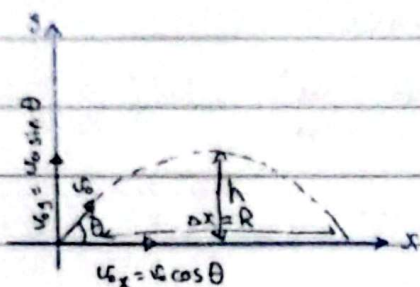


* Projectile Motion [object 102] *



* $a_x = 0$

* $a_y = -g$

x $\Delta x = v_{0x} t + \frac{1}{2} a_x t^2$
 $\Delta x = v_{0x} t$

$\Delta x = v_0 t \cos \theta$

$t = \frac{\Delta x}{v_0 \cos \theta}$

y $\Delta y = v_{0y} t - \frac{1}{2} g t^2$

الارتفاع الذي يسقط عن
 نقطة البداية $\leftarrow 0 = v_{0y} t - \frac{1}{2} g t^2$

$v_0 t \sin \theta = \frac{1}{2} g t^2$

$v_0 \sin \theta = \frac{1}{2} g t$

$t = \frac{2 v_0 \sin \theta}{g}$

$t_x = t_y$

$\frac{\Delta x}{v_0 \cos \theta} = \frac{2 v_0 \sin \theta}{g}$

$\Delta x = \frac{2 v_0^2 \sin \theta \cos \theta}{g}$

$\Delta x = R = \frac{v_0^2 \sin 2\theta}{g}$ Range
 $\sin 2\theta = 2 \sin \theta \cos \theta$ maximum

Height (max height)

$\Delta y = v_{0y} t - \frac{1}{2} g t^2$

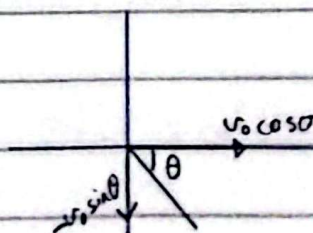
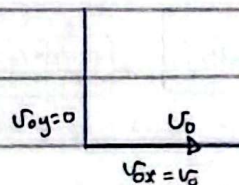
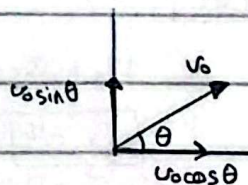
$h = v_0 t \sin \theta - \frac{1}{2} g t^2$

$t = \frac{v_0 \sin \theta}{g}$

$h = (v_0 \sin \theta) \left(\frac{v_0 \sin \theta}{g} \right) - \frac{1}{2} \left(\frac{v_0 \sin \theta}{g} \right)^2$

$= \frac{v_0^2 \sin^2 \theta}{g} - \frac{1}{2} \frac{v_0^2 \sin^2 \theta}{g}$

$h = \frac{v_0^2 \sin^2 \theta}{2g}$



* maximum Range *

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

we need to make $R_{\max} \Rightarrow \sin 2\theta = 1$

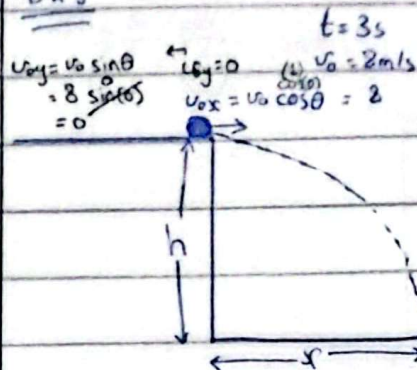
$$2\theta = 90$$

$$\theta = 45^\circ$$

* ملاحظة * $\theta = 45^\circ$

الزاوية المطلقة للزاوية 90° يعطوا نفس Range

Ex 8



$$\textcircled{1} x = ??$$

$$\textcircled{2} h = ??$$

$$\textcircled{1} \Delta x = v_{0x} t$$

$$= 8(3)$$

$$\Delta x = 24 \text{ m}$$

$$\textcircled{2} h = \Delta y$$

$$\Delta y = v_{0y} t - \frac{1}{2} g t^2$$

$$= -\frac{1}{2} (9.8) (3)^2$$

$$\Delta y = -44.1 \text{ m}$$

problems

Q15 page: 102

A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. what is the angle of projection?

$$R = 3h_{\max}$$

$$h_{\max} = \frac{v_0^2 \sin^2 \theta}{2g}$$

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

$$\frac{v_0^2 \sin \theta \cos \theta}{g} = \frac{3 v_0^2 \sin^2 \theta}{2g}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{4}{3} \Rightarrow \tan \theta = \frac{4}{3}$$

$$\theta = \tan^{-1} \frac{4}{3} \Rightarrow \theta = 53.13^\circ$$

* Homework *

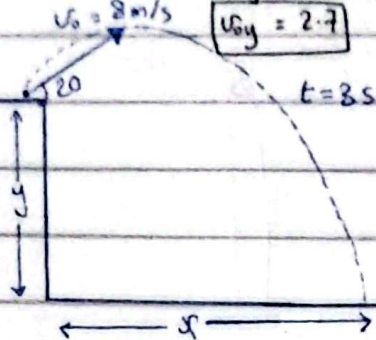
$$v_{0x} = v_0 \cos 20$$

$$= (8)(0.93)$$

$$v_{0x} = 7.5$$

$$v_{0y} = v_0 \sin 20$$

$$v_{0y} = 2.7$$



$$x = ?? \quad y = ??$$

$$\textcircled{1} \Delta x = v_{0x} t$$

$$= 7.5(3)$$

$$\Delta x = 22.5$$

$$\textcircled{2} \Delta y = v_{0y} t - \frac{1}{2} g t^2$$

$$= (2.7)(3) - \frac{1}{2} (9.8)(3)^2$$

$$= 8.1 - 44.1$$

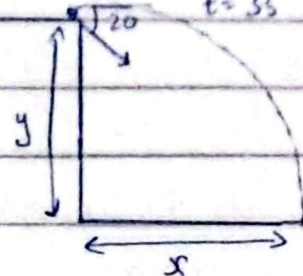
$$\Delta y = -36$$

$$v_{0x} = v_0 \cos 20 = 7.5$$

$$v_{0y} = v_0 \sin 20 = -2.7$$

$$v_0 = 8 \text{ m/s}$$

$$t = 3 \text{ s}$$



$$x = ?? \quad y = ??$$

$$\textcircled{1} \Delta x = v_{0x} t = 22.5$$

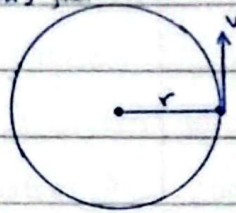
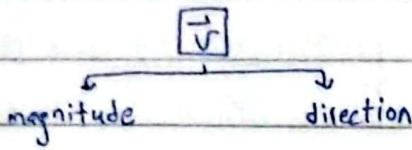
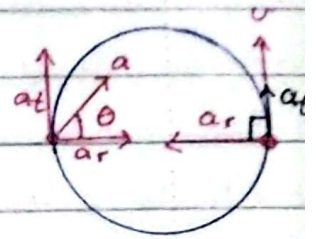
$$\textcircled{2} \Delta y = v_{0y} t - \frac{1}{2} g t^2$$

$$= (-2.7)(3) - \frac{1}{2} g t^2$$

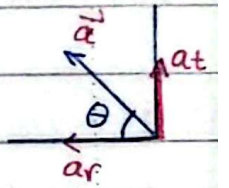
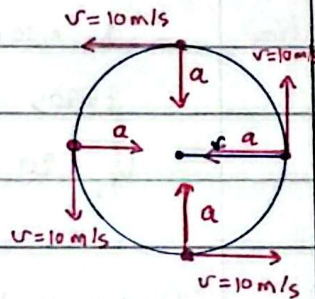
$$\Delta y = -52.2$$

[جسيم في حركة دائرية منتظمة]

* Particle in A uniform circular motion

② magnitude of v is variable $r = \text{radius}$  $a_t = \text{tangential acceleration}$ * Change in magnitude or/and direction
acceleration

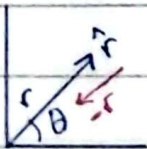
$$a = \sqrt{(a_t)^2 + (a_r)^2}$$

① magnitude is const
 $|\vec{v}|$ is const.

$$\theta = \tan^{-1}\left(\frac{a_t}{a_r}\right)$$

a ^{نقطة} point to the center

$$|a_r| = \frac{v^2}{r} \leftarrow a_c = \text{centripetal}$$



$$a_r = -\frac{v^2}{r} \hat{r}$$

نحو المركز

Q5 A golf ball is hit off a tee at the edge of cliff. Its x and y coordinates as functions of time are given by $x = 18t$ and $y = 4t - 4.9t^2$ is in seconds.

(a) write a vector expression for the ball's position as a function of time, using the unit vectors $\hat{i}$ and $\hat{j}$.

position : $\vec{r} = x\hat{i} + y\hat{j}$

$$\vec{r}(t) = (18t)\hat{i} + (4t - 4.9t^2)\hat{j}$$

b) the velocity vector $\vec{v}$ as a function of time.

* velocity vector :

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{v}(t) = 18\hat{i} + (4 - 9.8t)\hat{j}$$

c) the acceleration vector $\vec{a}$ as a function of time.

* acceleration vector : $\vec{a} = \frac{d\vec{v}}{dt}$

$$\vec{a}(t) = 0\hat{i} + (0 - 9.8)\hat{j}$$

$$\vec{a}(t) = -9.8\hat{j}$$

d) Next use unit-vector notation to write expressions for the position, the velocity, and the acceleration of the golf at $t = 3s$.

$$\vec{r}(3) = 18\hat{i} + -2.54\hat{j} \quad \vec{v}(3) = 54\hat{i} + -32.1\hat{j}$$

$$\vec{a}(t) = -9.8\hat{j}$$

Q9 A fish swimming in a horizontal plane has velocity $\vec{v}_0 = (4\hat{i} + 1\hat{j})$ m/s at a point in ocean where position relative to a certain rock is $\vec{r}_0 = 10\hat{i} - 4\hat{j}$ m. After the fish swims with constant acceleration for 20s, its velocity is $\vec{v}_f = 20\hat{i} - 5\hat{j}$.

X	$v_{0x} = 4 \text{ m/s}$	{	Y	$v_{0y} = 1 \text{ m/s}$
	$x_0 = 10 \text{ m}$			$y_0 = -4 \text{ m}$
	$t = 20 \text{ s}$			$t = 20 \text{ s}$
	$v_{fx} = 20 \text{ m/s}$			$v_{fy} = -5 \text{ m/s}$

(a) what are the components of the acceleration?

$$\vec{a} = ?$$

$$v_{fx} = v_{0x} + a_x t \Rightarrow a_x = \frac{v_{fx} - v_{0x}}{t}$$

$$a_x = \frac{20 - 4}{20} = 0.8 \text{ m/s}^2$$

$$a_y = \frac{v_{fy} - v_{0y}}{t} = \frac{-5 - 1}{20} = -0.3 \text{ m/s}^2$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

$$\vec{a} = 0.8\hat{i} - 0.3\hat{j}$$

(b) what is the direction of its acceleration with respect to unit vector? $(0.8, -0.3)$

$$\phi = \tan^{-1}\left(\frac{-0.3}{0.8}\right) = 20.5$$

$$\theta = 360 - 20.5 = 339.5^\circ$$

(c) If the fish maintains constant acceleration, where is it at $t=25\text{ s}$ and in what direction is it moving? $\vec{r}_f = ??$

$$\Delta x = v_{0x}t + \frac{1}{2}a_x t^2$$

$$x_f - x_0^{(10)} = (4)(20) + \frac{1}{2}(0.8)(20)^2$$

$$\boxed{x_f = 250\text{ m}}$$

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$$

$$y_f - y_0^{(-4)} = 1(20) + \frac{1}{2}(-0.3)(20)^2$$

$$\boxed{y_f = -44\text{ m}}$$

$$\vec{r}_f = x_f \hat{i} + y_f \hat{j}$$

$$\boxed{\vec{r}_f = 250\hat{i} - 44\hat{j}}$$

$$v_{fx} = v_i$$

$$v_{fy} = 0$$

$$\Delta y = \cancel{v_{iy}t} - \frac{1}{2}gt^2$$

$$-40 = -\left(\frac{1}{2}\right)(9.8)t^2$$

$$\boxed{t_p = 2.86\text{ s}}$$

$$t_s = t - t_p$$

$$= 3 - 2.86$$

$$\boxed{t_s = 0.14\text{ s}}$$

$$d = v_s t_s = 343(0.14) = 48.02\text{ m}$$

$$v_i = ?? \text{ u.s. } *$$

$$\Delta x = v_{ix}t_p + \cancel{\frac{1}{2}at^2}$$

problems
page 103+104

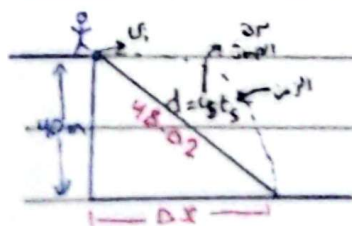
$$v_{iy} = 0$$

$$v_{ix} = v_i$$

$$26.6 = v_0 \cdot 2.86$$

$$\boxed{v_0 = 9.3\text{ m/s}}$$

Q27 A soccer player kicks a rock horizontally off a 40 m high cliff into a pool of water. If the player hears the sound of splash $\overset{t}{3\text{ s}}$ later, what was the initial speed given to the rock? Assume the speed of sound in air is $\overset{v_s}{343\text{ m/s}}$



$$t = t_s + t_p$$

Sound projectile

$$\Delta x = \sqrt{d^2 - y^2} = \sqrt{(48.02)^2 - (40)^2} = \sqrt{705}$$

$$\Delta x = 26.6\text{ m}$$