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Probability and Statistics

Chapter 5

Chapter 5

Some Discrete Probability Distributions

1) Discrete Uniform Distribution

Example 5.1: When a light bulb is selected at random from a box that contains a 40-watt bulb, a 60-watt bulb, a 75-watt bulb, and a 100-watt bulb, each element of the sample space $S = \{40, 60, 75, 100\}$ occurs with probability 1/4. Therefore, we have a uniform distribution, with

$$f(x;4) = \frac{1}{4}, \quad x = 40, 60, 75, 100.$$

number of sample space point

يعني ثابتة مهما تغير المتغير العشوائي

Example 5.2: When a fair die is tossed, each element of the sample space $S = \{1,2,3,4,5,6\}$ occurs with probability 1/6. Therefore, we have a uniform distribution with

$$f(x;6) = \frac{1}{6}, \quad x = 1, 2, 3, 4, 5, 6.$$

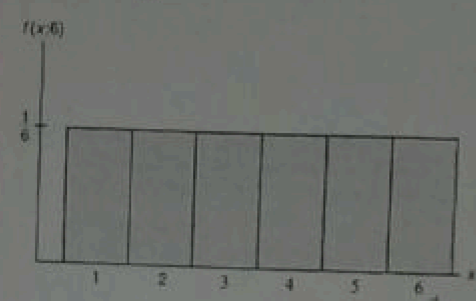


Figure 5.1: Histogram for the tossing of a die.

Binomial distribution
كيف نميز سؤال
إذا السؤال ذكر التالي
1) احتمالية جزء من كل (مثلا 10 من 100)
2) الاحتمالية ثابتة وموجودة بالسؤال
3) الناتج رح يكون نجاح أو فشل
4) كل تجربة مستقلة عن الأخرى

2) The Bernoulli Process

Strictly speaking, the Bernoulli process must possess the following properties:

1. The experiment consists of repeated trials. → أيقاف / لما عنا تكرار
2. Each trial results in an outcome that may be classified as a success or a failure. (70% / 20%) → تصنيف

success ← اللي بنركز عليه بالسؤال

3. The probability of success, denoted by p, remains constant from trial to trial.
4. The repeated trials are independent.

Consider the set of Bernoulli trials where three items are selected at random from a manufacturing process, inspected, and classified as defective or nondefective. A defective item is designated a success. The number of successes is a random variable X assuming integral values from 0 through 3. The eight possible outcomes and the corresponding values of X are

75%

$\frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{64}$

Outcome	NNN	NDN	NND	DNN	NDD	DND	DDN	DDD
x	0	1	1	1	2	2	2	3

Binomial distribution

Since the items are selected independently and we assume that the process produces 25% defectives, we have

$$P(NDN) = P(N)P(D)P(N) = \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) = \frac{9}{64}.$$

$P(NNN) = P(N)^3 = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$

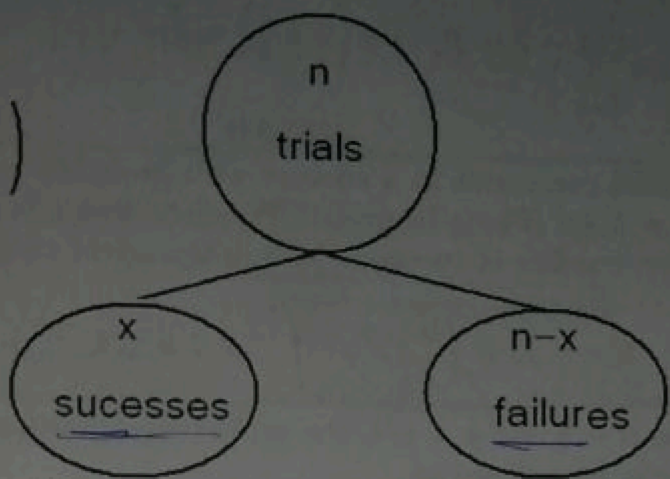
Similar calculations yield the probabilities for the other possible outcomes. The probability distribution of X is therefore

x	0	1	2	3
$f(x)$	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

حاصل جمع كل اللي عنصر واحد

حاصل جمع اللي عنصر 2

$\binom{n}{x}$



Probability of successes	Probability of failures
$\underbrace{p\,p \ldots p}_{x \text{ times}} = p^x$	$\underbrace{(1-p) \ldots (1-p)}_{(n-x) \text{ times}} = (1-p)^{n-x}$

Outcome	Probability	x
NNN $\frac{3}{4}$	$\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$	0
NND	$\frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{9}{64}$	1
NDN	$\frac{3}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{9}{64}$	1
NDD	$\frac{3}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{3}{64}$	2
DNN	$\frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{9}{64}$	1
DND	$\frac{1}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{3}{64}$	2
DDN	$\frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{64}$	2
DDD	$\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$	3

The probability distribution of X is

x	$f(x)=P(X=x)$
0	$\frac{27}{64}$
1	$\frac{9}{64}+\frac{9}{64}+\frac{9}{64}=\frac{27}{64}$
2	$\frac{3}{64}+\frac{3}{64}+\frac{3}{64}=\frac{9}{64}$
3	$\frac{1}{64}$

2) **Binomial Distribution**

The number X of successes in n Bernoulli trials is called a **binomial random variable**. The probability distribution of this discrete random variable is called the **binomial distribution**, and its values will be denoted by $b(x; n, p)$ since they depend on the number of trials and the probability of a success on a given trial. Thus, for the probability distribution of X, the number of defectives is

$$P(X = 2) = f(2) = b\left(2; 3, \frac{1}{4}\right) = \frac{9}{64}.$$

Binomial Distribution A Bernoulli trial can result in a success with probability p and a failure with probability $q = 1 - p$. Then the probability distribution of the binomial random variable X, the number of successes in n independent trials, is

number of trial

probability

exact Value

$$\boxed{b(x; n, p) = \binom{n}{x} p^x q^{n-x}}, \quad x = 0, 1, 2, \ldots, n.$$

Note that when $n = 3$ and $p = 1/4$, the probability distribution of X, the number of defectives, may be written as

$$b\left(x; 3, \frac{1}{4}\right) = \binom{3}{x}\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^{3-x}, \quad x = 0, 1, 2, 3,$$

n

rather than in the tabular form on page 144.

$\left(1, 3, \frac{1}{4}\right)$

$\boxed{P = \frac{3}{4}}$

Example 5.1: The probability that a certain kind of component will survive a shock test is 3/4. Find the probability that exactly 2 of the next 4 components tested survive.

X

Solution: Assuming that the tests are independent and $p = 3/4$ for each of the 4 tests, we obtain

pass — fail

$\frac{3}{4}$ — $\frac{1}{4}$

$$b\left(2; 4, \frac{3}{4}\right) = \binom{4}{2}\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2 = \left(\frac{4!}{2!\,2!}\right)\left(\frac{3^2}{4^4}\right) = \frac{27}{128}.$$

Example 5.2 The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that (a) at least 10 survive, (b) from 3 to 8 survive, and (c) exactly 5 survive?

Solution: Let X be the number of people who survive.

$n = 15$
$x \geq 10$
$P = 0.4$

(a) $P(X \geq 10) = 1 - P(X < 10) = 1 - \sum_{x=0}^{9} b(x; 15, 0.4) = 1 - 0.9662$

$= 0.0338$

نستخدم table أسهل

(b) $P(3 \leq X \leq 8) = \sum_{x=3}^{8} b(x; 15, 0.4) = \sum_{x=0}^{8} b(x; 15, 0.4) - \sum_{x=0}^{2} b(x; 15, 0.4)$

$= \underline{0.9050 - 0.0271} = 0.8779$

x تمثل r

(c) $P(X = 5) = b(5; 15, 0.4) = \sum_{x=0}^{5} b(x; 15, 0.4) - \sum_{x=0}^{4} b(x; 15, 0.4)$

$\hookrightarrow F(5) - F(4) \boxed{= 0.1859}$

بتحدد n و p
بنقاطع x

ال table تمثل F(x)
وتمثل الأقل من

الي هون بتمثل المنطقة $\frac{1}{4}$

Example 5.2: The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that (a) at least 10 survive, (b) from 3 to 8 survive, and (c) exactly 5 survive?

Solution: Let X be the number of people who survive. $b(x; n, p) = \binom{n}{x} p^x q^{n-x}$

(a) $P(X \geq 10)$ P(X=10)+P(X=11)+P(X=12)+P(X=13) +P(X=14)+P(X=15)

$\binom{15}{10}(0.4)^{10}(0.6)^5 + \binom{15}{11}(0.4)^{11}(0.6)^4 + \binom{15}{12}(0.4)^{12}(0.6)^3 + \binom{15}{13}(0.4)^{13}(0.6)^2 + \binom{15}{14}(0.4)^{14}(0.6)^1 + \binom{15}{15}(0.4)^{15}(0.6)^0$

(b) $P(3 \leq X \leq 8) =$ P(X=3)+P(X=4)+P(X=5)+P(X=6) +P(X=7)+P(X=8)

$\binom{15}{3}(0.4)^3(0.6)^{12} + \binom{15}{4}(0.4)^4(0.6)^{11} + \binom{15}{5}(0.4)^5(0.6)^{11} + \binom{15}{6}(0.4)^6(0.6)^{11} + \binom{15}{7}(0.4)^7(0.6)^8 + \binom{15}{8}(0.4)^8(0.6)^7$

(C) (P X=5)= $\binom{15}{5}(0.4)^5(0.6)^{10}$ = 0.1859

1) $P(X \geq 10) = 1 - P(x < 10)$
$= 1 - 0.9662$
$=$

$\Sigma P = 1$

(0-9) 10 15

داخل
هون ما بنقدر
نطلع ال < نستخدم
الأكل

Example 5.3: A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%.

$n=20$

$p=0.03$

(a) The inspector randomly picks 20 items from a shipment. What is the probability that there will be at least one defective item among these 20? $P(x \geq 1)$

(b) Suppose that the retailer receives 10 shipments in a month and the inspector randomly tests 20 devices per shipment. What is the probability that there will be exactly 3 shipments each containing at least one defective device among the 20 that are selected and tested from the shipment?

Solution: (a) Denote by X the number of defective devices among the 20. Then X follows a $b(x; 20, 0.03)$ distribution. Hence,

$1-P(\ldots)$ دائماً الأكبر من ال ... !! Complement $P(x<1)$

$$P(X \geq 1) = 1 - P(X = 0) = 1 - b(0; 20, 0.03)$$
$$= 1 - (0.03)^0(1 - 0.03)^{20-0} = 0.4562.$$

(b) In this case, each shipment can either contain at least one defective item or not. Hence, testing of each shipment can be viewed as a Bernoulli trial with $p = 0.4562$ from part (a). Assuming independence from shipment to shipment and denoting by Y the number of shipments containing at least one defective item, Y follows another binomial distribution $b(y; 10, 0.4562)$. Therefore,

$$P(Y = 3) = \binom{10}{3} 0.4562^3(1 - 0.4562)^7 = 0.1602.$$

$n=10$

$x=3$

شو احتمالية كل شحنة يطلع فيها defective — من الفرع الاول constant probability

20 بفحص shipment لكل

Example 5.4: It is conjectured that an impurity exists in 30% of all drinking wells in a certain rural community. In order to gain some insight into the true extent of the problem, it is determined that some testing is necessary. It is too expensive to test all of the wells in the area, so 10 are randomly selected for testing.

p $n=10$ x

(a) Using the binomial distribution, what is the probability that exactly 3 wells have the impurity, assuming that the conjecture is correct?

(b) What is the probability that more than 3 wells are impure?

Solution: (a) We require

$$b(3; 10, 0.3) = \sum_{x=0}^{3} b(x; 10, 0.3) - \sum_{x=0}^{2} b(x; 10, 0.3) = 0.6496 - 0.3828 = 0.2668.$$

$n = 10 \rightarrow P(x=3) \rightarrow F(3) - F(2) =$

(b) In this case, $P(X > 3) = 1 - 0.6496 = 0.3504$.

$1 - P(X \leq 3)$

(a) $\binom{10}{3}(0.3)^3(0.7)^7 = 0.2668$

$$\boxed{P(X > 3) = 1 - P(X \leq 3)} = 1 - 0.6496$$

$$\boxed{= 1 - \sum_{x=0}^{3} b(x; 10, 0.3)}$$

$$\boxed{1 - \{0.0282 + 0.1211 + 0.2335 + 0.2668\}}$$

Since the probability distribution of any binomial random variable depends only on the values assumed by the parameters n, p, and q, it would seem reasonable to assume that the mean and variance of a binomial random variable also depend on the values assumed by these parameters. Indeed, this is true, and in the proof of Theorem 5.1 we derive general formulas that can be used to compute the mean and variance of any binomial random variable as functions of n, p, and q.

Theorem 5.1: The mean and variance of the binomial distribution $b(x; n, p)$ are

$$\boxed{\mu = np \text{ and } \sigma^2 = npq.}$$

sheet $\times$

Example 5.5: Find the mean and variance of the binomial random variable of Example 5.2, and then use Chebyshev's theorem (on page 137) to interpret the interval $\mu \pm 2\sigma$.

Solution: Since Example 5.2 was a binomial experiment with $n = 15$ and $p = 0.4$, by Theorem 5.1, we have

$$\mu = \overset{n}{(15)}\overset{p}{(0.4)} = 6 \text{ and } \sigma^2 = \overset{n}{(15)}\overset{p}{(0.4)}\overset{(1-p)}{(0.6)} = 3.6.$$

Taking the square root of 3.6, we find that $\sigma = 1.897$. Hence, the required interval is $6 \pm (2)(1.897)$, or from 2.206 to 9.794. Chebyshev's theorem states that the number of recoveries among 15 patients who contracted the disease has a probability of at least 3/4 of falling between 2.206 and 9.794 or, because the data are discrete, between 2 and 10 inclusive.

There are solutions in which the computation of binomial probabilities may allow us to draw a scientific inference about population after data are collected. An illustration is given in the next example.

~~Example 5.2: The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that (a) at least 10 survive, (b) from 3 to 8 survive, and (c) exactly 5 survive?~~

Example 5.6: Consider the situation of Example 5.4. The notion that 30% of the wells are impure is merely a conjecture put forth by the area water board. Suppose 10 wells are randomly selected and 6 are found to contain the impurity. What does this imply about the conjecture? Use a probability statement.

ماذا تعني هذه الفرضية

Solution: We must first ask: "If the conjecture is correct, is it likely that we would find 6 or more impure wells?"

عشان اكبر

$= 1 - P(X<6)$

$$P(X \geq 6) = \sum_{x=0}^{10} b(x; 10, 0.3) - \sum_{x=0}^{5} b(x; 10, 0.3) = 1 - 0.9527 = 0.0473.$$

طلع الاقل من 6 طلوب

As a result, it is very unlikely (4.7% chance) that 6 or more wells would be found impure if only 30% of all are impure. This casts considerable doubt on the conjecture and suggests that the impurity problem is much more severe.

As the reader should realize by now, in many applications there are more than two possible outcomes. To borrow an example from the field of genetics, the color of guinea pigs produced as offspring may be red, black, or white. Often the "defective" or "not defective" dichotomy is truly an oversimplification in engineering situations. Indeed, there are often more than two categories that characterize items or parts coming off an assembly line.

exact Value

اذا قال بالضبط $= \binom{n}{x} p^x q^{n-x}$ استخدم الطريقة

Suppose a large urn contains 400 red marbles and 600 blue marbles.

A random sample of 10 marbles is drawn without replacement. What is the probability exactly 3 are red?

R	B
400	600

10 ~ اسحب

3 R

$\frac{n}{N} =$

$$P(X=3) = \frac{\binom{400}{3}\binom{600}{7}}{\binom{1000}{10}} = 0.2155$$

Hypergeometric Distribution → grouping يتميز بوجود

Hypergeometric Distribution The probability distribution of the hypergeometric random variable X, the number of successes in a random sample of size n selected from N items of which k are labeled **success** and $N-k$ labeled **failure**, is

$$h(x; N, n, k) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}, \quad \max\{0, n-(N-k)\} \le x \le \min\{n, k\}.$$

sample size

عدد الكل في الحالة

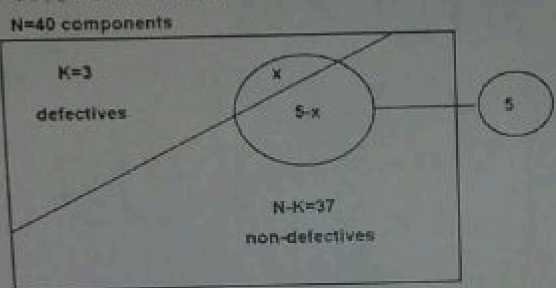
Example 5.9: Lots of 40 components each are deemed unacceptable if they contain 3 or more defectives. The procedure for sampling a lot is to select 5 components at random and to reject the lot if a defective is found. What is the probability that exactly 1 defective is found in the sample if there are 3 defectives in the entire lot?

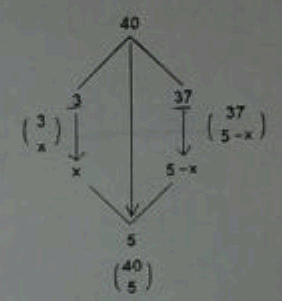
Solution: Using the hypergeometric distribution with $n = 5$, $N = 40$, $k = 3$, and $x = 1$, we find the probability of obtaining 1 defective to be

$$h(1; 40, 5, 3) = \frac{\binom{3}{1}\binom{37}{4}}{\binom{40}{5}} = 0.3011.$$

Once again, this plan is not desirable since it detects a bad lot (3 defectives) only about 30% of the time. ■

40 → 5





Exercise 5. 31

A homeowner plants 6 bulbs selected at random from a box containing 5 tulip bulbs and 4 daffodil bulbs. What is the probability that he planted 2 daffodil bulbs and 4 tulip bulbs?

$\boxed{4 \quad 5}$ 6 نسحب 9

$$h(2; 9, 6, 4) = \frac{\binom{4}{2}\binom{5}{4}}{\binom{9}{6}} = \frac{5}{14}.$$

اسحب 4 ← $\boxed{8 \quad 7_{N} \quad 10}$

Exercise 5.32

From a lot of 10 , 4 are selected at random and fired. If the lot contains 3 defective missiles that will not fire, what is the probability that (a) all 4 will fire? (b) at most 2 will not fire?

(a) h(4;10,4,7) = 1/6 $\qquad$ D $\qquad \dfrac{\binom{7}{4}\binom{3}{0}}{\binom{10}{4}} = 1/6$

(b) $\sum_{x=0}^{2} h(x; 10, 4, 3) = \frac{29}{30}$. $= \frac{n}{N}$ $P(x \leq 2)$

$\dfrac{\binom{3}{2}\binom{7}{2}}{\binom{10}{4}} = \dfrac{3}{10}$

$P(x=0) + P(x=1) + P(x=2)$

$\dfrac{\binom{7}{4}}{\binom{10}{4}} + \dfrac{\binom{3}{1}\binom{7}{3}}{\binom{10}{4}} + \dfrac{\binom{3}{2}\binom{7}{2}}{\binom{10}{4}}$

$= \dfrac{29}{30}$

Theorem 5.2: The mean and variance of the hypergeometric distribution $h(x; N, n, k)$ are

$$\mu = \frac{nk}{N} \text{ and } \sigma^2 = \frac{N-n}{N-1} \cdot n \cdot \frac{k}{N}\left(1 - \frac{k}{N}\right).$$

Example 5.10: Let us now reinvestigate Example 3.4 on page 83. The purpose of this example was to illustrate the notion of a random variable and the corresponding sample space. In the example, we have a lot of 100 items of which 12 are defective. What is the probability that in a sample of 10, 3 are defective?

سحب 10

***Solution*:** Using the hypergeometric probability function, we have

$\boxed{\begin{matrix} 88 & 12 \\ N & D \end{matrix}}$

$$h(3; 100, 10, 12) = \frac{\binom{12}{3}\binom{88}{7}}{\binom{100}{10}} = 0.08.$$

$\mu = \dfrac{12 * 10}{100} = 1.2$

Example 3.4: Statisticians use sampling plans to either accept or reject batches or lots of material. Suppose one of these sampling plans involves sampling independently 10 items from a lot of 100 items in which 12 are defective.

Let X be the random variable defined as the number of items found defective in the sample of 10. In this case, the random variable takes on the values $0, 1, 2, \ldots, 9, 10$.

J

$\sigma^2 = \left(\dfrac{100-10}{100-1}\right) \cdot 10 * \dfrac{12}{100}\left(1 - \dfrac{12}{100}\right) = \dfrac{24}{25}$

Example 5.11: Find the mean and variance of the random variable of Example 5.9 and then use Chebyshev's theorem to interpret the interval $\mu \pm 2\sigma$.

Solution: Since Example 5.9 was a hypergeometric experiment with $N = 40$, $n = 5$, and $k = 3$, by Theorem 5.2, we have

$$\mu = \frac{(5)(3)}{40} = \frac{3}{8} = 0.375,$$

and

$$\sigma^2 = \left(\frac{40-5}{39}\right)(5)\left(\frac{3}{40}\right)\left(1 - \frac{3}{40}\right) = 0.3113.$$

Taking the square root of 0.3113, we find that $\sigma = 0.558$. Hence, the required interval is $0.375 \pm (2)(0.558)$, or from -0.741 to 1.491. Chebyshev's theorem states that the number of defectives obtained when 5 components are selected at random from a lot of 40 components of which 3 are defective has a probability of at least 3/4 of falling between -0.741 and 1.491. That is, at least three-fourths of the time, the 5 components include fewer than 2 defectives. ∎

Example 5.9: Lots of 40 components each are deemed unacceptable if they contain 3 or more defectives. The procedure for sampling a lot is to select 5 components at random and to reject the lot if a defective is found. What is the probability that exactly 1 defective is found in the sample if there are 3 defectives in the entire lot?

Solution: Using the hypergeometric distribution with $n = 5$, $N = 40$, $k = 3$, and $x = 1$, we find the probability of obtaining 1 defective to be

$$h(1; 40, 5, 3) = \frac{\binom{3}{1}\binom{37}{4}}{\binom{40}{5}} = 0.3011.$$

$\boxed{\frac{N(D)}{37 \mid 3}}$ 4

21

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4) **Negative Binomial Distribution** If repeated independent trials can result in a success with probability p and a failure with probability $q = 1 - p$, then the probability distribution of the random variable X, the number of the trial on which the kth success occurs, is → number of suc

مقابله عدد محاولات

the probability that, the k'th success occurs on the xth trial. Experiments of this kind are called negative binomial experiments.

$$b^*(x; k, p) = \binom{x-1}{k-1} p^k q^{x-k}, \quad x = k, k+1, k+2, \ldots.$$

Constant تحوي كلها probability

Example 5.14: In an NBA (National Basketball Association) championship series, the team that wins four games out of seven is the winner. Suppose that teams A and B face each other in the championship games and that team A has probability 0.55 of winning a game over team B.

$k = 4$

7

احتمالية تحقيق النجاح

(a) What is the probability that team A will win the series in 6 games?

(b) What is the probability that team A will win the series?

(c) If teams A and B were facing each other in a regional playoff series, which is decided by winning three out of five games, what is the probability that team A would win the series?

number of trial vs number success

Solution: (a) $b^*(6; 4, 0.55) = \binom{5}{3} 0.55^4 (1 - 0.55)^{6-4} = 0.1853$

(b) P(team A wins the championship series) is

$$b^*(4; 4, 0.55) + b^*(5; 4, 0.55) + b^*(6; 4, 0.55) + b^*(7; 4, 0.55)$$
$$= 0.0915 + 0.1647 + 0.1853 + 0.1668 = 0.6083.$$

(c) P(team A wins the playoff) is

$$b^*(3; 3, 0.55) + b^*(4; 3, 0.55) + b^*(5; 3, 0.55)$$
$$= 0.1664 + 0.2246 + 0.2021 = 0.5931.$$ ∎

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5) Geometric Distribution

If repeated independent trials can result in a success with probability p and a failure with probability $q = 1 - p$, then the probability distribution of the random variable X, the number of the trial on which the first success occurs, is

$$g(x; p) = pq^{x-1}, \quad x = 1, 2, 3, \ldots$$

الوقت المحتاج لتلبية الخرجة ← النجاح الكلمة المفتاحية

الخرج: إعطاء خصوصية للمرة الأولى.

Example 5.15: For a certain manufacturing process, it is known that, on the average, 1 in every 100 items is defective. What is the probability that the fifth item inspected is the first defective item found?

$p = \frac{1}{100}$

Solution: Using the geometric distribution with $x = 5$ and $p = 0.01$, we have

$$g(5; 0.01) = (0.01)(0.99)^4 = 0.0096.$$

بمعنا نعرف متى يطلع أول defective في الإنتاج الأولى

Example 5.16: At a "busy time," a telephone exchange is very near capacity, so callers have difficulty placing their calls. It may be of interest to know the number of attempts necessary in order to make a connection. Suppose that we let $p = 0.05$ be the probability of a connection during a busy time. We are interested in knowing the probability that 5 attempts are necessary for a successful call.

إعطاء الخرجة من أول مرة يا normal لازم تكون على

بعد 5 رنات ليرد !!

Solution: Using the geometric distribution with $x = 5$ and $p = 0.05$ yields

$$P(X = x) = g(5; 0.05) = \underbrace{(0.05)(0.95)^4}_{pq^{x-1}} = 0.041.$$

Quite often, in applications dealing with the geometric distribution, the mean and variance are important. For example, in Example 5.16, the *expected* number of calls necessary to make a connection is quite important. The following theorem states without proof the mean and variance of the geometric distribution.

Theorem 5.3: The mean and variance of a random variable following the geometric distribution are

$$\boxed{\mu = \frac{1}{p} \text{ and } \sigma^2 = \frac{1-p}{p^2}.}$$

The *geometric* distribution is the distribution of the number of trials needed to get the first success in repeated independent Bernoulli trials.

The *negative binomial* distribution is the distribution of the number of trials needed to get the rth success.

The *binomial* distribution is the distribution of the number of successes in a fixed number of independent Bernoulli trials. n

The *negative binomial* distribution is the distribution of the number of trials needed to get a fixed number of successes. r