

ch - 26

"The capacitance"

المواصفات

the capacitance definition



we define the capacitance as

$$C = \frac{\text{charge}}{V} = \frac{Q}{V}$$

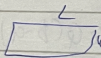
$$C = \frac{Q}{V}$$

calculating the capacitance

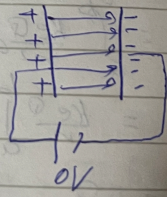
a) parallel plate capacitor

المكثفات ذات الصفائح المتوازية

 $A = L^2$

 $A = Lw$

 $A = \pi R^2$



$$C = \frac{Q}{\Delta V} \quad ; \quad \Delta V = \frac{Q}{A}$$

$$\Delta V = - \int E dx \quad ; \quad - \int_{x=0}^d \frac{Q}{\epsilon_0 A} dx = - \frac{Q}{\epsilon_0} d$$

$$C = \frac{\frac{Q}{\epsilon_0 A}}{\frac{Q}{\epsilon_0 d}} = \frac{\epsilon_0 A}{d}$$

$$C_{\text{Parallel}} = \frac{\epsilon_0 A}{d}$$

(b) The spherical Capacitor

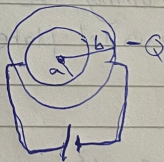
$$C = \frac{Q}{\Delta V_{ab}}$$

$$\Delta V_{ab} = - \int E dr$$

$$= - \int_a^b \frac{k_e Q}{r^2} dr$$

$$= \frac{k_e Q}{r} \Big|_a^b = k_e Q \left[\frac{1}{b} - \frac{1}{a} \right] = \frac{k_e Q (b-a)}{ab}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{k_e Q (b-a)}{ab}}$$



$$C_{\text{ph}} = \frac{4\pi \epsilon_0 ab}{b-a}$$

③ Cylindrical capacitor

$$E = \frac{2Ke\lambda}{r} \quad a \leq r \leq b$$

$$C = \frac{Q}{\Delta V}$$

$$\Delta V = - \int E dr = - \int_a^b \frac{2Ke\lambda}{r} dr$$

$$= -2Ke\lambda \int_a^b \frac{dr}{r} = -2Ke\lambda \ln \left| \frac{b}{a} \right|$$

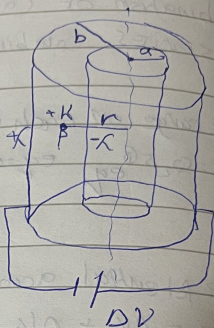
$$\Delta V = -2Ke\lambda \ln \left| \frac{a}{b} \right|$$

$$|\Delta V| = 2Ke\lambda \ln \left| \frac{a}{b} \right|$$

$$C = \frac{Q}{\Delta V} = \frac{\lambda L}{\Delta V} = \frac{\lambda L}{2Ke\lambda \ln \left| \frac{b}{a} \right|}$$

$$C_{cy} = \frac{2\pi \epsilon_0}{\ln \left| \frac{b}{a} \right|} L$$

the capacitance per unit length $\frac{C_{cy}}{L} = \frac{2\pi \epsilon_0}{\ln \left| \frac{b}{a} \right|}$



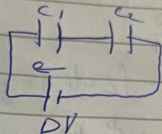
Combination of Capacitance

a) series combination

توصيل التوالي

① The charge on each capacitor

$$Q_1 = Q_2 = Q_{eq} \quad eq = \text{equivalent}$$



② The potential across each capacitor

$$\Delta V = \Delta V_1 + \Delta V_2$$

③ The equivalent capacitance

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

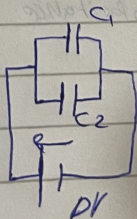
b) parallel combination

توصيل التوازي

$$Q_{eq} = Q_1 + Q_2$$

$$\Delta V_1 = \Delta V_2 = \Delta V$$

$$C_{eq} = C_1 + C_2$$



Ex: $C = 2 \mu F$
 $C_1 = \frac{4}{3}, C_2 = \frac{7}{3}, C_{eq} = \frac{21}{23} \quad Q_1 = 42$

① Find the equivalent capacitance?

$$\frac{1}{C_1} = \frac{1}{C} + \frac{1}{2C} = \frac{1}{2} + \frac{1}{4} = \boxed{\frac{3}{4} \mu F}$$

$$C_2 = C_1 + \frac{1}{2}C = \frac{4}{3} + 1 = \boxed{\frac{7}{3} \mu F}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_2} + \frac{1}{3C} + \frac{1}{C} = \frac{\frac{3}{7} + \frac{1}{6} + \frac{1}{2}}{1} = \frac{18 + 7 + 21}{42} = \frac{46}{42} = \boxed{\frac{23}{21} \mu F}$$

$$\rightarrow Q_{eq} = C_{eq} \cdot DV$$

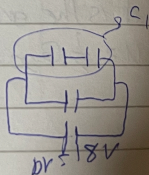
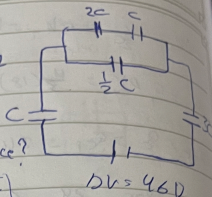
$$= \frac{21}{23} (46) = 42 \mu C$$

$$\rightarrow Q_C = Q_1 = Q_{3C} = 42 \mu C$$

$$\rightarrow DV_C = \frac{Q_C}{C} = \frac{42}{2} = 21V$$

$$\rightarrow DV_{C_2} = \frac{Q_{C_2}}{C_{C_2}} = \frac{42}{\frac{7}{3}} = 18V$$

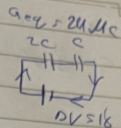
$$\rightarrow DV_{3C} = \frac{Q_{3C}}{C_{3C}} = \frac{42}{6} = 7V$$



$$\Delta V_1 = \Delta V_2 = \Delta V = 18$$

$$Q_{C1} = C_1 \Delta V_1 = \frac{4}{3}(18) = 24 \mu C$$

$$Q_{2C} = Q = Q_1 = 24 \mu C$$



$$\Delta V_C = \frac{Q_{2C}}{C_2} = \frac{24}{2} = 12 V, \quad \Delta V = \frac{Q}{C} = \frac{24}{2} = 12 V$$

The energy stored in the capacitor

$$U = \frac{1}{2} Q \Delta V, \quad Q = C \Delta V$$

$$= \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} \frac{Q^2}{C}$$

What is the energy stored in the system of capacitors?

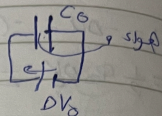
$$U = \frac{1}{2} Q_{eq} \Delta V = \frac{1}{2} (42)(18)$$

What is the energy stored in $3C$?

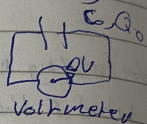
$$U = \frac{1}{2} Q_{3C} \Delta V_{3C} = \frac{1}{2} (42)(7)$$

is the capacitance dielectric
 a) charging the capacitor with a battery
 while it's filled in air

$$Q_0 = C_0 \Delta V_0$$



b) disconnecting the capacitor from the battery



c) Filling the capacitor with dielectric material

(quartz, plastic)

dielectric material



$$C = K C_0$$

$$Q = Q_0$$

$$\Delta V = \frac{\Delta V_0}{K}, K \geq 1$$

$$K = \frac{\Delta V_0}{\Delta V}$$

before | After

$$Q_0 \quad | \quad Q = Q_0$$

$$\Delta V_0 \quad | \quad \Delta V = \frac{\Delta V_0}{K}$$

$$C_0 \quad | \quad C = K C_0$$

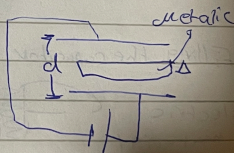
what is the work (energy) required to insert the dielectric material inside the capacitor

$$W = -\Delta U = -\left(U_f - U_0 \right) = U_0 - U_f = \frac{1}{2} Q_0 V_0 - \frac{1}{2} Q_0 V$$

$$= \frac{1}{2} Q_0 V_0 - \frac{1}{2} Q_0 \frac{V_0}{K} = \frac{1}{2} Q_0 V_0 \left(1 - \frac{1}{K} \right)$$

$$= U_0 \left(1 - \frac{1}{K} \right)$$

Note: Inserting metallic slab inside the capacitor =
what is the new capacitance



$$C = \frac{\epsilon_0 A}{D-d}$$

D is the metallic slab thickness

Example: $d = 5 \text{ cm}$, $A = 2.6 \text{ cm}^2$, $D = 2 \text{ mm}$

what is the capacitance?

$$C = \frac{\epsilon_0 A}{D-d} = \frac{8.85 \times 10^{-12} \times 2.6 \times 10^{-4}}{(50 - 2) \times 10^{-3}}$$

Q₁) $C = 3 \mu F$, $Q = 27 \mu C$, $0V$ s?

$$Q = C \Delta V \rightarrow 0V = \frac{Q}{C} = \frac{27}{3} = 9V$$

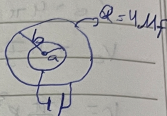


Q₄) $a = 7cm$, $b = 14cm$

$$C = \frac{4\pi\epsilon_0 ab}{b-a}$$

$$= \frac{4\pi(8.85 \times 10^{-12})(7 \times 10^{-2})(14 \times 10^{-2})}{(14 - 7) \times 10^{-2}}$$

$$= 15.6 \text{ pF}$$

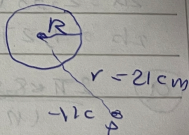


$$\Delta V = \frac{Q}{C} = \frac{4 \times 10^{-6}}{15.6 \times 10^{-12}} = 256 \text{ KV}$$

Q₁₁) $R = 12cm$, $E_p = 4.9 \times 10^4 \text{ V/C}$

$$E = \frac{kqQ}{r^2}$$

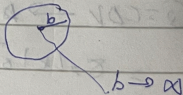
$$4.9 \times 10^4 = \frac{9 \times 10^9}{0.12^2} Q \rightarrow Q = 0.24 \mu C$$



$$C = \frac{4\pi\epsilon_0 ab\epsilon_0}{b-a}$$

$$C = 4\pi\epsilon_0 a$$

Isolated sphere or droplet



$$C = 4\pi\epsilon_0 a = 4\pi(8.85 \times 10^{-12})(0.12)$$

Sigma

water droplet



$$C_0 = 4\pi\epsilon_0 R_0$$

$$V_0 = \frac{4\pi}{3} R_0^3$$

$$V = \frac{4\pi}{3} R^3 = 3 \left(\frac{4\pi}{3} R_0^3 \right)$$

$$R = \sqrt[3]{3} R_0$$

$$C = 4\pi\epsilon_0 R = 4\pi\epsilon_0 (\sqrt[3]{3} R_0)$$

$$C = \frac{4\pi\epsilon_0 R_0 \sqrt[3]{3}}{C_0}$$

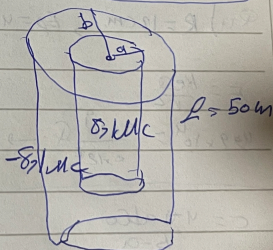
$$\text{So } \frac{C}{C_0} = \sqrt[3]{3}$$

$$2a = 2,58 \text{ mm}$$

$$2b = 7,27 \text{ mm}$$

$$a) \frac{2\pi \times 8,85 \times 10^{-12} (50)}{\ln \left| \frac{7,27}{2,58} \right|}$$

$$b) Q = C \Delta V \rightarrow \Delta V = \frac{Q}{C} = 8,1 \times 10^{-6}$$



$$C = \frac{2\pi\epsilon_0 l}{\ln \left| \frac{b}{a} \right|}$$

↓
 $\ln \left| \frac{2b}{2a} \right|$ نفس الشيء

$$Q47) d = 1.5 \text{ cm} = 1.5 \times 10^{-2} \text{ m}$$

$$A = 25 \text{ cm}^2 = 25 \times 10^{-4} \text{ m}^2$$

$$DV_0 = 250 \text{ V}$$

$$K_{\text{water}} = 80$$

distilled
water



$$C = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 25 \times 10^{-4}}{1.5 \times 10^{-2}} = 1.48 \text{ pF}$$

$$C_0 = \epsilon_0 DV_0 = (1.48)(250) = 370 \text{ pC}$$

$$DV_0 = 250$$

$$\frac{C}{K_{\text{water}}} = KC_0 = 80 \times 1.48 \text{ pF} = 118 \text{ pF}$$

$$Q = Q_0 = 370 \text{ pC}$$

$$DV = \frac{QV_0}{K} = \frac{250}{80} = 3.125 \text{ V}$$

$$Q21 \quad C_P = 100, C_S = nC_S$$

$$C_P = C + C + \dots = nC$$

$$\frac{1}{C_P} = \frac{1}{C} + \frac{1}{C} + \dots = \frac{n}{C} \rightarrow C_S = \frac{C}{n}$$

$$C_P = 100 C_S$$

$$nC = 100 \frac{C}{n} \rightarrow n^2 = 100 \rightarrow n = 10$$

$$\text{So } n = \sqrt{N}$$

$$C_1, C_2$$

$$C_p = C_1 + C_2 = 9$$

$$C_p = 9 \text{ pF} \rightarrow C_s = 2 \text{ pF}$$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2}$$

$$\frac{C_1 + C_2}{C_1 C_2} = \frac{1}{2}$$

$$2(C_1 + C_2) = C_1 C_2$$

$$C_1 C_2 = 2(9) = 18$$

$$C_1 (9 - C_1) = 18$$

$$-C_1^2 + 9C_1 - 18 = 0$$

$$C_1^2 - 9C_1 + 18 = 0$$

$$(C_1 - 3)(C_1 - 6) = 0$$

$$C_1 = 3 \text{ or } 6$$

$$C_2 = 6 \text{ or } 3$$