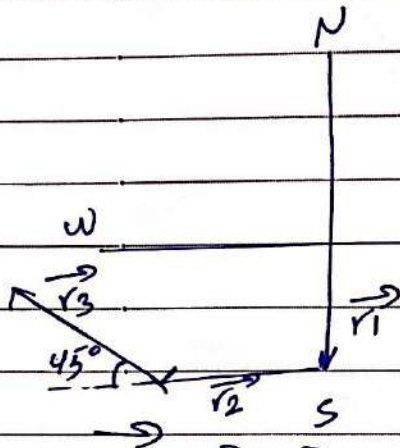
$$= 2 * \frac{1}{2} = 1$$

* problem 1

Velocity = 20 m/s for 3 minutes south

= 25 m/s for 2 = west

= 30 m/s for 1 = Northwest



$$|\vec{r}_1| = v \cdot t = 20 \cdot 180 = 3600$$

$$|\vec{r}_2| = v \cdot t = 25 \cdot 120 = 3000$$

$$|\vec{r}_3| = v \cdot t = 30 \cdot 60 = 1800$$

(a) $R = \vec{r}_1 + \vec{r}_2 + \vec{r}_3$

$$= \left(-3000 - \frac{1800}{\sqrt{2}}\right) \hat{i} + \left(-3600 + \frac{1800}{\sqrt{2}}\right) \hat{j}$$

(b) average speed = $\frac{d}{\Delta t}$

$$= \frac{3600 + 3000 + 1800}{6 \cdot 60} \text{ m/s}$$

$$r_{1x} = 0 / r_{1y} = 3600$$

$$r_{2x} = -3000 / r_{2y} = 0$$

$$r_{3x} = r_3 \cos 45 = \frac{1800}{\sqrt{2}}$$

(c) $v_{avg} = \frac{R}{\Delta t}$

$$\frac{\left(-3000 - \frac{1800}{\sqrt{2}}\right) \hat{i} + \left(-3600 + \frac{1800}{\sqrt{2}}\right) \hat{j}}{6 \cdot 60}$$

$$r_{3y} = r_3 \sin 45 = \frac{1800}{\sqrt{2}}$$

* Problem 68 -

$$\vec{a} = 3\hat{j} \text{ m/s}^2$$

$$\vec{v}_i = 5\hat{i}$$

(a) find the vector position at any time

$$d\vec{r} = \vec{v}_i t + \frac{1}{2} \vec{a} t^2$$

$$\vec{r}(t) = 5t\hat{i} + \frac{3}{2} t^2 \hat{j} \quad * \text{ Position at any time} *$$

$$\textcircled{b} \vec{v} = \frac{d\vec{r}}{dt} = 5\hat{i} + 3t\hat{j}$$

$$\textcircled{c} \vec{r}(t=2) = 10\hat{i} + 6\hat{j}$$

$$(10, 6)$$

$$\textcircled{d} \vec{v}(t=2) = 5\hat{i} + 6\hat{j}$$

$$\text{speed} = \sqrt{25 + 36} = 7.8$$

* Problem 98 -

$$\vec{v}_i = 4\hat{i} + \hat{j}$$

$$\vec{r}_i = 10\hat{i} - 4\hat{j}$$

t = 20 sec / accel = constant

$$\vec{v}_f = 20\hat{i} - 5\hat{j}$$



a) What is the direction of the accel with respect to ...

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

$$20\hat{i} - 5\hat{j} = (4\hat{i} + \hat{j}) + 20\vec{a}$$

$$16\hat{i} - 6\hat{j} = 20\vec{a} \Rightarrow \vec{a} = \frac{16}{20}\hat{i} - \frac{6}{20}\hat{j}$$

c) $\vec{dr} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$

$$\vec{r}_f - \vec{r}_i = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

$$\vec{r}_f = \vec{v}_i t + \vec{r}_i + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a} \cdot \vec{dr}$$

$$\vec{v}_{fx}^2 = \vec{v}_{ix}^2 + 2a_x \cdot \Delta x$$

$$\vec{v}_{fy}^2 = \vec{v}_{iy}^2 + 2a_y \cdot \Delta y$$

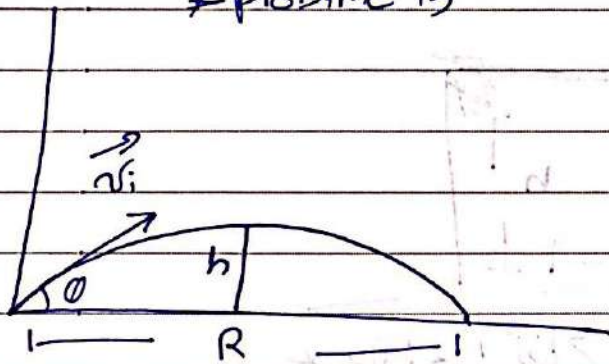
$$\Delta x = x_f - x_i$$

$$\Delta y = y_f - y_i$$

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

$$= (4\hat{i} + \hat{j}) + \left(\frac{20}{10}\hat{i} - \frac{6}{20}\hat{j}\right) * 25$$

* probleme 15



$$R = 3h$$

$$v_{fy} = v_{iy} - gt$$

$$0 = v_i \sin \theta - 10t \quad \Rightarrow \quad t = \frac{v_i \sin \theta}{10}$$

$$dy = h = v_{iy} \sin \theta t - \frac{1}{2} g t^2$$

$$= v_i \sin \theta \left(\frac{v_i \sin \theta}{10} \right) - 5 * \left(\frac{v_i^2 \sin^2 \theta}{100} \right)$$

$$= \frac{v_i^2 \sin^2 \theta}{20} \quad \text{--- (1)}$$

$$T = 2t = \frac{v_i \sin \theta}{5}$$

$$R = v_i \cos \theta * T = v_i \cos \theta * \frac{v_i \sin \theta}{5}$$

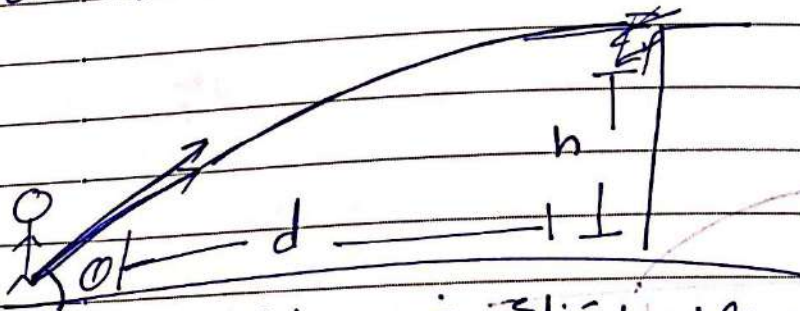
$$3h = R = \frac{v_i^2 \sin \theta \cos \theta}{5} \quad \text{--- (2)}$$

$$3 = \frac{\cos \theta}{\sin \theta} \frac{1}{5} = 4 \cot \theta$$

$$\frac{1}{20}$$

$$\tan \frac{4}{3} \Rightarrow \theta = 40^\circ$$

* probleme 21 8-



* (ب) اى ارتفاع ضرب باليب الجيب *

$$dy = h = v_i y t - \frac{1}{2} g t^2$$

دوة الارتفاع

بالاته v_i, θ, d

$$= v_i \sin \theta t - 5 t^2 \quad (1)$$

$$x = R = v_i x t$$

$$d = v_i \cos \theta t \quad (2)$$

from (2) $t = \frac{d}{v_i \cos \theta}$

$$h = \frac{v_i \sin \theta * d}{v_i \cos \theta} - 5 \left(\frac{d^2}{v_i^2 \cos^2 \theta} \right)$$

$$h = d \tan \theta - \frac{5 d^2}{v_i^2 \cos^2 \theta}$$

* probleme 41 8-

$$v_i = 90, t = 15$$

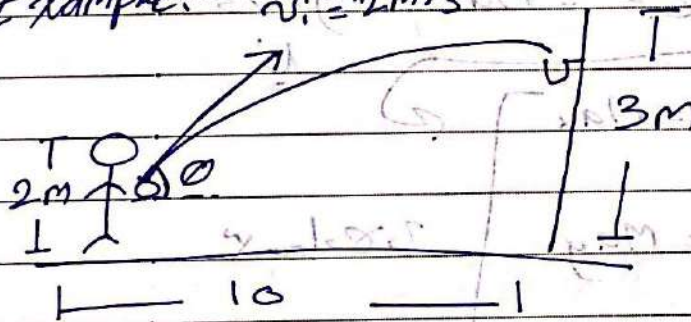
$$v_f = 50, r = 150$$

$$a_c = \frac{v^2}{r} = \frac{2500}{1.5} \text{ km/h}^2$$

$$a_t = \frac{dv}{dt} = \frac{50 - 90}{15} = \text{km/h}^2$$

$$a = \sqrt{a_c^2 + a_t^2}$$

Example: - $v_i = 2 \text{ m/s}$



$$y = v_i y t - \frac{1}{2} g t^2, \quad 1 = \frac{2 \sin \theta \cdot 5 - 5 \left(\frac{25}{\cos^2 \theta} \right)}{\cos \theta}$$

$$1 = v_i \sin \theta t - \frac{1}{2} g t^2, \quad 1 = 10 \tan \theta - \frac{125}{\cos^2 \theta}$$

$$1 = 2 \sin \theta t - 5 t^2 \quad \text{--- (1)}$$

$$1 \cos^2 \theta = 10 \sin \theta \cos \theta - 125$$

$$x = v_i x t$$

$$= v_i \cos \theta t$$

$$10 = 2 \cos \theta t$$

$$t = \frac{5}{\cos \theta}$$

$$\text{--- (2)}$$

* Newton's laws of motion :-

* First law :-

The motion state of an object stays the same unless a resultant force acts on it (with respect to initial frame of references)

* \vec{v} , \vec{a} constant, \vec{p} constant

* Second law :-

$$\vec{a} \propto \sum \vec{F} \propto \vec{a}$$

$$\sum \vec{F} = \text{constant } \vec{a} \rightarrow \boxed{\sum \vec{F} = M\vec{a}} \quad \left(\frac{\text{kg m}}{\text{s}^2} \right)$$

N

Mass
 M

$$[1] \sum F_x = Ma_x$$

$$[2] \sum F_y = Ma_y$$

$$[3] \sum F_z = Ma_z$$

* Third law :-

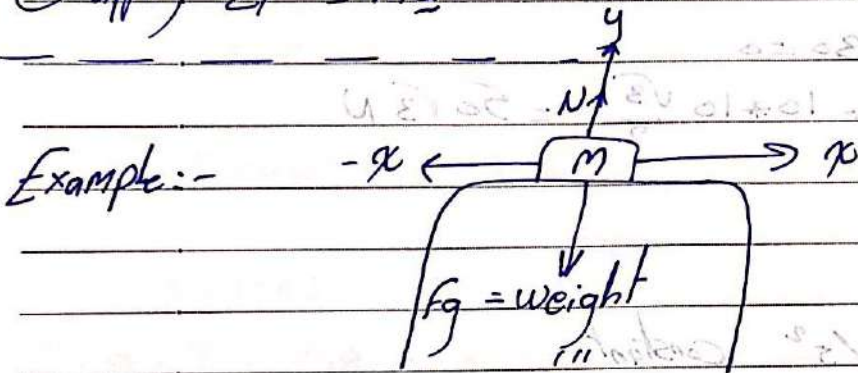
For every action there is a reaction equal in magnitude and opposite in direction.

* How to solve problems using newton's law.

① do free body diagram (all forces)

② analyze the forces

③ apply $\sum \vec{F} = M\vec{a}$



$$\sum F_y = N - mg$$

$$N = mg$$

Consider the traffic light in the figure if ropes 1 and 2 stands too N. does the traffic light remain hanging

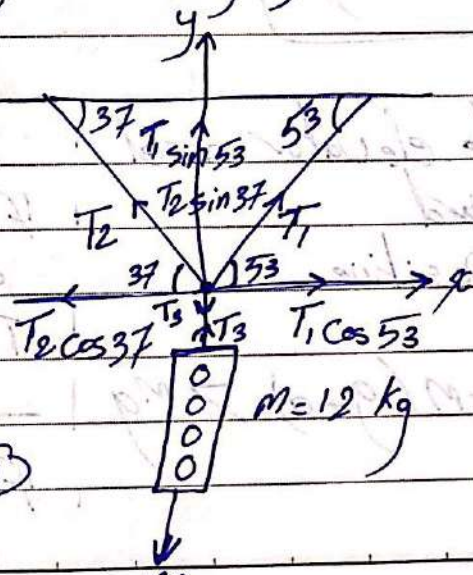
$$\sum F_{\text{traffic light}} = 0$$

$$T_3 - mg = 0$$

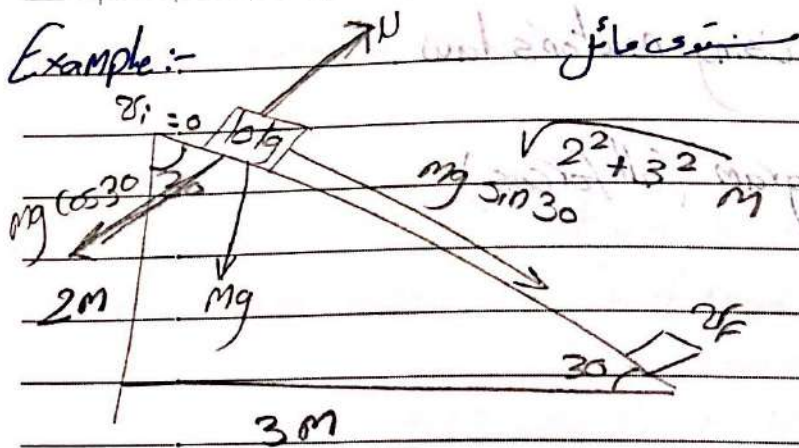
$$T_3 = mg = 12 * 10 = 120$$

$$\sum F_x = T_1 \cos 53 - T_2 \cos 37 = 0 \quad (1)$$

$$\sum F_y = T_1 \sin 53 + T_2 \sin 37 - T_3 = 0 \quad (2)$$



Example:-



$$\sum F_y = 0 = N - Mg \cos 30 = 0$$

$$N = Mg \cos 30 = 10 * 10 \frac{\sqrt{3}}{2} = 50\sqrt{3} \text{ N}$$

$$\sum F_x = Ma_x$$

$$Mg \sin 30 = Ma_x$$

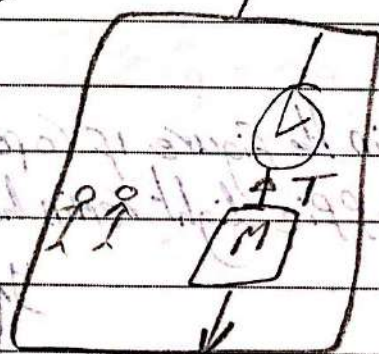
$$a_x = 10 * \frac{1}{2} = 5 \text{ m/s}^2 \text{ constant}$$

Example:- Weighing a fish in elevator:-

$$\sum F = ma$$

$$T - Mg = ma$$

$$T = M(a + g)$$



* If the elevator accel up ward

$a = \text{positive}$

$$T = M(g + a) > Mg$$

If elevator accel down ward

$a = \text{negative}$

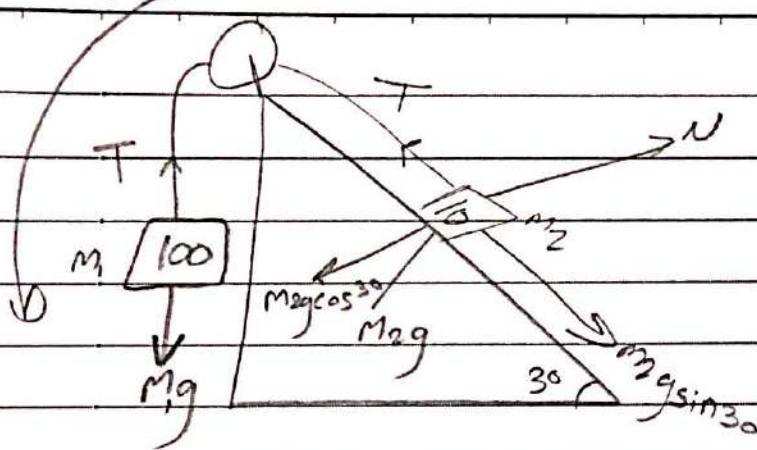
$$T = M(g - |a|) < Mg$$

If the accel constant speed

$a = 0$

$$T = Mg$$

Example:-



$$N - m_2 g \cos 30 = 0$$

$$N = m_2 g \cos 30 = 50\sqrt{3} \text{ N}$$

$$T - m_2 g \sin 30 = m_2 a \quad \text{--- (1)}$$

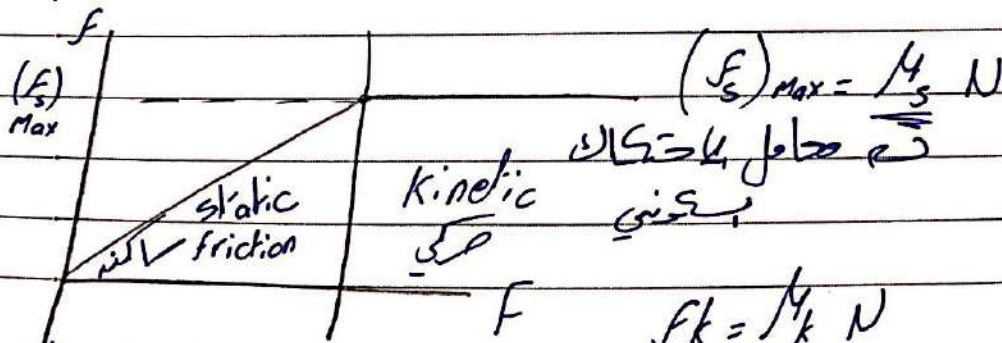
$$m_1 g - T = m_1 a \quad \text{--- (2)}$$

$$m_1 g - m_2 g \sin 30 = (m_1 + m_2) a$$

$$a = \frac{m_1 g - m_2 g \sin 30}{m_1 + m_2}$$

$$= \frac{1000 - 100 \times 1/2}{110} = \frac{950}{110} = \frac{95}{11} \text{ m/s}^2$$

* Forces of friction:-

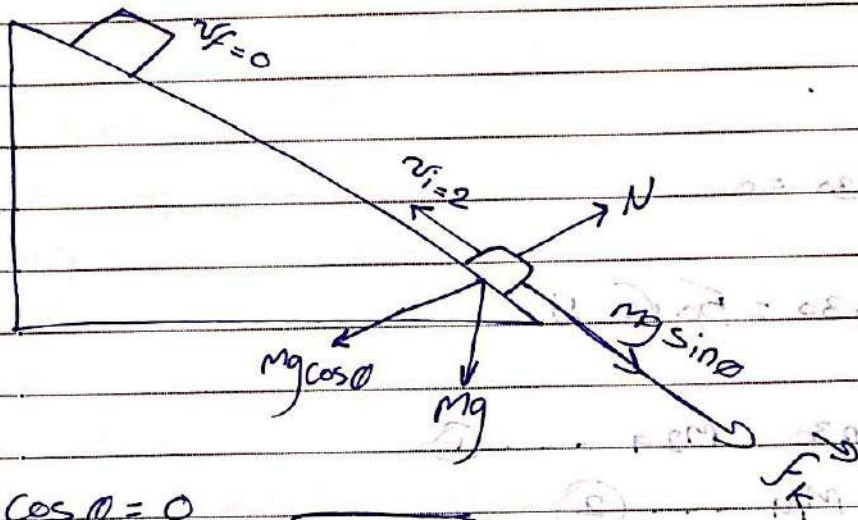


$$\mu_s > \mu_k$$

$$0 \leq \mu_s, \mu_k \leq 1$$

تھوڑا سا لڑائی
بہتر

Example:- if toss an object of mass 10 kg up an incline with coefficient of friction 0.5 with initial velocity 2 m/s. How much distance the object reach until it stops. The angle of the incline is 30°



$$N - Mg \cos \theta = 0$$

$$N = Mg \cos \theta \rightarrow \boxed{50\sqrt{3} \text{ N}}$$

$$-f_k - Mg \sin \theta = Ma$$

$$- \mu_k Mg \cos \theta - Mg \sin \theta = Ma$$

$$a = -0.5 * 10 * \frac{\sqrt{3}}{2} - 10 * \frac{1}{2} =$$

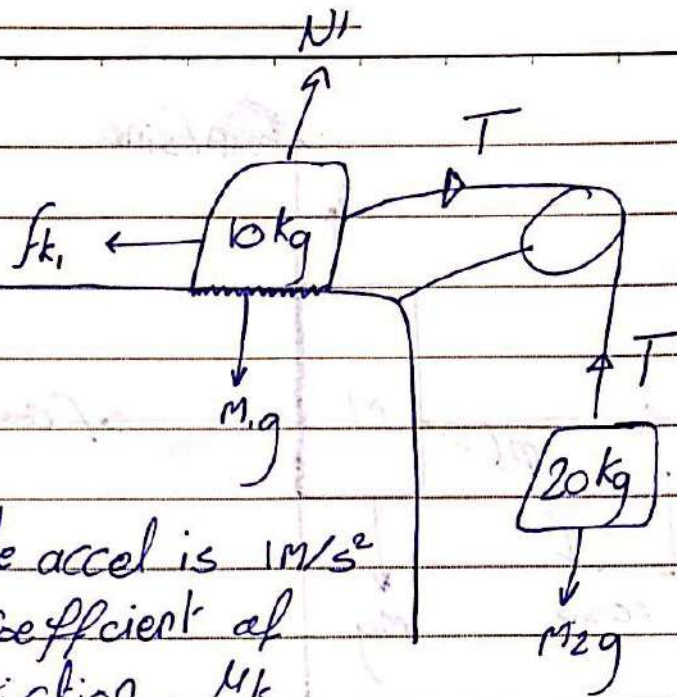
$$= -\left(\frac{5\sqrt{3}}{2} + 5\right) = \boxed{-9.3 \text{ m/s}^2}$$

$$v_f^2 = v_i^2 + 2a \cdot d$$

$$0 = 4 + 2 * -9.3 \cdot d$$

$$\boxed{d = \frac{4}{2 * 9.3}}$$

Example:-



suppose the accel is 1 m/s^2
find the coefficient of kinetic friction μ_k

$$N_1 - m_1 g = 0 \quad \Rightarrow \quad N_1 = m_1 g = \boxed{100 \text{ N}}$$

$$T - f_k = m_1 a$$

$$T - \mu_k m_1 g = m_1 a \quad \text{--- (1)}$$

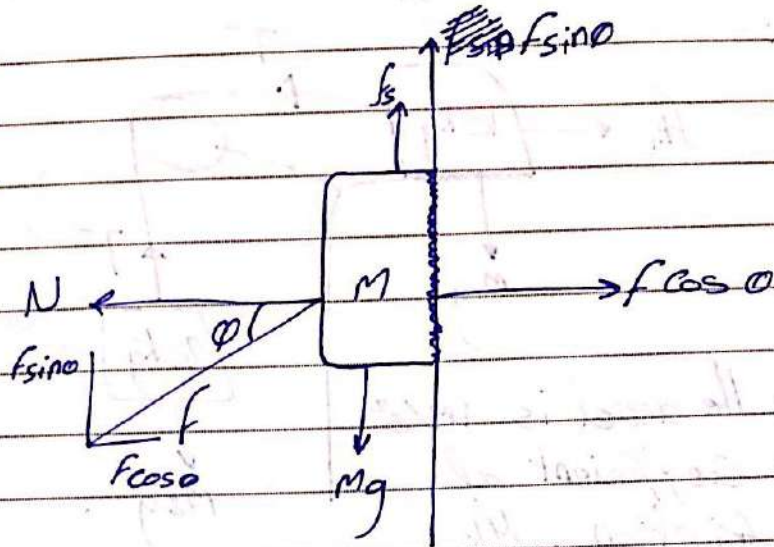
$$m_2 g - T = m_2 a \quad \text{--- (2) } \oplus$$

$$m_2 g - \mu_k m_1 g = (m_1 + m_2) a$$

$$\mu_k = \frac{(m_1 + m_2) a - m_2 g}{-m_1 g}$$

$$= \frac{30 - 200}{-100} = \frac{-270}{-100} = \boxed{2.7}$$

Example:-



$$N - f \cos \theta = 0$$

$$Mg - fs - f \sin \theta = 0$$

$$f_{\min} = \frac{Mg - fs}{\sin \theta}$$

$$f_{\min} = \frac{Mg - \mu_s f_{\min} \cos \theta}{\sin \theta}$$

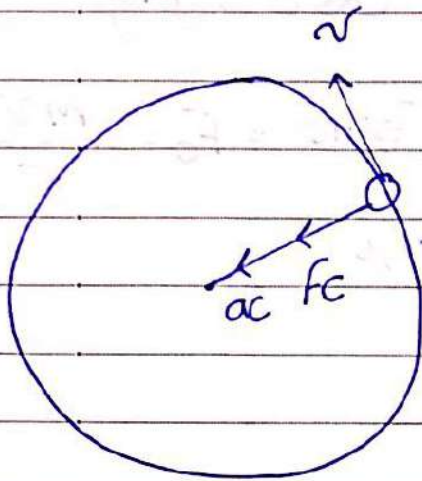
$$f_{\min} + \mu_s f_{\min} \cos \theta = \frac{Mg}{\sin \theta}$$

$$f_{\min} = \frac{Mg}{\sin \theta (1 + \mu_s \cos \theta)} = \frac{Mg}{\sin \theta + \mu_s \cos \theta}$$

Chapter 6

6-1

Uniform circular motion



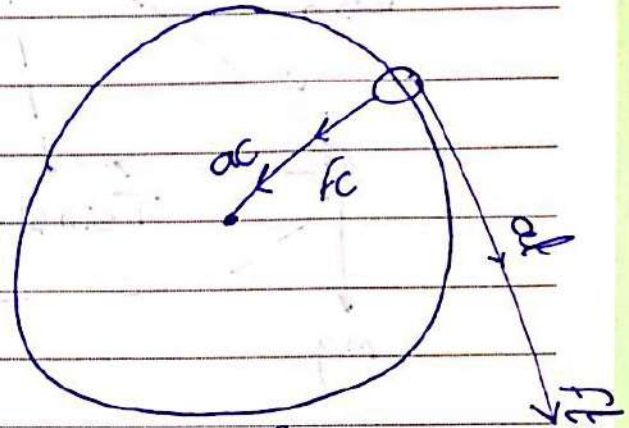
$$a_c = \frac{v^2}{r}$$

$$F_c = \frac{mv^2}{r}$$

central force قوت مرکزی

6-2

Non uniform circular motion

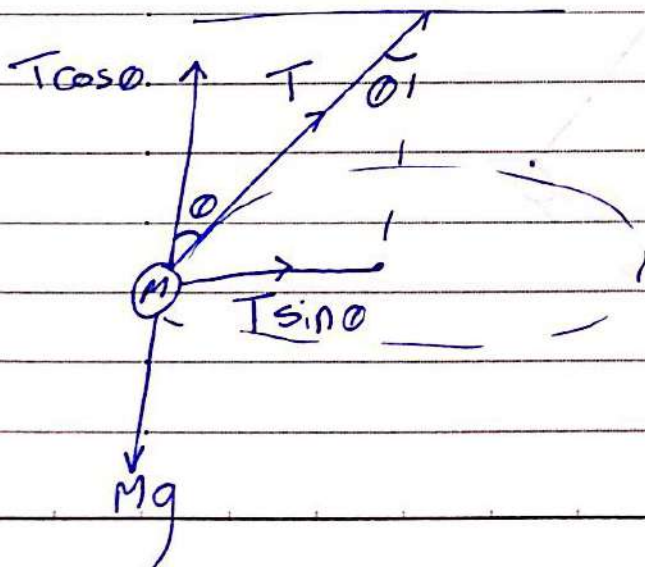


$$F_c = \frac{mv^2}{r}$$

$$F_t = ma_t$$

$$F = \sqrt{F_c^2 + F_t^2}$$

Example:- Conical pendulum



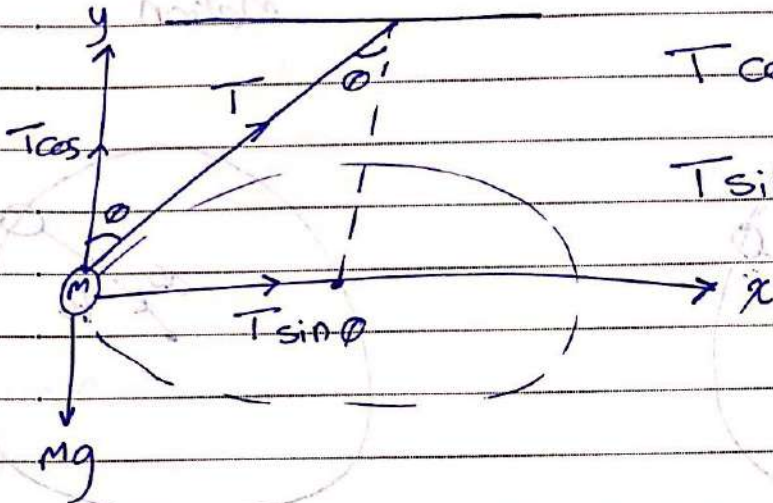
$$T \cos \theta = mg$$

$$T \sin \theta = F_c = \frac{mv^2}{r}$$

$$T = \frac{mv^2}{r \sin \theta}$$

*Application on circular motion :-

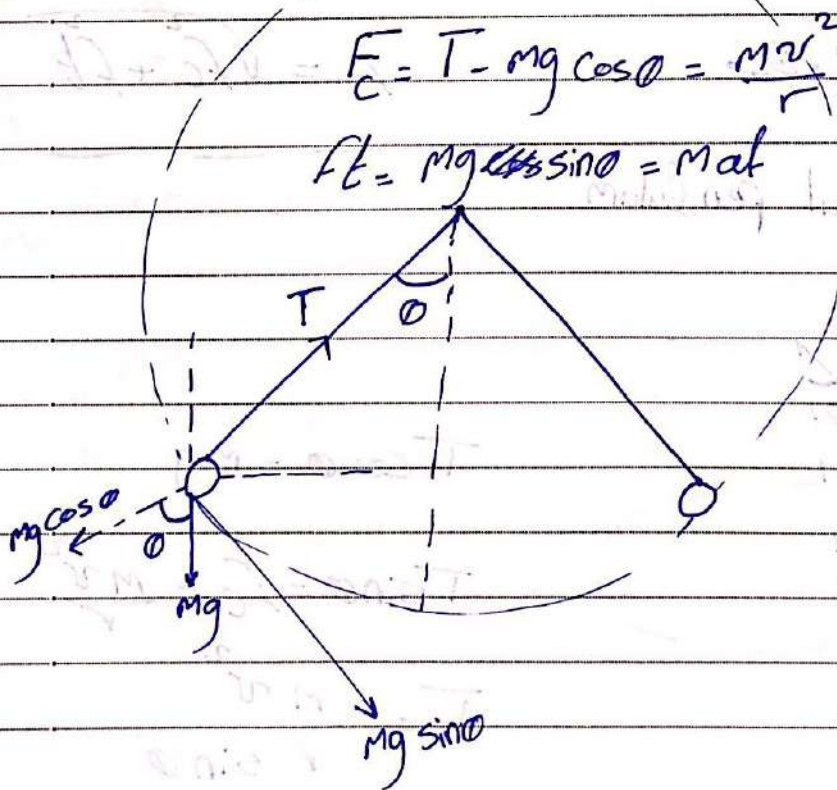
① Conical pendulum :-



$$T \cos \theta = mg$$

$$T \sin \theta = F_c = \frac{mv^2}{r}$$

② regular pendulum :-

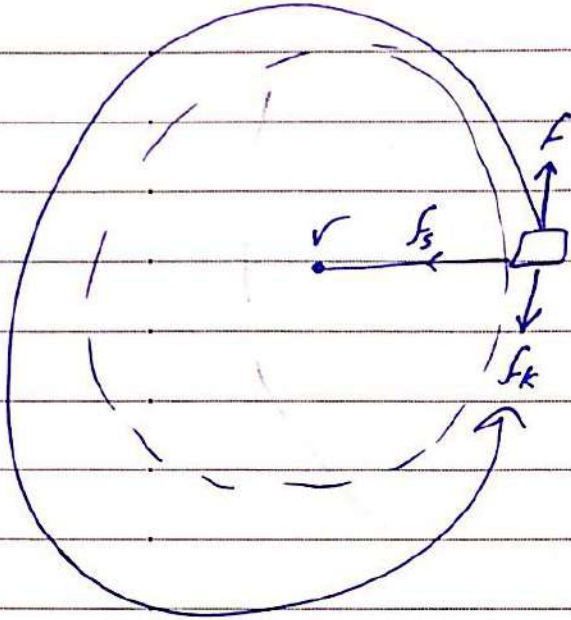


$$F_c = T - mg \cos \theta = \frac{mv^2}{r}$$

$$F_t = mg \sin \theta = ma_t$$

③ Banking angle :-

* if there is no banking angle :-

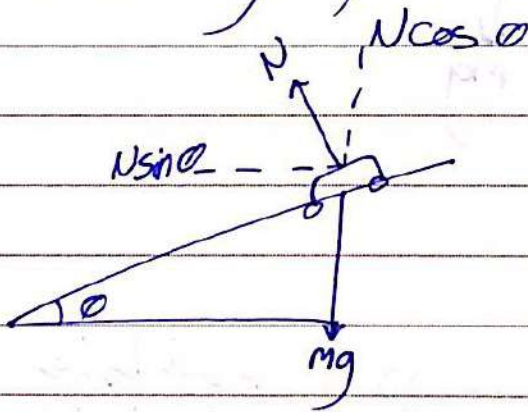


$$N = mg$$

$$F - f_k = ma_c$$

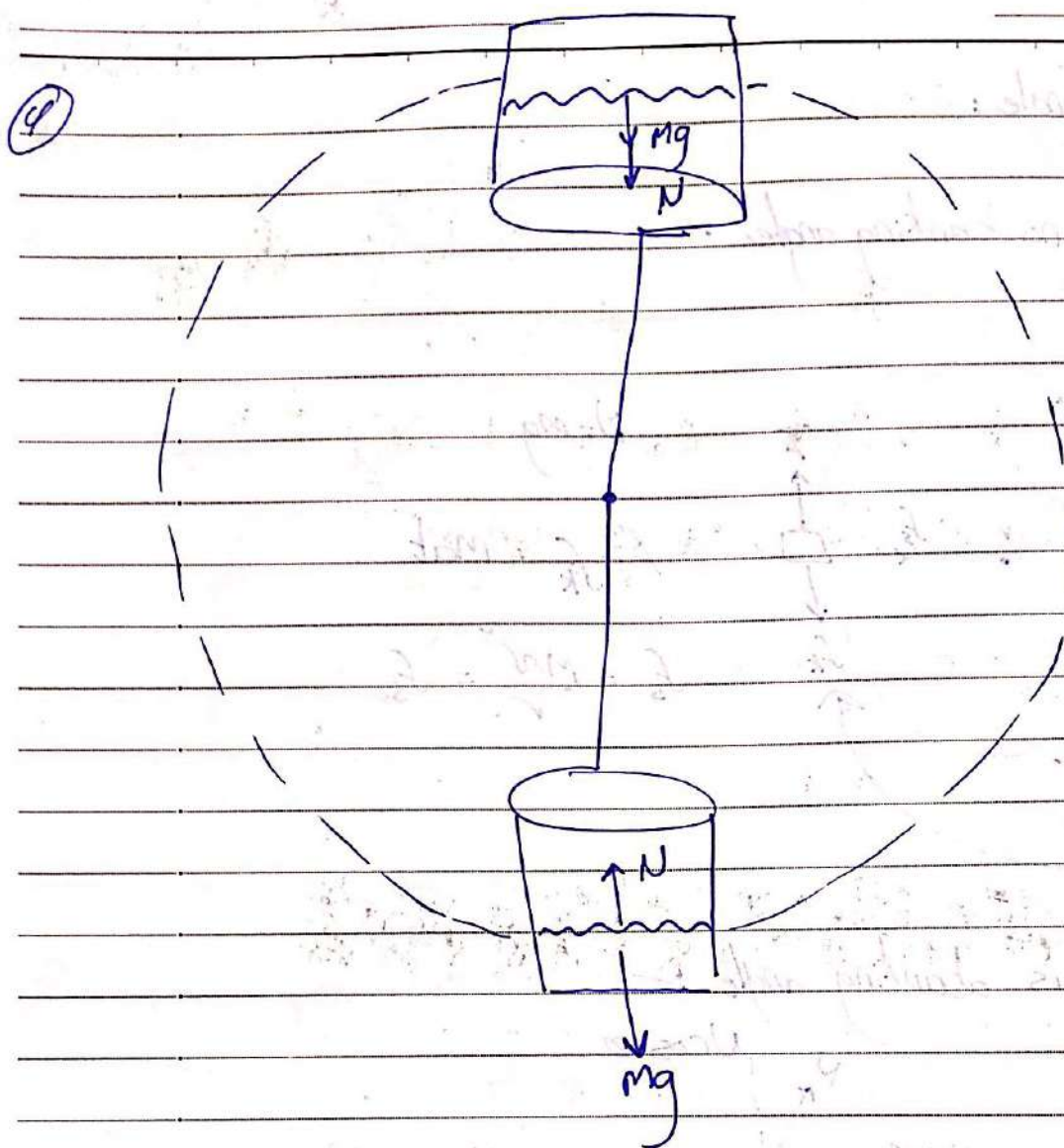
$$f_s = \frac{mv^2}{r} = f_s$$

* if there is a banking angle :-



$$mg = N \cos \theta$$

$$f_c = N \sin \theta = \frac{mv^2}{r}$$



$$F_c = N + mg = \frac{mv^2}{r}$$

$$N = \frac{mv^2}{r} - mg$$

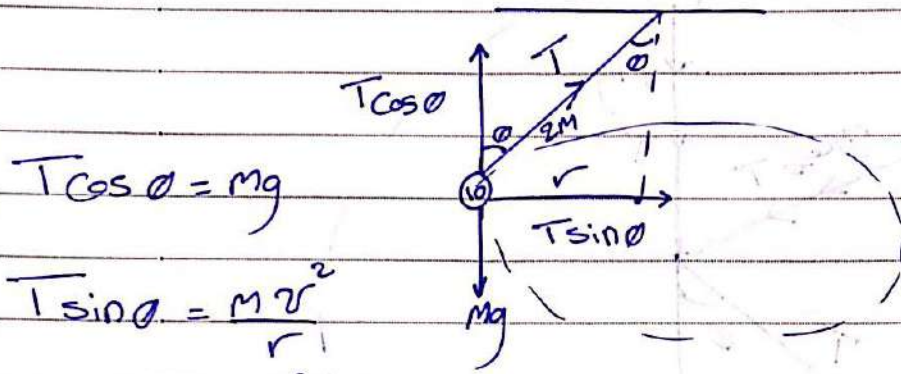
نورس با س
الدله في اشارة

~~#~~ if $N=0 \rightarrow v = \sqrt{gr}$

$$F_c = N - mg = \frac{mv^2}{r}$$

نورس با س
الدله في اشارة

Example:- For a conical pendulum if the length of the rope connected is 2 meters and the mass connected is 10 kg and the velocity is 2 m/s find the angle of the pendulum.



$$T \cos \theta = mg$$

$$T \sin \theta = \frac{mv^2}{r}$$

$$\tan \theta = \frac{v^2}{rg} = \frac{4}{10 \times 2 \sin \theta}$$

$$\sin \theta = \frac{r}{2} \rightarrow r = 2 \sin \theta$$

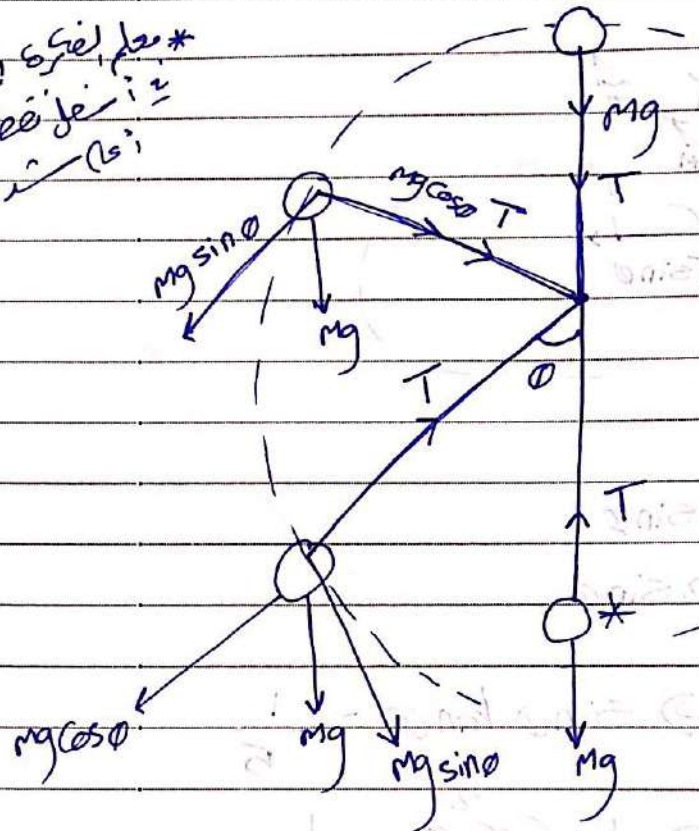
$$\tan \theta = \frac{4}{20 \sin \theta} \rightarrow \sin \theta \tan \theta = \frac{1}{5}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{1}{5} \rightarrow \frac{1 - \cos^2 \theta}{\cos \theta} = \frac{1}{5}$$

$$1 - \cos^2 \theta = \frac{1}{5} \cos \theta$$

Example:- for a regular pendulum with length 2m, if the rope stands 100N and mass of 8 kg is connected does the pendulum break? assume the velocity of the lowest point is 2m/s.

بسیار عالی است
 بسیار عالی است
 بسیار عالی است



$$T - mg = \frac{mv^2}{r}$$

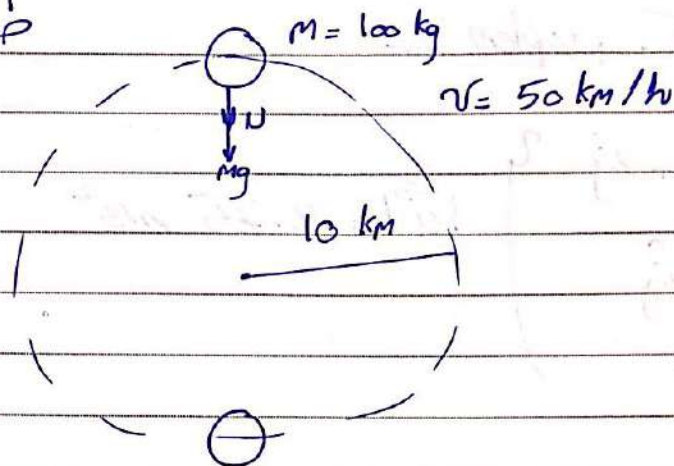
$$T = mg + \frac{mv^2}{r} \rightarrow 8 \times 10 + \frac{8 \times 4}{2}$$

$$80 + 16 = 96 < 100$$

بسیار عالی است
 بسیار عالی است
 بسیار عالی است

Example :-

σ, L^1

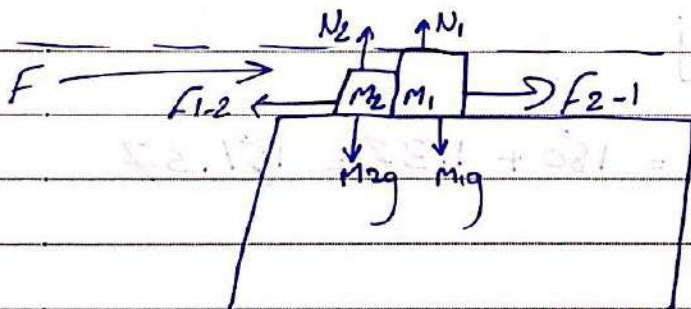


$$N + mg = \frac{mv^2}{r}$$

$$N = \frac{mv^2}{r} - mg \rightarrow \frac{100 * 50^2}{10} - 100 * 10 * \frac{1}{1000 * \left(\frac{1}{60 * 60}\right)}$$

$$= < 0$$

$$> 0$$



$$F = (m_1 + m_2) a$$

$$a = \frac{F}{m_1 + m_2}$$

$$N_1 = m_1 g$$

$$f_{2-1} = m_1 a = \frac{m_1 F}{m_1 + m_2}$$

$$f_{1-2} = f_{2-1} = \frac{m_1 F}{m_1 + m_2}$$

Chapter 5. problem 22.

$$\begin{aligned} \vec{f}_1 &= -2\hat{i} + 2\hat{j} \\ \vec{f}_2 &= 5\hat{i} - 3\hat{j} \\ \vec{f}_3 &= -45\hat{i} \end{aligned} \quad \left. \vphantom{\begin{aligned} \vec{f}_1 &= -2\hat{i} + 2\hat{j} \\ \vec{f}_2 &= 5\hat{i} - 3\hat{j} \\ \vec{f}_3 &= -45\hat{i} \end{aligned}} \right\} |\vec{a}| = 3.75 \text{ m/s}^2$$

- ① what is the direction of \vec{a}
- ② what is the mass
- ③ if $\vec{v}_i = 0$, find the speed after 10s
- ④ what is the component of velocity after 10s

$$\begin{aligned} 1- \vec{F} &= m\vec{a} \\ \vec{F} &= \vec{f}_1 + \vec{f}_2 + \vec{f}_3 = -42\hat{i} - \hat{j} \end{aligned}$$

$$\theta = \tan^{-1} \frac{-1}{-42} = \tan^{-1} \frac{1}{42} = 180 + 1.37 = 181.37$$

$$2- |\vec{F}| = |m\vec{a}| = m|\vec{a}|$$

$$\sqrt{42^2 + 1^2} = m * 3.75$$

$$m = \frac{\sqrt{42^2 + 1^2}}{3.75} = \boxed{11.2}$$

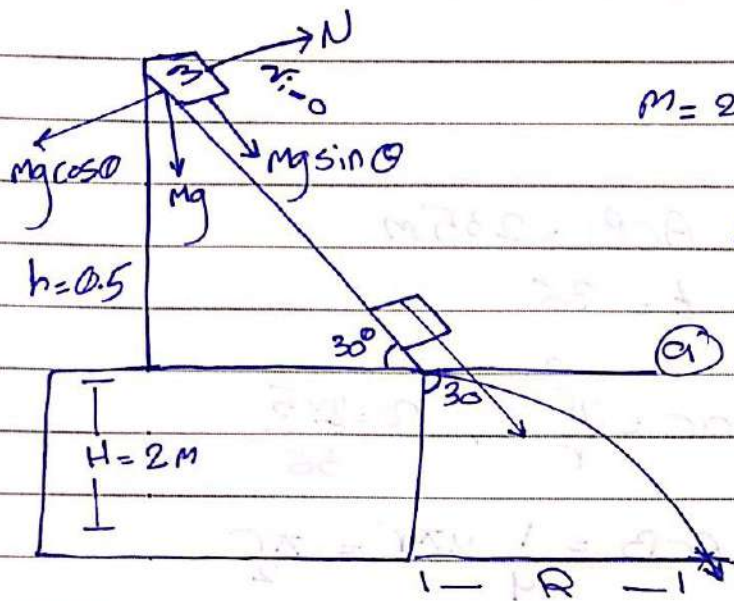
$$v_x = \frac{-420}{11.2}$$

$$3- \vec{v}_f = \vec{v}_i + \vec{a} * t$$

$$v_y = \frac{-10}{11.2}$$

$$= 0 + \frac{\vec{F}}{m} * t = \frac{-42\hat{i} - \hat{j}}{11.2} * 10$$

$$\text{Speed} = |\vec{v}_f| = |\vec{a}| t = 3.75 * 10$$



$$m = 2 \text{ Kg}$$

$$\sum F_x = ma$$

$$\textcircled{a} \quad mg \sin \theta = ma$$

$$a = g \sin \theta$$

$$= 10 \times \frac{1}{2} = 5 \text{ m/s}^2$$

$$\textcircled{b} \quad v_f^2 = v_i^2 + 2a\Delta h$$

$$= 0 + 2 \times 5 \times 0.5 \times \frac{1}{2}$$

$$v_f^2 = 2.5 \Rightarrow v_f = \sqrt{2.5}$$

$$R = v_x t$$

$$= v_i \cos 30^\circ t$$

$$\sqrt{2.5} \times \frac{\sqrt{3}}{2} \times t$$

$$Ry = v_{iy} t - \frac{1}{2} g t^2$$

$$-2 = -v_i \sin 30^\circ t - 5t^2$$

$$\textcircled{t} =$$

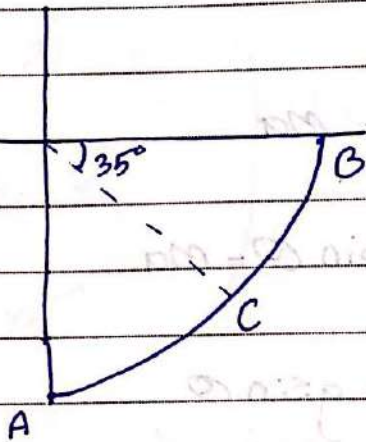
$$t = \sqrt{\frac{1}{10} + t}$$

$$0x = v_i t + \frac{1}{2} a t^2$$

$$0.5 \times \frac{1}{2} = 0 + \frac{1}{2} \times 5 t^2$$

$$t^2 = \frac{1}{10} \Rightarrow t = \sqrt{1/10}$$

chapter 6 . problem 6



$$ACB = 235 \text{ m}$$

$$t = 36$$

$$a_c = \frac{v^2}{r}, \quad v = \frac{235}{36}$$

$$ACB = \frac{1}{2} 2\pi r = \frac{\pi r}{2}$$

$$235 = \frac{\pi r}{2} \Rightarrow r = \frac{470}{\pi}$$

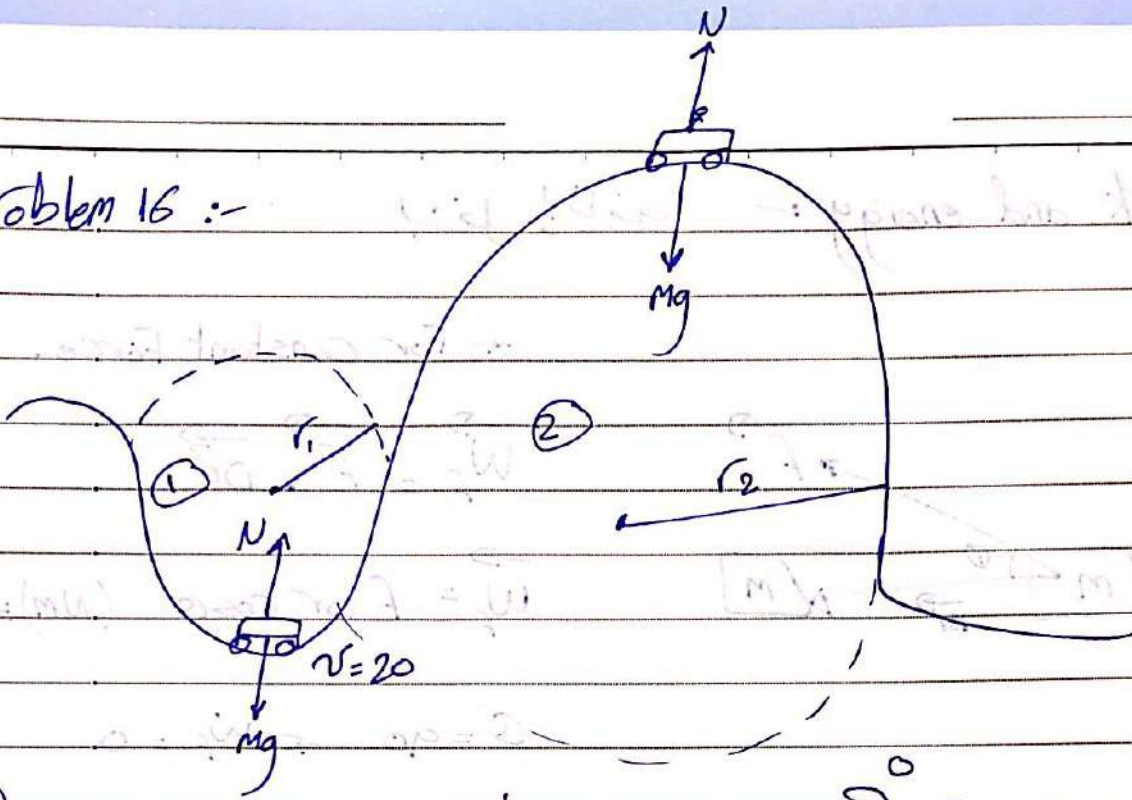
$$a_c = \frac{\left(\frac{235}{36}\right)^2}{\frac{470}{\pi}}$$

$$a_x = a_c \cos 145$$

$$a_y = a_c \sin 145$$

$$\vec{a}_{avg} = \frac{\Delta v}{\Delta t} \Rightarrow \vec{a}_{avg} = \frac{235}{36} (j - i)$$

Problem 16 :-



$$\textcircled{1} \quad N - Mg = \frac{Mv^2}{r} \quad | \quad \textcircled{2} \quad Mg - N = \frac{Mv^2}{r}$$

$$N = Mg + \frac{Mv^2}{r}$$

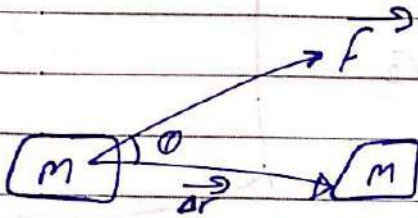
$$v_{\min} = \sqrt{rg}$$

$$= 5000 + 5000 * \frac{20^2}{10}$$

$$= \sqrt{15 * 10} = \boxed{12.2}$$

* Work and energy :- بندگی و سکتی

* For constant Force.



$$\vec{W}_F = \vec{F} \cdot \vec{dr}$$

$$\vec{W}_F = F dr \cos \theta \quad (\text{Nm}) \equiv \text{J}$$

$$\theta = 90 \rightarrow W_F = 0$$

(eg)