27/10

No.

* upper triangular Matrix :

$$A = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix} , \quad B = \begin{bmatrix} 1 & 4 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 6 \end{bmatrix}$$

* lower triangular Matrix :

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 8 & -1 & 0 \\ 7 & 6 & 8 \end{bmatrix}$$

* diagonal Matrix D :

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

* Scalar Matrix S :

$$S = \begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix}$$

$\therefore$ c = Scalar (ثابت).

* Unite Matrix OR Identity Matrix :

$$I_{2\times2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} ,$$

$$I_{3\times3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[AI = IA = A]$$

7.3) Linear system of eq. Gauss-elimination.

let linear system with - m - eq - n - unknowns

$$a_{11}x_1 + \_\_\_ + a_{1n}x_n = b_1$$
$$a_{21}x_1 + \_\_\_ a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{1m}x_1 + \_\_\_ + a_{mn}x_n = b_m .$$

No.

1. linear system $\Rightarrow x_1 x_2 \_ x_3$ of Order 1
2. for linear system with all (b_j) are zero $\Rightarrow$ [homogeneouse L.S]
3. for linear system with at least one (b_j) are not zero $\Rightarrow$ [Non - Homogeneouse L.S]

* For linear system $Ax = b$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$\tilde{A} = [A \vdots b]$

* for linear system $Ax = b$ has three possible Causes:

1. prencisely on solution if the line Intersect.
2. Infinitly Many soln. if the lines coincide
3. No- soln if the lines are parrallel

* Gauss-elimination & back soln:

$2x_1 + 5x_2 = 2$ $\Rightarrow$ $Ax = b$

$+13x_2 = -26$

$$\begin{bmatrix} 2 & 5 \\ 0 & 13 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -26 \end{bmatrix}$$

$x_2 = -2$

نعوضها بالمعادلة الأولى

$x_1 = 6$

EX: $2x_1 + 5x_2 = 2$

$-4x_1 + 3x_2 = -30$

find x_1 & x_2 ?

1. الطريقة الأولى (حذف وتعويض)

$[2x_1 + 5x_2 = 2] \times 2$

$-4x_1 + 3x_2 = -30$

$13x_2 = -26$

$x_2 = -2$

$x_1 = 6$

smile

No.

2 الطريقة الثانية

Using G-E

① $AX+b$

$$\begin{bmatrix} 2 & 5 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -30 \end{bmatrix}$$

② $\tilde{A} = [A \vdots b]$

$$\begin{bmatrix} 2 & 5 & \vdots & 2 \\ -4 & 3 & \vdots & -30 \end{bmatrix} \Rightarrow \text{pivot}$$

$$= \begin{bmatrix} 2 & 5 & \vdots & 2 \\ 0 & 13 & \vdots & -26 \end{bmatrix} \to R_2 + 2$$

$2x_1 + 5x_2 = 2$

$+13x_2 = -26$

$\Rightarrow x_1 = 6,\ x_2 = -2$

EX: With one solution $\Rightarrow$ find x_1, x_2, x_3

$x_1 - x_2 + x_3 = 0$

$-x_1 + x_2 - x_3 = 0$

$+10x_2 + 25x_3 = 90$

$20x_1 + 10x_2 = 80$

① $AX = b$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 0 & 10 & 25 \\ 20 & 10 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 90 \\ 80 \end{bmatrix}$$

② $\tilde{A} = [A \vdots b]$

$$\text{pivot} \Leftarrow \begin{bmatrix} 1 & -1 & 1 & \vdots & 0 \\ -1 & 1 & -1 & \vdots & 0 \\ 0 & 10 & 25 & \vdots & 90 \\ 20 & 10 & 0 & \vdots & 80 \end{bmatrix} \begin{matrix} \\ \Rightarrow R_2 + R_1 \\ \text{كما هي} \\ \Rightarrow R_4 - 20R_1 \end{matrix}$$

$$\text{pivot} \rightarrow \begin{bmatrix} 1 & -1 & 1 & \vdots & 0 \\ 0 & 0 & 0 & \vdots & 0 \\ 0 & 10 & 25 & \vdots & 90 \\ 0 & 30 & -20 & \vdots & 80 \end{bmatrix} \begin{matrix} \\ \\ \Rightarrow R_3 - 3R_2 \\ \\ \end{matrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 & \vdots & 0 \\ 0 & 10 & 25 & \vdots & 90 \\ 0 & 0 & -95 & \vdots & -190 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$$

$x_1 - x_2 + x_3 = 0$

$10x_2 + 25x_3 = 90$

$[x_3 = 2,\ x_2 = 4,\ x_1 = 2] \Leftarrow -95x_3 = -190$

smile for life

No.

* elementary Raw operation:

① Inter change of Two Raws

② Addition of constant multiple to on Row to another

③ Multiplication of one Row by non-zero constant

has no effect on the soln.

Note ⇒

① over determined linear system if it has more eq. than unknown

② determined L.S if the number of eq. equal number of unknowns

③ underdetermined L.S if the system has fewer eq than unknowns

④ consistent L.S if it has at least one soln.

⑤ In-consistent L.S if it has no-soln

① $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

② $\begin{bmatrix} 5a_{11} & 5a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5b_1 \\ b_{12} \end{bmatrix}$

③ $\begin{bmatrix} a_{21} & a_{22} \\ a_{11} & a_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_2 \\ b_1 \end{bmatrix}$

1+2+3 ⇒ Has soln.

No.

Ex: with infinitly Many soln:

$3x_1 + 2x_2 + 2x_3 - 5x_4 = 8$

$0.6x_1 + 1.5x_2 + 1.5x_3 - 5.4x_4 = 2.7$

$1.2x_1 - 0.3x_2 - 0.3x_3 + 2.4x_4 = 2.1$

Find x_1, x_2, x_3, x_4 ?

① $Ax = b$

$$\begin{bmatrix} 3 & 2 & 2 & -5 \\ 0.6 & 1.5 & 1.5 & -5.4 \\ 1.2 & -0.3 & -0.3 & 2.4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 8 \\ 2.7 \\ 2.1 \end{bmatrix}$$

② $\tilde{A} = [A \vdots b]$

$$\left[\begin{array}{cccc|c} 3 & 2 & 2 & -5 & 8 \\ 0.6 & 1.5 & 1.5 & -5.4 & 2.7 \\ 1.2 & -0.3 & -0.3 & 2.4 & 2.1 \end{array}\right] \to \text{pivot}$$

$$= \left[\begin{array}{cccc|c} 3 & 2 & 2 & -5 & 8 \\ 0 & 1.1 & 1.1 & -4.4 & 1.1 \\ 0 & -1.1 & -1.1 & 4.4 & -1.1 \end{array}\right] \begin{array}{l} \Rightarrow \text{pivot} \to R_2 - 0.2R_1 \\ \to R_3 - 0.4R_1 \end{array}$$

$$= \left[\begin{array}{cccc|c} 3 & 2 & 2 & -5 & 8 \\ 0 & 1.1 & 1.1 & -4.4 & 1.1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \to R_3 + R_2$$

$3x_1 + 2x_2 + 2x_3 - 5x_4 = 8$

$1.1x_2 + 1.1x_3 - 4.4x_4 = 1.1$

$x_2 + x_3 = 1 + 4x_4$

$3x_1 + 2[1 + 4x_4] - 5x_4 = 8$

$$\left(\begin{array}{ll} x_1 = 2 - x_4 & x_3 = \text{arbitary} \\ x_2 = 1 + 4x_4 - x_3 & x_4 = \text{arbitary} \end{array} \right)$$

No.

ex: with no - soln

$3x_1 + 2x_2 + x_3 = 3$

$2x_1 + x_2 + x_3 = 0$

$6x_1 + 2x_2 + 4x_3 = 6$

Find x_1, x_2, x_3

① $Ax = b$

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}$$

② $\tilde{A} = [A \,\vdots\, b]$

pivot $\leftarrow$

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 3 \\ 2 & 1 & 1 & 0 \\ 6 & 2 & 4 & 6 \end{array}\right] = \text{pivot} \rightarrow \left[\begin{array}{ccc|c} 3 & 2 & 1 & 3 \\ 0 & -\frac{1}{3} & \frac{1}{3} & -2 \\ 0 & -2 & 2 & 0 \end{array}\right]$$

$R_3 - 2R_1$ $\quad$ $R_2 - \frac{2}{3} R_1$

$$= \left[\begin{array}{ccc|c} 3 & 2 & 1 & 3 \\ 0 & -\frac{1}{3} & \frac{1}{3} & -2 \\ 0 & 0 & 0 & 12 \end{array}\right] \rightarrow R_3 - 6R_2$$

$3x_1 + 2x_2 + x_3 = 3$

$-\frac{1}{3} x_2 + \frac{1}{3} x_3 = -2$

$0 = 12 \Rightarrow$ false - statement, no - soln

* Row echelon form & information from it :

let L.S $Ax = b$ argmented Matrix $\tilde{A} = [A \,\vdots\, b]$ of the Gauss - elimination.

$$[R \,\vdots\, F] = \left[\begin{array}{ccccc|c} r_{11} & r_{12} & \cdots & r_{1n} & & f_1 \\ & r_{22} & \cdots & r_{2n} & & f_2 \\ & & \vdots & & & \vdots \\ & r_{rr} & \cdots & r_{rn} & & f_r \\ & & & & & \vdots \\ & & & & & f_m \end{array}\right]$$

No.

① $r_{11} \neq 0$

② all element in the tringle & rectangle is zero

ⓐ no-soln if $r < m$ & at least one number $f_r, \dots f_m$ is not zero

$$[R \mid F] = \left[\begin{array}{ccc|c} 3 & 2 & 1 & 3 \\ 0 & -1/3 & 1/3 & -2 \\ 0 & 0 & 0 & 12 \end{array}\right]$$

$r_{11} \neq 0 = 3$

$r_{23} = \frac{1}{3} = 0.$ $r_{rn} < \begin{matrix} r = 2 \\ n = 3 \end{matrix}$

$f_m = f_3 = 12$

$m = 3$

No - Soln.

ⓑ One-soln: if $r = n$ & all numbers $f_r \dots f_m$ are zero

$$[R \mid F] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 0 & -95 & -190 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$r_{11} = 1 \neq 0$

$r_{rn} = r_{33} = -95$, $r = 3$, $n = 3$

$f_m = f_4 = 0$

$(m = 4)$

ⓒ infinitly Many soln if $r < m$ & all numbers $f_{r+1} \dots f_m$ are zero

$$[R \mid f] = \left[\begin{array}{cccc|c} 3 & 2 & 2 & -5 & 8 \\ 0 & 1.1 & 1.1 & -4.4 & 1.1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$r_{11} = 3 \neq 0$

$r_{nn} = r_{24} = -4.4 < \begin{matrix} r = 2 \\ n = 4 \end{matrix}$

$f_m = f_3 = 0 \Rightarrow m = 3$

No.

7.4) linear Indep. of vector & Rank of Matrix

let : $a_1, a_2 \dots a_m$ are m-vectors

$c_1, c_2 \dots c_m$ are m-scalar

then $c_1 a_1 + c_2 a_2 + \dots + c_m a_m = 0$ eq (1)

(1) If eq (1) holds For all (c_j) are zero then a's are linearly Indep.

(2) If eq (1) holds for at least one (c_j) isn't zero a's are linearly dep.

Ex: let $a_1 = [3 \ 0 \ 2 \ 2]$

$a_2 = [-6 \ 42 \ 24 \ 54]$

$a_3 = [21 \ -21 \ 0 \ -15]$

are a_1, a_2, a_3 linearly dep ??

$3c_1 - 6c_2 + 21c_3 = 0$

$0c_1 + 42c_2 - 21c_3 = 0$

$2c_1 + 24c_2 + 0c_3 = 0$

$2c_1 + 54c_2 - 15c_3 = -1$ $\Rightarrow$ $c_1 = 6, c_2 = \frac{1}{2}$, $c_3 = -1$

$\therefore$ linearly dep.

* Rank of A : is the Max number of linearly Indep. Row vector of A

Ex: let $A = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$ find Rank A?

pivot

$$= \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 42 & 28 & 58 \\ 0 & -21 & -14 & -29 \end{bmatrix} \begin{matrix} \\ \to R_2 + 2R_1 \\ \to R_3 - 7R_1 \end{matrix} = \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 42 & 28 & 58 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(Rank = 2)

عدد الصفوف اللي ما صارت صفر

$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 5 \\ 0 & 0 & 7 \end{bmatrix}$

Rank = 1 Rank = 3