Subject Wednesday 29/10/2025 Date No.

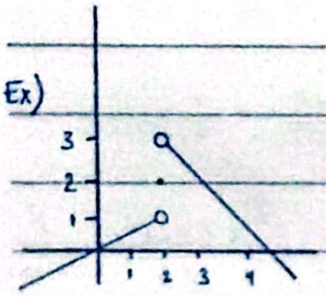
* limits :

* Defn : $\lim_{x \to a} f(x) = l$

- if $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = l$ then $\lim_{x \to a} f(x) = l$ "exist"

- if $\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x)$ then $\lim_{x \to a} f(x)$ does not "exist" [d.n.e]

Ex)



is $\lim_{x \to 2} f(x)$ exist ?

$\lim_{x \to 2^-} f(x) = 1$

$\lim_{x \to 2^+} f(x) = 3$

$\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$

$\therefore \lim_{x \to 2} f(x)$ does not exist .

• Thrm : if $\lim_{x \to a} f(x) = M$, $\lim_{x \to a} g(x) = L$ then

① $\lim_{x \to a} (f \pm g)(x) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = M \pm L$

② $\lim_{x \to a} (f \cdot g)(x) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = M \cdot L$

③ $\lim_{x \to a} \left(\frac{f}{g}\right)(x) = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{M}{L}$, $L \neq 0$

④ $\lim_{x \to 0} (f(x))^n = \left(\lim_{x \to a} f(x)\right)^n = (L)^n$

⑤ $\lim_{x \to a} c = c$ "c is constant

Ex) if $\lim_{x \to -1} f(x) = 3$, $\lim_{x \to -1} g(x) = 2$ find

① $\lim_{x \to -1} f(x) + g(x) = \lim_{x \to -1} f(x) + \lim_{x \to -1} g(x)$

$= 3 + 2 = 5$

② $\lim_{x \to -1} f(x) - \frac{1}{g(x)} = 3 - \frac{1}{2} = 1.5$

Ex) find $\lim_{x \to 1} (2x^5 - 4x + 5) = 2(1) - 4(1) + 5$

$= 3$

* note that :

$\boxed{1}$ $\lim \frac{f(x)}{g(x)} = \frac{\text{number}}{\text{number}}$

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Ex) $\lim_{x\to 4} \frac{3x+5}{\sqrt{x}+3} = \frac{17}{5}$

$\boxed{2}$ $\lim \frac{f(x)}{g(x)} = \frac{0}{\text{number}} = 0$

Ex) $\lim_{x\to \frac{1}{2}} \frac{2x-1}{2x} = \frac{1-1}{1} = \frac{0}{1} = 0$

$\boxed{3}$ $\lim \frac{f(x)}{g(x)} = \frac{0}{0} \Big\}$. نحلل

Ex) $\lim_{x\to 3} \frac{x^2-9}{x-3} = \frac{3^2-9}{3-3} = \frac{0}{0} = 0$

* الاختصار فقط في lim *

$\lim_{x\to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = \lim_{x\to 3}(x+3) = 3+3 = 6$

Ex) $\lim_{x\to -4} \frac{2x+8}{x^2+x-12} = \frac{-8+8}{16-4-12} = \frac{0}{0} = 0$ اذا الجواب صفر نحلل

$\lim_{x\to -4} \frac{2\cancel{(x+4)}}{(x-3)\cancel{(x+4)}} = \lim_{x\to -4} \frac{2}{(x-3)} = \frac{2}{-7}$

Ex) $\lim_{x\to 1} \frac{x^{\frac{3}{2}}-x}{x^{\frac{1}{2}}-1} = \frac{0}{0}$

$\lim_{x\to 1} \frac{x\cancel{(x^{\frac{1}{2}}-1)}}{\cancel{(x^{\frac{1}{2}}-1)}} = 1$

Ex) $\lim_{x\to 2} \frac{\frac{5}{3}-\frac{5}{x+1}}{x-2} = \frac{\frac{5}{3}-\frac{5}{3}}{0} = \frac{0}{0} = 0$

$= \lim_{x\to 2} \frac{\frac{5x+5}{3x+3}-\frac{15}{3x+3}}{x-2} = \lim_{x\to 2} \frac{\frac{5x-10}{3x+3}}{x-2}$

$= \lim_{x\to 2} \frac{5x-10}{(3x+3)\cancel{(x-2)}} = \lim_{x\to 2} \frac{5\cancel{(x-2)}}{(3x+3)\cancel{(x-2)}}$

$= \lim_{x\to 2} \frac{5}{(3x+3)} = \frac{5}{9}$

Ex) $\lim_{x\to 0} \frac{(x-2)^2-4}{x} = \frac{0}{0}$

$\lim_{x\to 0} \frac{x^2-4x+\cancel{4}-\cancel{4}}{x} = \frac{\cancel{x}(x-4)}{\cancel{x}} = \lim_{x\to 0} x-4$

$= -4$

Ex) $\lim_{x\to 4} \frac{|x-4|}{x-4} = \frac{0}{0}$

$-(x-4)$ $\quad - \quad 4 \quad + \quad$ $(x-4)$

$$\lim \begin{cases} \frac{x-4}{x-4} = 1, & x \geq 4 \\ \frac{-(x-4)}{x-4} = -1, & x < 4 \end{cases}$$

$= \lim_{x\to 4^+} f(x) = 1$

$\lim_{x\to 4^-} f(x) = -1$

$\therefore \lim_{x\to 4^-} f(x) \neq \lim_{x\to 4^+} f(x)$

$\therefore \lim_{x\to 4} f(x) = \text{d.n.e}$

لم ب بالمرافق

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Ex) $\lim_{x\to 3^+} \frac{|x-3|}{2x-6} = \frac{0}{0}$

$= \lim_{x\to 3^+} \frac{x-3}{2x-6} = \frac{\cancel{x-3}}{2(\cancel{x-3})} = \frac{1}{2}$

Ex) $\lim_{x\to 1} \frac{\sqrt[4]{x+15} - 2}{x-1} = \frac{0}{0}$

* لما يكون دليل الجذر

$\sqrt[4]{x+15} = y \Rightarrow x+15 = y^4$ اكثر من 2 أو 3 بفضل نعوض.

$\therefore \boxed{x = y^4 - 15}$

If $x \to 1 \Rightarrow y\ \sqrt[4]{1+15} = \sqrt[4]{16} = 2$

$\therefore \lim_{y\to 2} \frac{y-2}{y^4-15-1} = \lim_{y\to 2} \frac{y-2}{y^4-16} = \lim_{y\to 2} \frac{(y-2)}{(y^2+4)(y^2-4)}$

$\lim_{y\to 2} \frac{\cancel{(y-2)}}{(y^2+4)\cancel{(y-2)}(y+2)} = \frac{1}{32}$

* note that :

① $a^2 - b^2 = (a-b)(a+b)$

② $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

③ $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

لما يكون جذر تربيعي

Ex) $\lim_{x\to 1} \frac{2-\sqrt{x+3}}{x^2+2x-3} = \frac{0}{0} = 0$

نضرب بالمرافق

$\lim_{x\to 1} \frac{2-\sqrt{x+3}}{x^2+2x-3} \times \frac{(2+\sqrt{x+3})}{(2+\sqrt{x+3})}$

$\lim_{x\to 1} \frac{4-(x+3)}{x^2+2x-3\ (4)}$ نعوضها ب 1

$= \frac{1}{4} \lim_{x\to 1} \frac{4-x-3}{(x+3)(x-1)}$

$= \frac{1}{4} \lim_{x\to 1} \frac{\cancel{(1-x)}\ -1}{(x+3)\cancel{(x-1)}} = \frac{1}{4} \lim_{x\to 1} \frac{-1}{x+3}$

$= \frac{1}{4} \cdot \frac{-1}{4} = \frac{-1}{16}$

Ex) $\lim_{x\to 3} \frac{3-\sqrt[3]{2x+21}}{x^2-9} = \frac{0}{0}$

$\lim_{x\to 3} \frac{3-\sqrt[3]{2x+21}}{x^2-9} \times (9 + 3\sqrt[3]{2x+21} + \sqrt[3]{2x+21}$
$)$

$\lim_{x\to 3} \frac{9-(2x+21)}{x^2-9\ \ (6)}$

$\frac{1}{6} \lim_{x\to 3} \frac{2(x- \quad 2x-12}{(x+3)(x-3)}$

الصفحة البعد

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Ex) $\lim_{x\to3} \dfrac{3-\sqrt[3]{2x+21}}{x^2-9} = \dfrac{0}{0}$

$$\lim_{x\to3} \frac{3-\sqrt[3]{2x+21}}{x^2-9}\cdot\frac{\left(9+3\sqrt[3]{2x+21}+\left(\sqrt[3]{2x+21}\right)^2\right)}{\left(9+3\sqrt[3]{2x+21}+\left(\sqrt[3]{2x+21}\right)^2\right)}$$

نعوض 3

$$= \lim_{x\to3} \frac{(27-(2x+21))}{(x^2-9)\quad(27)}$$

$$= \frac{1}{27}\lim_{x\to3}\frac{\overset{-2(\cancel{3-x})^{-1}}{(6-2x)}}{\cancel{(x-3)}(x+3)} = \frac{1}{27}\lim_{x\to3}\frac{-2}{(x+3)}$$

$$= \frac{1}{27}\cdot\frac{-\cancel{2}}{\cancel{6}\,3} = \frac{-1}{81}$$

Ex) $\lim_{x\to6} \sqrt{x-2} = \sqrt{6-2} = \sqrt{4} = 2$ ✓

Ex) $\lim_{x\to1} \sqrt{x-3} = \sqrt{1-3} = \text{d.n.e}$ ← سالب داخل الجذر التربيعي

Ex) $\lim_{x\to2} \sqrt{2-x} \neq 0$

$+ \quad | \quad -$
$\quad\;\; 2$

$$\lim_{x\to2^-} \sqrt{2-x} = 0$$

$$\lim_{x\to2^+} \sqrt{2-x} = \text{d.n.e}$$

$$\therefore \lim_{x\to2^-} \neq \lim_{x\to2^+} = \lim_{x\to2^+} \text{d.n.e}$$

*لما نعوض داخل الجذر التربيعي وبطلع

موجب ← مش مشكلة

سالب : d.n.e

صفر : ناخذ يمين وشمال

Ex) $f(x) = \begin{cases} \sqrt{x-4}, & x>4 \\ 8-2x, & x\le4 \end{cases}$

① find $\lim_{x\to0} f(x) = \lim_{x\to0} 8-2x = 8$

② find $\lim_{x\to4} f(x)$ (٤) نقطة تشعب
باخذ يمين وشمال

$$\lim_{x\to4^+} \sqrt{x-4} = 0$$

$- \quad | \quad +$
$\quad\;\; 4$

عكس

$$\lim_{x\to4^-} 8-2x = 0$$

$$\therefore \lim_{x\to4} f(x) = 0 \overset{\text{لأنه}}{\Rightarrow} \lim_{x\to4^+} f(x) = \lim_{x\to4^-} f(x)$$

Ex) $f(x) = \begin{cases} kx^2, & x\le2 \\ 2x+k, & x>2 \end{cases}$

find k such that $\lim_{x\to2} f(x)$ exist

$$\lim_{x\to2^+} f(x) = \lim_{x\to2^+} f(x)$$

$$\lim_{x\to2^+} 2x+k = \lim_{x\to2^-} kx^2$$

$$4+k = 4k$$

$$4 = 4k-k \Rightarrow 4 = 3k$$

$$\boxed{k=\frac{4}{3}}$$

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4. $\lim_{x \to a} \frac{f(x)}{g(x)}$, $f(a) \neq 0$, $g(a) = 0$

*دائماً لما يكون تعويض $\frac{\text{عدد}}{0}$

ثلاث اجوبة .
- ∞
- $-\infty$
- d.n.e

Ex) $\lim_{x \to 2^+} \frac{x+1}{x^2-4} = \frac{3}{0}$

① نأخذ اصفار البسط واصفار المقام وندرس الإشارة .

نعوض رقم أكبر من 2 في الاقتران لحتى نطلع الاشارة

هاي الجهة +

-2 صفر المقام , -1 صفر البسط , 2 صفر المقام

$= +\infty$

لا يوجد اصفار للبسط

Ex) $\lim_{x \to 0^-} \frac{3}{x^2 - x} = \frac{3}{0}$

$x^2 - x = 0$ اصفار المقام

$x(x-1) = 0$

$\boxed{x=0}$ / $\boxed{x=1}$

هاي الجهة +

0 , 1

$= +\infty$

لا يوجد اصفار للبسط [لا يحلل] .

Ex) $\lim_{x \to 0} \frac{x^4 + 5}{x^2} = \frac{5}{0}$

$+$ $+$

0 صفر مقام

$\lim_{x \to 0^+} \frac{x^4+5}{x^2} = +\infty$

$\lim_{x \to 0^-} \frac{x^4+5}{x^2} = +\infty$

$\therefore \lim_{x \to 0} \frac{x^4+5}{x^2} = +\infty$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

Ex) $\lim_{x \to 1} \frac{x}{x^2-1} = \frac{1}{0}$

$-$ $+$

-1 صفر مقام , 0 صفر البسط , $+1$ صفر مقام

$\lim_{x \to 1^+} \frac{x}{x^2-1} = +\infty$

$\lim_{x \to 1^-} \frac{x}{x^2-1} = -\infty$

$\lim_{x \to 1^+} \frac{x}{x^2-1} \neq \lim_{x \to 1^-} \frac{x}{x^2-1}$

$\therefore \lim_{x \to 1} \frac{x}{x^2-1} = d.n.e$

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Ex) $\lim_{x \to 2} \frac{x+3}{x-2} = \frac{5}{0}$

$-$ $+$

-3 (صفر بسط) 2 (صفر مقام)

$\lim_{x \to 2^+} \frac{x+3}{x-2} = +\infty$

$\lim_{x \to 2^-} \frac{x+3}{x-2} = -\infty$

$\lim_{x \to 2^+} \frac{x+3}{x-2} \neq \lim_{x \to 2^-} \frac{x+3}{x-2}$

$\lim_{x \to 2} \frac{x+3}{x-2} = d.n.e$

Ex) $\lim_{x \to -3} \frac{1}{(x+3)^2} = \frac{1}{0}$

$+$ $+$

-3

$\lim_{x \to -3^+} \frac{1}{(x+3)^2} = +\infty$

$\lim_{x \to -3^-} \frac{1}{(x+3)^2} = +\infty$

$\therefore \lim_{x \to -3} \frac{1}{(x+3)^2} = +\infty$

Ex) $\lim_{x \to 6} \frac{-1}{(x-6)^2} = \frac{-1}{0}$

$-$ $-$

6

$\lim_{x \to 6^+} \frac{-1}{(x-6)^2}$

$\lim_{x \to 6^-} \frac{-1}{(x-6)^2}$

$\therefore \lim_{x \to 6} \frac{-1}{(x-6)^2} = -\infty$

Ex) $\lim_{x \to 4} \frac{2-x}{x^2-2x-8} = \frac{-2}{0}$

$(x-4)(x+2)$

$+$ $-$

-2 2 4

$\lim_{x \to 4^+} \frac{2-x}{x^2-2x-8} = -\infty$

$\lim_{x \to 4^-} \frac{2-x}{x^2-2x-8} = +\infty$

$\therefore \lim_{x \to 4} \frac{2-x}{x^2-2x-8} = d.n.e$

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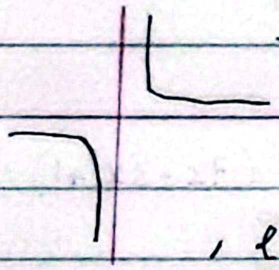
Limits of infinity

* $\lim_{x \to \infty} f(x)$ means that the behaviour $f(x)$ when x becomes very large +ve number

1



$\frac{1}{x}$ $0 = \frac{\text{عدد}}{\infty / -\infty}$ lim*

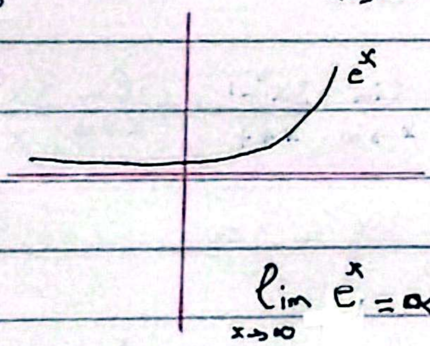
$\lim_{x \to \infty} \frac{1}{x} = 0$

$\frac{\infty}{\text{عدد موجب}} = \infty$

$\frac{\infty}{\text{عدد سالب}} = -\infty$

, $\lim_{x \to -\infty} = \frac{1}{x} = 0$

2



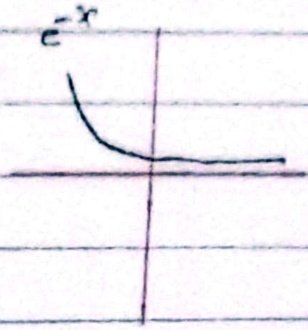
$\lim_{x \to \infty} e^x = \infty$

$\lim_{x \to -\infty} e^x = 0$

* if $a > 1$, $\lim_{x \to \infty} a^x = \infty$, $\lim_{x \to -\infty} a^x = 0$

Ex) $\lim_{x \to \infty} 2^x = \infty$

$\lim_{x \to -\infty} 2^x = 0$ $\frac{1}{2^\infty}$



$\lim_{x \to \infty} e^{-x} = 0$

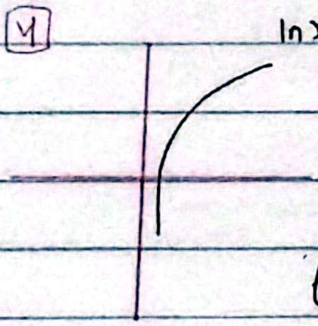
$\lim_{x \to -\infty} e^{-x} = \infty$

* if $0 < a < 1$, $\lim_{x \to \infty} a^x = 0$,

$\lim_{x \to -\infty} a^x = \infty$

Ex) $\lim_{x \to \infty} 2^{-x} = 0$, $\lim_{x \to -\infty} 2^{-x} = \infty$

4



$\lim_{x \to \infty} \ln x = \infty$

$\lim_{x \to 0^+} \ln x = -\infty$

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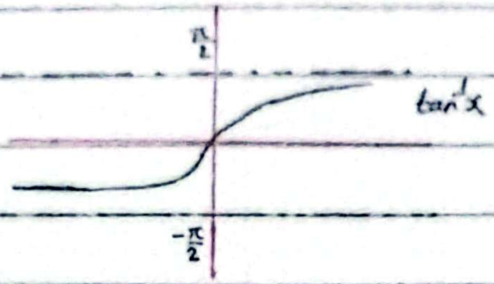
$\boxed{5}$



$$\lim_{x\to\infty} \tan^{-1}x = \frac{\pi}{2}$$

$$\lim_{x\to-\infty} \tan^{-1}x = -\frac{\pi}{2}$$

Ex) $\lim_{x\to\infty} x^3 = \infty$

Ex) $\lim_{x\to\infty} -x^2 = -\infty$

Ex) $\lim_{x\to-\infty} 3x^5 = -\infty$

* note that : $\lim_{\substack{x\to\infty \\ x\to-\infty}} (a_0 + a_1x + a_2x^2 + a_nx^n)$

$= \lim_{\substack{x\to\infty \\ x\to-\infty}} a_n x^n$

* دائماً في polynomials بناخذ $\lim_{\substack{x\to\infty \\ x\to-\infty}}$ هو أكبر اس في polynomial مع معامله.

Ex) $\lim_{x\to\infty} (3x - 4x^3 + 5) = \lim_{x\to\infty} -4x^3$

$= -\infty$

Ex) $\lim_{x\to\infty} (3x^4 + 3x^2 - 10) = \lim_{x\to\infty} 3x^4 = \infty$

Ex) $\lim_{x\to-\infty} (7x - 3x^6 + 1) = \lim_{x\to-\infty} -3x^6 = -\infty$

Ex) $\lim_{x\to\infty} \frac{3x^2-1}{x+7} = \lim_{x\to\infty} \frac{3x^{\cancel{2}}}{\cancel{x}} =$

$\lim_{x\to\infty} 3x = \infty$

Ex) $\lim_{x\to-\infty} \frac{7-2x+x^3}{3x^3-5}$

$= \lim_{x\to-\infty} \frac{\cancel{x^3}}{3\cancel{x^3}} = \lim_{x\to\infty} \frac{1}{3}$

Ex) $\lim_{x\to\infty} \frac{7-x^2}{5+3x^5} = \lim_{x\to\infty} \frac{-\cancel{x^2}}{3x^{\cancel{5}\,3}}$

$= \lim_{x\to\infty} \frac{-1}{3x^3} = 0$ $\qquad 0 = \frac{\text{عدد}}{\pm\infty}$

* $\lim_{\substack{x\to\infty \\ x\to-\infty}} \frac{\text{poly}}{\text{poly}}$

1) درجة البسط أقل من درجة المقام دائماً جوابي صفر.

2) درجة البسط تساوي درجة المقام ومعامل أكبر اس للبسط على معامل أكبر اس للمقام.

3) درجة البسط أكبر من درجة المقام جوابي ∞ او $-\infty$.

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* note that:

* $\sqrt{x^2} = |x| \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

* $\sqrt{x^4} = |x^2| = x^2$

* $\sqrt{x^6} = \left(\sqrt{x^2}\right)^3 = (|x|)^3 = \begin{cases} x^3, & x \geq 0 \\ -x^3, & x < 0 \end{cases}$

Ex) $\lim_{x\to\infty} \frac{\sqrt{5x^2-2}}{x+3}$ $\quad \lim_{x\to-\infty} \to -x \quad \lim_{x\to\infty} \to +x$

$= \lim_{x\to\infty} \frac{\sqrt{5x^2}}{x} = \lim_{x\to\infty} \frac{\sqrt{5}\,|x|}{x}$

$= \lim_{x\to\infty} \frac{\sqrt{5}\,\cancel{x}}{\cancel{x}} = \sqrt{5}$

---

* lim ندخل تحت الجذر اذا كان fn كلو تحت الجذر ,

Ex) $\lim_{x\to\infty} \sqrt{\frac{5x^2-2}{x+3}}$

$= \sqrt{\lim_{x\to\infty} \frac{5x^2-2}{x+3}} = \sqrt{\lim_{x\to\infty} \frac{5x^{\cancel{2}}}{\cancel{x}}}$

$= \sqrt{\lim_{x\to\infty} 5x} = \infty$

بعوض بالأول

Ex) $\lim_{x\to\infty} \sqrt{x^6-5} - x^3 = \infty - \infty = 0$?

$\lim_{x\to\infty} \sqrt{x^6-5} - x^3 \frac{\left(\sqrt{x^6-5} + x^3\right)}{\left(\sqrt{x^6-5} + x^3\right)}$

$= \lim_{x\to\infty} \frac{\cancel{x^6} - 5 - \cancel{x^6}}{\sqrt{x^6+5} + x^3} = \lim_{x\to\infty} \frac{-5}{\sqrt{x^6+5} + x^3}$

$= \lim \frac{5}{\sqrt{x^6}} = \frac{5}{\infty} = 0$

Ex) $\lim_{x\to\infty} \frac{e^{-x} - 3e^{2x}}{e^{-2x} + 5e^{x}}$

$= \lim_{x\to\infty} \frac{\cancel{e^{-\infty}} - 3e^{2\infty}}{\cancel{e^{-2\infty}} + 5e^{\infty}} = \lim_{x\to\infty} \frac{-3e^{\cancel{2}x}}{5\cancel{e^{x}}}$

$= \lim \frac{-3}{5} e^{x}_{(\infty)} = \infty$

Ex) $\lim_{x\to-\infty} \frac{5e^{-2x} - 3e^{x}}{e^{2x} - 4e^{-2x}}$

$= \frac{5e^{-\infty} - 3e^{\infty}}{e^{\infty} - 4e^{-\infty}} = \lim_{x\to-\infty} \frac{+3e^{x}}{+4e^{2x}}$

$= \frac{3}{4\,e^{x}} =$

ل الصفحة البعد

N O T E B O O K

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Ex) $\lim_{x \to -\infty} \dfrac{5e^{-2x} - 3e^{x}}{e^{2x} - 4e^{-2x}}$

$= \dfrac{5e^{\infty} - \cancel{3e^{-\infty}}}{\cancel{e^{-\infty}} - 4e^{\infty}} = \lim_{x \to -\infty} \dfrac{5e^{-2x}}{-4e^{-2x}}$

$= \dfrac{-5}{4}$

مابصير كذا لازم احلل

Ex) $\lim_{x \to \infty} \dfrac{\ln 2x}{\ln 3x} = \dfrac{\infty}{\infty} =$

$= \lim_{x \to \infty} \dfrac{\ln 2 + \ln x}{\ln 3 + \ln x}$

$= \lim_{x \to \infty} \dfrac{\cancel{\ln x}\left(\frac{\ln 2}{\ln x} + 1\right)}{\cancel{\ln x}\left(\frac{\ln 3}{\ln x} + 1\right)}$

$= \lim_{x \to \infty} \dfrac{0 + 1}{0 + 1} = 1$

Ex) $\lim_{x \to \infty} \ln x = \infty$

حفظ

$\boxed{1}$ $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$

$\boxed{2}$ $\lim_{x \to 0} \dfrac{\sin ax}{bx} = \dfrac{a}{b}$

$\boxed{3}$ $\lim_{x \to 0} \dfrac{bx}{\sin ax} = \dfrac{b}{a}$

$\boxed{4}$ $\lim_{x \to 0} \dfrac{\tan ax}{bx} = \dfrac{a}{b}$

$\boxed{5}$ $\lim_{x \to 0} \dfrac{\sin ax}{\tan bx} = \dfrac{a}{b}$

Ex) $\lim_{x \to 0} \dfrac{\tan 3x}{\sin 4x} = \dfrac{3}{4}$