Second and third order determinent

$\det A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $D = \det[A] = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

$= a_{11}a_{22} - a_{12}a_{21}$

Cramer Rule:

For second order linear system

$AX = b$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$X_1 = \frac{D_1}{D}$ $X_2 = \frac{D_2}{D}$

$$D_1 = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{21} \end{vmatrix} = b_1 a_{22} - a_{12} b_2$$

$$D_2 = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix} = a_{11} b_2 - b_1 a_{21}$$

EX: let $4X_1 + 3X_2 = 12$ $AX = b$

$2X_1 + 5X_2 = -8$ $\xrightarrow{\text{sol.}}$ $\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \end{bmatrix}$

Find X_1 X_2 ??

$$D = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = 20 - 6 = 14$$

$$D_1 = \begin{bmatrix} 12 & 3 \\ -8 & 5 \end{bmatrix} = 60 + 24 = 84$$

$$D_2 = \begin{bmatrix} 4 & 12 \\ 2 & -8 \end{bmatrix} = -32 - 24 = -56$$

$X_1 = D_1/D = 84/14 = 6$ $X_2 = D_2/D = -56/14 = -4$

* Third order determinent:

for linear system $AX = b$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$X_1 = D_1/D$ $X_2 = D_2/D$ $X_3 = D_3/D$

$$D = \det[A] = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

zoom

$$D = +a_{11}\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13}\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$= +a_{11}[a_{22}a_{33} - a_{23}a_{32}] - a_{12}[a_{21}a_{23} - a_{23}a_{31}] + a_{13}[a_{21}a_{32} - a_{32}a_{31}]$$

$$D_1 = \begin{bmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{bmatrix} \qquad D_2 = \begin{bmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{bmatrix}$$

$$D_3 = \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{bmatrix}$$

EX: find D for A if $A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix}$

$$D = +1\begin{bmatrix} 6 & 4 \\ 0 & 2 \end{bmatrix} - 3\begin{bmatrix} 2 & 4 \\ -1 & 2 \end{bmatrix} + 0\begin{bmatrix} 2 & 6 \\ -1 & 0 \end{bmatrix}$$

$$= 1[12] - 3[4+4] \Rightarrow -12$$

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

The Minor Matrix of A is M:-

$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$ $\quad M_{11} = \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} = a_{22}a_{33} - a_{23}a_{32}$

$M_{12} = \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} \cdots\cdots \quad M_{32} = \begin{bmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{bmatrix}$

$M_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

The cofactor Matrix of A is C:

$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$ $\quad C_{jk} = [-1]^{j+k} M_{jk}$

$C = \begin{bmatrix} M_{11} & -M_{12} & M_{13} \\ -M_{21} & M_{22} & -M_{23} \\ M_{31} & -M_{32} & M_{33} \end{bmatrix}$

Theorem:-

1) Interchange of two rows Multiply the value of determinent by [-1].

2) addition of mutiple of Row to another row does not alt the value of determent.

3) multiplication of row by non-zero constant [C] multiplies the value of determinant by [C]

4) det [CA] = C^n det [A]

n = number of row in A.

$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $|A| = 1 - 0 = 1$

$B = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ $|B| = 0 - 1 = -1$

$C = \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix}$ $|C| = 2$

$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ $|A| = 3 - 2 = 1$

$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \rightarrow R_2 - R_1$

$|D| = 1 - 0 = 1$

$|cA| = c^n A$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \quad |A| = 3 - 2 = 1$$

$$|3A| = \begin{bmatrix} 3 & 6 \\ 3 & 9 \end{bmatrix} = 3(9) - 3(6) = 3[3] = 9 \qquad [3A] = 3^2 |A| = 9(1) = 9$$

7.6 Second and third order determinant

let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $\quad D = \det[A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$= a_{11}a_{22} - a_{12}a_{21}$

Cramer Rule:-

For second order linear system

$AX = b$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$X = D_1/D \quad X_2 = D_2/D$

$$D_1 = \begin{bmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{bmatrix} = b_1a_{22} - a_{12}b_2$$

$$D_2 = \begin{bmatrix} a_{11} & b_1 \\ a_{22} & b_2 \end{bmatrix} = a_{11} b_2 - b_1 a_{21}$$

EX= let $4x_1 + 3x_2 = 12$ find x_1, x_2?

$2x_1 + 5x_2 = -8$

$Ax = b$

$$\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \end{bmatrix}$$

$$D = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = 20 - 6 = 14$$

$$D2 = \begin{bmatrix} 4 & 12 \\ 2 & -8 \end{bmatrix} = -32 - 24 = -56$$

$X_1 = D_1/D = 84/14 = 6$

$X_2 = D_2/D = -56/14 = -4$

7.6 second and third order determinent:

let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $D = \det[A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

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Cramer Rule:-

for second order linear system

$$AX = b \quad \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$X_1 = D_1/D$, $X_2 = D_2/D$

$$D_1 \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} = b_1 a_{22} - a_{12} b_2$$

$$D2 = \begin{bmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{bmatrix} = a_{11} b_2 - b_1 a_{21}$$

The Minor Matrix of A is M

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

$$M_{11} = \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} = a_{32}a_{33} - a_{23}a_{32}$$

$$M_{12} = \begin{bmatrix} a_{31} & a_{23} \\ a_{31} & a_{23} \end{bmatrix} \cdots\cdots \quad M_{32} = \begin{bmatrix} a_{11} & a_{13} \\ a_{31} & a_{23} \end{bmatrix}$$

$$M_{33} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

The Cofactor Matrix of A is C:

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \qquad C_{jk} = [-1]^{j+k} M_{jk}$$

$$\begin{bmatrix} M_{11} & -M_{12} & M_{13} \\ -M_{21} & M_{22} & -M_{23} \\ M_{31} & -M_{32} & M_{33} \end{bmatrix}$$

Theorem:

1) Interchange of two rows Maltely the value of determid by [-1].
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n : number of Raws in A.

Ex: let $4x_1 + 3x_2 = 12$ find x_1, x_2?

$2x_1 + 5x_2 = -8$

$AX = b$ $\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \end{bmatrix}$

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$X_1 = D_1 / D = 84/14 = 6$

$$X_2 = \frac{D_2}{D} = -\frac{56}{14} = -4$$

ZOOM Notebook