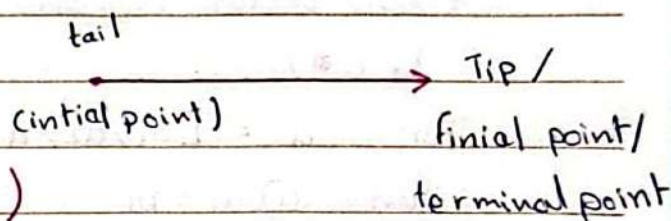


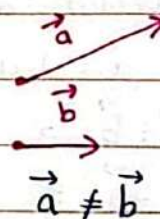
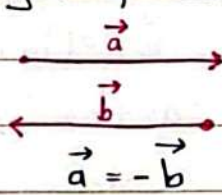
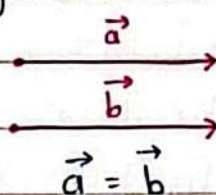
New chapter:

9.1) vector in 2-space & 3-space

- ① **Scalar**: quantity that is determined by its magnitude like [time, length]
- ② **vector**: quantity that is determined by magnitude & direction like [force]

For vector $\vec{a} = \vec{a}$  $(|\vec{a}| = \text{length} = \text{norm} = \text{Magnitude})$ if $|\vec{a}| = 1$ $\vec{a}$ is unit vector

Def: Two vectors $\vec{a}$ & $\vec{b}$ are equal so $\vec{a} = \vec{b}$ iff they have same length & same direction

**Component of vectors:**

for vector $\vec{a}$ has initial point $P: (x_1, y_1, z_1)$ & terminal point $Q: (x_2, y_2, z_2)$

$$a_1 = x_2 - x_1, \quad a_2 = y_2 - y_1, \quad a_3 = z_2 - z_1$$

$$\vec{a} = [a_1, a_2, a_3], \quad |\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Ex: for vector $\vec{a}$ with initial point $P: (4, 0, 2)$ & terminal point $Q: (6, -1, 2)$ find vector component + $|\vec{a}|$?

$$a_1 = 6 - 4 = 2 \quad \vec{a} = (2, -1, 0)$$

$$a_2 = -1 - 0 = -1 \quad |\vec{a}| = \sqrt{4 + 1 + 0} = \sqrt{5}$$

$$a_3 = 2 - 2 = 0$$

* **position vector** ($\vec{r}$) at point A: (x, y, z) is a vector with origin $(0, 0, 0)$ as initial point & A as terminal point.



* **zero vector** :- has length zero & no-direction



for $\vec{a} = [a_1, a_2, a_3]$ & $\vec{b} = [b_1, b_2, b_3]$ if $\vec{a} = \vec{b}$

then ① $a_1 = b_1$

② $a_2 = b_2$

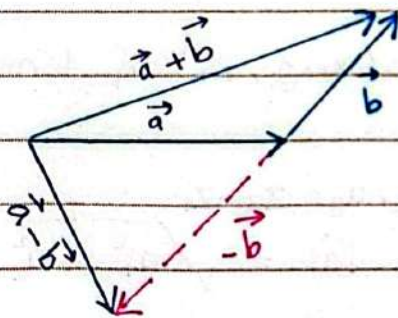
③ $a_3 = b_3$

* **Vector addition & scalar multiplication** for

$$\vec{a} = [a_1, a_2, a_3], \vec{b} = [b_1, b_2, b_3]$$

$$\vec{a} + \vec{b} = [a_1 + b_1, a_2 + b_2, a_3 + b_3]$$

$$\vec{a} - \vec{b} = [a_1 - b_1, a_2 - b_2, a_3 - b_3]$$



* **Rules** :

$$① \vec{a} + \vec{b} = \vec{b} + \vec{a}$$

$$② [\vec{u} + \vec{v}] + \vec{w} = \vec{u} + [\vec{v} + \vec{w}]$$

$$③ \vec{a} + 0 = \vec{a}$$

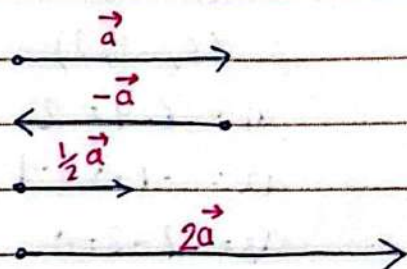
$$④ \vec{a} + (-\vec{a}) = \text{zero}$$

* **Scalar multiplication**:

$$\vec{a} = [a_1, a_2, a_3]$$

$$C\vec{a} = [Ca_1, Ca_2, Ca_3]$$

C: Scalar



Rules :

① $c[\vec{a} + \vec{b}] = c\vec{a} + c\vec{b}$

④ $1\vec{a} = \vec{a}$

② $(c+k)\vec{a} = c\vec{a} + k\vec{a}$

⑤ $0\vec{a} = \vec{0}$

③ $c[k\vec{a}] = (ck)\vec{a}$

⑥ $-1\vec{a} = -\vec{a}$

Ex: let $\vec{a} = [4, 0, 1]$ & $\vec{b} = [2, -5, \frac{1}{3}]$

① $-\vec{a} = [-4, 0, -1]$

② $7\vec{a} = [28, 0, 7]$

③ $\vec{a} + \vec{b} = [6, -5, \frac{4}{3}]$

④ $2[\vec{a} - \vec{b}] = [4, 10, \frac{4}{3}]$

Unit vector : $\hat{i}, \hat{j}, \hat{k}$ for any vector $\vec{a} = [a_1, a_2, a_3]$ we can write it as

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\hat{i} = [1, 0, 0] \quad , \quad \hat{j} = [0, 1, 0] \quad , \quad \hat{k} = [0, 0, 1]$$

Ex:

$$\vec{a} = [0, 0, -3] \Rightarrow \vec{a} = -3\hat{k}$$