13.7] logarithm – General power & principle value:

$z = x + iy = re^{i\theta}$

$$\ln|z| = \ln|x+iy| = \ln|re^{i\theta}|$$
$$= \ln r + \ln e^{i\theta}$$
$$= \ln r + i\theta\,[\underbrace{\ln e}_{=1}]$$
$$= \ln r + i\theta$$

$r = \sqrt{x^2+y^2}$ $\qquad$ $\theta = \tan^{-1}\left(\frac{y}{x}\right) \pm 2\pi n$

Ex: Find $\ln[3-4i] = ?$

$x = 3,\ y = -4$

$r = 5,\ \theta = -0.93$

$$\ln[3-4i] = \ln r + i\theta$$
$$= \ln 5 + (-0.93 + 2\pi n)i$$

No.

* Principle value of $\ln(z)$ is $Ln(z)$

[1] $Ln(z) = \ln(z) + i\,Arg(z)$

[2] $\ln(z) = L(z) \pm 2\pi n i$

At $n=0$

$\ln(z) = Ln(z)$

- $\ln(z_1 z_2) = \ln z_1 + \ln z_2$
- $\ln\left(\frac{z_1}{z_2}\right) = \ln z_1 - \ln z_2$
- $[\ln(z)]' = \frac{1}{z}$

Ex: Find $Ln(3-4i)$

$1.61 + i[-0.93]$

* General power :

$[z]^c = e^{\ln[z]^c}$

$= e^{c\ln z}$

$\Rightarrow c = \pm 1, \pm 2 \cdots$

$c = 2/3, 3/4, \cdots$

$c = 1/2, 1/3, \cdots$

$c = 1+i, x+iy$.

Ex: Find $(i)^i$ in simple Form.

$(i)^i = e^{\ln(i)^i}$

$= e^{i\ln i} \rightarrow \ln i = \ln r + i\theta$

$= \cancel{\ln 1} + i\left[\frac{\pi}{2} + \frac{2\pi n}{?}\right]$

$r = 1$

$\theta = \frac{\pi}{2}$

$(i)^i = e^{i[i[\pi/2 + 2\pi n]]}$

$= e^{-\pi/2 \pm 2\pi n}$

Ex: Find $(1+i)^{2-i}$ in simple Form :

Sol:

$(1+i)^{2-i} = e^{\ln(1+i)^{2-i}}$

$= e^{(2-i)\ln(1+i)} \rightarrow$

$= e^{(2-i)[\ln\sqrt{2} + i\pi/4 + 2\pi n i]}$

$\ln(1+i) = \ln r + i\theta$

$r = \sqrt{2}$ $\quad = \ln\sqrt{2} + i\left(\frac{\pi}{4} + 2\pi n\right)$

$\theta = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$

تكملة السؤال.

* principle value for General power : $[n=0]$

Ex: Find the principle value For $(i)^i$?

$= e^{-\pi/2}$

[New chapter]

No.

7.1 Matrix, vectors addition & scalar Multiplication:

Matrix ⇒ is a rectangular array of numbers or function which we will enclose in brackets.

$$A = \begin{bmatrix} 0.3 & 1 & -5 \\ 0 & -0.2 & 10 \end{bmatrix}, \quad B = \begin{bmatrix} x & xy \\ \sin x & \cos y \end{bmatrix}$$

الأرقام
elements, entery

* linear system:

$$4x_1 + 6x_2 + 9x_3 = 6$$
$$6x_1 + 0x_2 - 2x_3 = 20$$
$$5x_1 - 8x_2 + x_3 = 10$$

1. $Ax = b$

$$\begin{bmatrix} 4 & 6 & 9 \\ 6 & 0 & -2 \\ 5 & -8 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 20 \\ 10 \end{bmatrix}$$

2. Augmenting Matrix $\tilde{A} = [A \vdots b]$

$$\tilde{A} = \left[\begin{array}{ccc|c} 4 & 6 & 9 & 6 \\ 6 & 0 & -2 & 20 \\ 5 & -8 & 1 & 10 \end{array}\right]$$

$$x = \begin{bmatrix} 3 \\ 1/2 \\ -1 \end{bmatrix}$$

* For Matrix $A = [a_{jk}]$ has $m \times n$ Matrix size of $A = \underset{\text{Row}}{m} \times \underset{\text{Column}}{n}$

* square Matrix if Row & column are equal.

⊛ If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Main diagonal element $= a_{11}, a_{22}, a_{33}$

⊛ **Vectors:** is a Matrix with one Row OR One Column.

$a = [a_1 \quad a_2 \quad a_3] \rightarrow$ size 1×3

$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \rightarrow$ size 3×1

⊛ **equality of Matrix:**

Def: Two matrix $A = [a_{jk}]$ & $B = [b_{jk}]$ are equal iff

① have same size $m \times n$

② corresponding element are equal, if $A \neq B$ then A & B are different

[1] do not have same size

[2] correspounding element are not equal.

Ex: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$C = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 6 \end{bmatrix}$

[1] A & B are equal — Same size & elements

[2] $A \neq C$ — different element $1 \neq -1$

[3] $A \neq D$ — different size

No.

⊛ Addition of Matrix:

Def: the sum of Two matrix $A=[a_{jk}]$ & $B=[b_{jk}]$ of same size is written $A+B=[a_{jk}+b_{jk}]$ & obtained by adding the coresponding element.

note: Matrix with different size cannot added.

Ex: let ⇒

$$A=\begin{bmatrix}-4 & 6 & 3\\ 0 & 1 & 2\end{bmatrix},\quad B=\begin{bmatrix}5 & -1 & 0\\ 3 & 1 & 0\end{bmatrix}$$

$$C=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$$

1. $A+B=\begin{bmatrix}1 & 5 & 3\\ 3 & 2 & 2\end{bmatrix}_{2\times3}$

2. $A-B=\begin{bmatrix}-9 & 7 & 3\\ -3 & 0 & 2\end{bmatrix}_{2\times 3}$

3. $A+C=$ can not be added (different size).

⊛ Scalar Multiplication:

Def: The product of any $m\times n$ Matrix $A=[a_{jk}]$ & any scalar (C) written CA & it's $m\times n$ Matrix $CA=[Ca_{jk}]$ obtained by multipling each element of A by C.

Ex: let $A=\begin{bmatrix}2.7 & -1.8\\ 0 & 0.9\\ 9 & -4.5\end{bmatrix}$

Find 1. $-A$ 2. $\frac{10}{9}A$ 3. $0A$

$$-A=\begin{bmatrix}-2.7 & 1.8\\ 0 & -0.9\\ -9 & 4.5\end{bmatrix}\qquad \frac{10}{9}A=\begin{bmatrix}3 & -2\\ 0 & 1\\ 10 & -5\end{bmatrix}\qquad 0A=\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}$$

No.

- **Rules of addition:**

① $A+B = B+A$

② $[A+B]+C = A+[B+C] = A+B+C$

③ $A+0=A$ ④ $A+(-A)=0$

- **Rules of scalar Multiplication:**

① $C[A+B] = CA+CB$

② $(C+K)A = CA+KA$

③ $C[KA] = [CK]A$

④ $1A = A$

7.2) Matrix Multiplication:

Def: The product $C=AB$ of an $(m\times n)$ Matrix $A=[a_{jk}]$ times an $(r\times p)$ Matrix $B=[b_{jk}]$ is definied iff $n=r$ & the $m\times p$ matrix $C=[c_{jk}]$ with element

$$c_{jk} = \sum_{l=1}^{n} a_{jl}\, b_{lk} = a_{j1}b_{1k} + \dots + a_{jn}b_{nk}$$

(صف ضرب عامود واجمع)

$j = 1, 2, \dots m$

$k = 1, 2 \dots p$

$$\underbrace{\underset{m\times n}{A}\ \underset{r\times p}{B}}_{\text{equal}} = \underset{m\times p}{C}.$$

Ex: let $A = \begin{bmatrix} 3 & 5 & -1 \\ 4 & 0 & 2 \\ -6 & -3 & 2 \end{bmatrix}$ find AB?

$A\times B = C \longrightarrow 3\times4$

$\underbrace{3\times3 \quad 3\times4}_{\text{equal}}$

$B = \begin{bmatrix} 2 & -2 & 3 & 1 \\ 5 & 0 & 7 & 8 \\ 9 & -4 & 1 & 1 \end{bmatrix}$

$C = \begin{bmatrix} 22 & -2 & 43 & 42 \\ 26 & -16 & 14 & 6 \\ -9 & 4 & -37 & -28 \end{bmatrix}$

$C_{11} = (3)(2) + 5(5) + (-1)(9)$

صف من A ضرب عامود من B واجمع

No.

Ex: let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

$AB = ??$

$$AB = \begin{bmatrix} a_{11}(b_{11}) + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

* $\underset{(2\times 2)}{\begin{bmatrix} 4 & 2 \\ 1 & 8 \end{bmatrix}} \underset{(2\times 1)}{\begin{bmatrix} 3 \\ 5 \end{bmatrix}} = \underset{(2\times 1)}{\begin{bmatrix} 22 \\ 43 \end{bmatrix}}$

* $\underset{(1\times 3)}{\begin{bmatrix} 3 & 6 & 1 \end{bmatrix}} \underset{(3\times 1)}{\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}} = \underset{(1\times 1)}{\begin{bmatrix} 19 \end{bmatrix}}$

* $\underset{(3\times 1)}{\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}} \underset{(1\times 3)}{\begin{bmatrix} 3 & 6 & 1 \end{bmatrix}} = \begin{bmatrix} 3 & 6 & 1 \\ 6 & 12 & 2 \\ 12 & 24 & 4 \end{bmatrix}$ (3×3)

$\therefore$ $AB \neq BA$ (علاقة غير تبادلية)

• Rules ⇒
1. $(KA)B = K(AB)$
2. $A(BC) = (AB)C$
3. $(A+B)C = AC + BC$
4. $C(A+B) = CA + CB$

$AC \neq CA$ (for 3, 4)

توزيع

* product in terms of Row & columns vector (طريقة ثانية)

Ex: let $\underset{(3\times3)}{A} = \begin{bmatrix} - & a_1 & - \\ - & a_2 & - \\ - & a_3 & - \end{bmatrix}$, $\underset{(3\times4)}{B} = \begin{bmatrix} | & | & | & | \\ b_1 & b_2 & b_3 & b_4 \\ | & | & | & | \end{bmatrix}$

$$\Rightarrow \underset{(3\times4)}{AB} = \begin{bmatrix} a_1b_1 & a_1b_2 & a_1b_3 & a_1b_4 \\ a_2b_1 & a_2b_2 & a_2b_3 & a_2b_4 \\ a_3b_1 & a_3b_2 & a_3b_3 & a_3b_4 \end{bmatrix}$$

- **Parrallel processing of product:**

$$AB = A\begin{bmatrix} | & | & \cdots & | \\ b_1 & b_2 & & b_n \\ | & | & & | \end{bmatrix} = \begin{bmatrix} | & | & \cdots & | \\ Ab_1 & Ab_2 & & Ab_n \\ | & | & & | \end{bmatrix}$$

Ex: $\underset{(2\times2)}{\begin{bmatrix} 4 & 1 \\ -5 & 2 \end{bmatrix}} \underset{(2\times3)}{\begin{bmatrix} 3 & 0 & 7 \\ -1 & 4 & 6 \end{bmatrix}} = \underset{(2\times3)}{\begin{bmatrix} 11 & 4 & 34 \\ -17 & 8 & -23 \end{bmatrix}}$

$$Ab_1 = \begin{bmatrix} 4 & 1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 11 \\ -17 \end{bmatrix}$$

$$Ab_2 = \begin{bmatrix} 4 & 1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$Ab_3 = \begin{bmatrix} 4 & 1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ 6 \end{bmatrix} = \begin{bmatrix} 34 \\ 23 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 4 & 34 \\ -17 & 8 & 23 \end{bmatrix}$$

* Transposition :

the transepose of $(m \times n)$ Matrix $A = [a_{jk}]$ in $(n \times m)$ Matrix A^T (A transpose) that has the first Row of A as its first column & second Row of A as its second column & so on .

[1] $A = \begin{bmatrix} 5 & -8 & 1 \\ 4 & 0 & 0 \end{bmatrix}_{2\times3} \Rightarrow A^T = \begin{bmatrix} 5 & 4 \\ -8 & 0 \\ 1 & 0 \end{bmatrix}_{3\times2}$

[2] $A = [6\ 2\ 3]_{1\times3} \Rightarrow A^T = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}_{3\times1}$

Rules:

① $(A^T)^T = A$

② $(A+B)^T = A^T + B^T$

③ $(CA)^T = CA^T$

④ $(AB)^T = B^T A^T$

- $(ABC)^T = C^T B^T A^T$

$\bar{A}$ ← مصفوفة .

- symmetric Matrix if

$A^T = A$

- skew – symmetric Matrix

$A^T = -A$

Ex: let $A = \begin{bmatrix} 20 & 120 & 200 \\ 120 & 10 & 150 \\ 200 & 150 & 30 \end{bmatrix}_{3\times3}$

$A^T = \begin{bmatrix} 20 & 120 & 200 \\ 120 & 10 & 150 \\ 200 & 150 & 30 \end{bmatrix}$

$A = A^T \Rightarrow A$ is symmetric

Ex: let $A = \begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix}$

$A^T = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & 2 \\ -3 & -2 & 0 \end{bmatrix}$

$-A = \begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix} \Rightarrow -A = A^T$ (A is skew - symmetric)