* Higher Order (DE) (Linear homog. & non-homog.)

Ex: Find the general solution to:

① $y''' - 8y = 0$

soln:

$\sqrt{4\times3}\, i = 2\sqrt{3} i$

$r^3 - 8 = 0$

$(r-2)(r^2+2r+4) = 0 \Rightarrow r = \frac{-2 \pm \sqrt{-12}}{2} = -1 \pm \sqrt{3} i$

α β

$r_1 = 2$

$\Delta = b^2 - 4ac$

$= 4 - (4\times1\times4)$

$= 4 - 16 = -12 < 0$

$y_1 = e^{2x}, \; y_2 = e^{-x}\cos(\sqrt{3}x).$

$y_3 = e^{-x}\sin(\sqrt{3}x)$

G.Soln $\Rightarrow y = c_1 e^{2x} + C_2(e^{-x}\cos\sqrt{3}x) + C_3(e^{-x}\sin(\sqrt{3}x)).$

② $y''' + y = 0$

Soln:

$r^3 + 1 = 0 \rightarrow \Delta = b^2 - 4ac \Rightarrow r = \frac{1 \pm \sqrt{-3}}{2}$

$(r+1)(r^2 - r + 1) \quad = 1 - (4\times1\times1) \quad = \frac{1}{2} \pm \frac{\sqrt{3} i}{2}$

$r_1 = -1 \quad = 1 - 4 = -3$

α β

$y_1 = e^{-x}, \; y_2 = e^{\frac{1}{2}x}\cos\left(\frac{\sqrt{3}}{2}x\right), \; y_3 = e^{+\frac{1}{2}x}\sin\left(\frac{\sqrt{3}}{2}x\right).$

G.Soln $\Rightarrow y = c_1 e^{-x} + c_2\left(e^{\frac{1}{2}x}\cos\left(\frac{\sqrt{3}}{2}x\right)\right) + c_3\left(e^{\frac{1}{2}x}\sin\frac{\sqrt{3}}{2}x\right)$

③ $y^{(4)} - y^{(2)} = 0$

$r^4 - r^2 = 0$

$r^2(r^2 - 1) = 0$

G.Soln $\Rightarrow y = c_1 + c_2 x + c_3 e^{x} + c_4 e^{-x}$

$r_1 = 0, r_2 = 0, r_3 = 1, r_4 = -1$

$y_1 = 1, \; y_2 = x, \; y_3 = e^{x}, \; y_4 = e^{-x}$

④ $y^{(4)} - 8y^{(2)} + 16y = 0$

Soln :

$r^4 - 8r^2 + 16 = 0$

$(r^2 - 4)(r^2 - 4) = 0$

$r = \pm 2 \qquad r = \pm 2$

$y_1 = e^{2x}, \quad y_2 = e^{-2x}, \quad y_3 = xe^{2x}, \quad y_4 = xe^{-2x}$

G.Soln $\Rightarrow$ $y = C_1 e^{2x} + C_2 e^{-2x} + C_3 e^{2x} x + C_4 x e^{-2x}$

⑤ $y^{(4)} + 10y^{(2)} + 25y = 0$

Soln $r^4 + 10r^2 + 25 = 0$

$(r^2+5)(r^2+5) = 0$

$r = \pm\sqrt{-5} \qquad r = \pm\sqrt{-5}$

$= \pm\sqrt{5}i \qquad = \pm\sqrt{5}i$

$\alpha = 0 \quad \beta \qquad \alpha = 0 \quad \beta$

G.Soln $\Rightarrow$

$y = C_1 \cos(\sqrt{5}x) + C_2 \sin(\sqrt{5}x) + C_3 x \cos(\sqrt{5}x) + C_4 x \sin\sqrt{5}x$

$y_1 = \cos(\sqrt{5}x) \qquad y_3 = x\cos(\sqrt{5}x)$

$y_2 = \sin(\sqrt{5}x) \qquad y_4 = x\sin(\sqrt{5}x)$

⑥ $y^{(4)} = 0$

Soln: $r^4 = 0$

$r_1 = 0, \quad r_2 = 0, \quad r_3 = 0, \quad r_4 = 0$

$y_1 = 1, \quad y_2 = x, \quad y_3 = x^2, \quad y_4 = x^3$

G.Soln $\Rightarrow$ $y = C_1 + C_2 x + C_3 x^3 + C_4 x^3$.

⑦ $y^{(5)} - 4y^{(3)} = 0$

Soln:

$r^5 - 4r^3 = 0$

$r^3 (r^2 - 4) = 0 \rightarrow r^2 - 4 = 0$

$\Downarrow$

$r_1 = 0,\ r_2 = 0,\ r_3 = 0,\quad r = \pm 2$

$y_1 = 1,\ y_2 = x,\ y_3 = x^2$

$y_4 = e^{2x},\ y_5 = e^{-2x}$

G.Soln $\Rightarrow$ $y = C_1 + C_2 x + C_3 x^2 + C_4 e^{2x} + C_5 e^{-2x}$

⑧ $y^{(3)} + 3y^{(2)} - 4y = 0$

الطريقة الأولى

Soln $\Rightarrow$ $r^3 + 3r^2 - 4 = 0$

$r = 1$ بالتجريب

$(1)^3 + 3(1)^2 - 4 = 0$

$4 - 4 = 0$ ✓

	r^3	r^2	r	الثابت
	1	3	0	-4
$\boxed{1}$	↓	↓	↓	↓
	↓	1	4	4
	1	4	4	0

الباقي

$r_1 = 1 \quad (r^2 + 4r + 4) \Leftarrow \therefore r^2 + 4r + 4 = 0$

$(r+2)(r+2) = 0$

$r_2 = -2 \quad r_3 = -2$

الطريقة الثانية

*	r^2	$4r$	4	
r	r^3	$4r^2$	$4r$	$\boxed{0}$
-1	$-r^2$	$-4r$	-4	

$-r^2 + \square = 3r^2$

$-4r + \square = 0$

$\therefore r^2 + 4r + 4 = 0$

$y_1 = e^x,\ y_2 = e^{-2x},\ y_3 = e^{-2x}(x).$

G.Soln. $\Rightarrow$

$y = c_1 y_1 + c_2 y_2 + c_3 y_3$

$= c_1 e^x + c_2 e^{-2x} + c_3 x e^{-2x}.$

⑨ If xe^{3x} , $e^{2x}\cos(5x)$ are solns to the linear homog. (DE) $L[y]=0$ with constant coefficients. Find this (DE).

Soln ⇒ since xe^{3x} (مكرر)

$r_1 = 3$, $r_2 = 3$

$$(r-r_1)(r-r_2)=0 \Rightarrow (r-3)^2 = r^2-6r+9=0$$

& since $e^{2x}\cos(5x)$, α, β

$$r = 2 \pm 5i$$

$$(r-2)^2 = (\pm 5i)^2$$

$$(r^2-4r+4) = -25 \rightarrow r^2-4r+29=0$$

$$\therefore (r^2-6r+9)(r^2-4r+29)=0$$

$$r^4 - 4r^3 + 29r^2 - 6r^3 + 24r^2 - 174r + 9r^2 - 36r + 261$$

$$r^4 - 10r^3 + 62r^2 - 210r + 261 = 0$$

$$\left[\ y^{(4)} - 10y^{(3)} + 62y^{(2)} - 210y^{(1)} + 261y = 0\ \right]$$

Ex: If x^3 , x^3e^x are solns to the linear homog. DE $L[y]=0$ with constant coefficients Find this (DE).

Soln: since x^3

$r_1 = 0$, $r_2 = 0$, $r_3 = 0$, $r_4 = 0$

$[\ r^4 = 0\ ]$

& x^3e^x مكرر

$r_4 = 1$, $r_5 = 1$, $r_6 = 1$

$r_7 = 1$

$$(r-1)^4$$

$$(r-1)^2(r-1)^2 = 0$$

$$\rightarrow (r^2-2r+1)(r^2-2r+1)$$

$$r^4 - 2r^3 + r^2 - 2r^3 + 4r^2 - 2r + r^2 - 2r + 1$$

$$(r^4 - 4r^3 + 6r^2 - 4r + 1)r^4$$

$$y^{(8)} - 4y^{(7)} + 6y^{(6)} - 4y^{(5)} + y^{(4)} = 0$$

Ex: Find the form of particular soln to

① $y''' - 2y'' + y' = x + 2e^x - \cos(3x)$.

[1] Homog. $y^{(3)} - 2y^{(2)} + y^{(1)} = 0$

$r^3 - 2r^2 + r = 0$

$r(r^2 - 2r + 1) = 0$

$r_1 = 0 \quad (r-1)(r-1) = 0$

$r_2 = +1 \quad r_3 = +1$

$y_1 = 1,\ y_2 = e^x,\ y_3 = e^x(x)$.

[2] $g(x) = \underbrace{x}_{} + \underbrace{2e^x}_{} - \underbrace{\cos(3x)}_{}$

$y_p = (Ax+B)x^1 + (Ce^x)x^2 + (D\cos(3x) + H\sin(3x))x^0$

$= Ax^2 + Bx + Cx^2e^x + D\cos(3x) + H\sin(3x)$.

② $y^{(4)} + 4y^{(2)} = x^2 + e^{2x} - 5\sin(2x)$.

[1] Homog. $r^4 + 4r^2 = 0$

$r^2(r^2+4) = 0$

$r_1 = 0,\ r_2 = 0 \quad r^2 = -4$

$r = \pm 2i$

$y_1 = 1,\ y_2 = x,\ y_3 = \cos(2x)$

$y_4 = \sin(2x)$

[2] Non-Homog. $g(x) = x^2 + e^{2x} - 5\sin(2x)$.

$y_p = (Ax^2 + Bx + C)x^2 + (De^{2x})x^0 + (H\sin(2x) + G\cos(2x))x^1$

$y_p = Ax^4 + Bx^3 + Cx^2 + De^{2x} + Hx\sin(2x) + Gx\cos(2x$.

③ $y^{(4)} + 2y^{(3)} + 2y^{(2)} = 2x + 1 + e^{-x}\cos x + \sin x$

① Homog. $\quad y^{(4)} + 2y^{(3)} + 2y^{(2)} = 0$

$$r^4 + 2r^3 + 2r^2 = 0$$

$$r^2(r^2 + 2r + 2) = 0$$

$r_1 = 0, r_2 = 0 \qquad \Delta = b^2 - 4ac$

$$= 4 - (4 \times 1 \times 2) = 4 - 8 = -4$$

$$r = \frac{-2 \pm 2i}{2} = \underbrace{-1}_{\alpha} \pm \underbrace{i}_{\beta = 1}$$

$y_1 = 1, \ y_2 = x, \ y_3 = e^{-x}\cos(x), \ y_4 = e^{-x}\sin(x).$

② Non-Homog.

$g(x) = 2x+1 + e^{-x}\cos x + \sin x$

$y_p = (Ax+B)x^2 + (Ce^{-x}\cos x + De^{-x}\sin x)x^1 + (H\sin x + E\cos x)x^0$

$y_p = Ax^3 + Bx^2 + Cxe^{-x}\cos x + Dxe^{-x}\sin x + H\sin x + E\cos x$

④ $y^{(3)} + 6y^{(2)} + 9y' + 4y = e^{-x}$

① Homog.

$$r^3 + 6r^2 + 9r + 4 = 0$$

بالتجريب $[r_1 = -1]$

*	r^2	$5r$	4	=
r	r^3	$5r^2$	$4r$	0
1	r^2	$5r$	4	=

$r^2 + \square = 6r^2$

$4r + \square = 9r$

$\therefore r^2 + 5r + 4 = 0$

$(r+1)(r+4)$

$[r_2 = -1] \quad [r_3 = -4]$

$y_1 = e^{-x}, \ y_2 = xe^{-x}, \ y_3 = e^{-4x}$

② Non-Homog. $\quad g(x) = e^{-x}$

$$y_p = (Ae^{-x})\,x^2$$

$$\left[\,y_p = Ax^2e^{-x}\,\right]$$

Ex: Find the homog. (DE) with constant coefficients of at least order with has the soln.

$$y = \underline{7x^2} + \overline{9xe^{3x}} - x\,\underline{\sin(2x)}.$$

$r^3 = 0$ $\qquad$ $r = 3, 3$ $\qquad$ $\alpha = 0,\ \beta = 2$

$(r-3)^2$ $\qquad$ $r = \pm 2i$

$r^2 = -4 \rightarrow (r^2+4)$

$\therefore\ r^3(r-3)^2(r^2+4)$

$r^3(r^2-6r+9)(r^2+4)$ ... نكمل الحل

Ex: If $-x$, $\frac{x^2}{2} - x$ are solns for the (DE)

$$x^3y''' + f(x)y'' - y = g(x)$$, find $f(x)$ & $g(x)$.

Soln: $y = -x$

$y' = -1$ $\qquad$ put this in DE

$y'' = 0$ $\qquad$ $x^3(0) + f(x)(0) - (-x) = g(x)$

$y''' = 0$ $\qquad$ $\left[\,g(x) = x\,\right]$

$y_2 = \frac{1}{2}x^2 - x \quad \rightarrow$

$$x^3(0) + f(x)(1) - \left(\tfrac{1}{2}x^2 - x\right) = x$$

$y' = x - 1$

$y'' = 1$ $\qquad$ $\left[\,f(x) = \frac{1}{2}x^2\,\right]$

$y''' = 0$

$y_h \qquad y_p$

Ex: Find a, b, c & d so that $y = e^{-x} + 2e^{2x} + \sin(4x)$ is a soln to the DE:

$$y'' + ay' + by = [c\sin(4x) + d\cos(4x)]$$

$$g(x)$$

① $y_h = e^{-x} + 2e^{2x}$

$r_1 = -1 \qquad r_2 = 2 \qquad \therefore a_1 = -1, \; b = -2$

$(r - r_1)(r - 2) = 0$

$(r + 1)(r - 2) = 0$

$r^2 - 2r + r - 2 = 0$

$r^2 - r - 2 = 0$

$y'' - y' - 2y = 0$

② y_p (Non-Homog) $= \sin(4x)$

$\alpha = 0 \quad \beta = 4$

$r = \pm 4i \qquad\qquad y_p = \sin(4x)$

$r^2 = -16 \rightarrow r^2 + 16 = 0 \qquad y_p' = 4\cos(4x)$

$y_p'' = -16\sin(4x)$

$-16\sin(4x) - 4\cos(4x) - 2\sin(4x) = C\sin(4x) + d\cos(4x)$

$-18 = C \qquad -4 = d$

$(C = -18) \qquad (d = -4)$

Ex: Given the auxiliary eq. $(r+4)(r^2-1)^2(r^2+4) = 0$ for the Non-homog. DE with constant coefficient $L[y] = 0$

1 Find the order of this DE?

Soln:

order (7).

Chapter (5) : Laplace Transforms :

Def : let $F(t)$ be a function define on $[0,\infty)$ then the laplace transform for $f(t)$ is given as:

$$L[f(t)] = \int_0^\infty f(t) \cdot e^{-st}\, dt = F(s)$$

Provided that the integral converges.

Ex : Find $L[3]$

Soln. $\Rightarrow$ $f(t) = 3 \Rightarrow L\{f(t)\} = L\{3\} = \int_0^\infty 3 \cdot e^{-st}\, dt$

$$= \lim_{h\to\infty} \int_0^h 3e^{-st}\, dt = \lim_{h\to\infty} \frac{3e^{-st}}{-s}\Big]_0^h$$

$$= \frac{-3}{s} \lim_{h\to\infty} \left(e^{-sh} - e^{0}\right)$$

$$= \frac{-3}{s}\left[e^{-\infty} - 1\right] = -\frac{3}{s}(0-1) = \frac{3}{s}$$

Remark :

$f(t)$	$L\{f(t)\} = F(s)$
a	$\frac{a}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$

$[n = 1, 2, 3, \dots]$

Remark: $L\{\alpha f(t) \pm \beta g(t)\} = \alpha L\{f(t)\} \pm \beta L\{g(t)\}$

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Ex: Evalute

$\boxed{1}$ $L\{5\} = \frac{5}{s}$ $\qquad$ $\boxed{2}$ $L\{\cos(3t)\} = \frac{s}{s^2+9}$

$\boxed{3}$ $L\{2\sin(5t) - 4e^{2t} + 10t^5 - 2\}$

$= 2L\{\sin(5t)\} - 4L\{e^{2t}\} + 10L\{t^5\} - L\{2\}$

$= 2\left(\frac{5}{s^2+25}\right) - 4\left(\frac{1}{s-2}\right) + 10\left(\frac{5!}{s^6}\right) - \frac{2}{s}$

$= \frac{10}{s^2+25} - \frac{4}{s-2} + \frac{1200}{s^6} - \frac{2}{s}$

$\boxed{4}$ $L\{\sinh(3t)\} = L\left\{\frac{e^{3t}-e^{-3t}}{2}\right\}$

$= \frac{1}{2}L\{e^{3t}-e^{-3t}\} = \frac{1}{2}\left(L\{e^{3t}\} - L\{e^{-3t}\}\right)$

$= \frac{1}{2}\left(\frac{1}{s-3} - \frac{1}{s+3}\right)$

(Remark)

$\sinh\alpha = \frac{e^{\alpha}-e^{-\alpha}}{2}$

$\cosh\alpha = \frac{e^{\alpha}+e^{-\alpha}}{2}$

$\boxed{5}$ $L\{\sin(2t+2\pi)\} = L\{\sin(2t)\}$

$= \frac{2}{s^2+4}$

Remark:

① $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$

② $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$ ← اذا كان ناقص يبقى زائد والعكس

③ $\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha-\beta) + \sin(\alpha+\beta)\right]$

④ $\sin\alpha\sin\beta = \frac{1}{2}\left[\overset{\cos}{\cancel{\sin}}(\alpha-\beta) - \overset{\cos}{\cancel{\sin}}(\alpha+\beta)\right]$

⑤ $\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$

⑥ $\cos\left(\frac{\pi}{2}-\alpha\right) = \sin\alpha$,,, $\sin\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha$

⑦ $\sin^2\alpha = \frac{1}{2} - \frac{1}{2}\cos(2\alpha)$

⑧ $\cos^2\alpha = \frac{1}{2} + \frac{1}{2}\cos(2\alpha)$

Ex: Evalute:

1. $L\{\sin(3t-\frac{\pi}{6})\} = L\{\sin 3t \cdot \underbrace{\cos\frac{\pi}{6}}_{=\frac{\sqrt{3}}{2}} - \cos 3t \underbrace{\sin\frac{\pi}{6}}_{=\frac{1}{2}}\}$

$= \frac{\sqrt{3}}{2} L\{\sin 3t\} - \frac{1}{2} L[\cos 3t]$

$= \frac{\sqrt{3}}{2}\left(\frac{3}{s^2+9}\right) - \frac{1}{2}\left(\frac{s}{s^2+9}\right)$

2. $L\{\sin^2(3t)\} = L\{\frac{1}{2} - \frac{1}{2}\cos \cancel{8}t\}$

$L\{\frac{1}{2}\} - \frac{1}{2}\{\cos(6t)\} = \frac{\frac{1}{2}}{s} - \frac{1}{2}\left(\frac{s\cancel{6}}{s^2+36}\right)$

3. $L\{\cos(2t)\cdot\cos(5t)\} = L\{\frac{1}{2}(\cos(-3t) + \cos(7t)\}$

$\frac{1}{2}[L\{\cos(-3t)\} + L\{\cos(7t)\}]$

$\frac{1}{2}\left(\frac{s}{s^2+9} + \frac{s}{s^2+49}\right)$

4. $\int_0^\infty e^{-st+10t}\,dt = \int_0^\infty e^{10t}\cdot e^{-st}\,dt = L\{e^{10t}\}$

$= \frac{1}{s-10}$

5. $\int_0^\infty e^{-2t}\cos(3t)\,dt = L\{\cos(3t)\}_{\underline{s=2}}$

(s)

$= \frac{s}{s^2+9} = \frac{2}{13}$

Remark: (shifting property)

$$L\{e^{at} \cdot f(t)\} = L\{f(t)\}\Big|_{s \to s-a} = F(s-a)$$

Ex: Evaluat

① $L\{e^{2t}\cos(5t)\} = \frac{s}{s^2+25}\Big|_{s \to s-2} = \frac{s-2}{(s-2)^2+25}$

② $L\{t^3 e^{-4t}\} = L\{t^3\}\Big|_{s \to s+4} = \frac{3!}{s^4}\Big|_{s \to s+4} = \frac{\overset{6}{3!}}{(s+4)^4}$

③ $L\left\{\frac{\sin 3t}{e^{2t}}\right\} = L\{e^{-2t}\sin 3t\} = L\{\sin 3t\}\Big|_{s \to s+2}$

$$= \frac{3}{s^2+9}\Big|_{s \to s+2} = \frac{3}{(s+2)^2+9}$$

Remark:

$$L\{t^n \cdot f(t)\} = (-1)^n \frac{d^n}{ds^n}\left[L\{f(t)\}\right]$$

Ex: ① $L\{t^{\overset{n}{2}}\cos(2t)\} = (-1)^2 \frac{d^2}{ds^2}\left[L\{\cos(2t)\}\right]$

$$= \frac{d^2}{ds^2}\left[\frac{s}{s^2+4}\right] = \frac{d}{ds}\left[\frac{1 * (s^2+4) - (s * 2s)}{(s^2+4)^2}\right]$$

$$\frac{d}{ds}\left[\frac{4-s^2}{(s^2+4)^2}\right] = \frac{-2s(s^2+4)^{\cancel{2}} - (4-s^2) \times 2\cancel{(s^2+4)} * 2s}{\cancel{(s^2+4)^4}}$$

$$= \frac{-2s^3 - 8s - 16s + 4s^3}{(s^2+4)^3}$$

$$= \frac{2s^3 - 24s}{(s^2+4)^3}$$

② $$L\{ t^{20} \cdot e^{3t} \} = L\{ t^{20} \} \Big|_{s \to s-3} = \frac{20!}{s^{21}} \Big|_{s \to s-3} = \frac{20!}{(s-3)^{21}}$$

③ $$L\{ t e^{4t} \cos(3t) \} = L\{ t \cos(3t) \} \Big|_{s \to s-3}$$

$$= \left[(-1)^1 \frac{d}{ds} \left[L\{ \cos(3t) \} \right] \right] \Big|_{s \to s-4}$$

$$= \left[- \frac{d}{ds} \left(\frac{s}{s^2+9} \right) \right] \Big|_{s \to s-4}$$

$$= \left[- \left(\frac{1 \times s^2+9 - (s \times 2s)}{(s^2+9)^2} \right) \right] \Big|_{s \to s-4}$$

$$= \frac{s^2 - 9}{(s^2+9)^2} \Big|_{s \to s-4} = \frac{(s-4)^2 - 9}{((s-4)^2+9)^2}$$

* Invers. laplace Transform:

Def: If. $L\{f(t)\} = F(s)$, then the inverse laplace is given as : $L^{-1}\{F(s)\} = f(t)$

$F(s) = L\{f(t)\}$	$L^{-1}\{F(s)\} = f(t)$	
$\frac{a}{s}$	a	
$\frac{1}{s-a}$	e^{at}	
$\frac{1}{s^2+a^2}$	$\frac{\sin at}{a}$	
$\frac{s}{s^2+a^2}$	$\cos at$	
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$	$n = 1, 2, 3, \ldots$

Remark:

① $L^{-1}\{F(s-a)\} = e^{at} L^{-1}\{F(s)\}$

② $L^{-1}\{F(s)\} = \frac{-1}{t} L^{-1}\{F'(s)\}$

③ $L^{-1}\{\alpha F(s) \pm \beta G(s)\} = \alpha L^{-1}\{F(s)\} \pm \beta L^{-1}\{G(s)\}$

Ex: Evaluat:

1. $L^{-1}\left(\frac{5}{s}\right) = 5$
2. $L^{-1}\left\{\frac{1}{s^2+9}\right\} = \frac{\sin(3t)}{3}$
3. $L^{-1}\left\{\frac{s}{s^2+16}\right\} = \cos(4t)$
4. $L^{-1}\left\{\frac{3}{s^9} - \frac{2s}{s^2+4} + \frac{2}{s} + \frac{4}{s-6}\right\} = 3\left(\frac{t^8}{8!}\right) + -2\cos(2t) + 2 + 4\left(e^{6t}\right)$

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$\boxed{5}$ $L^{-1}\left[(s+7)^{-5}\right] = L^{-1}\left\{\frac{1}{(s+7)^5}\right\} = e^{-7t} \cdot L^{-1}\left\{\frac{1}{s^5}\right\} = e^{-7t} \cdot \frac{t^4}{4!}$

$\boxed{6}$ $L^{-1}\left[\frac{7}{(s-3)^2+25}\right] = 7\, e^{3t}\, L^{-1}\left\{\frac{1}{s^2+25}\right\} = 7e^{3t}\, \frac{\sin(5t)}{5}$

$\boxed{7}$ $L^{-1}\left\{\frac{s-2}{(s-2)^2+16}\right\} = e^{2t}\, L^{-1}\left\{\frac{s}{s^2+16}\right\} = e^{2t}\cos(4t)$.

$\boxed{8}$ $L^{-1}\left\{\frac{s}{(s+3)^2+25}\right\} = L^{-1}\left\{\frac{s+3-3}{(s+3)^2+25}\right\} = L^{-1}\left\{\frac{s+3}{(s+3)^2+25}\right\} - L^{-1}\left\{\frac{3}{(s+3)^2+25}\right\}$

$= e^{-3t}\, L^{-1}\left\{\frac{s}{s^2+25}\right\} - 3e^{-3t}\, L^{-1}\left\{\frac{1}{s^2+25}\right\} = e^{-3t}\cos(5t) - 3e^{-3t}\,\frac{\sin(5t)}{5}$

$\boxed{9}$ $L^{-1}\left\{\frac{2}{s^2-4s+5}\right\} \Rightarrow \frac{2}{(s-5)(s+1)} = \frac{A}{s-5} + \frac{B}{s+1}$

كسور جزئية

في حال كان هناك عبارة تربيعية في المقام =>
① تحلل اذا نستخدم كسور جزئية
② لا تحلل (إكمال مربع)

$2 = A(s+1) + B(s-5)$

$s=-1 \Rightarrow 2 = 0 + -6B \quad \left(B = \frac{-1}{3}\right)$

$s=5 \Rightarrow 2 = 6A + 0 \quad \left(A = \frac{1}{3}\right)$

$L^{-1}\left\{\frac{\frac{1}{3}}{s-5} + \frac{-\frac{1}{3}}{s+1}\right\} = \frac{1}{3}L^{-1}\left\{\frac{1}{s-5}\right\} - \frac{1}{3}L^{-1}\left\{\frac{1}{s+1}\right\}$

$= \frac{1}{3}e^{5t} - \frac{1}{3}e^{-t}$

كسور جزئية

* طريقة أخرى :-

$\boxed{10}$ $L^{-1}\left\{\frac{3}{s^2+4s+3}\right\} \Rightarrow 3 \cdot \frac{1}{(s+1)(s+3)}$

يحلل ← كسور جزئية

$= \frac{3}{-2} \cdot \frac{\overbrace{(s+1)-(s+3)}^{=-2}}{(s+1)(s+3)} = -\frac{3}{2}\cdot\left[\frac{\cancel{s+1}}{\cancel{(s+1)}(s+3)} - \frac{\cancel{(s+3)}}{(s+1)\cancel{(s+3)}}\right]$

$= -\frac{3}{2}L^{-1}\left[\frac{1}{s+3} - \frac{1}{s+1}\right] = -\frac{3}{2}\left(e^{-3t} - e^{-t}\right)$

11 $L^{-1}\left\{\frac{2}{s^2+4s+13}\right\} \Rightarrow$ إكمال مربع $\quad s^2+4s+13 = \boxed{s^2+4s+4}\boxed{-4+13}$

$4 = \left(\frac{4}{2}\right)^2 = \left(\frac{s}{2}\right)^2$ معامل

$= (s+2)^2 + 9$

$= 2\,L^{-1}\left\{\frac{1}{(s+2)^2+9}\right\} = 2e^{-2t}\cdot L^{-1}\left\{\frac{1}{s^2+9}\right\}$

$= 2e^{-2t}\cdot\frac{\sin(3t)}{3}.$

12 $L^{-1}\left\{\frac{2}{(s^2+9)(s^2+4)}\right\} = 2\,L^{-1}\left\{\frac{1}{(s^2+9)(s^2+4)}\right\}$

$= \frac{2}{5}L^{-1}\left\{\frac{(s^2+9)-(s^2+4)}{(s^2+9)(s^2+4)}\right\}$ 5

$= \frac{2}{5}L^{-1}\left[\frac{\cancel{(s^2+9)}}{\cancel{(s^2+9)}(s^2+4)} - \frac{\cancel{(s^2+4)}}{(s^2+9)\cancel{(s^2+4)}}\right]$

$= \frac{2}{5}L^{-1}\left\{\frac{1}{s^2+4} - \frac{1}{s^2+9}\right\} = \frac{2}{5}\left(\frac{\sin(2t)}{2} - \frac{\sin(3t)}{3}\right)$

13 $L^{-1}\left\{\ln\left(\frac{s+4}{s+5}\right)\right\}$ (Remark ② استخدام)

$= L^{-1}\{\ln(s+4) - \ln(s+5)\}$

$= \frac{-1}{t}L^{-1}\left[\frac{1}{s+4} - \frac{1}{s+5}\right] = \frac{-1}{t}\left[e^{-4t} - e^{-5t}\right]$

(H.W) ① $L^{-1}\left[\frac{s+8}{(s+3)^2+81}\right]$

② $L^{-1}\left[\frac{s}{s^2-4s+5}\right]$.

③ $L^{-1}\left[\frac{2}{s^2+3s-4}\right]$.

④ $L^{-1}\left[\frac{3}{(s^2+1)(s^2+5)}\right]$.

⑤ $L^{-1}\left[\ln\left(\frac{s^2}{s^2+16}\right)\right]$.

⑥ $L^{-1}\left[\frac{2}{s^3+3s^2+2s}\right]$

Unit - Step Function :

$$u(t-a) = \begin{cases} 0 & 0 \leq t < a \\ 1 & t \geq 0 \end{cases}$$

Remark :

1. $L\left[u(t-a)\right] = \frac{e^{-as}}{s}$

2. $L\left[u(t-a)\,f(t)\right] = e^{-as}\,L\left[f(t+a)\right]$

3. $L^{-1}\left[e^{-as}F(s)\right] = u(t-a)\cdot L^{-1}\left[F(s)\right]_{t\to t-a}$