

التباديل (Permutation) : يوجد بها ترتيب

لـ والثَّقَا السُّؤَالِ يَحْتَوِي عَلَى different

Position: الاختلاف هو الذي يهتج الباديل

arrangement

Permutation

Circular

$$(n-1)!$$

بجوں اسی دائرے

يَمْنِي وَلِيْنَا مَا وَجِدْتَ خَلْفَهُ

♡ Circular محل تبادل ۱۱

linear

ترتيب - مختلفات 1) nobject

مثل جماعة تحتوي على حروف ضغامة

2) n object $\longleftrightarrow$ kein γ , nicht

$$nPr = \frac{n!}{(n-r)!}$$

3) n object $n!$

مثلاً جملہ حروف تہجی کے ساتھ

4) n object

عنا جبروتیہ بنانا اور عام علی group

$$\frac{n!}{n! | n_2! \dots}$$

- التوافيق (Combinations) $\frac{n!}{r!(n-r)!}$

50 - بدي اختار منها 2 portion، ~~الم~~ واحد

Just selection

ncv

Keyword: (select / elect / to get)

Circular

Ex.2.44

In how many ways can a caravan of 8 covered wagons from Arizona be arranged in a circle?

ترتيب
 $(n-1)! = 7!$

5040

3 حالات

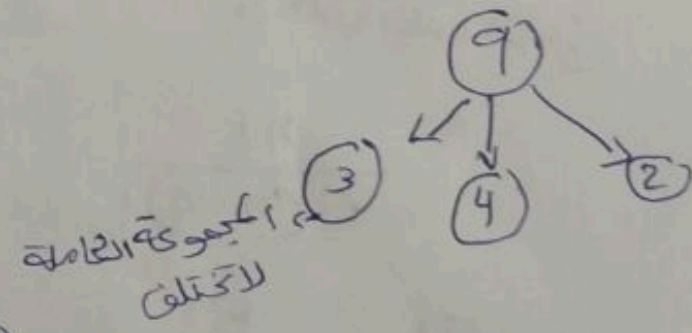
Ex 2.46

In how many ways can 3 oaks, 4 pines, and 2 maples be arranged along a property line if one does not distinguish among trees of the same kind?

1260

$$\frac{n!}{n_1! n_2! \dots}$$

$$= \frac{9!}{3! 4! 2!} = 1260$$



Ex. 2.47

12 مباريات

A college plays 12 football games during a season. In how many ways can the team end the season with 7 wins, 3 losses, and 2 ties?

الكافة 4

ليس هو مجموع
العدد
الموجود في البسط

$$\frac{12!}{7! 3! 2!} = 7920$$

47

Ex 2.49

How many ways are there to select 3 candidates from 8 equally qualified recent graduates for openings in an accounting firm

8 طلاب زعموا انهم

$$\frac{n!}{r!(n-r)!}$$

56 ways

$$\boxed{8P_3} = \frac{8!}{3!(8-3)!}$$

48
permutation

- 2.36 (a) How many three-digit numbers can be formed from the digits 0, 1, 2, 3, 4, 5, and 6, if each digit can be used only once? (b) How many of these are odd numbers? c) How many are greater than 330

0 1 2 3 4 5 6
بدون 0 = 6 + 5 + 3

a) 180 (6 * 5 * 4)

0 1 2 3 4 5 6

b) 75 (5 + 5 + 3)

5 + 5 + 3

c)

0 1 2 3 4 5 6

تباين أول مرة

الأرقام 330

49 ① 3 5 = 90

③ 1 1 6 = 6

② 1 3 5 = 15

90 + 15 + 6 = 111

330 (مجموع)

Definition 2.9:

جزء من كل

The probability of an event A is the sum of the weights of all sample points in A . Therefore,

0 ≤ P(A) ≤ 1, P(∅) = 0, and P(S) = 1.

Furthermore, if $A_1, A_2, A_3, \dots$ is a sequence of mutually exclusive events, then

$P(A_1 \cup A_2 \cup A_3 \cup \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$

Example 2.24: A coin is tossed twice. What is the probability that at least 1 head occurs?

Solution: The sample space for this experiment is

$S = \{HH, HT, TH, TT\}$

If the coin is balanced, each of these outcomes is equally likely to occur. Therefore, we assign a probability of ω to each sample point. Then $4\omega = 1$, or $\omega = 1/4$. If A represents the event of at least 1 head occurring, then

$A = \{HH, HT, TH\}$ and $P(A) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

If more than one head = $\frac{1}{4}$

Example 2.25: A die is loaded in such a way that an even number is twice as likely to occur as an odd number. If E is the event that a number less than 4 occurs on a single toss of the die, find $P(E)$.

even = 2 odd

$\sum p = 1 = 9w$ $w = \frac{1}{9}$

1 2 3 4 5 6
 w $2w$ w $2w$ w $2w$

Solution: The sample space is $S = \{1, 2, 3, 4, 5, 6\}$. We assign a probability of w to each odd number and a probability of $2w$ to each even number. Since the sum of the probabilities must be 1, we have $9w = 1$ or $w = 1/9$. Hence, probabilities of $1/9$ and $2/9$ are assigned to each odd and even number, respectively. Therefore,

1 2 3 < 4

$E = \{1, 2, 3\}$ and $P(E) = \frac{1}{9} + \frac{2}{9} + \frac{1}{9} = \frac{4}{9}$

$E = \left\{ \begin{array}{c} 1 \\ w \\ 2 \\ 2w \\ 3 \\ w \end{array} \right\}$
 $\frac{1}{9} + \frac{2}{9} + \frac{1}{9} = \frac{4}{9}$

$E \rightarrow$ more than 5

$E = \{6\}$
 $2w = \frac{2}{9}$

51

$\{1, 2, 3, 4, 5, 6\}$
 w $2w$ w $2w$ w $2w$

$A = \{2, 4, 6\}$ $P_A = \frac{6}{9}$

Example 2.26: In Example 2.25, let A be the event that an even number turns up and let B be the event that a number divisible by 3 occurs. Find $P(A \cup B)$ and $P(A \cap B)$.

Solution: For the events $A = \{2, 4, 6\}$ and $B = \{3, 6\}$, we have

$\frac{7}{9} \leftarrow P(A \cup B) = \frac{2}{9} + \frac{1}{9} + \frac{2}{9} + \frac{2}{9} = \frac{7}{9}$ and $P(A \cap B) = \frac{2}{9}$

By assigning a probability of $1/9$ to each odd number and $2/9$ to each even number, we have

$\{1, 2, 3, 4, 5, 6\}$
 $B = \{3, 6\}$
 $P_B = \frac{1}{9} + \frac{2}{9} = \frac{3}{9}$

$P(A \cup B) = \frac{2}{9} + \frac{1}{9} + \frac{2}{9} + \frac{2}{9} = \frac{7}{9}$ and $P(A \cap B) = \frac{2}{9}$

If the sample space for an experiment contains N elements, all of which are equally likely to occur, we assign a probability equal to $1/N$ to each of the N points. The probability of any event A containing n of these N sample points is then the ratio of the number of elements in A to the number of elements in S .

$P(A \cup B) = P(A) + P(B) - (P(A \cap B))$
 $= \frac{6}{9} + \frac{3}{9} - \frac{2}{9} = \frac{7}{9}$

Rule 2.3: If an experiment can result in any one of N different equally likely outcomes, and if exactly n of these outcomes correspond to event A , then the probability of event A is

$$P(A) = \frac{n}{N}.$$

Example 2.27: A statistics class for engineers consists of 25 industrial, 10 mechanical, 10 electrical, and 8 civil engineering students. If a person is randomly selected by the instructor to answer a question, find the probability that the student chosen is (a) an industrial engineering major and (b) a civil engineering or an electrical engineering major.

Solution: Denote by I , M , E , and C the students majoring in industrial, mechanical, electrical, and civil engineering, respectively. The total number of students in the class is 53, all of whom are equally likely to be selected.

- (a) Since 25 of the 53 students are majoring in industrial engineering, the probability of event I , selecting an industrial engineering major at random, is

$$P(I) = \frac{25}{53}.$$

- (b) Since 18 of the 53 students are civil or electrical engineering majors, it follows that

$$C \cup E$$

$$P(C \cup E) = \frac{18}{53} = \frac{8+10}{53} = \frac{18}{53} \quad \blacksquare$$

$$P(C) + P(E) - P(C \cap E) = 8 + 10 - 0$$

$$\begin{aligned} I &= 25 \\ M &= 10 \\ E &= 10 \\ C &= 8 \\ &= 53 \\ &\text{والم} \end{aligned}$$

Theorem 2.7: If A and B are two events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

$\rightarrow = 0$ if disjoint event

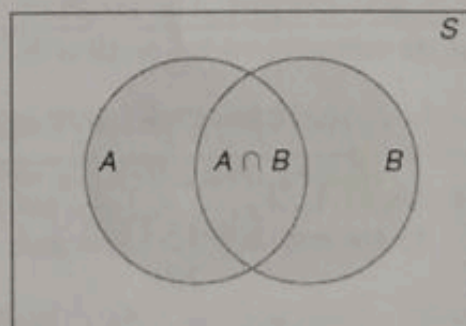


Figure 2.7: Additive rule of probability.

Example:

John is going to graduate from an industrial engineering department in a university by the end of the semester. After being interviewed at two companies he likes, he assesses that his probability of getting an offer from company A is **0.8**, and the probability that he gets an offer from company B is **0.6**. If on the other hand, he believes that the probability that he will get offers from both companies is **0.5**, what is the probability that he will get at least one offer from these two companies?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.8 + 0.6 - 0.5 = 0.9.$$

يعني اتحاد
يا إما الشركة
الأولى تقبل
أو الثانية

$$\text{both} = 0.5 = \cap$$

Corollary 2.1: If A and B are mutually exclusive, then

$$P(A \cup B) = P(A) + P(B).$$

disjoint
event

Corollary 2.1 is an immediate result of Theorem 2.7, since if A and B are mutually exclusive, $A \cap B = \emptyset$ and then $P(A \cap B) = P(\emptyset) = 0$. In general, we can write Corollary 2.2.

Corollary 2.2: If $A_1, A_2, \dots, A_n$ are mutually exclusive, then

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = P(A_1) + P(A_2) + \dots + P(A_n).$$

A collection of events $\{A_1, A_2, \dots, A_n\}$ of a sample space S is called a **partition** of S if $A_1, A_2, \dots, A_n$ are mutually exclusive and $A_1 \cup A_2 \cup \dots \cup A_n = S$. Thus, we have

Corollary 2.3: If $A_1, A_2, \dots, A_n$ is a partition of sample space S , then

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = P(A_1) + P(A_2) + \dots + P(A_n) = P(S) = 1.$$

As one might expect, Theorem 2.7 extends in an analogous fashion.

Theorem 2.8: For three events A, B , and C ,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

Example 2.30: What is the probability of getting a total of 7 or 11 when a pair of fair dice is tossed?

سؤال: ما هو احتمال الحصول على مجموع 7 أو 11 عند رمي زوج من النرد؟

الحل: Let A be the event that 7 occurs and B the event that 11 comes up. Now, a total of 7 occurs for 6 of the 36 sample points, and a total of 11 occurs for only 2 of the sample points. Since all sample points are equally likely, we have $P(A) = 1/6$ and $P(B) = 1/18$. The events A and B are mutually exclusive, since a total of 7 and 11 cannot both occur on the same toss. Therefore,

$$P(A \cup B) = P(A) + P(B) = \frac{1}{6} + \frac{1}{18} = \frac{2}{9}$$

This result could also have been obtained by counting the total number of points for the event $A \cup B$, namely 8, and writing

$$P(A \cup B) = \frac{n}{N} = \frac{8}{36} = \frac{2}{9}$$

7 → A
11 → B

(2,5) (5,2)
(6,1) (1,6)

$$P(A) = \frac{n}{N} = \frac{6}{36}$$

$$P(B) = \frac{2}{36}$$

$$\frac{2}{36} + \frac{6}{36} = \frac{8}{36}$$

Example 2.31: If the probabilities are, respectively, 0.09, 0.15, 0.21, and 0.23 that a person purchasing a new automobile will choose the color green, white, red, or blue, what is the probability that a given buyer will purchase a new automobile that comes in one of those colors?

الحل: Let G , W , R , and B be the events that a buyer selects, respectively, a green, white, red, or blue automobile. Since these four events are mutually exclusive, the probability is

$$P(G \cup W \cup R \cup B) = P(G) + P(W) + P(R) + P(B) = 0.09 + 0.15 + 0.21 + 0.23 = 0.68$$

Often it is more difficult to calculate the probability that an event occurs than it is to calculate the probability that the event does not occur. Should this be the case for some event A , we simply find $P(A')$ first and then, using Theorem 2.7, find $P(A)$ by subtraction.

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Theorem 2.9: If A and A' are complementary events, then

$$P(A) + P(A') = 1.$$

Proof: Since $A \cup A' = S$ and the sets A and A' are disjoint,

$$1 = P(S) = P(A \cup A') = P(A) + P(A').$$

$$A = \{2, 4, 6\}$$

$$\frac{3}{6}$$

$$A' = 1 - \frac{3}{6} = \frac{3}{6}$$

Example 2.32: If the probabilities that an automobile mechanic will service 3, 4, 5, 6, 7, or 8 or more cars on any given workday are, respectively, 0.12, 0.19, 0.28, 0.24, 0.10, and 0.07, what is the probability that he will service at least 5 cars on his next day at work?

$$P(E) = P(5) + P(6) + P(7) + P(8)$$

$$P(E) = 0.28 + 0.24 + 0.10 + 0.07$$

$$P(E) = 0.69$$

OR

Let E be the event that at least 5 cars are serviced. Now, $P(E) = 1 - P(E')$ where E' is the event that fewer than 5 cars are serviced. Since

$$P(E') = 0.12 + 0.19 = 0.31,$$

it follows from Theorem 2.9 that

$$P(E) = 1 - 0.31 = 0.69.$$

$$1 - P(3) - P(4)$$

$$P(A \cap B) = P(A' \cup B')$$

$$P(A' \cap B') = 1 - P(A \cup B)$$

2.58 An automobile manufacturer is concerned about a possible recall of its best-selling four-door sedan. If there were a recall, there is 0.25 probability that a defect is in the brake system, 0.18 in the transmission, 0.17 in the fuel system, and 0.40 in some other area.

(a) What is the probability that the defect is the brakes U or the fueling system if the probability of defects in both systems simultaneously is 0.15?

(b) What is the probability that there are no defects in either the brakes or the fueling system?

$$P(\text{brake system}) = 0.25$$

$$P(\text{transmission}) = 0.18$$

$$P(\text{other area}) = 0.4$$

$$P(\text{fuel}) = 0.17$$

$$(a) = 0.27 \rightarrow P(B \cup F) = P(B) + P(F) - P(B \cap F)$$

$$= 0.25 + 0.17 - 0.15 = 0.27$$

$$(b) = 0.73$$

$$P(B' \cap F') = 1 - P(B \cup F)$$

$$= 1 - 0.27$$

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Example 2.33: Suppose the manufacturer's specifications for the length of a certain type of computer cable are 2000 ± 10 millimeters. In this industry, it is known that small cable is just as likely to be defective (not meeting specifications) as large cable. That is,

[1990 - 2010]

the probability of randomly producing a cable with length exceeding 2010 millimeters is equal to the probability of producing a cable with length smaller than 1990 millimeters. The probability that the production procedure meets specifications is known to be 0.99.

(a) What is the probability that a cable selected randomly is too large?

(b) What is the probability that a randomly selected cable is larger than 1990 millimeters?

Solution: Let M be the event that a cable meets specifications. Let S and L be the events that the cable is too small and too large, respectively. Then

$$(a) P(M) = 0.99 \text{ and } P(S) = P(L) = (1 - 0.99)/2 = 0.005.$$

(b) Denoting by X the length of a randomly selected cable, we have

$$P(1990 \leq X \leq 2010) = P(M) = 0.99.$$

$$\text{Since } P(X \geq 2010) = P(L) = 0.005,$$

$$P(X \geq 1990) = P(M) + P(L) = 0.99 + 0.005 = 0.995.$$

This also can be solved by using Theorem 2.9:

$$P(X \geq 1990) + P(X < 1990) = 1.$$

$$\text{Thus, } P(X \geq 1990) = 1 - P(S) = 1 - 0.005 = 0.995.$$

Definition 2.10:

The conditional probability of B , given A , denoted by $P(B|A)$, is defined by

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, \text{ provided } P(A) > 0.$$

B given that A / if

Table 2.1: Categorization of the Adults in a Small Town

	Employed	Unemployed	Total
Male	460	40	500
Female	140	260	400
Total	600	300	900

$$P(M|E)$$

$$P(M|E) = \frac{P(E \cap M)}{P(E)}$$

$$P = \frac{n}{N}$$

احتمالية E لوحدوا $\leftarrow P(E) = \frac{600}{900} = \frac{2}{3}$ and $P(E \cap M) = \frac{460}{900} = \frac{23}{45}$

$$P(M|E) = \frac{460}{600} = \frac{23}{30} = \frac{\frac{460}{900} + \frac{900}{600}}{\frac{23}{30}}$$

$$P(F|U) = \frac{260}{900} + \frac{900}{300}$$

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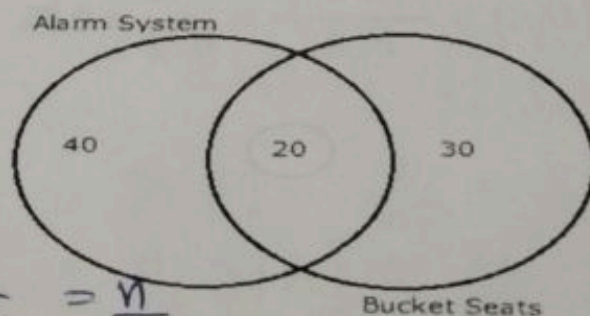
Conditional Probability

Question:

In a group of 100 sports car buyers, 40 bought alarm systems, 30 purchased bucket seats, and 20 purchased an alarm system and bucket seats. If a car buyer chosen at random bought an alarm system, what is the probability they also bought bucket seats?

$$A \cap B = 20$$

استري
alarm
system
ادل



$$P(B|A)$$

$$\frac{P(A \cap B)}{P(A)}$$

$$P(A) = \frac{40}{100} = \frac{n}{N}$$

$$P(A \cap B) = \frac{20}{100} = \frac{n}{N}$$

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Answer:

$$P(B|A) = P(A \cap B) / P(A) = 0.2 / 0.4 = 0.5$$

The probability that a buyer bought bucket seats, given that they purchased an alarm system, is 50%

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Example 2.34: ^{تعلق على الوقت} The probability that a regularly scheduled flight departs on time is $P(D) = 0.83$; ^{تعلق على الوقت} the probability that it arrives on time is $P(A) = 0.82$; and the probability that it departs and arrives on time is $P(D \cap A) = 0.78$. Find the probability that a plane

(a) arrives on time, given that it departed on time, and (b) departed on time, given that it has arrived on time. ^{مقدرة على الوقت}

Solution: Using Definition 2.10, we have the following.

(a) The probability that a plane arrives on time, given that it departed on time, is

$$P(A|D) = \frac{P(D \cap A)}{P(D)} = \frac{0.78}{0.83} = \underline{0.94}.$$

(b) The probability that a plane departed on time, given that it has arrived on time, is

$$P(D|A) = \frac{P(D \cap A)}{P(A)} = \frac{0.78}{0.82} = 0.95.$$

الجواب يستند على المقام

Consider an industrial process in the textile industry in which strips of a particular type of cloth are being produced. These strips can be defective in two ways, length and nature of texture. For the case of the latter, the process of identification is very complicated. It is known from historical information on the process that 10% of strips fail the length test, 5% fail the texture test, and only 0.8% fail both tests. If a strip is selected randomly from the process and a quick measurement identifies it as failing the length test, what is the probability that it is texture defective?

العملية التي قبل

السؤال في مسألة

Solution: Consider the events

L : length defective, T : texture defective.

Given that the strip is length defective, the probability that this strip is texture defective is given by

$$P(L) = 0.1$$

$$P(T) = 0.05$$

$$P(T \cap L) = 0.008$$

$$P(T|L) = \frac{P(T \cap L)}{P(L)} = \frac{0.008}{0.1} = 0.08.$$

بالسؤال لم تذكر given that
لكن ذكر length قبل
Texture

حرف
قبل (T | L)

بمجرد احتمالين

$$P(A) \neq P(B)$$

Definition 2.11: Two events A and B are independent if and only if

$$[1] P(B|A) = P(B) \text{ or } P(A|B) = P(A), \rightarrow \frac{P(A)}{P(B)}$$

assuming the existences of the conditional probabilities. Otherwise, A and B are dependent.

Theorem 2.10: If in an experiment the events A and B can both occur, then

$$[2] P(A \cap B) = P(A)P(B|A) \text{ provided } P(A) > 0. \text{ dependent}$$

Example 2.36: Suppose that we have a fuse box containing 20 fuses, of which 5 are defective. If 2 fuses are selected at random and removed from the box in succession without replacing the first, what is the probability that both fuses are defective?

Solution: We shall let A be the event that the first fuse is defective and B the event that the second fuse is defective; then we interpret $A \cap B$ as the event that A occurs and then B occurs after A has occurred. The probability of first removing a defective fuse is $1/4$; then the probability of removing a second defective fuse from the remaining 4 is $4/19$. Hence,

D → 5
Non D → 15
P N
5 15
20

$$P(A \cap B) = \left(\frac{1}{4}\right) \left(\frac{4}{19}\right) = \frac{1}{19}$$

dependent

$$P(A)P(B|A)$$

$$P(A) = \frac{5}{20} = \frac{1}{4}$$

$$P(B|A) = \frac{4}{19}$$

Slide 66

$$P(R) = \frac{3}{6}$$

B	Red
0	0
0	0
0	0

$$P(R) = \frac{3}{6}$$

احتمالية

احتمالية

الحدث الأول يؤثر على الحدث الثاني $\rightarrow$ dependent

$$P(R) = \frac{3}{6}$$

 $\rightarrow$ Independent

مرتين متتاليتين مع نتائج مختلفة (احتمالية مختلفة)

الحدث الأول يؤثر على الحدث الثاني $\rightarrow$ dependent

The probability that an American industry will locate in Shanghai, China is 0.7, the probability that it will locate in Beijing, China is 0.4 and the probability that it will locate in either Shanghai or Beijing or both is 0.8, What is the probability that the industry will locate (a) in both cities? b) in neither city?

$$P(S) = 0.7$$

$$P(B) = 0.4$$

$$P(S \cup B) = 0.8$$

$$a) P(S \cup B) = P(S) + P(B) - P(S \cap B)$$

$$0.8 = 0.7 + 0.4 - P(S \cap B)$$

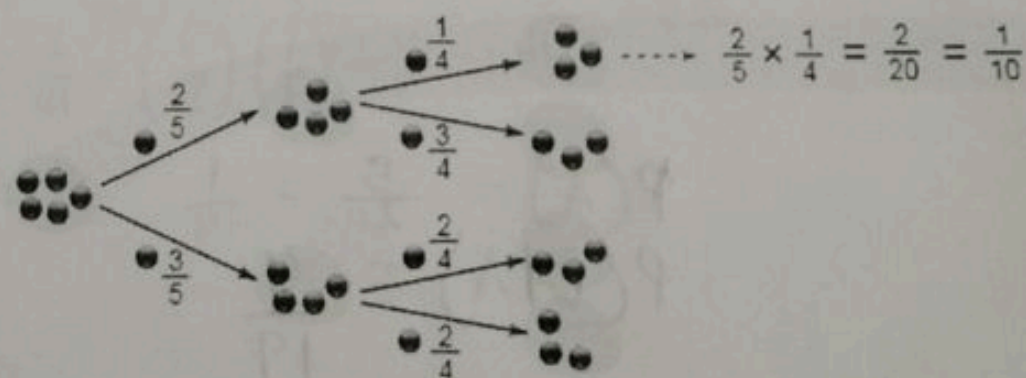
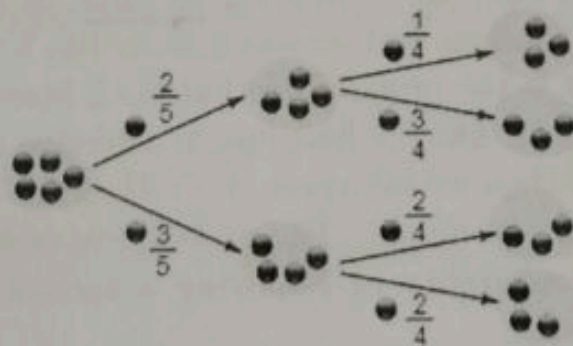
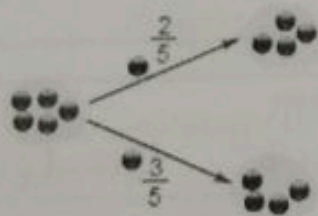
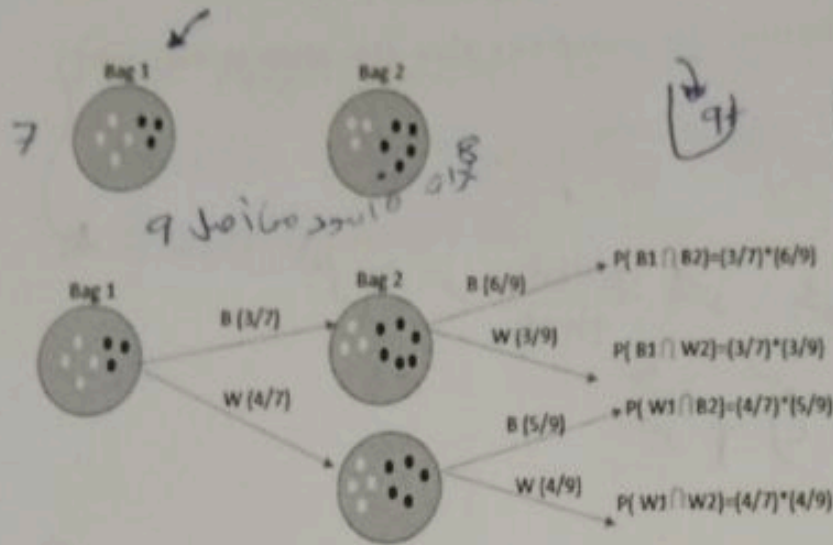
$$P(S \cap B) = 0.3$$

$$b) P(S' \cap B') = 1 - (S \cup B) = 1 - 0.8 = 0.2$$

Example 2.37: One bag contains 4 white balls and 3 black balls. One ball is drawn from the first bag and placed in the second bag. What is the probability that a ball now drawn from the second bag is black?

Solution: Let B_1 , B_2 , and W_1 represent, respectively, the drawing of a black ball from bag 1, a black ball from bag 2, and a white ball from bag 1. We are interested in the union of the mutually exclusive events $B_1 \cap B_2$ and $W_1 \cap B_2$. The various possibilities and their probabilities are illustrated in Figure 2.8. Now

$$P[(B_1 \cap B_2) \text{ or } (W_1 \cap B_2)] = P(B_1 \cap B_2) + P(W_1 \cap B_2) \\ = P(B_1)P(B_2|B_1) + P(W_1)P(B_2|W_1) \\ = \left(\frac{3}{7}\right)\left(\frac{6}{9}\right) + \left(\frac{4}{7}\right)\left(\frac{5}{9}\right) = \frac{38}{63}$$



Two events A and B are independent if and only if

$$P(A \cap B) = P(A)P(B).$$

Therefore, to obtain the probability that two independent events will both occur, we simply find the product of their individual probabilities.

Example 2.38: A small town has one fire engine and one ambulance available for emergencies. The probability that the fire engine is available when needed is 0.98, and the probability that the ambulance is available when called is 0.92. In the event of an injury resulting from a burning building, find the probability that both the ambulance and the fire engine will be available, assuming they operate independently.

Solution: Let A and B represent the respective events that the fire engine and the ambulance are available. Then

$$P(A \cap B) = P(A)P(B) = (0.98)(0.92) = 0.9016.$$

$$P(A) = 0.98$$

$$P(B) = 0.92$$

c) 1 - التماثل
 $1 - P(A_1 \cup A_2)$
 $1 - P(T_1 \cup T_2)$

② $P(A_1, T_2) =$
 $P(A_1, L_2)$
 $P(T_1, A_2)$
 $P(T_1, L_2)$

Ex. 2.95

$$\begin{array}{|c|c|} \hline 2 & 3 \\ \hline A & T \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline 3 & 2 \\ \hline A & T \\ \hline \end{array}$$

One overnight case contains 2 bottles of aspirin, and 3 bottles of thyroid tablets. A second tote bag contains 3 bottles of aspirin, 2 bottles of thyroid tablets and 1 bottle of laxative tablets. If 1 bottle of tablets is taken at random from each piece of luggage, find the probability that:

- (a) both bottles contain thyroid tablets: $P(T_1 \cap T_2)$
 (b) neither bottle contains thyroid tablets; $P(T_1^c \cup T_2^c)$
 (c) the 2 bottles contain different tablets. $= P(T_1^c) \cup P(T_2^c)$
 $\frac{2}{5} \text{ and } \frac{4}{6}$

② $P(T_1^c \cap T_2^c) = 1 - P(T_1 \cup T_2)$

Answer Ex. 2.95:

A1: aspirin tablets are selected from the overnight case,

A2: aspirin tablets are selected from the tote bag,

L2: laxative tablets are selected from the tote bag,

T1: thyroid tablets are selected from the overnight case,

T2: thyroid tablets are selected from the tote bag.

$$(a) P(T1 \cap T2) = P(T1)P(T2) = (3/5)(2/6) = 1/5.$$

$$(b) P(T'1 \cap T'2) = P(T'1)P(T'2) = (2/5)(4/6) = 4/15.$$

$$(c) 1 - P(A1 \cap A2) - P(T1 \cap T2) = 1 - P(A1)P(A2) - P(T1)P(T2) \\ = 1 - (2/5)(3/6) - (3/5)(2/6) = 3/5.$$

Example 2.39: An electrical system consists of four components as illustrated in Figure 2.9. The system works if components A and B work and either of the components C or D works. The reliability (probability of working) of each component is also shown in Figure 2.9. Find the probability that (a) the entire system works and (b) the component C does not work given that the entire system works. Assume that the four components work independently.

Solution: In this configuration of the system, A , B , and the subsystem C and D constitute a serial circuit system, whereas the subsystem C and D itself is a parallel circuit system.

(a) Clearly the probability that the entire system works can be calculated as

$$P[A \cap B \cap (C \cup D)] = P(A)P(B)P(C \cup D) = P(A)P(B)[1 - P(C' \cap D')] \\ = P(A)P(B)[1 - P(C')P(D')] \\ = (0.9)(0.9)[1 - (1 - 0.8)(1 - 0.8)] = 0.7776.$$

The equalities above hold because of the independence among the four components.

(b) To calculate the conditional probability in this case, notice that

$$P = \frac{P(\text{the system works but } C \text{ does not work})}{P(\text{the system works})} \\ = \frac{P(A \cap B \cap C' \cap D)}{P(\text{the system works})} = \frac{(0.9)(0.9)(1 - 0.8)(0.8)}{0.7776} = 0.1667.$$

$$P(C'/S) = \frac{P(S \cap C')}{P(S)}$$

$$P(A \cap B \cap D)$$

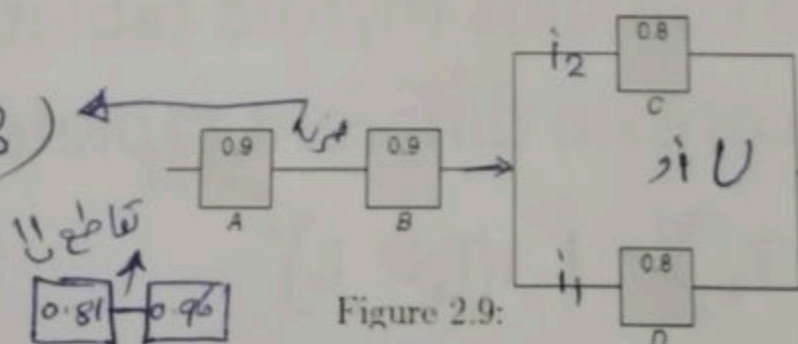


Figure 2.9:

$$P(C \cup D) = P(C) + P(D) - P(C \cap D) \\ = 0.8 + 0.8 - 0.64 = 0.96$$

مضان (A ∩ B)
لازم ہونا
72 سوا

$$= 0.7776$$

Ex. 2.99

A circuit system is given in Figure 2.11. Assume the components fail independently.

(a) What is the probability that the entire system works?

(b) Given that the system works, what is the probability that the component A is not working?

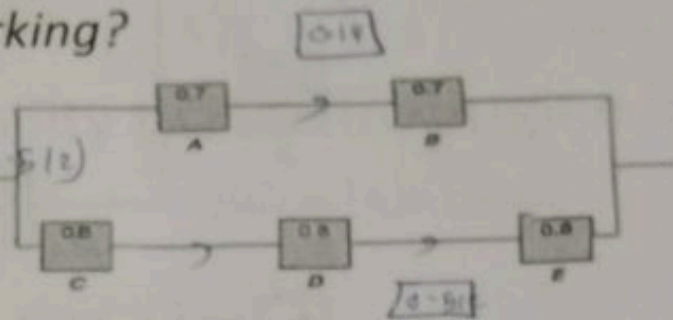


Figure 2.11: Diagram for Exercise 2.99.

$$a) P(S) = 0.49 + 0.512 - (0.49 \times 0.512)$$

$$b) P(A' | S) = \frac{P(S \cap A')}{P(S)}$$

$$= \frac{P(C \cap D \cap E \cap A')}{P(S)}$$

$$= \frac{0.8 \times 0.8 \times 0.8 \times (1 - 0.7)}{0.75112} = 0.8 \times 0.8 \times 0.8 \times 0.3$$

Answer Ex. 2.99

(a)

$$P = 1 - [1 - (0.7)(0.7)][1 - (0.8)(0.8)(0.8)] = 0.75112.$$

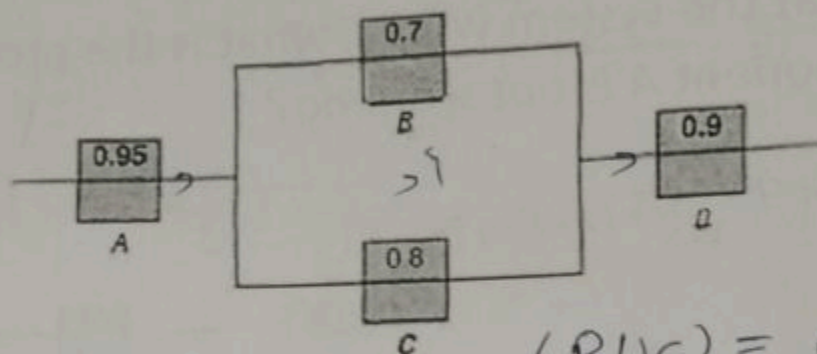
$$P = 0.75112$$

(b) =

$$P = \frac{P(A' \cap C \cap D \cap E)}{P_{\text{system works}}}$$

$$= \frac{(0.3)(0.8)(0.8)(0.8)}{0.75112} = 0.2045$$

- 2.98 Suppose the diagram of an electrical system is given in Figure 2.10. What is the probability that the system works? Assume the components fail independently.



$$\boxed{A} - \boxed{B \parallel C} - \boxed{D}$$

0.95 0.94 0.9

$$(B \cup C) = 0.7 + 0.8 - (0.7 \times 0.8) = 0.94$$

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if $\rightarrow P(B'/S) = \frac{P(A \cap B' \cap C \cap D)}{P(S)} = \frac{0.95 \times 0.8 \times 0.9}{0.94}$

system given then

EX:2.98

$$P = (0.95)[1 - (1 - 0.7)(1 - 0.8)](0.9) = 0.8037.$$

EX. 2.93

A town has 2 fire ^{A & B} engines operating independently. The probability that a specific engine is available when needed is 0.96.

- (a) What is the probability that neither is available when needed? $P(A' \cap B') = 0.04 \times 0.04 = 0.0016$
- (b) What is the probability that a fire engine is available when needed? $\text{على الأقل واحد مشغال}$

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$$(a) P(A' \cap B') = P(A')P(B') = (0.04)(0.04) = 0.0016.$$

$$(b) P(A \cup B) = 1 - P(A' \cap B') = 1 - 0.0016 = 0.9984.$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

EX. 2.89 The probability that a doctor correctly diagnoses a particular illness is 0.7. Given that the doctor makes an incorrect ^{complete} diagnosis, the probability that the patient enters a law suit is 0.9. What is the probability that the doctor makes an incorrect diagnosis and the patient sues?

$$P(L|D') = 0.9$$

$$P(L) = 0.9$$

$$P(D' \cap L)$$

$$P(L|D') = \frac{P(D' \cap L)}{P(D')}$$

$$P(D' \cap L) = 0.3 * 0.9 = 0.27$$

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Answer of EX. 2.89:

$$P(A' \cap B) = P(A')P(B | A') = (0.3)(0.9) = 0.27$$

$P(A_1 \cap A_2 \cap \dots \cap A_k)$ if $A_1, A_2, \dots, A_k$ can occur, then
 $= P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2) \dots P(A_k|A_1 \cap A_2 \cap \dots \cap A_{k-1})$.
 If the events $A_1, A_2, \dots, A_k$ are independent, then
 $P(A_1 \cap A_2 \cap \dots \cap A_k) = P(A_1)P(A_2) \dots P(A_k)$.

Bayes' Rule

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Table 2.1: Categorization of the Adults in a Small Town

	Employed	Unemployed	Total
Male	460	40	500
Female	140	260	400
Total	600	300	900

36 of Employees and
 12 of Unemployed are
 members of the
 Rotary Club

Find the probability of
 the event A that the
 individual selected is a
 member of the Rotary
 Club?

$$A = E \cap A \cup E' \cap A$$

$$P(A) = P[(E \cap A) \cup (E' \cap A)] = P(E \cap A) + P(E' \cap A)$$

$$= P(E)P(A|E) + P(E')P(A|E')$$

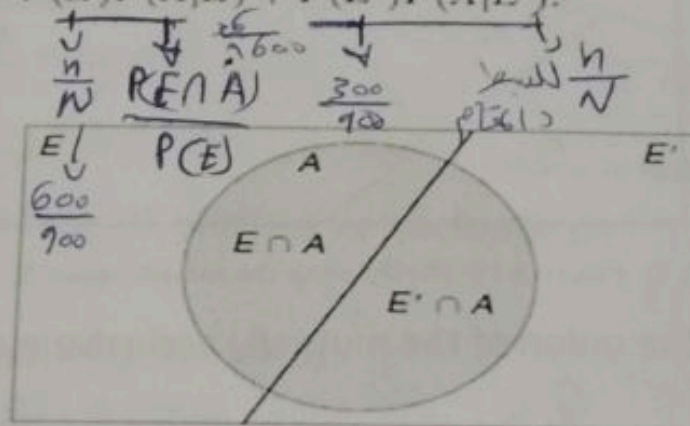


Figure 2.12: Venn diagram for the events A, E, and E'.

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 مشتركة
 بالتالي
 dependent

	Employed	Unemployed	Total
Male	400	40	440
Female	140	260	400
Total	540	300	840

$$P(E) = \frac{440}{840} = \frac{11}{21}, \quad P(A|E) = \frac{36}{440} = \frac{9}{110}$$

$$P(E^c) = \frac{400}{840} = \frac{10}{21}, \quad P(A|E^c) = \frac{12}{400} = \frac{3}{100}$$

36 of Employees and
12 of Unemployed are
members of the
Rotary Club

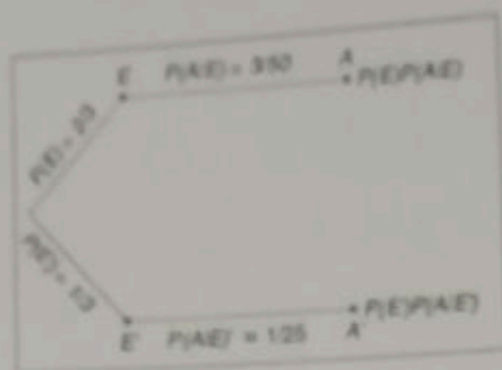


Figure 2.13: Tree diagram for the data on page 63, using additional information on page 72.

the probability $P(E^c)P(A|E^c)$, it follows that

$$P(A) = \left(\frac{11}{21}\right)\left(\frac{9}{110}\right) + \left(\frac{10}{21}\right)\left(\frac{3}{100}\right) = \frac{4}{75}$$

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Theorem 2.13: If the events $B_1, B_2, \dots, B_k$ constitute a partition of the sample space S such that $P(B_i) \neq 0$ for $i = 1, 2, \dots, k$, then for any event A of S ,

$$P(A) = \sum_{i=1}^k P(B_i \cap A) = \sum_{i=1}^k P(B_i)P(A|B_i)$$

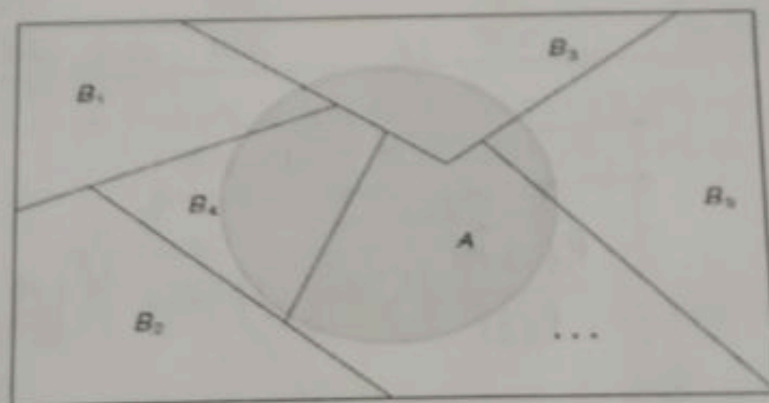


Figure 2.14: Partitioning the sample space S .

The event A is seen to be the union of the mutually exclusive events

Proof: Consider the Venn diagram of Figure 2.14. The event A is seen to be the union of the mutually exclusive events

$$B_1 \cap A, B_2 \cap A, \dots, B_k \cap A;$$

that is,

$$A = (B_1 \cap A) \cup (B_2 \cap A) \cup \dots \cup (B_k \cap A).$$

Using Corollary 2.2 of Theorem 2.7 and Theorem 2.10, we have

$$\begin{aligned} P(A) &= P[(B_1 \cap A) \cup (B_2 \cap A) \cup \dots \cup (B_k \cap A)] \\ &= P(B_1 \cap A) + P(B_2 \cap A) + \dots + P(B_k \cap A) \\ &= \sum_{i=1}^k P(B_i \cap A) \\ &= \sum_{i=1}^k P(B_i)P(A|B_i). \end{aligned}$$

Example 2.41: In a certain assembly plant, three machines, B_1 , B_2 , and B_3 , make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected. What is the probability that it is defective?

Solution: Consider the following events:

A : the product is defective,

B_1 : the product is made by machine B_1 ,

B_2 : the product is made by machine B_2 ,

B_3 : the product is made by machine B_3 .

Applying the rule of elimination, we can write

$$P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3).$$

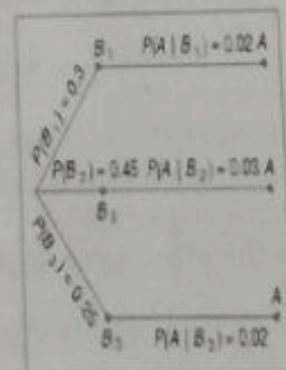


Figure 2.15: Tree diagram for Example 2.41.

Referring to the tree diagram of Figure 2.15, we find that the three branches give the probabilities

$$\begin{aligned} P(B_1)P(A|B_1) &= (0.3)(0.02) = 0.006, \\ P(B_2)P(A|B_2) &= (0.45)(0.03) = 0.0135, \\ P(B_3)P(A|B_3) &= (0.25)(0.02) = 0.005, \end{aligned}$$

and hence

$$P(A) = 0.006 + 0.0135 + 0.005 = 0.0245.$$

$$\begin{aligned} B_1 &= 0.3 \\ B_2 &= 0.45 \\ B_3 &= 0.25 \end{aligned}$$

B_1	B_2	B_3
0.3	0.45	0.25
2%	3%	2%
0.006	0.0135	0.005

$$P(A) = P(A|B_1) + P(A|B_2) + P(A|B_3)$$

احتمالية D من الآلة الأولى

كم منتج + كم بطلع

حساب

$$0.006 + 0.0135 + 0.005 = 0.0245$$

ننتقل على الفرع الأول

إذا لم يكن لدينا خبر

$$P(B_1|D) = P(D \cap B_1) = 0.006$$

Theorem 2.14: (Bayes' Rule) If the events $B_1, B_2, \dots, B_k$ constitute a partition of the sample space S such that $P(B_i) \neq 0$ for $i = 1, 2, \dots, k$, then for any event A in S such that $P(A) \neq 0$,

$$P(B_r|A) = \frac{P(B_r \cap A)}{\sum_{i=1}^k P(B_i \cap A)} = \frac{P(B_r)P(A|B_r)}{\sum_{i=1}^k P(B_i)P(A|B_i)} \text{ for } r = 1, 2, \dots, k.$$

Proof: By the definition of conditional probability,

$$P(B_r|A) = \frac{P(B_r \cap A)}{P(A)}$$

and then using Theorem 2.13 in the denominator, we have

$$P(B_r|A) = \frac{P(B_r \cap A)}{\sum_{i=1}^k P(B_i \cap A)} = \frac{P(B_r)P(A|B_r)}{\sum_{i=1}^k P(B_i)P(A|B_i)}$$

which completes the proof. $\blacksquare$

Example 2.42: With reference to Example 2.41, if a product was chosen randomly and found to be defective, what is the probability that it was made by machine B_3 ?

Solution: Using Bayes' rule to write

$$P(B_3|A) = \frac{P(B_3)P(A|B_3)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)}$$

and then substituting the probabilities calculated in Example 2.41, we have

$$P(B_3|A) = \frac{0.005}{0.006 + 0.0135 + 0.005} = \frac{0.005}{0.0245} = \frac{10}{49}$$

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Σ total

In view of the fact that a defective product was selected, this result suggests that it probably was not made by machine B_3 . $\blacksquare$

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Example 2.43: A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plans 1, 2, and 3 are used for 30%, 20%, and 50% of the products, respectively. The defect rate is different for the three procedures as follows:

$$P(D|P_1) = 0.01, \quad P(D|P_2) = 0.03, \quad P(D|P_3) = 0.02,$$

where $P(D|P_j)$ is the probability of a defective product, given plan j . If a random product was observed and found to be defective, which plan was most likely used and thus responsible?

Solution: From the statement of the problem

$$P(P_1) = 0.30, \quad P(P_2) = 0.20, \quad \text{and} \quad P(P_3) = 0.50,$$

we must find $P(P_j|D)$ for $j = 1, 2, 3$. Bayes' rule (Theorem 2.14) shows

$$P(P_1|D) = \frac{P(P_1)P(D|P_1)}{P(P_1)P(D|P_1) + P(P_2)P(D|P_2) + P(P_3)P(D|P_3)} = \frac{(0.30)(0.01)}{(0.30)(0.01) + (0.20)(0.03) + (0.50)(0.02)} = \frac{0.003}{0.019} = 0.158. \quad \frac{n}{N}$$

Similarly,

$$P(P_2|D) = \frac{(0.03)(0.20)}{0.019} = 0.316 \quad \text{and} \quad P(P_3|D) = \frac{(0.02)(0.50)}{0.019} = 0.526. \quad \frac{n}{N}$$

The conditional probability of a defect given plan 3 is the largest of the three; thus a defective for a random product is most likely the result of the use of plan 3. $\blacksquare$

Using Bayes' rule, a statistical methodology called the Bayesian approach has attracted a lot of attention in applications. An introduction to the Bayesian method will be discussed in Chapter 18.

P_1	P_2	P_3
30%	20%	50%
0.01	0.03	0.02
0.003	0.006	0.01
$\Sigma = 0.019$		

P_3