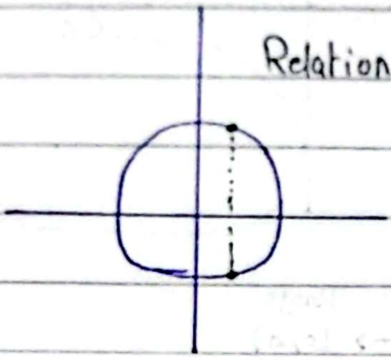
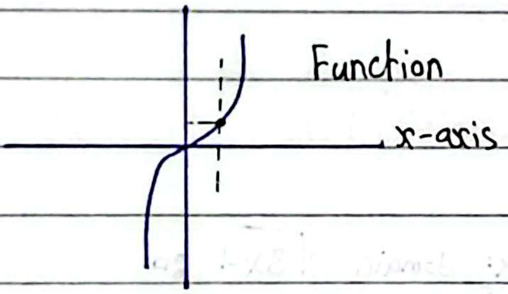
Monday 6/10/2025

Relation

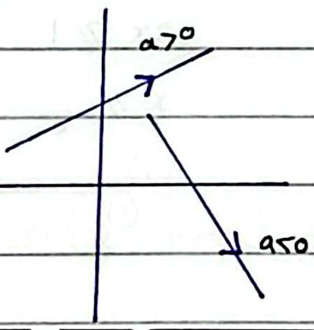


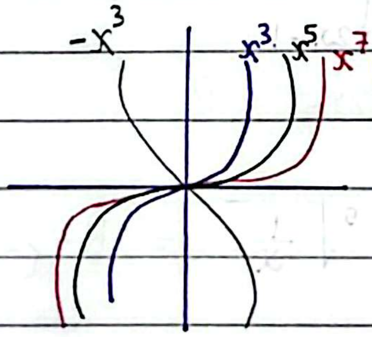
Function



* $f(x) = \overset{\text{slope}}{a}x + b$

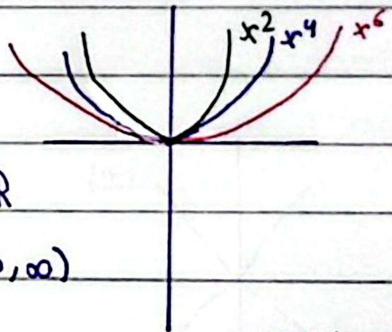
$\underset{\text{Dom}}{R} \longrightarrow \underset{\text{Range}}{R}$





$\underset{\text{Dom}}{R} \longrightarrow \underset{\text{Rang}}{R}$

Relation علاقة
Function اقتران
Domain مجال
Range مدى
polynomieals كثيرات الحدود



Domain : R

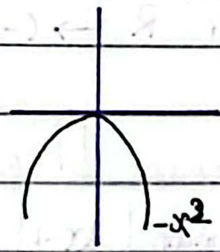
Range : $[0,\infty)$

* Domain ← محور x

* Range ← محور y

Domain : R

Range : $(-\infty, 0]$



Monday 6/10/2025

* polynomials : Dom $\to \mathbb{R}$

$f(x) = ax^2 + bx + c$

$a<0$: $(-\infty, f(\frac{-b}{2a})]$ $\qquad$ $a>0$: $[f(\frac{-b}{2a}), \infty)$

* find domain and Range :

$\mathbb{R}$

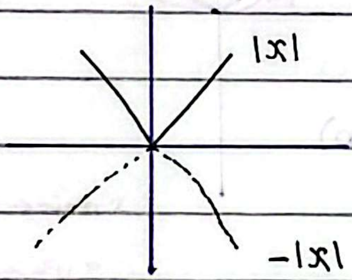
* $f(x) = \overset{b}{2x} - \overset{a}{x^2}$

Range

$\frac{-b}{2a} = \frac{-2}{2(-1)} = 1$

Range $(-\infty, f(1)] \Rightarrow (-\infty, 1]$ Range

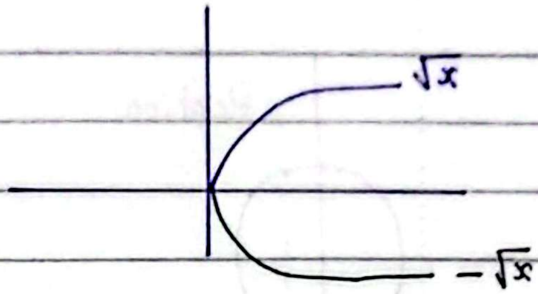
$|x|$



	Domain	Range
$\lvert x\rvert$:	$\mathbb{R} \to$	$[0, \infty)$
$-\lvert x\rvert$:	$\mathbb{R} \to$	$(-\infty, 0]$

* Domain $|f(x)|$ = dom $f(x)$

domain $|3x-1| = \mathbb{R}$



	Domain	Range
$\sqrt{x}$:	$[0, \infty) \to$	$[0, \infty)$
$-\sqrt{x}$:	$[0, \infty) \to$	$(-\infty, 0]$

* Domain $\sqrt[n]{f(x)}$ = $f(x) \geq 0$ (زوجي)

Ex: domain $\sqrt{3x-1}$:

$3x-1 \geq 0$

$3x \geq 1$

$x \geq \frac{1}{3}$ $\qquad [\frac{1}{3}, \infty)$

* Domain $\sqrt[n]{f(x)}$ = domain $f(x)$ (فردي)

Ex: domain $\sqrt[3]{2x-1}$

Domain $2x-1 = \mathbb{R}$

Ex: Domain $\sqrt[5]{\frac{1}{x}} = \mathbb{R} - \{0\}$

NOTEBOOK

Monday 6/10/2025

* $Dom \{ f \pm_{*} g \}$

$Dom\ f \cap Dom\ g$

* $Dom\ \frac{f}{g} \Rightarrow Dom\ f \cap Dom\ g - \{g(x)=0\}$

$|x| = a \Rightarrow x = \pm a$

$|x| < a = -a < x < a$

$|x| \geq a = x \leq -a$ or $x \geq a$

$\sqrt{x^2} = |x|$

Ex: Find Domain and Range:

1) $\sqrt{4-x^2}$

$4 - x^2 \geq 0$

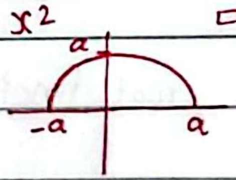
$-x^2 \geq -4$

$\sqrt{x^2} \leq \sqrt{4}$

$|x| \leq 2$

$-2 \leq x \leq 2$

$\sqrt{a^2 - x^2}$ ملاحظة مهمة :



ex: $\sqrt{25 - x^2}$

Domain: $[5, 5]$

Range: $[0, 5]$

* Chapter 1: Functions and models

Definition: $f: A \to B$, f is a function from A to B "f is a relation that assigns every of A "$a \in A$" to a Unique element "$b \in B$" denoted by y $f(x)$

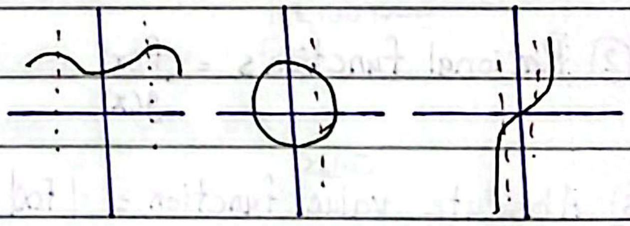
* الاقتران هو علاقة بحيث يرتبط بعنصر واحد فقط بالمدى

* vertical line test:

if the vertical line intersects the graph at exactly one point then y is a function

* Example:

which of the following is function?

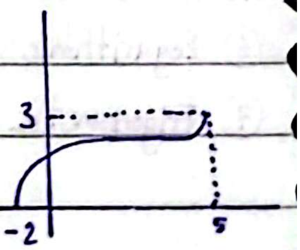


Function relation function

* Example:

Domain: $[-2, 5]$

Range: $[0, 3]$



Subject Wednesday 8/10/2025

Date

(*) Note that:

Domain : (Dom) : the set of all possible in put (x-axis)

Range: the set of all possible out put (y-axis)

• Types of functions :-

① polynomials كثيرات الحدود

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots$

$n = 1, 2, 3 \dots$ +ve integers : عدد صحيح موجب

$a_n \in R$

R = عدد حقيقي

② Rational functions الاقترانات النسبية $= \frac{f(x)}{g(x)}$

③ Absolute value function المطلق $= |f(x)|$

④ root functions الجذر , $f(x) = \sqrt[n]{g(x)}$

⑤ Exponential functions الأسي , $f(x) = a^x$

⑥ Logarithmic functions , $f(x) = \log_a x$

⑦ Trigonometic functions الاقترانات الدائرية

vertical line test اختبار الخط العمودي

polynomials كثيرات الحدود

integers عدد صحيح

Rational functions $\frac{f(x)}{g(x)}$ الاقترانات النسبية

Absolute value function اقتران القيمة المطلقة

root functions الجذور

Exponential function الاقتران الأسي

Logarithmic functions اللوغاريتمي

Trigonometic functions الاقترانات الدائرية

* Example : classify each functions :-

① $f(x) = 3x^8 - 7x^2 + x - 1$ polynomial function of degree 8

② $f(x) = x^2 - 2x + 5$ poly, Quadratic function

③ $f(x) = 2x + 1$ poly, linear.

④ $f(x) = x^3 - 2x^2 + 1$ poly, Cubic fun.

⑤ $f(x) = \frac{3x - 1}{x^2 + 1}$ Rational function.

⑥ $f(x) = \sqrt{3x + 1}$ root function.

⑦ $f = |x - 1|$ Absolut value function.

NOTEBOOK

Subject Wednesday 8/10/2025 Date No.

(*) Even and odd functions :-

* Definition :

① A function f is said to be an even function , if: $f(-x) = f(x)$

Ex:) $f(x) = x^2$

$f(x) = x^4$

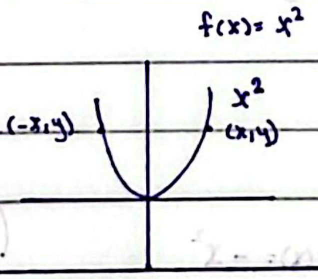
$f(x)$ x^{n} even

$f(x) = |x|$

$f(x) = \cos x$ $\cos(-x) = \cos x$

* note that :

even function متماثل symmetic about y- axis.



② A function f is said to be an odd function if $f(-x) = -f(x)$.

Ex:) $f(x)=x$

$f(x) = x^3$

$f(x) = x^{n} \rightarrow$ odd

$f(x) = \sin x$ $\sin(-x) = -\sin(x)$

Quadratic function اقتران تربيعي

linear function اقتران خطي

Cubic function اقتران تكعيبي

Even function اقتران زوجي

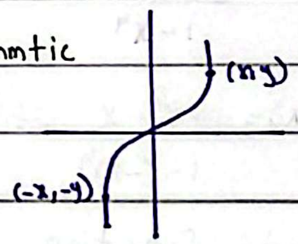
odd function اقتران فردي

symmetic متماثل

* not that :

$f(x) = x^3$

odd function symmtic about y=x "origin".



(*) Rule:

Even $\overset{*}{\div}$ Even = Even

Even $\overset{*}{\div}$ odd = odd

odd $\overset{*}{\div}$ Even = odd

odd $\overset{*}{\div}$ odd = Even

* Example: Determic whether each of the following function is even or odd or neither even nor odd

① $f(x) = \dfrac{3-x^4}{|x|}$

$f(-x) = \dfrac{3-(-x)^4}{|-x|} = \dfrac{3-x^4}{|x|}$ even function

Subject Wednesday 8/10/2025 Date No.

تابع للسؤال الأول في الصفحة

② $f(x) = x^3 + x$

$f(-x) = -x^3 - x$ odd function

③ $f(x) = \dfrac{\sin x + x^3}{\cos x}$

$f(-x) = \dfrac{\sin(-x) + -x^3}{\cos(-x)} = \dfrac{-\sin(x) - x^3}{\cos(x)}$

$= -\left(\dfrac{\sin x + x^3}{\cos(x)}\right)$

odd function

④ $f(x) = \dfrac{x^5 + x}{1 - x^4}$

$f(-x) = \dfrac{-x^5 - x}{1 - (-x)^4} = \dfrac{-x^5 - x}{1 - x^4} = \dfrac{-(x^5+x)}{1-x^4}$

odd function

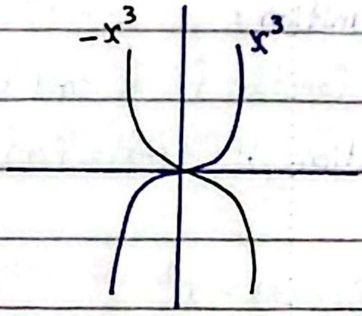
⑤ $f(x) = x + 5$

$f(-x) = -x + 5 \Rightarrow f(-x) = -(x - 5)$

$\neq -f(x)$

neither even nor odd

② cubic function:

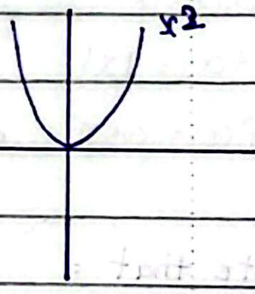


Dom: R

Rang: R

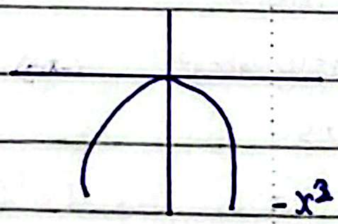
③ Quadratic function:

$f(x) = x^2$



Dom: R

Range: $[0, \infty)$



$f(x) = -x^2$

Dom: R

Rang: $(-\infty, 0]$

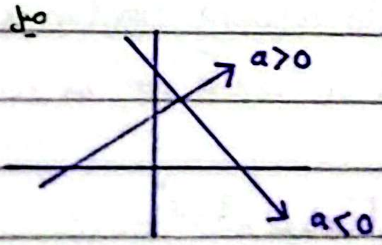
(*) Domain and Range :-

① linear function / $y = ax + b$

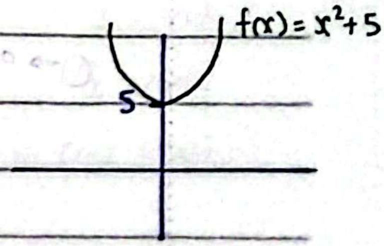
a = slope ميل

Dom: R

Rang: R



Ex ① $f(x) = x^2 + 5$

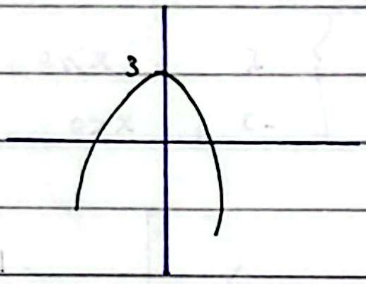


Dom: R

Range: $[5, \infty)$

Subject Wednesday 8/10/2025 Date No.

Ex) ② $f(x) = -x^2 + 3$



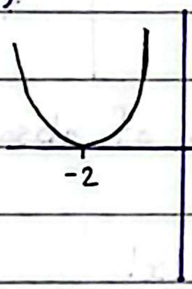
Dom: R

Range: $(-\infty, 3]$

③ $f(x) = (x+2)^2$

$x+2=0$

$x=-2$



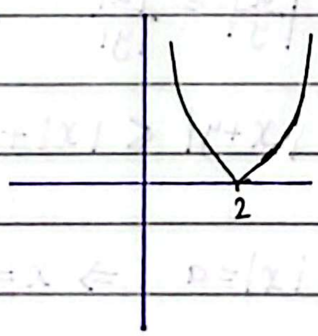
Dom: R

Range: $[0, \infty)$

④ $f(x) = (x-2)^2$

$x-2=0$

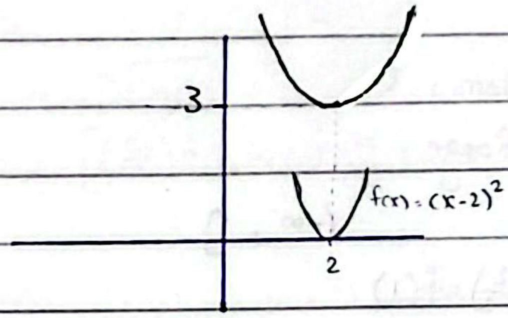
$x=2$



Dom: R

Range: $[0, \infty)$

⑤ $f(x) = (x-2)^2 + 3$

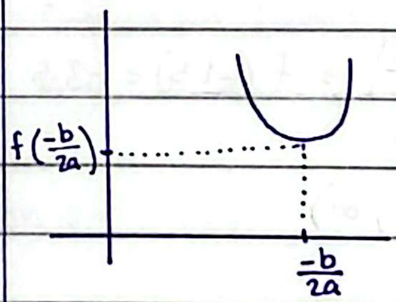


Dom: R

Range: $[3, \infty)$

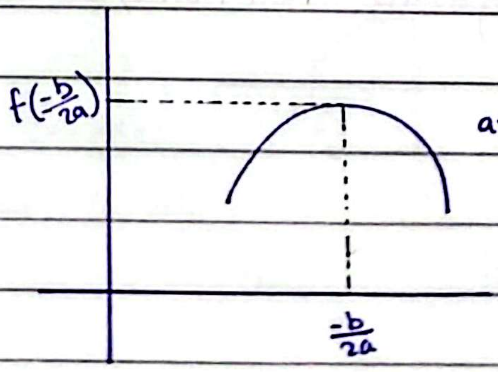
(*) note that: to find the range of quadratic function

$f(x) = ax^2 + bx + c$, $a \neq 0$



$a > 0$

Range $= [f(\frac{-b}{2a}), \infty)$



$a < 0$

Range $= (-\infty, f(\frac{-b}{2a}))$

Wednesday 8/10/2025

Ex) ① $f(x) = 2x - x^2$

Dom: R

Range: $(-\infty, f(\frac{-b}{2a})]$

$(-\infty, 1]$

$f(\frac{-2}{-2}) = f(1)$

$f(1) = 2-1 = 1$

② $f(x) = 2x^2 + 6x + 1$

Dom: R

Range: $[f(\frac{-b}{2a}), \infty)$

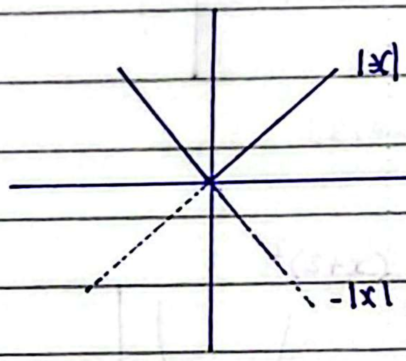
$f(\frac{-b}{2a}) = f(\frac{-6}{4}) = f(-1.5) = -3.5$

Range: $[-3.5, \infty)$

* Polynomial function ⇒ Dom = R *

⊛ Absolute value function

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



Dom: R
Range: $[0, \infty)$

Dom: R
Range: $[-\infty, 0]$

⊛ properties of absolute value :-

① $\sqrt{x^2} = |x|$

② $|-x| = x$

③ $|xy| = |x||y|$

④ $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

⑤ $|x+y| \leq |x| + |y|$

⑥ $|x| = a \Rightarrow x = \pm a$

⑦ $|x| \leq a \Rightarrow -a \leq x \leq a$

$\therefore x \in [-a, a]$

⑧ $|x| > a \Rightarrow x > a$ or $x < -a$

$(a, \infty) \cup (-\infty, -a)$

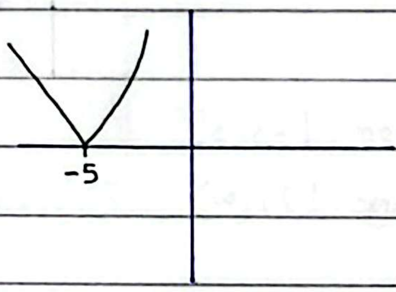
Subject Wednesday 2/10/2025 Date No.

* Example : Find dom and Range :-

① $f(x) = |x+5|$

$x+5=0$

$x=-5$



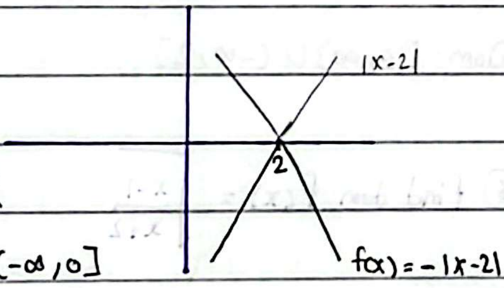
Dom : $\mathbb{R}$

Rang : $[0, \infty)$

② $f(x) = -|x-2|$

$x-2=0$

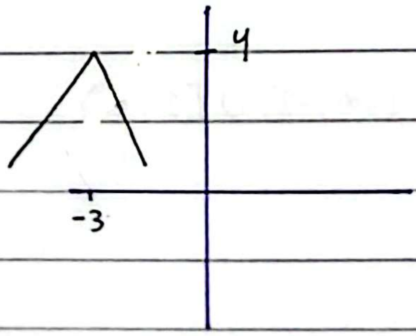
$x=2$



Dom : $\mathbb{R}$

Range : $(-\infty, 0]$

③ $f(x) = 4 - |x+3|$



Dom : $\mathbb{R}$

Range : $(-\infty, 4]$

④ Root function :

Rule:

① $f(x) = \sqrt[n]{g(x)}$ n: even number

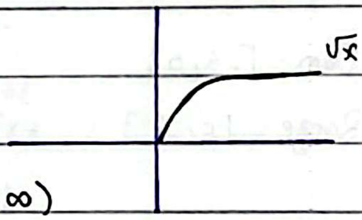
$dom(f) = \{x : g(x) \geq 0\}$

② $f(x) = \sqrt[n]{g(x)}$ n: odd number

$dom(f) = dom(g)$

* Example : Find dom and range

① $f(x) = \sqrt{x}$



Dom : $x \geq 0$ $[0, \infty)$

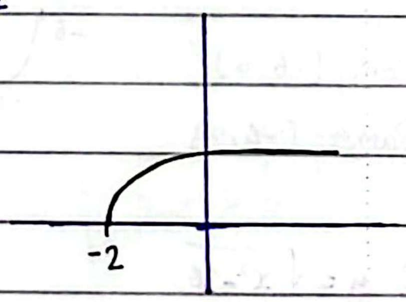
Range : $[0, \infty)$

② $f(x) = \sqrt{x+2}$

Dom : $x+2 \geq 0$

$x \geq -2$

$[-2, \infty)$



Rang : $[0, \infty)$

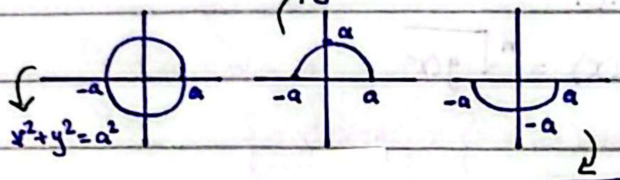
N O T E B O O K

Wednesday 8/10/2025

* Note:

نصف دائرة علوي

$y=\sqrt{a^2-x^2}$



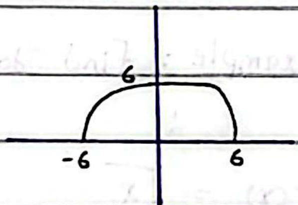
$x^2+y^2=a^2$

$y=-\sqrt{a^2-x^2}$

نصف دائرة سفلي

* Example: Find dom and range:

① $y=\sqrt{36-x^2}$



Dom: $[-6,6]$

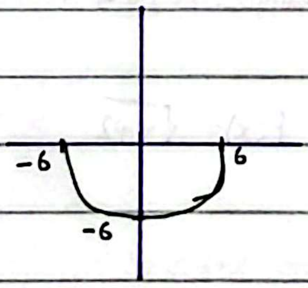
Range: $[0,6]$

$36-x^2 \geq 0$

$-x^2 \geq 36 \Rightarrow \sqrt{x^2} \leq \sqrt{36}$

$|x| \leq 6 \Rightarrow -6 \leq x \leq 6$

② $y=-\sqrt{36-x^2}$



Dom: $[-6,6]$

Range: $[-6,0]$

③ $y=\sqrt{x^2-36}$

$x^2-36 \geq 0$

$\sqrt{x^2} \geq \sqrt{36}$

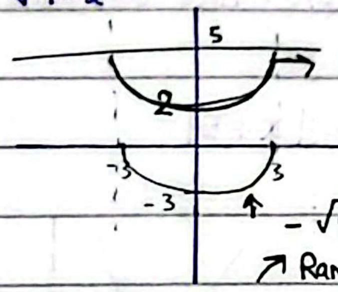
$|x| \geq 6$

$x \geq 6$ or $x \leq -6$

Dom: $[6,\infty) \cup (-\infty,6]$

range: $[0,\infty)$

④ $y=5-\sqrt{9-x^2}$



$5-\sqrt{9-x^2}$

$-\sqrt{9-x^2}$

Range: $[-3,0]$

+5 +5

Range: $[2,5]$

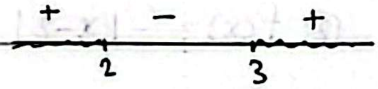
Dom: $[-3,3]$

Range: $[2,5]$

⑤ find dom $f(x)=\sqrt{x^2-5x+6}$

$x^2-5x+6 \geq 0$

$(x-3)(x-2) \geq 0$



Dom: $[3,\infty) \cup (-\infty,2]$

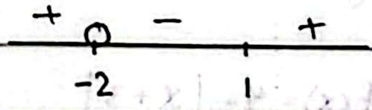
⑥ find dom $f(x)=\sqrt{\dfrac{x-1}{x+2}}$

[illegible]

$\dfrac{x-1}{x+2} \geq 0$

دائماً المقام

مفتوح 2



Dom: $(-\infty,-2) \cup [1,\infty)$

Subject Wednesday 8/10/2025 Date No.

⑦ find Dom $f(x) = \sqrt[3]{x-2}$

Dom $f(x)$ = Dom $(x-2)$ = R

⑧ find dom $f(x) = \sqrt[5]{\dfrac{2}{x-1}}$

Dom $f(x)$ = Dom $\left(\dfrac{2}{x-1}\right)$

$= R - \{1\}$

* find dom $f(x) = \sqrt[4]{|x+2|-5}$

$|x+2| - 5 \geq 0$

$|x+2| \geq 5$

$x+2 \geq 5$ or $x+2 \leq -5$

$x \geq 3$ or $x \leq -7$

Dom: $[3, \infty) \cup (-\infty, -7]$

⊛ New function from old go

→ Domain Rules:

① Dom $\{(f \pm g)(x)\}$ = Dom $f(x) \cap$ Dom $g(x)$

② Dom $\{(f\,g)(x)\}$ = Dom $f(x) \cap$ Dom $g(x)$

③ Dom $\left\{\left(\dfrac{f}{g}\right)(x)\right\} = \{\text{Dom } f(x) \cap \text{Dom } g(x)\} - \{x : g(x) = 0\}$

* Example: find domain:

① $f(x) = \overset{R}{\uparrow}{7} - \underbrace{\sqrt{x-2}}$

$x - 2 \geq 0 \Rightarrow x \geq 2$

Dom: $R \cup [2, \infty)$

② $f(x) = \sqrt{x+2} + \dfrac{1}{\sqrt{3-x}}$

$x+2 \geq 0$

$x \geq -2$

$[-2, \infty)$

لانه ضمن مقام
ما حطينا مساواة

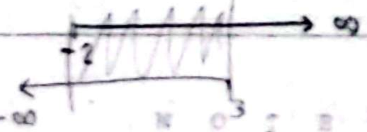
$3 - x > 0$

$x < 3$

$(-\infty, 3)$

Dom:

$[-2, \infty) \cup (-\infty, 3)$



Dom: $[-2, 3)$

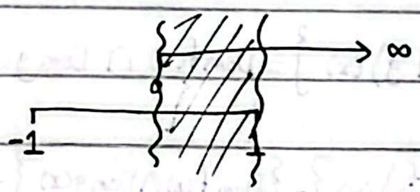
Subject Monday 13/10/2025 Date No.

③ $f(x) = \dfrac{\sqrt{x}}{1+\sqrt{1-x^2}}$

$\sqrt{x}$ → $x \geq 0$ $[0,\infty)$

denominator → R

$\sqrt{1-x^2}$ → نصف دائرة سفلي $[-1,1]$



Dom: $[0,1] - \{$ اصفار المقام $\}$

↳ $1+\sqrt{1+x^2} = 0$

ϕ $-1 \neq$ جذر → $\sqrt{1-x^2} = -1$

Dom: $[0,1]$

④ $f(x) = \dfrac{(x-1)^2}{x-1}$ $(x-1)^2 \to R$

$x-1 \to -\{1\}$

Dom: $R-\{1\}$.

⑤ $f(x) = \dfrac{x}{|x-1|-1}$ $x \to R$

$|x-1|-1 \to R-\{$ صفر المقام $\}$

$|x-1|-1=0$

$|x-1|=1$

$x-1=1$ or $x-1=-1$

$\boxed{x=2}$ $\boxed{x=0}$

Dom: $R-\{2,0\}$

⑥ $f(x) = \sqrt{|x-1|-4} + \dfrac{\sqrt{2x-1}}{3-|x|}$

$2x-1 \geq 0$

$2x \geq 1$

$x \geq \frac{1}{2}$

$3-|x| \to R-\{3,-3\}$

$3-|x|=0$

$|x|=3$

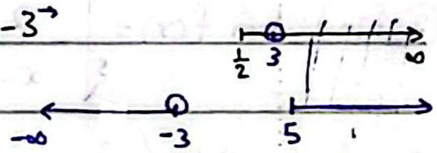
$x=3$ or $x=-3$

$|x-1|-4 \geq 0$

$|x-1| \geq 4$

$x-1 \geq 4$ or $x-1 \leq -4$

$x \geq 5$ او $x \leq -3$

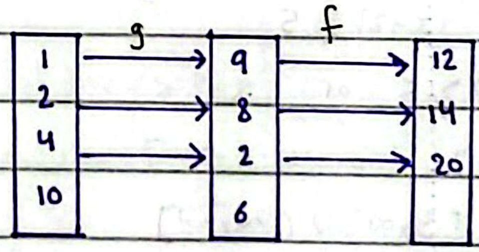


Dom: $[5,\infty)$

(*) تركيب composition of functions :

Given two functions f and g , the composite (fog) defined by $fog(x) = f(g(x))$;

fog: f circle g



$fog(1) = f(g(1)) = f(9) = 12$

$fog(2) = f(g(2)) = f(8) = 14$

$fog(4) = f(g(4)) = f(2) = 20$

$fog(10) = f(g(10)) = f(6) =$ undefined

NOTEBOOK

Subject Monday 13/10/2025

*** Example:** $f(x) = 10 - x^2$, $g(x) = \sqrt{x-1}$

find $f \circ g(x)$, $g \circ f(x) = ?$

① $f \circ g(x) = f(g(x)) = 10 - (\sqrt{x-1})^2$

$= 10 - x + 1 = 11 - x$

② $g \circ f(x) = g(f(x)) = g(10 - x^2)$

$= \sqrt{(10-x^2)-1} = \sqrt{9 - x^2}$

* متى يكون $g \circ f(x)$ ← $g(x) = f(x)$ عندما تكون دوال واحدة

$g(x) = f(x)$

انعكاس لها. ← $\sqrt[3]{x}$, x^3

x x

*** Example:** $f(x) = x^2 - 1$, $g(x) = \sqrt{3-x}$

Find

① $f \circ g(-1) = f(g(-1)) = f(2)$

$= 3$

② $g \circ f(1) = g(f(1)) = g(0) = \sqrt{3}$

very important **Example:** let $f \circ g(x) = 6x^2 - 10x + 5$, $f(x) = 2x + 1$

find $g(0) = ?$

$f(g(x)) = 6x^2 - 10x + 5$

$2g(x) + 1 = 6x^2 - 10x + 5$

$2g(0) + 1 = 5 \Rightarrow 2g(0) = 4$

$\boxed{g(0) = 2}$

very important

*** Example:** if $f(3x+5) = 6x + 11$

then $f(x) = ??$

$3x + 5 = y \Rightarrow 3x = y - 5$

$\boxed{x = \frac{y-5}{3}}$

$f(y) = \cancel{6}^{2}\left(\frac{y-5}{\cancel{3}}\right) + 11$

$f(y) = 2y - 10 + 11$

$f(y) = 2y + 1$ ← فقط استبدال

$f(x) = 2x + 1$

*** Example:** if $f \circ g(x) = x^2 + 6x + 6$ and $g(x) = x + 1$ find $f(x) = ??$

$f(g(x)) = x^2 + 6x + 6$

$f(x+1) = x^2 + 6x + 6$

$y = x + 1$

$\boxed{x = y - 1}$

$f(y) = (y-1)^2 + 6(y-1) + 6$

$f(y) = y^2 - 2y + 1 + 6y \cancel{-6} \cancel{+6}$

$f(y) = y^2 + 4y + 1$

$f(x) = x^2 + 4x + 1$

NOTEBOOK

Subject Monday 13/10/2025 Date No.

* Example: if $fog(x) = 3x^2+3x+2$, $f(x) = 3x+5$ find $g(x)$??

$f(g(x)) = 3x^2+3x+2$

$3(g(x))+5 = 3x^2+3x+2$

$3g(x) = 3x^2+3x-3$

$\boxed{g(x) = x^2+x-1}$

* Dom $fog(x)$

* Dom $(fog(x)) = \{x : x \in \text{Dom } g(x)$ and $g(x) \in \text{Dom } f(x)\}$ *

* Example: if Dom $f(x) = [1,4]$ then Dom $\{f(\underbrace{3x+4}_{g(x)})\}$ is

Dom $\{3x+4\} = R$

$(3x+4) \in [1,4]$

$1 \leq 3x+4 \leq 4$
$-4 \quad -4 \quad -4$

$\frac{-3}{3} \leq \frac{3x}{3} \leq \frac{0}{3}$

$-1 \leq x \leq 0$

Dom: $[-1,0]$

* Example: if $f(x) = \sqrt{1-x}$, $g(x) = 2x$ find dom $fog(x)$.

① $fog(x) = f(2x) = \sqrt{1-2x}$

Dom $(fog(x)) = 1-2x \geq 0$

$-2x \geq -1$

$x \leq \frac{1}{2}$ $\quad [-\infty, \frac{1}{2}]$

② Dom $g(x) = 2x$: R

$R \cap (-\infty, \frac{1}{2}]$

Dom: $(-\infty, \frac{1}{2}]$

* Example: if $f(x) = x^2-1$, $g(x) = \sqrt{3-x}$ find dom $fog(x)$ and dom $gof(x)$

① Dom $fog(x)$: $\to f(\sqrt{3-x}) = 3-x-1 = 2-x$

Dom: Dom$[2-x] \cap$ Dom $\sqrt{3-x}$

: $R \cap 3-x \geq 0 \Rightarrow x \leq 3$ $(-\infty, 3]$

Dom: $(-\infty, 3]$

② Dom $gof(x) = g(x^2-1) = \sqrt{3-x^2+1} = \sqrt{4-x^2}$

Dom $\sqrt{4-x^2} \cap$ Dom(x^2-1)

$[2,-2] \cap R$

Dom: $[2,-2]$

Subject Monday 13/10/2025

* Example: find dom $f(x) = \sqrt{4-\sqrt{x}}$ ($\sqrt{x}$ ↑ $g(x)$)

$f\circ g(x) = \sqrt{4-\sqrt{x}} \Rightarrow$ Dom: $4-\sqrt{x} \geq 0$

$-\sqrt{x} \geq -4$

$(\sqrt{x})^2 \leq (4)^2$

$x \leq 16 \quad (-\infty, 16]$

Dom: Dom $(4-\sqrt{x}) \cap$ Dom $\sqrt{x}$

$(-\infty, 16] \cap \quad x \geq 0 \ [0, \infty)$

Dom: $[0, 16]$

---

* Example: $f(x) = \dfrac{1+x}{1-x}$, $g(x) = \dfrac{x}{1-x}$

find dom $f\circ g(x)$

① $f\circ g(x) = f\left(\dfrac{x}{1-x}\right) = \dfrac{\frac{(1-x)}{(1-x)} + \frac{x}{1-x}}{\frac{(1-x)}{1-x}1 - \frac{x}{1-x}} =$

$= \dfrac{\frac{1}{\cancel{1-x}}}{\frac{1-2x}{\cancel{1-x}}} = \dfrac{1}{1-2x}$

Dom $\left(\dfrac{1}{1-2x}\right)$ → R ; → $R - \{$اصفار المقام$\}$; $1-2x=0$, $2x=1$, $\boxed{x=\frac{1}{2}}$

$R - \{\frac{1}{2}\}$

Dom $\left(\dfrac{x}{1-x}\right)$ → R ; → $R - \{1\}$

Dom $f\circ g(x) =$ Dom $\left(\dfrac{1}{1-2x}\right) \cap \left(\dfrac{x}{1-x}\right)$

$= R - \{\frac{1}{2}\} \cap R - \{1\}$

$= R - \{\frac{1}{2}, 1\}$

Wednesday 15/10/2025 Date No.

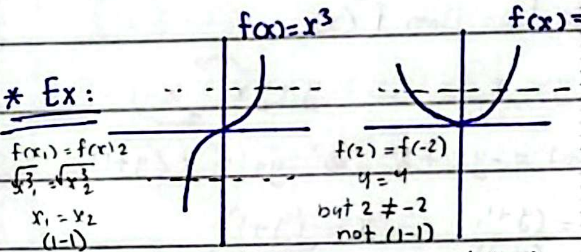
⊛ one to one function:

A function f is called a one to one function (1-1) if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

اختبار الخط الافقي

→ by Horizontal line test, A function is 1-1 iff no horizontal line intersects its graph more than once.

* Ex:



one to one function   not (1-1)

---

* Thrm: if $f(x)$ 1-1 function $\Leftrightarrow f(x)$ has inverse.

* inverse function $\equiv f^{-1}(x)$.

* $f^{-1}(x) \neq \dfrac{1}{f(x)}$.

Ex: if $f(x) = 3x - 5$ find $f^{-1}(x)$

$y = 3x - 5$

$3x = y + 5$

$\boxed{x = \dfrac{y+5}{3}}$

بس استبدال

$f^{-1}(x) = \dfrac{x+5}{3}$

N O T E B O O K

wednesday 15/10/2025

Subject Date No.

(*) Note that:

① $f \; : \overset{Dom}{A} \longrightarrow \overset{Range}{B}$

$f^{-1} : \overset{Dom}{B} \longrightarrow \overset{Range}{A}$ $\quad Dom\, f^{-1}(x) = Rang\, f$

* **Ex**: find the rang of $f(x) = \dfrac{x-1}{1+x}$

* range $f(x) =$ Dom $f^{-1}(x)$

① $y = \dfrac{x-1}{1+x} \Rightarrow y + yx = x - 1$

$y + 1 = -yx + x \Rightarrow y + 1 = x(1-y)$

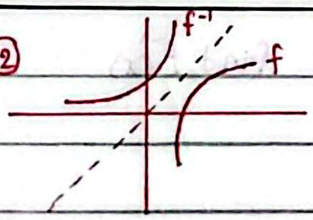
$x = \dfrac{(y+1)}{(1-y)}$

$f^{-1}(x) = \dfrac{1+x}{-x+1} \; \to R$ $\quad \leftarrow R - \{1\}$

Dom $f^{-1}(x)$: $R - \{1\}$

Rang $f(x) =$ Dom $f^{-1}(x) = R - \{1\}$

②



f & f^{-1} symmetic about $y = x$

③ $f \circ f^{-1}(x) = x \qquad f(f^{-1}(x)) = x$

$f^{-1} \circ f(x) = x \qquad f^{-1}(f(x)) = x$

* **Ex**: let $f(x) = x^3 + 4x + 1$, if $f^{-1}(c^3) = c$

the $c = ??$

$\cancel{f}(\cancel{f^{-1}}(c^3)) = f(c)$

$c^3 = \underset{\text{نعوض}}{f(c)}$

$c^3 = c^3 + 4c + 1$

$4c + 1 = 0 \Rightarrow \boxed{c = \frac{-1}{4}}$

Ex: let $f(x) = \dfrac{2x^3}{x^4+1}$, find x such that $1 = f^{-1}(x)$

$1 = f^{-1}(x) \Rightarrow f(1) = \cancel{f}(\cancel{f^{-1}}(x))$ نركب $f(x)$ للطرفين عشان نتخلص من $f^{-1}(x)$

$f(1) = x$

$\boxed{1 = x}$

Ex: $f(x) = x^3 + 4x + 1$ find $f^{-1}(6)$ * أكثر سؤال بيتكرر في الامتحان

صيغة السؤال * بنحلها بالتجريب $f(?) = 6$

$f(1) = 6$

$f^{-1}(6) = 1$

OR

$\cancel{f}(\cancel{f^{-1}}(6)) = f(y)$

$6 = f(y)$

$6 = y^3 + 4y + 1$

$y^3 + 4y - 5 = 0$

هذه الطريقة ما بحل زيها .

القسمة

$y - 1 \;)\; y^3 + 4y - 5$

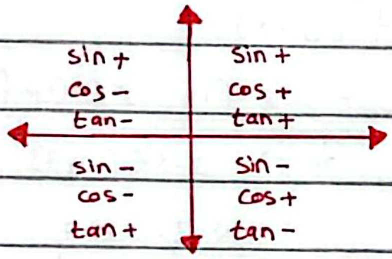
* حل اسئلة زي هيك فقط بالتجريب

ما بحلها زي هيك

Subject wednesday 15/10/2025 Date No.

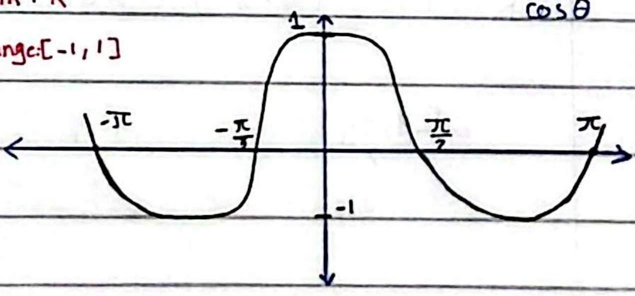
* Trigonometric functions *

0	$\frac{\pi}{4}$ 45	$\frac{\pi}{3}$ 60°	$\frac{\pi}{6}$ 30°	الزاوية
0	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\cos\theta$
1	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sin\theta$
2π 360°	$\frac{3\pi}{2}$ 270°	π 180°	$\frac{\pi}{2}$ 90°	الزاوية
1	0	−1	0	$\cos\theta$
0	−1	0	1	$\sin\theta$



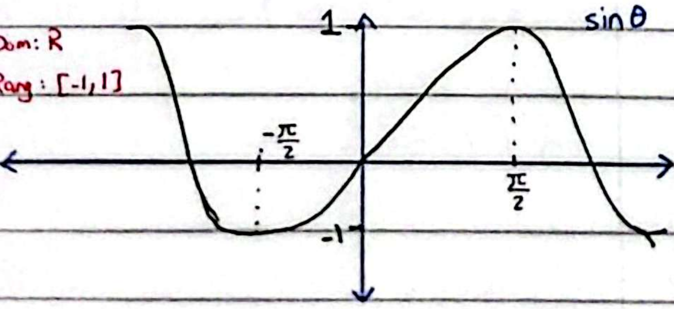
Dom : R

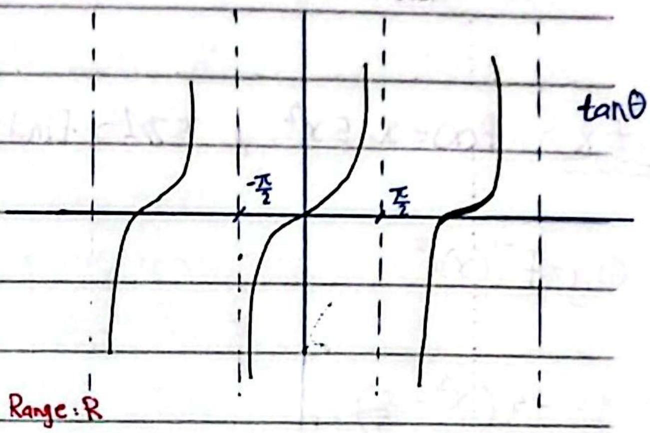
Range: [−1, 1]



Dom: R

Rang: [−1, 1]





Range: R

Dom: $R - \{\frac{\pi}{2} + n\pi\}$

$\{n = 0, \mp 1, \pm 2, \dots\}$

Ex: Find, dom $\{\cos(2x-1)\}$ composition function

$f(x) = \cos(x)$, $g(x) = 2x - 1$

$f \circ g(x) = \cos(2x-1)$

$R \cap R = R$

Ex: Find dom $\{\sin(\sqrt{x-5})\}$

Dom $\{\sqrt{x-5}\}$ $\cap$ Dom(sin)

$x - 5 \geq 0$ R

$x \geq 5$

$[5, \infty)$

Dom $\sin(\sqrt{x-5}) = [5, \infty)$

Subject wednesday 15/10/2025 Date No.

Ex: Find the range of

ⓐ $f(x) = \frac{3}{5+\cos x}$

$-1 \le \cos x \le 1$
$+5 \quad +5 \quad +5$

$4 \le \cos x + 5 \le 6$

نقلب المتباينة

$\frac{1}{4} \ge \frac{1}{\cos x + 5} \ge \frac{1}{6}$ $\quad \times 3 \quad \times 3 \quad \times 3$

$\frac{3}{4} \ge \frac{3}{\cos + 5} \ge \frac{3}{6}$

Range: $[\frac{3}{6}, \frac{3}{4}] = [\frac{1}{2}, \frac{3}{4}]$

ⓑ $f(x) = 2\sin^2 x + 3$

$(-1 \le \sin x \le 1)^2$

$0 \le \frac{\sin^2 x}{\cos^2 x} \le 1$

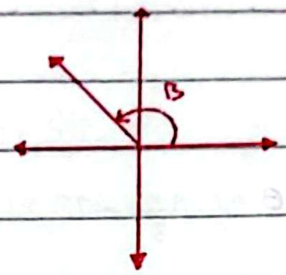
$0 \le \left|\frac{\sin x}{\cos x}\right| \le 1$

$2\times[\ 0 \le \sin^2 x \le 1$

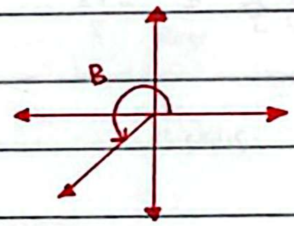
$3+\ (0 \le 2\sin^2 x \le 2)$

$3 \le 2\sin^2 x + 3 \le 5$

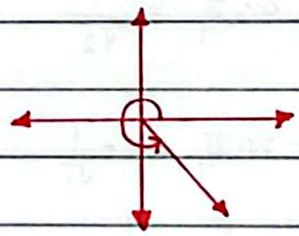
Range: $[3, 5]$



الربع الثاني $Q_2 \rightarrow Q_1$ الربع الأول $\qquad \theta = \pi - B$



$Q_3 \rightarrow Q_1 \qquad \theta = B - \pi$



$Q_4 \rightarrow Q_1 \qquad \theta = 2\pi - B$

Ex: Find $\sin B$ and $\cos B$ for:

(1) $B = \frac{5\pi}{6}$ الربع الثاني

$\theta = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$

$\cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = \frac{-\sqrt{3}}{2}$

$\sin\frac{5\pi}{6} = \sin\frac{\pi}{6} = \frac{1}{2}$

Wednesday 18/10/2025

الربع الثالث

② $B = \frac{4\pi}{3}$

$\theta = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$

$\cos\frac{4\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$

$\sin\frac{4\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

③ $B = \frac{7\pi}{4}$ الربع الرابع

$\theta = 2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$

$\cos\frac{7\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\sin\frac{7\pi}{4} = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

* Identities المتطابقات

* $\sin^2 x + \cos^2 x = 1$
* $\sin 2x = 2\sin x\cos x$
* $\cos 2x = \cos^2 x - \sin^2 x$ / $\cos 2x = 1 - 2\sin^2 x$ / $\cos 2x = 2\cos^2 x - 1$
* $\sin^2 x = \frac{1}{2}(1-\cos 2x)$
* $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

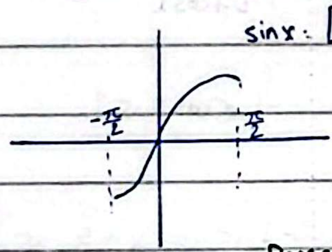
$\sin(x \mp y) = \sin x\cos y \mp \cos x\sin y$

$\cos(x \mp y) = \cos x\cos y \pm \sin x\sin y$

* $1 + \tan^2 x = \sec^2 x$
* $1 + \cot^2 x = \csc^2 x$

* Inverse trigonometric functions.

$\sin x : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$ (Dom → Range)

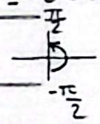


① $\sin^{-1} x : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (Dom → Range)

$\therefore$ Dom $\sin^{-1} x = [-1, 1]$

Rang of $\sin^{-1} x = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

الربع الاول والربع الثاني :



Ex: find domain of $f(x) = \sin^{-1}(2x+1)$ composition function

$2x+1 \in$ Dom $\sin^{-1}$ $[-1, 1]$

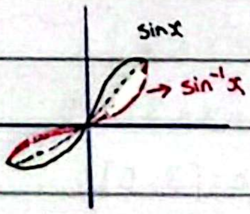
$-1 \le 2x+1 \le 1$

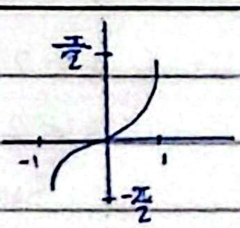
$\frac{-2}{2} \le \frac{2x}{2} \le \frac{0}{2}$

$-1 \le x \le 0$

$\therefore$ Dom $[-1, 0] \cap \mathbb{R}$

②





③ $f(f^{-1}(x)) = x$, if $x \in$ Dom $f^{-1}(x)$

$f^{-1}(f(x)) = x$, if $x \in$ Dom $f(x)$

Subject Wednesday 18/10/2025 Date No.

حفظ ∴ $\sin(\sin^{-1}(x)) = x$ if $x \in [-1,1]$

بلعبوا بعض اذا كان داخل ضمن الفترة المحددة $\sin^{-1}(\sin(x)) = x$ if $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ مجاله هذه

④ $\sin^{-1}(x)$ is odd function since

$\sin^{-1}(-x) = -\sin^{-1}x$

Ex: find the exact value of so

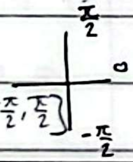
① $\sin^{-1}(\frac{1}{2}) = \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$= \frac{\pi}{6}$

② $\sin^{-1}(-1) = -\frac{\pi}{2}$ ما بصير اكتب $\frac{3\pi}{2}$ لانها خارج المجال $\frac{3\pi}{2} \notin [-\frac{\pi}{2}, \frac{\pi}{2}]$

or

$-\sin(1) = -\frac{\pi}{2}$



③ $\sin^{-1}(\frac{-\sqrt{3}}{2}) = -\sin^{-1}(\frac{\sqrt{3}}{2})$

$= -\frac{\pi}{3}$

④ $\sin^{-1}(\sin(\frac{\pi}{6})) = \frac{\pi}{6}$ الربع الاول مجاله

بيلغوا بعض لانها داخل المجال $\in [-\frac{\pi}{2}, \frac{\pi}{2}]$

⑤ $\sin(\sin^{-1}(\frac{1}{2})) = \frac{1}{2}$ مجاله

$\in [-1,1]$

⑥ $\sin^{-1}(\sin(\frac{2\pi}{3}))$ ② الربع الثاني ② $\theta = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

لا تنتمي $\notin [-\frac{\pi}{2}, \frac{\pi}{2}]$ ما تتلغى $\sin\frac{2\pi}{3} = +\sin\frac{\pi}{3}$

$\cancel{\sin^{-1}}(\cancel{\sin}(\frac{\pi}{3}))$

$= \frac{\pi}{3}$

* في حال كانت الزاوية مش بالربع الاول ؟؟
① الربع
② ...
③ ...

⑦ $\sin^{-1}(\sin(\frac{5\pi}{4}))$ خارج المجال $\notin [-\frac{\pi}{2}, \frac{\pi}{2}]$ الربع الثالث

$\theta = \frac{5\pi}{4} - \pi = \frac{\pi}{4}$

$\sin\frac{5\pi}{4} = -\sin\frac{\pi}{4}$

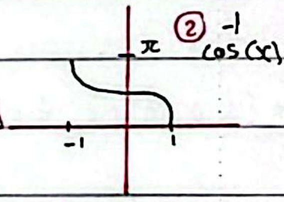
$\cancel{\sin^{-1}}(-\cancel{\sin}(\frac{\pi}{4})) = -\frac{\pi}{4}$

* $\cos^{-1}(x)$:

① $\cos x : [0,\pi] \to [-1,1]$ (Dom, Range)

$\cos^{-1}x : [-1,1] \to [0,\pi]$ (Dom, Range)

② $\cos^{-1}(x)$



③ $\cos(\cos^{-1}(x)) = x$, if $x \in [-1,1]$ مجاله هذه Dom $\cos^{-1}(x)$

$\cos^{-1}(\cos x) = x$, if $x \in [0,\pi]$

④ $\cos^{-1}(x)$ is neither even nor odd

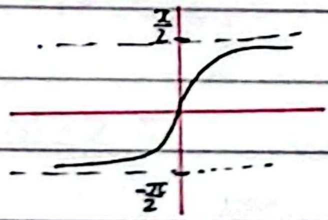
حفظ [$\cos^{-1}(-x) = \pi - \cos^{-1}(x)$

* $\tan^{-1}x$

① $\tan x : (-\frac{\pi}{2}, \frac{\pi}{2}) \to R$

$\tan^{-1}x : R \to (-\frac{\pi}{2}, \frac{\pi}{2})$

②



Wednesday 15/10/2025

③ $\tan(\tan^{-1}x) = x$, if $x \in \mathbb{R}$

$\tan^{-1}(\tan x) = x$, if $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$

④ $\tan^{-1}x$ is odd function

$\tan^{-1}(-x) = -\tan^{-1}x$

Ex: find dom $\cos^{-1}(3x-1)$

$-1 \leq 3x-1 \leq 1$
$+1 \quad\quad +1 \quad\quad +1$

$0 \leq 3x \leq 2$
$\div 3 \quad\quad \div 3 \quad\quad \div 3$

$0 \leq x \leq \frac{2}{3}$

$\therefore$ Dom $[0, \frac{2}{3}]$

Ex: Find the range of the function

$f(x) = \pi + 2|\tan^{-1}x|$

$-\frac{\pi}{2} < \tan^{-1}x < \frac{\pi}{2}$ مطلق $0 < |\tan^{-1}x| < \frac{\pi}{2}$

$(0 < |\tan^{-1}x| < \frac{\pi}{2}) \times 2$

$0 < 2|\tan^{-1}x| < \pi$
$+\pi \quad\quad +\pi \quad\quad +\pi$

$\pi < 2\tan^{-1}x + \pi < 2\pi$

Range: $(\pi, 2\pi)$

الفترة بتكون مفتوحة لأنه tan

Ex: find the exact value of:

لازم تكون الإجابة في الربع الأول والثاني

① $\cos^{-1}(\frac{1}{2}) = \frac{\pi}{3} \in [0, \pi]$

② $\tan^{-1}(\frac{1}{\sqrt{3}}) = \frac{\pi}{6} \in (-\frac{\pi}{2}, \frac{\pi}{2})$

③ $\cos^{-1}(\frac{-1}{\sqrt{2}}) = \pi - \cos^{-1}(\frac{1}{\sqrt{2}})$

$= \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

*ملاحظة - اذا كان داخل $\cos^{-1}$(سالب) ← $\pi - \cos^{-1}$(الزاوية)

④ $\sec^{-1}(-2) =$

Note * $\sec^{-1}(x) = \cos^{-1}(\frac{1}{x})$

$= \cos^{-1}(-\frac{1}{2}) = \pi - \cos^{-1}(\frac{1}{2})$

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

⑤ $\tan^{-1}(-1) = -\tan^{-1}(1) = -\frac{\pi}{4}$

⑥ $\cos^{-1}(\cos(\frac{12\pi}{7}))$

الربع الرابع

صار؟ $\notin [0, \pi]$ لأ

$\theta = 2\pi - \frac{12\pi}{7} = +\frac{2\pi}{7}$

$\cos^{-1}(\cos(\frac{12\pi}{7})) = \frac{2\pi}{7}$

tan داخلها مجال $\frac{\pi}{8}$

⑦ $\tan^{-1}\left(\tan\left(\frac{\pi}{8}\right)\right) = \frac{\pi}{8}$

$\in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ هل تنتمي لمجال

مفتوحة

الربع الثالث

⑧ $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right) = \cos^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right)$

$\notin [0, \pi]$

$= \pi - \cancel{\cos^{-1}}\left(\cancel{\cos}\left(\frac{\pi}{6}\right)\right)$

$\theta = \frac{7\pi}{6} - \pi = \frac{\pi}{6}$

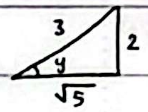
$= \pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$

② $\cot\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$

① $\sin\left(\sin^{-1}\frac{2}{3}\right) = \sin y$

$\frac{\text{المقابل}}{\text{الوتر}} = \frac{2}{3} = \sin y$



$\cot y = \frac{\text{المجاور}}{\text{المقابل}}$

$\tan y = \frac{\text{المقابل}}{\text{المجاور}}$

$\boxed{\cot y = \frac{\sqrt{5}}{2}}$

* Identites for inverse Ingonometic functions

Ex: Find the exat value for 80

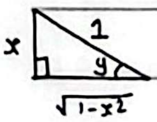
طريقة الحل [شرط انها تنتمي لل Dom]

① $\cos(\sin^{-1} x)$

① $\sin^{-1} x = y$

نركب sin للطرفين ② $\cancel{\sin}\left(\sin^{-1}(x)\right) = \sin y$

③ $\sin y = x$



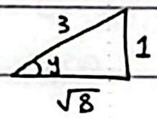
$\frac{\text{المقابل}}{\text{الوتر}} = \frac{x}{1} = \sin y$

$\cos(y) = \sqrt{1-x^2} = \frac{\text{المجاور}}{\text{الوتر}}$

③ $\tan\left(\sin^{-1}\left(-\frac{1}{3}\right)\right) = -\tan\left(\sin^{-1}\left(\frac{1}{3}\right)\right)$

$\sin\left(\sin^{-1}\left(\frac{1}{3}\right)\right) = \sin y$

$\frac{\text{المقابل}}{\text{الوتر}} = \frac{1}{3} = \sin y$



$= -\tan(y) = -\frac{\text{المقابل}}{\text{المجاور}}$

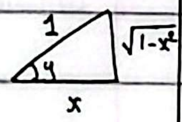
$= -\frac{1}{\sqrt{8}}$

Subject Wednesday 15 / 10 / 2025 Date No.

④ $\sin(2\cos^{-1}x)$

$\cos(\cos^{-1}x) = \overset{\cos}{y}$

$\dfrac{\text{المجاور}}{\text{الوتر}} = \dfrac{x}{1} = \cos y$



$\rightarrow \sin(2y) = 2\sin y\cos y$

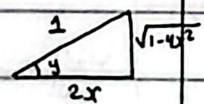
$= 2\left(\dfrac{\sqrt{1-x^2}}{1}\right)\left(\dfrac{x}{1}\right)$

$\boxed{\sin(2y) = 2x\sqrt{1-x^2}}$

⑤ $\sin(\pi + \cos^{-1}(2x))$

$\cos y = \overset{\cos}{\cos^{-1}}(2x)$

$\cos y = \dfrac{2x}{1} = \dfrac{\text{المجاور}}{\text{الوتر}}$



$\rightarrow \sin(\pi+y) = \overset{(0)}{\sin\pi}\cancel{\cos y} + \sin y\overset{(-1)}{\cos\pi}$

$= -\sin y$

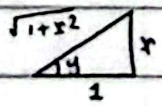
$= -\sqrt{1-4x^2}$

⑥ $\sec(\tan^{-1}x)$

$\cos = \dfrac{\text{المجاور}}{\text{الوتر}}$

$\tan(\tan^{-1}x) = \overset{\tan}{y}$

$\dfrac{\text{المقابل}}{\text{المجاور}} = \dfrac{x}{1} = \tan y$



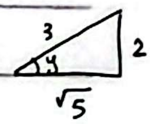
$\rightarrow \sec(y) = \dfrac{\text{الوتر}}{\text{المجاور}}$

$= \sqrt{1+x^2}$

⑦ $\sin\left(2\sin^{-1}\left(\frac{2}{3}\right)\right)$

$\sin\left(\sin^{-1}\left(\frac{2}{3}\right)\right) = \overset{\sin}{(y)}$

$\dfrac{\text{المقابل}}{\text{الوتر}} = \dfrac{2}{3} = \sin y$



$\rightarrow \sin(2y) = 2\sin y\cos y$

$= 2\left(\frac{2}{3}\right)\left(\frac{\sqrt{5}}{3}\right)$

$\sin(2y) = \dfrac{4\sqrt{5}}{9}$

NOTEBOOK