

Table 1.6: Double-Stem-and-Leaf Plot of Battery Life

Stem	Leaf	Frequency
1-	69	2
2*	2	1
2-	5669	4
3*	001111222333444	15
3-	5567778899	10
4*	11234	5
4-	577	3

Table 1.4: Car Battery Life

2.2	4.1	3.5	4.5	3.2	3.7
3.4	1.6	3.1	3.3	3.8	3.1
2.5	4.3	3.4	3.6	2.9	3.3
3.3	3.1	3.7	4.4	3.2	4.1
4.7	3.8	3.2	2.6	3.9	3.0

Table 1.5: Stem-and-Leaf Plot of Ba

Stem	Leaf
1	69
2	25669
3	0011112223334445567778899
4	11234577

Table 1.6: Double Stem-and-Leaf Plot of Battery Life

Stem	Leaf	Frequency
1-	69	2
2*	2	1
2-	5669	4
3*	001111222333444	15
3-	5567778899	10
4*	11234	5
4-	577	3

(5-9)

max
avg = 3.4
(mean)
أكثر

singel

max avg = 3.9

singel
بالتالي mean
له 3.9

stem
1* → (0-4)
2* → (5-9)

2* → (0-4)
2* → (5-9)

3* → (0-4)
3* → (5-9)

double stem إلى

* الأكثر انتشاراً

3) Histogram

stem-leaf
1.5 + 1.9 / 2
leaf

Table 1.7: Relative Frequency Distribution of Battery Life

Class Interval	Class Midpoint	Frequency, f	Relative Frequency
1.5-1.9	1.7	2	2/40 = 0.050
2.0-2.4	2.2	1	1/40 = 0.025
2.5-2.9	2.7	4	4/40 = 0.100
3.0-3.4	3.2	15	0.375
3.5-3.9	3.7	10	0.250
4.0-4.4	4.2	5	0.125
4.5-4.9	4.7	3	0.075

Mean of Histogram

$$\text{Mean} = \frac{\sum \text{Class Midpoint} \times f}{\sum f}$$

$$= \frac{(1.7 \times 2) + (2.2 \times 1) + \dots + 4.7 \times 3}{2 + 1 + \dots + 3}$$

$$= 3.4125$$

total
A
B

A و B

$$\sum f = 40$$

$$\sum 1$$

$$1 - 0.050$$

$$\frac{B}{40} = 0.12$$

$$- \frac{1.6}{-0.1} = 1.5$$

نجد 1.6 داخل الفترة

(1.5, 1.9)

فترة

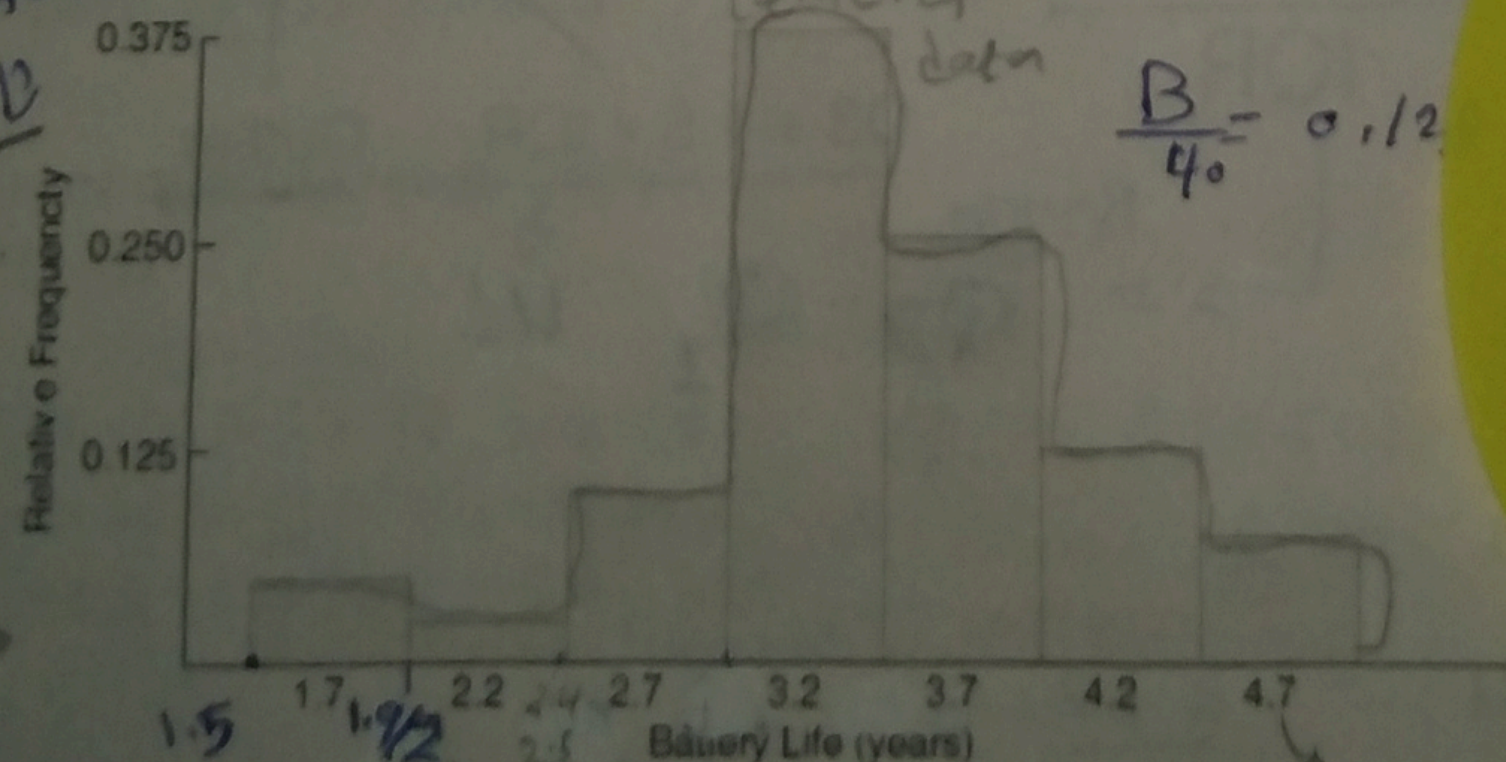


Figure 1.6: Relative frequency histogram.

الفرق بين
0.4
Histogram

54
Histogram
من 2.04
4.9

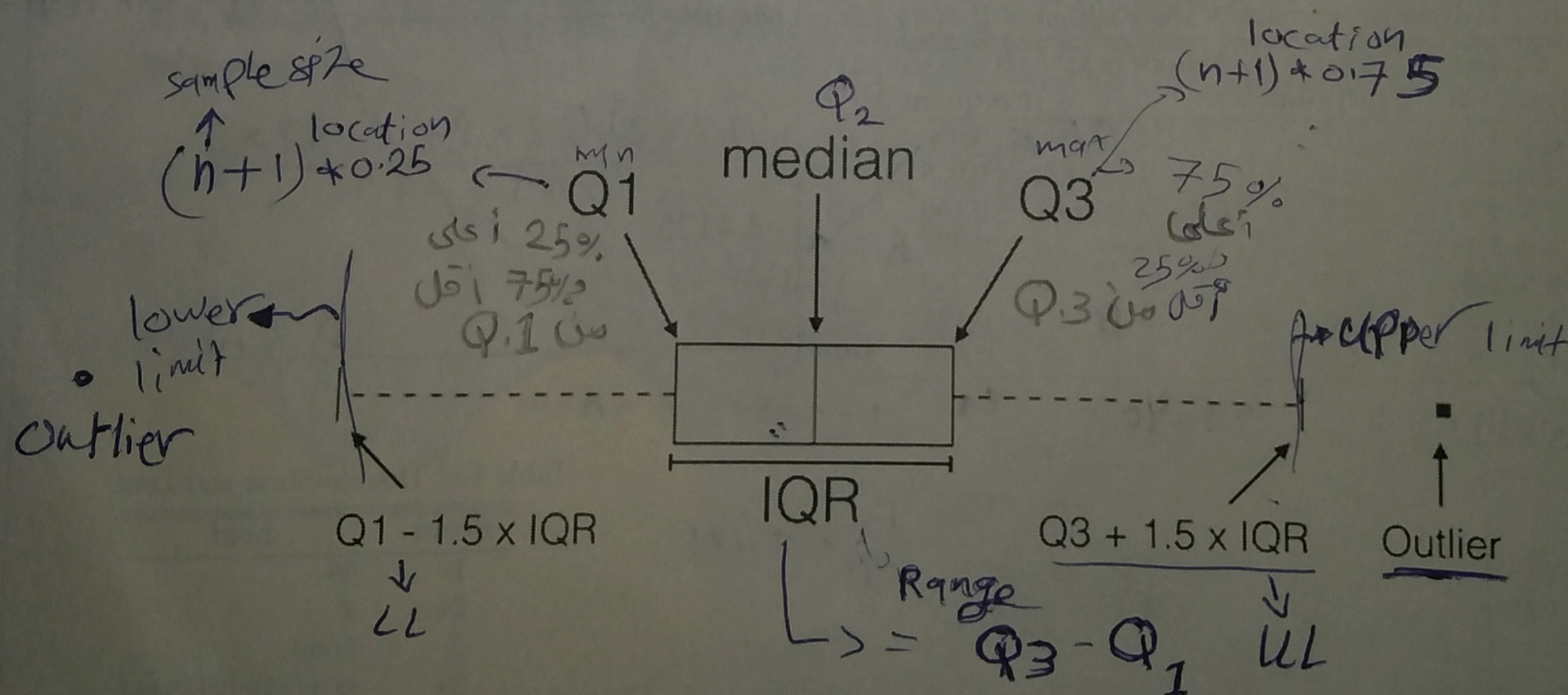
4- Box plot

First Quartile and Third Quartile

Definitions:

- The **lower half** of a data set is the set of all values that are to the left of the median value when the data has been put into increasing order.
- The **upper half** of a data set is the set of all values that are to the right of the median value when the data has been put into increasing order.
- The **first quartile**, denoted by Q_1 , is the median of the *lower half* of the data set. This means that about 25% of the numbers in the data set lie below Q_1 and about 75% lie above Q_1 .
- The **third quartile**, denoted by Q_3 , is the median of the *upper half* of the data set. This means that about 75% of the numbers in the data set lie below Q_3 and about 25% lie above Q_3 .

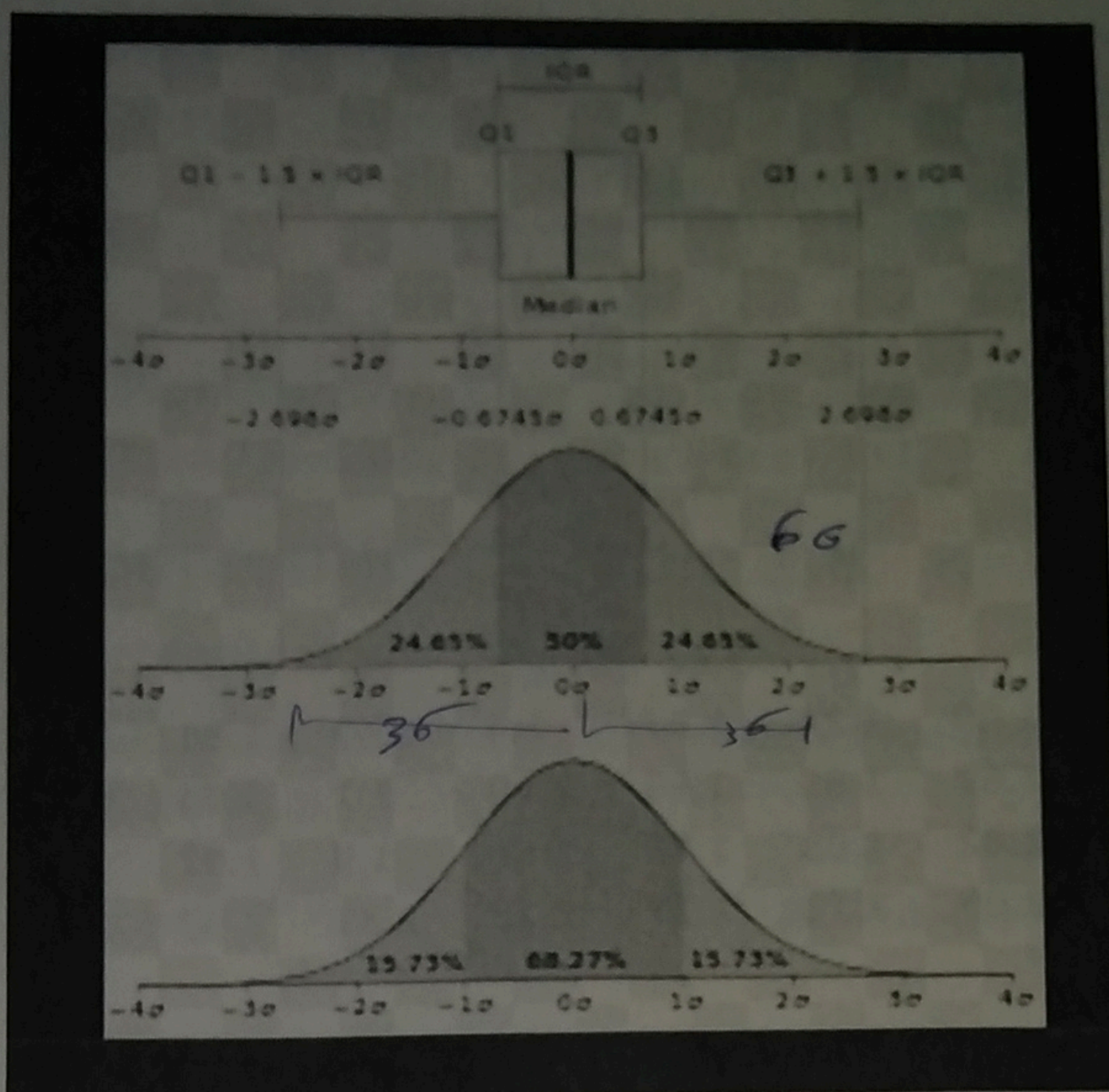
23



$$Q_2 \rightarrow 1 \ 2 \ 3 = 2$$

$$Q_2 \rightarrow 1 \ 2 \ 3 \ 4 = \frac{3+2}{2} = 2.5$$

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1.5
IQR → 36

3σ = 3σ
يسار يمين
تحقق عند 1.5

25

$(n+1) \cdot 0.25$
Location Value

Q1 Q3

Find the first and third quartiles of the data set {3, 7, 8, 5, 12, 14, 21, 13, 16, 18}.

in increasing order: 3, 5, 7, 8, 12, 13, 14, 16, 18, 21.

ترتيب

Location of Q1: $(10+1) \cdot 0.25 = 2.75$

Interpolation

Q1 = value of location 2 + $0.75 \cdot (\text{value of location 3} - \text{value of location 2})$

Q1 = $5 + 0.75 \cdot (7 - 5) = 6.5$

Location of Q2: $(10+1) \cdot 0.5 = 5.5$

Q2 = $(12+13)/2 = 12.5$

Location of Q3: $(10+1) \cdot 0.75 = 8.25$

Q3 = value of location 8 + $0.25 \cdot (\text{value of location 9} - \text{value of location 8})$

Q3 = $16 + 0.25 \cdot (18 - 16) = 16.5$

Inter quartile range (IQR) =

Q3 - Q1

Q3 - Q1

IQR = $16.5 - 6.5 = 10$

UL = $16.5 + (1.5 \cdot 10) = 31.5$

LL = $6.5 - (1.5 \cdot 10) = -8.5$

Q1

Q3 → $L(n+1) \cdot 0.75$

$11 \cdot 0.75 = 8.25$

Q1
2 → 5
2.75 → x
3 → 7
2.75
5 + 0.75(7-5)

26
8 → 16

8.25 → x

9 → 18

data Value
outlier
point
16 + 0.25(18-16)

الفرق - الفرق الكبير

4) Box-and-Whisker Plot or Box Plot

-You have to know to estimate the percentile and quartile

e.g., Nicotine content was measured in a random sample of 40 cigarettes. The data are displayed in Table 1.8.

Table 1.8: Nicotine Data for Example 1.5

1.09	1.92	2.31	1.79	2.28	1.74	1.47	1.97
0.85	1.24	1.58	2.03	1.70	2.17	2.55	2.11
1.86	1.90	1.68	1.51	1.64	0.72	1.69	1.85
1.82	1.79	2.46	1.88	2.08	1.67	1.37	1.93
1.40	1.64	2.09	1.75	1.63	2.37	1.75	1.69

In order

0.72	0.85	1.09	1.24	1.37	1.4	1.47	1.51	1.58	1.63
1.64	1.64	1.67	1.68	1.69	1.69	1.7	1.74	1.75	1.75
1.79	1.79	1.82	1.85	1.86	1.88	1.9	1.92	1.93	1.97
2.03	2.08	2.09	2.11	2.17	2.28	2.31	2.37	2.46	2.55

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Q.1
 Location of Q1: $(40+1)*0.25=10.25$
 Q1: $1.63+0.25*(1.64-1.63)=1.6325$

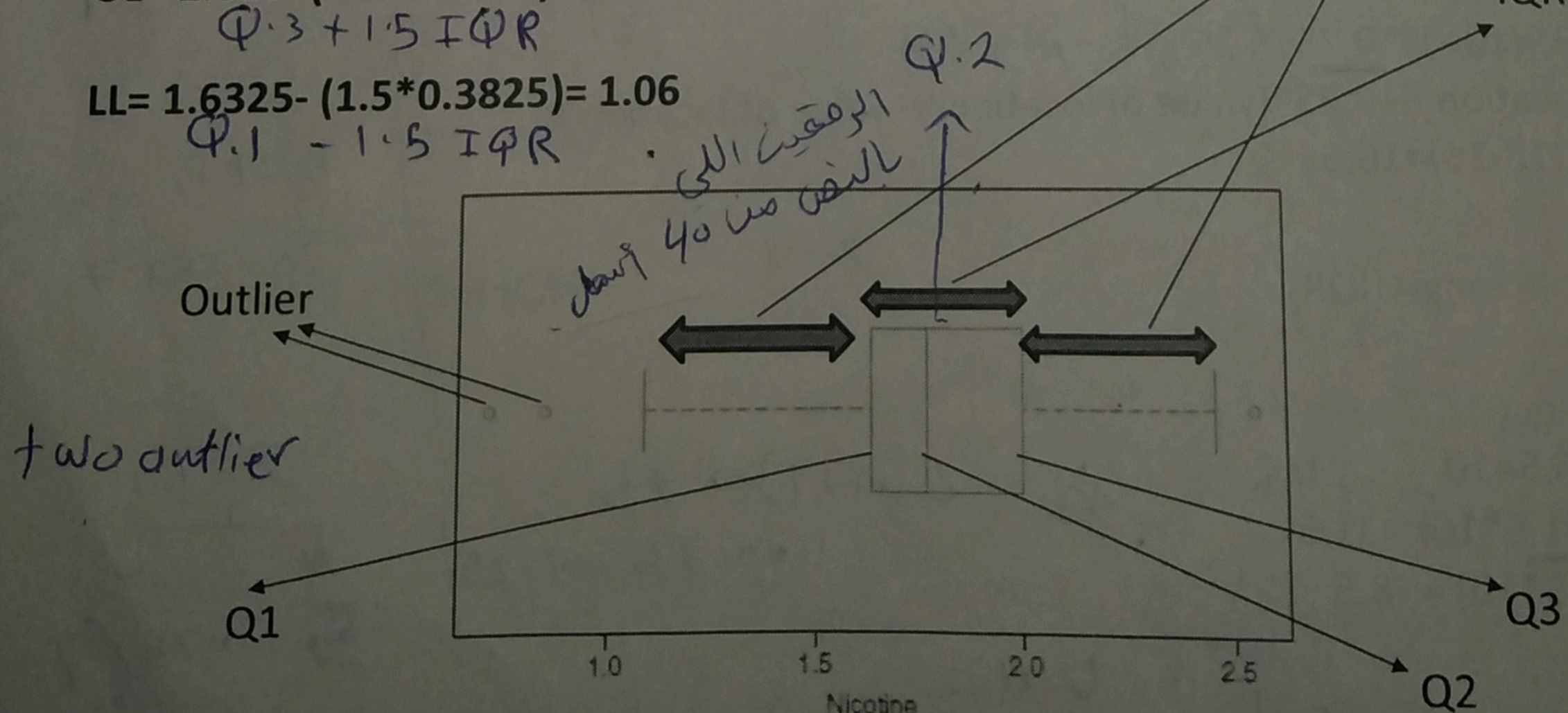
Q2: $(1.75+1.79)/2=1.77$

Location of Q3: $(40+1)*0.75=30.75$
 Q3: $1.97+0.75*(2.03-1.97)=2.015$

IQR: $2.015-1.6325=0.3825$

UL: $2.015+(1.5*0.3825)=2.58875$

LL: $1.6325-(1.5*0.3825)=1.06$



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Figure 1.9: Box-and-whisker plot for Example 1.5.

Percentile

أي نسبة
مستوى النسب (النسبة)

و هذا الـ 10
عليه 11

- Find 62% percentile of {3, 7, 8, 5, 12, 14, 21, 13, 16, 18}.

$n=10$

3 / 5 / 7 / 8 / 12 / 13 / 14 / 16 / 18 / 21
6 7

Location of 62% percentile is $(n+1)*0.62=6.82$

62% percentile = Value of location 6 + 0.82 * (Value of location 7 - Value of location 6) = $13 + 0.82 * (14 - 13) = 13.82$

$13 + 0.82 * (14 - 13)$

- Find 29% percentile of Table 1.8

$n=40$

Location of 29% percentile is $(40+1)*0.29=11.89$

29% percentile = Value of location 11 + 0.89 * (Value of location 12 - Value of location 11) = $1.64 + 0.29 * (1.64 - 1.64) = 1.64$

$1.64 + 0.89 * (1.64 - 1.64)$

هنا يدرك أن ϕ موجودة عند العتبة
نفسها بشكل مستمر
بالتالي الجواب يبقى
قيمة ϕ داخل العتبة

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Slide 22

الأسئلة تكون على شكل
إيجاد قيمة frequency

مقدراً

$$1) \frac{f}{\sum f} = 0.125 \rightarrow R$$

$$2) \frac{f}{\sum f} \text{ أول إيجاد قيمة الـ } R$$

$$A = 1 - 0.5 - 0.1 \dots$$

Layan

$$Q.1 \xrightarrow{\text{Location}} (n+1) * 0.25$$

$$\rightarrow \text{Value} \xrightarrow{\text{الفئة 1}} + \text{الفئة 2} * \left(\frac{\text{الفئة 1} - \text{الفئة 2}}{2} \right)$$

الفئة 1: المركز الثاني
الفئة 2: المركز الثالث
الفئة 1: x_1
الفئة 2: x_2
بناها القيمة 1

$$Q.3 \xrightarrow{\text{Location}} (n+1) * 0.75$$

$$\xrightarrow{\text{Value}} \text{الفئة 1} + \text{الفئة 2} * \left(\frac{\text{الفئة 1} - \text{الفئة 2}}{2} \right)$$

الفئة 1: المركز الثاني
الفئة 2: المركز الثالث
الفئة 1: x_1
الفئة 2: x_2
بناها القيمة 1

$$Q.2 \xrightarrow{\text{Location}} (n+1) * 0.5$$

$$\xrightarrow{\text{Value}} \text{الفئة 1} + \text{الفئة 2} * \left(\frac{\text{الفئة 1} - \text{الفئة 2}}{2} \right)$$

الفئة 1: المركز الثاني
الفئة 2: المركز الثالث
الفئة 1: x_1
الفئة 2: x_2
بناها القيمة 1

$$LL = Q.3 + 1.5 IQR$$

$$LL = Q.1 + 1.5 IQR$$



Probability and Statistics

حساب احتمالية
حدث معين

Chapter 2

Department of Industrial Engineering

Sample space هي التي نتج

تجربة إحصائية ← أي تجربة تعطي output مثل رمي حجر نرد

- Statisticians use the word experiment to describe any process that generate a set of data.
- Example of a statistical experiment is the tossing of a coin.

$S \leftarrow$ Sample Space

الفضاء العيني

$$S = \{ \dots \}$$

مجموعة نتيجاتها من خلالها element التجربة الإحصائية

3

Definition 2.1: The set of all possible outcomes of a statistical experiment is called the sample space and is represented by the symbol S .

Each outcome in a sample space is called an element or a member of the sample space, or simply a sample point. If the sample space has a finite number of elements, we may list the members separated by commas and enclosed in braces. Thus, the sample space S , of possible outcomes when a coin is flipped, may be written

$$\text{Coin} \rightarrow S = \{H, T\}$$

where H and T correspond to heads and tails, respectively.

N نواتج محاسبها

مثلاً عدد طلاق الهندسة الصناعية

10 من أصل 50

$$\frac{10}{50} = \frac{n}{N} = \text{الاحتمالية}$$

Example 2.1: Consider the experiment of tossing a die. If we are interested in the number that shows on the top face, the sample space is

$$S_1 = \{1, 2, 3, 4, 5, 6\}$$

If we are interested only in whether the number is even or odd, the sample space is simply

$$S_2 = \{\text{even}, \text{odd}\}$$

**** More than one sample space can be used to describe the outcomes of an experiment.**

In some experiment it is helpful to list the elements of the sample space by means of a tree diagram.

نستخدمها عندما يكون مثالاً sample space complicated

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Example 2.2: An experiment consists of flipping a coin and then flipping it a second time if a head occurs. If a tail occurs on the first flip, then a die is tossed once. To list the elements of the sample space providing the most information, we construct the tree diagram of Figure 2.1. The various paths along the branches of the tree give the distinct sample points. Starting with the top left branch and moving to the right along the first path, we get the sample point HH , indicating the possibility that heads occurs on two successive flips of the coin. Likewise, the sample point $T3$ indicates the possibility that the coin will show a tail followed by a 3 on the toss of the die. By proceeding along all paths, we see that the sample space is

$$S = \{HH, HT, T1, T2, T3, T4, T5, T6\}.$$

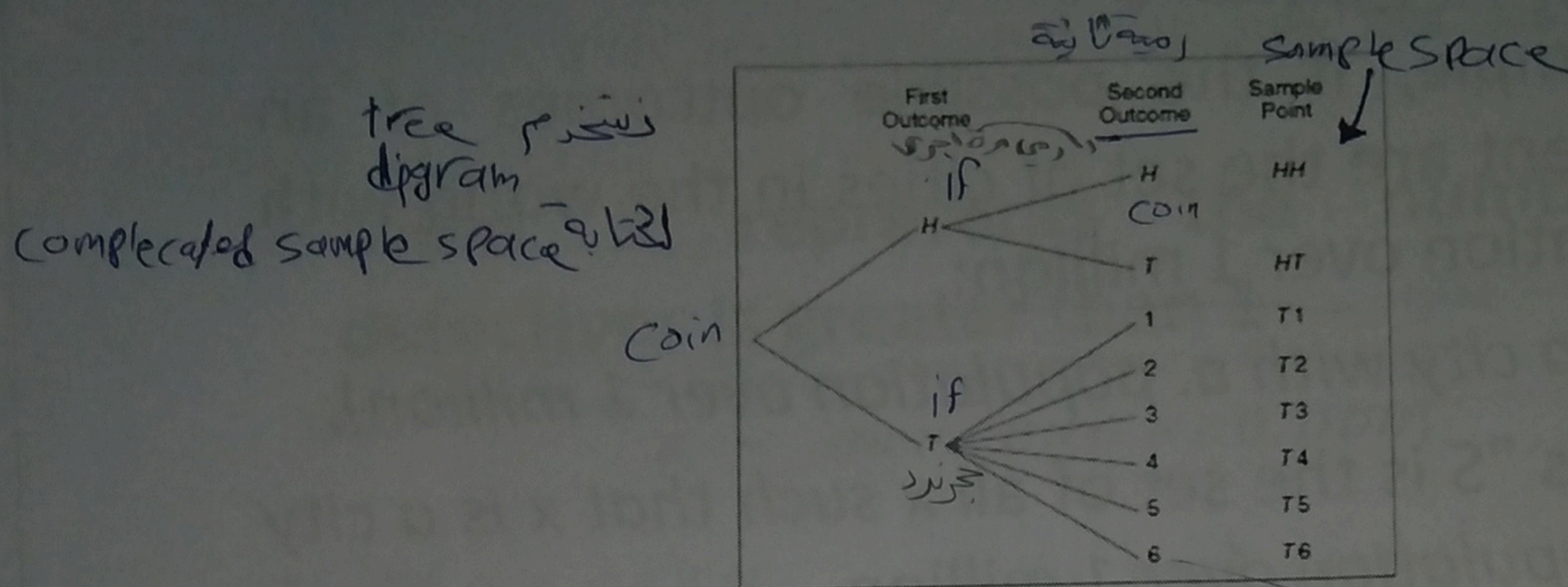


Figure 2.1: Tree diagram for Example 2.2.

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Probability : احتمال وجوده داخل
Sample Space

Example 2.3: Suppose that three items are selected at random from a manufacturing process. Each item is inspected and classified defective, D , or nondefective, N . To list the elements of the sample space providing the most information, we construct the tree diagram of Figure 2.2. Now, the various paths along the branches of the tree give the distinct sample points. Starting with the first path, we get the sample point DDD , indicating the possibility that all three items inspected are defective. As we proceed along the other paths, we see that the sample space is

$$S = \{DDD, DDN, DND, DNN, NDD, NDN, NND, NNN\}.$$

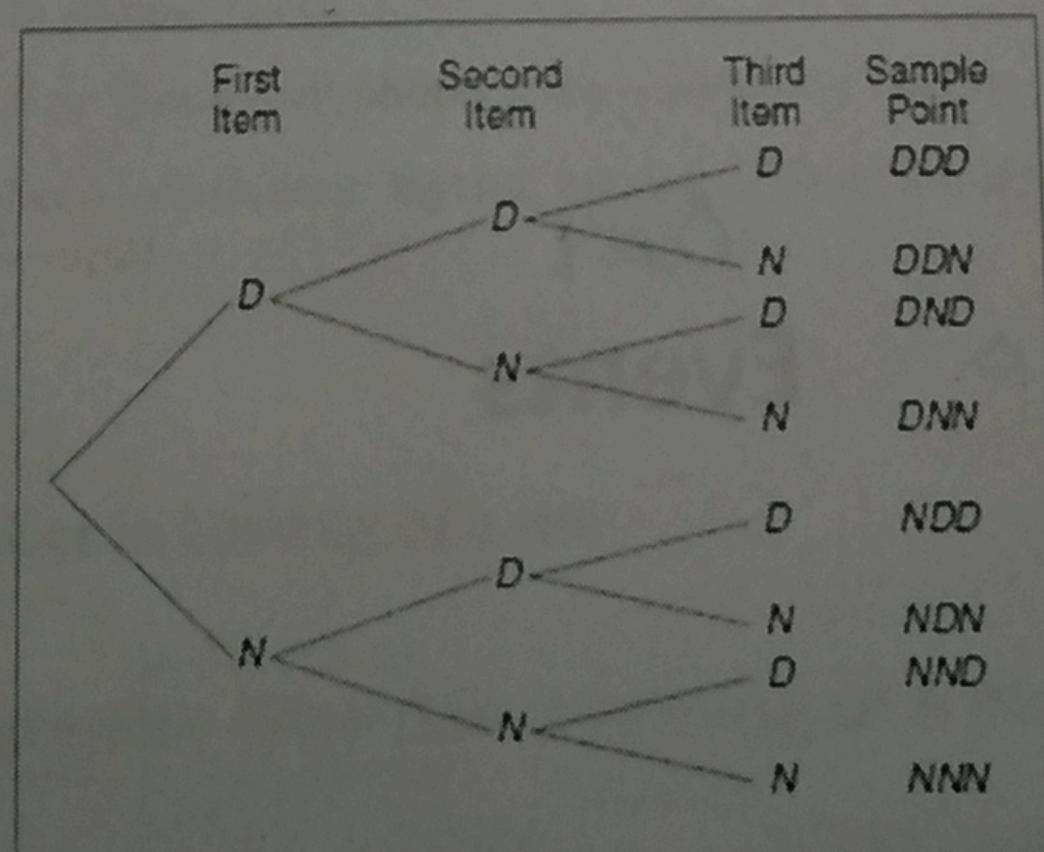


Figure 2.2: Tree diagram for Example 2.3.

for 3 items

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الاستقافه عن نتائج element باستخدام rule

- Sample spaces with a large or infinite number of sample points are best described by a **statement** or **rule**.
 بدل ما نكتب element داخل الفضاء العيني

- For example, if the possible outcomes of an experiment are the set of cities in the world with a. population over 1 million:

$S = \{x \mid x \text{ is a city with a. population over 1 million}\}$,
 such that

statement ← which reads "S is the set of all x such that x is a city with a population over 1 million.

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هذا شرط
 is a city with a

أو مثلاً عدد الطلاب الذي جابوا A+

أكثر
 حدث داخل sample space Events

- **Definition 2.2:** An event is a subset of a sample space.

$$S = \{DDD, DDN, DND, DNN, NDD, NDN, NND, NNN\}.$$

ببقر نسبيما أي شيء

- **Event B** represents the number of defectives is greater than 1 → يعني

like more than

$$B = \{DDN, DND, NDD, DDD\}$$

$$S = \{1, 2, 3, 4, 5, 6\}$$

at least one → على الأقل واحد

$$A = \{3, 6\}$$

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at least two A

$$C = \{DNN, NDN, NND, NNN\}$$

السؤال

rule

الجواب rule

لا يوجد عقر بالسالب

Example 2.4: Given the sample space $S = \{t \mid t \geq 0\}$ where t is the life in years of a certain electronic component, then the event A that the component fails before the end of the fifth year is the subset $A = \{t \mid 0 \leq t < 5\}$.

$$A = \{2, 4, 6\}$$

$$A' = \{1, 3, 5\}$$

Definition 2.3:

The complement of an event A with respect to S is the subset of all elements of S that are not in A . We denote the complement of A by the symbol A' .

Example 2.6: Consider the sample space

$$S = \{\text{book, cell phone, mp3, paper, stationery, laptop}\}.$$

Let $A = \{\text{book, stationery, laptop, paper}\}$. Then the complement of A is $A' = \{\text{cell phone, mp3}\}$.

$$S = \{1, 2, 3, 4, 5, 6\}$$

If event A is the even number in tossing of a die

$$A = \{2, 4, 6\}, A' = \{1, 3, 5\}$$

If event B is a number greater than 3 in a tossing of a die

$$B = \{4, 5, 6\}, B' = \{1, 2, 3\}$$

more than

أكثر من 3

at least 3

أعلى 3 تكون من 3 فصاعدا

Definition 2.4:

The intersection of two events A and B , denoted by the symbol $A \cap B$, is the event containing all elements that are common to A and B .

Example 2.7: Let E be the event that a person selected at random in a classroom is majoring in engineering, and let F be the event that the person is female. Then $E \cap F$ is the event of all female engineering students in the classroom.

Example 2.8: Let $V = \{a, e, i, o, u\}$ and $C = \{l, r, s, t\}$; then it follows that $V \cap C = \emptyset$. That is, V and C have no elements in common and, therefore, cannot both simultaneously occur.

For certain statistical experiments it is by no means unusual to define two events, A and B , that cannot both occur simultaneously. The events A and B are then said to be **mutually exclusive**. Stated more formally, we have the following definition:

Definition 2.5:

Two events A and B are **mutually exclusive**, or **disjoint**, if $A \cap B = \emptyset$, that is, if A and B have no elements in common.

Definition 2.6:

The **union** of the two events A and B , denoted by the symbol $A \cup B$, is the event containing all the elements that belong to A or B or both.

Example 2.10: Let $A = \{a, b, c\}$ and $B = \{b, c, d, e\}$; then $A \cup B = \{a, b, c, d, e\}$.

$$A \cap B = \{b, c\}$$

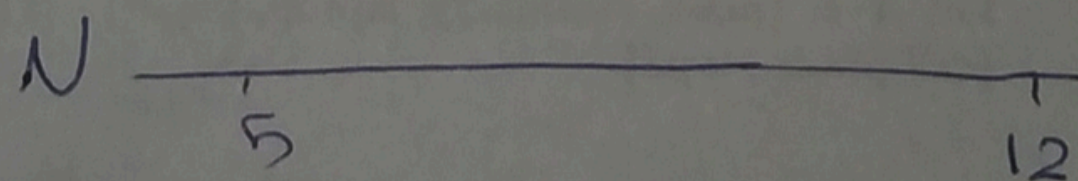
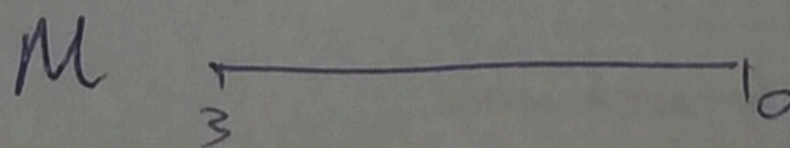
Independent event / disjoint event

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Example 2.12: If $M = \{x | 3 < x < 9\}$ and $N = \{y | 5 < y < 12\}$, then

$$M \cup N = \{z | 3 < z < 12\}$$

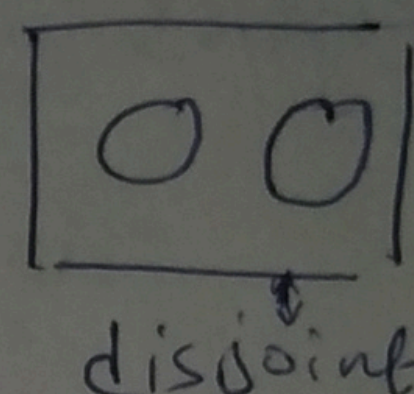
$$M \cap N = \{z | 5 < z < 9\}$$



* disjoint event →
أحداث لا تتلحق في نفس الوقت

* Independent →
أحداث تتلحق في نفس الوقت لكن
لا يؤثر أحدهما على الآخر

- The relationship between events and the corresponding sample space can be illustrated graphically by means of Venn diagrams.



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The relationship between events and the corresponding sample space can be illustrated graphically by means of Venn diagrams. In a Venn diagram we let the sample space be a rectangle and represent events by circles drawn inside the rectangle. Thus, in Figure 2.3, we see that

$$A \cap B = \text{regions 1 and 2,}$$

$$B \cap C = \text{regions 1 and 3,}$$

$$A = \{1, 2, 4, 7\}$$

$$B = \{1, 2, 3, 6\}$$

$$C = \{1, 3, 4, 5\}$$

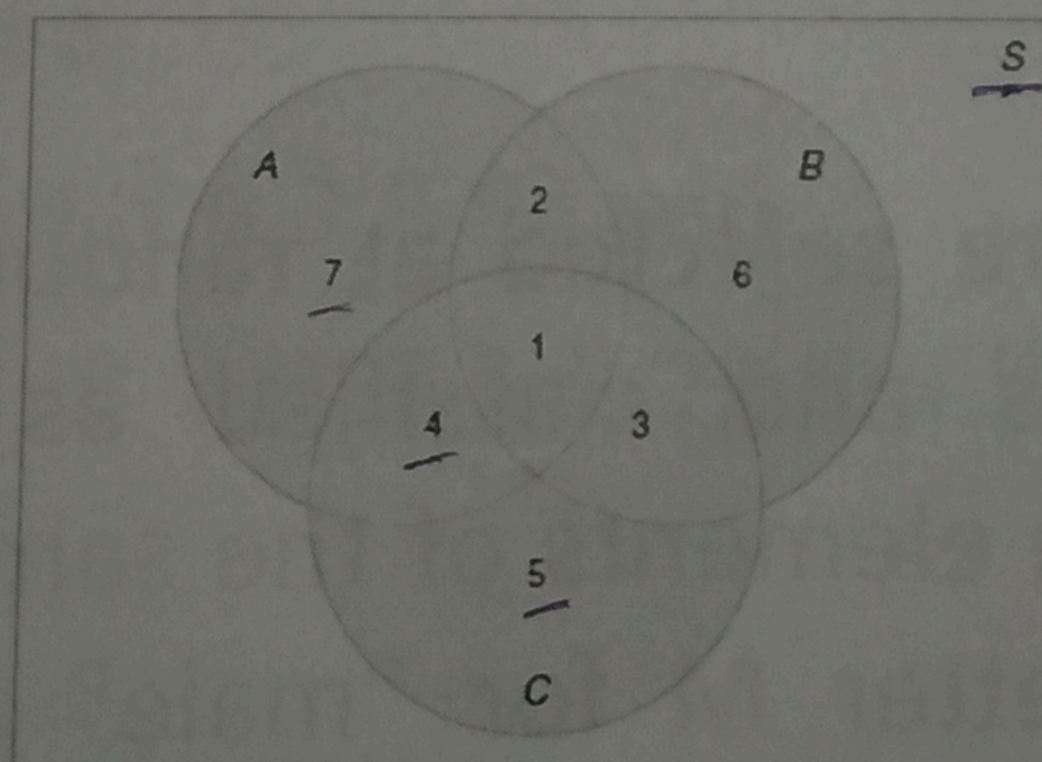


Figure 2.3: Events represented by various regions.

الدوائر تمثل الأحداث

$$A \cup C = \text{regions 1, 2, 3, 4, 5, and 7.}$$

$$B' \cap A = \text{regions 4 and 7,}$$

$$A \cap B \cap C = \text{region 1,}$$

$$(A \cup B) \cap C' = \text{regions 2, 6, and 7,}$$

{ 4, 5, 7 } أي منطقة خارجها

$$\{1, 2, 4, 7, 3, 6\} \cap \{2, 7, 6\}$$

$$= \{2, 6, 7\}$$

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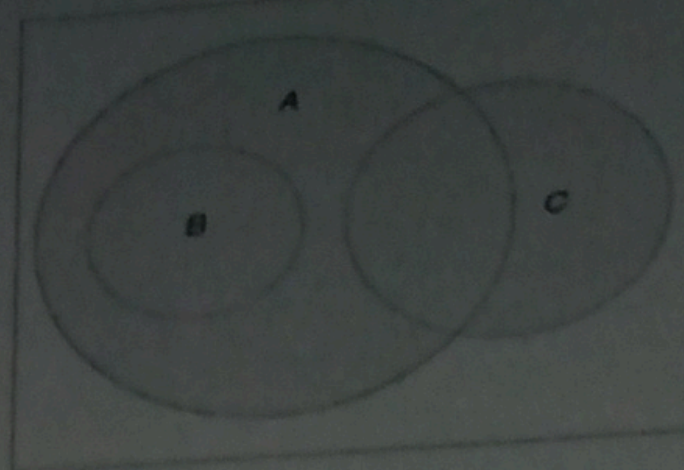


Figure 2.4: Events of the sample space S .

In Figure 2.4, we see that events A , B , and C are all subsets of the sample space S . It is also clear that event B is a subset of event A ; event $B \cap C$ has no elements and hence B and C are mutually exclusive; event $A \cap C$ has at least one element; and event $A \cup B = A$. Figure 2.4 might, therefore, depict a situation where we select a card at random from an ordinary deck of 52 playing cards and observe whether the following events occur:

A : the card is red.

B : the card is the jack, queen, or king of diamonds.

C : the card is an ace.

Clearly, the event $A \cap C$ consists of only the two red aces.

Several results that follow from the foregoing definitions, which may easily be verified by means of Venn diagrams, are as follows:

1. $A \cap \phi = \phi$.

2. $A \cup \phi = A$.

3. $A \cap A' = \phi$.

4. $A \cup A' = S$.

5. $S' = \phi$.

6. $\phi' = S$.

7. $(A')' = A$.

8. $(A \cap B)' = A' \cup B'$.

9. $(A \cup B)' = A' \cap B'$.

• Exercise 2.7:

Four students are selected at random from a chemistry class and classified as male or female. List the elements of the sample space S_1 using the letter M for "male" and F for "female." Define a second sample space S_2 where the elements represent the number of females selected.

$S_1 = \{MMMM, MMMF, MMFM, MFMM, FMFM, FFMM, FMFF, FFMF, FFFM, FFFF\}$

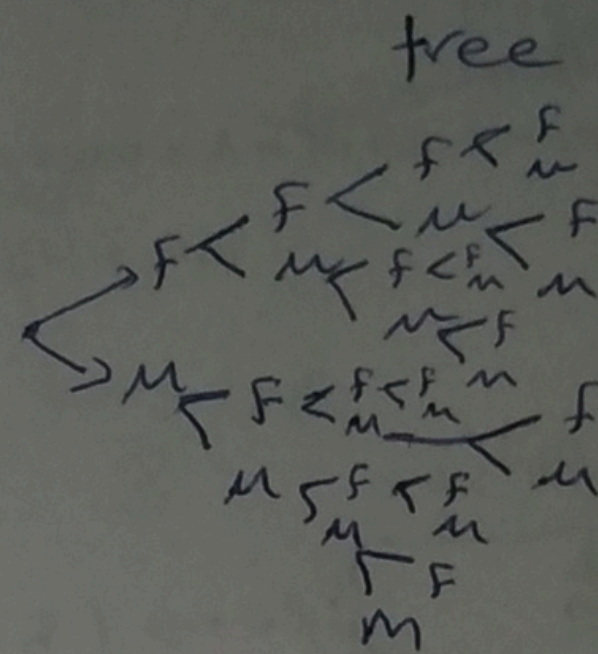
Handwritten numbers above the strings: 0, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 4.

Handwritten number 16 in a box.

$S_2 = \{0, 1, 2, 3, 4\}$

sample space
جمله من S_1

احتمالية
وجود female



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• Exercise 2.14:

If $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and

$A = \{0, 2, 4, 6, 8\}$, $B = \{1, 3, 5, 7, 9\}$, $C = \{2, 3, 4, 5\}$, and

$D = \{1, 6, 7\}$

(a) $A \cup C = \{0, 2, 3, 4, 5, 6, 8\}$.

(b) $A \cap B = \emptyset$.

(c) $C' = \{0, 1, 6, 7, 8, 9\}$.

(d) $(C' \cap D) \cup B$; $C' \cap D = \{1, 6, 7\}$, $(C' \cap D) \cup B = \{1, 3, 5, 6, 7, 9\}$

(e) $(S \cap C)'$; $C' = \{0, 1, 6, 7, 8, 9\}$. $(S' \cup C')$

(f) $A \cap C \cap D' = \{2, 4\}$, so $A \cap C \cap D' = \{2, 4\}$.

Rule 2.1: If an operation can be performed in n_1 ways, and if for each of these ways a second operation can be performed in n_2 ways, then the two operations can be performed together in $n_1 n_2$ ways.

Example 2.13: How many sample points are there in the sample space when a pair of dice is thrown once?

Solution: The first die can land face-up in any one of $n_1 = 6$ ways. For each of these 6 ways, the second die can also land face-up in $n_2 = 6$ ways. Therefore, the pair of dice can land in $n_1 n_2 = (6)(6) = 36$ possible ways.

Example 2.14: A developer of a new subdivision offers prospective home buyers a choice of Tudor, rustic, colonial, and traditional exterior styling in ranch, two-story, and split-level floor plans. In how many different ways can a buyer order one of these homes?

Solution:

Since $n_1 = 4$ and $n_2 = 3$, a buyer must choose from

$$n_1 n_2 = (4)(3) = 12 \text{ possible homes.}$$

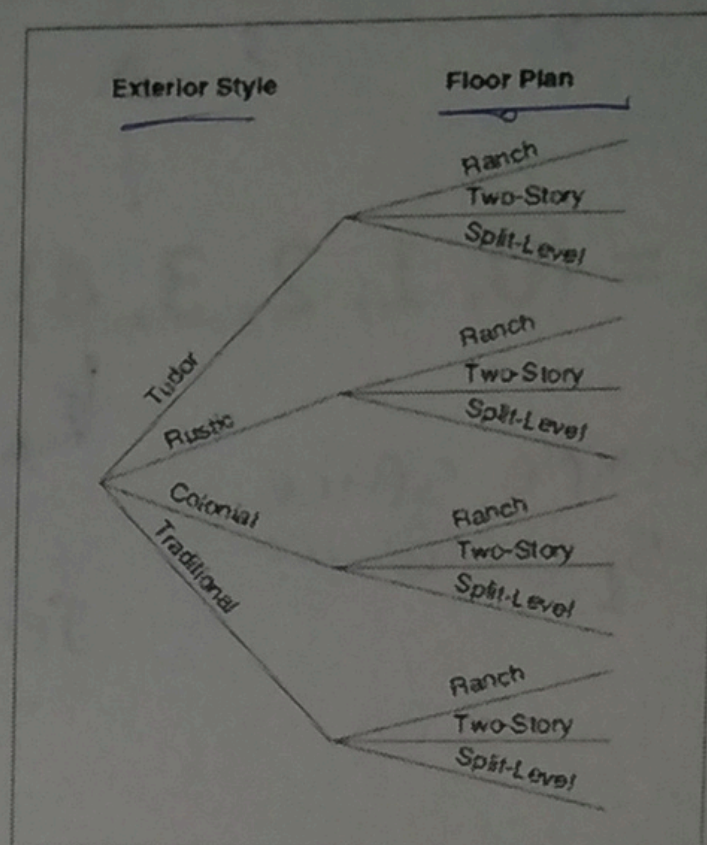
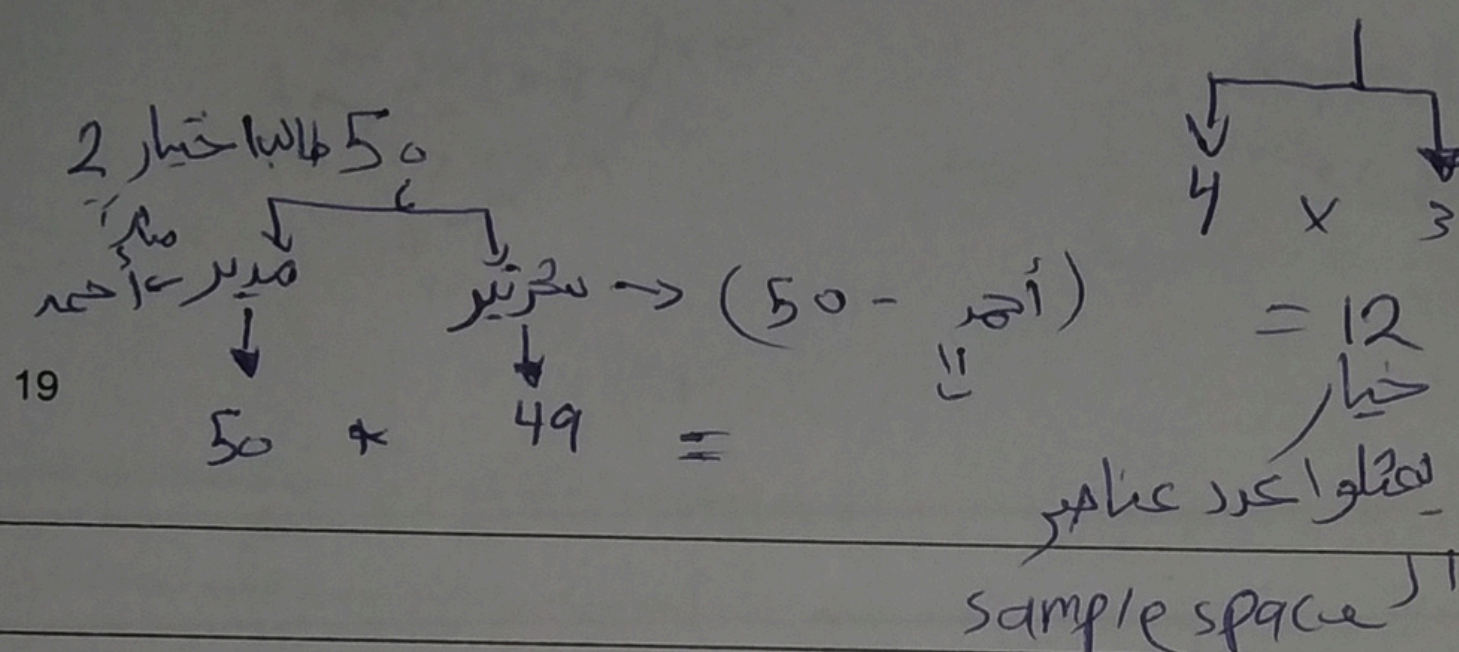


Figure 2.6: Tree diagram for Example 2.14.



Q.1

Example 2.15: If a 22-member club needs to elect a chair and a treasurer, how many different ways can these two to be elected?

Solution: For the chair position, there are 22 total possibilities. For each of those 22 possibilities, there are 21 possibilities to elect the treasurer. Using the multiplication rule, we obtain $n_1 \times n_2 = 22 \times 21 = 462$ different ways.

The multiplication rule, Rule 2.1 may be extended to cover any number of operations. Suppose, for instance, that a customer wishes to buy a new cell phone and can choose from $n_1 = 5$ brands, $n_2 = 5$ sets of capability, and $n_3 = 4$ colors. These three classifications result in $n_1 n_2 n_3 = (5)(5)(4) = 100$ different ways for a customer to order one of these phones. The **generalized multiplication rule** covering k operations is stated in the following.

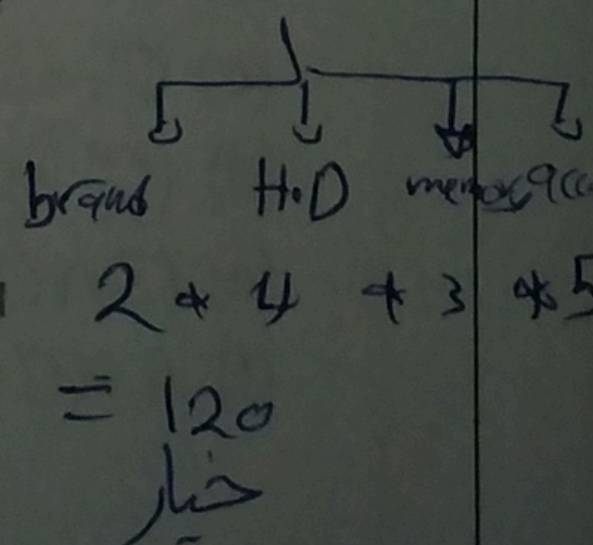
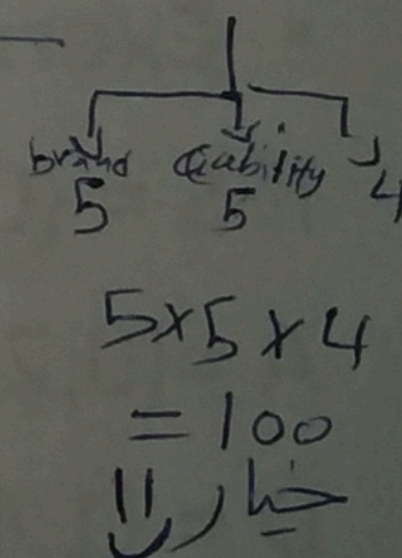
Rule 2.2: If an operation can be performed in n_1 ways, and if for each of these a second operation can be performed in n_2 ways, and for each of the first two a third operation can be performed in n_3 ways, and so forth, then the sequence of k operations can be performed in $n_1 n_2 \cdots n_k$ ways.

Example 2.16: Sam is going to assemble a computer by himself. He has the choice of chips from two brands, a hard drive from four, memory from three, and an accessory bundle from five local stores. How many different ways can Sam order the parts?

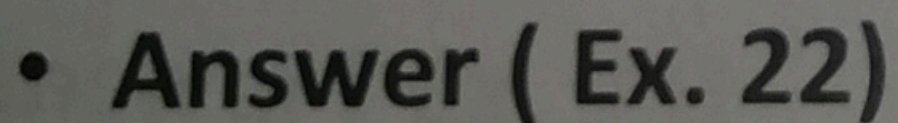
Solution: Since $n_1 = 2$, $n_2 = 4$, $n_3 = 3$, and $n_4 = 5$, there are

$$n_1 \times n_2 \times n_3 \times n_4 = 2 \times 4 \times 3 \times 5 = 120$$

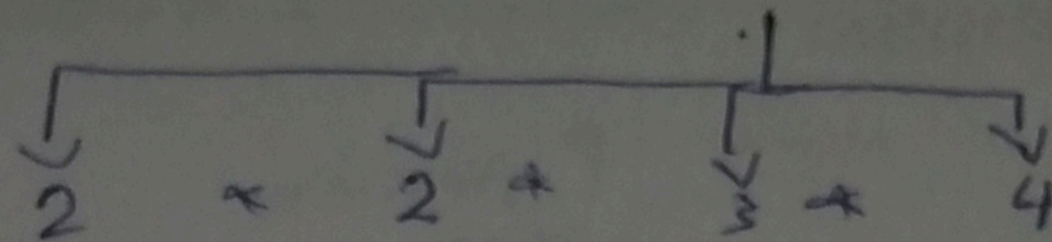
different ways to order the parts.



21



2.27 A developer of a new subdivision offers a prospective home buyer a choice of 4 designs, 3 different heating systems, a garage or carport, and a patio or screened porch. How many different plans are available to this buyer?



= 48
in sample space

- Answer (Ex. 27)

$$n_1 n_2 n_3 n_4 = 4 * 3 * 2 * 2 = 48 \text{ different house plans}$$

عدد حادث وهران ١١

2.33 A witness to a hit-and-run accident told the police that the license number contained the letters RLH followed by 3 digits, the first of which is a 5. If the witness cannot recall the last 2 digits, but is certain that all 3 digits are different, find the maximum number of automobile registrations that the police may have to check.

3 مختلفان

RLH 5 | - -

5 مسبقا

9 خياران 8 خياران 0 1 2 3 4 5 6 7 8 9

= 72

sample space

إذا كانتا مختلفتان 10 9 8
25
similar

Answer (Ex. 33)

$$n_1 n_2 = 9 * 8 = 72 \text{ registrations to be checked.}$$

Example 4

A president and a treasurer are to be chosen from a student club consisting of 50 people. How many different choices of officers are possible if

- (a) There are no restrictions?
- (b) A will serve only if he is president?
- (c) B and C will serve together or not at all?

Solution:

$$(a) {}_{50}P_2 = \frac{50!}{(50-2)!} = \frac{50!}{48!} = \frac{50 \cdot 49 \cdot 48!}{48!} = 50 \cdot 49 = 2450$$

(b) Since A will serve only if he is president, we have two situations here: A is selected as the president, which yields 49 possible outcomes for the treasurer's position, or officers are selected from the remaining 49 people without A, which has the number of choices ${}_{49}P_2 = 49 \times 48 = 2352$. Therefore, the total number of choices is $49 + 2352 = 2401$.

(c) The number of selections when B and C serve together is 2. The number of selections when both B and C are not chosen is ${}_{48}P_2 = 2256$. Therefore, the total number of choices in this situation is $2 + 2256 = 2258$.

2 - 23

Example

- **A president and a treasurer are to be chosen from a student club consisting of 50 people. How many different choices of officers are possible if**

– (c) **B and C will serve together or not at all:**

(1) B president, C - Treasurer →
C President, B - Treasurer →
(2)

(17) $n = 48, r = 2$

$${}_{48}P_2 = \frac{48!}{(48-2)!} = \frac{48 \times 47 \times 46!}{46!} = 48 \times 47 = 2256$$

$$2 + 2256 = 2258$$

نصف ولد نصف بنت ولد
 أحمد / محمد / فادي / رامي
 واحد 2.37 In how many ways can 4 boys and 5 girls sit in
 a row if the boys and girls must alternate?

حدا الاوى
 اختيار البنات للانا عدد هم ايجبروا

$$\frac{5}{5} \times \frac{4}{4} \times \frac{4}{4} \times \frac{3}{3} \times \frac{3}{3} \times \frac{2}{2} \times \frac{1}{1} \times \frac{1}{1} = 2880$$

Example:

How many (even three digit) numbers can be performed from the digits 1,2,5,6 and 9 if each digit can be used only once?

لكل رقم يستخدم مرة واحدة
 رقم زوجي 3 منازل

$$n_1 n_2 n_3 = (4)(3)(2) = 24$$

$$\frac{9/6}{3 \times 4 \times 2} = 24$$

 خيار

إذا سمح التكرار

$$\frac{5}{5} \times \frac{5}{5} \times \frac{2}{2}$$

$$= 50$$

 خيار

(موضوع تفرع في منطقة الأرقام) - 5 3 0/2/6

0/1/2/5/6/9

Example 2.17: How many even four-digit numbers can be formed from the digits 0, 1, 2, 5, 6, and 9 if each digit can be used only once?

Solution: Since the number must be even, we have only $n_1 = 3$ choices for the units position. However, for a four-digit number the thousands position cannot be 0. Hence, we consider the units position in two parts, 0 or not 0. If the units position is 0 (i.e., $n_1 = 1$), we have $n_2 = 5$ choices for the thousands position, $n_3 = 4$ for the hundreds position, and $n_4 = 3$ for the tens position. Therefore, in this case we have a total of

$$3 \times 4 \times 5 \times 1 = 60$$

$$n_1 n_2 n_3 n_4 = (1)(5)(4)(3) = 60$$

even four-digit numbers. On the other hand, if the units position is not 0 (i.e., $n_1 = 2$), we have $n_2 = 4$ choices for the thousands position, $n_3 = 4$ for the hundreds position, and $n_4 = 3$ for the tens position. In this situation, there are a total of

$$n_1 n_2 n_3 n_4 = (2)(4)(4)(3) = 96$$

Since the above two cases are mutually exclusive of each other, the total number of even four-digit numbers can be calculated by $60 + 96 = 156$

Definition 2.7: A permutation is an arrangement of all or part of a set of objects.

تباديل (تبديل عندي وجوه اختلاف)

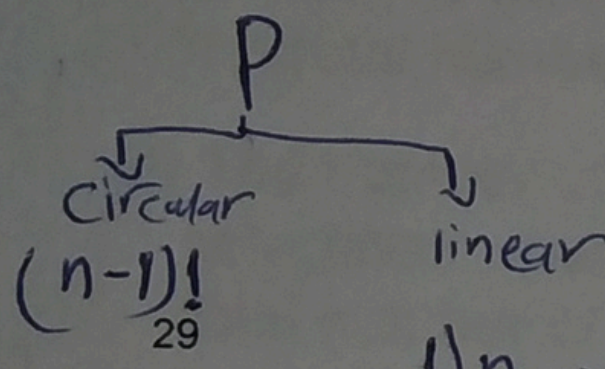
Consider the three letters a, b , and c . The possible permutations are abc, acb, bac, bca, cab , and cba . Thus, we see that there are 6 distinct arrangements. Using Rule 2.2, we could arrive at the answer 6 without actually listing the different orders by the following arguments: There are $n_1 = 3$ choices for the first position. No matter which letter is chosen, there are always $n_2 = 2$ choices for the second position. No matter which two letters are chosen for the first two positions, there is only $n_3 = 1$ choice for the last position, giving a total of

$$n_1 n_2 n_3 = (3)(2)(1) = 6 \text{ permutations}$$

by Rule 2.2. In general, n distinct objects can be arranged in

$$n(n-1)(n-2) \cdots (3)(2)(1) \text{ ways.}$$

There is a notation for such a number.



1) n object $n!$ $\rightarrow abc$

2) n object $\rightarrow$ بديلات $\rightarrow 22$

3) n object $\rightarrow$ $\frac{n!}{(n-r)!}$ $\rightarrow$ 21 different object

4 4 3 2
ص الف
ليون
المفرح يبي بالنهن

$$3 \times 2 \times 1 = 3!$$

Theorem 2.1: The number of permutations of n objects is $n!$.

The number of permutations of the four letters a, b, c , and d will be $4! = 24$. Now consider the number of permutations that are possible by taking two letters at a time from four. These would be $ab, ac, ad, ba, bc, bd, ca, cb, cd, da, db$, and dc . Using Theorem 2.1 again, we have two positions to fill with $n_1 = 4$ choices for the first and then $n_2 = 3$ choices for the second for a total of

$$n_1 n_2 = (4)(3) = 12$$

permutations. In general, n distinct objects taken r at a time can be arranged in

$$n(n-1)(n-2) \cdots (n-r+1)$$

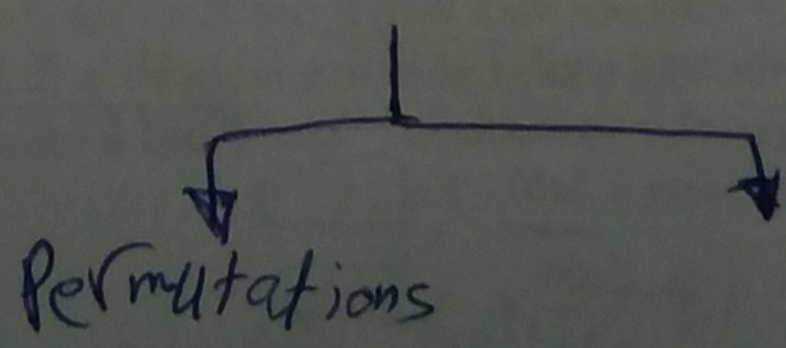
ways. We represent this product by the symbol

$${}_n P_r = \frac{n!}{(n-r)!}$$

As a result we have the theorem that follows.

Theorem 2.2: The number of permutations of n distinct objects taken r at a time is

$${}_n P_r = \frac{n!}{(n-r)!}$$



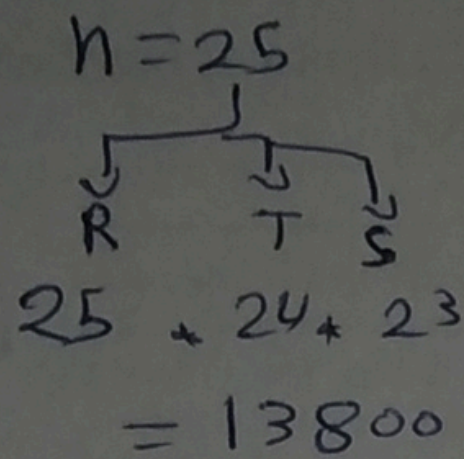
واحد كذا في الكل

Example 2.18: In one year, three awards (research, teaching, and service) will be given to a class of 25 graduate students in a statistics department. If each student can receive at

most one award, how many possible selections are there?

Solution: Since the awards are distinguishable, it is a permutation problem. The total number of sample points is

$${}_{25}P_3 = \frac{25!}{(25-3)!} = \frac{25!}{22!} = (25)(24)(23) = 13,800.$$



$$\frac{25!}{23!} = \frac{25 \times 24 \times 23!}{23!}$$

31

و لاهو

+ لاهو

Example 2.19: A president and a treasurer are to be chosen from a student club consisting of 50 people. How many different choices of officers are possible if

(a) there are no restrictions;

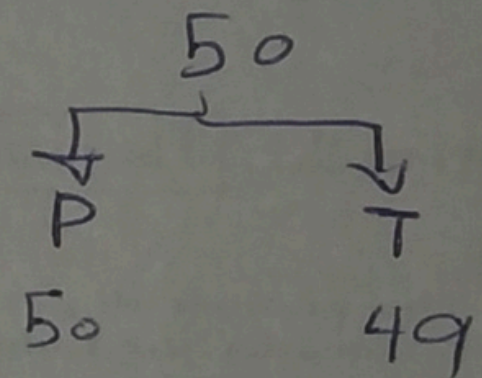
(b) A will serve only if he is president; → *president*

(c) B and C will serve together or not at all;

(d) D and E will not serve together?

Solution: (a) The total number of choices of officers, without any restrictions, is

$${}_{50}P_2 = \frac{50!}{48!} = (50)(49) = 2450.$$



(b) Since A will serve only if he is president, we have two situations here: (i) A is selected as the president, which yields 49 possible outcomes for the treasurer's position, or (ii) officers are selected from the remaining 49 people without A, which has the number of choices ${}_{49}P_2 = (49)(48) = 2352$. Therefore, the total number of choices is $49 + 2352 = 2401$.

(c) The number of selections when B and C serve together is 2. The number of selections when both B and C are not chosen is ${}_{48}P_2 = 2256$. Therefore, the total number of choices in this situation is $2 + 2256 = 2258$.

(d) The number of selections when D serves as an officer but not E is $(2)(48) = 96$, where 2 is the number of positions D can take and 48 is the number of selections of the other officer from the remaining people in the club except E. The number of selections when E serves as an officer but not D is also $(2)(48) = 96$. The number of selections when both D and E are not chosen is ${}_{48}P_2 = 2256$. Therefore, the total number of choices is $(2)(96) + 2256 = 2448$. This problem also has another short solution: Since D and E can only serve together in 2 ways, the answer is $2450 - 2 = 2448$.

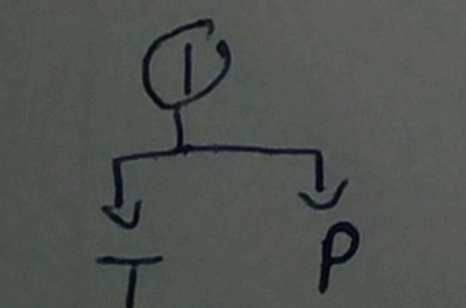
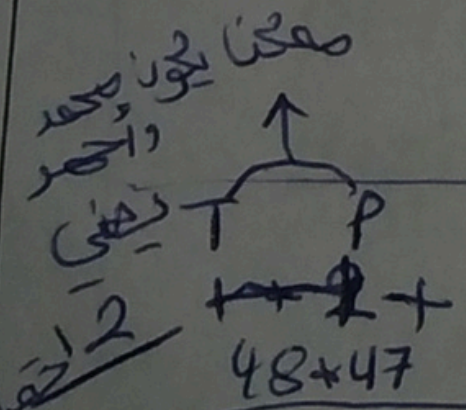
Start again with the 2450 permutations. If B, other than C who might be treasurer. And if E other than C who might be president. We must also eliminate the 96 outcomes in which C will serve. $2450 - 96 - 96 = 2258$

$$\rightarrow 2 \times 48$$

$$\rightarrow 2 \times 48$$

$$48 \times 47$$

$$2 \times 4$$



$$1 + 1 = 2$$

$$2 \times 48$$

32

$$2 \times 48$$

$$2 \times 48$$

Theorem 2.3: The number of permutations of n objects arranged in a circle is $(n-1)!$.

So far we have considered permutations of distinct objects. That is, all the objects were completely different or distinguishable. Obviously, if the letters b and c are both equal to x , then the 6 permutations of the letters a, b , and c become $axx, axx, xax, xax, xxa, xxa$, of which only 3 are distinct. Therefore, with 3 letters, 2 being the same, we have $3!/2! = 3$ distinct permutations. With 4 different letters, 2 being the same, we have 24 distinct permutations. If we let $a = b = x$ and $c = d = y$, we can list only the following distinct permutations: $xyxy, xyxy, yxyx, yxyx, xyxx, xyxx$, and $yxyx$. Thus, we have $4!/(2!2!) = 6$ distinct permutations.

Theorem 2.4: The number of distinct permutations of n things of which n_1 are of one kind, n_2 of a second kind, ..., n_k of a k th kind is

$$\frac{n!}{n_1! n_2! \cdots n_k!}$$

$\downarrow \quad \downarrow \quad \downarrow$
 $2! \quad 1! \quad 1!$

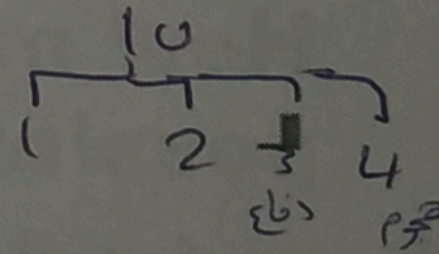
1 البسطة = 1
 2! 1! 1!

35

Example 2.20: In a college football training session, the defensive coordinator needs to have 10 players standing in a row. Among these 10 players, there are 1 freshman, 2 sophomores, 4 juniors, and 3 seniors. How many different ways can they be arranged in a row if only their class level will be distinguished?

Solution: Directly using Theorem 2.4, we find that the total number of arrangements is

$$\frac{10!}{1! 2! 4! 3!} = 12,600.$$



Theorem 2.5: The number of ways of partitioning a set of n objects into r cells with n_1 elements in the first cell, n_2 elements in the second, and so forth, is

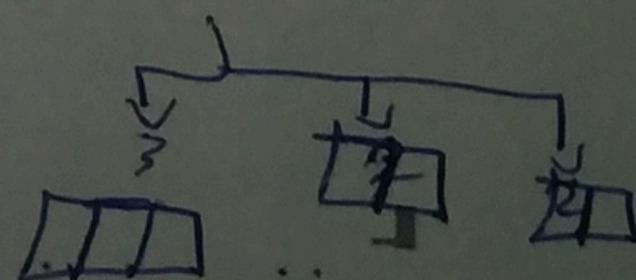
$$\binom{n}{n_1, n_2, \dots, n_r} = \frac{n!}{n_1! n_2! \cdots n_r!},$$

where $n_1 + n_2 + \cdots + n_r = n$.

Example 2.21: In how many ways can 7 graduate students be assigned to 1 triple and 2 double hotel rooms during a conference?

Solution: The total number of possible partitions would be

$$\binom{7}{3, 2, 2} = \frac{7!}{3! 2! 2!} = 210.$$



36

ما عدد الترتيبات المختلفة التي يمكن أن تكون من الحروف في الكلمة STATISTICS?

Example 2.23: How many different letter arrangements can be made from the letters in the word STATISTICS?

Solution: Using the same argument as in the discussion for Theorem 2.6, in this example we can actually apply Theorem 2.5 to obtain

$$\binom{10}{3, 3, 2, 1, 1} = \frac{10!}{3! 3! 2! 1! 1!} = 50,400.$$

Here we have 10 total letters, with 2 letters (S, T) appearing 3 times each, letter I appearing twice, and letters A and C appearing once each. On the other hand, this result can be directly obtained by using Theorem 2.4. $\blacksquare$

SSS TTT II AC

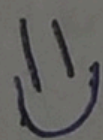
$$\frac{10!}{3! 3! 2! 1! 1!}$$

In many problems, we are interested in the number of ways of selecting r objects from n without regard to order. These selections are called **combinations**. A combination is actually a partition with two cells, the one cell containing the r objects selected and the other cell containing the $(n - r)$ objects that are left. The number of such combinations, denoted by

$$\binom{n}{r, n-r}, \text{ is usually shortened to } \binom{n}{r},$$

since the number of elements in the second cell must be $n - r$.

Theorem 2.6: The number of combinations of n distinct objects taken r at a time is



select
elect

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Example 2.22: A young boy asks his mother to get 5 Game-Boy™ cartridges from his collection of 10 arcade and 5 sports games. How many ways are there that his mother can get 3 arcade and 2 sports games?

Solution: The number of ways of selecting 3 cartridges from 10 is

$$\binom{10}{3} = \frac{10!}{3!(10-3)!} = 120.$$

The number of ways of selecting 2 cartridges from 5 is

$$\binom{5}{2} = \frac{5!}{2! 3!} = 10.$$

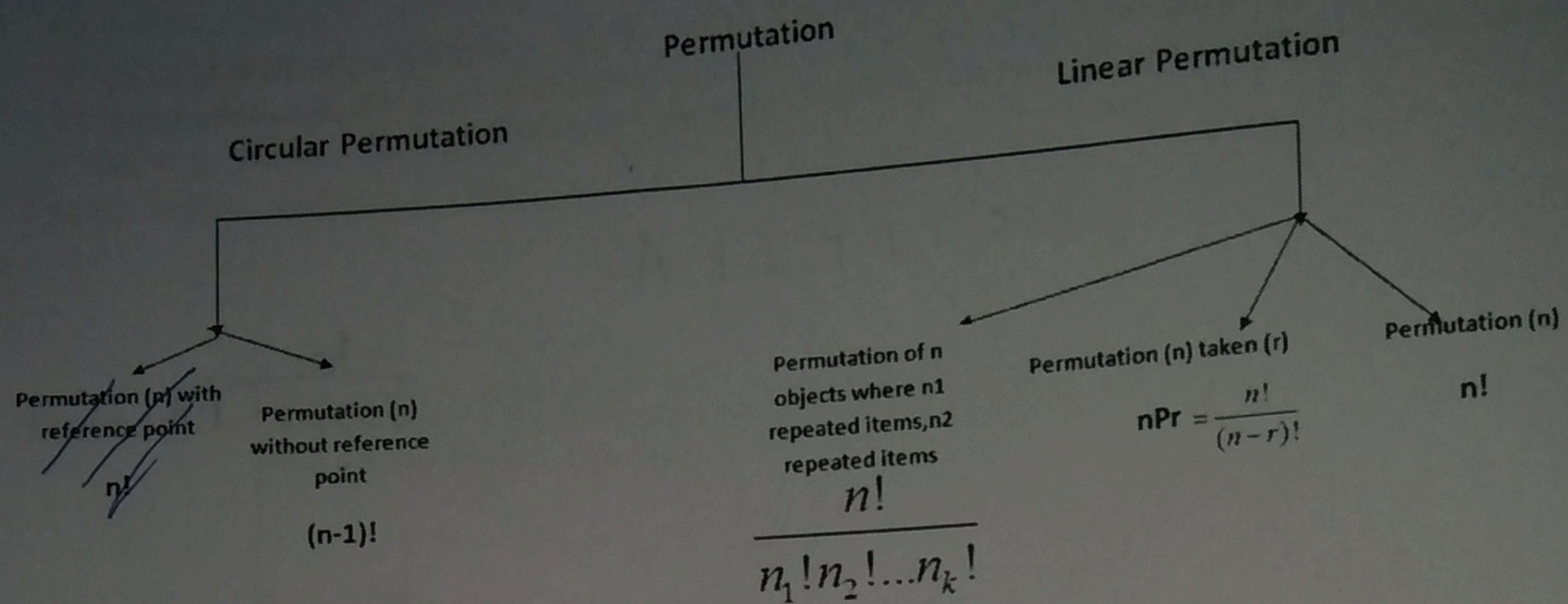
Using the multiplication rule (Rule 2.1) with $n_1 = 120$ and $n_2 = 10$, we have

$$(120)(10) = 1200 \text{ ways.}$$

Rule 2.1: If an operation can be performed in n_1 ways, and if for each of these ways a second operation can be performed in n_2 ways, then the two operations can be performed together in $n_1 n_2$ ways.

بعض الألعاب
لأنه احتمالي
مكونوا مع بعض عادي

10 arcade
3
5 sport
2



Case 3+4

39

Permutations and Combinations

Number of permutations (order matters) of n things taken r at a time:

$$P(n, r) = \frac{n!}{(n-r)!}$$

Number of combinations (order does not matter) of n things taken r at a time:

$$C(n, r) = \frac{n!}{(n-r)!r!}$$

Number of different permutations of n objects where there are n_1 repeated items, n_2 repeated items, ... n_k repeated items

$$\frac{n!}{n_1!n_2!\dots n_k!}$$

40

- If an experiment consists of throwing a die and then drawing a letter at random from the English alphabet, how many points are there in the sample space?

(1) للعدد 6
 (2) للأحرف 26
 $6 \times 26 = 156$

Ex 2.32:

- a) How many distinct permutations can be made from the letters of the word columns?

5040

7! no object
1 2 3 4 5 6 7

- b) How many of these permutations start with the letter m?

بدون m في columns

$6! = 720$

1 a b c d
 2 a b c d
 3 a b c d
 4 a b c d
 5 a b c d

EX. 2.31

If a multiple-choice test consists of 5 questions each with 4 possible answers of which only 1 is correct,

(a) In how many different ways can a student check off one answer to each question?

$$(4^5) = 1024$$

(b) In how many ways can a student, check off one answer to each question and get all the answers wrong?

$$(3)^5 = 243$$

Ex. 2.40

In how many ways can 5 starting positions on a basketball team be filled with 8 men who can play any of the positions?

different

$$6720$$

$$8-5 = 3!$$

$$\frac{8!}{3!} =$$

$$8 \rightarrow 5$$