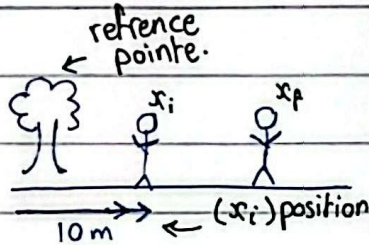


* chapter (2) *

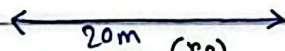
* Motion in 1 Dimension [1D]

Kinematics

① position :-



② Displacement $[\pm]$: $(\Delta x = x_f - x_i)$



③ Distance $[\pm]$ (d = total distance)

④ velocity $[\pm]$ $\rightarrow \bar{v} = \frac{\Delta x}{\Delta t}$ average

$$v = \frac{x_f - x_i}{t_f - t_i}$$

[Instantaneous] $v = \frac{dx}{dt}$

⑤ speed $[\pm]$

$$S = \frac{\text{total distance}}{\text{total time}}$$

Kinematic دراسة حركة الجسم

reference point نقطة مرجعية

position موقع

Displacement الإزاحة

Distance المسافة

velocity السرعة

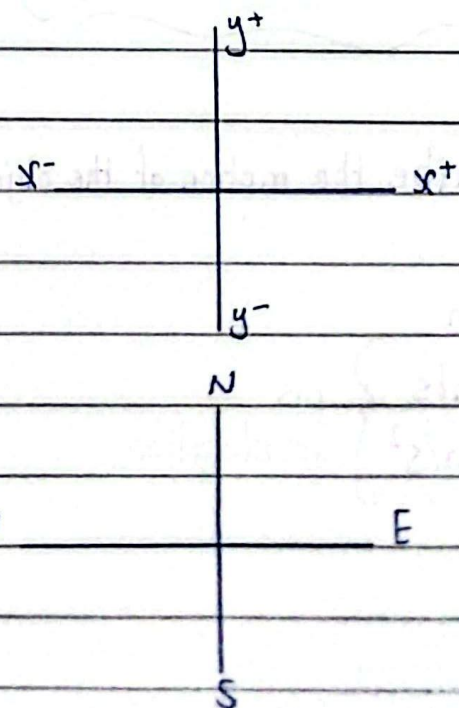
instantaneous السرعة اللحظية

Acceleration التسارع

⑥ Acceleration $[a]$ $[\pm]$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

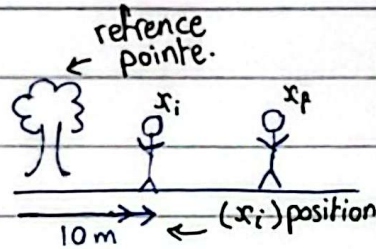


* chapter (2) *

* Motion in 1 Dimension [1D]

Kinematics

① position :-



② Displacement $\Delta x = x_f - x_i$ (x_f)

③ Distance (d = total distance)

④ velocity $\bar{v} = \frac{\Delta x}{\Delta t}$ average

$$v = \frac{x_f - x_i}{t_f - t_i}$$

[Instantaneous] $v = \frac{dx}{dt}$

⑤ speed $\bar{v}$

$$S = \frac{\text{total distance}}{\text{total time}}$$

Kinematic دراسة الحركة

reference point نقطة مرجعية

position موقع

Displacement الإزاحة

Distance المسافة

velocity السرعة

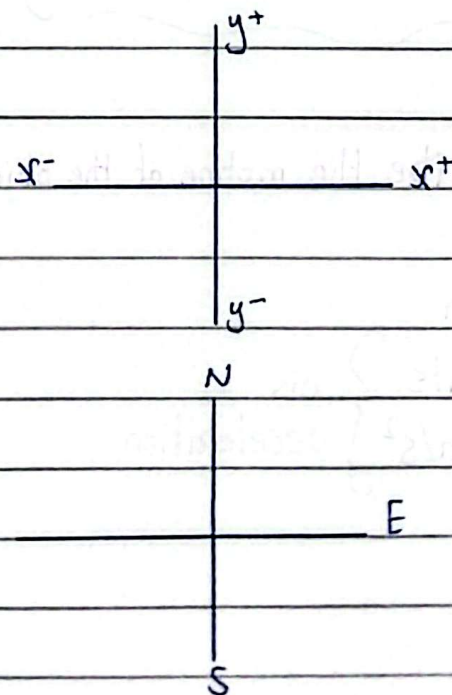
instantaneous السرعة اللحظية

Acceleration التسارع

⑥ Acceleration [a] $\pm$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$



$$\Delta x = +, -$$

$$v = +, -$$

$$a = +, -$$

$a, v \Rightarrow$ Same sign

$\left. \begin{array}{l} a+, v+ \\ a-, v- \end{array} \right\} \Rightarrow$ تسارع acceleration

$a, v \Rightarrow$ opposite sign

$\left. \begin{array}{l} a+, v- \\ a-, v+ \end{array} \right\} \Rightarrow$ تباطؤ deceleration

* S.I units :

$$[x] = m \quad [a] = m/s^2$$

$$[v] = m/s \quad m, s, kg$$

Ex: describe the motion of the object:

$$\Delta x = 3m$$

$$\left. \begin{array}{l} v = 7m/s \\ a = -2m/s^2 \end{array} \right\} \text{deceleration}$$

deceleration تباطؤ

Equation of motion
معادلات الحركة

[aftkar academy] مرجع للدراسة
Casio 991ES نوع الآلة الحاسبة

Ex: $\Delta x = 4m$

$$\left. \begin{array}{l} v = -8m/s \\ a = -10m/s^2 \end{array} \right\} \text{acceleration}$$

* Equation of motion :

$$v_f = v_i + at$$

$$v_f^2 = v_i^2 - 2a \Delta x$$

$$\Delta x = v_i t + \frac{1}{2} at^2$$

$$\bar{x} = \frac{x_f - x_i}{2}$$

$$\bar{v} = \bar{x} t = \left(\frac{x_f - x_i}{2} \right) t$$

Subject

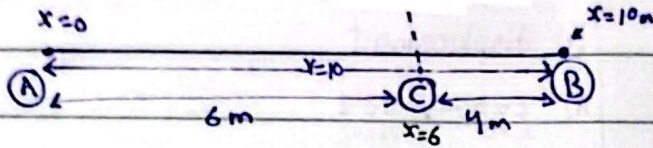
wednesday

8/10/2025

Date

No.

Ex:



① Find distance from A → C

$$d = 10 - 6 = 4 \text{ m}$$



② Find displacement from A → C

$$\Delta x = x_f - x_i$$

$$= 6 - 0 = 6 \text{ m}$$

③ Average acceleration from $t=0 \rightarrow t=1$

قيمة التسارع

$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{13 - 5}{1 - 0} = 8 \text{ m/s}^2$$

Ex:

$$x = 4t^2 + 5t + 6$$

① displacement: $t=0 \rightarrow t=1 \text{ s}$
 $x_i = 6$ $x_f = 15$

$$\Delta x = x_f - x_i$$

$$= 15 - 6$$

$$\Delta x = 9 \text{ m}$$

② Average velocity. from $t=0 \rightarrow t=1$
 $x_i = 6$ $x_f = 15$

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{15 - 6}{1 - 0} = 9 \text{ m/s}$$

or

$$\bar{v} = \frac{v_1 + v_2}{2} = \frac{5 + 13}{2} = 9 \text{ m/s}$$

$$v = \frac{dx}{dt} = 8t + 5$$

$$v(0) = 5$$

$$v(1) = 13$$

distance المسافة

Graphical relationship

العلاقة البيانية

Freely falling السقوط الحر

Lunched إطلاق

return العودة

or

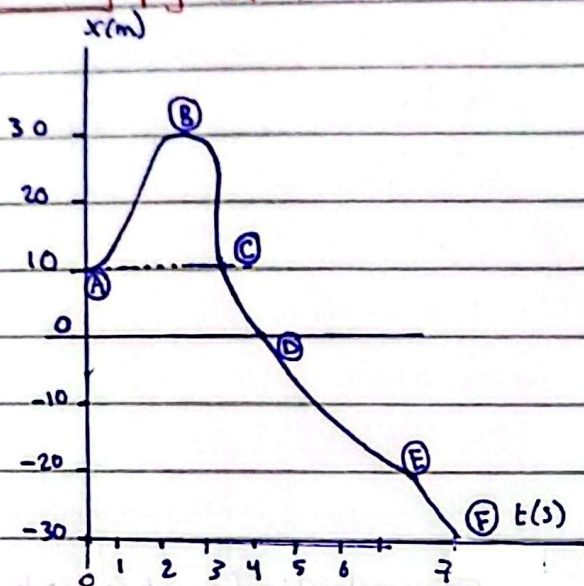
$$\bar{a} = \frac{a_1 + a_2}{2}$$

$$a = \frac{dv}{dt} = 8$$

$$\bar{a} = \frac{8 + 8}{2} = 8 \text{ m/s}^2$$

NOTED BOOK

Ex: 2.1 page: 24



① $\Delta x = ??$ A $\rightarrow$ F

$$\Delta x = x_f - x_i$$

$$= -30 - 10$$

$$= -40 \text{ m}$$

② $\bar{v} = ??$ A $\rightarrow$ F

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{-30 - 10}{7 - 0} = \frac{-40}{7} \text{ m/s}$$

③ speed = ?? A $\rightarrow$ F

سرعته المتوسطة

$$S = \frac{\text{total distance}}{\text{total time}} = \frac{80}{7} = 11.4 \text{ m/s}$$

Ex: 2.3 / page: 27

$$x(t) = -4t + 2t^2$$

① displacement

A) $t=0 \rightarrow t=1$
 $x_i=0 \quad x_f=-2$

$$\Delta x = x_f - x_i = -2 - 0 = -2 \text{ m}$$

B) $t=1 \rightarrow t=3$
 $x_i=-2 \quad x_f=6$

$$\Delta x = x_f - x_i$$

$$= 6 - (-2) = 8 \text{ m}$$

② average velocity * مطلوب له بطريقته هيرى الركون بده

A) $t=0 \rightarrow t=1$
 $x_i=0 \quad x_f=-2$

(1) $\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{-2 - 0}{1 - 0} = -2 \text{ m/s}$

(2) $\bar{v} = \frac{v_1 + v_2}{2} \quad v = \frac{dx}{dt} = -4 + 4t$
 $= \frac{0 + (-4)}{2} \quad v_1(0) = -4$
 $= -2 \text{ m/s} \quad v_2(1) = 0$

③ instantaneous velocity at $t=2.5$

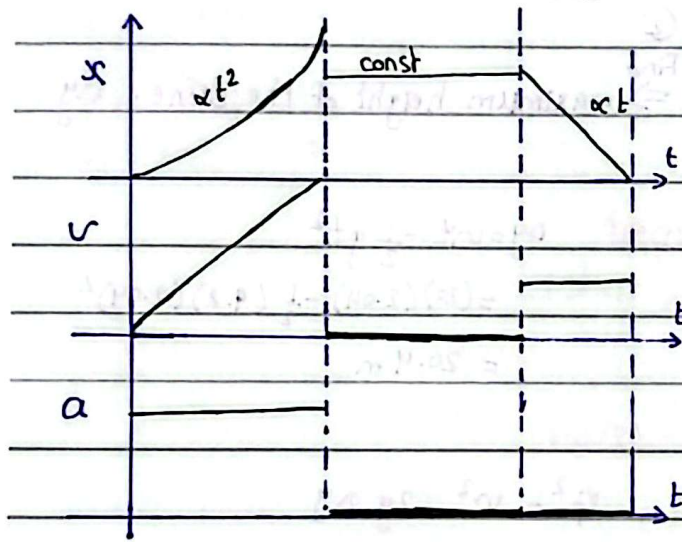
$t=2.5$

$$v = \frac{dx}{dt} = -4 + 4t = 6 \text{ m/s}$$

* Graphical Relationship

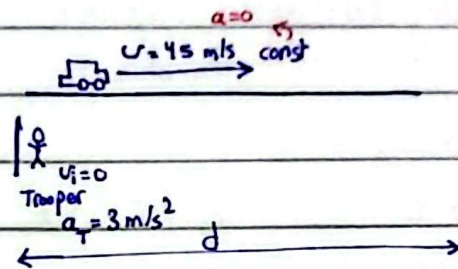
$$\boxed{x} \xrightarrow{\frac{dx}{dt}} \boxed{v} \xrightarrow{\frac{dv}{dt}} \boxed{a}$$

$$\xleftarrow{\int v dt} \quad \xleftarrow{\int a dt}$$



very important
Ex: 2.8

page: 39



* How long does it take $\rightarrow$ (t) Time

$$\Delta x_{car} = v_i t + \frac{1}{2} a t^2$$

$$\boxed{\Delta x_{car} = v_i t} \rightarrow \boxed{\Delta x_{car} = 45 t}$$

$$\Delta x_{Trooper} = v_i t + \frac{1}{2} a t^2$$

$$\boxed{\Delta x_{Trooper} = \frac{1}{2} a t^2}$$

$$\boxed{\Delta x_{Trooper} = \frac{3}{2} t^2}$$

* But Trooper starts after 1s

$$\boxed{\Delta x_{car} = \Delta x_{Trooper}}$$

* Equation of motion

* * a [acceleration] must be constant.

$$\boxed{1} \quad v_f = v_i + at$$

$$\boxed{2} \quad v_f^2 = v_i^2 + 2a \Delta x$$

$$\boxed{3} \quad \Delta x = v_i t + \frac{1}{2} a t^2$$

$$\boxed{4} \quad \bar{v} = \frac{v_i + v_f}{2}$$

$$\boxed{5} \quad \Delta x = \bar{v} t = \left(\frac{v_i + v_f}{2} \right) t$$

$$45t = \frac{3}{2} t^2$$

$$\boxed{t = 30}$$

* But Trooper start after 1s.

$$t = 30 + 1 \rightarrow \boxed{t = 31 s}$$

Freely falling so

$$\Delta x \rightarrow \Delta y$$

$$t \rightarrow t$$

$$v_x \rightarrow v_y$$

$$a \rightarrow -g$$

$$g = 9.8 \text{ m/s}^2$$

$$1) v_f = v_i - gt$$

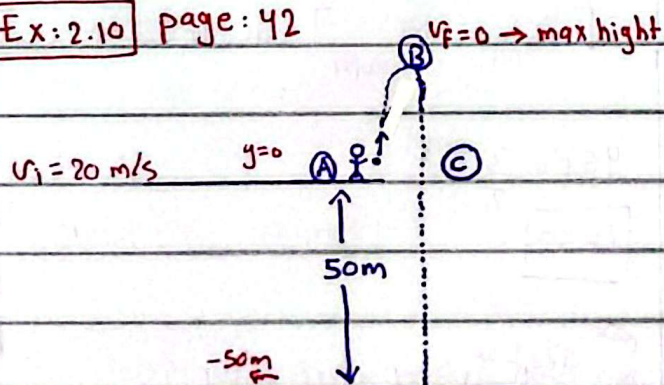
$$2) v_f^2 = v_i^2 - 2g \Delta y$$

$$3) \Delta y = v_i t - \frac{1}{2} g t^2$$

$$\bar{v} = \frac{v_i + v_f}{2}$$

$$\Delta y = \left(\frac{v_i + v_f}{2} \right) t$$

* Ex: 2.10 page: 42



* كيف عرفت إنه الحجر تعمريه للأعلى وليس للأسفل : صون

بإشارة v بتحدد ، إذا كانت (+) الحجر نرمي للأعلى ، إذا

كانت (-) الحجر نرمي للأسفل . [في تمارع السقوط الحر]

①
Find $t = ?? \Rightarrow \text{max height } (v_f = 0)$

$$v_f = v_i - gt$$

$$0 = 20 - (9.8)t$$

$$t = 2.04 \text{ s}$$

②
Find $\Rightarrow \text{maximum height of the stone. } \Delta y$

$$\begin{aligned} \Delta y &= v_i t - \frac{1}{2} g t^2 \\ &= (20)(2.04) - \frac{1}{2} (9.8)(2.04)^2 \\ &= 20.4 \text{ m} \end{aligned}$$

(2) طريقة

$$\begin{aligned} v_f^2 &= v_i^2 - 2g \Delta y \\ 2g \Delta y &= v_i^2 - v_f^2 \\ \Delta y &= \frac{v_i^2 - v_f^2}{2g} \end{aligned}$$

$$\Delta y = \frac{(20)^2 - (0)}{2(9.8)} = 20.4 \text{ m}$$

* ملاحظة عند الحل إذا كان فيه معادلة فيها جميع المعطيات فداخ
السؤال هذا المعطيات التي بيدي أجاب عليه بوظيف استخدام المعطيات
استخدام معادلة ثانية يكون أنا طلعت واحد من المعطيات في
نزع قبل .

مراجعة

Ex. 8-10
تعالج المسألة

الارتفاع المتجهة
الارتفاع

5) velocity before reach the ground.
 $\Delta y = -5 \text{ m}$

3) At point C find $v_f = ??$ $\Delta y = 0$

$$v_f^2 = v_i^2 - 2g\Delta y$$

$$v_f^2 = v_i^2 - 2g\Delta y$$

$$= (20)^2 + 2(9.8)(+50)$$

$$= 1380 \text{ m/s}$$

$$v_f^2 = (20)^2$$

$$\sqrt{v_f^2} = \sqrt{400}$$

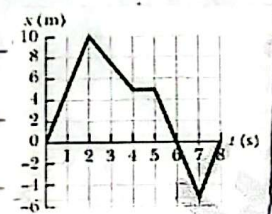
$$v_f = \pm 20 \rightarrow v_f = +20 \text{ (Lunched)}$$

$$v_i = -20 \text{ (return)}$$

⊕ تكون $\uparrow$ Lunched
⊖ تكون $\downarrow$ return
الارتفاع المتجهة velocity وتكون إطلاق
الارتفاع velocity وتكون عودة

* أسئلة على Chapter 2 من الكتاب 8

1) The position versus time for a certain particle moving along the x-axis is shown in figure, Find the average velocity in the time intervals



4) Find the velocity and position of stone at $t = 5 \text{ s}$

$$\Delta y \left. \begin{array}{l} \\ v \end{array} \right\} t = 5 \text{ s}$$

$$v_f = v_i - gt$$

$$= 20 - 9.8(5)$$

$$= -29 \text{ m/s}$$

$$\Delta y = v_i t - \frac{1}{2} g t^2$$

$$= 20(5) - \frac{1}{2} (9.8)(5)^2$$

$$= -22.5 \text{ m}$$

(a) 0 to 2s

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{10 - 0}{2 - 0} = 5 \text{ m/s}$$

(b) 0 to 4s :- $\bar{v} = \frac{5 - 0}{4 - 0} = 1.25 \text{ m/s}$

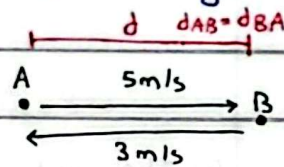
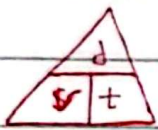
(c) 2s to 4s :- $\bar{v} = \frac{5 - 10}{4 - 2} = -2.5 \text{ m/s}$

(d) 4s to 7s :- $\bar{v} = \frac{-5 - 5}{7 - 4} = -3.3 \text{ m/s}$

(e) 0 to 8s :- $\bar{v} = \frac{0 - 0}{8 - 0} = 0 \text{ m/s}$

3 A person walks first at a constant speed of 5 m/s along a straight line from point A to point B and then back along the line from B to A at a constant speed of 3 m/s.

(a) what is her average speed over the entire trip? (b) what is her average velocity over the entire trip?



(a) average speed = $\frac{\text{total distance}}{\text{total time}}$

$$S = \frac{d_{AB} + d_{BA}}{t_{AB} + t_{BA}} = \frac{2d}{\left(\frac{d}{v_{AB}} + \frac{d}{v_{BA}}\right)}$$

$$S = \frac{2d}{\frac{d}{v_{AB}} + \frac{d}{v_{BA}}} = \frac{2d \cdot (v_{AB} \cdot v_{BA})}{d(v_{BA} + v_{AB})}$$

$$S = \frac{30}{8} = 3.75 \text{ m/s}$$

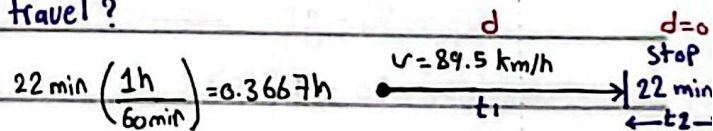
(b) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0-0}{\Delta t} = 0$

4 A particle moves according to the equation $x = 10t^2$, where x is in meters and t is in seconds. (a) Find the average velocity for the time interval from 2s to 3s. (b) Find the average velocity for the time interval from 2s to 2.10s.

$$(a) \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{90 - 40}{3 - 2} = 50 \text{ m/s}$$

$$(b) \bar{v} = \frac{44.1 - 40}{2.10 - 2} = 41 \text{ m/s}$$

13 A person takes a trip, driving with a constant speed of 89.5 km/h for a 22 min rest stop. If the person's average speed is 77.8 km/h, (a) how much time is spent on the trip and (b) how far does the person travel?



(b) $S = \frac{d_1 + 0}{t_1 + t_2}$

$$77.8 = \frac{d_1}{\frac{d_1}{89.5} + 0.3667} \Rightarrow 77.8 = \frac{89.5 d_1}{d_1 + 32.82}$$

$$d_1 = 219$$

(a) $t_1 = \frac{d_1}{v} = \frac{219}{89.5} = 2.45 \text{ h}$

[15] A velocity-time

graph for an object moving along the x-axis is shown



in figure. (a) Plot a graph of the acceleration versus time. Determine the average acceleration of the object (b) in the time interval $t = 5s$ to $t = 15s$ and (c) in the time interval $t = 0$ to $t = 20s$.

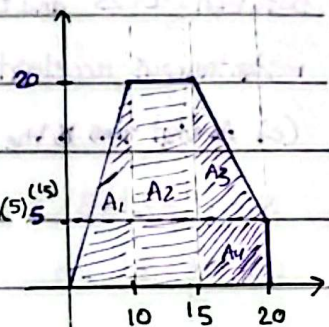
$$S_3 = (-3)(5) = -15$$

$$\text{at } t = 10s \Rightarrow S = 20 \text{ m/s}$$

$$\text{at } t = 20 \Rightarrow S = S_1 + S_2 + S_3 = 20 + 0 + -15 = 5 \text{ m/s}$$

(b) distance from $t = 0 \rightarrow t = 20$ Area

under V curve.

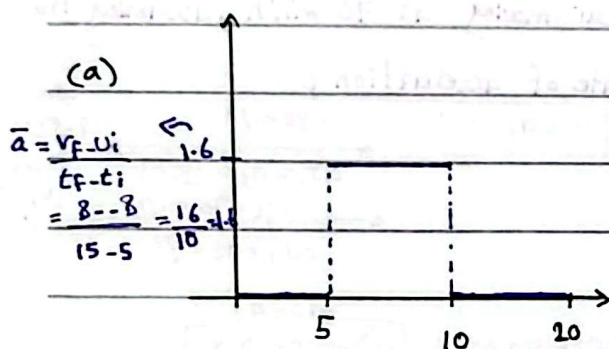


$$d = A_1 + A_2 + A_3 + A_4$$

$$= \frac{1}{2}(10)(20) + (20)(5) + \left(\frac{1}{2}\right)(5)(15) + (5)(5)$$

$$= 100 + 100 + 37.5 + 25$$

$$d = 262.5 \text{ m}$$



$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{8 - -8}{15 - 5} = \frac{16}{10} = 1.6$$

$$(b) \bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{8 - -8}{15 - 5} = 1.6 \text{ m/s}^2$$

$$(c) \bar{a} = \frac{8 - (-8)}{20 - 0} = 0.8 \text{ m/s}^2$$

* 100 *

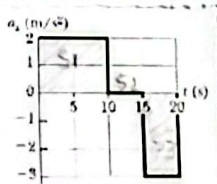
[19] A particle starts from rest $v_i = 0$

and accelerates as shown in

Figure. Determine (a) the particle's

speed at $t = 10s$ and at $t = 20s$ and (b)

the distance traveled in the first 20s.



(a) Speed $\Rightarrow$ Area under curve.

$$S_1 = (2)(10) = 20 \text{ m/s}$$

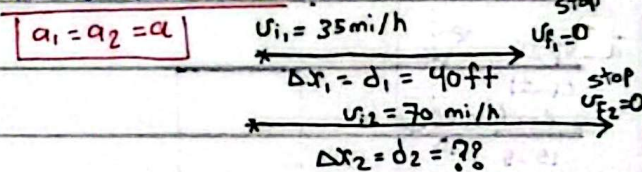
$$S_2 = 0$$

[20] An object moves along the x axis according to the equation $x = 3t^2 - 2t + 3$, where x is in meters and t is in seconds. Determine

(e) $v = 0 \Rightarrow t = ?$

$$0 = 6t - 2 \Rightarrow 6t = 2 \Rightarrow t = \frac{1}{3}$$

* [24] The minimum distance required to stop a car moving at 35 mi/h is 40 ft. what is the minimum stopping distance for the same car moving at 70 mi/h, assuming the same rate of acceleration?



$$a_1 = a_2 = a$$

$$v_{f1}^2 = v_{i1}^2 + 2a_1 \Delta x_1$$

$$v_{f1}^2 = -2a \Delta x_1$$

$$v_{f2}^2 = v_{i2}^2 + 2a_2 \Delta x_2$$

$$v_{f2}^2 = -2a_2 \Delta x_2$$

$$\frac{v_{i1}^2}{v_{i2}^2} = \frac{-2a_1 \Delta x_1}{-2a_2 \Delta x_2} \Rightarrow \Delta x_2 = \frac{\Delta x_1 v_{i2}^2}{v_{i1}^2}$$

$$\Delta x_2 = \frac{40 (70)^2}{(35)^2}$$

$$\Delta x_2 = 160 \text{ ft}$$

(a) the average velocity between $t = 2$ s and $t = 3$ s, (b) the instantaneous velocity at $t = 2$ s and $t = 3$ s, (c) the average acceleration between $t = 2$ s and $t = 3$ s and (d) the instantaneous acceleration at $t = 2$ s and $t = 3$ s

(e) At what time is the object at rest?

$$x = 3t^2 - 2t + 3 \quad x(2) = 11 \quad / \quad x(3) = 24$$

$$v = 6t - 2 \quad v(2) = 10 \quad / \quad v(3) = 16$$

$$a = 6$$

$$(c) \bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{16 - 10}{3 - 2} = 6 \text{ m/s}^2$$

$$(d) a = \frac{dv}{dt} = 6$$

$$a(2) = 6 \text{ m/s}^2$$

$$a(3) = 6 \text{ m/s}^2$$

$\Rightarrow$ constant acceleration

$$(a) \bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{24 - 11}{3 - 2} = 13 \text{ m/s}$$

$$(b) v = \frac{dx}{dt}$$

$$v(2) = 10 \text{ m/s}$$

$$v(3) = 16 \text{ m/s}$$

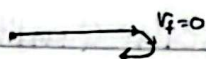
[38] A particle moves along the x-axis. Its position is given by the equation $x = 2 + 3t - 4t^2$, with x in meters and t in seconds. Determine (a) its position when it changes direction and (b) its velocity when it returns to the position it had at $t=0$.

(a) $0 = 3 - 8t$

$8t = 3 \Rightarrow t = 0.375$

$x = 2 + 3(0.375) - 4(0.375)^2$

$x = 2.56 \text{ m}$



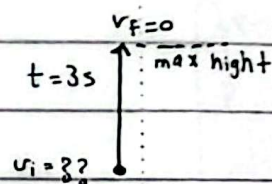
(b) $v_f = ??$ when $\Delta x = 0$ [return to initial position]

$t=0$ $x(0) = 2\text{m}$
 $v(0) = 3$
 $a(0) = -8$

$v_f^2 = v_i^2 + 2a\Delta x$

$v_f^2 = (3^2) \Rightarrow \sqrt{v_f^2} = \sqrt{9} \Rightarrow v_f = \pm 3$
 $v_f = 3$ or $v_f = -3$
 [return]

[42] A baseball is hit so that it travels straight upward after being struck by the bat. A fan observes that it takes 3s for the ball to reach its maximum height. Find (a) the ball's initial velocity and (b) the height it reaches.



(a) $v_i = ??$

$v_f = v_i - gt$

$0 = v_i - (9.8)(3)$

$v_i = 29.4 \text{ m/s}$

(b) $\Delta y = ??$

$\Delta y = v_i t - \frac{1}{2} g t^2$
 $= (29.4)(3) - \frac{1}{2} (9.8)(3)^2$

$\Delta y = 88.2 - 44.1$

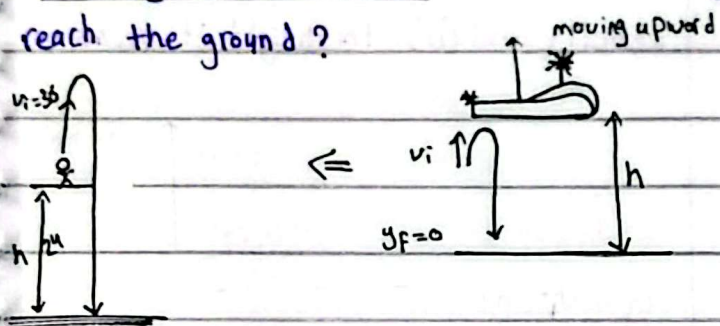
$\Delta y = 44.1 \text{ m/s}$

Chapter 3 vectors

50 The height of a helicopter above the ground is given by $h = 3t^3$, where h is in meters and t is in seconds. At $t = 2$ s.

The helicopter releases a small mailbag.

How long after its release does the mailbag reach the ground?



$$h = 3t^3 \text{ at } t=2$$

$$v = 9t^2$$

$$v(2) = 9(2)^2 = 36 \text{ m/s}$$

$$h(2) = 3(2)^3 = 24 \text{ m}$$

$$\Delta y = v_i t - \frac{1}{2} g t^2$$

$$-24 = 36t - \frac{1}{2}(9.8)t^2$$

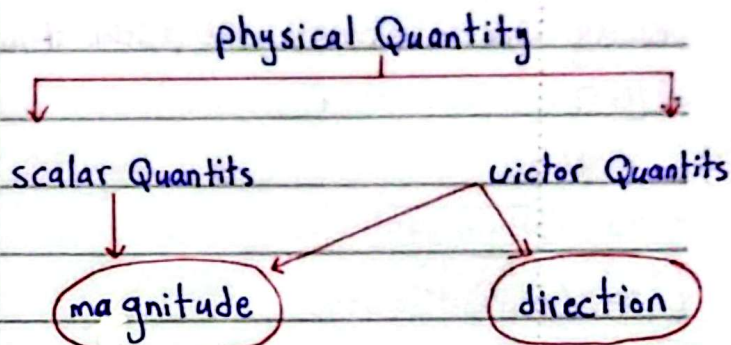
$$4.9t^2 - 36t - 24 = 0$$

$$t_1 = 7.96 \text{ s}$$

* تكبير على SETUP رقم 5 و 0

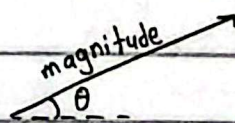
رقم 3

$$t_2 = -0.61$$



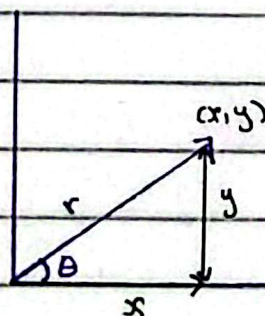
* Scalar: Temperature / mass / Volume
work / Time / speed / distance.

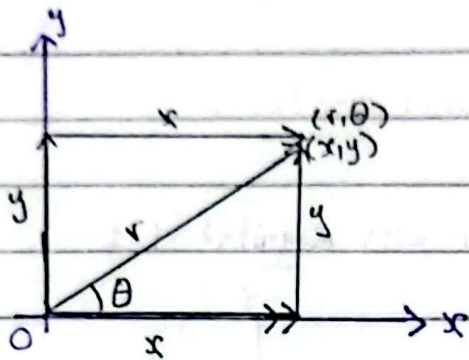
* Vectors: Force / accelerats / velocity
displacement / momentum / pressure.



(x, y) = cartesian

(r, θ) = Polar





$$\sin \theta = \frac{y}{r}, \quad \cos \theta = \frac{x}{r}$$

$$y = r \sin \theta$$

$$x = r \cos \theta$$

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

$$\tan \theta = \frac{y}{x}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

* ملاحظة: لا نطلع قيمة زاوية كذا إلا في الحالة يجب تحويلها إلى الحالة الجبرية النظام
Ex: polar cartesian

$$p(5, 30^\circ) \rightarrow (x, y)$$

$$x = r \cos \theta = 5 \cos 30 = 4.33$$

$$y = r \sin \theta = 5 \sin 30 = 2.5$$

$$(4.3, 2.5)$$

$$\text{Ex: } p(4, 3) \rightarrow (r, \theta)$$

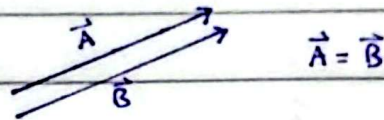
$$r = \sqrt{x^2 + y^2} = \sqrt{(4)^2 + (3)^2} = 5$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.8^\circ$$

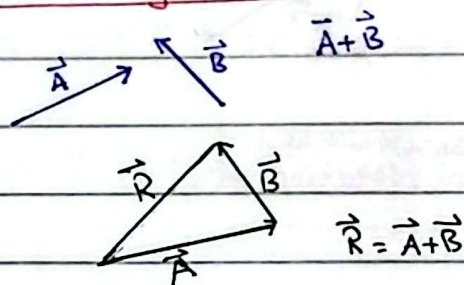
$$p(5, 36.8^\circ)$$

* some vectors properties go

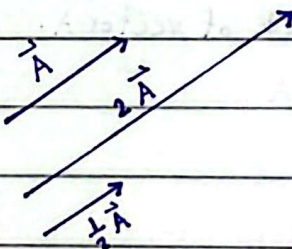
1 Equality :-



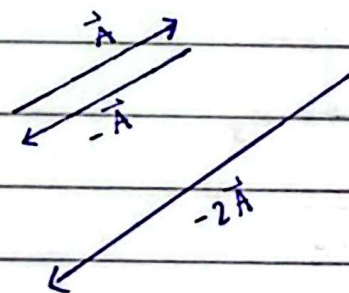
2 Adding vectors

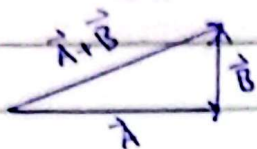


3 multiplication with Constant: (scalar)



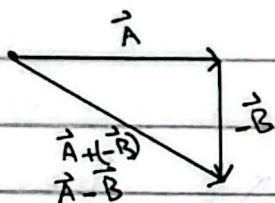
4 Negative vector





$$\vec{A} - \vec{B}$$

$$\vec{A} + (-\vec{B})$$



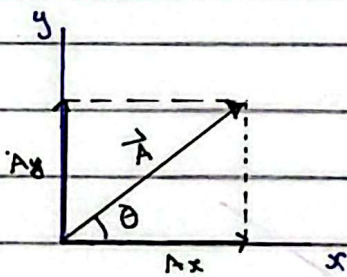
* Vector Notation :-

$\vec{A}$ = The vector A

$|\vec{A}|$ = The magnitude of vector A.

A = magnitude of A

* Component of vectors :-

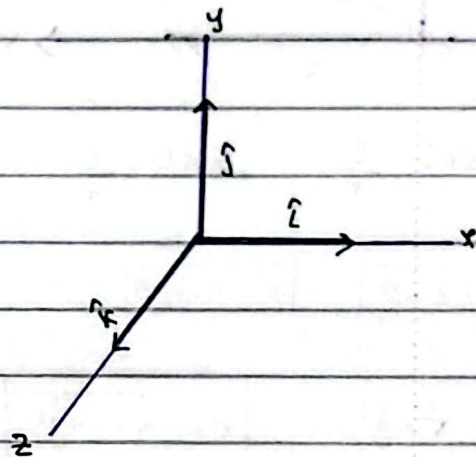


$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \tan^{-1} \left(\frac{A_y}{A_x} \right)$$

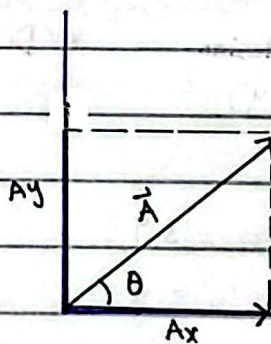
* unit vector so

vector with magnitude of 1



$\hat{i}, \hat{j}, \hat{k}$ = unit vector

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$



Properties of unit vectors

multiplication

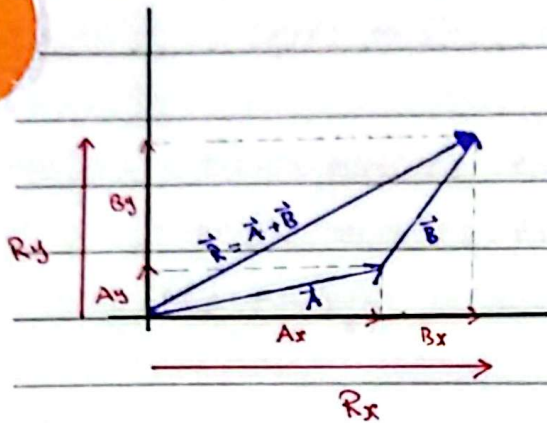
vector Notation

حساب ضرب متجهات / طريقة المتجهات

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \tan^{-1} \left(\frac{A_y}{A_x} \right)$$



$$\vec{R} = R_x \hat{i} + R_y \hat{j}$$

$$\vec{A} + \vec{B} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j}$$

$$|\vec{A} + \vec{B}| = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

Ex: $\vec{A} = 5\hat{i} + 6\hat{j}$

$$\vec{B} = 3\hat{i} - 4\hat{j}$$

$$\vec{A} + \vec{B} = (5+3)\hat{i} + (6-4)\hat{j} = 8\hat{i} + 2\hat{j}$$

(+)
دائماً

دائماً +
بعض
ملاحظات

$$\vec{A} - \vec{B} = (5-3)\hat{i} + (6-(-4))\hat{j} = 2\hat{i} + 10\hat{j}$$

(+)
دائماً

$$\vec{B} - \vec{A} = (3-5)\hat{i} + (-4-6)\hat{j} = -2\hat{i} - 10\hat{j}$$

$$\vec{A} - 2\vec{B} = (5-2(3))\hat{i} + (6-2(-4))\hat{j} = -1\hat{i} + 14\hat{j}$$

تجاه

$$1) |\vec{A} + \vec{B}| = \sqrt{2^2 + 4^2} = 2.9$$

$$\theta = \tan^{-1}\left(\frac{2}{4}\right) = 26.5$$

$$2) |\vec{A} - \vec{B}| = \sqrt{(2)^2 + (10)^2} = 10.2$$

$$\theta = \tan^{-1}\left(\frac{10}{2}\right) = 78.6$$

الزاوية
(2, 10)

الزاوية
(-2, -10)

$$3) |\vec{B} - \vec{A}| = \sqrt{(-2)^2 + (-10)^2} = 10.2$$

$$\theta = \tan^{-1}\left(\frac{10}{-2}\right) = 78.6 + 180 = 258.6$$

$$4) |\vec{A} - 2\vec{B}| = \sqrt{(-1)^2 + (14)^2} = 14.04$$

(-1, 14)
الزاوية

$$\theta = \tan^{-1}\left(\frac{14}{-1}\right) = 85.9$$

$$\theta = 180 - 85.9 = 94.1$$

*) $\vec{A} \cdot \vec{B}$ dot product chapter 7

*) $\vec{A} \times \vec{B}$ cross product physics 2

*) Dot product 80

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\hat{i} \cdot \hat{i} = 1$$

$$\hat{j} \cdot \hat{j} = 1$$

$$\hat{k} \cdot \hat{k} = 1$$

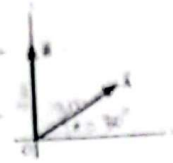
$$\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = 0$$

$$\hat{j} \cdot \hat{k} = \hat{j} \cdot \hat{i} = 0$$

$$\hat{k} \cdot \hat{i} = \hat{k} \cdot \hat{j} = 0$$

$$\boxed{\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z}$$

11 The displacement vector $\vec{A}$ and $\vec{B}$ shown in Figure both have magnitudes of 3m. The direction

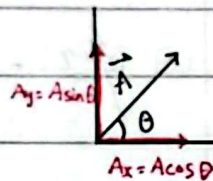


of vector $\vec{A}$ is $\theta = 30^\circ$, find graphically (a) $\vec{A} + \vec{B}$, (b) $\vec{A} - \vec{B}$, (c) $\vec{B} - \vec{A}$, and (d) $\vec{A} - 2\vec{B}$. [Report all angles counterclockwise from positive x-axis].

$$\vec{A} = 3\cos 30^\circ \hat{i} + 3\sin 30^\circ \hat{j}$$

$$\vec{A} = 2.6\hat{i} + 1.5\hat{j}$$

$$\vec{B} = 0\hat{i} + 3\hat{j}$$



$$(a) \vec{A} + \vec{B} = 2.6\hat{i} + 4.5\hat{j}$$

$$(b) \vec{A} - \vec{B} = 2.6\hat{i} - 1.5\hat{j}$$

$$(c) \vec{B} - \vec{A} = -2.6\hat{i} + 1.5\hat{j}$$

$$(d) \vec{A} - 2\vec{B} = 2.6\hat{i} - 4.5\hat{j}$$

12 Three displacements are $\vec{A} = 200\text{m}$ due south,

$\vec{B} = 250\text{m}$ due west, and $\vec{C} = 150\text{m}$ at 30° east of

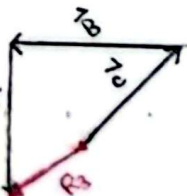
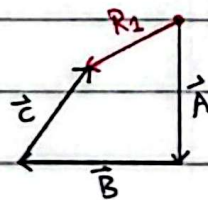
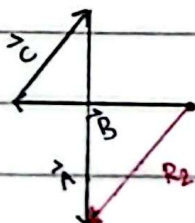
north. (a) Construct a separate diagram for each

of the following possible ways of adding these

vectors: $\vec{R}_1 = \vec{A} + \vec{B} + \vec{C}$, $\vec{R}_2 = \vec{B} + \vec{C} + \vec{A}$,

$\vec{R}_3 = \vec{C} + \vec{B} + \vec{A}$. (b) Explain what you can conclude

from comparing the diagrams.



13 A vector has an x component of -25 units and a y component 40 units. Find the magnitude and direction of this vector.

$$\vec{A} = -25\hat{i} + 40\hat{j}$$

point
(-25, 40)

$$|\vec{A}| = \sqrt{(-25)^2 + (40)^2} = 47.2$$

$$\theta = \tan^{-1}\left(\frac{40}{-25}\right) = 57.9$$

$$\theta = 180 - 57.9 = 122.01$$

23 Consider the two vectors $\vec{A} = 3\hat{i} - 2\hat{j}$ and $\vec{B} = -\hat{i} - 4\hat{j}$. Calculate (a) $\vec{A} + \vec{B}$, (b) $\vec{A} - \vec{B}$ (c) $|\vec{A} + \vec{B}|$, (d) $|\vec{A} - \vec{B}|$, and (e) the direction of $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$.

$$(a) \vec{A} + \vec{B} = 2\hat{i} - 6\hat{j}$$

$$(b) \vec{A} - \vec{B} = 4\hat{i} + 2\hat{j}$$

$$(c) |\vec{A} + \vec{B}| = \sqrt{(2)^2 + (-6)^2} = 6.3$$

$$(d) |\vec{A} - \vec{B}| = \sqrt{(4)^2 + (2)^2} = 4.5$$

(e) direction $\vec{A} + \vec{B}$

point
(2, -6)

$$\theta = \tan^{-1}\left(\frac{-6}{2}\right) = 71.6$$

$$\theta = 360 - 71.6 = 288.4^\circ$$

direction $\vec{A} - \vec{B}$

point
(4, 2)

$$\theta = \tan^{-1}\left(\frac{2}{4}\right) = 26.6^\circ$$

Direction of $\vec{A} + \vec{B}$ is 288.4°
Direction of $\vec{A} - \vec{B}$ is 26.6°

22) Given the vectors $\vec{A} = 2\hat{i} + 6\hat{j}$ and

$\vec{B} = 3\hat{i} - 2\hat{j}$ (a) draw the vector sum $\vec{C} = \vec{A} + \vec{B}$

and the vector difference $\vec{D} = \vec{A} - \vec{B}$. (b) Calculate

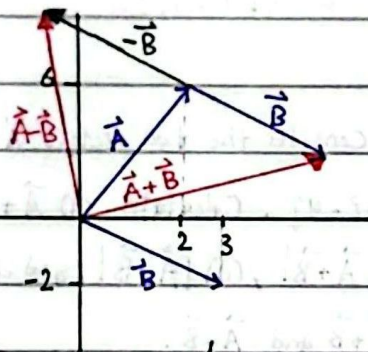
$\vec{C}$ and $\vec{D}$, in terms of unit vector. (c) calculate

$\vec{C}$ and $\vec{D}$ in terms of polar coordinates, with

angles measured with respect to the positive

x-axis.

(a)



$$(b) \vec{C} = \vec{A} + \vec{B} \Rightarrow \boxed{\vec{C} = 5\hat{i} + 4\hat{j}} \quad / \quad \vec{D} = \vec{A} - \vec{B} \Rightarrow \boxed{\vec{D} = -\hat{i} + 8\hat{j}}$$

$$(c) \vec{C} \Rightarrow r = \sqrt{(5)^2 + (4)^2} = 6.4 \quad \text{جوابه} \quad (5,4)$$

$$\theta = \tan^{-1}\left(\frac{4}{5}\right) = 38.7$$

$\vec{C} (6.4, 38.7^\circ) \rightarrow$ polar coordinates

$$\vec{D} \Rightarrow r = \sqrt{(-1)^2 + (8)^2} = 8.1 \quad \text{جوابه} \quad (-1, 8)$$

$$\phi = \tan^{-1}\left(\frac{8}{-1}\right) = 82.9$$

$$\theta = 180 - 82.9 = 97.1$$

$\vec{D} (8.1, 97.1^\circ) \rightarrow$ polar coordinates

31) Consider the three displacement

vectors $\vec{A} = 3\hat{i} - 3\hat{j}$, $\vec{B} = \hat{i} - 4\hat{j}$, and $\vec{C} = -2\hat{i} + 5\hat{j}$.

Use the component method to determine

(a) the magnitude and direction of $\vec{D} = \vec{A} + \vec{B} + \vec{C}$

and (b) the magnitude and direction of $\vec{E} = -\vec{A} - \vec{B}$

$$(a) \vec{D} = 2\hat{i} + 2\hat{j}$$

$$|\vec{D}| = \sqrt{(2)^2 + (2)^2} = 2.8 \quad \text{جوابه} \quad (2, -2)$$

$$\phi = \tan^{-1}\left(\frac{2}{2}\right) = 45$$

$$\theta = 360 - 45 = 315^\circ$$

$$(b) \vec{E} = -6\hat{i} + 12\hat{j}$$

$$\text{جوابه} \quad (-6, 12)$$

$$|\vec{E}| = \sqrt{(-6)^2 + (12)^2} = 13.4$$

$$\phi = \tan^{-1}\left(\frac{12}{-6}\right) = 63.4$$

$$\theta = 180 - 63.4 = 116.6^\circ$$

34) Vector $\vec{B}$ has x, y and z components of 4, 6, and 3 units, respectively. Calculate (a) the magnitude of $\vec{B}$ and (b) the angle that $\vec{B}$ makes with each coordinate axis.

$$a) \vec{B} = 4\hat{i} + 6\hat{j} + 3\hat{k}$$

$$|\vec{B}| = \sqrt{(4)^2 + (6)^2 + (3)^2} = 7.8$$

b)

[37] (a) Taking $\vec{A} = 6\hat{i} - 8\hat{j}$ units, $\vec{B} = -8\hat{i} + 3\hat{j}$ units, and $\vec{C} = 26\hat{i} + 19\hat{j}$ units, determine a and b such that $a\vec{A} + b\vec{B} + \vec{C} = 0$. (b) A

$$(a) \quad (6a - 8b + 26 = 0) \times 8$$

$$(-8a + 3b + 19 = 0) \times 6$$

$$48a - 64b + 208 = 0$$

$$-48a + 18b + 114 = 0$$

$$-46b + 322 = 0$$

$$b = 7$$

$$a = 5.4$$

$$(b) \quad A = \sqrt{(6)^2 + (-8)^2} = 10$$

$$\vec{R} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 + \vec{d}_4$$

$$= (100 + -300 + -130 + -100)\hat{i} + (-300 + -75 + +173)\hat{j}$$

$$= -130\hat{i} - 202\hat{j}$$

$$|\vec{R}| = \sqrt{(-130)^2 + (-202)^2} = 240 \text{ m}$$

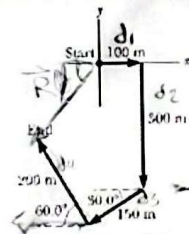
$$\theta = \tan^{-1}\left(\frac{202}{130}\right) = 57.2$$

$$\tan^{-1}\left(\frac{-202}{-130}\right)$$

$$\theta = 57.2 + 180$$

$$\theta = 237.2$$

[51] A person going for a walk follows the path shown in Figure. The total trip consists of four straight line



paths. At the end of the walk, what is the person's resultant displacement measured from the starting point?

$$d_1 = 100\hat{i}$$

$$d_2 = 300 \cos 90^\circ \hat{i} + 300 \sin 90^\circ \hat{j} \Rightarrow d_2 = -300\hat{j}$$

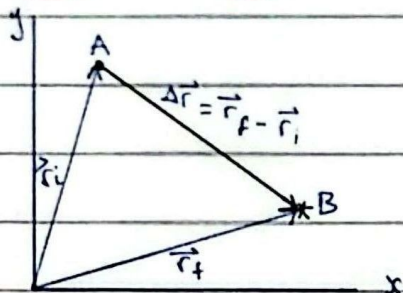
$$d_3 = 150 \cos 30^\circ \hat{i} + 150 \sin 30^\circ \hat{j} \Rightarrow d_3 = -130\hat{i} - 75\hat{j}$$

$$d_4 = 200 \cos 60^\circ \hat{i} + 200 \sin 60^\circ \hat{j} \Rightarrow d_4 = 100\hat{i} + 173\hat{j}$$

$$-x \text{ o.w. } 100\hat{i}$$

Chapter 4

Motion 122 D



① displacement ∴

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

$$\Delta \vec{r} = \Delta x \hat{i} + \Delta y \hat{j}$$

$$\vec{r} = x \hat{i} + y \hat{j}$$

② velocity

$$\vec{V}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

$$= \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j}$$

$$\vec{V}_{avg} = \vec{v}_x \hat{i} + \vec{v}_y \hat{j}$$

$$\vec{V} = \frac{d\vec{r}}{dt}$$

$$= \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j}$$

③ Acceleration

$$\vec{a} = \frac{d\vec{V}}{dt}$$

$$= \frac{\Delta v_x}{\Delta t} \hat{i} + \frac{\Delta v_y}{\Delta t} \hat{j}$$

$$\vec{a} = \frac{d\vec{V}}{dt}$$

$$= \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j}$$

* speed ∴ $|\vec{V}| = \text{speed}$

④ Equation of motion ∴

a must be constant:

$$\vec{v}_f = \vec{v}_0 + \vec{a}t \quad \begin{cases} v_{fx} = v_{0x} + a_x t \\ v_{fy} = v_{0y} + a_y t \end{cases}$$

$$\vec{v}_f^2 = \vec{v}_0^2 + 2\vec{a} \Delta \vec{r} \quad \begin{cases} v_{fx}^2 = v_{0x}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{0y}^2 + 2a_y \Delta y \end{cases}$$

$$\Delta \vec{r} = \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 \quad \begin{cases} \Delta x = v_{0x} t + \frac{1}{2} a_x t^2 \\ \Delta y = v_{0y} t + \frac{1}{2} a_y t^2 \end{cases}$$

$$\vec{V}_{avg} = \frac{1}{2} (\vec{v}_0 + \vec{v}_f)$$

$$\Delta \vec{r} = \frac{1}{2} (\vec{v}_0 + \vec{v}_f) t$$

$$\vec{v}_f \begin{cases} v_{fx} \\ v_{fy} \end{cases}$$

$$v_0 \begin{cases} v_{0x} \\ v_{0y} \end{cases}$$

$$\vec{a} \begin{cases} a_x \\ a_y \end{cases}$$

$$\Delta \vec{r} \begin{cases} \Delta x \\ \Delta y \end{cases}$$

t is the same.

→ angle the velocity vector makes with the x -axis:

$$\theta = \tan^{-1} \left(\frac{v_{fy}}{v_{fx}} \right) = \tan^{-1} \left(\frac{-15}{40} \right) = -21$$

$$\rightarrow \text{speed} = |\vec{v}| = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(40)^2 + (-15)^2}$$

$$|\vec{v}| = 43 \text{ m/s}$$

Ex 8 4.1 / page: 83

A particle moves in xy plane, starting from the origin at $t=0$ with an initial velocity having an x component of 20 m/s and a y component of -15 m/s .

The particle experiences an acceleration in the x direction, given by $a_x = 4 \text{ m/s}^2$.

A) Determine the total velocity vector at any time.

$\vec{v}_f(t)$ at any time $t=t$

$$v_{fx} = v_{0x} + a_x t$$

$$v_{fy} = v_{0y} + a_y t$$

$$v_{fx} = 20 + 4t$$

$$v_{fy} = -15$$

$$\text{OR } \vec{v}_f = \vec{v}_i + \vec{a}t$$

$$\vec{v}_f = (20\hat{i} - 15\hat{j}) + 4t\hat{i}$$

$$\vec{v}_f(t) = (20 + 4t)\hat{i} - 15\hat{j}$$

$$\vec{v}_f(t) = (20 + 4t)\hat{i} - 15\hat{j}$$

B) Calculate the velocity and speed of particle at $t=5 \text{ s}$ and the angle the velocity vector makes with the x -axis.

$$\vec{v}(5) = (20 + 4(5))\hat{i} - 15\hat{j} = 40\hat{i} - 15\hat{j}$$

c) Determine the x and y coordinate of the particle at any time t and its position vector at this time.

$$\Delta x = v_{0x}t + \frac{1}{2}a_x t^2$$

$$x_f - x_i =$$

$$x_f = v_{0x}t + \frac{1}{2}a_x t^2$$

$$= 20t + \frac{1}{2}(4)t^2$$

$$x_f = 20t + 2t^2$$

$$y_f - y_i = v_{0y}t + \frac{1}{2}a_y t^2$$

$$y_f = -15t$$

→ position vector at this time.

$$\vec{r}_f = x_f\hat{i} + y_f\hat{j}$$

$$\vec{r}_f = (20t + 2t^2)\hat{i} + (-15t)\hat{j}$$