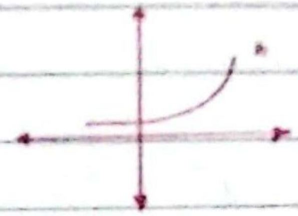
Monday 27/10/2025

Range $(0, \infty)$

* Exponential function

Defn: $f(x) = b^x$, $b > 0$, $b \neq 1$ is called exponential function

$\therefore$



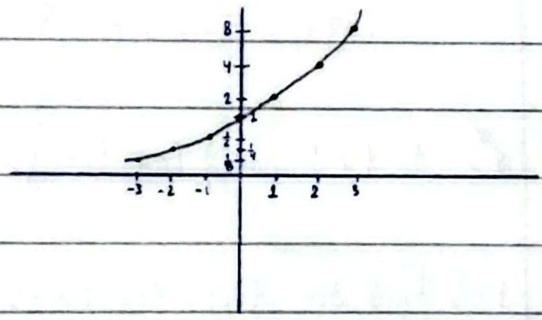
$f(x) = b^x$, $b > 1$

Dom: $\mathbb{R}$, Range: $(0, \infty)$

لايساوي صفر

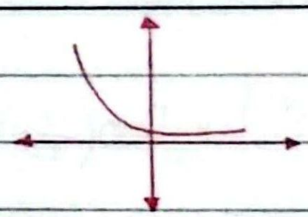
Ex) sketch $f(x) = 2^x$

x	-3	-2	-1	0	1	2	3
f(x)	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



$f(x) = b^x$

$0 < b < 1$

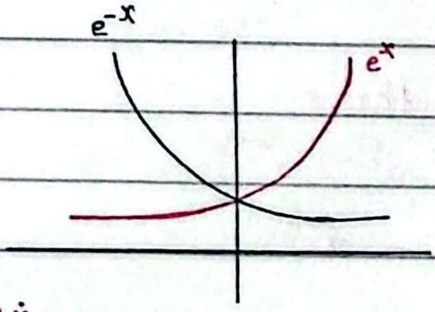


Dom: $\mathbb{R}$

Range: $(0, \infty)$

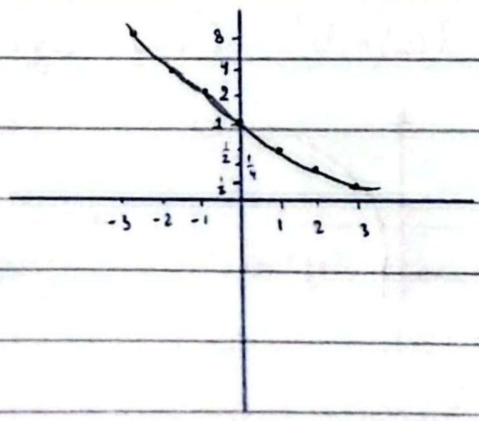
Defn: The natural exponential function التزايد الأسي الطبيعي

$f(x) = e^x$ where $e = 2.718 \ldots$



Ex: sketch $f(x) = \left(\frac{1}{2}\right)^x$

x	-3	-2	-1	0	1	2	3
f(x)	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$



$\therefore$ مهم جداً

* note that: $\text{Dom}\{b^{f(x)}\} = \text{Dom}\{f(x)\}$

$$\text{Dom}\{e^{f(x)}\} = \text{Dom}\{f(x)\}$$

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Ex: find dom:

① $f(x) = 3^{x+1}$

$Dom\left(3^{(x+1)}\right) = Dom(x+1) = \mathbb{R}$

② $f(x) = 2^{\frac{1}{x-1}}$

$Dom\left(2^{\frac{1}{x-1}}\right) = Dom\left(\frac{1}{x-1}\right) = \mathbb{R} - \{1\}$

③ $f(x) = e^{\sqrt{x-1}}$

$Dom\left(e^{\sqrt{x-1}}\right) = Dom\ \sqrt{x-1} = \quad x-1 \geq 0$

$x \geq 1$

$Dom = [1, \infty)$

* note that:

1. $a^x \cdot a^y = a^{x+y}$
2. $\frac{a^x}{a^y} = a^{x-y}$
3. $(a^x)^y = a^{x \cdot y}$
4. $a^{-x} = \frac{1}{a^x}$
5. $(a \cdot b)^x = a^x \cdot b^x$
6. $a^{x/y} = \sqrt[y]{a^x}$
7. $a^0 = 1$

* note that:

$f(x) = b^x \Rightarrow f^{-1}(x) = \log_b x$

∴ Logarithmic function: $f(x) = \log_b x$

$b > 0,\ b \neq 1$

Now ① $b^x : \underset{Dom}{\mathbb{R}} \rightarrow \underset{Range}{(0, \infty)} \quad f$

$\log_b x : \underset{Dom}{(0, \infty)} \longrightarrow \underset{Range}{\mathbb{R}} \quad f^{-1}$

*** to find dom $\{\log_b f(x)\}$: set $f(x) > 0$

طريقة الحل

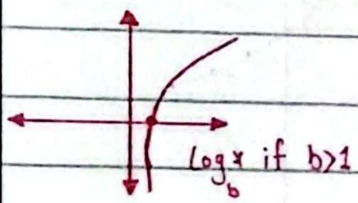
Ex) Find dom $f(x) = \log_3 (x+5)$

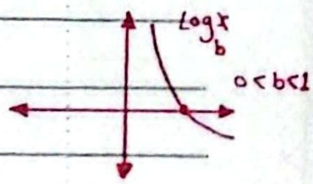
$x + 5 > 0$

$x > -5$

Dom $(-5, \infty)$

② $f(x) = b^x$, $f^{-1}(x) = \log_b x$





NOTEBOOK

Subject Monday 27/10/2025

③ $f \circ f^{-1}(x) = x$ $f = b^x$, $f^{-1} = \log_b x$

$b^{\log_b x} = x$

$f^{-1} \circ f(x) = x$

$\log_b b^x = x$

* note that :

① $\log_{10} x = \log x$

② $\log_e x = \ln x$ " natural log "

* Algebric properties of Logarithms [خصائص اللوغاريتم]

if $b > 0$ & $b \neq 1$, $x, y > 0$ and $r \in \mathbb{R}$. Then

① $\log_b (xy) = \log_b x + \log_b y$

$\log(x \pm y) \neq \frac{\log(x)}{\log(y)}$

② $\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$

③ $\log_b x^r = r \log_b x$

④ $\log_b \frac{1}{y} = \log_b y^{-1} = -\log_b y$

⑤ $\log_y x = \frac{\log_b x}{\log_b y}$, $b \neq 1$

$= \frac{\ln x}{\ln y} = \frac{\log x}{\log y}$

⑥ $\log_b b = 1$

⑦ $\ln e = \log_e e = 1$

⑧ $\log_b 1 = 0$

⑨ $\ln 1 = 0$

* note that:

$f(f^{-1}(x)) = x$

now $f(x) = e^x$, $f^{-1}(x) = \ln x$

$e^{\ln x} = x$ / $\ln e^x = x$

find $x = ??$

Ex)) $\ln x = 5$

$\cancel{e}^{\ln x} = e^5$

$x = e^5$

Ex: $\ln e^x = \ln 5$

$\cancel{\ln e}^x = \ln 5$

$x = \ln 5$

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Ex: if $f(x) = 2^{3x-1}$ Find

① $f(0) = 2^{-1} = \frac{1}{2}$

② $f(2) = 2^{5} = 32$

③ $f(\frac{1}{3}) = 2^{0} = 1$

Ex: if $g(x) = e^{2x}$ find

① $g(\frac{1}{2}) = e^{1} = e = 2.178...$

② $g(0) = e^{0} = 1$

③ $g(-1) = e^{-2} = \frac{1}{e^2}$

Ex: Find the value of :

① $\log_2 16 = \log_2 2^4 = 4 \log_2 2 = 4$

OR : $\log_2 16 = x \Rightarrow 2^x = 16 \Rightarrow 2^x = 2^4$
$x = 4$

② $\log_4 4 = 1$

③ $\log \frac{1}{1000} = \log_{10} 10^{-3} = -3 \log_{10} 10 = -3$

④ $\log_5 1 = 0 \Rightarrow \log_5 1 = x \Rightarrow 5^x = 1 \Rightarrow \boxed{x=0}$

⑤ $\log_9 3 = x \Rightarrow 9^x = 3 \Rightarrow x = \frac{1}{2}$

$(3^2)^x = 3$
$3^{2x} = 3$
$2x = 1$
$x = \frac{1}{2}$

⑥ $\ln e^3 = 3$

⑦ $e^{\ln 5} = 5$

⑧ $\ln \sqrt[3]{e} = \frac{1}{3} \ln e = \frac{1}{3}$

⑨ $\ln \frac{1}{e} = \ln e^{-1} = -\ln e = -1$

⑩ $e^{-2\ln 3} = e^{\ln 3^{-2}} = \frac{1}{9}$

⑪ $\log_2 6 - \log_2 15 + \log_2 20$

$\log_2 (\frac{6}{15}) + \log_2 20$

$\log_2 (\frac{6}{15} * 20) = \log_2 8 = \log_2 2^3$

$= 3 \log_2 2 = 3$

NOTEBOOK

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Ex8 find the domain of

① $f(x) = e^{\sqrt{x^2-9}}$ $=$ Dom $\sqrt{x^2-9}$

$x^2-9 \geq 0 \Rightarrow \sqrt{x^2} \geq \sqrt{9} \Rightarrow |x| \geq 3$

$x \geq 3$ or $x \leq -3$

Dom $= [3, \infty) \cup (-\infty, -3]$

② $f(x) = 3^{\frac{7-x}{2-x}}$ Dom $\left(\frac{7-x}{2-x}\right)$ $\leftarrow R$, $\leftarrow R-\{2\}$

$\therefore R-\{2\}$

③ $f(x) = \text{Log}(x^2-25)$

$x^2-25 > 0$

$\sqrt{x^2} > \sqrt{25}$

$|x| > 5$ $x > 5$ or $x < -5$

$\therefore (5, \infty) \cup (-\infty, -5)$

④ $f(x) = \ln(7-x)$

$7-x > 0$

$x < 7$ $\therefore (-\infty, 7)$

⑤ $f(x) = \frac{1}{\ln(4-x)}$ $\leftarrow R$

$4-x > 0$

$x < 4$ $(-\infty, 4)$

$\ln(f(x)) = 0$, $f(x) = 1$

$R \cap (-\infty, 4) - ($ $\ln(4-x) = 0$

$4-x = 1$

$\boxed{x = 3}$

$\therefore R \cap (-\infty, 4) - \{3\}$ $\therefore (-\infty, 4) - \{3\}$

⑥ $f(x) = \ln(x^2+4)$

$x^2+4 > 0$ $\therefore$ Dom R

⑦ $f(x) = \ln|x|$

$|x| > 0$ $|x| = 0 \rightarrow x = 0$

$\therefore R-\{0\}$

⑧ $f(x) = \ln(x^2-4)$

$x^2-4 > 0$

$\sqrt{x^2} > \sqrt{4}$

$|x| > 2$

$x > 2$ or $x < -2$

$\therefore (2, \infty) \cup (-\infty, -2)$

NOTEBOOK

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⑨ $f(x) = \ln(\ln(x))$

$x > 0 \qquad \therefore (0,\infty) \cap (1,\infty)$

$\ln x > 0 \Rightarrow e^{\ln x} > 1$

$x > 1 \qquad (1,\infty)$

Dom $(1,\infty)$

---

⑩ $f(x) = \dfrac{1}{e^{x} - e^{2x}}$ $\begin{array}{l} \leftarrow R \\ \leftarrow R - \{\text{الاصفار}\} \end{array}$

$e^{x} - e^{2x} = 0$

$e^{x}(1 - e^{x}) = 0$

$e^{x} \neq 0 \quad \Big| \quad 1 - e^{x} = 0 \Rightarrow \ln e^{x} = \ln 1$

$\ln 1 = x \qquad \boxed{x = 0}$

$\therefore R - \{0\}$

---

⑪ $f(x) = \log_{x} 3 = \dfrac{\log 3}{\log x}$ $\begin{array}{l} \leftarrow R \\ \leftarrow x > 0 \end{array}$

مش كده

$(0,\infty) - \log x \neq 0$

$\boxed{x = 1}$

$\therefore (0,\infty) - \{1\}$

Ex: Find the range of

① عن طريق الرسم

② او عن طريق $f^{-1}(x)$

① $f(x) = e^{2x+10}$

$\ln y = \ln e^{2x+10} \Rightarrow \ln y = 2x + 10$

$2x = \ln y - 10$

$x = \dfrac{\ln y - 10}{2}$

$f^{-1}(x) = \dfrac{\ln x - 10}{2}$

داخل ln

$x > 0$

Dom f^{-1} = Range $f(x) = (0,\infty)$

---

② $f(x) = 2^{x} + 5$

$y = 2^{x} + 5$

$\log_2 (y - 5) = \log_2 (2^{x})$

$\log_2 (y-5) = \log_2 2^{x}$

$\log_2 (y-5) = x$

$f^{-1}(x) = \log_2 (x - 5)$

Dom $f^{-1}(x)$ = Range $f(x)$ = $x - 5 > 0$

$x > 5$

$\therefore (5,\infty)$

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Ex: Solve for x

① $\dfrac{(x^2-9)(e^{-x}-2)\log(x-6)}{\log(x-2)} = 0$

$R \qquad R$

*طريقة الحل ⁘

① اول خطوة نطلع Dom :

② نجد قيم x ونستثني القيم الخارج المجال

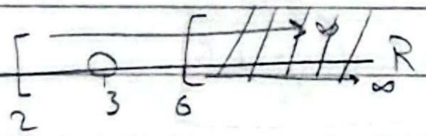
$\log(x-6) \leftarrow x-6>0 \quad x>6 \quad \therefore (6,\infty)$

$\log(x-2) \leftarrow x-2>0 \quad x>2 \quad \therefore (2,\infty) - \quad \log(x-2)=0$

$x-2=1$

$\boxed{x=3}$

$(6,\infty)\cap(2,\infty)-\{3\}$



Dom $\therefore (6,\infty)$

$\rightarrow x^2-9=0 \Rightarrow x^2=9 \Rightarrow \boxed{x=\pm 3}$ X لانه خارج مجال $\notin (6,\infty)$

$\rightarrow e^{-x}-2=0$

$e^{-x}=2 \Rightarrow \ln e^{-x}=\ln 2 \Rightarrow -x=\ln 2$

$\boxed{x=-\ln 2}$ X $\notin (6,\infty)$

$\rightarrow \dfrac{\log(x-6)}{\log(x-2)\rightarrow \neq 0} = 0 \Rightarrow$ $\boxed{\dfrac{a}{b}=0 \text{ if } a=0}$

$\log(x-6)=0$

$x-6=1 \Rightarrow \boxed{x=7}$ ✓ $\in (6,\infty)$

الحل $\boxed{x=7}$

② $\dfrac{1}{3-e^{2x}} = 4$

$1 = 12-4e^{2x} \qquad \therefore$ Dom R

$4e^{2x}=11$

$\ln e^{2x} \overset{\ln}{=} \dfrac{11}{4} \Rightarrow 2x = \ln\dfrac{11}{4}$

$\dfrac{2x}{2} = \dfrac{\ln 11 - \ln 4}{2}$

$x = \dfrac{1}{2}\ln(11/4)$

$x = \ln\sqrt{\ln(11/4)}$

③ $3e^{-2x}=5 \qquad \therefore$ Dom $= R$

$\ln e^{-2x} \overset{\ln}{=} \dfrac{5}{3}$

$-2x = \ln(5/3)$

$x = \dfrac{\ln(5/3)}{-2}$

$x = \dfrac{-1}{2}\ln(5/3)$

مهم الفكرة هاي

$= \dfrac{1}{2}\ln(5/3)^{-1} = \dfrac{1}{2}\ln\left(\dfrac{3}{5}\right)$ or

$= \ln\sqrt{3/5}$

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④ $x^2 \ln x - 16 \ln x = 0$

Dom $\therefore (0,\infty)$

$\ln x \,(x^2 - 16) = 0$

$\ln x = 0$

$\boxed{x = 1}$

تحقق $\in (0,\infty)$

$x^2 - 16 = 0$

$x^2 = 16$

$x = \pm 4$

$-4 \notin (0,\infty)$ / $+4 \in (0,\infty)$

الجواب $\boxed{x = 1}$ $\boxed{x = 4}$ $x \in \{1, 4\}$

⑤ $\log x^{\frac{3}{2}} - \log \sqrt{x} = 5$ Dom $\therefore (0,\infty)$

$\log_{10} \dfrac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}} = 5$

$\log_{10} x^{\frac{3}{2} - \frac{1}{2}} = 5$

$\log_{10} x = 5 \Rightarrow \boxed{x = 10^5} \in (0,\infty)$ الجواب

⑥ $e^{-2x} - \dfrac{1}{9} = 0$ Dom $\therefore \mathbb{R}$

$e^{-2x} = \dfrac{1}{9}$

$\ln e^{-2x} = \ln\left(\dfrac{1}{9}\right)$

$-2x = \ln\left(\dfrac{1}{9}\right)$

$x = -\dfrac{1}{2} \ln\left(\dfrac{1}{9}\right)$

$x = \ln\sqrt{9}$

$\boxed{x = \ln 3}$

⑦ $2^{x-5} = 3$ Dom $\therefore \mathbb{R}$

$\log 2^{x-5} = \log 3$

$(x-5)\log 2 = \log 3$

$(x-5) = \dfrac{\log 3}{\log 2}$

$\boxed{x = \dfrac{\log 3}{\log 2} + 5} \in \mathbb{R}$

$\boxed{x = \log_2 3 + 5}$

8) $e^{2x} - e^{x} = 6$ Dom $\therefore \mathbb{R}$

$e^{2x} - e^{x} - 6 = 0$

$(e^x + 2)(e^x - 3) = 0$

$e^x - 3 = 0$

$e^x = 3$

$\boxed{x = \ln 3}$

الحل

$e^x + 2 = 0$

$e^x \neq -2$