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evaluation of determinent by reduction to trianglur from Gauss elimination:

$$D = \begin{vmatrix} 2 & 0 & -4 & 6 \\ 4 & 5 & 1 & 0 \\ 0 & 2 & 6 & -1 \\ -3 & 8 & 9 & 1 \end{vmatrix} \rightarrow \text{Pivot}$$

$$= \begin{array}{l} \\ \text{Pivot} \rightarrow \\ \\ \\ \end{array} \begin{vmatrix} 2 & 0 & -4 & 6 \\ 0 & 5 & 9 & -12 \\ 0 & 2 & 6 & -1 \\ 0 & 8 & 3 & 10 \end{vmatrix} \begin{array}{l} \\ \rightarrow R_2 - 2R_1 \\ \\ \rightarrow R_4 + 1.5R_1 \end{array}$$

$$= \begin{array}{l} \\ \\ \text{Pivot} \rightarrow \\ \\ \end{array} \begin{vmatrix} 2 & 0 & -4 & 6 \\ 0 & 5 & 9 & -12 \\ 0 & 0 & 2.4 & 3.8 \\ 0 & 0 & -11.4 & 29.2 \end{vmatrix} \begin{array}{l} \\ \\ \rightarrow R_3 - 0.4R_2 \\ \rightarrow R_4 - 1.6R_2 \end{array}$$

$$= \begin{vmatrix} 2 & 0 & -4 & 6 \\ 0 & 5 & 9 & -12 \\ 0 & 0 & 2.4 & 3.8 \\ 0 & 0 & 0 & 47.25 \end{vmatrix} \begin{array}{l} \\ \\ \\ \rightarrow R_4 + 4.75R_3 \end{array}$$

$D = [2]\,[5]\,[2.4]\,[47.25] = 1134$

Theorem:

1) Interchange of two columns Mutiply the value of determinamt by [-1].

2) addition of multiple of colum to anather colum doesnot alter the value of determinant.

3) Multiplication of colum by non-zero Constant [C] Multiple the value of det by [C]

4) $\det[A] = \det[A^t]$

5) zero Row or zero colum resaltent the value of determnt to zero

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6) ProPortional Rows or colums render the value of det to zero

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \quad |A| = 0$$

7.8 Inurse of Matrix : Gauss Jordan elemenation

- For square Matrix $n \times n$ A its inverse is A^{-1}

$$A^{-1}A = AA^{-1} = I$$

note: 1) if A has invers then A is non-Singular

2) if A has no-invers then A is Singular

For system $AX = b$ we can find X using A^{-1}

$AX = b$

$A^{-1}AX = A^{-1}b$

$IX = A^{-1}b$

$X = A^{-1}b$

Theorem: the inverse A^{-1} of an $n \times n$ Matrix A exist iff

1) Rank $(A) = n$
2) $\det[A] \neq 0$
3) A is non-singular

* if Rank $(A) < n$ then
  → A is singular
  → A has no-inverse

Finding inverse of Matrix by Gauss-elimination

Ex find A^{-1} if $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$.

1) $\tilde{A} = [A : I]$

$$\left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 3 & -1 & 1 & 0 & 1 & 0 \\ -1 & 3 & 4 & 0 & 0 & 1 \end{array}\right] \rightarrow \text{Pivot}$$

$$= \begin{array}{r} \\ \text{Pivot} \rightarrow \\ \\ \end{array} \left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 7 & 3 & 1 & 0 \\ 0 & 2 & 2 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \rightarrow R_2+3R_1 \\ \rightarrow R_3-R_1 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 7 & 3 & 1 & 0 \\ 0 & 0 & -5 & -4 & -1 & 1 \end{array}\right] \begin{array}{l} \\ \\ \rightarrow R_3-R_2 \end{array}$$

$$= \begin{array}{r} \\ \\ \text{Pivot} \rightarrow \end{array} \left[\begin{array}{ccc|ccc} 0 & -1 & -2 & -1 & 0 & 0 \\ 0 & 1 & 3.5 & 1.5 & 0.5 & 0 \\ 0 & 0 & 1 & 0.8 & 0.2 & -0.2 \end{array}\right] \begin{array}{l} \rightarrow -R_1 \\ \rightarrow 0.5R_2 \\ \rightarrow -0.2R_3 \end{array}$$

$$= \begin{array}{r} \\ \text{Pivot} \rightarrow \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0.6 & 0.4 & -0.4 \\ 0 & 1 & 0 & -1.3 & -0.2 & 0.7 \\ 0 & 0 & 1 & 0.8 & 0.2 & -0.2 \end{array}\right] \begin{array}{l} \rightarrow R_1+2R_3 \\ \rightarrow R_2-3.5R_3 \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -0.7 & 0.2 & 0.3 \\ 0 & 1 & 0 & -1.3 & -0.2 & 0.7 \\ 0 & 0 & 1 & 0.8 & 0.2 & -0.2 \end{array}\right] \begin{array}{l} \rightarrow R_1+R_2 \\ \\ \\ \end{array}$$

$\rightarrow [I : A^{-1}]$

$$A^{-1} = \begin{bmatrix} -0.7 & 0.2 & 0.3 \\ -1.3 & -0.2 & 0.7 \\ 0.8 & 0.2 & -0.2 \end{bmatrix}$$

Theorem : inverse of Matrix by determinant
the inverse of a non singular $n \times n$
Matrix . $A = [a_{jk}]$ is given by

$$A^{-1} = \frac{1}{|A|} C^{T}$$

$$A \rightarrow M \rightarrow C \rightarrow C^{T}$$

$$C_{jk} = (-1)^{j+k} M_{jk}$$

Ex : let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ find A^{-1}?

$A^{-1} = \frac{1}{|A|} C^T$ $\quad |A| = a_{11}a_{22} - a_{12}a_{21}$

$$C^T = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

Ex: let $A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$ find A^{-1}?

$$A^{-1} = \frac{1}{|A|} C^T = \frac{1}{10} \begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 0.4 & -0.1 \\ -0.2 & 0.3 \end{bmatrix}$$

let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ $\quad A^{-1} = \frac{1}{|A|} C^T$

$$|A| = +a_{11} \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12} \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{32} \end{bmatrix} + a_{13} \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$C^T \to C \to M \to A$

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \qquad C^T = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{32} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

$$C_{11} = (-1)^2 M_{11} = M_{11} = \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}$$

EX: let $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ find A^{-1}?

$$A^{-1} = \frac{1}{|A|} C^T$$

$$|A| = -1 \begin{bmatrix} -1 & 1 \\ 3 & 4 \end{bmatrix} - 1 \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix} + 2 \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$$

$$= 6$$

$$C^T = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$C_{11} = (-1)^2 M_{11} = \begin{bmatrix} -1 & 1 \\ 3 & 4 \end{bmatrix} = -7$$

$$C_{12} = -\begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix} = -13$$

$$C_{13} = (-1)^4 \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} = 8 \qquad C_{21} = (-1)^3 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = 2$$

$$C_{22} = (-1)^4 \begin{bmatrix} -1 & 2 \\ -1 & 4 \end{bmatrix} = -2 \qquad C_{23} = (-1)^5 \left| \begin{matrix} -1 & 1 \\ -1 & 3 \end{matrix} \right] = 2$$

$$C_{31} = (-1)^4 \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = 3 \qquad C_{32} = (-1)^3 \begin{vmatrix} -1 & 2 \\ 3 & 1 \end{vmatrix} = 7$$

$$C_{33} = (-1)^6 \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} = -2$$

$$C^T = \begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix} \quad A^{-1} = \frac{1}{10} C^T \begin{bmatrix} 0.7 & 0.2 & 0.3 \\ -1.3 & -0.2 & 0.7 \\ 0.8 & 0.2 & -0.2 \end{bmatrix}$$

EX: let $A = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ find A^{-1}?

$$A^{-1} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rules:
1) $[AC]^{-1} = C^{-1}A^{-1}$
2) $[A^{-1}]^{-1} = A$
3) $(A^{-1})^T = (A^T)^{-1}$

* Un usaul Properties of Matrix Multiplicatio
1) $AB \neq BA$
2) $AB = 0 \rightarrow$ it dose not generally imply $A = 0$ or $B = 0$

Ex: let $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$

2×2 2×1

equal

Theorem: detominet of product of Matrix

For any $n\times n$ Matrix A, B

$\det[AB] = \det[BA] = \det[A]\det[B]$

Ex: let $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ find $[AB]$?

$= \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$ $7-8=-1$

$|AB| = |A||B| = -1\times 1 = -1$

adjoint Matrix:-

1) adj $[A] = C^T$

2) $[A][\text{adj}(A)] = [\text{adj}[A]][A] = \det(A) I$

$A \to M \to C \to C^T$

scalar Matrix $= S =$
$$\begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & S \end{bmatrix}$$
scalar

2) $AC = AD$ it does not generly imdy that $C = D$

Theorems can cellation laws
let A, B, C, be $n \times n$ matrix then

a) if rank $(A) = n$ and $AB = AC$ then $B = C$

b) if rank $A = n$ and $AB = A0$ then $B = 0$

c) if $AB = 0$ and $A \neq 0$, $B \neq 0$ then
Rank $(A) < n$, Rank$(B) < n$

d) if A is sigular then BA, AB are singular

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theorom: determineat of Product of Matrix for any $n \times n$ Matrix A, B

$$\det[AB] = \det[BA] = \det[A]\det[B]$$

Ex = let $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

find $\det[AB]$?

$|AB| = \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}$ $7 - 8 = -1$

$|AB| = |A||B| = -1 \times 1 = 1$