Theorem : Row equivelent Matrix:

① Matrix A_1 is a row equivelent to Matrix A_2 if A_1 can be obtained from A_2 by elementary Row operation.

② Row-equivelent Matrix has same Rank.

Theorem :

Consider p-vector that each have n-component then, these vectors are linearly Indep. if Matrix formed with these vectors as Row vector has Rank P, however these vectors are linearly dep. if Matrix formed has Rank less than P

⊛ Rank (A) = Rank (A^T).

Theorem:

consider p-vector each have n-component if $n<P$ then these vectors are linearly dep.

Ex: let P_1 $[1,2]$ ⇒ are p's linearly dep ??

P_2 $[3,4]$

P_3 $[5,6]$

Soln : $p=3$, $n=2$ ⇒ $n<p$ ∴ linearly dep.

Theorem: Soln of linear system existance & uniqueness for $Ax=b$ with n-unknown , m-eq.

$$\tilde{A} = [A \vdots b]$$

① **Existence :** linear system has soln iff [Rank(A) = Rank$(\tilde{A})$]

② **uniqueness :** linear system has one soln iff [Rank(A) = Rank$(\tilde{A}) = n$]

③ **infinitly Many soln :** iff [Rank$(A) < n$, Rank$(\tilde{A}) < n$]

④ **No-Soln if :** [Rank $(A) \neq$ Rank $(\tilde{A})$]

No.

7.6) Second & third order det. let:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad D = \det[A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$= a_{11}a_{22} - a_{12}a_{21}$$

Ex: let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ find D ??

$D = (1)(3) - (2)(1) = 1$

* Cramer Rules: for second order linear system

$Ax = b$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$x_1 = \frac{D_1}{D}, \quad x_2 = \frac{D_2}{D}$$

$$D_1 = \begin{bmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{bmatrix} = b_1 a_{22} - a_{12} b_2$$

$$D_2 = \begin{bmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{bmatrix} = a_{11} b_2 - b_1 a_{21}$$

Ex: let $4x_1 + 3x_2 = 12$, find x_1, x_2 ??

$2x_1 + 5x_2 = -8$

① $Ax = b$

$$\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \end{bmatrix}$$

$$x_1 = \frac{D_1}{D} = \frac{\begin{bmatrix} 12 & 3 \\ -8 & 5 \end{bmatrix}}{\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}}$$

$$x_2 = \frac{D_2}{D} = \frac{\begin{bmatrix} 4 & 12 \\ 2 & -8 \end{bmatrix}}{\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}}$$

$$x_1 = \frac{12(5) - (3)(-8)}{4(5) - (3)(2)} = \frac{84}{14} = 6$$

$$x_2 = \frac{4(-8) - (12)(2)}{4(5) - (3)(2)} = \frac{-56}{14} = -4$$

No.

* Third order det:

For linear system $Ax = b$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$x_1 = \frac{D_1}{D}, \quad x_2 = \frac{D_2}{D}, \quad x_3 = \frac{D_3}{D}$$

$$D = \det[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= +a_{11} \begin{bmatrix} a_{22} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} - a_{12} \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13} \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$= a_{11}[a_{22}a_{33} - a_{23}a_{31}] - a_{12}[a_{21}a_{33} - a_{23}a_{31}] + a_{13}[a_{21}a_{32} - a_{22}a_{31}]$$

Ex: let $A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix}$, find $\text{Det}[A]$??

$$= 1 \begin{bmatrix} 6 & 4 \\ 0 & 2 \end{bmatrix} - 3 \begin{bmatrix} 2 & 4 \\ -1 & 2 \end{bmatrix} + 0 \begin{bmatrix} 2 & 6 \\ -1 & 0 \end{bmatrix} = -12$$

$$D_1 = \begin{bmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{bmatrix}, \quad D_2 = \begin{bmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{bmatrix}$$

$$D_3 = \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{bmatrix}$$

No.

let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

* The Minore Matrix of A is M:

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

$$M_{11} = \begin{bmatrix} M_{22} & M_{23} \\ M_{32} & M_{33} \end{bmatrix} = M_{22}M_{33} - M_{23}M_{32}$$

$$M_{12} = \begin{bmatrix} M_{21} & M_{23} \\ M_{31} & M_{33} \end{bmatrix}$$

$\vdots$

$$M_{32} = \begin{bmatrix} M_{11} & M_{13} \\ M_{21} & M_{33} \end{bmatrix} , \quad M_{33} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

* The Cofactor Matrix of A is C:-

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \Longrightarrow \left[C_{jk} = (-1)^{j+k} M_{jk} \right]$$

$C_{11} = (-1)^2 M_{11} = M_{11}$

$C_{12} = (-1)^3 M_{12} = -M_{12}$

$$C = \begin{bmatrix} M_{11} & -M_{12} & M_{13} \\ -M_{21} & M_{22} & -M_{23} \\ M_{31} & -M_{32} & M_{33} \end{bmatrix}$$

No.

Theorem: (a) Interchange of two Rows Multiply the value of determined by [-1]

let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $|A| = 1$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ $|B| = -1$

(b) addition of multiple of Row to another Row doesn't alter the value of determinte

let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ $|A| = 1$, $B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $|B| = 1$

(c) multiplication of Row by non-zero constant [C] multiply the value of det by [C]

(d) $\det[CA] = C^n \det[A]$, n = number of Row in A

EX: let $A = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$ find $|A|$, $|3A|$??

$|A| = 4 - 2 = 2$, $|3A| = \begin{bmatrix} 3 & 6 \\ 3 & 12 \end{bmatrix} = 3(12) - (3)(6) = 18$

$|3A| = 3^2 \det[A] = 9(2) = 18$

smi)e for life